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An Investigation Of Students' Problem Solving Skills In An Introductory Physics Class

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AN INVESTIGATION OF STUDENTS' PROBLEM SOLVING SKILLS IN AN INTRODUCTORY PHYSICS CLASS

by

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ABSTRACT

The purpose of this study was to document the specific errors that introductory physics students make in each phase of the solution of Force and Motion problems. A mixed methods design was used to identify those errors, and it was determined that the errors which students made the most frequently were the omission of $mgcos\theta$, $mgsin\theta$, and the lack of a clearly defined coordinate system as part of the free-body diagram. Additionally, there was a negative statistically significant relationship between the quality of the free-body diagram and the quality of equations that were produced to describe the object's motion. The results indicate that students do not have a full understanding of the role of a free-body diagram or its relationship to the system of equations that are generated as a result of the application of Newton's Second Law to the free-body diagram.

This document is dedicated to my son, Marquise, and Sabrina for their support and patience throughout my entire graduate career.

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CHAPTER ONE: INTRODUCTION

Students who have completed an introductory course in physics should be able to, at minimum, "think logically and coherently about technical issues, and be able to apply basic concepts of physics to more complex problems" (Knight, 2004, p. 10). However, an investigation into students' conceptual understanding by Dufresne, Gerace, and Leonard (1997), using the Force Concept inventory, revealed that "many of these students completed their introductory physics courses with a shallow understanding of concepts, and with a narrow set of problem-solving skills" (Dufresne, Gerace, and Leonard, 1997, p. 2). The Force Concept Inventory (FCI), designed by Hestenes, Wells, and Swackhamer, (1992), is a multiple choice assessment designed to test students' understanding of basic physics concepts and their ability to translate physics concepts to similar problems which are placed in a different context. According to Hestenes, Wells, and Swackhamer,(1992), the factors which contributed to the deficiency in students' quantitative problem solving skills and their conceptual understanding of physics concepts need to be understood and documented, since the quality of a student's conceptual knowledge affects the development of their problem solving skills and how to best apply those skills (Knight, 2004).

The National Research Council(NRC) argued that "the ability to apply knowledge to novel situations, that is, transfer of learning, is affected by the degree to which students learn with understanding" (Bransford, Brown, Cocking, Donovan, and Pellegrino, 2000). The extent to which this transfer takes place depends on the students' conceptual understanding of physics concepts. Therefore, instructional strategies must be designed in such ways that facilitate a student's conceptual growth. The most effective form of instruction takes student misconceptions into account (Heller and Reif, 1984). There has been a considerable amount of research on student problem solving ability using multiple choice questions. However, in order to get the most comprehensive characterization of the state of a student's understanding, non-multiple choice problems must be used in addition to the multiple choice test, since students perform differently on them (Berg and Smith, 1994). Therefore, the major focus of this study was to characterize the quality of student understanding of physics concepts, by identifying the specific errors they made on their solution to non-multiple choice problems, since these problems allowed for a deeper analysis of the solutions. These misconceptions can be used to inform instructional strategies used by educators so that they can model their instruction to address students' misconceptions, which is likely to result in a deeper conceptual understanding.

Physics is filled with many different classes of problems. However, Force and Motion problems (F&M), which formed the basis of this study, rendered themselves as the best choice since they lay the foundation for other topics that students will encounter later in their physics course and other science courses. Approximately 40% of all publications in Physics Education Research has been focused on Force and Motion problems (Duit, Niedderer, and Schecker, 2007; Etkina, Van Heuvelen, White-Brahmia, Brookes, Gentile, Murthy, Rosengrant, & Warren, 2006; Rosengrant, Van Heuvelen, & Etkina, 2005; Rosengrant, 2007; Rosengrant, Van Heuvelen, & Etkina, 2009; Heller & Reif,1984). In addition, force and motion problems allowed for the investigation of a more detailed analysis of the quality of a student's understanding of broad physics concepts, as their conceptual understanding is a major factor in the development of their overall problem solving skills (Knight, 2004), and their solution process is well defined in three distinct stages (Heller & Reif, 1984):

1. The generation of an initial problem description and qualitative analysis designed to facilitate the subsequent construction of a problem solution
2. The generation of the actual solution by methods which facilitate the decisions making required for efficient search
3. The assessment and improvement of this solution

The first phase was of particular important since "the initial representation of the word problem ultimately determines how easily a problem is solved and what is learned in the process" (Dufresne, Gerace, & Leonard, 1997, p. 2). Therefore, by helping students increase their ability to perform well in phase one, their learning gains will likely be increased. This study focused on a specific representation in phase one, known as the free-body diagram (FBD), which is a diagrammatic representation that is used to show the forces acting on an object. The second phase required the student to apply a specific physics concept know as Newton's Second Law (NSL) to the representation in phase one. Unlike multiple choice problems, the results of each of these two phases of F&M problems provided more insight into student's misconceptions which may ultimately affect their conceptual understanding of physics. The combination of the results of non-multiple choice and multiple choice problems provided a more complete description and understanding of the misconceptions that students have and the errors they made when they solved the problems.

Purpose

The purpose of this study was to identify the conceptual difficulties that may hinder students as they solve non-multiple choice problems, by identifying the specific errors they made in the different phases of the Force and Motion problem solution.

Research Questions

1. What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?
2. What types of errors do students make in phase II of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?
3. Does quality of performance in Phase I predict the quality of performance in Phase II?

Conceptual Framework

The conceptual framework was couched in two major foci: 1) students' conceptual understanding of force and motions concepts, and 2) error identification as it related to students' demonstrated problem-solving performance assessed with the Force Concepts Inventory (FCI). The National Research Council (NRC) argued that "the ability to apply knowledge to novel situations, that is, transfer of learning, is affected by the degree to which students learn with understanding" (Bransford, Brown, Cocking, Donovan, and Pellegrino, 2000). The extent to which this transfer takes place depends on the students' conceptual understanding of physics concepts. Therefore, instructional strategies must be designed in such a way that facilitates a student's conceptual growth. The most effective form of instruction takes student misconceptions into account (Heller and Reif, 1984). There has been a considerable amount of research on student problem solving ability using multiple choice questions. However, in order to get the most comprehensive characterization of the state of a student's understanding, it was necessary to use non-multiple choice problems, in addition to the multiple choice test, since students perform differently on them (Berg and Smith, 1994). Therefore, the major focus of this study was to characterize the quality of student understanding of physics concepts, by identifying the specific errors they make on their solution to non-multiple choice problems. Non-multiple choice problems allow for a more in depth analysis of students' solutions. Any identified student misconceptions can then be used to inform instructional strategies used by educators so that they can model instruction, which addressed these misconceptions into account. By doing so, the conceptual knowledge and understanding of the students would likely increase, in addition to their ability to transfer the physics concepts to other problems.

Overview of Research Design

The study used a mixed methods design in order to uncover the specific misconceptions that students had, which were identified by assessing the errors they made in each phase of the force and motion solution process. A rubric was designed to assess the overall quality level of students' solutions at each phase. Using a rubric, errors identified within each phase were assigned a numeric rating according to the completeness and accuracy. The numbers were summed to obtain a total score that was used to define the quality level. In order to determine if a statistically significant relationship existed between the phases that defined the process of obtaining the solution, a Linear Mixed Effects Model was used.

Rationale

The National Research Council(NRC) argued that "the ability to apply knowledge to novel situations, that is, transfer of learning, is affected by the degree to which students learn with understanding"(Bransford, Brown, Cocking, Donovan, & Pellegrino, 2000). Therefore, it is imperative for students to obtain an adequate understanding of the conceptual ideas in physics if learning is to take place. Learning is defined as a measure of a student's ability to solve problems which are similar but may be situated in a different context. The current state of physics students has been described by Dufresne, Gerace, and Leonard (1997),who asserts that the majority of students complete their physics course with problem solving skills that are not well defined and leave the course lacking a conceptual understanding of concepts. Until students' understanding is increased, they will continue to leave the physics class with poor problem solving skills. This assertion was based upon the results of student performance on the Force Concept Inventory (FCI) (Hestenes, Wells, & Swackhamer,1992), which is a multiple-choice test designed to assess a student's conceptual understanding of physics concepts.

In order to ensure that those students who leave the introductory physics classroom do so with a high degree of understanding, the instructional strategies that occur in the classroom must be such that they are defined within the context of student misconceptions. Heller (1984) notes instructional strategies must take student misconceptions, which are identified in the research, into account in order to be the most effective. Many of the misconceptions which have been identified are defined in terms of how students perform on multiple choice problems (Rosengrant, 2007). However, since students perform differently on multiple-choice and non-multiple choice problems (Berg & Smith, 1994), the analysis of the students performance on non-multiple choice problems needed to be documented as well (Rosengrant, 2007). Therefore, in order to obtain a more complete understanding of student misconceptions related to physics concepts, student performance on both multiple-choice and non-multiple choice problems needs to be investigated.

Significance of Study

Much of the previous research has focused on the quality of student problem solving skills in physics. This study was significant since it contributed to filling the gap in the literature concerning how students perform on non-multiple choice problems. This contribution has provided a more complete description of the problem solving behavior of students who enroll in a traditional introductory physics course. By outlining the specific errors students made when they drew non-mathematical representations, and applied Newton's Second Law in component form to the non-mathematical representation, a better understanding of student's misconceptions was identified.

Assumptions

This study assumed that 1 hour was an adequate amount of time for students to solve the Force and Motion problems. Since the students did not receive credit (grade, points, etc) for the problems, it was assumed that they would have performed in the same way if credit were given.

Limitations

The research was done during the summer session. In this case, the number of students who enrolled during the summer was fewer, which reduced the sample size "*N*". Also, the amount of time the students spend in lecture was less during the summer than the time spent by students who take the course in the fall or spring semester.

Terms and Definitions

This section provided a list of the terms and definitions that were be used throughout the dissertation. However, they are explained in more detail in the appropriate chapters.

Expert vs. Novice

While there is not a single definition which defines an expert or novice problem solver, there is a set of characteristics which contrasts the two groups. The table below presents these characteristics (Gerace, Dufresne, Leonard, & Mestre, 2001)

Table 1: Expert vs. Novice Characteristics

Expert	Novice
Conceptual knowledge impacts problem solving	Problem solving largely independent of concepts
Often performs qualitative analysis, especially when stuck	Usually manipulates equations
Uses forward-looking concept-based strategies	Uses backward-looking means-ends techniques
Has a variety of methods for getting unstuck	Cannot usually get unstuck without outside help
Is able to think about problem solving while problems solving	Problem solving uses all available mental resources
Is able to check answer using an alternative native method	Often has only one way of solving problem

Newton's Second Law

Force is defined as a push or pull on an object which results in a change in the object's motion. By definition, a change in an object's velocity provides acceleration to that object. Newton's Second Law states that this acceleration, which the object experiences, is directly proportional to the net force acting on the object, and inversely proportional to its mass. The law can also be written in mathematical form as:

$$a = \frac{\Sigma F}{m} \tag{1}$$

Here, a is the acceleration, F is the force, and m is the mass. This expression can also be separated into x and y components so that the motion along each of these directions can be analyzed separately.

$$a_x = \frac{\Sigma F_x}{m} \tag{2}$$

$$a_y = \frac{\sum F_y}{m} \tag{3}$$

Free-Body Diagram

A free-body diagram (FBD) is a type of representation used to show the forces acting on a body by other objects. The construction of the FBD is often considered as the first and most important step in the solution to problems involving the application of Newton's Second Law, as it is useful in helping to solve the problem. There are many different ways to construct the free- body diagram, but this study focused on the standard construction of it which was shown in Figure 3, where it showed a car resting on an incline, and its corresponding free- body diagram. The black dot represented the car, and the lines with the arrow tips (vectors) represented the directions of the various forces acting on the car.

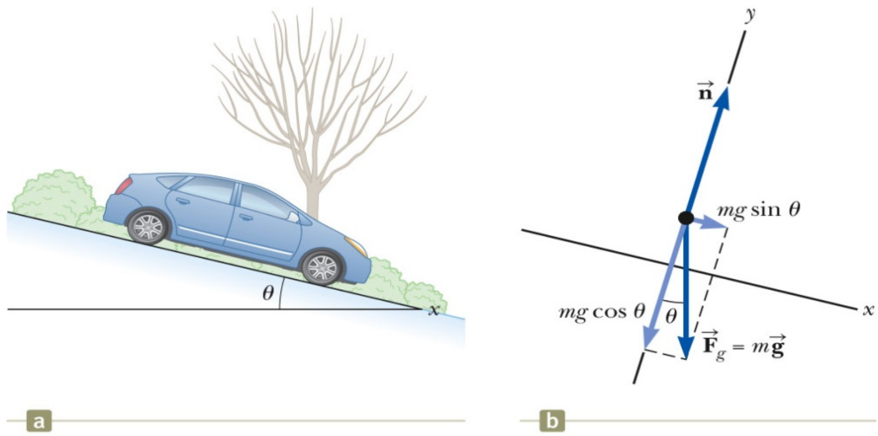


Figure 1: Car resting on incline and the corresponding free-body diagram

Problem Category

The problems in this study were grouped into one of two categories. Category one consisted mainly of objects moving along an incline as shown in Figure 2a, and those in category two consisted of pulleys connected to objects which may or may not move along an incline as shown in Figure 2b. These two categories best represented many of the types of problems, and various combinations of them, students are required to solve in the standard introductory physics class which require the application of Newton's Second Law.

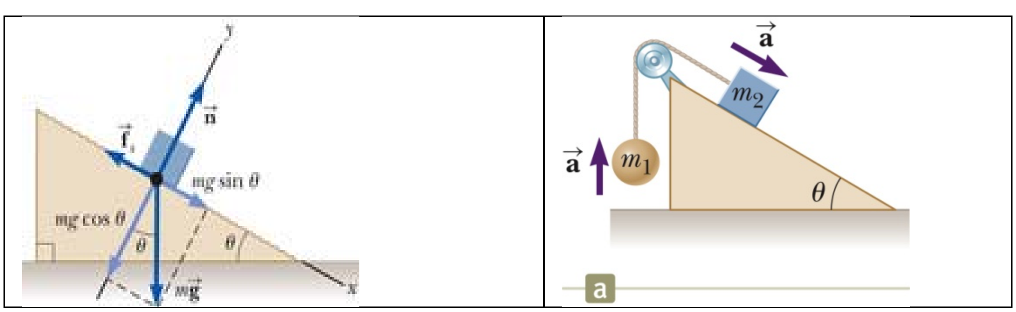


Figure 2: a. Incline problem b. Pulley problem

Problem Solution

The general solution to the type of problems in this study can be separated into three general phases (Heller and Reif, 1984):

1. The generation of an initial problem description, and qualitative analysis, designed to facilitate the subsequent construction of a problem solution
2. The generation of the actual solution by methods which facilitate the decisions making required for efficient search
3. The assessment and improvement of this solution.

Phase one required the construction of a free body diagram and phase two required the application of Newton's Second Law of Motion (NSL) in component form, which produced a system of equations that were used to solve for the desired variable asked for in the word problem. Figure 3 shows a typical solution to the type of problem that was addressed within this study. The solution contains all of the essential elements which indicated that the student has a firm understanding of the concepts and of the mathematical skills that are necessary to successfully solve the problem. The solution to the problem was separated into three major phases which include the physical representation (phase I), mathematical representation (phase II), and the mathematical manipulation (phase III). However, for purposes of this study, the focus was on phases I and II.

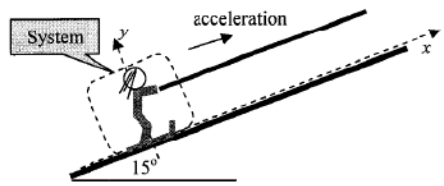
The problem: A rope parallel to a 15° ski slope pulls an 80-kg skier. Assume that the slope is very slippery and that we can ignore friction. The skier's speed increases by 0.50 m/s each second. Describe the motion with a picture, a free-body diagram, and mathematically.

The solution:
Picture and Translate
Sketch the situation described in the problem and include all of the known information.

Choose a system object. List objects interacting with the system.

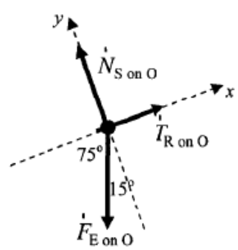
$m_{\text{person plus skis}} = 80$ kg

$a_{\text{skier}} = 0.50$ m/s^2



Simplify
We simplify the system as a point particle and ignore friction.

Represent Physically
Construct a free-body diagram for the skier and skis. Include perpendicular *x*- and *y*-axes—the *x*-axis is parallel to the ski slope. Note subscripts for forces: S = surface, O = skier, R = rope, and E = Earth.



Represent mathematically
Apply Newton's second law in component form to the situation shown in the free-body diagram. Be sure that you have the same number of components in the equation (some components may be zero) as you have force arrows in the diagram.

$$\sum F_x = m\,a_x$$
$$T_{\text{R on O}} \cos 0^\circ + N_{\text{S on O}} \cos 90^\circ - F_{\text{E on O}} \cos 75^\circ = m\,a_x$$
$$T_{\text{R on O}} + 0 - F_{\text{E on O}}\,(0.26) = m\,a_x$$

$$\sum F_y = m\,a_y$$
$$T_{\text{R on O}} \sin 0^\circ + N_{\text{S on O}} \sin 90^\circ - F_{\text{E on O}} \sin 75^\circ = m\,0$$
$$0 + N_{\text{S on O}} - F_{\text{E on O}}(0.97) = 0$$

Figure 3.2.
Sample Problem solved following Problem Solving Strategy (Van Heuvelen and Etkina, 3-23, 2006)

Figure 3: Sample solution of Force and Motion problem

Assessment Rubric

The first skill that the student was required to master was an adequate drawing of a free body diagram. Etkina et al., 2006 developed a rubric to assess the quality of the free body diagram. It was be used in this study. The values range from zero, indicating that the student showed no evidence of a free body diagram to three which indicated that an adequate free body diagram exist. The important components that students should be able to include in the free body diagram during this phase are summarized in the table below (Etkina et al., 2006)

Table 2: Scoring rubric for free-body diagram

Scientific ability	0 (Missing)	1 (Inadequate)	2 (Needs improvement)	3 (Adequate)
Is able to construct a free-body diagram	No free-body diagram is constructed	FBD is constructed but contains major errors such as incorrect force vectors; length of vectors; wrong direction; extra incorrect force vector, or missing vector.	FBD contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts; vectors are not drawn from a single point; or axes are missing.	The diagram contains no errors and each force is labeled so that it is clear what each force represents.

Summary

Chapter 1 provided an overview of the context for this study, the research questions that were answered, and the limitations of the study. An overview of the relevant research, and results were presented in Chapter 2. The methodology with an explanation of the instruments used in this study was discussed in Chapter 3. Chapter 4 provided the analysis of the data and results. A detailed discussion of the results, and their implications was presented in Chapter 5 along with future directions for research.

CHAPTER TWO: LITERATURE REVIEW

Introduction

There are three basic areas of empirical research as it relates to problem solving identified in the literature (Chi, Feltovich, & Glaser, 1981). The first area includes the identification of methods used for the investigation of the cognitive domain and the way that knowledge is structured in the minds of problem solvers (Shavelson, 1975;1975). The second area of research focused on the effect of prior knowledge on a student's problem solving ability (McCloskey, Caramazza, and Green, 1980; Halloun and Hestenes, 1985). The third area of research, which provided the context for this study, focused specifically on the identification of strategies and processes that students use as they solve problems (Siegler, 1978).

It was determined by Duit (2007) that in order to become a successful problem solver in physics, the problem solver was required to have both a conceptual understanding of the physics concepts, in addition to an understanding of the specific processes of how to solve the problem. However, many students who have taken physics do not have basic conceptual understanding (FCI), nor do they have an adequate set of technical problem solving skills (Dufresne, 1997). While there are many factors which affect their conceptual understanding of physics concepts, the main factor that determines the strategies and decisions students make during the problem solving process are generally made within the context of what they experience in their everyday lives. The major problem with this is that, not only are these beliefs, which are a function of their experiences, inconsistent with physics concepts, they are resistant to change (Duit and Schecker, 2007). In order to work towards changing the ideas that students have, instructional strategies must be aligned with them (Heller & Reif, 1984).

Two major perspectives on teaching and learning are situated within the context of acknowledging that students bring their own ideas in to the classroom, and that they are difficult to change (Maloney, 1994). However, in order for the instruction to be relevant and effective, it must be based on a comprehensive understanding of the misconceptions that students have when they enter the classroom (Heller and Reif, 1984), which was addressed in this study.

Problem Solving

Approximately forty percent of all publications in Physics Education Research has been focused on Mechanics; specifically with forces and kinematics (Duit, Niedderer, and Schecker, 2007; Etkina et al., 2006; Rosengrant et al., 2005; Rosengrant et al., 2009; Heller and Reif, 1984; Reddish, 1994). One of the major findings that came out of this research was the characterization of the novice and expert problem solver. The major differences are outlined in Table 3.

Table 3: Expert vs. Novice characteristics

Expert	Novice
Conceptual knowledge impacts problem solving	Problem solving largely independent of concepts
Often performs qualitative analysis, especially when stuck	Usually manipulates equations
Uses forward-looking concept-based strategies	Uses backward-looking means-ends techniques
Has a variety of methods for getting unstuck	Cannot usually get unstuck without outside help
Is able to think about problem solving while problems solving	Problem solving uses all available mental resources
Is able to check answer using an alternative native method	Often has only one way of solving problem

Table 3 provides the basic differences that exist in the methods that the novice and expert use when they solve problems. While this list is not exhaustive, it does, however, provide the major differences that exist between the two groups of problem solvers. It is important to note that although these differences between the two groups have been identified, a few of them have been under debate as to whether or not they exist. When an [expert] is asked to solve a problem that they are unfamiliar with, their problem solving ability becomes more like that of the novice (Singh, 2002). Also, Priest (1992), argues that the expert may not use forward inferences when they solve problems. Other strategies which have been identified that are used by the expert, or the more successful problem solver, include think-aloud techniques (Dhillon, 1998; Simon & Simon, 1978), chunking (Larkin and Reif, 1979), self-monitoring and transfer skills (Smith, 1991). These strategies identified are in addition to those described by Gerace et al.(2001).

The ability to solve problems that require the transfer of concepts in a systematic way is one of the most challenging skills for students (Dufresne, 1988), including those who have completed a traditional introductory physics class. An objective of this research is to identify the errors that problem solvers make as they solve a problem in order to inform the physics community. In this way, the instructional strategies can be modified to take them into account, because instruction that is centered on difficulties that students have is the most effective form of instruction (Heller and Reif, 1984). It has been documented that when students enter the physics classroom, they have many "common-sense" ideas about the physical world which are incompatible with physics concepts (Sherin, 2006). However, this incompatibility that exists between the students common-sense ideas of the physical world, and physics concepts, can be used to promote the necessary state of cognitive dissonance which promotes learning (Piaget, 1928). Piaget (1928) defines this state as being necessary as it allows the students to explore the differences between their current beliefs about phenomena and the extent to which it differs from the correct interpretation. Here, the student is afforded the opportunity to attempt to consolidate the discrepancy that exists between any misconceptions they may have and the related physics concept. The degree of difference that exists between these misconceptions, which consequently affect their problem solving ability, and the physics concepts can be defined in terms of the Zone of Proximal Development (ZPD) (Vygotskiĭ and Cole, 1978). The ZPD is a qualitative measure of the difference between the actual problem solving ability of the student, and the desired problem solving ability that best represents a firm qualitative and quantitative understanding of physics concepts. In order to minimize the difference which may exist between the current and the desired state of a students' problem solving ability, it is important to fully understand and characterize the factors that prevent this minimization. These factors were identified as errors that students made in this study.

Approximately 40% of the research in physics, and the misconceptions associated with it, has centered on mechanics (Duit et al., 2007). Specifically, this research is in the area of Force and Motion, where the main focus in the investigation deals with the ability of students to transform mechanics problems from a verbal representation into a non-mathematical representation and the factors which affect their ability to do so (Hinrichs, 2005). The ability of a student to transfer the word problem representation to the non-mathematical representation, in particular, has been discussed with respect to the quality of the non-mathematical representation and the relationship that it has to the final solution of the problem (Rosengrant et al., 2005). It was documented that the representation is necessary but not sufficient for problem solving success, so an investigation of the entire problem solving process is necessary. The problem solving process is well defined, and consists of three different phases. Although there are many different representations that students may choose, the most common representation, and the one that has received the most attention is the free body diagram. This is a diagrammatic representation showing the forces acting on an object, and the quality of it has been correlated to how successful the student is at arriving at a correct solution (Rosengrant et al., 2005; Rosengrant, van Heuvelen, and Etkina, 2006; Rosengrant, 2007). It was found that students who drew high quality free-body diagrams were more successful, in general than those who did not (Rosengrant, 2007).

Many of the studies which have been done regarding problem solving were performed using multiple choice problems. This study extended the investigation to increase the lack of focus that has been placed on non-multiple choice problems (Rosengrant, 2007). It is important to include them since the performance of students vary depending on whether or not the question is a multiple choice or non-multiple choice format (Berg and Smith, 1994) and the way the questions are phrased (Meltzer, 2005; Beichner, 1994). The characterization of student problem solving abilities may provide a more comprehensive set of difficulties. This does not, however, suggest that this list of difficulties be created and used as a way to "teach to" the problems, but rather as a guide to help ensure that the most common difficulties are addressed. Independent of the problem format, the ability of the student to construct adequate representations is an important skill to develop.

Representations play a critical role in the physics classroom and can be used in many different ways during instruction (Dufresne et al., 1997)

1. As a means to elucidate a problem, as occurs when a student draws a sketch of a physical situation and provides a summary of given information
2. As the focus of a problem, as occurs when a student is explicitly asked to draw a graph or find the value of a physics quantity from a given graph
3. As a step in a formal procedure, as occurs when students are required to draw a free-body diagram as one of the initial steps in applying Newton's laws to solve a problem

However, mode three described above is best aligned with the strategies and instruction that instructors use in the physics classroom; therefore, it forms the basis of this study. In particular, the free-body diagram was used as the vehicle through which the student's conceptual understanding was explored as it related to the first step in the solution process of the class of problems that were explored in this study. The solution process to these problems is well defined (Heller & Reif, 1984) and is shown below.

1. The generation of an initial problem description, and qualitative analysis, designed to facilitate the subsequent construction of a problem solution
2. The generation of the actual solution by methods which facilitate the decisions making required for efficient search
3. The assessment and improvement of this solution.

The three phases above in the solution process document the class of word problems that were investigated in this study. The first phase in the process is considered the most important phase due to its requirement for the student to translate the initial word problem into a non-mathematical representation. The second phase of the problem solving process requires the student to translate the non-mathematical representation from phase I into a mathematical representation. This is accomplished by using a specific physics concepts known as Newton's Second Law (NSL). Force is defined as a push or pull on an object which results in a change in the object's velocity. A change in an object's velocity provides acceleration to that object. By definition NSL states that this acceleration, which the object experiences, is directly proportional to the net force acting on the object, and inversely proportional to its mass. The law can also be written in mathematical form as:

$$a = \frac{\Sigma F}{m} \tag{4}$$

where a is the acceleration, F is the force, and m is the mass. This expression can also be separated into x and y components so that the motion along each of these directions can be analyzed separately.

$$a_x = \frac{\Sigma F_x}{m} \tag{5}$$

$$a_y = \frac{\sum F_y}{m} \tag{6}$$

The application of NSL to the free-body diagram, which was generated in phase I requires a firm conceptual understanding of NSL and other physics concepts. Due to the conceptual understanding that is required in these two steps, they were both used to explore the difficulties that students have with problem solving.

The third phase of the problem solving process requires the student to generate the actual solution to the problem by solving the mathematical equations which are the result of phase II. Since this phase requires the assessment of the mathematical skills of the student before a conclusion regarding their ability to successfully solve the problem in its entirety, it was not addressed in this study. This study was more focused on the conceptual difficulties that students have as they progress through the solution phase, and not necessarily their mathematical ability with respect to solving a system of equations; therefore, the study focused on phases I and II. The lack of attention to phase III is not meant to suggest that mathematical skills are not important. Champagne and Kloper (1980) defines math as the language of science and it is an important element in the process of generating the solution.

The most common difficulty among most students, related to problem solving, centers on their inability to transfer concepts from one problem to another (Dufresne, 1988; Savelsbergh, 19997), including those who have completed a traditional introductory physics class. Phases I and II were used to help identify the difficulties and errors that affect students' successful transfer of basic physics concepts to different word problems. The ability to transfer concepts from familiar situations to those which are unfamiliar serves as the "meter stick of learning." It is a measure of how efficiently a student can take the skills that they have learned in the class and apply both qualitative and quantitative skills to various different problems. When students are able to successfully transfer the learning and apply the aforementioned skills, it can be said that "learning" has occurred. This philosophy is echoed by the National Research Council (NRC) which reports, "The ability to apply knowledge to novel situations, that is, transfer of learning, is affected by the degree to which students learn with understanding"(Bransford et al., 2000).

Good problem solving skills are not only important in physics but other areas as well, due to the major qualitative reasoning skills which are necessary to become an adequate problem solver (Ploetzner, Fehse, Kneser, and Spada, 1999; Clement, 1982a). Some general factors that affect the ability to solve problems in an efficient way have been documented and include students' inability to see the patterns that exist from other problems they may have solved (Larkin, McDermott, Simon, and Simon, 1980). This is, perhaps, due to the way the information is organized, indexed in their brain, and structured; the organization determines the ease in which information can be retrieved (Simon & Simon, 1978; Chi et al., 1981).

In order to help students become better problem solvers it has been suggested that they be required to explain problems in multiple ways and from different perspectives. Studies have shown that students who are able to explain problems from multiple perspectives are more likely to solve word problems correctly compared to those who may only be able to see the problem from a single perspective (Chi, Bassok, Lewis, Reimann, and Glaser, 1989). Additionally, group discussions, which are conceptual in nature rather than mathematical, should be encouraged. This discussion should also include different strategies on how to categorize problems beyond their surface features (Hardiman, Dufresne, and Mestre, 1989; Hegarty, 1995), which has been shown to help increase the problem solving skills of students (Heller, 1992a; 1992b; Linder, 1996; Van Domelen and Van Heuvelen, 2002). An environment should also be created that allows for the discussion of students' alternative views of the problem (Tao, 2001). These suggestions along with providing the student with a specific set of problem solving instructions (Zhaoyao, 1993), which focus on their problem solving difficulties, may ensure that they have a firm basis of strategies, tools, and resources that they can use to help solve problems in an array of different contexts.

Multiple Representations

The first step in obtaining the solution to the problems in this study was the translation of the verbal representation of the word problem into a non-mathematical representation (Heller & Reif, 1984). The translation of word problems into this more useful format can be accomplished by using any one of the many different representations, which are collectively referred to as multiple representations (MRs). They refer to "any of the widely diverse forms in which physics concepts may be understood and communicated (Meltzer, 2005), and are useful in solving an array of problems in all areas of physics. Although this is a very important step in the process, many students struggle with this stage of the process. Perhaps they struggle due to the conceptual demands of this stage, and are consequently less likely to get the problem correct. Additionally, much of the focus in the traditional classroom setting is on the quantitative aspect of the problem solving process, and less focus is placed on the necessary conceptual aspect of problem solving process. In this case, attention needs to be given to the traditional classroom and ensure that the classroom is aligned with needs of the students, especially focused on the conceptual aspect of the problem solving process. Although the discussion of problem solving should include the use of representations in the problem solving process, it is important for the instructor to realize that this discussion cannot begin with an assumption that students necessarily understand them (Beichner, 1994) or how to use them, because many students do not. Instructors often begin the discussion of the problem solving process by drawing a free-body diagram, for example, and many of the students in the class copy down the free body diagram without understanding its meaning. This practice can lead to the inability of the student to make connections to other problems which may require the same process to solve them. The students need to view the free body diagram as a tool, rather than a formal step in the process that is disconnected from the problem; the free-body diagram "is the problem," it is just represented in a more useful format. The inability to view the free-body diagram as such hinders students' ability to make the connection between the diagram and phenomena that occur in the real world (. McDermott, Rosenquist, and van Zee, 1987), and leads to the student only being able to view the representation and problem in terms of surface features (Kozma, 2003). The surface view also promotes the creating of modified forms of representations which points to a fragmented understanding of concepts which are embedded in the free-body diagram (Greeno & Hall, 1997).

It is difficult to find a physics professor who does not rely on the use of a particular representation to explain concepts when they teach physics, yet only 10-20% of students use them when they solve word problems (Van Heuvelen, 1991). There may be several reasons for this, including students who do not see the role of the representation or understanding the meaning of the descriptors that are used in the diagram (Dufresne et al., 1997), or they don't have a basic understanding of the variable and descriptors that are used in the diagram and their relationship to the verbal word problem (Van Heuvelen, 1991; McDermott, 1984). For the students who do use them, the lack of conceptual understanding they have with kinematics in general (McDermott, 1984) directly affects their ability to successfully generate an adequate representation of the verbal problem. While there are numerous factors that affect students' ability to successfully generate and use representations, conventional instruction does very little to increase their ability to do so (Halloun & Hestenes, 1985; Van Heuvelen, 1991). However, the emphasis placed on the use of multiple representations needs to be such that they are actually explained and students are shown how they help facilitate the problem solving process. They are not just multiple ways of solving the same problems but rather they should be used to show, for example, if a student is having trouble with understanding one of the concepts from the perspectives of one of the MRs, the other representations can be used to help the student get a firmer grasp on the concept with which they are struggling with (Ainsworth, 1999).

Multiple representation are also useful in other courses that students may take (Paivio, 1971;Janvier, 1987;Lesh, Post, and Behr, 1987), which include calculus (White and Mitchelmore, 1996) and other science courses where their use has also been shown to increase a student's conceptual understanding (Cheng, 1999), such as chemistry (Kozma and Russell, 1997). Hinrichs (2005) documented how the pre and post test scores of students who were in an environment that focused on the use of MRs were different, in a significant way, from the scores of the students who were not in the modified classroom environment. The pre and post test results of those who used MR's, and those who did not, respectively, were 1.1 ± 1.0 pretest and 3.7 ± 0.8 post, and 1.2 ± 1.0 pretest and 2.8 ± 1.2 post. The results show the power of representations. de Leone and Gire (2006) also showed that students who used more representations when solving problems solved a greater number of problems correctly compared to those students who used a fewer number of representations to solve word problems. It was concluded that the more ways a student is able to represent a problem, the better their conceptual understanding. The free body diagram was used as a specific representation in this study to explore the quality of student problem solving.

Free-body diagram

What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The first step in the problem solving process, of the problems in this study, was the translation of the word problem into a non-mathematical representation. Here, the non-mathematical representation was free body diagram. The free body diagram, shown in Figure 4 is a representation that shows all of the forces acting on an object.

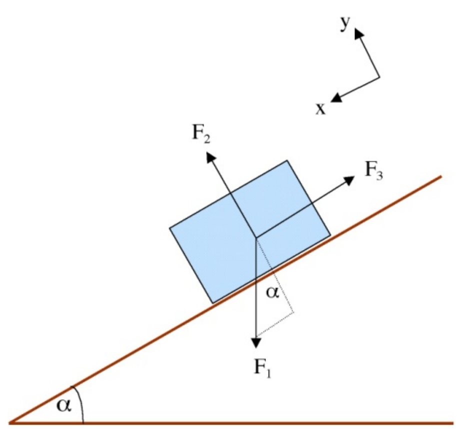


Figure 4: Free body diagram

There are many different ways to construct the free body diagram (Fisher, 1999; Hinrichs, 2005; Lane, 1993; Maloney, 1990; Mattson, 2004; Puri, 1996; Sperry, 1994). However, we wish to focus on the basic qualitative features outlined in the rubric defined by Etkina (2006). Problem solvers who are more successful during the problem solving process construct the free-body diagram that is more closely related to the actual physics concepts and the specific factors that affect the motion of the object, whereas the novice draws a diagram that is more closely related to the general or naïve components of the free body diagram. This difference in conceptualization causes some difficulties for the novice as they attempt to solve problems. The purpose of the free body diagram is to provide problem solvers with a tool that they can use to help organize the physics concepts in the problem, which ultimately makes the problem solving process easier. However, the free-body diagram, while it is a very useful tool, requires that the student employs a variety of skills to construct one that is of high quality. The difficulties that students have with free-body diagrams vary, and range from not knowing how to construct them, to not knowing what to do with them once they are constructed. This creates a huge problem for instructors since it is understood as a matter of common practice to create a non-mathematical representation as a first step to solving a word problem in physics, yet only 10-20% of students do so when they solve problems in the traditional introductory physics classroom setting.

In order to understand why such a small percentage of students use the free-body diagram when they solve problems, researchers have conducted studies to investigate this question. The research that has been done on the free-body diagram can be separated into two major lines of research (Rosengrant, 2007; Rosengrant et al., 2005; 2007;2009). These lines include

1. What effect does the free-body diagram have on the likely hood of the student soling a problem correctly?
2. What was the thought process of the student as they used the free-body diagram to solve their problem?

The major results of these questions concluded that students who construct free-body diagrams when they solve problems have a greater likelihood of correctly solving a problem compared to those who do not construct one (Rosengrant et al., 2006; Rosengrant et al., 2005; Rosengrant et al., 2006). Additionally, it was found that the student who constructed a free-body diagram that was incorrect was less likely to get the problem correct than a student who did not draw one (Rosengrant et al., 2005; Rosengrant, 2007). With respect to the thought process, it was found that students, independent of the instructor they had still drew pictures, even if they knew credit was not given for the drawing. However, only the high achieving student drew correct free body diagrams and used them to evaluate their final solution to the problem (Rosengrant et al., 2006).

Many of the studies that have been done, were done so in a modified classroom environment (Rosengrant et al., 2005; Rosengrant et al., 2006; Rosengrant, 2007; Rosengrant, 2007; Rosengrant et al., 2009). Here, there was a specific and intentional emphasis placed on the use of the free-body diagram during the lecture, and other activities in the class. Also, the questions given to the students during these studies were multiple choice questions. However, the research needs to be extended to include the performance of students who are in a traditional introductory class (Rosengrant, 2007), to determine if they perform differently in the traditional classroom setting when given non-multiple choice problems as opposed to multiple choice problems (de Leone and Gire, 2006).

Kotovsky (1985) investigated whether students used free body diagrams when they solved problem by comparing students in a traditional course, where the emphasis is placed on mathematical manipulation, to the students who were enrolled in a reformed course, where the use of multiple representations were explicitly taught. They found that when the students in each class were given a multiple choice test, approximately 58% of the students in the reformed course chose to use free body diagrams to help solve the problems, and only 15% of the students in the traditional course used a free body diagram to help solve the problems. While diagrams are important in helping to solve problems de Leone and Gire, (2006) conclude that simply constructing the free-body diagram is not enough to ensure successful completion of the problem; the student must understand what to do with it. Rosengrant, et al (2009) found that high achieving students used the diagrams to help solve the problems and as a tool to evaluate their work while low achieving students only used representations as aids in the problem-solving process.

In order to help with some of the difficulties that students have with the conceptual understanding, it has been suggested that there be some modifications to the classroom environment, such as a stronger focus on helping the students develop a better conceptual understanding (Linder, 1996). Additionally, the students should be required or encouraged to view each problem from a different perspective; when they are successful at it, this shows evidence of a much deeper understanding of the problem (Tao, 2001). Also, students who were involved in an inquiry based lab (Thornton and Sokoloff, 1998) that allowed them to investigate the physics concepts in a laboratory setting, Constructing and Applying Concepts of Physics (CACP)((Van Domelen and Van Heuvelen, 2002), developed a deeper conceptual understanding. Currently, instructional strategies which do not allow for the student to get adequate exposure to the multiple skills that are available to solve problems on a deeper level, and consequently students perform poorly on assessments that measure their thinking skills (Schoenfeld, 1992).

Application of Newton's Second Law

What types of errors do students make in phase II of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The second phase of the problem solving process requires the student to translate the non-mathematical representation from phase I into a mathematical representation. This is accomplished by using a specific physics concept known as Newton's Second Law (NSL). Force is defined as a push or pull on an object which results in a change in the object's velocity. It is well known in the science community that a change in an object's velocity provides acceleration to that object. By definition NSL states that this acceleration, which the object experiences, is directly proportional to the net force acting on the object and inversely proportional to its mass. The law can also be written in mathematical form as:

$$a = \frac{\Sigma F}{m} \tag{7}$$

where a is the acceleration, F is the force, and m is the mass. This expression can also be separated into x and y components so that the motion along each of these directions can be analyzed separately.

$$a_x = \frac{\Sigma F_x}{m} \tag{8}$$

$$a_y = \frac{\sum F_y}{m} \tag{9}$$

The application of NSL to the free-body diagram, which was generated in phase I, requires a firm conceptual understanding of NSL and other physics concepts. Due to the deep conceptual understanding that is required in these two steps, they were used to explore the difficulties that students have with problem solving. It has determined that a correlation exists between the quality of the free-body diagram a student drew and their final solution (Rosengrant et al., 2006; Rosengrant, 2007). However, there have been no reported detailed analyses of how the student progresses through each stage of the problems solving process other than the work done by Heller and Reif, (1984) to create a model of what factors should be included to replicate good problem solving skills. This dissertation included the study of how students progress through the different stages of obtaining the solution to the problem.

Math in Physics

The final phase of the problem solving process, phase III, requires the student to solve the system of equations that are generated in phase II. Equations 10 and 11 represent a system of equations that a student is likely to find in an algebra course.

$$2x + 5y = 7 \tag{10}$$

$$\frac{3}{7}y + 5 = 6x \tag{11}$$

It has been determined that many students who are in science courses cannot solve basic algebra equations (Clement, 1982b), such as those shown in Equations 10 and 11. They also struggle to define and identify variables in the algebraic equations, which is a necessary skill in order to solve algebraic equations (Rosnick, 1981). Students view the process of solving the equation for a given variable as being isolated from the problem, rather than viewing the variable as representing a physical quantity (Van Heuvelen, 1991; Clement, 1982b). Many students also struggle to solve for variables in the equations, especially when there are not any numbers to plug into the equation, and if variables other than the variable *x* are present. They are more comfortable plugging in numbers since it reduces the cognitive load when solving the equation (Hsu, 2004).

Equations 14 and 15 represent two typical equations that students are likely to encounter in their physics course during phase II of the problem solving process, and they also represent the types of equations that students are expected to solve in phase III. Comparing Equations 12 and 13 with Equations 14 and 15 reveal that their form is strikingly similar. Therefore, it is feasible to assume that the skills that are necessary to solve Equations 12 and 13 are the same that is required to solve Equations 14 and 15. In addition to the "algebraic skills" that are required to solve Equations 12 and 13, Equations 14 and 15 require an additional skill set in order to solve them.

$$2x + 5y = 7 \tag{12}$$

$$\frac{3}{7}y + 5 = 6x \tag{13}$$

$$mg \sin\theta - \mu_k F_N = ma_x \qquad (14)$$

$$\mathrm{F_N - mg\cos\theta = ma_y} \qquad (15)$$

Equations 14 and 15 contains $\sin\theta$ *and* $\cos\theta$, respectively. They also contain a_x and a_y. Therefore, the additional skills needed to solve the equations are such that the student has to identify those variables from the word problem in order for the student to be able to solve the equations. However, the variable may not be explicitly defined in the problem. The word problem may describe an object moving, for example, at a *constant speed*. In this case, the student must recall that acceleration is defined as the change in velocity per unit time as shown in Equation 16

$$a_x = \frac{\Delta v}{\Delta t} \qquad (16)$$

However, since the speed is constant, the acceleration is zero; $a_x = 0$. Therefore, math plays a fundamental role in physics and may affect how successful a student is solving problems. for he purpose of this study, phase III was not addressed.

Summary

Problem solving skills are, perhaps, the most important skills for a student to develop in their physics class. Not only are these skills important for physics, but they are useful in other science courses as well. In order to be successful, the student must have mastered both the quantitative and qualitative skills that are a necessary part of the problem solving process. Research is currently underway to understand the specific difficulties that students have as they solve problems in order to help ensure that their classroom experience provides them with all of the necessary tools to maximize their learning gains. There are many different types of problems that the student will encounter on their physics journey. However there is a specific class of problems that provides the best representation for, and lays the foundation for, many other problems they will encounter. These problems are in the area of mechanics and have a solution process that is well defined. Independent of the type of problem that a student encounters in physics, or any other science, there are certain basic skills that are required in order to be successful. One of the most important skills requires the student's ability to represent the problem in as many different ways as possible, which is one of the major ways that the student is able to show that he or she has obtained a firm conceptual understanding of the content. However, there may be gaps in the student's conceptual understanding as it relates to problem solving. This study sought to identify the factors which contribute to the gap in the student's understanding. Chapter three discussed the methods, research design and data collection process that were used to identify the conceptual difficulties that students have as they solve word problems.

CHAPTER THREE: METHODOLOGY

Introduction

This study investigated the specific errors that students who were enrolled in an introductory physics class made as they solved force and motion problems. The solution to the problems in this study consisted of three phases. This chapter discussed the research design and the quantitative and qualitative methods that were used to collect and analyze the data. The chapter outlined and explained how the instruments were used within the context of an actual problem solved by a student who was part of this study.

Research Design

This study used a mixed methods design. Each student was given 1 hour to solve problems whose solution consisted of three phases. They were give separate sheets of paper that was used to solve the problems. Quantitative methods were necessary in this study since each phase was analyzed separately to determine if a significant correlation existed between the quality of the free-body diagram and the quality of the mathematical equations, and also if the performance in one of the phases could be used to predict the performance in one of the other phases. In order to investigate if a prediction can be made, it was appropriate to use linear mixed effects model. Qualitative methods were appropriate since this study also sought to identify the specific errors that students make within each of the phases.

Setting

The study was conducted in a calculus-based physics course, during the summer at a large research based university located in the south east. The average enrollment during the summer in a calculus based physics course was approximately 180 students. The number of students who participated in this study was N = 65, and were enrolled in the calculus based section of physics. The majority of the students in the class were mostly males. The physics courses, at this university, were taught in the traditional way, meaning that there was a major focus on problem solving with an emphasis placed mostly on the quantitative aspect. The students had 3 exams, including a midterm and final. The problems on the exams were the standard problems that are generally given during the first semester of physics, which are primarily modeled after the end of chapter problems in the text. The course included a lecture and lab component. The lab component, which was taught by Graduate Teaching Assistant, to the course met once each week for three hours. The first hour of the lab component was used to allow students an opportunity to get additional help with homework questions if they needed it, or to ask questions about general questions they may have had. IRB permission was requested and granted to carry out this study. (Appendix A).

Research Questions

1. What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?
2. What types of errors do students make in phase II of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?
3. Does quality of performance in Phase I predict the quality of performance in Phase II?

<u>Questions and Solution</u>

The questions in this study were in one of two categories. Category one consisted mainly of objects moving along an incline as shown in Figure 5a, and those in category two consisted of pulleys connected to objects which move along an incline as shown in Figure 5b. These two categories best represent many of the types of problems, and various combinations of them, students are required to solve in the standard introductory physics class. These problems required the students to apply Newton’s Second Law in component form.

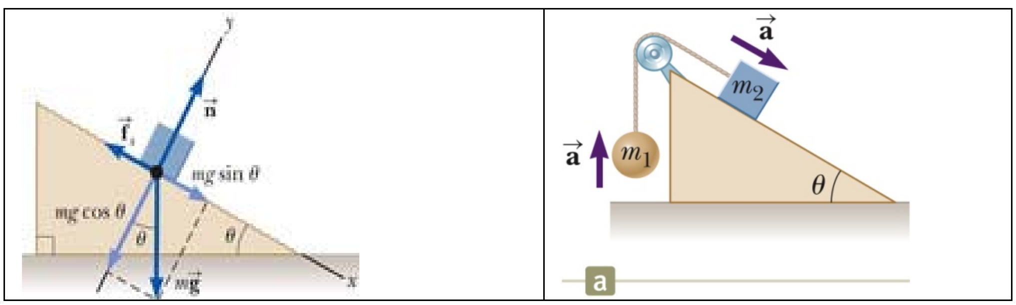


Figure 5: a. Incline b. Pulley problems

The question below is a sample of the type of question and its solution that were used in this study:

“A block of mass m has just begun descending a slope at an angle θ. Assuming the coefficient of kinetic friction is μ_k, what is the blocks acceleration?”

Following the phases as outline by (Heller & Reif, 1984), the first step is to draw the free body diagram, as shown below

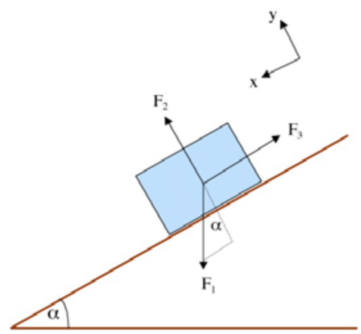


Figure 6: Free body diagram

The construction of the free –body diagram, shown in Figure 6 is followed by the application of Newton's second Law in component form to the free body diagram that resulted in a system of equations, shown below

Applying Newton's second Law in the x direction, we obtain:

$$\Sigma F_x = ma_x \tag{17}$$

$$mg \sin\theta - \mu_k F_N = ma_x \tag{18}$$

Applying Newton's Second Law to the y-direction, we obtain

$$\Sigma F_y = ma_y \tag{19}$$

$$\mathrm{F_N - mg\cos\theta = ma_y} \tag{20}$$

Thus, the resulting two equations of this phase II are shown below

$$mg \sin\theta - \mu_k F_N = ma_x \tag{21}$$

$$F_N - mg\cos\theta = ma_y \tag{22}$$

These two phases were analyzed using the instruments discussed in the following section.

Instruments

The main focus in this category had to do with the quality of the free body diagram in phase I above. The quality of the free-body diagram was evaluated using the rubric, shown in Table 4, created by (Etkina et al., 2006). The reliability and validity of the rubric was addressed in an article by (Etkina et al., 2006) as well. The rubric provided a way to systematically assess the quality of the free body diagrams that students draw as a first step in the solution of word problems. The scale ranges from zero, which indicated that the student did not draw a free body diagram as part of their solution process to three, indicating that not only did the student draw the diagram, but it was adequate; it had all of the necessary components and was clear what each of them represented.

Table 4: Free body diagram rubric

Scientific ability	0 (Missing)	1 (Inadequate)	2 (Needs improvement)	3 (Adequate)
Is able to construct a free-body diagram	No free-body diagram is constructed	FBD is constructed but contains major errors such as incorrect force vectors; length of vectors; wrong direction; extra incorrect force vector, or missing vector.	FBD contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts; vectors are not drawn from a single point; or axes are missing.	The diagram contains no errors and each force is labeled so that it is clear what each force represents.

Heller and Reif (1984) created a list of scientific abilities, that best describes the problem solving behaviors of a student who has a good conceptual understating of physics concepts. These abilities were validated with human problems solvers to ensure reliability of the results of the rubric. In order to assess the major scientific abilities (Heller & Reif, 1984) that are required to produce a high quality set of equations in phase 2 of the solution process were identified. Table 5 is a summary of the abilities.

Table 5: Scientific Abilities

Scientific Ability
Adequacy of Motion: Was information about the magnitude and direction of each system's acceleration correctly included in the equations? (i.e. did they explicitly show $a_y = 0$ and $a_x \neq 0$)
Adequacy of Interaction: Were all required forces included in the equations with the correct magnitude?
Adequacy of Equations: Were the number of equations correct?
Adequacy of direction: Was the direction of the force consistent with the coordinate system provided?

A more detailed explanation of how these rubrics were used will be provided in a different section.

Methods

Data Collection and Analysis

The data collected were analyzed using two rubrics (Heller & Reif, 1984; Etkina, 2006), due to the two different phases that define the solution process. The first step in solving a word problem is the generalization of an initial problem description, and qualitative analysis, designed to facilitate the subsequent construction of a problem solution (Heller & Reif, 1984) this initial description helped highlight the major details in the problem. Representations play a critical role in the physics classroom and can be used in many different ways during instruction (Dufresne et al., 1997)

1. As a means to elucidate a problem, as occurs when a student draws a sketch of a physical situation and provides a summary of given information
2. As the focus of a problem, as occurs when a student is explicitly asked to draw a graph or find the value of a physics quantity from a given graph
3. As a step in a formal procedure, as occurs when students are required to draw a free-body diagram as one of the initial steps in applying Newton's laws to solve a problem

However, mode three described above is best aligned with the strategies and instruction that instructors use in the physics classroom; therefore, it forms the basis of this study. In particular, the free-body diagram was used as the vehicle through which the student's conceptual understanding was explored as it related to the first step in the solution process of the class of problems that were explored in this study. The solution process to these problems is well defined (Heller & Reif, 1984) and is shown below.

1. The generation of an initial problem description, and qualitative analysis, designed to facilitate the subsequent construction of a problem solution
2. The generation of the actual solution by methods which facilitate the decisions making required for efficient search
3. The assessment and improvement of this solution.

The three phases above in the solution process document the class of word problems that were investigated in this study. The first phase in the process is considered the most important phase due to its requirement for the student to translate the initial word problem into a non-mathematical representation. The second phase of the problem solving process requires the student to translate the non-mathematical representation from phase I into a mathematical representation. This is accomplished by using a specific physics concepts known as Newton's Second Law (NSL). Force is defined as a push or pull on an object which results in a change in the object's velocity. A change in an object's velocity provides acceleration to that object. By definition NSL states that this acceleration, which the object experiences, is directly proportional to the net force acting on the object, and inversely proportional to its mass.

The diagrams in Figure 11 show an example of two free body diagrams of a block sliding down an incline. The methods that may be used to construct them are numerous (Fisher, 1999;

Heller and Reif, 1984; Lane, 1999; Maloney, 1990, Mattason, 2004; Newburgh, 1994; Puri, 1996; Van Heuvelen 1991b), but the qualitative features of an adequate drawing should be the same. In this case the rubric that was used was modified slightly to take into account more details of the free body diagram. The free body diagram serves two main purposes. The first was to allow the problem solver with a consistent way to identify all of the forces acting on the object in the problems. These forces are indicated by the arrow tipped lines in the figures. The second purpose was to identify a coordinate as it provided a way for the problem solver to identify if the arrow tipped line is pointing in the positive or negative direction. Although there are many techniques that may be used to construct these free-body diagrams, only the major qualitative requirements were used during the analysis, which are to identify the forces of the object and to define a system for identifying the directions of the force. Using the rubric by (Etkina et al., 2006) two guiding factors were used to determine the quality of the free body diagram. In general, when and object is resting on a surface, the force of gravity acts directly downward. If that same object is placed on an incline, gravity is still acting downward on the object, however the force has to be "resolved into components" to reflect the fact the object is on an incline and the weight is distributed in a direction along the incline (parallel to it) and into the incline (perpendicular to it), which was indicated in Figure 7.

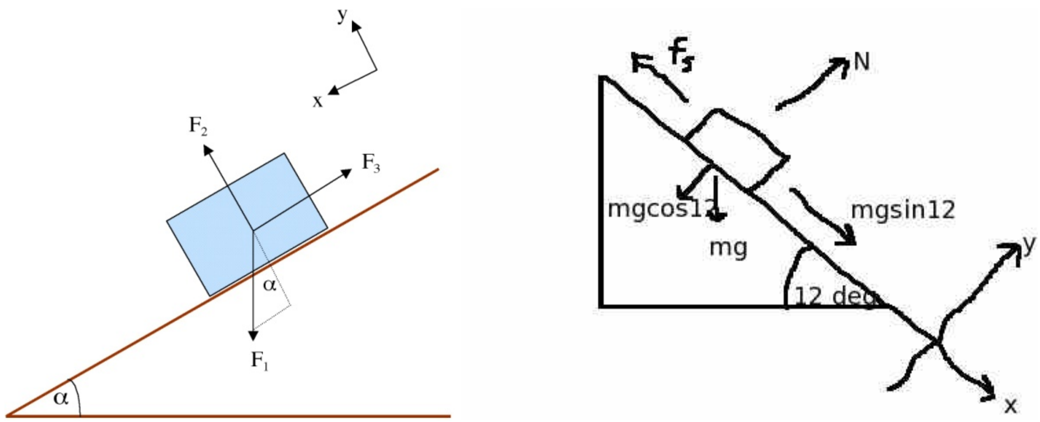


Figure 7: Free body diagram and forces

This was indicated by the F_1 in Figure 7 along with the two lighter grey lines which form a small triangle. The angle in that same small triangle indicated by α, is correlated with the angle to which the incline itself is raised, which is also indicated by α. An *x* and *y* axis coordinate system was draw above the incline. The coordinate system indicated that an arrow-tipped line directed downward along the incline will be identified as directed in the positive *x* direction, and likewise an arrow tipped line directed upward, but perpendicular to the surface of the incline is said to be directed in the positive *y* direction. Comparing this to diagram in the second figure, we have a similar situation, with the exception that the coordinate system is part of the incline drawn at the base of the incline, and the components that result in the resolution of the force of gravity are label as $mgsin12$ and $mgcos12$; these two values correspond to the *two lighter grey lines* in the first figure. At this point, the decision has to be made on how to assess the quality for the free body diagram using the rubric in order to differentiate collected data. We must first ask, if there was a coordinate system? In both of these cases, we find that the students drew a proper coordinate system. The second question assessed whether or not the students drew all of the necessary forces; for both examples the answer is yes, including the fact they resolved force of gravity into its proper components. Using the rubric, the first diagram received a score of 2 according to the rubric. It did not receive a 3 since the components of the weight are not labeled. The second diagram, where the force arrows are somewhat unconventional still received a score of 3. The diagram demonstrated that the student understands the concept of resolving force and the relationship that they resolved force has with the angle of inclination. In this case, since the student had identified all of the forces, provided a proper coordinate system, and labeled all of the forces correctly, the student was awarded a quality level of three. These two examples were used to guide the analysis of all students' responses.

Phase I Data Collection and Analysis

Research Question one: What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The research questions and study design were submitted to university's Institutional Review Board (IRB). After a thorough review of the documentation, the study received approval to be carried out at the university by the IRB office. The first research question that was addressed in this study was:

To answer this question, I used the rubric in Table 6 (Etkina,2006), along with the coding scheme in Table 7. Table 6 was designed to assess the quality level of the free-body diagram. The values range from zero, indicating that the student showed no evidence of a free body diagram to three which indicates that an adequate free body diagram existed. Since this study was concerned with identifying the specific errors that students enrolled in an introductory calculus based physics class made, when they constructed a free-body diagram, it was necessary to create Table 7.

Table 6: Free-body diagram rubric

Scientific ability	0 (Missing)	1 (Inadequate)	2 (Needs improvement)	3 (Adequate)
Is able to construct a free-body diagram	No free-body diagram is constructed	FBD is constructed but contains major errors such as incorrect force vectors; length of vectors; wrong direction; extra incorrect force vector, or missing vector.	FBD contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts; vectors are not drawn from a single point; or axes are missing.	The diagram contains no errors and each force is labeled so that it is clear what each force represents.

Table 7 is essentially a more detailed version of Table 6. According to Table 6, a score of 3, indicating the highest quality, indicates that the student created a free-body diagram free of errors. In order to received a score of three, the free body diagram must exceed the descriptions which correspond to a score of 0,1 or 2 in Table 6. For example, the free-body diagram must have a clearly defined coordinate system, all forces must be included, and all forces must be pointed in the correct directions, etcetera, according to Table 6. On the other hand, if the student does not have a clearly defined coordinate system, or they did not included all of the forces, or some of the forces they included were not pointing in the correct directions, these were identified as errors. Table 7 shows all of the possible errors that the student could make, based on Table 6, that would ultimately reduce the quality level of their free body diagram. Each of the possible errors was assigned a code that was used to track the errors. So, if a student drew a free-body diagram that did not included *mg*, that error was coded as "1"; if they free body diagram included "irrelevant forces", that error was coded as "7", etcetera.

Table 7: Coding Scheme for errors

Code	Error
1	Missing mg
2	Missing $mgcos\theta$
3	Missing $mgsin\theta$
4	Missing F_N
5	Missing coordinate system
6	Missing F_k
7	Addition of extra forces(irrelevant forces)
8	Forces pointed in wrong direction
9	Missing free-body diagram

A description of how this rubric was applied to assess the quality of the free-body diagram was described below. Figure 9 shows a free body diagram that was constructed by one of the students.

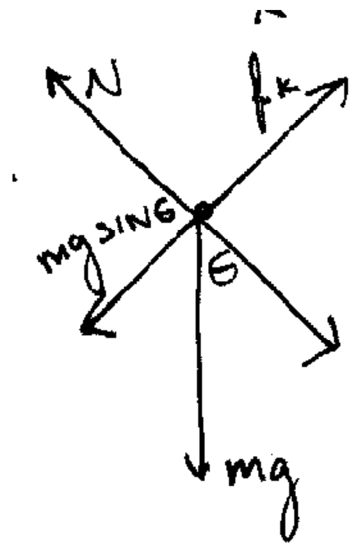


Figure 8: Sample free body diagram

In order to assess the quality level of the free body diagram in Figure 8, the rubric in Table 6 and Table 7 were used. An excel spread sheet was used to track the errors that the student made in this diagram. Table 8 shows a sample of the Excel spreadsheet that was used to track the errors.

Table 8: Sample Excel sheet used to track errors

Participant	Problem ID	FBD error	FBD Score	Error1	Error2	Error 3	Error 4	Error 5	Error 6
3	2	2,5	1		2			5	

Table 8 consists of two columns. A description of each column is shown in Table 9.

Table 9: Explanation of columns in Table 6

Column Label	Explanation
Participant	This refers to the student ID
Problem ID	This refers to the problem number
FBD error	This refers to the errors made based on Table 8
FBD score	This refers to the quality of the FBD diagram in Figure 5 based on Table 9
Error 1	A number appears here only if it appears in the FBD error column, which is repeated

This method was repeated for each participant, N = 65 and all four problems.

Phase II Data Collection and Analysis

Research Question 2: *What are the specific errors that that students make in phase 2 of the problem solving process?*

In order to answer the second research question, methods similar to those used in the Phase I Data Collection section were employed. The main difference was in the rubric that was used. Figure 9 showed an example of a set of equations that were produced by one of the students.

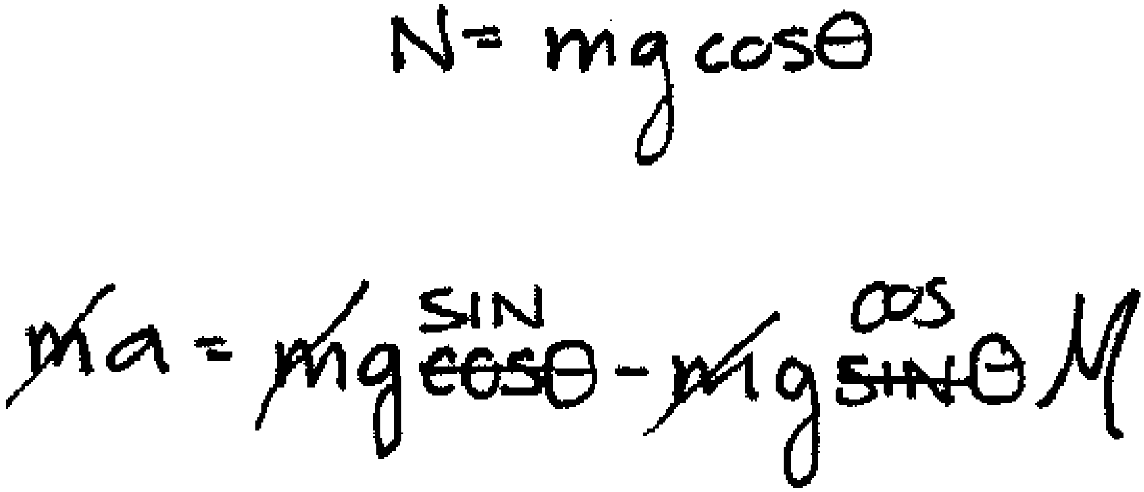


Figure 9: Sample student equations in phase II

To assess the quality of the equations in Figure 9, the rubric in Table 9 was used which identifies the major scientific abilities (Heller and Reif, 1984) that are necessary to understand in order to produce a high quality set of equations. Table 10 summarizes these abilities:

Table 10: Scientific Abilities

Scientific Ability
Adequacy of Motion: Was information about the magnitude and direction of each system's acceleration correctly included in the equations? (i.e. did they explicitly show $a_y = 0$ and $a_x \neq 0$)
Adequacy of Interaction: Were all required forces included in the equations with the correct magnitude?
Adequacy of Equations: Were the number of equations correct?
Adequacy of direction: Was the direction of the force consistent with the coordinate system provided?

The four scientific abilities outlined in Table 10 were recoded to make them more useful in the data collection. Table 10 thru Table 14 show the coding that was used to assess each of the abilities defined in Table 10. If the student provided the correct number of equations in their solution, they received the corresponding score. In order to track the specific errors students made in the Phase II, Table 11 was created.

Table 11: Error Codes for Phase II

Code	Error
1	Did not identify magnitude of acceleration
2	Did not have correct number of equations
3	Incorrect magnitudes
4	Inconsistent use of coordinate system

Table 12: Equation Scientific Ability

Scientific ability	0	1
Adequacy of equations: Were the number of equations correct	No	Yes

Table 13: Forces Scientific Ability

Scientific ability	0	1	2
Adequacy of interaction information: Were all required forces and correct magnitudes included in the equations?	No equations	No	Yes

Table 14: Direction Scientific Ability

Scientific Ability	0	1	2
Adequacy of direction: Was the direction of the force consistent with the coordinate system provided	No coordinate system provided	No	Yes

Table 15 showed a sample of the Excel spreadsheet that was used in order to track the errors each student made. Table 16 is a description of the columns in Table 15.

Table 15: Excel Sheet to track equation errors

Participant	Problem ID	Equation error	Error1	Direction	Equation
14	1	2,5	3	0	1

Table 16: Explanation of columns in Table 13

Column Label	Explanation
Participant	This refers to the student ID
Problem ID	This refers to the problem number
Equation error	This refers to the errors made based on Table 10
Equation	This refers to the error made based on 11
Error 1	A number appears here only if it appears in the Equation error column, which is repeated

This method was repeated for all 65 students.

Linear Mixed Methods Design

Research Question 3: Can the quality of a student's performance in phase 1 be used to predict their performance in phase 2?

To answer this question, it was necessary to determine if there was a significant relationship between the quality of the free-body diagram in phase I, and the quality of the equations in Phase 2. This analysis required the comparison between two numbers; the numerical quality of the free-body diagram and the numerical quality of the equations. These numerical quality values were obtained from the Excel spreadsheets created in each section above. This data were then computed in PASW Predictive Analytics SoftWare (PASW).

Summary

In Chapter 3 a review of research questions, research design, methods, data collection and analysis were presented. In order to answer the first two questions the solutions that students provided had to be coded into a form that could be analyzed more easily. It was necessary since, the research questions sought to identify the specific errors that students make when they solve Force and Motion problems. A Linear Mixed Model was necessary in order to answer the third research question since it sought to determine if a relationship existed between two categories. The results of the analysis were reported in the Chapter 4.

CHAPTER FOUR: DATA ANALYSIS

Introduction

This chapter discussed the mixed methods design that was used to document the difficulties that students have with word problems, and the potential factors that may affect their ability to master physics concepts and translate concepts to problems that are presented in a different context. The solution process of the problems in this study consisted of three phases. Each phase was analyzed separately to determine the qualitative features of the solution, which were subsequently used to characterize the quality level of each phase. After determining the quality level of each phase, a linear mixed effects model was used to determine if a significant relationship existed between the quality of the phases.

Free-Body Diagram

What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The first step in the problem solving process required the student to express the initial word problems in the form of a non-mathematical representation, known as a free-body diagram. In order to assess the quality level of these free body diagrams, the rubric in Table 17 was used.

Table 17: Free body diagram rubric

Scientific ability	0(missing)	1(Inadequate)	2(Needs improvement)	3 (Adequate)
Is able to construct a free-body diagram	No free-body diagram is constructed	FBD is constructed but contains major errors such as incorrect forces vectors; length of vectors; wrong direction; extra incorrect force vector, or missing vector	FBD contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts; vectors are not drawn from a single point or axes are missing	The diagram contains no errors and each force is labeled so that it is clear what each force represents

Table 17 was expanded into a format that was more suitable for tracking the specific errors that students made as they solve word problems. For example, a score of 2, according to Table 17, indicates that the free-body diagram needs improvement. The free-body diagram lacks features such as the inclusion of all of the necessary forces to fully characterize the word problem. However, since this study was focused on the specific errors, we needed a way to identify the particular force, or any other error, that the student neglected to include in their free-body diagram. Table 18 showed the coding that was used to accomplish this. The first column represents the code that was used to identify the specific errors, which are located in column 2.

Table 18: Coding rubric for error types

Code	Error
1	Missing *mg*
2	Missing *mgcosθ*
3	Missing *mgsinθ*
4	Missing F_N
5	Missing coordinate system
6	Missing F_k
7	Addition of extra forces(irrelevant forces)
8	Forces pointed in wrong direction
9	Missing free-body diagram

Each student was given a problem and their solution was analyzed using the rubric in Table 17 and Table 18. Figure 13 is a free body diagram submitted by one of the students who participated in this study.

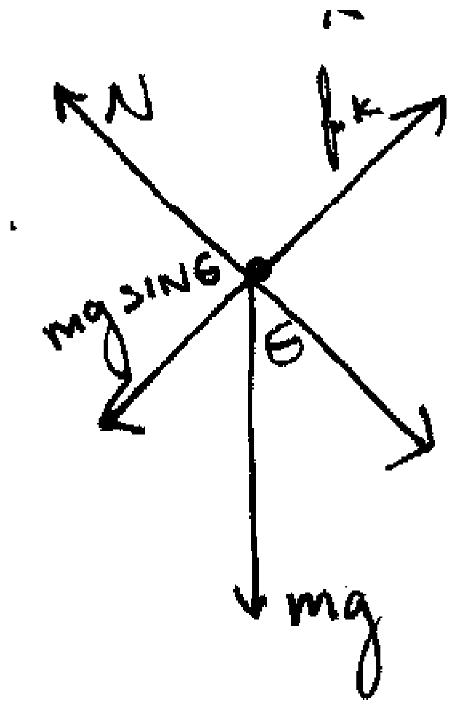


Figure 10: Sample student free-body diagram of quality 1

Figure 11 showed the free-body diagram that one of the students drew. Here, the student received a score of 1, according to the rubric in Table 17. This free-body diagram received a score of 1 since the student did not included all of the relevant forces in the diagram, nor does it have a clearly identifiable coordinate system. In addition to assigning it a quality score of 1, the specific errors needed to be identified. Referring to Table 18, since the free body diagram is missing a force, in particular $mgcos\theta$, and it was missing a clearly defined coordinate system, it was coded 2(missing $mgcos\theta$) and 5 (missing coordinate system).

The free-body diagram in Figure 14 is an example student response that would receive a score of 3 since it has all of the appropriate forces labeled, and has a clearly identifiable coordinate system.

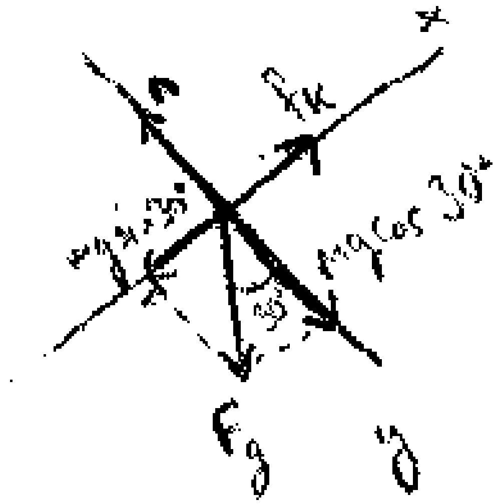
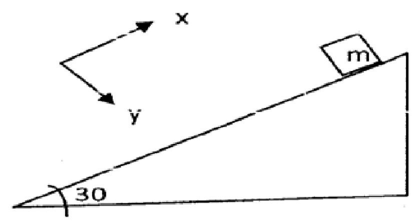


Figure 11: Sample student free-body diagram of quality 3

Problem 1

1. The block in the figure below has just begun descending the 30° slope. The x and y axis coordinat system are parallel and perpendicular, respectively, with the incline. Assuming the coefficient of kinetic friction is 0.10



a. Draw the free-body diagram

b. Calculate the block's acceleration

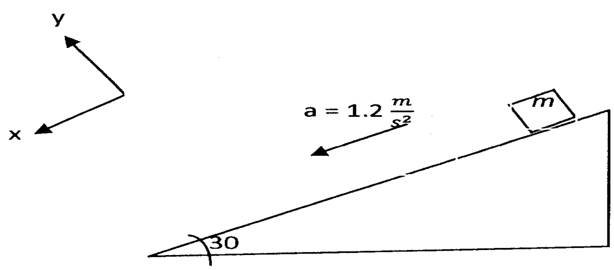
Table 19: Error Types on Problem #1

Error Type	1	2	3	4	5	6	7	8
Total number of error type for problem #1	6	49	46	7	45	7	16	13
% of student who answered making a specific error type	9	75	71	11	69	11	24	20
overall % of each error type	3	26	24	4	24	4	8	7

Table 19 shows that 75% of the students did not include the *mgcosθ* vector in their free-body diagram, which accounted for 26% of the errors. The next most frequent type of error, which was the omission of *mgsinθ,* which accounted for 24% of the errors was made by 71% of the students. Sixty-nine percent of the student did not include a clearly identifiable coordinate system, which accounted for 24% of the errors. Nine percent of the students did not did not include the *mg* vector in their diagram, which accounted for 3% of the errors.

Problem 2

2. The x and y axes are parallel and perpendicular to the incline respectively.



a. Draw the Free body diagram

b. Determine the coefficient of kinetic friction between the 2.2 *kg* box and the ramp.

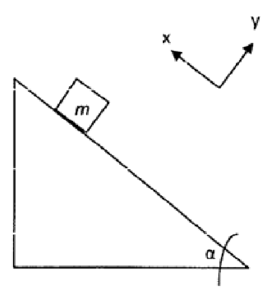
Table 20 shows that 71% of the students did not include the *mgcosθ* vector in their free-body diagram, which accounted for 21% of the errors. The omission of *mgsinθ* also accounted for 21% of the errors and was also made by 21% of the students. The next most frequent type of error, which was the omission of coordinate system, accounted for 23% of the errors and was made by 77% of the students. Only 11% of the students omitted the normal force (F_n) from their diagram.

Table 20: Error Types on Problem #2

Error Type	1	2	3	4	5	6	7	8
total number of error type for problem #2	16	46	46	7	50	14	21	16
% of student who answered making specific error type	25	71	71	11	77	21	32	25
Overall % of each error type	7	21	22	33	23	6	10	7

Problem 4

4. A block of mass *m* slides down an incline of angle α Using the diagram below,



a. Draw the free body diagram

b. Determine an equation for the acceleration of the mass along the incline

Table 21 shows that 66% of the students did not include *mgcosθ*, which accounted for 25% of the total number of errors. The omission of *mgsinθ* and corrdinate system accounted for 22% and 58% of the total number of errors, respectively. Twenty-three percents of the students had forces pointed in the wrong direction, which accounted for 9% of all the errors. The error type that occurred the least was the omission of F_k ,which accounted for 7% of the errors, and was made by 5% of the students

Table 21: Error Types on Problem #4

Error Type	1	2	3	4	5	6	7	8
TOTAL NUMBER OF ERROR TYPE FOR PROBLEM #4	10	43	38	10	38	3	12	15
% of student who answered making specific error type	15	66	58	15	58	5	18	23
overall % of each error type	6	26	22	6	22	2	7	9

Problem # 5 was analyzed in the same way except that since it is a category II problem, not all of the error types were relevant. Table 22 shows the relevant possible errors for problem 5.

Table 22: Category II problems

Code	Error
1	Missing *mg*
5	Missing coordinate system
7	Addition of extra forces(irrelevant forces)
8	Forces pointed in wrong direction

Table 23 shows that most of the students did not included a coordinate system when on their free-body diagram, which accounted for approximately 70% of the total number of errors, and were made by 40% of the students. Irrelevant forces errors account for 22% of the errors, and was made by 12% of the students. The least number of errors accounted for 8 percent of the total number of errors and were made by 5% of the students.

Table 23: Error for problem #5

Error Type	1	5	7	8
TOTAL NUMBER OF ERROR TYPE FOR PROBLEM #5	0	26	8	3
% of student who answered making specific error type	0	40	12	5
overall % of each error type	0	70	22	8

Application of Newton's Second Law

What types of errors do students make in phase II of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The second phase of the problem solving process requires the student to use the Newton's Second Law in component form to a set of equations, which is a mathematical representation of the problem. Each of the problems were analyzed against the rubric in Table 24.

Table 24: Rubric for phase II

Code	Error
1	Did not identify magnitude of acceleration
2	Did not have correct number of equations
3	Incorrect magnitudes
4	Inconsistent use of coordinate system

Table 25 shows that an inconsistent use of the coordinate system accounted for 62% of the total number of errors, and were made by 95% of the students. Very few students identified the magnitude of the acceleration; this error accounted for 61% of the errors, and was made by 94% of the students. The least number of errors accounted for 42 % of the total number of errors and were made by 66% of the students.

Table 25: Error Types on Problem #1

Equation Error Type for problem 1	1	2	3	4
TOTAL NUMBER OF ERROR TYPE FOR PROBLEM #1	61	42	43	62
% of student who answered making specific error type	94	65	66	95
overall % of each error type	29	20	21	30

Table 26 shows that 94% of the students did not identify the magnitude of the acceleration, which accounted for 61% of the total number of errors. Additionally incorrect magnitude also contributed to the total errors, they accounted for 52% of the errors, and was made by 80% of the students. 30 % of the total number of errors was due to incorrect magnitudes of the force , and was made by 46% of the students.

Table 26: Error Types on Problem #2

Equation Error Type for problem 2	1	2	3	4
TOTAL NUMBER OF ERROR TYPE FOR PROBLEM #2	61	42	30	52
% of student who answered making specific error type	94	65	46	80
overall % of each error type	33	23	16	28

Table 27 shows 98% of the students did not identify the correct magnitude for the acceleration, which accounted for 64% of the total number of errors. Additionally, many students did not correctly use the coordinate system, which accounted for 61% of the errors, and was made by 94% of the students. Incorrect magnitudes of the forces were identified 31% of the time and accounted for 31 % of the total number of errors and were made by 48% of the students

Table 27: Error Types on Problem #4

Equation Error Type for problem 4	1	2	3	4
TOTAL NUMBER OF ERROR TYPE FOR PROBLEM #4	64	58	31	61
% of student who answered making specific error type	98	89	48	94
overall % of each error type	30	27	14	29

Table 28 shows, 26% of the total number of errors we due to forces being labeled with the incorrect magnitude were made by all of the students. Students did not identify the correct magnitude of the acceleration where those errors accounted for 25% of the errors, and was made by 94% of the students. Although a greater number of students did not have the correct number of equation, which occurred the least, account accounted for 31 % of the total number of errors and were made by 48% of the students.

Table 28: Error Types on Problem #5

Equation Error Type for problem 5	1	2	3	4
Total number of error type for problem #5	64	59	65	65
% of student who answered making specific error type	94	91	100	100
Overall % of each error type	25	23	26	26

Qualitative Analysis

A linear mixed effects model was used in order to determine if a significant relationship existed between the quality of the free-body diagram and the quality of the equations that were produced in phase 2 of the solution process. It was determined that there was a statistically significant relationship, $t(245) = -4.48$, $p < .001$, $B = -0.153$, $SE = .034$, between the quality of the free-body diagram and the quality of the equations that were generated as a result of the application of Newton's Second Law in component form. Since the relationship was significant, we conclude that students who obtained a high score on the quality on the free-body diagram obtained a low on the quality of the equations. In order to gain more insight into the relationship between the equations and the free-body diagram, It was also determined that there is a statistically significant relationship, $t(207) = 3.45$, $p = .001$, $B = .649$, $SE = .188$, between the quality of the equations produced in pulley type problems and non-pulley type problems. However, the quality of the equations that were produced in the pulley type problems was higher than the quality of the equations that were produced in the non-pulley type problems, so students performed better on the problems that involved pulleys. Additionally, it was determined that there was a statistically significant difference, $t(126) = -10.3$, $p < .001$, $B = -1.42$, $SE = .139$,in the quality of the equations that are generated by the students depending on if the problem asks for a numerical computation as opposed to a purely abstract representation of the answer. The statistical significance showed that the predictive model was such that students who perform well on problems that require numerical computations, will likely perform poorly on problems that require and abstract representation.

Summary

Chapter 4 presented a detailed analysis of students' errors and results. The mixed methods revealed several findings. The errors that students made have been identified, and the errors that occur the most frequently in problems that were on an incline included the omission of $mg\cos\theta$, $mg\sin\theta$, and a clearly defined coordinate system. While the students made other errors, these errors were made the most frequently. Additionally, there was a negative statistically significant relationship between the free-body diagram and the equations that produced in phase 2 of the solution process. With respect to the quality of the equations, the types of errors that students made seem to make up on average 20-30 % of the total errors on each of the problems. The implications and a more detailed discussion of these results were reported in chapter 5.

CHAPTER FIVE: CONCLUSION

Introduction

It was determined by Duit (2007) that in order to become a successful problem solver in physics, the problem solver was required to have both a conceptual understanding of the physics concepts, in addition to an understanding of the specific processes of how to solve the problem. In order to work towards changing the ideas that students have, instructional strategies must be aligned with them, and it must be based on a comprehensive understanding of the misconceptions that students have when they enter the classroom (Heller and Reif, 1984). This chapter outlined the major findings related to the errors that students made as they solved Force and Motion problems and gave a brief discuss of the implications of the findings. This chapter concluded with some possible directions for future research.

Research Question1

What types of errors do students make in phase I of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The results of the study indicated that students consistently make certain errors when they solve Force and Motion problems. However, it was determined that certain errors occurred more frequently than others. These errors include the omission of two particular forces; namely $mg\cos\theta$, $mg\sin\theta$. In addition to these forces students did not include a clearly defined coordinate system as part of their free-body diagram.

In almost every case, the student included the *mg* vector as part of their diagram; however they did not resolve the force into the appropriate components. The resolution of *mg* into components serve as the fundamental distinction between objects that move along on a flat surface ($\theta = 0$), and objects that move along an incline ($\theta \neq 0$), where θ is the angle of the incline. Since the students included all of the other forces on the diagram, but not the components of the weight, it appears that, perhaps, they view these components as being "understood" to be part of the free-body diagram and which do not need to explicitly labeled; in some cases the "vectors" were included but not labeled. This interpretation, as a reason to explain the omission of these forces, leads to a more general conclusion, and that is the student does not understand the function of the free-body diagram, or their understanding is partial. This partial understanding ultimately creates a challenge when students are given similar problems and asked to solve them. An example of this could include the application of a force that is applied in a direction that is at an angle α relative to the direction of motion. In this case, the student may not be able to answer any quantitative or qualitative questions which require them to focus on the components of the force, rather than the magnitude of the force.

The lack of clearly defined coordinate systems was another component of the free-body diagram that many students did not include. This is another indication that the student has a partial understanding of the definition of a free-body diagram. Although, it may even point out the lack of a conceptual understanding with respect to the definition of a vector; they have both magnitude and direction. However the operative part of that definition is the "direction". If the student did not include a coordinate system, this is, perhaps, an indication that they do not understand the importance of specifying the direction of a vector. The coordinate system can be viewed as the "road map" of the free body diagram; it is essentially the connecting factor that bridges phase I and II together in the problem solving process. Without a coordinate system, the transition from phase I to phase II is based on the random assignment of directions to the forces, which result from the application of Newton's Second Law in component form. In cases where the assignment of the direction of these forces is not random, they may be based on "recall" of what they have seen in the lecture. Since many of the free-body diagrams that students encounter in class consist of a coordinate system which choose +x in the direction of motion of the object and +y in the direction that is perpendicular to the direction of motion, or in a direction parallel to the normal force.

The analysis of one question in particular led to interesting results. Question # 4 was similar to the other questions in the same category. The major difference was that instead of asking for a numerical computation, the students were asked to solve the problem in an abstract way, expressing the answer in terms of variables rather than using numbers. There was one error that contributed significantly to the overall errors made in the problem, which was the addition of an extra force. The specific force that was added was friction f_k. The addition of this force was significant since the problem never mentioned friction. However, 72% percent of the students added this extra frictional force to the free body diagram. The data do not provide a definitive reason as to why so many students added this extra force. However, it may provide some insight into the possibility of a behavior that is consistent with a similar behavior displayed by students when they solve word problems. It has been document that when students are presented with a word problem that required the use of an equation in order to solve the problem, to reduce their cognitive load, they simply write down several equations, even if they are not related to the problem (Hsu. 2004). The results of question four may point to a similar strategy displayed by students as they construct free body diagram; the inclusion of unrelated forces, instead of equations, to reduce the cognitive load that they may experience as they create free body diagrams. Another explanation may be that some sort of a "carry-over" or "recency" effect is at play. The problems that preceded question four were all similar in what was asked in the problem, except that those problems required the inclusions of a frictional force in the free body diagram. Perhaps, it was just natural to include friction in the free-body diagram due to the "habit" of doing so in the previous problems. A final interpretation may be that when students are presented with several problems that differ in a minor way, the quality of the attention that is given to the question stem begins to decrease with an increase in the number of questions which are similar. However, each of these explanations warrants further investigation.

Research Question 2

What types of errors do students make in phase II of a Force and Motion physics problem that focuses on pulley and non-pulley type problems?

The purpose of the free body diagram was to identity all of the forces acting on an object in a compact and convenient way that can be used with Newton's Second Law in component form to generate a set of mathematical equations that can be used to facilitate in the generation of the solution of the word problem. Therefore, it was a reasonable conclusion to assume that if the student truly has an understanding of all the appropriate forces in the free body diagram, the relevant forces would also appear in the equations that are generated in phase II of the process of obtaining the solution. However, after analyzing the quality of the equations that students generated, they were found to be inconsistent with what was represented in the free body diagram.

The fact many of the students made these errors raises a serious question as to what role the free body diagram plays in the solution of the problem. In particular, many students did not resolve *mg* into it appropriate components in the free-body diagram, however they included the same components as terms in the equation.

Research Question 3

Does quality of performance in Phase I predict the quality of performance in Phase II?

The general result of this study indicated that students who create high quality equations, were likely to produce low quality free-body diagrams or vice versa. This negative relationship was somewhat counterintuitive since the equations that are generated in phase II of the problem solving process result from the application of Newton's Second Law in component form to the free-body diagram in phase one. This discrepancy, between these two phases, point to the need to investigate the reason that the negative correlation existed.

Discussion

Students who are enrolled in an introductory physics calculus based physics class consistently make errors when they solve word problems. Therefore, instructional strategies need to be such that they focus on the errors since they may interfere with students' ability to learn. In particular a focus needs to be placed on the role that the free body diagram serves, and it must include a clear definition of how to properly construct one. This discussion must include a specific explanation of the role that the coordinate system plays in the process of obtaining the solution. Additionally, a connection needs to be made between the equations in phase two and the free-body diagram. Students appear to view them as two separate problems rather than connected parts of the same problem. In order to make the connection stronger, the coordinate system can be embedded within the discussion of phase I and II. In this way, the equations in phase II can be seen as having two parts, which are the "magnitudes of the forces" and "the signs of the magnitudes", where the signs of the magnitudes of the forces can be connected to the coordinate system. By connecting all three of these components the student may get a holistic view of the problem solving process rather than an isolated discussion of each. It seems that the fragmented discussion seems to encourage misconceptions that students hold about the problem solving process and the role that each component plays.

Areas of Future Research

There needs to be more research into the relationship that each phase on the final solution on the problem. Additionally, more work needs to be done to find out what additional factors contribute to the negative relationship that existed between phase I and phase II of the problem solving process. Also, the investigation into a possible relationship between the phase II and the final solution should also be investigated. The investigation of different instructional strategies that may help to improve student's understanding of the coordinate system should also be explored, which should include an analysis of student understanding of vectors.

Conclusion

The results of this study identified the specific problem solving errors made by students as they solve physics problems. These errors hinder the process of transferring basic physics concepts to other problems which are situated within a different context. Since the ability to do so is a measure that learning has taken place, it is important to ensure that the classroom environment is such that students are provided with an opportunity to reflect upon these misconceptions. Therefore, educational researchers must continue to work teachers to ensure they are aware of the difficulties that students have with problem solvers, since effective instruction takes them into account, and also with curriculum developers to ensure that the specific misconceptions are problem solving errors are addressed within the development of the curriculum.

APPENDIX A: SUBJECT CONSENT LETTER

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Frank C. McDonald**

Date: **July 21, 2010**

Dear Researcher:

On 7/21/2010, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	Initial Review
Project Title:	An investigation of the problem solving skills of students in an introductory physics class
Investigator:	Frank C McDonald
IRB Number:	SBE-10-07029
Funding Agency:	None

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Joseph Bielitzki, DVM, UCF IRB Chair, this letter is signed by:

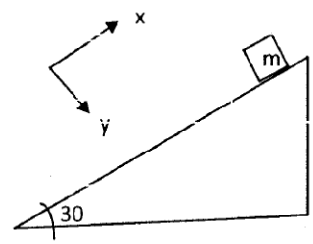
Signature applied by Janice Turchin on 07/21/2010 11:35:57 AM EDT

IRB Coordinator

APPENDIX B: PHSYICS PROBLEMS

ID:__________________ class/lab section__ ____

1. The block in the figure below has just begun descending the 30° slope. The x and y axis coordinate system are parallel and perpendicular, respectively, with the incline. Assuming the coefficient of kinetic friction is 0.10



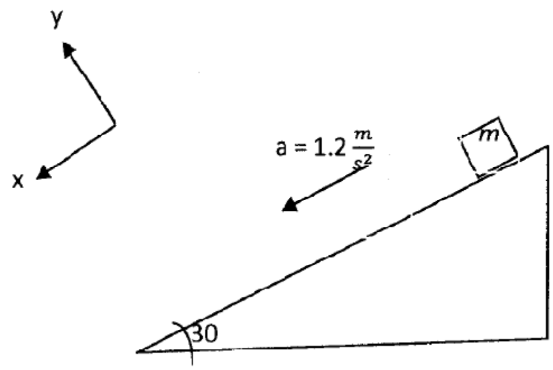
   a. Draw the free-body diagram

   b. Calculate the block's acceleration

   c. Suppose that the slope is covered with syrup and the block moves down the 30° slope at constant speed. What is the coefficient of friction, μ_k?

ID:__________________ class/lab section_________

2. The x and y axes are parallel and perpendicular to the incline respectively.



a. Draw the Free body diagram

b. Determine the coefficient of kinetic friction between the 2.2 kg box and the ramp.

ID:____________________ class/lab section__________

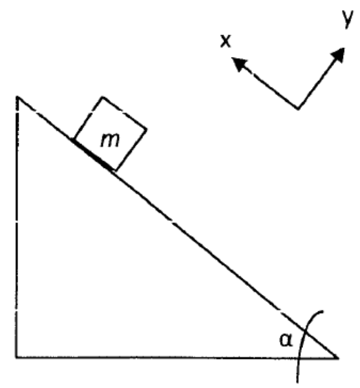
3. Solve the system of equations below for q and r(show all work).

$$2.35q + 3r = 21$$

$$\frac{3}{4}r - 10 = 14q - 5$$

ID:____________________ class/lab section________

4. A block of mass *m* slides down an incline of angle α Using the diagram below,



a. Draw the free body diagram

b. Determine an equation for the acceleration of the mass along the incline

c. Determine an equation for the coefficient of kinetic friction between the mass and the ramp.

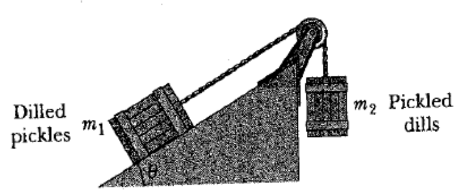
ID:________________   class/lab section________

6. The figure below shows a crate of dilled pickles with mass m_1, which moves along a plane that makes an angle of θ with the horizontal. That crate is connected to a crate of pickled dills with mass m_2 by a taut, massless cord that runs over a frictionless, massless pulley. The dills descend with constant velocity.

   a. Find an expression for the magnitude of the frictional force on the pickles from the plane.

   b. What is the direction of the frictional force on the pickles from the plane?

   c. Find and expression for the coefficient of kinetic friction.

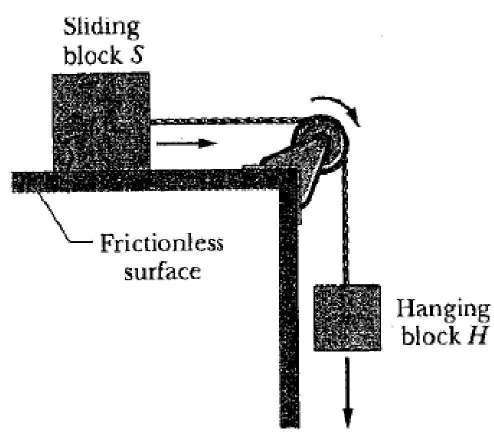


7. The figure below shows a block S(the *sliding block)* with mass M. The block is free to move a horizontal frictionless surface such as an air table. The first block is connected by a cord th wraps over a frictionless pulley to a second block H (the *hanging block)*, with mass m. The c and pulley are massless. The hanging block H falls as the sliding block S accelerates to the ri

   a. Find an expressing for the acceleration of the sliding block

   b. Find an expression for the acceleration of the hanging block

   c. Find an expression for the acceleration of the tension in the cord.



REFERENCES

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*(2-3), 131-152.

Beichner, R. J. (1994). Testing student interpretation of kinematics graphs. *American Journal of Physics, 62*(8), 750-62.

Berg, C. A., & Smith, P. (1994). Assessing students' abilities to constructt and interpret line graphs: Disparities between multiple-choice and free response instruments. *Science Education, 78*, 527-554.

Bransford, J., Brown, A. L., Cocking, R. R., Donovan, M. S., & Pellegrino, J. W. (2000). *How people learn : Brain, mind, experience, and school* (Expa ed.). Washington, D.C.: National Academy Press.

Champagne, A. B., & Klopfer, L. E. (1980). Factors influencing the learning of classical mechanics. *American Journal of Physics, 48*(12), 1074-1079.

Cheng, P. C. -. (1999). Unlocking conceptual learning in mathematics and science with effective representational systems. *Computers & Education, 33*(2-3), 109-130.

Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5*, 121-152.

Chi, M. T., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 13*(2), 145-182.

Clement, J. (1982a). Students' preconceptions in introductory mechanics. *American Journal of Physics, 50*(1), 66-71.

Clement, J. (1982b). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*(1), pp. 16-30.

de Leone, C. J., & Gire, E. (2006). Is instructional emphasis on the use of non-mathematical representations worth the effort? *AIP Conference Proceedings, 818*(1), 45-48.

Dhillon, A. S. (1998). Individual differences within problem-solving strategies used in physics. *Science Education, 82*(3), 379.

Dufresne, R. J., Gerace, W. J., & Leonard, W. J. (1997). Solving physics problems with multiple representations. *Physics Teacher, 35*(5), 270.

Dufresne, R. J. (1988). *Problem solving: Learning from experts and novices* Retrieved from

Duit, R., Niedderer, H., & Schecker, H. (2007). Teaching physics. In S. K. Abel, & N. G. Lederman (Eds.), *Handbook of research on science education* (pp. 599-629)

Etkina, E., Van Heuvelen, A., White-Brahmia, S., Brookes, D. T., Gentile, M., Murthy, S., Rosengrant, D., & Warren, A. (2006). Scientific abilities and their assessment. *Physical Review Special Topics - Physics Education Research, 2*(2), 020103-1--020103-15.

Fisher, K. (1999). Exercises in drawing and utilizing free-body diagrams. *Physics Teacher, 37*(7), 434-35.

Gerace, W. J., Dufresne, R. J., Leonard, W. J., & Mestre, J. P. (2001). *Problem solving and conceptual understanding*. Paper presented at the Proceedings of the 2001 Physics Education Research Conference, Rochester, NY:

Greeno, J. G., & Hall, R. P. (1997). Practicing representation. *Phi Delta Kappan, 78*(5), 361.

Halloun, I. A., & Hestenes, D. (1985). The initial knowledge state of college physics students. *American Journal of Physics, 53*(11), 1043-1055.

Hardiman, P. T., Dufresne, R., & Mestre, J. P. (1989). The relation between problem categorization and problem solving among experts and novices. *Memory and Cognition, 17*(5), 627-638.

Hegarty, M. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology, 87*, 18.

Heller, J. I., & Reif, F. (1984). Prescribing effective human problem-solving processes: Problem description in physics. *Cognition & Instruction, 1*(2), 177.

Heller, P. (1992a). Teaching problem solving through cooperative grouping. part 1: Group versus individual problem solving. *American Journal of Physics, 60*(7), 627.

Heller, P. (1992b). Teaching problem solving through cooperative grouping. part 2: Designing problems and structuring groups. *American Journal of Physics, 60*(7), 637.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *Physics Teacher, 30*(3), 141.

Hinrichs, B. E. (2005). Using the system schema representational tool to promote student understanding of Newton's third law. *AIP Conference Proceedings, 790*(1), 117-120.

Janvier, C. (1987). *Problems of representation in the teaching and learning of mathematics*. Hillsdale, NJ: L. Erlbaum Associates.

Knight, R. D. (2004). Five easy lessons: Strategies for successful physics teaching. *American Journal of Physics, 72*(3), 414.

Kotovsky, K. (1985). Why are some problems hard? evidence from tower of hanoi. *Cognitive Psychology, 17*(2)

Kozma, R. B., & Russell, J. (1997). Multimedia and understanding: Expert and novice responses to different representations of chemical phenomena. *Journal of Research in Science Teaching, 34*(9), 949-968.

Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction, 13*(2), 205-226.

Lane, B. (1993). Why can't physicists draw FBD's? *Physics Teacher, 31*(4), 216-17.

Larkin, J. H., & Reif, F. (1979). Understanding and teaching problem-solving in physics. *European Journal of Science Education,* Larkin, J., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science, 208*(4450), pp. 1335-1342.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: L. Erlbaum.

Linder, C. J. (1996). Teaching by conceptual exploration: Insights into potential long-term learning outcomes. *The Physics Teacher, 34*(6), 332.

Maloney, D. P. (1990). Forces as interactions. *Physics Teacher, 28*(6), 386-90.

Mattson, M. (2004). Getting students to provide direction when drawing free-body diagrams. *Physics Teacher, 42*(7), 398-399.

McCloskey, M., Caramazza, A., & Green, B. (1980). Curvilinear motion in the absence of external forces: Naive beliefs about the motion of objects. *Science, 210*(4474), pp. 1139-1141.

McDermott, L., Rosenquist, M. L., & van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics
. *American Journal of Physics, 55*(6), 503-513.

McDermott, L. C. (1984). Research on conceptual understanding in mechanics. *Physics Today, 37*(7), 24.

Meltzer, D. E. (2005). Relation between students' problem-solving performance and representational format. *American Journal of Physics, 73*(5), 463-473.

Paivio, A. (1971). *Imagery and verbal processes*. New York: Holt, Rinehart and Winston.

Piaget, J. (1928). *Judgment and reasoning in the child : International library of psychology*. London: Routledge.

Ploetzner, R., Fehse, E., Kneser, C., & Spada, H. (1999). Learning to relate qualitative and quantitative problem representations in a model-based setting. *Journal of the Learning Sciences, 8*(2), 177.

Priest, A. G. (1992). New light on novice expert differences in physics problem solving. *British Journal of Psychology, 83*(3), 389.

Puri, A. (1996). The art of free-body diagrams. *Physics Education, 31*(3), 155-57.

Reddish, E. F. (1994). Implications of cognitive studies for teaching physics. *American Journal of Physics,* 62, 796-803.

Rosengrant, D. (2007). *Multiple representations and free-body diagrams : Do students benefit from using them?*. (Doctoral Dissertation, Rutgers, The State University of New Jersey). , 1-140. (3274182)

Rosengrant, D., Van Heuvelen, A., & Etkina, E. (2005). Free-body diagrams: Necessary or sufficient? *AIP Conference Proceedings, 790*(1), 177-180.

Rosengrant, D., van Heuvelen, A., & Etkina, E. (2006). Case study: Students' use of multiple representations in problem solving. *AIP Conference Proceedings, 818*(1), 49-52.

Rosengrant, D. (2007). An overview of recent research on multiple representations. *AIP Conference Proceedings, 883*, 149.

Rosengrant, D., Van Heuvelen, A., & Etkina, E. (2009). Do students use and understand free-body diagrams? *Phys. Rev. Spec. Top. Phys. Educ. Res., 5*, 010108. doi:10.1103/PhysRevSTPER.5.010108

Rosnick, P. (1981). Some misconceptions concerning the concept of variable. are you careful about defining your variables? *Mathematics Teacher, 74*, 418-420.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334-370). NY: MacMillan.

Shavelson, R. J. (1974). Methods for examining representations of a subject-matter structure in a student's memory. *Journal of Research in Science Teaching, 11*(3), 231.

Shavelson, R. J., & SHAVELSON. (1975). Construct validation: Methodology and application to three measures of cognitive structure. *Journal of Educational Measurement, 12*(2), 67.

Sherin, B. (2006). Common sense clarified: The role of intuitive knowledge in physics problem solving. *Journal of Research in Science Teaching, 43*(6), 535.

Siegler, R. (1978). *Children's thinking : What develops?*. Hillsdale, N.J.; New York: L. Erlbaum Associates; distributed by Halsted Press Division of Wiley.

Simon, D. P., & Simon, H. A. (1978). Individual differences in solving physics problems. In *Children's thinking: What develops?* (pp. 325-348). Hillsdale, N.J.; New York: L. Erlbaum Associates; distributed by Halsted Press Division of Wiley.

Singh, C. (2002). When physics intuition fails. *American Journal of Physics, 70*(11), 1103-1109.

Smith, M. U. (1991). *Toward a unified theory of problem solving : Views from the content domains*. Hillsdale, N.J.: L. Erlbaum Associates.

Sperry, W. (1994). Placing the forces on free-body diagrams. *Physics Teacher, 32*(6), 353.

Tao, P. K. (2001). Developing understanding through confronting varying views: The case of solving qualitative physics problems. *International Journal of Science Education, 23*(12), 950-963.

Thornton, R. K., & Sokoloff, D. R. (1998). Assessing student learning of newton's laws: The force and motion conceptual evaluation and the evaluation of active learning laboratory and lecture curricula. *American Journal of Physics, 66*(4), 338-352.

Van Domelen, D. J., & Van Heuvelen, A. (2002). The effects of a concept-construction lab course of FCI performance. *American Journal of Physics, 70*(7), 779-780.

Van Heuvelen, A. (1991). Learning to think like a physicist: A review of research-based instructional strategies. *American Journal of Physics, 59*(10), 891-897.

Vygotskiĭ, L. S., & Cole, M. (1978). *Mind in society : The development of higher psychological processes*. Cambridge: Harvard University Press.

White, P., & Mitchelmore, M. (1996). Conceptual knowledge in introductory calculus. *Journal for Research in Mathematics Education, 27*(1), pp. 79-95.

Zhaoyao, M. (1993). Difficulties in teaching and learning mechanics: A consideration of three problems. *Physics Education, 28*(6), 371.