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Fostering Teacher's Conceptual Understanding Of Ordering, Adding, And Subtracting Fractions Through School-based Professional Development

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FOSTERING TEACHER'S CONCEPTUAL UNDERSTANDING OF ORDERING, ADDING, AND SUBTRACTING FRACTIONS THROUGH SCHOOL-BASED PROFESSIONAL DEVELOPMENT

by

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ABSTRACT

In an attempt to examine my practice of providing conceptually-based professional development on fractions to fifth grade teachers, I conducted a series of four one hour professional development workshops. I focused on the conceptual understanding of ordering, adding, and subtracting fractions. I examined the solution process that teachers used to solve fraction problems and their abilities to explain and justify their solutions in an attempt to interpret their understanding. My data showed the effects of this workshop series. The study helped determine the effects of conceptually-based professional development on fractions as demonstrated in the teachers' discussions, participation, and written explanations.

I dedicate this thesis to all of those who love me and all of those who I have loved.

I couldn’t have done this without you.

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CHAPTER 1: INTRODUCTION

Rationale

According to *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, proficiency with fractions is a major goal for K-8 mathematics education (National Mathematics Advisory Panel, 2008). The problem arises however in that teachers are not proficient with fractions themselves (Fuller, 1996; Lester, 1984; Ma, 1999, Tirosh, Fischbein, Graeber, & Wilson, 1998). Providing professional development for K-8 teachers, specifically in fraction concepts, is therefore important and necessary.

I am currently the Mathematics Coach at a public elementary school located in a low socioeconomic sector of a large urban school district in the southeast United States. This study was conducted at that school. As a Mathematics Coach, it is my duty to assist in improving the mathematical practices of the teachers at the school through modeling, co-teaching, mentoring, and providing professional development.

Throughout my own journey of professional development, I have experienced the importance of constructing conceptual understanding of mathematical ideas and have improved my own teaching practice. After four years of university study and three years in the classroom, I was enrolled in a mathematics course in graduate school. This class became the most challenging and also the most rewarding of my academic career. This class was structured to take us back to the very basics of arithmetic. Its focus was to solve those basic problems with an emphasis on conceptual understanding and being able to provide an explanation and justification for the answer and not just the solution itself. As children, we were all asked to solve problems such as: Which is greater 1/6 or 1/8. We were taught a procedure of how to solve the problem but maybe not taught the explanation and justification behind the procedure.

In this class we used the example of drawing two pizzas. One was cut into six slices the other into eight. The next step was to do a visual comparison of the two and show that the slice of the pizza cut into six pieces was larger than the one from the pizza cut into eight pieces. This can allow the solution to the problem to be viewed in a context rather than in purely abstract terms. The ability to help students visualize the solution can increase their ability to understand the problem and to find the proper solution.

Purpose

Recent reform documents continue to call for instruction that emphasizes mathematical inquiry and conceptual understanding (National Council of Teachers of Mathematics, 2006; National Mathematics Advisory Panel, 2008). Research indicates however that teachers' have a lack of conceptual understanding in mathematics (Brover, Deagan, & Farina, 2001; Cohen & Hill, 2000; Cooney, 1994; Fuller, 1996; Koency & Swanson, 2000; Ma, 1999; Weissglass, 1994). Among many questions that Ma asked 23 United Sates elementary school teachers in her widely cited research study, she asked them to solve 1 3/4 divided by 1/2. Out of the 23 teachers, only 21 attempted to solve the problem and only nine completed the problem and reached the correct answer. This study illustrates the lack of conceptual understanding among these teachers. A lack of conceptual understanding prohibits teachers from providing instruction that focus on conceptual understanding for their students. After all, one cannot teach students to develop a deep understanding of mathematics if they do not fully understand the concepts themselves (Ma, 1999; Stoddart, Connell, Stofflett, & Peck, 1993; Wheeldon, 2008). This deficiency can be overcome with the implementation of professional development focused on content.

The purpose of this action research project was to determine the effects of my professional development practice on elementary school teachers' conceptual understanding of fractions. Teachers' mathematical knowledge has been linked to students' achievement, thus, the importance of teachers' content knowledge cannot be understated (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Hill, Rowan, & Ball, 2005; National Mathematics Advisory Panel, 2008). This thesis documents my efforts to instruct teachers who voluntarily attended my workshop series. My goal was to further develop their conceptual understanding of ordering, adding, and subtracting fractions.

Research Question

How does my practice of providing conceptually-based professional development related to fractions affect teachers' abilities to explain and justify procedures related to ordering, adding, and subtracting fractions?

Significance

The United States of America continues to be outperformed in mathematics compared to other nations (Koency & Swanson, 2000). This was made apparent in the 1999 Video Study conducted by the Third International Mathematics and Science Study (TIMSS). This study revealed that students in the United States are not learning mathematics as deeply as in other nations (Hiebert, 2003). Schmidt, Houang, and Cogan (2002) attribute this deficiency to the fact that teachers in the United States teach a curriculum that is "a mile wide and an inch deep," (p. 3) covering many mathematical concepts but none of them in depth. Sigford (2006) suggests teaching a curriculum that is "a mile deep and an inch wide," (p. 88) covering fewer mathematical concepts but teaching more in depth.

This study is relevant because there are teachers both locally and nationally who are inadequately prepared to teach mathematics. One reason for this is that teachers do not have enough mathematical background to engage students in meaningful mathematical discourse (Cohen & Hill, 2000; Koency & Swanson, 2000). Meaningful mathematical discourse includes establishing social norms like the ability to explain and justify solutions, make sense of explanations given by others, and ask clarifying questions when confused (Yackel & Cobb, 1996). The recommendation is that teachers need to know the mathematical content they teach in greater detail and at a more advanced level as well as knowing the content both prior to and beyond what they teach (National Mathematics Advisory Panel, 2008; Saxe, Gearhart, & Nasir, 2001).

Teachers' mathematical content knowledge has been linked to students' achievement thus emphasizing the importance of teachers' content knowledge (Darling-Hammond et al., 2009; Hill, Rowan, & Ball, 2005; National Mathematics Advisory Panel, 2008). Teachers who possess conceptual understanding of fundamental mathematics are able to provide more rigorous instruction for their students which lead to higher student achievement in mathematics (Koency & Swanson, 2000). The benefits to students who are taught to develop conceptual understanding include proficiency of problem solving, abstract reasoning, the ability to generalize knowledge to new situations, and the ability to make connections to related information (Sigler & Saam, 2006).

One way to help teachers who lack conceptual knowledge is to provide continuing professional development focused on improving content knowledge of mathematics (Brover et al., 2001; Cohen & Hill, 2000; Kennedy, 1999; Koency & Swanson, 2000; Mistretta, 2005; Schmidt et al., 2002). Research suggests that teachers should participate in ongoing professional development focusing on the content that their students are to learn, how they are to teach the content, and how students are to learn it (Cohen & Hill, 2000; Kennedy, 1999). In order to be effective however, teachers should not be mandated to attend professional development, it should be their choice (Castle & Aichele, 1994).

Conclusion

I was interested in studying my practice of providing conceptually-based professional development to the fifth grade teachers at my school. I wanted to provide the most effective form of professional development given the circumstances. I would provide teachers with an opportunity to develop conceptual understanding and they would gain the content knowledge needed to teach their students conceptual understanding.

The review of literature examines students' and teachers' conceptual understanding of fractions, strategies that promote conceptual understanding, and the most effective characteristics of professional development that can prepare teachers to instruct their students. The review of literature provided a framework for the methods I used in planning my professional development for this study, which are presented in chapter three. The data from my research are presented in chapter four and discussed along with my conclusions and recommendations in chapter five.

CHAPTER 2: LITERATURE REVIEW

Introduction

The knowledge and understanding of mathematics that a teacher brings to the classroom plays an important role in the successes and failures of their students (National Mathematics Advisory Panel, 2008). Knowing this, what knowledge and understanding of mathematics do elementary school teachers need to provide quality mathematics instruction for our students? The purpose of this literature review is to examine teachers' and students' conceptual understanding of fractions and effective forms of professional development.

Suggestions have been provided through recent publications regarding what students should know about fractions at the elementary school level (National Council of Teachers of Mathematics, 2006; National Mathematics Advisory Panel, 2008). The recommendations are consistent that by the end of grade 5 students should be able to compare, add, and subtract fractions with proficiency (National Council of Teachers of Mathematics; National Mathematics Advisory Panel). The recommendations stem from a connection between students' understanding of fractions and their later achievement in algebra courses (National Mathematics Advisory Panel). Knowing the recommendations and the connection to later achievement, how do students reach this goal?

Teaching based on conceptual understanding is the key to preparing students to achieve mathematical success. Conceptual knowledge provides the understanding behind procedures, and if a student is to be successful year after year, then they must understand the concepts behind the procedures (Dixon, 2005). However, conceptual understanding has traditionally not played an important role in elementary education. A traditional mathematics teacher emphasizes procedures with little attention to explanations and justifications (Ball, 1991).

A review of the literature suggests that conceptual understanding is the important foundation that is lacking from both our teachers and students (Hiebert & Stigler, 2004; Koency & Swanson, 2000). We need teachers who help students develop conceptual understanding by having them explain and justify their mathematical thinking (Ball, 1991). In order to better prepare our students our teachers need to be better prepared to teach them. One way to prepare our teachers is through professional development. Research says, "professional development needs to be situated in the teachers' practice, connected to the curriculum, focused on clear student learning goals and student thinking, and continuing over time" (Hiebert & Stigler, 2004, p. 14).

Professional Development

In order to improve the quality of mathematics education for our students we need to improve the quality of our teachers' mathematical knowledge (Ma, 1999). The research has shown that teachers need a deeper understanding of fundamental mathematics (Brover et al., 2001; Cohen & Hill, 2000; Ma, 1999; National Council of Teachers of Mathematics, 2006; Schmidt et al., 2002). Fuller (1996) examined teachers' content knowledge of fractions and found 5 out of 54 teachers believed that 1/2 plus 1/3 equals 2/5. This study demonstrates that there are teachers that do not have the mathematical content knowledge they need to teach students.

Through continuous professional development focused on content areas however, teachers can have the opportunity to learn the mathematical content knowledge they need (Darling-Hammond et al., 2009; Koency & Swanson, 2000). All too often the topics of professional development include assessment, classroom management, and other topics not specific to the content areas that the teachers teach and to how students learn. "Programs that focus on subject matter knowledge and on student learning of particular subject matter are likely to have larger positive benefits for student learning than programs that focus mainly on teaching behaviors" (Kennedy, 1999, p.4). It is the recommendation of the National Mathematics Advisory Panel (2008) that "teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior to and beyond the level they are assigned to teach" (p.37).

Teachers with limited mathematical content knowledge lack the confidence and ability to deliver quality mathematics instruction (Brover et al., 2001). Therefore, comprehensive and ongoing professional development that engages teachers in doing mathematics is necessary for teachers to be able to understand, apply, and teach mathematics (Brover et al.). Teachers need to be taught in a way that reflects the methods they are being asked to teach (Cooney, 1994; Wheeldon, 2008). Research has found that teachers are more likely to use practices that have been modeled for them in professional development (Darling-Hammond et al., 2009). Teachers should have time in-between professional development sessions to try out ideas in their classroom and then have time to dialogue and reflect as they attempt to make sense of their observations (Darling-Hammond et al.; Zech, Gause-Vega, Bray, Secules, & Goldman, 2000). Research suggests that less than 14 hours of professional development does not have an effect on student learning while professional development with longer-durations does have positive and significant effects on student achievement (Darling-Hammond et al.). Studies have indicated that professional development which is sustained over a longer period of time produces positive results for both teachers and their students.

In research conducted by Saxe et al. (2001) they looked at 23 elementary classroom teachers and their students dividing them into three different groups to examine the influence of professional development and curriculum. Two groups implemented a unit on fractions which emphasized problem solving and conceptual understanding. The first group of teachers, Integrated Mathematics Assessment (IMA), implemented the unit on fractions and participated in a professional development that was designed to enhance teachers' understanding of fractions, students' thinking, and students' motivation. They attended a five day summer institute and participated in 13 meetings approximately every two weeks during the year. The second group of teachers was known as the Collegial Support (SUPP). They too implemented the unit on fractions but instead of participating in professional development, they met nine times during the year to discuss strategies in regards to implementing the curriculum. The third group of teachers, known as the traditional group (TRAD), valued and used the textbooks and did not implement the unit on fractions, received no professional development support, and did not meet as a group throughout the year.

Before and after the teachers taught fractions, the children were given a pre-test and a post-test. This test consisted of two types of questions, those that would show computational skills and those that would show conceptual understanding. The items were constructed so that the computational items could be easily solved with a procedure or memorized facts while the conceptually oriented items could not be as easily solved with procedures and would require knowledge of mathematical relations involving fractions. To analyze the data, the pre-test and post-test were compared among the three groups. The results showed that the IMA group achieved greater conceptual gains on the post-test than the SUPP and TRAD groups. There was actually no difference on the post-test in conceptual gains between the SUPP and TRAD groups. The researchers attribute these results to the fact that the IMA group provided opportunities for the teachers to enhance their understanding of mathematics and the ways that children make sense of mathematics. Looking at the computational gains, the post-test revealed no difference in the IMA and TRAD group, although the TRAD group did achieve greater scores than the SUPP group. Their results indicate that when curriculum is implemented and includes support for teacher knowledge that it may lead to gains in students' conceptual understanding greater than with just traditional practices and that at the same time it may not lead to a decrease on performance with computational skills. Conceptual understanding will allow students to be successful year after year (Dixon, 2005).

Conceptual Understanding of Fractions

"No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions" (Smith, 2002, p. 3). This is made evident when many students enter high school and still have a lack of proficiency with fraction concepts (Brown & Quinn, 2006). Siebert and Gaskin (2006) describe just how complicated fractions are:

> When we add or subtract fractions, we have to find a common denominator, but not when we multiply or divide. And once we get a common denominator, we add or subtract the numerators, but not the denominators, despite the fact that when we multiply, we multiply both the numerators and the denominators, and when we divide, we divide neither the numerators nor the denominators. (p. 394)

One reason for students' lack of proficiency with fractions is that many teachers do not have the conceptual understanding of fractions to effectively teach their students (Fuller, 1996; Lester, 1984; Ma, 1999; Saxe et al., 2001; Tirosh et al., 1998). In a study conducted by Fuller she examined the content knowledge of novice and experienced elementary teachers and found that both groups primarily possessed procedural knowledge of fractions, meaning they understood the required steps to solve a problem (Hecht, 1998). Conceptual knowledge, or knowing the meaning of mathematical symbols (Hecht), however is important to know and needs to be taught before procedures. Once conceptual understanding is developed, procedures become easier to understand (Niemi, 1996). According to National Council of Teachers of Mathematics (NCTM):

> A conceptual approach enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving. (1989, p. 17)

Cramer, Post, and delMas (2002) conducted a study looking at two different approaches to teaching fractions to elementary school students. They used a commercial curriculum (CC) and a curriculum called the Rational Number Project (RNP). The CC curriculum focused on developing operational skills and procedures for fraction skills. The RNP curriculum focused on developing an understanding of the meaning of fractions on a conceptual level. They integrated two different methodologies in their study to examine information gathered from a large number of students that could be generalized to a larger population and information gathered from a smaller group of students that could provide a humanistic perspective. In the former methodology, interpretation and meaning is accounted for in analysis of data (Brown, Cooney, & Jones, 1990).

After an average of 29 days of instruction some of their results are discussed. One hundred and eleven fourth grade students were asked which is larger, 4/5 or 11/12. Of the 53 students who received the RNP curriculum, 62% of them found the correct answer and 40% of those who answered correctly used conceptual understanding to solve the problem. Of the 57 students who received the CC curriculum, 67% of them found the correct answer and 16% of those who answered correctly used conceptual understanding to solve the problem. While the percentage of students who found correct answers is close for both groups, the percentage of students who used conceptual knowledge differs greatly. A smaller group consisting of 17 students was asked to estimate the answer of 3/12 plus 2/3. The results showed that of the eight students who received instruction using the RNP curriculum, 75% of them found the correct answer and all 100% of them used conceptual understanding to solve the problem. Of the nine students who received instruction using the CC curriculum, 44% of them found the correct answer and 0% of them used conceptual understanding. These results show that students who used conceptual understanding had a higher accuracy rate of being able to estimate a reasonable answer to an addition of fractions problem.

A larger group of 83 students was asked to estimate the answer of 11/12 minus 4/6. The results showed that of the 49 students who received instruction using the RNP curriculum, 76% of them found the correct answer and 41% of those students did so using conceptual understanding. Of the 34 students who received instruction using the CC curriculum, 47% of them found the correct answer and only 6% of them did so using conceptual understanding. Again, the results show that students who used conceptual understanding had a higher accuracy rate by being able to estimate a reasonable answer to a subtraction of fractions problem. Overall, the results from this study show that students whose instruction focused on conceptual understanding outperformed students whose instruction focused on procedures.

When focusing on evaluating conceptual understanding, representational knowledge, justifications, and explanations can provide useful and valid information regarding students' knowledge (Niemi, 1996). Their use of mathematical representations and language is an important element in assessing conceptual understanding (Niemi). Focusing on the explanation of how they interpret the problem and how they found the solution is essential (Yackel & Cobb, 1996). One should be able to explain and justify solutions, make sense of explanations given by others, and ask clarifying questions when confused (Yackel & Cobb). Those who cannot effectively explain and justify mathematical symbols, concepts, and operations cannot be judged to know mathematics (Niemi). It is for these reasons that my research on professional development focused on teachers' abilities to provide explanations and justifications, hoping to assess teachers' conceptual understanding of fractions. Demonstrating a proper explanation and justification includes using conceptually-based strategies such as benchmarks fractions and pictures to demonstrate the solution process.

Fraction Strategies

Comparing Fractions

If students were asked to compare 5/8 with 4/9 most would attempt to find a common denominator to convert both fractions to equivalent forms (Reys, Kim, & Bay, 1999). However, the Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence, by the National Council of Teachers of Mathematics (NCTM) advocate solving fraction problems that involve comparing and ordering fractions by "using models, benchmark fractions, or common numerators or denominators" (2006). Applying these strategies shows evidence that students understand fraction concepts related to ordering fractions.

In a classroom focusing on conceptual understanding, students would be taught to compare the size of fractions with the use of benchmarks such as 0, 1/2, and 1 (Dixon, 2008; Reys et al., 1999). For example, take the comparison of 5/8 to 4⁄9. Half of eight is four; therefore, 4/8 equals one half. Since 5/8 is greater than 4/8, 5/8 is more than one half. Similarly looking at 4/9, half of nine is four and a half. Four is less than four and a half so 4/9 is less than one half. This strategy of comparing fractions allows students to think about the part to whole relationship and relative size of fractions without the need to find common denominators. Using this strategy demonstrates conceptual understanding of comparing fractions.

Another way suggested by NCTM to compare fractions is to use models. For example, when trying to compare 4⁄5 to 11⁄12 both fractions are one away from a whole. If a model was drawn to show 4⁄5 and 11⁄12 it would help make sense of the missing pieces. Since the size of the pieces divided into fifths are larger, the one missing piece is larger than the one missing piece from the twelfths. 4⁄5 is therefore smaller than 11⁄12.

The last strategy suggested by NCTM to compare fractions is to look for common numerators or denominators. If two fractions have common numerators such as 3⁄4 and 3⁄8 one should think about the size of the denominators. Fourths are larger pieces than eighths so three pieces of fourths would be more than 3 pieces of eighths making 3⁄4 larger than 3⁄8. Similarly, if two fractions have common denominators such as 3⁄5 and 4⁄5, one should realize that the size of the pieces are equal and therefore three pieces is less than four pieces making 4⁄5 larger than 3⁄5. Using these strategies recommended by NCTM show that students have developed a conceptual understanding of comparing fractions. After students understand the size of fractions, they are ready to transfer their knowledge to adding and subtracting fractions.

Adding and Subtracting Fractions

When students are asked to add fractions such as 3/5 plus 2/3 and get 5/8 they are showing the common add across error (Smith, 2002). While this error is easy to detect, it is difficult to remedy (Brown & Quinn, 2006). Students who make this mistake show no evidence of understanding that the fractions are of unequal size and no evidence of being able to estimate a reasonable answer. When this mistake is made they are attributing the rules of addition of natural numbers to fractions. Another error that can be made when adding 3⁄5 plus 2⁄3 is that 2⁄3 may become 4⁄5 by adding two to the numerator and denominator. An incorrect answer of 7⁄5 would show evidence of misconceptions related to equivalent fractions and no evidence of reasonable estimates.

Similar misunderstandings are found when students are asked to subtract fractions. Dixon (2005) identifies two strategies that are useful for modeling addition and subtraction of fractions in her professional development for mathematics on fraction operations. She recommends the use of linear and area models for the following reasons. Fractions are viewed as regional parts of a whole using the area model and manipulatives such as fraction circles and pattern blocks work well with this model. In the linear model, fractions are represented as lengths of a segment and manipulatives such as fraction bars work well. These models illustrate that finding common denominators first is not necessary. Using these models reminds students that addition is combining and subtraction is removal in regards to parts in context of the whole. They also build estimation skills by visualizing benchmark values. Once students are comfortable with these concepts, the set model, where fractions are based on parts of a group, is ready to be introduced. Work with the set model will eventually lead to learning and understanding the standard algorithms.

Conclusion

The review of literature suggests that a conceptual understanding of fractions is needed for both teachers and students. Support for teachers and adequate resources, including ongoing professional development, must be available to ensure that teachers gain the knowledge and skills they need to provide quality mathematics instruction. By providing quality professional development, which is focused on developing conceptual understanding, students in the United States may begin to perform as well in mathematics as students in other countries.

It is my desire to provide effective professional development for the teachers at my school. By focusing on conceptual understanding of fractions, it was hoped that teachers could explain and justify ordering, adding, and subtracting fractions. I used an action research-based, qualitative approach as I addressed this issue with the teachers. I planned a professional development that was based on constructivist views of learning (Baviskar, Hartle, & Whitney, 2009). First, prior knowledge was activated through the use of a pre-interview and pre-test. Next, cognitive dissonance was created through problematic situations. Third, application of new knowledge with feedback was accomplished through discussions. Lastly, reflection on learning was assessed through the post-interview and post-test. Through this learning theory it was my goal to create a context in which the learner was motivated to learn.

In my concluding three chapters I explain the methodology, data analysis, and my conclusions. In my next chapter, I explain the methods used for my study. My question, *how does my practice of providing conceptually-based professional development related to fractions affect teachers' abilities to explain and justify ordering, adding, and subtracting fractions*, is explored in greater detail in the following chapters.

CHAPTER 3: METHODS

Introduction

In order to examine teachers' abilities to explain and justify ordering, adding, and subtracting fractions, I conducted a series of four one-hour professional development workshops. In this chapter, I describe the setting and the methods I used to gather the information necessary to answer my question as well as plans for data analysis. Because my goal is to improve teachers' conceptual understanding of fractions, I conducted my action research study with a group of teachers at my school. In this qualitative research, it was my goal to describe the improvement in the conceptual understanding of fractions among the teachers by analyzing their ability to explain and justify their solutions.

Design of Study

The design of my study was qualitative. Data collection included interviews, tests, and work samples. The emphasis of the data collection was focused on the mathematical processes in addition to the solutions. The goal was to know what the participants were thinking and why they were thinking what they did.

Setting

School Setting

The school at which I am a Mathematics Coach is a public elementary school located in a low socioeconomic sector of a large urban school district in the southeast United States. Free or reduced lunches are offered at the school and more than 90% of the students qualify. Of the 930 students at this school, 48% are Hispanic, 43% are African-American, 4% are White, and 5% are other. Sixty percent of the population was English language learners.

Professional Development Setting

My action research was conducted with 3 of the 5 fifth grade teachers at the school. All fifth grade teachers were invited to participate in my series of four one-hour professional development workshops due to the curriculum they teach and the fraction expectations for fifth grade students set by the National Mathematics Advisory Panel, National Council of Teachers of Mathematics, and state standards. Three of the five teachers agreed to participate in the study. The group consisted of two female teachers and one male teacher. All of the participants were Caucasian and held a Bachelor's degree in Elementary Education. Their teaching experience ranged from 10 years to 39 years. Interviews and assessments were given during their 45 minute planning time. The professional development workshops were held on Wednesday afternoons from 2:30 pm – 3:30 pm at the school in my office.

Methods

Data Collection

After receiving Institutional Review Board (IRB) approval (Appendix A), District Approval (Appendix B), and Principal approval (Appendix C), I distributed consent forms to all five fifth grade teachers (Appendix D). Three of the five teachers consented to participate in the study. Before I began collecting data each teacher chose a number unique to them for coding purposes. The master list with teacher names and corresponding numbers was kept locked in a filing cabinet to which only I had access. The list was then shredded after the study. I assigned pseudonyms to each participant for the purpose of reporting the data.

Interviews: Before the series of professional development workshops began teachers were individually interviewed to determine how they currently teach fraction concepts and why they choose those methods (Appendix E). The first question asked how they teach students to compare and order 1/4, 5/8, 8/9 and why. These fractions were chosen because they represent fractions less than 1⁄2, greater than 1/2, and almost 1 whole. They were also chosen because finding an exact answer using a common denominator algorithm would have been time consuming. The second question asked was how do you teach students to add 2/3 and 1/2 and why. These fractions were chosen because they had unlike denominators. The last question asked was how do you teach students to compute 5/6 minus 1/3 and why. These fractions were again chosen because they had unlike denominators. After the series of professional development workshops ended, teachers were interviewed again to determine how they planned to teach ordering fractions, addition of fractions, and subtraction of fractions in the future and why (Appendix F). The fractions in the problems remained the same as in the pre-interview but the focus of the questions this time was on how the teachers planned to teach students and why.

Pre-Test ⁄ Post-Test: Teachers were administered a written assessment both before and after the series of professional development workshops to determine their conceptual understanding of how to compare and order fractions, add fractions, and subtract fractions (Appendix G). I used information from a conceptually based fraction workshop that I attended to guide my selection of questions (Dixon, 2005). There were a total of six fraction problems. The first two problems focused on addition of fractions. The teachers were asked to explain and justify the solutions to 1⁄5 plus 4/6 and 4/10 plus 1 2/5. The next two problems focused on subtraction of fractions. They asked the teacher to explain and justify the solutions to 4/5 minus 1/9 and 1 2/3 minus 4/6. All of the fraction pairs for the four problems were chosen because they had unlike denominators. The last two problems focused on ordering fractions. They asked the teacher to explain and justify how to compare and order 95/94, 23/26, 19/71, 101/104, 53/62 and 9/16, 24/22, 18/20, 15/17, 2/7. These fractions were chosen because they had unlike denominators and represented fractions less than 1/2, more than 1/2, almost 1 whole, and more than 1 whole. They were also chosen because two fractions in each problem were missing one piece of the whole and in general, finding an exact answer using a common denominator algorithm would have been time consuming. This assessment was not tested for reliability but was validated by peers and experts in the field.

Teacher Work Samples: During the series of professional development workshops I collected teacher work samples (Appendix H-J). There were work samples for each session: ordering fractions, adding fractions, and subtracting fractions. Each work sample consisted of four or five problems similar to the pre-test. These work samples were completed as a group during the professional development sessions as we discussed how to teach for conceptual understanding.

Procedures

Two weeks prior to beginning the professional development series, I conducted individual pre-interviews with the participants during their 45 minute planning time. The interview questions focused on how they currently teach fraction concepts and why. The day before the series of professional development workshops began I gave the teachers the pre-test in a group setting during their planning time. The next day I began my first one hour professional development workshop focused on ordering fractions. See Table 1 for an outline of the workshop series.

Table 1 Mathematical Content Addressed in the Professional Development Workshop

Professional Development Timeline	Topic
2 weeks prior to the 1st workshop Time: 45-minutes	Pre-interview on ordering fractions, addition of fractions, and subtraction of fractions
1 day prior to the 1st workshop Time: 45-minutes	Pre-test on ordering fractions, addition of fractions, and subtraction of fractions
1st professional development workshop Time: 1 hour	Ordering fractions
2nd professional development workshop Time: 1 hour	Addition of fractions
3rd professional development workshop Time: 1 hour	Subtraction of fractions
4th professional development workshop Time: 1 hour	Review of ordering fractions, addition of fractions, and subtraction of fractions
1 day after the last workshop Time: 45-minutes	Post-test on ordering fractions, addition of fractions, and subtraction of fractions
2 weeks after the last workshop Time: 45 minutes	Post-interview on ordering fractions, addition of fractions, and subtraction of fractions

I started by asking teachers how to compare and order 1/4, 5/8, 8/9. The teachers worked together to solve the problem. We then shared our ideas and discussed how using an algorithm to solve the problem was time consuming and did not show evidence of conceptual understanding. We then discussed how using benchmarks of 0, 1/2, and 1 along with knowledge of the part-whole relationship was more time effective and showed evidence of conceptual understanding. Teachers then worked together to explain and justify the answers to the ordering fraction problems on their handout (Appendix H). At the close of the first workshop teachers were asked to incorporate into their classroom the strategies that they had learned related to ordering fractions. Teachers were requested to collect unidentifiable student samples related to the implementation of the strategy to share at our next meeting.

The next week we met for the second workshop. I started by asking teachers to share and discuss the unidentifiable student samples they brought showing how students compared and ordered fractions. We discussed how students arrived at their answers and discussed which students showed evidence of conceptual understanding. I then asked them how to add 2/3 plus 1/2. The teachers worked together to solve the problem. We then shared our ideas and discussed how using an algorithm to solve the problem did not show evidence of conceptual understanding. We then discussed how to properly use drawings to help teach fraction concepts with conceptual understanding. We discussed that if students did not have conceptual understanding and forgot the algorithm for adding fractions that they were likely to come up with the wrong answer. Teachers then worked together to explain and justify the answers to the addition of fraction problems on their handout (Appendix I). At the close of the workshop teachers were asked to incorporate into their classroom the strategies that they had learned related to how to add fractions. They were requested to collect unidentifiable student samples related to the implementation of this strategy to share at our next meeting.

Two weeks passed before we met for our third workshop. I started by asking teachers to share and discuss the unidentifiable student samples they brought showing how students added fractions. Teachers shared with each other their student samples discussing who provided evidence of conceptual understanding. I then asked the teachers how to subtract 5/6 minus 1/3. They worked together to solve the problem. We then shared our ideas and discussed how using an algorithm to solve the problem did not show evidence of conceptual understanding. We then discussed how using drawings can show evidence of conceptual understanding. Teachers worked together to explain and justify the answers to the subtraction of fraction questions on their handout (Appendix J). At the close of the workshop teachers were asked to incorporate into their classroom the strategies that they had learned related to subtracting fractions. Teachers were requested to collect unidentifiable student samples related to the implementation of this strategy to share at our next meeting.

The next week we met to finish our series of professional development workshops. I started by asking teachers to share and discuss the unidentifiable student samples they brought showing how students subtracted fractions. We then focused on reviewing the prior three workshops by discussing what the teachers had learned, what they felt was useful and what was not. We brought all of the information together and discussed the differences between procedural knowledge and conceptual knowledge and the need for both (Hecht, 1998).

The next day I gave the teachers the post-test (Appendix G) in a group setting during their planning time. Within the following two weeks after the series of professional development workshops, I individually interviewed the teachers again during their planning time (Appendix F). This concluded the data collection phase of my study.

Data Analysis

I first began by examining the pre-test, post-test, and teacher work samples for correct and incorrect solutions to the problems. I wanted to know if teachers could arrive at correct solutions to fraction problems regardless of how they solved them. It was during this time that I looked for possible fraction misconceptions.

I then looked at each fraction concept and data source looking to see whether the questions or problems were answered using procedural knowledge or conceptual knowledge. I wanted to see if a pattern existed in which the participants tended to use one type of knowledge versus the other. Did it vary from data source?

The use of drawing pictures was then examined. I looked to see where pictures were drawn. I also looked to categorize the picture as either representing the solution process or the solution. Take 1⁄2 plus 1⁄4 for example. When a picture is drawn to represent the solution process you can visually see how they used their drawing to find the solution. See figure one for an example.

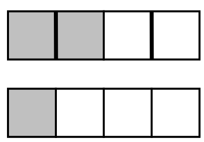


Figure 1 Picture representing the solution process

In this figure you can see the darker lines that were drawn to change the two 1⁄2 pieces into four 1⁄4 pieces. By doing this, equal pieces were made and then added to find a solution of 3⁄4. When a picture is drawn however to represent the solution there is no distinction to show how they solved the problem. See figure two for an example.

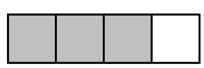


Figure 2 Picture representing the solution

In this drawing no thought process is shown, the picture represents only an answer with no indication on how it was solved.

Finally, I looked at the data from each participant from the beginning to the end. I started by looking at their pre-interview and examined all of their work till the post-interview looking for a change in their ability to explain and justify their solutions. I attempted to examine how each participant evolved during the professional development. Of course however, while examining the data, there were some assumptions.

Assumptions

It was assumed that the participants would begin the professional development with limited conceptual understanding of fractions and participate in all four workshops. It was also assumed that through the professional development they would construct conceptual understanding and solve the problems using that conceptual knowledge instead of using procedural knowledge. It was anticipated that they would take their knowledge back to their classrooms and incorporate it into their own practice. A further assumption was that teachers would ask questions for clarification and for help if needed. Other important assumptions were that teachers would be honest about their own learning and the learning of their students.

Limitations

A limitation to the study was that the professional development was held on Wednesday afternoons, a shortened school day, and only lasted only an hour each session. Due to the time constraints however, extending the time provided for this action research was not an option. Another limitation was that the teachers who participated were only from one school. Since the participants were my peers and volunteered to participate, it limited my ability to push them to provide explanations and justifications. There were no benefits such as a stipend or grade to motivate them. The last limitation was that fractions concepts were not addressed in the county pacing guide during the semester in which the research was conducted; they would have been addressed in the following semester. Therefore, when the teachers were asked to collect student samples, they had to teach mini lessons that were not included in the pacing guide. These limitations effected my professional development.

Conclusion

The qualitative methodology provided a context for me as I set out to describe in detail the affects of providing conceptually-based professional development on fractions to the fifth grade teachers at my school. The action research we participated in as a group provided data that helped identify these affects. Interpretation of the data is discussed in chapter four, Data Analysis. A thorough analysis of the data describes our experiences.

CHAPTER FOUR: DATA ANALYSIS

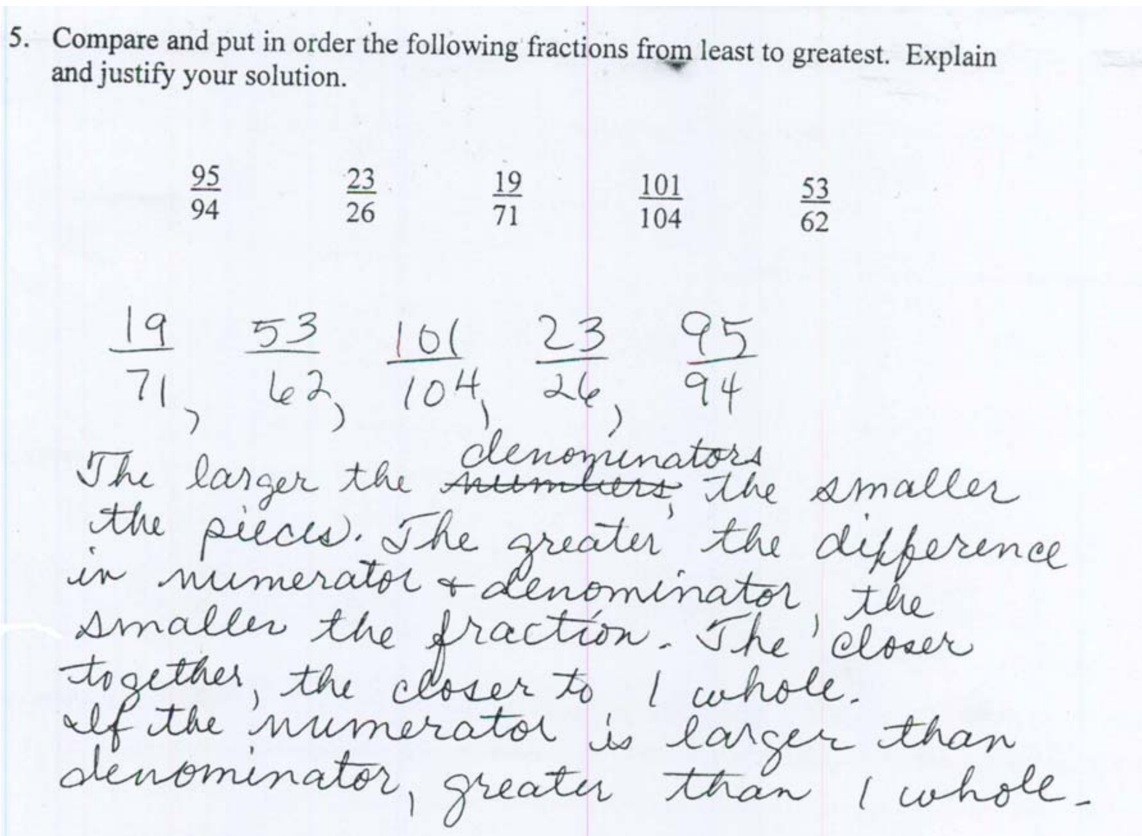
Introduction

After conducting a series of four one-hour professional development workshops with three fifth grade teachers, I began to analyze the data I collected. I wanted to know how my practice of providing conceptually-based professional development related to fractions affected teachers' abilities to explain and justify procedures related to ordering, adding, and subtracting fractions. The first observation I made was related to one participants' reoccurring misconception that did not allow them to correctly solve ordering fraction problems on the pre-test. In addition to this observation, two themes emerged. The first theme was that all of the participants were able to give conceptually based answers to the questions on the post-interview regardless of whether or not they demonstrated conceptual knowledge during the professional development. The second theme that emerged was that when participants gave conceptual solutions to the problems, they all included a drawing that demonstrated their thinking. The following narrative demonstrates how my practice of providing conceptually-based professional development increased the participants' conceptual understanding of fractions. Achieving this deeper understanding was the primary purpose of my professional development series and it was the determining factor of its success.

Misconceptions

Before trying to decide if the participants demonstrated procedural or conceptual knowledge of fractions, I first decided to find out if they were able to correctly answer the problems. I looked at the problems that the participants solved incorrectly to determine if computational errors were made or if misconceptions were present. The computational errors were minor and will not be discussed. The one misconception that was observed is discussed below.

Jessie's misconception came from the pre-test prior to participating in the professional development. She missed both of the problems having to do with ordering fractions. The first problem asked her to compare and put in order 95/94, 23/26, 19/71, 101/104, and 53/62 from least to greatest and she answered 19/71, 53/62, 101/104, 23/26, and 95/94 (see figure 3).


5. Compare and put in order the following fractions from least to greatest. Explain and justify your solution.

$\frac{95}{94}$ $\frac{23}{26}$ $\frac{19}{71}$ $\frac{101}{104}$ $\frac{53}{62}$

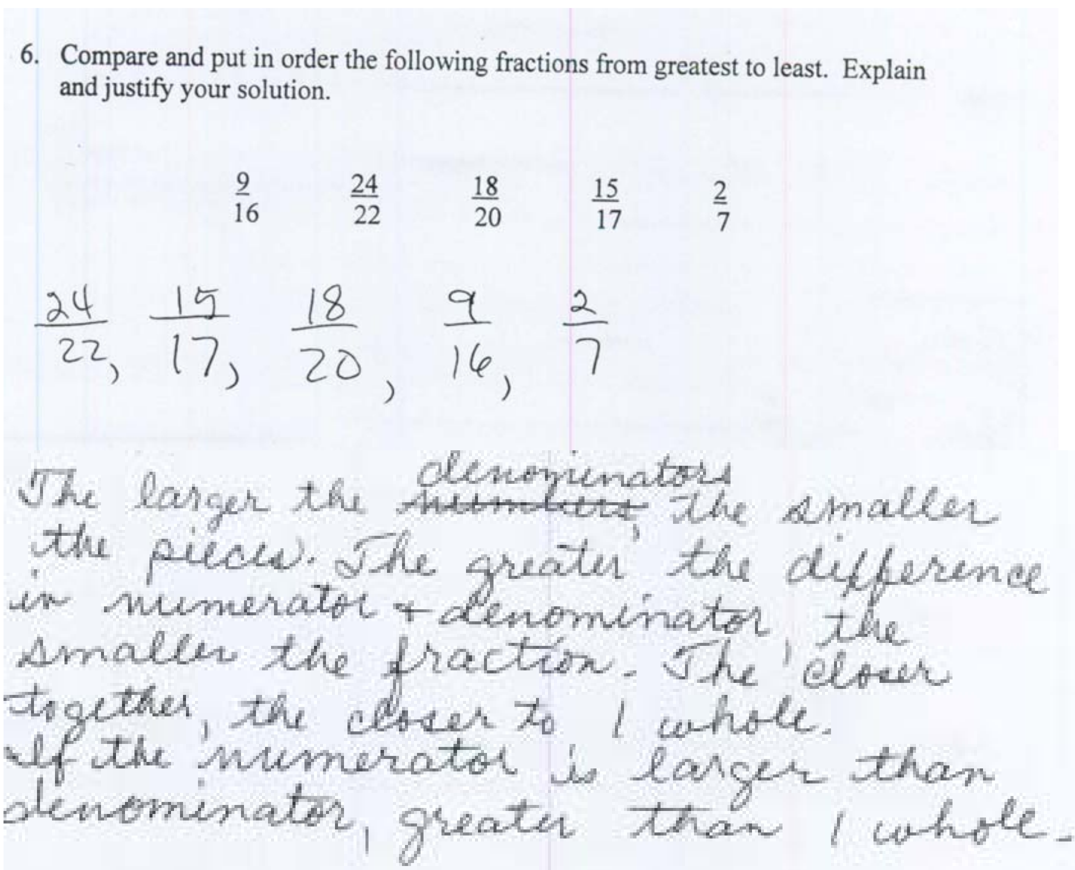
$\frac{19}{71}$, $\frac{53}{62}$, $\frac{101}{104}$, $\frac{23}{26}$, $\frac{95}{94}$

The larger the ~~numbers~~ denominators, the smaller the pieces. The greater the difference in numerator + denominator, the smaller the fraction. The closer together, the closer to 1 whole. If the numerator is larger than denominator, greater than 1 whole.

Figure 3 Jessie's solution to problem 5 on the pre-test

Her explanation was that the larger the denominator, the smaller the pieces. She explained that the greater the difference in the numerator and denominator, the smaller the fraction and that the closer together, the closer the fraction is to one whole. She ended in saying that if the numerator is larger than the denominator, it is greater than one whole. Analyzing her explanation she was incorrect in her thinking that the greater the difference in the numerators and denominators, the smaller the fraction. When she went to compare two fractions that were both three away from the denominator she was missing a concept about the size of the piece that prevented her from correctly ordering the fractions. She did not take into consideration when comparing 101⁄104 and 23⁄26 that they were both three away from a whole and that the smaller pieces in 101⁄104 meant that there were three smaller pieces missing from the whole compared to the three larger pieces missing from 23⁄26.

The second problem she answered incorrectly asked her to compare and put in order 9/16, 24/22, 18/20, 15/17, and 2/7 from greatest to least. She answered 24/22, 15/17, 18/20, 9/16, and 2/7 (see figure 4).


6. Compare and put in order the following fractions from greatest to least. Explain and justify your solution.

$\frac{9}{16}$ $\frac{24}{22}$ $\frac{18}{20}$ $\frac{15}{17}$ $\frac{2}{7}$

$\frac{24}{22}$, $\frac{15}{17}$, $\frac{18}{20}$, $\frac{9}{16}$, $\frac{2}{7}$

The larger the ~~numbers~~ denominators the smaller the pieces. The greater the difference in numerator + denominator the smaller the fraction. The closer together, the closer to 1 whole. If the numerator is larger than denominator, greater than 1 whole.

Figure 4 Jessie's solution to problem six on the pre-test

Her explanation for this problem was the same as for the problem before. Again, she had a misconception and was not able to correctly order two fractions that were both two away from a whole. These incorrect answers showed that Jessie did not have an understanding that the smaller the denominator, the larger the size of the pieces, making the missing pieces bigger in size.

Procedural or Conceptual Knowledge

Knowing that the participants could correctly solve the majority of the problems, I looked to see specifically how they solved the problems First, I reviewed the participants and their work as a whole. Did they use procedural knowledge or conceptual knowledge to solve the problems? Did it vary from activity? (see table 2).

Table 2 Use of Procedural Knowledge versus Conceptual Knowledge

	Prior to Professional Development		During Professional Development		After Professional Development	
Fraction Concept	Procedural Knowledge	Conceptual Knowledge	Procedural Knowledge	Conceptual Knowledge	Procedural Knowledge	Conceptual Knowledge
Ordering	37%	63%	66%	34%	22%	78%
Adding	100%	0%	28%	72%	67%	33%
Subtracting	100%	0%	80%	20%	67%	33%

In order to categorize the participants' solutions to the problems as either showing evidence of procedural or conceptual knowledge I looked at their explanations and justifications. If they correctly described the solution process without using an algorithm then I determined that the problem was solved conceptually. If they did not describe the solution process and/or used a algorithm I determined that the problem was solved procedurally.

When it came to examining ordering fractions the percent of the problems that were solved conceptually varied based on the activity. Prior to the professional development, 63% of problems were solved using conceptual knowledge. This implies that the participants began the workshop with some conceptual understanding of how to order fractions. During the professional development however the percent decreased and only 34% of the problems were solved using conceptual understanding. I attribute this drop to a participant who had a dominating personality and convinced the other participants to order the fractions by converting them to percents. After the professional development the percent increased again and 78% of the problems were solved showing conceptual understanding. This 78% includes all three participants' answers to the post-interview in regards to ordering fractions. This leads me to believe that the participants had some knowledge of conceptual understanding prior to the professional development and that while they did not use their conceptual knowledge as much as their procedural knowledge during the professional development, they did show an increase of understanding based on the activities after the professional development which included the post-interview.

Examining the activities before and after the professional development that dealt with addition of fractions showed a preferred use of procedural knowledge to solve the problems. None of the problems prior to the professional development were solved conceptually. During the professional development however, 72% of the problems were solved conceptually and after the professional development 33% of the problems were also solved conceptually. The 33% includes all three participants' answers to the post-interview in regards to addition of fractions. While the percent of problems that were solved conceptually decreased after the professional development, these results lead me to believe that the participants could solve these problems conceptually as they were able to do so during the professional development and on the post-interview. I attribute the decline in using conceptual understanding to the fact that solving the problems procedurally was more efficient in regards to time.

Finally, when looking at subtracting fractions, procedural knowledge was used more than conceptual knowledge during all the activities. None of the activities before the professional development, 20% of the activities during the professional development, and 33% of the activities after the professional development were all solved conceptually. These results originally led me to believe that the participants did not have conceptual understanding of how to subtract fractions but on the post-interview all three participants' answers in regards to subtraction of fractions were conceptually based. Again, it appears that the participants preferred to use procedures even though they had conceptual understanding.

Use of Pictures

When addition and subtraction of fractions were taught in this professional development the use of drawing pictures was encouraged to show evidence of conceptual understanding. When examining the addition and subtraction of fraction problems, it was noticed that not all answers included pictures and that those that did have pictures showed evidence that some were drawn representing the solution process (see figure 6) whereas others showed that they were drawn to represent the solution (see figure 7).

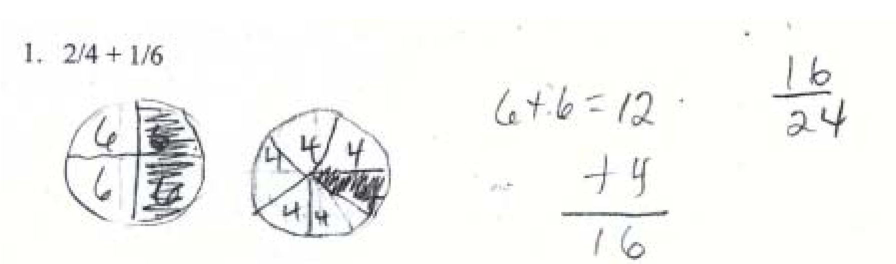


Figure 5 Picture representing the solution process

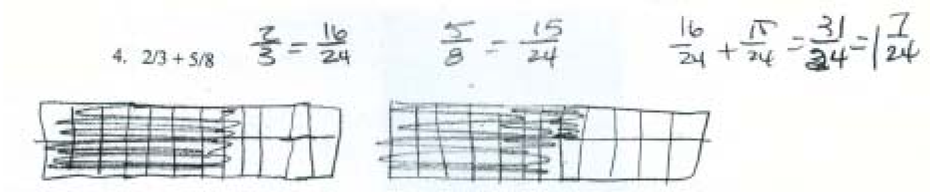


Figure 6 Picture representing the solution

In figure 6 the picture shows how the participant first drew and shaded in 2⁄4 of a circle and then 1⁄6 of another circle representing the problem. They then showed how they made the pieces in both circles equal by “dividing” each 1⁄4 piece into 6 pieces and each 1⁄6 piece into 4 pieces. By doing this 2⁄4 became 12⁄24 and 1⁄6 became 4⁄24. 12⁄24 plus 4⁄24 is 16⁄24. This picture shows the participants solution process.

In figure 7 the picture does not show the participants solution process. They drew two rectangles and divided each into 24 pieces. They then shaded 16 pieces in one rectangle and 15 pieces in the other. There is no evidence that the pictures once represented 2⁄3 and 5⁄8. This therefore leads me to believe that the participant drew the picture after finding the solution.

Of the 21 pictures drawn for addition problems, 12 were pictures that represented the problem. Of the 17 pictures drawn for subtracting problems, six were pictures that represented the problem. In examining the drawings that answered the problems it was noticed that all of those problems were all solved conceptually.

Participants’ Individual Growth

Alex

Alex began the professional development with some conceptual understanding of how to compare and order fractions and strictly procedural knowledge of how to add and subtract fractions. Her use of drawings was limited and they represented the solutions not the problems.

During the session on ordering fractions, she solved all five problems using procedural knowledge. Then, during the session on adding fractions, she used conceptual knowledge to solve 2 of the 5 problems and her drawings for those two problems represented the solution to the problems. Next, with subtraction of fractions, she used conceptual knowledge to solve 1 of the 5 problems. Her drawing for the one problem she solved conceptually did represent the solution to the problem. After the professional development, Alex used procedural knowledge to answer 4 of the 5 problems on the post-test. The only time she used conceptual understanding was when she put fractions in order. It is obvious from the overall data that Alex prefers to use procedural knowledge to solve the fraction problems. It is also obvious from her post-interview that she has conceptual understanding of fractions as she answered all three questions conceptually with drawings that matched the solution process.

Terri

Terri began the professional development with mostly procedural knowledge. She answered only one ordering problem using conceptual knowledge. She did use drawings frequently. However, the drawings described her answers not the solutions to the problems. She solved 2 of the 4 problems during the ordering fractions sessions using conceptual knowledge. During the session on addition of fractions, she solved all five of the problems conceptually using drawings for four of them that represented the solution to the problem. The last session on subtracting fractions she solved 1 of the 5 problems conceptually. While she continued to use drawings, they reflected her answers and not the solutions to the problems. After the professional development, she solved all of the addition and subtraction problems on the post-test using procedural knowledge. The only problems she answered using conceptual knowledge were the ones that involved ordering fractions. Terri was able to use conceptual knowledge and drawings that match the problem to the answer for all the questions on the post-interview.

Jessie

Like the other participants, Jessie began the professional development with mostly showing evidence of procedural knowledge of fraction concepts. The only question she answered with conceptual knowledge prior to the professional development was the question on the pre-test that asked how do you teach students to compare and order 1⁄4, 5⁄8, and 8⁄9 and why. During the ordering fraction session she answered all of the problems correctly and for 2 of the 4 problems she used conceptual understanding. If you recall, Jessie was the participant who had a misconception about the size of the missing pieces when comparing two fractions that were missing the same number of pieces. During this activity she solved two questions conceptually in which see used the model of missing pieces to find her solution (see figure 7 & 8).

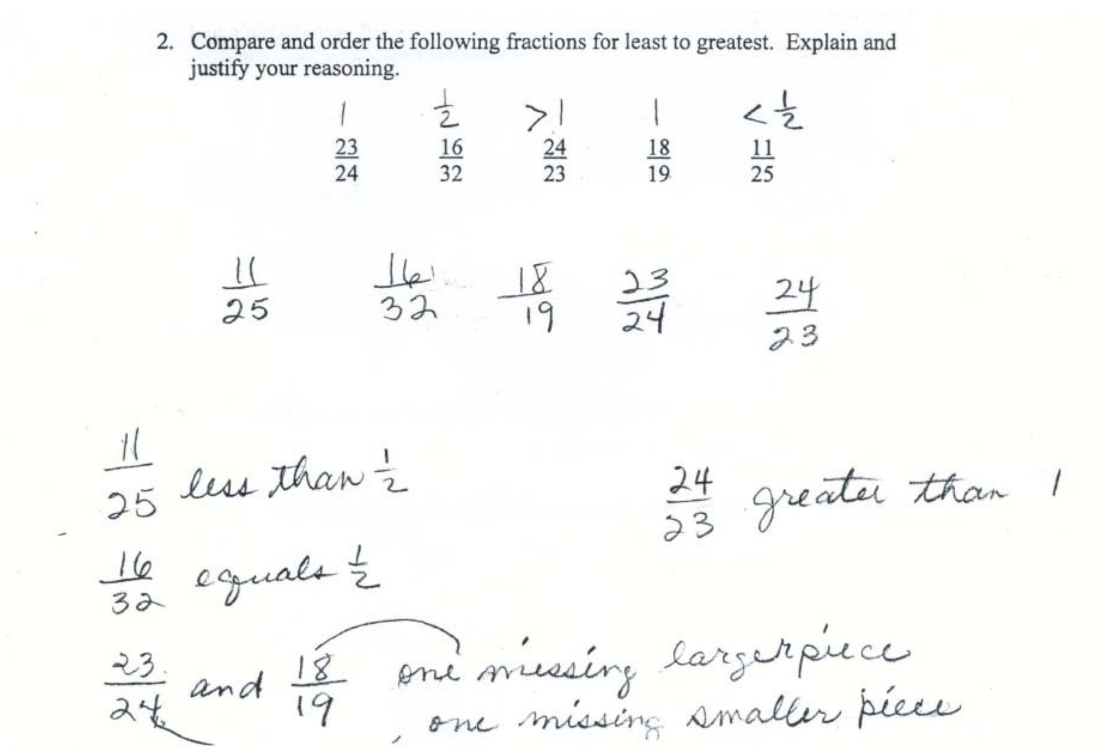


Figure 7 Jessie's solution to problem 2 on the ordering fractions teacher work sample.

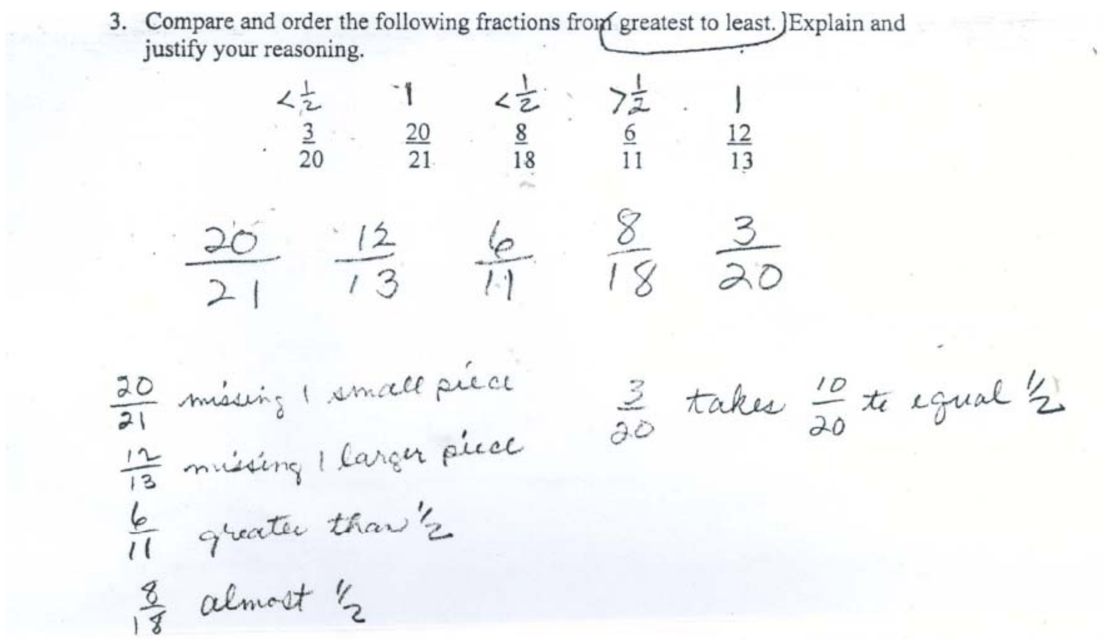


Figure 8 Jessie's solution to problem 3 on the ordering fractions teacher work sample.

In both of these figures you can see how she identified each of the fractions that were one away from a whole by describing them as missing either one small or large piece. This is evidence that she had overcome her prior misconceptions.

During the addition of fractions session, Jessie answered 3 of the 5 problems conceptually and her drawings matched the solution process. During the last session on subtracting fractions, she answered 1 of the 5 problems conceptually and her use of drawings was limited. On the post-test she answered all 6 of the problems procedurally yet during the post-interview she was able to give conceptual answers for all three questions with drawings to match the solution process.

Conclusion

The three sources I used to collect data for this research were pre- and post- interviews, pre- and post-tests, and teacher work samples. The themes that emerged from my research included an increase in teachers' conceptual knowledge related to fractions and drawings that demonstrated conceptual understanding. Overall the themes that emerged in my study were trustworthy and supported by the research conducted by others. In chapter five I will discuss in further detail how my themes are supported by the literature review.

CHAPTER 5: CONCLUSION

Introduction

I was curious to see what would happen when I conducted conceptually-based professional development related to fractions with the fifth grade teachers at my school. I was able, through this action research study, to answer my question, *how does my practice of providing conceptually-based professional development related to fractions affect teachers' abilities to explain and justify ordering, adding, and subtracting fractions*? In this chapter I will discuss my findings, connect my research study to the literature, and offer recommendations for future studies.

Findings

My initial plan when designing the professional development was to model the graduate class I participated in that focused on conceptual understanding and forced me to explain and justify my solution process. In this class, social norms were established from the very first day that we had to explain and justify our solution process using words and pictures. While this was the atmosphere I intended to create, I found that I did not establish these necessary norms. By not establishing these norms, the explanations and justifications I asked my participants to provide were not as detailed as I had envisioned them to be.

To examine the explanations and justifications that the participants provided I intended to examine the pre- and post-interviews, pre- and post-tests, and the teacher work samples in an attempt to categorize each individual participant's level of conceptual understanding. I thought that I could score each of their responses on a scale of 0-3 and average them together to determine their overall conceptual understanding of fractions. What I did not realize was that attempting to categorize the participants into different levels of conceptual understanding would not show me my participants' solution processes and how those processes may or may not have changed over the course of the professional development. When I realized the flaws in my original plan I abandoned the notion of scoring their responses on a scale but at the same time I still had to make a value judgment when I categorized their solutions as either showing evidence of procedural or conceptual knowledge. In making this judgment I assumed that if they solved the problems procedurally or did not explain their solution process that they did not have conceptual understanding. However, as my results indicated, just because the participants solved problems procedurally did not mean that they did not have conceptual understanding.

Connections to the Literature

Research indicates that teachers' have a lack of conceptual understanding in mathematics (Brover et al., 2001; Cohen & Hill, 2000; Cooney, 1994; Fuller, 1996; Koency & Swanson, 2000; Ma, 1999; Weissglass, 1994). This was evident in the data I collected prior to conducting my professional development. The teachers that participated in my study resorted to solving fraction problems based on procedures with little understanding of the concepts during both the pre-interview and pre-test. The research also suggests that teachers are not proficient with fractions (Fuller, 1996; Lester, 1984; Ma, 1999, Tirosh et al., 1998). An example of this emerged when one of the teachers had a misconception of fractions and did not understand that the smaller the denominator, the larger the size of the pieces, making the missing pieces bigger in size.

Research indicated however that through providing continuing professional development focused on content that teachers' could increase their conceptual understanding of mathematics (Brover et al., 2001; Cohen & Hill, 2000; Kennedy, 1999; Koency & Swanson, 2000; Mistretta, 2005; Schmidt et al., 2002). This too was evident in the data I collected. After the teachers participated in my professional development they were able to solve fraction problems using drawings to represent their thinking. Research from Niemi (1996) indicates that representational knowledge is one effective way to evaluate conceptual understanding.

Implications

The results of my action research study support the current research that teachers can acquire conceptual understanding of mathematics through ongoing professional development focusing on the content that their students are to learn, how they are to teach the content, and how students are to learn it (Cohen & Hill, 2000; Kennedy, 1999). School districts and administrators need to realize the importance of professional development specifically related to concepts. More time and resources need to be devoted to preparing our teachers. Professional development needs to be focused on content, well planned, continued for an extended period of time, allow for collaboration, and provide time for reflection. When the time and money is spent preparing our teachers, students will begin to receive the quality education they deserve and need.

Recommendations

The results of my study provide support that professional development can increase teachers' conceptual understanding of fractions. I found with the group of three teachers that I worked with that their conceptual understanding of fractions began to evolve with even minimal time for professional development. If I was to conduct this research again however I would make some adjustments. The first thing I would change would be to increase the number of participants. I feel that conducting this research with a larger sample size would allow me to generalize the results to other teachers. I would also like to provide the professional development for a longer period of time and overlap the professional development with when fractions are addressed in the curriculum. Providing this professional development during the classroom instruction on fractions would be more appropriate. Also, by increasing the time allotted for the professional development I could establish the norms of explaining and justifying the solution process to the extent in which I had originally anticipated. Extending the allotted time would also allow me to tie the research to student achievement. Research suggests that when professional development occurs for more than 14 hours that there are positive and significant effects on student achievement (Darling-Hammond et al., 2009).

Ideally, I believe that the best way to conduct this research would be to have a larger number of teachers who possess procedural knowledge of fractions participate. I would divide the teachers into two groups. One group would receive conceptually-based professional development focused on ordering, adding, and subtracting fractions while the other group would receive no professional development. Their students would then be administered a pre-test and post-test based on ordering, adding, and subtracting fractions prior to and after instruction from their teachers. The post-test results would then be compared based on the two groups of teachers to identify if one group possessed higher levels of conceptual understanding than the other. If the results showed positive students' achievement from the teachers who received the conceptually-based professional development then the other group of teachers would receive the professional development too. By making these changes, I feel that my study can be improved.

Conclusion

This action research has allowed me to examine my practice of providing conceptually-based professional development related to ordering, adding, and subtracting fractions. Through this journey I have been able to reflect on my own learning experiences and provide conceptually-based instruction for my fellow colleagues. I learned that while I set out to push the participants to explain and justify their solution processes that I failed to accomplish this. When working with my peers I felt uncomfortable to push them from their comfort zone to give the explanations I was looking for. In order to be a better mathematics coach I need to be able to gain the confidence to push teachers just the right amount. As a mathematics leader for my county I will continue to improve my practice of providing conceptually-based instruction so that teachers can gain the knowledge needed to provide quality instruction for their students.

APPENDIX A: INSTITUTIONAL REVIEW BOARD

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901, 407-882-2012 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Notice of Expedited Initial Review and Approval

From : **UCF Institutional Review Board**
FWA00000351, Exp. 6/24/11, IRB00001138

To : **Jessica Maguhn**

Date : **August 26, 2008**

IRB Number: **SBE-08-05757**

Study Title: **Conceptually based professional development on fractions.**

Dear Researcher:

Your research protocol noted above was approved by **expedited** review by the UCF IRB Vice-chair on 8/26/2008. **The expiration date is 8/25/2009.** Your study was determined to be minimal risk for human subjects and expeditable per federal regulations, 45 CFR 46.110. The category for which this study qualifies as expeditable research is as follows:

> 7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

The IRB has approved a **consent procedure which requires participants to sign consent forms.** **Use of the approved, stamped consent document(s) is required.** Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

To continue this research beyond the expiration date, a Continuing Review Form must be submitted 2 – 4 weeks prior to the expiration date. Advise the IRB if you receive a subpoena for the release of this information, or if a breach of confidentiality occurs. Also report any unanticipated problems or serious adverse events (within 5 working days). Do not make changes to the protocol methodology or consent form before obtaining IRB approval. Changes can be submitted for IRB review using the Addendum/Modification Request Form. An Addendum/Modification Request Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at http://iris.research.ucf.edu .

Failure to provide a continuing review report could lead to study suspension, a loss of funding and/or publication possibilities, or reporting of noncompliance to sponsors or funding agencies. The IRB maintains the authority under 45 CFR 46.110(e) to observe or have a third party observe the consent process and the research.

On behalf of Tracy Dietz, Ph.D., UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 08/26/2008 01:27:06 PM EDT

Janice m Turchin

IRB Coordinator

APPENDIX B: DISTRICT APPROVAL

Submit this form and a copy of your proposal to:
Accountability, Research, and Assessment
P.O. Box 271

RESEARCH REQUEST FORM

Your research proposal should include: Project Title; Purpose and Research Problem; Instruments; Procedures and Proposed Data Analysis

Requester's Name: Jessica Maguhn　　Date: 6/30/08

Address: Home　　Phone

Business　　Phone

Project Director or Advisor: Dr. Juli Dixon　　Phone: 407-823-4140

Address: College of Education, PO Box 161250 Orlando, FL 32816-1250

Degree Sought: (check one)　☐ Associate　☐ Doctorate　☐ Bachelor's　☐ None　☑ Master's　☐ Specialist

Project Title: Conceptually Based Professional Development On Fractions

ESTIMATED INVOLVEMENT

PERSONNEL/CENTERS	NUMBER	AMOUNT OF TIME (DAYS, HOURS, ETC.)	SPECIFY/DESCRIBE GRADES, SCHOOLS, SPECIAL NEEDS, ETC.
Students			
Teachers	6	11 hours	5th grade teachers @
Administrators			
Schools/Centers			
Others (specify)			

Specify possible benefits to students/school system: Possible benefits include an increase in teacher content knowledge and an increase in student learning.

ASSURANCE　　▸ RECEIVED JUL 03 2008

Using the proposed procedures and instrument, I hereby agree to conduct research in accordance with the policies of the Public Schools. Deviations from the approved procedures shall be cleared through the Senior Director of Accountability, Research, and Assessment. Reports and materials shall be supplied as specified.

Requester's Signature: Jessica Maguhn

Approval Granted: ☒ Yes　☐ No　　Date: 7-7-08

Subject to principal approval.

Signature of the Senior Director for Accountability, Research, and Assessment: Lee Baldwin

NOTE TO REQUESTER: When seeking approval at the school level, a copy of this form, signed by the Senior Director, Accountability, Research, and Assessment, should be shown to the school principal.

Reference School Board Policy GCS, p. 249

7PL202　　FORM ID #GB0103/23-1/1FY　REV 1/04

APPENDIX C: PRINCIPAL APPROVAL

June 19, 2008

IRB Office
Office of Research & Commercialization
University of Central Florida
12201 Research Parkway, Suite 501
Orlando, FL 32826

To Whom It May Concern:

I, Daniel Merchant, give Jessica Maguhn, graduate student at UCF, permission to conduct her action research project on conceptually based professional development at Elementary for the school year of 2008-2009.

Sincerely,

Daniel Merchant
Principal

APPENDIX D: CONSENT FORM

Informed Consent for:

Conceptually Based Professional Development on Fractions

Researchers at the University of Central Florida (UCF) study many topics. To do this we need the help of people who agree to take part in a research study. You are being invited to take part in a research study which will include about six people. You can ask questions about the research. You can read this form and agree to take part right now, or take the form home with you to study before you decide. You will be told if any new information is learned which may affect your willingness to continue taking part in this study. You have been asked to take part in this research study because you are a teacher. You must be 18 years of age or older to be included in the research study and sign this form.

The person doing this research is Jessica Maguhn of the UCF Teaching and Learning Principles Department. This research is a Masters Thesis requirement. Because the researcher is a graduate student, she is being guided by Dr. Juli Dixon, a UCF faculty supervisor in the Teaching and Learning Principles Department.

Study title: Conceptually based professional development on fractions

Purpose of the research study: The purpose of this study is to determine the effects of professional development practice on elementary school teachers' conceptual understanding of fractions.

What you will be asked to do in the study: Participants will be asked to take part in four, one hour, professional development sessions over the course of two months. In addition, an individual pre- and post interview consisting of 3 questions related to current teaching practices of adding, subtracting, and ordering fractions will be conducted. A pre- and post test consisting of 6 questions relating to conceptual understanding of adding, subtracting, and ordering fractions will be administered in a group setting. During the professional development sessions, teacher work samples will be collected from the participants. Participants will be expected to reflect in a journal writing one or more paragraphs on their experiences after the first 3 sessions, incorporate the learned strategies in their teaching within a week, and provide unidentifiable student samples showing the implementation of the learned strategies in their classroom. The interviews, tests, teacher work samples, and journals will be collected, analyzed, and included in my research findings.

Voluntary participation: You should take part in this study only because you want to. There is no penalty for not taking part, and your job as a teacher will not be affected whether you participate or not. You have the right to stop at any time. Just tell the researcher or a member of the research team that you want to stop. You will be told if any new information is learned which may affect your willingness to continue taking part in this study. Results of this study will not be shared with or the principal.

Location:

Time required: This study will involve 4 professional development sessions to be conducted over the span of two months. 1 ½ hours total for the pre- and post test during planning time. 30 minutes total for the pre- and post interview during planning time. 4 hours total for the professional development sessions after school on Wednesday afternoons. An hour or less for reflections during personal time. 4 hours or less for implantation of strategies in the classroom.

Audio or video taping: This study does not include any audio or video taping.

Risks: There are no expected risks for taking part in this study. You do not have to answer every question or complete every task. You will not lose any benefits if you skip questions or tasks. You do not have to answer any questions that make you feel uncomfortable.

Benefits: Benefits for participating in this study include professional development points and learning more about how research is conducted.

Compensation or payment:
There is no compensation or other payment to you for taking part in this study.

Confidentiality: The researcher will make every effort to prevent anyone who is not on the research team from knowing that you gave us information, or what that information is. For example, your information will be assigned a code number and there will be no list connecting you to your code. Your information will be combined with information from other people who took part in this study. When the researcher writes about this study in her thesis to share what was learned with other researchers, she will write about this combined information. Your name will not be used in any report, so people will not know how you answered or what you did. This research does require some group work during the professional development sessions and confidentiality among the group members is expected but once group members leave the group, confidentiality cannot be guaranteed.

Study contact for questions about the study or to report a problem: Jessica Maguhn, Graduate Student, Teaching and Learning Principals, College of Education, (407) 933-0789 or Dr. Juli Dixon, Faculty Supervisor, Department of Teaching and Learning Principals at (407) 823-2233 or by email at jkdixon@mail.ucf.edu.

IRB contact about your rights in the study or to report a complaint: Research at the University of Central Florida involving human participants is carried out under the oversight of the Institutional Review Board (UCF IRB). For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901.

How to return this consent form to the researcher: Please sign and return this consent form to Jessica Maguhn. A second copy is provided for your records. By voluntarily agreeing to take part in this study, you give me permission to report your responses anonymously in the final manuscript to be submitted to my faculty supervisor as part of my course work.

☐ I have read the procedure described above

☐ I voluntarily agree to take part in the study

☐ I do not agree to take part in the study

☐ I am at least 18 years of age or older

____________________________ ____________________________ ________

Signature of participant Printed name of participant Date

______________________________________ ____________

Principal Investigator Date

APPENDIX E: PRE-INTERVIEW

Pre-Interview Questions

How do you teach students to compare and order 1/4, 5/8, 8/9? Why?

How do you teach students to add 2/3 + 1/2? Why?

How do you teach students to subtract 5/6 – 1/3? Why?

APPENDIX F: POST-INTERVIEW

Post Interview Questions

How will you teach students to compare and order 1/4, 5/8, 8/9? Why?

How will you teach students to add 2/3 + 1/2? Why?

How will you teach students to subtract 5/6 – 1/3? Why?

APPENDIX G: PRE-TEST/POST-TEST

Pre-Test / Post-Test

Explain and justify 1/5 + 4/6

Explain and justify 4/10 + 1 2/5

Explain and justify 4/5 – 1/9

Explain and justify 1 2/3 – 4/6

Compare and put in order the following fractions from least to greatest. Explain and justify your solution.

$$\frac{95}{94} \qquad \frac{23}{26} \qquad \frac{19}{71} \qquad \frac{101}{104} \qquad \frac{53}{62}$$

Compare and put in order the following fractions from greatest to least. Explain and justify your solution.

$$\frac{9}{16} \qquad \frac{24}{22} \qquad \frac{18}{20} \qquad \frac{15}{17} \qquad \frac{2}{7}$$

APPENDIX H: ORDERING FRACTIONS TEACHER WORK SAMPLE

Ordering of Fractions

Compare and order the following fractions from least to greatest. Explain and justify your reasoning.

$$\frac{1}{3} \qquad \frac{4}{5} \qquad \frac{5}{7} \qquad \frac{6}{7} \qquad \frac{3}{6}$$

Compare and order the following fractions from least to greatest. Explain and justify your reasoning.

$$\frac{23}{24} \qquad \frac{16}{32} \qquad \frac{24}{23} \qquad \frac{18}{19} \qquad \frac{11}{25}$$

Compare and order the following fractions from greatest to least. Explain and justify your reasoning.

$$\frac{3}{20} \qquad \frac{20}{21} \qquad \frac{8}{18} \qquad \frac{6}{11} \qquad \frac{12}{13}$$

Compare and order the following fractions from greatest to least. Explain and justify your reasoning.

$$\frac{15}{27} \qquad \frac{12}{18} \qquad \frac{11}{23} \qquad \frac{22}{27} \qquad \frac{12}{15}$$

APPENDIX I: ADDITON OF FRACTIONS TEACHER WORK SAMPLE

Addition of Fractions

Solve each problem. Explain and justify your reasoning.

2/4 + 1/6

3/8 + 3/6

4/10 + 1 3/5

2/3 + 5/8

1 2/4 + 2/5

APPENDIX J: SUBTRACTION OF FRACTIONS TEACHER WORK SAMPLE

Subtraction of Fractions

Solve each problem. Explain and justify your reasoning.

6/8 – 1/2

4/5 – 2/8

5/6 – 2/4

1 2/3 – 3/4

1 2/4 – 5/8

REFERENCES

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Teacher's knowledge of subject matter as it relates to their teaching practice* (Vol. 2). Greenwich, CT: JAI Press.

Baviskar, S.N., Hartle, R.T., & Whitney, T. (2009). Essential criteria to characterize constructivist teaching: Derived from a review of the literature and applied to five constructivist-teaching method articles. *International Journal of Science Education, 31(4)*, 541-550.

Brover, C., Deagan, D., & Farina, S. (2001). *Why understanding 1 ¾ divided by ½ matters to math reform: ABE teachers learn the math they teach*. (ERIC Document Reproduction Service No. ED482956) Retrieved March 31, 2008, from ERIC database.

Brown, G., & Quinn, R. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher, 62(4),* 28-40.

Brown, S. I., Cooney, T. J., & Jones, D. (1990). Mathematics teacher education. In W. R. Houston (Ed.), Handbook of research on teacher education (pp. 639-656). New York: Macmillan.

Castle, K., & Aichele, D. B. (1994). Professional development and teacher autonomy. In D. Aichele & A. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 1-8). Reston, VA: National Council of Teachers of Mathematics.

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record*, *102(2),* 294-343.

Cohen, D. K., & Hill, H. C. (2001). *Learning policy*. New Haven, CT: Yale University Press.

Cooney, T. J. (1994). Teacher education as an exercise in adaptation. In D. B. Aichele &A. F. Coxford (Eds.), Professional development for teachers of mathematics. Reston, VA: National Council of Teachers of Mathematics.

Cramer, K. A., Post, T. R., & delMas, R. C. (2002). Initial fraction learning by fourth- and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education, 33(2),* 111-144.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the United States and abroad*. Dallas, TX: National Staff Development Council.

Dixon, J. K. (2005). *Professional development for math: Fraction Operations*. Orlando, FL: Harcourt School Publishers.

Dixon, J. K. (2008). Understanding the K-8 mathematics standards: An example of depth: Comparing fractions. Dimensions in Mathematics, 28(1), 5-8.

Fuller, R. (1996). *Elementary teachers' pedagogical content knowledge of mathematics.* (ERIC Document Reproduction Service No. ED401270) Retrieved January 21, 2009, from ERIC database.

Hecht, S. A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90(3),* 545-559.

Hiebert, J., & Stigler, J. (2004). A world of difference. *JSD, 25(4),* 10-15.

Hiebert, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: National Center for Education Statistics.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42(2)*, 371-406.

Kennedy, M. (1999). Form and substance in mathematics and science professional development. *NISE Brief, 3(2),* 1-7.

Koency, G., & Swanson, J. (2000). *The special case of mathematics: Insufficient content knowledge a major obstacle to reform*. (ERIC Document Reproduction Service No. ED444943) Retrieved March 31, 2008, from ERIC database.

Lester, F. K. (1984). Preparing teachers to teach rational numbers. *Arithmetic Teacher,31*(6), 54-56.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Mistretta, R. (2005). Mathematics instructional design: Observations from the field. *The Teacher Educator, 41(1),* 16-33.

National Council of Teachers of Mathematics (2006). *Curriculum focal points forK-8*. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the national mathematics advisory panel*, U.S. Department of Education: Washington, DC.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89(6),* 351-363.

Reys, B., Kim, O., & Bay, J. (1999). Establishing fraction benchmarks. *Mathematics Teaching in the Middle School*, *4(8)*, 530-32.

Saxe, G. B., Gearhart, M., & Nasir, N. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education, 4*, 55-79.

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator*, *26(2),* 1-18.

Smith, J. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions* (pp.3-17). Reston, VA: National Council of Teachers of Mathematics.

Siebert, D. & Gaskin, N. (2006). Creating, naming, and justifying fractions. *Teaching Children Mathematics, 12(8)*, 394-400.

Sigford, J. L. (2006). *The effective school leader's guide to management.* Thousand Oaks, CA: Corwin Press.

Sigler, E. A., & Saam, J. (2006). Teacher candidates' conceptual understanding of conceptual learning: From theory to practice. *Journal of Scholarship of Teaching and Learning, 6,* 118-126.

Stoddart, T., Connell, M., Stofflett, R., & Peck, D. (1993). Reconstructing elementary teacher candidates' understanding of mathematics and science content. *Teacher & Teacher Education, 9(3)*, 229-241.

Tirosh, D., Fischbein, E., Graeber, A. O., & Wilson, J. W. (1998). Prospective elementary teachers' conceptions of rational numbers. Retrieved March 23, 2009, from http://jwilson.coe.uga.edu/Texts.Folder/Tirosh/Pros.El.Tchrs.html

Weissglass, J. (1994). Changing mathematics teaching means changing ourselves: Implications for professional development. In D. Aichele & A. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 67-78). Reston, VA: National Council of Teachers of Mathematics.

Wheeldon, D. (2008). Developing mathematical practices in a social context: An instructional sequence to support prospective elementary teachers' learning of fractions. Ed.D. dissertation, University of Central Florida, United States Florida. Retrieved March 22, 2009, from Dissertations & Theses @ University of Central Florida-FCLA database. (Publication No. AAT 3319283).

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458-477.

Zech, L. K., Gause-Vega, C. L., Bray, M. H., Secules, T., & Goldman, S. R. (2000). Content-based collaborative inquiry: A professional development model for sustaining educational reform. *Educational Psychologist, 35(3)*, 207-217.