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Levels Of Line Graph Question Interpretation With Intermediate Elementary Students Of Varying Scientific And Mathematical Knowle

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LEVELS OF LINE GRAPH QUESTION INTERPRETATION WITH INTERMEDIATE ELEMENTARY STUDENTS OF VARYING SCIENTIFIC AND MATHEMATICAL KNOWLEDGE AND ABILITY: A THINK ALOUD STUDY

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ABSTRACT

This study examined how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs and how their interpretations were affected by graph question levels. A purposive sample of 14 6th-grade students engaged in think aloud interviews (Ericsson & Simon, 1993) while completing an excerpted Test of Graphing in Science (TOGS) (McKenzie & Padilla, 1986). Hand gestures were video recorded. Student performance on the TOGS was assessed using an assessment rubric created from previously cited factors affecting students' graphing ability. Factors were categorized using Bertin's (1983) three graph question levels. The assessment rubric was validated by Padilla and a veteran mathematics and science teacher. Observational notes were also collected. Data were analyzed using Roth and Bowen's semiotic process of reading graphs (2001).

Key findings from this analysis included differences in the use of heuristics, self-generated questions, science knowledge, and self-motivation. Students with higher prior achievement used a greater number and variety of heuristics and more often chose appropriate heuristics. They also monitored their understanding of the question and the adequacy of their strategy and answer by asking themselves questions. Most used their science knowledge spontaneously to check their understanding of the question and the adequacy of their answers. Students with lower and moderate prior achievement favored one heuristic even when it was not useful for answering the question and rarely asked their own questions. In some cases, if students with lower prior achievement had thought about their answers in the context of their science knowledge, they would have been able to recognize their errors. One student with lower prior achievement motivated herself when she thought the questions were too difficult.

In addition, students answered the TOGS in one of three ways: as if they were mathematics word problems, science data to be analyzed, or they were confused and had to guess. A second set of findings corroborated how science background knowledge affected graph interpretation: correct science knowledge supported students' reasoning, but it was not necessary to answer any question correctly; correct science knowledge could not compensate for incomplete mathematics knowledge; and incorrect science knowledge often distracted students when they tried to use it while answering a question. Finally, using Roth and Bowen's (2001) two-stage semiotic model of reading graphs, representative vignettes showed emerging patterns from the study.

This study added to our understanding of the role of science content knowledge during line graph interpretation, highlighted the importance of heuristics and mathematics procedural knowledge, and documented the importance of perception attentions, motivation, and students' self-generated questions. Recommendations were made for future research in line graph interpretation in mathematics and science education and for improving instruction in this area.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE: INTRODUCTION

Graphing is a key component of higher-order thinking activities within mathematics and science education, and it "should come as no surprise that graph-related activities have taken an important place in recent reform efforts in mathematics and science education" (Roth & Bowen, 2001, p. 159). The creation and interpretation of graphs have been cognitive and psychomotor tools for several parts of the scientific process, including collecting, analyzing, transforming, and presenting data (Latour & Woolgar, 1979). Students collect data, manipulate them into information, and present what hopefully becomes knowledge claims (Roth & McGinn, 1997). These "chains of inscriptional practices" have been central to the practices of scientists, mathematicians, and engineers (Roth & McGinn, 1998, p. 166). As elementary students progress towards middle school and high school, science teachers assume students are able to read and understand graphs (Roth & Bowen, 1994).

Elementary school provides opportunities for students to engage in concrete experiences; whether in mathematics class with manipulatives (Cobb, 2002) or science class with inquiry labs (Marx, Blumenfeld, Krajcik, & Soloway, 1997). After their initial encounter, another connection can be made to data analysis and representation. Students can learn to take their concrete experiences and transfer them to graphic representations; therefore, elementary school provides the foundation with many opportunities for future mathematics and science learning (McClain & Cobb, 2001). Pinker asserted,

> A striking fact about human cognition is that we like to process quantitative information in graphic form. One only has to look at the number of ways in which information is depicted in pictorial form—line, bar, and pie graphs, Venn diagrams, flow charts, tree structures, node networks, to name just a few. (1990, p. 73)

Therefore, providing elementary students with graphing activities that are conceptually and intellectually sound will prove beneficial when they attend middle school and high school (AAAS, n.d.). Specifically, science knowledge learned during the elementary school years will play a more vital role in graph interpretation, a critical science skill, than previously identified (McKenzie & Padilla, 1986). However, the roles that mathematics and science background knowledge played during line graph interpretation was not adequately documented in earlier studies and needed to be more fully understood.

Statement of the Problem

As intermediate elementary teachers try to support and improve students' abilities to interpret graphic information in mathematics and science, it is important to understand how students of varying abilities approach line graphs of differing levels of difficulty and how students integrate their mathematics and scientific background knowledge (McKenzie & Padilla, 1986; Roth & Bowen, 1994; Wu & Krajcik, 2006). Line graphs for this study were chosen for their importance in mathematics and science education. "Line graphs display the relationship between two continuous variables in pictorial form. These graphs are thought to promote the communication of complex concepts and ideas" (McKenzie & Padilla, 1986, p. 571). Learning complex ideas within project-based science education is often challenging for students, and learning to communicate about graphs during projects helps students to better understand these challenging ideas (Marx, Blumenfeld, Krajcik, & Soloway, 1997; Roth & Bowen, 1994; Wu & Krajcik, 2006). "Inscriptions are signs that are materially embodied in some medium, such as paper or computer monitors…Knowledgeability with respect to inscriptions is indicated by the degree to which individuals participate in purposive, authentic, inscription-related activities"

(Roth & McGinn, 1998, p. 37). These sociocultural researchers argued that in order for students to become adept at graphing tasks, they must first become engaged in meaningful community-centered graphing activities within the school classroom.

Purpose and Overview of Methodology

The purpose of this study was to understand how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs. In addition, the purpose was to understand how students' interpretations were affected according to graph question level. The research methods for this study were primarily qualitative, though quantitative data were used to help with sample selection, categorization of students into subgroups, and analysis of data across the sample (Creswell, 2003). The qualitative data were collected using think aloud interviews (Ericsson & Simon, 1993), video, and observational notes. Two quantitative measures were used. One was the 2006 5th Grade Florida Comprehensive Assessment Tests (FCAT) of Mathematics and Science used to create a purposive sample using maximum variation sampling (Patton, 2002) to capture the variability of sixth grade students at one school. For the purpose of this study, students' achievement on the FCAT was assumed to be an appropriate predictor of student background knowledge and ability in mathematics and science (Florida Department of Education, n.d.). The other quantitative measure was the excerpted Test of Graphing in Science (TOGS) instrument developed by McKenzie and Padilla (1986). The TOGS measured students' ability to interpret line graphs (Appendix C). A scoring rubric (Appendix A) to assess student performance on the TOGS was created by the researcher and validated by Prof. Michael Padilla (Appendix F) using previously cited factors from the literature affecting students' graphing ability (Appendix E). These factors were categorized using

Bertin’s stages of the reading process and three graph question levels (1983). Data were also analyzed using Roth and Bowen’s two-stage semiotic process of reading graphs (2001).

Importance of Study and Conceptual Framework

It has always been critical for teachers to spend each instructional minute in the classroom engaging students in worthwhile activities using rigorous and academic standards and curriculum, and the FCAT in Mathematics and Science have made student achievement in these areas even more prominent (Marzano, Pickering, & Pollock, 2001). Data collected from the FCAT indicated that intermediate elementary students across Florida struggled with the data analysis strand in mathematics and the nature of science strand in science (Florida Department of Education, 2006). Teachers and school district leaders used these data to improve student performance (Marzano et al., 2001) but have had difficulty selecting instructional activities to foster success within the data analysis and nature of science Sunshine State Standards (SSS) strands (Florida Department of Education, 2006). Activities that meaningfully integrate mathematics and science would help students to better understand both strands (Roth & Bowen, 1994). Singer, Marx, Krajcik, and Chambers (2000) insisted, “[T]o deeply understand the principles of a discipline, students must actively see how knowledge or skills function within the context of the disciple” (p. 166).

Finally, it is crucial for students to develop the ability to think in abstract ways (AAAS, n.d.). Abstract thinking enables students to take lessons learned from concrete situations into other concrete situations. This allows students to transfer skills and knowledge in order to make generalizations (Roth, Pozzer-Ardenghi, & Han, 2005). Too often, students view science education as a list of discrete facts to be learned and mathematics education as an inventory of complicated procedures to be memorized (AAAS, n.d.). When students are forced to memorize long lists of remote facts, comprehension suffers. Marx, Blumenfeld, Krajcik, and Soloway resisted the "memorization of discrete facts" and called them "isolated and inert;" they argued for the "flexible access and use of knowledge so that the learner can ask and answer questions, draw analogies to new situations, and solve problems" (1997, p. 342). Graphing is a generalizable skill used across a large numbers of domains of both academic and professional activities (Roth & Bowen, 2001), and enables students to see deeper connections and patterns when they engage in well-constructed pedagogies (Singer et al., 2000).

Conceptual Framework

The conceptual framework for this study was a synthesis of two constructs published in the literature on graphing. These constructs were Bertin's (1983) distinctions among three graph question levels and Roth and Bowen's (2001) two-stage semiotic process of reading graphs.

Bertin's Three Graph Question Levels

The distinctions among three levels of questions (Bertin, 1983) while interpreting graphs enabled the researcher to categorize previously cited factors affecting students' ability to interpret line graphs and to classify the primary focus of the questions used in the think aloud interviews. Specifically, the existing literature suggested a number of specific problems that prevented students from interpreting a graph in the way the graph's designer intended. Some of these documented problems included: reading individual points on a graph, integrating information across data points (Friel, Curcio, & Bright, 2001), searching for spatial locations of specifiers, encoding the values of specifiers, performing arithmetic operations on the encoded values (Gillan & Lewis, 1994), and knowing graph features (Carpenter & Shah, 1998).

The idea of dividing graph interpretation into three levels emerged from Bertin's reading of C.S. Peirce (Buchler, 1978). Peirce argued for three interacting components of the interpretation of any sign (1978). The referent is the thing the sign refers to; the sign represents something not itself present, a word, symbol, or inscription created to represent the referent; and the interpretant is the understanding an individual has of the referent from perceiving the sign (see also Roth, Pozzer-Ardenghi, & Han, 2005).

Bertin (1983) called his three levels of graph questions elementary, intermediate, and overall. These three levels each involved different cognitive processes. Elementary level questions, according to Bertin (1983), involved data extraction, location, and translation (Curcio, 1987). A conceptual connection was made between Bertin's elementary level questions (Roth & Bowen, 2001) and Peirce's notion of sign (Buchler, 1978). Intermediate level questions involved trends seen in parts of the data (1983). Curcio (1987) referred to this as reading between the data and included the skills of interpolation, extrapolation, describing the trends seen in the graph, and describing qualitative or quantitative differences between data points (Appendix A). Again, a connection existed between Bertin and Peirce's notion of interpretant (Buchler, 1978; Roth & Bowen, 2001). Finally, overall level questions involved an "understanding of the deep structure of the data being presented in their totality, usually comparing trends and seeing groupings" (Bertin, 1983, p. 16). The goal of answering an overall level question, according to Curcio (1987), was to read beyond the data, including generating ideas and predicting outcomes.

Care had to be taken, however, not to interpret these question levels as increasing levels of difficulty. "Although these levels of questions involve an increasingly broad understanding of the data, they do not necessarily imply an increase in the empirical difficulty of the question"

(Wainer, 1992, p.16). Padilla, however, disagreed and suspected the data would show the increasing levels were more difficult (personal communication, December 4, 2006).

Roth and Bowen's Two-Stage Semiotic Process of Reading Graphs

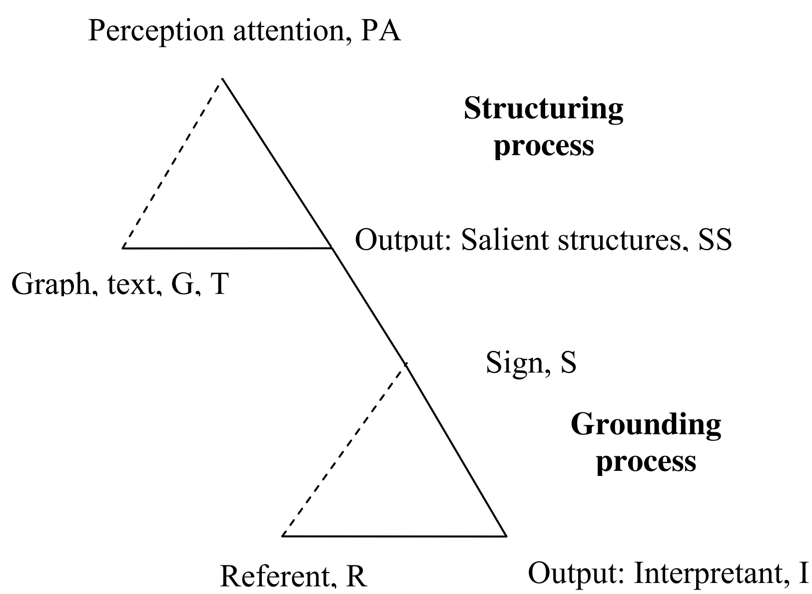

Figure 1: Roth and Bowen's two-stage semiotic process of reading graphs

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The semiotic model of reading graphs (Roth & Bowen, 2001) provided a comprehensive framework for describing and interpreting students' cognition and behavior while engaged in line graph interpretation. Based on their analysis of students engaged in scientific inquiry using a variety of data, scientific texts, and field experiences, they argued, “The move from a text (word problem) to a more powerful inscription, such as a data table, graph… is quite complex…. Much as in hermeneutics, secondary texts elaborate the meaning of primary texts, they become con-text” (Roth et al., 2005, p. 81).

Their qualitative research was primarily focused on the grounding side of the model (Roth, Pozzer-Ardenghi, & Han, 2005). The students and scientists in their studies needed to learn to coordinate the graphical representations of their data with the published information and their own experiences in the field. What a person comes to understand a graph to signify is in turn affected by experiences with the referent and the theories or ideas that form interpretants—what Roth called grounding (bottom triangle in Figure 1). Like Bertin (1983), the grounding side of Roth and Bowen’s model (2001) also borrowed Peirce’s theory of semiotics which included signs, objects and interpretants (Buchler, 1978).

However, Roth and colleagues (2005) argued that Peirce’s semiotics was inadequate and claimed that signs never simply stood on their own in a clean, unambiguous relationship to a referent (citing Ricœur, 1991). Instead, they argued, signs should always be understood in the context of other signs (Roth & Bowen, 2001). When a person tried to interpret a graph, what she or he understood the graph (text) to signify was affected by perception attentions (i.e., knowledge, skills, and habits). Roth called the experience of making meaning from the graph structuring (top triangle in Figure 1). That is, the process of structuring the information in the graph was facilitated by the knowledge, skills, and habits previously learned (Roth, Pozzer-Ardenghi, & Han, 2005).

Research Questions

This research project was designed to answer the following questions:

1. How do student behaviors observed during think aloud interviews vary during line graph interpretation in science across mathematics and science achievement levels?
2. How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)?
3. Drawing from these data, what patterns emerge from student thinking during line graph interpretation given their prior performance in mathematics and science?

Personal Biography

For 12 years, I have been teaching intermediate elementary school children. These experiences provided a foundation for the tacit theory grounding this research project (Polanyi, 1967). In Polanyi's theory of knowledge, he argued that tacit knowing "account[s] for three things: 1) valid knowledge of a problem, 2) scientist's capacity to pursue it, guided by his sense of approaching its solution, and 3) valid anticipation of …indeterminate implications of the discovery arrived at in the end" (p. 145). In this section, Polanyi's third definition of tacit knowing was used. Of particular importance here were experiences teaching elementary school students who did well with graphing tasks in mathematics but were seemingly unable to use their graphing skills in science inquiry activities (Parmar & Signer, 2005). When studying mathematics, these students were often able to interpret correctly graphs given in the textbook or create graphs when categorical data were collected inside the classroom (e.g., favorite food, favorite sports team, favorite color). Yet, these same students often had trouble in science class discerning errors within the data during graph construction tasks when irregularities occurred or using science knowledge when interpreting graphs. In Leinhardt, Zaslavsky and Stein's (1990) exhaustive review of the research on graphing, they argued a directionality division between mathematical and scientific presentations of graphing.

> The mathematical presentation is usually from an algebraic function rule to ordered pairs to a graph, or from a data table of ordered pairs to a graph. The scientific presentation, on the other hand, most often proceeds from observation, to data array, to ordered pairs of data, to selection of axis labels, to scale construction, to graph and (maybe) to function. Often students who can solve graphing or function problems in mathematics seem to be unable to access their knowledge in science. (Leinhardt et al., 1990, p. 3)

Although students have difficulty with graphing in science, I am still a proponent of hands-on, inquiry based science learning. I find myself paying even greater attention to students who have difficulty transferring the graphing skills learned during mathematics instruction to the science tasks of data analysis and representation. This study has made me more sensitive to this issue.

Another issue was the importance of teaching graphing in science classrooms. McKenzie and Padilla's (1986) research addressed this very topic:

> In science, more than in any other subject, students should be involved in predicting relationships between variables and attempting to quantify these relationships. Line graph construction and interpretation are very important to science instruction because they are an integral part of experimentation, the heart of science. (p.572)

In my classroom, for example, one class collected data during a soil experiment activity using a GrowLab® system to grow morning glory plants inside the classroom. For a month, students in cooperative groups daily measured and recorded on a data table the height of these plants in four different types of soil—top soil, sand, clay (kitty litter), and gravel. When it was time to graph the data at the end of the month, some groups had produced line graphs showing more growth in top soil. Then, strangely, one of the groups showed their line graph for top soil decreasing precipitously to a minimal height of 0 cm during the last week. Instead of using their background knowledge learned during this unit or discussing why a plant would shrink from 30 cm to 0 cm in a day, they simply plotted the data points as 30 cm on day 25 and 0 cm on day 26. Their final line graph had little to no effect on this group's comprehension of their data, and they were unable to explain their results. The group did not even mention that an error could have occurred while measuring and recording the data, which would have explained the drastic descent of their line within their graph. It was apparent this group was not thinking about their experience collecting the data to validate their conclusive representation of the experiment (i.e., the line graph).

Like the classroom example with the GrowLab® experiment, McKenzie and Padilla (1986) found that although these graphing skills were of utmost importance, "research indicate[d] many students ha[d] not acquired these skills. In several studies, line graphs were found to be the most difficult type of graph to interpret" (p. 572). Being sensitive to struggling learners who were trying to transfer skills across academic subjects as well as teaching graphing in both mathematics and science classes were two situations I had to acknowledge during this study. The classroom experiences along with the literature on graphing led me to anticipate that a detailed study of the relationship between students' background knowledge and their ability to interpret line graphs would yield knowledge that would help my practice and the practice of other teachers.

Delimitations of Study

This study was conducted with a purposive sample of 14 sixth grade students from one elementary school in Central Florida. The research setting and sample choice affected the robustness of the results in one way. Not using a probability sample may have resulted in an undetected sampling bias.

Limitations of Study

The graphing tasks used in the think aloud protocol were challenging for academically low-performing students (see sampling matrix Table 1). In addition, students in this age range have had relatively little formal science education and relatively little education about line graphs. As a result, it was difficult to interpret what the results of this study may imply for students with more experience with science and graphing. As a result, these students' verbal reports had to be treated cautiously.

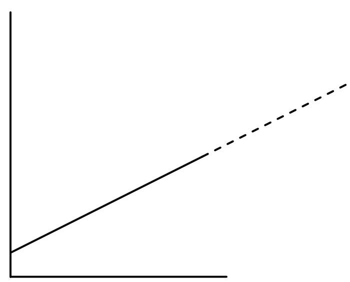
In addition to cognitive challenges facing students with low prior achievement, the school from which the sample was drawn had relatively few students with low math and low science abilities who were willing to participate in this study. As a result, the study lacked a robust sample of low-performing students to provide an adequate comparison to the larger number of more able students.

The pilot study revealed that the data collected using the TOGS questions elicited relatively little background science knowledge. Attempts were made to address this limitation with added verbal prompts to elicit further science background knowledge where appropriate. However, even with these modifications, the TOGS questions still elicited relatively little science background knowledge.

Definition and Description of Terms

External Identification Stage – While reading a graph the "reader must identify, *in the mind*, the invariant and components involved in the information… External identification relies on acquired habits… It permits [the reader] to isolate, from the vast realm of human knowledge, the precise domain treated by the [graph]" (Bertin, 1983, p. 140).

Extrapolate – To estimate a value by following a pattern and going beyond values already known.



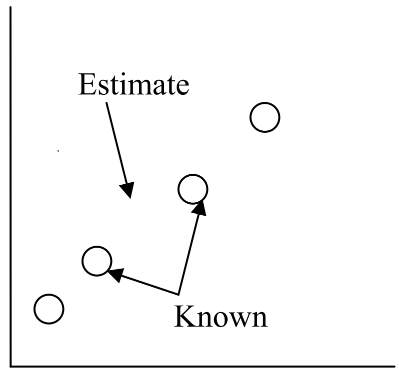
Florida Comprehensive Assessment Test (FCAT) - Currently, public school students in Florida are required to take the FCAT, administered to students in Grades 3-11. This achievement test contains "two basic components: criterion-referenced tests (CRT), measuring selected benchmarks in Mathematics, Reading, Science, and Writing from the Sunshine State Standards (SSS); and norm-referenced tests (NRT) in Reading and Mathematics, measuring individual student performance against national norms" (Florida Department of Education, 2006, p. 1). Mathematics FCAT student scores are reported by achievement levels, scale scores, and developmental scale scores. The scale scores, which range from 100 to 500, are divided into five categories, from 1 (lowest) to 5 (highest), called achievement levels. Science FCAT student scores are reported only by achievement level and scale scores (Florida Department of Education, n.d.). Students' 5th grade 2006 SSS FCAT Mathematics and Science scores were used to select the purposive sample for this study. For the purpose of this study, students' achievement on the FCAT was assumed to be an appropriate predictor of student background and ability in mathematics and science (Florida Department of Education, n.d.). The 2006 FCAT Mathematics and Science Test was able to accurately and consistently categorize students by grade level and subject area into one of five levels; accuracy = 0.961; consistency = 0.932 (HumRRO, 2007). The 2006 5th grade FCAT in Mathematics received a classical reliability score of 0.87 (Cronbach's Alpha) for the Sunshine State Standards test and a 0.91 (KR-20) for the norm-referenced test. No reliability measure was available for the FCAT Science since no state-wide norm-referenced science tests were given across the state of Florida (FDOE, 2007).

Grounding process – Grounding is the second process in Roth and Bowen's two-stage semiotic process of reading graphs model (Roth & Bowen, 2001) where the learner "connect[s] sign[s] and referent[s] (familiar things in the world) in reading" (Roth, Pozzer-Ardenghi, & Han, 2005, p. 17). This process is a part of the model which contributes to this study's conceptual framework and was used to analyze the data in the study.

Heuristic –Strategy used to solve a problem; examples include drawing a picture, answer elimination, estimating, mnemonics, working backwards, using an easier problem, guess and check (Po'lya, 1945; Schoenfeld, 1992).

Internal identification stage – The second stage of the graph reading process, according to Bertin, when the reader recognizes the components of the graph and their visual arrangement. It is the identification of the points that are represented on the graph, how they relate to each other, and the patterns that are formed (Bertin, 1983).

Interpolate – To calculate or estimate values between two known values in a set of data.



Interpretants – Theories about phenomena during the graph interpretation process (Roth & Bowen, 2001). Interpretants can either be formal scientific theories or informal theories; interpretants can either be theories directly about the phenomena or theories borrowed from analogous phenomena. “Interpretants are commentaries on the sign, definitions or glosses on the sign in its relation to the referent object” (Roth, Pozzer-Ardenghi, & Han, 2005, p. 15).

Maximum variation sampling – In order to “document unique” and “diverse variations that have emerged in adapting to different conditions,” this specific purposive sampling strategy is used in studies to choose a wide range of cases “to get variation on dimensions of interest” (Patton, 2002, p. 243).

Perception attention (PA) – These are characteristics the learner brings to the graphic situation that affect the structuring process (Roth, 2008, in press); perception attention is located in the top triangle of Roth and Bowen’s two-stage semiotic process of reading graphs model (Roth & Bowen, 2001) where the learner perceives certain aspects of the graph and the relationships between those aspects. This perception is framed by his or her prior background knowledge and socialization. This construct is a part of the Roth and Bowen model which contributes to this study's conceptual framework and was used to analyze the data in the study.

Purposive sampling – In a study with a small sample size, selecting "*information-rich cases* for study in depth" for the purpose of answering the research questions (Patton, 2002, p. 46) was important to ensure trustworthiness.

Referent (R) – This construct is a part of the Roth and Bowen model which contributes to this study's conceptual framework and will also be used to analyze the data in the study.

Salient Structures (SS) – The superficial order inferred in the graph during the interpretation process, such as a trend on the graph or an outlying point, without connection to the phenomena that the graph was intended to represent. This construct is a part of the Roth and Bowen model which contributes to this study's conceptual framework and was used to analyze the data in the study.

Sign (S) – A sign "stands for something to the idea which it produces, or modifies. Or it is a vehicle conveying into the mind something from without" (Buchler, 1978, p. 80). The sign is located on the bottom triangle in Roth and Bowen's two-stage semiotic process of reading graphs model (Roth & Bowen, 2001) where the learner relates the structuring process to the grounding process. This construct is a part of the Roth and Bowen model which contributes to this study's conceptual framework and was used to analyze the data in the study.

Strategy – see Heuristic.

Structuring process – Structuring is the first process in Roth and Bowen's two-stage semiotic process of reading graphs model (Roth & Bowen, 2001). It is during this first process when looking at an unfamiliar inscription that potential signs are created that come to be "related to possible references in and through the (mutually exclusive) grounding process" (Roth, 2008, in press, p. 1). This process is a part of the model which contributes to this study's conceptual framework and was used to analyze the data in the study.

Test of Graphing in Science (TOGS) – Quantitative instrument developed by McKenzie and Padilla (1986). The TOGS measured students' ability to interpret line graphs (Appendix C). For the purposes of this study, 13 questions out of 22 questions were excerpted. These 13 questions were developmentally appropriate for sixth graders; the eight questions omitted were not.

Think aloud – Informational processing model where participants verbalize "out loud whatever they are saying silently to themselves" (Ericsson & Simon, 1993, p. 226) and the researcher, then, may observe the participants' thoughts "recently acquired by the central processor (i.e., brain)" and "kept in STM (i.e., short term memory)" (1993, p. 11) in the moment of activity. These thoughts are assumed to be "directly assessable for further processing" like for creating think aloud reports (1993, p. 11).

There are three levels of verbalization within the think aloud protocol:

- Level 1 verbalization – The first level is considered the most direct level where participants reproduce information in the exact manner in which it was heeded and "expend no special effort to communicate [their] thoughts…[and] there are no intermediate processes" (Ericsson & Simon, 1993, p. 79). Extra time is usually not required for Level 1 think aloud verbalizations.
- Level 2 verbalization – At the second level, participants use description of thought content but "do not bring new information" into their focus that could possibly "change the structure of the process for performing the main task" (Ericsson & Simon, 1993, p. 79). At this level, "one or more mediating

processes occur[ed] between attention to the information and its delivery" (1993, p. 16). When the mediating process was the translation of an internal representation of a non-verbal code into a verbal code where no communication to another person took place, a Level 2 verbalization occurred (Ericsson & Simon, 1993). Due to the increased cognitive activity during Level 2 verbalizations, participants may require extra time using these verbalizations (1993). For the purposes of this study, Level 2 verbalizations were preferred.

- Level 3 verbalization – During Level 3 verbalizations, participants were explaining their thoughts, recoding them from short term memory and also "linking this information to earlier thoughts and information attended to previously" (Ericsson & Simon, 1993, p. 79). The danger in these behaviors was the likelihood of "changing the structure of the thought process" (1993, p. 80). Level 3 verbalizations were avoided in this study by facing the participants toward the wall in the classroom to avoid communication with the researcher.

To review, Level 1 and Level 2 verbalizations were preferred in this study because they did not allow the participants to change their thoughts in short term memory (Ericsson & Simon, 1993). "[W]ith Level 1 and Level 2 verbalization the sequence of heeded information remains intact and no additional information is heeded…Level 3 verbalization requires attention to additional information and hence changes the sequence of heeded information" (1993, pp. 18-19).

<u>Triangulation</u> - Patton (2002) cites Denzin (1978) to define one type of triangulation as "methodological triangulation, the use of multiple methods to study a single problem or

program" (Patton, 2002, p. 247). For this study, the multiple methods used were the TOGS (McKenzie & Padilla, 1986) and think aloud interviews (Ericsson & Simon, 1993). In addition, on the TOGS, most graph interpretation skills were assessed by two question items (Appendix A).

Two-stage semiotic process of reading graphs – Developed by Roth and Bowen (2001), this semiotic model of graph reading includes two stages called structuring and grounding. Structuring is the first process the learner engages when he or she is unfamiliar with the graph (Roth & Bowen, 2001). "The process of structuring…yields specific features that subsequently become the signs for the second process [i.e., grounding]" (p. 165). Grounding is the second process where the learner relates the graph to his or her prior background knowledge (Roth & Bowen, 2001). "During the grounding process, signs and familiar things in the world (referents) are mutually stabilized" (Roth, Pozzer-Ardenghi, & Han, 2005, p. 16). This model is a major part of this study's conceptual framework and was used to analyze the data in the study.

Ethical Considerations

To protect the rights of all participants in this study and to guarantee ethical considerations were observed throughout the entire process, the researcher took a utilitarian viewpoint (Miles & Huberman, 1994). Four main actions were taken to protect participants: the research protocol was reviewed by the Institutional Review Boards (IRB) of the university and school district, participants were recruited with informed consent and assent letters, fieldwork was conducted so as "to avoid harm to others," and participants' confidentiality was maintained through the use of pseudonyms (p. 289).

Some specific procedures were used to protect the participants during the study. First, all participants were reminded of their rights to withdraw from the study at any time. Although it affected the study, one participant chose to withdraw after her parents consented and she assented. She was reassured of her rights and the researcher expressed appreciation for her attempt to contribute to the study. Second, participants' confidentiality was protected through the use of selective videoing. Participants' hand gestures during the think aloud interviews were the only body parts recorded because they would not lead to student identification after the study. Finally, once identifying information was no longer needed, the participants were only discussed by their pseudonyms and all nominal data were kept in a locked filing cabinet. Through careful consideration, no harm was done to participants during this study and ethical considerations were made throughout the entire process using a utilitarian viewpoint (Miles & Huberman, 1994).

This chapter introduced the reader to the research study including the problem, purpose, importance, conceptual framework, research questions, delimitations, limitations, definition and description of terms, and ethical considerations. Chapter 2 reviews the conceptual and empirical literatures on graph interpretation. Chapter 3 explains the research methods used in this study. Chapter 4 presents the data analysis and summary of findings of the research questions and Chapter 5 connects the literature review and methodology with the summary of findings in a conclusion and discussion section.

CHAPTER TWO: REVIEW OF THE LITERATURE

In this literature review, the research and scholarship on the challenges students encounter while graphing in mathematics and science education are reviewed and critiqued. Although psychological research on graphing has examined the factors that influence graph comprehension, these studies have not directly reviewed students' cognition during graphing tasks. As such, this literature review provides additional insight into the methodological limitations of the psychological literature and suggests think aloud protocols as a viable alternative (Ericsson & Simon, 1993, 1998). The analytical focus on students' mathematics and science background knowledge provides another insight (Hawkins, 1965; Kerslake, 1981; Lehrer & Romberg, 1996). The idea of graph question levels, adopted from Bertin (1983), highlights the role of formal and informal knowledge during graph interpretation. In addition, although some studies have identified the importance of graphing as a social and communicative activity in science practice, such studies have not been successful at capturing students' cognition during these tasks (Marx, Blumenfeld, Krajcik, & Soloway, 1997; Wu & Krajcik, 2006). This issue is addressed by demonstrating that Roth and Bowen's (2001) two-stage semiotic process of reading graphs provides a conceptual framework for integrating the social and psychological aspects of graph interpretation.

In this chapter, a conceptual framework is presented and explained, then supported through a literature review which links psychological and sociocultural approaches to studying graph interpretation. Then, a representative sample (Cooper, 2003) of earlier studies are summarized and critiqued to show both what is known about how students interpret graphs and what studies have not yet been able to address. Finally, the research methods that have been used to study graphing are reviewed and critiqued, and think aloud research methods are presented as a means of overcoming the limitations of earlier research methods.

Literature Review of Conceptual Framework

Graphing, in general, has been studied for at least two centuries (Costigan-Eaves & Macdonald-Ross, 1990), and graphing in mathematics and science education has been studied for several decades (Friel, Curcio, & Bright, 2001; Shah & Hoeffner, 2002). Not surprisingly, the perspective that researchers brought to their studies reflected their assumptions about the value of graphing and challenges that people faced when they interpreted graphs (Leinhardt, Zaslavsky, & Stein, 1990). The focus of this research project was on the challenges that students face while they interpreted graphs, similar to many psychological studies of graphing (Aberg-Bengtsson & Ottosson, 2006; Friel, Curcio, & Bright, 2001; Wainer, 1992; Wittrock, 1992). At the same time, it recognized that students should learn to graph to help with data collection, analysis, and presentation, similar to many sociocultural studies of graphing (Marx, Blumenfeld, Krajcik, & Soloway, 1997; Roth & Bowen, 1994; Roth & McGinn, 1998; Wu & Krajcik, 2006). A conceptual framework emerged from a synthesis of two constructs published in the literature on graphing. These constructs were Bertin's (1983) distinction among three graph question levels and Roth and Bowen's (2001) two-stage semiotic process of reading graphs.

Bertin's Distinctions among Three Graph Question Levels

Graphs have been used to analyze volumes of information and communicate ideas (Barab, Hay, & Yamagata-Lynch, 2001; Cobb, 2002). It was in the context of his semiotic analysis of graphs that Bertin developed five aspects of graph quality. Only the first two, stages of the reading process and level of the question or reading level, were used for the purpose of this study. The other three stages, definition of an image, construction of an image, and limits of an image, did not apply and were, therefore, not used in this study (1983).

Stages of the Reading Process

Bertin divided the stages of the reading process into three successive operations. First, "before all else the reader must identify, in the mind, the invariant [*sic*] and components involved in the information" (1983, p. 140). He called this the external identification stage. For example, when students read line graphs, they identified the labels of the x- and y-axes, units, intervals, scale, etc. This information was necessary to identify as precisely as possible what the graph was referring. Like Roth and Bowen (2001), Bertin stressed that external identification relies on acquired habits—i.e., perception attentions. Second, the reader recognized the components of the graph and their visual arrangement. He called this the internal identification stage. When students read a line graph, they identified the points represented on the graph, how they related to each other, and what patterns were formed. Third, after the reader had undergone the external and internal identification stages, she or he would be ready to perceive pertinent correspondences, "which the drawing isolates from the vast number of possible correspondences" (1983, p. 140). A well-constructed graph, according to Bertin, used legends and notes to explain the connection between the external identification stage and the internal identification stage. Indeed, Roth and Bowen's research extended Bertin's claim by showing that in scientific articles, the interpretation of the graph was woven into the text of the article and constructed to ensure that the reader came to only one possible conclusion about the meaning of what was represented in the graph (Roth & Bowen, 2001; Roth, Pozzer-Ardenghi, & Han, 2005).

In addition, Bertin stressed that the reader's perceptions of the meaning of pertinent information was shaped by the question or questions the reader brought to the interpretive process (Bertin, 1983, 2001).

Level of the Question or Reading Level

Bertin (1983) divided his graph questions into three levels which he called elementary level, intermediate level, and overall level questions. To distinguish among them, he specified certain characteristics of each that made them unique. For elementary level questions, he described them as being "introduced by a single element of the component and resulting in a single correspondence" (p. 141). This type of question required the student to extract a single piece of information that could be found on one location on the graph. Then, the student needed to use the external identifications from the graph to understand what the single piece of information represented. An elementary level question may ask students to find how tall the morning glory plant grew on the fifth day in all four soil types (see GrowLab® experiment under Personal Biography section). To answer this question, students would need to be familiar with line graph components, like the x- and y-axes, and be able to correctly identify the corresponding point where the soil type and the fifth day intersected.

Intermediate level questions, by contrast, were "introduced by a group of elements or categories and result[ed] in a group of correspondences" (Bertin, 1983, p. 141). For these particular questions, students had to notice patterns among groups of elements on the graph, gleaning information from several places on the graph and consolidating information into a more general statement (2001). Such questions, Bertin emphasized, tended to reduce the amount of information "in order to discover, from within the information, groups of elements or homogeneous categories which are less numerous than the original categories and consequently easier to understand and memorize" (Bertin, 1983, p. 141). An intermediate level question may ask students to find the difference in height on day 5 between morning glory plants grown in top soil and morning glory plants grown in sand (GrowLab® experiment). Here, students would need to access two data points on the line graph and then use the correct computation procedure (i.e., subtraction) to find the answer.

The last type of question, called overall level questions, required students to use their background mathematics and science knowledge and experiences to help them develop a theory to explain what they saw in the graph (Bertin, 1983). Roth and Bowen referred to students' background knowledge and experiences as referents and theories as interpretants (2001). Overall level questions required using the information on the line graphs along with additional background knowledge to sufficiently answer the question and asked the reader to "reduce all the information to a single, ordered relationship among the components…[and]… enable[d] the reader to retain the whole of the information and compare it to other information" (Bertin, 1983, p. 141). An example of an overall level question from the GrowLab® experiment was, "Why did the morning glory plants grown in top soil grow taller than plants grown in sand?" Students needed to use their mathematics and science background knowledge (referent) and develop a theory (interpretant) to answer this question correctly. While all of these examples were of someone else asking students questions, it is worth noting that students can also ask their own questions (Roth, Pozzer-Ardenghi, & Han, 2005).

Roth and Bowen's Two-Stage Semiotic Process of Reading Graphs

Introduced in Chapter 1, Roth and Bowen developed their two-stage semiotic process of reading graphs to provide a descriptive lens that linked together the components of graph interpretation (2001). In this section, each vertex on the model is described, then each of the two triangles is explained and linked to Bertin's (1983) processes and questions.

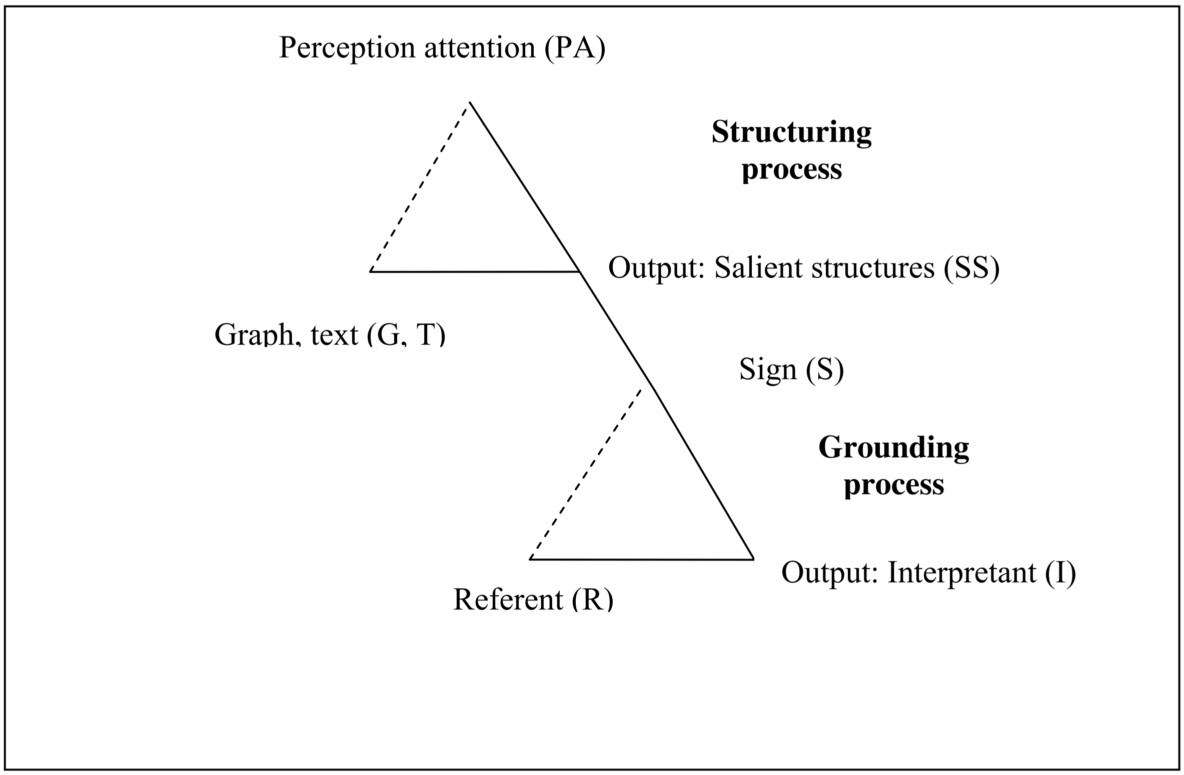


Figure 2: Roth and Bowen's two-stage semiotic process of reading graphs.

Structuring Process

The process that Roth and Bowen called structuring in their two-stage semiotic process of graph reading paralleled what Bertin (1983) called the internal identification stage. In both theories, this was the process or stage wherein the reader related and understood the significance (sign) of the text or graph using their cognitive and memory processing abilities (perception attentions). Most important among these abilities were their learned mathematical skills, but Roth's language also suggested personality factors like curiosity, attentiveness, and persistence (perception attention) (Roth, 2008, in press). While perception attention has received relatively little explicit attention in Roth's research on graphing, he wrote of it most clearly when he referred to scientists using the perception attentions they were socialized into during their apprenticeship in their discipline and field (Roth & Bowen, 2001; Roth & McGinn, 1997). Referring to students in school, he wrote that "[T]hese dispositions generate patterned (i.e., structured) perceptions and with it the field of possible (material, discursive, etc.) patterned actions, that is the practices characteristic of a field" (Roth, Pozzer-Ardenghi, & Han, 2005, p. xii). It is important to note that even when Roth wrote about this psychological development, his focus was on the social origins and interactions that led to these psychological attainments (Roth & Bowen, 2001; Roth & McGinn, 1998; Roth & McGinn, 1997).

Moreover, while perception attentions has received relatively little attention in Roth and Bowen's work on this topic, their emphasis on critical graphicacy foregrounds the centrality of certain kinds of dispositions in addition to the elementary mathematical and scientific skills (Roth et al., 2005). Too often, they argued, traditional science and mathematics instruction have not encouraged students to critically evaluate and reflect upon the knowledge claims being presented (Roth & Bowen, 1994, 1995). The unfortunate result, Roth concluded, was that students "will always be subject to some form of indoctrination" without the ability to "question the different power relations that are thereby constructed" (Roth et al., 2005, p. xiii). Hence, perception attentions in graphical interpretation cannot and should not be reduced to merely cognitive and memory processes (i.e., mathematical skills), but should also include factors like curiosity, attentiveness, and persistence (2005). Therefore, while there were clear parallels between Bertin's and Roth and Bowen's models, Bertin did not make any normative claims about the relationship between the social roles of graphing while Roth and Bowen clearly did (Bertin, 1983; Roth & Bowen, 2001; Roth, Pozzer-Ardenghi, & Han, 2005).

Grounding Process

The process that Roth and Bowen (Roth & Bowen, 2001) called grounding paralleled what Bertin (1983) called the external identification stage. Again, in both theories, the reader needed to connect their knowledge and experiences (referents) the graph was referring to with the signs and symbols presented within the graph. Roth and Bowen's model highlighted the reader's experiences (referent) with the phenomena presented in the graph, and while Roth and Bowen emphasized the role of direct personal experience, there was no reason to believe the reader could not also use vicarious experiences during interpretation (Roth & Bowen, 1994, 2001; Roth & McGinn, 1998). Presenting students with direct personal experience of scientific phenomena has long been one of the justifications of activity-based science education (Hawkins, 1965) and project-based science education (Marx, Blumenfeld, Krajcik, & Soloway, 1997). However, more recently, the increasing use of educational technologies and media have made it possible for teachers to present students with vicarious experiences that would be otherwise inaccessible, too time-consuming, expensive, or dangerous (Linn, 1998). Whether personal or vicarious, Roth and Bowen's (2001) notion of referent within the grounding process was more specific than Bertin's (1983) notion of external identification stage.

In addition, Roth and Bowen (2001) were also more specific about the role of theories (interpretants) in graph interpretation than was Bertin (1983). Interpretants are theories about phenomena, and they can either be formal scientific theories or informal theories, and they can either be theories directly about the phenomena or theories borrowed from analogous phenomena (Buchler, 1978; Roth & Bowen, 2001).

To better understand these ideas, recall the GrowLab® example presented in the Personal Biography section in Chapter 1. As the students were looking at the final graph they had created about the growth of morning glory plants over three weeks, they failed to connect all of the information they had acquired. They had personal experiences (referents) which had accrued over three weeks of watching the plants grow and had charted the height of the plant every few days. In the lessons and activities that accompanied this experiment in class, they also learned knowledge and theories about plant growth (interpretants). Yet, when they looked at the final graph they had created from their data and saw the precipitous drop in plant height on the last day, they did not use either their experiential knowledge (referent) or their scientific knowledge (interpretant) to recognize (perception attention) that an error had been made in graphing their own data.

Findings from Earlier Empirical Studies

A number of representative empirical findings were important to consider in relation to this study's major findings. These empirical studies came from two perspectives in the literature: the psychological point of view and the sociocultural standpoint.

Psychological Factors that Influence Graph Interpretation

A review of the psychological literature revealed many factors that contributed to student difficulties during graph interpretation tasks. These potential problems included students' background knowledge, developmental readiness, learner characteristics, graph features (like axes, their scales and variables), prediction tasks (like interpolation and extrapolation), and iconic or literal interpretation tasks.

Students' background knowledge

Students' science background knowledge has been found to be a contributing factor that affects their ability to interpret graphs. Specifically, Shah and Hoeffner (2002) found that students' knowledge of graph content influenced their interpretation of the data and their ability to remember it (see also Carpenter & Shah, 1998). Students were also better able to recognize problems in studies where the results were inconsistent with their prior beliefs (1998). For example, if a student who likes cats was presented with data showing that dogs were more intelligent, she or he would be more critical of the data and how they were collected since it contradicted her or his belief. In another study, 91 4th- and 5th-grade students with and without learning disabilities were given line graphs and asked to write a narrative about what the graph represented (Parmar & Signer, 2005). Their research revealed that students did not use their science background knowledge during graph interpretation tasks and 25% of the students with LDs wrote narratives that did not correspond to the line graph (2005). In this study, graph question level (Bertin, 1983) was also used to describe students' graph interpretation responses in their final analysis (Parmar & Signer, 2005).

Students' knowledge of mathematics has also been found to affect their ability to interpret graphs correctly. Not surprisingly, students who were less capable and less experienced with mathematics generally had more difficulty with graph interpretations tasks (Curcio, 1987; Gal, 1993; Thomas, 1933). Students who were less skilled at graphing in general had more difficulty inferring trends in graphs that were less familiar (Shah & Hoeffner, 2002).

Developmental readiness

Several authors have tried to relate specific cognitive difficulties to developmental stage theories to provide a broader conceptual framework that projects a typical developmental trajectory (Friel, Curcio, & Bright, 2001; Parmar & Signer, 2005). For example, Vessey (1991) developed a theory using "fairly simple information evaluation tasks" with a goal to describe what the relationship was between graphs and tables (p. 220). When a problem representation aid like a graph supported the task, complexity of the task was greatly reduced. This was called, "cognitive fit" (p. 220). However, when students do not have cognitive fit,

> [A] mismatch occurs between [the] problem representation and task, similar processes cannot be used to both act on the problem representation and solve the problem, and problem solvers will therefore no longer be guided in their choice of problem-solving processes. They will either formulate a mental representation based on the problem representation, in which case they will need to transform it to derive a solution to the problem; or they will formulate a mental representation based on the task, in which case they will need to transform the data derived from the problem representation into the mental representation suitable for task solution. In either case, performance will be worse than if the problem solver had been supplied a representation emphasizing the type of information that best supported the task solution. (p. 221)

Taking a different development perspective, Berg and Phillips (1994) found that students who had developed Piagetian mental structures did better than students who did not. In other words, students seemed to need a number of mathematical-logical reasoning skills before they could do more advanced graph interpretation tasks (Berg & Phillips, 1994).

Learner characteristics

Beyond science and mathematics background knowledge, a number of learner characteristics have also been found to affect students' ability to interpret graphs correctly. In their study of students with LDs, Parmer and Signer (2005) found that these students could rarely answer questions more complicated than those at the elementary level, they tended to ignore graph labels and axes, they rarely checked their answers, and more generally they had difficulty understanding the purpose of the graphs. Interestingly, however, while it might be assumed that maturation significantly affects graph interpretation ability, Berg and Philips (1994) found that students in grades 7, 9, and 11 differed little in most graph interpretations tasks. The only exceptions were that the older students were better able at tasks that required them to order and scale the axes, and they were better able to interpret distance-time graphs than younger students. In addition, younger students have consistently been found to confuse abstract representations with iconic representations (Shah & Hoeffner, 2002).

Graph features like axes, their scales and variables

Graph features have also challenged students' abilities to interpret graphs correctly (Carpenter & Shah, 1998; Leinhardt, Zaslavsky, & Stein, 1990; Parmar & Signer, 2005; Preece, 1983). When students are working with scales and axes, they have to pay close attention to units of measurement (Leinhardt et al., 1990). In their research study, Parmer and Signer (2005) found that most 4th- and 5th-grade students had difficulty interpreting scale and axis labels. What made interpreting graphs difficult for students working with scale was that depending on the scale of a graph, the shape and size of the graph could change (see also Leinhardt et al., 1990). In Leinhardt's research with her colleagues, they observed how difficult it was for students to transfer graphing skills learned in mathematics class to science class.

> It is interesting that scale is an issue when using graphs for scientific data analysis…but usually is not an issue when introducing graphing in mathematics classes. It may be that, because the scale is often assumed or given in mathematics instruction (normally the scale is the same on each axis), it then becomes difficult to use or access later in science classes. (p. 17)

The variable of time was another graph feature that affected students' ability to interpret graphs correctly. In Shah's (2002) research with colleagues, students were found to be better able to work with graphs when time was one of the graph variables. Janvier also researched the variable of time and found that students who had the "familiarity of time plus its unidirectionality" were better able to work with only one variable changing (the non-time variable) (1987, p. 28).

Prediction tasks involving interpolation and extrapolation

A few studies have examined the difficulties students have when they are asked to find a coordinate between points on a graph (interpolation) or beyond the points on a graph (extrapolation). When students were asked to predict during graph interpretation tasks, they had to either estimate a value that was not present, detect patterns using visual cues given on a graph, or "conject[ure] a rule given a number of its instances (e.g., 'guess my rule')" (Davis, 1982; Leinhardt, Zaslavsky, & Stein, 1990). Bell and Janvier (1981) gave students a line graph with weight in kilograms on the y-axis and age in years on the x-axis. The first question asked students to find the average weight of boys at age nine. Since the numeral "9" was not present on the x-axis, students had to infer its location and use estimation to identify the unknown point. This skill of interpolating the correct coordinates on the line graph was difficult for students when they had to estimate to answer the question correctly (Bell & Janvier, 1981). Additionally, in a 1989 study by Stein and Leinhardt (cited in Leinhardt, Zaslavsky, & Stein, 1990), students were unable to interpolate points on a graph when they did not have a procedure to use or an equation. The review of literature did not identify any earlier empirical studies that investigated the challenges faced by intermediate elementary students when asked to extrapolate beyond the data presented in a graph.

Iconic or literal interpretation tasks

One of the problems that students have had when they focus too "broadly on the overall shape of the graph or parts of the graph" (Leinhardt et al., 1990, p. 37) has been mistakenly interpreting the graph as a "literal picture of that situation" (p. 39). Kerslake (1981) researched how students understood travels paths. Students looked at three distance-time graphs that represented journeys and were asked to describe the motion of the object being represented in each graph. The key finding was that students were unable to identify graphs that were physically impossible because they represented instantaneous motion (moving from one place to another with no time passage) or moving backwards in time. Instead, these students interpreted the graphs as literal paths that the object traveled. For example, if there was a vertical line, the student would claim the object was traveling upwards; if the line was turned towards the x-axis, the student would interpret this as the object returning to its origin. Using Bertin's (1983) language, these students were failing to connect the internal processing (i.e., the shape of the graph) with the external processing (i.e., the labels on the axes). Additional examples of iconic or literal interpretation task errors by students included Janvier's studies (1981; Janvier, 1987) with speed-distance graphs where students confused the graph for the shape of the track. The last example was Clement's (1989) research which examined students' interpretations of the meaning of a velocity-time graph with two crossing lines, each representing a car. Students inferred the point of intersection as the path on the racetrack as the spot where the two cars collided (1989).

The focus of the psychological literature upheld how individuals' perceptions, cognitive and memory processing, and mathematical abilities affected their performance on graphing tasks (perception attentions) (Simkin & Hastie, 1987; Vessey, 1991; Wainer, 1984, 1992). Researchers frequently noted that students seemed to lack the mathematical skills and knowledge to interpret even simple graphs, let alone interpret more complex graphs (Berg & Smith, 1994; Friel, Curcio, & Bright, 2001; Parmar & Signer, 2005). Specific factors like students' background knowledge, developmental readiness, learner characteristics, graph features (like axes, their scales and variables), prediction tasks (like interpolation and extrapolation), and iconic or literal interpretation tasks were found in a review of the psychological literature to contribute to student difficulties during graph interpretation tasks.

Heuristics

One finding that emerged from this study was the prominent use of mathematical heuristics while the participants were interpreting graphs. However, the review of the literature did not reveal any prior studies that discussed the role of heuristics during graph interpretation, though it has been extensively studied and discussed in the mathematics problem solving literature (Schoenfeld, 2006). The idea of heuristics is most closely associated with the work of George Pólya and his influential book, *How to solve it: A new aspect of mathematical method* (1945). In this book he described a general four-process model of problem solving, very similar to Dewey's earlier analysis of reflective problem solving (Dewey, 1933). In addition, Pólya introduced a "short dictionary of heuristic" (1945, p. 37) which encompassed most of the book. These included now familiar recommendations like "draw a figure," (p. 99) "check the result,"

(p. 59) and “try to solve a variation of the problem” (p. 209). Intended for undergraduate mathematics students and professors, the book and its heuristics have gone on to have a tremendous impact on many fields of research and scholarship (Sharps, Hess, Price-Sharps, & Teh, 2008), and its emphasis on problem solving influenced efforts to reform mathematics education at all levels (Schoenfeld, 2006).

Indeed, the idea of problem solving has been the guiding idea of mathematics education reform for the last three decades in the United States (Schoenfeld, 2008). From the 1970s through the early 1990s, problem solving was the main focus of mathematics education research. Successive generations of research have documented the characteristics of more and less challenging problems, the strategies used by more and less successful problem solvers, and the effects of metacognition, beliefs, and affect on problem solving ability. These studies have documented the effectiveness of heuristics and metacognitive training on problem solving ability (Schoenfeld, 1985, 2008; Sharps, Hess, Price-Sharps, & Teh, 2008). Between this research and the emphasis on problem solving in the NCTM *Standards* (2000), it is not surprising that heuristics are now taught extensively in K-12 education and included in most mathematics textbooks (Schoenfeld, 2008).

Current research emphasized the value of teaching these “general cognitive strategies” because they can guide thinking and “reduce cognitive strain” (Sharps, Hess, Price-Sharps, & Teh, 2008, p. 73). For example, Sharps and colleagues studied the role of heuristics to understand the significant discrepancies in the passing rates on the California Basic Educational Skills Test between “European Americans” and other ethnic and cultural groups. They found that “heuristic competency in mathematics was associated with better scores in science and mathematics” (p. 71). The problem, they argued, was that:

> The analytic and recognition skills involved in heuristic processing are not explicitly taught in the classroom, as are the steps to algorithmic solutions in the same domains; they are arrived at through experience with contexts and problem spaces similar to the ones of interest. (p. 74)

Given the importance and prevalence of research on heuristics, it was interesting that no prior studies have examined the role of heuristics during graph interpretation, probably because no one has associated graph interpretation with problem solving or because earlier research methods did not allow fine-grained attention to how the graph was being interpreted.

Sociocultural Perspective on Graph Interpretation Tasks

The differences between the psychological and sociocultural perspectives were revealed by the ways the research was reported. The sociocultural researchers on graph interpretation viewed learning to interpret graphs in science education differently than the psychological researchers. Whereas psychological researchers attempted to examine and analyze the component skills, knowledge, and dispositions needed to interpret graphs correctly, sociocultural researchers were instead interested to understand the relationships among the component aspects of graph interpretation. For example, Roth and Bowen's two-stage semiotic model of graph interpretation (2001) highlighted various aspects of the graph interpretation process (referent, interpretant, perception attention, etc.), but the goal was not to situate problems of graph interpretation in any one aspect of the model. Instead, their goal was to highlight the interconnected nature of these aspects in order to describe and reinterpret what happened when people tried to interpret a graph (Roth & McGinn, 1997). Whereas the psychological studies sought to enumerate the individual student deficits, the sociocultural studies, instead, demonstrated graph learning as a form of enculturation (Roth, Pozzer-Ardenghi, & Han, 2005; Wu & Krajcik, 2006).

The sociocultural literature borrowed ideas from social studies of science and mathematics[1] and focused on graphing as a social and communicative practice (Roth, Pozzer-Ardenghi, & Han, 2005). Graphs have been seen by many as central artifacts to mathematics and science communities that provide cultural scaffolding for new members (Brown, Collins, & Duguid, 1989; Roth & McGinn, 1998). When novice members of these communities need to learn how to think and act like seasoned members, cultural tools such as graphs provide a physical mechanism to think with (Brown, Collins, & Duguid, 1989; Roth & Bowen, 1994). Learning to use them carries the "wisdom and hidden assumptions that went into their design" (Salomon & Perkins, 1998, p. 5). Artifacts such as books and videotapes within a classroom "tacitly embody shared cultural understandings" (p.5). Specific tools and symbol systems, such as line graphs, represent a "language of thinking" (p.5).

For an example that is easier to understand than a line graph, Roth and McGinn (1998) recounted an episode in a multi-grade 6-7 classroom. A teacher captured his students' attention by setting up a pulley system in the classroom and then challenged his 20 students to a friendly

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[1] Social studies of science and mathematics use sociological and anthropological research methods to study the practices of scientists, mathematicians, and engineers. While a fairly broad field, these researchers are interested to understand how knowledge is created and validated. For example, the anthropologist of science, Bruno Latour, studied microbiologists and the Salk Institute to describe how they collected and analyzed data, argued about its meaning, and published scientific articles (Latour & Woolgar, 1979). In the United States, the Society for Social Studies of Science is the main research organization for such researchers.

game of tug-a-war. After easily beating them, the teacher used a diagram of a pulley drawn on the chalkboard to demonstrate and discuss the physics principle he wanted to emphasize.

> The students had come to the instructional situation with their pre-unit physics discourses related to pulleys and forces; the teacher, a trained physicist, commanded the linguistic repertoires of canonical physics. In their interactions over and about the drawings, teacher and students negotiated new forms of talk about pulleys that were more appropriate from a canonical physics perspective. (p. 37)

This diagram became a focal object in the conversation shared between the teacher and his 20 students.

Another study involved middle school students and curriculum materials designed to incorporate technology that engaged students in standards-based science inquiry (Singer, Marx, Krajcik, & Chambers, 2000). Throughout the 4-year study, six extended inquiry projects were piloted focusing specifically on physical science, chemistry, geology, and biology strands. The learning technologies enabled the students to analyze and represent their data findings more easily than paper and pencil would have allowed. What the researchers found at the end of this study was that students had a difficult time with discourse in the science classroom, even when supplemental support was given.

Conversely, in an 8-month research study focusing on students' use of data tables and graphs involving two 7th-grade classrooms where the students were very inexperienced with the scientific inquiry process, two classroom teachers had to scaffold their 27 students (i.e., designing experiments and using inscriptions) (Wu & Krajcik, 2006). Students in this naturalistic approach (Lincoln & Guba, 1985) were guided by the "driving question approach" (Marx et al., 1997) that began the unit within a contextualized setting and working collaboratively with peers for a "long-term investigation of the driving question and its related sub-questions" (Wu & Krajcik, 2006, pp. 66-67). These students, with the support of their teachers, were interested in the specific driving question of the health of the stream behind their school. This question was contextualized to their situation and the effects were meaningful to them. Most important in this research study, the supplemental teacher support had a strong positive effect on student learning at the end of the study. One outcome was that increasing student involvement with inscriptional practices and inquiry made them become more competent in interpreting and reasoning about inscriptions.

> As students became more competent in interpreting and reasoning about inscriptions, they expressed more opinions or comments on the design of interpretations or on the conclusions drawn from inscriptions. They also developed more coherent arguments in their writing about data and inscriptions. (p. 90)

This study highlighted the possibility that intensive interventions that focused on improving students' understanding of inquiry and data analysis methods can yield significant improvements. The classroom activity focused on classroom practices that emphasized scientific inscriptions such as data tables and graphs, rather than mental structures and thinking skills. The result of this study was that participants moved towards more sophisticated activities that involved creating, interpreting, and critiquing inscriptions.

In a somewhat similar study, McClain and Cobb (2001) worked with a 7th-grade class over the course of a full school year. They focused on helping the students to interpret statistical data and graphical representations by developing classroom social norms, socio-mathematical norms, and classroom mathematical practices that enabled students to engage in relatively sophisticated analyses. They found that students eventually developed their collective ability to analyze the statistical data to develop and support data-based arguments.

In a study that contrasted the interpretive practices of undergraduates and practicing scientists, Bowen, Roth, and McGinn (1999) presented both groups with the same graphs in a think aloud study (Ericsson & Simon, 1993). The scientists' interpretations were scaffolded by the concerns and perception attentions that characterized their disciplines. They were also helped by having extensive field- and lab-based experiences (i.e., referents) and the interpretive theories (i.e., interpretants) of their disciplines. Students, by contrast, did not have robust vocabularies, the experiential base, or knowledge of specific organisms to help them when interpreting the graphs. In addition, because students were mainly concerned with earning a good grade, they did not develop or deploy more general graph interpretation skills and instead used their professors' interpretations of the graphs (Bowen, Roth, & McGinn, 1999). The students' behavior seemed to indicate that they were mainly consumed with the structuring process and did not have the interpretive resources to enter the grounding process (Roth & Bowen, 2001).

Critique of Earlier Studies

The conceptual framework for this study provided an analytic lens to assess the existing literature. It was helpful to separate the literature on graphing in education research into two traditions to more easily compare and contrast their merits and problems. The first and older tradition dates back to the 1960s and presented graphing as a psychological activity which can be taught as a decontextualized body of skills and knowledge, assuming transferability. This perspective is predominantly found in the psychological and mathematical research literature with several key pieces found in the science education literature (Friel, Curcio, & Bright, 2001;

McKenzie & Padilla, 1986). Unfortunately, most prior research on graphing in science in the psychological tradition has assessed only mathematics skills and knowledge using tests and surveys, requiring no formal science skills and background knowledge to obtain high scores (Aberg-Bengtsson & Ottosson, 2006; Jungwirth, 1990; McKenzie & Padilla, 1986; Svec, n.d.; Tairab & Khalaf Al-Naqbi, 2004).

In contrast, the second more recent sociocultural research tradition presents graphing as a socially situated activity that is inexorably contextualized and does not expect transferability (Roth & Bowen, 2001). This perspective is most often found in the science education research literature and only infrequently found in the mathematics education research literature (Barab, Hay, & Yamagata-Lynch, 2001; Greeno & Hall, 1997; Wu & Krajcik, 2006). Topics other than graphing, such as fractions or calculus, were more widely examined when studying mathematics as a socially situated activity (Cobb, 2002; McClain & Cobb, 2001). The anthropological methods used in these sociocultural studies did not enable researchers to isolate student deficits in graphing skills and knowledge. Because of the disjuncture between the psychological and sociocultural literatures, there was a chasm between the scientific theories being taught in the upper elementary school classroom and the mathematics skills students needed to analyze and understand scientific theories (Barab, Hay, & Yamagata-Lynch, 2001; Kerslake, 1981; Leinhardt, Zaslavsky, & Stein, 1990; Roth, 1996; Roth & Bowen, 1995).

Roth (2005) critiqued the psychological literature's omission of the importance of science background knowledge. His sociocultural research program highlighted the effects of social context and professional socialization on perception and meaning. Professional scientists were affected by the same psychological factors as students and novices. "[E]ven highly trained individuals may find themselves in situations where they get stuck in perceptually dissecting a graph without ever connecting it to some external referent… [the] deficit model [of cognitive psychology] does not satisfactorily explain" (p.15). In short, both the psychological and sociocultural literatures highlighted important factors affecting student graphing success.

The psychological literature focused on general factors, including skills, knowledge, and dispositions that inhibited students' ability to interpret graphs correctly (e.g., Berg & Smith, 1994; Buchler, 1978; Friel, Curcio, & Bright, 2001; Peirce, 1940; Shah & Hoeffner, 2002). The sociocultural literature, by contrast, presumed that learning to graph was a socialization process and that graphing is highly contextually dependent (i.e., not easily transferable) (e.g., Janvier, 1981; Marx, Blumenfeld, Krajcik, & Soloway, 1997; Roth & McGinn, 1998; Wu & Krajcik, 2006). While the sociocultural literature placed a strong emphasis on scientific knowledge in the graph interpretation process, thus yielding sophisticated descriptions of student learning, those studies did not produce findings that helped teachers to understand the learning needs of individual students. The psychological literature placed a strong emphasis on finding generalizable findings, but those studies were not built upon robust theories of the interpretive process that would allow the findings to be easily integrated with one another.

The psychological literature on graphing provided insights on the specific skills and knowledge people needed to construct and interpret graphs as mathematical objects (e.g., axes, points, lines—information involved in elementary level and intermediate level questions) (Bertin, 1983). However, this literature was less effective when describing how people related the mathematical objects to the science ideas they signified (overall level questions) (1983). The sociocultural literature did a good job describing how scientists and science students used graphs to collect, analyze, and interpret data during scientific inquiry (Brown, Collins, & Duguid, 1989). It failed to explain, however, the challenges students faced and the discrete skills necessary while learning to reach this sophisticated stage. Ignoring these challenges became detrimental to students who found these mathematical skills and knowledge difficult to comprehend (i.e., unable to correctly answer an overall level question) (Bertin, 1983).

Review of Research Methods Used to Study Graphing

Sociocultural Research Methods Used to Study Graphing

Most sociocultural research in science education that has looked at graphing has used data collected either through ethnographic inquiry or design experiments. By default, such data necessarily restricted the researchers to examine socially situated aspects of these phenomena. What these researchers have failed to acknowledge was that their data did not help us to understand the impact of psychological processes on students' learning (Brown, Collins, & Duguid, 1989; Janvier, 1981; Krajcik, Czerniak, & Berger, 1999).

Psychological Research Used to Study Graphing

In contrast, the psychological research in science education that has analyzed graphing has used multiple choice and open response graphing tests to assess students' graphing abilities (Berg & Smith, 1994; McKenzie & Padilla, 1986; Tairab & Khalaf Al-Naqbi, 2004). These studies, however, did not focus on science background knowledge during graph interpretation. Additionally, their tests did not enable the researchers to understand the cognitive problems occurring simultaneously as the participant answered the graph questions. Instead, qualitative data were needed to better understand student cognitive processes, and since these data needed to be collected in a controlled environment, clinical interviews were the most direct way to accomplish this task (Roth & Bowen, 2001). To meet this need, a think aloud protocol was chosen for this study to answer the research questions (Ericsson & Simon, 1993, 1998).

Think Aloud Protocol Analysis to Study Graphing

Think aloud verbal protocols provided a systematic way to overcome the limitations to testing methods used in prior psychological research and to previous ethnographic methods used in sociocultural research (Ericsson & Simon, 1993). Overcoming prior research limitations while maintaining a methodical way to document student thinking was important to understanding how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs. Think aloud protocols enabled students to share their mathematics and science reasoning while engaged with the TOGS instrument (Ericsson & Simon, 1993; McKenzie & Padilla, 1986). With this technique, students answered test questions while sharing their thought processes, leaving less room for the researcher to make dubious inferences like in previous research studies where only surveys, tests, or non-participant observations had been used (Ericsson & Simon, 1998). Actually allowing students to reveal what was occurring during the act of cognition made it easier to understand why mistakes were taking place. This was possible using think aloud student interviews (Ericsson & Simon, 1993).

Think aloud research methods may not be familiar to all educational researchers; however, these methods have been used in psychology and many related fields for decades (de Groot, 1965; Duncker, 1945; Newell, Shaw, & Simon, 1958; Newell & Simon, 1956). Some opponents (e.g., Smagorinsky, 1998) have raised concerns about the validity and reliability of these methods of data collection and analysis. However, by the late 1990s, Ericsson and Simon (1998) defended their think aloud methodology with research from their earlier studies and claimed,

> Today it is relatively uncontroversial that thinking can be represented as a sequence of thoughts…interspersed by periods of processing activity. The main methodological

> issues have been to determine how to gain information about the associated thought states without altering the structure and course of the naturally occurring thought sequences. The primary focus of our work has been to identify the circumstances where individuals could verbalize their thoughts without any, or at worst with minimal, reactive influences on their thinking. Perhaps the single most important precondition for successful direct expression of thinking is that the participants are allowed to maintain undisrupted focus on the completion of the presented tasks. (p. 180)

With some training and prompting, their research participants were able to verbalize their thought processes while engaged in cognitively demanding tasks. The result was often marked by incomplete sentences and uncorrected errors, indicating to the researcher that the participants were not engaged in monitoring their cognitive processes. Ericsson and Simon inferred from this that the think aloud methods were not interrupting participants' natural thought processes and could be used as a method to collect data on student cognition in the moment the thoughts were occurring. Think aloud protocols would allow access to student thinking in the moment the student was interpreting a line graph question in order to record verbal responses for later data analysis transcriptions and coding.

In this chapter, the conceptual framework was developed and explained to link psychological and sociocultural approaches to studying graph interpretation. A representative sample of studies were summarized and evaluated to show what was known about students' graph interpretation abilities. The limitations of the current literature were also presented. Finally, the research methods that have been used to study graphing were reviewed and analyzed, and think aloud research methods were presented as a means of overcoming these limitations.

CHAPTER THREE: METHODOLOGY

Introduction

The purpose of this study was to understand how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs. In addition, the purpose was to understand how students' interpretations were affected according to graph question level. The research methods for this study were primarily qualitative, though quantitative data were used to help with sample selection, categorization of students into subgroups, and analysis of data across the sample (Creswell, 2003). The qualitative data were collected using think aloud interviews (Ericsson & Simon, 1993), video, and observational notes. Two quantitative measures were used. One was the 2006 5th Grade Florida Comprehensive Assessment Tests (FCAT) of Mathematics and Science used to create a purposive sample using maximum variation sampling (Patton, 2002) to capture the variability of 6th-grade students at one Central Florida elementary school. For the purpose of this study, students' achievement on the FCAT was assumed to be an appropriate predictor of student background knowledge and ability in mathematics and science (Florida Department of Education, n.d.). The other quantitative measure was the excerpted Test of Graphing in Science (TOGS) instrument developed by McKenzie and Padilla (1986). The TOGS measured students' ability to interpret line graphs (Appendix C). A scoring rubric (Appendix A) to assess student performance on the TOGS was created by the researcher and validated by Padilla (Appendix F) and a veteran mathematics and science teacher. For the rubric, previously cited factors affecting students' graphing ability (Appendix E) were synthesized and categorized using Bertin's three graph question levels (1983). Data were also analyzed using Roth and Bowen's two-stage semiotic process of reading graphs (2001). The setting, participants, design, instruments, data collection, analysis, and assurance of the trustworthiness of the study were reported in this chapter.

Rationale for Research Design

The purpose and methods of this study presumed a realist epistemological position (Patton, 2002) assuming, for example, that students' background knowledge and experiences were real and their knowledge and experiences affected their behaviors while taking the TOGS (McKenzie & Padilla, 1986). This post-positivist stance had a variety of implications for the methods of data collection, analysis and verification. This epistemology presumed there was a reality to be explained. In turn, these assumptions led the researcher to use the think aloud data collection method because it could better capture students' cognition in a reliable manner and better enable valid inferences to be made compared to data collected using ethnographic methods or tests alone without the think aloud method (Ericsson & Simon, 1993).

However, these realist assumptions did not lead the researcher to believe that access to the truth would be available, as logical positivists would assume (Patton, 2002). Instead, the researcher proceeded "recogniz[ing] that discretionary judgment is unavoidable in science, that proving causality with certainty in explaining social phenomena is problematic…and that all methods are imperfect" (p. 92, citing Campbell and Russo, 1999). Throughout this study the researcher had to make choices that diverged somewhat from orthodox think aloud methods, and these choices were made in an effort to better capture what was emerging from the data during the interpretive process. Nevertheless, the methods used in this research study provided insights into the nature of student graph interpretation that likely could not have been captured by following an orthodox methodology. Additional methods of data and analytic verification were used to ensure the trustworthiness of the findings. These included triangulation, negative case analysis, external auditing, and peer debriefing.

Population and Sample

The school from which the sample was drawn was built in the early 1960s during a period of rapid population growth supporting the development of the US Space Program. It is located in an area of relatively affluent and well-educated residents. The enrollment for the school was approximately 900 students ranging from Kindergarten to 6th-grade (School Advisory Council, 2006). The demographics for this school were 85% White, 5% Hispanic, 5% multiethnic, 3% Asian-Pacific Islander, and 2% Black. Of these students, 18% qualified for Free and Reduced Lunch, 17% of students qualified for Exceptional Education services (not Gifted), 5% qualified for Gifted Education services, and less than 1% needed English Speakers of Other Languages (ESOL) services. The teachers at this school have an average tenure of 19 years.

This school has been recognized as an “A” school by the Florida Department of Education for the last ten years. While the factors and weighting of those factors have changed from year to year, the grading of Florida public schools is based mainly on FCAT student achievement data (FDOE, 2008). The population of this school is relatively stable, providing easy access to students’ FCAT data.

The selection of participants were based on previous student FCAT scores in mathematics and science. For the purposes of this study, the FCAT was an adequate means of differentiating able from less able students. It was important to use a trustworthy measure to answer research questions 1 and 3. Without using previous FCAT data, it would have been difficult to infer students' mathematics and science achievement levels with any other measure available in elementary school.

The parents and guardians of approximately 125 6th-grade students at one Central Florida elementary school were asked to consent to their child's participation in this study. Students were also asked to assent. Consent and assent were obtained from 37 and 33 parents and students respectively. Attempts were made to ensure the sample was representative of the school population with regard to gender and ethnicity. Roughly the same number of males and females returned the consent and assent forms. One Asian American student had consent and assent to participate but did not fit the sampling strategy noted in the next paragraph. One Hispanic student fit the sampling strategy and had both consent and assent but moved away before the interview took place.

The sampling strategy was intended to select students who demonstrated a variety of behaviors during graphing tasks. By selecting a purposive sample of 14 sixth graders of varied levels of mathematics and science achievement (2006 5th Grade FCAT Mathematics and Science levels respectively), an attempt was made to examine claims made in the existing literature (i.e., that mathematics performance on a standardized test would predict graphing ability) and to study the influence of science knowledge on graphing ability which has been scarce in the literature as described in Chapter 2. The qualitative data of one participant was not used in the analysis because it was collected during the pilot study before changes were made, reducing the final sample to 13 participants.

After gaining consent and assent to participate from parents and selected students and reviewing standardized test scores in cumulative folders, a purposive sample of 14 6th-grade students using maximum variation sampling (Patton, 2002) was selected following the criteria in Table 1.

Table 1

Table 1 Sampling Matrix: Pseudonyms for Participants Selected by 2006 5th Grade FCAT[a] *Mathematics and Science Levels*

Science \ Mathematics	Low	Medium	High
Low	**Lydia Lynn**		**Hester Luck** **Hodge Leader**
Medium	**Linda Mills** **Lucy Mag**	**Matt Maples** **Misty Murphy** **Megan Mason**	**Heather Miller**
High			**Hans Hazel** **Hugh Hickson** **Henrietta Harmon** **Huck Handy** **Hyde Hegel**

Note. The first letter of the first name denotes the math level (i.e., L=low level=1 or 2, M=medium level=3, H=high level=4 or 5), and the first letter of the last name denotes the science level (i.e., L=low level=1 or 2, M=medium level=3, H=high level=4 or 5). [a]Florida Comprehensive Assessment Test.

The pool of available students had a disproportionate number of high achievement levels. The following choices were made to ensure the research questions could be answered. First, out of three identified low math-low science students who had given consent and assent, one moved away in the middle of the school year and one withdrew from the study prior to the interview.

Since no other low math-low science students were available, only one low math-low science student was able to be interviewed. Second, there were no students in the pool who fit under the category low math, high science. Therefore, two students were chosen under the category low math, medium science. Third, one of the high math-high science students provided seemingly incomplete verbal protocols despite repeated prompting. As a result, an additional student was easily identified and sampled. Fourth, due to a disproportionate number of male students in the high math-high science category and no more high math-high science females available, a high math-medium science female student was sampled.

To maintain students' confidentiality, once the interviews were transcribed, each participant was assigned a pseudonym (Tables 1 and 4). To help the reader remember each participant's prior achievement, the first name of their pseudonym started with an H, an M or an L. These denoted high, medium, or low achievement on the 2006 5th Grade FCAT Mathematics Test. With a first name starting with the letter H, the student had scored either a Level 4 or a 5 on the mathematics test. With an M, they had scored a Level 3. With an L they had scored a Level 1 or 2 (Tables 1 and 4). Similarly, the last name of the participants' pseudonym denoted their achievement on the 2006 5th Grade FCAT Science Test. Participants whose last pseudonym started with an H had scored a Level 4 or 5 on the FCAT science. With an M, they had scored a Level 3. With an L, they had scored a Level 1 or 2 (Table 1). Please note that with this system of pseudonyms and levels, LM does not represent "low mathematics" but, instead, represents "low" mathematics "medium" science.

Instruments

Florida Comprehensive Assessment Test

Currently, public school students in Florida are required to take the FCAT, administered to students in Grades 3-11. This achievement test contains "two basic components: criterion-referenced tests (CRT), measuring selected benchmarks in Mathematics, Reading, Science, and Writing from the Sunshine State Standards (SSS); and norm-referenced tests (NRT) in Reading and Mathematics, measuring individual student performance against national norms" (Florida Department of Education, 2006, p. 1). Mathematics FCAT student scores are reported by achievement level, scale score, and developmental scale score. The scale scores, which range from 100 to 500, are divided into five categories, from 1 (lowest) to 5 (highest), called achievement levels. Science FCAT student scores are reported only by achievement level and scale scores (Florida Department of Education, n.d.). Student FCAT Mathematics and Science scores were used to select the sample. The 2006 FCAT Mathematics and Science Test was able to accurately and consistently categorize students by grade level and subject area into one of five levels; accuracy = 0.961; consistency = 0.932 (HumRRO, 2007). The 2006 5th Grade FCAT in Mathematics received a classical reliability score of 0.87 (Cronbach's Alpha) for the SSS test and a 0.91 (KR-20) for the NRT. No reliability measure was available for the 2006 FCAT Science since no state-wide NRT in science was given across the state of Florida (FDOE, 2007).

Test of Graphing in Science

Questions were excerpted from the TOGS (Appendix C) (McKenzie & Padilla, 1986) to understand students' elementary level and intermediate level (Bertin, 1983) line graph interpretation abilities in the think aloud protocol. McKenzie and Padilla developed the multiple choice test of graphing skills to assess “specific graphing abilities” (p. 572) and to establish “a base line of information on this skill” (p. 572). They established content validity by submitting their 26-item test to a panel of reviewers who had 94% agreement between test items and objectives. However, as mentioned above, the TOGS only assessed Bertin’s elementary level and intermediate level graph interpretation skills and did not assess Bertin’s overall level graph interpretation skills. Therefore, drawing on this previously validated instrument, for the purposes of the study, an excerpted version of 13 questions with additional prompts was used in order to select developmentally appropriate questions for sixth graders. To ensure completeness of the verbal protocols, prompting questions were added to two questions (questions 1, 8) to better understand participants’ abilities to discriminate among multiple choice questions.

Certain line graphs on the TOGS had multiple questions associated with them. After the last question for each graph (questions 5 and 13), an overall level question (Bertin, 1983) was asked by the researcher to assess comprehension of the situation described in the question. Additionally, these questions provided supplementary data on participant interpretations of graphing in science (Research Question 1). Moreover, questions prompting the participant to remember personal background experience with each situation were also asked after the last question for each graph (questions 5, 8, and 13). When prompting by the researcher occurred prior to participant responses, the code P was inserted on the scoring rubric (Appendix A) to differentiate an unprompted response from a prompted one.

Prof. Padilla, former President of the National Science Teacher Association (NSTA) and co-creator of the TOGS, granted the researcher permission (Appendix B) to use and adapt his TOGS (1986). The adaptation of this test and the creation of the accompanying scoring rubric using Bertin’s (1983) three graph question levels took almost a year of planning. (A list of research considered when creating the scoring rubric but not necessarily used can be found in Appendix E.)

Graph Interpretation Scoring Rubric

In order to have a structured observation tool to record participant behaviors during graph interpretation, a comprehensive assessment rubric was created. The development of this instrument was based on the synthesis of the literature on the challenges previously identified in the research (Appendix E). Factors were included in the assessment rubric if they focused on graph interpretation (versus graph construction) and skills appropriate for sixth graders. Factors related to intelligence and academic achievement were excluded because they were part of the sampling method. The included factors were synthesized and then categorized under the headings taken from Bertin's (1983) three graph question levels in a scoring rubric to be used during student interviews (Appendix A). Items on the rubric were scored as either correctly answered, incorrectly answered, or omitted. Each question on the TOGS was evaluated to determine the primary focus of the question and highlighted in yellow. Secondary possible indicators for each TOGS question were highlighted in blue.

This assessment rubric was validated in two ways. First, the TOGS (1986) designer, Prof. Padilla, reviewed the draft rubric and approved the correlation of Bertin's (1983) graph question levels to the questions on the TOGS using email and phone correspondences (Appendix F). Second, during the pilot study, a draft of the assessment rubric was used by two raters independently scoring student responses to the TOGS. After the first pilot study, a minor problem, discussed in the next section, was identified and addressed; this problem was found to be eliminated after the second pilot study.

Data Collection Procedures

Pilot Study

Two students participated in a pilot study on February 5, 2007, and March 13, 2007, respectively, to ensure validity within the TOGS scoring rubric (Appendix A). In addition to the researcher, another teacher listened to the digital recordings of the pilot interviews and scored the student responses using the rubric. After the first pilot, a problem with the scoring rubric was identified and resolved—secondary indicators needed to be included by highlighting them in blue. In addition, problems were found with implementation procedures like hand gestures that needed to be video recorded for additional information, supplementary questions that needed to be added to prompt student thinking to further elicit science background knowledge, and students who needed time to read the questions silently prior to reading them aloud. Changes were implemented for the second pilot study and the follow-up scoring of both teachers aligned the second time.

The additional questions added to the TOGS allowed the second pilot student an opportunity to think aloud and share more science background knowledge than the first pilot study student. On account of this, the supplemental questions were included in the rest of the study. Since changes were made after the first pilot study, only the quantitative data from the first pilot were included in the final data analysis section, whereas the qualitative data were not used. No changes were made after the second pilot study; therefore, all of the data from the second pilot study were included in the data analysis.

Participant Recruitment and Preparation

Two weeks prior to data collection, letters were sent home to parents/guardians for consent and to participants for assent. These letters described the intent and purpose of the study and explained the rights of both the parents and students (e.g., participation was voluntary and withdrawal was allowed at any time during the study). Permission was sought to access student cumulative folders for sampling purposes and for participation in the clinical interviews if chosen for the sample. An explanation by way of classroom visits was given to students in order to get their assent to participate.

Once the sample of students was selected and they affirmed their willingness to participate in the study, mutually convenient times were arranged to meet with them individually after school in a classroom. When the students arrived, their willingness to participate was confirmed, and they were reminded they could withdraw consent at any time. No one withdrew consent. A standard script (see Appendix G) was used to explain the study purpose and procedure for the think aloud protocol, emphasizing the importance of talking during the entire interview without needing to explain their thinking to the interviewer. As Ericsson and Simon (1993) asserted,

> A second level of verbalization involves description, or rather explication of the thought content. We assign to this level verbalizations that do not bring new information into the focus of the subject's attention, but only explicate or label information that is held in a compressed internal format or in an encoding that is not isomorphic with language. (p. 79)

Students needed to describe what they were thinking without worrying about the conventions of communicating their thinking to observers. For this reason, the participant desk was in the front of the room facing the wall so that no personal communication would be anticipated or expected.

Before the interview began, Erickson and Simon's (1993) think aloud methods were demonstrated using a recorded example of the researcher on an unrelated topic (the difference between thinking aloud while shopping at grocery store and explaining what occurred after shopping at the grocery store) so that the students understood the process. The importance of verbalizing their thinking was emphasized. The participants were asked to do two practice questions; one was a computation problem involving addition with carrying and the second, a question asking them to count the number of windows in their home. The researcher provided immediate feedback (i.e., "Nice job thinking aloud," when the participant verbalized his thinking, and "Keep talking," when the participant remained silent for more than three seconds.) and prompted the participants to encourage them to verbalize their thinking without trying to explain their thinking to the interviewer.

TOGS Completion

After the practice session, participants were given an excerpted portion of the TOGS and asked to begin (see Appendix C). Each participant was asked to answer the same 13 questions excerpted from the TOGS during the clinical interview. These think aloud sessions were audio and video recorded to document participants' verbalizations and hand gestures for transcription and coding at a later date. The video recorder focused only on the desk, TOGS page and participant's hand movements; their faces were never recorded.

While the students were thinking aloud, a coding sheet (see Appendix A) of factors identified in previously published research was used to track participant actions while taking the TOGS. The researcher focused on highlighted areas on the coding sheet. Additionally, any behavior not accounted for on the list of factors was noted at the bottom of the coding sheet for further examination during data analysis. While listening to the participant's think aloud, participant questions asked for clarification of test procedures were answered immediately (e.g., "Do I write on the test?"). Any participant questions asked of the researcher that could have invalidated the test were not answered (e.g., "Did I get that question right?").

Participants were prompted regardless of the accuracy of their answer. Before the participants left, they were asked if they had any questions about the study and were thanked for their time and participation.

Role of the Researcher

The researcher was a 5th-grade mathematics and science teacher at the school where the research study was taking place. The participants for this study were 6th-grade students of whom several had been science students of the researcher the previous year (she only taught science and social studies that year). This study was not an Action Research project because the researcher was not directly studying the effects of her teaching on the participants (Patton, 2002).

Data Analysis

The data analysis explored the individual behaviors of students with different competencies as identified by their 2006 5th Grade FCAT Mathematics and Science levels. This purposive sampling strategy using maximum variation sampling (Patton, 2002) selected students with different competency levels who demonstrated a variety of behaviors during graphing tasks to capture the variability of 6th-grade students at one Central Florida elementary school. Having selected students with a range of mathematics and science ability levels, the data analysis discussed in this section showed how the researcher represented what the existing literature would predict (i.e., that mathematics performance on a standardized test would predict graphing ability). The data analysis methods described in this section also showed how the researcher analyzed the data to understand the influence of science knowledge on participants' graphing ability.

Transcription and Preliminary Coding

The audio-taped TOGS sessions were transcribed. The verbatim transcriptions included all student utterances except when unintelligible; in these cases, notes were recorded in parentheses within the transcriptions. Standard US spelling was used throughout except when students used relatively common colloquialisms (e.g., 'cuz). The natural flow of speech was transcribed as spoken, including pauses, non sequiturs, and run-on sentences. Pauses were transcribed with ellipses (i.e., "…"), but no effort was made to record the length of the pause. Only when students spoke in complete sentences were they transcribed and punctuated as such. To avoid redundancy, when students read the question or answer choices, this was noted in a parenthetic insertion (e.g., Read question aloud).

In addition, other pertinent information was inserted in parentheses in the transcription. These parenthetic insertions included: clarifying text was inserted to elucidate what the student was referring to when the object of an utterance was unclear, hand gestures or drawing that occurred simultaneously with the think aloud, and the starting and stopping time on the digital recorder to make it easier to return to specific locations on the audio recording. However, when students stated or read graph coordinates, these were transcribed using proper coordinate notion, i.e., (x, y).

Transcriptions were completed in a standard two-column template. The header of each page included the pseudonym of the participant, interview date, and mathematics and science ability levels (see Table 1). The columns were divided into 13 rows, one for each TOGS question with the questions pre-typed. Transcriptions were recorded in the left column; the right column was used to record codes from the Graph Interpretation Scoring Rubric, emergent codes, and researcher notes.

During the transcription process, codes from the Graph Interpretation Scoring Rubric collected during the observation of the think aloud protocol were inserted in the right-hand column. These codes included the primary focus and secondary possible indicator for each TOGS question (see Appendix A). In subsequent analyses, these codes were used to determine if participants correctly answered the TOGS questions and which graphing skills they overtly demonstrated while answering the questions. Emergent codes like *drawing on the graph* and *answer elimination* were also recorded in the right-hand column during transcription and salient excerpts of the transcriptions were highlighted.

Recategorization of Data

Once the transcriptions were completed and the data were coded using the Scoring Rubric, the researcher attempted to analyze the data using descriptive and inferential codes. While the descriptive coding proceeded with relative ease, the inferential coding was stymied. Research Questions 1 and 2 required that the data be analyzed to illuminate differences in the ways that participants approached the graph interpretation tasks according to their prior academic achievement levels and by the graph question levels. However, the ways that the data were initially organized made it very difficult to separate the effects of ability from question level. First, some participants who were initially grouped together by prior achievement level on the FCAT did not behave in similar ways while completing the TOGS. Second, with the participants sorted into the cells of the sampling matrix, the analysis was convoluted for having too many small groups of participants. Third, the data for some of the elementary level questions were quite different than the other elementary level questions. For these reasons, a new graph question level was created and the participants were recategorized into a smaller number of groups.

Recategorized Question Levels

In Chapter 2, elementary, intermediate, and overall level questions were defined under the Stages of the Reading Process: Level of the Question or Reading Level. A reanalysis of the questions showed that questions 1, 8, and 9 were about the graphs themselves. Specifically, question 1 assessed students' abilities to choose the best range and intervals for a set of data and questions 8 and 9 asked students to determine the independent and dependent variables. By contrast, questions 3, 6, 7, and 10 focused on truly elementary level questions, like the ability to identify a point on the graph or the coordinates of a point (Bertin, 1983).

A more detailed analysis of the transcripts and a reexamination of Bertin's ideas (1983) suggested why some of the questions were different. Bertin's three question levels required the students to interpret, one way or another, the meaning of data on a graph. Questions 1, 8, and 9, by contrast, focused students' attention on the conventions of line graphs in science and their ability to recognize graphs that were inconsistent with those conventions. Reviewing Bertin's ideas, it became clear that these questions were assessing what he called the external identification stage in the graph reading process. Such questions were not designed to assess students' ability to read and interpret a point or a set of points on a graph, as the graph question levels would. Instead, they were intended to assess the skills of graph construction and to recognize a faultily constructed graph. From this analysis, the researcher decided to recategorize TOGS questions 1, 8, and 9 as external identification stage questions. Table 2 presented the recategorized TOGS questions.

Table 2

Table 2 Recategorized TOGS Questions with Question Ease and Primary Focus Objectives

Ques. Level	Ques. Number	Ques. Ease (%)	Primary Focus[a]
External Identification Stage	1	43	1.6 Select an appropriately scaled set of axes for a set of data (correct range and interval)
	8	57	1.5 Associate the x-axis with the independent (causal) variable and the y-axis with the dependent (effected) variable
	9	79	1.5
Elementary	3	86	1.3 Select a corresponding value of X (or Y) for a value of Y (or X) on a given graph
	10	93	1.3
	6	71	1.1. Read “x-y coordinates” of point on a graph
	7	71	1.2 Find point of given “x-y coordinates”
Intermediate	2	64	2.3 Infer an omitted point between data points (interpolation)
	12	86	2.3
	4a	79	2.5 Describe a relationship between the x and y variables
	11a	86	2.5
	5	93	2.4 Infer a point beyond the plotted data points (extrapolation)
	13	93	2.4

Note. Students’ behavior on overall level questions could not be analyzed in this table and section. These data were analyzed in Research Question 3. [a]Primary Focus is the main objective being assessed for each question on the TOGS. See Appendix A.

Recategorized Participant Groups

While analyzing the data it became clear that the original matrix used to sample students (Tables 1 and 4) was not effective for the purpose of answering the research questions. At first, the organization of the sample was focused on students' FCAT data but these data did not neatly align with students' behavior. Some students in the same cell of the matrix behaved differently than other students within the same cell. Likewise, students in different cells behaved similarly to one another. For the sake of disaggregating prior student achievement and graph question levels, the sample was reorganized to group students whose behaviors were most similar (see Table 3). It is important to note that within each FCAT achievement level there is significant variability (HumRRO, 2007), and FCAT achievement alone could not predict how students behaved on the TOGS.

Initially, the data were analyzed to discern patterns in the question levels that students answered incorrectly. This analysis suggested that although Henrietta Harmon was a high-high student, she was the only high-high student to answer elementary level questions incorrectly—three in total; she also answered two of the external identification stage questions incorrectly. This pattern suggested that her behavior was similar to the two high-low students, Hester Luck and Hodge Leader. Closer analysis of the transcripts further suggested that these three students also shared a certain reticence to pose their own questions while answering the test questions. This group was referred to as the Compliant and Reserved group.

Similarly, while Heather Miller was a high-medium student, her test score and her behavior suggested that she should be placed among the remaining high-high students: Hans Hazel, Hyde Hegel, Hugh Hickson, and Huck Handy. Heather, along with Hugh and Huck, answered question 1 incorrectly, whereas Hans and Hyde answered all questions correctly. In addition, analysis of the transcripts revealed that while Hugh and Huck each answered an additional question incorrectly, those were simply careless mistakes. More importantly, this group of students, except Huck Handy, displayed inquisitiveness. Hans, Hugh, Huck, and Heather, unlike any other students in the sample, asked their own questions while answering the TOGS questions. These self-generated questions led them to a deeper understanding of the original TOGS item, often leading them to anticipate the answers to later questions before they were asked. For example, all four of these students found the answer to question 4b while answering question 2. By contrast, Huck was considerably less verbal than the other members of his group and he took the shortest amount of time to answer the test items. Even though he answered all questions correctly, his verbal protocols were terse and brief; however, a review of the transcripts and video revealed that he made extensive use of hand gestures and drawings on the graph. Their strategies were different, but all five of these students displayed a marked degree of Inquisitiveness and Independence.

Matt Maples' score could have placed him among the higher groups, but he lacked the inquisitiveness or independence of the Independent and Inquisitive group. However, he also demonstrated a great deal more science knowledge than the students in the Compliant and Reserved group. Indeed, he demonstrated at least as much science knowledge as any other student in the study, despite only scoring a Level 3 on the 2006 5th Grade Science FCAT (refer to Table 1). The combination of extensive science knowledge during the think aloud interview but his lack of inquisitiveness suggested that he should be analyzed separately from the Independent and Inquisitive and Compliant and Reserved groups.

Examining the remaining four students to see if any others exhibited similar behaviors to Matt yielded the unlikely choice of Misty Murphy. While Misty's cumulative score of 69% on the TOGS excerpt suggested that she may have been grouped with the lowest students, throughout the think aloud interview she demonstrated robust informal science knowledge, similar to Matt's. This similarity suggested that Misty and Matt might suitably be analyzed together and referred to as the Few Strategies but Decent Science group. The remaining three students were not similar to Misty or Matt in these respects.

The final three students answered questions from the external identification stage incorrectly and questions from each of the three levels incorrectly. While Linda Mills and Lucy Mag answered the same number of questions correctly as did Misty Murphy, they did not demonstrate any substantive science knowledge in their think aloud interviews. For this reason they were analyzed in a different group than Misty. Finally, Lydia Lynn's behavior was very similar to Linda and Lucy's behavior. Although Lydia struggled more with the test than any other student and answered more questions incorrectly, there was no reason to differentiate her from Linda and Lucy. For these reasons, Linda, Lucy, and Lydia were analyzed together and referred to as the Earnest but Confused group.

Research Question 1 asked, How do student behaviors observed during think aloud interviews (Ericsson & Simon, 1993) vary during line graph interpretation in science across mathematics and science achievement levels? For the sake of sampling the population, the students' FCAT Mathematics and Science scores generated appropriate diversity within the sample. However, preliminary analysis of the data suggested that FCAT scores alone were not a sufficient means of grouping the students for the sake of further analysis. For this reason, the four recategorized groups outlined in this section were used to answer Research Question 1 (Table 3).

Table 3

Table 3 Analysis of Student Graph Interpretations by Prior Student Achievement Level

Recategorized Group Names	Students	Average Score	Characteristic Student Graph Interpretation Behaviors
Independent and Inquisitive Group	Hans Hazel	100	• Made a few careless errors • Hugh, Huck, and Heather also answered question 1 incorrectly
	Hyde Hegel	100	• All students demonstrated automaticity with elementary and intermediate level graph questions
	Hugh Hickson	85	• Extensive use of heuristics like answer elimination, mnemonics, drawing with pencil and tracing with finger, question asking, pattern seeking, and estimation
	Huck Handy	85	• Had some difficulty answering external identification stage questions correctly
	Heather Miller	92	• All students except Huck were inquisitive and asked their own questions • Recognize salient features of the graph
Compliant and Reserved Group	Henrietta Harmon	62	• Answered most external identification stage questions incorrectly and a disproportionate number of elementary level questions incorrectly • All three confused the order of proper coordinates (x, y)
	Hester Luck	85	• Used heuristics: Hester used drawing extensively, Hodge almost always used answer-checking (able to self-correct on question 3), Henrietta used answer elimination, answered automatically or used no strategy
	Hodge Leader	69	• Never explored beyond what was required to answer questions
Few Strategies but Decent Science Group	Matt Maples	85	• Answered questions from the external identification stage and intermediate level incorrectly • Used informal science knowledge to assist with TOGS questions • Lack of self-generated questions
	Misty Murphy	69	• Few heuristics used
Earnest but Confused Group	Linda Mills	69	• Answered questions from the external identification stage and all three question levels incorrectly
	Lucy Mag	69	• Unawareness of breakdowns (comprehension errors, lack of self-monitoring) • Lack of confidence (negative self-talk, saving face behaviors)
	Lydia Lynn	54	• Procedural problems (applied incorrectly, poor choices)

Group Characteristics

The Independent and Inquisitive group members made few careless errors while answering the TOGS questions. They did, however, have some difficulty answering the external identification stage questions correctly. All of them demonstrated automaticity with elementary and intermediate level graph questions and made extensive use of heuristics like answer elimination, mnemonics, drawing with pencil and tracing with finger, question asking, pattern seeking, and estimation. The Independent and Inquisitive members were able to easily recognize salient features of the graph. As a group, these students were in many ways similar to the practicing scientists studied by Roth and Bowen (2001) insofar as they demonstrated curiosity and procedural independence while interpreting graphs. In addition, both this group of students and the practicing scientists were able to recognize when their understanding of the referent differed from their interpretation of the graph and reexamine their understanding of the salient structures in the graph (Roth, Pozzer-Ardenghi, & Han, 2005). Finally, as Roth and Bowen saw on a few occasions, the practicing scientists and this group of students both analyzed the graphs without referents to the physical situation they represented.

The Compliant and Reserved group members answered most external identification stage questions incorrectly and a disproportionate number of elementary level questions incorrectly. They all used heuristics but used a fewer number and variety than the Independent and Inquisitive group. Additionally, these participants never explored beyond what was required to answer the TOGS questions. These students were similar to the undergraduate students described by Bowen, Roth, and McGinn (1999). Like the undergraduates, the students in this group demonstrated little substantive knowledge of the science behind the graphs and were primarily concerned to answer the questions correctly. They did not, however, demonstrate any curiosity about the science involved in the question or independent judgment about concerning the graph they were interpreting.

The Few Strategies but Decent Science group used informal science background knowledge to assist with answering the TOGS questions, similar to the Independent and Inquisitive group but in contrast to the other two groups. However, they did not self-generate questions to help themselves when their thinking broke down, used few heuristics, struggled with the external identification stage and intermediate level questions, which differentiated them from the Independent and Inquisitive group. This pattern of behavior has not been reported in the published literature of graph interpretation.

The Earnest but Confused group struggled with external identification stage questions and questions from all three levels. They were seemingly unaware of these lapses in their thinking and did not seem able to monitor themselves to know when they were not making progress answering a question correctly. They also lacked confidence in their abilities, as seen in their negative self-talk and face-saving behavior. Finally, they exhibited frequent procedural problems, such as completing a procedure incorrectly or choosing the wrong procedure for the problem. The students in this group were similar to the students with learning disabilities described by Parmar and Signer (2005), although the disability status of the students in this study from any of the groups was not known. Like the students with LDs, the students in this group seemed to have significant difficulty with mathematical procedures and with connecting their background knowledge to the graph.

Descriptive and Inferential Coding and Analysis

Research Question 1

Research Question 1 asked: How do student behaviors observed during think aloud interviews (Ericsson & Simon, 1993) vary during line graph interpretation in science across mathematics and science achievement levels? The analysis to answer this question first focused on inductive descriptive coding (Miles & Huberman, 1994; Rossman & Rallis, 2003). These descriptive codes were then sorted into four categories: use of heuristics, self-generated questions, use of science knowledge, and motivation. The data were then re-analyzed using these descriptive codes and categories. This process led to the addition of codes to the categories but did not challenge the categories themselves.

Patterns within and among the categories were then analyzed across student achievement levels. Inferences were made about the relationships and data were recoded a third time to identify affirmative and contradictory examples from the data. When contradictory examples were found, the inferences were reconsidered, adapted, elaborated, or rejected. These inferences informed the presentation of the data in Chapter 4.

Research Question 2

Research Question 2 asked: How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)? To answer this question, the data were reanalyzed deductively using Roth and Bowen's two-stage semiotic process of reading graphs (2001). A set of transcripts, selected to represent a range of student achievement levels and question levels, were coded to determine whether Roth and Bowen's model could adequately describe participants' behavior. It could. These analyses were then sorted to see whether Roth and Bowen's model revealed any patterns in how students' responses varied according to graph question level. It did.

This analysis suggested general patterns in the ways students responded. Additional transcripts were recoded to verify and test these finding which led to a deeper understanding of the roles that science knowledge played while answering the questions. These inferences informed the presentation of the data in Chapter 4.

Research Question 3

Research Question 3 asked: Drawing from these data, what patterns emerge from student thinking during line graph interpretation given their prior performance in mathematics and science? To answer this question, the findings from Research Questions 1 and 2 were synthesized and used in conjunction with Roth and Bowen's two-stage semiotic process of reading graphs (2001). A representative participant from each group was selected. Each vignette was written to highlight the factors identified in the first two Research Questions and the relationships among those factors. These vignettes were illustrated with analyzed transcriptions to show differences between groups in the ways they engaged in line graph interpretation.

Trustworthiness of Data and Analysis

A variety of methods were used to assure the trustworthiness of the data and the analysis (Creswell, 2003). These methods included: triangulation; thick, rich description; negative case analyses; and external auditing and peer debriefing (Denzin, 1978; Miles & Huberman, 1984; Patton, 2002).

Triangulation

Triangulation, for Miles and Huberman, “is not so much a tactic as a way of life” (1994, p. 267). Throughout this study, the researcher attempted to “collect and double-check findings, use multiple sources and modes of evidence” (p. 297). Patton (2002) cites Denzin (1978) to define one type of triangulation as “methodological triangulation, the use of multiple methods to study a single problem or program” (Patton, 2002, p. 247). For this study, the multiple methods used were the TOGS (McKenzie & Padilla, 1986) and think aloud interviews (Ericsson & Simon, 1993). Whenever possible multiple data sources were used: FCAT scores, verbal protocols, video recordings of hand gestures, observational notes, and samples of student inscriptions. In addition, on the TOGS, most graph interpretation skills were assessed by two question items (Appendix A). Finally, an external auditor and peer debriefer was used to verify the instruments and the coding of the data. Multiple methods, multiple data sources, and multiple researchers all worked together to corroborate and validate the researcher’s findings and interpretations (Denzin, 1978).

Negative Cases Analysis

Throughout the coding and analysis process, all inferences and interpretations about the data were tested against the data to determine whether the generalizations were correct. Whenever counter-examples could not be found, the researcher felt more confident in her findings. When counter-examples were found, however, the inferences were expanded, modified, or rejected in favor of more encompassing findings.

External Auditing and Peer Debriefing

The same teacher who validated the scoring rubric in the two pilot studies served as an external auditor to verify the coding of student data. She was provided with representative samples of four complete transcriptions and scoring rubrics. Student pseudonyms were used to preserve and protect confidentiality. After completing the assessment rubrics using the transcriptions, the raters agreed on all but one indicator across all four students. The disagreement focused on whether participants had to overtly demonstrate their ability to read the (x, y) coordinates (indicator 1.1 Appendix A) to be recognized for doing it correctly. This disagreement also clarified that only overt behaviors would be coded. Besides this minor detail, both readers agreed on all other codes.

This teacher also served as a trusted reader as the research study developed. Because both the descriptive and inferential coding were open to multiple interpretations, it was important to establish trustworthiness of the coding and analysis within the study and maintain that reasonable inferences were made about the codes on the transcripts. When the analysis was unclear, she asked clarifying questions that led to data reexamination or clarification in the data presentation. Student pseudonyms were used during this process to preserve and protect confidentiality.

Member Checking

Member checking involves returning either the raw or analyzed data to the participant to verify the accuracy of the transcription or the appropriateness of the interpretations. Even normally, member checking would be of limited use in a think aloud study; Ericsson and Simon (1993) never mention it as a normal procedure. This is probably because it would be very difficult to believe that participants have any privileged understanding or perspective on their behavior during the tasks that are typically studied using think aloud methods. In addition, in this study, member checks could not be performed due to the young age of the participants.

In this chapter, the rationale for the research design was explained, and the methods of data collection were reviewed, including the sampling methods, data collection procedures, and instruments. Methods of data analysis and verification for each question followed. The next chapter will present the analysis of the data collected in this research study.

CHAPTER FOUR: DATA ANALYSIS

Purpose and Summary of Methods

The purpose of this study was to understand how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs. In addition, the purpose was to understand how students' interpretations were affected according to graph question level. The research methods for this study were primarily qualitative, though quantitative data were used to help with sample selection, categorization of students into subgroups, and analysis of data across the sample (Creswell, 2003). The qualitative data were collected using think aloud interviews (Ericsson & Simon, 1993), video, and observational notes. Two quantitative measures were used. One was the 2006 5th Grade Florida Comprehensive Assessment Tests (FCAT) of Mathematics and Science used to create a purposive sample using maximum variation sampling (Patton, 2002) to capture the variability of 6th-grade students at one Central Florida school. For the purpose of this study, students' achievement on the FCAT was assumed to be an appropriate predictor of student background knowledge and ability in mathematics and science (Florida Department of Education, n.d.). The other quantitative measure was the excerpted Test of Graphing in Science (TOGS) instrument developed by McKenzie and Padilla (1986). The TOGS measured students' ability to interpret line graphs (Appendix C). A scoring rubric (Appendix A) to assess student performance on the TOGS was created by the researcher and validated by Padilla (Appendix F) and a veteran mathematics and science teacher. For the rubric, previously cited factors affecting students' graphing ability (Appendix E) were synthesized and categorized using Bertin's three graph question levels (1983). Data were also analyzed using Roth and Bowen's two-stage semiotic process of reading graphs (2001).

Description of Sample

Fourteen 6th-grade students at one Central Florida elementary school volunteered to participate in this study. As shown in Tables 1 and 4, four were academically high in both mathematics and science (high = Level 4 or 5 on the 2006 FCAT), two were high in mathematics but low in science (low = Level 1 or 2 on the 2006 FCAT), one was high in mathematics and medium in science (medium = Level 3 on the 2006 FCAT), three were both medium in mathematics and science, two were low in mathematics and medium in science, and one was low in both mathematics and science. This sample provided adequate variability to answer the research questions. One student participated in the first pilot study; as discussed in Chapter 3, her data revealed that changes in the instrument and data collection methods were needed. These changes meant that her quantitative data were included in Tables 5 and 6 but her qualitative data were not included in the study. The result was that 14 participants were included in the descriptive statistics in Tables 5 and 6, but that 13 participants were included in the main part of the study.

Overview of Data Analysis

Research Question 1 asked, How do student behaviors observed during think aloud interviews vary during line graph interpretation in science across mathematics and science achievement levels? The analysis of data revealed major findings related to how students used heuristics, self-generated questions, science knowledge, and motivation. Research Question 2 asked, How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)? From the analysis of data emerged findings related to students approaching the TOGS questions as mathematics word problems versus approaching them as science data to be analyzed. Research Question 2 also led to additional findings related to how students used science knowledge during graph interpretations tasks. In the data, participants who only made reference to the elements of Roth and Bowen’s top triangle (i.e., structuring) approached the questions as mathematics word problems. Participants who made references to elements of Roth and Bowen’s top and bottom triangles (i.e., structuring and grounding) approached the questions as science data to be analyzed. Research Question 3 asked, Drawing from these data, what patterns emerge from student thinking during line graph interpretation given their prior performance in mathematics and science? The major findings from Research Questions 1 and 2 were used in conjunction with Roth and Bowen’s model to answer Research Question 3. Vignettes were created for representative participant(s) in each group that illustrated and related the patterns that emerged in how students interpreted line graphs in science.

Table 4

Table 4 Sampling Matrix: Participant Pseudonyms Organized by 2006 5th Grade FCAT Mathematics and Science Levels

	Low FCAT Math Level (Levels 1 or 2)	Medium FCAT Math Level (Level 3)	High FCAT Math Level (Levels 4 or 5)
Low FCAT Science Level (Levels 1 or 2)	**Lydia Lynn** (low math- low science) **LL**		**Hester Luck** **Hodge Leader** (high math- low science) **HL**
Medium FCAT Science Level (Level 3)	**Linda Mills** **Lucy Mag** (low math- medium science) **LM**	**Matt Maples** **Misty Murphy** **Megan Mason** (medium math – medium science) **MM**	**Heather Miller** (high math- medium science) **HM**
High FCAT Science Level (Levels 4 or 5)			**Hans Hazel** **Hugh Hickson** **Henrietta Harmon** **Huck Handy** **Hyde Hegel** (high math- high science) **HH**

Preliminary Analysis

A preliminary analysis of the data suggested that two important changes were needed before data could be analyzed to answer the research questions. First, the preliminary analysis of the data revealed some peculiar patterns that suggested that a new graph question level had to be introduced. Three of the questions were placed in this new category (Table 2). Second, while the sampling strategy was effective for ensuring adequate diversity in the sample (Table 4), it proved too cumbersome for analyzing the data. For this reason, the students were recategorized into four new groups based upon their performance on the TOGS and their behaviors during the think aloud protocol (Table 3).

Recategorization of Question Levels

A detailed analysis of the questions, explained in Chapter 3, revealed that these three questions assessed students' abilities to recognize deviations from the conventions of scientific line graph presentations. Questions 1, 8, and 9 were different than the other four elementary level questions that focused on students' abilities to identify or label a single point on a line graph. A review of Bertin's (1983) theory of graph interpretation showed that questions 1, 8, and 9 were more appropriately interpreted as part of the first "stage in the reading process" and labeled as "external identification" questions (p. 140). This could not have been determined prior to data collection. The recategorized questions were used in the remaining analysis.

Table 5

Table 5 Assessment of Students' Correctness for Each TOGS Question

Student	Hans Hazel	Hugh Hickson	Henrietta Harmon	Huck Handy	Hyde Hegel	Hester Luck	Hodge Leader	Heather Miller	Matt Maples	Misty Murphy	Megan Mason	Linda Mills	Lucy Mag	Lydia Lynn	Ques Ease[a]
Math Ability	H[b]	H	H	H	H	H	H	H	M[c]	M	M	L[d]	L	L	
Science Ability	H	H	H	H	H	L	L	M	M	M	M	M	M	L	
Ques 1	1[e]	0[f]	0	0	1	1	1	0	1	0	0	0	0	1	43
Ques 2	1	1	1	1	1	1	1	1	0	0	1	0	0	0	64
Ques 3	1	1	0	1	1	1	1	1	1	1	1	1	0	1	86
Ques 4a	1	1	1	1	1	1	1	1	1	0	0	1	0	1	79
Ques 5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	93
Ques 6	1	1	0	1	1	0	0	1	1	1	1	1	1	0	71
Ques 7	1	1	0	1	1	0	0	1	1	1	1	1	1	0	71
Ques 8	1	1	1	0	1	1	0	1	0	1	0	0	1	0	57
Ques 9	1	1	0	1	1	1	0	1	1	1	0	1	1	1	79
Ques 10	1	0	1	1	1	1	1	1	1	1	1	1	1	1	93
Ques 11a	1	1	1	1	1	1	1	1	1	1	0	1	1	0	86
Ques 12	1	1	1	1	1	1	1	1	1	0	0	1	1	1	86
Ques 13	1	1	1	1	1	1	1	1	1	1	1	0	1	1	93
Average Score[g]	100	85	62	85	100	85	69	92	85	69	54	69	69	54	

[a]Percentage of TOGS questions that each student answered correctly. [b]H=High. [c]M=Medium. [d]L=Low. [e]Students who answered the TOGS question correctly received a 1 in the table cell. [f]Students who answered the TOGS question incorrectly received a 0 in the table cell. [g]Percentage of students in the sample who answered the question correctly.

Table 6

Table 6 Students' Incorrect Answer Choices for Each TOGS Question

Student	Hans Hazel	Hugh Hickson	Henrietta Harmon	Huck Handy	Hyde Hegel	Hester Luck	Hodge Leader	Heather Miller	Matt Maples	Misty Murphy	Megan Mason	Linda Mills	Lucy Mag	Lydia Lynn	Ques Ease
Math Ability	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**M**	**M**	**M**	**L**	**L**	**L**	
Science Ability	**H**	**H**	**H**	**H**	**H**	**L**	**L**	**M**	**M**	**M**	**M**	**M**	**M**	**L**	
Ques 1		A[a]	D	D				D		A	A	A	A		43
Ques 2									A	D		D	C	A	64
Ques 3			A										B		86
Ques 4a										A	C		C		79
Ques 5														D	93
Ques 6			D			D	D							D	71
Ques 7			B			B	B							B	71
Ques 8				B			B		D		B	B		B	57
Ques 9			A				A				A				79
Ques 10		D													93
Ques 11											D			D	86
Ques 12										B	B				86
Ques 13												D			93
Average Score	100	85	62	85	100	85	69	92	85	69	54	69	69	54	

[a]Students' incorrect answer choices on the TOGS; correct answers omitted.

Recategorization of Students

The analysis of the student groupings first focused on participants' quantitative performance on the TOGS, both overall and among the graph question levels. However, as detailed in Chapter 3, students were initially regrouped by the kinds of questions they answered incorrectly. In addition, the transcripts of the members of these new groups were compared for similarities and differences in their approaches to the TOGS questions, also explained in Chapter 3. This analysis led to the creation of four new groups with members who approached the TOGS in similar ways and with significant inter-group differences. These new groups were used in the remaining analysis.

Group Characteristics

The Independent and Inquisitive group members made few careless errors while answering the TOGS questions. They did, however, have some difficulty answering the external identification stage questions correctly. All of them demonstrated automaticity with elementary and intermediate level graph questions and made extensive use of heuristics like answer elimination, mnemonics, drawing with pencil and tracing with finger, question asking, pattern seeking, and estimation. The Independent and Inquisitive members were able to easily recognize salient features of the graph. As a group, these students were in many ways similar to the practicing scientists studied by Roth and Bowen (2001) insofar as they demonstrated curiosity and procedural independence while interpreting graphs. In addition, both this group of students and the practicing scientists were able to recognize when their understanding of the referent differed from their interpretation of the graph and reexamine their understanding of the salient structures in the graph (Roth, Pozzer-Ardenghi, & Han, 2005). Finally, as Roth and Bowen saw on a few occasions, the practicing scientists and this group of students both analyzed the graphs without referents to the physical situation they represented.

The Compliant and Reserved group members answered most external identification stage questions incorrectly and a disproportionate number of elementary level questions incorrectly. They all used heuristics but used a fewer number and variety than the Independent and Inquisitive group. Additionally, these participants never explored beyond what was required to answer the TOGS questions. These students were similar to the undergraduate students described by Bowen, Roth, and McGinn (1999). Like the undergraduates, the students in this group demonstrated little substantive knowledge of the science behind the graphs and were primarily concerned to answer the questions correctly. They did not, however, demonstrate any curiosity about the science involved in the question or independent judgment about concerning the graph they were interpreting.

The Few Strategies but Decent Science group used informal science background knowledge to assist with answering the TOGS questions, similar to the Independent and Inquisitive group but in contrast to the other two groups. However, they did not self-generate questions to help themselves when their thinking broke down, used few heuristics, struggled with the external identification stage and intermediate level questions, which differentiated them from the Independent and Inquisitive group. This pattern of behavior has not been reported in the published literature of graph interpretation.

The Earnest but Confused group struggled with external identification stage questions and questions from all three levels. They were seemingly unaware of these lapses in their thinking and did not seem able to monitor themselves to know when they were not making progress answering a question correctly. They also lacked confidence in their abilities, as seen in their negative self-talk and face-saving behavior. Finally, they exhibited frequent procedural problems, such as completing a procedure incorrectly or choosing the wrong procedure for the problem. The students in this group were similar to the students with learning disabilities described by Parmar and Signer (2005), although the disability status of the students in this study from any of the groups was not known. Like the students with LDs, the students in this group seemed to have significant difficulty with mathematical procedures and with connecting their background knowledge to the graph.

Research Question 1: Analysis of Data by Student Prior Achievement Level

Research Question 1 asked, How do student behaviors observed during think aloud interviews vary during line graph interpretation in science across mathematics and science achievement levels? The students' interpretations of graphing in science were first analyzed for the four recategorized groups (Table 3). The researcher focused on reading the transcripts and identifying interesting sections (inductive descriptive coding) (Miles & Huberman, 1994; Rossman & Rallis, 2003). These descriptive codes were then sorted into four categories: use of heuristics, self-generated questions, use of science knowledge, and motivation. The data were then reanalyzed using these descriptive codes and categories. This process led to the addition of codes to the categories but did not challenge the categories themselves. Next, the relationship among these major themes was identified and exemplified using transcriptions from the think aloud interviews and copies of student work. This analysis was used to answer Research Question 1

As the analysis will show, of the four participant groups, every student used a heuristic on almost every question, though their number, type, and usefulness varied significantly across prior achievement levels. Across these groups, the remaining three themes were much less frequent and unevenly distributed. The variance of heuristic usage, incidents of science knowledge, application of self-generated questions, and motivation suggested that these may explain aspects of the differences in performance on the TOGS questions. Inferences were made about the relationships and data were recoded a third time to identify affirmative and contradictory examples from the data. When contradictory examples were found, the inferences were reconsidered, adapted, elaborated, or rejected. This analysis informed the presentation of the data that follows.

Independent and Inquisitive Group's Graph Interpretation

The analysis of the transcripts revealed that, as a group, the Independent and Inquisitive members had little difficulty quickly and correctly answering the elementary level questions. Indeed, their verbal protocols for these questions were often short and terse, perhaps indicating that these students were well-practiced at these skills.

All of the Independent and Inquisitive group members answered the intermediate level questions with 100% accuracy. In addition, these students used many of the same strategies as one another to answer the intermediate level questions. They often used multiple strategies within the same question. These observed strategies included:

- Answer elimination
- Drawing line segments or tracing with finger on the graph
- Pattern seeking and identification
- Self-generated question asking
- Mnemonics

- Procedure following
- Answer checking
- Estimating
- Arithmetic

Table 7

Table 7 Independent and Inquisitive Group's Approaches to Answering TOGS Questions

	ExtId	Interm	Elem	Interm	Interm	Elem	Elem	ExtId	ExtId	Elem	Interm	Interm	Interm
	Ques1	Ques2	Ques3	Ques4a	Ques5	Ques6	Ques7	Ques8	Ques9	Ques10	Ques11a	Ques12	Ques13
Hans Hazel	Ans Elim	Draw	Auto	Ans Elim	Ans Elim	Auto	Auto	Ans Elim, Draw	Ans Elim	Draw, Auto	Ans Elim	Draw	Draw, Auto
Hugh Hickson	Ans Elim, Ques Asking, **Incorrect**	Ans Elim	Auto	Ans Elim	Auto	Mnem, Ans Elim	Mnem, Ans Elim	Ans Elim, Ques Asking	Ans Elim, Auto	Initially Auto, then Arithm **Incorrect**	Arithm, Ans Elim	Arithm	Auto
Huck Handy	Auto **Incorrect**	Auto	Draw, Auto	Ans Elim	Auto	Mnem	Mnem	Ans Elim **Incorrect**	Ans Elim	Draw, Auto	Ans Elim	Draw	Draw
Heather Miller	Ques Asking, **Incorrect**	Arith	Arith, Auto	Ans Elim	Auto	Mnem	Mnem	Personal Exp	Ans Elim	Auto	Ans Elim	Auto	Patt Seek, Ans Elim
Hyde Hegel	Ans Elim	Estim	Ans Elim	Ans Elim	Ans Elim	Proced, Ans Elim	Proced, Ans Elim	Mult Ques Asking	Ans Elim	Mult Ques Asking	Ans Elim	Self-Corrects Auto	Patt Seek,

Note. ExtId = External Identification Stage Question; Interm = Intermediate Level Question; Elem = Elementary Level Question; Ans Elim = Answer Elimination; Draw = Drawing; Auto = Automatic; Ques Asking= Question Asking; Mnem = Mnemonic; Arith = Arithmetic; Personal Exp = Personal Experience; Patt Seek = Pattern Seeking; Estim = Estimating; Proced = Procedural Knowledge; Incorrect = Participant answered the question incorrectly.

Inquisitiveness

Hyde Hegel was one of the most inquisitive members of this group. He was very comfortable thinking aloud and asked himself many questions during the TOGS session. After rereading question 8 a second time, Hyde thought aloud about how to correctly label the axes of a line graph after time had been measured to heat various amounts of water to boiling:

> So let's see, if he puts the time in minutes then to heat various amounts of water to boiling, hmm, so you could have the time on the bottom and the temperature…*do we want the amount*, heat various amounts of water so the temperature of water would probably go somewhere else. You probably need time maybe…you're definitely going to need time and you're going to need various amounts but you're going to need… Oh! Time needed to heat various amounts… Whoa! Whoa! Whoa! *What would be the heat? How high is he going to heat it to?* That would be a good thing for them to put in. They didn't put it in, though…(emphasis added)

Hyde asked his own questions in order to understand what the question was initially asking. At first, he misunderstood and had to go back and clarify his thinking. He continued on and asked more questions in order to answer the question.

> Time it took to heat the amount of water, I was thinking, *do you want time on the side*? Just thinking. All I have to have a time maybe a 1 minute, 2 minutes, 3 minutes, 4 minutes, 5 minutes, 6 minutes, 7 minutes (says this as he writes these numbers on the y-axis of answer choice A). Then we could have 1 milliliter, 3 milliliters, 5 milliliters, 7 milliliters, 9 milliliters, 11 milliliters (says this as he writes these numbers on the x-axis of answer choice D). *Or, do we want to have it like that, or do we want to have the*

milliliters on the side and time on the bottom (reversed example in answer choice D)? (emphasis added)

Hyde considered what he had written and the questions he had asked and finished answering the question:

> That'd be bad. Need to variate them. 1, 3, 5, 7, 9 And of course you can't have no zeros. We've gotta have a zero, so, *do we want to like that or do want it on the bottom*? Probably thinking we'd have to put this on the bottom. Forgot 11 (wrote 11 about the 9 on the y axis of answer choice D). So, um, see we have time minutes, amount of water in milliliters, hmm, what could set time on the side…water…say it took 3 minutes (plotted points on graph) and 7 you have time on the bottom and amount of water. I'm thinking A. (emphasis added)

Hugh Hickson, although not as inquisitive as Hyde, asked himself questions when parts of the test question became unclear. By asking questions, he was able to clarify the meaning of TOGS question 8 for himself.

> Looking at A, that one might work because it has time labeled and if you're timing things, that…actually it couldn't work because on the left side, on the horiz…on the vertical side we're looking for things that make a bar graph or line graph, things that can go up to…And the question says the time needed to heat various amount of water… The time needed to heat various amounts of water, to heat, to heat… *Does that mean to? How much you should put on the stove or various amount of water? How much needed to heat various? So would it be?…*(long delay)…various, so you put the heat on the bottom, I know it's either A or C. (emphasis added)

Hugh correctly identified time and incorrectly identified temperature of water as labels to be placed on the graph and then further struggled with his final decision upon which axis each label went. He luckily chose the correct label which happened to have been placed on the correct axis, too:

> …but I don't know if on the horizontal axis it should amounts of water or temperature of water just because heat various amounts of water so would you use the same heat, making it A or would you use, it says various amount of water so it's going to have to be A just because it says various amounts of water. That's my answer.

While in the end Hugh was helped more by luck than by skill, this example, nevertheless, demonstrated that it was the inquisitiveness of this group that usually led them to a deeper understanding of the graphs. In turn, this inquisitiveness and their deeper understanding afforded them a greater degree of independence.

Independence

Hyde Hegel excelled in using clarifying questions to monitor his own comprehension while interpreting line graphs on the TOGS. This question-asking and self-monitoring enabled him, as with the other students in this group, to approach the TOGS questions with a greater degree of independence compared to the other groups. On question 10, Hyde was initially confused about what the question was asking him to do. Right after he generated his own question, he independently offered a better wording that would have been clearer:

> How much did he use to drive 1 km at 60 km per hour? 60 km per hour, hmm, how much gas in liters were used to drive 1 km at 60 km per hour? *Do they mean*, hmm, I think they mean that how much gas allowance, they don't have 1 km on there, they have 60 km

> per hour, hmm, I bet they mean to word to it like, 'How much gas in liters were used to drive 1 km [at] 60 km per hour?' (emphasis added)

By rephrasing the question, Hyde clarified for himself the intent of the question, improving his comprehension and enabling him to get the right answer: "I guess they could mean it like 60 km per hour then like the speed they were going…when they're going 60 km per hour. So, it shows here the dot is at 0.07…so C."

Heather Miller showed similar independence by trusting her own answer and asserting its accuracy even when it was not one of the four multiple choice options on TOGS question 2.

> Well, on the graph, 140 would be in between 120 and 160 and so with 120 ml of water it grew 20 cm and with 160 ml of water it grew 10 cm, there is a number 10 difference between those two numbers. So with 140 being right in between that (120 and 160), I would say that the answer would be 15. The plant would grow 15 cm.

When asked which multiple choice answer she wanted, she replied, "I didn't see the multiple choice (giggles). I'm sorry. Well, I'm guessing it would be 15 cm, but because 15 is not on the multiple choice (test) and the number 16 is right above it, that's what I'd probably guess. (answer choice) B." Even though she was forced to choose 16, she would have been satisfied keeping her original answer of 15 cm which she had determined was more correct.

Automaticity

Hans Hazel's protocol for question 3 was typical: "So it's pretty much reading the graph, it's much easier. 40 ml… okay, here's how it grew 10 cm… there's the plant, it's going to be 160 ml. C (answer choice)." While Hans was talking, he drew a horizontal line across from the y-axis at 10 cm over to the data point on the graph and then a vertical line down to the x-axis (see

Figure 3), giving him the answer of 160 ml. He then saw that 160 ml was a choice on the list of possible multiple choice answers, corresponding to answer C. Hans' protocols for the other three elementary level questions were similarly short and direct.

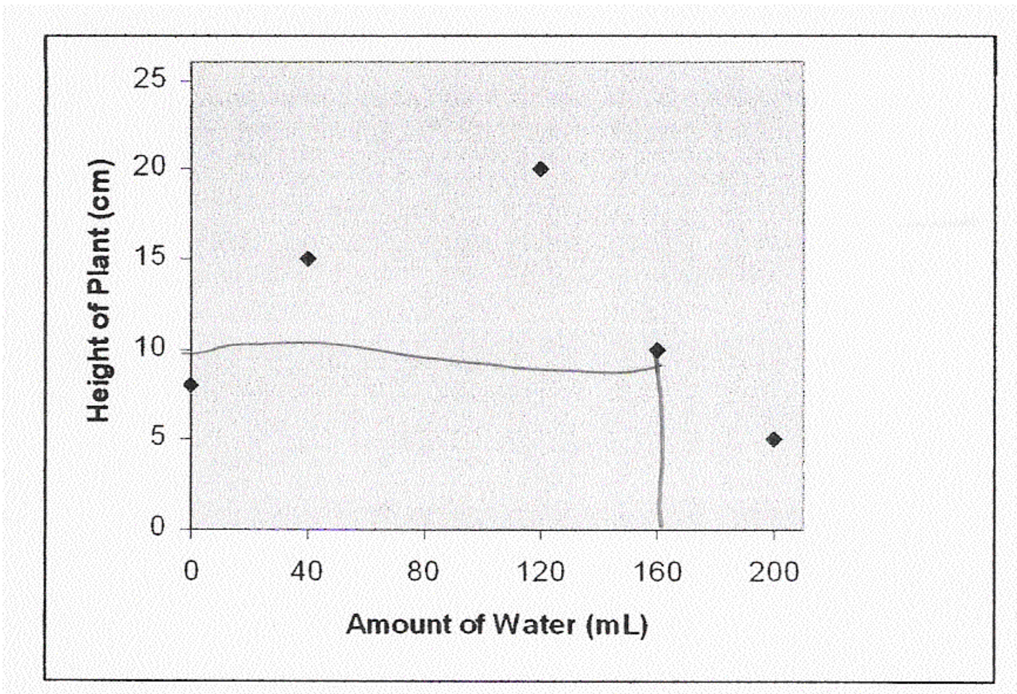


Figure 3: Hans Hazel's drawing for TOGS question 3.

Likewise, Huck Handy answered all of the elementary level questions correctly and his verbal protocols were terser than Hans'. For question 3 he said, "Now 10 is right there… 160, C." When asked to describe his thinking he said, "Well, I was thinking where I saw 10 cm, and then went over to where I saw a dot close to it and just went down below it, and I saw the number (160)." His brief answers suggested these skills were so automated that they required little thought and effort.

Use of Heuristics

Mnemonics

Hugh Hickson's answer to question 6 was about twice as long and somewhat less automated. He also used the mnemonic, "healthy vitamins," to remember that in proper coordinate notation, the horizontal number (x-axis) is first, and the vertical number (y-axis) is second. Then, Hugh eliminated incorrect answers from among the multiple choices and finished by checking his answers:

> You can use "healthy vitamins," horizontal then vertical…appropriate for point a… (Reads answer choice A) If anything's near that, that would be (answer choice) B…that's not really it. (Reads answer choice B) If you go all the way to 20 and then go up to 20, 12, that is it for point A. Let's just check the other answers. (Reads answer choice C) Nothing's really there. (Reads answer choice D) That's all. The answer would be B, (20, 12).

In this answer, Hugh used a combination of strategies: mnemonic and the test taking skill of eliminating poor answer choices. A similar pattern emerged in two of the other three elementary level answers. In contrast to Hans, who apparently has these elementary level skills well practiced and automated, Hugh's answers were less practiced, and he had to rely on other heuristics.

Another Independent and Inquisitive group member to use the mnemonic heuristic was Heather Miller. She answered questions 6 and 7 with this strategy very skillfully and independently:

> On the vertical axis are 0, 3, 6, 9, 12, 15 and on the horizontal axis the numbers are 0, 5, 10, 15, 20, and 25. Now, when I was in fourth grade, I heard this trick called "healthy vitamins." It means you go horizontal first and then vertical, horizontal stands for "healthy" and vertical stands for "vitamins" (giggles). So if I was going horizontal first for point A, point A would be on the 20 mark and then I'd go vertical and that would be on the 12 mark. So the coordinates would be (20, 12) and that would be answer B.

Again, for question 7, Heather used this mnemonic to remember in which order to read the axis coordinates:

> Back to the healthy vitamins thing, if you go horizontal to 15, the only dot on the 15 line is C, and you'd find there isn't an 8 on the vertical axis, but C is in between 6 and 9, a little bit below 9, so that would probably be an 8. So the answer would be point C, which is C.

Answer elimination

Hyde Hegel presented an interesting example of automaticity and the selective use of heuristics. He did not have to rely on the use of a mnemonic while answering questions 6 and 7. Instead, he recalled the procedural knowledge, "I know you have to go sideways first then up" to identify the x- and y-coordinates. However, he relied on an answer elimination strategy to work systematically through the answer choices:

…so A, (9, 12). A's all the way near 20 so it'd have to be 20 and probably 12 there's also 20 and then 8 (answer choice C) but 8 is down here. You can't do (9, 12) (answer choice A) because 9 is way over here. And 12 is right here but 8 is over here. You can't do (20, 8). The only logical answer is (20, 12).

One common strategy used by the Independent and Inquisitive group members among the intermediate level indicators was answer elimination. No Independent and Inquisitive group member used this strategy for question 2 which required participants to interpolate data. All four Independent and Inquisitive group members used the answer elimination strategy on questions 4 and 11a which asked them to describe a relationship between the x- and y-variables. For example, in question 4, Hugh Hickson thought aloud:

(After reading answer choice A) A can't be right because 120 ml was the tallest and over 120 has decreased. (After reading answer choice B) I am not sure what that asks, what that answer reads… (Reads answer B again) that can't be right because there's no 120 cm. (Reads answer choice C) That can't be right because there's no graph on how much, what was the speed of the thing and there is, yeah. (Reads answer choice D) That has to be right because they increased once they got to 120 ml and jumped up to 20 cm and then when they got greater than 120 ml it went down back to 116 (unclear)…it decreased. Answer D.

In the transcript, Hugh demonstrated a methodical consideration of each answer choice while he evaluated how it fit with the graph. Hans Hazel also considered each answer choice in question 4 but with less precision than Hugh:

(Reads answer choice A aloud) Uhhh….not so sure about that one. (Reads answer choice B aloud) Hmm, that maybe a good one. (Reads answer choice C aloud) No…no, no. (Reads answer choice D aloud) Alright, let's go over and look at B and D, they're my best guesses. (Reads answer choice B aloud again) No, that is not correct. The correct answer is D.

Answer elimination was a necessary strategy for these kinds of questions that asked students to determine the best description of the relationship between the variables. Determining the best answer required students to compare answers and eliminate poor answer choices.

Line segments drawn with a pencil or fingers used to trace over the graph

Students were more comfortable tracing the graph with their finger than using their pencil to draw lines. However, on questions requiring extrapolation and interpolation–questions 4, 5, 12, and 13–students were more likely to use the strategy of drawing lines on the graph. For a typical example, Hans Hazel on question 2 thought aloud while drawing a point on the graph at (140, 15) and then circling the two x-axis coordinates, 120 and 160, as he said them aloud:

> Want to pour 140 ml of water daily for the next three weeks…so that would be 140…be somewhere right in between here (draws on graph)…height of plant…one plant up…given uh next three weeks, the time…after three weeks, um, this is a hard one, obviously 120 is the best so far…got the graph aid to put it in there and pretty much in between 120 and 160 (draws on graph).

Drawing and circling on the graph provided Hans a visual aid to focus his attention on salient graph features and keep track of his thinking.

Huck Handy used a similar strategy on question 13 when he extended the x-axis when the question required him to extrapolate beyond the printed graph, "Eighty would be out here (draws a horizontal line segment extending the x-axis) and then right here it seems to be going up so I think C, nine hundredths (.09)." He then used his finger to extend the general direction of the imaginary line graph to about where it would intercept 80 on the x-axis. Notice that he was able to visualize the extended y-axis without either drawing with his pencil or his finger. Drawing in the graph, whether with finger or pencil, was a very important strategy for answering the interpolation and extrapolation questions when the students needed to visualize a line or curve.

Few Careless Errors Made

Hugh's answer to question 10 provided an interesting example of how a student's thinking can break down. The objective of this question was similar to question 6 and could easily have been answered using the same method, but Hugh struggled with this question and chose a method that led to the wrong answer:

> Gas in liters so liters…you have…speed of automobile, km per hour, he goes 30 it goes 0.4, it uses 0.4 (mistake) it goes 60 you…the answers are (reads answer choices). And tell you the truth; I really don't know how to figure this. Liters aren't really my favorite thing, how would I do this? You would…speed of the automobile, 60 miles per, 60 km per hour.

So, instead of simply reading the y-coordinate of the point on the graph that corresponds with 60 km per hour, Hugh instead started to focus on the fact that the x-axis is a measure of rate:

> This is going 60 km per hour…and so if you were going 60 km per hour, you're going a kilometer every…no…you'd go 60 kilometers in an hour so that would be one kilometer a minute.

This resulted in Hugh inferring that the rate of consumption must be continuous:

In gas, it takes 20, it goes 30 km for every 4, so it would have to go 8, you'd have to use 0.08 km, I mean liters of gas so it'd be on the graph it says…huh!... (he read the graph and saw that the correct coordinate should be 0.07 and expressed surprise) I'd have to go with the math so it has to be 0.08 liters so the answer is D.

In his last segment, Hugh converted what should have been a task reading the coordinates of the point on a graph into an arithmetic question. He figured that if the car consumed 0.04 l of gas when it went 30 km/h, then the amount of gas consumed would double when the car goes 60 km/h 0.08 l rather than the correct answer from the graph of 0.07 l. If the graph had been perfectly linear, then his calculation would have been correct. Instead of simply selecting a corresponding value of y for a given value of x on a given graph (item 1.3, Appendix A), he relied on simple multiplication. It is important to note that Hugh did answer question 3 correctly, which assessed the same objective as question 10; moreover, question 10 could have been answered using the same method as question 3. The only difference between questions 3 and 10 was that in question 3, the y-coordinate was given in the question, while in question 10, the x-coordinate was given.

Recognition of Salient Features of the Graph

The salient features of the graphs on the TOGS included, but were not limited to, the axes labels, the independent and dependent variables, the size of the intervals, and the units of the intervals. In the think aloud protocols, the Independent and Inquisitive students seemed to have little difficulty recognizing the salient graph features and, as a result, commented upon them fairly rarely. In other words, their ability to recognize the salient features of the graphs appeared automatic.

Several noteworthy examples appeared in the think aloud protocols. When Huck Handy started to work on question 2 which required him to interpolate between two points on the graph, the first things he did were to read the graph labels and the size of the intervals on the x-axis. "(Reads graph labels) So it goes up by 40's (x-axis)..." Having this information was important for him to then recognize that 140 was halfway between two labeled intervals, 120 and 160, "…so the one given (in the question) 140…" He then quickly inferred that the point (140, 15) would be about on the imaginary line connecting the two points on the graph, (120, 21) and (160, 11). He continued, "…would probably be about 15, in the middle, so I think it'd be B, 16 because it was just about in the middle of 21 and 11 (y-axis)." This example showed how quickly Huck was able to recognize a number of salient features of the graph and used that information to answer the question correctly.

A similar approach was seen in Hans Hazel's answer to question 5, which required him to extrapolate beyond the graph. The first things he thought aloud were the intervals, units, and scale of the x-axis: "Okay, they're in intervals of 40 ml of water (x-axis) so it'd be really close to 200 (the scale of the x-axis)…" With this basic information in place, he was also quickly able to correctly answer the question.

A more elaborate example occurred when Hugh Hickson answered question 12, which required him to interpolate two points on the graph. He immediately inferred that "…55, that's in between 50 and 60," meaning that the value of x that he needed was between two intervals on the graph. He was then distracted by the abrupt increase in y-values in that range of the graph:

> Huh, that's going to have to be in between, that is really strange that it goes from 40 km to 50 km and it only jumps up one-thousandth or a couple-thousandth…just jump up if

you go 10 km faster. Just to go from 50 to 60 it takes like 2 whole hundredths of liters of gas…

By noticing this feature of the graph, he recognized that the y-values in that range increased by two one-thousandths, so the mid-point between the points on the graph would be an increase of one one-thousandths from the point (50, 0.05). "…[S]o you're going to have to put it in between which is the answer C, at 0.06 liters of gas for 55 km per hour." His ability to recognize these salient graph features with relative ease seemed to make it fairly easy for him to correctly answer the question.

From this analysis, several patterns emerged regarding the ways the Independent and Inquisitive students approached the TOGS questions. Most of these group members approached the elementary level questions in an automatic fashion. For more challenging questions, they used a variety of strategies: habituated procedures, mnemonics assisted to aid memory, drawing to externalize thinking and to aid memory, arithmetic, and answer elimination. Most importantly, both these strategies and their learned perception attentions encouraged them to monitor their own thinking and pose questions of themselves to keep themselves on track. In turn, these behaviors served as an interesting comparison when the other groups were analyzed.

Compliant and Reserved Group's Graph Interpretation

As a group, the Compliant and Reserved group were marked by the difficulty they displayed with external identification stage and elementary level questions. All three of these students had scored Levels 4 or 5 on the 2006 5th Grade FCAT in Mathematics and they might have been expected to excel in questions that seemingly focused on low-level graphing tasks.

These skills included determining the range and intervals for a graph, determining the order of proper coordinates (x, y), and finding a point on a graph. Given their difficulties with external identification stage and elementary level questions, it was noteworthy that these three students correctly answered all intermediate level questions correctly and chose the same incorrect answer choices for questions 6 and 7. Henrietta and Hodge chose the same incorrect answer choice for question 9, as well.

Table 8

Table 8 Compliant and Reserved Group's Approaches to TOGS Questions

	ExtId	Interm	Elem	Interm	Interm	Elem	Elem	ExtId	ExtId	Elem	Interm	Interm	Interm
	Ques1	Ques2	Ques3	Ques4a	Ques5	Ques6	Ques7	Ques8	Ques9	Ques10	Ques11a	Ques12	Ques13
Henrietta Harmon	Ans Elim **Incorrect**	Auto	Ans Check **Incorrect**	Ans Elim	Auto	Auto, No Strategy **Incorrect**	Auto, No Strategy **Incorrect**	Ans Elim	Auto **Incorrect**	Auto	Ans Elim	Pattern Seek	Auto
Hester Luck	Ans Elim	Draw, Estim	Draw	Draw, Ans Elim	Draw, Ans Check	Draw, Ans Check **Incorrect**	Draw, Ans Comp, Auto **Incorrect**	Auto, Ans Check	Ans Elim	Draw, Ans Check	Ans Elim	Draw, Ans Check	Draw, Ans Elim
Hodge Leader	Ans Elim	Ans Elim	Ans Elim	Ans Elim, Self-Correct	Auto, Ans Check	Ans Elim **Incorrect**	Auto **Incorrect**	Ans Elim **Incorrect**	**Incorrect**	Ans Elim	Auto	Ans Elim	Ans Elim

Note. An empty box denotes no strategy used.

Use of Heuristics

The Compliant and Reserved group used fewer heuristics than the Independent and Inquisitive group and their use was not as helpful to them. The analysis showed these group members may not have understood as many strategy approaches or the appropriate choice of heuristic for a given problem. Most of the students in this group relied on one or two strategies throughout the entire TOGS.

Line segments drawn on graph and answer checking

Hester Luck was the only group member to draw line segments on the graph and use answer checking. She and Matt used the drawing strategy more than anyone else in the entire study. In fact, Hester drew on the graphs for nine of the 13 test questions. Her drawings were often rudimentary and her think aloud accompanied her drawings as if she was reminding herself of the mathematics procedures she was engaged. For question 3, she thought aloud and drew simultaneously:

> Well, if she gave it 140 then the height that's a lot so if we look at the graph, there isn't a 140 so I'm just going to estimate to right there. If we bring it up we have to draw (draws on graph) that so the line it's probably going to be a height probably of 16 an estimate probably 16 because it looks like it's the closest. Well, if I looked at centimeters I should look to the side (y-axis) where it says Height of Plant centimeters. So, if I draw a line to the centimeters I'm going to connect the graph dots again then you draw a big line it will go to probably about 160. Probably around 160. So it's probably C (answer choice).

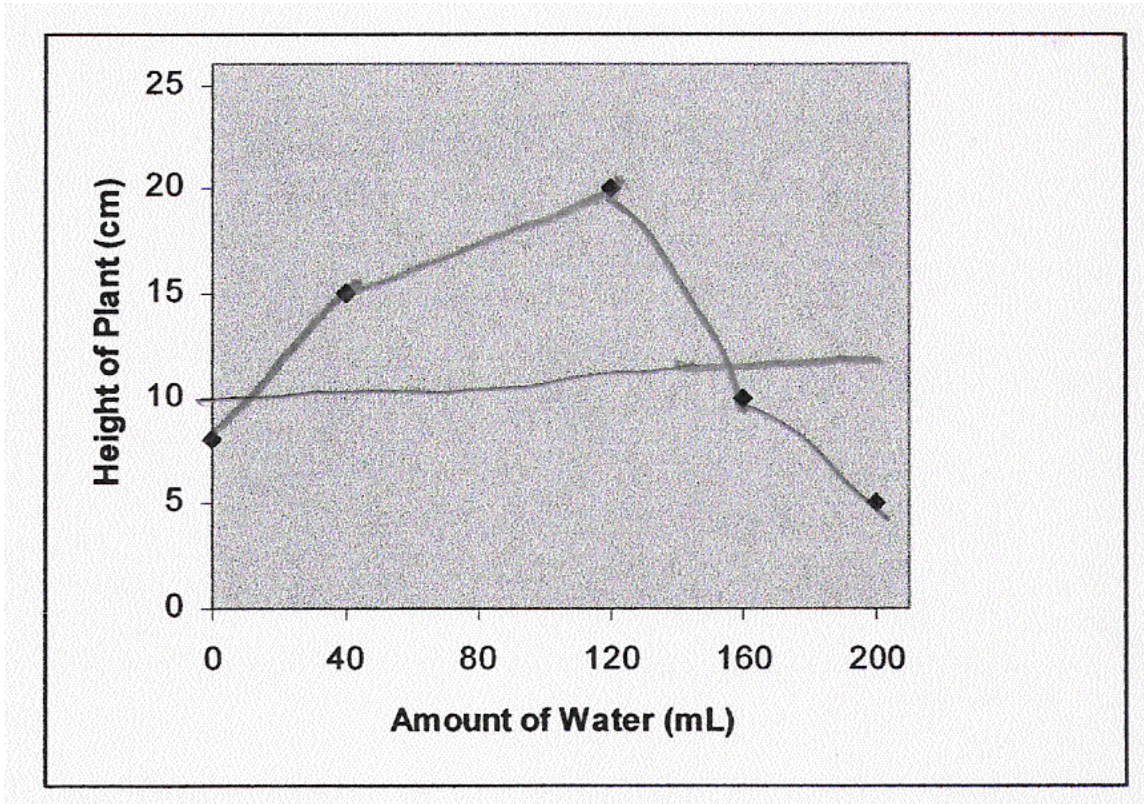


Figure 4: Hester Luck's graph drawing on TOGS question 3.

Again, on question 6 Hester thought aloud and drew. She also checked her answer by going back over the other multiple choice answers to discredit them.

> So, we want the proper coordinates for point a. Well, I'm going to draw a line all the way to (point) a from 12 which goes to the side it's 12 and at the bottom it's 20. So, I'm probably going to say D, (12, 20), I mean (20, 12). But A (answer choice) says (9, 12) which I know can't be right or 'cuz there's no possible answer unless we put (point) a lower. And (answer choice) B is close but I need to make sure first. And C is (20, 8) but it would have to be lower then. D is (12, 20) and I better make sure I'm making sure it's on the right thing. Which I'm not so it'd be (12, 20) which I'm going to say it's D.

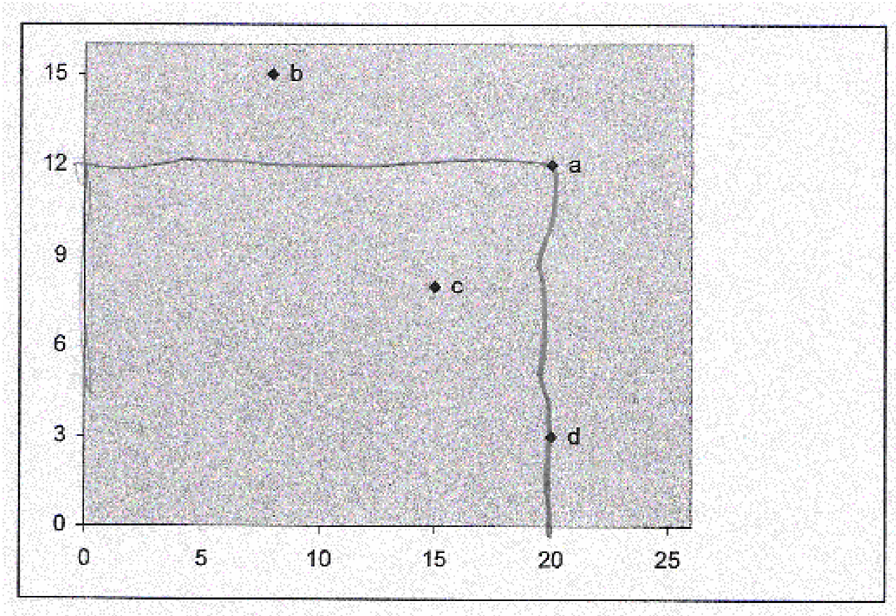


Figure 5: Hester Luck's graph drawing on TOGS question 6.

For question 13, Hester once again drew her way through the think aloud. She eliminated incorrect answer choices as she went and narrowed down her selection until she settled on C:

> So the car is going 80 which is not on the graph, so let's just say like here was 80 and draw a big line (off the graph) and connect all the dots, so I think, and if we're looking at this it would have to be over 0.08 because these 70 miles per hour is going out (last dot going off the graph), so I can know it can't be A (answer choice) 'cuz that's exactly what it is, it's less than .08. And B is exactly .08 so it's between C and D. So if I'm looking at this I should see 30 km per hour at .04, 40 at .05 and 50 it was a little above that and 60 it went a big change, it went to .07 and 70 went to .08 so if I'm looking at this it's probably going to be .09 'cuz that's what it looks like 'cuz .10 would be probably too high so I'm going to go with C.

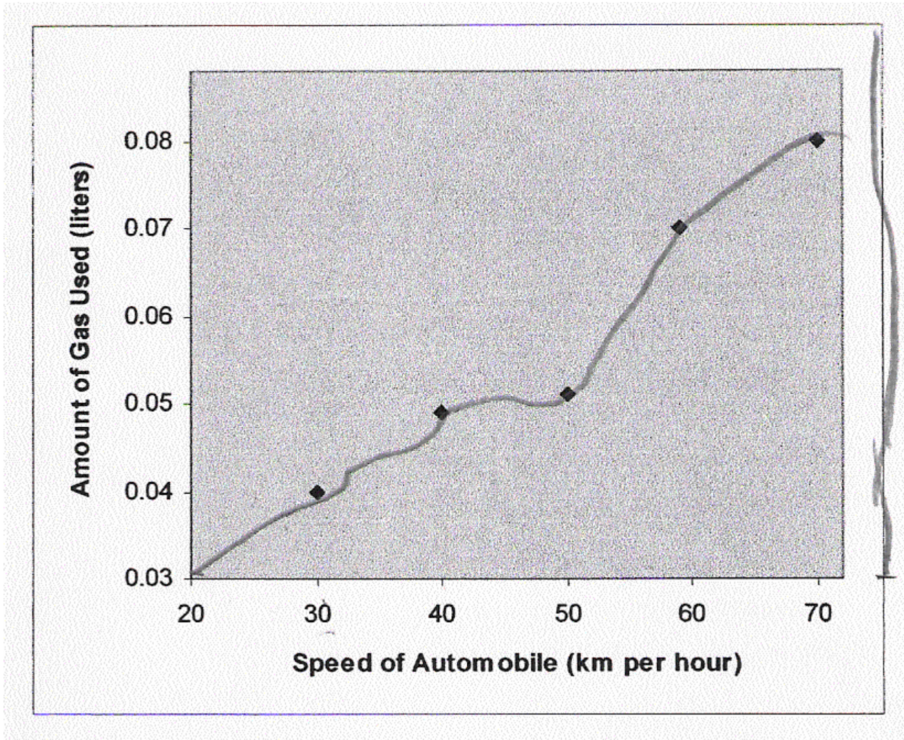


Figure 6: Hester Luck's graph drawing on TOGS question 13.

Answer elimination

All three group members used the heuristic of answer elimination to reduce unreasonable answer choices before choosing a final answer. Unfortunately, there were times when using answer elimination did not enable them to answer the question correctly. This happened with Henrietta and Hodge. Hester used answer elimination less often than the other two members because she often used answer checking instead.

On question 1, Henrietta almost answered automatically and then decided to use answer elimination to narrow down the options. However, she did not notice that answer choice D, her choice, had an incorrect scale on the y-axis:

> The amount of fertilizer is in grams, going up to 100, like it, um…I would say the answer would, well, um…the amount of fertilizer it could be wait, no, it couldn't be, I don't think it would be A because not everything is going up by twos, um, B it probably wouldn't be because it's going up by twos all the way. Um, let's see, C could, yea, it could be. I think the answer would be D because the answer is 3 and then goes up by ones (see y-axis on answer choice D) and most of them are in the number, the number shell.

As this example made clear, answer elimination was only helpful when the student understood the intention of the question and had the knowledge and skills to eliminate incorrect answers.

Hodge incorrectly answered question 6 while using answer elimination. The difficulty Hodge encountered was not remembering the correct order of coordinates of a point on a line graph. Instead of the correct order (x, y), Hodge reversed it and asserted it to be (y, x):

> (Reads answer choices.) A, I don't think is true because you always go left or right and so it can't be (9, 12). B, it can't be (20, 12) because it doesn't go that high. C can't because it doesn't go that high. D is the answer because it goes up 12 and over 20. The answer is D.

Hodge answered question 4 using the same heuristic as question 6 but with a very different outcome. In fact, the process of going through each question choice may have enabled Hodge to self-correct and repair his mistake. The other behavior that Hodge displayed that helped him self-correct was reading the answer choice and interpreting the implication for the shape of the graph. When he saw a breakdown between his choice C and the graph, he reevaluated and made a different choice. He also used his science knowledge to verify that he had selected the correct answer:

> (Reads all answer choices aloud.) Well A is not right because as the water increased to 120 ml it did not decrease. It increased by 5 cm. This is B, both the amounts of water and the height…no, because the amount of water in ml raised 40 ml but the height did decrease so I think it is C, the amount of water…it did, it started to, no, um, it didn't, I don't think it's C because it didn't keep increasing and D, (reads aloud) I think it is D and I think it happened because the more water might have slowed down the growth.

Because he understood the intention of the question and he used the correct knowledge and skills, Hodge was able to use answer elimination effectively.

Incorrect Procedural Knowledge

All three Compliant and Reserved students answered questions 6 and 7 incorrectly, and all chose the same incorrect answer, because all three relied on the same incorrect procedural knowledge to answer the question. In addition, none of them used a mnemonic or other strategy to verify they used the correct procedural knowledge. These questions simply asked students to read "x-y coordinates" of a point on a graph to find a point of given "x-y coordinates" (Elementary Level Indicators 1.1 and 1.2, respectively). Question 6 asked, "What are the proper coordinates for point a?" and provided a graph with four discrete points that were labeled a, b, c, and d. The transcriptions were all fairly similar to Hodge's:

> A, I don't think is true because *you always go left or right* and so it can't be (9, 12). B, it can't be (20, 12) because it doesn't go that high. C can't because it doesn't go that high. D is the answer because *it goes up 12 and over 20*. The answer is D. (emphasis added)

The emphasized phrases highlighted Hodge's procedural knowledge, embodied in the phrases "you always go left or right" and "it goes up… and over…." At first it seemed that perhaps he was not able to remember the order of the procedure. Later, he reversed the order of the procedure. On question 7 his answer was much terser, "Um, up 15, over 8, so B." With the procedural knowledge at hand, albeit incorrect, he did not need to think through the solution.

Lack of mnemonics

The lack of correct procedural knowledge was made worse because the Compliant and Reserved students also did not use a mnemonic strategy to recall the correct procedure. Whereas the Independent and Inquisitive group members answered questions 6 and 7 easily, quickly, and correctly, Henrietta answered these questions easily and quickly, but incorrectly. In response to this question, Henrietta tersely thought aloud, "Got to go up so up 12 and down 20 so it'd be (12, 20), wait, yeah, (12, 20), D." Unfortunately, she approached the question in an automated fashion and identified the y-coordinate, 12, first, and then identified the x-coordinate, 20, second. This led to her answer, "…(12, 20)…" which was provided as answer D as a distracter. She answered question 7 incorrectly for the same reason. Again, without using a mnemonic she was unable to remember the order of coordinates in bracket notation; but in this case, she could not see there was a discrepancy on account of the test constructors having used this error as a distracter.

Lack of Self-Generated Questions

The Compliant and Reserved group did not ask their own questions while answering the TOGS questions and analyzing the graphs, in contrast to the Independent and Inquisitive group. In all of the transcriptions, never once did any of these three students pose their own question or explore the situation beyond what was immediately required. The result was that this group demonstrated less comprehension of the material than did the Independent and Inquisitive group, which suggested that the Independent and Inquisitive students' ability to pose their own questions encouraged deeper understanding of the graphs they were interpreting.

In question 3, Henrietta focused on the left-hand side of the graph instead of the right and treated it as an interpolation question (see questions 2 and 12—both of which she answered correctly). For question 3, she said,

> Somewhere probably around 10…it would probably be 20 or wait no that's not one of the choices. Um, okay, 10 cm would be about um, 40 and 80 would be closer than 120 and 160, 200. So, I'm going to probably guess and say A because it's closer to 40 and 80.

While she was speaking, she traced her finger from 10 on the y-axis over to the imaginary line connecting points (8, 0) and (15, 40) leading her to say, "…it would probably be 20…"

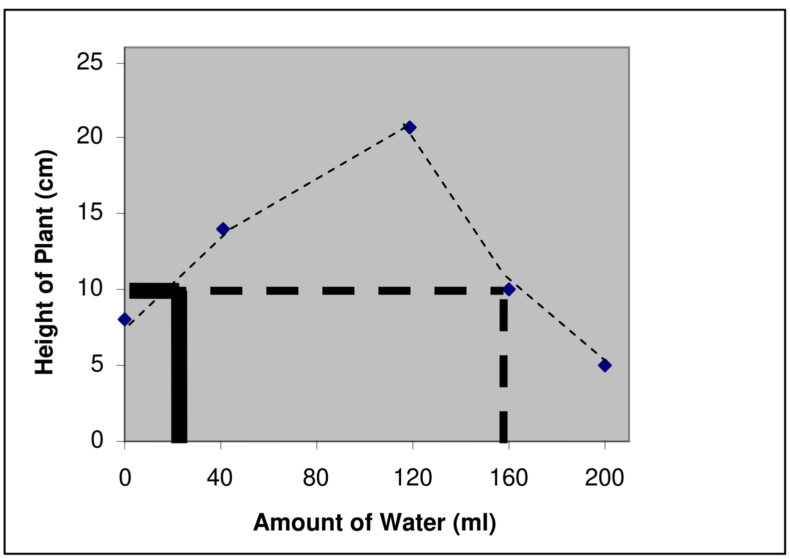


Figure 7: A representation of Henrietta Harmon's approach to TOGS question 3.

The thick solid line segments represent Henrietta's finger tracing a line on the graph from point (0, 10), across to (20, 10), and then down to (20, 0) as the interpolated between points (0, 8) and (40, 15) on the graph. The thick dashed line segments represent the path she should have drawn as the identified the point (10, 160). The thin dashed lines represent the imaginary lines between the points on the graph.

In other words, she inferred that this was an interpolation question (Intermediate Level Indicator 2.3: Infer an omitted point between data points). Instead, the intent of question 3 was to select a corresponding value of x for a given value of y on a given graph (Elementary Level Indicator 1.3). The result was that she decided the answer was 20 ml, when the answer choices provided were 120 ml, 140 ml, 160 ml, and 180 ml. When she saw her answer was not among the choices or even close to any of the choices, she first equivocated that the answer "would be, um, 40 and 80 would be closer than 120." She then said, "I would probably guess and say A (120) because it's closer to 40 and 80," and moved on to question 4. Rather than questioning herself when she arrived at an answer that was not even close to any of the answer choices provided, she, instead, simply chose the closest possibility.

Her behavior was in stark contrast to the Independent and Inquisitive students. When Heather was confused about the intent of question 3, she said, "I'm guessing that..." but she then went back and reread the question until she clarified for herself how to proceed. On question 2, when Heather arrived at an answer of 15 and saw that the closest answer was 16, she declared, "Well, I'm guessing it would be 15 cm, but because 15 is not on the multiple choice and the number 16 is right above it, that's what I'd probably guess (answer choice) B." Finally, on question 8, Heather asked the researcher if it would be okay to guess the answer. Rather than simply guessing, she proceeded to reread and think through the question and her background knowledge, a total of 483 words on the think aloud transcript, until she arrived at the correct answer. Whereas Henrietta Harmon was quite willing to settle with a guess when her answer choice was not among the available options, every time that Heather Miller said that she needed to guess, she initiated an examination of her thinking and her approach until she was completely satisfied and both times, Heather ended up choosing the correct answer after using the extra time. The result of Henrietta's guessing was that she had the second lowest score of all the students in the sample, 62%. Only Lydia Lynn (54%) had a lower score than Henrietta.

From this analysis, question asking may be a strategy helpful for preventing and identifying errors as well as encouraging a deeper understanding of the TOGS material. These students appeared compliant and reserved in their outward appearance because they were reluctant or unable to ask their own questions while answering the test questions. Even when they produced answers that were clearly incorrect, they seemed satisfied to merely proceed with the test instead of taking the time to go back and generate their own questions to confirm the accuracy of their answers.

Few Strategies but Decent Science Group's Graph Interpretation

Matt Maples and Misty Murphy were initially grouped together because they shared a few obvious characteristics—both students scored a Level 3 on the 2006 5th Grade FCAT in Mathematics and Science, both students answered all elementary level questions correctly; both answered one external identification question incorrectly; and the remaining questions they answered incorrectly were intermediate level questions. Closer analysis of the transcripts revealed more interesting similarities that led to them being analyzed together. Both displayed significant informal science knowledge, neither displayed any inquisitiveness or independence, neither posed their own questions, and besides Matt's extensive use of drawing (nine of 13 questions), neither used many strategies (see Table 9) when solving the TOGS problems. While this was the smallest of the four groups, these two participants fit more closely with one another than with any other group members.

Table 9

Table 9 Few Strategies but Decent Science Group's Approaches to TOGS Questions

	ExtId	Interm	Elem	Interm	Interm	Elem	Elem	ExtId	ExtId	Elem	Interm	Interm	Interm
	Ques1	Ques2	Ques3	Ques4a	Ques5	Ques6	Ques7	Ques8	Ques9	Ques10	Ques11a	Ques12	Ques13
Matt Maples		Draw **Incorrect**	Draw	Draw, Ans Elim		Draw, Proced	Draw, Proced, Estim	Draw, Ans Elim **Incorrect**	Ans Elim	Draw, Proced		Draw	Draw
Misty Murphy	Ans Elim **Incorrect**	Draw **Incorrect**	Auto	Ans Elim **Incorrect**	Auto	Mnem(2)	Self-Correct, Auto, Proced		Self-Correct	Proced	Ans Elim	Draw **Incorrect**	Auto

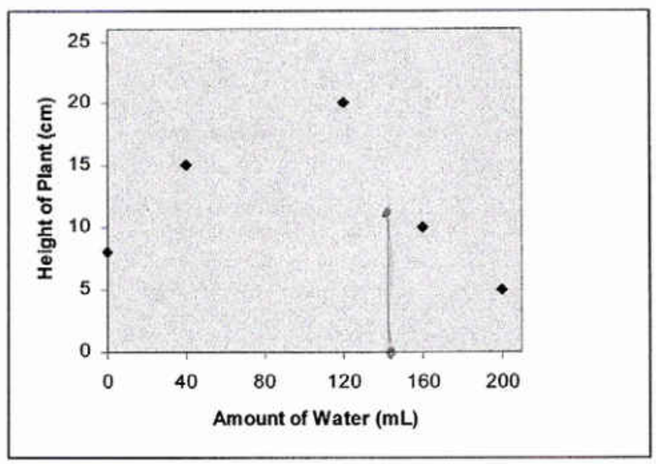
Relatively Few Heuristics

Table 9 delineated the heuristics used by this group. Matt and Misty each used fewer strategies than did each member of the Compliant and Reserved group and, again, many of these strategies were not helpful (per student comparison). Matt drew, almost habitually, on nine of the 13 questions (commensurate with Hester); while this strategy was often helpful, on a few occasions it seemed superfluous and on two occasions it did not help him to answer the questions correctly. Beyond drawing, he only used strategies on six other questions, often in conjunction with drawing. Misty used a greater variety of strategies and used a strategy on all but one question. However, these strategies did not help her on the four questions she answered incorrectly. In sum, the students in this group used fewer strategies and the ones they did use were less helpful.

Answered Intermediate Level Questions Incorrectly

Students in this group struggled mainly with the intermediate level questions. These questions required them to coordinate and calculate multiple pieces of information from the graph. This problem was seen most clearly in Matt's answer to question 2 and Misty's answer to question 12— they were qualitatively correct, noticing the trend, but quantitatively incorrect.

For question 2, which required him to interpolate, Matt placed the point first at 12 and then drew up from 140 to his point. Students who answered this question correctly, implicitly or explicitly, drew two line segments—one between the two points on the graph and another up from 140 to the first line. Unfortunately, Matt only drew one line segment where he should have drawn two and seen where they intersected. While using a drawing strategy was an appropriate heuristic for this question, his use of the strategy suggested that he did not fully understand how this strategy should have been used for an interpolation question.



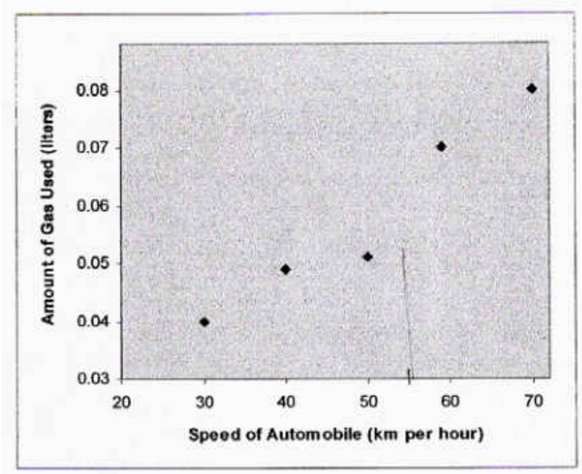


Figure 8: Matt and Misty's incorrect interpolation drawings on TOGS questions 2 and 12. Figure on left: Matt's drawing for question 2. Figure on right: Misty's drawing for question 12.

Matt answered question 12 correctly by using the correct procedure. While Matt attempted to defend his answer, saying that the plant height would have dropped more quickly after 120 ml of water, he did not explicitly argue against the test as Heather Miller did. This question was also interesting because Matt used his science knowledge to verify his answer choice, which was qualitatively correct. Misty made exactly the same mistake on question 12, suggesting that she also did not understand how to use the drawing strategy to answer an interpolation question.

Misty's answer to question 2 suggested a different kind of problem with answering intermediate level questions. After she read the question which asked her to identify where 140 ml would be on the x-axis, she marked the location with a point (see Figure 8). Her behavior suggested that she believed the trend of the graph continued upward after 120 ml and ignored the implied downward trend of the graph. Her incorrect assumption led her to treat this as an extrapolation question and then continued to estimate the answer to be 23 cm. However, in contrast to Matt, who actively sought to understand the physical reality behind this graph, Misty did not spontaneously check her answer against her understanding of the situation represented or her science knowledge and, as a result, her error went uncorrected.

However, when Misty tried to answer question 4, she seemingly recognized that the graph went downward after 120 ml, but made no mention of her earlier mistake (and never went back to change her answer to question 2). However, while she was able to correctly describe the shape of the graph verbally, she was unable to choose the correct answer from among the multiple choice answers. This seemed to be a simple reading comprehension problem caused by misreading the words, "increasing" and "decreasing." Her ability with this skill was further suggested by the fact that she was able to answer question 11 correctly, which assessed the same skill as question 4. Taken together, her incorrect answer for question 2 combined with her verbally correct answer for question 4 suggested that Misty lacked the spontaneous question asking tendencies of the Independent and Inquisitive group and instead behaved more like the Compliant and Reserved group.

Answered External Identification Stage Questions Incorrectly

The Compliant and Reserved group's tendencies were also observed during Misty's think aloud protocols for question 1, which required her to closely examine 4 alternative graphs as the most appropriate for a given set of data. Her review of the alternatives was best described as cursory, though she seemed to be using an answer elimination strategy and, in the end, guessed at an answer. Her verbal protocol implied that she did not notice the problems with the ranges and intervals on three of the choices, and by using words like “maybe” and “probably,” this strongly suggested she was uncertain of her answer choice. While she used answer elimination as a strategy, which would have been appropriate, she did not have the ability to eliminate incorrect answers.

By contrast, Matt’s answer for question 8 implied that he simply lacked the procedural knowledge or a mnemonic to determine independent and dependent axes. Indeed, his thinking on this question was impressive. His think aloud protocol was very thorough and he created a detailed drawing that showed his reasoning about the relationship between amount of water and time to boil (Figure 9). He also eliminated two of the incorrect answer choices because he knew that water always boils at 212˚F – one of the few students to explicitly state this knowledge— so he knew there would be no reason to measure the temperature of the water in this question.

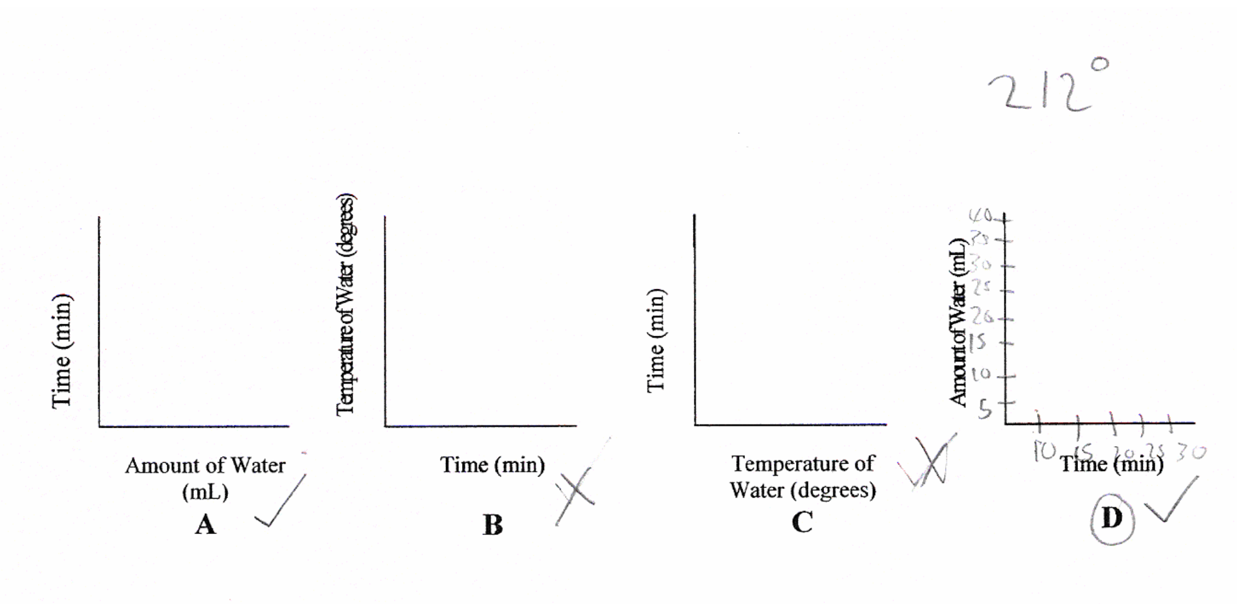


Figure 9: Matt’s drawing for TOGS question 8.

As he drew he also displayed fairly detailed knowledge of heat transfer for a sixth grader. Unfortunately, he answered the question incorrectly because he did not have the main knowledge and skill assessed by the question— knowing the convention of independent and dependent variables. He demonstrated this skill on question 9, however, making it unclear why exactly he answered question 8 incorrectly.

Informal Science Knowledge

One of the factors that distinguished this group from the Compliant and Reserved group was their relatively robust informal science knowledge, though they demonstrated it in somewhat different ways from one another. Matt evoked his knowledge spontaneously on several occasions, either to help him to answer a question or as a quick check on his answer selection. Misty, by contrast, only discussed her science knowledge when prompted by the researcher to elaborate her thinking.

Matt's science knowledge helped him on questions 2 though 5 to see and understand the trend in the graph, at least qualitatively. On question 2 Matt spontaneously mentioned, "A, that's right over 160 cm and because um, more water was given except for120 it was going down and 40 and 0, so I thought that 140 would still be going down, so I didn't put it at 16, I put it at 11, A." On question 4 he indicated that answer choice C was incorrect, "because after 120 ml the plants started decreasing." When he answered 4b he said, "I think that happened because the plant was getting too much water and could not take all of it in so it started shrinking or shriveling." For question 5 he indicated, "I would say A less than 5 cm because it looks like it is decreasing the height of the plant, decreasing after 120 cm." When prompted by the researcher after answering question 5 he was able to give a fairly robust explanation of the science involved in the question.

> Rose was conducting…she gave more water to the… more or less water to the plant would it grow higher or would it not grow at all. After she watered the plants everyday and after three weeks she would measure the height of the plant…. She found out that 120 ml of water a day, if you wanted to grow a very tall plant, that would be the best reasonable answer for this problem 'cuz it is 20 cm and all the rest are 15, 10, 7, and 5.

It seems that on several occasions while answering questions 2 through 5 he used his understanding of the science behind the graph to guide his answers.

A more direct example of his science knowledge was seen on question 8, which was analyzed in the last section. He clearly used his knowledge of the properties of water and thermodynamics to eliminate the incorrect answer choices and verify for himself that his choice was correct (even though it was not).

> I would not choose B because it has temperature of water and Danny does not want to know that…*212 degrees is boiling…should already know that*. And C is time in minutes and temperature of water, no, 'cuz it's the same but swapped. Check for amount of water. I might choose D because it goes amount of water and time in minutes and that has what he wants to know. (emphasis added)

Unfortunately, he was confused about the external identification stage knowledge needed to correctly differentiate the independent and dependent variables:

> I showed that because A with the amount of water on the bottom and time going up, and I think it would be more reasonable for time to be on the bottom and amount of water to be going up.

After prompted by the researcher why he did not choose answer A, he explained:

> I didn't choose A because time is, on most things like the thing that kind of the most important and in this case it would be time, usually goes on the bottom like what color a person likes best like on the bottom it would be pink, blue, purple or green and on the top it would be 5, 10, 15, and 20. So I usually think the most important thing would go on the bottom and on this one, A, that is not how it is…the most important thing is on the side.

He ended by elaborating on his experiences boiling water while cooking and had already noticed that, "It would take longer because in a small pan it's *less substance to heat up* to boil, and in a big pot, *there's more substance to heat up and boil*." (emphasis added) While the science knowledge that he used in this question was still best described as informal, it was, nevertheless, robust enough to help him.

Interestingly, however, in question 9 he continued to explain his procedural knowledge for determining the x- and y-axes, stating:

> I think I would choose B because as I said in the previous question I like to put the most important thing on the bottom and in this case it would be weight of chickens because you want to know if you should get a fatter chicken and maybe it would have more eggs or just a normal size chicken and it would have more eggs.

Unfortunately for him, his reasoning in question 8 was incorrect whereas his reasoning in question 9 was correct. In other words, his simple strategy for determining independent and dependent variables did not consistently help him to answer the questions correctly.

While answering question 11a he spontaneously offered, “On the first one it’s 30 km only 4 hundredths so it obviously increases the speed of the automobile and the amount of liters used.” This suggested that he implicitly understood that increased speed led to increased fuel consumption. He confirmed this understanding when answering part b:

> I think this happened because the car going faster would need to use more fuel and going farther so the engine would have to run…would have to work harder to get the wheels moving faster and farther so the engine would need more fuel for the car to keep going.

His answer provided the most sophisticated and articulate understanding of the science behind fuel consumption of any student in the study.

On question 13, by contrast, his science understanding broke down. When the researcher asked Matt what Lynn found out in her study, he answered:

> I think she found out that you don’t have to put in 50 liters to go 50 km, you only have to put in about five hundredths of a liter to go that far…you don’t have to put in very much to go far.

Although he demonstrated informal science knowledge while answering many of the TOGS questions and understood that Lynn only traveled 1 kilometer, he nevertheless misunderstood the difference between speed (rate) and distance.

Overall, Misty Murphy displayed less spontaneous science knowledge than did Matt Maples. For example, while answering questions 2 through 4a, Misty did not spontaneously offer any science knowledge, but when answering 4b she was able to explain that, “Maybe because they were getting too much water,” the plants’ growth was being affected. After answering question 5, however, in response to questions from the researcher, she explained that she had considerable experience growing plants with her family,

> My mom’s growing strawberries right now and tomatoes. Her neighbor’s strawberries are growing better. I think it’s because we’re not giving the strawberries as much water. Yeah. They water theirs each day with a watering pot thing. We just take the hose and water ours. But if you give them too much water they don’t grow good. That too much water is bad, but in the middle is best.

Unfortunately, she did not spontaneously offer this information while answering the questions and, while she answered the questions correctly, there was no clear evidence that her background knowledge helped her.

Her science knowledge seemed to help her on question 8, however. Before even reading the answer choices, she noted that, “Time should probably go...Danny measured the time needed to heat various amounts of water so *amounts of water should go on the bottom* and then time” (emphasis added). When prompted by the researcher, she was able to elaborate that she understood the science behind her answer.

Finally, Misty seemed to use her science knowledge in question 11 to recognize the correct description of the data in the graph. She was able to briefly summarize the main point by stating, “Because the faster you go the more gas you use.”

Matt and Misty’s informal science knowledge seemed to help them as they answered the TOGS questions. Matt clearly used his understanding of the physical situation and science theories represented in the graph to guide his thinking and to check his answer choices. Misty made a less than overt use of her science knowledge, but it nevertheless seemed to help her on a few occasions. That said, neither student displayed obvious curiosity nor the independence displayed by the Independent and Inquisitive group members, though the reason for this was unclear.

Lack of Self-Generated Questions

The two students in the Few Strategies but Decent Science group were also characterized by not self-generating questions while answering the TOGS. In lieu of providing evidence of something that did not occur, the analysis focused on occasions when these students would have helped themselves by pausing to think. Such occasions could have included asking themselves if they understood the question, if their approach was appropriate, or if their answer made sense.

Such an occasion was seen when Misty failed to notice the incongruence between questions 2 and 4. In question 2 she believed the graph continued upwards and in question 4 she recognized that the graph went up then down. This incongruence did not lead her to question either answer. Misty never once spontaneously made a connection to her science knowledge nor questioned her incorrect thinking.

The differences between how Matt and Hyde used their science knowledge was also suggestive. Like Hyde, Matt spontaneously mentioned science information along the way, such as in question 2 and question 11 in the previous two sections, as a way of verifying his interpretation of the question. This was similar to Hyde Hegel on question 2. However, Matt answered the question without excitement, unlike Hyde.

Earnest but Confused Group's Graph Interpretation

The Earnest but Confused group earned some of the lowest scores on the TOGS and each member answered questions at every level incorrectly. Lydia and Lucy each answered four questions in a row incorrectly. This group also exhibited the most frequent and debilitating examples of significant cognitive breakdowns and outright guessing. On some occasions they seemed completely unaware of how to even approach the TOGS questions. When they did use a strategy, they often chose approaches that could not work for the particular question they were on, perhaps suggesting they had few strategies to choose from. Other times, they executed the strategy incorrectly, yet they seemed unaware that it was not working. Finally, all three verbalized their lack of confidence, albeit in different ways.

Table 10

Table 10 Earnest but Confused Group's Approaches to TOGS Questions

	ExtId	Interm	Elem	Interm	Interm	Elem	Elem	ExtId	ExtId	Elem	Interm	Interm	Interm
	Ques1	Ques2	Ques3	Ques4a	Ques5	Ques6	Ques7	Ques8	Ques9	Ques10	Ques11a	Ques12	Ques13
Linda Mills	**Incorrect**	**Incorrect**	Auto **Incorrect**	Ans Elim	Ans Elim	Proced, Auto	Proced, Auto	Ans Elim **Incorrect**	Draw, Ans Elim	Proced, Auto	Auto	Draw, Ans Elim	Ans Elim **Incorrect**
Lucy Mag	**Incorrect**	Arith **Incorrect**	**Incorrect**	**Incorrect**	Ans Elim	Proced, Auto	Proced, Auto	Auto, Ans Elim	Draw, Ans Elim	Draw, Ques Asking	Ans Elim, Self-Correct	Draw, Ans Elim	Ans Elim
Lydia Lynn		Arith **Incorrect**	Auto, Draw	Ans Elim,	Arith, Draw **Incorrect**	Draw, Proced **Incorrect**	Auto, Draw **Incorrect**	Proced **Incorrect**	Auto		**Incorrect**	Estim	Draw

Unawareness of Breakdowns

One of the defining characteristics of the Earnest but Confused group was their inability to monitor and correct their thinking when they did not understand what they were doing and when their strategies were not effective.

Comprehension errors

A simple example of a verbal nonsense was Linda Mills reviewing the wrong answer choices in question 11, "I think D is wrong (reads it aloud) because increase means going down and decrease means going up so it (choice D) would be incorrect." She had already answered question 11 correctly so this mistake did not affect her answer but did show an unawareness of her incorrect thinking.

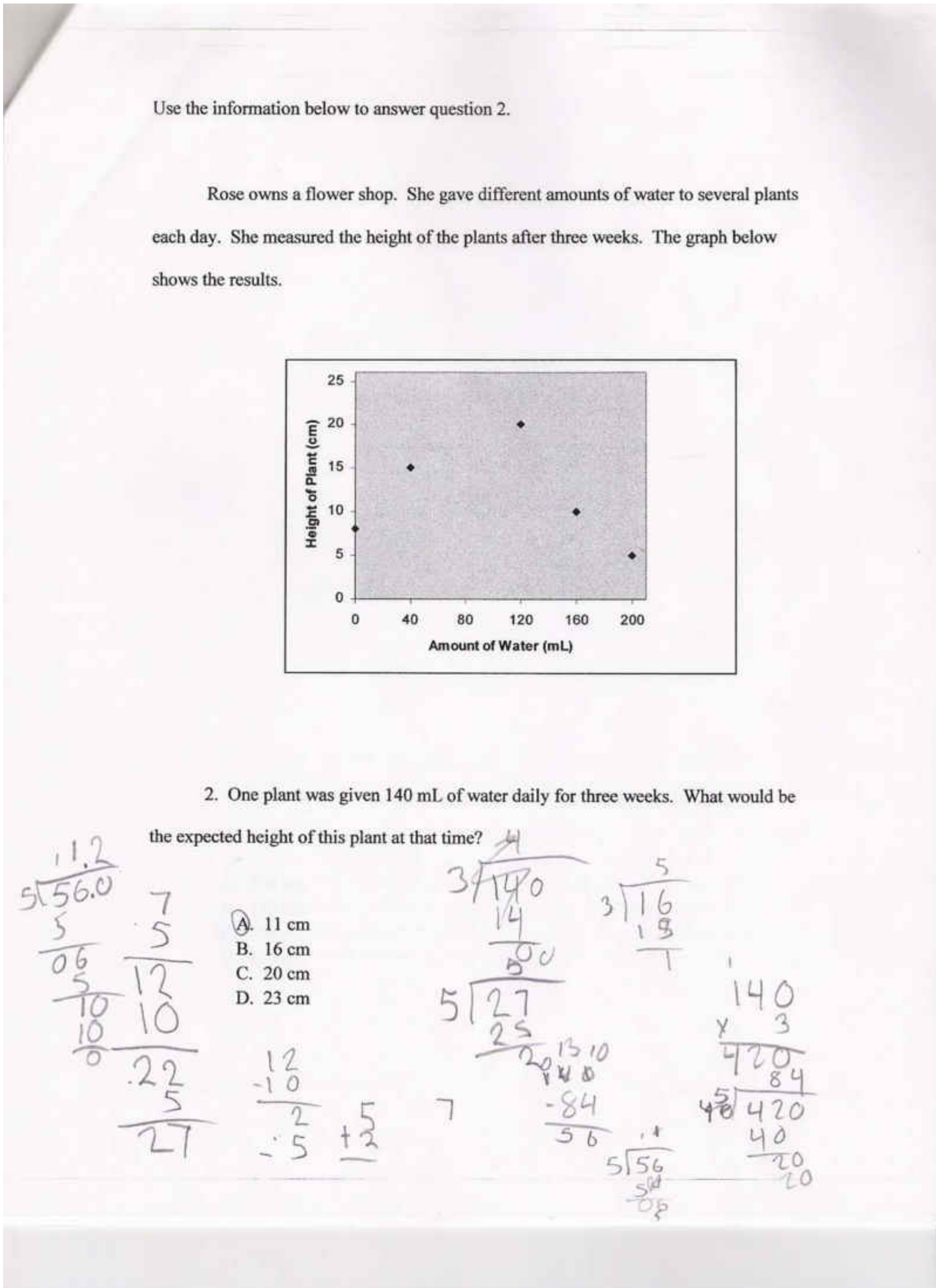
Lack of self-monitoring

While answering question 2, Lydia initially identified the data points of the graph by their y-values; however, she then proceeded, seemingly, to add the differences between the successive y-coordinates. Lydia Lynn exhibited a complete lack of self-monitoring, in addition to her significant breakdown in thinking, while answering question 2. She spent over seven minutes meandering through this question and, for no obvious reason, calculated the sum of the absolute differences between the y-values of each point in the graph.

> What I did first was I count 8 and 15 which I got 7 so I do 15 to 20 which is 5 first I'll add those, 7, 8, 9, 10, 11, 12. It'll be 12. 20 round to 10, so 20 to 10, you got to add that, 22.

She occasionally reread parts of the question aloud and proceeded to add those values and attempted to divide by 5, the number of data points (see Figure 10). She eventually settled on answer choice A when the last calculation, 56 divided by 5, gave her 11.2 which was closest to 11 cm.

Her purpose in this activity was unclear. She could have been attempting to calculate a mean, vaguely remembering a procedure learned in class. Or, she may have been unaware of what procedure to follow but wanted to "save face" and chose to perform calculations until she stumbled upon something close to one of the four answer choices. When she found herself in trouble, she either did not recognize her peril or lacked the awareness or strategies like answer elimination, mnemonics, or answer checking to help herself.


Use the information below to answer question 2.

Rose owns a flower shop. She gave different amounts of water to several plants each day. She measured the height of the plants after three weeks. The graph below shows the results.

2. One plant was given 140 mL of water daily for three weeks. What would be the expected height of this plant at that time?

A. 11 cm
B. 16 cm
C. 20 cm
D. 23 cm

Figure 10: Lydia Lynn’s computations while answering TOGS question 2.

There was no evidence in the verbal protocol that Lydia developed any understanding of what her final answer for question 2 signified. At the end of her calculations, she had a number (11. 2) that was close enough to answer choice A (11 cm) for her to decide that she had already completed enough calculations. She seemingly made no effort to check whether her answer choice made sense on the graph nor did she attempt to interpret her answer choice in the context of Rose's experiment. Lydia's struggle to choose a helpful strategy led her to be mired in an arithmetic procedure and lose sight of the purpose of the question. She lacked the perception attention to relate the problem solving activity to the purpose of the problem.

Lack of Confidence

Another characteristic of this group was that they lacked confidence in their abilities. This was seen in the frequent use of negative self-talk and face-saving verbalisms.

Negative self-talk

Lucy Mag presented a very interesting case of the relationship among negative self-talk, affect, and achievement. Through the first four questions on the TOGS she appeared to be expending little effort on the test. On questions 1, 3, and 4, she seemed to simply guess at an answer and provided little evidence of the thinking that supported her answer. On question 2, discussed above, she used an inappropriate arithmetic procedure. In addition, in questions 2 and 3, she made verbalizations that suggested that she was confused. On question 2, she began her answer by commenting how weird the question appeared and went on to use an incorrect procedure. When prompted by the researcher why she thought the question was weird, she replied, "Because it has like a lot of the milliliters and that kind of stuff and I'm not very good at that stuff so I thought it was weird." In question 3, after making a perfunctory effort she stated, “So, okay, well, this is a hard question because the wording is a little bit tricky…” She then ended her answer abruptly stating, “…but I think I have to say 140, B.”

However, from question 5 onward, there was a gradual shift in her affect as she began to recognize some of the question types from her mathematics class and books. On question 5 she was finally able to use a procedure to answer the question and she answered the question correctly. By question 6 she explicitly noted the connection between the question and math class, “Okay, it’s just like what we’re learning in math now. So, point a’s right there, 20, go up to 12, so it’d be (20, 12), B.” Once that connection was made, she applied the correct procedural knowledge with automaticity. This continued on to question 7 when she ended by answering the researcher’s prompt for her thinking by stating, “I was thinking this was a really easy question because this is what we’re learning in math right now.”

On question 10, which assessed the same skill as question 3 which she had answered incorrectly, she started with a little self-pep-talk. “Okay, I don’t like these either. I’m going to try my hardest to think of these kinds of things.” She worked through the answer for awhile and was able to correctly read the individual points on the graph. However, she struggled to connect that information with what the question was asking her to do. She then posed a question for herself, paraphrasing the test question – the only instance of a student not in the Independent and Inquisitive group to do this. “Okay, so how much gas did she take to drive 60 miles per hour? How much gas in liters?” She then engaged in another bout of negative self-talk, “Well, the wording is a little bit confusing in my brain to process...” But unlike the beginning of the test, she was now able to refocus and then had a revelation that helped her to correctly answer the question:

> ...used to drive, okay so if she wants 60 down here then 1 here I think it would be about, Oh! Oh! ‘cuz 70 was Oh! Okay, so then since 60 was about in this range (points to the graph) so probably be C.

The language she used here was almost identical to that used by Hyde Hegel in question 2 when he used self-questioning and, consequently, finally understood the intent of the question and its relationship to the graph.

It seemed that at the beginning of the test, Lucy Mag was unsure of herself and feeling anxious, perhaps because of her poor mathematics achievement in the past. But whether because she started to see that she was able to answer the questions correctly or because she saw the connections with her math class, as her affect improved her negative self-talk all-but-disappeared and she was able to answer all of the remaining TOGS questions correctly.

Saving face

In contrast to Lucy Mag’s negative self-talk, Linda Mills and Lydia Lynn never explicitly said that they lacked confidence in their answers or their ability to answer the TOGS questions. Instead, they used vacuous phrases to save face and to avoid being seen as unable to answer the TOGS questions. Their verbal protocols were short and terse, so the researcher frequently needed to prompt them to recall their thinking while answering the TOGS questions. Linda engaged in face saving frequently during the test, while Lydia engaged in it only a few times. It was noteworthy that the less able student who answered more questions incorrectly felt the need to save face less often.

When prompted to recall her thinking during question 1, Lucy stated, “I remember thinking that was the best choice and that it had the most right information and it was the correct answer." After question 2 she stated, "I was thinking that it was the most…what's the word I'm looking for…the most reasonable answer because the others were too low." By question 9 the researcher felt the need to remind Lucy to recall rather than explain the answer during the think aloud. Nevertheless, Lucy responded vapidly, "What was going on in my head was that it was the only possible choice and that the other answers didn't seem right." These vapid non-explanations were pervasive throughout her protocol, even when she answered the questions correctly and when her protocol indicated that she probably did understand what she was doing. Rather than saying that she did not have anything to add or did not know how to answer the researcher's prompts, she said nothing eloquently.

In contrast to Linda's frequent face saving, Lydia participated in face saving only twice. After answering question 1 incorrectly, she was asked by the researcher to clarify the reason she rejected answer D. "Because I think that you should have more room at the top than having no room." This was a reasonable answer to ensure that the range on the y-axis was great enough. However, when asked to think aloud what she meant by "having more room" she replied, "By having more room as in between here to show everybody how much room that it took and how much length." This answer suggested that she could not explain her reasoning but was unwilling to simply say so.

Lydia used this approach again on question 9, which she answered incorrectly. When asked by the researcher to recall the reason she rejected answer choice A, she replied, "It's that I don't really think that they want to know to put it in the other order. I think they should put it in the order like this, like B instead of doing it like A." Unfortunately, this explanation explained nothing. When further prompted by the researcher, she replied with a pseudo-procedural explanation:

> *What I usually do is I don't usually* put the weight of chickens like I put like this then *I put* the grams like underneath there and *I usually put* like number like this number of eggs and then I put the height of chickens. See you can usually have more room to put all the numbers and you can just put it down like that. (emphasis added)

This pseudo-procedural explanation seemed intended to add authority to her answer, suggesting that she frequently constructed graphs about chickens and eggs.

Procedural Problems

The final defining characteristic of this group was that they had difficulty choosing the correct procedure and had difficulty executing procedures correctly. More than anything, this led to their low scores on the TOGS.

Procedures applied incorrectly

In question 1, Lucy Mag mistakenly found the misplaced interval spacing within the table instead of the graphs themselves and verbalized the pattern she saw. What she did not notice, unfortunately, was the incorrect interval on the y-axis of answer choice A:

> Find the pattern (in the data table), 3 to 4 would be one, 4 to 6 would be two, 6 to 8, 6, 7, 8, 8, 8…okay, look at the graph and see which one has the best results to give me my answer. (Read the y-axes on answer D then answer C, and then answer A, then answer B.) I think it would be A because of the differences that there was, A.

When asked why she chose answer choice A instead of B, Lucy replied, "I chose A instead of B because I figured because of the differences between the numbers of tomatoes produced and the graph and the numbers, I thought it'd be A rather than B." Her procedure of looking for differences was correct. The problem was that she was looking for differences within the table instead of on the axis label of the graphs.

Lydia Lynn used arithmetic (see Figure 11) to solve question 5 which was an extrapolation question. In order to solve this problem, Lydia wrote the numeral 7 at location (40, 10), 5 at (80, 17), 10 at (140, 15), and 5 at (180, 8). She seemed to be estimating the distances between data points on the graph. She calculated three addition problems on her test page and came up with the final answer of 27. Lydia appeared very steadfast in her approach and she never went back to the graph to check her answer choice. Had she checked and compared her result to the points on the graph, she may have seen that her answer choice was unreasonable:

> (Reads the answer choices) By adding this before, what I've got was, let's see, well that would be 8, no 7, that would be 5 right then and there, that's 20 that's 10 so that would be 10 (the difference) then 10 to 5 would be 5. That would be 15 that would be 20 then 27. So 7 plus 5 plus 10 plus 5. 7 and 5 would make 12 make that 22 and make that 27 so what I think the answer would be is D, more than 20 cm.

When asked by the researcher what she was thinking that made her pick answer choice D, Lydia replied, "Well, if you add up all of the heights and the growths, um, you'd see that if you add 7 and 5, 10 and 5 (refer to the graph where she wrote these amounts) you'd equal 27 cm."

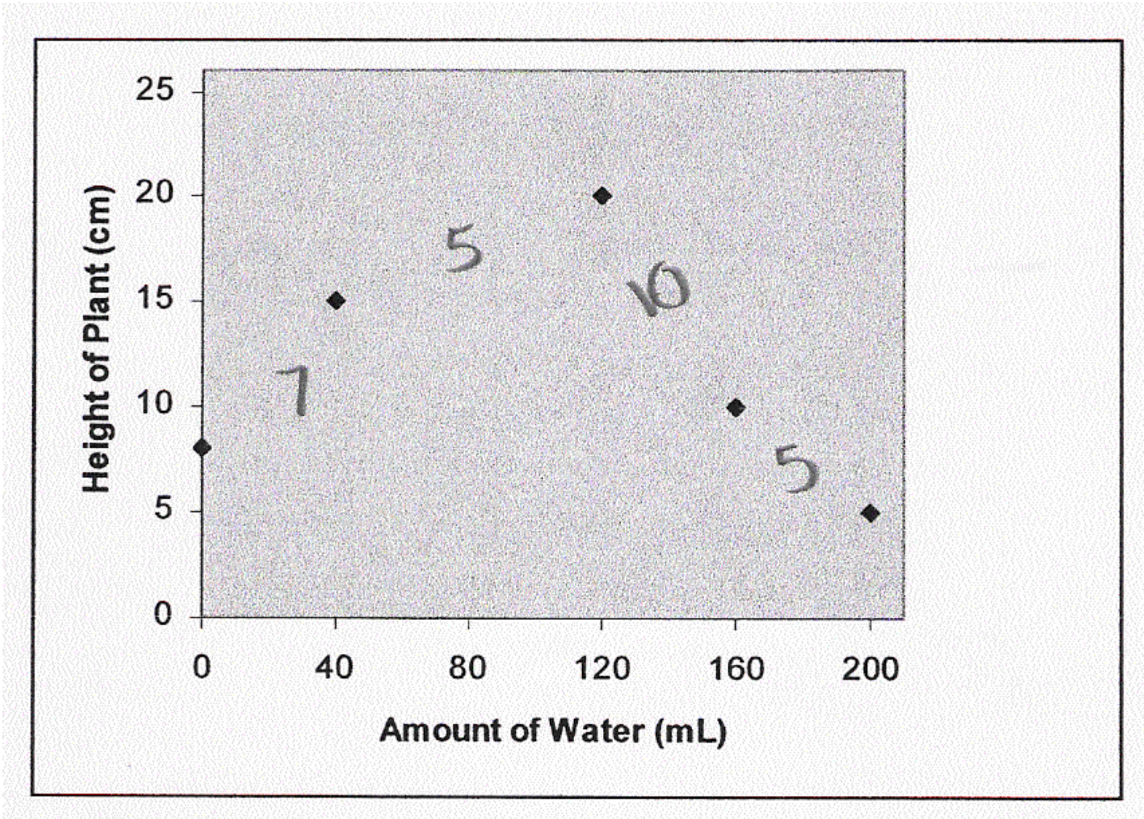


Figure 11: Lydia Lynn's drawing on TOGS question 5.

Lydia made the same procedural mistakes answering questions 6 and 7. While answering question 6 she drew a horizontal line from (0, 12) over to point a (20, 12) and a vertical line from point a down to (20, 0) (see Figure 12). This made a rectangle. She then labeled the x-axis "x 2" and the y-axis "y 1." Instead of correctly identifying the coordinates for point a using (x, y) notation, Lydia reversed the order and incorrectly used the coordinates (y, x):

> Well, point a is that you need to go up, you go up the y axis and then the x axis. So this would be 1 (labels the y-axis) and that would be 2 (labels the x axis) so it'd be (12, 20) and that would be answer D.

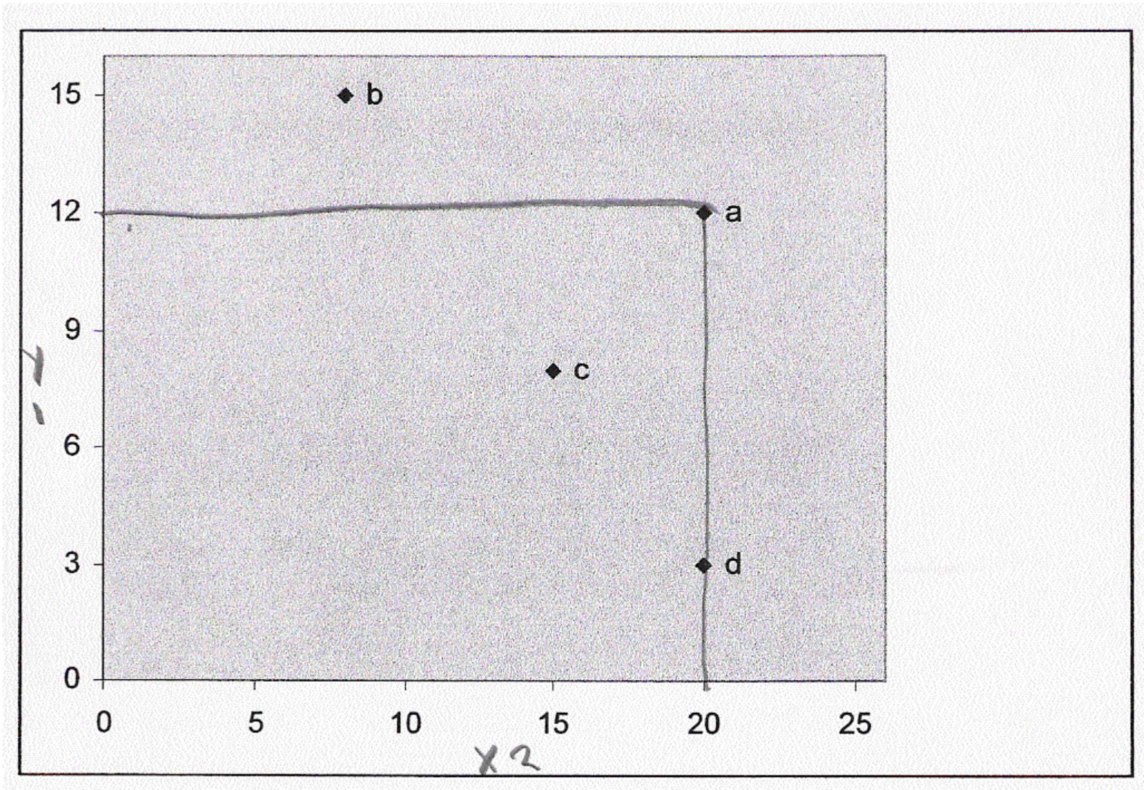


Figure 12: Lydia Lynn's drawing for TOGS question 6.

Although she had a procedure for finding the location of a point on a graph, she did not have the correct procedure. Had Lydia used another heuristic like a mnemonic to help her remember the correct order, as other participants did, she may have remembered the correct procedure.

Poor choice of procedures

Lucy Mag tried to answer question 2 using her multiplication tables. She noticed a pattern of the y-values and was lost from that moment on:

> Weird question…reminds me of some of the stuff that would be on my math tests…okay, so the height of the plant, the amount of water, so kind of weird 'cuz you kind of have to measure these things but that one is 15, this one is 0, this one goes up to 20, this one goes

> all the way back to 10, and this one goes to about 5 so I think it would be 20, C. I was thinking that this is kind of a weird problem but if I focus on my thoughts of my 5 times tables I could maybe get this right because the amounts, the heights were in the fives and the differences were going between the fives so I figured…

In this example, Lucy seemed to be at a loss for how to approach the question and impulsively chose to focus on the first pattern she noticed. She seemed unnerved to be working on a math problem, “Weird question…reminds me of some of the stuff that would be on my math tests.” She tried to calm herself by focusing on her “thoughts,” which became a motivational strategy for her later in the test. She noticed that y-values are approximately intervals of five and inferred that it was a regular sequence. There was nothing redeeming about her approach to the problem.

The Earnest but Confused group had the lowest prior achievement scores on the FCAT and the TOGS. While they did manage to answer many of the questions correctly and to demonstrate reasonably good understanding at times, it was their cognitive breakdowns, poor strategy usage, and frequent inability to self-monitor their performance that characterized their performance on the TOGS most acutely.

Summary of Findings for Research Question 1

Research Question 1 asked: How do student behaviors observed during think aloud interviews vary during line graph interpretation in science across mathematics and science achievement levels? To answer this question, 13 6th-grade students of varying math and science abilities were asked to think aloud while answering 13 questions excerpted from the Test of

Graphing in Science (McKenzie & Padilla, 1986). Participants were recategorized into four groups based on their prior academic achievement and behaviors observed during the think aloud interviews (Table 6). Only two participants were placed with members of different prior science achievement on account of the similarities in their performance on the TOGS. These two participants were Heather Miller and Henrietta Harmon.

The analysis of students' interpretations of graphing yielded key findings that included differences in the use of: heuristics, self-generated questions, science knowledge, and self-motivation.

Use of Heuristics

1. Students with higher prior achievement used a greater number and variety of strategies while answering the TOGS questions.
2. Students with higher prior achievement more often chose strategies that helped them to correctly answer the questions.
3. Students with lower and moderate prior achievement tended to favor one particular strategy most often, even when that strategy was not particularly useful for answering the question.

Use of Self-Generated Questions

4. Students with higher prior achievement monitored their understanding of the question, the adequacy of their strategy, and the adequacy of their answer by asking themselves questions.
5. Students with lower prior achievement rarely monitored themselves while answering the questions by asking themselves questions.

Use of Science Knowledge

6. Most students with higher prior achievement and one student with moderate prior achievement used their science knowledge spontaneously while answering the questions to check their understanding of the question and the adequacy of their answer.
7. A few students with higher prior achievement used science knowledge beyond what was required to answer the question, demonstrating inquisitiveness.
8. Students with lower prior achievement and some students with higher and moderate prior achievement did not spontaneously evoke or use their science knowledge while answering the questions, even though they could discuss their knowledge when prompted by the researcher.
9. In many cases, students with lower prior achievement would have been able to recognize their errors if they had thought about their answers in the context of their science knowledge.

Motivation

10. Two students, one with higher prior achievement and one with lower prior achievement, expressed excitement while answering the questions when they overcame their confusion.
11. One student with lower prior achievement seemed to motivate herself while answering the questions when she thought the questions were too difficult and when her understanding was breaking down.

These findings provided the context and background to be used for Research Questions 2 and 3 which analyzed student responses according to graph question level and patterns emerging from student thinking during line graph interpretation given their prior performance in mathematics and science.

Research Question 2: Analysis of Data by Graph Question Level

The primary focus of this section was to answer Research Question 2: How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)? To answer this research question, Roth and Bowen's (2001) two-stage semiotic process of reading graphs (see Figure 13) was used to reanalyze students' thinking while they interpreted line graphs. In the analysis for Research Questions 2 and 3, references in the think aloud protocols were coded as follows:

Aspect of Roth & Bowen's Model	**Codes**
Perception attention	PA
Graph	G
Text (in the question)	T
Salient structures	SS
Sign	S
Referent	R
Interpretant	I

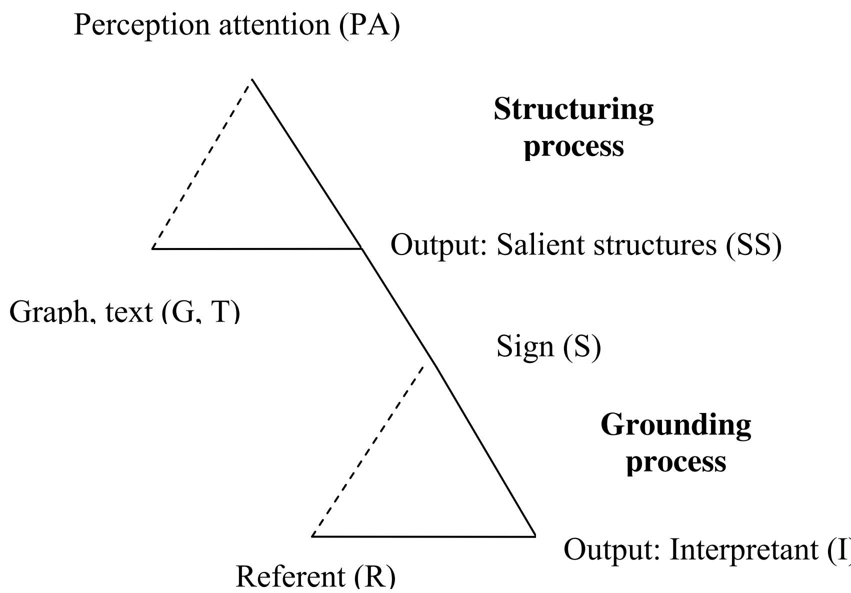


Figure 13: Roth and Bowen's two-stage semiotic process of reading graphs and codes used to analyze think aloud protocols.

This coding scheme showed how the aspects of Roth and Bowen's model appeared and interrelated in the think aloud protocols.

This analysis highlighted two main findings in the data. One key finding from this analysis was that students approached the TOGS questions as if they were mathematics word problems, science data to be analyzed, or they were confused and they guessed. In the data, participants who only made reference to the elements of Roth and Bowen's top triangle (i.e., structuring) approached the questions as mathematics word problems. Participants who made references to elements of Roth and Bowen's top and bottom triangles (i.e., structuring and grounding) approached the questions as science data to be analyzed.

A second key finding was the roles that background knowledge played while answering these questions. Correct science knowledge bolstered students' reasoning but was not necessary to answer the TOGS questions correctly. In addition, correct science knowledge could not compensate for incomplete mathematics knowledge. Finally, incorrect science knowledge often distracted students when they tried to use it while answering a question.

Analysis of Intermediate Level Questions

Intermediate level questions afforded students the greatest variety of approaches. Specifically, students used three general methods when answering the intermediate level TOGS questions. They either answered these questions as if they were mathematics word problems, science data to be analyzed, or with confusion about how to approach the problem. The intermediate level TOGS questions were 2, 4a, 5, 11a, 12, and 13. There was relatively little difference among the questions themselves, so this analysis focused on question 2.

Using Roth and Bowen's two-stage semiotic process of reading graphs model (2001), this analysis showed that students who treated the problems as mathematics word problems stayed in the top triangle whereas students who treated the problem as science data to be analyzed engaged in both the top and bottom triangles of the model.

Treating an Intermediate Level Question like a Mathematics Word Problem

Huck Handy answered TOGS question 2 in less than a minute and answered it so automatically that he needed no heuristic to assist in his cognition. Notice that Huck remained in the top triangle of Roth and Bowen's model:

> (Reads graph labels) (PA) So it goes up by 40's (x-axis) (G) so the one given 140 (G) would probably be about 15, in the middle (SS), so I think it'd be B, 16 (T) because it was just about in the middle of 21 and 11 (y-axis) (SS).

To Huck, this question was simply a mathematics word problem asking him to interpolate between two data points. For Huck, this skill was automated because he answered it very quickly without relying on the use of a deliberate strategy. Huck answered this problem and remained in the top triangle of Roth and Bowen's model.

Hester Luck also treated question 2 as a mathematics word problem and, like Huck, remained in the top triangle of the Roth and Bowen model:

> Well, if she gave it 140 (T) then the height that's a lot so if we look at the graph (G), there isn't a 140 (PA) so I'm just going to estimate to right there (pointed to the graph). If we bring it up we have to draw (draws on graph) that so the line it's probably going to be a height probably of 16 (SS) an estimate probably 16 because it looks like it's the closest.

Hester's performance was not automated like Huck's but instead she used heuristics to solve the question like a mathematical word problem using estimating and drawing strategies. Nevertheless, Hester remained in the top triangle of the Roth and Bowen model.

Treating an Intermediate Level Question like Science Data to be Analyzed

Hyde Hegel answered question 2 differently than Huck and Hester. Hyde treated question 2 like a science problem with data to be analyzed. He first read the y-axis coordinates of the points on the graph prior to reading the question aloud (PA). "7, 7.5, 15, 20, 10, or 11, and 5 (G)." Only then did he read the question being asked (T), "One plant was given 140 ml of water daily for three weeks. What would be the expected height of this plant at that time?" He then demonstrated automaticity as he read the x- and y-coordinates of the points in the region of interest on the graph (G), "Let's see, with 120 ml of water, it came up to about 20 (G). If that was 20 and then160 ml was 10 (G)." Then, Hyde Hegel spontaneously interpreted (I) the line graph, "Seems like that one [pointed to the data point (160, 10)] was overdosed (I) with water which means that one [the data point (120, 20)] had just the right amount." In other words, without prompting, Hyde sought to understand the meaning (S) of what the graph represented, and inferred the physical situation (i.e., over-watering) that led to the graph (G) that he was seeing. It could be inferred that for some Independent and Inquisitive (PA) students like Hyde, this need to understand (S) the meaning and physical situation (R) behind the graph (G) was itself a perception attention (PA)—he was curious.

After he had temporarily satisfied his curiosity, he returned to answer the question. He then used answer elimination to rule out incorrect answer choices (PA). "I'm estimating, hmm, it would probably not be 11 'cuz that's around (160, 20) was about the amount of (120, 23) is higher than 120 which I think it would go decreasing (SS) instead of increasing over here which it did." As he did so, he tried to understand (S) the relationship between each choice of the y-coordinate (T) and the graph (G). Hyde then had an epiphany, "Oh! So, it increases then decreases 'cuz of too much water, too less water (R, I)." Here he again inferred the relationship

(I) between plants and the amount of water provided each day (R), developing a deeper understanding (S) of what was signified in the graph (G) and the text (T). Finally, he answered the intermediate level question stipulated in the text (T), "Huh, so that means the only other valuable choice would be B, 16 cm."

Hyde's curiosity (PA) encouraged him to understand and engage with question 2, posing a theory (I) to describe the data and then test his theory against the data and his understanding of the physical situation (R). The analysis using Roth and Bowen's model showed how his interpretation traversed and coordinated both triangles until he understood (S) what was represented.

Treating an Intermediate Level Question with Confusion

Lydia Lynn answered question 2 in a unique way. Had it not taken her almost eight minutes to solve, it would have simply been coded as treating the intermediate level question like a mathematics word problem. But she took almost eight minutes to answer it, and she used the most unusual arithmetic procedures that could only lead one to conclude that Lydia did not understand what question 2 was asking her to do. Unlike other participants, she did not self-question in order to clarify the problem. She also did not offer a different wording of the question like another participant had done. She just kept on calculating until she came to an answer that was closest to one of the multiple choice options (see Figure 10):

> What I did first was I count 8 and 15 which I got 7 so I do 15 to 20 which is 5 first I'll add those, 7, 8, 9, 10, 11, 12. It'll be 12. 20 round to10, so 20 to 10, you got to add that, 22… (She continues doing seemingly random arithmetic calculations for over seven minutes. She sometimes rereads parts of the question aloud. She settled on answer

choice A when her last calculation she did, 56 divided by 5 gave her 11.2 which was closest to 11 cm.)

Lydia's confusion and lack of self-correction led her to guess at the procedures. She was unable to relate this problem to one she knew how to solve, therefore, she focused on the incorrect salient features and applied an incorrect procedure that could not be helpful.

Another student who answered intermediate level question 2 with confusion was Linda Mills. She answered very automatically and used no heuristic to assist her. She articulated the salient structures (SS) of the graph and correctly identified 140 to be located between 120 and 160. Her confusion lay with understanding what the question was asking. Linda simply was confused and thought she was being asked to give the y-coordinate for 140 without noticing the abrupt change in decreasing slope of the graph:

> 140…I think maybe D, 23 ml, cm (self-corrects) because the scale goes 0, 40, 80, 120, 160, 200 (SS) (x axis) then going up it goes 0, 5, 10, 15, 20, 25 (SS) and 160, it's between that and 120 so that (D, 23 ml) would be the correct answer.

When prompted right after giving her answer as to what she was just thinking, Linda replied, "I was thinking that it was the most…what's the word I'm looking for…the most reasonable answer because the others were too low." She still failed to see that question 2 was an interpolation question instead of a coordinate question.

Analysis of Elementary Level Questions

Elementary level questions required the students to extract a single piece of information that could be found on one location on the graph (Bertin, 1983). Almost all students answered these questions as mathematics word problems and stayed on the top triangle of Roth and Bowen's model. Only Hugh tried to use his science knowledge to replace procedural knowledge, but it did not help him. On account of the straightforwardness of the questions, students had two choices of approaches with the elementary level questions. They could either answer them using a procedure or they could answer them without a procedure. Those who used their procedures correctly answered the questions correctly and those who answered quickly and correctly without an obvious procedure were said to have used automaticity.

The elementary level TOGS questions were 3, 6, 7, and 10. Participants across ability levels did not explore the bottom triangle of Roth and Bowen's model while answering elementary level questions. There was relatively little difference among the questions themselves so for the purpose of this analysis question 6 and 10 were selected for analysis in this section.

Using Procedures

Question 6 required students to give the proper coordinates of a point on a graph. Many students used a mnemonic to remember the correct order of the x- and y-coordinates. For students with automaticity using no mnemonics and no obvious verbal procedural knowledge, it was clear that the procedures were memorized. Other students verbalized their procedures and answered correctly when the procedures were also correct; conversely, when the procedure was verbalized but incorrect the answer was also incorrect.

Lucy Mag had her procedures memorized on question 6 but was still able to articulate enough of them for the researcher to get a glimpse into her mathematical thinking. "Okay, it's just like what we're learning in math now. So, point a's right there, 20 (G), go up to 12, so it'd be (20, 12), B (T)." Lucy correctly went over to the x-axis coordinate before going up to the y-axis coordinate. Even though Lucy had a low FCAT achievement score in math, the fact that she was working with this procedural knowledge in class may have helped her recall it quickly as she was able.

In contrast, Henrietta Harmon used a procedure on question 6 but the incorrect procedure was used. “Got to go up so up 12 and down 20 so it’d be (12, 20) (G), wait, yeah, (12, 20), D (T). Having scored a high achievement level on FCAT math did not help to answer question 6 correctly. None of these students explored the referents (R) or the interpretants (I) while answering the elementary level questions.

No Procedures Used

Hugh Hickson answered question 10 in a different manner than Lucy and Henrietta answered their elementary level question. He spent time with his think aloud and verbalized his dislike of the metric unit of liters. Although he thought this elementary level problem through, without the use of a procedure, he was unable to come to an understanding of what the question was asking and consequently answered it incorrectly:

> Gas in liters so liters…you have…speed of automobile (R), km per hour, he goes 30 it goes 0.4, it uses 0.4 (G) (mistake) it goes 60 you…the answers are (reads answer choices). And tell you the truth; I really don’t know how to figure this. Liters aren’t really my favorite thing, how would I do this? You would…speed of the automobile (R), 60 miles per, 60 km per hour. This is going 60 km per hour…and so if you were going 60 km per hour, you’re going a kilometer every…no…you’d go 60 kilometers in an hour so that would be one kilometer a minute (SS).

Until this point, Hugh was randomly searching for salient information and patterns that might help him. Then, he noticed that at 30 km per hour, the car consumed .04 liters of gasoline. He then incorrectly inferred that if the speed were doubled the amount of gasoline consumed would also be doubled.

> In gas, it takes 20, it goes 30 km for every 4, so it would have to go 8, you'd have to use 0.08 km, I mean liters of gas so it'd be on the graph it says… I'd have to go with the math so it has to be 0.08 liters so the answer is D.

Had he recognized that this was a simple graph reading task, he could have checked his answer. Instead, he relied on arithmetic to help him complete the rest of the problem. When Hugh was unable to understand what the question was asking him, he briefly descended into the lower triangle of Roth and Bowen's model to see if understanding the referent (R) could help him. This differentiated him from the other participants who used their procedures and answered the elementary questions as mathematics word problems. By acknowledging the referent (R), Hugh demonstrated that he may have been attempting to understand the graph and the physical situation (R) associated with the graph more deeply. Unfortunately, without the use of procedures, he was unable to answer the question correctly.

Analysis of External Identification Stage Questions

TOGS questions 1, 8, and 9 made up the group of external identification stage questions. These questions focused on assessing students' understanding of the conventions of line graphs in science and their ability to recognize graphs that were inconsistent with those conventions. Participants across ability levels had a difficult time with these questions. In fact, question 1 turned out to be the most challenging question on the TOGS for the participants. Students' responses to questions 1, 8, and 9 varied significantly; therefore, question 1 was analyzed by itself and questions 8 and 9 were contrasted. On question 1, the students either approached it with automaticity or were confused and guessed. On question 8, several students needed to understand the science involved in the question to answer it correctly, though a few answered it as a mathematics word problem. On question 9, by contrast, the students were mostly able to treat it as a mathematics word problem, although they made references to the physical situation (R) involved in the question.

Analysis of TOGS Question 1

Rather interestingly, question 1 was the most challenging question on the TOGS for students to answer correctly, with only 43% (6 of 13) answering it correctly (students had a 25% chance of guessing correctly). The question was designed to primarily assess students' ability to, "Select an appropriate scaled set of axes for a set of data (correct range and interval)." The question was presented to students as a story problem about choosing a set of scaled axes for a data set. However, three of the four answers could have been eliminated without reading the problem or data set by anyone understanding the conventions for constructing a line graph: graph A had irregular intervals on its y-axis (0, 2, 4, 5, 7, 10) and its x-axis (0, 100, 150, 250, 450, 500); the range of the x-axis for graph C started at 100 rather than 0; and graph D also has irregular intervals on its y-axis (0, 3, 5, 6, 7, 8). If students had recognized these problems, the only choice would have been graph B. Probably for this reason, answer elimination was the preferred heuristic, used by seven of 13 students, though it did not lead all of them to answer the question correctly. Very few students even noticed all of the problems with answer choices A, C, and D.

Every student was distracted by the question prompt and accompanying data table. Some treated it as if it were a word problem, trying to fit the data from the table into each graph. These students focused on the smallest and largest data points on the table and imagined if they would fit within the graph's axes. This approach distracted the students from being able to notice the problems with the ranges and intervals on the axes. The remaining students seemingly had no approach and seemed to move randomly through the question.

Among the six students who answered the question correctly, only Hans was able to eliminate all of the incorrect answer choices for the correct reasons. Of the remaining students who answered the question correctly, three also used answer elimination but did not always eliminate answer choices for the most important reasons. Hyde and Hester, for example, eliminated answer choice C because the y-axis went up to 600, not because it started at 100. Matt chose the correct answer B because it had the best range, but ignored that answer choice A had the same range. Overall, with the exception of Hans, their answer elimination strategy helped them to eliminate a few of the incorrect choices but not all incorrect choices. As a result, they answered the question correctly at least partially by luck. Finally, Lydia answered the question correctly, but she used no strategy and gave no meaningful reasons for her choice; she guessed correctly.

The seven students who answered the question incorrectly had many of the same problems as the students who answered it correctly; they were unlucky. Because almost all of the students were unable to focus on the salient features of the graphs to just eliminate the incorrect answers, a few eliminated answer choice B for the wrong reason. For example, Hugh Hickson quickly moved past B even though he gave no reason to eliminate it, and settled on choices A or D as viable options. He never returned to B. Other students simply seemed lost and at times eliminated answer choices for the right reasons but at other times eliminated answer choices for the wrong reasons.

The analysis of question 1 revealed that most of the students in this sample had great difficulty recognizing all of the salient features affecting the choice of a set of scaled axes. More specifically, only one was able to recognize all of the problems with the incorrect answer choices. Students in this sample lacked these skills and perception attention (PA).

Analysis of TOGS Questions 8 and 9

Questions 8 and 9 ostensibly assessed the same skill; "associate the x-axis with the independent (causal) variable and the y-axis with the dependent (effected) variable" (see Appendix A). Only 57% of students answered question 8 correctly while 79% of students answered question 9 correctly, and the students' ability to answer the question correctly was poorly predicted by either prior mathematics or science achievement.

In addition to students who could not consistently determine independent and dependent variables, there seemed to have been two other differences that accounted for the disparity. First, question 8 relied on students' abilities to coordinate multiple abstract concepts (time, temperature, heat, and heat transfer) whereas question 9 required students to relate concrete measures (mass and number). Second, question 8 also required students to remember that the temperature at which water boils was invariant (for the sake of this question). It is worth noting that ability level did not determine whether students chose to approach these questions as science data analysis or math word problems, though the students with higher prior achievement were more successful with either approach.

Treating External Identification Stage Questions like Science Data to be Analyzed

The TOGS external identification stage questions were not intended to assess students' background knowledge, so it was interesting to see that for seven of the students, background knowledge played a prominent role while they answered questions 8 and 9. Correct science background knowledge helped two students to identify the independent and dependent variables in these questions. Incorrect science knowledge, however, distracted several students. Even correct science background knowledge, by itself, could not help students to overcome their misunderstanding of the conventions of placing dependent and independent variables on the x- and y-axes.

Linda Mills typified students who answered question 8 incorrectly but answered question 9 correctly. She showed her preference for relatively concrete variables when she stated, "I think B because the amount of water (R) doesn't really have any effect on it if it's boiling or not and time does (I) and that is A. B is time and the temperature of water (G) and that seems the most reasonable…" After she had answered the question, she was prompted why she did not choose answer A (the correct answer) and she stated, "…option A is wrong because it has the amount of water in it and *the amount of water doesn't have any effect on if or how long it takes to boil* (I). It just the boil on time and the temperature of the water. The amount of water has no effect on (I), it could be a whole lot of water or only a little and it could still be boiling (I)" (emphasis added). By stating that the amount of water did not affect the amount of time it would take to boil, she also showed that she did not understand the science (I) behind the question.

By contrast, when Linda answered question 9 she seemed much more comfortable handling the concrete variables:

> I think B is the correct answer because number of eggs on the bottom for A (T) and on the top row it has weight of chickens (G) and B on the bottom row column has weight of chickens and the top one is number of eggs and C is weight of eggs which has nothing to do with it and number of eggs which has nothing to do with it. D is weight of eggs grams and weight of chickens grams which is in the wrong order plus the weight of the eggs has nothing to do with it.

Lydia Lynn and Huck Handy demonstrated a similar pattern in their answers, being confused by the science knowledge of boiling water related to answer question 8 but were aided by their understanding of chickens and eggs (R).

Hyde Hegel and Hans Hazel, however, were able to use their science background knowledge to help them determine the independent and dependant variables in both questions, but both also needed to connect their background knowledge to the graph by drawing a set of scales on the graphs. Because they approached this as a science question, both students initially struggled. Hans' answer was the briefer of the two:

> You need to measure various amounts of water boiling (T)…alright, (reads answer choice labels) time (G) minutes, amount of water, temperature of water, degrees , no, because we pretty much, it wouldn't be this one (answer choice B) because *all the water should be at the same amount of degrees* (I).

By recognizing that it did not make sense to measure the temperature of the water because water always boils at the same temperature, he could eliminate answer choices B and C. "It comes down to either A or D (T).” He then guessed that it should be answer A, but drew on the graphs to verify his guess.

> It’s obvious it’s, it’s, okay I got it…it’s A because the time minutes (G) should have it going up and then say amount of water, 100 ml (writes this example on the x-axis and draws a bar going up on answer choice A) to boiling point (R) (recognizes that there is one point for this to happen) 200 ml, it wouldn’t be right going up (I) ‘cuz then you’d say 1 minute, 2 minutes (writes these examples on x-axis on answer choice D) and then say 1 minute went up to 100 degrees but what if you didn’t have 1 minute? I believe that A is the better answer.

Specifically, he first drew a bar graph on graph A (“100 ml”) and imagined what would have happened for 200 ml. He then eliminated graph D by stating “it wouldn’t be right going up” while he drew time intervals on graph D (“you’d say 1 minute, 2 minutes”). Thus, similar to the way Hyde approached his answer, Hans started with his understanding of the science (R, I) and then had to draw on the graph to imagine which variable should have gone on which axis. Hans was working between both triangles in Roth and Bowen’s model and his drawing helped him to develop his theory (I) and his referents (R) to better understand (S) what was happening.

Correct science knowledge did not help Matt Maples, however. He correctly articulated a salient piece of science knowledge while answering question 8, “I would not choose B because it has temperature of water (G) and Danny does not want to know that…212 degrees is boiling (R)…should already know that.” Even with this knowledge, however, he could not correctly determine the causal and effected variables, or his firmly held belief that time would usually be placed on the x-axis:

> And so time in minutes and amount of water (G), amount of water and time in minutes (comparing answer choices A and D which have the same labels but in switched locations). I think I would choose D because amount of water is going up so it would be like 5 minutes, 10 min., 15, 20, 25, 30, 35, and 40 minutes. And then time on the bottom will go 10 minutes, 15 minutes, 20 minutes, 25 minutes, 30 minutes. I showed that because (answer choice) A with the amount of water on the bottom and time going up, and I think *it would be more reasonable for time to be on the bottom* and amount of water to be going up. (emphasis added)

Lacking another means of determining which variable to place on which axis, he simply determined that "it would be more reasonable" to place time on the x-axis. Similarly, on question 9 he was able to use his knowledge of chickens and eggs to determine the relationship between the two, but that reasoning did overcome his inability to determine which variable to place on which axis.

Finally, Hodge Leader also approached questions 8 and 9 as if they were science problems, but he was confused both about the relevant science knowledge and about independent and dependent variables. During question 8 he demonstrated his confusion when he asserted, "I think it is B because the *temperature of the water decreases* (R) so it will be hotter so it will get to boiling faster (I)." (emphasis added) When asked whether he had personal experience boiling water, he replied, "No, not really." However, the problem was not just his lack of science knowledge. When answering question 9, he was better able to deal with the concrete referents assumed in the question. While he initially rejected answer A, as he thought about it he recognized that it might be plausible,

> I don't think it's A because the weight of the chickens. I think it is A because the weight of the chickens might produce more eggs, so the weight of the chickens… the more grams maybe the more eggs are produced.

While the researcher did not ask whether he had personal experience with chickens, he was seemingly able to imagine that the mass of the chickens might affect the number of eggs they produce. His ability to imagine chickens (R) enabled him to develop a plausible theory (I) about the relationship which in turn enabled him to determine the correct variables for the graph (G). In the end, however, Hodge answered question 9 incorrectly because he guessed the causal and effected variables incorrectly, contrary to his high mathematics prior achievement but consistent, nonetheless, with his low mathematics prior achievement.

Treating External Identification Stage Questions like Mathematics Word Problems

While seven of the students treated questions 8 and 9 like science data to be analyzed, the remaining six students treated them as if they were mathematics word problems. The way Heather Miller approached question 9 was typical. After reading the question, she used an answer elimination strategy and discarded answers that did not contain the phrases present in the question:

> The weight of the eggs is on the horizontal axis. I don't think that would be right because he doesn't want to know the weight of the eggs so I would say that C wouldn't be one of the answers. And once again it asks for the weight of eggs but I really don't think he wants to find out the weight of the eggs so I would say that (answer choices) C and D would be wrong.

This left her to try to decide between answer choice A and B. Then she used her preference for counted variables on the horizontal axis:

> So that's (answer choices) A and B. I think that normally I see the number of something, like the number of eggs, would be on the vertical axis and the other would be on the horizontal. I would probably think that the answer to that one would be B.

In other words, she had no appropriate means of determining the x- and y-axis and simply got lucky with her guess. At no time while answering this question did Heather try to understand the relationship among the variables. Consequently she stayed in the top triangle of Roth and Bowen's model.

Henrietta Harmon typified students who were able to answer question 9 correctly but answered question 8 incorrectly because she understood the science behind question 9 but not the science behind question 8. While answering question 8, she quickly determined the correct phrases that needed to be found among the multiple choice options. "Time, so if he's measuring amounts of water, like the time to boil water he'd have to have a time bar (G) and he'd have to have the amount of water (T) and the temperature of the water." She then corrected herself, though her reasoning was unclear from the transcript. "No. Um, yeah, he'd need a variable." It is not clear why she did not think that time and amount of water were acceptable variables but she corrected herself by using an answer elimination strategy:

> Okay, A (answer choice) has time and amount of water (T). B has water and time so it can't be B. Temperature of water degrees and it has time and amount of water. That is switching with A and C and D is the same thing. It's hard to look at.

Finally, by rereading the question she was able to hone in on the correct phrases and the relationship between them to determine the correct graph to chose. "Time, amount of water, so water, so various amounts of water to boil so we need amounts of water. And then that would be on [unclear] and then time to boil water so it's A, the answer is A." Again, at no time while answering this question did she inquire into the meaning of the variables or what she thought might be relationship among them. For Henrietta, the meaning of this problem was unimportant; for her, it was simply an opportunity to apply well-practiced mathematics and reading skills.

With her skills improved from answering question 8, Henrietta was able to quickly dispatch question 9: "I would probably choose A because number of eggs would probably be at the bottom (G) and you'd have to list the weight of chickens and so you'd see how many eggs that chicken lay (T)." However, still without any means of correctly judging which variable to place on the x- and y-axes, her quick answer was wrong.

One of the few students to treat questions 8 and 9 as mathematics word problems and understand how to place the variables on the x- and y-axes was Hugh Hickson. He was explicit about his treatment of the variables as simple phrases to be placed on the graph: "…we're looking for things correctly labeled for time needed to heat various amount of water to boiling (T)." He then used an answer elimination strategy to discard the graphs that did not have those phrases (G). By treating it as a mathematics word problem, he was quickly able to reduce the number of choices. This did not help him to finish the question, however, because he still needed to place the variables on the axes:

> Looking at A, that one might work because it has time labeled (G) and if you're timing things that…actually it couldn't work because on the left side, on the horiz…on the

> vertical side we're *looking for things* that make a bar graph or line graph, things *that can go up*. (emphasis added)

While he did not use the vocabulary of dependent and independent variables, he did have the tacit knowledge evoked in the phrase "looking for things… that can go up." At first he was inclined (PA) to place time on the x-axis, but quickly corrects himself. "And not just, and the time would have to be on the bottom (G) if you're timing things or no, no, time on the vertical bar is correct 'cuz you're not timing certain amounts of times." In contrast to several students, he recognized that just because time was being measured it did not need to be placed on the x-axis. Instead, the purpose of the experiment was to measure the amount of time needed to boil various amounts of water.

> You're just saying how much time it takes to boil something. And the question says (T) the time needed to heat various amount of water. Amount of water could be right on the bottom (G) so it can't be B because the time on the bottom.

He could then eliminate answer choice B because it had time on the bottom. He then reminded himself why answer choice C was out of contention, "and then, just because the time on the bottom, and C could be time on the side, but *it doesn't matter which is the temperature of water*." Notice, however, that he was only saying that temperature did not matter because it was not mentioned in the question (T), not because water boils at an invariant temperature. After deliberating for a while longer about how to "put the heat on the bottom" of the graph, he finally framed the dilemma for himself. "I know it's either A or C but I don't know if on the horizontal axis it should be amounts of water or temperature of water just because heat various amounts of water." He finally realized that, "so would you use the same heat." This, finally, allowed him to determine the correct answer, "making it A or would you use, it says various amount of water so it's going to have to be A just because it says various amounts of water. That's my answer." Hugh never attempted to understand the physical situation being described (R) or the relationship among the variables (I). Hugh treated this problem as a mathematics word problem and was able to answer it correctly. He withheld his science knowledge and never made any effort to move into the bottom triangle of Roth and Bowen's model.

Summary of Findings for Research Question 2

Research Question 2 asked: How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)? To answer this question, 13 sixth grade students of varying mathematics and science abilities were asked to think aloud while answering 13 questions excerpted from the Test of Graphing in Science (McKenzie & Padilla, 1986). One key finding from this analysis was that students answered the TOGS questions as if they were mathematics word problems, science data to be analyzed, or they were confused and they guessed. Students used all three approaches to answer the external identification stage and intermediate level questions, but only approached the elementary level question as if they were mathematics word problems. In the data, participants who only made reference to the elements of Roth and Bowen's top triangle (i.e., structuring) approached the questions as mathematics word problems. Participants who made references to elements of Roth and Bowen's top and bottom triangles (i.e., structuring and grounding) approached the questions as science data to be analyzed.

A second set of findings added to the roles that science background knowledge played during graph interpretation:

- Correct science knowledge supported students' reasoning, but it was not necessary to answer any question correctly.
- Correct science knowledge could not compensate for incomplete mathematics knowledge.
- Incorrect science knowledge often distracted students when they tried to use it while answering a question.

These findings expanded those already identified in Research Question 1.

Research Question 3

The primary focus of this section was to answer Research Question 3: Drawing from these data (i.e., the data and findings of Research Questions 1 and 2), what patterns emerge from student thinking during line graph interpretation given their prior performance in mathematics and science? Data analyzed during Research Question 1 led to findings related to how students used heuristics, self-generated questions, science knowledge, and motivation. From the analysis of Research Question 2 emerged findings related to students approaching the questions as mathematics word problems versus approaching them as science data to be analyzed. Research Question 2 also led to additional findings related to how students used science knowledge during graph interpretations tasks.

To answer Research Question 3, the researcher wrote vignettes using Roth and Bowen's (2001) two-stage semiotic model of reading graphs to reanalyze a small, illustrative sample of students' answers to TOGS questions. This reanalysis served to amplify and clarify some of the patterns that emerged during Research Questions 1 and 2. In addition, this analysis examined the role of the two overall level TOGS questions which, until this point, had not been explicitly examined. During the pilot study, the researcher found that students' verbalizations were sparse and sometimes did not include spontaneous scientific thinking. As a result, the researcher added supplementary verbal questions for students to answer (see Appendix C for these verbal questions). With these verbal questions given after the student had completed addressing the multiple choice questions, there were no effects on student thinking. These responses provided additional information about students thinking, especially the roles that science knowledge played during graph interpretation. The two overall TOGS questions were not the only prompts for the students to provide their scientific thinking during this study. As the previous analysis showed, some students engaged in overall level interpretation when unprompted.

Independent and Inquisitive: Science Data to be Analyzed

The members of the Independent and Inquisitive group demonstrated one primary pattern while answering the TOGS questions. When answering the questions as science data to be analyzed they:

- were motivated to understand the physical situation being represented by the question and graph (i.e., they were inquisitive),
- perceived questions as science data to be analyzed,
- used science knowledge as a resource to understand the questions and check their answers,
- were confident and independent with procedural knowledge,
- used several different heuristics and chose them appropriately,
- used their confidence to enable them to ask questions to check their understanding.

This pattern was seen in all five group members in varying degrees and was exemplified by Hugh Hickson.

Throughout the think aloud protocol, Hugh Hickson often seemed more interested in understanding the science behind the question and on several occasions answered the question almost as an afterthought. This inquisitiveness often meant that he took longer to answer the questions than some of the other students and he sometimes seemed to struggle in the middle of his answers, but he demonstrated the mathematical and scientific procedural independence to extricate himself.

Using the language from major findings from Research Questions 1 and 2, he seemed to be primarily motivated by his desire to understand than his desire to answer the question correctly. This motivation, in turn, led him to often approach the TOGS questions as if they were science data to be analyzed rather than mathematics word problems. For this reason, his perception attention (PA) often led him to focus on the data in the graph first and relate what he saw in the data to his science knowledge. On several occasions his science knowledge was helpful to him while answering the question. When he ran into difficulties understanding the data, he was willing to ask himself questions to clarify whether he understood the question, whether his approach was on the right path, and whether his answer was adequate. He was willing to do so because he was confident in his procedural knowledge; consequently, he effectively used six different heuristics while answering the TOGS question. This pattern was seen in his approach to TOGS question 2.

After reading the question (T), he demonstrated automaticity as he read the x- and y-coordinates of the points in the region of interest on the graph (G),

> On 0 grams, I mean 0 ml of water, it went up to 8 cm (G), just because the plant grew for 3 weeks (R, T) and on 40 ml it looked like it was 15 cm (G) and on 80 there's nothing there (G) which is weird (PA). On 120 it went to 20 cm (G) and on 160 it went to 10 cm (G) and on 200 it went to 5 cm (G).

While reading the points on the graph (G), Hugh also tried to understand the physical situation (R) behind the graph. Because this was an intermediate level question, he could have answered it without understanding the science implied in the graph; however, he was motivated to understand , not just to answer the question correctly. When his understanding broke down or became unclear, he reacted accordingly (e.g., "which is weird"). Noting that something was weird in the data suggested that he was willing to question the discrepancy and acknowledge that he did not understand something.

Hugh then answered the question, almost in passing, before returning to better understand the science:

> If I was going to guess, not guess (PA), my answer would be 16 (T) just because it looks like 160 (ml) (G) was a little too over much water (R, I) and it looks like 120 (G) was a good amount (I) and 200 ml (G) of water was too much (I) so was 160 (G). Just to put it in the middle (SS) just because it would be a little too much but a little (I), but not the right amount, I mean just a little bit too much (I), it would be 16 cm (T) and that would be my answer (S).

Hugh showed that understanding the physical situation (R) of the graph prior to answering the question was important to him.

Question 2 only required an intermediate level answer, meaning that the Hugh could have restricted his thinking to structuring (Roth & Bowen, 2001) and still have answered the question correctly. However, Hugh was curious (PA) while answering question 2. His curiosity encouraged him to try to understand the physical situation (R) being represented in the graph and text (T), and to try to understand the relationship being represented in the graph. The result was that he demonstrated a more complete understanding of the situation (S), even when not prompted by the question.

A few minutes later Hugh answered question 4b, an overall level question: Why do you think this happened? By this time, he had not only given his answer to question 2, but also to questions 3 and 4a also. It was understandable that his answer was rather brief:

> I think this happened because 120 ml is probably the correct amount of water (R, I) which makes it go up to 20 cm (G) and 160 ml and 200 ml (G) was over watering and flooding (R) the plant which gave it too much water and it pretty much choked (S, I).

Especially because he had already answered question 2 as if it were science data to be analyzed, he had already said everything he knew about the causes of the relationships seen in the graph. He was still able to briefly summarize his understanding, coordinating and relating his mathematical knowledge from the structuring triangle with the scientific knowledge from the grounding triangle (Roth & Bowen, 2001).

Independent and Inquisitive: Mathematics Word Problems

In addition to the pattern described in the previous section, the members of the Independent and Inquisitive group occasionally demonstrated a second pattern while answering the TOGS questions. When answering the questions as word problems, they:

- were motivated to answer the question correctly and quickly,
- perceived questions as mathematics word problems,
- did not spontaneously use science knowledge but did understand when prompted,
- focused narrowly on the information needed to answer the question and did not ask their own questions,
- were confident and independent with procedural knowledge, and
- used several different heuristics and chose them appropriately.

This pattern was seen occasionally in all five group members and was exemplified by Hugh Hickson.

Huck Handy answered question 2 on the TOGS very differently than Hugh Hickson. He also had a high level of prior academic achievement on FCAT in both Mathematics and Science. However, while working on the TOGS, Huck did not verbalize or demonstrate any curiosity (PA). He was, however, very methodical and goal oriented and often answered the questions quite swiftly. He rarely appeared confused.

Using the language from the major findings of Research Questions 1 and 2, he seemed to be primarily motivated to answer the questions correctly. When prompted, he demonstrated that he understood the science behind the graphs and the questions, but he rarely mentioned it spontaneously or inquired about it while answering the questions. This motivation was likely the reason that he approached the TOGS questions as if they were mathematics word problems rather than science data to be analyzed. For this reason, his perception attention (PA) led him to focus narrowly on the information and patterns he needed to answer the question. He was usually able to quickly choose an appropriate heuristic to answer the question and he demonstrated his facility with seven of them. He was also quite confident in his abilities; as soon as he found an answer that he was secure in, he moved on to the next question without feeling the need to double-check his answer or verify it using another means, such as checking the science behind his answer. Between his surety in this approach, his confidence in his answers, or his lack of interest in the science behind the questions, he never asked a question of himself during the think aloud. This pattern was seen in his approach to TOGS question 2.

After reading question 2 aloud, Huck read the graph labels first (G, T) and then quickly noted the interval on the x axis, "So it goes up by 40's (G)." He then focused on the relevant region of the graph, "So the one given 140 (T) would probably be about 15, in the middle (SS)." With this, he answered the question and provided a brief explanation, "So I think it'd be B, 16 (T) because it was just about in the middle of 21 and 11 (G)." The transcript was only 39 words, and it took him less than a minute.

Huck's perception attention (PA) was very different from Hugh's. His verbal protocol indicated that his thinking was entirely in the structuring triangle (Roth & Bowen, 2001) and he never made any effort to ground the graph and text in a physical situation (R) or theory (I). As a result, Huck showed no attempt at understanding what was going on in the graph. When prompted in question 4b, Huck demonstrated little difficulty articulating the physical reality described in the test question, "I think this happened because it's good to water your plants (I)…Your plants need water but too much water could kill them (I)...She (Rose) has tried to find how much water is good for a plant (S)." Huck had the ability to understand the referent (R) and the interpretant (I), but his perception attention (PA) did not encourage him to explore it until prompted.

Compliant and Reserved: Mathematical Word Problems without any Science

The members of the Compliant and Reserved group demonstrated a fairly consistent pattern while answering the TOGS questions. They:

- were motivated to answer the questions correctly,
- perceived questions as mathematics word problems,
- focused narrowly on the information needed to answer the question and did not ask their own questions,
- used several different heuristics and chose them appropriately,
- were confident with procedural knowledge but not confident with science knowledge,
- did not spontaneously use science knowledge and demonstrated little understanding when prompted, and
- saved face when prompted about science or when having difficulty.

This pattern was seen in all three group members and was exemplified by Hester Luck.

Throughout the think aloud protocol, Hester Luck was motivated to get the right answer, which she was able to do more quickly than many of the other participants in the study. This may be due to her high achievement in mathematics on FCAT. Her frequent use of the answer-checking heuristic demonstrated her desire to get the right answer and reassure the researcher that she was not being impetuous, not because she was really confused or really wanted to understand the science. Otherwise, she never asked her own questions (PA), took the extra time to understand the physical situation (R) represented in the graph, or explore the interpretant (I).

Hester Luck never used science knowledge spontaneously. In 2006, she scored a low prior achievement level on the FCAT in Science and when prompted, could say relatively little about the science – on question 11b she evoked the authority of her parents because she lacked confidence in her own science knowledge: "I think it's that because I remember when I was little I heard my parents talking about that and they are smarter than me so I picked that." This technique of bringing in "someone smarter than me" allowed Hester to avoid answering the question, save face and rely on her strength—her mathematics ability.

Although Hester used a variety of heuristics—six different kinds in all—she approached all of the questions as mathematics word problems and never used any science knowledge to help when her thinking broke down. Hester Luck, like Huck, answered question 2 very quickly (PA). She did not read the answer choices (T) like Hugh and instead answered succinctly, rather like Huck:

> Well, if she gave it 140 (G) then the height that's a lot so if we look at the graph (G), there isn't a 140 so I'm just going to estimate to right there. If we bring it up we have to draw (draws on graph) that so the line (SS) it's probably going to be a height probably of 16 (G) an estimate probably 16 because it looks like it's the closest.

Although she used no science referents (R), Hester was able to come up with a theory (I) for why the plant's growth was diminishing according to the graph she had drawn on. For the overall level question 4b, Hester reasoned aloud the relationship of why the increase in the amount of water up to 120 ml would cause the plant height to increase but with amounts greater than 120 ml plant heights would decrease:

> I think this happened because it got over-watered with all the water put in (I) and I think 120 was just right but once you put 160 and 200 in it in milliliters, it just went and overflowed it so it just was not going to grow at all (I).

What was notable in Hester's answer to the overall level question was her reliance on her strong mathematics ability to assist her in answering the question. She gave a theory (I) without giving any referents (R) and showed no understanding of the physical situation in the question but, instead, offered an interpretant (I) ("it got over-watered with all the water put in").

Previously, when prompted on question 2, she also demonstrated little interest in the physical situation or the relationship being represented in the graph. Hester's response was similar to Huck Handy's on question 2 in both brevity and terseness, showing little curiosity compared to Hugh Hickson's. By herself on question 4b, Hester was able to offer an interpretant, although it was lacking the science referents that other participants had offered while trying to understand the physical situation of the graph. Hester never tried to engage in the bottom triangle of Roth and Bowen's two-stage model; she was more comfortable using her mathematical strength and treating the TOGS questions like mathematical word problems.

While not demonstrated in her answer to question 2, Hester's final distinguishing characteristic was being the only participant in the sample to use an answer checking heuristic rather than the more common answer elimination heuristic. On question 5, for example, she almost immediately answered that the answer will be "less than 5." But in contrast with Huck who would have simply moved on to the next question, Hester spent about a minute checking each of the other answer choices and explained why they would not be appropriate. She used this answer checking strategy a total of five times, and not once did it reveal that her original answer was incorrect. It seemed that she used this strategy as the result of an abundance of caution.

Hester Luck was motivated to answer the TOGS questions correctly but did not use any science knowledge to help herself. Her skill using a variety of heuristics enabled her to align the appropriate strategy with the need of the question. She treated each TOGS question as a mathematics word problem instead of science data to be analyzed. This was most likely due to her high prior achievement in mathematics and her lower prior achievement in science.

Few Strategies but Decent Science: Data Analysis without Procedures

The members of the Few Strategies but Decent Science group demonstrated a different pattern while answering the TOGS questions. They:

- were motivated to understand the physical situation,
- perceived questions as science data to be analyzed,
- used science knowledge to answer questions,
- were not confident in procedural knowledge, often answering without a clear procedure and making procedural errors,
- used a few heuristics, but did not always choose them appropriately, and
- did not ask their own questions.

This pattern was seen in both group members and was exemplified by Matt Maples.

Matt Maples was motivated to answer the TOGS questions correctly and used his science knowledge to answer 85% of the TOGS questions correctly. Unlike many of the other participants, Matt approached most of the questions as science data to be analyzed. He also used the heuristic of drawing to assist him in answering many of the questions. Matt did not rely on procedures when he answered the TOGS questions and never asked his own questions. It was his use of science knowledge, though, that helped him most effectively on several occasions by allowing him time to think through the questions to better understand what the physical situations involved. Engaging in both triangles of Roth and Bowen's model was common for Matt even if he used informal science knowledge to develop his interpretants (I).

Matt Maples answered the overall level question 11b very similarly to the way he answered many of the other TOGS questions—using his science knowledge. He had already answered the first part of question 11 correctly and when asked why he thought the speed of the car increased as the amount of the gas increased, Matt answered:

> I think this happened because the car going faster would need to use more fuel (I) and going farther so the engine (R) would have to run…would have to work harder (I) to get the wheels (R) moving faster and farther (S) so the engine would need more fuel for the car to keep going.

Matt had two referents (R) in his answer, the wheels and the engine which were not mentioned in the question. For Matt, he had made this physical situation real enough that he understood "the engine (R) would have to…work harder (I) to get the wheels (R) moving faster and farther (S)." The TOGS question did not provide this information; Matt provided this information himself on account of his perception attention (PA) which enabled him to use his science knowledge when answering the TOGS questions.

Similarly, Matt used his science knowledge when he answered question 5. Although he did not use a heuristic, he relied on many referents to the physical situation of the problem. When the researcher asked Matt how Rose could have improved her experiment, Matt introduced the vocabulary "control" and "variable." Henrietta was the only other participant that also used the word "variable" in her think aloud. No one else used the word, "control" in the study, and when the researcher asked Matt to clarify what he meant by that term, he was able to:

> Rose could have gave fertilizer to some plants as a variable and these five plants could have been the control. She would have had a variable and a control and maybe one would, one of the fertilizers would grow taller or maybe it wouldn't grow well at all…

> Control, I mean by they did not change anything to it, they gave the same only water and the variable something different from all the others like fertilizer.

Later the researcher asked Matt if he had any personal experiences growing plants that would help him to better understand the graph. Not surprisingly, he had many:

> I sometimes grow gardens with my mom. We usually grow basil, flowers, roses, broccoli, uh, and lots of other herbs…I can better understand this graph 'cuz once I like drowned the plant and it died and…like I was watering all the plants and I saw one in the pot so I drowned it in water like an inch of water in the pot. It ended up the plant dying 'cuz it had too much water. It died.

Matt's experiences with his mom with many different types of plants added to his repertoire of informal science knowledge. These experiences helped him answer TOGS questions 1, 3, 4 and 5.

However, Matt's science knowledge could not help him to overcome his lack of systematic procedural skills. He answered question 2 incorrectly, even though he recognized the trend in the graph. He understood (S) that it was an interpolation question but he lacked the skill to calculate the exact numerical value. Instead of placing the point at the intersection of the vertical line up from 140 and between the points (120, 21) and (160, 11), he simply placed a dot somewhere between the points. Matt made a sloppy procedural mistake on question 2, one that his science knowledge was unable to help him recover from.

Matt Maples was motivated to answer the TOGS questions correctly and used his science knowledge to help him accomplish this task. Instead of approaching the questions as mathematical word problems, Matt treated most of the questions as science data to be analyzed.

His approach of using his science knowledge helped him on many occasions to think through the questions to better understand what the physical situations (R) involved. Engaging in both triangles of Roth and Bowen's model was common for Matt even if he used informal science knowledge to develop his interpretants (I). While he frequently used the heuristic of drawing to assist him in answering many of the questions, his lack of mathematics procedural skill hampered him on a few occasions. He also answered many of the questions without any clear procedure; perhaps for this reason he never felt the need to ask his own questions.

Earnest but Confused: No Lifesavers

The members of the Earnest but Confused group demonstrated a consistent pattern while answering the TOGS questions. They:

- were motivated to answer the questions correctly, but
- often saved face to disguise their lack of procedural and science knowledge,
- perceived questions as mathematics word problems,
- used a limited repertoire of heuristics and demonstrated shallow science knowledge, and often
- guessed instead of asking their own questions.

This pattern was seen in all three group members and was exemplified by Linda Mills.

Each of the three students in the Earnest but Confused group was unique, but they all shared a common frailty. They demonstrated less background knowledge and used fewer strategies to help themselves get started on a question or recover once they ran into trouble. Linda Mills managed to answer nine of the 13 questions correctly in all question levels, so she clearly was not without ability. What placed her in this group, however, was her limited repertoire of heuristics (two) and shallow science knowledge. When she had difficulty answering a question she could not switch to another heuristic or try to understand the science represented in the questions. She could only guess. She was motivated to answer the questions correctly and approached all of the questions as if they were mathematics word problems. When she ran into difficulty, however, her motivation changed to face saving. When prompted, she tried to hide her lack of understanding behind empty verbalisms.

When attempting to answer question 2, she made the simple mistake of interpreting the graph as continuing on an upward curve instead of curving downwards. Had it been an extrapolation question, the rapidity with which she answered would have rivaled Hester Luck's impressive speed. "140…I think maybe D, 23 ml (T), cm (self-corrects)" And, like Hester, she proceeded to review her thinking and check her answer, "because the scale goes 0, 40, 80, 120, 160, 200 (G, x axis) then going up it goes 0, 5, 10, 15, 20, 25 and 160, it's between that and 120 so that would be the correct answer." However, unlike Hester she did not engage in explicit answer checking because she did not systematically identify why each of the other answers was incorrect. If she had, she might have noticed that her assumption that the curve continued upward was incorrect.

The shallowness of her processing was demonstrated when prompted by the researcher to elaborate on her thinking, she stated, "I was thinking that it was the most…what's the word I'm looking for…the most reasonable answer because the others were too low." Had the graph continued on an upward curve after point (120, 20), then her interpolation would have been reasonable, so this does not appear to be a problem with that skill. Instead, it seemed that she was unfamiliar with line graphs that are non-linear.

However, a few minutes later she was able to correctly answer question 4, which required her to choose the correct description for the curve of the graph.

> No, I think D because as the amount of water increased to 120 ml, the height of the plants increased (T, repeated the wording of the answer choice) and with amounts greater than 120 ml, the height of the plants decreased.

While she chose the correct description of the graph, she did not comment on the incongruence of her answers to question 4 and question 2. Moreover, she indicated that she had no experience with growing plants but understood the effect of varying amounts of water on plant growth, "I think this happened because the person gave the plant too much water (R) and too much water is kind of bad for a plant (I). That's why I think that happened." She also had a basic understanding of the requirements for plant growth, "Plants need nutrients, water and soil and sunlight to grow (I). If not, they are going to wither and die (I). That's all I know." Her word choice suggested that this may have been information previously memorized, rather than a robust understand of the science of plant growth.

The shallowness of Linda's knowledge and procedural skill was evident throughout her transcript. It was difficult to code any part of Linda's verbal protocol for question 4 with an S (Sign) because so much of her answer was either directly quoting or paraphrasing the TOGS questions and answer choices— she rarely formed a complete thought using her own words. She never spontaneously mentioned the referents or interpretants in the grounding triangle. In order to code Linda for understanding the significance of a Sign, the researcher would have had to see some effort to coordinate the graph and text with the referent and the interpretant. Compare Linda Mill's verbal protocols for question 2 and 4 with Hyde Hegel's for question 2. While Hyde spontaneously wanted to coordinate what he saw in the graph and text with his math and science background knowledge, Linda was not inclined to do anything more than what was directly asked of her as she tried to solve these mathematics word problems.

This section answered Research Question 3: Drawing from these data (i.e., the data and findings of Research Questions 1 and 2), what patterns emerge from student thinking during line graph interpretation given their prior performance in mathematics and science? To answer Research Question 3, the major findings from Research Questions 1 and 2 were reanalyzed and presented as vignettes for a representative sample of students across ability levels to show the relationships among the findings. These vignettes showed how students used heuristics, self-generated questions, motivation, how they approached the questions, and how students used science knowledge during graph interpretation tasks.

In this chapter, the preliminary data analysis was explained and how it led to the reorganization of the TOGS questions and the student groups. Next, the data were analyzed to answer Research Questions 1, 2, and 3, yielding a number of findings. In the next chapter, these findings will be discussed, as well as the implications for future research and for practice.

CHAPTER FIVE: CONCLUSION AND DISCUSSION

Purpose and Overview of Methodology

The purpose of this study was to understand how intermediate elementary students' mathematics and science background knowledge affected their interpretation of line graphs. In addition, the purpose was to understand how students' interpretations were affected according to graph question level. The research methods for this study were primarily qualitative, though quantitative data were used to help with sample selection, categorization of students into subgroups, and analysis of data across the sample (Creswell, 2003). The qualitative data were collected using think aloud interviews (Ericsson & Simon, 1993), video, and observational notes. Two quantitative measures were used. One was the 2006 5th Grade Florida Comprehensive Assessment Tests (FCAT) of Mathematics and Science used to create a purposive sample using maximum variation sampling (Patton, 2002) to capture the variability of sixth grade students at one school. For the purpose of this study, students' achievement on the FCAT was assumed to be an appropriate predictor of student background knowledge and ability in mathematics and science (Florida Department of Education, n.d.). The other quantitative measure was the excerpted Test of Graphing in Science (TOGS) instrument developed by McKenzie and Padilla (1986). The TOGS measured students' ability to interpret line graphs (Appendix C). A scoring rubric to assess student performance on the TOGS was created by the researcher and validated by Padilla (Appendix F) using previously cited factors from the literature affecting students' graphing ability (Appendix A). These factors were categorized using Bertin's stages of the reading process and three graph question levels (1983). Data were also analyzed using Roth and Bowen's two-stage semiotic process of reading graphs (2001).

Table 11

Table 11 Concise Summary of Research Findings

Research Questions	Concise Findings
1. How do student behaviors observed during think aloud interviews vary during line graph interpretation in science across mathematics and science achievement levels?	The findings for students' prior achievement were divided among four general areas: 1. Students with higher prior achievement used a greater variety of *heuristics and strategies*, and more often chose appropriate strategies. 2. Students with higher prior achievement monitored their understanding and progress by asking themselves *questions*. 3. Students with higher prior achievement used their *science knowledge* spontaneously while answering the questions to check their understanding of the question and the adequacy of their answer. Several also inquired into the science behind the questions, even when unprompted. In several cases, students with lower prior achievement could have recognized their errors if they had used their science knowledge. 4. One student with lower prior achievement *motivated* herself when she thought the questions were too difficult. Both she and one student with higher prior achievement expressed excitement while answering the questions when they overcame their confusion.
2. How do student responses during think aloud interviews vary according to graph question level (Bertin, 1983)?	1. Students answered the external identification stage and intermediate level questions as if they were mathematics word problems, science data to be analyzed, or they were confused and they guessed. They approached the elementary level questions as if they were mathematics word problems. In addition, the analysis showed that: 2. Correct science knowledge supported students' reasoning, but it was not necessary to answer any question correctly. 3. Correct science knowledge could not compensate for incomplete mathematics knowledge. 4. Incorrect science knowledge often distracted students when they tried to use it while answering a question.
3. **Drawing from these data, what patterns emerge from student thinking during line graph**	*Independent and Inquisitive Group, Pattern 1* • were motivated to understand the physical situation being represented by the question and graph (i.e., they were inquisitive), • perceived questions as science data to be analyzed, • used science knowledge as a resource to understand the questions and check their answers, • were confident and independent with procedural knowledge,

interpretation given their prior performance in mathematics and science?	• used several different heuristics and chose them appropriately, • used their confidence to enable them to ask questions to check their understanding. *Independent and Inquisitive Group, Pattern 2* • were motivated to answer the question correctly and quickly, • perceived questions as mathematics word problems, • did not spontaneously use science knowledge but did understand when prompted, • focused narrowly on the information needed to answer the question and did not ask their own questions, • were confident and independent with procedural knowledge, and • used several different heuristics and chose them appropriately. *Compliant and Reserved Group* • were motivated to answer the questions correctly, • perceived questions as mathematics word problems, • focused narrowly on the information needed to answer the question and did not ask their own questions, • used several different heuristics and chose them appropriately, • were confident with procedural knowledge but not confident with science knowledge, • did not spontaneously use science knowledge and demonstrated little understanding when prompted, and • saved face when prompted about science or when having difficulty. *Few Strategies but Decent Science Group* • were motivated to understand the physical situation, • perceived questions as science data to be analyzed, • used science knowledge to answer questions, • were not confident in procedural knowledge, often answering without a clear procedure and making procedural errors, • used a few heuristics, but did not always choose them appropriately, and • did not ask their own questions. *Earnest but Confused Group* • were motivated to answer the questions correctly, but • often saved face to disguise their lack of procedural and science knowledge, • perceived questions as mathematics word problems, • used a limited repertoire of heuristics and demonstrated shallow science knowledge, and often • guessed instead of asking their own questions.

Conceptual Framework

The conceptual framework for this study was a synthesis of two constructs published in the literature on graphing. These constructs were Bertin's (1983) distinction among three graph question levels and Roth and Bowen's (2001) two-stage semiotic process of reading graphs.

The distinctions among three levels of questions (Bertin, 1983) while interpreting graphs enabled the researcher to categorize previously cited factors affecting students' ability to interpret line graphs and to classify the primary focus of the questions used in the think aloud interview. Bertin (1983) called his three levels of graph questions elementary, intermediate, and overall. These three levels each involved different cognitive processes. Elementary level questions, according to Bertin, involved data extraction, location, and translation. Intermediate level questions involved trends seen in parts of the data. Curcio (1987) referred to this as reading between the data and included the skills of interpolation, extrapolation, describing the trends seen in the graph, and describing qualitative or quantitative differences between data points (Appendix A). Finally, overall level questions involved an "understanding of the deep structure of the data being presented in their totality, usually comparing trends and seeing groupings" (Bertin, 1983, p. 16). The goal of answering an overall level question, according to Curcio (1987), was to read beyond the data, including generating ideas and predicting outcomes.

The semiotic model of reading graphs (Roth & Bowen, 2001) provided a comprehensive framework for describing and interpreting students' cognition and behavior while engaged in line graph interpretation. Based on their analysis of students engaged in scientific inquiry using a variety of data, scientific texts, and field experiences, they argued that students and scientists in their studies needed to learn to coordinate the graphical representations of their data with the published information and their own experiences in the field. What a person comes to understand a graph to signify is in turn affected by experiences with the referent and the theories or ideas that form interpretants—what Roth called grounding (bottom triangle in Figure 1). However, they argued, signs should always be understood in the context of other signs (Roth & Bowen, 2001). When a person tried to interpret a graph, what she or he understood the graph (text) to signify was affected by perception attentions (i.e., knowledge, skills, and habits). Roth called the experience of making meaning from the graph structuring (top triangle in Figure 1). That is, the process of structuring the information in the graph was facilitated by the knowledge, skills, and habits previously learned (2005).

Summary and Discussion of Major Findings

Use of Science Knowledge

The analysis of Research Question 1 yielded four major findings related to the role of science background knowledge during graph interpretation:

1. Most students with higher prior achievement and one student with moderate prior achievement used their science knowledge spontaneously while answering the questions to check their understanding of the question and the adequacy of their answer.
2. A few students with higher prior achievement used science knowledge beyond what was required to answer the question, demonstrating inquisitiveness.
3. Students with lower prior achievement and some students with higher and moderate prior achievement did not spontaneously evoke or use their science knowledge while answering the questions, even though they could discuss their knowledge when prompted by the researcher.

4. In many cases, students with lower prior achievement would have been able to recognize their errors if they had thought about their answers in the context of their science knowledge.

In addition, the analysis of Research Question 2 yielded three more major findings related to the role of science background knowledge during graph interpretation:

5. Correct science knowledge supported students' reasoning, but it was not necessary to answer any question correctly.
6. Correct science knowledge could not compensate for incomplete mathematics knowledge.
7. Incorrect science knowledge often distracted students when they tried to use it while answering a question.

These findings confirmed several issues already identified in the literature and suggest that as a whole, although students' science knowledge was not as helpful when their mathematics procedures were incomplete, students' erroneous science knowledge was very distracting when students were treating the TOGS questions as mathematics word problems.

A few earlier studies (Aberg-Bengtsson & Ottosson, 2006; Leinhardt, Zaslavsky, & Stein, 1990; Parmar & Signer, 2005) have examined the role of content knowledge during graph interpretation. Unlike these earlier studies, the research methods used in this current project enabled the researcher to more closely examine the particular role of science knowledge, adding to the literature of content knowledge. An example of a study examining social science content knowledge was Shah and Hoeffner's reseach (2002) where they found that subject matter knowledge influenced interpretation of data. In this project, using social science data, undergraduate students were better able to recognize problems in graphs when those graphs portrayed data that were inconsistent with their prior beliefs. Several participants in this study were also able to recognize inconsistencies between their understanding of the situation and their interpretation of the graph. The students with higher prior achievement were more likely to be able to do this.

However, several of the students with lower prior achievement were not able to recognize such inconsistencies. The finding that students with lower prior achievement were less able to relate their knowledge to their interpretation of the graph was reminiscent of Parmar and Signer's (2005) finding that intermediate students (fourth and fifth graders) with learning disabilities did not use their science background knowledge while engaged in graph interpretation tasks. Because LDs were not a focus in this study, the researcher did not have access to this data, though it was possible that some of the participants may have had labeled or unlabeled disabilities. Nevertheless, even some of the participants with prior higher achievement in the sample had difficulty relating their background knowledge to the graph, so it is possible that this problem is more pervasive than is currently recognized.

Beyond science and mathematics background knowledge, learner characteristics have also been found to affect their ability to correctly interpret graphs. Not surprisingly, less intelligent students according to an Intelligent Quotient measure have consistently been found to have more difficulty with graph interpretation than more intelligent students (Vernon, 1946). More pertinently, in their study of students with learning disabilities, Parmer and Signer (2005) found that students with LDs could rarely answer questions more complicated than those at the elementary level. While this study did not collect data about participants' disability status, it did find that some participants struggled with answering questions from all graph question levels.

Use of Heuristics and Mathematical Procedural Knowledge

The analysis of Research Questions 1, 2, and 3 yielded a number of findings related to how participants used heuristics during graph interpretation:

1. Students with higher prior achievement used a greater number and variety of heuristics while answering the TOGS questions.
2. Students with higher prior achievement more often chose heuristics that helped them to correctly answer the questions.
3. Students with lower and moderate prior achievement tended to favor one particular heuristic most often, even when that heuristic was not particularly useful for answering the question.

In addition, procedural knowledge played an important role. Students who had procedures for mathematical questions like finding the location of a point on a graph were less likely to confuse the x- and y-coordinates. These students also were more likely to answer the TOGS questions in an automated fashion when they had mathematical procedures to use.

Much of the prior research has focused on the role of mathematical ability in general and, not surprisingly, people with lower mathematical ability have often been found to be less capable on graph interpretation tasks (Curcio, 1987; Gal, 1993; Thomas, 1933). However, these studies have provided less detail about the specific mathematical methods that participants used or failed to use. These studies used paper and pencil tests and merely noted the correlation between mathematical ability and graph interpretation ability.

By contrast, the think aloud methods used in this study highlighted the role of specific heuristics and mathematical procedural knowledge that participants used during graph interpretation tasks. The pervasive use of heuristics by participants in this study clearly suggests their importance. The differences between ability levels and heuristic choices and usage suggests that performance on the TOGS often depended on students' abilities to choose and use the appropriate heuristic which were often omitted in earlier research studies.

In addition, mathematical procedural knowledge also played an important role in students' abilities to correctly answer the TOGS questions. Students who had correct procedural knowledge that was automated were at a clear advantage. Students who used incorrect procedures or used no procedures were at a clear disadvantage. Bell and Janvier (1981) found that students had to estimate when they were doing interpolation or extrapolation problems. Some participants in this current study also took that approach. However, other participants in this sample calculated the interpolation point using one of two methods: arithmetic mean or intersecting lines. These students who calculated were more successful than the participants who either estimated, used no procedure, or used the incorrect procedure. Likewise, in a 1989 study, Stein and Leinhardt (cited in Leinhardt, Zaslavsky, & Stein, 1990) found that students were unable to interpolate a point with the absence of a known procedure. In addition, Shah and Hoeffner (2002) found that students who were less skilled at graphing in general had more difficulty inferring trends in graphs that were less familiar.

Participants in this study also struggled with other procedural knowledge. Parmer and Signer found that most 4th- and 5th-grade students had difficulty interpreting scale and axis labels. By contrast, most of the sixth grade participants in this study could interpret the scales and axis labels with facility, so long as those interpretation tasks were embedded in elementary or intermediate level graph questions. However, participants in this study struggled with the external identification stage tasks which required them to deal with axes and labels directly. One inference was that the elementary and intermediate level questions provided enough additional context clues to scaffold participants, but when left without those context clues, they struggled.

In the same study, Parmer and Signer (2005) also found that students with LDs tended to ignore graph labels and axes, rarely checked their answers, and more generally had difficulty understanding the purpose of graphs. This research study did not collect data about participants' disability status, but did find that participants across prior achievement levels also struggled with graph labels and axes and that many did not check their answers. However, none of the participants could have been identified with not understanding the purpose of graphing.

Reinterpreting these data from a sociocultural perspective, it could be inferred that the participants who had higher prior achievement may have had more or better prior experiences with graph interpretation. This would be consistent with the finding that as students gained more experience with inscriptional practices and inquiry they became more competent in interpreting and reasoning about inscriptions. Wu and Krajcik found that the iterative processes of inquiry, inscription creating, and interpretation, enabled their participants to use increasingly sophisticated practices and develop gradually more complex interpretations (2006).

Students' choices of which heuristics to use in certain situations were determined largely by what they were attending to on the graphs and in the questions themselves. These perception attentions have been ingrained and acculturated by years of schooling and experiences that students connect with the graphs themselves (Roth, 2005; Roth & Bowen, 2001). The analysis of the use of heuristics revealed three patterns in how students approached the data in the graphs. As the next section explains, another pattern that emerged was how participants' individual perception attentions and motivation affected their choices of heuristics in the TOGS questions.

Perception Attention and Motivation

Throughout this study the construct of perception attention has been used to code students' habituated patterns of perceptions, seen in the aspects of the graphs and texts that they have focused on and the ways they focused their attention during the tasks. In turn, their perception attention structured how they approached the questions, including the procedures and heuristics they used (or failed to use), their enthusiasm or reticence, and even how they spoke about the task and themselves doing the task. "[T]hese dispositions generate patterned (i.e., structured) perceptions and with it the field of possible (material, discursive, etc.) patterned actions, that is the practices characteristic of a field" (Roth, Pozzer-Ardenghi, & Han, 2005, p. xii). While this construct remains imprecise, it nevertheless emerged as an important factor to understand how these participants interpreted the TOGS questions. Prof. Roth was kind enough to respond to a request (see Appendix I) for clarification of the construct of perception attentions by sending an in-press chapter (Roth, 2008, in press). However, the goal of Roth's paper was not to provide an operational definition of perception attentions. Therefore, the Roth and Bowen (2001) definition remains the clearest definition for the use and purpose of this study.

The analysis of the data for Research Question 2 revealed that students approached each TOGS questions in one of three ways: as if they were mathematics word problems, as if they are science data to be analyzed, or as if they were confused and had to guess. Students used all three approaches to answer the external identification stage and intermediate level questions, but only approached the elementary level question as if they were mathematics word problems. By coding the data using Roth and Bowen's (2001) two-stage semiotic process of graph reading, it became evident that participants who only made reference to the elements of the top triangle (i.e., structuring) approached the questions as mathematics word problems. Participants who also made references to the referents and interpretants implied by the graph and question in the bottom triangle (i.e., grounding) approached the questions as science data to be analyzed.

The difference between interpreting a graph as a mathematical task or a scientific task has been noted in the literature but from the perspective of the teachers. When graphing is used in mathematical tasks, the purpose is to teach or assess students' understanding of mathematical principles and the physical situation that is referred to in the question is only intended to motivate the student (Putnam, Lampert, & Peterson, 1990). In science, by contrast, the purpose is to use data analysis and graphing to represent real phenomena and patterns. However, Leinhardt and colleagues (1990) noted that students often struggle to use their mathematics knowledge while interpreting science graphs.

In this study, it seemed that most students habitually saw the questions as mathematics word problems and a few habitually saw the external identification and intermediate questions as science data to be analyzed. The literature review did not identify any previous studies in which this difference emerged, most likely because either the questions did not afford the option to choose the approach or because the research method was not capable of capturing the method of approach.

When comparing the interpretative practices of scientists and biology undergraduate students, Bowen, Roth, and McGinn (1999) noted that the scientists' interpretations were scaffolded by the concerns that characterize their disciplines. In addition, scientists were also helped by having extensive field- and lab-based experiences and the interpretive theories of their discipline. Undergraduate biology students, by contrast, did not have robust vocabularies, the experiential base, or detailed knowledge of specific organisms to help them when interpreting the graphs. In addition, because students were mainly concerned with earning a good grade, they did not develop or deploy more general graph interpretation skills and instead used their professor's interpretations of the graphs.

The Bowen, Roth, and McGinn (1999) article suggested a connection between perception attention and motivation. While the data presented in Research Question 1 about motivation were sparse, they nevertheless were suggestive:

1. Two students, one with higher prior achievement and one with lower prior achievement, expressed excitement while answering the questions when they overcame their confusion.
2. One student with lower prior achievement seemed to motivate herself while answering the questions when she thought the questions were too difficult and when her understanding was breaking down.

Again, somewhat surprisingly, the literature reviewed for this study did not reveal any studies that explicitly examined the role of student motivation during graph interpretation tasks. The data presented in the analysis of Research Questions 3 suggested, like Bowen, Roth, and McGinn's study that it is not merely a question of being motivated or not. Instead, the important question is what motivates the participant. Participants' behavior varied greatly depending on whether they were motivated to understand, answer the question correctly, or to save face. Within this study, most participants seemed to be motivated to answer the question correctly. However, this motivation to answer the question correctly also discouraged them from trying to understand the science behind the question and consequently, sometimes led to face saving.

It is noteworthy that the same participants who were most often motivated to understand the science behind the questions were the ones who were labeled as Independent and Inquisitive. These participants often were the ones who asked questions of themselves when they were unsure of their understanding of the question, the correctness of their procedure, or the correctness of their answer.

Use of Self-Generated Questions

The analysis of Research Question 1 yielded two major findings related to the use of self-generated questions during graph interpretation:

1. Students with higher prior achievement monitored their understanding of the question, the adequacy of their strategy, and the adequacy of their answer by asking themselves questions.
2. Students with moderate and lower prior achievement rarely monitored themselves while answering the questions by asking themselves question.

Since the study of graph interpretation would seem to presume an interest in people's ability to ask questions while interpreting graphs, it was rather surprising that the literature review did not reveal any prior empirical studies related to the kinds of questions people asked while interpreting graphs nor the roles that those questions played during the interpretive process. Bertin's (1983) phenomenological theory of graph interpretation was based on question asking and several of the studies used his framework (Curcio, 1987; Friel, Curcio, & Bright, 2001; Parmar & Signer, 2005; Wainer, 1984). Yet these studies have not examined how the participants' questions affected their graph interpretation. Instead, these studies have always supplied the participants with the questions that they needed to answer and the research methods employed were not able to capture the questions the participants may have asked themselves during the tasks.

Only Roth's studies (Roth & Bowen, 1994, 1995, 2001; Roth & McGinn, 1997) have tangentially examined the role of question asking in authentic inquiry contexts, and his recent work on critical graphicacy (Roth, Pozzer-Ardenghi, & Han, 2005) has accentuated the importance of being able to ask good questions while interpreting science data in graphs. The findings in this current study suggest that the ability to ask questions during the graph interpretation process played a few important roles: ensuring that the participant understood the intent of the question, that they had chosen an appropriate method, and that their answer was reasonable. Moreover, the fact that only a few of the participants with higher prior achievement and that none of the participants with moderate and lower prior achievement engaged in question asking further suggests that it may be related to academic achievement. However, the nature of this relationship could not be illuminated with the research methods used in this study.

Summary of Contributions to the Literature

The discussion of the research findings suggests that this research project offered several insights into science graph interpretation with intermediate level elementary students. First, this research project suggested a more complicated role for science background knowledge during graph interpretation. Within the psychological research on graph interpretation, the role of content knowledge has been largely ignored and superficially treated, and within the sociocultural literature the role of science content knowledge has not been adequately separated from the inquiry process.

Second, this research project is the first study of graphing to note the importance of heuristics and procedural knowledge. Considering the prevalence of studies of heuristics in the mathematics problem solving literature it was surprising that no prior studies were identified that examined its importance. However, this study made clear that very often the participants succeeded or failed because of their ability to choose an appropriate heuristic and execute it properly. In addition, this study also noted how students' success depended on their ability to use procedural knowledge correctly.

While only preliminary, this study also documented the importance of perception attentions and motivation during graph interpretation. The roles of perception attentions were noted throughout the data analysis, often in conjunction with other behaviors that led to the success and depth of interpretation. Most fundamental was the distinction between approaching these questions as if they were mathematics word problems or science data to be analyzed. Moreover, perception attentions were noted throughout the interpretive process. However, while Roth has written several pieces that included discussions of perception attentions (Roth, 2008, in press; Roth & Bowen, 2001; Roth, Pozzer-Ardenghi, & Han, 2005), the construct still needs further clarification and operationalization before its full import can be understood. In addition, while the current study only noted two of the participants' motivational behavior,, their data suggested that motivation can play an important role when participants are confused about or lacking confidence during a graphing task. However, the existing literature on graph interpretations has not noticed the importance of motivation.

Finally, the data from this study also suggest that the ability to ask self-generated questions enabled students to monitor their understanding of the task, their approach to the task, and the adequacy of their answer. Again, it was somewhat surprising that this phenomenon had not been noted in the existing literature about graph interpretation.

Taken together or individually, each of these contributions to the literature suggests directions for future research. Especially considering the recent efforts to develop data analysis skills in the reform of elementary science and mathematics education (AAAS, n.d.; NCTM, 2000; NRC, 1996, 2000), more research on these topics is timely and needed.

Limitations of Study and Recommendations for Future Research

With the robust findings from this study, it would be easy to lose sight of the limitations of this research. While several were noted in the Introduction, there are few that should be reiterated. With the acknowledgment of these limitations come suggestions for future research that could broaden this study. This research study suggested how a group of intermediate level participants interpreted graphing items on a multiple choice test. Future research on graph interpretation might make several extensions.

First, while the test was called the Test of Graphing in Science, it was, nevertheless, primarily a mathematics test. The data from the study showed that most participants treated a majority of the questions as mathematics word problems instead of science data to be analyzed, and it was possible to answer every question correctly without references to science knowledge. One of the intentions of the researcher was to better understand how students' science background knowledge affected their ability to interpret line graphs. Even with the addition of two questions and several prompts intended to encourage students to use their science knowledge, the TOGS was of limited use in understanding the role of science background knowledge. In addition, the fact that the TOGS questions used a multiple choice format encouraged students to use answer elimination and limited the amount of inquiry and open-ended thinking. At least including some open-ended response questions would be preferable to an entire multiple choice test.

Second, the use of a think aloud interview in this study was very successful for accessing student thinking while participants were completing the TOGS, but it was very difficult to generalize from the findings of this test how students would behave in different academic settings or activities. While it may not be difficult to assume that students' behavior and thinking during the think aloud interview would be somewhat similar to how they would behave in other testing environments with similar questions, it would be difficult to assume that these findings would generalize to students' behavior and thinking during more authentic inquiry activities or during collaborative activities. The result of using individual interviews was a decrease in ecological validity, approximating the real-life situation that is being studied in the use of methods, materials, and setting (Shadish, Cook, & Campbell, 2002). Future research might, instead, use dyads instead of single test takers in order to capture the behavior they might exhibit during collaborative tasks but still be controlled enough for systematic data collection (i.e., think aloud).

Third, among the volunteers that participated in the study, there were certain cells within the sampling matrix (see Table 1) that were underrepresented; therefore, the sample was heavily weighted toward students with higher prior achievement in both mathematics and science. In addition, there were not enough volunteers that fell within the middle prior achievement level in the sampling matrix. Once the students were recategorized, the two students with moderate achievement in mathematics and science remained in a group of two by themselves. These two students were similar enough to be analyzed together but their differences were still rather pronounced. The inclusion of more students with low and moderate prior achievement would have provided more insight into the difficulties these students experience when they interpret graphs.

Practical Implications

This study has some important implications for educators who teach mathematics and science to intermediate level students. First, teachers need to find opportunities to employ the think aloud procedure, even informally, inside the classroom to observe student thinking. Being able to capture cognitive interruptions as soon as they occur allows teachers to better understand why mistakes are happening and leaves less room for inferences that often become relied on from multiple choice assessments. It is crucial for teachers to be able to "see in the moment" when a breakdown ensues to provide clarifying instruction. This enables the teacher to immediately modify the student's weak spot and strengthen it instead of lingering until further misconceptions build. Waiting to see a student's answer on paper is sometimes too late. It would be more effective if the teacher could hear the thinking as the brain is composing it. Besides, students are often able to say a lot more than they are willing to write. Adding the think aloud procedure to the classroom repertoire would by no means replace paper and pencil assessments. This strategy would make teachers more effective by giving them a glimpse into student thinking the moment it occurs, allowing them to make instructional and curricular changes to improve student performance.

Second, educators need to teach students how to ask their own questions for when their thinking breaks down. While initially the most able students' independence and inquisitiveness seemed to be the result of their high achievement, it now appears that their question asking served both meta-cognitive and meta-motivational purposes that helped them to clarify the intention of the test questions and to keep themselves engaged in the task. The origin of this behavior cannot be addressed by these data, but it does suggest that these high achieving students have come to the test with a set of skills that may enable them to better succeed in school. For example, Hans Hazel and Hugh Hickson had both been very successful on standardized achievement tests (see Table 1). Hans Hazel answered all of the questions correctly and was described as being automated and well-practiced because of not using any strategy during his think aloud to eliminate poor answer choices. He also did not rely on mnemonic aids to help his memory like three of the other Independent and Inquisitive group members chose. Hugh Hickson, on the other hand, almost seemed less capable than Hans having used heuristics and test-taking strategies and then having not scored as well. One thing to think about was the awareness Hugh displayed by thinking aloud his one particular strategy. He was quite sure that his answer choice was correct since the mnemonic, healthy vitamins (used to remember the correct order of the horizontal x-axis and the vertical y-axis when plotting points on a graph),, was strongly entrenched in his memory. Hans Hazel, however, just answered and perhaps relied on his high level of math skills, but if he had guessed incorrectly, he would have had few ways to help himself. Teachers need to be cautious about which behaviors are rewarded and to encourage students who ask questions and think more deeply—not just those students who quickly answer the question correctly.

Third, the study suggests that these intermediate level students would have profited from more instruction in a number of areas. The students knew heuristic strategies but were not always proficient at selecting the appropriate one. In some textbooks (e.g., MacMillan/McGraw-Hill), heuristics are taught in isolation in each chapter but then never brought back together to enable students to understand how to select from among the alternatives. Practice with this skill may help students on standardized tests as well as in problem solving situations in the classroom and in real life situations. In addition to becoming more comfortable with selecting the appropriate heuristic, students also need more time becoming adept at mathematical procedures to have automaticity on graphing tasks like the ones these participants experienced. Having automaticity frees up cognitive resources that allows for creativity and sophisticated thinking. Finally, students need more practice with nonsense graphs as a means of focusing their science background knowledge with skills of data analysis instead of teaching data analysis in isolation. This transformation will encourage students to look at graphs, identify their referents, and learn to develop sophisticated interpretants which emulate the practices of real scientists.

These instructional changes will assist intermediate elementary students in approaching graphing questions with added confidence, using their science and mathematics background knowledge more effectively. Some students may approach graphing problems as mathematics word problems, others may treat them as science data to be analyzed. Hopefully, it is the goal of all educators to encourage students to understand their choices and the consequences of their selections to help them become independent learners. For this, the researcher is grateful for the time spent on this project and the knowledge gained.

APPENDIX A: GRAPH INTERPRETATION SCORING RUBRIC

Elementary Level Indicators *Reading the data*[a] (i.e., locating, translating)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
1.1. Read "x-y coordinates" of point on a graph													
1.2. Find point of given "x-y coordinates"													
1.3. Select a corresponding value of X (or Y) for a value of Y (or X) on a given graph.													
1.4. Read the value of a point using an axis and a label													
1.5. Associate the x-axis with the independent (causal) variable and the y-axis with the dependent (effected) variable													
1.6 Select an appropriately scaled set of axes for a set of data (correct range and interval)													
1.7 Know that in a coordinate pair "(x, y)" the x is the horizontal axis coordinate, and the y is the vertical axis coordinate													
Intermediate Level Indicators *Reading between the data* (i.e., integrating, interpreting)													
2.1. Make a relative comparison (qualitative) between data points (i.e., higher, taller, lower, smaller, etc.)													
2.2 Calculate the comparative (quantitative) difference between data points													
2.3. Infer an omitted point between data points (interpolation)													
2.4 Infer a point beyond the plotted data points (extrapolation)													
2.5. Describe a relationship between the x and y variables				a							a		
2.6. Identify a relationship between two graphs													
2.7. Select an appropriate graph to display the data													
Overall Level Indicators *Reading beyond the data* (i.e., generating, predicting)													
3.1. Identifies a bias about the physical situation the graph describes that affects graph interpretation													
3.2. Identifies preconceived notions about the physical situation the graph describes that affects graph interpretation													
3.3. Identifies scientific knowledge about graph content that affects graph interpretation				b							b		
3.4. Identifies personal experience about graph content that affects graph interpretation													

Notes. Questions 4 and 11 have two parts which are noted by the letters a and b, not to be confused with a footnote. For scoring, x = incorrectly answered, √ = correctly answered, o = omitted, p = answered with prompting. [yellow] = primary focus of question, [light blue] = secondary possible indicator.
[a]Italicized text is Curcio's (1987) language.

APPENDIX B: PERMISSION TO USE AND ADAPT THE TEST OF GRAPHING IN SCIENCE (TOGS)

YAHOO! MAIL Classic

Print - Close Window

From:	Michael Padilla [SMTP:mpadilla@uga.edu]
To:	'Keller, Stacy@
Cc:	valkil@uga.edu
Subject:	RE: Request for a copy of TOGS
Sent:	1/20/2006 1:23 PM **Importance:** Normal

You have my permission to use or adapt the TOGS materials as necessary. I will ask my assistant to send you a copy of the materials. There will be a $20 charge for copying and mailing. Good luck in your dissertation.

____

Please note my new email address: mpadilla@uga.edu

Michael J. Padilla
Director of Partnerships
President, National Science Teachers Association
315 Aderhold Hall
College of Education
University of Georgia
Athens, GA 30602-7101
706 542-1686

-----Original Message-----
From: Keller, Stacy@]
Sent: Monday, January 02, 2006 10:15 PM
To: 'mpadilla@uga.edu'
Subject: Request for a copy of TOGS

Dear Dr. Padilla,

I am in the midst of writing my dissertation proposal and would be very much obliged to you if you could send me a copy of the TOGS.

It is possible that I may want to adapt some of your questions to give my fifth grade students. May I have your permission to look into this further?

Thank you for your time, Dr. Padilla,

Stacy Keller
UCF Doctoral Candidate and 5th Grade Teacher

APPENDIX C: TEST OF GRAPHING IN SCIENCE (TOGS) EXCERPT

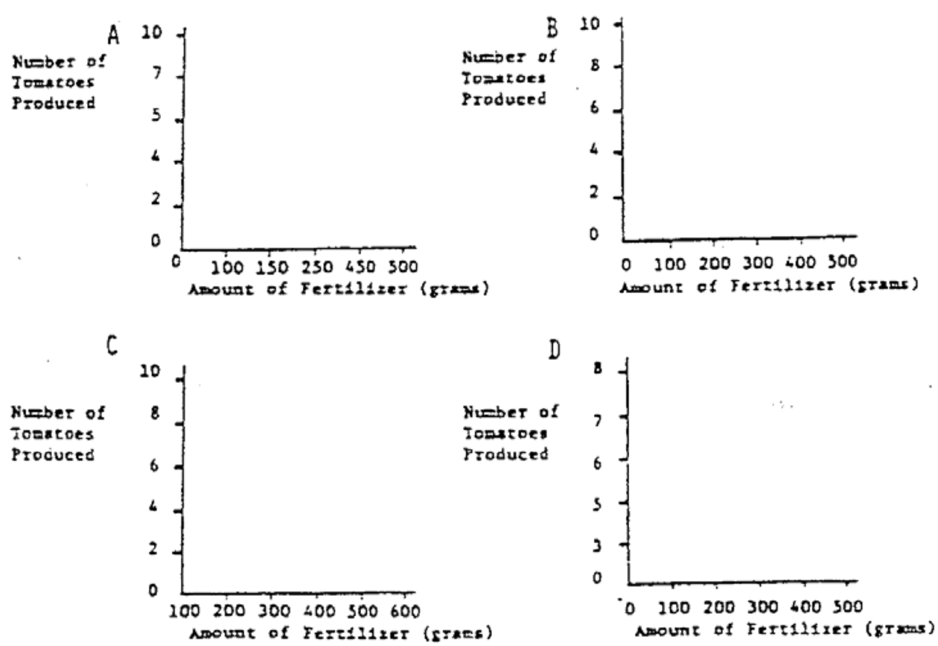
Excerpts from Prof. Michael Padilla's ***Test of Graphing in Science (TOGS)***, 3rd version.

Directions: This is a test of how well you understand graphs. Read each question carefully. (Questions inside the textbox written in italics were read orally by the researcher and not written on the participant's test copy.)

1. Tom wanted to know how much fertilizer he should give his tomato plants. He gave different amounts of fertilizer to his plants. He then counted the number of tomatoes grown on each plant. His results are in the data table below.

Amount of Fertilizer (grams)	Number of Tomatoes Produced
0	3
100	4
150	6
250	8
450	8

Which set of axes below is the <u>best</u> to use for graphing his results?

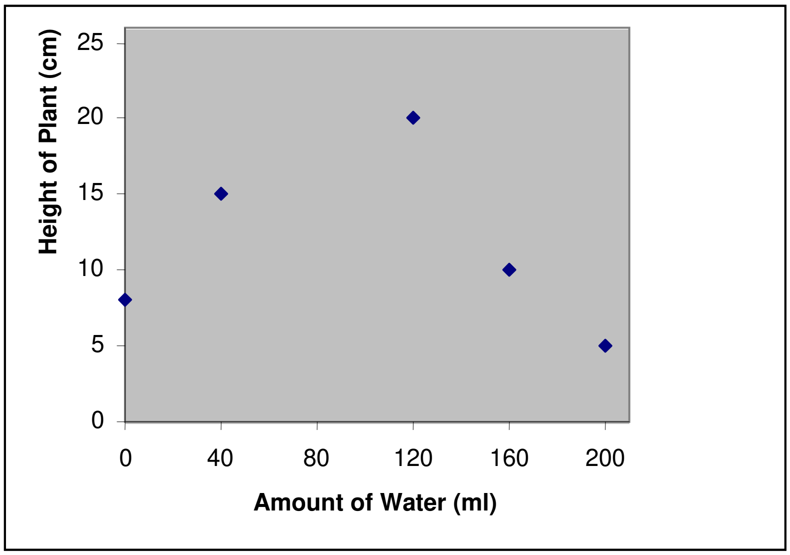


Why did you choose [B] instead of D?

1

Use the information below to answer question 2.

Rose owns a flower shop. She gave different amounts of water to several plants each day. She measured the height of the plants after three weeks. The graph below shows the results.

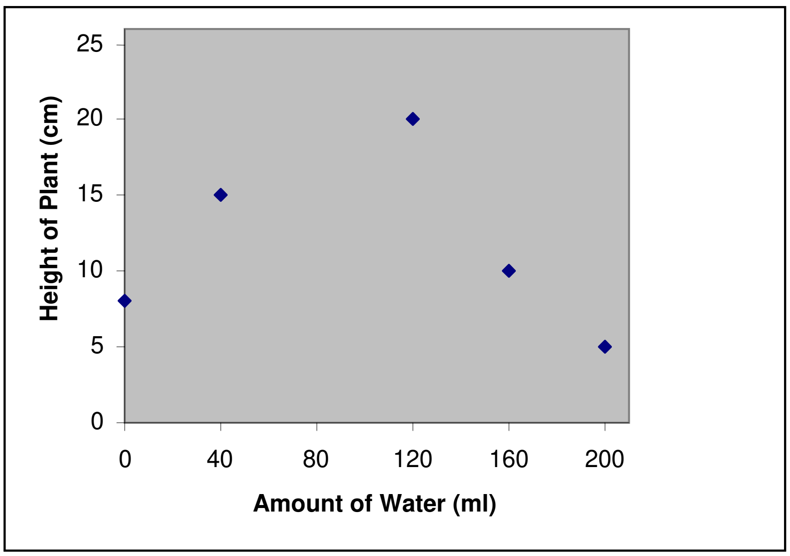


2. One plant was given 140 ml of water daily for three weeks. What would be the expected height of this plant at that time?

A. 11 cm
B. 16 cm
C. 20 cm
D. 23 cm

Use the information below to answer question 3.

Rose owns a flower shop. She gave different amounts of water to several plants each day. She measured the height of the plants after three weeks. The graph below shows the results.

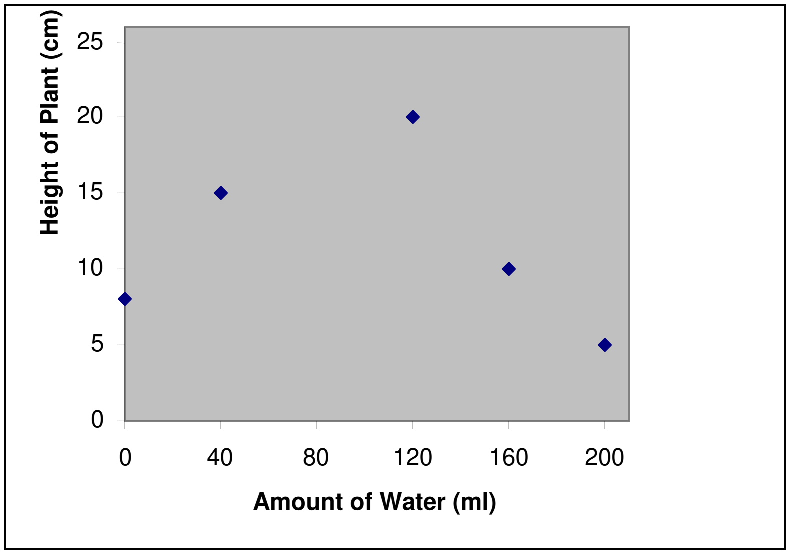


3. How much water was given each day to the plant that grew 10 cm tall?

A. 120 ml
B. 140 ml
C. 160 ml
D. 180 ml

Use the information below to answer question 4.

Rose owns a flower shop. She gave different amounts of water to several plants each day. She measured the height of the plants after three weeks. The graph below shows the results.



4. The following statements describe the relationship between the amount of water given and the height of the plant. [a]Which is the best description? [b]Why do you think this happened?

   A. As the amount of water increased to 120 ml, the height of the plants decreased. With amounts greater than 120 ml, the height of the plants increased.
   B. Both the amount of water and the height of the plants increased up to 120 ml. Then they both decreased.
   C. As the amount of water increased to 120 ml, the plant growth quickly increased. After 120 ml of water the plant growth increased more slowly.
   D. As the amount of water increased to 120 ml, the height of the plants increased. With amounts greater than 120 ml, the height of the plants decreased.

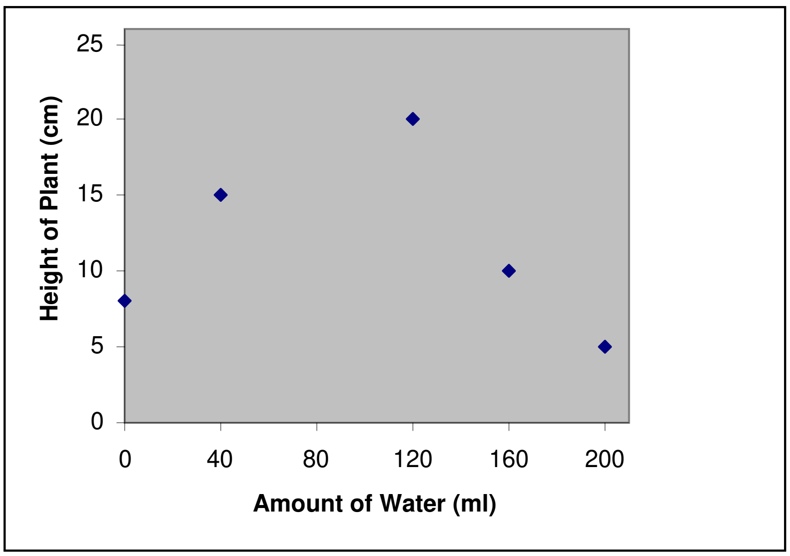
In your own words, describe the experiment Rose was conducting. What did she find out?

How could she have improved her experiment?

Do you have any similar personal experience growing plants to help you better understand the graph?

Use the information below to answer question 5.

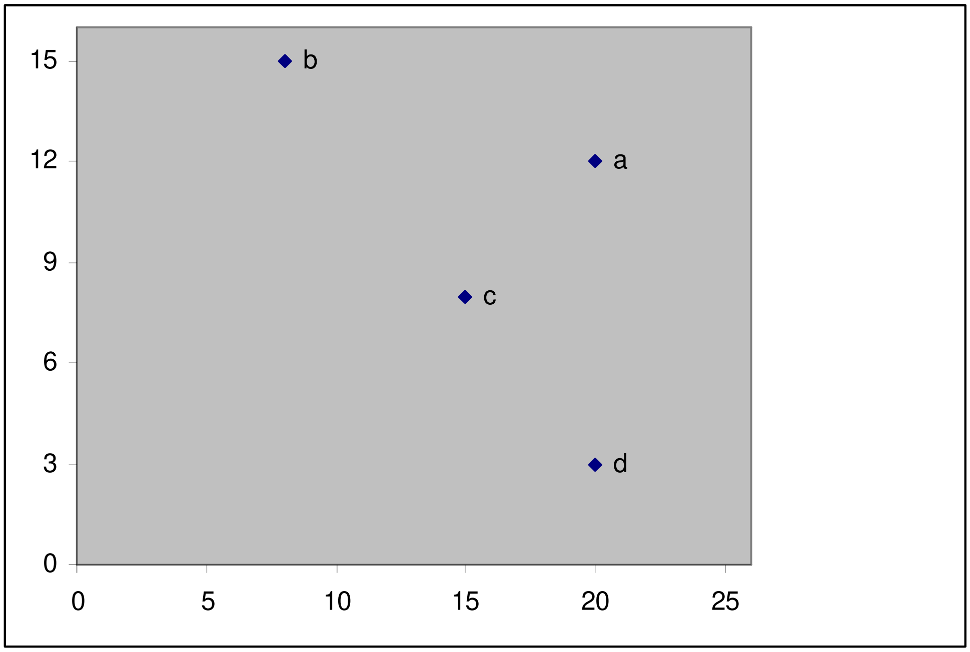
Rose owns a flower shop. She gave different amounts of water to several plants each day. She measured the height of the plants after three weeks. The graph below shows the results.



5. How tall would you expect plants to grow if given 205 ml of water each day?

A. less than 5 cm
B. 8 cm
C. 10 cm
D. more than 20 cm

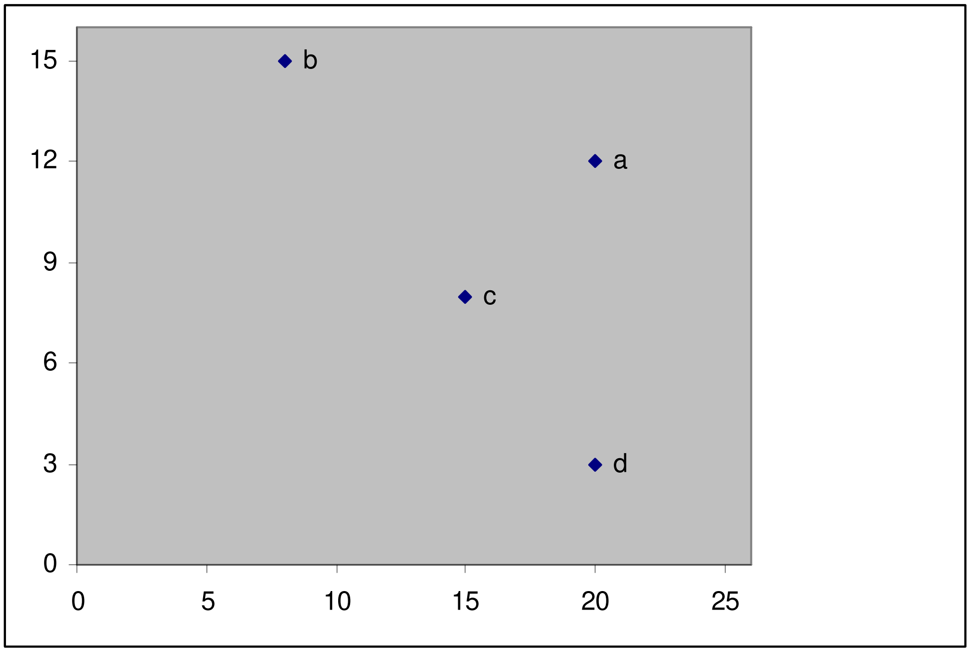
Use the graph below to answer question 6.



6. What are the proper coordinates for point **a**?

A. (9,12)
B. (20,12)
C. (20,8)
D. (12,20)

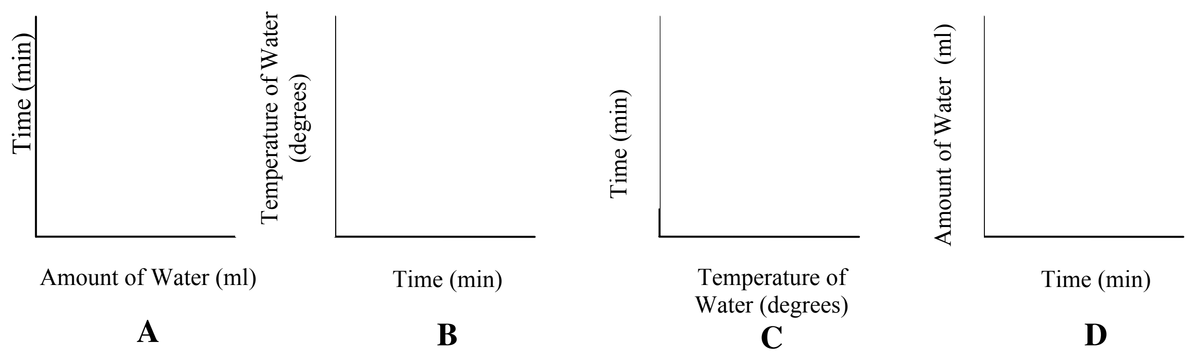
Use the graph below to answer question 7.



7. Which point is identified by the coordinates (15, 8)?

A. a
B. b
C. c
D. d

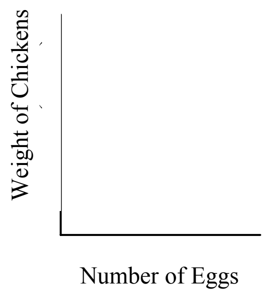
8. Danny measured the time needed to heat various amounts of water to boiling. Which of the following is correctly labeled for showing the results?



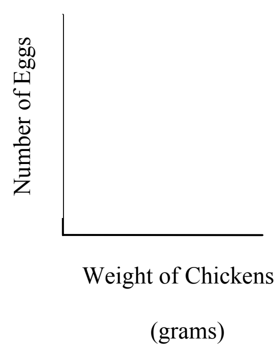
What's wrong with answer choice B?

Do you have any personal experience boiling water that could help you better understand this question?

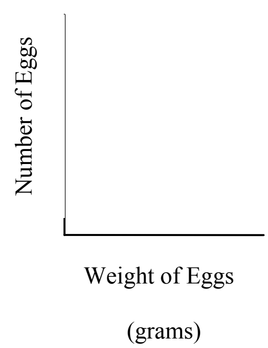
9. Mike wanted to know if the weight of chickens affected the number of eggs they laid each day. Which of the following is correctly labeled for showing his results?



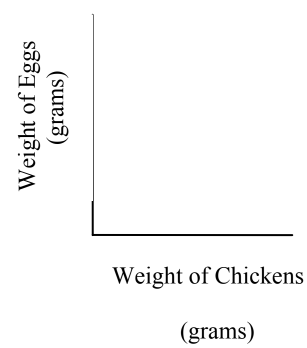
A



B



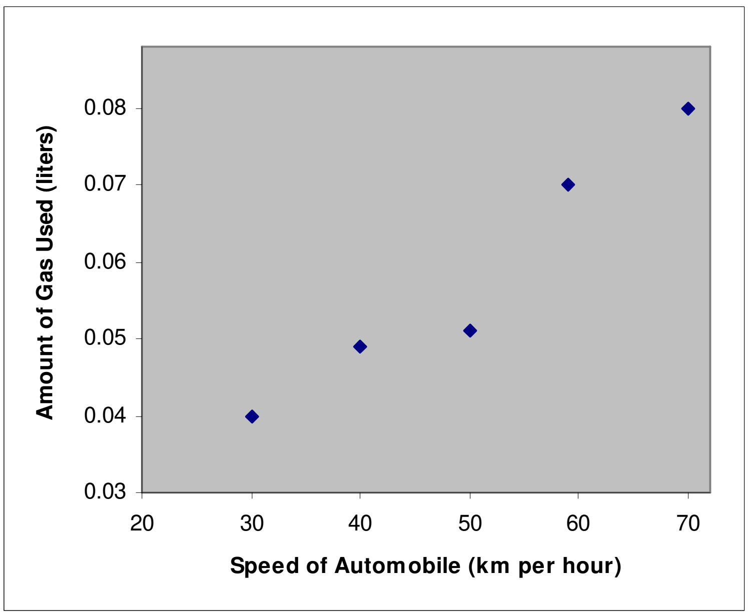
C



D

Use the information below to answer question 10.

Lynn measured the amount of gas needed to drive one km at different speeds. Her results are plotted below.

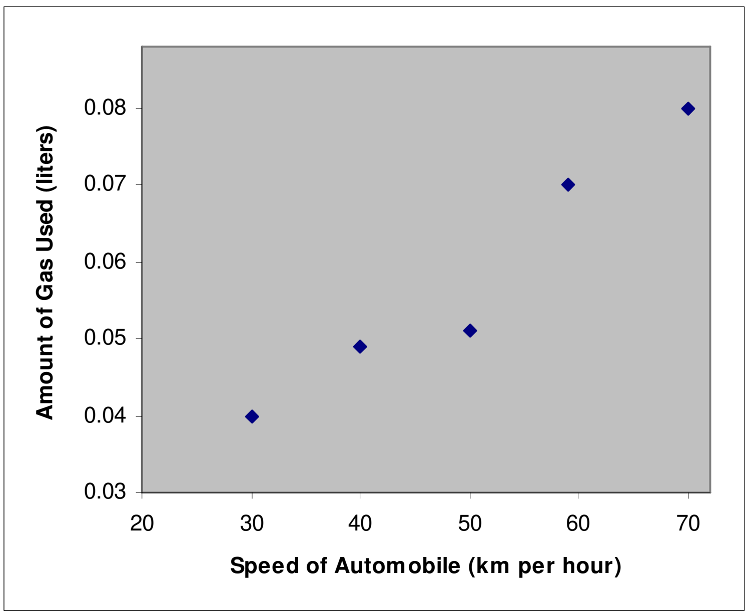


10. How much gas (liters) was used to drive one km at 60 km per hour?

A. .05
B. .06
C. .07
D. .08

Use the information below to answer question 11.

Lynn measured the amount of gas needed to drive one km at different speeds. Her results are plotted below.

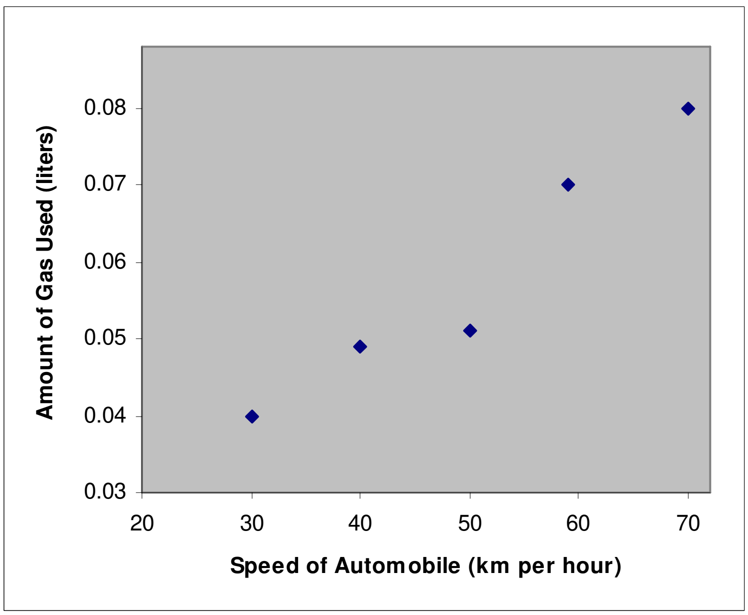


11. [a]Which of the following is the <u>best</u> description of the relationship shown on the graph? [b]Why do you think this happened?

A. As the speed of the car increases, the amount of gas used also increases.
B. As the speed of the car decreases, the amount of gas used increases.
C. The amount of gas used increases as the speed of the car decreases.
D. The amount of gas used decreases as the speed of the car increases.

Use the information below to answer question 12.

Lynn measured the amount of gas needed to drive one km at different speeds. Her results are plotted below.

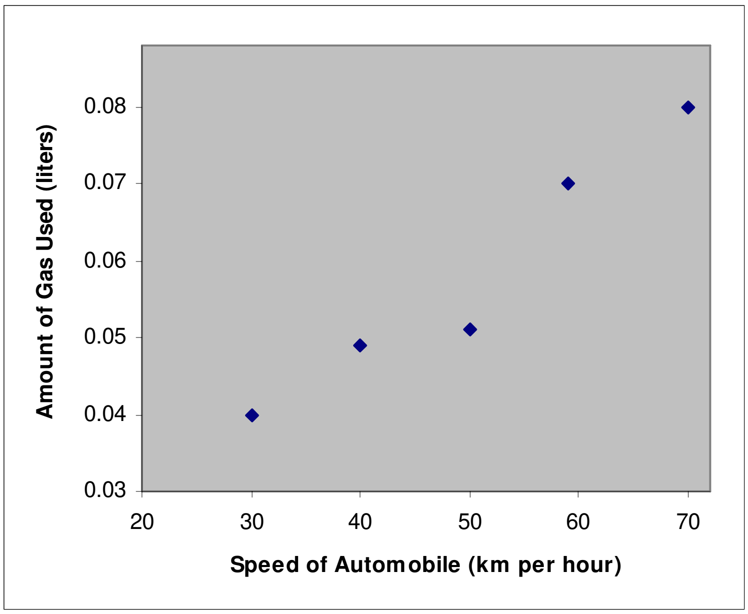


12. At 55 km per hour, how much gas (liters) would the car use?

A. .04
B. .05
C. .06
D. .07

Use the information below to answer question 13.

Lynn measured the amount of gas needed to drive one km at different speeds. Her results are plotted below.



13. At 80 km per hour, how much gas (liters) would the car use?

A. .07
B. .08
C. .09
D. .10

In your own words, describe what Lynn did. *What was the purpose of the experiment?* *What did Lynn find out?*

APPENDIX D: IRB APPROVAL LETTERS, LETTERS OF CONSENT AND ASSENT, PLUS ADDENDUMS

Parent Informed Consent Letter

Dear Parent or Guardian of 6th Grade Student,

Your child has an opportunity to participate in a study being conducted in the College of Education, University of Central Florida. In addition to being a fifth grade teacher at , Stacy Keller is a doctoral candidate at UCF under the supervision of Professor Karen Biraimah. Her proposed dissertation is called "Levels of line graph interpretation with intermediate elementary students of varying scientific and mathematical knowledge and ability: A think aloud study." The purpose is to document and describe children's experiences while interpreting graphs. It is important to identify what students can do easily while graphing and what they struggle with. The results of this study will be published in Ms. Keller's dissertation. The results may also be presented at education conferences or in journal articles. This study may assist other educators to develop instructional strategies that help students better understand graphs.

The parents and guardians of all sixth grade students at are being asked for consent for their children to participate in this study. Ms. Keller is requesting permission to read your child's fifth grade Math and Science FCAT test score. These scores will be used to identify approximately ten students representing a variety of abilities to participate in interviews about graphing. These interview sessions with Ms. Keller will take no more than 2 hours in a classroom at
If your child is selected to participate in one of these sessions, they will be asked to talk out loud while answering a series of questions about graphs and asked questions to ensure Ms. Keller understood what they were thinking.

When considering whether you will consent to allow your child to participate, please consider the following points:

- There are no known risks or immediate benefits to the participants in this study.
- You and your child have the right to withdraw consent for your child's participation at any time without consequence. Your child may refuse to answer any questions without consequence. Your child will be reminded of these rights prior to the interview.
- If you do not provide consent or if your child does not assent, your child's cumulative folder will not be read and your child will not be asked to participate in the after-school interview session.
- The only purpose of accessing the 5th grade Math and Science FCAT data is to select participants for interview sessions.
- The purpose of audio recording this session is only to allow for accurate transcriptions of the interviews. The only people with access to the audio tapes will be Ms. Keller and the University faculty who supervise her research. Tapes will be stored in a locked cabinet at her home and will be destroyed soon after the research is complete.
- When the results of this research are published, Ms. Keller may discuss the data of individual students. These individual data may provide an important way of understanding how students understand graphs. However, in those reports and publications, your child's name, the names of his/her teachers, and the name of your child's school will be kept confidential to the extent allowed by law. All identifying information will be concealed with alternative names.
- No compensation is offered for participation.
- Your child's participation or non-participation in this study will not affect his or her grades or relationship with his or her teacher in any way.

If you have any questions about the purpose or procedures of the study you may contact Ms. Keller at or email at You may also contact Ms. Keller's professor, Dr. Karen Biraimah, at or by email at about the study.

This research study has been reviewed by the UCF Institutional Review Board. Questions or concerns about research participants' rights may be directed to the UCF IRB office, University of Central Florida, Office of Research & Commercialization, University Towers, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246, or by campus mail 32816-0150. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The telephone number is (407) 823-2901.

APPROVED BY
University of Central Florida
Institutional Review Board
10-13-06
IRB Designated Reviewer

Sincerely,

Stacy K. Keller

Stacy K. Keller
Doctoral Candidate, University of Central Florida

Parent Signature Page

Please check all that apply, write in your child's first and last name, and then provide a signature and date.

____ I have read the procedure described on the parent informed consent letter.

____ I have received a copy of the informed consent letter and a copy of the parent signature page to keep for my records.

____ I will sign and return the additional copy of the parent signature page to Stacy Keller.

____ I give consent for Stacy Keller, the researcher, to have access to my child's cumulative folder to access fifth grade Math and Science FCAT scores.

____ If selected, I give consent for my child to participate in the after-school interview.

I voluntarily give my consent for my child, ______________________________, to participate in Stacy Keller's study entitled, "Levels of Line Graph Interpretation with Intermediate Elementary Students of Varying Scientific and Mathematical Knowledge and Ability: A Think Aloud Study" and to be interviewed by Ms. Keller during non-instructional time.

______________________________ /__________

Parent/Guardian Date

Please sign and return one copy of this page within one week.

APPROVED BY
University of Central Florida
Institutional Review Board
10-13-06
IRB Designated Reviewer

Revised Parent Informed Consent Letter

Dear Parent or Guardian of [redacted] 6th Grade Student,

Thank you for previously consenting to allow your child to participate in a study to document and describe children's experiences while interpreting graphs. This study may assist other educators to develop instructional strategies that help students better understand graphs.

After conducting a pilot study to practice the think aloud protocol, it became apparent that it was important to video record student hand gestures while answering graph questions. Without this video data it was difficult to interpret student explanations using only the audio data. At no time during the video were student faces recorded.

I would like to be able to use the video of your child's hand gestures in future presentations of this research. The future audiences for these presentations will be my dissertation committee and other researchers interested in math and science education. The video will only be of your child's hands pointing to the graph examples on the paper and will not include your child's face. The people listening to the presentation will be able to hear your child's voice thinking aloud and will see hand movements but will not be able to identify your child.

When considering whether you will consent to the additional video data collection, please consider the following points:

- There are no known risks or immediate benefits to the participants in this study.
- You and your child have the right to withdraw consent for your child's participation at any time without consequence. Your child may refuse to answer any questions without consequence. Your child will be reminded of these rights prior to the interview.
- The purpose of audio recording this session is only to allow for accurate transcriptions of the interviews.
- The purpose of video recording student hand gestures is to better understand student thinking about graph interpretation.
- Video clips of your child's hand gestures may be used in future research presentations. However, in those presentations and later publications, your child's name, the names of his/her teachers and the name of your child's

school will be kept confidential to the extent allowed by law. All identifying information will be concealed with alternative names.

- No compensation is offered for participation.
- Your child's participation or non-participation in this study will not affect his or her grades or relationship with his or her teacher in any way.

If you have any questions about the purpose or procedures of the study you may contact Ms. Keller at [redacted] or email at [redacted]. You may also contact Ms. Keller's professor, Dr. Karen Biraimah, at [redacted] or by email at about the study.

This research study has been reviewed by the UCF Institutional Review Board. Questions or concerns about research participants' rights may be directed to the UCF IRB office, University of Central Florida, Office of Research & Commercialization, University Towers, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246, or by campus mail 32816-0150. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The telephone number is (407) 823-2901.

Sincerely,

Stacy K. Keller

Doctoral Candidate

University of Central Florida

Revised Parent Signature Page

Please check all that apply, write in your child's first and last name, and then provide a signature and date.

____ I have read the procedures described on the parent informed consent letter.

____ I have received a copy of the informed consent letter and a copy of the parent signature page to keep for my records.

____ I will sign and return the additional copy of the parent signature page to Stacy Keller.

____ I give consent for Stacy Keller, the researcher, to audio and video record my child in an after-school interview.

I voluntarily give my consent for my child, ____, to participate in Stacy Keller's study entitled, "Levels of Line Graph Interpretation with Intermediate Elementary Students of Varying Scientific and Mathematical Knowledge and Ability: A Think Aloud Study" and to be interviewed by Ms. Keller during non-instructional time.

____/______________

Parent/Guardian Date

Please sign and return one copy of this page within one week.

Student Assent Script

Read to all 6th grade students at

My name is Ms. Keller. I am a 5th grade teacher here at , I am also a graduate student at the University of Central Florida. I would like to read your school record to look at your 5th grade Science and Math FCAT scores Will you let me do this?

After I read these records I will ask some of you to come to interviews after school. I am doing a research project on 6th graders using graphs, data tables, and charts. I am interested in how students like you learn and grow in their use of these picture tools. These interviews will take about two hours and ask you to talk out loud while you are answering some questions about graphs. I will record your voice while you are doing this. Will you do one of these interviews if I ask you?

Only Dr. Biraimah, my professor at UCF, and I will listen to these tapes. I will destroy the tapes at the end of the study. All names will be changed so that nobody will know it was you in my study. It will not affect your grades or relationship with your teachers if you decide you don't want to do this. You can stop participating at any time. You will not be paid for doing this. Would you like to take part in this research project?

____ I want to participate in Ms. Keller's research project.

____ She can read my school record to get my FCAT scores.

____ I want to participate in the interviews about graphs after school if selected.

______________________ ______

Student's Signature Date

____

Student's Printed Name

APPROVED BY
University of Central Florida
Institutional Review Board
IRB Designated Reviewer

Revised Student Assent Script

Read to all 6th grade students at [redacted] Who Have Previously Given Assent

Thank you for agreeing to participate in my study of 6th graders using graphs, data tables, and charts. I would like for you to come to an interview after school that will take about an hour. I will ask you to think out loud while you are answering questions about graphs. I will record your voice and video your hand gestures while you are answering the graph questions. Will you let me do this?

I would like to be able to use the video of your hand gestures in future presentations of this research. The future audiences for these presentations will be my dissertation committee and other researchers interested in math and science education. The video will only be of your hands pointing to the graph examples on the paper and will not include your face. The people listening to the presentation will be able to hear you thinking aloud and see your hand movements but will not be able to identify you.

Your name and your school name will be changed so that nobody will know it was you in my study. It will not affect your grades or relationship with your teachers if you decide you don’t want to do this. You can stop participating at any time. You will not be paid for doing this. Would you like to take part in this research project?

____ I want to participate in the interview about graphs after school.

____ I allow Ms. Keller to both audio tape my voice and video my hand gestures during the think aloud interview.

_____I give Ms. Keller permission to use the video of my hand gestures in future research presentations.

______________________________ _______

Student's Signature Date

____

Student's Printed Name

University of Central Florida

Office of Research & Commercialization

October 13, 2006

Stacy Keller
[redacted]

Dear Ms. Keller:

With reference to your protocol #06-3848 entitled, "Levels of Line Graph Interpretation with Intermediate Elementary Students of Varying Scientific and Mathematical Knowledge and Ability: A Think Aloud Study," I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. **This study was approved on 10/12/06. The expiration date for this study will be 10/11/2007.** Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Joanne Muratori
UCF IRB Coordinator
(FWA00000351 Exp. 5/13/07, IRB00001138)

Copies: IRB File
Karen Biraimah, Ph.D.

JM:jt

12201 Research Parkway • Suite 501 • Orlando, FL 32826-3246 • 407-823-3778 • Fax 407-823-3299

APPENDIX E: SYNTHESIS OF PRIOR RESEARCH USED TO DEVELOP THE GRAPH INTERPRETATION SCORING RUBRIC

Table 12

Table 12 Synthesis of Prior Research to Develop the Graph Interpretation Scoring Rubric

	Scoring Rubric
McKenzie, D. L., & Padilla, M. J. (1986). The construction and validation of the test of graphing in science (TOGS). ***Journal of Research in Science Teaching, 23*****(7), 571-579.**	
1. Given a description of an investigation and/or a completed data table, the student will select an appropriately scaled set of axes.	**1.6**
2. Given a set of coordinates (or a point on a graph) the student will locate the corresponding point on a graph (or select the set of coordinates).	**1.1**
3. Given a description of an investigation and/or a completed data table, the student will identify graphs with the manipulated and responding variables appropriately assigned.	**1.5**
4.Given a series of graphs the student will select the graph with the most appropriate best fit line	**Excluded**
5. Given a description of an investigation and/or a completed data table the student will select a graph that correctly displays the data.	**2.7combination of 1.5 & 1.6**
6. Given a graph and a value for X (or Y) the student will select the corresponding value of Y (or X).	**1.3**
7. Given a graph and a situation requiring interpolation and/or extrapolation the student will identify trends displayed in the set of data.	**2.3, 2.4**
8. Given a graphed relationship (linear or curvilinear) the student will select an appropriate description of the relationship.	**2.5**
9. Given graphs of two related relationships the student will identify a generalization that appropriately interrelates the results of the two graphs.	**2.6**
Friel, S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. ***Journal for Research in Mathematical Education, 32*****(2), 124-158.**	
• Correctly reading individual points on a graph	**1.1, 1.7**
• Correctly interpreting information across data points (e.g., performing computations, comparisons, or trend)	**2.1, 2.2**
Gillan, D. J., & Lewis, R. (1994). A componential model of human interaction with graphs: 1. Linear regression modeling. ***Human Factors, 36*****, 419-440.**	
• Search for spatial locations of specifiers	**1.2**
• Encoding the values of specifiers (using an axis & associated labels)	**1.4, 1.7**
• Performing arithmetic operations on the encoded values	**2.2**

• Making spatial comparisons among specifiers (relative heights or lengths)	**2.1**
• Responding with an answer	**Excluded**
Carpenter, P. A., & Shah, P. (1998). A model of the perceptual and conceptual processes in graph comprehension. ***Journal of Experimental Psychology Applied, 4*****, 75-10.**	
• Awareness of preconceived notions about graph & reader context	**3.2**
• Awareness of bias affecting interpretation	**3.1**
• Knowledge of graph features	**1.5, 1.6**
Beeby & Taylor (1977)	
• Misunderstanding of scale	**1.6**
Roth, W. M., & McGinn, M. (1998). Inscriptions: Toward a theory of representing as social practice. *Review of Educational Research, 68*(1), 35-39.	
• Background knowledge of the phenomenon	**3.3**
• Experience with the phenomenon	**3.4**
Vernon, M. D. (1946). Learning from graphical material. ***British Journal of Psychology, 36*****, 145-158.** **Winn, W. (1991). Learning from maps and diagrams.** ***Educational Psychology Review*****,** ***3*****, 211-247.**	
• General learner characteristics (general intelligence)	**Excluded**
Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education*, *18,* 382-393. Gal, I. (1993). Reaching out: Some issues and dilemmas in expanding statistics education. In L. Pereira-Mendoza (Ed.), Introducing data-analysis in the schools: Who should teach it and how? (pp.189-203). Voorburg, The Netherlands: International Statistics Institute. Thomas, K. C. (1933). The ability of children to interpret graphs. In G. M. Whipple (Ed.), *The teaching of geography.* 32nd Yearbook of the National Society for the Study of Education (pp. 492-494). Bloomingdale, IL: Public School.	
• Math knowledge and experience	**Excluded**

Russell, S. J. (1991). Counting noses and scary things: Children construct their ideas about data. In D. Vere-Jones (Ed.), *Proceedings of the third international conference of teaching statistics* (Vol. 1, pp. 158-164). Voorburg, The Netherlands: International Statistics Institute.	
• Number knowledge	**Excluded**
Gal, I. (1993). Reaching out: Some issues and dilemmas in expanding statistics education. In L. Pereira-Mendoza (Ed.), Introducing data-analysis in the schools: Who should teach it and how? (pp.189-203). Voorburg, The Netherlands: International Statistics Institute.	
• Proportional concepts	**Excluded**
Bertin, J. (1983). *Semiology of graphics* (2nd ed., W.J. Berg, Trans.). Madison: University of Wisconsin Press. (Original work published 1967)	**Categorical Headings**

APPENDIX F: CORRESPONDENCE WITH PROF. MICHAEL PADILLA FOR SCORING RUBRIC VALIDATION

Spoke to Dr. Padilla on the phone just now (2006, Dec. 4th from 4:02-4:11PM). He asked me to call him (***********) to explain what I was asking him to do in my earlier email. He told me right away that the TOGS was 20 years old and was a project he worked on with one of his students back then. Although his TOGS test is dated, I told him I was unable to find a better graphing test for the purposes of my study. He agreed that there was not much out there. I then asked him if he was still conducting research on graphing in math and science and he responded no, he was not. He is still interested in the subject but is just too busy.

I explained how I used Bertin's three levels to categorize the literature's documented problems that students have when graphing. I asked him to verify that I placed the indicators under the correct heading on my rubric. He said he would try but may not be able to get back to me right away. I thanked him for speaking to me and he wished me good luck. Very nice man.

December 4, 2006

Dear Professor Padilla,

I really appreciate you taking the time to speak to me this afternoon. I understand that you are pressed for time so I am going to quickly summarize Bertin's three levels. I copied the following two paragraphs from my proposal, Dr. Padilla. I hope they will be useful in explaining the difference between Bertin's three levels of graph questions.

After reading this brief explanation, please scan the attached rubric to see if the categories listed fit appropriately under each of the three levels. Like you said on the phone, if there are any you feel uncomfortable verifying, please indicate that in your response.

Bertin separated his three levels of questions into elementary, intermediate, and overall level questions and pointed out that "although these levels of questions involve an increasingly broad understanding of the data, they do not necessarily imply an increase in the empirical difficulty of the question" (Wainer, 1992, p.16). Elementary level questions, according to Wainer's interpretation of Bertin (1992), "involve data extraction" (p.16). A conceptual connection can be made between Bertin's elementary level questions and Peirce's notion of sign. Intermediate level questions "involve trends seen in parts of the data" (p.16). Again, a connection exists between Bertin and Peirce's notion of interpretant. Finally, overall level questions "involve an understanding of the deep structure of the data being presented in their totality, usually comparing trends and seeing groupings" (p.16). These may be connected to the relationship among Peirce's signs and interpretants.

Specific examples of elementary, intermediate, and overall level questions using the previous science classroom example (i.e., found under Practical Importance: Tacit Theory) will clarify these ideas. Recall these data collected during a plant growth experiment comparing the differences in height of morning glory plants in four different soils. An elementary level question might ask students to find how tall the morning glory plant grew on the fifth day in all four soil types. To answer this question, students would need to be familiar with line graph components, like the x and y axes, and be able to correctly identify the corresponding point where the soil type and the fifth day intersect. An intermediate level question may ask students to find the difference in height on day five between morning glory plants grown in top soil and morning glory plants grown in sand. Here, students would need to access two data points on the line graph and then provide the correct computation (i.e., subtraction) to find the answer. Then, an overall level question might ask students to describe why at the end of the experiment the morning glory plants were, on average, tallest in top soil. This kind of question would require the use of the line graph along with additional background knowledge to sufficiently answer it.

I am looking forward to your insight. Any help you provide will be greatly appreciated.

Sincerely,

Stacy K. Keller

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You replied on 12/6/2006 9:25 AM.

From: Michael Padilla [mpadilla@uga.edu]

Sent: Tue 12/5/2006 4:39 PM

To: 'Keller, Stacy@ '

Cc:

Subject: RE: Graphing assessment rubric validation of TOGS excerpt

Attachments:

View As Web Page

The paragraphs you provided help me to better understand the three levels offered by Bertin. While you caution not to interpret these as difficulty levels, I believe, in fact that they are correlated to difficulty. That is, the first level would typically be the easiest and those in the last level would be the most difficult. I do not feel you should interpret difficulty within any of the levels, however.

Your correlation of Bertin's levels to questions on TOGS matches mine for the first 7 items, then we diverge and do not match at all. I am wondering if you have the same copy of the test I have. My test has 26 items. If you do not have this copy, I will mail you one.
1-7 Look great
8 - Not sure of the match but I can live with it.
9 - The items asks for a best fit line. I don't know where this matches.
10 - This item asks for the same info as #8
11 - Looks ok
12 - I would say it is 1.3
13 - I would say 3.3
14-26 Not mentioned

Let me know where we go from here.

-----Original Message-----
From: Keller, Stacy@]
Sent: Monday, December 04, 2006 5:45 PM
To: 'mpadilla@uga.edu '
Subject: Graphing assessment rubric validation of TOGS excerpt

Dear Professor Padilla,

I really appreciate you taking the time to speak with me this afternoon. I understand that you are pressed for time, so I am going to quickly summarize Bertin's three levels. I copied the following two paragraphs from my proposal, Dr. Padilla. I hope they will be useful in explaining the differences between Bertin's three levels of graph questions.
After reading this brief explanation, please scan the attached rubric to see if the categories listed fit appropriately under each of the three levels.
Like you said on the phone, if there are any you feel uncomfortable verifying, please indicate that in your response.

Bertin separated his three levels of questions into elementary, intermediate, and overall level questions and pointed out that "although these levels of questions involve an increasingly broad understanding of the data, they do not necessarily imply an increase in the empirical difficulty of the question" (Wainer, 1992, p.16). Elementary level questions, according to Wainer's interpretation of Bertin (1992), "involve data extraction" (p.16). A conceptual connection can be made between Bertin's elementary level questions and Peirce's notion of sign. Intermediate level questions "involve trends seen in parts of the data" (p.16). Again, a connection exists between Bertin and Peirce's notion of interpretant. Finally, overall level questions "involve an understanding of the deep structure of the data being presented in their totality, usually comparing trends and seeing groupings" (p.16). These may be connected to the relationship among Peirce's signs and interpretants.

Specific examples of elementary, intermediate, and overall level questions using the previous science classroom example (i.e., found under Practical Importance: Tacit Theory) will clarify these ideas. Recall these data collected during a plant growth experiment comparing the differences in height of morning glory plants in four different soils. An elementary level question might ask students to find how tall the morning glory plant grew on the fifth day in all four soil types. To answer this question, students would need to be familiar with line graph components, like the x and y axes, and be able to correctly identify the corresponding point where the soil type and the fifth day intersect. An intermediate level question may ask students to find the difference in height on day five between morning glory plants grown in top soil and morning glory plants grown in sand. Here, students would need to access two data points on the line graph and then provide the correct computation (i.e., subtraction) to find the answer. Then, an overall level question might ask students to describe why at the end of the experiment the morning glory plants were, on average, tallest in top soil. This kind of question would require the use of the line graph along with additional background knowledge to sufficiently answer it.

I am looking forward to your insight. Any help you provide will be greatly appreciated.

Sincerely,

Stacy K. Keller

Due to Florida's broad public records law, most written communications to or from government employees regarding public education are public records. Therefore, this e-mail communication may be subject to public disclosure.

APPENDIX G: THINK ALOUD PROTOCOL INSTRUCTIONS

THINK ALOUD PROTOCOL INSTRUCTIONS

In this study I am interested in what you are thinking as you find the answers to questions I give you. In order to do this, I am going to ask you to “THINK ALOUD” as you work on the task at hand. What I mean by think aloud is that I want you to tell me EVERYTHING that you are thinking and doing from the time you first read the question until you have completed your answer. I would like you to talk aloud CONSTANTLY from the time you hear the question until you feel that you have completed your answer. I do not want you to feel as if you have to plan what you are going to say or that you have to explain what you have said. Act as if I am not in the room and you are here speaking out loud and working on the questions by yourself. It is important that you keep talking at all times. If you are silent for a length of time, I will prompt you to keep talking. Do you understand what I have asked of you?
Good.

Now let’s begin by practicing on a sample question. Remember to think aloud as you answer the question. Tell me everything that you are thinking and doing from the moment you first read the question. What is the result of adding 1584 + 426? Good.

Now I want to hear how much you can remember about what you were just thinking from the time you read the question until you gave your answer. I am interested in what you can actually REMEMBER rather than what you think you must have thought. If possible, I would like you to tell about your memories in the sequence they happened while you were working on the question. Please tell me if you are uncertain about any of your memories. I don’t want you to rework the problem, just report all you can remember thinking about when answering the question. Now, tell me what you remember. Good.

Now, I will give you one more practice problem before we proceed to the main activity. I want you to do the same thing for this problem as you just did. I want you to think aloud as before as you think about the question and after you have answered it, I will ask you to report all that you can remember about your thinking. Any questions? Here is your next problem. How many windows are there in your parents' house? Good.

Now I want to hear how much you can remember about what you were just thinking from the time you read the question until you gave your answer. I am interested in what you can actually REMEMBER rather than what you think you must have thought. If possible, I would like you to tell about your memories in the sequence they happened while you were working on the question. Please tell me if you are uncertain about any of your memories. I don't want you to rework the problem, just report all you can remember thinking about when answering the question. Now, tell me what you remember. Good.

Now we are ready to move onto the graphing test. During these questions, you will continue to use the same protocol as you did for your two sample questions. Each question is on a separate page. Read each question silently to yourself and then reread it aloud the second time. As you think aloud, please feel free to write on your test copy and be sure to circle the correct answer when you are done.

When you finish with one question, I may ask you to remember what you were thinking while answering the question. If I am not going to ask you this, I will simply tell you to turn the page. This will be your cue to turn the page and move on to the next question. Remember to think aloud as you answer the questions. Tell me everything that you are thinking and doing from the moment you first read the question.

APPENDIX H: CORRESPONDENCE WITH THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM) TO REQUEST AND OBTAIN COPYRIGHT PERMISSION

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Subject:	RE: Dissertation copyright permission request for JRME
Date:	Tue, 24 Jun 2008 11:46:39 -0400
From:	"permissions" <permissions@nctm.org>
To:	"Stacy Keller" < >

Dear Stacy,

This e-mail constitutes our permission for you to use Figure 1 from Roth, W.-M., and Bowen, G. M. (2001). Professionals read graphs: A semiotic analysis. Journal for Research in Mathematics Education, 32(2), 159-194.

Please use the following credit line to accompany this material:

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Thank you for your request, and best of luck on your dissertation!!

Ramona.

Ramona Grewal, Permissions Editor
Publications Department
National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 20190
permissions@nctm.org

From: Stacy Keller [mailto:]
Sent: Monday, June 23, 2008 6:34 PM
To: permissions
Subject: Dissertation copyright permission request for JRME

Dear Colleagues,

I need your help. I will be completing my dissertation entitled: Levels of Line Graph Question Interpretation with Intermediate Elementary Students of Varying Scientific and Mathematical Knowledge and Ability: A Think Aloud Study at the end of the summer at the University of Central Florida in Orlando.

I am citing the following article in my dissertation:
Roth, W.-M., and Bowen, G. M. (2001). Professionals read graphs: A semiotic analysis. Journal for Research in Mathematics Education, 32(2), 159-194.

I would like to request permission to use their Figure 1: The two-stage semiotic process of reading graphs (p. 162). I tried doing this through the Copyright Clearance Center but was not successful. Would you be able to grant me permission over email or clarify this process? I need help.

Thank you so much.

Stacy K. Keller
UCF Doctoral Candidate
6-23-2008

APPENDIX I: CORRESPONDENCE WITH PROF. WOLFF-MICHAEL ROTH FOR ASSISTANCE WITH THE TWO-STAGE SEMIOTIC PROCESS MODEL OF READING GRAPHS

Yahoo! Mail Classic

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Date:	Wed, 18 Jun 2008 20:24:24 -0700
From:	"Wolff-Michael Roth" <mroth@uvic.ca>
Subject:	Re: Note from UCF Doctoral Student about Perceptual Dispositions
To:	"Stacy Keller" < >

Hi Stacy, here is a chapter in an upcoming book. You find editor, book information on bottom of first page. Don't know yet exact page numbers. Let me know should you have more questions. Cheers, Michael PS: The other is a paper I had, and you find more stuff on making perceptual distinctions.

On 16-Jun-08, at 3:06 PM, Stacy Keller wrote:

Thank you so much. I look forward to reading your papers. Enjoy the rest of your vacation, Dr. Roth.

Stacy Keller

Wolff-Michael Roth <mroth@uvic.ca> wrote:

I am on vacation in Ireland right now. When I get home I can send you
another paper or two on the topic of making perceptual distinctions.
I will be back home in a couple of days. I hope this will work for
you. Sincerely, Michael

Print - Close Window

On 15-Jun-08, at 2:05 PM, Stacy Keller wrote:

Dear Dr. Roth,

I'm a doctoral student at the University of Central Florida and am using your Critical Graphicacy book as a framework for my dissertation. In my data, students' perceptual dispositions are emerging as a focus of the analysis. However, I want to be certain how you meant the notion of perceptual dispositions within your hermeneutic semiotic model. I've read what you have written on the topic but remain concerned that I may be using the concept too broadly. Can you help me?

Thank you so much,

Stacy Keller

Attachments

Files:

Roth2a.pdf (507k)

052102_Roth.pdf (1.1MB)

APPENDIX J: WRITTEN PERMISSION FROM THE AUTHOR TO REPRINT THE TEST OF GRAPHING IN SCIENCE (TOGS)

YAHOO! MAIL Classic

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To:	"Stacy Keller" < >
Subject:	Re: Copyright permission request from Stacy Keller, UCF Doctoral Candidate
From:	padilla@clemson.edu
Date:	Wed, 25 Jun 2008 18:54:05 +0000

You have my permission. Best of luck.

Sent from my Verizon Wireless BlackBerry

-----Original Message-----
From: "Stacy Keller" < >

Date: Wed, 25 Jun 2008 13:19:53
To:<mpadilla@uga.edu>
Subject: Copyright permission request from Stacy Keller, UCF Doctoral Candidate

6-25-08

Dear Prof. Padilla,

Two years ago you gave me permission to use and adapt your Test of Graphing in Science. Then, you also helped me validate my scoring instrument which I so greatly appreciated. The dissertation is nearing completion but my committee members are having difficulty understanding my analysis of the TOGS interviews without seeing the test items the students worked on. To remedy this problem, I would like to include the 13 excerpted test items in the data analysis chapter and appendix sections of my dissertation with your permission. Would this be okay?

Sincerely,

Stacy Keller
UCF Doctoral Candidate

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