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Developing Fourth Graders' Proficiency In Basic Multiplication Facts Through Strategy Instruction

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DEVELOPING FOURTH GRADERS' PROFICIENCY IN BASIC MULTIPLICATION FACTS THROUGH STRATEGY INSTRUCTION

by

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ABSTRACT

The purpose of this action research study was to evaluate my own practice of teaching basic multiplication facts to fourth graders. I wanted to see how focusing my instruction on strategies would help my students develop proficiency in basic multiplication facts. I chose this topic because Florida was in the process of shifting to new standards that encourage teaching for deeper meaning. I hoped this research would give my students the opportunity to make sense of multiplication on a deeper level, while giving me insight into how students learn multiplication. Through this study, I learned that students initially find multiplication to be very difficult, but they can solve basic facts with ease when using strategies. Students did become more proficient with basic multiplication facts, and they were also able to apply basic fact strategies to extended facts and other multidigit multiplication problems. There is a limited amount of research on how students acquire basic multiplication fact proficiency; however, this study offers more insight to teachers and the research community.

This study is dedicated to students everywhere
who are struggling to learn multiplication facts.

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CHAPTER 1: INTRODUCTION

Rationale

The teaching and learning of mathematics in the United States is undergoing change. With new standards shaping the mathematics curriculum in the United States, there is much more emphasis on the importance of deeper understanding of mathematics (NCTM, 2006). Recently, the National Council of Teachers of Mathematics (NCTM) published *Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics: A Quest for Coherence* in the hopes that instruction would be focused on a "target" and that concepts would be explored with much more depth (NCTM, 2006).

In 2007, Florida adopted new standards modeled after the national focal points, known as the Next Generation Sunshine State Standards (FLDOE, 2007). One of the three focal points for fourth grade is that students develop quick recall of multiplication facts and related division facts and fluency with whole number multiplication (NCTM, 2006). This has affected me greatly as a teacher. With multiplication being such a crucial skill in fourth grade, I realized the way I have taught it in the past might need to change. For years I have used practice drills and songs that repeated basic facts in hopes of getting my students to develop quick recall of multiplication. From my experience, most students find multiplication to be very difficult and few students develop quick recall in the short time frame of a school year. Even if students have memorized their basic facts, they are often unsure of what it really means to multiply and how to apply that knowledge to other types of problems where multiplication is needed. With the standards shifting, I wanted to see how changing my instruction would help my students develop proficiency in multiplication, which is so much more than just memorizing facts.

Proficiency

As defined by Kilpatrick, Swafford, and Findell (2001), proficiency encompasses "five strands that are interwoven and interdependent including: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition" (p. 116). I focused much of my research on procedural fluency, also known as computational fluency and conceptual understanding; however, the other three components are also integral parts of proficiency that cannot be ignored.

Computational fluency can be defined as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick, Swafford, & Findell, 2001, p. 116). Mathematical flexibility as it relates to computation is commonly defined as selecting a rational choice between mental calculation strategies based on the nature of the problem being faced (Threlfall, 2002). For example, not all multiplication problems require a paper and pencil algorithm. Mathematical fluency involves knowing what strategy to use and when to use it.

Conceptual understanding is defined as the "comprehension of mathematical concepts" (Kilpatrick et al., 2001, p. 116). This is linked very closely to computational fluency because in order for students to learn a skill they must have some understanding of it (Kilpatrick et al., 2001). For instance, if students are solving 8 x 4 and they use the problem 4 x 4 and double it, they must have an understanding that multiplying half of one factor (in this case 8) by the other factor, then doubling, gives them the same result as the original factors.

Kilpatrick et al. (2001) define strategic competence as the "ability to formulate, represent, and solve mathematical problems" (p. 116). For my study, I refer to this as problem solving. Children's fluency and understanding are built up when they are given the opportunity to act out or model a situation in a problem-solving context (Kilpatrick et al., 2001). Eventually, the direct modeling procedures students use are replaced by more efficient calculations (Kouba, 1989).

Adaptive reasoning is the "capacity to think logically about the relationships between concepts and situations" (Kilpatrick et al., 2001, p. 129). It also includes the ability to explain and justify the reasoning behind the method for finding a solution (Kilpatrick et al., 2001). A large component of my study involved students sharing their explanations of the strategies they used when solving multiplication problems and why their strategies worked.

The last component of proficiency is productive disposition. "The term *disposition* should not be taken to imply a biological or inherited trait. As used here, it is more akin to a *habit* of thought, one that can be learned and, therefore, taught" (Resnick, 1987, p. 41). Productive disposition is the inclination to have a positive attitude towards mathematics, to recognize the value of it, and to see oneself as a capable learner (Kilpatrick et al., 2001). Teachers play an integral role in helping students see the usefulness of mathematics and can shape students attitudes about it.

Strategies

Kilpatrick et al. (2001) state, "When instruction emphasizes thinking strategies, children are able to develop the strands of proficiency in a unified manner" (p. 7). Much like addition and subtraction, students learn multiplication by progressing through the use of calculating methods such as: counting, recognizing number patterns, repeated addition, and mixed strategies such as using a known fact and counting on (Cooney, Swanson, & Ladd, 1988; Anghileri, 1989; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005). Research indicates that computational fluency and problem solving skills develop with greater understanding when students begin learning operations by modeling real world situations (Fuson, 2003). Researchers support the idea that practice will have its greatest effect when basic facts are not treated in isolation, but when these number triads (two factors and a product, e.g. 3, 5, and 15) are continually linked to meaningful examination of patterns and strategies such as counting and or using a known fact and a calculation (Sherin & Fuson, 2005).

For my study, I use the term strategy to refer to a calculation method used to solve a basic multiplication problem. A strategy refers to a counting procedure or "the construction of a sequence of transformations of the number problem in order to arrive at a solution" (Threlfall, 2001, p. 30). Direct modeling is also a strategy used for solving basic multiplication facts (Anghileri, 1989).

The opposite of a calculation strategy would be direct retrieval. Retrieval is defined as being able to retrieve an answer to a number combination from long term memory (Steel & Funnell, 2001). Anghileri (1989) also refers to retrieval as using a known number fact. Students who use known facts can generally verbalize the answer to a basic fact very rapidly without the use of finger counting, drawing, or calculating (Sherin & Fuson, 2005).

Research Questions

After years of focusing my multiplication instruction on rote memorization with little results and with the state of Florida transitioning into new standards, I realized it was time to make a change in my instruction. Through my research, I explored two main questions:

Question 1: How can focusing my instruction on multiplication strategies help fourth grade students develop proficiency with basic multiplication facts?

Question 2: How can strategies with single digit multiplication be applied to extended facts?

Significance of the Study

With the reform taking place in mathematics, teachers are being challenged to change the way they teach mathematics. This study guides teachers toward developing new ideas about how students learn and how to facilitate instruction that emphasizes deeper understanding. There is an abundance of research on how students learn addition and subtraction (Baroody, 1984; Brownell & Chazal, 1935; Fuson, 2003; Kilpatrick et al., 2001; Isaacs & Carroll, 1999; Tournaki, 2003), but much less on multiplication. This study offers insight to the research community about the way students learn multiplication and how to teach it in a way that will lead to proficiency.

If students do not learn multiplication during the elementary school year they will have difficulty being successful in middle school and even high school (NMAP, 2008). Many middle and high school teachers reported that students are lacking necessary basic skills and strategies to be successful with upper level mathematics (NMAP, 2008). Students must be proficient with basic fact multiplication in order to carry out more complex algorithms (Fuson, 2003). One study further supports this notion in that the students who used effective strategies for multiplication performed better on other types of mathematics tests (Steel & Funnell, (2001). The elementary years are the prime time for students' to become proficient in multiplication.

Conclusion

The state of Florida is currently in the process of implemented the Next Generation Sunshine State Standards for Mathematics in response to the nation's call for reform, which emphasizes teaching for deeper meaning (NCTM, 2006). Because of this transition, teaching for meaning has become an exciting endeavor. Furthermore, the ultimate goal is for my students to reach proficiency in multiplication that will carry over into other aspects of mathematics. I hope to use my research to revolutionize my instruction in mathematics and inspire my students to become deeper thinkers. Finally, my research will be beneficial to other teachers and researchers who want to help students understand multiplication as more than just the facts. In Chapter 2, I review the literature that supports teaching multiplication strategies to increase proficiency in multiplication. I also unpack the importance of computational fluency in multiplication and how it works together with the other strands of proficiency.

CHAPTER 2: LITERATURE REVIEW

Introduction

Traditionally in the United States, learning basic facts has been predominantly characterized by drill methods focused on memorization (Baroody, 1984, Kilpatrick et al., 2001). Fuson (2003) explains that through this type of learning students memorize a number pair and operation, such as 4 x 5, and then associate the answer of 20 in response to the problem. Furthermore, drill methods rely on repetition and require little understanding.

The drill and practice technique has been criticized because it focuses on rote memorization verses deeper understanding and thinking strategies (Baroody, 1984; Erenberg, 1995; Isaacs & Carroll, 1999; Fuson, 2003; Kilpatrick et al., 2001; and Tournaki, 2003). With mathematics education undergoing reform, the National Council of Teachers of Mathematics (NCTM) is calling for instruction that supports the development of deeper understanding (NCTM, 2006). Researchers argue that it is imperative for children to develop the concept that arithmetic is more problem solving and strategic reasoning over simply getting quick answers (Steffe, Cobb, & Glassersfeld, 1988).

A review of the literature suggests teaching basic facts through rote memorization does not foster the development of proficiency. There is a large body of research that supports a different approach. The purpose of this literature review is to explore the research on strategy-based instruction for basic multiplication facts. The literature review unpacks how teaching multiplication for meaning through strategies can develop the strands of proficiency. Two strands, computational fluency and conceptual understanding, are given more attention; however, strategic competence, adaptive reasoning, and productive disposition will also be addressed throughout the review. Additionally, the strategies through which students developmentally progress while learning multiplication are discussed. Finally, the review of the literature addresses how classroom instruction that allows students to explore and investigate strategies can have a tremendous impact on students' success with mathematics.

The Call for Proficiency

Recently, the United States has called for a reform in the Education System. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008) urges that action be taken to improve mathematics education, and that schools should place more emphasis on basic skills. This reform also calls for the necessity of number proficiency as defined by the National Research Council in *Adding It Up* (2001). According to *Adding It Up* (Kilpatrick et. al, 2001), there are five building blocks of proficiency with numbers which include conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and procedural disposition. Each strand is dependent on the others. "Conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations together support effective and efficient problem solving" (NMAP, 2008, p. 26). Simply put, proficiency is more than just memorizing mathematics facts, and the National Council of Teachers of Mathematics (NCTM) emphasizes the importance of deeper understanding of mathematics in the United States (NCTM, 2006).

There is a dire need for students to be proficient in multiplication if they are to be successful in other areas of mathematics (NMAP, 2008). Basic facts are required in estimation and mental computation (Isaacs & Carroll, 1999). Isaacs and Carroll (1999) ask, "How can students use 80 x 40 to estimate 84 x 41 if they do not know 8 x 4?" (p.1).

Teachers in the upper grades agree that most students lack proficiency with basic computations. The National Mathematics Advisory Panel (NMAP) surveyed Algebra I teachers, and most agreed that students are not adequately prepared by the time they get to Algebra I during middle school. Some of the survey responses are provided here

1. "Students need to be better prepared in basic math skills and not be quite so calculator dependent. Also, more training in thinking skills."

2. "Make sure the 1st–8th grade teachers teach the foundations of mathematics and that the students know their basic skills." (NMAP, 2008, p. 9)

With reform underway in mathematics education, it is imperative to acknowledge what research says about how students learn basic multiplication facts. Instruction should be focused on developing all five strands of proficiency.

The Strands Work Together

According to Fuson (2003), "Fluency with computational methods is the heart of what many people in the United States and Canada consider to be the elementary mathematics curriculum" (p. 71). Interwoven through the NCTM's focal points for grades 3–5 are three crucial mathematical themes--multiplicative thinking, equivalence, and computational fluency (NCTM, 2006). Indeed, computational fluency is a vital part of the upper elementary curriculum.

Much like a cornerstone in a house, fluency with multiplication is a key component in a young person's acquisition of more challenging arithmetic. According to the NGSSS, students are taught multiplication concepts in third grade and are expected to develop quick recall of the basic facts by fourth grade (FLDOE, 2007). However, as teachers have observed, fifth grade students are still struggling to be fluent with the basic facts needed to build further mathematical development and often rely on multiplication charts and counting manipulatives (Wallace & Gurganus, 2005).

Fluency and conceptual understanding are intertwined, and one strand cannot be fostered without the other. Wu (1999) describes learning that prioritizes one against the other as a bogus dichotomy. According to Kilpatrick et al. (2001)

> Understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting. By the same token, a certain level of skill is required to learn mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding. (p. 122)

Students who have conceptual understanding often have less to learn because they can see relationships among numbers and situations (Kilpatrick et. al, 2001). For example, students who understand the commutative property of multiplication can reduce the amount of facts to learn nearly in half (Fuson, 2003).

All five strands of proficiency are equally important. Kilpatrick et al.(2001) state, “If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out” (p. 131). When productive disposition develops and students are given opportunities to make sense of mathematics, all the strands can be cultivated (Kilpatrick et. al, 2001).

Benefits of Strategy Instruction

For decades, children have been taught basic facts through rote practice, timed drills, and memorization (Brownell & Chazal, 1935; Kilpatrick, et. al, 2001). Some researchers claim that drilling students on their basic facts leads to quick recall (Burns, 2005; Poncy, Skinner, & Jaspers, 2006; Wong & Evans, 2007). Although many support drill and practice, there is ample support for strategy based instruction over drill and practice in basic facts. There is a tremendous amount of research that supports thinking strategies for the learning of the basic addition and subtraction facts (Baroody, 1984; Fuson, 2003; Isaacs & Carroll, 1999; Kilpatrick et al., 2001; Steffe et al., 1988); however, there is much less on multiplication, especially multidigit multiplication (Cooney, et al., 1988; Fuson, 2003; Kilpatrick et al, 2001; Sherin & Fuson, 2005). What we do know about single digit addition and subtraction, however, can also be applied to the learning of multiplication (Anghileri, 1989; Mulligan & Mitchelmore, 1997).

Researchers believe teaching through drill and practice can have negative effects. Timed tests and pressure to memorize can cause anxiety in students, which can lead to a lack of motivation and a bad attitude towards mathematics (Isaacs & Carroll, 1999). Worse yet is that it cultivates the belief that mathematics is more memorizing than thinking and problem solving (Isaacs & Carroll, 1999; Sherin & Fuson, 2005). Steffe, Cobb, and Glassersfeld (1988) argue, "Remembering certain connections should not be discouraged, but children should not develop the concept that arithmetic is devoted to answer getting rather than to problem solving and strategic reasoning" (p. 322).

In an article from Teaching Children Mathematics, the authors (both teachers) make a bold statement in regards to the importance of teaching students to think strategically: "Children with deep conceptual understanding of multiplication will have an advantage when faced with a forgotten multiplication fact" (Wallace & Gurganus, 2005, p.32). For example, if a student forgets the product of eight and seven, they can use 4 x 7 and double it. A student would only be able to use this strategy if they have a conceptual understanding of multiplication as well as keen number sense as to the ways the number 8 can be broken apart. Students also tend to have an easier time multiplying lower factors (1-5) and can use those facts to solve problems with higher factors (Anghileri, 1999). Strategy instruction broadens students' computational resources (Sherin & Fuson, 2005).

A recent multiplication study showed that students receiving both strategy instruction and timed practice versus students who only participated in timed multiplication drills, performed slightly higher on tests with two digit times one digit and estimation problems (Woodward, 2006). The students in the strategy-based group had the opportunity to see and discuss connections between basic facts, extended facts (such as 3 x 25), and methods for estimating answers to multiplication problems.

In a 2003 experimental study in which one group of students received "traditional" basic fact instruction in addition through drills and another group of students received explicit strategy instruction, the data revealed the practice of thinking strategies was more beneficial than rote practice of basic facts (Tournaki, 2003). The students who received strategy instruction performed better on a transfer task in which they had to add three digits instead of two. It is important to note that some of these participant included students with learning disabilities. Surprisingly, the students with learning disabilities solved basic addition facts faster than the students who had no learning disabilities in the opposing group (who learned by drill only).

Although, this study was on addition, it clearly exhibits evidence to support strategy instruction. Because research on multiplication is limited in comparison to addition, studies such as this can give great insight into other basic fact operations, including multiplication.

The Challenges of Learning Multiplication

One reason learning basic multiplication facts are so challenging is the sheer amount students are expected to learn. When looking at a multiplication table, it is easy to see why memorizing all the basic facts is so difficult for students (Fuson, 2003). There are one hundred number combinations, or facts, to learn. Looking for patterns among basic facts alleviates some of the difficulties of learning basic multiplication facts (Fuson, 2003).

Not only are there so many facts, but multiplication itself requires high-level thinking. The process of learning multiplication is comparable to learning addition, but is much harder. Anghileri (1989) explains addition and subtraction are unary operations with each input representing the same kind of element. For example, 3 + 4 could be thought of as three blocks added to four blocks. On the contrary, multiplication should be viewed as a binary operation with two unique inputs (Anghileri, 1989). For the groups of objects interpretation of multiplication, the first input, or factor, represents the number of sets or groups and the second factor represents the number in each group. In this way, the two numbers represent distinct elements of the multiplication process. In contrast to the previous block problem, three times four would represent three stacks with four blocks in each stack. Multiplicative thinking is clearly distinguishable from additive thinking because the meaning of the numbers is different (Clark & Kamii, 1996). To illustrate this further, when adding 4 + 6, all you have to do is think about putting two sixes together. On the other hand, 4 x 6 is like adding 6 six four times. There is a level of abstraction with one factor, which makes it difficult.

In one study, only 28% of fourth graders and about half of fifth graders interviewed demonstrated consistently solid multiplicative thinking (Clark & Kamii, 1996). The children were given three different sizes of wooden models that represented fish labeled A, B, and C and 100 chips/counters to represent fish food. They were told that fish B eats two times as much food as fish A, and that fish C eats three times as much as fish A. Students were asked questions like, "If this fish (A) gets 1 chip of food, how many chips of food would you feed the other two fish?" (Clark & Kamii, 1996, p.3). Many students often chose answers representative of addition, such as 7, 8, and 9 for fish A, B, and C over a multiplicative answer (4, 8, 12).

In a study of 241 children in England, ages 8-12, most children said that learning multiplication was difficult, especially those problems with higher factors, from 7 to 12. The children reported practicing multiplication facts by coloring in tables of squares, investigating patterns in numbers, doing workbook pages, taking tests, and by writing down the facts and continually saying them (Steel & Funnell, 2001).

Another study of 4th, 5th, and 6th graders, showed that most children could solve a multiplication problem; however, 74% of the fourth graders showed no multiplicative context when asked to write a story problem for a simple problem such as 6 x 3, despite having had at least two years of multiplication instruction in the classroom (O'Brien & Casey, 1983). This clearly shows that students may have facts memorized, yet have no conceptual understanding of how to represent multiplication as a situational problem.

These three studies reveal just how challenging learning single digit multiplication can be. Even after much practice and instruction in the classroom, children found multiplication to be difficult. It is not clear whether the students in the previously mentioned studies received drill or strategy instruction. What is certain is that learning multiplication is a difficult task. One way to ease this challenge is to allow students the opportunity to explore the many strategies or methods for multiplying. The following section describes the ways students learn multiplication through the early stages of concrete examples, to more complex thinking strategies.

Strategy Development

Research supports the notion that proficiency with multiplication is developed over time, and that students do not learn basic facts without progressing through easier to more advanced methods (Thornton, 1978; Cooney et al., 1988; Anghileri, 1989; Erenberg, 1995; Mulligan & Mitchelmore, 1997; Isaacs & Carroll, 1999; Fuson, 2003; Kilpatrick et al., 2001; Sherin & Fuson 2005). As children get older they begin to use strategies more quickly and efficiently (Baroody, 1999). Sherin and Fuson (2005) studied the literature on those strategies and forged a consensus on the taxonomy of strategies for multiplication. Their investigation was built upon earlier research (Anghileri, 1989; Mulligan & Mitchelmore, 1997) supporting the notion that children move through strategies for learning basic facts ranging from using different forms of counting, adding, and then on to using known facts.

Much like the process of learning addition, students can begin to multiply by making equal groups and counting all the objects. Students may model a multiplication problem with manipulatives or draw a picture (Sherin & Fuson, 2005). For instance, a student solving a problem involving 3 x 4 could illustrate this by drawing 3 circles with 4 small circles in each and counting all of the small circles. Researchers claim that a counting strategy such as this underlies the initial acquisition of mental multiplication (Cooney et al., 1988). Other studies have been conducted on counting procedures for multiplication, and the youngest children were observed to count each individual item in the product set (Anghileri, 1989; Mulligan & Mitchelmore, 1997). This approach of acting out or modeling a problem is a powerful way to keep computational fluency connected to conceptual understanding and strategic competence (Kilpatrick et al., 2001). Also, Kouba (1989) suggests that over time, the direct modeling approach is replaced with more efficient counting methods, such as counting, repeated addition, or using a derived fact. Beginning with simple word problems gives children the opportunity to practice direct modeling, while building their understanding and fluency (Kilpatrick et al., 2001).

Once students become proficient in the count all procedure, they generally move to additive calculation. Sherin and Fuson (2005) explain that to solve 3 x 4 with an additive calculation, a student would first add 4 + 4 to get 8, then 8 + 4 to get 12. Most students begin formal instruction of multiplication with the conceptual understanding of addition; thus they already possess the essential conceptual capabilities required for understanding multiplication (Sherin & Fuson, 2005). For years, multiplication has been taught as repeated addition, however research shows that multiplication requires higher-order multiplicative thinking, which the child develops out of addition (Clark & Kamii, 1996).

The literature suggests that as children progress in their multiplication strategies, they may use rhythmic counting or the count-by procedure (Anghileri, 1989; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005). For example, if a student is solving 3 x 6, the student would count by six three times. This strategy may also be classified as finding a number pattern or skip counting. Children may keep track of their counting through a variety of ways, such as counting on their fingers or using a sheet of paper to keep record of the number of groups that have been counted. The count-by technique was demonstrated to be beneficial in improving fluency with basic multiplication facts even in students with learning disabilities (McIntyre, Test, Cooke, & Beattie, 1991). Sherin and Fuson (2005) imply that the count-by strategy is most often used with lower factors including 2, 3, and 4.

Another way children solve basic fact problems is to use derived facts (Erenberg, 1995; Isaacs & Carroll, 1999; Fuson, 2003; Kilpatrick et al., 2001; Sherin & Fuson 2005; Steel & Funnell, 2001). There are many ways students use derived facts for multiplication. For example, one method is to use a doubling strategy which a student might solve 8 x 6 by using the product of 4 x 6 and doubling it. Focusing instruction on using derived facts with doubles, such as using 3 x 3 and doubling the product to solve 6 x 3 may be easy strategy (Isaacs & Carroll, 1999). Students might also use a combination method in which they use a known fact plus a calculation to get the product. In this way, a student might use 5 x 7, then add 7 more to the initial product to solve 6 x 7.

Many textbooks begin early instruction on basic facts multiplication with pictorial representations of multiplication, specifically the array. Barmby, Harries, Higgins, and Suggate (2008) found that the array representation for multiplication supports calculation strategies such as identifying equal groups within in the array (possibly recognizing the distributive properties of the array) and moving parts of the array around or completing the array to make the calculation easier.

Researches have discovered that most children use calculation strategies over "just knowing" when solving basic facts. In Anghileri's 1989 study of young children's (ages 4-12) understanding of multiplication, results showed that only above average children appear to use a known number fact, whereas 81% of test items solved showed students used either a calculation strategy or direct modeling. Another study showed most eight and nine year olds used a mix of strategies including retrieval, calculation using a known fact, and counting to solve multiplication facts. Also, children ages 10-12 were more likely to use retrieval than the eight and nine year olds (Steel & Funnell, 2001). As children get older and therefore have more experience, they begin to shift toward the use of more efficient strategies (Cooney et al., 1988; Steel & Funnell, 2001).

It is clear from the research that most elementary students are using some strategies over just knowing their multiplication facts, even when instruction was focused on drill and practice (Cooney et al, 1988). Research not only supports the notion that students will use different strategies for different operands, but also suggests that strategies will vary across classrooms depending on the instruction that has taken place (Sherin & Fuson, 2005). Therefore, classroom instruction plays a key role in students' development of computational resources.

Teacher's Role

Teachers should address basic fact multiplication instruction in a way that encourages students to develop the strands of proficiency. Sherin and Fuson (2005) believe that students will learn some basic facts (like 5's or 10's because of their ability to count by 5 and 10) on their own, but other higher factors, such as 6, 7, 8, and 9 especially require explicit instruction.

The following instructional sequence is based on Isaacs and Carroll's progression for teaching basic addition facts (1999); however, it is also synthesized with the research supporting the progression learning basic multiplication facts (Thornton, 1978; Cooney et al., 1988; Anghileri, 1989; Erenberg, 1995; Mulligan & Mitchelmore, 1997; Fuson, 2003; Kilpatrick et al., 2001; Sherin & Fuson 2005). Basic fact instruction should begin with real world multiplication situations that students can model with manipulatives and count all the objects. Drawing pictures, such as equal groups and arrays, should also be a part of the early instruction of basic facts. From there, students should be given the opportunity to relate the equal groups to repeated addition or an additive calculation.

Classroom instruction should then move toward the exploration of count-bys or skip counting, such as 3, 6, 9, 12, and so on for 3s facts. Kilpatrick et al. (2001) suggest that children need much experience in producing count-by lists and exploring patterns. Students should also be given the opportunity to analyze patterns or rules for basic facts, such as 0's, 1's, 9's, and 10's (Sherin & Fuson, 2005). Eventually, instruction should promote the use of a combination of strategies, such as splitting a factor and using a known fact or additive calculation (Sherin & Fuson, 2005).

A crucial part of basic fact instruction involves discourse among students. The benefits of class discussion of strategies are twofold. First, students can sharpen their communication skills; and second, they can learn strategies from their peers (Isaacs & Carroll, 2005). Research also suggests creating a classroom atmosphere where children can describe to one another the strategies they use while the teacher can support their descriptions with explicit language and proper ways of recording (Threlfall, 2002).

Exposing children to different strategies allows them the opportunity to discover and internalize relationships among numbers (Baroody, 1984). Sherin and Fuson (2005) agree that practice will have its greatest effect when "facts" are not treated in isolation, but when number combinations are continually associated with meaningful examination of patterns and strategies. For instance, it is highly unlikely that students will recognize patterns in multiples of nines if they are not addressed through classroom instruction (Sherin & Fuson, 2005). Thornton (1978) agrees that children do seem to adopt strategies that are explicitly taught, encouraged, or otherwise suggested during instruction.

Becoming proficient with multiplication involves much more than just the teacher explaining procedures and students practicing for memorization. Kilpatrick et al. (2001) state

> "It involves students- with support from learning materials, teachers, and peers-inventing, understanding, and practicing methods; trying to learn and use concepts that look easy to adults but are challenging to children; and gradually increasing their mathematical proficiency by continuing to make sense of number and numerical operations." (p. 217)

Lastly, teachers should not be disappointed when a child does not adopt more efficient strategies immediately because each student will develop at their own pace (Isaacs & Carroll, 1999). A classroom that fosters the development of computational resources will help children form a rich network of strategies to use when solving a problem (Sherin & Fuson, 2005). Fuson (2003) claims that regardless of instruction, students invent their own methods of calculating; however, instruction that is centered on teaching for understanding will help students develop a wider range of effective methods with fewer errors. Through this type of instruction, proficiency can be cultivated.

Conclusion

A review of the literature supports the need for instruction that manifests the strands of proficiency in multiplication (Kilpatrick et al., 2001). Teachers play an active role in helping their students become more proficient with basic multiplication facts. The strands of proficiency can be facilitated by allowing students to progress through the stages of developmental strategies (Thornton, 1978; Cooney et al., 1988; Anghileri, 1989; Erenberg, 1995; Mulligan & Mitchelmore, 1997; Isaacs & Carroll, 1999; Fuson, 2003; Kilpatrick et al., 2001; Sherin & Fuson 2005).

I was interested in getting my students to a higher level of proficiency with multiplication. I wanted to explore the research on basic fact instruction through strategies. Ultimately, I hoped to improve my instruction so that my students would develop fluency with facts, conceptual understanding, better problem solving skills, reasoning skills, and a positive attitude toward multiplication, which are the five strands of proficiency.

In the next three chapters, I discuss the methodology I chose, the analysis of the data, and my conclusions. My questions, “How can focusing my instruction on multiplication strategies help fourth grade students develop proficiency with basic multiplication facts?” and “How can strategies for single digit multiplication be applied to extended facts?” are investigated further in these chapters.

CHAPTER 3: METHODOLOGY

Introduction

The topic of my study was the development of proficiency in basic multiplication facts in fourth grade students. More specifically, I wanted to know if focusing my instruction on multiplication strategies would help my students develop proficiency with basic facts and if those strategies could be applied to extended facts. In this chapter, I describe the classroom setting and the methods used to discover the answer to my questions.

Design of Study

In order to study my own instructional methods in the classroom and how they impacted my students, I conducted a qualitative type of research called Action Research. Action Research is defined as a form of research done by an individual in an attempt to improve one's practice (McNiff, Lomax, & Whitehead, 1996). It was my goal to examine my instructional methods for teaching multiplication in the classroom, and how I could improve my instruction in hopes of helping my students become proficient in multiplication. My research questions were:

Question 1: How can focusing my instruction on multiplication strategies help fourth grade students develop proficiency with basic multiplication facts?

Question 2: How can strategies with single digit multiplication be applied to extended facts?

Setting

School Setting

My school is located in central Florida and has been an A-rated school five out of six years. Sixty-five percent of the population is Caucasian, 14% is Hispanic, 7% is African American, 6% is Asian, and 7% is multiracial. Nineteen percent of the students are on free or reduced lunch.

Classroom Setting

My action research was conducted in a fourth grade classroom of twenty students. The students in my class were heterogeneously grouped and selected by my administration and guidance counselor. All of the parents gave parental consent for their children to be involved in the study; therefore, all twenty students participated in my research. My class consisted of ten girls and ten boys. Four students had active Individual Education Plans (IEP) and received support from the Exceptional Student Education (ESE) teacher in the classroom. The ESE teacher gave support facilitation two to three times a week for about 30 minutes each time. She strictly supported what the whole group was doing by working one on one with her students and acting as a coach. I also had two gifted students. Although academically diverse, the majority of the students were Caucasian and four were Hispanic.

Methods

Preliminary Action

I first obtained permission to conduct my study from through the Institutional Review Board (Appendix A) and my principal (Appendix B). During parent curriculum night, parental consent letters (Appendix C) were passed out and all were returned. In addition, I obtained county permission (Appendix D) before I proceeded with the study.

Once permission was granted, I was ready to begin the first step toward exploring my questions. Each student was asked to complete a written questionnaire during class. The questionnaire was a premade document with nine questions (Appendix E). The questionnaire enabled me to get an idea of whether my students liked or disliked mathematics in general. More specifically, it helped me to know how they felt about multiplication and if they spent time practicing their basic facts at home. Additionally, students were asked to express how they felt about timed tests, word problems, and if they thought learning multiplication was important.

Before any multiplication instruction began, all students were given a timed multiplication basic facts test (Appendix F) taken from our school's textbook series, *Scott Foresman-Addison Wesley Florida Mathematics* (Randall, Crown, & Fennell, 2005). Students were given 10 minutes to solve 50 basic facts. After collecting the questionnaire and the basic facts pretest, I began my instruction on multiplication, which I describe further in the procedures section.

Classroom Setup and Discussions

During this study, the students were given opportunities to work independently and within small groups. Students' desks were arranged into small groups of four so they would always be in close proximity of each other for ease of sharing and discussing strategies with each other. The groups mostly stayed the same, but a few individuals were switched around occasionally to help with behavior management. Whole class discussions were characterized both as student-led and teacher-led. There were some strategies that I explicitly taught to my students, but I also gave the students the opportunity to explain and justify how, when, and why they might use the strategy. During whole group discussions, I encouraged everyone to listen to each other and reminded them that they might learn how to apply a strategy from their peers. This was especially true when I did not tell students what strategy to use for a particular problem. I would often give the students a problem, such as 7 x 7, and have the students explain the strategy they used to find the product.

Procedures

After the questionnaires and basic facts pretests were administered, I embarked on the journey of teaching multiplication through strategy instruction. The students' daily routine did not change, and our mathematics time still consisted of a word problem of the day or review, around twenty minutes of instructional time, twenty minutes of individual or partner practice time, and ten to fifteen minutes of review with whole group discussion. I used the county's Mathematics Instructional Plan as my pacing guide as well the research on the progression of strategies as my guide. The textbook was never used during instructional time; however, it was used to give students practice problems.

On the first day of multiplication instruction, I gave each student a set of counters. I asked them to show me how they could use the counters to model 2 x 3. I allowed them the chance to discover all the ways they could demonstrate 2 x 3 with the counters. We discussed how 2 x 3 could be represented as a set (equal groups) or an array. After exploring other basic multiplication facts with the counters, students were asked to draw what they showed with the counters and to solve each basic fact. From there, I explained that each of their arrays or sets could also be represented as repeated addition. The Commutative Property was also introduced at this time.

During the next lesson we worked as a class to write different situations (story problems) for 4 x 6. I required students to illustrate the story problems as well as write the repeated addition and multiplication sentence. Then students were given the opportunity to practice writing story problems on their own for a basic multiplication fact.

Once students demonstrated they grasped that meaning of equal groups for multiplication, it was time to begin further exploration of the strategies that can be used to multiply. I began with the 2's, 5's, and 9's facts. Students were given a hundreds chart and they had to circle the multiples of two, five, and nine using a different color for each. I led the students in a discussion of the patterns they saw within the multiples and what strategies they could use to help them solve those basic facts. For example, for the 2's, they can count by two and they can count by fives for the 5's facts since they are familiar with this type of skip counting. We then explored the patterns within the multiples of nine and how the tens digit in the product is always one less than the other factor besides nine, such as 3 x 9 = 27. The two in the tens place is one less than 3. We also discussed that the sum of the digits of each product of nine is nine.

After about a week of focusing on meanings for multiplication through pictorial representations of sets and arrays, repeated addition, story problems, and patterns for 2's, 5's, 0's, 1's and 9's, it was time to begin teaching strategies for solving single digit multiplication problems with higher factors including six, seven, and eight. Because the research indicated these facts are more difficult for students, I wanted to help them develop the use of the break-apart strategy to make these facts easier. To begin with, students were given counters and I had them make an array for 6 x 8. I then had them discuss with their table groups the different ways they could break apart the arrays into two separate multiplication problems (modeling the distributive property) in order to find the solution more efficiently. This was introduced as a different approach than using repeated addition. Students shared their ideas among their small groups. While students were working with their manipulatives, I walked around the room and observed the students' arrays.

After observing the students, I had them share with the class different ways they split apart the array, and I modeled with overhead counters how they did it so the whole class could see. I then wrote the two multiplication facts, or partial products that could be used next to each smaller array. I pinpointed that only one of the factors is split apart. For example, with 6 x 8, you could think of it as 3 rows of 8 and 3 rows of 8. Only the six was split apart because it stands for "rows" or equal groups.

We practiced breaking apart arrays further by splitting even factors in half and doubling. To solve 8 x 8 this way, break apart an array to show 4 rows of eight and 4 rows of eight. Once students find the partial product of 4 x 8 they can double it (32 + 32). Through the array model, students also discovered ways to use fives facts to help them find a product. Using the problem 8 x 8 again, a student could find the partial product of 5 x 8 and add three more rows of eight (3 x 8) to find the solution. The next few days, students independently practiced breaking apart facts using array dot paper (Appendix G). The practice was always followed by a time of sharing and discussion as to how and why they split the arrays the way they did. After a few days, I had students practice using the break-apart method without the array representation.

So far we had moved through the strategies of arrays and models with counters, skip counting, patterns, repeated addition, and using derived facts (breaking apart). I then introduced the multiples of ten, eleven, and twelve, which are extended facts. Since I knew that my students already were familiar with the tens and elevens, not a lot of time was spent on those strategies. However, the multiples of twelve are much more difficult once students get higher than 12 x 4. I introduced how the students could break apart the twelve into a ten and two's fact. We practiced several problems using twelve as a factor, and I always modeled on the board how to write the two multiplication sentences and then add the products together.

After about two weeks of progressing through different strategies with multiplication, we transitioned into division concepts. Because division and multiplication are inverse operations, we continued practicing using multiplication strategies to help solve division problems. Again, students were given discussion time so they could share their thinking strategies with their classmates.

In order to continue putting the strategies learned into practice, students create a mini booklet out of plain white paper. Each day for the next week, I gave the students a basic fact to solve, usually a problem that included factors of 3 or higher. They were to write the problem and show the strategy they used. Students were then asked to share how they solved the problem with their peers. This took about ten minutes per day and the booklet was eventually used for solving multidigit problems as well.

I then focused instructional time on multiplying with factors greater than 12. We initially looked at patterns for multiplying by ten, a hundred, and thousands, such as 4 x 10, 4 x 100, and 4 x 1,000. Eventually, students moved into more difficult problems such as 55 x 10 or 55 x 100.

For these types of problems, I had students think about the tens and ones as separate multiplication problems, just like when we broke apart the basic facts. By doing this, I was hoping for students to gain understanding of what they were multiplying and not just focus on the traditional algorithm. Each day, I gave students two-digit times one-digit problems, such as 3 x 42, to solve without teaching them the standard algorithm. I wanted them to use strategies fluently and with understanding instead of just going through rote procedures. Students were continually encouraged to work in their small groups and share their strategies with each other. Throughout the study, students were expected to explain and justify the strategies they chose. The entire unit on multiplication concepts lasted for about four weeks.

As our curriculum transitioned into dividing with greater numbers, I continued to encourage discussion on how multiplication strategies could help with solving division. Finally, students were given a post questionnaire and a basic facts posttest.

Data Collection

I used several types of data collection during my study including a basic facts pre and posttest, a student questionnaire, student classwork and homework samples, informal interviews with students, observations with field notes, and teacher journaling. These instruments were used to provide triangulation in the data.

Students' classwork was collected in various ways, including index cards, the strategy booklet they added to daily, and practice worksheets. These resources provided a tremendous amount of information for me to see how students were using multiplication strategies and whether they were becoming more fluent in multiplication. Classroom observation also provided a great way for me to analyze students' conceptual understanding based on their explanations and justifications of strategies.

Student interviews were helpful in allowing me to gather data from individuals. Interviews were very informal and conversational. Most of the conversations took place while students were working independently or in small groups. The questions I asked often depended upon the type of strategy a student used or if they needed help when using a particular strategy. The following types of questions were asked:

- Do you like using thinking strategies to solve multiplication problems?
- Why did you solve that problem the way you did?
- Do you think it is easier to memorize or use a strategy to solve multiplication? Why?
- Why are some problems easier for you than others? Which ones are easier?

The pre and post questionnaires completed by students were slightly different, but both used open ended questions to allow for more individualized responses (Fraenkel & Wallen, 2009). The goal of the questionnaire was for students to express their thoughts and feelings about mathematics, multiplication, and classroom instruction. The final questionnaire (Appendix H) included an assessment-type question that asked students to write a story problem for 6 x 4.

Data Analysis

The data were analyzed based on a grounded theory study of Glaser and Strauss (1967). Fraenkel and Wallen (2009) describe this as a constant comparative method in which data are collected and analyzed and then the theory is suggested. The theory is later revised after more data are collected. It is important to mention that in the grounded theory approach, the researcher does not begin a study with a theory or claim, rather "he or she develops a theory out of the data that are collected" (Fraenkel & Wallen, 2009, p. 430).

Throughout the data collection phase, I continually looked for emerging patterns as a means to analyze the data. The data were categorized into themes such as multiplication strategies most commonly used by students, invented strategies, ability to transfer from basic facts to extended facts, and attitudes toward multiplication.

Validity and Reliability

Content validity of the pretest and posttest was upheld by using a basic multiplication facts test of fifty problems taken from the textbook series. The problems ranged from factors with zero to nine. The same test was used for both the pretest and the post test. The posttest was administered within four weeks of the basic multiplication fact instruction.

Summary

The qualitative methodology used in this study provided an appropriate format for me to examine my own practice of teaching basic multiplication fact strategies. What the data revealed is discussed in Chapter Four. A detailed analysis of the data describes the impact of my instruction as my class and I discovered strategies to solve multiplication facts.

CHAPTER 4: DATA ANALYSIS

Introduction

In the early planning stages of my action research, I was initially interested in how teaching multiplication strategies would help my students attain fluency with basic facts. Because fluency with multiplication is one of three big ideas in fourth grade based on the Next Generation Sunshine State Standards for Mathematics (FLDOE, 2007), I thought it would be an appropriate subject to research. However, although fluency is important, it is only one aspect of students' proficiency with multiplication. Ultimately, I wanted my students to build fluency with understanding, be able to choose appropriate strategies, develop problem solving skills related to multiplication, and deepen their confidence in themselves as being able to solve multiplication problems. All of what I was hoping to accomplish can be summarized in the word proficiency. Through qualitative research, I explored these questions:

Question 1: How can focusing my instruction on multiplication strategies help fourth grade students develop proficiency with basic multiplication facts?

Question 2: How can strategies with single digit multiplication be applied to extended facts?

Where Do They Stand?

At the beginning of the study, I wanted to get an idea about my students' thoughts, beliefs, and feelings toward mathematics and more specifically, multiplication. Students responded to a written questionnaire I had created. One question asked if they liked mathematics. There was a wide range of answers to this question. Many students reported they liked mathematics, but a few said “no” and “kind of.” I was glad to know most of my students replied positively, yet I hoped to change the minds of the others.

I also wanted to know if my students spent time outside of school practicing basic multiplication facts. The majority of students said they did not practice on their own. I was actually pleased to hear this because I felt it would really help me to see if my instruction truly was beneficial. If the students had come to my fourth grade class having already spent a great deal of time practicing multiplication facts they would have been less likely to engage in strategy instruction. From my experience as a teacher, when students already feel they “know” something they are less inclined to dive in and embrace what is being taught. Only a handful of students stated they did practice their facts with flash cards. One student wrote “I did in third grade…by reading over them to get them in my head.” That response was clearly the opposite of what I was hoping to do in my classroom.

The questionnaire also revealed that most students thought learning multiplication was important. Here are some of their responses to the question, “Do you think learning multiplication is important?”

- “Yes, because it will help me with division.”
- “Yes because it is always good to learn and a step further from adding and subtracting.”
- “Sometimes you need multiplication in science or other subjects.”
- “Yes, because it is much faster than plus and it is a fun challenge.”
- “Yes. Because it will make you smarter.”

Many of the students also wrote that they would need multiplication in life. It was interesting to see that the students already saw multiplication as useful. I wondered whether they had come to this notion themselves or if their parents and/or former teachers had influenced them.

One response to a question immediately caught my eye. When asked, “How do you feel about multiplication?” the student responded, “I feel proud.” That one response helped me see how important all strands of proficiency are. The fifth strand, productive disposition, was evident in that statement. I hoped that throughout the study all students would come to see themselves as capable learners and feel proud when they were able to tackle a problem. Overall, the questionnaire generally revealed that my students had a positive outlook on mathematics.

In addition to the questionnaire, students took a basic multiplication fact pretest. The students had ten minutes to answer fifty problems on the pretest. During the test, I observed students counting on fingers or using the back of the test to write repeated addition problems. I could hear murmurs of counting, such as 4, 8, 12, and so on. It was clear to me that the students’ strategies were very limited. The results showed that seventeen students scored below a 76% and six of those scored a 50% or below. Two students got more than 80% of the problems correct. The pretest showed my students did not already know all their basic facts, which was a good factor in my study. Had the students come in to fourth grade having memorized their facts, I do not believe my students would have engaged in the strategy instruction to the extent I was hoping.

Models, Patterns, and Strategies

For the very first instructional activity, I had the students use counters to represent 2 x 3. Most students showed an understanding that multiplication deals with equal groups. I observed students making two groups with three counters in each and even a few students made arrays. It was clear students grasped the idea that multiplication represents equal groups. It is likely they learned that concept in third grade. This activity helped me to see students' background knowledge of multiplication.

A valuable piece of information I learned in the beginning stages of the study was that students' understanding of number patterns for the nines was very limited. After working with hundred charts and circling the multiples of 2, 5, and 9 with different colors, students could easily explain the patterns for two and five. They could not, however, identify the patterns of nines right away. I thought perhaps it was because students had relied so greatly on the 9's "finger trick". For example, to solve 3 x 9, you hold up both hands with the back of your hands facing toward you. Starting with the pinky finger on the left hand, count three fingers over and bend the middle finger down. The fingers to the left represent tens (20) and fingers to the right of the bent finger represent ones (7). Using the "finger trick", students can easily get the answer of 27 for three times nine. Although an easy trick that gets a quick answer, the 9's finger trick diminishes the need to think about patterns with the nines and other strategies that could be used to figure out the products.

After giving the students time to look carefully at the 9's multiples, one or two students began to discover that the tens digit is one less than the other factor besides nine. They shared their responses with the class and others soon caught on. I also asked them to see if they could find any other patterns. Sometimes as a teacher we give our students opportunities to discover things on their own, but this was a time I needed to intervene. I felt it was my job to expose them to new strategies they might not have seen otherwise. I asked the class to pick one of the 9's products and find the sum of the digits. The students quickly found it to be nine. I encouraged them to try this on other multiples of nine and see what the sum of the digits would be. With a little help, students concluded that the digits within multiples of nine add up to nine. Students thought that was pretty amazing. Had I strictly breezed through the nines facts by giving the students the facts and products, students would have never gained more insight into these patterns. I soon noticed that students were trying to find patterns like this in other facts, so I had to correct that error and tell them this pattern was only true for the factors of nine.

Partial Products

My students had become familiar with equal groups, counting strategies, repeated addition, and arrays. I wanted to expose them to more efficient strategies, such as using derived facts. Since students were familiar with arrays, I decided I would use the array model to teach students this new strategy. I wanted them to understand that a multiplication fact could be broken apart into two partial products. I felt this was an important strategy for them to add to their toolbox of computation resources while laying down the foundations for the distributive property. At first, the students worked in pairs with counters to make arrays. The first problem I asked them to model was 3 x 4. I then asked them to break apart their array into two separate problems. The students had difficulty at first. Several added more rows and doubled the array to 6 x 4. Finally, a few students figured it out and shared how they could split apart the array. One pair of students modeled 2 x 4 and 1 x 4. Another pair of students modeled 3 x 2 and 3 x 2.

I asked them why it worked and the students explained it was still 3 x 4 because when they combined the two arrays they were still three rows of four, or 3 x 4. I could tell this was a brand new concept for the students, and I was excited to them deepen their understanding of multiplication through the activity.

For several days we continued to practice breaking apart arrays using one centimeter square dot paper. To do this, students had to draw lines around the array and then draw a line where they broke it up. I encouraged my students to choose how they would break apart different basic facts. To solve 8 x 8, one of the most common strategies was to split it in half and double 4 x 8. Others split 8 x 8 apart using 5 x 8 and 3 x 8. Eventually, they practiced using breaking apart a basic fact without the array representation. Based on class discussions, students were able to break apart a basic fact and use a combination of strategies. Sometimes students used a doubling strategy or partial products. Students also used derived facts and counted on.

Although students demonstrated the ability to use some of the new strategies they had learned, they preferred to use repeated addition. It was evident that students were still using repeated addition and even drawing arrays when examining their classwork and homework. They especially used these strategies when solving basic facts with higher operands. This was discouraging because of the discussions we had about using other strategies that are more efficient. One student explained to her classmates that if you are shopping in the grocery store and trying to do a calculation, you don’t want to have “whip out a notepad and pencil and start adding”. She was beginning to grasp the idea of using less cumbersome strategies to calculate.

Putting the Strategies to Use

When I asked the students to solve 7 x 7 on an index card for the purpose of data collection, it was very apparent repeated addition was the top choice of strategies. Four students used derived facts plus addition. Two of those four found the product of 5 x 7 and added the product of 2 x 7. One student found the product of 6 x 7 to be 42, and added 7 more. Another student used a mix of strategies. She used 3 x 7= 21, doubled 21, and then added 7 more to reach the final product of 49. Three students drew arrays, but only two out of the three counted correctly. Seven students solved 7 x 7 by using repeated addition.

Because I wanted to challenge my students to practice using some of the new strategies they had learned, I decided it was time to "outlaw" the use of the nine finger trick and repeated addition while working on basic facts in class. My students were not happy about the decision at first, but attitudes changed quickly when they started realizing they were in fact capable of using other strategies with just as much ease. I had students work in small groups and come up with all the ways they could solve 9 x 4 without using the finger trick, repeated addition, or drawing an array. Here are some of the calculation strategies they used.

1. 9 x 5 = 45, 45 – 9 = 36
2. 18 + 18 = 36
3. 9 x 2 = 18, 9 x 2 = 18, 18 + 18 = 36
4. 9 x 1 = 9, 9 x 2 = 18, 9 x 3 = 27, 9 x 4 = 36

In order to practice using different strategies, I had students make a "Strategies Book" where they could record their strategies. I challenged the students not to use repeated addition, and since some students still relied on drawing arrays, they could only use the array if they showed how it could be split apart into partial products. The children enjoyed working in their strategy books and were always excited for the next problem. They were also eager when it came time to share the strategies they used with their peers. They were beginning to enjoy challenging themselves. Discussion became much richer as students shared their strategies with the class. There would often be statements heard such, “I did it the same way” or “Wow, that is really interesting. I see why that works.” Students really began learning more from each other than just solving problem using my demonstrated strategies.

After looking back through the strategy books, it was evident that students were able to solve basic multiplication facts without relying on repeated addition. The strategies they used varied from one problem to the next and differed from one student to the next as well. For example, to solve 4 x 6 one girl added six plus six getting a sum of 12. She then doubled 12. (Figure 1).

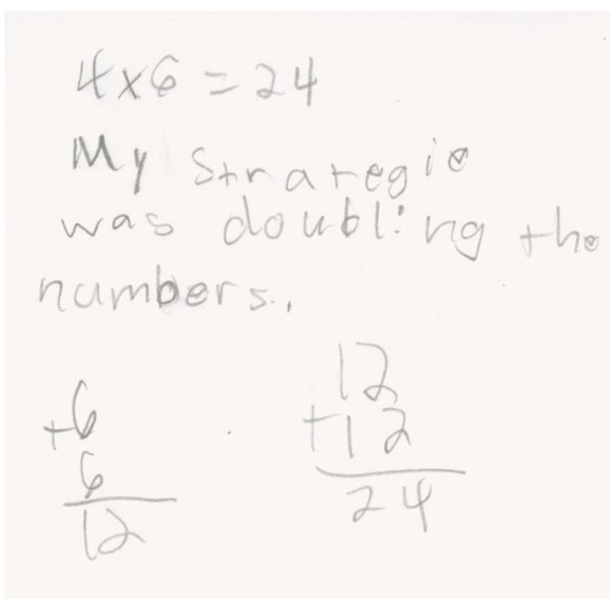

Figure 1: Doubling strategy for 4 x 6

Another student used a doubling strategy to solve the same problem. He drew an array to show how he found his partial products (Figure 2).

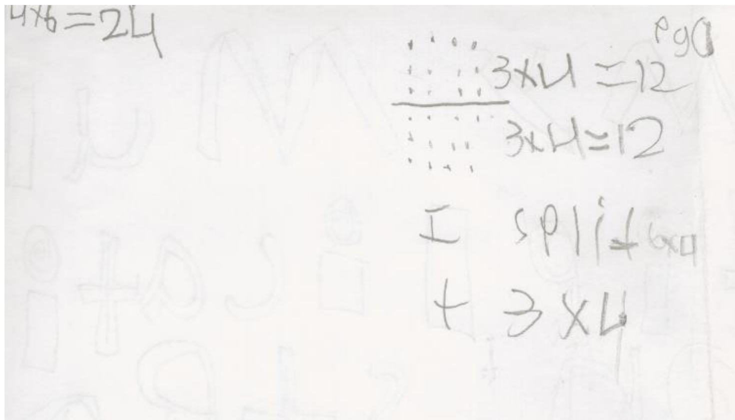

Figure 2: A student splits an array and uses a double to solve 4 x 6.

Similarly, one of the girls also used a double by finding 2 x 6 plus 2 x 6 (Figure 3). She explains that she "split the 4" then shows how she even uses adding to help get the answer.

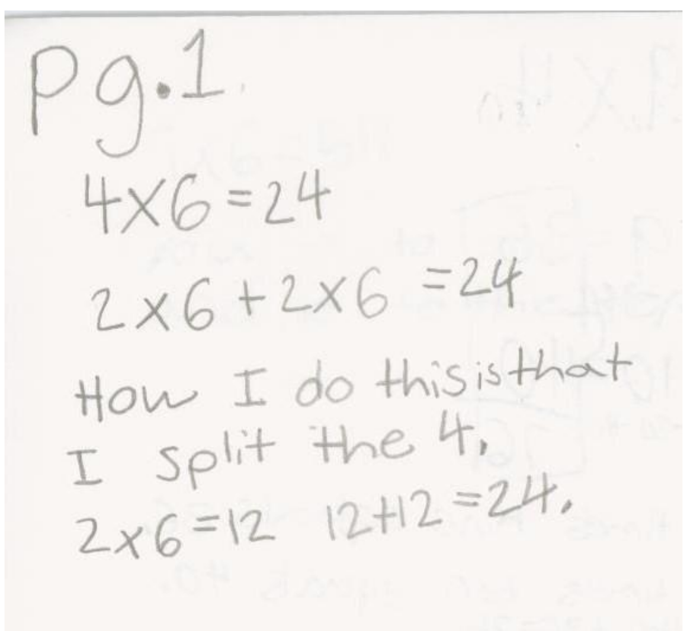

Figure 3: This student doubles 2 x 6 and writes her explanation.

An interesting strategy a student used to solve 6 x 6 was to add 6 + 6 + 12 + 12 (Figure 4). He begins by adding six plus six and then demonstrates an understanding of the need to add four more sixes if he were to continue with repeated addition. However, instead of adding six four more times, he adds twelve plus twelve. This shows multiplicative thinking in that he knows 2 x 6 = 12, and he needs to add 12 twice in order to make sure he has six sixes.

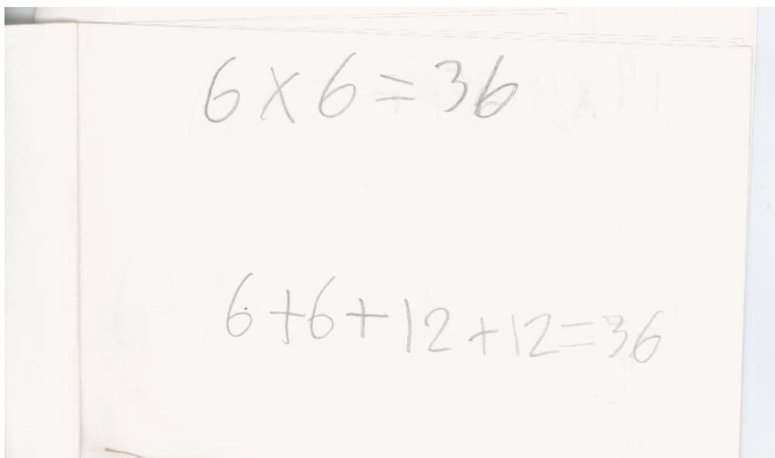

Figure 4: A student solves 6 x 6 by using adding and multiplicative thinking.

The students were beginning to choose more efficient strategies, but a handful of students were drawing arrays for facts with large operands. The students with learning disabilities in particular preferred to use the arrays over a calculation strategy (Figure 5). I encouraged the students to split arrays to help them develop the strategy of using derived facts.

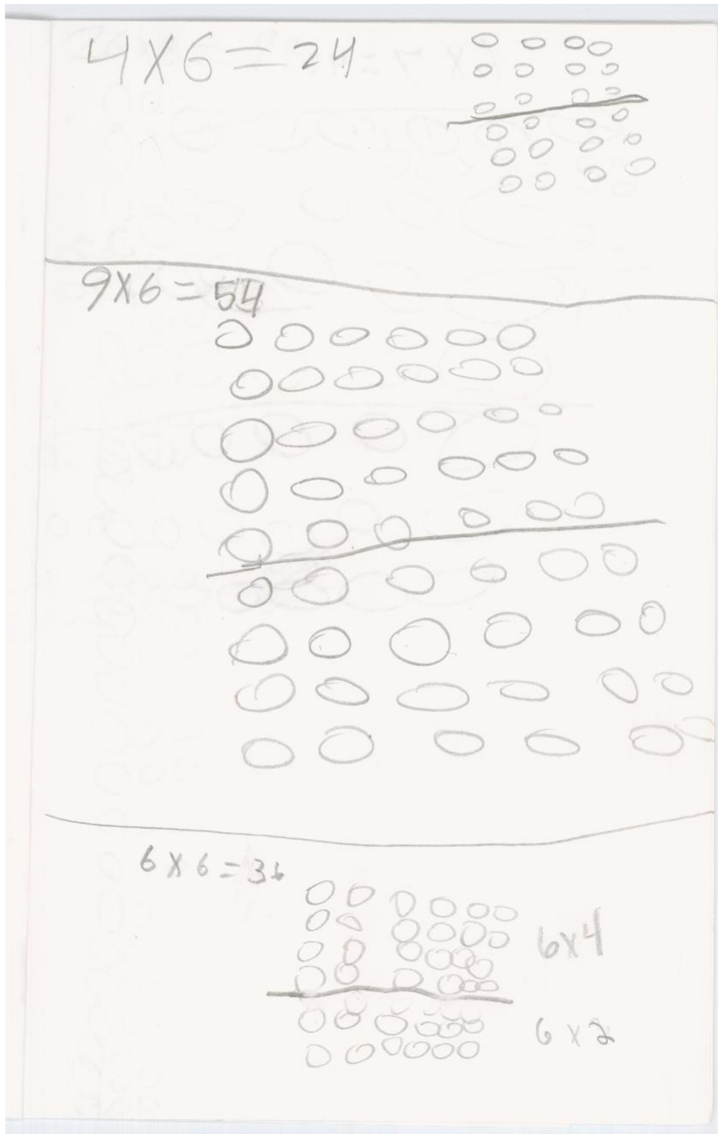

Figure 5: A student's sample of splitting arrays to solve basic facts.

Most Common Strategies

Throughout the study it seemed that students were using particular strategies for certain basic facts. For instance, the most popular strategy for the 9's was the finger trick. Even after examining patterns and other strategies for the 9's, students still preferred using their fingers. Many explained that it gives them the answer quicker than another strategy.

Students commonly used doubles for basic facts with 4, 6, or 8 as factors. For instance, if a student was solving 4 x 7, they would think of 2 x 7= 14 and then double 14. To find the product of 4 x 6, a student would either use 2 x 6 = 12 and double 12, or use 4 x 3 = 12 and double 12. I noticed that students were familiar with repeatedly adding 12 up to 48 fairly easily. Whenever a problem allowed them to get 12 as a partial product they would use that (12) to double. They used products of 12 for other problems too. For 6 x 3, students often used the Commutative Property and found 2 x 6 = 12, and then added 6 more. Generally, students were quick at solving facts such as 2 x 6, 3 x 4, and 4 x 6. It is not known whether students were getting these products through automatic retrieval or another counting strategy. Either way, they were fairly efficient with these facts and used them to solve problems with higher factors when possible.

Similarly to how students would double 3 x 4 =12 for 4 x 6, students also used doubles for problems with both factors being over five, such as 8 x 6. Just as students were able to quickly find products of 12, they were equally as quick when finding products of 24 which helped them solve problems with higher factors efficiently. For example, to solve 8 x 6, students would use 4 x 6 = 24, and then double 24.

Whenever 8 was a factor, students usually split the eight into four and four, since multiplying by lower factors proved to be easier for them. Even for a problem such as 8 x 7, students would split it apart into 4 x 7 and 4 x 7. In this case, students would use 2 x 7 plus 2 x 7, which is 28, and then double 28 to get the product of 56.

For facts involving factors of two, five, and ten, students generally relied on count-by strategies. Occasionally, students used a 5's facts and counted on for basic facts such as 6 x 7. Students using this strategy would first find 5 x 7= 35 and then add 7 more. Based on my observations and data collection, this strategy was less popular than using a double. Many students used repeated addition for all types of problems, but even more for higher factors and even for extended facts with 12 as a factor. Repeated addition was used less towards the end of the study than in the beginning. Most importantly, students were able to use reasoning skills by choosing strategies that were appropriate for each type of problem.

Inventive Strategies

Along with noticing that students used common strategies for particular facts, one of the most surprising findings was that students had their own strategies they used. Students were solving problems in ways that I had never even thought to do before. I was particularly impressed by one student's strategies for the nines' facts. In a one on- one interview she explained to me that she thinks of multiplying by 10 instead of nine and then subtracts. She said, "When I think of nine, I think of it is one less than 10. So if I am solving 2 x 9 that would be 2 less than 2 x 10, which is 18. Three times 9 would be 3 less than 30." She was able to justify exactly why she used that strategy. The neatest thing was that she said she had come up with this on her own. I was quite proud to see her really thinking about the numbers and using the strategy efficiently.

One day while the class was trying to compute 6 x 8, a student shared an extremely creative way to solve the eights' facts. She admitted that multiplying by eight was very difficult for her, so she had found a way that would help her. In comparison to my other student who used multiples of ten to help her solve the nines, this student also used a tens fact to help her with multiplying by eight. Going back to 6 x 8, she first used a fact that she knew, which was 5 x 8 = 40. From there, she added 10 getting a sum of 50. Last, she subtracted two and got 48.

I was astonished by this students' thinking, so I pulled her aside later to talk a little more about it. I asked her to show me how she would use the strategy for solving other problems with factors of eight. She explained and justified why it worked by showing me that it worked because ten minus two is eight. Going back to the original problem of 6 x 8, I was curious why she didn't just use the derived fact of 5 x 8= 40 and add eight more. I asked her, "Couldn't you just add eight?" She responded, "Yes, but it is easier for me to add ten and subtract two." That was an important conversation to my study. Through out my study, my goal had been for students to learn strategies that would help them solve basic multiplications with ease while developing proficiency. She had figured out a strategy of her own to help her do this. Students often used strategies they thought were easier verses ones I thought would be best to use.

Then, she shocked me even more with her understanding of numbers and multiplication. I asked her if she could show me how she would solve 9 x 8. She had already done 7 x 8 during our interview, so I think she chose to use that product and go from there. She began, "Seven times eight is fifty-six. Add 20 more and…" She suddenly got stuck so I questioned why she added 20. She told me that since we were trying to find two more groups of eight and she uses tens she would need to add two tens (20). "Then how much are you going to subtract?" I asked. She looked at me a little puzzled. Even though she had been using this strategy to add 10 and subtract 2, it was the first time she combined the tens. Wanting her to discover what the next step would be I guided her along. "Normally you add ten and take away two, but this time you add twenty. You doubled it." As a team, we figured out together that if she doubled ten she would also have to double the two that she subtracts. "So 56 plus 20 is 76. Take away four and that's 72."   This was an indicator that although she had a great strategy in place she was limited in how she used it. By working together, I believe I helped her see how her strategy could be even more effective for her. But more importantly, she taught me that students can think deeply and grasp a meaningful understanding of their strategies.

While working with students one on-one it helped me to see these two girls were not the only ones using their own strategies. One of my students was working on 8 x 7 and having difficulty. He was asking for help so I prompted him with the question, "What strategy could you use to solve it?" He proceeded to use his finger trick for the nines and multiplied 9 x 7. I was not quite sure why he was doing this, but then he subtracted seven from his original product of 63. I asked him why that was the strategy he chose and he said he always tries to find the easiest way to the solution. Just like the girls mentioned before, he also wanted a quick way to calculate the answer. Although, I had not taught the students to use a known fact and subtract, many had discovered it on their own or had adopted it after hearing their peers explain how they used it.

Extended Facts and Beyond

Students were able to solve extended facts with much more ease than I anticipated. They quickly realized that multiplying with factors of 12 can be made simpler by thinking of a ten's fact and a two's fact. Much to my surprise, students were even able to solve harder two-digit times one-digit facts, such as 3 x 46 and 7 x 36. Prior to any instruction on how to multiply with factors greater than twelve, the students were solving these types of problems by applying some of the strategies they had used for basic facts. For instance, one student solved 7 x 36 by first finding 7 x 30 = 210. He then multiplied 7 x 5 and 7 x 1. Finally, he added up all three partial products to get 252. This demonstrated deep understanding, fluency, and problem solving skills. To solve 4 x 27, a student used doubles by adding 27 and 27 and got 54. She then doubled the sum equaling 108. Without difficulty, she was able to apply the doubling strategy with numbers less than 10 to greater numbers.

For me, this was yet another astonishing moment of the study. The students had reasoning skills that allowed them to manipulate the numbers in a way that made sense. The most interesting part was that they were the ones teaching each other. The lesson was teacher facilitated, but I never once told them exactly how to solve these problems. I never would have believed that my students could solve extended facts and facts with double digits without using the standard algorithm, but they proved me wrong and amazed me with their level of understanding. Needless to say, it was a very exciting day in my room.

As we shifted instruction from basic facts to extended facts and then on to multiplying greater numbers, I continued to encourage students to use strategies that would help them solve multiplication problems. While helping one of my students with 80 x 60, I guided him towards finding the basic fact in the problem. He identified that 8 x 6 could be used to help him solve the problem. He was able to take over from that point. He used the Commutative Property and explained that he knew 2 x 8 = 16, so if he tripled 16, it would be the same as 8 x 6. He got 48 then added two zeros to show the shift in place value. Upon first glance of the problem, he was clearly frustrated and felt defeated, but after guiding him toward a strategy he beamed with pride knowing that he had really been the one to solve such a challenging problem. Examples such as this as well as observing students during instructional time and analyzing classwork helped me to see that students can use strategies even when solving problems beyond basic facts.

Posttest

After about four weeks of learning, examining, practicing, and sharing basic fact strategies it was time for the posttest. The students had ten minutest to complete the test, which was the same amount of time as the pretest. The outcome on the posttest was much more positive than the pretest. Only one student did not make gains from the pretest to the posttest. Three students got 70 % correct and all others scored 80% or higher. The posttest scores show that students are becoming proficient with basic facts; however, they were not all 100% proficient yet. The range in scores from the pretest to the posttest was extremely high for most students, demonstrating growth that took place.

Table 1 shows individual student scores for both the pretest and posttest. The numbers in the pretest and posttest columns represent the percent of basic facts answered correctly out of fifty problems. Students were randomly given a number to protect to their confidentiality.

Table 1: Percent Correct on Pretest and Posttest and Gains Made

Student	Pretest	Posttest	Gains
1	34	42	10
2	38	90	52
3	32	72	40
4	50	76	26
5	76	78	2
6	60	80	20
7	48	84	36
8	90	96	6
9	74	92	18
10	72	94	22
11	90	96	6
12	70	100	30
13	84	100	16
14	90	98	8
15	88	94	6
16	86	94	8
17	82	94	14
18	86	96	10
19	50	absent	
20	absent	absent	
Class Average	68.4	87.6	19.2

Some common patterns on the posttest were noticed. First, when students skipped problems altogether, they were basic facts containing higher numbers such as 6, 7, or 8. Students also made errors in their calculations and were often close to the correct product. For instance, a student wrote 55 as the product of 7 x 8. This student scribbled on her paper 16 + 16 + 16 + 7. She clearly made a mistake by adding a 7 instead of an eight. She had the right idea but went from groups of eight (doubling 8 to get 16) to a final 7. Several papers had addition problems written in the margins and all were the higher operands.

Initially, I thought ten minutes was ample time for students to demonstrate fluency; however, if they had been given more time to complete the posttest I believe they would have answered more problems correctly because they would have had time to put their strategies to use.

Word Problems

Toward the end of the study, I wanted to see if students were developing deeper understanding of multiplication so I asked them write story problems for 6 x 4. "A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes" (Kilpatrick et al. (2001, p. 119). I had begun my multiplication instruction by modeling story problems and having the students write story problems for basic facts, but it had been several weeks since then.

Below are some of their story problems that clearly demonstrated multiplicative thinking. Note that the responses were recorded exactly as the students wrote them, except for corrections in spelling or punctuation.

1. Lucky has 6 dog treats in one box. How many dog treats would lucky have if he had 4 boxes? 24 dog treats
2. There are 4 children. They each got 6 candies. How many candies in total?
3. There were six rows of 4 people and the movie person wants to know how many people were sitting in the 6 rows.
4. There were six kids. Each kid had four cards. How many cards are there in all?
5. There were 6 cupcakes and 4 times that in cookies. How many cookies were there?

6. There are 6 puppies. Each puppy gets 4 chew toys. How many toys are there in all? 24 toys

7. There were 6 bushes. Each bush had 4 Easter eggs hidden inside each bush. How many Easter eggs are in all?

These all came from students who are generally the higher level thinkers in the classroom. The majority of these stories also came from girls.

About half of the students were able to write a story problem that demonstrated multiplication. The other half of the students were not able to write a story problem. One student said, “I get division stories and multiplication stories mixed up.” Another student wrote a division story for 24 divided by 4. This could be due to the fact that the continuation of multiplication strategies overlapped with division instruction. On a positive note, these students showed they understood that multiplication and division are related.

One student wrote a story indicative of addition. The problem is recorded here:

There were 6 kids. Then 18 more showed up. How many were there? 24

This student’s problem is interesting. I did not tell the students the product was 24. He somehow made the connection that 6 x 4 = 24 because he chose to write an addition situation that added up to 24. This leads me to believe he had to have known the end product was 24, but he was unsure of how to demonstrate the multiplication through an appropriate story problem.

The other seven students either said they did not know, could not remember, or did not write anything. This was somewhat disheartening, since we had spent so much time on what it means to multiply. On the other hand, it reflected the notion that multiplication is a challenging skill. It was also important to take notice that all of my identified students who receive support from the ESE teacher were the ones who did not write a story problem. Interestingly, the students who did not write a story problem answered on average about 70% of the posttest facts correctly. All of the students with learning disabilities were not able to write a story problem. However, students who answered about 90% of the posttest problems correctly were the ones who wrote appropriate story problems, thus showing evidence of the correlation between fluency and understanding.

Final Questionnaire and Interviews

Based on the final questionnaire and interviews, most students reported to have much more difficulty with higher operands, in particular 6, 7, 8, and even 9. When solving problems with at least one of these numbers as a factor, students generally shifted toward the strategy of repeated addition. Even after allowing students to discover and practice the use of partial products and derived facts, many still opted for repeated addition. While a few students used the patterns involved with 9's, most students still preferred the nine's finger trick.

The questionnaire showed that students enjoyed learning strategies. One student mentioned she had already known some before I taught the class, but that she had fun learning more. Many students stated they loved learning strategies because it helped them solve harder problems. There were a couple of students who said the strategies can be confusing. One student said she just liked memorizing. I had not encouraged memorization in the classroom at all, so this told me that her parents had probably been working with her at home. Several students mentioned they "felt good" when they solved a problem correctly.

Proficiency Revealed

Each day during the study, we spent time discussing the strategies that could be used for solving different problems. I could tell the students started gaining a sense of pride when they would share their explanations with their peers. They often impressed me with their reasoning skills and the way they could articulate how, when, and why to use a certain strategy. Their ability to write story problems also demonstrated understanding of meanings for multiplication.

One of my students who groaned about how difficult multiplication was when we first began (her low pretest score was an indicator that she had little experience with multiplication and solved the majority of the problems with repeated addition) soon embraced new ways of thinking about multiplication. "This is fun!" she would say. She even mentioned how she liked learning strategies from another student in the class during discussions. Most students showed a deep understanding of multiplication and could fluently solve basic and extended facts.

Summary

Data were collected through questionnaires, interviews, observation, and a pre and posttest. The data revealed that students had come to fourth grade with a basic understanding that multiplication deals with equal groups. They also were familiar with repeated addition and counting strategies such as counting-by or counting on. Students were unfamiliar with patterns for solving nines, partitioning strategies, and other mixed strategies. It was evident from the data that students gained more computational resources for solving basic multiplication facts and extended facts. However, it seemed that students used doubles and repeated addition over other strategies and this was a reoccurring theme throughout the study. Another theme that emerged was that students had a much more difficult time solving basic facts with higher operands. In addition, the data showed that students invented their own strategies that were helpful to them. Many demonstrated the ability to manipulate numbers reasonably to solve problems. Overall, the students maintained positive feelings towards multiplication and enjoyed learning strategies. Finally, the pre and posttest scores revealed that after strategy instruction students' scores increased. The final chapter of this study explains the results of the study, implications for those involved with education, limitations, and recommendations.

CHAPTER 5: CONCLUSION

Introduction

As I began my action research study, I sought to explore what would happen if I changed my instruction for basic multiplication facts. My research questions were:

Question 1: How can focusing my instruction on multiplication strategies help fourth grade students develop proficiency with basic multiplication facts?

Question 2: How can strategies with single digit multiplication be applied to extended facts?

In this chapter I review the results of my study in relation to the literature. I also discuss implications, limitations, and recommendations for further research.

Results

Overall, I learned that basic multiplication fact proficiency is developed in an environment that focuses instruction on deeper meaning. My students were forced to think about numbers instead of being spoon fed answers to multiplication problems. Researchers argue that it is imperative for children to develop the concept that arithmetic is more problem solving and strategic reasoning over simply getting quick answers (Steffe, Cobb, & Glassersfeld, 1988). Research highly supports the use of strategy instruction within the classroom (Anghileri, 1989; Baroody, 1984; Cooney, et al., 1988, Fuson, 2003; Isaacs & Carroll, 1999; Kilpatrick et al., 2001; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005; Steffe et al., 1988).

In comparison to other studies, I also found that students had a more difficult time with higher operands (Anghileri, 1999; Steel & Funnell, 2001). Students tended to have an easier time with factors of 1-5 and 10. Factors including 6, 7, 8, and even 9 proved to be more difficult, and this was evident in the data. Students especially chose the strategy of repeated addition to solve problems with factors of 6 through 12 because they said it was easier for them. They also used doubles frequently and were efficient with this strategy. Students also invented their own strategies, which is consistent with Fuson's (2003) research. Fuson (2003) also suggests that students tend to have many errors in their invented strategies when learning has not been centered upon deeper understanding. This was evident in the data, and many times students just needed a little guidance in helping them use their strategies more efficiently.

According to the research, students with learning disabilities can have an even harder time learning multiplication, but teaching them strategies can prove to be effective (Tournaki, 2003). All students with learning disabilities demonstrated the ability to use strategies successfully while learning basic multiplication facts. These students did express that multiplication was challenging for them.

Similar to another study in which students were taught multiplication strategies, my students were able to perform calculations on extended facts with ease (Woodward, 2006). My instruction encouraged students to examine strategies for factors of twelve. Students were actually able to transfer that knowledge to other extended facts and even higher two digit problems, such as 7 x 36 or 3 x 46. This was an amazing realization, since I had not taught students any problems with factors higher than twelve.

Focusing my instruction on strategies created an atmosphere for students to have discussions about the strategies they used. As a result, this produced several positive outcomes. Students gained confidence in themselves as they explained and justified their own strategies.

This was evident in their smiling faces and their eagerness to continue sharing. Their communication skills were built up while becoming a community of learners. All of these findings correlate to the belief about the importance of developing students' adaptive reasoning skills and productive disposition (Kilpatrick et al., 2001). Had I not spent time teaching strategies and allowing the students to share their strategies, I do not believe these strands would have been fostered. In the past, I have enforced memorization and provided activities such as timed multiplication drills which do not promote discussion. This was a valuable lesson for me that has already impacted how I teach mathematics.

Implications

Since data showed that learning multiplication facts, especially those with higher operands, is a challenging task for elementary students, perhaps more attention should be paid to these types of problems.

Teachers across all grade levels should allow their students the chance to practice solving mathematics problems through different strategies. The Next Generation Sunshine State Standards, Florida's new mathematics standards based on NCTM's *Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics: A Quest for Coherence,* require teachers to change their instruction to be more meaningful (FLDOE, 2007; NCTM, 2006). Teaching strategies for all the basic facts is one way to do this. It fosters the notion that mathematics is much more than just memorizing facts. Strategy instruction promotes thinking, reasoning, and problem solving. While the strands of proficiency cannot be mastered in a short time, they can certainly be developed through strategy instruction.

Because of the new mathematics standards, both third and fourth grade curricula are centered on multiplication. The NGSSS pertaining to multiplication for third grade and fourth grade are as follows:

> MA.3.A.1.1: Model multiplication and division including problems presented in context: repeated addition, multiplicative comparison, array, how many combinations, measurement, and partitioning.
>
> MA.3.A.1.2: Solve multiplication and division fact problems by using strategies that result from applying number properties.
>
> MA.4.A.1.1: Use and describe various models for multiplication in problem-solving situations, and demonstrate recall of basic multiplication and related division facts with ease.
>
> MA.4.A.1.2: Multiply multi-digit whole numbers through four digits fluently, demonstrating understanding of the standard algorithm, and checking for reasonableness of results, including solving real-world problems (FLDOE, 2007).

These standards cannot be achieved without looking deeper into multiplication. Students in third grade should be exposed to numerous strategies which will give them a foundation for fourth grade and lead them on the path to proficiency. This study gives valuable insight to third and fourth grade teachers who will be teaching multiplication for meaning in the coming years.

Limitations

One major limiting factor in this study was time. With so many skills to teach prior to state's standardized testing, time is of the essence. I spent much more time on basic multiplication facts than is recommended by both my county's scope and sequence and the textbook. Even though I spent a great deal of time (more than I have ever spent) on multiplication facts, I still believe it was not enough. I was eventually forced to move on due to the realization that I had so many other mathematics skills to teach in order to prepare my students for standardized testing. Another factor of time to be considered was on the pretest and the posttest. If the students had been given more time than the allotted ten minutes it is possible they would have answered more problems correctly because they would have had more time to think about their strategies.

If I had been able to devote more classroom instruction time specifically to basic multiplication facts, I believe the students would have benefited more. Research supports the notion that proficiency with multiplication is developed over time (Thornton, 1978; Cooney et al., 1988; Anghileri, 1989; Erenberg, 1995; Mulligan & Mitchelmore, 1997; Isaacs & Carroll, 1999; Fuson, 2003; Kilpatrick et al., 2001; Sherin & Fuson 2005). In addition, Baroody (1999) argued that students begin to use strategies much more efficiently as they get older. It would be interesting to study the same students over the course of an entire school year or even two school years, particularly third and fourth grade.

Another factor to consider is the population of my study. My students generally come from homes that support their students' learning. These children may have an advantage over other students who do not have the same level of parental support.

Recommendations

Because the research on multiplication is limited, much more is required (Cooney et al., 1988; Fuson, 2003; Kilpatrick et al, 2001; Sherin & Fuson, 2005). With the education system undergoing reform, teaching is not the same as it once was. Educators need to have access to research based information on how best to teach students. There have been a few studies devoted strictly to the types of strategies students use (Anghileri, 1999; Steel & Funnell, 2001), but there is still much to be learned about how students learn multiplication so that educators can know how to best meet the needs of their students.

If I were to do this study again, I would change some of my data collection techniques. Specifically, I would use video to capture the students' rich discussions and their excitement toward learning new strategies. I would have also recorded interviews for ease of analyzing data. Additionally, focusing the study on specific factors would be valuable. Since the higher operands prove to be more challenging for students, it would be interesting to do a study strictly on factors of six, seven, eight, and nine.

Summary

I began my action research in hopes of learning more about how my fourth grade students learn multiplication, and how I could improve my teaching in that area. I wanted to know if focusing my instruction on basic multiplication fact strategies would help fourth graders develop proficiency with basic and extended facts. While multiplication is a challenging skill for most fourth graders, strategies did make solving multiplication problems much easier for students and many used more efficient strategies as the study progressed.

I chose to study this topic because of the reform taking place in mathematics throughout the country, as well as in my state. Since the new standards are calling for deep understanding of mathematics, I realized teaching multiplication through rote practice may not develop the strands of proficiency. Rather than focusing on repetition, I sought to give my students the experience of examining patterns, meanings, representations, and eventually calculation strategies for basic multiplication facts. Students demonstrated fluency and understanding with both basic and extended facts, but still had room to improve. They developed quick retrieval with some of the facts, but needed more practice with higher operands. They also demonstrated reasoning skills in their strategy choices for different facts. Overall, my fourth graders' feelings towards multiplication were positive, and they enjoyed sharing their strategies with the class.

It is imperative for students to develop all five strands of proficiency in the elementary years, and giving students an opportunity to look deeper into multiplication does lead students on the path to proficiency. On a personal note, I can still remember practicing and memorizing basic multiplication facts in my elementary days. While practice is important, I often wonder how differently I would have thought of mathematics had I been given the chance to think instead of only memorize at such an early age. I believe I learned just as much as my students about the ways in which number facts can be manipulated and the strategies that can be used to solve them. To all teachers, the way we learned multiplication may not be the best way. It is time we help our students develop proficiency and help them dive deeper into mathematics.

APPENDIX A: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Stacey Braddock**

Date: **September 18, 2009**

Dear Researcher:

On 9/18/2009, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	Initial Review
Project Title:	The effect of teaching multiplication strategies on fourth graders' computational fluency.
Investigator:	Stacey Braddock
IRB Number:	SBE-09-06389

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Joseph Bielitzki, DVM., UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 09/18/2009 08:21:22 AM EDT

Janice M. Turchin

IRB Coordinator

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APPENDIX B: PRINCIPAL APPROVAL

August 11, 2009

Institutional Review Board
University of Central Florida
Office of Research & Commercialization
12201 Research Parkway Suite 501
Orlando, FL 32826-3246

Dear IRB Coordinator,

Miss Stacey Braddock has notified me about the action research study she would like to implement with her 4th grade class during the 2009-2010 school year. She will be studying the effects of strategy instruction in multiplication and computational fluency.

I have reviewed the letter that will be sent home to parents for permission. I have also read the child assent letter.

I authorize Miss Braddock to conduct her proposed research project with her students beginning in September of 2009.

Sincerely,

[redacted]

[redacted]

Principal

APPENDIX C: PARENTAL CONSENT FORM

Dear Parents, September, 2009

As part of my master's program at the University of Central Florida (UCF), I am completing an action research study on my teaching practice. I need the help of students who agree to take part in this research study. You are being asked to allow your child to participate in this study which will include the students in my 2009-2010 fourth grade class. This project is being guided by Dr. Juli K. Dixon.

The purpose of this study is to explore the effects of multiplication strategy instruction on children's computational fluency. In other words, will spending allotted time on multiplication strategies each day over several weeks help students to solve multiplication problems with greater accuracy and understanding?

All students will receive the same instruction; however, only those children with parental consent will participate in gathering the data that will be used to measure whether the strategy instruction is successful. Strategy instruction on multiplication will take place for 15-20 minutes a day during the first and possibly second trimester. Instruction will be given to the whole group and will be part of the regular school day routine. Dr. Dixon will be observing the class occasionally and taking field notes. I will be interviewing each child, as well as collecting work samples. All students will be given pseudonyms to protect confidentiality.

There are no anticipated risks, only the benefits of the students learning more about the research process and building their multiplication skills. Students will not receive extra credit or compensation for taking part in this study.

Research at the University of Central Florida involving human participants is carried out under the oversight of the Institutional Review Board (UCF IRB). This research has been reviewed and approved by the IRB. For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901.

It is not mandatory to participate in the study and students' grades will not be affected if you choose not to participate. You can also withdraw your child at any time. If you have any questions you may contact me at [redacted] or Juli K. Dixon at [redacted]

Sincerely,

Miss Braddock

Your signature below indicates your permission for the child named below to take part in this research.

____

____

Name of participant

UCF University of Central Florida IRB
IRB NUMBER: SBE-09-06389
IRB APPROVAL DATE: 9/18/2009

Signature of parent or guardian

Date

☐ Parent
☐ Guardian (See note below)

Printed name of parent or guardian

Note on permission by guardians: An individual may provide permission for a child only if that individual can provide a written document indicating that he or she is legally authorized to consent to the child's general medical care. Attach the documentation to the signed document.

THIS SECTION FOR USE BY UCF STUDY PERSONNEL:

If signature of second parent not obtained, indicate why: (select one)

- ☑ IRB determined that the permission of one parent is sufficient
- ☐ Second parent is deceased
- ☐ Second parent is unknown
- ☐ Second parent is incompetent
- ☐ Second parent is not reasonably available
- ☐ Only one parent has legal responsibility for the care and custody of the child

Assent
- ☐ Obtained
- ☑ Not obtained because:
  - ☑ IRB determined that assent of the child was not a requirement
  - ☐ The capability of the child is so limited that the child cannot reasonably be consulted.

Stacey Braddock

Printed name of person obtaining consent and assent

Stacey Braddock 9/24/09

Signature of person obtaining consent and assent Date

My signature and date indicates that the information in the consent document and any other written information was accurately explained to, and apparently understood by, the participant or the participant's legally authorized representative, and that informed consent was freely given by the participant or the legally authorized representative.

UCF University of Central Florida IRB
IRB NUMBER: SBE-09-06389
IRB APPROVAL DATE: 9/18/2009

APPENDIX D: COUNTY APPROVAL

August 28, 2009

Ms. Stacey Braddock
[redacted]

Dear Ms. Braddock:

I am in receipt of the proposal and supplemental information that you submitted for permission to conduct research in the [redacted] County Public Schools. After review of these documents, it has been determined that you are granted permission to conduct the study described in these documents under the conditions described herein.

Your school principal has the authority to decide if she wishes to participate in your study. Therefore, if you have not already done this, your first order of business is to contact [redacted] and explain your project and seek permission to conduct your research at [redacted] If necessary you are expected to make appointments in advance to accommodate the administration and/or staff for research time.

Please forward a summary of your project to my office upon completion.
Good Luck!

Sincerely,

[redacted]

[redacted]
Executive Director
Secondary Education

[redacted]
cc [redacted]

APPENDIX E: INITIAL QUESTIONNAIRE

Math Questionnaire

1. Do you like math? ____

2. What is your favorite part of learning math? ___________________________________

   ____

3. How do you feel about timed math tests? ____________________________________

   ____

4. Do you practice multiplication on your own? Yes or No? ________________________

5. If yes to # 4, how do you practice on your own? _______________________________

   ____

6. How do you feel when you come to a word problem in math? ____________________

   ____

7. Do you think it is important to learn multiplication? Explain.______________________

   ____

8. What is your favorite way to solve a multiplication problem? ____________________

   ____

9. How do you feel about multiplication? _______________________________________

   ____

APPENDIX F: PRETEST AND POSTTEST

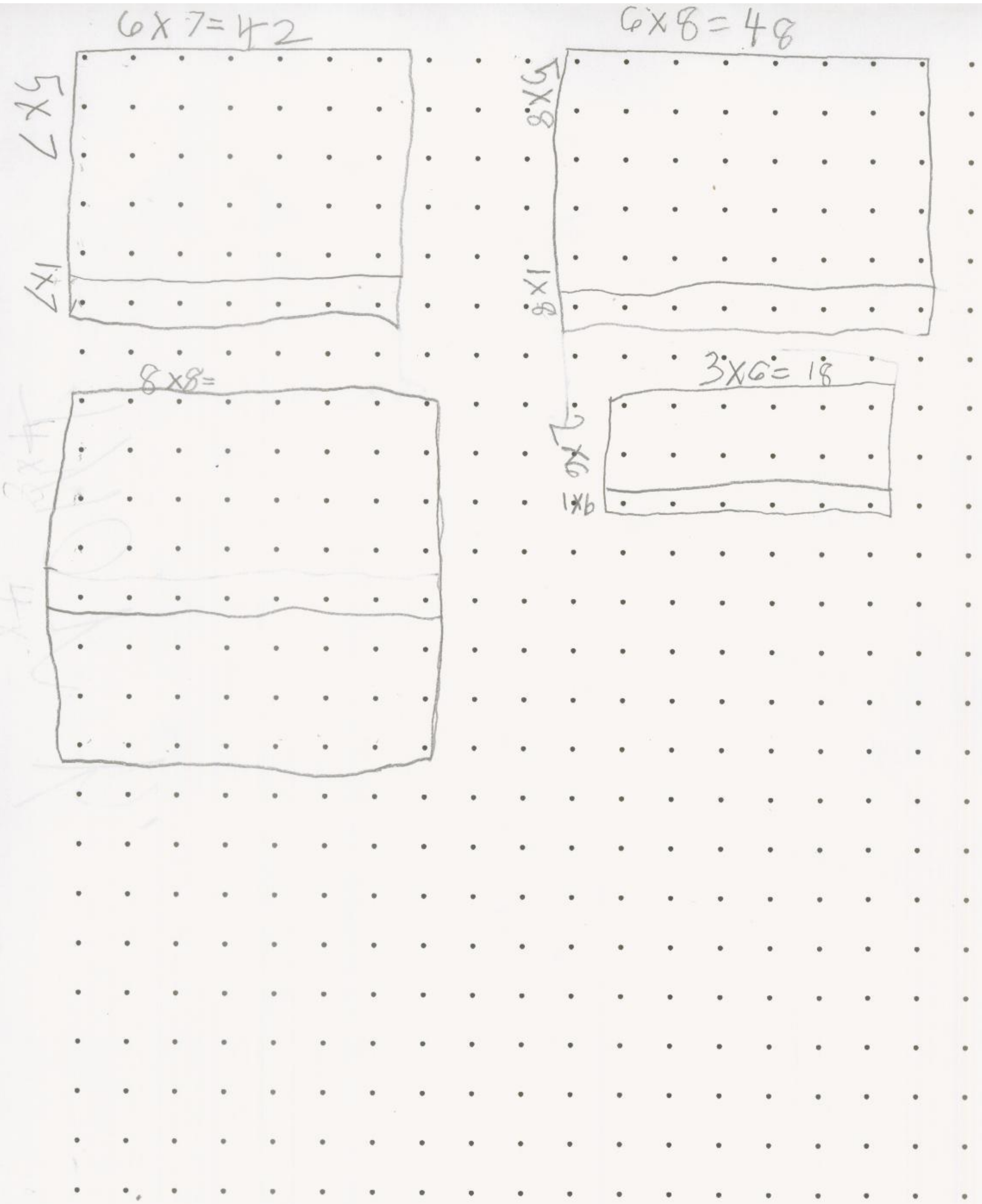
Name____

Basic-Facts Timed Test 5

Give each answer.

1. 4 × 3 = ______
2. 7 × 3 = ______
3. 5 × 5 = ______
4. 8 × 1 = ______
5. 3 × 3 = ______
6. 8 × 6 = ______
7. 9 × 2 = ______
8. 3 × 4 = ______
9. 5 × 8 = ______
10. 9 × 3 = ______
11. 2 × 8 = ______
12. 3 × 6 = ______
13. 2 × 5 = ______
14. 0 × 7 = ______
15. 5 × 6 = ______
16. 2 × 9 = ______
17. 8 × 2 = ______
18. 5 × 4 = ______
19. 9 × 6 = ______
20. 2 × 7 = ______
21. 8 × 3 = ______
22. 7 × 2 = ______
23. 3 × 8 = ______
24. 6 × 7 = ______
25. 7 × 4 = ______
26. 5 × 3 = ______
27. 1 × 4 = ______
28. 7 × 6 = ______
29. 6 × 3 = ______
30. 7 × 8 = ______
31. 8 × 4 = ______
32. 6 × 2 = ______
33. 4 × 9 = ______
34. 5 × 7 = ______
35. 3 × 2 = ______
36. 6 × 9 = ______
37. 9 × 7 = ______
38. 2 × 6 = ______
39. 8 × 5 = ______
40. 6 × 5 = ______
41. 4 × 0 = ______
42. 9 × 8 = ______
43. 5 × 2 = ______
44. 7 × 7 = ______
45. 5 × 9 = ______
46. 2 × 4 = ______
47. 9 × 9 = ______
48. 4 × 8 = ______
49. 6 × 6 = ______
50. 7 × 9 = ______

APPENDIX G: SAMPLE OF DOT PAPER WITH ARRAYS



1-cm square dot grid—33

APPENDIX H: FINAL QUESTIONS

Final Questions

1. How do you feel about multiplication?

2. How would you solve 7 x 7?

3. Did you like learning strategies for multiplication?

4. Write a story problem for 4 x 6.

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