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DEVELOPMENT OF SEVENTH GRADE PRE-ALGEBRA STUDENTS'
MATHEMATICAL PROBLEM SOLVING THROUGH WRITTEN
EXPLANATIONS AND JUSTIFICATIONS

by

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A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Education
in the Department of Teaching and Learning Principles
in the College of Education
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ABSTRACT

In this action research, the interactions of seventh grade pre-algebra students in a mathematics classroom shared their explanation and justification processes through group work. Prior to the start of the study students were given a written pre-test to determine current conceptual thinking in mathematics. Over the next nine weeks, the teacher engaged the students in problem solving activities that included reasoning skills, communication and making connections through discussion with their peers. Following nine weeks of written and verbal discourse, students were provided a post-test to determine changes in their conceptual thinking. Overall students' grades, journal writings and test scores showed positive gains with the greatest changes occurring in written explanations of their conceptual thinking in mathematics.

I dedicate this thesis to my amazing students. Your support and patience through this process has meant the world to me. You are our future and I am proud to have been a part of it. Always remember to learn something new today and come back tomorrow a little bit smarter. I will continue to sit on your shoulder while you reach for your dreams.

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CHAPTER 1 INTRODUCTION

The purpose of this action research study was to determine how the use of verbal and written explanations and justifications in a seventh grade pre-algebra class impacted student outcomes. This study attempted to answer the following two action-research questions.

1. How can problem solving enhance students' written explanations and justifications in mathematics?

2. How did students' communication in a problem-solving climate impact their written explanations and justifications in mathematics?

The researcher, in the past, worked with students considered gifted mathematically over the last ten years. Each year in the past, a form of discourse took place in the classroom as the students who were labeled gifted worked in small groups during problem solving. The students consistently gained a high level of conceptual understanding as shown by the students' standardized tests. The researcher wanted to look at this practice in her new role to see if these high learning gains were from best practices or if the students, being gifted, learned naturally.

This year, the researcher was given an assignment to work with students in a pre-algebra class rather than her typical class assignment of gifted students. Therefore, the action research study focused on if students who were not labeled gifted in a pre-algebra classroom could learn through classroom discussions and discourse focused on problem solving.

Based upon recommendations from the National Council of Teachers of Mathematics (NCTM) students were provided opportunities to work together cooperatively in both large and small groups on significant problems. Students questioned, discussed, made mistakes, listened to others' ideas and provided constructive criticism throughout the communication process. The students were encouraged to think and reason as the teacher served as the facilitator of learning and communication.

The Principles and Standards for School Mathematics (2000) authored by the National Council of Teachers of Mathematics (NCTM) emphasizes a need for teachers to establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. In addition, according to the standard of communication, "Instructional programs from pre-kindergarten through grade 12 should enable all students to...communicate their mathematical thinking coherently and clearly to peers, teachers, and others" and students need to learn "what is acceptable as evidence in mathematics" (NCTM 2000, p. 60). In this study the researcher was working to create this physical setting, while refining learning in a problem solving atmosphere and understanding what was acceptable and what was not.

The focus of this study is in alignment with both state and national standards that call for more conceptual learning and understanding in mathematics. In today's classrooms teachers need to develop new and innovative ways to improve student learning. Johnson (2006) defines conceptual understanding as comprehension of mathematical concepts, operations, and relations. According to Pourdavood (2003), rather than memorizing inflexible procedures provided by a teacher or textbook, students

seem to learn best by constructing their own mathematics. In addition, in a pedagogical problem-solving context, students are given opportunities to design, plan, evaluate, recommend, review, critique, explain, and make situations problematic. Problem solving engages students in the development of their conceptual understanding of mathematics and is a process through which mathematics becomes meaningful and relevant (Johnson, 2003).

To become effective problem-solvers, students should engage in mathematics beyond the superficial level of rote learning. According to Marrone (2004) “social constructivist theories of learning call for students to be active participants in their own learning” (p. 20). These theories also call for teachers to design activities that facilitate students’ development of knowledge by involving the students in conversation that stretches their current boundaries. Johnson (2006) states, “A well-conceived lesson plan can clarify all of these and lay a foundation for a class lesson in which students learn meaningful and engaging mathematics” (p. 60). Through participation in dialogic interactions, children observe, experience, try out, and eventually internalize various “psychological tools” that advance their cognitive development to higher levels (Vygotsky, 1982). Conceptual understanding reveals itself in students in ways such as a student’s ability to explain and justify why particular relationships hold in a problem and why certain operations or procedures are used in a problem.

Mathematics teachers develop and maintain the mathematical and pedagogical knowledge they need to teach their students well. In addition, NCTM explains effective teaching as observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions. Research

has clearly found that not all students learn the same way; few students find the same approaches in mathematics persuasive, and few students benefit from a single approach to mathematical concepts or processes (Johnson, 2006). Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm (NCTM, 2000).

The literature supports that through students communicating in a classroom they learn from one another's explanations helping them gain a better understanding of mathematical concepts. According to NCTM (1989), "Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking" (p. 79). Bicknell (1999) stresses that discussion, amongst students and with a teacher, provides the student with opportunities for social interaction, and for shared understandings to be negotiated and developed. Current inquiry into the practice of mathematics concludes there is not one mathematical practice, one way of understanding mathematics, one way of thinking about mathematics, or one way of working in mathematics (Burton, 1999). To increase mathematical academic performance, NCTM has outlined four standards that underlie all mathematical skills and concepts: problem solving, reasoning, communicating, and connecting. These standards are intertwined and overlap one another through the problem solving process in this study.

Definitions

In this study the following terms were used as they pertain to mathematics as defined by the action researcher:

AVID: Advancement Via Individual Determination (AVID) is a program designed to help underachieving middle and high school students prepare for and succeed in colleges and universities. (AVID Website). Students in the program commit themselves to improvement and preparation for college.

Communities: A classroom where the teacher and students are working and learning together. A place where students ask questions, develop and share their own mathematical thinking, work in small groups and listen to one another describing the process they went through to solve a problem.

Communication: Students may use verbal language to communicate their thoughts, extend thinking, and understand mathematical concepts. They may also use written language to explain, reason, and process their thinking of mathematical concepts. Communication is a tool, which can help students to form questions or ideas about concepts (Hatano and Inagaki 1991).

Concepts: The understanding of basic mathematical skills involving measurement, classification, conservation, ordering and one-to-one correspondence, the transition from purely manipulative to rigorous mathematics and having innate number sense.

Connections: The ability to understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Constructivism: How one learns to construct their own knowledge, which is tied to the exposure to new experiences.

Group Work: A balance of students in a small group context working on an activity or problem together. Generally, students are given certain tasks to do, but for this study, it only involves small groups of students working together to discuss and solve a problem.

Discourse: A general term for a number of approaches to analyzing written, spoken or signed language use. It refers to the ways of representing, thinking, talking, questioning, agreeing and disagreeing during problem solving and classroom discussions.

Explanation: The process one goes through to solve a problem and then explains to step by step what he did to solve or workout the problem.

Exceptional Student Education (ESE): Refers to any student with a learning impairment or disability, exceptional education where students learn differently.

English Speaking Students of Other Languages (ESOL): English Speaking Students of Other Languages

Inquiry: Active participation in authentic practice of mathematics where discussion takes place. It includes the behaviors involved in the struggles for reasonable explanations of what we are curious about learning. It involves the observation of patterns, testing of conjectures and estimation of results.

Justification: To develop a strategy to explain the solution, to develop a logical conclusion, to compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems. The “why” explained mathematically.

Modeling: Clearly demonstrate the relevance of target mathematics concept/skill/strategy to the authentic context. Provide students with the opportunity to practice the target mathematics concept/skill/strategy within authentic contexts while monitoring students as they practice, providing them specific corrective feedback.

Precision: Precision is the use of terms and symbols, consistent with mathematical definitions, in ways appropriate for students at particular grade levels.

Problem Solving: Problem solving is the application of previously learned fact and computation skills within some organized framework of thinking in order to understand some previously unknown situation. It is a process through which mathematics becomes meaningful and relevant (Johnson, 2003).

Pedagogy: The term generally refers to strategies of instruction, or a style of instruction referred to as the correct use of teaching strategies harbored and governed by the pupil's background knowledge and experiences, personal situations, and environment, as well as learning goals set by the student and teacher.

Socio-mathematical norms: A set of mathematical rules that are specific to the field of mathematics, such as to what constitutes a proof.

Standards: Standards are descriptions of what mathematics instruction should enable students to know and do. They specify the understanding, knowledge, and skills that students should acquire from pre-kindergarten through grade 12. (NCTM)

Overview of the Study

During this action research, the interactions of seventh grade pre-algebra students shared their explanation and justification processes for their work as it pertained to pedagogical intentions of the class. As the dialogue was occurring, the explanations and

justifications that emerged from the discussions were measured through students' daily writing journals. Students also were administered a pre and post-test to determine growth in their written explanations and justification.

Throughout the study the researcher attempted to develop an environment conducive to problem solving using dialogue with intentions of enriching students written explanations and justifications in mathematics.

The students' written work showed positive changes from the dialogue within the classroom. While the results are promising for the students in this classroom, this study represents only the findings of this teacher with this unique population of students currently in her pre-algebra class.

CHAPTER 2 LITERATURE REVIEW

In this action research study, students in a seventh grade pre-algebra mathematics classroom interacted and shared their explanations and justifications for problem solving in mathematics. Changes were measured through students' written products. The theoretical framework for this study was based upon a recommendation from NCTM that, "Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one where students assume more responsibility for validating their own thinking" (p. 79). The emphasis in the current literature on students communicating to increase their mathematical understanding provided the foundation for this action-research study. This chapter provides a review of the literature on the relationship of problem solving to students' written explanations and justifications and the importance of creating a climate for communication in mathematics' classrooms.

Problem Solving and Students' Written Explanations and Justifications

Mathematics teaching and learning has moved away from a mechanical view, to one with an emphasis on problem solving, understanding and communicating with others. Spikell (1993) sums it up as students today have a need and desire for practical mathematics. Therefore, mathematical instruction needs to be relevant to students' daily lives. The nature of traditional mathematics teaching and learning, based on customary rules and algorithms taught by rote skills and learned through practice and memorization should be a practice of the past (NCTM, 2000). Today, research shows that to learn

mathematics, students must be engaged in exploring, conjecturing, and thinking rather than, engaged only in rote learning of rules and procedures (NCTM, 2000).

At the core of today's classroom, is problem-solving. Problem-solving as defined by Szetla (1992) is the process of confronting a novel situation, formulating connections between given facts, identifying the goal, and exploring possible strategies for reaching the goal. According to NCTM (1991), centering mathematics instruction on problem solving can help all students learn key concepts and skills within motivation contexts. Historically, problem solving and communication in mathematics was taught by rote memory, with very little engagement by the teacher of students (Pourdavood, 2003). The primary goals of mathematics are understanding and problem solving (Lester, 2003) which in the revised mathematics standards are a core principal to be taught in every classroom.

Problem solving is the application of previously learned fact and computation skills within some organized framework of thinking in order to understand some previously unknown situation (Johnson, 2006). Most historical views of problem solving make a distinction between acquiring knowledge and applying knowledge. Applying knowledge through problem solving in different contexts is a primary goal of mathematics education. The core value of students being engaged in problem solving stems from studies that have shown students learn best when they are active rather than passive learners (Spikell, 1994). NCTM cited, "Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning" (p. 52). Marrone (2004) states, "Social constructivist theories of learning call for students to be active participants in their own learning" (p. 20). Martinez (2001)

noted, “Problem solving, followed by students’ explanations rather than teacher explanations, allows a student to derive concepts actively from their work rather than passively from teacher or a textbook” (p. 119). Problem solving engages students in the development of their conceptual understanding of mathematics, a process through which mathematics becomes meaningful and relevant. Turner (2002) suggested that asking students to explain their mathematical understanding conveys the message that the teacher believes that learning is important and that all students can be successful. Stiff (1993) states, “Strengthening problem-solving and reasoning skills empowers students to handle real-life problems, which promotes a better understanding of mathematical skills and concepts” (p. 4).

The NCTM also argued that problem solving should become “the focus” of mathematics in school (1989). According to NCTM, centering mathematics instruction on problem solving can help all students learn key concepts and skills within motivating contexts. According to Lawson (2000), a major aim of mathematics education is to devise ways of encouraging students to take more active roles in acquiring, experimenting with, and using the mathematical ideas and procedures that are included in the school curriculum. Lawson (2000) asks, “Instead of having students complete meaningless exercises and memorize what the teacher tells them, why not have students learn key mathematical ideas while solving interesting problems?” (p. 28)

Changes in Practice

This new approach to mathematics instruction in the 21st century, according to Boyer (1995) should include a shift from acquisition models of learning to student mathematical empowerment through life-long learning. “Empowering students

mathematically includes providing opportunities for students to develop their “abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems” (NCTM, 1998 p. 5).

However, with current mathematics practices in the United States in need of change, or reform, NCTM took a stance on improving mathematics for all students. Their vision was to produce a document, *Principles and Standards for School Mathematics* (2000), which would outline the expectations for a rigorous curriculum where all students would have access to rich and meaningful mathematical opportunities for conceptual understanding. For students to learn the mathematics necessary to compete in the 21st century, students need to become flexible and resourceful problem solvers, work in groups and communicate their ideas effectively. The NCTM has remained committed to the view that standards can play a leading role in guiding the improvement of mathematics education. “As an organization representing teachers of mathematics, NCTM shares with students, school leaders, and parents and other caregivers the responsibility to ensure that all students receive a high-quality mathematics education” (p. 4).

In addition to the NCTM vision for a need for continued improvement of mathematics instruction, The 2003 Rand Study states, “The recent legislation entitled “No Child Left Behind” has committed the nation to ensuring that all children meet high standards of mathematical proficiency. As workplaces evolve, the mathematical ideas that students need on the job change, and people must be prepared to learn, analyze, and use mathematical ideas they have never encountered in school or used before” (Ball, 2003).

As a part of retraining a workforce for today's society, students need to learn the reasoning behind mathematics. This level of reasoning can be achieved through classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge (NCTM 2000). In addition, procedural fluency and conceptual understanding can be developed through problem solving activities centered on student communication.

This change in the way students learn mathematics may show a shift from emphasis on rules and routine problem solving dominated by teacher talk and passive learning, to active student participation, in which reasoning and communication are stressed. Heibert (1999) concludes that we have quite a traditional way of teaching mathematics, which places the emphasis on teaching and computation procedures and places little attention to helping students develop conceptual ideas. Therefore, problem solving should be a part of all aspects of mathematics activity, because being mathematically literate means being a good problem solver (Heibert 1999).

Classroom-based Research

Research studies on problem solving have been conducted. Lubiensky (2000) reported results of a study after examining 30 seventh-grade students' experiences with a problem-centered curriculum involving learning mathematics through problem solving and discourse. This study was conducted in a socio-economically diverse school located in a medium-sized Midwestern city. The study compared the problem solving growth of students defined as slow learners and students defined as regular learners based on their

economic background. Most of the problem solving activities stemmed from the CMP project; a middle school curriculum-development project funded by the National Science Foundation to create problem-centered materials aligned with NCTM Standards. The curriculum was organized around rich problem settings. Writing prompts were added to the project, which helped students as they summarized their mathematics units. The teacher launched the problem and facilitated the class as the students explored the problem and summarized their solutions. Each lesson was followed by class discussion.

To gather data, the teacher conducted interviews with the students, observed student class work and homework, read journal entries, and analyzed audio recordings to compare participation from the students during whole class discussions. While the results did not show significant growth in basic skills for the students at the lower achievement levels, the researcher did demonstrate positive results related to problem solving. Lubiensky reported the apathetic students became engaged in problem exploration. The study showed that students had an appreciation of the open problems in relation to more computation-oriented curricula. Lubiensky noted that the average students preferred the problem-solving curriculum compared to the traditional mathematics lesson. In addition, from the interviews, she found that in the past students of lower economic backgrounds generally received rote instruction and became followers whereas students with average economic backgrounds were actively involved in problem solving and became leaders. In most cases, economics were tied to the student's prior achievement levels.

Similarly, Bottge (2001) reported positive results from a study involving 75 eighth-grade students from different learning levels, as defined by prior standardized test scores, in a rich problem-solving environment in the upper Midwest. The students were

either in a remedial mathematics class or one of the three pre-algebra classes involved in the study. The purpose of the study was to compare the performance of a remedial class to the general class as concepts emerged from the instructional settings. Determining whether students with low achievement in mathematics could match the performance of students in pre-algebra classes on problem-solving curricula aligned to NCTM standards for middle school was the secondary goal. Video instruction and teacher led instruction with problem solving, group work and discussions took place.

Students were assessed before and after the study began on both computation and problem solving skills. The testing involved a WRAT-III Arithmetic Subtest, a Problem Solving Test, and a Maintenance Test. Students' interviews were audio taped and analyzed. Field notes and classroom video were compared.

The findings of this study supported engaging students in challenging and meaningful problems through problem solving as the post-test showed growth in this area. The students from the low-achieving class scored as well as the average students and won several of the competitive events involving car racing. In addition, several of the low-achieving students' classroom grades moved from the "D" level to the "A" level from the beginning to the conclusion of the problem-solving unit. The challenges sustained the interest of the students just as Dewey suggested it would. Students' conversations revealed that students used math as a tool for solving problems that interested them. The findings of this study support engaging students in challenging and meaningful problems.

In both studies the researchers found that in a pedagogical problem-solving context, students are given opportunities to design, plan, evaluate, recommend, review,

critique, explain, and make situations problematic. According to Forman (2000) these pedagogical problem-solving contexts may motivate students to link meaning with mathematics. Supporting mathematics problem solving contexts are approaches to teaching that encourage risk-taking in a supportive environment. Problem solving is the central goal of the mathematics curriculum, and word problems make up an important part of this goal (Bebout, 1993).

Lester (2003) noted that students need to understand that answers alone are not sufficient. A significant part of understanding mathematics comes from an analysis of the thinking that went into a solution. The connection between solving problems and deepening understanding is symbiotic. Teachers want students to be able to solve problems in mathematics and in the real world.

Because students' opportunities to develop mathematical proficiency are shaped within classrooms through their interaction with teachers and specific content and materials, the RAND Mathematics Study Panel selected mathematical thinking and problem solving as one of the three domains in which proficiency and equity can be achieved (Ball, 2003). The ultimate goal in mathematics is to create an environment in which students can grow as problem solvers. Good problem solvers regularly monitor their thinking and are aware of when they should rethink the problem or switch strategies (Roberts & Tayeh, 2006). Through problem solving, students can experience the power and utility of mathematics. According to NCTM (2000), "Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas" (p. 256). It is through problem solving all mathematical parts are tied together. Bruner (1986), a cognitive

psychologist, maintained, “We teach a subject not to teach little living libraries on the subject, but rather to get a student to think mathematically for himself...to take part in the process of knowledge-getting. Knowledge is a process not a product” (p. 72). And, according to Lester (2003), mathematics is certainly a discipline whose principal component is thinking. Knowing how to execute procedures does not ensure that students understand what they are doing. “Students develop, extend, and enrich their understandings by solving problems” (p. 53).

Justification

In many middle school mathematics classrooms, students are often asked to explain their understanding and reasoning of mathematical concepts. NCTM (2000) shares, “By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense” (p. 56). Mathematical reasoning is fundamental to the learning of mathematics. When reasoning is effectively and routinely promoted and fostered in the classroom through justifying solutions, developing ideas, predicting results, or making sense of observed phenomena, students can develop a deeper understanding of mathematical ideas. As NCTM (2000) pointed out in the Principles and Standards, “Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts” (p. 56). To improve understanding, students must take responsibility for sharing the results of their inquiries and for explaining and justifying their methods (Hiebert, 1996).

According to the Connected Mathematics Program (1995), “Students solve problems and in doing so they observe patterns and relationships; they conjecture, test, discuss, verbalize, and generalize these patterns and relationships. Through this process they discover the salient features of the pattern or relationship; construct understandings of concepts, processes, and relationships; develop a language to talk about the problem; and learn to integrate and discriminate among patterns and relationships” (p. 24).

Allowing students to work in small groups fosters problem-solving skills as the groups lend themselves to opportunities for the students to explain and justify their thought processes by using quality, questioning techniques. Moreover, Mack (1990) states, “In such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning, and argumentation” (p. 21). In addition, NCTM (2000) states, “Students can learn about reasoning through class discussion of claims that other students make” (p. 57).

Marrone stated (2004), in learning through problem solving, students not only have more opportunity to express their ideas and justify their answers verbally, but also have more opportunity to pose and respond to cognitively demanding questions. In addition, rather than memorizing inflexible procedures provided by a teacher or textbook, students seem to learn best by constructing their own mathematics (Pourdavood, 2003). In the Rand Study of 2003, the panel found that mathematical justification involves reasoning that is more general than what we typically call “proof” (p. 38). The Rand Study (2003) describes the importance of the justification process. The study states that new curricula and standards have paid more attention to processes such as problem solving and justifying. Justification centers on how mathematical knowledge is certified and established as “knowledge”. The study also mentions that justification is a practice supported by both intellectual tools and mental “habits” (p. 37). In addition, one of the

strands of proficiency, as outlined by Kilpatrick (2001) stated a student must have the capacity for logical thought, reflection, explanation and justification to be mathematically proficient. (Kilpatrick, Swaddord & Findell, 2001, RAND 2003).

We look for problems that stimulate discussion, require students to justify their answers, and are accessible for students at different stages in their development as problem solvers (Roberts, 2007). The focus on explanation and justification illuminates the necessity for argumentation in the classroom (Pourdavood, 2003). Students should discuss their reasoning on a regular basis with the teacher and with one another, explaining the basis for their conjectures and the rationale for their mathematical assertions. Wheatley (1997) stated, “The difference between good and poor problem solvers is often the extent to which they use imagery” (p. 295).

Written Language

Writing in mathematics also can help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson. Johanning (2000) added that writing allows students to view the concept or problem in a new perspective since they must understand each step of their thinking. Writing also provides teachers with a “window” of student thinking and understanding, addressing the “what” and “why” of their misconceptions (Pugalee, 2005). Journals can be very helpful in getting reluctant students to participate in class as described in research conducted by Reilly (2007).

Langston (1997) reported in her study on how students displayed their communication and thinking in mathematics through learning logs. The study took place with two seventh-grade classrooms over a two-year span. Most of the students were from

military families and were grouped heterogeneously which represented a similar population of the small, 300 student populated school. One of the two classes sat in straight rows and was taught in a traditional style while the second class sat in groups, collaborated their solutions with one another and wrote their solutions in learning logs.

Over the two-year period, Langston collected data on the classes through teacher observation, field notes, student interviews, teacher projects, student work samples, learning logs and standardized tests. Final work was analyzed with respect to both written and pictorial forms of communication. The problems were designed to focus on conceptual knowledge. The results of the study showed huge differences in the conceptual development of the classes. The class that was taught with a traditional method had 24% of the students show conceptual understanding, as they tended to show procedures. In contrast, the class that wrote in learning logs showed 89% of the students demonstrated a depth of conceptual knowledge in mathematics through their written work and interviews.

Roberts (2006) tells us that when students write about and reflect on their own thinking, it makes a significant impact on their ability to solve problems now and in the future. Encouraging students to write about their thinking can provide unique insight into the way students are thinking about the mathematics they are investigating. The Curriculum and Evaluation Standards for School Mathematics, published in 1989 by NCTM, placed an emphasis on the importance of communication for learning and doing mathematics. Writing is one way to achieve this valuable communication in the classroom. NCTM (2000) states, "Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their own work and

clarify their thoughts about ideas developed in the lesson” (p. 61). Steele (2007) says, asking students to write provides a way for teachers to understand the depth of their knowledge. Students benefit through writing by actively engaging in the process of clarifying and critically reflecting on their thoughts and developing deeper understanding of the topic or concept, which enhances students’ ability to effectively solve various problems, they encounter in mathematics (Pugalee, 2005).

Research conducted by Klishis (2003) investigated the relationship between instructional strategies and student understanding in mathematics. Specifically, the study explored whether the combination of journal writing and discourse were effective instructional strategies to improve student achievement in mathematics. The study consisted of 39 fifth grade students who were part of a total population of 79 students in three intact classrooms. All students received traditional mathematics instruction that differed only in the inclusion of writing or writing and discourse in each of the classrooms.

Achievement was measured by three comprehensive tests, as this was the only consistent instrument used in all three classrooms. Results suggested a major performance advantage for the journal discourse group as demonstrated by their higher test scores on the post-test. The study also showed that while the journal-writing group performed better on the tests, the journal writing responses lacked substantial elaboration. More research was needed to see if the lack of elaboration was due to insufficient modeling that the students only engaged in discourse and writing twice a week.

Creating a Climate for Communication in Mathematics

“Learning to think mathematically is essential in the world in which we live and will be even more important in the world where our student will work” (Santiago & Spanos, 1993 p. 134). By explaining and justifying ideas, students are able to make new connections that form a mathematics community focused on the development of mathematical ideas (Cassel, 2002). According to Johnson (2006) Middle school students, by their very nature, need to explore, analyze, create, discover prove, and disprove conjectures. They need to discuss, explore, and participate in the construction of the rules.

Yackel and Cobb (1996) suggest that social norms for genuine problem solving include expecting students to be able to do four things: (1) explain and justify solutions, (2) attempt to make sense of explanations, (3) agree and disagree, and (4) ask clarifying questions in situations that need to be better understood. The mathematics teacher should strive to establish a communication-rich classroom in which students are encouraged to share their ideas and seeks successful clarification. Teachers who do not share the responsibility for classroom explanations may be shortchanging students by not allowing them the experience that comes with equitable classroom discussions. NCTM (2000) wrote, “In such a classroom community, communication is central to teaching and learning mathematics and to assessing student’s knowledge” (p. 271). According to Willoughby (1990), communication is, and always has been, an important part of mathematical problem solving. Classroom communication can take on several faces. Communication may be oral, written, or it may take other forms such as building a model or drawing a picture. Communication takes place when students work together in a non-

risk environment. Each of these forms allows students to think through problems, formulate explanations, and explain and justify their solutions, which are important reasoning skills as well as communication processes. However, according to NCTM (1989), “Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking” (p. 79). Vygotsky (1982) says that through participation in these dialogic interactions, children have the opportunities to observe, experience, try out, and eventually internalize various “psychological tools” that advance their cognitive development to higher levels. The connection between students’ ownership of mathematics and classroom communication is the teacher.

Shaughnessy (1994) showed the results of a study conducted in a school district of Portland on verbal and written forms of communication. There were 18 middle schools in the district, all serving sixth, seventh and eighth graders involved in the study. Teachers repeatedly stated that they valued the students' thinking and their ability to explain their thinking. Teachers made a point of not asking students to state the correct answer. Instead, the teachers constantly requested that students communicate the process they used to solve the problems. Each student’s journal included written explanations of problem solving and documentation of times that students explained their thinking to other students. The data showed there were significant improvements in three areas: student abilities in problem solving, student placement into high school courses, and student beliefs about mathematics. Data was compiled from a citywide math test given in

the spring to place eighth grade students into general math, pre-algebra and algebra classes in high school. The percentage of students scoring high enough for placement in the more challenging courses showed a dramatic increase.

Studies of classroom discourse provide important information regarding the presence and development of dialogic exchanges in a classroom. Similarly, Liotta (2002) conducted a research study on assessing students through communication in problem solving activities in a sixth grade classroom. The students' scores showed significant improvement from the pre-test to the post-test in communication along with an improvement in attitudes towards mathematics.

Studies also describe specific methodological strategies and technological tools useful to capturing and representing important aspects of naturally occurring discourse. Establishing and maintaining an environment that is conducive to learning is a priority. Stiff states (1993), "The importance that teachers place on student discourse is connected to the quality of student-teacher interpersonal communications" (p. 4). Instead of managing to keep students quiet and attentive to the teacher, a classroom might be managed to enable students to talk with one another and utilize collaborative learning strategies, (Lorsbach, 2006). Legitimate student participation in mathematical discussions requires that the student first learn how to use the language of classroom discourse (Zevenbergen, 2000). The difficulty of assessing complex processes necessary for problem solving is exacerbated by the failure of students to communicate clearly what they have done or what they are thinking.

According to Palincsar (2002), the conversations that take place during classroom discussions provide the means to enhance higher-order-thinking. These research

outcomes call for teachers to design activities that facilitate students' development of knowledge by involving the students in conversations that stretch the boundaries of their knowledge. Communication is an essential part of mathematics and mathematics education through sharing ideas and clarifying understanding. Out of communication come ideas, which become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. NCTM, (2000) states, "When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing" (p. 60). Communicating is a broad goal in mathematics learning that provides the means to develop and share understanding.

Teachers should analyze their processes and get students to communicate their thinking. (Szetela, & Nicol, 1992). Discussion, among students and with a teacher, provides for opportunities for social interaction and for shared understandings to be developed (Bicknell 1999). Rubentein (2007) suggests communication improves when mathematical vocabulary develops through cooperative learning, using journal writing and implementing open-ended assessments that require explanations and justifications. Ball (2005) states that mathematics is communicated by means of a powerful language whose vocabulary must be learned. The ability to reason about and justify mathematical statements is fundamental, as is the ability to use terms and notations with appropriate degrees of precision. The National Board for Professional Teaching Standards' Ways of Thinking Mathematically standard for adolescence and young adulthood (National Board for Professional Teaching Standards, 2004) asserts, "Accomplished mathematics teachers

develop students' abilities....to justify and communicate their conclusions, and to question and extend those conclusions.”

Communication through Group Work

Researchers have studied peer-based learning approaches and established that working collaboratively with others can increase academic performance (Slavin, 1990). Student collaboration in a dialogic problem-solving process, aids in the development of critical thinking through discussion, clarification of ideas, and evaluation of others' ideas (Gokhale, 1995). Maher and Martino (1996) state, “When students are allowed to work collaboratively in groups and offer “proof and justification” for their answers, disparate and distinct structures of knowledge interact and eventually become integrated” (p. 197). Glasser (1993) wrote, “When students help each other, whether they do it informally as members of a small working group or as teaching assistants, they learn far more than if they just do their own work and do not teach or help others” (p. 99).

Instead of managing to keep students quiet and attentive to the teacher, a classroom might be managed to enable students to talk with one another and utilize collaborative learning strategies, (Lorsbach, 2006). Allowing students to work in small groups often is a very effective approach to ensuring that the discourse contributes to the mathematics learning of the group members. These groups must provide opportunities for the students to explain and justify their thought processes by using quality, questioning techniques as research shows. During group work, the students are responsible for analyzing the problem, proposing possible solutions, deciding what additional information to obtain and revising the group's solution.

Student collaboration provides an opportunity to observe how students learn. They need to talk, read, write, and explain to others in order to reach understandings. Cooperative groups provide students a chance to exchange ideas, to ask questions freely, to explain to one another, to clarify ideas in meaningful ways and to express feelings about their learning. Powell (1998) reminds educators that student interaction makes cooperative learning powerful. To accomplish their group's task, students must exchange ideas, make plans and propose solutions. Thinking through an idea and presenting it in a way that can be understood by others is intellectual work and will promote intellectual growth. The exchange of alternative ideas and viewpoints enhances that growth and stimulates broader thinking. It is the teacher's job to encourage such exchanges and structure the students' work so their communication is on-task and productive.

There is no one "right way" to develop cooperative learning. Teachers must choose models and methods that match their particular teaching styles, students, and lesson content on the classroom. NCTM (2000) recommends that students be provided opportunities to work together cooperatively in large and small groups on significant problems-problems that arise out of their experiences and frames of reference. Students should question, discuss, make mistakes, listen to the ideas of others, provide constructive criticism and summarize discoveries.

Problem Solving

NCTM (2000) concludes, "Problem solving...can serve as a vehicle for learning new mathematical ideas and skills....A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need

to understand and use various strategies, mathematical properties, and relationships” (p. 182).

According to Lester (2003), the key to fostering understanding is engaging students in trying to make sense of problematic tasks in which the mathematics to be learned is embedded. An essential ingredient of teaching mathematics through problem solving is “listening” to students as they do mathematics (Yackel 1996). In addition, Lester (2003) states, “Teaching mathematics through problem solving involves substantive changes in the nature of classroom activity and discussion, as well as changes in what is expected of both students and teachers, teachers should establish and sustain a risk-free classroom environment in which students’ reasoning, not just answers, is valued” (p. xiv).

To be able to solve problems, one must have deep, conceptual understanding of the mathematics involved. If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential. Mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments, are important because they serve as a basis for developing new insights and promoting further study.

NCTM (2000), “Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking. Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning” (p.52). “In the middle-grades mathematics classroom, young adolescents should regularly engage in thoughtful activity tied to their emerging

capabilities of finding and imposing structure, conjecturing and verifying, thinking hypothetically, comprehending cause and effect, and abstracting and generalizing” (p. 211).

Summary

In conclusion, there is an abundance of research that supports problem solving as a tool to help students as they engage in explaining and justifying mathematical thinking. In addition, studies show that students’ communication also impacts this process.

The researcher intended to promote explanations and justifications through problem solving and communication in a seventh grade pre-algebra course. Chapter three provides the methodology used to develop a classroom community rich in dialogue and the impact of this structure on the written explanation and justification of these students.

CHAPTER 3 METHODOLOGY

The purpose of this study was to investigate how the use of verbal and written explanations and justifications in a seventh grade pre-algebra class impacted student outcomes. The researcher focused on the following questions.

1. How can problem solving enhance students' written explanations and justifications in mathematics?

2. How did students' communication in a problem-solving climate impact their written explanations and justifications in mathematics?

These research questions were analyzed through an action research project used a mixed-methods design. Data were collected from teacher observations, daily journals, audio recordings and pre/post test. Daily journals were scored using a validated rubric to determine growth in written explanations and justifications. Data were triangulated from audio recordings, observations and pre-post tests to provide a summary of the impact of the action-research study on students' written explanations and justifications in mathematics.

School Setting

This middle school is located on the east side of a large urban school district in Florida. The school belongs to the 11th largest public school district in the nation and is within the 3rd largest public school district in the state of Florida. The school is located in a middle/low socioeconomic area with a 31% mobility rate. Free and reduced lunch is offered to 71.6 % of the students. Of the 1150 students at the school, 25.4 % are speakers

of other languages (ESOL), 4.7% are enrolled in the gifted program, and 32.5% have disabilities (ESE). The school has a principal, two assistant principals, and two deans. Each grade level has a grade-level counselor. The school has one police resource officer and a SAFE coordinator (a staff member the students can confide in for personal problems, home troubles, or money issues). The students have a variety of electives to choose from to enhance their academic performance. The students with low reading scores, as denoted by the yearly state achievement test, take a reading class as one of their electives.

This year, the staff in this school moved into a brand new, three-story building, which has the latest technology as well as all new classroom structures. This technology allowed students the best learning environment the school district had to offer. All of the students in the ESOL program are taught either by an ESOL teacher or are on monitor, which allows the students to be in a regular classroom. ESE classes are taught through the inclusion, co-taught model where two teachers teach in the same class at the same time. One of the teachers is a regular content teacher and the other teacher specializes in special education. In the co-taught classes there are both regular education students and students who meet the qualifications to be in ESE classes.

Classroom Setting

The action research study was conducted with a diverse group of seventh grade, pre-algebra mathematics students who belong to the AVID program. AVID: Advancement Via Individual Determination (AVID) is a program designed to help underachieving middle and high school students prepare for and succeed in colleges and universities. Students in the program commit themselves to improvement and preparation

for college. These students were selected to be in the AVID program by the current AVID teachers and administrations after their applications were reviewed. The AVID program is a rigorous program designed to give average students the opportunity to see that college is a possibility. Most of these students come from families where their parents did not attend college.

Though the students belonged to the AVID program, they were placed heterogeneously in the class. There were 4 boys and 14 girls. This diverse group of students consisted of 5 Caucasian, 9 Hispanic, 2 African American, and 2 students from other ethnic backgrounds, which reflects the traditional makeup of the school population. Three of the participants were above average and the remaining 15 participants consisted of average and below average ability (as denoted by scoring below the average mean on the state assessment). More than 50% of the class fell into the below average range. The ages of the students were twelve and thirteen year olds. None of the students in the group had been diagnosed as gifted or having a learning disability. Only three of the students fell into the ESOL category, but were serviced on a monitor status and received no additional help within their classes other than reading.

Procedures

After receiving Institutional Review Board (IRB) approval (see Appendix A) and principal approval (see Appendix B), the researcher sent home both the parent consent form, in English and Spanish (see Appendix C and D) and the student assent form (see Appendix E). Each student involved in the study returned both the parent consent form and the student assent form. During the open house, the teacher shared with the parents what the action research would entail and gained a working partnership and approval

before sending home the forms. While the school has a 31% mobility rate, none of the participants moved during this study. Only one student moved into the class during the middle of the first marking period.

Instruments

The following instruments were used to collect data related to each of the research questions. An overview of each instrument is provided along with how it was used and the reliability and validity of the data collection tool.

Pre-Post Test: A 5-item test (see Appendix F and G) was used to measure student performance on solving problems before the action research study began and upon the conclusion of the nine-week period. The researcher developed the test in the following way. First, two of the problems were identified and matched to item formats in popular mathematics textbooks, which aligned with the state benchmarks the students used in class for the seventh grade curriculum. One problem was selected that required students to calculate and predict speed based on times and distances using prior mathematical knowledge and connecting concepts. One problem was selected to reach higher-level thinking of a concept not yet taught. Both of these concepts are a part of the pre-algebra benchmarks. The last question selected was a problem-solving question involving thinking rather than computational type skills. The items were validated based upon the textbook company and that they mirrored items from the state assessment. The teacher used expert opinion to select items she felt would reflect the purpose of the action research study. The reliability of data collection and scoring procedures was calculated with a score of .80 being acceptable. An independent rater was asked to rate 25% of the

items to ensure accuracy of data entry. Based on the work and explanations given by the students four points per question were given using the rubric in Appendix H.

Rubric: When students wrote solutions to problems, they were assessed with a rubric (Appendix H) that focused on three areas: (1) mathematical knowledge, strategic knowledge, and (3) explanation (Illinois School Board of Education, 2005, p. 111). The rubric's highest score was Score Level 4: (1) shows complete understanding of the problem's mathematical concepts and principals, (2) uses appropriate mathematical terminology and notation including labeling answer if appropriate, (3) executes algorithms and computations completely, (4) identifies all important elements of the problem and shows complete understanding of the relationships among elements, (6) shows complete evidence of an appropriate strategy that would correctly solve the problem (7) gives a complete written explanation of the solution process; clearly explains what was done had why it was done, and (8) may include a diagram with a complete explanation of all its elements. The lowest ratings appeared on Score Level 0: (1) no answer, strategy or explanation attempted. Students understood that they had to explain their problem-solving approach and justify why that approach made sense. This rubric was used for all writings. Permission (see Appendix I) to use the rubric was granted by the School Board of Illinois who reported construct validity of the instrument to be used to assess the pre-post test and all journal entries. Another teacher of mathematics conducted reliability checks using the rubric finding that seventeen of the eighteen pre-tests and post-tests had the same score. The grading difference of one test had a one-point difference.

Audio taping: The researcher used audio taping on three occasions to monitor teacher-questioning techniques, student communication, and to gain field notes of problem solving through the explanation and justification process. The teacher would ask the students to find a solution to a problem by asking “what is the problem asking us to find”, “what prior knowledge do we know that will aid us in solving this problem”, “who can explain how to find a solution to this problem”, “what can be added to the explanation to justify the solution”? Students were heard talking to one another and disagreeing with each other on how they were to work the problem.

Daily Procedures

Students involved in the research study completed a pre-test (see Appendix F) and post-test (see Appendix G) to determine their ability to solve problems through the explanation and justification process in written form and if any learning gains had been achieved related to the problem-solving process. Pre and Post-test measures of content knowledge were collected and given a score by comparing them to a pre-determined rubric (see Appendix H). The class observations and data collection lasted for nine weeks. The researcher decided to collect data on all students and triangulated the data from all instruments for 6 students: two low achieving, two high achieving and two average students as indicated by past performance on the state assessment to be discussed further in chapter 4.

The observation instruments used were teacher observation, a pre-test and post-test, and journal entries. The students were placed at tables, instead of desks, which is more conducive for working together. Students worked in small groups of three or four to collaborate with one another during the problem solving process. However, due to

absences, groups were often made up of two or three students, which may have altered the learning experiences as students sometimes, had to work with fewer members than other groups.

The data collected from the pre and post-tests were compared and analyzed using the rubric in Appendix H to determine the amount of growth occurred in written explanations and justifications over the nine-week period. To triangulate any changes observed on the pre and post data from students' daily journals were analyzed, to also validate changes in their written explanations and justifications. Students were given 15 minutes daily to write in their journals and if they were not finished, written work was completed the written work was completed as homework. Any student absent the day the teacher collected the journal writing was responsible for completing the problem at home and returning for credit as with any other missed work due to absences. If the work was not turned in, a zero was placed in the grade book for the assignment. As always, students were allowed to turn in the missing assignment for partial credit.

To validate the communication occurring in the classroom and to further enrich the triangulation of data collected from the writing process, classes were audio taped every three weeks. To assure the privacy and anonymity of the students no one listened to these tapes except the researcher. The tapes were used to see if the researcher was asking the number and level of probing questions needed to facilitate problem-solving and oral communication within the classroom and to observe specific information on the six target students that will be discussed in chapter 4. The researcher gathered anecdotal notes on the changes in all students with specific information documented on the six target students.

During the first marking period prior to the start of the study, students were given word problems to work on as a class each day as they entered the classroom. The students were encouraged to talk through the problems and to draw pictures of the problems to better see what the problem was asking. Then, the students were to talk about the problem with the other members at the table to see if together, an answer or solution to the problem could be found. This process was followed so the researcher could begin to teach the students how to work with each other as the students engaged in a discourse manner. Many of the students had come from previous classrooms as noted by the students where the students sat in straight rows and stated they were only allowed to talk during a project or playing a game of some sort. The researcher needed to model how discourse takes place. This process allowed the students to think through the problem and question each other. The researcher also wanted the students to get comfortable working together and feeling like the classroom was a non-risk environment.

To stay consistent with the research study, a typical day in the action-research study consisted of the teacher following a daily routine as outlined in her lesson plans. The students entered the class, got out their journals, did the problem solving entry, discussed the solution, went over homework and then worked on the new mathematics instruction for the day using different instructional tools as needed for the new lesson. Each problem solving entry was done in groups so the students could communicate with one another until the solution was found.

The researcher then began asking the students to orally explain the process of solving the problems and justifying why their solutions worked mathematically. This technique was used to teach the process of discourse. However, the researcher was unable

to ascertain the true depth of mathematical understanding from each student. The classroom discourse only allowed several students the opportunity to respond to each question until a solution was found. The researcher needed a better way to collect data on each student to ensure understanding of mathematical concepts took place. Thus began the journal writing where each student was responsible for explaining and justifying their mathematical solutions to all problems.

The journals were for the students to write their explanations and justifications to the problems that were solved. As the researcher walked around the room, students would read the explanations to the class. Since there is not just one-way to solve a problem, the teacher asked for volunteers to read the written explanations if the solution or explanation was different. From the researchers expertise this was a positive outcome. As the students began explaining the solutions better, the researcher began the justification process by asking the students “why” the work was done the way it was explained. The class discussed what was said and pulled it together through oral communication and problem solving. The verbal information given to the teacher was demonstrated on the board for the visual learner to grasp. Depending on the skill, a manipulative was occasionally used to enhance the discourse process. The problem-solving process took 25 minutes of the class time each day. The remainder of class was spent going over homework and beginning instruction on another skill.

Twice a week, on Tuesdays and Thursdays, the researcher collected the mathematics writing journals consisting of word problems to read and grade (see Appendix J). The grades assigned to each writing entry used a rubric (see Appendix H) chosen to objectively assess each student’s ability to explain and justify the work

alongside monitoring the growth of mathematical and strategic knowledge. The mathematics studied during the nine-week marking period was a mixture of the seventh grade and pre-algebra benchmarks as required by the school district and state requirements as shown in Table 1. The weeks are listed as weeks of instructional units and are not consecutive Monday-Friday weeks due to holidays, county level benchmark testing and other school events.

The entire nine-week marking period is listed in the table to show what skills were taught each week. Each of these instructional units was taught during the second part of each instructional day, after the problem solving and journal writing took place. A mixture of traditional and contemporary instruction, where students actively constructed knowledge in contexts that they found meaningful and motivating, was incorporated into the second part of the class time. During instruction, the teacher used a variety of instructional tools and materials including the doc-cam, notes, discourse and manipulatives as needed to move the students from a concrete level into an abstract level.

Table 1: Mathematical Content Covered During Study

Week 1: Rational Numbers. Students reviewed adding, subtracting, multiplying and dividing fractions in real-world problems using both like and unlike denominators. Also, students wrote fractions as terminating or repeating decimals.

Week 2: Ratio, Proportion and Percent. Students determined unit rates and solved proportions in real-world problems.

Week 3: Ratio, Proportion and Percent. Students used percent proportion to solve problems. Also, students learned to find percent increase and decrease in real-world application.

Week 4: Positive and Negative Exponents Using Scientific Notation. Students solved expressions using positive and negative exponents listing the answer as a fraction and a decimal. Students expressed numbers in both standard and in scientific notation

Week 5: Pythagorean Theorem. Students used the Pythagorean Theorem to find the length of a side or the hypotenuse of a right triangle

Week 6: Area Formulas. Students used the formulas to find the area of a square, rectangle, triangle and trapezoid in real-world problems.

Week 7: Area and Circumference. Students used the formulas for finding the area and circumference of a circle in real-world problems.

Week 8: Volume. Students used the formulas for finding the volume of cylinders and prisms in real world-problems.

Week 9: Surface Area. Students used the formulas for finding total surface area in cylinders and prisms in real-world problems.

The teacher gave the students problems to solve, which primarily centered on the week of study as listed in Table 1. In addition to the problems the teacher graded the problem solving journals (see Appendix J) other problems were given to review prior benchmarks for the students to stay abreast of the previous taught skills through a spiraling effect. In other words, the students did not solve problems that only practiced the topic studied for the week. The teacher used problems for students to pull from prior mathematics knowledge as well as skills not yet learned. The problems given to the students incorporated the skills and concepts taught for the state assessment test while incorporating the explanation and justification process for problem solving.

In addition to the class from which the data were being collected, consistency in lessons took place so all students would have the same educational opportunity in each class. The students entered the classroom, got out the needed tools, listened to the teacher explain or read the problem and then began the problem-solving process. When solutions were found, the students had to explain and justify the answers both in written and verbal

form even though their journal entries were not being scored for this data collection. Their grades were only to monitor student academic performance and to place grades in the grade book. Following this activity, homework was reviewed and graded, followed by a new skill if time permitted. The teacher would either teach the skill in a traditional setting or through a form of discovery using manipulatives as shown in Table 2.

Table 2 outlines the marking period, grouped into three weeks of study at a time. These weeks were set-aside for the teacher to reevaluate what was happening in the classroom so audio taping could take place and for assessment purposes.

Table 2: Weekly Lesson Plans

Opening Week: The teacher discussed discourse and the expectations of how the groups would work together in the problem-solving process. The students were rearranged to ensure groups of three or four students were seated at each table for equitable collaboration to take place. Students took a pre-test.

Week One-Week Three: Class began each day with a problem to be solved. The teacher modeled explanations and used discourse to engage students in verbal communication. Students were encouraged to draw pictures of the problem and use the reference sheet if applicable. The teacher facilitated the classroom asking probing questions at each table. Whole class discussions took place after solutions were complete. Students were reminded to think through the problems and to listen to the other students in the group. Journals were collected on both Tuesday and Thursday for the teacher to grade. The teacher audiotaped one class discussion. A quiz was given once a week covering the weekly benchmarks.

Week Four – Week Six: Class began each day with a problem to be solved. The teacher read the problems to the students. The teacher facilitated the classroom asking probing questions. Whole class discussions took place after solutions were complete. Students shared their solutions and the justification process was review and discussed. Students were instructed to listen for the justification and decide whether it was there or not. Journals were collected on both Tuesday and Thursday for the teacher to grade. The teacher audiotaped one class discussion. A quiz was given once a week covering the weekly benchmarks

Week Seven–Week Nine: Justifications Emphasized at this interval due to the lack of the “why Class began each day with a problem to be solved. The teacher read the problems to the students. The teacher facilitated the classroom asking probing questions. Whole class discussions took place after solutions were complete. Students shared their

solutions and the justification process was review and discussed. Students were instructed to listen for the justification and decided whether it was there or not. Journals were collected on both Tuesday and Thursday for the teacher to grade. The teacher audiotaped one class discussion. A quiz was given once a week covering the weekly benchmark.

Closing Week: Teacher discussed what had been observed over the last nine weeks. Students took a post-test.

As shown in the table, each class period was delivered in a consistent format to keep all classes learning the same curriculum. The changes were related to the topic and what the teacher emphasized for the explanation and justification process. The first few weeks, students began the communication process and writing down solutions. Then, the teacher emphasized the explanation process by modeling examples of “what” was done to find the solution and had students discuss orally what they had done to find their solutions. When the students accomplished this part of the process, the teacher moved the emphasis on the justification process or the “why” it was done that way mathematically. These intervals allowed the students to pull it altogether to make the connections to solve the problems. In addition, this process gave the teacher the opportunity to address common mistakes students had made.

Data Collection and Analysis

Throughout the nine-week marking period, students were reminded to write in their journals and to provide explanations and justifications for their work verbally and in writing. On occasion, the researcher would notice students having conversations unrelated to mathematics and have to pull the students back to the problem at hand. After implementing lots of consistent instructions on what the teacher’s expectations were, the

groups began to work much better together through the collaboration and problem-solving process.

As time progressed, the researcher began to ask the class to listen for both the explanation and justification that each group presented. The students listened to the explanation and would discuss if the group had explained thoroughly what was done to solve the problem and ways to make it clearer. Then, the students would listen to the justification and see if it both existed and was complete. In many of the written solutions, the justification was missing. The teacher led students into discussions regarding the problems they had solved. When needed, the teacher allowed students to go to the board and show step-by-step how they had found their solution while they explained and justified their reasoning.

Summary

In this action research study the teacher examined the impact of student communication related to explanations and justifications on students written outcomes in mathematics. Data were triangulated for six students using a pre-post test, journal writing, audio recordings and teacher field notes. The results for these six students as well as an overall discussion of the entire class are provided in chapter 4 as they relate to the explanation and justification process of mathematical problem solving.

CHAPTER 4 RESULTS

The researcher investigated how the use of verbal and written explanations and justifications in her seventh grade pre-algebra class impacted student written outcomes in mathematics. The instruments used to address the action research questions were teacher observation, pre-test and post-test, journal writing and audio recordings. The researcher used each of these tools to reflect upon the following research questions throughout the study.

1. How can problem solving enhance students' written explanations and justifications in mathematics?
2. How did students' communication in a problem-solving climate impact their written explanations and justifications in mathematics?

The first section of this chapter provides an overview of the instruments used for data collection. The second section discusses how these instruments related to each research question. The third section discusses the reliability and validity of the testing instruments with the fourth section providing a summary of the chapter.

Research Instruments

Teacher Observation

During weeks one to nine, the researcher provided time for students to talk on a daily basis. As noted in chapter 3, a consistent structure was used daily to increase participation and improve the climate of the classroom. An overall theme from the

observation field notes was that the classroom appeared to provide a climate that supported and encouraged focused and productive dialogue amongst the students. As the students felt comfortable in the classroom, students were more engaged in the problem solving process and shared their solutions in a non-risk environment. Communication developed in both verbal and written form. Students actively engaged in conversations and helped each other. The students began to explain and justify their solutions as denoted in the field notes, audiotapes, the post-test and journal writing.

Pre-Test and Post-Test

A pre-test was administered at the beginning of the action research study consisting of five problems as described in chapter 3 (See Appendix F). The test contained two problems the teacher felt the students could answer using prior mathematical knowledge, one problem that showed conceptual knowledge as the marking period went by and one question the teacher thought the students could think through well enough to put down some information using a chart or at least provide a guess. The last question was a think type question used to assess all students, regardless of prior mathematical knowledge. The rationales for the selection of problems in this instrument are provided in chapter 3.

A Mathematics Scoring Rubric: A Guide to Scoring Extended-Response Items (See Appendix H) was used to score each of the five questions on the test as explained in chapter 3. The researcher found many of the questions on the pre-test were left blank by the students, had no explanation or showed a lacking of mathematical knowledge making the pre-test scores very low.

The eighteen students' total class score for the pre-test was 51 points using the rubric. Since each student had the possibility of making 20 points per test, the 51 points was a low number out of the total 360 available points. After the nine week marking period was over, the teacher administered the post-test consisting of the same five problems. The total score derived from using the rubric to grade all eighteen students for the post-test was 114 points. These points reflected only seventeen out of the eighteen students, as one student did not take the post-test due to a series of absences.

Table 3 provides an individual summary of each students' increase in both rubric scores related to all writings as well as semester grades. See for Table 3 for student's scores. The first column identifies each student using a letter of the alphabet. The second and third columns show the score on the pre and post-test followed by the positive or negative change. The fifth and sixth columns show the scores obtained on the first and last journal entries for the marking period followed by the seventh column showing the positive or negative change. The eighth and ninth columns show the grade increase or decrease from the first making period through the end on the second marking period. As shown in the rows per student, sixteen out of eighteen students made improvements on the post-test.

Table 3: Student Scores

Student	Pre-Test	Post-Test	Change	First Journal	Last Journal	Change	First Report Card	Second Report Card	Change
1-A	3	9	6	2	3	1	80	82	2%
2-B	2	11	9	2	4	2	93	95	2%
3-C	1	4	3	1	2	1	62	73	11%
4-D	2	11	9	2	3	1	78	90	12%
5-E	1	3	2	1	2	1	65	71	16%
6-F	6	9	3	2	3	1	88	93	5%
7-G	3	4	1	2	3	1	60	71	11%
8-H	4	5	1	2	4	2	77	83	6%
9-I	2	5	3	1	2	1	60	72	12%
10-J	2	11	9	2	3	1	77	93	16%
11-K	6	8	2	3	4	1	88	94	6%
12-L	1	3	2	0	1	1	50	64	14%
13-M	2	5	3	2	3	1	72	74	2%
14-N	4	2	(- 2)	2	3	1	59	70	11%
15-O	3	4	1	2	3	1	70	80	10%
16-P	4	10	6	2	4	2	61	75	14%
17-Q	2	Did not Take	NA	1	3	2	85	67	(-18%)
18-R	3	10	7	2	4	2	87	75	(-12%)
Average or Change	2.8	6.7	3.9	1.7	2.9	1.2			

Journal Writings

The use of journal writings for this action research was not used as a reflection of the daily lessons, but a notebook to collect the daily word problems as part of the class work (See Appendix J). The journal provided the teacher with the data necessary to compare growth in the student's ability to provide a written explanation of the solution process with a justification for the solution. As the nine-week period progressed, the researcher was able to see a growth in the explanation process. Information recorded in the journals was available for students to read and the teacher discussed it with the students individually as part of the instructional process of the class.

The fifth and sixth columns of Table 3 showed the scores obtained on the first and last journal entries using the rubric for the marking period followed by the seventh column showing the positive or negative change. All students showed a growth in the journal writing based upon their rubric scores.

Questioning through Audio Tapings

The fourth instrument used in the data collection process was audio tapings of the questioning techniques used by the teacher. This instrument was used once every three weeks on three different occasions. From these tapes the researcher reviewed the tapes to determine if the six target students were increasing in their participation and if the researcher was providing a climate for verbal dialogue in the class discussions. Although no formal tool was used to analyze the data from reflection of the researcher, individual students level of participation did increase and the climate based on master level teacher reflection appeared to allow for rich mathematical dialogue.

Written Explanations and Justifications in Mathematics

The outcomes related to problem solving were derived from triangulation of students' pre/post test scores, daily journal entries and students' verbal participation from observations and audiotapes. From the teacher observation field notes the gain in student learning was evident and reflected in the students' discussion about solving mathematical problems. Table 3 shows where the students began the marking period and where they ended up at the end of the research project showing that all students but two increased in their rubric scores as well as their overall letter grades. The researcher, based on teaching expertise, felt the climate of the classroom was established to allow for discourse and problem solving through the teacher's questioning, facilitation and engaging of students in verbal and written communication. During the communication process the students appeared to be able to use reasoning skills to facilitate their learning and conceptual growth as shown by an increase in their post-test results and journal entries.

Three students in particular stood out to model the level of growth the researcher hoped to observe in this project as shown in the problem solving area. These students, as classified on the state assessment, ranged from the lowest level, considered "low performing students" to the above average level. These examples show growth for these three students in a problem-solving environment of discourse. Each of the examples provided show the students applying previously learned facts and computational skills along side procedural steps. These students attempted to explain each final solution, but there is no evidence of the justification process. While the pre-test and post-test data may not show individual growth for each of these students, the researcher provides the

following examples and reflects on the growth of each student based upon the research data.

In Figure 1, Student C, an ESOL student on monitor, showed growth in understanding the problem's mathematical concepts and the ability to write an explanation over the nine-week period of study. The researcher selected this work as a good example of problem solving strategies being used after the student provided procedures, a formula and a picture in the second problem showing growth in problem solving. This student also showed changes in verbal communication as shown from the audiotape recordings through frequent participation.

In the first problem, the student was trying to solve a problem regarding the price of going horseback riding, The student saw three numbers and multiplied them together just using an algorithm. The student offered a minimal explanation, but failed to justify why the strategy was used. The second problem asked the student to find the height of a television without telling the student to use the Pythagorean Theorem. The problem gave the hypotenuse as the diagonal of the television and the student had to use the converse of the Pythagorean Theorem to answer the problem correctly. The evidence showed a complete understanding of the problem's mathematical concepts and principles. The student executed algorithms and computations completely and correctly.

In addition, the student showed evidence of using a strategy as shown in the picture drawn by the student. The student gave a written explanation of the solution process by explaining the steps used in problem solving. The student attempted to insert some terminology as shown with the words equation, square rooted and substituted. The researcher would have liked to see the student justify why the Pythagorean converse was

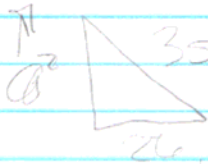
used, but felt the amount of growth in conceptual knowledge was shown. The researcher attributed this growth from one small algorithm to a picture and written explanation as a result of triangulation of the dialogue this student engaged in during class (as observed by the teacher and in the audiotapes) the weekly journal entries that supported the students writing their responses and that this student's grade changed from a 62% to a 73% for the marking period. In addition, from the pre-test to the post-test the student earned three additional points on the rubric used for scoring. As the researcher scored the journal entries, the explanation and justification grew as shown in the chart by an increase in one point overall showing at least some growth over this nine-week study.

$215.2.4$
 $212.15.4$
 answer = 23.4

I did this problem
 by using the
 formula $a^2 + b^2 = c^2$
 which is $26^2 + a^2 = 35^2$
 $5^2 + a^2 = 26^2$. An
 got my answer

answer = 23.4 inches

23.4



First I substituted
 the value.
 Then I did the
 arithmetic.
 After that I
 solved the
 equation by
 subtracting. Finally
 I squared root of
 my # and
 got my answer
 which was
 23.4 inches

$a^2 + b^2 = c^2$
 $a^2 + 26^2 = 35^2$
 $a^2 + 676 = 1225$
 -676
 $a^2 = 549$
 $a = 23.4$

Figure 1: Student Problem Solving

In Figure 2, Student D, an average student, as identified by the state assessment test, showed growth in conceptual knowledge by making connections to both the real world and prior mathematical knowledge. The student used formulas, procedures and explanations in her problem solving techniques.

The student was to compare the volume of a can and a box on the pre-test. The first attempt, on the pre-test, showed no work from the student. The student wrote im not

sure of how to do this. However, after nine-weeks of problem solving activities, when the post-test was given, the student took time to write down the formulas to solve the problem, then showed the substitution process and accurately answered the problem. This written product showed evidence of using an appropriate strategy to solve the problem and connections were made. In addition to correctly executing the steps and computation, the student was able to explain what was done. The student failed to justify the written response, but was able to use correct terminology in naming the can as a cylinder. From triangulation of the data related to the increase in this student's dialogue in class (as observed by the teacher and in the audiotapes), the increased scores on her rubric related to her weekly journal entries and that she earned a 78% the first marking period and had a grade of 90% at the end of the study. This student's ability to problem-solve and justify her answers in writing did increase.

1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

im not sure of how to do this
no picture
no attempt

$$V = \pi r^2 h$$

$$V = 3.14 \cdot 8^2 \cdot 24$$

$$V = 4823.04$$

$$V = lwh$$

$$V = 16 \cdot 12 \cdot 24$$

$$V = 4608$$

1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

First I found the volume of the can by using the formula $V = \pi r^2 h$. When I found the volume which is 4823.04 I moved on to finding the volume of the box by using $\text{Volume} = lwh$. when I used the formula I got 4608. Then I compared the two answers and found out that the cylinder had a greater volume.

Figure 2: Student Problem Solving

In Figure 3, Student K, an above average student, as identified by the state assessment test, wrote only the word First followed by Jars = 25 cents on the pre-test. The student offered no written explanation nor showed any apparent strategy being used to obtain the cost of the jar of paint. Her response did not contain any algorithmic or computational notations. The solution left the teacher to believe that the answer was a good guess for one of the prices to be found or that the student didn't feel comfortable writing and explaining her solution.

When the student took the post-test, the student offered a written explanation that clearly demonstrated her use of the problem solving process that she went through to find a solution. The student identified most of the important elements of the problem and

showed a general understanding. In addition to the written explanation, the student showed understanding of the problem's mathematical concepts and principles. However, there was a minor computational error in the final step in finding the cost of the jar. The student showed 2 jars at 50 cents and forgot to list one jar at 25 cents. This error allowed the student to use an incorrect substitution value, which then obtained an incorrect final answer. Had the student substituted the correct value, the cost of the brush would have been found to be \$2.55, which was correct.

The researcher believed the work showed an abundance of conceptual knowledge and growth, as the class had not learned about systems of equations. The researcher, from triangulation of data related to the student's growth in dialogue (as observed by the teacher and in the audiotapes), weekly journal entries and increase in her overall grade to her growth in thinking and writing was able to show a richer explanation and justification on the post-test.

The student was able to use all of the strategies learned during the marking period through the problem solving process including discourse and reasoning skills. The student's grade improved from 88% to 94% during the course of the action research study and was clearly identified in review of the audiotapes as being able to converse mathematically in the class and worked with other students very well. Even though the student was a great student at the beginning of the year, the student's depth of understanding grew as shown in the figure through the explanation process.

5. A beginner's artist kit costs \$6.35 and contains 2 brushes and 5 jars of paint. The standard kit has 4 brushes and 12 jars of paint and costs \$13.20. Assuming that all the brushes cost the same amount and the jars of paint cost the same amount, how much would you expect to pay for a deluxe kit containing 7 brushes and 20 jars of paint?

First

Jars = 25¢ yes!

can you explain

5. A beginner's artist kit costs \$6.35 and contains 2 brushes and 5 jars of paint. The standard kit has 4 brushes and 12 jars of paint and costs \$13.20. Assuming that all the brushes cost the same amount and the jars of paint cost the same amount, how much would you expect to pay for a deluxe kit containing 7 brushes and 20 jars of paint?

To solve this problem I knew I had to get the price of 1 jar and 1 brush. standard kit subtract on artist kit and get 2 Brushes + 7 Jars = \$6.85. Then I take away another beginners kit and get 1 jar equals 25¢. Then I do 2 brushes plus 5 jars. Then I fill in the jars and do 2 brushes plus 2.50 equals 6.35, subtract 2.50 from both sides and get 2 brushes equals 3.85.

divide by 2 and get

$$\begin{array}{r}
 4B \quad 12J \quad 2B + 5J = 6.35 \quad 13.20 \\
 - 2B \quad 6J \quad 4B + 12J = 13.20 \quad 6.35 \\
 \hline
 2B \quad 6J = 6.85 \\
 - 2B \quad 5J = 6.35 \\
 \hline
 \quad \quad J = 0.50 \\
 \quad \quad 2 = 1.00
 \end{array}$$

1 brush equals \$1.92.

$$\begin{array}{r}
 2B + 5J = 6.35 \\
 2B + 2.50 = 6.35 \\
 \hline
 \quad \quad 3.85 \\
 \quad \quad 2B = 3.85 \\
 \quad \quad \quad 1.92
 \end{array}$$

Figure 3: Student Problem Solving

Related to the first research question, these three students all showed an increase in their use of terminology and classroom dialogue as well as academic performance. The lowest student began to draw visual images to explain her work, while the average

student began to use more appropriate terminology and the high achieving student provided richer explanations and justifications in her work. Although each student had unique changes overall, the method of problem solving provided in this classroom did appear to have a positive impact on these students written explanations and justifications.

Communication in a Problem-Solving Climate

The findings related to communication in this action-research project emerged from triangulation of audiotapes, journal writings and observations. A communication-rich classroom is one in which students are encouraged to share their ideas and to seek clarification until they understand.

The teacher spent most of first marking period getting the students comfortable working together and communicating about mathematics within the classroom. The establishment of socio-mathematical norms took much longer than the researcher thought it would. Creating a classroom where students felt comfortable in a non-risk environment was of utmost importance to build a foundation for this action research.

To create the climate of the classroom to meet this objective, the teacher used hands-on activities, oral discussions, and informal assessments throughout the first marking period to actively engage students in classroom activities. While students were working, the teacher circulated around the room asking questions and getting to know the students on a more personal level. One of the informal assessments used during instruction allowed students the opportunity to work on small hand-held whiteboards the teacher had made for the class. Students would answer problems and hold the answer up in the air. This traditional type of assessment only captured the right or wrong answer, but gave students the opportunity to answer a problem quietly, holding up the answer

where no one but the teacher could assess, and know by the shake of the teacher's head that the answer was correct or not. The action took place too quickly for students to know who was right and who was wrong. The teacher believed this activity aided in the climate of the classroom comfort zone. The development of such environment was documented in the teacher observation notes where the class climate changed and students started taking risks to talk about the mathematical process in both verbal and written form.

Evidence of a communication rich classroom was evident in the structure of the classroom. The class began with a word problem on the board or doc cam each day. The climate of the classroom began each day with students actively engaging in communicating mathematically with one another and taking risks in front of their peers. The researcher observed more discourse taking place within the group settings as the nine-weeks progressed as evident from a review of the number of questions students asked of each other from the audiotapes. Over time, in each audiotape, students were discussing with each other more and explaining to one another how to solve problem independent of the teacher.

Another data point that validated the richness of students communicating was from researcher observations. The researcher observed students raising hands, taking risks by answering more questions and completing more homework than during the first marking period. As the school year progressed, the students conversed with the teacher more, stopped by to say hello between classes, and appeared more comfortable talking to the teacher about issues. In addition, the influence of this dialogue was further validated in that grades improved from the first marking period to the second period.

The evidence of an increase in written dialogue clearly emerged in the final post-test data. Examples of students increasing their communication in a mathematical community are best represented in the work of the following three students. The researcher selected these three students because they represented the mixed ability levels in the classroom (low, middle and high achievers based upon the state assessment). Also, these students in the audio recordings emerged as increasing their communication during group work and classroom discussions, which appears to have impacted their written justifications on the post-test.

In Figure 4, Student I, a low performing student on the state assessment, wrote, I tried but IDK on the pre-test at the beginning of the marking period. There is no evidence of the student taking a risk at attempting to solve the problem with any computations, drawings or mathematical language being used. While the student is low, the teacher would have expected something to be written down. However, the student elected no form of any kind of solution. At the time of the pre-test, the student was quiet and a loner. The student had a hard time communicating with the other students in the group. The student was low achieving the first marking period. As the classroom climate changed, the student began to become more engaged, as noted through teacher observation and audio recordings, in the justification and problem-solving processes in class.

As shown in the second attempt at finding a solution to the problem in Figure 4, the student answered the problem by the drawing (a form of communication) and writing the steps used to solve the problem. The student took time to draw the pictures, which indicated an understanding of the problem. The procedure used to answer the problem was incorrect as the student works with one formula for surface area and a mixture of

formulas for finding volume and surface area on the second figure showing a limited understanding in mathematical concepts. Despite the work being incorrect, the researcher attributed this growth to the dialogue this student engaged in during class (as observed by the teacher and in the audiotapes) the weekly journal entry scores increasing and that the student attempted to answer the problem by drawing pictures, one of the forms of communication learned through classroom discussion and group work. In addition, the student has shown growth over the semester by grade averages going from 60% to 72% providing for triangulation of data to demonstrate that the classroom did increase students' communication over time in this class.

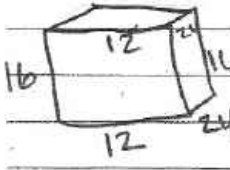
1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

I tried
but I DK (LOL)

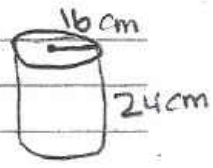
did you
draw a
picture.

1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

A box.



$16 \times 12 = 192 \times 2 = 384$
 $12 \times 24 = 288 \times 2 = 576$
 $24 \times 16 = 384 \times 2 = 768$
1298



Ctd = 24
 $3.14 \times 16 = 24 = 205.7$
1205.9

Figure 4: Written Communication

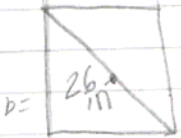
Another student who increased her skills in mathematics communication was Student H. In Figure 5 is an example of the increase in written communication for Student H, an average student on the state assessment test, who on the pre-test was asked

to determine the height of a television set when given the diagonal and the length of the screen. The student recognized the problem was one that used the Pythagorean theorem but failed to see the converse was needed in order to find the solution. The student showed some understanding of the problem's mathematical concepts and principles, identified some of the important elements of the problem as shown by the drawing and strategy used, and gave some written explanation of the solution process by explaining what was done. However, the student labeled the picture incorrectly leading to an inappropriate use of the formula. The student wrote in the explanation that a model was drawn to show the angles. The problem did not require any knowledge of angles. The student did not justify any part of the written explanation. The researcher did not feel the student was making full connections to prior knowledge. Due to the lack of communication that had not taken place in the classroom at the time of the pre-test, the student was unable to apply any reasoning skills to solve the problem or justify the solution. The student used procedures, algorithms and a brief explanation.

This student was observed and identified in the audio records over the nine-week period as actively participating in classroom dialogue and was improving in her daily journal writing. In the second problem, the student was asked to find the area a squirrel could cover. The problem never mentioned that a circle was needed in order to find the solution. In the written explanation the student gave, there was a complete written explanation and justification for the problem's mathematical concepts. The student included some mathematical terminology, drew a picture as a useful communication strategy, and showed the computation needed to find the solution. The student gave a complete written explanation of the rational and steps toward the solution process. The

student was an average student whose reasoning ability grew through classroom discourse, group work and the justification process. The conceptual development of the student moved from average to high end during the marking period. It was as if the light bulb turned on and no batteries were needed to keep the student learning. The confidence level was astounding as observed by the teacher and mentioned by the student in that she shared she was much better in mathematics. The student was engaged at all times, worked well in the group, offered many great explanations, both in verbal and written form, to the class and went to the board to show work. The student moved from a 77% to a 83% from one marking period to the next. The student showed evidence of making connections to the real world and using prior mathematical knowledge. The data all indicated that this student was actively engaged in the written and verbal communication process in this classroom. This engagement impacted her post-test scores, daily journal entries and her overall grades along with her confidence from being in a communication rich classroom.

Solve



$$a^2 + b^2 = c^2$$

$$35^2 + 26^2 = c^2$$

$$1225 + 676 = c^2$$

$$c^2 = \sqrt{1901} = 43.60$$

explain

First, I drew a model to show the angles. Then, I squared 35 and 26. After, I added my answers, 1225 and 676, together. My answer now was 1901 but I squared rooted it to 43.6.

Explain and justify your solution. Use all strategies learned this marking period. A California ground squirrel usually stays within 150 yards of its burrow. Find the area of a California ground squirrel's world.



To solve this answer I drew a picture. I drew a small circle and labeled it "burrow" and drew lines coming out of the "burrow". The squirrel could go out from each part of the circle, 150 yards and form another circle. Therefore, I needed to know the formula for finding the area of a circle and put the radius as 150. Now, I needed to evaluate, 3.14×150^2 . Then, I got 70650. So, the California ground squirrel's world area 70,650 yards.

Figure 5: Written Communication

The final student, Student B, demonstrated his increase in his written communication skills as seen in Figure 6. Student B was an above average student on the state assessment, and during a journal writing entry was asked to find the height of two brothers when a total height was given and the difference in the height was five inches. The student was able to reach the correct answer using the strategy of guess and check but with minimal written explanations. He failed to mention the five-inch difference in the height of the two brothers. The mathematical terminology also was missing and so

was the justification. By using a guess and check method, where a lack of explanation was not fully executed, the researcher felt connections of prior mathematical skill were lacking.

The last journal entry problem asked the student to find the amount of material needed for the lining of a swimming pool. This problem required the student to understand how to find the surface area of a cylinder. However, the pool did not have a lid, so the student had to recognize that the full formula was not needed in order for the solution to be found. The work shown on this problem indicated the student understood the problem's mathematical concepts and principles. The student did not draw a picture of the pool with the dimensions, but was able to compute the surface area correctly without being told to find the surface area. The student used the appropriate formulas and executed the algorithms and computations completely and correctly. The student indicated that the pool had no lid and took out the extra measure for the lid. The student used correct terminology as indicated by the word surface area, cylindrical, cylinder and equation, which were key words of communication throughout the nine-week period.

The researcher noted that the student's daily classroom work and written journal entries showed a growth in conceptual development as well as the ability to explain and justify a solution. This student was an above average student who was very quiet at the beginning of the year. The student had a difficult time learning to speak in the class, but became one of the most affluent speakers in the classroom by the end of the nine-week study. The student was a leader in the group and was able to explain and justify all problems by the end of the marking period. The student was continuously engaged in class, worked well with his group, offered many explanations and justifications for his

work, both verbal and written form, throughout the nine-week period. The student moved from a 93% to a 95% over the grading period. The data all indicated that this student was actively engaged in the written and verbal communication process in this classroom and that it impacted his post-test scores, daily journal entries and his overall grades along with his confidence from being in a communication rich classroom.

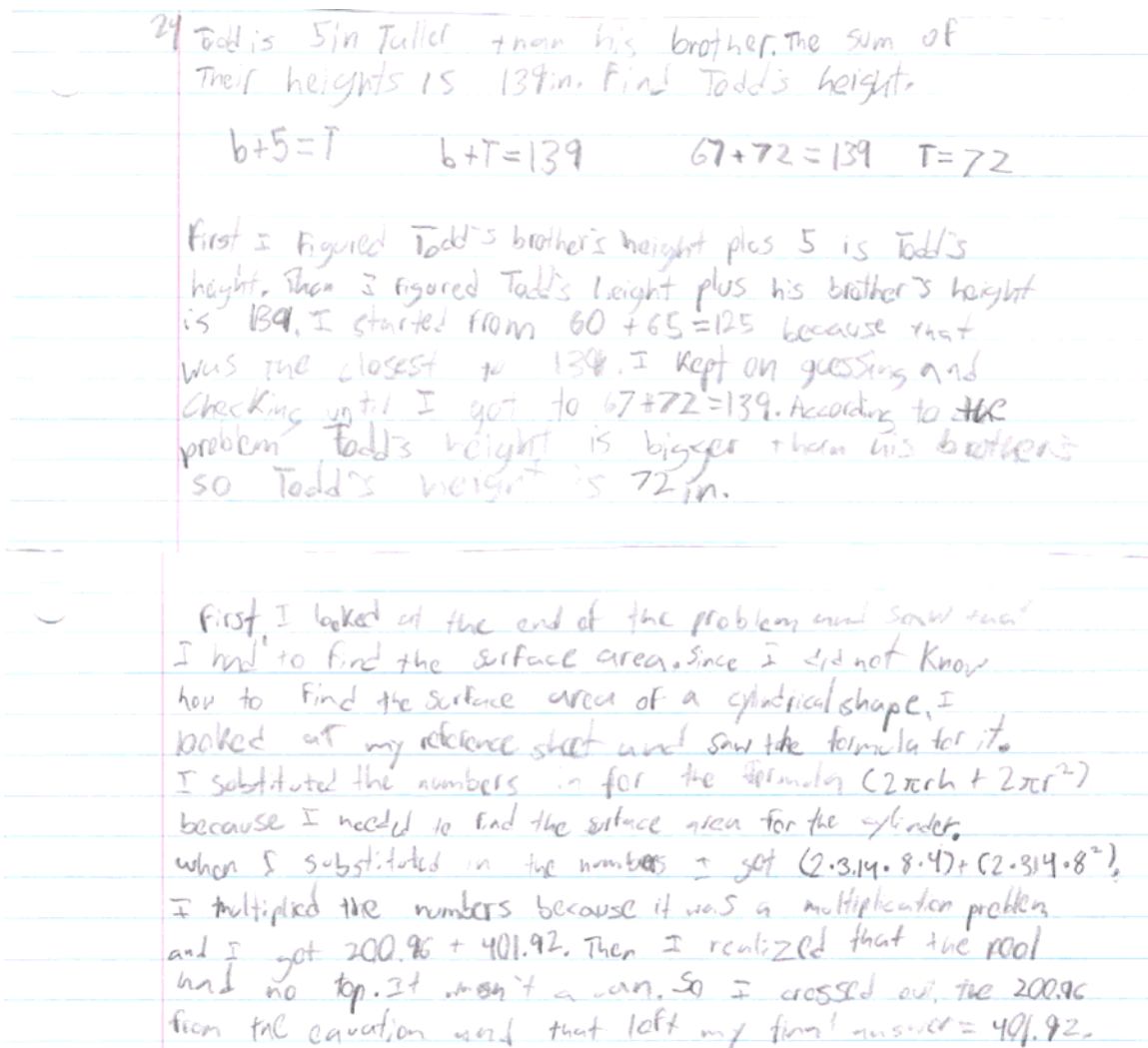


Figure 6: Written Communication

As the researcher compared the growth of the three students, it is evident through triangulation of journal writings, teacher observation and pre-post test scores these

students' skills in communication in mathematics did grow. Overall there was not a specific measure provided of communication but the evidence from students drawing pictures, growth in written explanations and justifications as well as the teacher observing the growth of these students' dialogue as a professional educator and in the audio recordings provides indications of a dialog rich setting. Examples of the type of dialogue that provides further evidence of the communication rich classroom came from the audiotape recordings. The next section provides a snapshot of the classroom and an example of the daily dialogue occurring in the classroom.

Audio-taping

A snap shot of one of the lessons is shown below from one of the audiotaped problems. The problem had the students calculate the amount of bags of seed needed to fertilize the lawn around a building. The building was in the shape of a rectangle and the yard was in the shape of a trapezoid. The building occupied part of the area of the trapezoid. Students needed to make several calculations and then bring in the amount of square feet a bag of fertilizer covered to finally calculate the amount of bags needed. The teacher facilitated conversations like this on a daily basis to model questioning techniques and the thinking process needed in problem solving.

Teacher: "I see some of you agree and some of you do not. Who can tell me why you disagree?"

Student G: "The building"

Teacher: "What do you mean the building?"

Student G: "It's in the way"

Teacher: “What do you mean it’s in the way?”

Student G: “The building is in the way of the grass”

Teacher: “Can someone tell me what he means that it’s in the way of the grass?”

Teacher: “Who can tell me what I need to do with the building?”

Student H: “Take it out”

Teacher: “Can you better explain what you mean?”

Student H: “First you need to find the area of the trapezoid and subtract out the area of the building”

Teacher: “Ok. Good job. Do you agree with this?”

Teacher: “Ok. What else do we need to do to answer the question being asked?”

Student I: “Read it”

Teacher: “Can you read it for us?”

Student I: “How many bags of fertilizer need to be purchased?”

Teacher: “How do we answer this question?”

The researcher felt the ongoing questioning and student dialogue kept the students engaged with the problem. However, from this teacher’s past experience some of the students would have given up without this type of questioning if they did not understand the problem. The researcher felt the audio-tapings were a good instrument for gathering data on the communication used in the classroom. The researcher observed from reviewing the tapes and making anecdotal notes about the students that this level of questioning and communication added discussion of the mathematics in the problem. In addition, the students modeled the process through discussions; questioning and written

examples to demonstrate to their peers through communication their justification and explanation for how they solved the problem. Creating a classroom climate rich in communication through drawing, writing and talking that appeared to have an impact on student achievement over the nine-week action research study.

Reliability and Validity

The researcher felt the climate of the classroom was set where these students felt comfortable in taking risks, working together and growing as communicators in mathematics. Then, through the modeling of questioning and the facilitation of discourse, engaging communication took place between these students. During the communication process the students were able to use reasoning skills to facilitate their learning and conceptual growth.

As shown in the table of student scores, each student grew from the beginning of the research project to the end of the research project. The researcher saw the biggest gain from the pre-test to the post-test in student B by an increase in nine points, a tie for the highest gain in the class. In addition, the journal writings for each student increased at least one point. The researcher attributes this gain to the climate of communication used in classroom discussions and writings in the daily journals. The students began to write explanations and justifications for each problem given.

Reasoning is an integral part of doing mathematics and shown through communication as the students written justifications emerged Students use their reasoning skills by deepening their evaluations of their assertions and conjectures and using inductive and deductive reasoning to formulate mathematical arguments (NCTM, 2000).

The researcher observed the students as they began to develop the mathematical language and vocabulary needed to improve conceptual developments and connections in mathematics. Through the discourse happening at each table, the teacher observed reasoning skills as they took place. The students began to argue and back up their explanations with justifications even though they were not always a complete thought.

At first, from teacher observation field notes, some of the students only wrote the problem and the procedure used in answering the problems. For many of the students the algorithm was written, but no written explanation or justification was given in written form as shown in the problem solving section under question one. The researcher believed that the students lacking the confidence to write chose not to do the work or attempt the problem in fear of being judged as too low performing or in adequate problem solvers. As the marking period progressed, the researcher observed improvement in the explanations and justifications as the students wrote in the journals.

The researcher felt these findings support the data indicating the instruments used were a reliable source for collecting data as shown in the increased test scores and journal writings scores. The researcher observed an increase in confidence and the ability to make connections. The researcher felt the climate of the classroom aided in the growth of these students. The students had the opportunity to discuss mathematics, use reasoning skills and make connections to real world situation through problem solving opportunities.

Summary

Throughout this chapter, the researcher has shown the ways in which discourse and problem solving this action research study led to an increase in students taking risks,

developing conceptually in mathematics, and making connections through the problem solving process using the instruments the researcher selected for data collection. In order for students to regularly act in mathematical agentive ways, the students needed to have an atmosphere where they are expected to think for themselves. The researcher as documented in field notes and audio recordings was promoting discourse by modeling the questioning needed to spark thinking and communication within the classroom. In addition, by allowing students to work in small groups, the students were afforded the atmosphere conducive to learning in a non-risk environment. The conditions were set for learning to take place through the problem solving process as research suggested.

In chapter five, the researcher discusses the relationship of conclusions found in this action research study with current literature along with the implications of this study. Hence, providing a reflection on how these students' areas of growth reflect current practices in the field as well as how this study could further enhance with additional research.

CHAPTER 5 CONCLUSIONS

This chapter provides a summary of how the findings of this action-research study relate to the current status of communicating through written explanations and justifications in a mathematics class. The purpose of this action research study was to look at how the use of verbal and written explanations and justifications in a seventh grade pre-algebra class impacted student outcomes. This researcher focused on the following action-research questions.

1. How can problem solving help students written explanations and justifications in mathematics?

2. How did students' communication in a problem-solving climate impact students' written explanations and justifications in mathematics?

NCTM (2000) states, "Students are flexible and resourceful problem solvers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it" (p. 3). This statement on the way students need to learn mathematics is precisely the core value embracing this action research project. For this project the researcher wanted to investigate if the implementation of discourse in the researcher's classroom impacted the communication and learning outcomes of students who were in a pre-algebra classroom compared to her past experience with discourse with students who were considered gifted and talented.

The researcher was anxious to see what would occur when problem solving and written and verbal communication were introduced into the pre-algebra mathematics classroom where the traditional classroom norms of being simply a consumer of

mathematics were challenged for most of the students. Through this action research study, the researcher was able to gain insight into the question, “How the use of verbal and written explanations and justifications in a seventh grade pre-algebra class impacted student outcomes”.

Results

This chapter provides first a review of the results of the study in relation to current findings in the literature followed by a discussion on the implications and limitations of the study. The chapter ends with recommendations and suggestions for future research in the area of using verbal and written communication in a pre-algebra classroom.

After examining the overall outcomes of this action-research project, it became evident that over the nine-week period, changes occurred in the students’ quality of written explanations and justifications. Students began to use reasoning skills and make connections to other mathematical concepts as well as to be able to explain verbally in class and in written format within their journals through words and pictures. Mathematics educators, who subscribe to social constructivism, recognize that students learn and experience higher levels of understanding when they are stimulated by challenging activities in which they reason, conjecture, and explain their mathematical reasoning (Watson, 1995). This study appears to parallel the thought of Watson in that students did increase in their overall grades and in their written pre and post-test scores. The following findings are provided related to each of the research questions.

Implications

In relation to the first research question regarding how this study impacted students' problem solving and written explanations and justifications in mathematics a change observed. These findings reflect the observation by Witzel (2007) that students need opportunities to verbalize their thinking. Incorporating language experiences into mathematics study is a powerful tool for students to learn to rationalize mathematical processes. NCTM (2000) also placed an emphasis on the importance of communication for learning and doing mathematics, which reflects what occurred during this nine-week period.

In a pedagogical problem-solving context, students are given opportunities to design, plan, evaluate, recommend, review, define, critique, explain, and make situations problematic (Pourdavood, 2003). Teachers need to analyze their processes and get students to communicate their thinking (Szetela, 1992). "Problem solving engages students in the development of their conceptual understanding of mathematics. It is a process through which mathematics becomes meaningful and relevant" (Johnson, 2006, p. 94). In this study, the researcher found that the students actively engaged in the problem solving process as the climate in the classroom changed. Once the students felt they were in a non-risk environment they began to explain and justify their work solutions both verbally and in written form. The students were able to reason through their solutions with more mathematical sense. Once the students began to verbalize their findings, they could see and hear what else needed to be added to the step-by-step process. The communication going on around the room helped students move from a complete concrete level into a more abstract level as shown by the journal writings and

the difference in the class' scores on the pre-tests and post-tests administered over the nine weeks.

The researcher's findings support the importance of communication within the mathematics classroom and the need for a change in the role of the teacher and students as they move away from the traditional setting into a constructivist classroom where learning takes place through collaboration, problem solving and communication. This implication suggests that teachers should not be the deliverers of knowledge, depositing information into students (Freire, 1968), but that students should be engaged with their teacher and peers discussing and writing about their mathematical understanding.

In this action research study the results were favorable to indicate that a classroom filled with verbal and written communication enhanced the students' ability to explain and justify their work. Academic performance increased as indicated by grades as well as post-test analysis as students worked in small groups, engaged in dialogue and were required to explain and justify their solutions.

Effective teaching as described by NCTM (2000) states, "if students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential" (p. 18). Also, NCTM (2000) cites, "to support classroom discourse effectively, teachers must build a community in which students feel free to express their ideas" (p. 61). The teacher in this classroom firmly agreed with the stance NCTM had taken and worked hard to implement changes within the classroom by creating a comfortable, non-risk environment for the students. Lester

(2003) said, “to foster discussions that center on student’s solution processes, the teacher and students must create and sustain a risk-free environment in which students’ reasoning, not getting answers, is valued” (p. 149).

Mathematical Connections

Past research as well as the data from this study support a positive response to the question focused on students showing how connections have been made in the mathematics classroom.

Research reveals the importance of building conceptual understandings in relation to mathematics (Harbaugh, 2005; Hiebert, 1999; Johnson, 2006, Lampert, 1986; NCTM, 2000). Furthermore, research has shown that mathematical understanding is accelerated in a climate rich in student discourse (Lampert 1986; Santiago, 1993; Pourdavood, 2003; Reznitskaya 2005; Smith, 1998; Willoughby 1990).

If students’ understanding of mathematical concepts are improved through problem solving activities and discourse, then our traditional classrooms need to be re-evaluated or reformed. If our goal is to aid in the teaching and learning of mathematics in the US, then improvement must begin in our classrooms. In order to equip the students with the knowledge needed for the work place these changes must take place (Johnson, 2006; Marrone 2004).

The outcomes of this action research reflected a statement by Stallings (2007), “If students are pushed to think about mathematics more deeply and flexibly, they will, in effect, “see a different mathematics” (p. 212). Johnson (2006) states “problem solving engages students in the development of their conceptual understanding of mathematics. It

is through this problem solving process in which mathematics becomes meaningful and relevant” (p. 94). The researcher believed that the students in this study saw mathematics as relevant and important through the growth of all the work they demonstrated during this marking period both in written and verbal form.

The creation of a climate of communication also seemed to impact students’ written explanations and justifications in mathematics. The NCTM supports the need for students to communicate in an effective classroom. “Effective teaching requires continuing efforts to learn and improve. These efforts include learning about mathematics and pedagogy, benefiting from interactions with students and colleagues, and engaging in ongoing professional development and self-reflection” (NCTM, 2000 p.18). One way for teachers and schools to achieve an interactive climate rich in communication is through the implementation of cooperative learning, where groups of students explore mathematical ideas, form conjectures, discuss results, and compare mathematical strategies together (Rubel, 2006). In this action research study students worked in groups regularly to enrich their problem-solving and justification skills through dialogue with their peers.

The researcher observed that at times students had difficulty working with a partner and communicating mathematically. Even though the students were allowed to talk through the problems, most of the students chose to work independently. Students were encouraged to talk about procedural processes and steps involved in obtaining solutions. Answers were to be compared and steps revisited to find errors. The teacher went as far as to tell the students “two heads are better than one” to get the students more comfortable working together. Students had been conditioned to deposit information as

Freire (1968) described as the 'banking' concept of education, "whereas teachers merely deposit knowledge into "empty vessels" that are the students" (p. 59). Instead, in this study, the teacher attempted to get students to fill their own vessels through dialogue with their peers attempting to cement mathematical concepts.

At the beginning of the action research study, students were more likely to work independently. Students relied on algorithms or procedures in answering a problem. Many of the students would not attempt a problem and would leave answers blank on class work, homework, and tests if they were not sure of the procedure to use. If called upon, they would answer, "I don't know", or "I didn't get that one". Many of the students were quiet and shy. They felt as if talking would get them into trouble. They were not used to sitting at tables and working with other students.

For the students willing to share in front of their classmates prior to the study, explanations were only verbal and consisted of step-by-step procedures, but usually steps were left out. Several students could get correct solutions, but could not complete the process of explaining what was done to get to the final answer and no justifications were used. When the traditional norm was challenged, the dynamics of the classroom began to change. As the climate of the classroom changed, students began to take risks in both attacking problems and offering explanations in front of their peers. Students began to raise their hands and were more actively engaged in the problem solving process.

As the students began to see success, more students engaged in mathematical discourse rather than inappropriate talking at their tables. With the focus on the process of explaining their thinking and helping others to understand within their groups, the understanding of mathematical concepts began to emerge. These findings could suggest

that students were more involved in problem solving discourse to allow maximum learning to take place. The communication process also seemed to help students build meaning and permanence for ideas and makes them public. “When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing” (NCTM, 2000 p.60). The findings in this action research project relate to what Rubentein (2007) suggests that communication improves when mathematical vocabulary develops through cooperative learning, using journal writing and implementing open-ended assessments that require explanations and justifications.

When students learn to be involved in the problem solving process and are allowed to participate in communication with classmates, they can become empowered by knowledge. They can learn that “two heads are better than one”, and that sharing knowledge with one another can take learning to new heights.

Limitations

As with any study there are limitations to the outcomes. For this study those limitations include student absences, student motivation, class size, and professional roadblocks.

Throughout the study, groups changed during problem solving activities, which may have changed the direction of the discourse due to student absences. This factor must be taken into account when analyzing the data from this study. Although the students were put into groups of three and four, on occasion, one or two students within the group were absent, making a smaller group. Sometimes the smaller group was then

reorganized into another group where the comfort zone between students may have changed the direction of the communication process.

In addition, the students in the class were handpicked for the AVID program and are more serious about their education than many of the other seventh grade students, a criterion for participation. While their levels of ability are similar to the students outside the classroom, there is more parental involvement and inner drive is more apparent than students on other teams. Overall, these students seemed motivated to do their work more than other students in the building.

The class was made up of only eighteen students. This small class size may have had a direct impact on student academic performance. The teacher had the opportunity to facilitate the classroom more effectively asking and answering questions more often than in a larger classroom. Research shows that smaller classes learn more.

Summary

During this nine-week action research study, the researcher observed several important factors. One of these factors was that the teacher's style of instruction was validated as a positive method on student learning outcomes. The research study indicated the teacher's work with students who were considered gifted in the past did not just perform due to their intellect, but that both groups were able to progress in a communication rich classroom. The results of the study indicated that students in the AVID group who were not considered gifted, did learn through mathematical verbal and written communication, group work and problem solving activities. The teacher's style of using hands-on activities and students sitting at tables working together was conducive to learning as research shows. In addition, the study validated that students understand

mathematical concepts if they can explain it through the explanation and justification process.

Another factor the researcher observed was how many of the students' grades improved. There were two students who missed a lot of school and their scores did not rise like their peers. The researcher attributes the decrease in scores to too many absences and lost collaboration time with their peers.

The researcher believes further studies need to be conducted in different classroom settings to concur with these findings. The researcher also believes that the study should encompass a longer period of time for true conceptual growth to be better understood. The data collection methods need to have a wider range of instruments to further validate learning outcomes. Further research needs to be conducted with larger groups of students to ensure this method works across a larger group of students and teachers. Having both an experimental group to compare results with a controlled group involved in a quantitative research study would further validate the entire process.

Overall, during this action research study, students were more apt to ask questions and take risks in giving solutions in written and verbal form. As students built more confidence, they began to rely on prior mathematics knowledge to aid in their thinking process. The students overall became more resourceful and looked things up in the mathematics textbook to see if they could find resources to help find solutions. Pape states, "Self-Regulated learners are active participants in their own learning, are able to select from a repertoire of strategies and to monitor their progress in using these strategies towards a goal (2003 p. 185). That outcome is precisely the one desired and hopefully achieved for these 18 students in this study.

APPENDIX A: IRB APPROVAL

Central
Florida

Department of Health Services
1900 East Colonial Avenue, Suite 100
Orlando, Florida 32809

Dear Sir:

Reference is made to

your letter of

the above captioned subject, and in reply to inform you that the same has been forwarded to the

appropriate authorities for their consideration. The results of their action will be reported to you as soon as they are available.

Very truly yours,
[Signature]

Director, Department of Health Services

Enclosed for your information are copies of the report of the committee on the subject of the

above captioned subject, and a copy of the report of the committee on the subject of the

above captioned subject, and a copy of the report of the committee on the subject of the

above captioned subject, and a copy of the report of the committee on the subject of the

James M. ...

APPENDIX B: PRINCIPAL APPROVAL



UNION PARK MIDDLE SCHOOL
ORANGE COUNTY PUBLIC SCHOOLS
"A United Community Inspiring Excellence"

Kris S. Viles, Principal

Patrick Jones, Dean of Students
Jessica Abrew, Assistant Principal
Francisco Ramirez, Dean of Students
Dr. Monica Johnson, Assistant Principal

June 28, 2007

Dear IRB Coordinator,

Ms Rebecca Jones has informed me of the action research project she would like to implement in the fall of the 2007-2008 school year with her seventh grade, pre-algebra students at Union Park Middle School.

Ms. Jones will conduct a study on how the effects of her teaching practice in establishing a discourse community influences students' explanation and justification of mathematical concepts. This action research project is part of her Masters' program in the Lockheed Martin K-8 Mathematics and Science program at UCF.

Ms. Jones will video/audio tape her students working and discussing mathematics. In addition, she will gather class work samples of her student's work to use in her study. She has informed me that all collected data will be locked in her cabinet until her study is complete and then destroyed after ciphering through the material to collect her data.

I have reviewed the letter Ms. Jones will be sending home requesting parental and student consent for student participation. She does note that students' grades will not be affected should they elect to not participate in the study. Ms. Jones will not use any data from students not approved for the study. I see little to no risks involved towards her students as a result of her study.

I approve Ms. Jones proposed research project with her students beginning in August, 2007.

Sincerely,

Kris S. Viles
Principal

APPENDIX C: PARENT CONSENT ENGLISH

Informed Consent to Participate in Research Study

Dear Parent or Guardian,

My name is Rebecca Jones and I am delighted to be your child's pre-algebra, mathematics' teacher this year! I have spent time this summer planning for the coming school year, and I am confident that your child will have a positive learning experience in my class.

In addition to my responsibilities as your child's mathematics' teacher, I am also a graduate student in the Lockheed Martin K-8 Mathematics and Science Education Program at the University of Central Florida. I am currently planning a research project for my Master's thesis that will take place within my classroom from October until January.

The goal of my research is to study how my teaching practices in establishing a discourse community influences student's explanation and justification of mathematical concepts. I am interested in investigating the use of these teaching techniques and how they affect your child's understanding of mathematical concepts.

With your permission, I will video/audio tape your child taking part in whole class and small group discussions about their mathematical thinking and problem solving methods as well as interviews discussing his or her participation in class discussions. I will also collect samples of your child's class work in the mathematics' class. Participation in this study is completely voluntary. Compensation such as extra credit will not be provided, and participation will not affect your student's grades in any way.

Your child's identity will be kept confidential during this study. The purpose of this study is to analyze my teaching practices, not assess your student's mathematical ability. I do not anticipate any risks to your student during the course of the study, only the potential benefit of identifying more effective teaching and learning strategies for middle school mathematics. Upon completion of the project, any connection between your student and the data collected will be destroyed.

If you have any questions regarding this study, you may contact me at anytime at (407) 249-6309. You may withdraw consent at any time. Questions or concerns about participant's rights may be directed to the UCF IRB Office of Research and Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32806. The hours of operation are Monday through Friday, 8:00 a.m. – 5:00 p.m. The phone number is (407) 823-2901.

Thank you so much for your help in this process.

Sincerely,
Rebecca Jones
(407) 249-6309
jonesr9@ocps.net

I have read the procedure described above and understand what is being asked of my child as a participant of this research study. I voluntarily agree to allow my child to participate in the study and to be video/audio taped during class and interview sessions. I have received a copy of this form.

Informed Consent to Participate in Research Study

Name of child (Printed)

- I give consent for my child to be video/audio-taped during class time and interviewed.
 I would like more information about this study.

Name of Parent/Guardian (Printed)

Name of Researcher (Printed)

Name of Parent/Guardian (Signed)

Name of Researcher (Signed)

Date

Date

APPENDIX D: PARENT CONSENT SPANISH AND VERIFICATION

Informed Consent to Participate in Research Study

Querido Padre o Guardián,

¡Mi nombre es Rebecca Jones y me alegra ser la maestra de matemáticas, pre-algebra de su niño(a) este año! He estado planeando en este pasado verano para el próximo año escolar, y estoy confiada de que su niño(a) tendrá una experiencia de aprendizaje muy positiva en mi clase.

Además de mi responsabilidad como maestra de matemáticas de su niño(a), yo también soy una estudiante graduada en Pedagogía en Matemáticas y Ciencias para los grados K-8 en la Universidad de la Florida Central. En el momento estoy planeando un proyecto de investigación para mi tesis de Maestría que se llevará a cabo en mi salón desde octubre hasta enero.

La meta de esta investigación es la de estudiar como mis prácticas de enseñanza al establecer una comunidad de discusión de temas influencia las explicaciones y justificaciones de conceptos matemáticos de los estudiantes. Yo estoy interesada en investigar el uso de estas técnicas de enseñanza y como ellas afectan el entendimiento de conceptos matemáticos de su niño(a).

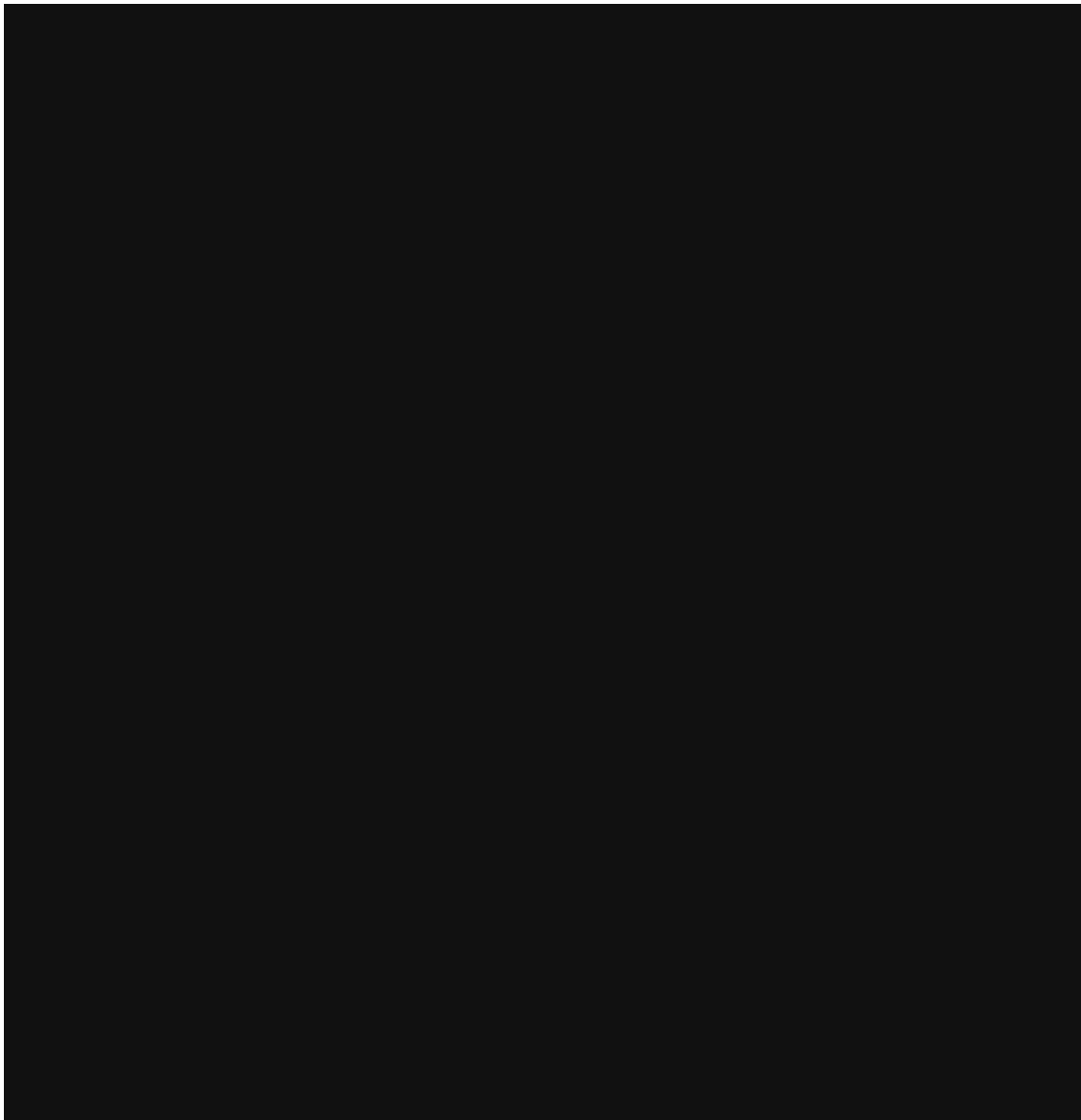
Con su aprobación, yo voy a grabar en video/audio a su niño(a) como parte de toda la clase y en discusiones de grupos pequeños acerca del pensamiento matemático y métodos de solución de problemas, al igual que entrevistas discutiendo su participación en clase. Yo también tomaré ejemplos del trabajo de su niño(a) en la clase de matemáticas. La participación en este estudio es completamente voluntaria. Compensación como crédito extra no será proveído, y la participación no afectará de ninguna manera las notas del estudiante.

La identidad de su niño(a) será mantenida en confidencia durante el estudio. El propósito del estudio es el de analizar mis prácticas de enseñanza como maestra, no es para evaluar la habilidad matemática del estudiante. Yo no anticipo ningún riesgo para los estudiantes durante el curso de este estudio, sólo el posible beneficio de identificar estrategias de enseñanza efectivas para las matemáticas en la escuela intermedia. Al terminar el proyecto, cualquier conexión entre su estudiante y la data colectada será destruida.

Si usted tiene cualquier pregunta relacionada con este estudio, usted puede contactarme en cualquier momento al (407) 249-6309. Usted puede retirar su consentimiento en cualquier momento. Preguntas o preocupaciones relacionadas con los derechos de los participantes pueden ser dirigidas a UCF IRB Office of Research and Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32806. Sus horas de operación son de lunes a viernes, 8:00 a.m. – 5:00 p.m. El número es (407) 823-2901.

Muchas gracias por su ayuda en este proceso.

Sinceramente,
Rebecca Jones
(407) 249-6309
jonesr9@ocps.net





APPENDIX E: STUDENT ASSENT

Student Assent to Participate in Research Study

Dear Student,

Welcome to my pre-algebra mathematics class! I am very excited about being your teacher this school year. This year is a very exciting one for me. I am a graduate student at the University of Central Florida working on getting a Master's degree in the Lockheed Martin K-8 Mathematics and Science Education Program. As a part of this program, I am conducting a research project that studies the way I teach mathematics.

During the study, I would like to video and audiotape our class to learn more about our class discussions. You may choose to not answer any of the questions asked.

You do not have to participate in this study if you don't want to. Remember that my goal in the study is to take a closer look at my teaching practices. It is not an assessment of your work in mathematics as a grade in the grade book. In fact, your name will not appear anywhere in my study. The only person who will see the videos and listen to the recordings will be me unless my supervisor is needed to help me decipher the data. There will be no extra credit given for participation in the study and your mathematics grade will not be affected in any way.

If you have any questions regarding this study, you may contact me at anytime at (407) 249-6309. You may withdraw consent at any time. Questions or concerns about participant's rights may be directed to the UCF IRB Office of Research and Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32806. The hours of operation are Monday through Friday, 8:00 a.m. – 5:00 p.m. The phone number is (407) 823-2901.

I look forward to getting to know you this school year. Thank you!

Sincerely,

Ms. Jones

By signing below, I am saying that I understand my role in the study to be conducted by my teacher. I have asked any questions that I may have had, and those questions were answered so that I understand what I will be expected to do. By signing, I am saying that I am willing and would like to participate in this study.

Student Name

Date

APPENDIX F: PRE-TEST

Problem Solving Pre-test

Name _____

Read each problem carefully. Decide what mathematics you know that will aid you in solving the problem. You may use the FCAT Reference sheet to help you on any formulas you need as well as a calculator. Explain the process you went through to obtain your solution and then justify your answer for each listed problem. Each problem gets progressively more difficult.

1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

2. The Lincoln Tunnel in New York is about twice as long as the Bayton Tunnel in Texas. Together they are about 12,300 ft long. About how long is each tunnel?

3. A balance scale has only two weights, 1 ounce and 4 ounces. In only three weightings split 180 ounces of sand into two bags of 40 and 140 ounces.

APPENDIX G: POST-TEST

Problem Solving Pre-test

Name _____

Read each problem carefully. Decide what mathematics you know that will aid you in solving the problem. You may use the FCAT Reference sheet to help you on any formulas you need as well as a calculator. Explain the process you went through to obtain your solution and then justify your answer for each listed problem. Each problem gets progressively more difficult.

- 1. Which has a greater volume: a can 16 cm in diameter and 24 cm high or a box 16 cm by 12 cm by 24 cm?

- 2. The Lincoln Tunnel in New York is about twice as long as the Bayton Tunnel in Texas. Together they are about 12,300 ft long. About how long is each tunnel?

- 3. A balance scale has only two weights, 1 ounce and 4 ounces. In only three weightings split 180 ounces of sand into two bags of 40 and 140 ounces.

4. Daniel can get from his house to Susan's house either by going 52 kilometers directly along the Farm Road or by going 43 kilometers down Highway 50, turning 90° , and going along State Road 419. He can average 1.4 kilometers per minute (km/min) on the highways, but only 0.9 km/min on the Farm Road. Which way is quicker? How much quicker?

5. A beginner's artist kit costs \$6.35 and contains 2 brushes and 5 jars of paint. The standard kit has 4 brushes and 12 jars of paint and costs \$13.20. Assuming that all the brushes cost the same amount and the jars of paint cost the same amount, how much would you expect to pay for a deluxe kit containing 7 brushes and 20 jars of paint?

APPENDIX H: RUBRIC

MATHEMATICS SCORING RUBRIC: A

	<p>MATHEMATICAL KNOWLEDGE: Knowledge of mathematical principles and concepts which result in a correct solution to a problem.</p>	<p>STRAT Identification and use that represent and inform solution (e.g., models</p>
<p>Score Level 4</p>	<ul style="list-style-type: none"> ◆ shows complete understanding of the problem's mathematical concepts and principles ◆ uses appropriate mathematical terminology and notations including labeling answer if appropriate ◆ executes algorithms and computations completely and correctly 	<ul style="list-style-type: none"> ◆ identifies all important elements among complete ◆ shows complete that would correct
<p>3</p>	<ul style="list-style-type: none"> ◆ shows nearly complete understanding of the problem's mathematical concepts and principles ◆ uses mostly correct mathematical terminology and notations ◆ executes algorithms completely; computations are generally correct but may contain minor errors 	<ul style="list-style-type: none"> ◆ identifies most of problem and shows relationships among ◆ shows nearly correct strategy for solving
<p>2</p>	<ul style="list-style-type: none"> ◆ shows some understanding of the problem's mathematical concepts and principles ◆ uses some correct mathematical terminology and notations ◆ may contain major algorithmic or computational errors 	<ul style="list-style-type: none"> ◆ identifies some important elements among them ◆ shows some evidence of problem
<p>1</p>	<ul style="list-style-type: none"> ◆ shows limited to no understanding of the problem's mathematical concepts and principles ◆ may misuse or fail to use mathematical terminology and notations ◆ attempts an answer 	<ul style="list-style-type: none"> ◆ fails to identify important elements among them ◆ reflects an inappropriate strategy for solving
<p>0</p>	<ul style="list-style-type: none"> ◆ no answer attempted 	<ul style="list-style-type: none"> ◆ no apparent strategy

APPENDIX I: RUBRIC PERMISSION



APPENDIX J: JOURNAL WRITING SCORING PROMPTS

Week 1: Astronaut Training: Approximately 98 astronauts train everyday at Kennedy Space Center. How many astronauts train in a week?

Savings Account: You have \$175.00 in your savings account. You started the account with \$25.00 and have deposited the same amount each Friday for 10 weeks. How much did you deposit each Friday?

Week 2: Horse Back Riding: Suppose you pay \$15 per hour to go horseback riding. You ride for 2 hours today and plan to ride 4 more hours this weekend. Find the total cost.

Ears of Corn: A bin contains 120 ears of white and yellow corn. Of these, 78 ears are yellow. What percent of the ears are white?

Week 3: Percent Increase: Suppose 36 videos were added to a video collection that has 24 videos. What is the percent of change?

Percent Decrease: In 1944, there were 2,372,292 active United States Air Force personnel. In 2001, there were 351,935. What was the percent of change in the United States Air Force?

Week 4: Manatee Springs: Manatee Springs in Levy County produces a water flow of up to 150,000,000 gallons of crystal clear water per day. What is the gallon flow per hour? Write this in scientific notation.

Todd's Height: Todd is five inches taller than his brother. The sum of their heights is 139 inches. How tall is Todd?

Week 5: Television Size: The length of the diagonal of a screen determines the size of a television set. If a 35-inch television screen is 26 inches long, what is its height to the nearest inch?

Triangle Height: Suppose a triangle has an area of 20 square inches and a base of 2.5 inches. What is the measure of the height?

Week 6: Flag Area: The flag shown at the right is the international signal for the number three. Find the area of the red region. (A trapezoid is drawn and divided into three parts. The various parts are labeled and the student must find the area of one part of the trapezoid)

Trapezoid Height: A trapezoid has an area of 54 square feet. What is the measure of the height if the bases measure 16 and 8 feet?

Week 7: Squirrel Area: A California ground squirrel usually stays within 150 yards of its burrow. Find the area of a California ground squirrel's world.

Ice cream Distance: Jack and Mayuko left school at the same time to run errands. Mayuko walked east 1,600 feet to the bookstore. Jack walked south 1,200 feet to the gym. Jack and Mayuko wanted to meet for ice cream half way between the bookstore and the gym. How many feet would each have to walk?

Week 8: Water Volume: If the full glass of water is poured into the rectangular container shown below, will the water overflow the container? (The glass was a cylinder with a diameter of 4 and a height of 6, the rectangular prism measured 5 by 5 by 3)

Phone Comparisons: Which phone company charges less for a 5-minute phone call between New York and Chicago? Which company is cheaper for a 20-minute phone call? At what point are they the same? The Dash Company charges 48 cents for the first minute and 19 cents for each additional minute. The TT and T Company charges 75 cents for the first minute and 16 cents for each additional minute.

Week 9: Pool Surface Area: Jose needs to put a liner in his above ground pool. It is a circular pool. The pool is four feet high and eight feet and sixteen feet across the pool. How much vinyl does Jose need to line the pool?

Coconut and Sailor: 3 sailors and a monkey are stranded on an island. The sailors walk around the island and collect all the coconuts they can find. They are so tired at the end of the day, they decide to go to bed and sort them in the morning. One sailor gets up in the middle of the night and divides the coconuts into three equal piles with one coconut left over. He gives it to the monkey. Then, he takes his portion and hides it. After moving the rest of the coconuts back into a pile he goes to sleep. The second sailor wakes up and does the same thing. Upon finishing he also goes back to bed. The third sailor wakes up and does the same thing as the first and second sailor and returns to sleep. The following morning, the sailors get up and divide the rest of the coconuts into three equal piles, which leaves one for the monkey. What is the lowest number of coconuts they could have started with?

REFERENCES

- Ball, D.L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R.J., Schmid, W., & Schaar, R. (October 2006). Reaching for Common Ground in K-12 Mathematics Education, *Notices of the AMS*, Volume 52, No. 9.
- Bebout, H. (1993). *Reaching All Students With Mathematics: Reaching All Students: A Vision of Learning Mathematics*, The National Council of Teachers of Mathematics, Reston, VA (p. 219).
- Bicknell, B. (1999). The writing of explanations and justifications in mathematics: differences and dilemmas. *Making the Difference: Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (pp. 75-83). Adelaide: Mathematics Education Research Group of Australasia.
- Bottge, B.A. (2001). Anchoring Adolescents' Understanding of Math Concepts in Rich Problem-Solving Environments. *Remedial and Special Education* (pp. 299-314). The H.W. Wilson Company.
- Bruner, J. (1986). *Actual Minds, Possible Worlds*. (p. 72). Cambridge, MA: Harvard University.
- Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics*, 37, 121-143.
- Faitel, T. (2007). Increasing student achievement and motivation on high stakes standardized tests. Ed.D. dissertation, Wayne State University, United States – Michigan. Retrieved March 30, 2008, from ProQuest Digital Dissertations database. (Publication No. AAT 3260782).
- Glasser, W. (1993). *Quality School Teacher*. New York, NY10022: Harper-Collin Publishers.
- Gokhale, A. (1995). Collaborative learning enhances critical thinking. *Journal of Technology Education* Volume 7, Number 1, Fall 1995.
- Harbaugh, A. (2005). *Authoritative Discourse in the Middle School Mathematics Classroom: A Case Study*. Dissertation: Texas A&M University.
- Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Wearne, D., Murray, H., Oliver, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.
- Johanning, D. J. (2000). An analysis of writing and postwriting group collaboration in middle school pre-algebra. *School Science and Mathematics*. 100(3), 151-157.
- Johnson, A., & Norris, K. (2006). *Teaching Today's Mathematics in the Middle Grades*. Pearson Education.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction*, 3(4), 305-342.
- Lampert, M. (1986). Teaching Multiplication. *Journal of Mathematical Behavior* 5, no. 3(December 1986): 241-80.
- Langston, B. (1997). *How Learning Logs Reveal the Connections Between Conceptual & Procedural Knowledge*. Dissertation, University of South Carolina. (UMI No. 9815520)
- Lawson, M., Chinnappan, M. (2000). Knowledge Connectedness in Geometry Problem Solving. *Journal for Research in Mathematics Education*, Vol 31, No. 1 26-43.
- Lester, F. K. (2003). *Teaching Mathematics through Problem Solving*. Reston, VA 20191-1502: The National Council of Teachers of Mathematics, Inc.
- Lorsbach, A., Tobin, K. (2006). *Constructivism as a Referent for Science Teaching: Institute for Inquiry*, November 4, 2006.
<http://www.exploratium.edu/IFI/resources/research/constructivism.html>
- Mack, N. (2001). Building on informal knowledge through instruction in a complex content domain: Partitioning, units, and understanding multiplication of fractions. *Journal for Research in Mathematics Education*, 32(3), 267-295.
- Marrone, A., Harkness, S., D'Ambrrosio B., Caulfield, R. (2004). Patterns of Instructional Discourse the Promote the Perception of Mastery Goals in a Social Constructivist Mathematics Course. *Educational Studies in Mathematics*, v56 n1, 19-38.
- Martinez, J. (2001). Exploring, Inventing, and Discovering Mathematics A Pedagogical Response to the TIMSS. *Mathematics Teaching in the Middle School*, vol. 7. No.2, 114-119.

- National Council, (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.
- National Council of Teachers of Mathematics, (1989). Curriculum and evaluation of standards for teaching mathematics, Reston, VA.
- National Council of Teachers of Mathematics, (1989). Professional Standards for School Mathematics, Reston, VA.
- National Council of Teachers of Mathematics, (1989). Principles and standards for school mathematics, Reston, VA.
- National Council of Teachers of Mathematics, (2000). Principles and standards for school mathematics. Reston, VA.
- Palincsar, A.S., & Herrenkohl, L.R. (2002). Designing collaborative learning contexts. *Theory Into Practice*, 41, 27-32.
- Pape, S.J., Bell, C.V., Yetkin, IE. (2003). Developing Mathematical Thinking and Self Regulated Learning: A Teaching Experiment in a Seventh-Grade mathematics Classroom. *Educational Studies in Mathematics*, v53 n3. 179-202.
- Pourdavood, R. Ph.D. (2003). Problem solving dynamics: Students nonroutine problem solving engagement. The University of Oklahoma.
- Powell, M. (1998) Cooperative Learning. *Classroom Compass*, Volume 1, Number 2.
- Pugalee, D. K. (2005). Writing for mathematical understanding. Norwood, MA: Christopher Gordon Publishers.
- RAND Mathematics Study Panel, (2003). Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education Arlington, VA: RAND Mathematics Study Panel, Deborah Loewenbug Ball, Chair.
- Reilly, E.M. (2007). Writing to learn mathematics: A mixed study. Dissertation.
- Reznitskaya, A. Thinking. *The Journal of Philosophy for children*, Volume 17, Number 4.
- Roberts, S., & Tayeh, C. (December 2006/January 2007). It's the Thought That Counts: Reflecting on Problem Solving. *Mathematics Teaching in the Middle School*, Vol. 12, No. 5.
- Rubel, L.H. (September 2006). M & M's, Rhinos, Cockroaches, and Cooperative Learning in Mathematics Classrooms. *Mathematics Teacher*, Vol. 100.

- Rubenstein, R.N. (November 2007). Focused Strategies for Middle-Grades Mathematics Vocabulary Development. *Mathematics Teaching in the Middle School*, Vol. 13, No. 4.
- Santiago, F., & Spanos, G. (1993). *Meeting the NCTM Communication Standards for All Students* Reston, VA: The National Council of Teachers of Mathematics.
- Slavin, R.E. (1990). *Cooperative learning: Theory, research, and practice*. Englewood Cliffs, NJ: Prentice Hall.
- Smith, C., (1998). A Discourse on Discourse: Wrestling with Teaching Rational Equation. *Algebra for All*, Vol. 91, No.9.
- Spikell, M. (1993). *Teaching mathematics with manipulatives: A resource of activities for the K-12 teacher*. New York: Allyn and Bacon.
- Stallings, L.L. (November 2007). See a Different Mathematics: Mathematics Teaching in The Middle School, Vol.13, No. 4, pp. 212-217.
- Steele, D.F. (September 2007). Understanding Students' Problem-Solving Knowledge through Their Writing. *Mathematics Teaching In The Middle School*, Vol. 13, No. 2.
- Stiff, L.V. (1993). *Reaching All Students With Mathematics: Reaching All Students: A Vision of Learning Mathematics* Reston, VA: The National Council of Teachers of Mathematics.
- Szetela, W., Nicol, C. (1992). Evaluating problem solving in mathematics. *Educational Leadership*, Vol.49, 42.
- Turner, J.C. (2002). The classroom environment and students' reports of avoidance strategies in mathematics: A multimethod study. *Journal of Educational Psychology*, 94, 88-106.
- Vygotsky, L.S. (1982-1984). *Collected Works*, 2 vols., vol. 2. Moscow. Moscow Progress Publishers.
- Watson, G. (1995). Middle School Mathematics Teacher Change: Social Constructivism Climbs a Step. *Mathematical Autobiographies: The Impact of Teachers' Early Experiences on Current Perceptions and Decisions..*
- Wheatley, G.H. (1997). Reasoning with images in mathematical activity. In *Mathematical Reasoning-analogies, metaphors and images*, Lyn English (ed), Mahwah, N.J., Erlbaum, L.

Willoughby, S.S. (1990). *Mathematics Education for a Changing World* Alexandria, VA: the Association for Supervision and Curriculum Development.

Witzel, B.S., & Allsopp, D. (November 2007). Dynamic Concrete Instruction in as Inclusive Classroom: Mathematics Teaching In The Middle School. Vol.13, No. 4, pp. 244-248.

Yackel, E. & Cobb, P. (1996). Social mathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458-477.

Zevenbergen, R. (2000). "Cracking the code" of mathematics classrooms: School success as a function of linguistic, social and cultural background. In Boaler, J. (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201-223). Westport, CT: Ablex Publishing.