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## The Influence of Choice in Manipulatives on Second Grade Students' Attitudes, Achievement, and Explanations of Two-Digit Addition Concepts

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THE INFLUENCE OF CHOICE IN MANIPULATIVES ON SECOND GRADE STUDENTS'  
ATTITUDES, ACHIEVEMENT, AND EXPLANATIONS OF TWO-DIGIT ADDITION  
CONCEPTS

by

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B.A. Salem College, 2007

A thesis submitted in partial fulfillment of the requirements  
for the degree of Master of Education  
in the School of Teaching, Learning, and Leadership  
in the College of Education  
at the University of Central Florida  
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## **ABSTRACT**

The purpose of this study was to improve my teaching practice, as well as my second grade students' success with two-digit addition concepts, by allowing them to choose the manipulative tools to explore problems and justify solutions. I examined how allowing my students this choice influenced their attitudes, achievement, and explanations of their thought processes. I found that allowing students to choose their own manipulatives had positive influences in all three areas. Pre- and post-test results showed an overall shift toward more positive mathematics attitudes, as well as increased academic achievement with two-digit addition concepts. Students also demonstrated changes in the ways they used the manipulatives, as well as how they explained their solutions to two-digit addition problems.

## ACKNOWLEDGMENTS

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## CHAPTER ONE: INTRODUCTION

### Study Rationale

In the summer of 2010, I began attending UCF as a student in the Lockheed Martin/UCF Academy, intending to complete a Master's Degree in K-8 Mathematics and Science Education. As I progressed through the program, I acquired a firm theoretical knowledge base, as well many strategies and ideas for classroom application. These theories and tactics have challenged me to become a better teacher, and have, in turn, pushed my students into deeper explorations of mathematics and science topics. This shift could not have come at a more opportune time, as the state of Florida has set the academic bar higher with the Next Generation Sunshine State Standards (Florida Department of Education [FDOE], 2008) and a move toward the Common Core State Standards (National Governor's Association [NGA] & Council of Chief State School Officers [CCSSO], 2011). "Drill-and-kill" methodology is not good enough; I worked diligently to help my students truly *understand* the curriculum concepts.

In recent years, I have learned a great deal about my own lack of deep conceptual knowledge of certain mathematical ideas. As that knowledge was enhanced, I was able to bring it back into the classroom and share it with my students. I learned that simple questions like, "How did you do that?" or "Why did you choose that strategy?" or "Who can show me another way to think about that?" can open the most profound windows into the thought process and understanding of any student. Even better, it got my students excited about mathematics. They took the expectation that they would have to justify and

explain their answers as a challenge, always trying to come up with as many different strategies as possible for each question posed.

However, as any teacher knows, honing one's craft is always a work in progress. As I spent the last year working harder than ever to teach my elementary students mathematics, I was pleased to watch them work through problems with appropriate procedures and strategies. However, as I tried to probe deeper into their thinking with informal interviews, questioning, and journaling, I realized that they were still somewhat lacking in the deep conceptual knowledge needed for full content mastery.

Then, as I began to consider topics for my action research, I began reading about Universal Design for Learning, or UDL. Universal Design for Learning involves setting up a classroom and curriculum that is equally accessible for all students, a large part of which involves student choice (Flores, 2008; Rose & Meyer, 2006; Rose & Meyer, 2002). Universal Design for Learning requires teachers to think about accommodations for different students *before* the lesson is designed, rather than as an afterthought. The principals of UDL also allow all students to choose a process of learning that is easiest for them, rather than having to change their own thought processes and preferences to fit the agenda of the teacher.

The more I pondered this idea, the more practical it seemed. If I allow my students more opportunity to enhance their learning of mathematical ideas, then it seems logical that they stand a better chance of fully grasping the inherent concepts. Furthermore, it then follows that these students will be much more confident in their

mathematical skills, and view the subject of mathematics as a whole in a more positive light. I made it my research goal to provide these opportunities to my students.

As the 2011-2012 school year began, I moved from teaching third grade to teaching second grade. It was a difficult transition. But then, after the shock wore off, I realized that teaching second-grade students had even more potential than teaching third-grade students. I could use my new knowledge and strategies to get students excited about mathematics content at a younger age, and hopefully do so in a lasting way. Furthermore, I could anticipate common mistakes (Lannin, Barker, & Townsend, 2007; Tucker, 1981), specifically with addition and subtraction regrouping, and put strategies into place to help reduce these misunderstandings. I made it my goal to help my young students see that mathematics can be fascinating, and that they have every capability to be successful mathematicians. I wanted my instruction to illuminate mathematics in a way that was meaningful and exciting, while still ensuring that students met the required state benchmarks. This desire was a driving force for my choice of research topics and methods.

### Purpose

The purpose of this research project is twofold. The first purpose is to improve myself as a teacher. I want to try something new and challenge myself to stretch and grow professionally. I am attempting to allow my students more control of their own learning through choice in their methods of determining and justifying problem solutions. I anticipate that this will be a positive experience for my students, and therefore encouraging to me as a teacher.

The second purpose of this project is to enhance my students' learning. Theoretically, I know that allowing students to construct their own knowledge is effective, especially in the area of mathematics (Lester & Charles, 2003). I am expecting that this will hold true in my research, and I will see improvement in both academics and student attitudes.

### Research Questions

My research was guided by the following questions:

1. How does student manipulative choice, using UDL, influence students' attitudes towards mathematics?
2. How does student manipulative choice influence student achievement with two-digit addition concepts?
3. How does the availability of manipulatives influence student explanations of two-digit addition concepts?
4. How do student manipulative choices and manipulative use change over time?

### Conclusion

In the chapters that follow, I examine the literature related to Universal Design for Learning, manipulative use in the mathematics classroom, students' acquisition of whole number addition concepts, and student attitudes toward mathematics. I illustrate the research conditions and study methodology. Finally, I present the data analysis and conclusions for the study.

## CHAPTER TWO: LITERATURE REVIEW

### Introduction

In 2000, the National Council of Teachers of Mathematics (NCTM) released its *Principles and Standards for School Mathematics*, a document that is crucial to the mathematics education community. In this document, NCTM laid out common principles and standards by which they believed mathematics education should be guided. A “principle” is an issue that may be present throughout all subject areas, yet is also “deeply intertwined with school mathematics programs” (National Council of Teachers of Mathematics [NCTM], 2000, p. 11). Conversely, “standards” are content-specific processes and concepts that NCTM advocates for students in specific grade levels (NCTM, 2000). More simply put, principles are overarching educational statements, while standards are content and grade level specific.

NCTM describes principles for equity, curriculum, teaching, learning, assessment, and technology. Teachers are responsible for maintaining all of these principles. However, with district and school policies, procedures, and resources, sometimes the control of curriculum, assessment, and technology is out of the teacher’s hands. In spite of this, the classroom teacher *does* have full control of the principles of equity, teaching, and learning, outlined below:

- Equity - excellence in mathematics education requires high expectations and strong support for all students.

- Teaching - effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning - students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (NCTM, 2000, p. 11)

It is interesting to note that the very first principle listed is “equity.” Clearly, NCTM regards equity as a major component of quality mathematics instruction. In fact, equity may be the most crucial principle that NCTM presents. A teacher can know her students, know what they need to learn, and attempt to engage them in actively pursuing that knowledge. Then again, if not all students have equal access to that knowledge, then the teacher is only doing part of the job.

This chapter begins with a discussion of Universal Design for Learning (UDL), a framework that attempts to meet the principle of equity and enhance the principles of teaching and learning. I discuss the theory behind UDL, as well as suggested classroom applications. The discussion is focused on how using UDL creates an equitable classroom environment for all students.

This chapter also focuses on aspects of student learning. I present research on the role that manipulatives play in the acquisition of number sense concepts, including place value and multidigit addition. Finally, I discuss the importance of good mathematics instruction as it relates to student attitudes towards mathematics.



## Universal Design for Learning

In the current age of Response to Intervention (RtI) and inclusion, regular education teachers are coming into contact with more and more diverse classrooms and learners than ever before, and the pressure for success is rising tremendously (Jimenez, Graf, & Rose, 2007; Rose & Meyer, 2002). With each different student comes a different set of background knowledge, interests, and abilities. Unfortunately, this can leave teachers wondering how to serve every student effectively. Universal Design for Learning (UDL) may hold the answer to this vital question.

### *Universal Design for Learning: A Theory*

UDL is a crossbreed of theories connecting the work of Vygotsky and Gardner with current neuroscience developments and theory (Rose & Meyer, 2006). In utilizing UDL, teachers must take into account that each person's brain is similar in structure, but vastly different in function. Each brain uses 3 "networks" to process information: The recognition network which identifies and assigns meaning to patterns, the strategic network which generates and monitors motor patterns in the body, and the affective network which evaluates patterns and attaches emotional significance (Rose & Meyer, 2002). For example, while a student is solving a two-digit addition problem, the recognition network identifies the digits in the numbers and the place value that each represents, while the strategic network determines the steps needed to solve the problem and controls the motor tasks involved in writing the solution. Meanwhile, the affective network is activated, and the student's attitude and motivation toward mathematics begins to come in to play. In any one person, one of these networks may be stronger than

others, and some components within networks may be stronger than others (Rose & Meyer, 2002). In short, “The materials and methods teachers use can either present students with barriers to understanding or enhance their opportunities to learn” (Rose & Meyer, 2002, p. 8).

So, how do teachers ensure that each student is presented with appropriate learning opportunities? That’s where the “universal design” of UDL comes into play. The structure of UDL stems from the engineering and architecture world, where products and services are often universally designed (Jimenez, Graf, & Rose, 2007, Rose & Meyer, 2002). When the Americans with Disabilities Act was signed into law, many buildings had to be retrofitted with elevators, ramps, and other adaptations in order to meet the requirements of the legislation. Engineers quickly realized that “it is better to anticipate the needs of all possible users before building something than to try and retrofit the same structure at a later date” (Ender, Kinney, Penrod, Bauder, & Simmons, 2007, p. 119), and therefore began designing buildings and other public spaces accordingly. They subsequently noticed that even though the designs were intended to support the disabled, the accommodations were useful to all sorts of people (Ender et al., 2007). Take, for example, curb cuts and wheelchair ramps. They are clearly intended for those who need wheelchairs and walkers to remain mobile. However, those without need for assistive mobility devices also frequently use curb cuts and ramps. Consider the parent pushing a stroller, the child on a skateboard, or the deliveryman toting a dolly full of boxes. Each one of these people could make good use of a curb cut or a ramp, even though it was not designed with them in mind (Ender et al., 2007).

UDL works in much the same way, although goods and services are replaced with lessons and classroom environments. In the classroom, UDL involves thinking about accommodations *first*, rather than as an *afterthought* for those learners who need differentiation (Lieberman, Lytle, & Clarcq, 2008). UDL is not a program or a curriculum; rather, it is an approach to classroom management and teaching that allows all students to choose their own methods for both gaining and expressing mastery of new knowledge (Rose & Meyer, 2006). Furthermore, it beautifully compliments ideas that are already prevalent in school systems, such as differentiated instruction and response to intervention (Basham, Israel, Gradin, Poth, & Winston, 2010; Rose & Meyer, 2006). However, the main difference between these ideas and UDL is that differentiated instruction and response to intervention involve adapting existing lessons and techniques for specific students and disabilities. On the other hand, UDL involves designing lessons with all learners in mind *from the very beginning of the planning stages*, because even classrooms “that might appear to be homogeneous are not” (Rose & Meyer, 2006, p. 35).

### *UDL in the Classroom*

While the theory behind UDL is intuitive and rational, teachers are generally concerned less with theory and more with day-to-day practice when it comes to pedagogical methods. Flores (2008) lays out the principles for UDL in the classroom very clearly:

- Materials are available for equitable use, meaning all students can use the technology and materials that are presented.

- Materials are flexible in use. Instruction and activities accommodate learning preferences and abilities through choice.
- Instruction is simple and intuitive, with background knowledge accounted for. Consistent terminology is used throughout multiple lessons on similar concepts.
- Material is perceptible for all students, meaning that it is presented in a way so that any student can take in the information (especially in regards to written information).
- Lessons and assessments allow tolerance for error, including revisions and editing.
- Use of materials, such as manipulatives, is achieved through low physical effort on the part of the student.
- The physical classroom setup allows enough space for all students to retrieve and use materials appropriately.

Essentially, UDL involves the teacher anticipating the needs of all her students, and then providing materials and lessons that meet those needs. The students are then free to use whichever ways of learning and expression they deem most appropriate for the ir own purposes and learning styles (Rose & Meyer, 2006). The most promising bit of UDL lies in this choice (Bray, 2010). Much like the example of curb cuts and ramps, an accommodation made with one student in mind may serve many other students as well (Lieberman, Lytle, & Clarcq, 2008). For example, an audio book provided for a visually

impaired student could also be beneficial to a student who is simply an auditory learner (Rose & Meyer, 2002).

Lieber, Horn, Palmer, and Fleming (2008) make the point that the idea of access must go further than simply mainstreaming and inclusion. Students must not only be present in the classroom, but fully engaged with what is going on during learning. Universal Design for Learning helps a teacher set up the classroom environment and plan lessons in a way that makes such engagement possible for *every* student. In mathematics, a simple way to begin implementing UDL is by allowing students to choose their own ways of expressing mathematical concepts through the use of manipulatives.

### Manipulatives

In the mathematics classroom, allowing students to choose their own manipulatives can easily incorporate the idea of student choice. However, not all teachers feel comfortable in the use of manipulatives. Uribe-Florez and Wilkins (2010) found that teachers in the primary grades (K-2) exhibited more manipulative use in the classroom than did intermediate teachers (3-5). Also, younger teachers tended to use manipulatives more often than older teachers, and, somewhat conversely, the more experienced a teacher was, the more likely they were to use manipulatives. However, these authors are careful to mention that the relationships described were merely correlational, not predictive (Uribe-Florez & Wilkins, 2010). Moyer and Jones (2004) found similar results in their study on teacher manipulative use. They found that teachers who express anxiety over the use of manipulatives tend to use these tools less frequently in the classroom, regardless of the experience level of the teacher.

That being said, in *Principles and Standards for School Mathematics*, NCTM (2000) presents the following standard for prekindergarten through second grade students: “All students should use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators” (p 70). Jacobs and Kusiak (2006) defined “tools” as anything (including fingers, manipulatives, or paper and pencil) that a student uses to aid in the solving of mathematics problems. Furthermore, in their year-long study of first grade students, they found that students were always able to come up with a correct answer when using tools, even in very complex problem-solving situations (Jacobs & Kusiak, 2006). In short, tools and manipulatives are absolutely vital for good mathematics instruction, especially in the case of young learners (NCTM, 2011; NGA & CCSSO, 2011).

#### *Manipulatives as a Tool for Creating Understanding*

Manipulative use can be as straightforward as a student counting beans or counters, or as involved as the trading and grouping of place value blocks. In fact, Schwerdtfeger and Chan (2007) found that the simple act of watching a child count could provide a great amount of information on his/her knowledge of number and problem solving strategies. They reported that noticing, questioning, and extending students’ counting strategies allowed teachers insight into what students knew about multiples and skip counting, as well as how capable specific students were in creating their own problem solution strategies.

When considering the use of manipulatives to teach number sense concepts, Van de Walle, Karp, and Bay-Williams (2012) outline three different models for classroom

use. The first are the groupable models, in which 10 ones may be physically grouped to make 1 ten, 10 tens grouped to make 1 hundred, and so on. This would include items such as snapping cubes, beans and cups, or straws and rubber bands. In this study, 2 groupable models were utilized: straws and rubber bands, and Unifix Cubes.

The second model Van de Walle et al. (2012) discuss are pregrouped or trading models. In these models, the pieces cannot be broken apart or combined. Instead, pieces must be traded: 1 ten for 10 ones, 10 tens for 1 hundred, etc. In this model, it is imperative that students understand the value of each piece relative to the other pieces, in order to avoid trading errors. The most common trading model is base-ten blocks, which was the pregrouped manipulative used in this study.

The final model Van de Wall et al. (2012) present are the nonproportional models. In these models, the sizes of the manipulatives are not proportional to their mathematical meanings; 1 hundred is not physically 10 times bigger than the ten, for example. These models include items such as money (pennies, dimes, and dollars). These models are not recommended for students who are in the beginning stages of understanding place value concepts, and were therefore not used in this study.

In an in-depth exploration of children's use of manipulatives, Sherman and Bisanz (2009) investigated children's ability to solve equivalence problems (such as  $4 + 3 = 5 + \underline{\quad}$ , where children had to fill in the missing number). Students who were initially presented with a nonsymbolic (or manipulative) representation solved the problems correctly more often than those who were presented with symbolic (or written) representations first. Even one week later, the students who had experienced nonsymbolic

representations before symbolic representations were more skillful in solving equivalence problems than those who had not.

In a related report, Manches, O'Malley, and Benford (2010), studied 4- to 8-year-olds' use of materials in partitioning, or regrouping, problems. Partitioning, which involves decomposing numbers in multiple ways, is a crucial prerequisite skill for addition and subtraction. Students were asked to partition numbers first with no aids, then with groupable manipulatives (Unifix cubes), and finally with a pictorial representation of the problem. Almost every student was able to come up with significantly more partitions when using the cubes than either of the other two methods. Skoumpourdi (2010) found similar results in a study of kindergarten students. One half of her subjects received pictures and cubes to manipulate, and the other half received only a number line. While solving identical ordering, addition, and subtraction problems, children were more apt to spontaneously use the blocks as an aid. In fact, Skoumpourdi (2010) found that students who were given a number line often never even referenced it, let alone attempted to use it as an aid.

Bebout (1990) discovered similar results in her study of addition and subtraction word problems. She found that students were better able to connect number sentences and word problems when they were required to represent their addition and subtraction strategies with concrete models. These differences remained significant even after they were disaggregated by students' overall mathematics ability.



### *Manipulatives and Addition Concepts*

Maurer (1998) defines an algorithm as, “a precise, systematic method for solving a class of problems” (p. 21). One inputs a number, goes through a series of steps, and receives an output (Maurer, 1998). Traditionally, mathematics instruction has focused on the mastery of singular algorithms and correct procedures, but this focus is shifting toward an emphasis on a holistic and flexible understanding of mathematics concepts (Lester & Charles, 2003).

One of the first algorithms children encounter during formal schooling is the addition algorithm, and it is here that the potential for unfounded procedural knowledge begins to surface. Lannin, Barker, and Townsend (2007) found that students’ mathematical errors tended to rise out of an overgeneralization of strategies and methods learned prior to the content at hand. Often, students use pre-taught or invented routines such as counting to solve problems, rather than using the context of the problem and their conceptual knowledge to assist in finding a solution (DeCorte & Verschaffel, 1981; Verschaffel, DeCorte, & Vierstraete, 1999). For example, a student might incorrectly assume that 47 plus 25 equals 612, because 7 ones plus 5 ones equals 12 ones, and 4 tens plus 2 tens equals 6 tens. This shows a lack of understanding that 12 ones can also be regrouped as 1 ten and 2 ones, for a correct sum of 7 tens and 2 ones, or 72. (Tucker, 1981).

In fact, Kamii and Dominick (1998) determined that the exclusive use of algorithms in the elementary grades is not only undesirable, but can also be harmful to students. Such practices rob students of independent thinking and shift their focus away

from the underlying place value concepts inherent in mathematical operations such as addition. Indeed, many leaders in the field insist that the teaching and learning of addition should be firmly rooted in an understanding of place-value concepts (Baroody, 1990; Carpenter, Fennema, & Franke, 1996; Fuson, 1990; Fuson et al., 1997; Lopez-Fernandez & Velazquez-Estrella, 2011).

According to Fuson (1990), the understanding of multidigit addition concepts is fully entrenched not only in place-value concepts, but also in the ability to compose and decompose (or regroup) multidigit numbers. She suggests that place-value concepts should be taught and reinforced concurrently with addition concepts, because the two are so deeply intertwined. Fuson and Briars (1990) found that first- and second-grade students showed significant increases in their regrouping and trading strategies after instruction with place-value blocks. This was true during both addition and subtraction problem-solving scenarios, even with numbers as large as four digits. The students' trading errors decreased after this instruction, while their scores on place value tests increased. Interestingly, Fuson and Briars (1990) also noted that students' ability to switch back-and-forth between word form and written form of numbers improved after manipulative instruction, as did students' ability to verbally identify the place value of given digits. Clearly, manipulatives are crucial in the creation of place-value concepts, which in turn make them a vital part of addition instruction.

Likewise, Carpenter and Moser (1984) found that students need concrete examples of addition, subtraction, and regrouping concepts, especially in the early stages of learning. During their two-year longitudinal study, they found that the younger (and

therefore the more inexperienced) the children in the study were, the more heavily they relied on concrete models. This was especially true as the numbers in the problems increased to 3 and 4 digits. As students gained knowledge of the concepts of addition, subtraction, and regrouping, they were able to switch between interchangeable strategies, even if their chosen strategy was not the most efficient (Carpenter & Moser, 1984).

To support that idea, Carpenter et al. (1996) posit that children come to school with some intuitive mathematical knowledge, which they can use to help solve basic problems. For example, take a simple word problem, such as, “Robin had 5 toy cars. Her friends gave her 7 more toy cars for her birthday. How many toy cars did she have then?” (Carpenter et al., 1996, p. 17). In such a situation, beginning mathematics students will most likely use manipulatives to act out the scenario, starting with 5 items, and then adding 7 more, finally counting all of the items to find a solution of 12. This strategy will often evolve into a counting strategy, where a student might start with 5, count up 7 more, and end on the solution of 12 cars. Finally, a student might then progress to using familiar facts, such as using  $5 + 5 = 10$ , plus 2 more is 12 in all. However, Carpenter et al. (1996) caution that while such methods for basic addition will most likely be spontaneously utilized, student success with multidigit operations (and eventually algorithms) is dependent on thorough instruction and student understanding of place value.

Fuson et al. (1997) observed stages in the understanding of numbers and operations that were very similar to those outlined by Carpenter et al. (1996). Fuson et al. (1997) also describe several different student conceptions of numbers within our base-ten

numeration system, all of which they believe should be supported and extended by the teacher as students explore operations such as addition. Furthermore, they explain that different students will use different addition strategies depending on their level of conceptual understanding of place-value. Although student methods become more complex, more abstract, and more efficient over time, the research shows that physical representations (manipulatives) are crucial for building the basic knowledge students need in order to acquire multidigit addition concepts (Carpenter, et al., 1996; Fuson et al., 1997). Moreover, students need time to connect their understanding of place value and their manipulative use to verbal explanations and algorithms (Fuson, 1990, Fuson et al., 1997). For this reason, manipulatives remain vital throughout elementary school mathematics instruction.

### *Classroom Use of Manipulatives*

At some point students should be able to “transform a physical artefact [sic] into a mental one” (Bussi, 2011, p. 97). Students have no choice but to eventually move from manipulatives to more rapid and symbolic mathematical processes. Keeping this development in mind, teachers should evaluate, choose, and utilize manipulatives carefully and effectively (Bussi, 2001).

First and foremost, manipulatives must be a tool, and “providing manipulatives does not automatically lead to student learning” (Puchner, Taylor, O’Donnell, & Fick, 2008, p. 324). Although students do require some instruction on appropriate use of manipulatives through “think-alouds” and demonstration (Moyer & Jones, 2004; Witzel & Allsopp, 2007), manipulatives can actually hinder student learning when correct usage

is the only goal and the focus is not on understanding the mathematical content (Puchner, et al., 2008). The manipulative use then becomes just another procedure to follow and it becomes devoid of mathematical content; the crucial step is connecting the models to the underlying mathematics concepts (Baroody, 1990). Carpenter et al. (1996) take it one step further, insisting that requiring students to verbalize their solutions and manipulations is a crucial step in connecting manipulative use to eventual algorithmic solutions.

Once students are able to appropriately use manipulatives, it may be beneficial to allow students a choice of when, how, and which manipulatives they use for a given task. In their study of student partitioning, Manches, O'Malley, and Benford (2010) found that students were not able to find as many solutions to a given problem when they had imposed restrictions on how they were allowed to manipulate their tools. This trend was visible regardless of whether children were using either physical or virtual manipulatives (Manches, O'Malley, & Benford, 2010).

Additionally, Moyer and Jones (2004) experimented with allowing students choice in manipulatives, asking students to choose first, whether or not they wanted to use manipulatives at all, and second, asking how they used any selected manipulatives. They found that when students had free choice of manipulatives, they spontaneously generated more answers during problem solving. This free choice led to more discussions among students and more student conceptual talk overall, which in turn lead to more student ownership of new knowledge (Moyer & Jones, 2004).

The literature on manipulative use is wide, but overlapping similarities are suggested for classroom application. These suggestions are outlined below:

- Manipulatives should be highly correlated to lesson objectives (Kurtz & Ross, 1993; Puchner et al., 2008).
- Student manipulatives should be multisensory whenever possible, especially when students with learning disabilities are involved (Thornton, Jones, & Tooney, 1983; Witzel & Allsopp, 2007).
- Materials should encourage students to create their own novel strategies for problem solving, rather than merely representing repetitions of known algorithms (Moyer & Jones, 2004; Puchner et al., 2008; Schwerdtfeger & Chan, 2007).
- Students should have varied experiences to familiarize them with manipulatives (Kurtz & Ross, 1993; Moyer & Jones, 2004; Witzel & Allsopp, 2007).
- Every student should be involved when using manipulatives in a lesson (Kurtz & Ross, 1993).
- Use of manipulatives should be followed by self-reflection and/or class comparisons of strategies (Jacobs & Kusiak, 2006; Kurtz & Ross, 1993)

Research has shown that manipulatives are absolutely critical to the development of young children's number sense skills. The use of manipulatives should be a common occurrence in elementary school and beyond, with teachers always ensuring that activities and materials are strongly linked to mathematical content and strategies (Puchner et al., 2008).

### Student Attitudes Towards Mathematics

Helping students cultivate positive attitudes in regards to mathematics is a vital part of mathematics instruction. Tymms (2001) empirically demonstrated what is intuitive to most teachers: The more academic achievement students experience, the more positive their attitudes become. Tymms (2001) examined seven-year-old children as test subjects, and determined that attitudes towards school are formed very early. Frenzel, Pekrun, and Goetz (2007) confirmed this notion, reporting that mathematics success was correlated with positive emotions, and mathematics failure was correlated with negative emotions.

This emphasis on student attitudes has proven to be especially crucial when considering girls in the mathematics classroom (Frenzel, Pekrun, & Goetz, 2007; Steffens, Jelenec, & Noack, 2010). Frenzel, Pekrun, and Goetz (2007) found that, although girls and boys performed equally well on tests of mathematics ability, girls had significantly more negative beliefs about mathematics and their capability with the subject. Steffens, Jelenec, and Noack (2010) extended this idea, reporting that girls stereotyped mathematics as a male subject and reading as a female subject as young as nine years old. Indeed, it seems that the younger students are, the more their attitude predicts mathematics performance, which can heavily influence school- and mathematics-related anxiety (Krinzinger, Kauffman, & Willmes, 2009). Clearly, a focus on attitudes towards mathematics is important for all teachers, even those who teach young students.

### *Self-Efficacy and Self-Concept*

Self-efficacy and self-concept are highly related ideas. While self-concept may be either domain-specific or global, self-efficacy is a self-evaluation of one's own abilities in a specific area (Schweinle & Mims, 2009). Sources of information that combine to form a child's self-efficacy are his/her personal experiences, observations of others, social interaction with peers and adults, and emotional and physical states when engaging in domain-specific activities (Usher & Pajares, 2009). Self-efficacy and self-concept both play a defining role in students' attitudes about mathematics.

Even in elementary school, students can experience great anxiety in mathematics, which often leads to poor mathematics self-efficacy (Passolunghi, 2011). Dermitzaki, Leondari, and Goudas (2009) studied first- and second-grade students during problem solving, and found a reciprocal relationship between students' mathematics self-concept and their motivation and persistence during mathematics tasks. Chouinard, Karsenti, and Roy (2007) illustrated the magnified effects of mathematics self-efficacy once students reach high school. They found that student self-perceptions were directly correlated to their beliefs about the utility of mathematics, as well as their engagement and effort when faced with mathematics tasks (Chouinard, Karsenti, & Roy, 2007). Likewise, Simpkins, Davis-Kean, and Eccles (2006) conducted a longitudinal study, which showed that young elementary school students who believed they were skilled in mathematics and science were significantly more likely to pursue classes and activities in these areas as adolescents. Students' self-concept was more predictive than past achievement, parent



beliefs about mathematics and science, or household income (Simpkins, Davis-Kean, & Eccles, 2006).

As one might imagine, self-concept and self-efficacy are particularly troublesome for students with disabilities. Passolunghi (2011) found that when students with learning disabilities were compared with normal achieving students in several domain-specific areas, the only area in which they showed significantly more anxiety was mathematics. Zeleke (2004) went even deeper, illustrating that students with learning disabilities have not only lower mathematics self-concepts, but also lower academic and overall self-concepts than their average- and high-achieving peers. Luckily for these students, UDL is gaining support in the educational system.

#### *Attitudes, Motivation, and the Classroom*

Student attitudes are obviously a substantial part of the success of mathematics instruction. Therefore, teachers need to create a positive classroom environment where students feel good about doing mathematics. A large part of creating such an environment is ensuring that students are focused on the process of mastering concepts, rather than just coming up with the “right” answer (Chouinard, Karsenti, & Roy, 2007).

De Corte, Verschaffel, and Depaepe (2008) conducted a study of fifth-grade students in which they discovered that children often saw mathematics as a boring subject forced upon them by adults. However, once the students received instruction via open-ended, real-life problems that included discussion of ideas, students began to take pleasure in mathematics and perform at significantly higher levels (De Corte, Verschaffel, & Depaepe, 2008). Metallidou and Vlachou (2010) also highlighted the

importance of meaningful work by reporting that students were more motivated to learn when they considered tasks useful and interesting.

In a study on classroom environment and motivation, Turner et al. (2002) found that in classrooms where teachers emphasized learning, understanding, enjoyment, and effort, students exhibited significantly fewer avoidance behaviors (i.e., lack of effort and resistance to new ideas). Mastery-oriented teachers addressed both the cognitive and the affective aspects of learning, and children responded accordingly (Turner et al., 2002). Students' beliefs in their own mathematics abilities are most heavily influenced by teacher support and mastery focus (Chouinard, Karsenti, & Roy, 2007). When students can agree with statements such as "An important reason I do my mathematics work is because I want to improve my skills" (Kenney-Benson, Pomerantz, Ryan, & Patrick 2006, p. 14), teachers will likely see an improvement in both mathematics attitudes and academic achievement (Kenney-Benson et al., 2006).

By focusing on mastery, teachers can change the meaning of "success" in a mathematics classroom, thereby ensuring that all students can experience mathematics-related positivity. When students believe that participation and discussion are crucial (but non-threatening), they are more likely to engage in conceptual mathematics talk (Jansen, 2008). When students are successful in this conceptual understanding, they are more likely to believe in their mathematics ability (Seegers, vanPutten, & deBrabander, 2002). When students constantly feel the threat of failure, however, they are immediately turned off by mathematics tasks. This is even true if students have had previous success with similar tasks (Seegers, vanPutten, & deBrabander, 2002).

## Conclusion

“Effective teaching requires a challenging and supportive classroom environment” (NCTM, 2002, p. 18). This bold statement from *Principles and Standards for School Mathematics* (NCTM, 2000) may seem overwhelming at first, but creating an effective environment is by no means an impossible feat. Current educational research is overflowing with strategies and suggestions for classroom improvement. In answering my research questions, I aim to reevaluate and strengthen my own classroom environment and teaching practice. By providing students with a UDL framework and choice in the powerful mathematics tools and manipulatives at their disposal, I expect to improve my students’ attitudes towards mathematics. As the literature suggests, providing my students with mathematics success may just be the key to their future endeavors.

## CHAPTER THREE: METHODOLOGY

### Problem and Rationale

In my five years of teaching, I have watched students work through mathematics problems with varying degrees of comprehension and ability. I have found that students who have trouble on classroom tasks, district assessments, and standardized tests have often been lacking conceptual knowledge on foundational and prerequisite skills. After moving from teaching third grade to teaching second grade, I realized that I have the potential to help students build that conceptual knowledge before they move on to intermediate grades. I made it my goal to do just that, and hopefully give my students an advantage as they move through their schooling. I also anticipate that a deeper conceptual knowledge base will lead to more students being successful in mathematics, improving their attitudes toward the subject as a whole. To that end, I conducted this study to investigate the role of student manipulative choice on second-grade students' attitudes towards mathematics and achievement in whole number addition concepts.

This study was conducted as an action research project in my regular education second grade classroom. The research questions were:

1. How does student manipulative choice, using UDL, influence students' attitudes towards mathematics?
2. How does student manipulative choice influence student achievement with two-digit addition concepts?

3. How does the availability of manipulatives influence student explanations of two-digit addition concepts?
4. How do student manipulative choices and manipulative use change over time?

### Setting

This research was conducted at a public elementary school in central Florida. The school serves students from prekindergarten to fifth grade, and the research was conducted in my regular education second-grade classroom. At the time of the study, the school had a total enrollment of 906 students, 424 (48% of total population) of which were female, and 482 (52%) of which were male. There were 292 (32%) White students, 317 (35%) Black students, 184 (20%) Hispanic students, 26 (3%) Asian students, 3 (1%) American Indian students, and 84 (9%) Multiracial students enrolled at the time of the study. One hundred fifty-two (18%) students received exceptional student education (ESE) services, and 98 (11%) students received ESOL services. Six hundred twenty-four (69%) students received free or reduced lunch.

My research was conducted using a group of 14 second-grade students for whom parental permission was obtained. Of these 14 students, 5 (36% of the study group) were girls and 9 (64%) were boys. Five (36%) were White, 2 students (14%) were Black, 6 (43%) were Hispanic, and 1 student (7%) was Asian. Three students (21%) were either receiving ESE services or in the Response to Intervention (RtI) process, and 3 (21%) received ESOL services. Eight (57%) students received free or reduced lunch.

This research took place during the confines of the regularly scheduled mathematics instructional block. This block was 60 minutes long, and it took place each

school day, immediately after lunch. The structure of work during the mathematics block was a mixture of whole-group, small-group, and individual problem solving. Students were seated at individual desks clustered into 4 “tables”. Each “table” consisted of a group of 4-5 students.

#### Data Collection Procedures

Upon receipt of IRB (Appendix A), principal (Appendix B), and county approval (Appendix C), parental consent forms were sent home to each of my 17 students. Since my school has a large bilingual population, parent consent forms were sent in both English (Appendix D) and Spanish (Appendix E) versions. Only students who received parental permission were included in data collection, which resulted in a study group of 14 students. Student assent was also obtained.

One week prior to the beginning of data collection, students were given an attitude scale (Doepken, Lawsky, & Padwa, 1993) (Appendix F) and a learning style survey (Cohen & Weaver, 2006) (Appendix G), to look for patterns among subjects. During the unit on place value and number composition/decomposition, which occurred prior to the unit studied in this research, students were instructed in appropriate use of Unifix cubes, base-ten blocks, and straws and rubber bands to model place value concepts. These manipulatives were chosen based on my preliminary research and informed by the literature.

Immediately prior to beginning the unit on addition, students were given the curriculum-based benchmark assessments (Appendix H). During any lesson in this unit where manipulatives were appropriate, students had a choice of which tools they used to

solve problems. One to two times per week, students were given a journal prompt that was scored using a rubric (Appendix I) in order to gain more information about how each student was processing the lessons.

Table 1 outlines the timeline and topics included in the research. Note that Week 1 included a County Workday, so it was only four days long. Week 3 was also four days long, due to grade-level holiday activities preempting the regular mathematics block. Additionally, there were 2 topics covered where manipulative use was not necessary, so manipulatives were not used on these days.

*Table 1: Study Timeline and Topics*

<b>Week</b>	<b>Day</b>	<b>Topic</b>	<b>Manipulatives Used?</b>	<b>Journal Prompt Given?</b>
1	1	Place Value/Basic Addition Review	Yes	Yes
1	2	Add on a Hundred Chart	No	No
1	3	Break Apart Ones to Add	Yes	Yes
1	4	Break Apart Ones to Add, Cont'	Yes	No
2	1	Break Apart Addends as Tens and Ones	Yes	No
2	2	Use Compensation	Yes	Yes
2	3	Draw a Diagram	Yes	No
2	4	Estimate Sums	No	No
2	5	Model Regrouping for Addition	Yes	Yes
3	1	Model and Record 2-Digit Addition	Yes	No
3	2	Record 2-Digit Addition	Yes	No
3	3	Rewrite 2-Digit Addition	Yes	No
3	4	Practice 2-Digit Addition	Yes	Yes
4	1	Concept/Chapter Review	Yes	No
4	2	Chapter Test: 2-Digit Addition	Yes	Yes

### *Research Tools*

The study included both quantitative and qualitative elements, in order to obtain a more accurate sense of what occurred. The quantitative data included student scores on

two items: A modified Fennema-Sherman Mathematics Attitude Scale (Doepken, et al., 1993) (Appendix F), and benchmark assessments from our countywide mathematics curriculum (Appendix H). Both of these tools were used with publisher permissions. Qualitative data included students' responses on a learning style inventory (Cohen & Weaver, 2006) (Appendix G), as well as their responses to journal prompts (Appendix I) and observations during daily class work.

The quantitative scales were included to measure changes in students' beliefs and mathematics comprehension from the beginning of the study to the end. The qualitative measures were included to allow me a deeper understanding of what happened over the course of the research. The learning style inventory was used to examine possible correlations between each student's learning style and subsequent manipulative choices. The journal entries and observational notes were employed to create a more robust picture of student understanding, as well as to allow me a window into the thought processes of my students. These particular research instruments were chosen because of their ability to quantify students' attitudes, explanations, and achievement while still being age-appropriate and understood by young children.

The mathematics attitude scale is a modified Fennema-Sherman Mathematics Attitude Scale (Appendix F), which was used with permission (Doepken, et al., 1993). It was modified by the author to include a length and style of questioning that was more appropriate for primary students, and each item was read aloud as students completed the scale. Students responded to positive and negative statements on a Likert-type scale, and responses were scored according to a guide. Possible responses ranged from A ("Strongly



Agree”) to E (“Strongly Disagree), with C in the middle (“Not Sure”). Positive statements were scored with a descending scale: A=5 points, B=4 points, C=3 points, D=2 points, and E=1 point. Negative items were scored on an ascending scale: A=1 point, B=2 points, C=3 points, D=4 points, and E=5 points. Higher point values indicated a “positive” attitude, and a lower point values indicated a “negative” attitude. Students’ attitudes were then classified as positive, neutral, or negative, depending on a combined score. A total score falling between 11 and 32 points indicated a “negative” overall attitude, a score of 33 indicated a “neutral” attitude, and a score falling between 34 and 55 points indicated a “positive” overall attitude.

The Learning Style Survey for Young Learners (Appendix G) has four subscales, wherein students respond to statements with a Likert-type scale, which were also read aloud as students completed the scale (Cohen & Weaver, 2006). For each statement, students chose one happy face (☺), two happy faces, or three happy faces, where three was considered the highest. I used only the first two subscales: “How I use my physical senses” and “How I expose myself to learning situations.”

The benchmark assessments came from the Go Math™ curriculum (Adams, Larson, Dixon, McLeod, & Leiva, 2011) (Appendix H). This curriculum is a countywide tool. Our county began using this curriculum in the 2010-2011 school year. The assessment given as part of my study was actually a combination of 3 “Mini-Assessments” from the Go Math™ second grade curriculum, which resulted in a total of 13 questions. Students were asked to read and complete this assignment independently, showing as much work and/or explanation as possible.

The student journal rubric (Appendix I) was used as a student guide for responding to journal prompts. The rubric was stated in child-friendly language, and was also posted in the classroom as a reference for the students and a reminder of the expectations. A rubric score of 1 was the lowest possible score, indicating missing or incorrect work. A rubric score of 2 indicated that the child included an explanation of his/her thinking along with a correct answer. A rubric score of 3 was the highest score possible, which required the student to answer the problem in more than one way.

#### *Classroom Procedures*

The attitude scale and the benchmark assessments were administered prior to starting the two-digit addition unit. After the completion of the unit, the attitude scale and the benchmark assessments were re-administered. The students' scores on these tests were compared to highlight any changes in attitude or learning. Qualitative data were also interpreted to look for any patterns over the course of the study.

Each of the mathematics lessons that comprised the study began with a short “warm-up” review, which each student completed independently in his/her mathematics journal. The review was taken directly from the Go Math™ curriculum (Adams et al., 2011), and typically contained 2 problems that reviewed concepts from the previous lesson, 2 problems from an earlier chapter, and 2 problems that reviewed first-grade benchmarks. The students completed the independent warm-up and answers were reviewed within the first 10-15 minutes of the lesson.

Once the warm-up was completed, each student was allowed to choose his/her manipulative for the lesson. Baggies with individual sets of manipulatives were placed on

a table along with place value charts, and students were allowed to go to the table and choose which manipulative they preferred for the lesson. The choices were Unifix cubes, place value blocks (tens and ones), and straws and rubber bands. Students also had the option of taking a place value chart with columns for hundreds, tens, and ones as an organizational aid. If a student changed his/her mind during the lesson, he/she was allowed to swap for a different manipulative, or retrieve/return a place value chart. Students were encouraged to use manipulatives, but were also permitted to complete the problems or lesson without them if they chose to do so.

It should be noted that this study did not include virtual manipulatives. Virtual manipulatives are becoming more and more popular and important, and I have used them in the past. However, the major focus of UDL is equal access for all students. My classroom resources would not have allowed all students simultaneous access to virtual manipulatives, and therefore my study only included physical manipulatives.

Once the manipulatives had been chosen, the lesson typically began with completing 1-2 problems together as a class, during which time I was leading the discussion and questioning. Then, the students typically worked on 2-3 problems with their partners and/or groups, as I circulated the room, probing students to explain what they were doing and why they were doing it. These pair/group problems were discussed as a class, with students explaining the reasoning behind their processes and solutions.

Next, students were given the opportunity to complete 2-4 problems on their own, showing work and using manipulatives as needed. Once the students had completed this part of the lesson, I chose one or two problems to review with the class. For each

problem, I would invite one student to come to the board and explain his/her solution. The other students were asked to revoice and/or add to the explanations as needed. The lesson ended with the students responding to a journal prompt on an index card that I collected and scored. The journal prompts were either mathematics problems (e.g., “What is  $56+29$ ? Explain how you found your answer.”), or more qualitative/conceptual questions (e.g., “Which manipulative do you like best and why?” or “Explain how you know when to regroup the ones.”).

#### Assumptions and Limitations

This research was conducted under the assumption that student choice and student success will lead to more positive attitudes toward mathematics. This assumption is based on the literature reviewed in my preliminary work. This initial research also formed the basis for my assumption that the use of manipulative tools will enhance student learning during mathematics lessons.

As this is a local action research project within a small sample, there are certainly limitations and threats to validity. The most blatant threat to internal validity is data collector bias, as I was the only researcher collecting data on my students.

I also realize that external validity is essentially nonexistent in this study. The sample is very small and non-random, and this sample does not represent a greater population. In this case, generalization is not advisable. However, this is not the *intent* of this study; rather, the intent is to inform and improve my personal teaching practice.

## Conclusion

This study was conducted to examine the influences that students' choice in manipulatives had on their attitudes and academic success. In the following chapter, I examine the results of the study through both qualitative and quantitative data. I also explore the changes in students' attitudes and academic achievements over the course of the study.

## CHAPTER FOUR: DATA ANALYSIS

### Introduction

This action research study was conducted to examine the effect that student choice in manipulatives would have on my second grade students' attitudes and achievements in mathematics. My time in the elementary classroom has illustrated the need for deeper conceptual knowledge of mathematics, rather than the simplistic procedural knowledge to which many students have become accustomed. It was my desire to implement methods that I believed would improve this conceptual knowledge, while subsequently creating positive student attitudes toward mathematics. By allowing students to choose the manipulatives they utilized in solving and explaining whole number addition with regrouping, I sought to fulfill these research goals.

This chapter examines and discusses the results uncovered by this study. This study was guided by four research questions, namely:

1. How does student manipulative choice, using UDL, influence students' attitudes towards mathematics?
2. How does student manipulative choice influence student achievement with two-digit addition concepts?
3. How does the availability of manipulatives influence student explanations of two-digit addition concepts?
4. How do student manipulative choices and manipulative use change over time?

Changes in student attitudes were examined via pre-test and post-test outcomes on the modified Fennema-Sherman Mathematics Attitude Scale (Doepken, et al., 1993)

(Appendix F). Changes in student achievement were described via a similar method, by examining the students' pre-test and post-test scores on the curriculum-based benchmark assessments (Appendix H). Students' explanations were examined through the use of the journal rubric (Appendix I), as well as teacher observational data. Finally, I examine the ways that students' choices changed over the course of the study, relying heavily on student journal responses and observational data. All student names used in this study are pseudonyms in order to protect student privacy.

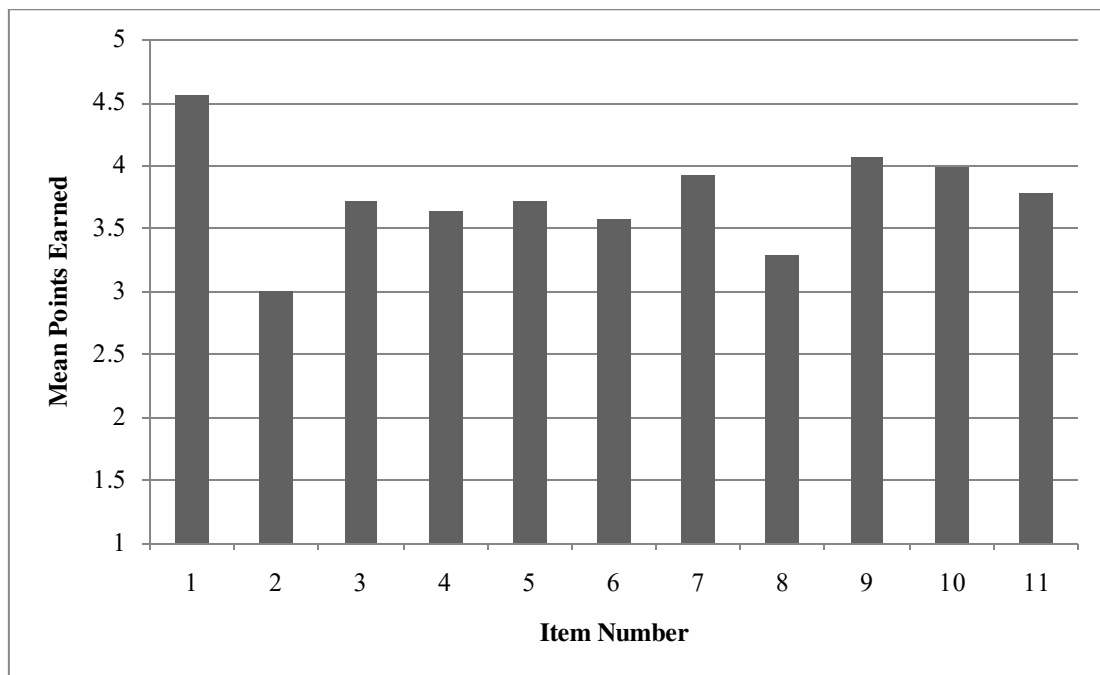
### Data Analysis

#### *Student Attitudes*

Student attitudes were determined using a Modified Fennema-Sherman Mathematics Attitude Scale (Doepken, et al., 1993) (Appendix F). This scale is comprised of 11 items scored on a Likert-type scale. Students followed along as each item was read aloud, and then chose a letter from A ("Strongly Agree") to E ("Strongly Disagree"), with C in the middle ("Not Sure"). Positive statements were scored with a descending scale: A=5 points, B=4 points, C=3 points, D=2 points, and E=1 point. Negative items were scored conversely, on an ascending scale: A=1 point, B=2 points, C=3 points, D=4 points, and E=5 points. Therefore, for each item, a higher point value indicates a more "positive" attitude, and a lower point value indicates a more "negative" attitude. An overall mathematics attitude is calculated by combining the point values of each item to create a total score. A total score in the range of 11-32 points indicates a "negative" overall attitude, a score of 33 indicates a "neutral" attitude, and a score in the range of 34-55 points indicates a "positive" overall attitude.

### *Pre-test Results*

On the pre-test, 79% of test subjects (11 out of 14) were identified as having an overall positive attitude towards mathematics. Seven percent of students (1 out of 14) were identified as having a neutral attitude, and 14% of students (2 out of 14) had a negative attitude. The mean total score of the study group was 41.3 points. These data imply an overall positive attitude within the study group, with only 21% of students (3 out of 14) not identified as such. Interestingly, the 3 students who presented a neutral or negative attitude were all either receiving ESE services or in the RtI process. Figure 1 outlines the mean of points earned for each item on the attitude scale pre-test.

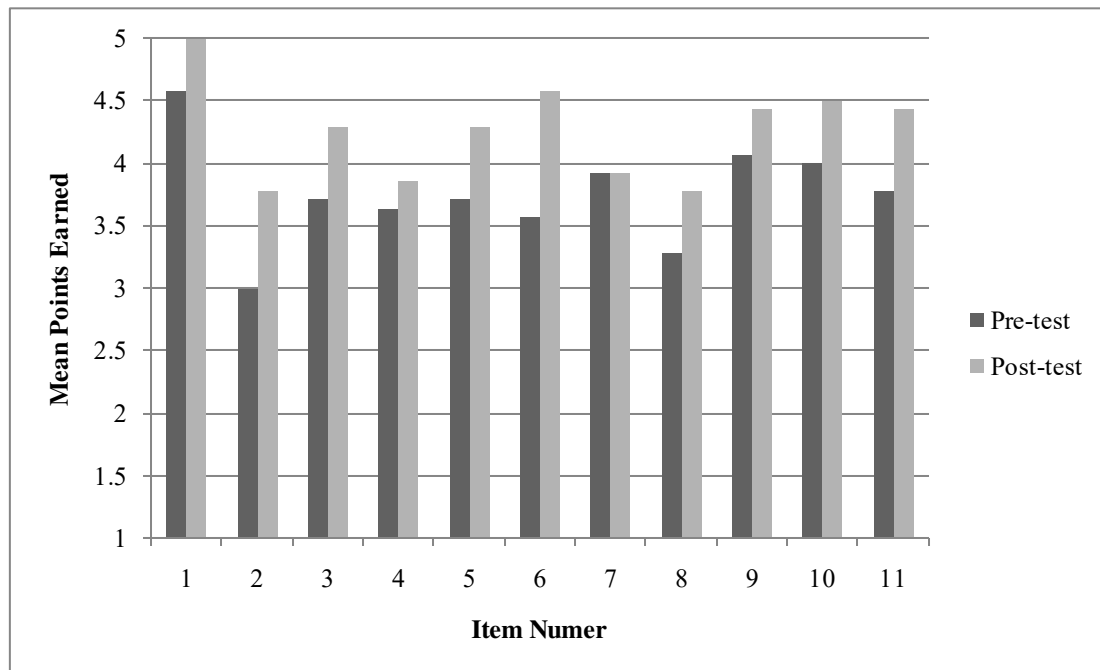


*Figure 1: Mean Points Earned for Each Attitude Scale Pre-Test Item*



### *Post-test Results*

After the completion of the unit, the Modified Fennema-Sherman Mathematics Attitude Scale (Doepken, et al., 1993) (Appendix F) was administered again in the same manner. All 14 subjects, or 100% of the group, were identified as having a positive overall mathematics attitude on the post-test. The mean total score for the study group increased to 46.9 points. Figure 2 outlines the mean of points earned on each item of the attitude scale post-test, along with a comparison between pre-test and post-test means for each item.



*Figure 2: Comparison Between Pre- and Post-Test Means for Attitude Scale Items*

### *Pre-test and Post-test Analysis*

Overall, students tended to move toward a more positive mathematics attitudes, both individually and as a group. The mean total score for the group increased by 5.6

points. The average amount of points earned also increased on 10 of the 11 individual questions. Seventy-nine percent (11 of 14) of individual student total scores increased from the pre-test to the post-test. While none of the students moved into the “neutral” or “negative” score ranges, the remaining 21% (3 students) did not show an increase in total scores from the pre-test to the post-test. Violet’s total score stayed constant at 40 points, though her answers on individual items did vary from the pre-test to the post-test. James’ total score decreased from 50 points to 40 points, and Jonah’s total score decreased from 44 points to 34 points. However, even with these decreases in total scores, these students still stayed in the point range that indicated a positive overall attitude.

As noted, three students scored in the neutral and negative attitude ranges on the pre-test. Mario had a total score of 33 points on the pre-test, which placed him in the neutral range. The point value of his individual answers increased on 6 of the 11 items on the post-test, raising his total post-test score from 33 to 49 points and placing him in the positive attitude range.

Cindy and Leon both scored in the negative range on the pre-test attitude scale. Cindy presented a total score of 20 points on the pre-test, which indicated a negative mathematics attitude. On the post-test, the point value of her individual answers increased on 9 out of 11 items, raising her total score to 45 points and placing her in the positive attitude range.

Leon presented a total score of 27 points on the pre-test, which indicated a negative mathematics attitude. On the post-test, the point value of his individual answers

increased on 8 out of 11 items. In fact, Leon earned 5 points on each item of the post-test, resulting in a total score of 55 points, which indicated a positive mathematics attitude.

### *Student Achievement*

Student achievement was measured using a curriculum-based benchmark assessment (Appendix H). This assessment was a combination of 3 “Mini-Assessments” from the Go Math™ second grade curriculum (Adams, et al., 2011), with a total of 13 questions. The questions were numbered sequentially on each page; i.e. page one was numbered 1-4, page two was numbered 1-4, and page three was numbered 1-5. For the purposes of this data collection, questions 1-4 are the questions on page one, questions 5-8 are the questions on page two, and questions 9-13 are the questions on page three. In other words, the questions are numbered from 1-13 in the order that they appeared to students, so that no question numbers were repeated.

Credit was given for a correct answer, regardless of the methods used to obtain it. This was done because part of the inherent design of this study was the notion that students would be allowed choice in how they decided to answer a question. Throughout the study, I was not as concerned about which method students chose, as long as they discovered a correct answer and were able to show and explain their methods. Therefore, even on questions, such as number thirteen, when the directions pointed students to a specific strategy (“ $17 + 4 = ?$  Show how to make one addend a ten.”), students were given credit for a correct answer regardless of their chosen method or strategy. Student scores were given as the number correct out of 13 problems.

### *Pre-test Results*

On the benchmark pre-test, the lowest score obtained by any subject was 4 correct answers, and the highest score obtained was 12 correct answers. The mean of all subjects' scores was 8 correct answers. Table 2 illustrates how many students answered each question correctly on the pre-test.

*Table 2: Number of Correct Answers on Benchmark Pre-test*

<b>Question Number</b>	<b>Concept Addressed</b>	<b>Number of Students with a Correct Answer</b>
1	Relation of addition and subtraction	14
2	One-digit addition with sum >10	13
3	One-digit addition with sum >10	9
4	Relation of addition and subtraction	5
5	Missing addend	10
6	Addition equivalence	10
7	Missing addend	11
8	Addition equivalence	9
9	Two-digit plus one-digit addition with regrouping	12
10	Two-digit addition with regrouping	6
11	Two-digit addition with regrouping	5
12	Two-digit addition with regrouping	6
13	Two-digit addition with regrouping	6

### *Post-Test Results*

On the benchmark post-test, the lowest score obtained by any subject increased to 5 correct answers, and the highest score obtained increased to 13 correct answers. The students with the lowest scores on the pre-test each increased their scores by 3 points. The mean of all subjects' scores increased from 8 to 9 correct answers. Table 3 illustrates how many students answered each question correctly on the post-test.

*Table 3: Number of Correct Answers on Benchmark Post-test*

<b>Question Number</b>	<b>Concept Addressed</b>	<b>Number of Students with a Correct Answer</b>
1	Relation of addition and subtraction	13
2	One-digit addition with sum >10	14
3	One-digit addition with sum >10	13
4	Relation of addition and subtraction	6
5	Missing addend	8
6	Addition equivalence	8
7	Missing addend	9
8	Addition equivalence	8
9	Two-digit plus one-digit addition with regrouping	13
10	Two-digit addition with regrouping	11
11	Two-digit addition with regrouping	11
12	Two-digit addition with regrouping	10
13	Two-digit addition with regrouping	8

*Pre-test and Post-test Analysis*

Eleven out of 14 students (79%) showed an increase in the number of correct answers from the pre-test to the post-test. The remaining 3 students (21%) showed no change from the pre-test to the post-test, with the same number of correct responses each time (it should be noted, however, that these subjects did not necessarily provide correct answers to the same items). None of the subjects showed a decrease in the number of correct responses on the post-test.

It is interesting to note which concepts showed positive changes or negative changes on the pre-test as compared with the post-test. Table 4 illustrates the change in number of correct responses for each item and its corresponding concept.

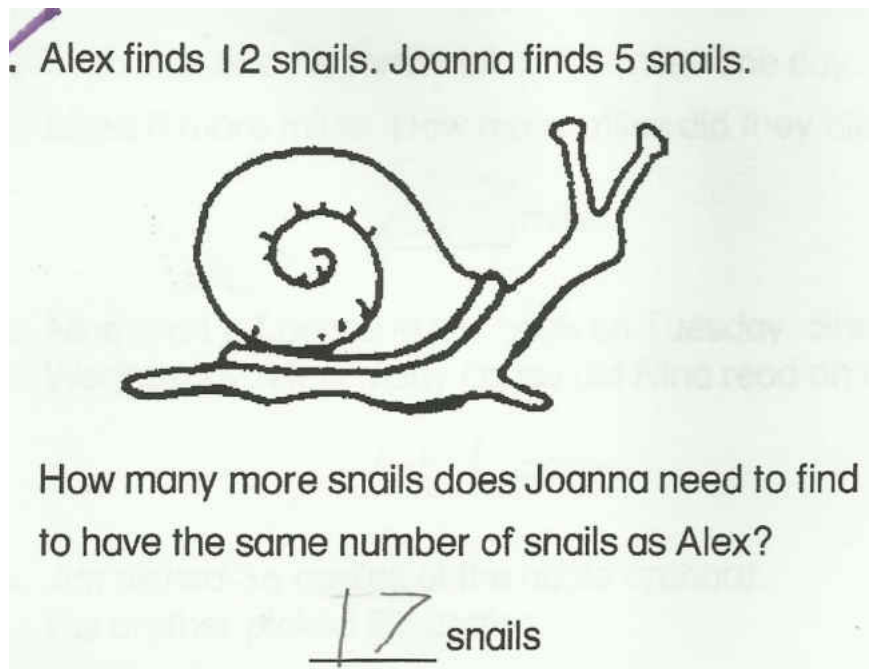
Table 4: Comparison of Benchmark Pre-test and Post-test

Question Number	Concept Addressed	Change in Number of Correct Answers
1	Relation of addition and subtraction	-1
2	One-digit addition with sum >10	+1
3	One-digit addition with sum >10	+4
4	Missing addend	+1
5	Missing addend	-2
6	Addition equivalence	-2
7	Missing addend	-2
8	Addition equivalence	-1
9	Two-digit plus one-digit addition with regrouping	+1
10	Two-digit addition with regrouping	+5
11	Two-digit addition with regrouping	+6
12	Two-digit addition with regrouping	+3
13	Two-digit addition with regrouping	+2

Overall, the students' performance shows an increase in correct responses when regrouping was involved. These questions included any problem where the sum was greater than 10, regardless if the addends were both one-digit numbers (e.g.,  $8+6$ ), one addend was a one-digit number and one addend was a two-digit number (e.g.,  $15+9$ ), or both addends were two-digit numbers (e.g.,  $13+18$ ). This finding suggests that, even though the unit of study included only two-digit addends, students experienced increased achievement with any problems that included regrouping.

The reason that the concepts of missing addends and addition equivalence were included in the data analysis was to investigate how students' experiences with manipulatives during instruction with two-digit addition concepts changed their approach to concepts taught in earlier lessons. As the data show, the group provided overall fewer correct answers to these types of questions. Most frequently, when a student gave an

incorrect answer on one of these items, it was because they simply added the two numbers in the problem. For example, take Piper's answer to question 5, shown in figure 3.



*Figure 3: Piper's Answer to Benchmark Post-test Question 5*

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The correct answer to the question is 7. However, Piper answered 17. This leads one to believe that instead of finding the missing addend, Piper generalized her recent experience with addition, and added the two numbers in the problem, as  $12+5=17$ . This was a common error on the post-test benchmark assessment.

#### *Student Explanations*

Student explanations were examined in two contexts: written explanations and verbal explanations. Data for written explanations were collected via journal entries and

the journal rubric (Appendix K). Data for verbal explanations were collected through teacher observations and notes, several of which had accompanying audio recordings. Both types of explanations are discussed in the following sections.

### *Written Explanations*

#### *First journal entry*

On the first day of the study, students were given the following problem and asked to write an explanation of how they came to their answer:

“Ms. Siegel has 8 pencils. Matt has 9 pencils. How many pencils do they have in all?”

Students had the potential to earn from 1 to 3 points on the journal rubric, with 1 being the lowest score, and 3 being the highest. All of the students scored a 1 (due to missing explanations) or 2 on the journal rubric, and all students had a correct answer. Many had drawings of a group of 9 items next to a group of 8 items, and/or the number sentence  $8+9=17$ . However, very few had any explanations of what exactly had been done beyond, “I did a[n] additoin [addition] problem  $9+8=17$ ,” or, “The anser [answer] is 17. I know becuase [because] I had count it up [no accompanying picture].” In fact, only James was able to offer any detailed insight into his thought process: “The answer is 17 because  $8+8=16$  if you just add one more it is  $8+9=17$ .” One can see that James was able to use a “doubles plus one” strategy, by adding one to both an addend and the sum.

#### *Midpoint journal entry*

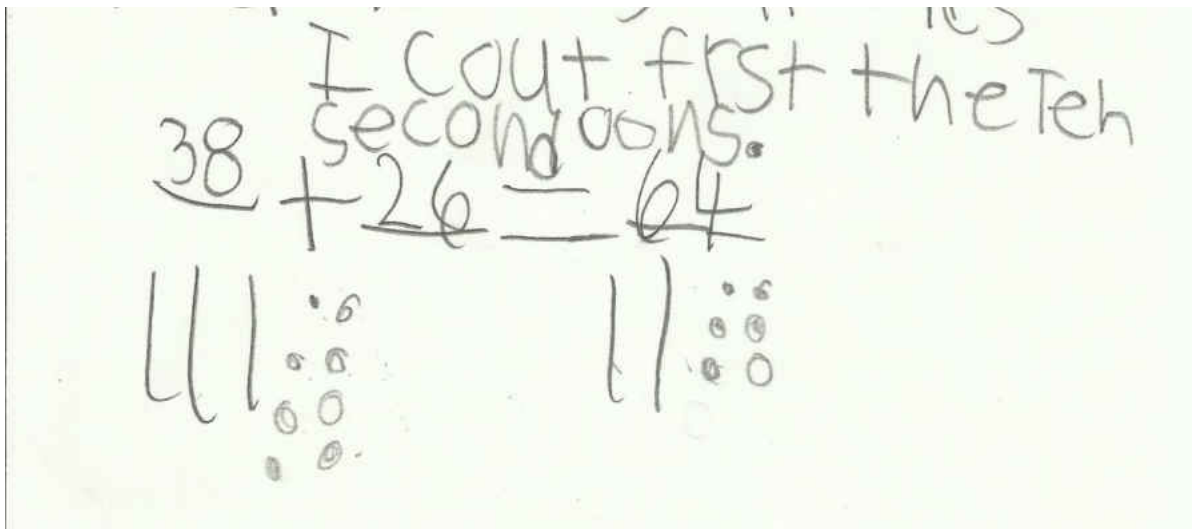
Near the midpoint in the study, students were given the following journal prompt:



“Ms. Siegel has 38 pieces of candy. Mrs. Davis has 26 pieces of candy.

How many pieces of candy do they have altogether?”

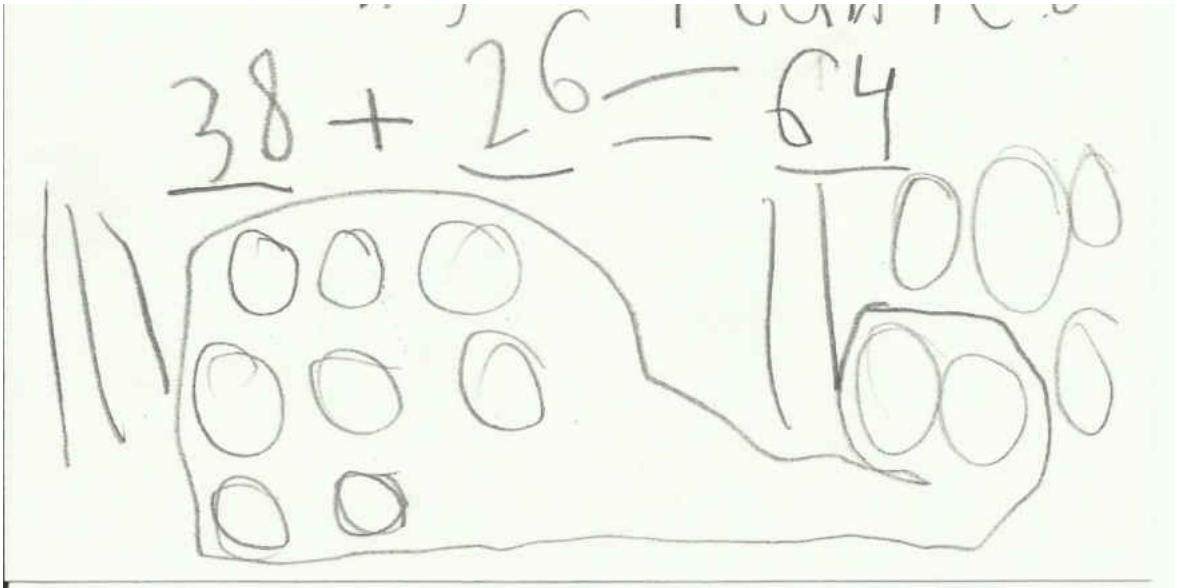
Again, all students scored a 1 or a 2 on the journal rubric. Eleven of the 14 subjects included a drawing similar to Teddy’s, shown below in Figure 4, where lines represented tens, and dots or x marks represented ones. Eleven out of 14 students also had the expression  $38+26$ , but only 9 had a correct answer. The remaining 5 students either had an incorrect answer, or no mathematical expression written down at all.



*Figure 4: Teddy's Response to Midpoint Journal Prompt*

Two students had only drawings, with no indication of how the problem was solved. Ten students had some variation of Teddy’s explanation that “I cout [count] first [first] the ten[s] second oons [ones],” indicating that they had counted the 5 tens to get to 50, then counted the 14 ones separately, adding the numbers somehow to find a sum of 64. Two students indicated through drawings that they had utilized the regrouping strategy of making a ten (see Figure 5), though there were no written explanations. Both

students showed that they grouped 10 ones together, regrouping the addends as 6 tens and 4 ones, or 64.



*Figure 5: Chad's Response to Midpoint Journal Prompt*

*Final journal entry*

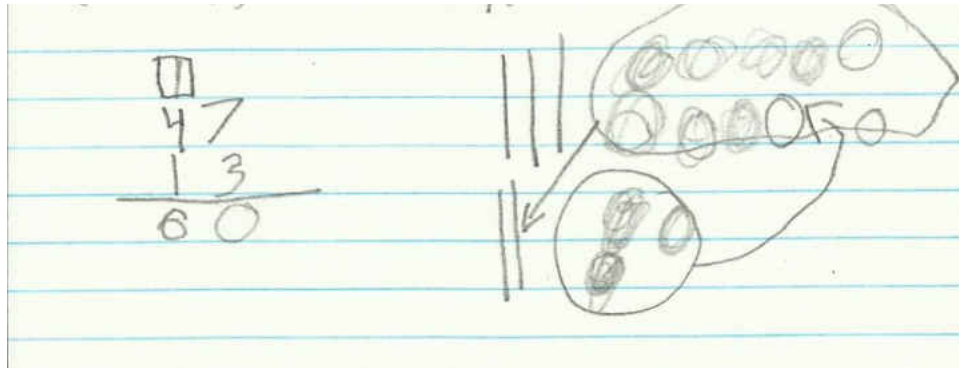
In the final journal entry, the students were given the following prompt:

“At a sports store, a baseball mask costs \$47. A soccer ball costs \$13.

How much money would you need to buy both items?”

At this point in the unit, the students had been practicing representing their drawings and manipulative use with the traditional algorithm. Again, all students scored either a 1 or 2 using the journal rubric. Twelve students answered the question correctly, one answered incorrectly, and one gave an explanation but no answer (“I made a number sentes [number sentence] and drew a piter [picture]!”). Seven students drew a picture to accompany their explanation, and 10 included the number sentence  $47+13=60$ . Only 3

students represented their process with the standard algorithm, similar to Mario's work shown in Figure 6.



*Figure 6: Mario's Response to Final Journal Prompt*

Nine students included written explanations, and of those, only 2 went any deeper than statements such as, “I regrouped,” or “I cant [count] tens first [first] I cant [count] the ones next.” Molly did not include a picture, but explained, “I saw  $4+1=5$  so  $40+10$  should be 50 but you have to conut [count] the ones. So  $7+3=10$  so  $40+10+10=60$ .” Mikey also omitted a picture, although he added the standard algorithm to his explanation: “First I counted the ones then I put 1 over the tens to make it one more ten.” Regardless of how the students solved the problem, only 2 out of the 14 subjects were unable to discern the correct solution to this question.

### *Verbal Explanations*

The data for verbal explanations comes from teacher observations and notes. There was one lesson in the midpoint of the study that also included audio recordings of student explanations. Overall, students were better able to explain their methods verbally, rather than through writing. This is not to say that all students were equally adept at

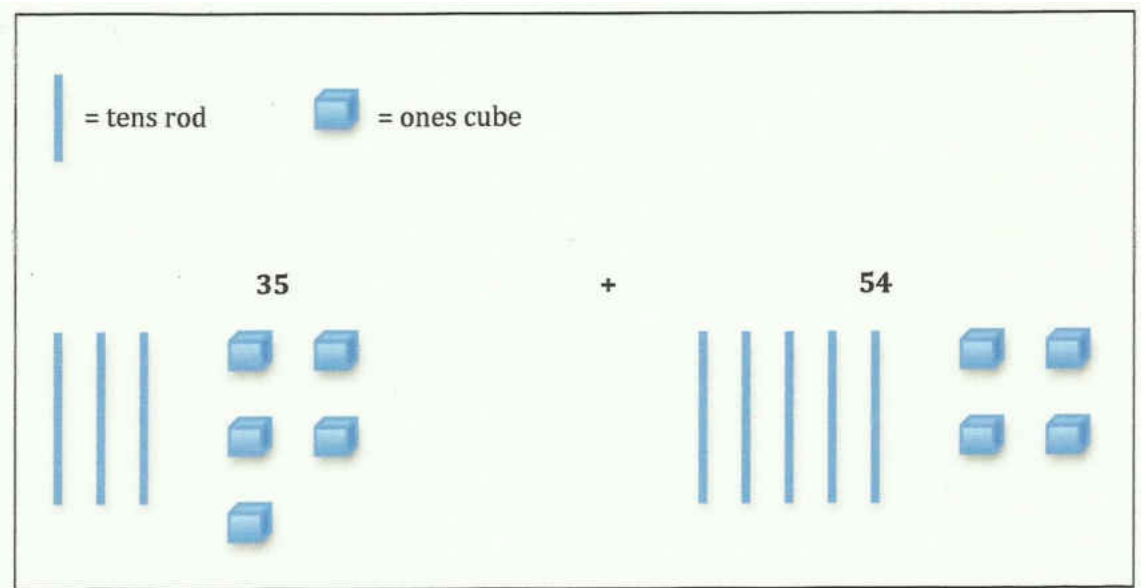
verbal explanations. However, in all cases this was the easier mode of communication, though one cannot be absolutely sure why. It is highly possible that the age of the subjects was largely at play in this situation, as this study took place at the beginning of the subjects' second grade year.

### *Week one*

In the early weeks of the study, students had limited strategies with which to solve the problems presented to them. The first strategies the students learned involved breaking apart addends into tens and ones, and then grouping and counting the tens and ones separately, finally adding those numbers together to find the sum. As the data shows, students eventually became more comfortable with this strategy, and began grouping and adding tens and ones. However, in the early stages of the study, most students simply represented each addend with manipulatives, and then counted the manipulatives without grouping or adding together the tens and ones.

For example, take Cindy's solution of the problem  $35+54$  illustrated in figure 7. Cindy represented both addends with place value blocks. On one half of the desk, she laid out 3 tens rods and 5 ones cubes, and on the other half she laid out 5 tens rods and 4 cubes. She did not group the tens rods together and the ones cubes together, to show  $30+50=80$  and  $5+4=9$ , for a total sum of 89. Instead, when I asked her to tell me how she found her answer, she simply counted the blocks while touching each manipulative. She started on the first half of the desk, touching and counting the tens rods, "10, 20, 30." She then switched to the other half of the desk and continued with those tens, "40, 50, 60, 70, 80." She moved back to the first half to count the ones cubes, "81, 82, 83, 84, 85."

Finally, she returned to the other half to finish counting those ones cubes, “86, 87, 88, 89.” This method was widely used by all students in the first week of the study, in both physical manipulations and drawings.



*Figure 7: Illustration of Cindy's Desk*

*Week two*

In the second week, most students tended to either continue using the tens-and-ones counting method, or switch over to a compensation/regrouping model. Students who used a compensation model often chose to represent this in one of three ways. The first was trading ten ones for one ten, or as Jonah put it, “There were more than ten ones on my whole desk, so I took ten of them and traded for a ten, with some left over.” In the second method, students configured ten ones to make them look like a ten, instead of physically trading blocks. This process meant either connecting ten Unifix cubes to make a tens rod, or lining up ten place-value cubes to make blocks look like a tens rod. Those

who chose to draw representations to either accompany or replace physical manipulatives used the final method. In this method, students drew a separate representation of each addend, using lines for tens and dots or x marks for ones. Then, they grouped ten ones by either circling a total of 10 ones, or erasing a total of 10 ones from both representations and drawing a new ten.

This method can be observed in the transcription of Mario's explanation below. The question posed was, "Marvin has 28 chocolates. Marion has 49 chocolates. How many chocolates are there in all?" As Mario counts and moves tens and ones, he draws the corresponding pictures on the whiteboard in front of the class.

Mario: In 28, I have 2 tens and... 8 ones. And on 49, we have-

...

Mario: 1, 2, 3, 4 tens.

Teacher: OK.

Mario: And then, on the ones we have... 1, 2, 3, 4, 5, 6, 7, 8, 9!

Teacher: OK.

Mario. Well I have 9 on this area, and 8 on this side. I can add a ten by grabbing, uh, 2 more from this side. So we take this one and this one away, add this one and this one here. Then we erase all of this into a ten, so we have 3 tens on this side and 4 tens on that side. No ones on this side, but 7 ones on this side. And yeah.

Teacher: [students start to clap]. Wait a minute, wait a minute. So how many do we have altogether? That's the big question.

Mario: So we have 1, 2, 3, 4 ones, I mean tens, on this side. 1, 2, 3 tens on this side. So, 1... 4 plus 3 equals 70.

Teacher: 4 *tens*...

Mario: 4 tens.

Teacher: Plus 3...?

Mario: Plus 3 tens is 30.

Teacher: 70.

Mario: Oh, yeah, 70.

Teacher: Can you write that on the board for us? 70... 70. And how many ones?

Mario: Uh... 7. Plus 7 equals (whispering)... 71, 72, 73, 74, 75, 76, 77. Equals 77!

Teacher: So how many chocolates are there in all?

Mario: 77.

Teacher: Excellent job. Thank you!

#### *Weeks three and four*

By the end of the study, most students had moved on to using the standard addition algorithm to explain their answers during class work, although the data previously discussed shows a lack of this in journal entries. There were a handful of students who still heavily relied on drawing pictures and regrouping tens and ones as they went, and several others who reverted to that if they became confused while using the

algorithm. However, it was more common to see and hear the explanations via the standard algorithms at this stage.

While this shift to the algorithm was widespread, students still explained their thinking in various ways. For example, take two students' explanations of the problem  $35+38$ . Chad explained his algorithm by reasoning, "I knew there was a 10 because 8 plus 2 equals 10, but there's a 5 there which is more than 2. So I put the 3 in the ones and put the 1 in the tens. One plus 3 plus 3 is 7, so it's 73." He then drew a picture to check his answer. Violet, on the other hand, drew no picture to check her reasoning. Instead, she stated, "I know that 3 plus 3 equals 6. But, 5 plus 8 has a ten, because 5 plus 8 equals 13. So I add a one to the tens, which is 7 tens. So it's 73."

#### *Changes in Manipulative Use*

At any time during the unit of study, students had their choice of manipulatives. Available manipulatives included Unifix cubes, place value blocks (tens and ones), or straws and rubber bands. Students also had the option of using a place value chart with columns for hundreds, tens, and ones to aid in their organization of materials. During the lessons, students were free to exchange or return manipulatives. Although manipulative use was encouraged, students also had the option to draw pictures instead, or solve

#### *Beginning of the Study*

Following the first lesson of the study, students were asked to respond to the initial qualitative journal prompt, "Which manipulative do you like using the best? Why?" The straws and rubber bands were a clear favorite, with 13 out of 14 subjects



choosing that manipulative as their preferred tool. It is interesting to note that none of the students who chose this manipulative indicated that mathematical reasoning influenced their choice. Each student justified their choice with reasons like, “I like using straws best because you get to pout [put] the rudrans [rubber bands] on it,” “I like using straws best because they are bendy,” or, “I like using the straws best because it is fun sumtime [sometimes].”

### *Changes Throughout the Study*

Even with such a strong initial draw, straws and rubber bands quickly fell out of favor as students realized their inefficiency. Violet commented, “This is just too hard to do and undo. It’s making me crazy!” before switching to Unifix cubes on Day 3 of the first week. On Day 4, James mumbled, “Confusing, confusing, confusing,” before also switching his straws for Unifix cubes. By Day 1 of the second week, Teddy was the only one in the group still using straws. The next day, he switched to place value blocks, stating that he “got too confused” with the straws. From that point on, the straws remained virtually untouched. For the remainder of the study, there was an approximate 2:1 ratio of students using place value blocks to students using Unifix cubes on any given day. Even when some students opted not to use manipulatives, the remaining students chose their manipulatives in this approximate ratio.

The number of students who actually used manipulatives of any sort decreased over time. As early as Day 1 of Week 2, some students chose to simply draw pictures without using any physical tools. Frequently throughout the study, some students would choose manipulatives and place them on their desks. Then, as they worked, they would

leave the manipulatives in their containers, working instead through pictures and/or the standard addition algorithm.

The closer the end of the unit came, the more frequently students solved problems without the use of manipulatives. By the last day of the study, only 3 students out of 14 (21% of the study group) actually utilized manipulatives. All of these students were either receiving ESE services or in the process of RtI, and they all chose place value blocks. One student took manipulatives to her desk, but never touched them, as she had done many times over the course of the study. Another took manipulatives and then returned them. When I asked him why he brought them back, he told me, “I just did it regular,” which I took to mean that he had decided to use only the standard algorithm.

It is worth noting that students’ self-identified learning styles (See Appendix H) did not seem to correlate to their choice of whether or not to use manipulatives (Carpenter & Weaver, 2006). All students identified themselves as introverted through the Part 2 of the scale (“How I expose myself to learning situations”), although their responses to Part 1 differed. Table 5 identifies each student’s learning style, as well as the number of days on which students chose to solve problems without any manipulatives at all (out of a total of 13 possible days). It is important to mention that students may have drawn pictures or used the standard algorithm on these days; this table only reflects days on which Unifix cubes, straws and rubber bands, or place value blocks were not chosen or used.

*Table 5: Comparison of Learning Styles and Manipulative Use*

<b>Student Pseudonym</b>	<b>Learning Style</b>	<b>Number of Days Without Manipulatives</b>
Susan	Auditory	5
Mario	Kinesthetic	3
Mikey	Auditory	6
Chad	Auditory	6
Piper	Auditory/Visual	5
Teddy	Auditory/Kinesthetic	5
Cindy	Kinesthetic	5
James	Visual	6
Leon	Auditory	6
Jonah	Visual/Kinesthetic	4
Joseph	Visual	6
Ricardo	Auditory	5
Violet	Kinesthetic	4
Molly	Kinesthetic	5

*End of the Study*

Even though many students stopped using manipulatives early on in the unit, all students were asked to respond to a final qualitative journal prompt on the last day of the study. This prompt was the same as the original prompt, “Which manipulative do you like using the best? Why?” Although the prompt was the same, the responses were much different at the conclusion of the unit.

One student chose straws and rubber bands, even though they had not been used by anyone since very early in the unit. Two students chose Unifix cubes. Nine students chose place value blocks, which was very consistent with the amount of times I had observed students using them throughout the study. Two students stated that they did not like any of the manipulative choices. Joseph wrote, “I don’t like cubes and blocks. Why

they are hard. That's why [I] hate cubes and block[s]." Chad wrote, "I dante [don't] like them because I know it I bont [don't] use them I know math I can add by my hands."

Clearly, attitudes and choices regarding manipulatives changed quite a bit over the course of the study. However, the rationale behind the choices remained similar in nature. Many students chose their favorite manipulative based on its ease of use, citing reasons such as, "they are fun to use," "they don't hurt my hands," "they don't have rumerbans [rubber bands]," or "there [they're] simpel [simple] to put away fast."

In fact, only 2 journal responses indicated that a choice was made based on how the manipulative might help the student solve a problem. Even then, the students' reasoning only vaguely hinted at this idea, without any specifics on how the manipulative helped solve problems. Jonah responded, "I like the blocks becus [because] when I'm cunfused [confused] I can use it." In a similar fashion, Joseph responded, "I yoys [use] a lot cyebs [Unifix cubes] is the best becasea [because] they cud [could] stick and I cud [could] make a ten and I cud [could] break into oens [ones]." Once again, it is hard to decipher whether this lack of mathematical reasoning is a gap in understanding, or simply due to immature writing capabilities.

### Conclusion

The purpose of this study was to discover how allowing my second grade students a choice in their manipulatives during a unit focused on two-digit addition would influence their attitudes and achievements in mathematics. As a group, students showed a marked improvement in mathematics attitudes, which was especially salient for those students who started in the neutral and negative ranges. There was also an increase in

student achievement when students were presented with addition problems that required regrouping. It is difficult to say whether student explanations improved, although it is clear that they evolved over the course of the study, especially when examining verbal explanations. Accordingly, how and when students chose to use manipulatives changed as the study progressed and the children became more comfortable and adept and utilizing the unit concepts.

In the following chapter, the data is discussed in more depth. I discuss my study questions in light of the data presented. Limitations, implications, and recommendations for possible future research are offered.

## CHAPTER FIVE: CONCLUSION

### Introduction

I began this study with a twofold purpose. The first was to improve myself as a teacher, and the second was to improve my students' learning and attitudes about mathematics. I chose to investigate these goals by allowing my students to choose which manipulative tools they utilized to explore and defend their solutions to two-digit addition problems. My research included the following questions:

1. How does student manipulative choice, using UDL, influence students' attitudes towards mathematics?
2. How does student manipulative choice influence student achievement with two-digit addition concepts?
3. How does the availability of manipulatives influence student explanations of two-digit addition concepts?
4. How do student manipulative choices and manipulative use change over time?

In this chapter, I review the results of my study. I also discuss the implications and limitations of my work, and suggest recommendations for further research.

### Results

#### *Student Attitudes*

Usher and Pajares (2009) explained that student attitudes are greatly influenced by a child's experiences and emotions when engaging in domain-specific activities such as the mathematical activities described in this study. When students are engaged in

positive mathematical experiences, they are more likely to persevere in their search for solutions and remain motivated (Dermitzaki, Leondari, & Goudas, 2009). Furthermore, when students are immersed in a positive classroom environment that focuses on learning and progress, rather than correct processes and answers, they are much more likely to view mathematics in a positive light (Chouinard, Karsenti, & Roy, 2007; Turner et al., 2002).

I found that my results in regards to student attitudes were concurrent with the research. During the study, my focus was on understanding how the students came to their solutions, and the students were reminded many times to concentrate on explaining and justifying their answers. Although many students presented a positive attitude before the study began, most students (79%) showed improved attitude scores at the conclusion of the study. After observing students' enthusiasm for using the manipulatives and the choices that they made to support their own understanding, I am led to believe that this freedom of choice influenced their successes in mathematics, and therefore their attitudes.

It is interesting to note that the Universal Design of this study seemed to have the greatest influence on the students with the most negative initial mathematics attitudes. It makes sense that the students with the most negative attitudes would have the most to gain, and my data seem to support that idea. As Tymms (2001) discovered, there is a correlation between student success and attitudes. Although it would not be appropriate to assume a causal relationship in this study, the data do seem to indicate that my

students' experiences with manipulative choice influenced their mathematics attitudes, especially in the cases where previous attitudes were particularly negative.

### *Student Achievement*

Many studies have illustrated the effectiveness of using manipulative tools during mathematics instruction (Bebout, 1990; Fuson & Briars, 1990; Manches, O'Malley, & Benford, 2010; Sherman & Bisanz, 2009). I decided to go a step further and allow my students freedom to choose their manipulatives, based on research that suggested beneficial effects in the classroom (Manches, O'Malley, & Benford, 2010; Moyer & Jones, 2004; Rose & Meyer, 2002). These studies show that manipulatives aided students in discovering correct solutions during mathematics problem solving, and my results were consistent with previous findings.

Again, it is not advisable that my data be interpreted in a causal fashion. However, my data clearly show that students experienced improved achievement with two-digit addition over the course of my study. My daily observation of the students indicated their increased understanding of and fluency with two-digit addition concepts. As expected, on the post-test, the number of correct responses increased on all questions involving regrouping. While I cannot explicitly say that the students' experiences with manipulatives *caused* this increase, the research would suggest that these experiences played a large part in my students' increased achievement with two-digit addition (Fuson & Briars, 1990; Manches, O'Malley, & Benford, 2010; Moyer & Jones, 2004).



### *Student Explanations*

Carpenter et al. (1996) described a series of developmental stages through which children progress as they develop an understanding of multidigit addition concepts. In the first stage, children rely heavily on manipulative tools, acting out the problem situation and simply counting the total number of manipulatives with one-to-one correspondence. Next, children tend to employ a counting strategy, starting with one number and then counting up to add, or down to subtract, landing finally on the correct solution. Subsequently, children progress to using familiar facts and number relationships to solve problems. Finally, with teacher support and a firm understanding of place-value, students should be able to move toward more abstract solution methods, such as using algorithms. Although my students showed little change in written responses, their verbal explanations of two-digit addition problems followed a similar path to the one that Carpenter et al. (1996) depicted in their research.

As noted, there was little change in my students' written explanations of their problem solutions. However, the change in their verbal solution descriptions leads one to believe that this finding is more influenced by my students' immature writing capabilities than their actual mathematical understanding. In fact, I was able to observe a marked difference in students' explanations from the beginning of the study to its conclusion.

Early in the study, most students spontaneously used the first of Carpenter et al.'s (1996) methods, and simply represented both addends and counted the total number of manipulatives. Later in the study, students tended to skip over the "counting up" stage and began to use number relationships and tens facts to solve problems. I would imagine

that this progression occurred because children were using manipulatives, which made it easier for them to physically show how they regrouped the addends. This ostensibly made counting up a less efficient strategy. Finally, students moved toward more consistent use of the abstract algorithm, although they tended to revert back to pictures or manipulatives when they became flustered or confused.

Although each student progressed at different rates, they all experienced development in their explanations and mastery of two-digit addition. Although reliance on manipulative tools was widespread during the initial stages of the study, this dependency decreased as students became more adept at solving two-digit addition problems. This progression was very much in-line with the research on manipulatives and children's learning (Carpenter et al., 1996; Fuson et al., 1997).

### Implications

NCTM (2000) encourages teachers to create an equitable environment in the classroom, in order to allow all students equal access to the mathematics concepts being presented. One way to do as NCTM suggests is through Universal Design for Learning (UDL) (Rose & Meyer, 2006). Research shows that students often need differentiated instruction to be successful in the classroom, and UDL is a way for teachers to proactively allow students to select the materials and strategies that they need to be successful (Lieberman, Lytle, & Clarcq, 2008; Rose & Meyer, 2002; Rose & Meyer, 2006). In my study, I implemented the theory of UDL by allowing my students to choose the manipulatives with which they solved and explained two-digit addition problems. This choice influenced both their academic success and personal satisfaction, and

therefore offers beneficial insight to teachers who are considering implementing principles of UDL in their own classrooms.

This study additionally supports the notion of utilizing manipulatives in the elementary classroom, which is also fully endorsed by NCTM (2000). Research shows that manipulatives are crucial in the development of place-value and multidigit operation concepts in young learners (Fuson & Briars, 1990; Manches, O'Malley, & Benford, 2010; Moyer & Jones, 2004). My students certainly benefited from the use of manipulatives. They were able to discover and explain solutions to two-digit addition problems, which led to more academic success and improved mathematics attitudes. This study was consistent with the existing research on the subject, and implies that teachers looking to incorporate manipulatives into the mathematics classroom will likely see positive results with their own students.

#### Limitations

This study is not an experimental one, so the results should not be viewed in a causal or generalized way. My sample was small and non-random, although it was representative of my particular school's makeup. Furthermore, I was the sole researcher and practitioner, and although I tried to remain as objective as possible, researcher bias is a factor in this study.

#### Recommendations

This study has the potential to inform future research. As noted, the development of place value concepts is crucial to the understanding of multidigit operations (Baroody,

1990; Carpenter et al., 1996; Fuson, 1990; Fuson et al., 1997; Lopez-Fernandez & Velazquez-Estrella, 2011). It would be interesting and informative to stretch my research questions over a longer period of time, allowing the researcher to investigate how student manipulative choice influences learning of place-value, addition, and subtraction concepts. Future research in this area would provide information on how students learn each concept individually, as well as how the learning of each set of concepts interacts with the others.

I would also liked to have been able to include virtual manipulatives in the study. While virtual manipulatives are similar to their physical counterparts, the way students engage with each type is very different (Jolicoeur, 2011). If this study could be altered to include access to virtual manipulatives for all students, the results could be vastly different from my own findings. Future researchers might consider comparing results using physical versus virtual manipulatives.

### Summary

I feel that my research has served both of the purposes for which it was intended. I certainly stretched myself as a teacher, and found success with a new teaching strategy. My students also benefited from the study conditions, as the data show improvements in both academics and attitudes. I wanted to see how allowing students control over their own learning and expression influenced them in the classroom, and it seems as though it was advantageous for each of my subjects.

I found it encouraging to observe my students' progress as I implemented my research. They began mathematics lessons with enthusiasm each day, and were eager to

solve problems and explain their thinking to me and to each other. Better yet, their daily class work and assessments showed that my students were able to deepen their understanding of two-digit addition concepts. Allowing students to choose their own manipulatives turned out to be a very positive experience for us all, and I plan to continue using and improving this method throughout my teaching career.

## **APPENDIX A: IRB APPROVAL**



University of Central Florida Institutional Review Board  
Office of Research & Commercialization  
12201 Research Parkway, Suite 501  
Orlando, Florida 32826-3246  
Telephone: 407-823-2901 or 407-882-2276  
www.research.ucf.edu/compliance/irb.html

### Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**  
**FWA00000351, IRB00001138**

To: **Aryn A. Siegel**

Date: **August 09, 2011**

Dear Researcher:

On 8/9/2011, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review: Exempt Determination  
Project Title: How does providing manipulative choice influence math attitudes and learning in second-grade students?  
Investigator: Aryn A. Siegel  
IRB Number: SBE-11-07781  
Funding Agency:  
Grant Title:  
Research ID: N/A

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Kendra Dimond Campbell, MA, JD, UCF IRB Interim Chair, this letter is signed by:

Signature applied by Joanne Muratori on 08/09/2011 08:33:21 AM EDT

IRB Coordinator

**APPENDIX B: PRINCIPAL APPROVAL**



July 26, 2011

To Whom It May Concern:

I give permission for Aryn Siegel to complete her action research project at our school.

In order for students to participate in the project, the student must receive parent permission. Ms. Siegel has completed the permission letter and it has been approved.

I fully support Ms. Siegel with this project.

Sincerely,

A handwritten signature in blue ink that reads "Carol Ann Darnell". The signature is written in a cursive style with a long horizontal flourish at the end.

Carol Ann Darnell

Principal

**APPENDIX C: COUNTY APPROVAL**

August 24, 2011

Ms. Aryn Siegel  
[REDACTED]

Dear Ms. Siegel,

I am in receipt of the proposal and supplemental information that you submitted for permission to conduct research in the [REDACTED] County Public Schools. After review of these documents, it has been determined that you are granted permission to conduct the study described in these documents under the conditions described herein.

[REDACTED] Elementary principal, Carol Ann Darnell, has the authority to decide if she wishes to participate in your study. Therefore, your first order of business is to contact Ms. Darnell and explain your project and seek permission to conduct your research. You are expected to make appointments in advance to accommodate the administration and/or staff for research time if necessary. If pertinent, the district does not allow the use of the [REDACTED] email system to disseminate your research information.

Please forward a summary of your project to my office upon completion.  
Good Luck!

Sincerely,

*Anna-Marie Cote*

Anna-Marie Cote, Ed.D.  
Deputy Superintendent  
Instructional Excellence and Equity

AMC/jr  
cc: Carol Ann Darnell

**APPENDIX D: ENGLISH PARENTAL CONSENT FORM**

Dear Parents,

August 2011

Hello! I am writing to request permission for your child to participate in a research study that I am conducting in our classroom at \_\_\_\_\_ Elementary this year. I am currently a student in the Lockheed Martin Academy at the University of Central Florida, working towards a Master's Degree in K-8 Mathematics and Science Education. This research project is being conducted as a part of my educational requirements.

My research will focus on allowing students choice in math manipulatives (place value blocks, Unifix cubes, etc.) and how that choice affects their learning and attitudes towards math. I will be conducting this research during our regularly scheduled math instruction, starting in mid-August and lasting through November. All students will be instructed using our county math curriculum, with additional methods designed to enhance their learning of math concepts. Research activities include: completion of a learning style inventory and student attitude survey, use of journal rubric for data collection, and occasional tape recording of student/teacher interactions.

There are no anticipated risks, only potential benefits from participation in a study designed to increase math understanding in our classroom. The identities of the students will be kept confidential in discussions with my advisor as well as the final research report. Student names will be removed from work samples, and student names will be changed in any written documentation. I will occasionally be using voice recording of students' responses and discussions. These recordings will only be heard by my advisor and myself, and will be destroyed at the conclusion of the study.

Participation is NOT mandatory, and your student's grades will not be influenced in any way regardless of your decision. Please know that you also have the right to withdraw your student from the study at any time. Unfortunately, I cannot offer any compensation, but I will be happy to share the results of the research with you once it has concluded.

If you have any questions, please feel free to call me at \_\_\_\_\_. You may also contact my faculty advisor, Dr. Janet Andreasen, at 407-823-5430. Any questions or concerns about participants' rights may be directed to the UCF Office of Research and Commercialization. Their address is 12201 Research Parkway, Suite 501, Orlando, FL 32826; their phone number is 407-823-3778.

If you do consent for your child to participate in this study, please sign and return this form to school as soon as possible. Please remember, there will be NO negative affects on your child's grades or treatment in the classroom if you decide not to consent.

Thank you,  
Ms. Siegel

\_\_\_\_\_ Yes, I have read the project description provided above.

\_\_\_\_\_ Yes, I give permission for my child  
\_\_\_\_\_ (name) to participate in Ms. Siegel's research  
project.

\_\_\_\_\_  
Parent/Guardian Signature

\_\_\_\_\_  
Date

**APPENDIX E: SPANISH PARENTAL CONSENT FORM**

Estimados padres,

Agosto 2011

¡Hola! Estoy escribiendo al permiso de la petición para que su niño/a participe en un estudio de la investigación que estoy conduciendo en nuestra sala de clase en la Escuela Elemental este año. Soy actualmente una estudiante en la Academia de Lockheed Martin en la Universidad de la Florida Central (UCF), trabajando hacia la Maestría en la Educación de las Matemáticas K-8 y de las Ciencias. Este proyecto de investigación se está conduciendo como parte de mis requisitos educativos.

Mi investigación se centrará en permitir a los estudiantes escogidos en los manipulativos de la matemáticas (bloques del valor de lugar, los cubos de Unifix, los etc.) y cómo esa opción afecta a su aprendizaje y actitudes hacia matemáticas. Conduciré esta investigación durante nuestra instrucción regularmente programada de la matemáticas, comenzando a los mediados de Agosto hasta el mes de Noviembre. A los estudiantes se les darán instrucciones usando nuestro plan de estudios de la matemáticas del condado, con los métodos adicionales diseñados para realzar su aprendizaje de los conceptos de la matemática. Las actividades de investigación incluyen: la terminación de una actitud de aprendizaje del inventario y del estudiante del estilo examina, uso de la rúbrica del diario para la colección de datos, y grabación ocasional de las interacciones del estudiante/ profesor.

No hay riesgos anticipados, solamente el potencial de beneficiarse de la participación en un estudio diseñado para aumentar mas interes en las matemáticas que aprenden en nuestra sala de clase. Las identidades de los estudiantes serán mantenidas confidenciales en las discusiones con mi consejero así como el informe final de la investigación. Los nombres del estudiante serán quitados de muestras del trabajo, y los nombres del estudiante serán cambiados en cualquier documentación escrita. Utilizaré de vez en cuando la grabación de la voz de las respuestas y de las discusiones de los estudiantes. Estas grabaciones serán oídas solamente por mi consejero y mi persona, y destruidas en la conclusión del estudio.

La participación NO ES OBLIGATORIA, y los grados de su estudiante no serán influenciados de ninguna manera sin importar su decisión. Sepa por favor que usted también tiene el derecho de retirar a su hijo/a del estudio en cualquier momento. Desafortunadamente, no puedo ofrecer ninguna remuneración, sino que me placeré compartir los resultados de la investigación con usted una vez que ha concluido.

Si usted tiene cualesquier pregunta, sienta por favor libre de llamarme al . Usted puede también entrar en contacto con mi consejera de la facultad, la Dra. Janet Andreasen, en 407-823-5430. Cualesquiera preguntas o preocupación por las derechas del participante se pueden dirigir a UCF Office of Research and Commercialization. Su direccion es 12201 Research Parkway, Suite 501, Orlando, FL 32826; y su telefono es 407-823-3778.

Si usted consiente para que su niño/a participe en este estudio, firme y envíe por favor este documento a la escuela cuanto antes. Recuerde por favor, no habrá efectos de la negativa en los grados de su niño/a o cambio de tratamiento en la sala de clase si usted decide no consentir.

Gracias,

Ms. Siegel

\_\_\_\_\_ Sí, he leído la descripción de proyecto proporcionada arriba.

\_\_\_\_\_ Sí, doy el permiso para que mi niño/a  
\_\_\_\_\_ (nombre) participe en el proyecto de investigación de  
Ms Siegel.

\_\_\_\_\_  
Firma del padre/del guarda

\_\_\_\_\_  
Fecha

**APPENDIX F: MODIFIED FENNEMA-SHERMAN MATHEMATICS  
ATTITUDE SCALE**



**From:** Marrero, Antoinette (Marrero@woodrow.org)  
**Sent:** Thu 7/21/11 3:56 PM  
**To:** ArynSiegel@knights.ucf.edu

Dear Mr. Siegel,

Thank you for your email regarding your wish to use the Modified Fennema-Sherman Attitude Scale found at <http://www.woodrow.org/teachers/math/gender/08scale.html>.

This modified scale was produced by a group of K-12 teachers taking part in a 1993 professional development institute hosted by the Woodrow Wilson National Fellowship Foundation. Please therefore be sure to attribute the scale to Diana Doepken, Ellen Lawsky, and Linda Padwa, as posted at the URL above by the Woodrow Wilson National Fellowship Foundation, 1993.

Finally, please bear in mind that no commercial publication or use is permitted, and that full responsibility for completeness, accuracy, and appropriate citations rests with the team who prepared these materials for the Leadership Program for Teachers.

Sincerely,

Antoinette Marrero

=====

Antoinette Marrero  
Communications Associate  
The Woodrow Wilson National Fellowship Foundation  
street address: 5 Vaughn Drive, Suite 300 | Princeton, NJ 08540  
mailing address: P.O. Box 5281 | Princeton, NJ 08543-5281  
telephone: 609-452-7007 x131 | fax: 609-452-0066  
[www.woodrow.org](http://www.woodrow.org)

NAME: \_\_\_\_\_ DATE: \_\_\_\_\_

### Mathematics Survey

Directions: Read the statement below. Then circle the letter that best responds to the statement.

**A= Strongly Agree**

**B= Sort of Agree**

**C= Not sure**

**D= Sort of Disagree**

**E= Strongly Disagree**

I am sure that I can learn math.	A	B	C	D	E
I don't think I could do advanced math.	A	B	C	D	E
Math is hard for me.	A	B	C	D	E
I am sure of myself when I do math.	A	B	C	D	E
I'm not the type to do well in math.	A	B	C	D	E
Math has been my worst subject.	A	B	C	D	E
I think I could handle more difficult math.	A	B	C	D	E
Most subjects I can handle OK, but I just can't do a good job with math.	A	B	C	D	E
I can get good grades in math.	A	B	C	D	E
I know I can do well in math.	A	B	C	D	E
I am sure I could do advanced work in math.	A	B	C	D	E

**Evaluating Process:**

Statements 1, 4, 7, 9, 10, 11 are positive

Statements 2, 3, 5, 6, 8 are negative

*Add of the total number of points according to the type of statement*

**Positive Statements**

A=5

B=4

C=3

D=2

E=1

**Negative Statements**

A=1

B=2

C=3

D=4

E=5

**Score Range: *x=student's score***

$55 \geq x > 33$  = Positive Attitude

$33 = x$  = Neutral Attitude

$33 > x \geq 11$  = Negative Attitude

## **APPENDIX G: LEARNING STYLES INVENTORY**

## Re: Message from the CARLA website

From: **Karin Larson** (larso205@umn.edu)  
Sent: Wed 7/20/11 2:38 PM  
To: arynsiegel@knights.ucf.edu

Dear Aryn:

Thank you for that detailed information. Dr. Cohen must have scanned and put up a copy of that survey unbeknownst to me. It is actually from the following print publication:

Cohen, A., & Weaver, S. (2006, April). *Styles- and strategies-based instruction: A teacher's guide*. (CARLA Working Paper Series). Minneapolis: University of Minnesota, Center for Advanced Research on Language Acquisition.

That said, you can probably see that on the footer we give blanket permission for the tool to be used for education purposes. If you were going to publish a paper in a book/journal and wanted to include the survey you would need to get specific permission from our office. For now you can feel free to copy it for use in your research.

Best,

**Karin Larson**

\*\*\*\*\*

Karin E. Larson  
Coordinator

The Center for Advanced Research on Language Acquisition (CARLA)  
Global Programs and Strategy Alliance  
University of Minnesota  
140 University International Center  
331 17th Avenue Southeast  
Minneapolis, MN 55414

CARLA Phone: (612) 626-8600  
Direct Phone: (612) 624-6022  
FAX: (612) 624-7514  
E-mail: [larso205@umn.edu](mailto:larso205@umn.edu)  
Web: [www.carla.umn.edu](http://www.carla.umn.edu)

*The Office of International Programs is now the Global Programs and Strategy Alliance.*

**Learning Style Survey for Young Learners:  
Assessing Your Own Learning Styles**

Andrew D. Cohen & Rebecca L. Oxford (2001)

**Purpose:**

The *Learning Style Survey for Young Learners* is designed to assess your general approach to learning. It does not predict your behavior in every instance, but it is a clear indication of your overall style preferences.

**Instructions:**

For each item, circle the response that best matches your approach. Complete all items. When you read the statements, try to think about what you generally do when learning.

For each item, circle your immediate response:

- ☺☺☺ = Often or always (3 pts)
- ☺☺ = Sometimes (2 pts)
- ☺ = Never or rarely (1 pt)

**Part 1: How I use my physical senses**

- |  |     |    |   |
|--|-----|----|---|
| I remember something better if I write it down.                  | ☺☺☺ | ☺☺ | ☺ |
| When I listen, I see pictures, numbers, or words in my head.     | ☺☺☺ | ☺☺ | ☺ |
| I highlight the text in different colors when I read.            | ☺☺☺ | ☺☺ | ☺ |
| I need written directions for tasks.                             | ☺☺☺ | ☺☺ | ☺ |
| I have to look at people to understand what they say.            | ☺☺☺ | ☺☺ | ☺ |
| I understand talks better when the speaker writes on the board.  | ☺☺☺ | ☺☺ | ☺ |
| Charts, diagrams, and maps help me understand what someone says. | ☺☺☺ | ☺☺ | ☺ |

A - Total \_\_\_\_\_

- |  |     |    |   |
|--|-----|----|---|
| I remember things better if I discuss them with someone.                   | ☺☺☺ | ☺☺ | ☺ |
| I like for someone to give me the instructions out loud.                   | ☺☺☺ | ☺☺ | ☺ |
| I like to listen to music when I study.                                    | ☺☺☺ | ☺☺ | ☺ |
| I can understand what people say even when I cannot see them.              | ☺☺☺ | ☺☺ | ☺ |
| I easily remember jokes that I hear.                                       | ☺☺☺ | ☺☺ | ☺ |
| I can tell who a person is just by their voices (e.g., on the phone).      | ☺☺☺ | ☺☺ | ☺ |
| When I turn on the TV, I listen to the sound more than I watch the screen. | ☺☺☺ | ☺☺ | ☺ |

B - Total \_\_\_\_\_

- |  |     |    |   |
|--|-----|----|---|
| I just start to do things, rather than paying attention to the instructions. | ☺☺☺ | ☺☺ | ☺ |
| I need to take breaks a lot when I study.                                    | ☺☺☺ | ☺☺ | ☺ |
| I need to eat something when I read or study.                                | ☺☺☺ | ☺☺ | ☺ |
| If I have a choice between sitting and standing, I'd rather stand.           | ☺☺☺ | ☺☺ | ☺ |

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30 · Styles- and Strategies-Based Instruction: A Teachers' Guide

- |  |     |    |   |
|--|-----|----|---|
| I get nervous when I sit still too long.                             | ☺☺☺ | ☺☺ | ☹ |
| I think better when I move around (e.g., pacing or my tapping feet). | ☺☺☺ | ☺☺ | ☹ |
| I play with or bite on my pens during talks.                         | ☺☺☺ | ☺☺ | ☹ |
| I move my hands a lot when I speak.                                  | ☺☺☺ | ☺☺ | ☹ |
| I draw lots of pictures in my notebook during class.                 | ☺☺☺ | ☺☺ | ☹ |

C - Total \_\_\_\_\_

**Part 2: How I expose myself to learning situations**

- |   |     |    |   |
|---|-----|----|---|
| I learn better when I study with others than by myself.       | ☺☺☺ | ☺☺ | ☹ |
| I meet new people easily by jumping into the conversation.    | ☺☺☺ | ☺☺ | ☹ |
| I learn better in the classroom than with a private tutor.    | ☺☺☺ | ☺☺ | ☹ |
| It is easy for me to talk to strangers.                       | ☺☺☺ | ☺☺ | ☹ |
| Talking with lots of other students in class gives me energy. | ☺☺☺ | ☺☺ | ☹ |

A - Total \_\_\_\_\_

- |  |     |    |   |
|--|-----|----|---|
| I prefer individual or one-on-one games and activities.            | ☺☺☺ | ☺☺ | ☹ |
| I only have a few interests, and I really concentrate on them      | ☺☺☺ | ☺☺ | ☹ |
| After working in a large group, I am really tired.                 | ☺☺☺ | ☺☺ | ☹ |
| When I am in a large group, I tend to keep silent and just listen. | ☺☺☺ | ☺☺ | ☹ |
| Before I try something, I want to understand it real well.         | ☺☺☺ | ☺☺ | ☹ |

B - Total \_\_\_\_\_

**Part 3: How I approach tasks**

- |  |     |    |   |
|--|-----|----|---|
| 1. I like to plan language study sessions carefully and do lessons on time or early. | ☺☺☺ | ☺☺ | ☹ |
| 2. My class notes, handouts, and other materials are carefully organized.            | ☺☺☺ | ☺☺ | ☹ |
| 3. I like to be certain about what things mean in the target language.               | ☺☺☺ | ☺☺ | ☹ |
| 4. I like to know how to use grammar rules and why I need to use them.               | ☺☺☺ | ☺☺ | ☹ |
| 5. I don't care too much about finishing assignments on time.                        | ☺☺☺ | ☺☺ | ☹ |
| 6. I have many piles of papers on my desk at home                                    | ☺☺☺ | ☺☺ | ☹ |
| 7. I don't worry about understanding everything in class.                            | ☺☺☺ | ☺☺ | ☹ |
| 8. I don't feel the need to come to quick conclusions in class.                      | ☺☺☺ | ☺☺ | ☹ |

B - Total \_\_\_\_\_

**Part 4: How I receive information**

- |   |     |    |   |
|---|-----|----|---|
| 1. I prefer short and simple answers rather than long explanations.           | ☺☺☺ | ☺☺ | ☹ |
| 2. I don't pay attention to details if they don't seem important to the task. | ☺☺☺ | ☺☺ | ☹ |
| 3. It is easy for me to see the overall plan or big picture.                  | ☺☺☺ | ☺☺ | ☹ |

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4. I get the main idea, and that's enough for me.	☺☺☺	☺☺	☹
5. When I tell a story, I forget lots of details.	☺☺☺	☺☺	☹
A - Total _____			
6. I need specific examples in order to understand fully.	☺☺☺	☺☺	☹
7. I pay attention to specific facts or information.	☺☺☺	☺☺	☹
8. I'm good at catching new phrases or words when I hear them.	☺☺☺	☺☺	☹
9. I enjoy activities where I fill in the blank with missing words I hear.	☺☺☺	☺☺	☹
10. When I tell a joke, I remember the details, but forget the punch line.	☺☺☺	☺☺	☹
B - Total _____			

**Understanding your Totals**

Once you have totaled your points, write the results on the blanks below. Circle the higher number in each part. If they are close, circle both and read about your learning styles on the next page.

**Part 1:**

- A \_\_\_\_\_ Visual
- B \_\_\_\_\_ Auditory
- C \_\_\_\_\_ Tactile/Kinesthetic

**Part 2:**

- A \_\_\_\_\_ Extroverted
- B \_\_\_\_\_ Introverted

**Part 3:**

- A \_\_\_\_\_ Closure-Oriented
- B \_\_\_\_\_ Open-Oriented

**Part 4:**

- A \_\_\_\_\_ Global
- B \_\_\_\_\_ Particular



## **APPENDIX H: CURRICULUM-BASED BENCHMARK ASSESSMENTS**



# HOUGHTON MIFFLIN HARCOURT

9400 South Park Center Loop  
Orlando, Florida 32819  
September 21, 2011

Ms. Arvn Siegel  
[REDACTED]

Dear Ms. Siegel:

Thank you for your inquiry dated July 26, requesting permission to include Benchmark Mini - Assessment, pages MA.2.A.2.1, MA.2.A.2.2, and MA.2.A.4.4 from *GO MATH!*, Florida, Grade 2, Assessment Guide in your research for your master's thesis titled "How does providing manipulative choice influence math attitudes and learning in second-grade students?"

We are pleased to grant your request on a one-time, nonexclusive, and nontransferable basis as stated, provided that you agree not to portray our copyrighted material in a negative manner, and that no deletions from, additions to, or changes in the material will be made without prior written approval of Houghton Mifflin Harcourt Publishing Company. This license only applies to use of our copyrighted material specified above as examples in your dissertation, and does not authorize mechanical or electronic reproduction in any form. Permission granted herein is limited to material owned by Houghton Mifflin Harcourt Publishing Company.

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Thank you for your interest in our publications.

Sincerely,

Mary Rodriguez  
Contracts Associate

Name \_\_\_\_\_

## Benchmark Mini-Assessment MA.2.A.2.1

1. Write the sum and the difference.

$$6 + 7 = \underline{\quad}$$

$$13 - 6 = \underline{\quad}$$

2. Emily scored 8 points in her first basketball game.  
She scored 6 points in her second basketball game.  
How many points did she score in all?

\_\_\_\_\_ points

3. Dennis has 8 books. His friend gives him 4 more books.  
How many books does Dennis have now?

\_\_\_\_\_ books

4. Brianna and Carly sold 15 cups of lemonade before lunch.  
They sold 9 cups of lemonade after lunch. How many more  
cups of lemonade did they sell before lunch than after lunch?

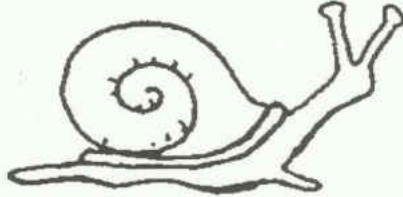
\_\_\_\_\_ more cups of lemonade

Use after Chapter 4 to assess mastery of MA.2.A.2.1.

Name \_\_\_\_\_

## Benchmark Mini-Assessment MA.2.A.4.4

1. Alex finds 12 snails. Joanna finds 5 snails.



How many more snails does Joanna need to find to have the same number of snails as Alex?

\_\_\_\_\_ snails

2. Write the number that will complete the number sentence.

$$8 + 6 = 7 + \square$$

3. Ella counted 11 fish in an aquarium. There were blue fish and orange fish. She counted 4 blue fish.

$$11 = 4 + \underline{\hspace{2cm}}$$

How many orange fish did Ella count?

\_\_\_\_\_ orange fish

4. Write the number that will complete the number sentence.

$$9 + \square = 7 + 8$$

Use after Chapter 4 to assess mastery of MA.2.A.4.4.

Name \_\_\_\_\_

## Benchmark Mini-Assessment MA.2.A.2.2

1. Nathaniel and his family biked 15 miles one day. The next day they biked 8 more miles. How many miles did they bike in all?

\_\_\_\_\_ miles

2. Nina read 23 pages in her book on Tuesday. She read 18 pages on Wednesday. How many pages did Nina read on those two days?

\_\_\_\_\_ pages

3. Jim picked 36 apples at the apple orchard. His brother picked 27 apples.



How many apples did they pick in all? \_\_\_\_\_ apples

4. Write the sum.

$$\begin{array}{r} 75 \\ + 26 \\ \hline \end{array}$$

5. Tyrone caught 17 fireflies. Maria caught 14 fireflies. How many did they catch in all?

$$17 + 14 = ?$$

Show how to make one addend a ten.

Complete the new sentence.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Use after Chapter 5 to assess partial mastery of MA.2.A.2.2.

**APPENDIX I: JOURNAL RUBRIC**

## **Math Journal Entry Rubric**

**1-** No work; work is incorrect

**2-** Number Sentence

Picture

An explanation of what you did/how you did it

**3-** Number Sentence

Picture with labels

An CLEAR explanation of what you did/how you did it

Another way you could have solved the problem

## LIST OF REFERENCES

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