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Examining Sociomathematical Norms Within The Context Of Decimals And Fractions In A Sixth Grade Classroom

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EXAMINING SOCIOMATHEMATICAL NORMS WITHIN THE CONTEXT OF DECIMALS AND FRACTIONS IN A SIXTH GRADE CLASSROOM

by

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ABSTRACT

Social norms are patterns of behavior expected within a particular society in a given situation. Social norms can be shared belief of what is normal and acceptable shapes and enforces the actions of people in a society. In the educational classroom, they are characteristics that constitute the classroom participation structure. Sociomathematical norms are fine-grained aspects of general social norms specifically related to mathematical practices. These can include, but are not limited to, a student-centered classroom that includes the expectation that the students should present their solution methods by describing actions on mathematical objects rather than simply accounting for calculational manipulations.

For this action research study, my goal was to determine if the role of the teacher would influence the social and sociomathematical norms in a mathematics classroom and in what ways are sociomathematical norms reflected in students' written work. I focused specifically on students' mathematics journal writing and taped conversations. I discovered that students tended to not justify their work. Also, I discovered that my idea of justification was not really justification. I learned from this and was able to change my idea of justification.

By encouraging the students to socialize in mathematics class, I found that the quality of their dialogue improved. Students readily discussed mathematical concepts within small groups and whole class discussions.

I dedicate this thesis to the two most important people in my life: Susan and Julie. My wife, Susan, has stood by me through this whole process. You have listened to my math and science-geek stories intently, you cried with me when I was ready to give up, you encouraged me to continue to work hard, and you comforted me when the stress levels peaked. You encouraged me constantly and made me feel special. I could not have done this without your love!

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CHAPTER 1: INTRODUCTION

Students must learn mathematics with understanding. Understanding is crucial because concepts that are understood can be used flexibly, adapted to new situations, and used to learn new concepts (Hiebert et al., 1997). "To know mathematics is to investigate and express relationships among patterns, to be able to discern patterns in complex and obscure contexts, to understand and transform relationships among patterns." (NCTM, 1990, p. 12). Researchers in mathematics education concur that (a) procedures learned by rote memorization are easily forgotten, error-prone, and resistant to transfer; and (b) procedural learning must be connected with conceptual knowledge to foster the development of understanding (Hiebert & Carpenter, 1992). There is little doubt that the rote execution of memorized procedures does *not* constitute mathematical understanding (Star & Seifert, 2006). The development of mathematical expertise involves not only learning to perform procedures accurately but also understanding the key concepts and principles that constitute these procedures (Gilmore, 2006). Bisanz and LeFevre (1992) proposed a framework that emphasized the need to consider the situation in which understanding is assessed. Children may show understanding of a concept in one situation but not another. These discrepancies may reflect potentially important differences in processes or representations. If a child demonstrates application for a procedure in only one situation, this may be an indication of rote memorization. A child may show an understanding of a concept if he/she is able to apply it in more than one situation. Problem solving comes into play with regard to understanding. When a child uses problem solving, he/she may be showing that he/she understand the mathematical concept, not just rote memorization. *The Principle and Standards for School Mathematics* (NCTM, 2000) states that problem solving means "engaging [students] in a task for which a solution method is not known in advance" (p. 52). Teachers should understand and be able to teach mathematical ideas in such a way that children make sense of them and explain, justify, predict, compare, and derive ideas and relate mathematical concepts to the world around them (O'Brien & Moss, 2004).

According to *The Principles and Standards for School Mathematics* (NCTM, 2000) to learn properly, teachers and students must work together in a risk-free environment. Students are more likely to have better attitudes toward problem-solving if they "play a role in establishing the classroom norms...where everyone's ideas are respected and valued" (p. 185). In classes where teachers describe their approach to teaching as having a focus on transmitting knowledge, students are more likely to report that they adopt a surface approach to the learning of that subject. Yet, in classes where students report adopting deeper approaches to learning, teachers report approaches to teaching that are more oriented towards students and to changing the students' conceptions (Trigwell, Prosser, & Waterhouse, 1999).

Classroom and social norms play a vital part in the dynamics of the classroom. Social norms describe attitudes and values that are appropriate, and behaviors for a given situation (Rutland, Cameron, Milne, & McGeorge, 2005). A typical social norm in a classroom is to always have a correct answer (Boaler & Humphreys, 2005). Children are not used to being able to give a wrong answer. They generally were raised in classrooms that are geared toward giving correct answers. Boaler & Humphreys (2005) saw a contradiction to this norm when discussing students' encouragement of other students in a classroom setting. They state “Cathy (Humphreys) encouraged listening by asking students to listen carefully and to keep their hands down when others were speaking. She encouraged them to build upon each other’s ideas by frequently asking questions such as ‘what do you think about what Alicia said?’” (p. 115). When a student builds upon the ideas of another student, he is not just thinking about the ideas put forward but thinking about constructing a representation of another person’s thoughts (Schwartz, 1999).

Sociomathematical norms are defined as “the normative aspects of whole-class discussions that are specific to students’ mathematical activity” (Cobb & Bauersfeld, 1995, p. 178) and “those norms that are of importance to the social environment in the mathematics classroom” (Lannin, 2002, p. 3). They are different from social or classroom norms in that they are unique to mathematics (Pang, 2001). For example, the sociomathematical norms in a student-centered classroom might include the expectation that the students are to present their methods of finding solutions by describing actions on mathematical objects instead of simply accounting for calculational methods (Pang, 2003). Students are often asked to collaborate with others in mathematics classrooms. Sharing ideas and working with other students is often the norm in a mathematics classroom. However, an advanced form of collaboration can be seen when students move beyond the act of communication to the goal of mutual understanding (Schwartz, 1999). Students can always share ideas, but if they have a concern for mutual understanding, they are actively working to help others understand.

Two mathematical practices are critical to the development of mutual understanding. They are justification and representation (Boaler & Humphreys, 2005). When students justify their thinking, they tend to help themselves learn better and provide support to other students (Boaler & Humphreys, 2005). Oftentimes, the validity of a student's justification is determined by social negotiation between the student and the teacher and student to student as the student describes his or her view of the validity of the justification (Lannin, 2002). Thusly, students must hold the belief that they are valued decision-makers, providing important contributions to the classroom culture (Lampert, 1990). When students justify their answers, they are not only giving a correct answer, they are demonstrating that they have a deeper understanding of the mathematical concept being taught. For example, a student may be able to add 1/2 and 1/4, but if he/she is able to apply this problem and understand what 1/2 + 1/4 means as well as justify his/her thinking, he/she will show a deeper understanding of the concept of adding fractions. An example of a real life situation could be recipes. If a child has a recipe that calls for 1/2 cup of flour while another recipe calls for 1/4 cup of flour and the child is able to tell his/her parents how much flour they need, he/she is applying the fraction addition concept. Students are able to justify their application if they can explain it to other peers or adults. Also, if they are able to transfer their understanding to some type of representation of the problem or show an understanding of the relationship between concrete and abstract thinking, they will show a better understanding.

A central goal of mathematics teaching is thus taken to be enabling students to pass from one representation to another without falling into contradictions (Hitt, 1998; Janvier, 1987). There are many different types of representations to promote understanding of concepts today (Gagatsis & Shiakalli, 2004). Some examples are visual-spatial representations (visualizing the mathematical problem in ones' head) and pictorial-schematic representations (drawing or creating a physical representation of a mathematical problem. The act of representing is very helpful in communicating ideas and supporting mutual understanding. When students engage in justification, they are articulating their ideas and interacting with the teacher and other students. They also provide support and understanding for their peers. It is also a critical process for students to learn so they may capture, edit, and create ideas with fluency (Eisner, 2004).

Rationale

As I began my seventh year of teaching sixth grade mathematics, I often found myself reflecting on my teaching practice. Although my students earned grades that were sufficient, I wondered if they were really grasping the concepts of what I taught. Research says that early adolescence is a very stressful time where youngsters are confronted with developmental changes, peer group identification, and role redefinition (Miller, 2001). When I teach my sixth grade students, it always seems as if their schoolwork, especially mathematics, is secondary to home life and, more importantly for them, socialization with their peers. I wondered if I could use that need to socialize to my advantage when teaching mathematics, and if peer-interactions in a cooperative group setting would be more effective than just presenting, modeling, and student practice.

Research suggests that students come to school with much informal knowledge about fractions that they can access (Empson, 1995; Mack, 1990; 1995). I have typically taught fractions and decimals using rote memory. For example, to change a fraction into a decimal, I always taught my students to divide the "top number by the bottom number and you get your decimal." This works great in the short term for passing a test, but what did my students actually learn? O'Brien and Moss (2004) state that although teachers care deeply about student achievement, often the emphasis in the classroom is on static facts and procedures while not enough time is spent on thinking and learning about and doing mathematics. I often remember one of my mentor teachers telling me when I first started teaching fractions that sometimes you just have to tell the students the procedure and forget about showing them why it is done. It has always bothered me. I wanted my students to understand why they used a specific procedure or algorithm. I also wanted them to become more efficient at solving mathematical expressions involving fractions and decimals. Children should be able to compute quickly, accurately, and usefully. However, mathematics is more than shopkeeper's arithmetic (O'Brien & Moss, 2004). It is involves understanding the concepts and being able to apply those concepts.

In August of 2005, I became a student in a master's level class that dealt with problem solving in mathematics. I was challenged to think not only about getting correct answers but to communicate *why* I got those correct answers. I, along with the other students in my class, became frustrated. Why did I have to explain myself; wasn't getting the correct answer enough? I started realizing those thoughts I was having about just wanting the correct answer were most likely what my students were thinking in my classroom. I would get upset when my students would not show their work or be able to explain to me why they got a specific answer on an assignment. Through this graduate level class I was able to somewhat understand what my students were going through. They functioned as if they were robots; conditioned to keep their mouths shut, do what they were told, and take a test on the subject matter. Of course, they would most likely not retain that information *after* the test. My students were not eager to learn mathematics. They just knew that it was something they were required to do.

Purpose of the Study

The purpose of this study was to investigate the affects of changing sociomathematical norms in a sixth grade classroom as it pertains to the teaching and learning of fractions and decimals.

Research Question #1

How does the role of the teacher influence the social and sociomathematical norms in a mathematics classroom?

Research Question #2

In what ways are sociomathematical norms reflected in students' written work?

Significance of the Study

> "All too often, children's disenchantment with mathematics begins late in elementary school or in middle school when, even after years of practice, they cannot remember how to 'do' fractions after summer vacation, or when they can perform steps, but are totally bored because they do not know what the steps mean or why they are doing them." (Lamon, 1999, p. xi).

Mathematics educational research has brought attention to the role of social context in the mathematics classroom and its impact on student learning (Cobb, 2000; Lerman, 1998). Typical practices that dominated teaching were periods of teacher modeling or demonstration during which interaction with pupils was generally in the form of testing (Wood, 1994). Teaching mathematics traditionally consisted of the teacher showing how a problem was solved, having the students practice that concept, and then the students demonstrate acquisition of that knowledge, typically using a multiple choice or short answer test as assessment. The class was traditionally taught as a whole and assessed as a whole. Yet our students are individuals, thus learning in different ways and at different paces. This whole group teaching can significantly affect how students learn mathematics, especially rational numbers. In a report from the Brown Center on Education Policy at the Brookings Institution, "an analysis shows that the National Assessment of Educational Progress (NAEP) math assessments rely on arithmetic skills that are far below the grade levels of the students being assessed. The analysis finds that almost all problem solving items use whole numbers and avoid fractions, decimals, and percentages – forms of numbers that students must know how to use to tackle higher order mathematics like algebra" (Bliss, 2004, p. 1). Rational number concepts are extremely important mathematical ideas that students will face prior to entering secondary school (Behr, Wachsmuth, Post, & Lesh, 1984; Charles & Nason, 2000). Children start learning basics, such as the meaning of fractions and decimals at an early age. These concepts continue to build into other concepts, such as fraction addition, decimal multiplication, and higher level algebra.

Summary

Students must have an understanding of mathematical concepts and be able to apply those concepts. Rote memorization of facts and algorithms will help a child "pass the test" but will not help a child understand why the concept is applied. Students who learn in a risk-free environment where they feel comfortable with others and their teacher may be more willing to be open to different ideas from other students. The classroom social and sociomathematical norms play an important role in a risk-free environment. Sixth grade children, as well as other children, identify with their peers and with the "rules" of the mathematic classroom as well as other classes (Cobb & Yackel, 1989; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990; Yackel & Rasmussen, 2002; Yackel, Rasmussen, & King, 2001). These unwritten rules drive the learning in a classroom. Children tend to "obey" these rules in all aspects of their classroom environments. I will discuss some of the research and literature on social norms, sociomathematical norms, and fractions and decimals in the next section.

CHAPTER 2: REVIEW OF LITERATURE

Research indicates that many students do not understand the meaning of symbolic representations such as fractions, decimals, or variables, thus performing mathematical operations with little understanding (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post, & Lesh, 1984; Mack, 1995; 1990; Post, 1981; Smith, Carey & Solomon, 2005). Traditional mathematics instruction can ignore a connection between the symbols taught in the classroom and the mathematical procedures (Leinhardt, 1988, Mack, 1990, 1995).

Students often memorize rote procedures and are never encouraged to "impose meaning on new material by attaching it to what is already known or altering what is already known" (Leinhardt, 1988, p. 122). If students understand and appreciate how and why mathematical procedures are performed, they will most likely improve conceptual knowledge and foster the development of understanding (Hiebert & Carpenter, 1992). The classroom environment plays an integral role in this learning.

In this chapter, I will discuss social norms and sociomathematical norms. I will also discuss fractions and decimals, and how students perceive them.

Social Norms

Social norms are defined as the many characteristics that constitute the classroom participation structure (Yackel & Cobb, 1996). One common social norm among teachers and students is when a student has a question, he/she is to raise their hand. It is the norm in most classrooms for a student to use a raised hand to get the attention of his/her teacher. This and many other social norms subtly dictate how students interact and react to each other. The vision to change classrooms into communities that are student-centered where learners engage in collaborative construction of knowledge is a hallmark of current efforts to reform mathematics education (NCTM, 2000). This vision cannot be actualized within a classroom unless norms for student and teacher expectations are made explicit and are consistent with creating this type of community of learners. According to Yackel (2001), a norm is "a sociological construct and refers to understandings or interpretations that become normative or taken-as-shared by the group" (p. 6). An example would be how Americans wait in line for an event. There is not written rule or posted law about standing in line for an event. Yet if someone were to go to the middle of a line and wait, others in the social group would most likely react negatively to that person.

The social norms that are constructed inside the classroom are constantly reconstructed and they do not exist apart from interactions that give rise to them (Yackel, 2001). For example, classrooms have a social "pecking order" among children. That is, popularity with others is typically more important to a child than academics. This "pecking order" can instantly change with the introduction of a new student, something unacceptable being said by a child, or a rumor. Norms can apply to any classroom, such as language arts, science, social studies, or mathematics. The typical social norm in many classrooms is teacher-centered, which refers to a teacher's explanations and ideas constituting the focus of the classroom practice. Student-centered environments refer to students' contributions and responses constituting the focus of classroom practice (Pang, 2003). Cobb and his colleagues have developed a theoretical framework that fits well with the reform agenda for instruction (Cobb & Bauersfeld, 1995). In this perspective, mathematical meanings are neither decided by the teacher in advance nor discovered by students; rather they emerge in a continuous process of negotiation through social interaction. Does this mean that a classroom is completely democratic with students making all the decisions of school life with the teacher? Should teachers no longer be authoritative, giving instructions to students and expecting them to complete assignments when at times they do not want to? Not at all. Direct guidance encompasses all parts of the communication process between adults and children (Taylor, Fall 1996). Children do not have all of the necessary knowledge to function in society. They need guidance from experienced individuals. Recent research suggests that a child's social and emotional adaptation and academic and cognitive development are enhanced by frequent opportunities to strengthen social competence during childhood. (Hartup & Moore, 1990; Ladd & Profilet, 1996; McClellan & Kinsey, 1987). For the purposes of this study, social norms will be defined as teacher-student and student-student interactions.

Children’s development is not only biological, but is social as well. Moll (1991) says that Vygotsky argues that “natural properties as well as social relations constrain-and therefore make possible-the social construction of a child’s higher psychological processes” (p. 1). This means that as a child develops in language and socialization, so does the complexity of his or her learning abilities. In early development, children tend to be very concrete in their thinking. The expression 1/2 + 1/2 will most likely not mean much to a child who is still developing his or her basic psychological functions. As the child continues to develop, he or she may be able to “see” that same mathematical expression in an abstract way.

According to Bodrova (2003) Montessori saw development as unfolding the sequence of stages preprogrammed in a human species. Both Montessori and Vygotsky are viewed as constructivists. These and other child development theorists concur that social interaction is a vital part of the development and learning of a child. Social interaction is key to a child's development, especially at the sixth grade level. At this age, children are highly involved with social aspects of appearances, speech, mannerisms, games, and relationships. Much of the time, school is not their top priority, let alone mathematics. As with social norms, sociomathematical norms relate to the normative structures of a classroom but in a mathematical setting.

A typical sixth grade classroom might have many social norms. For example, students might be grouped into four to six cooperative groups to facilitate learning. Both groups have different expectations. For instance, a group of four students might be expected to work well together and equally work on parts of a project. Yet in a group of six students, the dynamics change. It is easier for one student to not share the work load and become "lost" in the group. Higher-ability students can tutor, coach, and help lower-ability students in cooperative groups. Cooperative groups can also foster responsibility. A group of four students working on a project together can choose jobs (researcher, speaker, and writer) that may be tailored to their personal skills.

Sociomathematical Norms

Researchers differentiate general social norms as applicable to any subject matter area from sociomathematical norms which constitute the area of mathematics (Pang, 2001). Sociomathematical norms are more fine-grained aspects of these general social norms specifically related to mathematical practices (Yackel & Cobb, 1996). They are about the actual *process* by which both teacher and student contribute.

Some sociomathematical norms in the classroom are 1) the understanding of what counts as an acceptable mathematical explanation, 2) the understanding of what counts as an acceptable mathematical justification, and 3) the understanding of what constitutes mathematical difference (Yackel & Cobb, 1996).

Other sociomathematical norms in a student-centered classroom might include the expectation that the students should present their solution methods by describing actions on mathematical objects rather than simply accounting for calculational manipulations (Pang, 2003). Sociomathematical norms can be acceptable, justifiable, easy, clear, different, efficient, elegant, and sophisticated explanations of a mathematics problem (Bowers, Cobb, & McClain, 1999; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Yackel & Cobb, 1996).

Sociomathematical norms form when explanations and justification are made acceptable (Hershkowitz & Schwarz, 1999). "The acceptability itself is made possible when explanations and justifications can be interpreted by students in terms of actions on mathematical objects that were practiced" (Hershkowitz & Schwarz, 1999, p. 150). Justifications in the classroom are considered acceptable when they meet the criteria of the mathematics community (Lannin, 2005). Teachers need to not only give students multiple choice questions and answers, but have them justify their answers. This will help to affectively assess student understanding (Sweetman, 2002). Part of this justification can be convincing others (teachers included) how and why students solved the problem.

"Many believe that teaching and learning would be improved if classrooms were organized to engage students in authentic tasks, guided by teachers with deep disciplinary understandings. Students would conjecture, experiment, and make arguments; they would frame and solve problems; and they would read, write, and create things that mattered to them." (Ball, 1993, p. 3). Perhaps changing sociomathematical norms for a class that is learning fractions and decimals can help to provide these ways of accessing mathematics and improve the learning process.

Fractions and Decimals

Students sometimes tend to struggle with the idea of fractions, which seem to stem from students not possessing deep concept knowledge of fractions necessary for understanding and applying fraction principles (McGuire, 2004). "Not only do children fail at explicitly explaining the mathematical role of the numerator and the denominator in representing fractions, their lack of understanding is also revealed in simple ordering tasks, such as determining whether 1/56 is larger than 1/75" (Smith, Carney, & Solomon, 2005, p. 102). Many sixth grade students need to have a concrete perception of decimals and fractions. Students' mathematical interpretations, solutions, explanations, and justifications are not merely viewed as individual acts but, simultaneously, as acts of participating in collective or communal classroom processes (Bowers, Cobb, & McClain, 1999).

Russell (1924) suggests that a number such as three can be characterized as a set of all the sets of which can be put into a one-to-one correspondence with any constructed group of three. The word "three" is abstract to a child. The characteristic of "threeness" is abstract and relational in character and in making reference to any situation involving a three (Kieren, 1999). "In using symbols for three in action, a child will not be simply pairing the number word/symbol with a referent object in some way, but will be enacting a relationship involving correspondences, or successors for example, intertwining language use with his or her thought" (Kieren, 1999, p. 110). If this is the case for a whole number such as three, it surely is the case for fractions such a one-half. At an early age, children may have some type of concept of a fraction ("I ate one-half of the candy bar"), yet they typically do not have a concrete concept of what the fraction means. In using fractional number language, a child will be combining the mathematical language use with thoughts and actions which tend to be relational in character and involve what has been observed as constructive mechanisms in children's fractional numbers such as partitioning and splitting (Behr, Harel, Post, & Lesh, 1992; Confrey, 1998).

According to Smith, Carney, and Solomon (2005), students have difficulties in acquiring and understanding rational number concepts. Children continue to struggle with the relationship between fractions such as determining which fraction, 1/3 or 1/6 is larger (Wing & Beal, 2004). Research from other countries has shown that this difficulty of grasping the concept of fractions persists for some children into their high school years

(Behr, Wachsmuth, Post, & Lesh, 1984; Kerslake, 1986; Nesher & Peled, 1986). Lamon (1999) says that current instruction is not serving many students. Yet, in addition to having a need for change, one must have a direction for change. Research has shown some compelling reasons to change fraction and rational number instruction (Lamon, 1999).

Ball (1990) states, "a representation should make prominent conceptual dimensions of the content at hand, not just its surface or procedural characteristics" (p. 5). For example, 1/2 can represent several different meanings of fractions (Ohlsson, 1988). Many children might show a representation of 1/2 as a circle with half of the circle shaded in. Yet, the representation of 1/2 can also be a point halfway between 0 and 1, the ratio of one day of sunshine to every two days of clouds, or perhaps the probability of getting, or guessing, a correct answer on a true-false question. There is no perfect representational context. They are not static and do not stand alone. Representational context creates places in a student's mind for working on ideas (Ball, 1993). These places should be developed both by the student and the teacher. Social and sociomathematical norms can come into play here. Students should be allowed to explore different ways of solving fractional problems. They should also be encouraged to express and communicate to others different ways to solve equations involving fractions. This creates a feeling of "ownership" in students.

The concept of decimal numbers is considered to be of great significance especially due to its application and use in every day life. Decimal concepts need to be related to a variety of fraction ideas and to place value (Michaelidou, N., Gagatsis, A., & Pitta-Pantazi, D., 2004). Furthermore, teaching decimals in a comprehensive way should give students the opportunity to flexibly, utilize and link a variety of representations concerning the decimal numbers (Thomson & Walker, 1996). Decimals can be taught along with fractions. They are directly related to each other. Many of the misconceptions made with fractions may happen with decimals as well.

Summary

There has been a recent movement in mathematics education involving increased attention to the role of socialization in the classroom and its impact on learning (Cobb, 2000; Lerman, 1998). The social norms of a classroom can be influenced by correct and incorrect perceptions of how other people in our social group think and act (Berkowitz, 2004).

Sociomathematical norms are thematic patterns or procedures of interaction that are more specific to the mathematics classroom. These patterns or procedures usually occur when the teacher and the students routinely constitute a theme around some related issues (Voigt, 1989). Some typical sociomathematical norms in the classroom might be that students are expected to present their solution methods by describing actions on mathematical objects rather than simply accounting for calculational manipulations (Pang, 2003). “The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm” (Yackel & Cobb, 1996, p. 463).

Students tend to have difficulties in acquiring and understanding rational number concepts. How can these difficulties be overcome? Sixth grade students typically have been taught fractions and decimals by using rote memory. Students often memorize rote procedures and are seldom encouraged to "impose meaning on new material by attaching it to what is already known or altering what is already known" (Leinhardt, 1988, p. 122). When required to memorize algorithms or mathematical formulas, students typically are able to recall those concepts for a test and then quickly forget about them. My goal is to see if using and changing sociomathematical norms in my classroom will help improve my sixth grade students' learning and application of fractions and decimals.

CHAPTER 3: METHODOLOGY

Introduction

Research says that four problems have been identified with current teaching methods in the area of fractions and decimals. The first is that teachers often emphasize procedures in solving fractions and decimals at the expense of developing a strong sense of the meaning of rational numbers. The second problem identified is that teachers often take an adult-centered rather than a child-centered approach, emphasizing fully formed adult conceptions of rational numbers. The third issue is the problem of teachers using representations in which rational and whole numbers are easily confused. The fourth issue is problems in use of notation that can act as a hindrance to student development (Moss & Case, 1999). My action research was designed to examine the effects of changing sociomathematical norms in a sixth grade classroom on learning and comprehension of fractions and decimals. In the following chapter, I will discuss the methods of my study as well as my classroom and school setting, instruments used in the study, how I collected my data, and how I analyzed data.

Design of Study

This study is a convenience research study, as I was assigned these students to my classroom (Thyer, 2001). During the time of the study, I taught mathematics and science to two separate groups of students. Students were divided heterogeneously with respect to sex, reading ability, mathematics ability, and socialization to maintain balance between the classrooms. My teaching partner and I each have 24 students in our homeroom classes. I taught both groups mathematics and science while my team partner taught both groups language arts and social studies. Students were grouped by class with respect to boy/girl ratio, academic abilities, language, and learning abilities and disabilities.

Setting

School Setting

The school is located in east central Florida. At the time the study was conducted, it had a student population of 776 in grades pre-Kindergarten through sixth grade. The school first opened in the fall of 2003 in the central southwest portion of the county to alleviate overcrowding in the area. Most of the original students attended a nearby very highly regarded elementary school. There was much resistance to students being transferred to my school in 2003. Over the past three years, parents, students, and the community have come to love and respect the school and its teachers.

My school is located in an affluent area. Many of the students who attend my school are from two-parent households with only one parent working. Most are from middle to upper middle class families that live in “well-to-do” neighborhoods. Students are consistently well-dressed and well-groomed. They are typically respectful to other students and teachers. When they go home, they generally have help from one or both parents regarding homework. Their parents instill a work ethic in them that translates to the school setting.

The families of my students are generally very helpful and encouraging toward teachers. They will respond consistently to letters, emails, or phone calls regarding their child's work or behavior. They will support teachers in the classroom, in the volunteer workroom, or even financially to help purchase classroom materials.

Classroom Setting

Two sixth-grade classrooms were used for this study. The first class consisted of 24 students ranging in age from 11-13. The class consisted of 11 girls and 13 boys. The ethnicity for this class was as follows: 19 Caucasian students, 2 Hispanic students, 1 African American student and 2 mixed race students.

The second class used in the study consisted of 23 students. This class consisted of 13 girls and 10 boys. Five of these students were diagnosed with specific learning disabilities (SLD). They received extra help and guidance from an SLD teacher who came to my classroom daily. Three of the students in this class attended a gifted student program (GSP) every Tuesday. The ethnicity for this class was as follows: 16 Caucasian students, 3 Hispanic students, 1 African American student 2 mixed race students, and 1 Asian student.

These two groups of students were highly social. Research says that during adolescence the time spent with friends and intimacy of friendships increase substantially (Berndt & Perry, 1990). Also, adolescents increasingly view their friendships as supportive relationships that meet needs for companionship, help, and a feeling of being admired and respected by others (Buhrmester, 1996; Sullivan, 1953). These particular groups of students seemed to need to socialize, much more than students I have taught in the past. I have often had to discipline many of my students because they are either off task talking to their friends about events that happened over the weekend, or passing personal notes. After several discussions last year with the fifth grade teachers pertaining to this group, I realized that disciplining this group of students constantly for socializing would most likely not help them learn. I felt that I must use that tendency for socialization to my advantage in teaching.

Methods

Instruments

After receiving IRB (Institutional Review Board) approval (Appendix A) and principal approval (Appendix B), I sent home parent consent forms (Appendix C). Of the 47 students in my two classes, 46 returned parent consent forms to participate in my study. Of the 47 students who were given student assent forms (Appendix D), 38 chose to participate. The study lasted 16 weeks.

My data collection began in the Fall of 2006 by sending home a parent survey with my students (Appendix E). The survey consists of 13 questions based on a likert scale ranging from “strongly disagree” to “strongly agree”. The questions asked parents their feelings about how they perceive their children’s learning and liking mathematics, especially fractions and decimals. I then organized these data into a spread sheet so I could easily discern how the parents of my students felt about their children’s mathematics learning. At the end of my study, I gave the same survey to the same parents and used those data to compare and contrast their feelings about their children’s learning of fractions and decimals from the beginning to the end of the data collection period.

Crystal and Stevenson (1991) concluded, “Our findings suggest that U.S. parents tend to evaluate their children’s mathematics skills uncritically and that their lack of awareness of the frequency or severity of children’s problems reduces their effectiveness as a source of help to their children” (p. 375). I value my students’ parents’ opinion about how their child is learning. They are not a hindrance to my teaching but an ally. As stated previously by Crystal and Stevenson, parents tend to not be as critical in their evaluation of their student’s mathematics skills. By allying myself with parents, I feel I am not only educating my students more, but also educating their parents. This will help tremendously in teaching my students at school and at home.

Early in the school year, I gave my students a self-assessment (Appendix F). This test used a likert scale to determine each student’s self image as well as their view of themselves in a social setting. I then gave this same self-assessment to my students at the end of the study. I used the information from the pre-test and compared it to the post-test. This allowed me to better understand any change in my students’ self-image and how they feel about decimals, fractions, and mathematics in general. I created this test based on my knowledge of what students typically know and understand at this level of school. I included topics such as working by themselves, feeling nervous in front of others, group work, and responsibility. I felt changing a sociomathematical norm in my classroom might create changes in my students, both positive and negative. I was going to change the way my students were used to learning. This change incorporated a lot of socialization. For some students that created a problem. Children at this age may be shy and unwilling or unable to effectively speak in front of others. This can cause a great deal of stress in a child. I did not use the self-assessment as my only means of evaluating my students because their feelings are not an indicator of mathematical understanding by themselves. I used this assessment as a guide to help me understand my students and their views about mathematics before I began my study.

Data Collection

This study lasted for 16 weeks. I taught each of mathematics classes for a 90 minute block of time. I taught the same lesson for the first group as I taught to the second group. By teaching each group in a 90 minute uninterrupted block, I was able to focus on concepts longer than if I were to have smaller blocks of time. Two mathematics classes were studied; group A consisted of 24 students and group B consisted of 23 students. The demographics of my school did not change often, as my school did not have a large population of families that were mobile. Both groups consisted of a mixture of boys and girls and various academic levels.

A typical day in my mathematics classroom started with my students grouped in six cooperative groups of three to four students. I grouped them heterogeneously by ability, male-female, and socialization. I usually do not group students who will tend to socialize too much, but during this study I did. I wanted to know if I could use their need to socialize to my advantage in teaching. Some of my students were not students who typically socialize. They had their close nit group of friends and that was all. I felt that grouping these students together as well could prompt and encourage them to share their thoughts, ideas, and justifications with others.

I used a document camera to model mathematics problems in my classroom. I usually started off the day by showing a writing prompt on the document camera for my students to write about in their mathematics journals. After they wrote about the subject, they shared their ideas with other students at their table. My goals in using writing prompts were a) to assess my students' knowledge of the previous day's assignments, b) to have my students express their knowledge of the mathematical subjects in their own way, c) to give my students practice expressing themselves to others, d) to allow them to use different venues (verbalization, charts, sentences, etc.) that they are most comfortable with expressing themselves, and e) to allow them to experience other students' perceptions about how they viewed the specific mathematical writing prompt. I started at the beginning of the year with daily writing prompts related to the basics of fractions and decimals. Some examples included, "Explain what each place value in the number 6,345,086.34 means," "explain how to put the following numbers in sequential order: 346.23, 326.22, & 346.25," and "explain how to add and subtract numbers with decimals." As the study progressed, I used writing prompts dealing with adding and subtracting mixed numbers, multiplying fractions and decimals, and division of fractions and decimals. Each writing prompt dealt with a skill that had already been learned or with an application of that skill. I collected these writing prompts and used them to assess my students' knowledge and application.

I generally used mathematics manipulatives in my classroom as well. Before I started a lesson using them, I allowed my students to use the manipulatives to build, organize, play, or create anything they wanted for five minutes. During this time, they socialized with their friends as well. By allowing them to use the manipulatives any way they wanted and at their own pace, they could have fun and learn how to use them at their own discretion. They were also better able to visualize mathematical concepts and gain insights into fundamentals of mathematics (DeGeorge & Santoro, 2004). After free exploration time, I conducted a lesson by allowing and encouraging my students to use the manipulatives. A typical lesson might be for the students to work in groups and solve the problem 1/5 x 3/7. As they worked through the problem, they wrote their explanations and justifications in their mathematics journals. I then had them share their thoughts and ideas with their table partners.

After working with small groups and the whole group, my students completed an individual assignment, typically originating from the textbook. Students were encouraged to work with others in their group, or with me, during this time if they had trouble with the mathematics concept.

I encouraged my students to participate in classroom discussions often. I usually started this by allowing them to discuss mathematical topics in their own cooperative groups. This created a setting where the child felt less intimidated by the whole class and the teacher.

During classroom discussions, I frequently used a handheld tape recorder to record my students' conversations. By taping their conversations, I felt I could much more readily capture the true essence of their conversations. Only pseudonyms were used during this study.

Data Analysis

I analyzed the data collected from the students' pre- and post-tests to determine if their learning of fractions and decimals improved

By comparing the students' pre and post self-assessments, I determined if their self-efficacy and self-concepts changed. "Self-efficacy represents the judgment of confidence that individuals have in their abilities, while self-concept provides a description of the individual's own perceived self, accompanied by an evaluative judgment of self-worth." (Lackaye, Margalit, Ziv, & Ziman, 2006, p. 112). I feel it is very important for adolescents of this age to have a positive self-efficacy and self-concept.

I also compared and contrasted the parent surveys. My students' parents most likely had a preconceived notion of mathematics and how their children felt and performed. I used the post-survey to analyze how their perception of my teaching influenced their children's learning. Finally, I analyzed, compared, and contrasted students' journal entries, recordings of conversations, and discussions to show a change in learning. I used a small hand-held tape recorder to record teacher-student conversations and student-student conversations. The written transcripts of these conversations were used to analyze trends or changes in social and sociomathematical norms among students. Journal entries were compared and contrasted as well.

Summary

The qualitative tools discussed earlier helped me as I analyzed the effects of changing the sociomathematical norms in my classroom. The data I collected from this action research project helped me to identify the effects.

In the next chapter, I will discuss the results of my study, showing different themes that emerged from the data that I collected and analyzed.

CHAPTER 4: DATA ANALYSIS

Introduction

I started my study with the preconceived notion that my students would ultimately embrace this new norm of socializing and cooperative grouping in the classroom with open arms. My plan when starting this study was to examine the pre and post decimals and fractions tests of each student to determine if the they gained knowledge. This was my preconceived knowledge of how to evaluate students. It was simple in my mind: "Improvement on test scores shows a gain in knowledge and a lower score means that the students did not study." That thought was very egocentric on my part. I found myself often struggling in this area.

I also thought that sending home parent surveys (Appendix E) would help my students' parents "buy into this concept." I tended to try to please the parents of my students more than expected. I knew that using a parent survey would help to get the opinions of those closest to my students about their learning of decimals and fractions. I later realized that although it is good to know what parents perceive about their child's learning, it was not central to my study.

I struggled somewhat with allowing my students to have the freedom to talk during instruction time. Did I really want to have a "loud classroom" with students talking to others at their leisure? What would my principal and my students' parents think if they came into my classroom and saw students talking? I realized much of what was standing in my way was the word *my*. If I wanted my students to *want* to learn, I would have to get rid of the word *my* in “my classroom” and substitute the word *our*. This would become “*our* classroom.” I was to become a facilitator of conversations (mathematically based, of course), not just a presenter of knowledge.

I am and always have been the type of teacher that likes to control the classroom. I am very structured, always on time to lunch, and wanting of a quiet classroom. I have been complimented in the past on how well-behaved my students are inside the classroom and elsewhere in the school. I found myself wondering if my students were actually learning in this environment. They have always scored well on the state mandated test involving mathematics so, in my mind, I was doing my job. But was *doing my job* enough? Did my students understand why and how they scored well on tests?

As I read the first few transcripts of my students’ conversations, I realized they were “going through the motions.” That is, they were answering questions as they had been taught in the past. They seemed to know the routine: speak when spoken to, don’t argue, and the teacher is always correct. How would I overcome this obstacle and help my students become thinkers and risk-takers?

In this chapter, I will describe the patterns that I saw in the classroom as my students participated in classroom and small group discussions. First, I will discuss my teaching. Second, I will discuss how our classroom reacted to the new sociomathematical norm in our classroom. Finally, I reflected upon what I had learned from my study.

Barriers

Students

My students seemed to be "rule followers." What I mean by that is they have been conditioned since their first experience in school (most likely pre-kindergarten) to continually follow the rules, and that the teacher is always right. By the time these 11 and 12 year old children arrived in sixth grade, they may have been involved in six or seven years of this mindset. They expected to be taught "the facts" and then "regurgitate" their newly learned skills on a test of some sort. One of my students, Billy, was a prime example of this. His favorite phrase to say when he came up to me to ask a question was, "I don't get it." I became frustrated when he did this. I oftentimes said to him, "Did you read the directions on the previous page?" He responded with, "Oh yeah." As the study proceeded, Billy continued to say his favorite phrase, but he also became a leader in his group. He frequently had to be interrupted by his classmates because he became so excited about sharing his justifications.

Billy was an extreme example of my students not thinking for themselves and having the expectation that the teacher will always show them how to "understand the mathematics involved." Yet a good portion of my students had some degree of this thought. How would I be able to teach using a new sociomathematical norm involving justification and cooperative grouping if many of my students tend to not justify their mathematics in the first place? As I proceeded through the study, I realized that when given opportunities and encouragement, my students justified their mathematical reasoning often.

Parents

What would my students' parents think of this study? They most likely were taught in a "traditional setting," that is the teacher teaches and the student follows directions without needing to understand why. I was taught using that teaching philosophy. I feel that is why I tend to revert back to the traditional lecture, model, and practice method used often during my days as a student.

Of all my parent permission slips sent home, only one was returned not allowing the child to participate. I was pleased that all but one of my students' parents was willing to allow their children to participate in this study. I was even quite surprised by one response on the pre-survey from Cassie's mother. She wrote, "Absolutely great work on your side and I support you 100%!" Darbi's father has talked to me several times and told me how much he trusts me with his daughter's education using this study. I realized that parent participation was not actually as much of a barrier as I had thought but an asset to my study.

Myself

I had a few expectations about problems and challenges involved in my study as it pertained to the classroom. My biggest concern was giving my students that much freedom to socialize in class. I am a very structured teacher. I thought that if I allowed the students to talk, they would constantly get off task and discuss other topics besides the mathematics at hand. I tend to get upset when the classroom noise level gets too high. I realized that I must get past this tendency if I was to implement this new norm.

Throughout this study, I struggled to stay on task and not revert back to my traditional ways of teaching. I tended to "cop-out" when my students got overly loud and out of control (at least out of control in my mind). I made the students work silently as to alleviate the noise level. I realized that by squelching the "noise" I completely stopped the student interactions, thus most likely reducing the learning. My students seemed to *need* to socialize. If I stopped that, I took away a powerful means of teaching. My students most likely would perceive me as the teacher who rules the classroom, thus reverting back to the traditional teaching methods with which they were familiar.

Student Interactions

In the Beginning

When I first started allowing my students to discuss their ideas about a mathematical concept, they seemed to not know what to do. My goal was to encourage them not only to talk about mathematics and their justifications but to *want* to talk about it. I always gave them five to seven minutes to discuss their topic in small groups (usually 4 students with one table being six students). Most students just told the other students in their groups what they did while the others passively listened. No real dialogue was happening. There did seem to be excitement in their conversations. As I wandered around the room, listening to their discussions, I caught bits and pieces of non-mathematical conversations. I understood that this would happen. These children were naturally social. They just wanted to talk. The noise level became quite high. I separated the group of six into 2 groups of three. I noticed the dynamics of this large group. Arvis and Dennis, two boys who were considered average in mathematics, did not participate in the discussions. They realized that they could easily "blend in" to the group and let some of the other aggressive students dominate the conversation. By putting them in smaller groups and separating Arvis and Dennis from each other, they were essentially forced to participate in the group conversations.

After the Change

The process of encouraging my students to have a voice in their conversations was a gradual one. Many different personalities had to be dealt with. Some of my students, such as Alice, almost panicked in the classroom when called upon to talk. Alice was extremely shy and rarely made eye contact, except with her closest friends in a setting that was comfortable for her. Another student, Lou, had a wealth of knowledge and incredible abilities in mathematics. He loved solving problems, especially solving them in several different ways. Yet Lou, with all of his mathematical abilities, had trouble expressing himself. He struggled with telling others how he justified his work. I often told him, "Lou, you need to put what is up here (pointing to his head) down here (pointing to the table) so that others can understand it up there (pointing to the heads of his group members)." Lou continued to struggle throughout the study, but he learned to slow down with his explanations and try to justify his reasoning in a way that others could understand.

Student Dialogue

Student dialogues were quite easy to initiate. Quality discussions about mathematical justifications were not as easy to initiate. Our first attempt occurred during a lesson on place value. I wanted to establish a precedent for quality justification, correct vocabulary, and open discussions. The students were asked to explain how to solve 36 times 21. Most of my students solved this easily but did not justify their explanations. The following is a conversation involving Donna and myself. I dominated this conversation. I oftentimes elaborated on Donna's brief explanation instead of letting her or another student elaborate. Donna responded to my questions but did not speak unless spoken to. Donna was telling me the procedure for multiplying 2-two digit numbers.

Mr. N:	How do you multiply a two digit number by a two digit number?
Donna:	You line up the ones and the tens.
Mr. N:	OK, can you give me an example?
Donna:	36 times 21.
Mr. N:	She told me to line up the places, so I know to line them up like so with the 6 ones over the 1 one and the 3 tens over the 2 tens. Ok, so we have 36 times 21, what next Donna?
Donna:	Multiply the one times the six.
Mr. N:	Ok, one times six.
Donna:	is 7...oh (*Donna inadvertently added*).
Mr. N:	Ok, 1 one times 6 ones equals 6 ones, ok?

Donna:	And then one times six ones, and 1 one times 6 in the ones place and then one times 3 in the tens place. You have six there so you're going to put the three next to it.
Mr. N:	Ok, what next?
Donna:	Add a zero under the six.
Mr. N:	Ok, add a zero under the six.
Donna:	Then multiply the 2 tens and the 6 ones and you get 12.

This exchange between teacher and student shows that she had knowledge of how to solve the mathematical expression but she had trouble understanding why she used that specific procedure. Donna told me to multiply the one times the six. Donna knows the procedure for multiplying two digit numbers, but does not know why she multiplies the "2 and the 1."

I also learned something very profound from this conversation; I tend to dominate the conversation in my teaching. Donna basically answered my questions. No other students were involved in the discussion. My goal was to understand my students' thinking and justification, but by my own actions (dominating and leading the conversation) I seemed to give my students a reason *not* to justify their answers. I felt that I needed to stop dominating the conversation if I wanted my students to begin to justify their answers.

I began group discussions with addition of fractions. I wanted to see how the students interacted with each other while trying to solve a mathematical equation involving fractions. I gave the following problem to each group to solve together:

2/3 + 1/6. I knew of some dominant personalities within the groups. I was curious to see how they would interact with other personalities. Cooper was not one of those dominant personalities. He was shy and rarely participated in classroom discussions. I felt I put him on the spot during this dialogue. I wanted him to justify his reasoning, yet I did not allow him to do that. The dialogue and justification I had been hoping for may have been from Cooper but because I tended to dominate the conversation, I stifled him.

My goal was for my students to justify their thinking. In the conversation below with Cooper, I was not a good role model for that justification. He asked me for justification and I did not give it to him.

Mr. N: Ok, so you added together 2/3 and 1/6. How'd you add them together? What did you do....Daniel?

Daniel: I took 2/3 and multiplied the numerator and denominator by two and got 4/6 and I added the numbers 4 and 1 and it came out to be 5/6.

Mr. N: Ok, that sounds good. Kristina, how about you?

Kristina: I just put two and I found a common denominator between 1/3 and 1/6 which ended up being 6. So I put one mark on there and it was 2/6 and I added that one sixth and got 5/6.

Both Daniel and Kristina showed an understanding of how to make equivalent fractions with common denominators, and then add the numerators. Both students had scored well on the pre-test on addition of fractions. Yet both Daniel and Kristina did not justify *why* they changed the denominators. Both were using rote memory to solve the equation.

The conversation continued at this point using 2/3 + 1/6.

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Mr. N: Cooper, what about you?

Cooper: I did 1/3 and 1/3 as my denominator and got 2/3, which I basically doubled the 1/3 and then I....

Mr. N: Ok, you may not see what Cooper did...he did this...he had 2/3 and then he had 1/6 and he had a plus sign in between. He said, "What I did was I took the 1/3 and I changed it to 2/6", then he changed the other 1/3 to 2/6, and then had the other 1/6 and then, Cooper, what did you do next?

Cooper: And then I added the 2/6, the other 2/6 and the 1/6.

Mr. N: Ok, Cooper, one thing you said all right, basically you took the 1/3 and I doubled it. Ok, so you doubled 1/3. Tell me what you mean by "I doubled 1/3."

Cooper: I knew that 1/3 and 1/3 equals 2/6.

Mr. N: So did you double just 1/3 or did you double 1/3 and 1/3?

Cooper: I doubled just 1/3.

Mr. N: Ok, you want to say something about that Chet?

Chet: Uh, yeah. He doubled the numerator and the denominator to get 2/6 and then you have 2/6 plus 1/6.

Mr. N: Ok, so Cooper did you really double that 1/3?

Cooper: Yeah.

Mr. N: What did you really do? I know you are telling me what was supposed to be done, but what did you really do?

Cooper: I....(*Cooper had shut down. He seemed to feel as if he was being 'ganged up on'*.)

Mr. N: Ok, what do you think? (*speaking to the rest of the class*)

Mary: He simplified it?

Mr. N: He simplified it.

Mary: To make the denominators the same.

Mr. N: Simplified it, ok. What did you do, Betty?

Betty: (*Still reacting to what Cooper said*) I think he multiplied the numerator and the denominator by two to get 2/6.

Mr. N: Ok, he multiplied the numerator and denominator by 2 to get 2/6. Did he double the 1/3 though?

Betty: No.

Mr. N: What did he do, or...Cooper, what did you do?

Cooper: I added 1 and 1 and 3 and 3 and got 2/6 and I did the same to the other number.

Mr. N: Ok, Kitty?

Kitty: He unsimplified it.

Mr. N: Yeah, I don't know what other term to use other than 'he unsimplified it'. He didn't simplify it, he made it tougher, I don't want to say tougher. He didn't make it more complicated.

Peter?

Peter: He made an equivalent fraction.

Mr. N: Good, he made an equivalent fraction, not a double. If he doubled 1/3, he would get (*showing 1/3 on the document camera*) here's 1/3. If I double it I get 2/3, so it's a totally different amount. He just made it equivalent; same amount. Yes Kristina?

Kristina: I thought making a common denominator made it equivalent?

Mr. N: Ok, it does make it equivalent, good. So actually what we are doing is we take that one third and make its equivalent fraction. It's like you know, pizza. Here you go Anna (*speaking to a student,*) you can have 1/3 of the pizza. And she gets a big old slice. And she's sitting there (*saying,*) "oh my gosh, I'm gonna eat it all up." And I say, "Here you go Calle, you can have 2/6 of this pizza." And she gets 2 slices that are easier to pick up, but she holds them up to Anna's big old slice and it's the same amount. We're just renaming it.

Cooper: How do you know?

Mr. N: I've got 2/6 and 2/6. How do you know? I've got 2/6 and there's 2 of them. One, two, three, four, five, six…we get five sixths.

There were several things that I noticed about this conversation. I not only confused the other students, but confused Cooper very much! He was explaining his justification and I interrupted him and completely 'shut him down.' I wanted the students to justify their answers but, many times I did not allow them to do that!

The students were not freely conversing with me and others about mathematics. They were still just responding to my prompts. I was leading the conversations and in many cases I was explaining the solution to them. In the above conversation, I felt that I intimidated Cooper. He was trying to explain how he added 1/3 and 1/3 and I interrupted him. I then used him as an example of how not to add fractions. I felt as if I completely shut him down and perhaps embarrassed him in front of his peers.

I also noticed that while I was trying to get students like Cooper to justify their answers, *I* was not justifying *my* answers. I explained to Cooper about how he was making equivalent fractions with the pizza example but I was not justifying it. Cooper asked me, "How do you know?" He wanted me to justify my answer. I felt as if my answer to him was not justification enough. I felt that I had just resigned myself to thinking that Cooper did not understand the problem and I would just go on with my teaching. I should have stuck with Cooper or at least worked with him one on one. That was a great teaching opportunity that I did not take. How was I to get my students to think for themselves and take risks if I dominated the conversation? I would have to drastically change my style of teaching to affect the students' style of learning.

Another example of dialogue between students and me occurred a few days later. We had been studying exponents and using their applications with multiplication of decimals. The students worked in small groups to solve the following equation: Create a double line graph using the following 2 algebraic expressions: $y = 4^x$ and $y = 0.5^x$. The variable X will represent the exponent. My goal was to have my students discover the similarities and differences between multiplying whole numbers and multiplying rational numbers, in this case decimals. Lilly was having trouble creating her vertical axis for her line graph. She and her group have in the past created vertical axes on a graph that had consistent whole number intervals. They had created a whole number interval and realized after evaluating $y = 0.5^x$ they had to create an interval that allowed them to graph rational numbers. She was confused as to why the product of $y = 0.5^x$ would get smaller as her exponent got larger at first but began to understand as she continued. I did not seize a wonderful opportunity to ask her to explain why the product of $y = 0.5^x$ got smaller as the exponent got larger.

This group of students did not have much dialogue among their members. They had broken the assignment into two parts and paired up to solve one equation for each pair. During this first conversation, Lilly was involved but her other group members did not respond to any of the conversation.

Mr. N: Lilly, tell me something about 0.5 compared to 1.

Lilly: It's less than 1.

Mr. N: Is it less than 1?

Lilly: Yeah.

Mr. N: Is that going to go with the 4? (*I was pointing to the equation* $y = 4^x$)

Lilly: That is going to go lower, it's going to go even lower.

Mr. N: Ok, so 0.5 will increase the exponent number, the x will take the product lower?

Lilly: Yeah.

Mr. N: What are you going to do?

Lilly: and on this side (*pointing to the vertical axis*) maybe do something like .25: half.

Lilly had discovered that her group's original vertical axis intervals would not accommodate $y = 0.5^x$. I purposely gave them a whole number with an exponent and a rational number with an exponent. I was hoping that the students would be able to work together in their groups and come to a consensus about what interval to use on their vertical axis that would accommodate $y = 4^x$ and $y = 0.5^x$. Lilly needed a little persuasion in understanding that she needed to have intervals less than 1 as well. The conversation continued. At this point, Lilly was having trouble collaborating with one of her classmates, Kamila. Lilly felt that she completed her portion of the assignment and Kamila was not doing her share of the work. Lilly and Kamila were not working together, just working on separate parts of the assignment.

Lilly: Can I ask you a question?

Mr. N: What?

...

Lilly: ... I was doing the fours, right? (*she was referring toy =* 4^x). And she (*referring to Kamila*) was doing the 0.5 (*referring to* $y = 0.5^x$) so I did my part and then me and

Cassandra were just chitchatting and she's sitting there, "It's too hard." I'm getting frustrated over that.

Mr. N: Ok, so let's think about this. You multiplied whole numbers. Arvis and Kamila multiplied decimals. Which is harder? (*I was looking for their opinion*).

Arvis: Decimals.

Mr. N: Decimals, ok, so maybe she's having a little trouble with it. You have four people in the group. What can you do?

Lilly: Help each other?

Mr. N: Help each other. Now Kamila, you can't say, "I can't do it, Arvis you do it!"

Cassandra: Actually, she did say that.

Mr. N: Ok, so can you help each other?

After talking with this group, I realized even more that when dealing with individuals in a group setting, I must adapt to their own specific learning styles. This group's idea of sharing the work was "divide and conquer." In Lilly's mind, Cassandra and she were going to solve the algebraic equation $y = 4^x$ while Arvis and Kamila were going to solve $y = 0.5^x$. Other than that, they were not going to work together.

I found this trend in student behavior often. Many of the students seemed to be very self-centered. They were very concerned with completing tasks at hand and were not overly concerned about others, especially ones in their own group. I thought that because the students had a desire to socialize, that if I grouped them together they would socialize about mathematics. I didn't think that their conversations would stray away from the topic most of the time. Throughout the study, I encouraged the students to help each other in all areas of mathematics. I wanted them to share their knowledge with others in their groups. I felt this was an excellent way for them to verbalize their thinking. It also allowed for students to learn from a different person, not just me.

The following dialogue took place about two weeks after the previous one. A group of four students; Calle, Daniel, Arvis, and Kamila were discussing the problem $y = 8.5 \div 1/2$. They were using fraction bars to help them as well as paper and pencil. The conversation came in as Calle and Daniel were having a heated discussion. Calle was trying to explain her reasoning to Daniel and the rest of the group. Calle was trying to explain to her group how she is dividing 8 whole fraction bars and 1 half fraction bar in half. Both students (Calle and Daniel) were convinced that their method of solving the problem was correct. As they continue with the conversation, they began to respect each other's opinions. Also, my role began to change in this conversation. I was less of a teacher and more of a listener. Earlier, I had dominated most of the conversations. During this dialogue, I took on more of a role of listener and facilitator.

Mr. N: Ok, go ahead and try to explain it. Calle's going to explain.

Calle: A half of 8 is 4, wait, so like on this part (*she is pointing to 8 whole fraction bars.*) Eight wholes divided by one-half and then another half like right here (*pointing to a half fraction bar;*) one half and you have a whole bunch of wholes and you divide 8 wholes by one-half and then you take each whole and put a line through it meaning it's one half so then you count each half…wait. Count each half; 1, 2, 3,…16 and you add and then there is another half right here (*pointing to the half fraction bar*) so that one half and one half is one.

Kamila: What I don't get is why you changed it. If each one is a half….

Mr. N: Listen to Daniel, he wants to ask you (*Calle*) something.

Daniel: (*To Calle*) Why did you change that into a fraction?

Daniel did not remember that he could convert decimals to fractions if he wanted to. I had left the directions open so as to allow them to use a strategy that best suited them. Calle converted the decimal 0.5 into the fraction 1/2 because it best suited her in using the fraction bars. The conversation continued, mostly between Calle and Daniel.

Calle: Because 0.5 can be turned into a fraction so that we can divide a fraction into a fraction.

Daniel: Why don't you try changing that into a decimal and then…

Calle: (*interrupts*) Because then…..

Daniel: (*interrupting back*) ….find out how many places each divisor and the dividend and if you add the places there would be 2 places to the left and get 17 and move 2 places to the left and you'd have a decimal right there.

Daniel explained how he would solve the problem using decimals. It seemed to be that he was starting to defend his reasoning for solving the problem as superior to Calle's. The conversation continued with Daniel and Calle trying to convince each other that their method is correct.

Calle: But wait, this is fractions, not decimals. I have to talk to you about decimals (*meaning another time.*)

Daniel: But why don't you change it into a decimal like you had?

Calle: Sometimes if you write it down you can show it.

Daniel: (*Completely ignoring what Calle is saying and showing how he solved his problem*) Divided by…

Calle: Where did you get the 5 from?

Daniel: I had it there (*pointing to his paper*).

Mr. N: I think he was covering up the decimal there.

Calle: But would you originally already have the decimal right there?

Daniel: Yeah, that's it.

Calle: Yeah, but wouldn't it be, like, 4?

Arvis: Wait, we put 5 goes into 8 more than one.

Daniel: Each whole divided by a half is one half. One half and one half is one whole.

Calle: (*trying to think*) I know, wait.

Daniel: (*continuing*) So that would be 1, 2, 3…that would be 8 whole and 1 half. That would be 8 and a half.

Calle: We have 17 halves which is like 8 wholes and one half.

Kamila: You're multiplying aren't you? Wait. I put 17 halves instead of how many wholes it is. It equals 8 1/2.

Daniel: Ahhhh, then you divide 16 by 2. That would be 8 and one half which would be .5.

Calle: Sixteen and a half would be 8 1/2.

The conversation (especially between two dominant talkers like Calle and Daniel) was all over the place. Arvis and Kamila had trouble getting their points of view into the conversation. As the conversation went on, I could sense a tension between Calle and Daniel. Each wanted to discuss their views of the mathematics over the other. Yet as they progressed more into the conversation, both Calle and Daniel started respecting each other's views and even accepting their views in some cases. Each student started off the dialogue with the notion that "their way was best." As they progressed, each started to respect the other's views.

Calle was able to relate decimals to fractions. She was able to change the problem I gave her ($y = 8.5 \div 1/2$) into fraction form ($y = 8\ 1/2 \div 1/2$) to better suit her. She showed Daniel, Arvis, and Kamila that she was breaking her 8 whole groups in half, thus giving her 16 half groups. She then added the other half group to the 16 to get 17 half groups.

I felt that I had made a breakthrough in my teaching as well. I took on the role of listener and allowed the students to take control of the conversations. It has always been tough for me to give up "my power" to the students. This conversation helped me realize that I needed to give my students an arena to speak and justify comfortably without interference from me. Calle did not usually participate often in classroom discussions. I saw a different side of her during this dialogue with Daniel. She was confident that her problem solving technique was correct and she did not allow Daniel to sway her thinking. By allowing her to speak freely in a comfort zone that she liked, I felt Calle was able to show how much she really knows about decimals and fractions.

As the conversation came to the whole group, the groups worked on a different problem; $y = 8.5 \div 2$. Cassandra and Erica eagerly volunteered to show their work on the document camera in front of the rest of the class. Erica did not say much; she was at the document camera for support. Cassandra had started to explain how she and Erica had solved $y = 8.5 \div 2$. Cassandra understood that if she divided by a number she could also multiply by its reciprocal. She proceeded to use the reciprocal of 2, but she neglected to multiply instead of divide. Tim came into the conversation and addressed Cassandra's misconception. Tim was kind about how he addressed Cassandra. I had taken the role as listener again, not dominating the conversation.

Cassandra: We had 8.5 divided by 1/2, which equals, well, yeah, 8.5 divided by .5 so you…8, half of 8 is 4 and half of 50 is 25. (*Cassandra was getting confused in front of the whole class. She is still relating back to the previous problem of 8.5 ÷ 1/2. I let her continue.*)

John: 50?

Cassandra: 0.50 is 0.5, so then you have your answer then we'd say ok, that's that. So why don't we just do it the other way. So we had times 2 is 8.5 so you have to round the problem. Do you have a question besides that? Yeah?

Dallas: So you didn't times your answer…..

Cassandra: Well, 8.5 divided by 1/2 is 4.25 but we had to show you that if you divide it by half or multiply it by half you get the same number.

Cassandra was showing a misconception about multiplication and division of decimals and fractions. I let the conversation go on as not to influence their discussion.

Erica: Yeah!

Tim: Um, I think I know what you were trying to do. It's just… it's not half it's 2.

Cassandra: Huh?

Tim: I get that you're dividing it out, but you're saying you're dividing it by half, and if you divide by half, half is .5 or 1/2.

Cassandra: Oh no, we're dividing by……

Sallie: Yeah, you're dividing by it by 1/2.

Cassandra: Never mind, it makes sense. (*Cassandra looked frustrated.*)

Tim: I don't know if we used the correct sign that you used, but we meant 8.5, that way it's half of 8.5.

Cassandra: Yeah, ok.

Although Cassandra was confused about multiplying and dividing a whole number and its reciprocal, she took a risk in trying to justify. She got flustered often when it came to discussing a topic she was not familiar with (such as fractions and decimals.) I was pleased that she wanted to discuss this problem in front of the class. Tim politely disagreed with her and justified his response. Cassandra continued to volunteer to share her work with the class and justify it during the study.

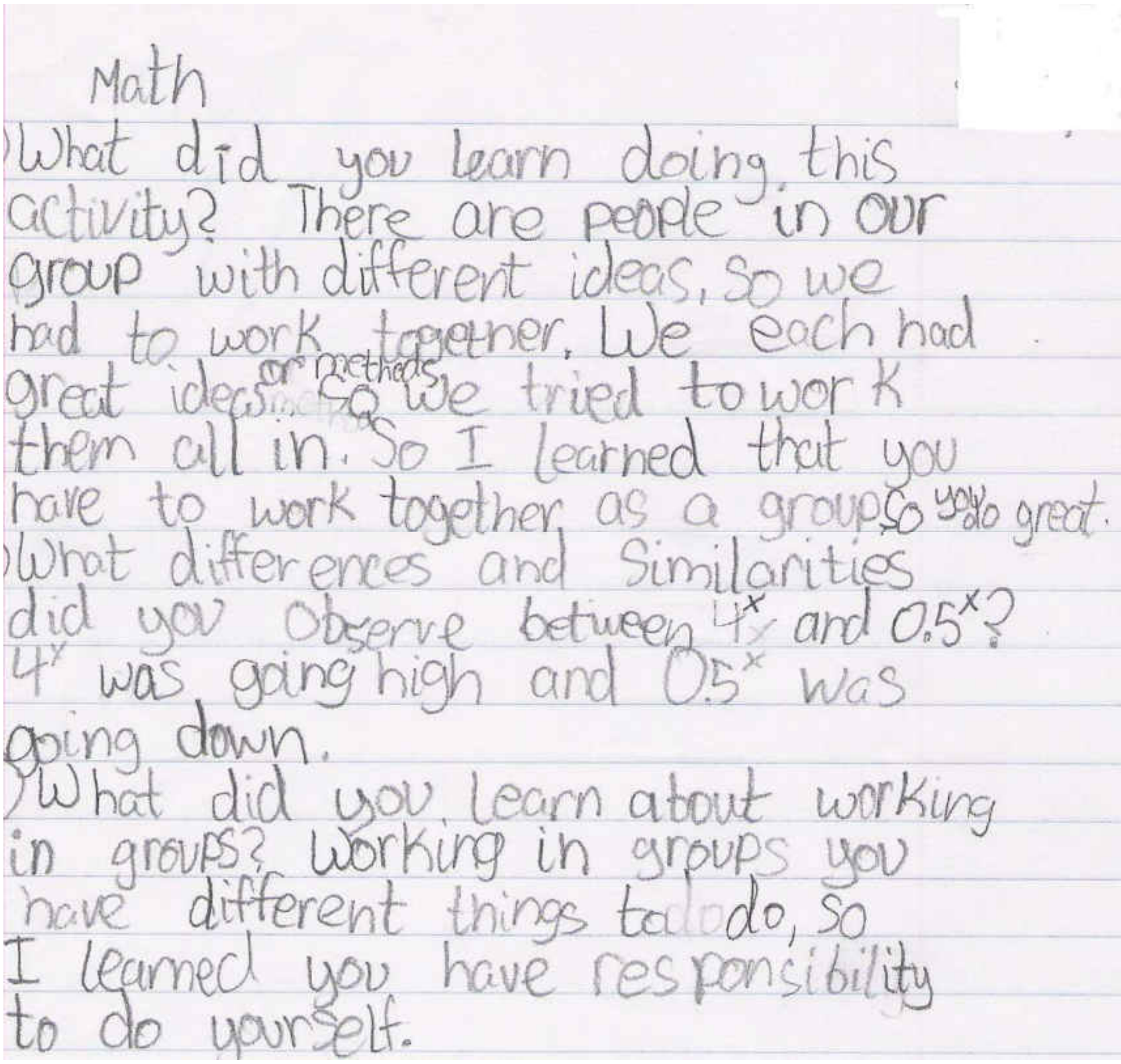
At this point in the study, this type of conversation was representative of most conversations in the class. Many students had a misconception of the relationship between fractions and decimals. I was completely unaware of this until I reviewed the transcripts of the students' conversations. I felt that I was not paying enough attention to my students' misconceptions. Because of this oversight on my part, I started teaching decimals and fractions together. I felt by doing this, students would be able to see more of a relationship between the two.

Journal Writing

I had not used journal writing as a means of mathematical expression prior to this study. I had always used the typical worksheets and questions from the textbook. Many of my colleagues had used journal writing but I had always shied away from it. I thought journal writing was for language arts classes. I used daily journal writing during this study. From the beginning of the year, as the students came into the classroom, I asked them to take out their mathematics journals, open to a clean page, and put the date at the top of their page. I put a mathematics journal topic on the overhead document camera for them to write about. I also put a countdown timer on the overhead as well so they could see how much time they had to write. Then I gave them five minutes to share with other group members. I expected them to explain their thoughts in their journals quite easily. I did not realize that their views of explaining were different than mine. I also thought that writing their thoughts down on paper would be easier for some children than speaking. In some cases it was.

Early in the study, I gave the groups a topic to work on; create a double line graph using the following 2 algebraic equations: $y = 4^x$ and $y = 0.5^x$ (this was discussed earlier in the dialogue section). I had them write in their journals the following; a) What did you learn doing this activity? b) What differences and similarities did you observe between $y = 4^x$ and $y = 0.5^x$? and c) What did you learn most about working in groups?

Darbi was an extremely shy student. She tended to be quiet in class, not answering questions for fear of embarrassment. She did not have much trouble with her journal writing (Figure 1). This example was reflective of some of my students, especially ones like Darbi who were verbally shy.

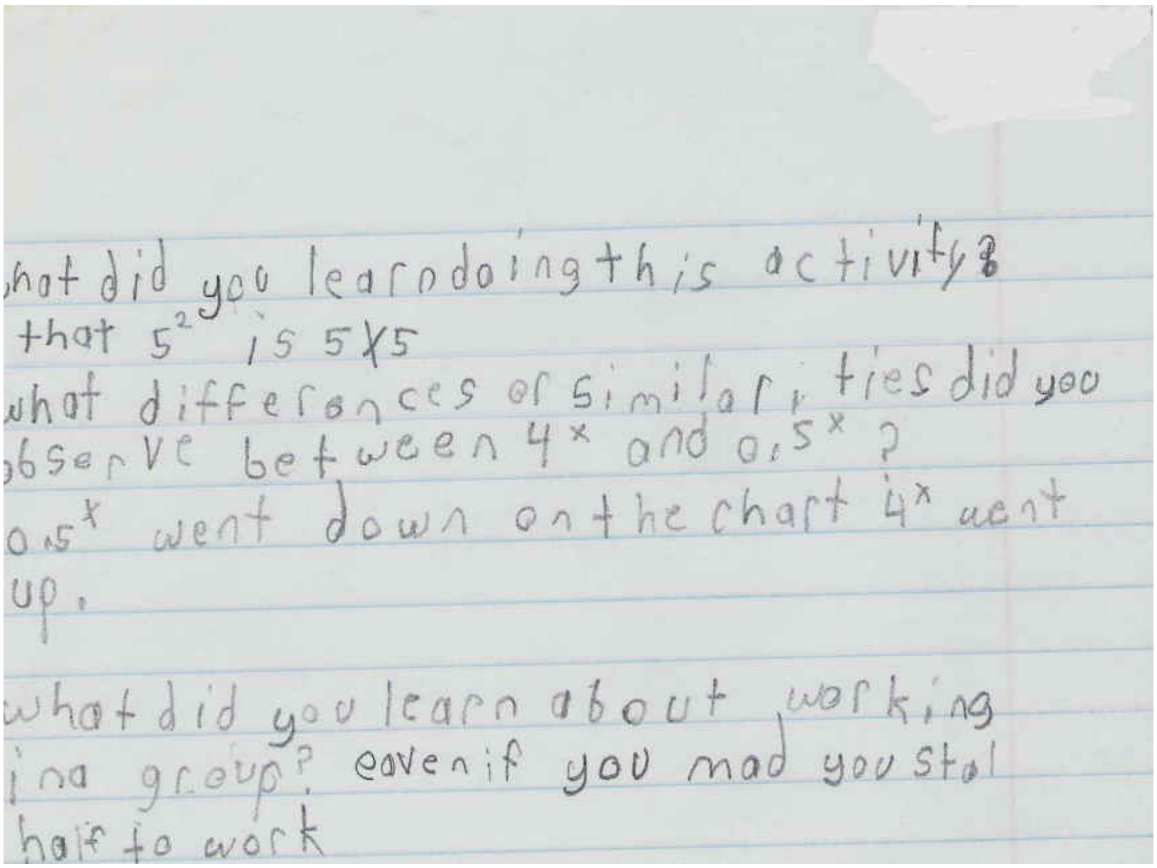


Math
)What did you learn doing this activity? There are people in our group with different ideas, So we had to work together. We each had great ideas or methods So we tried to work them all in. So I learned that you have to work together as a group so you do great.
)What differences and Similarities did you Observe between 4^x and 0.5^x? 4^x was going high and 0.5^x was going down.
)What did you learn about working in groups? Working in groups you have different things to do, So I learned you have responsibility to do yourself.

Figure 1: Darbi demonstrated her communication via journal writing.

Darbi had difficulty understanding mathematical concepts presented from me. She also struggled with presenting her own ideas to me or the whole group. Writing her thoughts and ideas in her journal seemed to help her.

Dennis continued to struggle presenting his ideas (Figure 2). His goal most of the time when he worked was to get his work finished. He rushed through with no concern for the quality of his work. Dennis' answers to the previous writing prompt were not detailed. He wrote "5 x 5 = 25" as his answer to the question, "What did you learn from this activity?" Dennis is a bright student but did not apply himself to the best of his abilities. Much of the time during group discussions he would allow others in his group to dominate the conversation so he would not have to participate. Many of his journal entries were similar to this one.



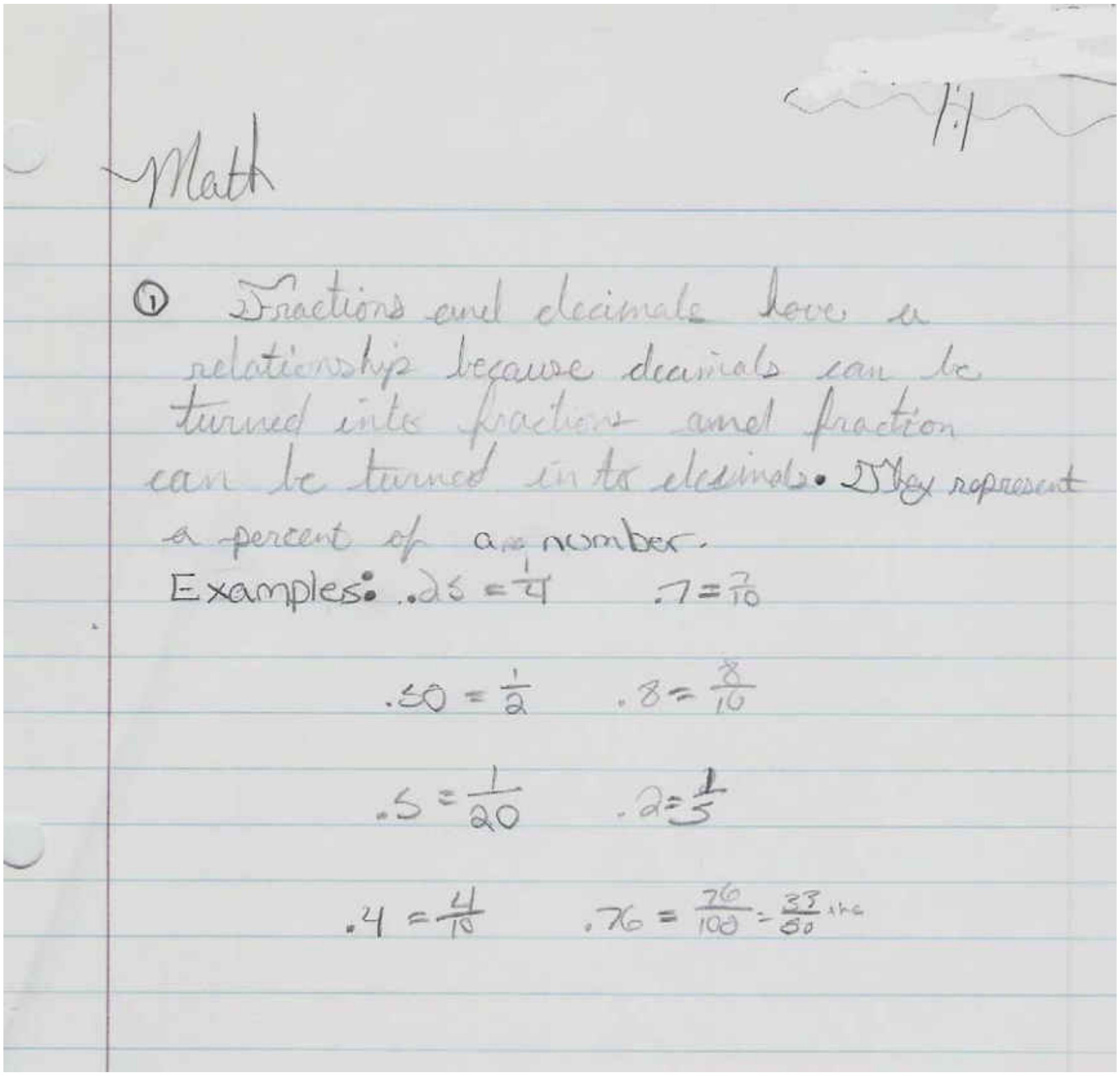
what did you learndoingthis activity?
that 5^2 is 5X5
what differances or similarities did you
observe between 4^x and 0.5^x ?
0.5^x went down on the chart 4^x went
up.

what did you learn about working
in a group? eaven if you mad you stol
half to work

Figure 2: Dennis's journal entry showed his struggle with writing.

Dennis continued to not elaborate with much of his journal writing, as evident from his answer, "0.5^x went down on the chart and 4^x went up." Dennis is a bright boy, but did not often elaborate in his writing. Several students' journals reflected this lack of elaboration as well.

As the study continued, students' journal entries became more detailed. This seemed to correlate with more detailed discussions as well. The following journal entry read, "Tell the relationship between fractions and decimals." Donna told of the relationship between decimals and fractions and even showed some examples (Figure 3). Yet she did not justify her explanation. She gave some examples of fraction-decimal equivalencies, but no reasoning as to why she wrote that. This non-justification appeared throughout most journal entries in the beginning of the study. The students seemed to just be going through the motions and writing to 'get the assignment finished.'


Math

① Fractions and decimals have a relationship because decimals can be turned into fractions and fraction can be turned in to decimals. They represent a percent of a number.

Examples: .25 = 1/4 .7 = 7/10

.50 = 1/2 .8 = 8/10

.5 = 1/20 .2 = 1/5

.4 = 4/10 .76 = 76/100 = 38/50

Figure 3: Donna showed the relationship between decimals and fractions.

Donna showed knowledge of certain decimals equaling fractions as evident from her writing .25 = 1/4. Yet she did not show any reasoning as to why “fractions can be turned into decimals and decimals can be turned into fractions.”

Donna and Dennis are just two examples of students who neglected to justify their reasoning in their journals in the beginning of this study. Several students did not explain their reasoning as to how decimals and fractions are related.

The students were asked daily to justify their answers and reasoning to other students, the whole group, and myself. As the study progressed, students became more proficient at justifying their answers. Students were asked to write directions and give examples on how to multiply mixed numbers. Helen had difficulty with this. She had a misconception about how to multiply 1 1/4 and 1 5/6 (Figure 4.) She related this skill (incorrectly) with adding mixed numbers.

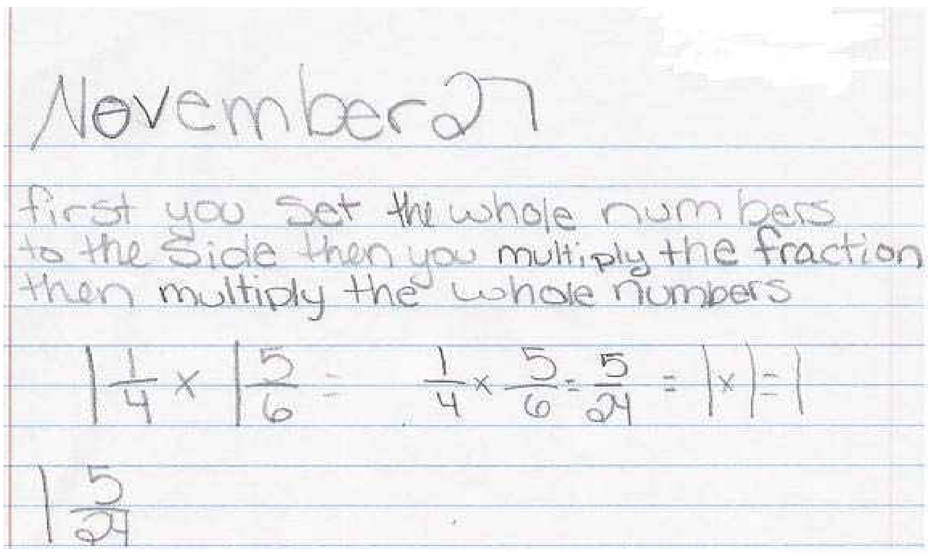


Figure 4 : Helen's misconception about multiplying mixed fractions.

Helen learned of her misconception from other students. Helen was receptive to their help. She came to understand the correct procedure to multiplying mixed numbers and more importantly *why* that is the procedure. Helen was able to relate her discussions with other students directly to her journal writing. As the study continued, students showed signs of more detailed journal writing as evident from one student, Julie.

Julie explained correctly how to multiply mixed numbers (Figure 5.) She even showed examples. Julie showed an understanding of the concept of mixed number multiplication. She readily explained to her peers how to multiply fractions. She still had not shown how she justified her reasoning. Her journal explained the procedure.

Ken also showed an understanding of mixed number multiplication (Figure 6). Ken wrote step by step directions as to how to change the mixed number into an improper fraction, simplify if necessary, multiply, and change the product back into a mixed number. Ken and Julie are prime examples during this study of how students explained their writings without actually explaining *why* these procedures work.

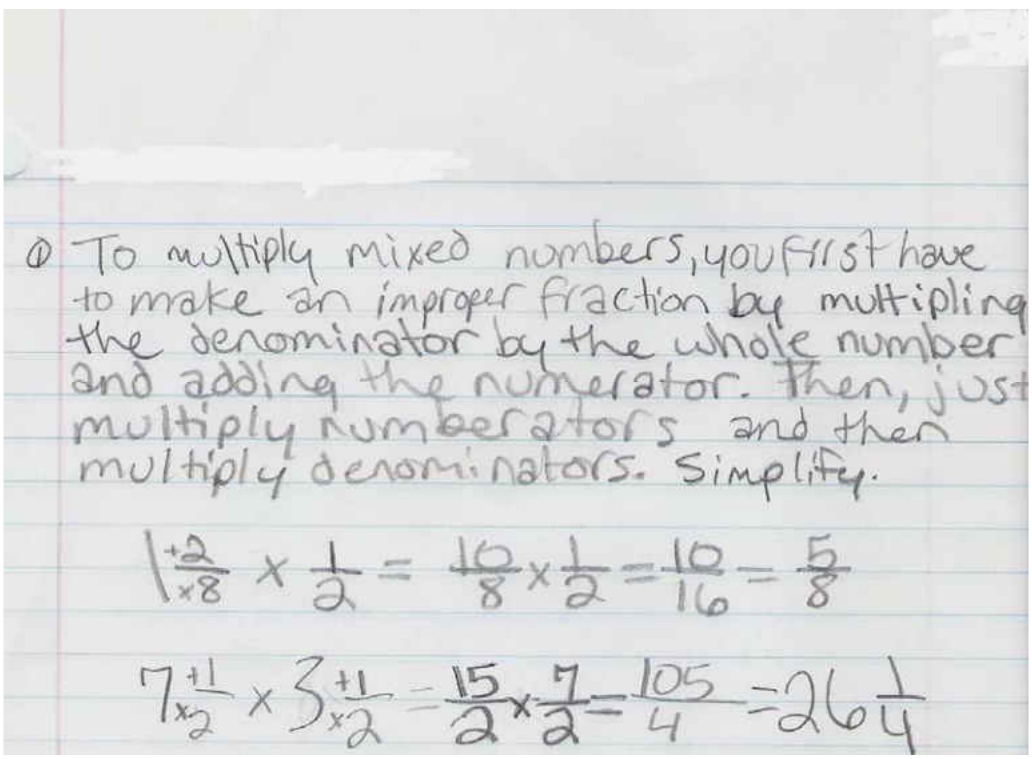


Figure 5 : Julie explained how to multiply mixed numbers.

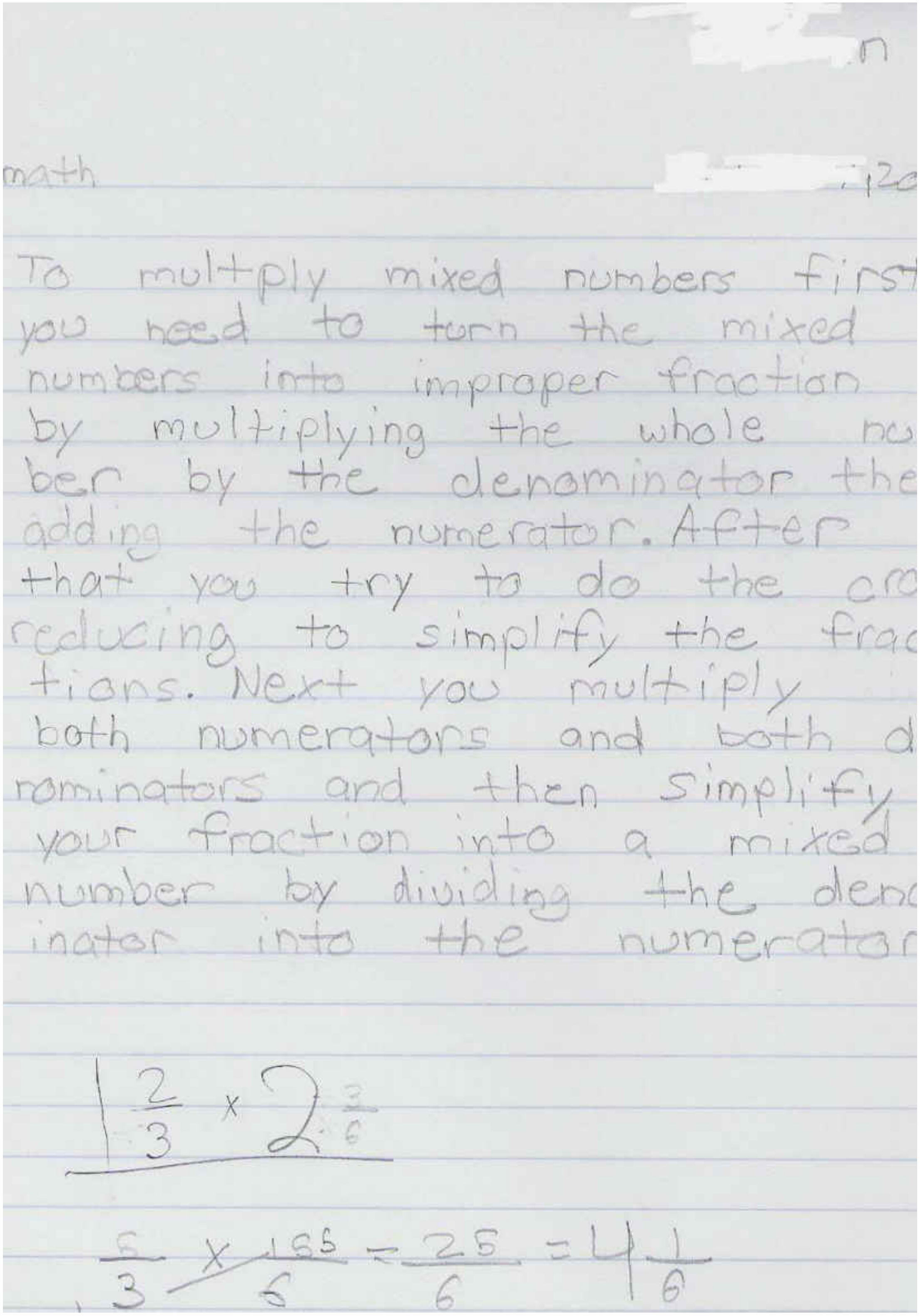


Figure 6: Ken demonstrated his knowledge of multiplying mixed numbers.

Ken, Julie, and Donna seemed to be combining their dialogue with their journal writing. This was the trend with most of the students involved in this study. Ken's journal entry (Figure 6) showed his writing as similar to his dialogue. He wrote in his journal as if he was talking to the reader. Julie seemed to write in dialogue form as well. By analyzing these journal entries and others in the class, it seemed as if the dialogues and discussions that took place daily in the mathematics class improved students' explanations with their journal entries.

Betty was one student who gave the impression that she understood why she multiplied. One of her journal entries said, "Show why 1 2/3 x 2 3/6 = 4 1/6." Betty showed the procedure for solving the problem (Figure 7). She changed mixed numbers into improper fractions, simplified, then multiplied. Her product was 25/6 which she converted to the mixed number 4 1/6. Yet Betty also showed her justification using fraction bars. She created 2 groups of 1 2/3 (she represented each group as 10/6). Then she created another 1/2 group, which she showed as 5/6. She added them together to get a sum of 25/6, or 4 1/6.

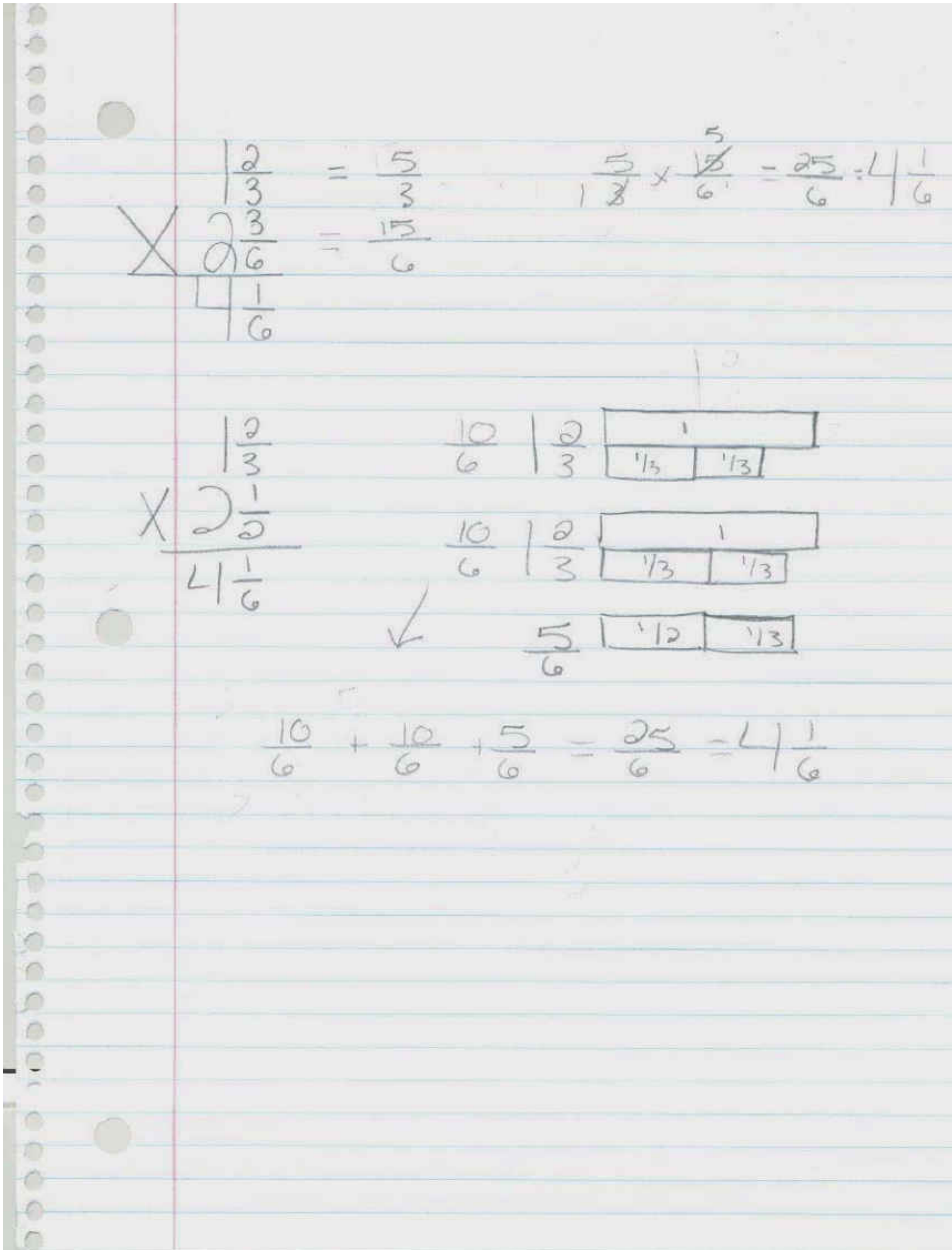


Figure 7: Betty multiplied mixed numbers with fraction bars.

Betty drew 2 3/6 groups (she actually told me she used 2 1/2 groups) of 1 2/3. She was able to justify to me and her group that she created 2 groups of 1 2/3 plus another half group of 1 2/3. She just "broke" the 1 in half and the 2/3 in half to give her a 1/2 fraction and a 1/3 fraction bar, which she grouped to make 5/6.

Students' became better adept at explaining and justifying their answers. Shy students such as Darbi, became better at explaining their reasoning, even in front of the class. I could see the struggle in her face as she tried to talk in front of the whole class but she persevered.

Students such as Mary, Peter, Betty, and Tim were able to explain their reasoning much better than others. For the most part, their socialization seemed to translate to their writing. Much of their journal writing seemed as if it were written in verbal language. Betty verbalized her reasoning for the following writing prompt, "Compare two fractions with the same denominator and different numerators. When you are finished, compare two other fractions with the same numerator and different denominators." She justified her thinking with explanations and pictures (Figure 8). She compared fractions and she even gave two examples of different numerators and two examples of different denominators.

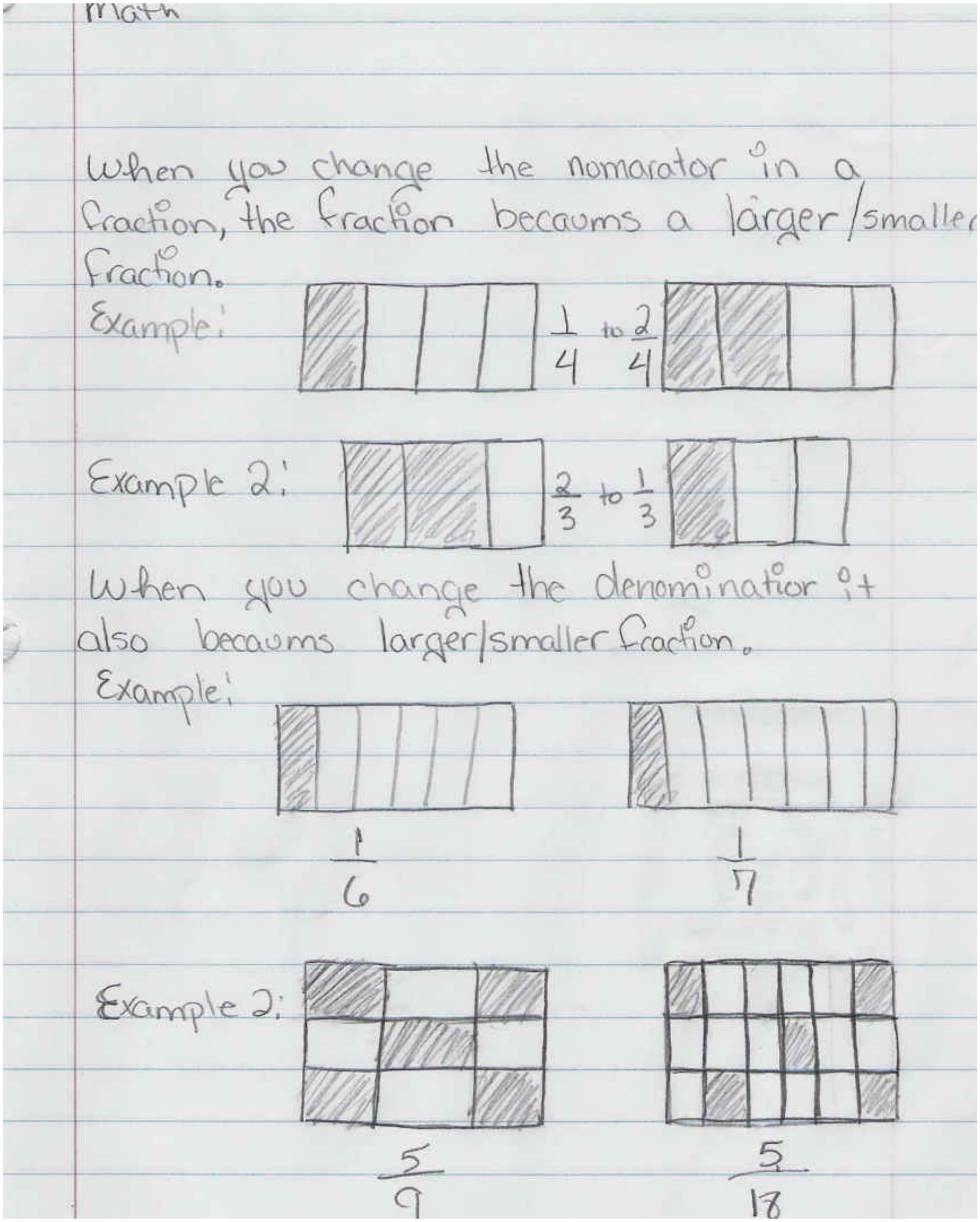


Figure 8: Betty showed knowledge of fraction relationships.

Betty compared two fractions with the same numerator (1/4 and 2/4.) She showed, by using drawings of fraction bars that when she changed the numerator and kept the denominator the same, the fraction size will increase. She also showed by using two pictures (1/6 and 1/7) and (5/9 and 5/18) that keeping the numerator the same and changing the denominator will decrease the fraction size. Betty was one of the first students in the class to start justifying her answers through pictures and not verbally. She showed an understanding of comparing fraction sizes by using pictures.

Summary

The data that I collected during this study showed me much about my students. My students' natural desire to socialize became a tool in their learning. They were able to work together in small groups and justify their thinking and reasoning. They became decision makers in their groups as well. Student dialogue translated directly to their journal writing. Journal writing became less of a chore to them as they became more proficient at it. I actually appeared as if they were enjoying their journal writing and sharing.

In the next chapter I will discuss key factors in the study. I will also review implications and recommendations for future studies.

CHAPTER 5: CONCLUSION

Introduction

I was curious to see if I could use the natural sixth-grade desire to socialize to help me teach decimals and fractions, commonly misunderstood concepts in sixth grade. Through this study, I was able to answer my research questions, "How does the role of the teacher influence the social and sociomathematical norms in a mathematics classroom?" and "In what ways are sociomathematical norms reflected in students' written work?" In this chapter I will discuss the results of this study, explore implications, and offer recommendations for future studies.

Results

One of my goals during this study was to see how the role of the teacher influenced the social and sociomathematical norms in the mathematics classroom? I knew from talking with the fifth grade teacher from the previous year that this group of students socialized a lot. I knew that I was an authoritative teacher who sis not like an "unruly" classroom. Yet I felt that it was not worth it for me to get upset when my students talked; they were going to do it anyway. I found that allowing, and even encouraging my students to socialize made an impact in our classroom. The quality of the dialogue in class improved. Students readily discussed mathematical concepts within small groups and within the whole class. Calle and Daniel, who discussed how to solve the problem 8.5 ÷ 1/2, are prime examples. Both showed an understanding of the concept of dividing fractions. This conversation was not prompted by me. Each of these students was able to discuss their reasoning as to why they thought their way of solving the problem was correct.

I also wanted to find out in what ways were sociomathematical norms reflected in students' written work. I encouraged students to write in their mathematics journals daily. I had never consistently used journal writing in my past classes because I thought they were used mostly for language arts class. I found that I was wrong in that assumption. Although Betty did not verbalize her justification for the topic, "compare two fractions with the same denominator and different numerators. When you are finished, compare two other fractions with the same numerator and different denominators" (Figure 8), she used pictures. She was able to convey her reasoning about that topic while not using words at all. I felt that this was a breakthrough in that it did not limit students to using just words for justification.

Students also discussed concepts among themselves, taking the initiative in those discussions. They learned that they were an integral and important part of our class and that their views and justifications were valued.

Implications

"Examination of the connection that students see between their generalizations and their justifications is important because these two components are closely linked—students' justifications provide a window for viewing the degree to which they see the broad nature of their generalizations and their view of what they deem as a socially accepted justification" (Lannin, 2005, p. 232.) To help support students in the building of their ideas, teachers should provide mathematical tasks that encourage student discourse in a small group setting and the classroom community, make tools available to enhance students' mathematical experiences, and closely monitor and facilitate the development of students' mathematical ideas (Martino & Maher, 1994.) Students need to be challenged in their mathematical thinking. They need to justify their reasoning. Students will need that ability to justify when they get into the workforce, whether it is in a technological field, education, or medicine.

The students tended to struggle with justifying their answers. Very seldom did they explain their reasoning. Much of their explanations dealt with algorithms or rote memory. Perhaps the students were taught in years past just procedures and algorithms. I felt as if I could have done a better job in modeling for them justification. Several of them seemed to have been almost understanding how to justify, but were still having trouble. I now know that justification should be an integral

In my study, I used what seemed to be a need of sixth grade students (socializing) to help me teach decimals and fractions. Students were encouraged to socialize with their peers but in a mathematical environment. I found that students became less worried about sharing their ideas and justifications in front of their peers and their teacher. Confidence built within them. Students who typically "sat by the wayside" and quietly completed their work became risk takers, trying to justify their responses.

I feel that I can no longer go back to traditional teaching; that is lecture, model, seatwork, and assess. I now believe that the classroom I teach in is not *my* classroom but *our* classroom. It needs to be a place where students feel comfortable in justifying their thinking and are willing and eager to take risks. Students need to be active participants in the classroom community of knowledge, experiencing new things and learning to their best abilities.

My subject matter in the study (justification, generalization, and sociomathematical norms) can be very subjective. What may be justification for one student may be completely different for another. Students vary from one class to another. I studied two different classes of roughly the same age, gender, and ethnicity. I found that most embraced mathematical socialization in the class while a few did not. These few students tended to be relatively quiet and were able to justify easier in writing. Because children differ considerably, this study may have produced different results with different groups of children.

This study would most likely produce different results with a different teacher. Teaching styles differ very from classroom to classroom. Some teachers are authoritative while others may be participatory. I know of teachers that would have a hard time teaching using this different sociomathematical norm. I was one of those teachers. I was able to change my classroom expectations. I struggled at times allowing my students the freedom to talk often in class. I know as a teacher, I must grow and adapt to my students, not demand that they adapt to me.

I found that students can learn mathematical concepts, specifically decimals and fractions, in a social setting. My students seemed to enjoy talking about their justifications. I saw the excitement in their eyes when they were called upon to share their work. I also found that risk taking increased. That is, more students volunteered to present their ideas to the class without fear of embarrassment. Teachers should involve students more in the teaching and learning process. I found that they have so much to say. They also have a way of communicating with their peers which differs from that of the teacher. Students are different and change during the course of the school year. Teachers need to constantly evaluate their teaching. They need to be flexible. This may involve changing their teaching style to accommodate student learning.

I have learned several things from completing this study.

1. I do not have to be such an authoritative teacher in the classroom. This will surely help me to 'reach' my students easier and hopefully they will learn from a less threatening environment.
2. I will use mathematical journal writing more often. After using this evaluation method, I felt as if I was able to understand students' reasoning and methods better if it was in their own words or pictures.
3. I will allow students to justify their problem solving their way. When students justify in their own words or pictures, they may be able to communicate their thoughts easier to others. This might also create a less threatening environment for the students.
4. I will not only allow dialogue in the classroom but encourage it. I found that by putting students into small groups and encouraging them to talk about mathematics, the students appeared to stay on task more than I expected. I had always been weary of giving students that much freedom in the classroom. Yet they for the most part stayed on task and learned from each other.

5. I will continue research on social and sociomathematical norms and how children learn mathematics, not just in the area of decimals and fractions. I have found that there is a plethora of research on mathematics that I can learn from. I can also use this wealth of knowledge to help other teachers too.

Summary

I have learned several things during this study. First, I learned that what I thought was justification was actually not justification. In my mind, if a student explained to me why they solved a problem (converting mixed numbers to improper fractions, multiplying and converting the product back to a mixed number) then that was justification. I learned later in this study that to justify, a student should show someone else their reasoning as to why they solved a problem. I felt after this study that for years I had not been encouraging my students to use justification. I had been "going through the motions," teaching the algorithms, and letting students pass the class with minimal work.

I also learned about myself. I have been for a long time an authoritative teacher. I am very structured and like "all my ducks in a row." I tended to inadvertently stifle my students' learning by demanding a quiet, orderly classroom. I learned that this philosophy caused much undue stress in my life and possibly the lives of my students. Why was I trying to control my students so much? I learned to embrace their strengths (socializing) and use that strength to help me teach. I also realized that we can learn so much from each other. I felt that my students became active learners and teachers in the classroom. They became better and more confident speakers as well. This skill will help them in the future in middle school, high school, and beyond. Most importantly I learned that students are people too. I need to treat them with respect and value their opinions. I learned so much from them during this study. I now feel that I am a better teacher for it.

APPENDIX A: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL

University of Central Florida

Office of Research & Commercializa

June 20, 2006

Marino Nardelli
4822 Buttonwood Drive
Melbourne, FL 32940

Dear Mr. Nardelli:

With reference to your protocol #06-3555 entitled, **"Will Changing Sociomathematical Norms in a Sixth Grade Classroom Change How Students Learn Decimals and Fractions?"** I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. **This study was approved on 6/16/06. The expiration date will be 6/15/07.** Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator. **Please notify the IRB office when you have completed this research study.**

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Joanne Muratori

Joanne Muratori
UCF IRB Coordinator
(FWA00000351 Exp. 5/13/07, IRB00001138)

Copies: IRB File
Juli Dixon, Ph.D.

JM: jm

12201 Research Parkway • Suite 501 • Orlando, FL 32826-3246 • 407-823-3778 • Fax 407-8

APPENDIX B: PRINCIPAL APPROVAL

Dear University IRB office:

I give Marino Nardelli permission to conduct research at [redacted] Elementary School in [redacted], Florida during the 2006-2007 school year. Mr. Nardelli has described the research he plans on conducting with his sixth grade students. I understand the scope of his research, and how he will be using the data collected. I understand that all data gathered will be done in a confidential and appropriate manner.

APPENDIX C: PARENTAL CONSENT FORM

Dear Parent or Guardian,

My name is Marino Nardelli and I am your child's teacher this year at [redacted] Elementary School. During this academic year, your son/daughter will be asked to take a number of assessments to measure the effects of different learning methods as it pertains to decimals and fractions. I am currently engaged in my Master's Program at the University of Central Florida. My thesis is based on the research in which your child will be participating.

Your son/daughter will be asked to take surveys before the implementation of the curriculum and after the curriculum has been taught. The information obtained through these assessments will be kept confidential. There will be no identification of the students. In addition to the surveys, your son/daughter will also participate in informal interviews with me during class. These interviews will be held as both group and individual discussions and will occur inside the confines of the class period. Students will also be videotaped and audio taped so that I can capture their thoughts, ideas, and collaborations with others. The notes, videotapes, and audiotapes taken during these discussions will be kept confidential and locked up when not in use. All data, including video and audio, will be destroyed at the end of the study. There will be no penalties for choosing not to participate in the research and there will be no rewards for students who do participate. Students may choose to stop participating in the study at any time. Data will be collected during regular math classes. Students who choose not participate will be engaged in the same classroom activities but no data will be gathered about them.

In order for your son/daughter to participate, I need a written release from the parents. Please fill out the form below and indicate whether you are giving permission for your child to participate or if you are denying permission.

I would like to thank-you for your help with this matter. If you have any questions, please feel free to contact me at (321) 242-1411 ext 5704.

Sincerely,

Marino Nardelli

Contact Information—UCF Faculty Advisor
Juli Dixon, Ph.D.
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Institutional Review Board
Tracy Dietz 6/14/06
CHAIRMAN

I have read the procedure described above. I voluntarily agree to participate in the procedure, and I have received a copy of this description.

I, ______________________________ have read and understand the letter for participation in Mr. Nardelli's research study on different learning methods as they pertain to fractions and decimals research study.

Student's Name ____

☐ Yes, I give permission to my child to participate.

(*Please check all that apply*)

_____ My child has permission to be audio taped during the study.

_____ My child has permission to be video taped during the study.

☐ No, I do not want my child to participate.

☐ I agree to participate in the pre and post survey about my child regarding his/her abilities and attitudes about decimals and fractions (each survey is 2 pages long and will take approximately 10 minutes to complete).

_____________________________ Parent Signature

_______________ Date

Information regarding your rights as a research participant may be obtained from:

Barbara Ward, CIM
Institutional Review Board (IRB)
University of Central Florida (UCF)
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: (407) 823-2901

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Tracy Dietz 6/14/06
CHAIRMAN

APPENDIX D: STUDENT ASSENT FORM

Student Assent Form

My name is Mr. Marino Nardelli and I am a student at the University of Central Florida. I would like to ask you some questions about how you feel about and learn about decimals and fractions. You may stop at any time and you will not have to answer any questions you do not want to answer.

As a way to study this, I would like to video tape and audio tape you during math class when you will be working in a cooperative learning group and on you own. At the end of the study, I may interview you on video tape by asking what you learned, how you chose certain graphs to use, and what you would change if you had the chance. Only Dr. Dixon, my professor at UCF, and I will see the video tapes. I will destroy the tapes at the end of the study. All names will be changed so that nobody will know it was you in my study. It will not affect your grade if you decide you don't want to do this. You can stop participating at any time. If you don't want to be video taped, you cannot be in the study and I will assign another activity for you. You will not be paid for doing this. Would you like to take part in this research project?

YES or NO (Circle one)

______________________________ ______________________

Print First and Last Name Date

____

Student Signature

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Institutional Review Board
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APPENDIX E: PARENT SURVEY

Parents' Decimals and Fractions Pre-Survey	Date

Student's Name: ____________________

Parent (s)' Name (s): ____________________

Dear Parents,

I would like to know your opinion about your student's interest in math, decimals, and fractions. Please circle the answer that best describes your student. Please answer each question. Do not think too much about each question; your initial thought is usually the most accurate. In the example below, if your student really enjoys math, you would circle strongly agree.

Example:

A. My student enjoys math. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

1. My student likes finding answers to math problems. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

2. My student does not have much interest in math. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

3. My student typically finds mathematical topics boring. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

4. My student looks forward to math lessons in school. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

5. My student would like to study math in more detail than he/she does now. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

6. My student's teachers have encouraged him/her to study more math. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

7. Decimals and fractions are hard for my student, even when he/she studies. *Strongly Disagree* *Somewhat Disagree* *Somewhat Agree* *Strongly Agree*

over →

8. My student is good at decimals and fractions. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

9. My student can add and subtract decimals. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

10. My student can multiply and divide decimals. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

11. My student can add and subtract fractions. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

12. My student can multiply and divide fractions. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

13. My student knows how to convert fractions to decimals. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

14. My student knows how to convert decimals to fractions. *Strongly Disagree Somewhat Disagree Somewhat Agree Strongly Agree*

APPENDIX F: STUDENT SELF ASSESSMENT

Student Self-Assessment

Name______________________ **Date**________________________

How Do I See Myself?

Check the appropriate box that corresponds with each statement.

1. I understand what a decimal is.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
2. I am able to add decimals well.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
3. I am able to subtract decimals well.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
5. I am able to multiply decimals well.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
6. I am able to divide decimals well.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
6. I know how to convert decimals to fractions and fractions to decimals.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
7. I work better on assignments in a group with other students.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
8. I work better on assignments alone.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
9. I feel nervous or scared speaking in front of the class.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree
10. I show reponsibility toward others and myself.	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree

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