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The Effects Of Problem Solving Strategy Instruction, Journal Writing And Discourse On 6th Grade Advanced Mathematics Student Per

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THE EFFECTS OF PROBLEM SOLVING STRATEGY INSTRUCTION, JOURNAL WRITING AND DISCOURSE ON 6TH GRADE ADVANCED MATHEMATICS STUDENT PERFORMANCE

by

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ABSTRACT

There are two purposes to this study. The first was for me, as a teacher, to try something new in my instruction and grow from it. The second purpose of this study focused on the students. I wanted to see what level of performance in problem solving my students are at currently, and how the use of journaling and discourse affected the students' problem solving abilities.

A problem-solving unit was taught heuristically in order to introduce students to the various strategies that could be used in problem solving. Math journals were also used for problem solving and reflection. Classroom discourse in discussion of problem solving situations was used as a means of identifying strategies used to solve the problem. Explanations and justifications were then used in writing and discourse to support students' solution and methods. An analytic problem-solving rubric was used to score the problems solved by the students. These scores, along with explanations and justifications, and discourse were used as data and analyzed for common themes.

The results of this study demonstrate overall improvement in student performance in problem solving. Heuristic instruction the students received on strategies in problem solving helped to improve their ability to not only select an appropriate strategy, but also implement it. This unit, along with the problem solving prompts solved in the journals, helped to improve the students' performance in explanations. It was discourse combined with all the previous instruction that finally improved student performance in justification.

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CHAPTER 1: INTRODUCTION

Rationale for the Study

"Do we have to do all four steps?" "Do we have to show our work?" "What other strategy could I use?" These are some of the common questions I hear from my students as I assign word problems for homework or group work. I have taught middle school mathematics for the last six years, and am still hearing these questions every time one of the problem solving lessons is taught. Problem solving is engaging in a task for which the solution as well as the method used to solve is not immediately known (NCTM, 2000). They have become the lessons that I do not look forward to teaching, not just because the students dislike it, but also because I have not found a good method of instruction and assessment for problem solving.

I have taught mathematics primarily from the Glencoe mathematics textbook series (Bailey *et al.*, 2006; Holliday *et al.*, 2005; Malloy *et al.*, 2005). Last year, the Saxon mathematics series became the textbook used for several of my mathematics classes (Hake, 2007). They each teach a version of Polya's four-step plan for problem solving. The Glencoe four-step plan begins with exploring the problem. Exploring includes reading the problem and identifying not only the important information but also the question that is asked. The second step is to plan. Students are to determine what steps they are going to take in determining the solution to the problem. This could include one or more problem solving strategies. These may be drawing a picture or diagram, finding a pattern, making a list, or working backwards. The plan may also include one of the basic operations of addition, subtraction, multiplication, or division. Once the students have determined a plan, they then can move on to step three, which is to carry out the plan that they wrote. Once the plan is carried out, the student should have a solution. The final step is to check, or examine, the answer. This includes ensuring that the solution actually answers the question and determining if the answer makes sense.

The Saxon text is similar in many ways to Glencoe in its plan for problem solving. It also has four steps to solving. These are to understand, plan, solve, and check. Even though the name of the steps varies slightly, the idea behind each step is the same. Both textbook series' four step plans are based off of Polya's (1957) problem solving plan.

Instruction of this material in each text is also different. Glencoe introduces problem solving in the first lesson of the textbook. One lesson that focuses on different problem solving strategies is then included in each chapter. These lessons do not appear in the pre-algebra or algebra texts, only in the texts for the general middle school courses. The four-step plan is to be used in each of these lessons, but it is never assessed. Word problems appear on the tests, but the students are not required or directed to follow the four steps in solving them.

Saxon also begins the middle school textbook with a pre-lesson on problem solving and its strategies and steps. If the Saxon curriculum is carried out as designed, students are supposed to solve a problem every day in class using the problem solving strategies that were taught. Problem solving and its steps are also used on every assessment written in the Saxon Curriculum. The students are expected and directed to follow the problem-solving plan as they solve. Two problems with this format for instruction are the length of the average class period and the information contained in the student textbook. Saxon lessons suggest 60 minutes of time from warm-ups to homework (Hake, 2007). Middle school class periods are often 50 minutes or less. Additionally, the solutions to the word problems they suggest using are included in the student textbooks.

The idea behind each of these texts' methods for problem solving instruction is to teach students better problem solving skills. "Writing down the thoughts and procedures involved for each of the steps of a problem solution adds yet another dimension to the processing" (Kenyon, 1989, p. 74) and solving of a problem. It is important that students are able follow these steps in order to improve their skills in problem solving (Higgins, 1997; Polya, 1957; Rickard, 2005). However, there are several problems that I have encountered as I teach problem solving using the four step plan prescribed by Polya and the strategies as written in these mathematics texts.

The first problem is based on the students and their respective backgrounds. Many students have not consistently received instruction in these methods, or if they have, all four steps have not been required. When teachers do require students to demonstrate all four steps in their work, they struggle with not only getting the students to follow it, but also in instructing them to do it well. As I stated earlier, my own students always question having to do the exploring and planning steps. They would rather just solve the problem. Part of this is due to lack of motivation, but it also is a result of them not really understanding how or why to do it this way. This issue stems from problem solving not being clearly defined (Rickard, 2005, p. 2). If problem solving is not clearly defined or explained to educators, then we cannot expect students to have a full understanding of it either. I also notice that my students prefer to use only operations in solving problems. Using another problem solving strategy is rarely an option for them. From my experiences, they also do not seem to understand many of these other strategies. If the student cannot determine how to solve it using an operation, then they either ask for help or do not answer the problem. They do not seem open to using more strategies. If students are taught heuristically, they tend to rely completely on the strategies taught. This heuristic instruction consists of students learning one problem-solving strategy at a time. The purpose is for students to make the connection between a type of word problem and an appropriate strategy. In heuristic instruction, only a variety of strategies are taught. When students are unable to determine new strategies, or combine them to find the solution, they exhibit less creativity in problem solving (Higgins, 1997).

The second problem I have found with the problem-solving plan is that no explanation or justification is required. Mathematical explanations are clarifications of thinking and methods used in problem solving and can be conducted in written or verbal form. Justification is reasoning behind the use of certain steps and strategies in problem solving.

Students need to be able to communicate about mathematics when working a problem (Countryman, 1992; NCTM, 2000). It is only recently that this idea has become more important in the mathematics classroom. Students should be able to explain their steps to a problem and justify why they chose those steps. This leads to the deeper understanding of the concepts being taught (NCTM, 2000). Explanations and justifications are part of effective communication, which is important in all subject areas.

The third problem I face with the instruction of problem solving is time. Saxon requires more time than I have in a period to teach problem solving. Due to this lack of time, I do not go over problem solving each day and the students see these problems only on their tests. As far as Glencoe, the mathematics lessons devoted to problem solving have too many problems to cover in one period. If there was more time in the class period, these lessons could be implemented in the way they were written to.

There are also problems with the teacher's role in instruction in a problem-solving lesson. Teachers are to demonstrate and model the method taught. I do not support these teaching methods completely, but since they are part of our current text, I use them. However, I do not always feel comfortable teaching in these ways. My mathematics classes as a child never included problem-solving plans. I was taught to simply solve the problem and doing nothing beyond finding the solution. In a way, this is still new for me. I also do not have a strong method for assessing the students using the four-step plan. For every method of instruction, there needs to be a means of assessing the students' level of understanding. Writing in mathematics is one means of assessment that is being suggested more and more (Burns & Silbey, 2001; Pugalee, 2001b), but the implementation of that in the classroom is still something I have not yet mastered.

Purpose

There are two purposes to this study. The first is for me, as a teacher, to try something new in my instruction and grow from it. A problem-solving unit will be taught heuristically in order to introduce students to the various strategies that could be used in problem solving. Math journals will also be used as a means of not only encouraging student reflection and communication regarding mathematics, but also as a place for them to practice problem solving and obtain feedback from me. Classroom discourse in discussion of problem solving situations will also be used as a means of identifying strategies used to solve the problem. Discourse consists of student discussion and writings pertaining to classroom situations, work, strategies, and solutions in a whole class and individual setting. Explanations and justifications will also be used in writing and discourse to support students' solution methods. It is not only important for the student to explain their own strategy but to see the strategies used by others. The students will then compare the strategies used. I want the students to be able to see that a variety of strategies can be used in solving one problem.

The second purpose of this study focuses on the 6th grade advanced mathematics students I teach. I want to see what level of performance in problem solving my students are at currently, and how the use of journaling and discourse affects the students' problem solving abilities.

Research Questions

1. How does heuristic problem solving instruction affect my 6th grade advanced mathematics students' performance on problem solving tasks?
2. How does journaling affect my 6th grade advanced mathematics students' performance on problem solving tasks?
3. How does discourse affect my 6th grade advanced mathematics students' performance on problem solving tasks?
4. How did this sequence of instruction and practices affect my 6th grade advanced mathematics students' performance on problem solving tasks?

Significance

The significance of this study is multifaceted. First it will help to fill the literature gap related to the use of and assessment of journal writing and classroom discourse in developing problem solving skills. Literature can be found on these topics related to the elementary level.

However, literature is lacking of articles that examine journaling in the middle school mathematics classroom as well as using journaling for problem solving.

According to Pugalee, research on writing in mathematics is inadequate (2001b). This is one reason that there has not been a great acceptance of writing in the mathematics classroom (Pugalee, 2004). Problem solving instruction has also not been implemented in effective ways in the classroom. This is partly due to problem solving instruction not being well described for teachers (Rickard, 2005). Teachers have not been taught how to teach it, so they have difficulty implementing it in their classrooms. This lack of research in both areas has impacted the instruction of problem solving and the use of writing in middle school mathematics.

In conducting this research, hopefully positive effects will be found that will encourage other mathematics teachers to improve their problem solving instruction, introduce writing in their classes, and instruct students in explaining and justifying their work. NCTM (2000) has also placed great importance on problem solving and communication. This research will hopefully also add to the research they have conducted in these areas and demonstrate their value and importance.

This study is also important because it will influence the students. The practices that I will use in this study will increase student writing in mathematics, allow them to use various strategies for solving problems, and allow them to discuss problems rather than just solving them. “Research on problem solving suggests that many students tend to give up rather quickly when presented with novel or unfamiliar problem solving tasks” (Doerr, 2006, p. 3). I am interested to see how journaling and introducing a strong method for problem solving will affect the student’s ability to solve problems. They will hopefully improve in these skills. This study will also introduce my students to the practice of writing and explaining in mathematics, which has become very important. With this study, my students will improve their mathematical literacy. It will also demonstrate to other educators the effects of writing in the mathematics classroom.

Some students are also afraid of mathematics. They view it as “a subject in which answers are right or wrong” (Rose, 1989, p. 15). This study will help students to see that the answer is not the only important part of mathematics. The process of getting a solution is just as important. Incorrect solutions and methods are just as important to discuss as correct ones.

In focusing on problem solving in this study, I will be meeting part of the national standards of mathematics. This will “help students to develop fluency with specific skills” and also “solidify and extend what they know” about mathematics (NCTM, 2000). Problem solving as such is a very important part of mathematics instruction.

This study also has direct significance to me and other mathematics teachers. Personally, it will show me how problem solving instruction, journals, and discourse affect my students. It will help me to determine if this is a practice that I will continue in the future. It is also a means for personal and academic growth. Other teachers will also be able to learn from my experience in this process.

Chapter two begins with a focus on writing in mathematics, and continues with discussions about research related to problem solving, discourse, explanation, and justification. The literature review will also consist of discussion of metacognition, social norms, and sociomathematical norms. Many of these topics are interrelated and are all important to the focus of this study.

CHAPTER 2: LITERATURE REVIEW

Introduction

Problem solving and communication are two aspects of the mathematics classroom that are viewed as important to instruction. The National Council of Teachers of Mathematics (NCTM) devotes two of its chapters in *Principal and Standards for School Mathematics* to the use of problem solving and communication in the classroom. In terms of problem solving, NCTM has set the following standards in the instructional programs from pre-kindergarten through grade twelve:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving (p. 256).

NCTM goes further into detail with respect to problem solving standards for the middle school grade levels. Problem solving should "build on and extend the mathematical understanding, skills, and language that students have acquired" (NCTM, 2000, p. 256).

According to NCTM (2000), "communication is an essential feature as students express the results of their thinking orally and in writing" (p. 268). Students should learn at all levels of their schooling to present and explain the strategy chosen and used. The students should also be able to "analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies" (NCTM, 2000, p. 268). As one can see, NCTM places great importance on communication. The standards for communication that have been listed for grades pre-kindergarten through grade twelve are:

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies to others
- Use the language of mathematics to express mathematical ideas precisely (p. 268).

This chapter begins with a discussion of social and sociomathematical norms, an important aspect of classroom interactions, that affects discourse and instructional decisions. Problem solving will then be addressed. I focus on the instruction of problem solving, which includes Polya's 4-step plan and strategy instruction. The method for teaching problem solving strategies is also described focusing primarily on the heuristic style. Metacognition and sociomathematical norms are also further addressed in terms of problem solving instruction.

This chapter also focuses on journaling, writing, and discourse in the mathematics classroom pertaining to problem solving. Definitions and descriptions of these models of instruction will be included. Teacher and student roles are defined throughout the various sections of this chapter as well as further discussion of norms relating to discourse. The value and importance of problem solving, writing, and discourse in the mathematics classroom is a strong theme in this chapter.

Social and Sociomathematical Norms

Norms are those activities that become "instrinsic aspects" (Yackel & Cobb, 1995, p.270) of the classroom in any subject area. For the purposes of this paper, two types of norms will be focused on: social and sociomathematical. Social norms are not specific to mathematics, but rather are found in any classroom. Sociomathematical norms are those established that pertain specifically to mathematics. Research related to these norms at higher levels of education is still lacking but has begun to be researched (Andreasen, 2006; Yackel, 2001).

Social norms that may be established in the classroom involve students giving explanations to solutions, offering different solutions, asking questions, and listening to other students' solutions (Yackel, 2001). Sociomathematical norms take these practices and apply them even more specifically to mathematics. These sociomathematical norms "include normative understandings of what counts as a different solution, a sophisticated solution, and an efficient solution and what counts as an acceptable explanation and justification" (Yackel & Cobb, 1995, p.264). Such norms cannot be taught through traditional instruction. Instead, they are "formed in and through the interactions of the participants in the classroom" (Yackel, 2001, p.7). Thus, students participate in classroom interactions, they are establishing and negotiating norms (Yackel, 2001; Yackel & Cobb, 1995).

In creating sociomathematical norms, one issue needs to be addressed first. Students, as well as teachers, have traditional views of the teacher and student roles in the classroom. The traditional view of the teacher's role has been the instiller of mathematical knowledge, concepts, and solutions. To establish norms in the mathematics classroom, this view needs to be overcome and replaced with new views (Whitenack *et al.*, 1995). As norms are negotiated, they are constrained by current beliefs, suppositions, and assumptions of the classroom participants (Whitenack et al., 1995; Yackel & Cobb, 1995, 1996). The renewal of these norms is based on the student taking a more active role and the teacher acting more as a guide in finding solutions and leading discussions. As such norms are established, students will "reorganize their beliefs about their own roles" (Whitenack et al., 1995, p.256).

Norms are "continually regenerated and modified by the students and the teacher through their ongoing interactions" (Yackel & Cobb, 1995, p.270). Every activity, discussion, and instruction plays a role in establishing both social and sociomathematical norms. Like social norms, sociomathematical norms are constantly evolving (Yackel & Cobb, 1995, 1996). As these norms evolve, so do the students' and teacher's beliefs about mathematical activity and learning.

Problem Solving

As stated earlier, problem solving is the process of finding a solution to a problem to which that solution is not immediately known. What needs to be understood is that there is a difference between problem solving and exercises (Rickard, 2005). Exercises are a procedural practice from a lesson or concept that has been taught. These problems do not require as much thought in how to solve them. The students have been instructed in a method and then practice it in the exercises. Problem solving however, can require the use of prior instruction, but "the person does not have an algorithm available to resolve the situation"(Kenyon, 1989, p. 75). It requires thought and planning, as well as previous knowledge and skills, which may not be immediately apparent to the student (Kenyon, 1989).

The instruction of problem solving has recently become a more important part of mathematics instruction. According to NCTM, learning problem solving results in students acquiring "ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom" (2000, p.52). What students learn in the mathematics classroom about problem solving is deemed so important because these skills can and will be used later in life outside of mathematics.

Though problem solving has become such an important aspect of mathematics instruction and has been adopted into many mathematics curriculums, it is still not always a part of instruction in the classroom. Even though problem solving became important in mathematics, it became apparent that a weakness in this reform. The problem is that problem solving itself is not well defined (Rickard, 2005). Some teachers do not understand enough about problem solving themselves to be able to change their practices to implement problem solving instruction in the classroom. Some aspects of the reform in problem solving instruction are being implemented, but this reform needs to involve a deeper understanding of mathematics and needs to be based more on inquiry (Kazemi & Stipek, 2001).

As the agreement on the importance of teaching problem solving has developed, so have differences in the best method for its instruction (Santos-Trigo, 1998). Polya is considered to have developed one of the first methods for problem solving instruction. His method has four stages to solving any problem (Polya, 1957). The first stage is to understand the problem. This involves not only reading the problem, but also looking for the important information. This stage also requires the student to look at the question being asked and make sure that they understand, or comprehend, it. Once they have read for understanding, students then move on to the second stage, which is to create a plan. In this step, students determine which strategy or strategies work best for the problem and how to carry them out. This leads to the third stage, solving the problem. The students use the plan created in stage three to solve the problem. The final stage is to look back. This includes review and discussion (Polya, 1957) of the solution as it relates to the problem. The students are checking to see that their solution is reasonable and answers the question asked. Thus, problem solving is a complex process that must be taught and modeled (Polya, 1957). Polya intended this process to show students how to think when problem solving. The problem is this process has been misinterpreted as steps. Textbooks base their problem solving instruction on Polya's ideas but teach them as steps to follow.

Polya's four stages to problem solving have been the basis for various other methods of problem solving instruction, including those methods included in textbooks (Niemi, 1996). Hohn and Frey developed another method of problem solving instruction for the purposes of their own research. Their method is called SOLVED, and it associates one step with each letter:

S – state the problem

O – options to use

L – links to the past

V – visual aid

E – execute answer

D – do check back (Hohn & Frey, 2002).

The similarities between the two indicate the value and importance of the steps that Polya developed.

Once a method of problem solving has been taught, either through Polya's or a variation of his method, students must learn the various strategies that are required. Such strategies become important in step two of Polya's method. These strategies include guess and check, finding a pattern, graphing, creating a table, eliminating possibilities, solving a simpler problem, and working backwards (Higgins, 1997). Other strategies include solving and equation, drawing a picture, making a list, and using logical reasoning. According to Higgins, these strategies are based on skills. The skills can be taught but not the strategies (Higgins, 1997). These skills could include teaching students how to create a table, but according to Higgins, the student has to determine when it can be used in a problem. The skills become strategies when a student selects one or more skills to use for solving a problem.

Another debate that has developed in the instruction of problem solving is whether to teach these strategies using heuristic methods. Teaching this heuristically involves focusing instruction on one strategy at a time. Students would be given multiple problems that would focus on a single strategy, thus giving the students plenty of repetition in practicing this strategy with the intent being that they would learn to recognize the problems that could be solved using this method.

There are positives and negatives to teaching heuristically. The positive side is that students have a strong understanding of the strategies taught and recognize problems that would use a given strategy. Higgins (1997) conducted a year-long study on problem solving instruction and its effects in middle school. The classes were split into two groups, those that learned problem solving heuristically and those that learned through the lessons taught in the textbook. After one year, Higgins found many benefits to teaching problem solving heuristically. Students had more confidence, it provided them with a place to start, students wanted more difficult problems, they were willing to spend more time on problems, and they were able to apply their knowledge outside of math class. Higgins also found a negative to this type of instruction. She found the students' problem solving skills and strategies lacked some creativity (Higgins, 1997).

Higgins, however, did not expand on how or where she saw this in her research, just stated that it occurred.

There are several reasons that the students may lack some creativity. If a problem requires a combination of strategies or a strategy that was not taught, the student may not be able to create a plan. They may not know where to begin in trying to solve the problem. They rely to heavily upon the strategies taught, which in turn, then limits their ability to solve the problem (Higgins, 1997).

Research tends to support the positives of teaching heuristically. According to De Corte (1995), "heuristic methods substantially increase one's probability of success in solving the problem" (p. 38). Hohn and Frey (2002) and Rickard (2005) also found that teaching problem solving strategies heuristically helped the students more than it hurt them.

Metacognition

Another idea presented in the instruction of strategies is the "use of both heuristics and metacognitive strategies to solve problems"(Santos-Trigo, 1998, p. 631). Metacognition is "thinking about thinking". It involves knowing one's own strengths and weaknesses (De Corte, 1995). Pugalee (2004) states that these metacognitive behaviors are apparent in every step of problem solving, which would include the steps that involve choosing and using a strategy. He goes on to state that metacognition is "essential to employing the appropriate information and strategies during problem solving"(Pugalee, 2001b, p. 237). This is important for teachers to realize in the instruction in problem solving. It is through metacognition that students are able to recollect previous problems and draw from those experiences to assist them with the current problem. Students may struggle with determining a strategy to use. This may be due to the student not having enough previous experience with that type of problem. This research is contradictory to Higgins (1997), who states that they struggle when the strategies they have learned do not apply.

While Pugalee states that metacognition is part of problem solving, Martinez states that problem solving is part of metacognition (Martinez, 2006). He states that metacognition is a conscious activity and that we use it daily. This process is not just used in problem solving in mathematics, but also in other areas of our lives. Martinez's views provide another support for problem solving instruction. Students learn to solve mathematical problems using a plan and strategy. This same practice could be applied outside of the mathematics classroom. If instruction of problem solving has taken place, students will be able to recollect how they solved other mathematical problems to help them solve problems in other situations.

Teacher Role

The teacher's role in problem solving and its instruction is very important. "One of the most important tasks of the teacher is to help his students"(Polya, 1957, p. 1). However, the teacher is not to help too much or too little. Polya states that it is important for the student to carry out a reasonable amount of the work. The question then arises as to what role the teacher should take. There are several parts to this role.

The teacher should assist the student when needed; however, they do need to learn when to step in so that it is not too soon or too late. According to Polya, the teacher should question the students. These questions should be general so that they will apply to any situation. They should also be asked repeatedly (Polya, 1957). Polya also stated that teachers need to know more about what they are teaching. This will help them to better assist and instruct the students.

Another responsibility of the teacher is to select the problems for the class to solve. Many people think that teachers should make understanding mathematics simple, others believe they should make it problematic (Hiebert & Wearne, 2003). Teachers need to select problems that are academically appropriate for the students but also challenging (Ediger, 2006; Pugalee, 2004). Again, the idea behind problem solving is that the solution is not immediately evident.

Finally, teachers need to model for students. Students will learn by "imitation and practice" (Polya, 1957, p. 5). Teachers need to model problem solving strategies for students, as this will help students to better understand the process. According to Martinez (2006), teachers also need to model metacognition. The teacher can accomplish this by speaking aloud all his thoughts while working through the problem solving process.

Student Role

The students have a role in problem solving instruction as well. As stated earlier, they need to practice it. The students should take on the right attitude towards problem solving. This needs to be demonstrated by the teacher as well (Polya, 1957). Students should be able to carry out the problem solving process on their own after they have been instructed in it. Knowing the process is not enough for students to solve a problem. They may need further guidance from the teacher. Students also need a strong conceptual understanding of the mathematics in order to determine a solution (Niemi, 1996).

Sociomathematical Norms

Also important to problem solving is social and sociomathematical norms. As stated earlier, social norms are aspects of social activity that become part of the routine of the classroom. Sociomathematical norms are those norms that relate directly to mathematics (Rasmussen *et al.*, 2003). Kazemi and Stipek (2001) further describe the differences between these two norms. Social norms include explaining your thinking, whereas the sociomathematical norm would be criteria for what counts as a good mathematical explanation. Another example of a social norm is to discuss the different strategies used. In sociomathematical norms, this would go further to compare the various strategies used in problem solving and the mathematical concepts behind the strategies. One final difference Kazemi and Stipek explain includes the norm of group work. Social norms can include any small group work. What makes group work a sociomathematical norm is when that group is required to come to a consensus on the solution to the problem (Kazemi & Stipek, 2001).

Acceptable explanations are an important sociomathematical norm in problem solving. Explanation should explain the act, but also go beyond a procedural description. There is a growing interest among researchers and teachers in investigating explanation and justification and how students learn these practices (Yackel, 2002). To teach students what an acceptable explanation is, they need to understand that the basis for their actions should be mathematical, which tends to be difficult because mathematics has been taught in a more procedural method (Yackel & Cobb, 1995). The purpose of explanation is to clarify aspects of mathematical thinking that may not be apparent to others (Yackel & Cobb, 1995). The student needs to explain their work and solution in a way that helps the other students to understand and also in a way that is mathematically correct. The collaboration of teachers and students together is what causes the ongoing establishment of acceptable explanations in the mathematics classroom (Yackel & Cobb, 1995). According to Kazemi and Stipek(2001), it should also consist of a "mathematical argument" .

Students should also be able to justify their solution and work. They should be able to explain why they solved it the way they did (Kazemi & Stipek, 2001). Students are not permitted to simply give an answer; they must also be able to explain to anyone in the class not only how they solved the problem, but also why they chose that method. This justification also includes the reasoning behind each step taken. Explanation and justification both help in developing stronger and deeper understanding of the concepts. "The problem solving process will be understood by the learner only when he or she can explain it to others"(Kenyon, 1989, p. 74).

Sociomathematical norms help student understanding. Individuals are "seen to develop their personal understandings as they participate in negotiating classroom norms" (Yackel & Cobb, 1995, p.266). Kazemi and Stipek's study gives credit to sociomathematical norms for promoting conceptual understanding as well. Students benefit greatly from the establishment and continual modification of sociomathematical norms.

Writing in Mathematics

There is a "growing interest in the relationship of writing to teach[ing] mathematics" (Rose, 1989, p. 17). There are many strategies that can be used in the classroom to promote a deeper conceptual understanding in the mathematics classroom. In some cases, these strategies focus on the solution to a problem. "Writing is a unique learning strategy in which process is at least as important as the product" (Rose, 1989, p. 15). Though NCTM and "literature asserts the power of writing as a learning tool advancing the writing across the disciplines movement, it has not gained wide acceptance in mathematics classrooms" (Pugalee, 2004, p. 28). The reason for this lies primarily with teachers. Teachers have resisted using writing in the mathematics classroom due to "the anticipated time to read and grade the work because they feel untrained as a composition teacher" (Rose, 1989, p. 27). The coursework for a mathematics educator focuses on instructing math. As writing becomes more important, it may become part of the curriculum at the college level as well. This would benefit future mathematics teachers to feel more trained in writing instruction and assessment.

Writing can also be difficult for the students. Writing "about what they think can be more difficult than describing a concrete action" (Burns & Silbey, 2001, p. 18). This relates back to the focus on the correct solution rather than the process. Our students are used to the solution being the important part. Writing about the process is new to them. They must be instructed in this and the teacher, as stated earlier, should model this for the students. The "students should be assisted to write well"(Ediger, 2006, p. 120). As students repeatedly practice this their comfort level in this will increase. It will then become a common practice to them, one that they expect, which would in turn make it a norm in the classroom. It takes on the form of a sociomathematical norm as the students write about the specific process of solving a mathematics problem as well as their reasoning behind the process (Rasmussen et al., 2003). Students would further be helped in this if writing was more "emphasized across the curricula" (Ediger, 2006, p. 120).

There are many benefits to including writing in the mathematics classroom. According to Pugalee's research "writing is posited as providing a level of reflection that promotes students' attending to their thinking about mathematical processes" (2004). Writing forces them to focus on their thinking, which involves the practice of metacognition. This helps mathematics to become about more than just the solution. It helps students to focus "on describing their thought processes while solving mathematics problems" (Pugalee, 2004, p. 32). Thinking about the process provides students with a better understanding. Writing about this "adds another dimension to processing" and helps to improve their cognitive skills (Kenyon, 1989).

Writing also allows students to be more active in the classroom. "Active students also have a different attitude towards their work" (Kenyon, 1989, p. 74). Students will become more confident in their work as they realize they not only have a solution but are able to communicate their solution and work to others. According to Baxter "communication is critical to the development of mathematical proficiency" (Baxter *et al.*, 2005, p. 120).

Journal Writing

Writing in the mathematics classroom should be done to achieve a goal (Ediger, 2006). There are many ways to implement writing into the mathematics classroom. One method in particular is journal writing. The use of journals depends on several factors (Burns & Silbey, 2001). The first is the purpose for the journals. Journals can be used to write the following: daily notes, problem solving, feelings, questions, descriptions of concepts learned or not understood, and reflections (Burns & Silbey, 2001; Rose, 1989). All of these various uses can be assigned for inside or outside of the classroom. Some of the uses could be made more personal, such as reflections. Others will remain more academic, like problem solving. The writing in journals can also be more focused or unstructured (Rose, 1989).

The second factor is the preference of the teacher. Journals do not have to be used for every one of the options above. Teachers can select those options that fit the goal that they would like to meet in the classroom.

The final factor is the age of the student. As students grow and develop their writing skills, the expectations of writing can increase. Teachers may also decide to try other journal writing options. Again, it is important for the teacher to select academically appropriate tasks.

For the purposes of this study, journal writing will focus on two types of journal writing: problem solving and reflection. Journals can provide a place for students to record their work and thoughts in how to solve a problem. This can be used throughout the problem solving process.

There are benefits to using problem solving as a part of journal writing. The journal becomes a means for students to record his or her strategies, thoughts, explanations, and justifications for his work (Pugalee, 2004). Writing also helps students to understand their work to the point that they can better defend it. In journal writing about problems students will "use writing and drawing to show why their solutions were correct" (Niemi, 1996, p. 354). It is very important for students to be able to explain and justify their work and solution to a problem. According to Kenyon (1989), this is the only way for students to completely understand the problem solving process. Writing their work in journals is one way this can be accomplished.

Journal writing can also be used as a means of reflection for the student. It can be used for "keeping a personal record of thoughts and reflections" (Kenyon, 1989, p. 84). Writing can be used at any point in the class. According to Baxter, Woodward, and Olson (2005), writing helps students to gather their thoughts. Students can write about what they learned (Ediger, 2006). It can also be an opportunity for the students to write about what they did not fully understand.

The writing done in the journal can assist the teacher in assessing student understanding (Baxter et al., 2005; Burns & Silbey, 2001; Doerr, 2006). Journal writing provides insight into student thoughts, the students' mathematical proficiency, and creates connections between the student and the teacher (Baxter et al., 2005). Baxter found this true especially for the girls in his study.

Based on research by Doerr (2006), teachers can be more effective in the classroom if they have an understanding of the student's mathematical thinking. Writing in problem solving and reflection formats provides this to the teacher. The teachers can then use this information to know how to proceed in their instruction to the class. Burns (2004, p.30), in a study of journal writing and assessment, found that "not only did I see how writing helped students think more deeply and clearly about mathematics, but I also discovered that students' writing was an invaluable tool to help me assess their learning". Pugalee (2001b) also found that student writing is a good means of assessment.

Teacher Role

The teacher's role in journal writing is important. The teacher determines the level and expectations of the writing. Some options include asking students for more than one strategy, having students write their own word problem, or requiring explanation and justification. These options all provide opportunities and means of assessment (Rose, 1989).

In the study conducted by Pugalee (2001), as students solved problems, they were instructed to write down every thought that came to mind. Teachers then needed to watch, upon making this requirement, that students actually did this. In Pugalee's study, if the students did not start writing, the teacher would remind them to write what they are thinking.

The teacher would also decide upon writing prompts for reflection. Various topics could include student descriptions of definitions, explaining what they understood about a lesson, or discussing what they did not understand about the lesson. Teachers should provide the prompt though, so that students have some direction in their writing.

Journal writing should not be one-sided, whether it is reflections or problem solving. The teacher should "make appropriate comments on written problem solutions" (Kenyon, 1989, p. 85). This includes providing written feedback to the student about misconceptions or asking for clarification. Pugalee (2004) also states that teachers should provide comments and questions on the students' entries, giving them feedback. As students are learning to write in mathematics, it is important for the teacher to write in the journal so that students will know if their explanations and justifications are clear enough. "Students report they find journals more beneficial when using them in dialogue with the teacher" (Rose, 1989, p. 25). Rose states that teacher writing also helps the students to be more consistent as students will know what is expected of them.

The benefits of writing in mathematics are numerous. The students are no longer passive learners, they become active in the process (De Corte, 1995). It demonstrates for teachers the students' mathematical reasoning and the metacognitive framework in the writing and solving process (Pugalee, 2001b). Writing helps the teacher to determine what the students do and do not understand. There are benefits for the students as well. Their problem solving and reflection skills also develop further (Burns & Silbey, 2001).

Discourse

As discussed, written communication is important in the mathematics classroom. Verbal communication is also very important. "There is an increasing appreciation for the important role that discourse plays in the development of conceptual thinking" (Pugalee, 1999, p. 21). Even though the role of discourse is so valued, it, like writing in the mathematics classroom, is not always included. Discourse should not only be included but should become an important aspect of every mathematics classroom.

"A class environment in which students are constantly asked to explain and communicate ideas to other students is an important feature of the class" (Santos-Trigo, 1998, p. 642). This skill leads students to participate in classroom discussion, where the process taken and the solution found may be further questioned or judged. By promoting such skills as explanation and justification in writing, these skills will then carry over and give students the confidence to explain the same things aloud. Discourse is a facilitator in the construction of knowledge and concepts in mathematics (Pugalee, 1999). Therefore, it is an integral part of instruction. In order for students to grow in their verbal communication abilities, teachers need to create "opportunities in the classroom for considering and discussing children's productions" (De Corte, 1995, p. 43).

There are issues that prevent the use of discourse in the classroom like there were for writing. Some teachers are unwilling or weary of beginning this practice in the classroom. Breyfogle (2005) places blame for this on teacher instruction. It is difficult for teachers to establish this in the classroom when they have received very little instruction in discourse. In order to convince teachers to establish this practice not only is instruction required but also a change in their beliefs and attitudes towards it is necessary (Breyfogle, 2005). If the teacher does not value discourse or see its importance, he or she will not change their instruction to include it.

Discourse has great value in the mathematics classroom. "We see discourse-oriented teaching as involving not only the establishment of social norms that empower students to discuss mathematics but also a belief that properly orchestrated discourse among students will result in the production of useful knowledge" (Williams & Baxter, 1996). Like problem solving instruction and journal writing, the teacher has a key role in establishing this practice so that it might become established as a norm for classroom interaction.

Teacher Role

The teacher's role in discourse is vitally important. In Nathan's (2003) study on classroom discourse, much detail was given to the role of the teacher. He describes the teacher as a "guide on the side"(Nathan, 2003, p. 176). The teacher should allow the students more freedom to discuss problems or questions. Teachers should be "…leading from behind, at times stepping in as a mathematical authority, and at other times carefully guiding the discussion and activities…" (Nathan, 2003, p. 176). He goes on further to describe other responsibilities of the teacher. As students give their input, the teacher may re-describe their words using more mathematical terms. In this, the teacher is then introducing mathematical terms for the students to learn and use the next time. The teacher may also point out important aspects of the student's contribution. This aspect may then lead to further and more in-depth discussion on that topic or lead to new ideas.

Nathan describes this teacher-student interaction as a part of social scaffolding. In this scaffolding, students are asked to "provide explanations for solutions to problems or eliciting contributions to whole class discussion" (Nathan, 2003, p. 179). This scaffolding can flow in several ways. It can flow vertically, which is from teacher to student. Information can also flow horizontally, or from student to student. In horizontal scaffolding it is imperative that teachers continually monitor the discourse. It is their responsibility to guide the discourse in a way that leads to whole class understanding.

There are a variety of ways for teachers to keep the discourse going in the right direction. Teachers can question students about their thinking (Burns & Silbey, 2001). This could include asking them to further explain a step or a solution in the problem solving, or it may be to ask them to explain why they took the steps they did. Teachers may question students when the solution is right or wrong. Either way, it leads to a greater understanding of the concepts involved. Another idea is for teachers to have students who solved the problem in different ways explain and show their work. This allows for the class to not only compare and contrast the methods, but also to discuss why they may or may not have worked. As students talk, the teacher may call on other students to restate what another student said. This allows students to put things into their own words and aids the teacher in assessing the level of understanding.

Williams and Baxter (1996) share some of Burns and Silbey's views on the teacher role. They also see the teacher as a facilitator. They believe that the teacher should talk less and listen more, placing more value on student talk than teacher talk. However, "the teacher is still ultimately in control of the knowledge that is produced" (Williams & Baxter, 1996). It is important that the teacher does not allow the students to accept or believe what is incorrect. The teachers need to make sure the students end up with proper understanding of the concept.

Like Burns and Silbey, Williams and Baxter also agree that we tend to rephrase students' thoughts using proper mathematical language. They also believe that we give cues to our students, and that students learn and watch for these cues. Teachers have to try not to continue this (Williams & Baxter, 1996).

Pugalee also has determined his views on the teacher's role based on his various studies. He discusses the issues that may arise and what teachers need to do in order to create an environment for discourse. He states that "adolescents are self-conscious and may be hesitant to expose their thinking to others; therefore, teachers should establish a classroom community that makes students feel free to share their thoughts without fear of ridicule" (Pugalee, 2001a, p. 298). The teacher must create an environment where there is trust and respect amongst the students. The teacher must also be an active facilitator. In this way, if things begin moving toward a lack of respect, the teacher can quickly step in.

Pugalee also discusses two other roles of the teacher. The first is that the teacher must ensure that the learning objectives are met. He also states that the teacher must select tasks that engage the students, which will make them more willing to discuss. These tasks must also require thought and reason and even the possibility of the use of multiple strategies (Pugalee, 2001a). By selecting problems like these, the teacher is creating a greater opportunity for discourse to occur and for it to occur more naturally.

Breyfogle (2005) points out one other important task of the teacher. He found in his research that "teacher reflection is a key component in changing teacher's practices" (p. 153). As teachers are carrying out discourse in their classroom, they should also reflect upon it. This will allow them to see not only how they can continue to better this practice, but also how important it is in the mathematics classroom.

Student Role

Students also have a role in discourse. Much of the research on the teacher role points out the importance of student-to-student discourse. They must be able to explain and justify their work to the whole class. Students must also be willing to question other students for clarification as well as on the correctness of their solution (Yackel, 2001). Students must also realize the value behind a wrong answer (Burns & Silbey, 2001). These discussions can also lead to a deeper conceptual understanding.

Horizontal scaffolding in discourse creates a framework for students to add on to their knowledge (Williams & Baxter, 1996). Students will hear and see other methods for solving and this will increase their own knowledge and understanding of other problem solving strategies.

Teachers do face some issues with the student role. Students may not ask productive questions or their explanations may be more procedural (Williams & Baxter, 1996). This is where the teacher's role as model and facilitator becomes important. Some students will not connect the discourse to the work and may see it as getting in the way of getting the answer (Williams & Baxter, 1996). Like the teacher, students will need to reflect to see the connection between discourse and problem solving. Again, the teacher may need to lead the discussion in that way.

Social and Sociomathematical Norms

Social norms and sociomathematical norms are also part of discourse. According to studies conducted by Yackel (2001), these norms help in clarifying the functions of and means by which explanation, justification, and argumentation could be fostered through discourse. Explanations and justifications are key parts of discourse. Yackel describes explanations as clarifications of aspects of mathematical thinking. Justification is described as coming in response to challenges to the solution or methods chosen.

Yackel also describes the differences between social and sociomathematical norms in discourse. Social norms include explaining and justifying thinking and solutions. It also includes listening and making sense of others' thoughts and solutions. Upon listening, students will then ask questions and raise challenges. Sociomathematical norms on the other hand occur when students discuss what is similar or different in their thinking and solutions. The discussion goes further to discuss which were more efficient (Yackel, 2001). Teaching students what acceptable explanations and justifications to a problem is also a part of establishing sociomathematical norms.

The main part of discourse is explanation and justification. This can be done in small groups or as a whole class. Students must learn as stated earlier, what an acceptable explanation and justification is. The questioning of a student's explanation begins as the teacher's role (Yackel & Cobb, 1995). This models for students how to ask classmates for clarification as well as showing how specific they themselves need to be in their own explanations. Eventually, students should take this role on themselves in looking for clarification and their own understanding (Yackel & Cobb, 1995). This is how mathematical meaning develops (Yackel, 2001). Students develop meaning and understanding based on their individual interpretation of the discussion. This leads students to ask questions and participate more, which in turn continues the establishment of sociomathematical norms (Yackel, 2001).

Besides this instruction, students also need to be able to recognize acceptable explanations and justifications given by other students. Instruction in this recognition leads students to “intellectual autonomy” (Yackel & Cobb, 1995, p.269). Autonomy is reached when a student relies on his or her own judgement of a mathematical explanation rather than the teacher’s (Cobb *et al.*, 2001). Upon reaching this autonomy, students will also be able to question and challenge other explanations without prompting from the teacher (Yackel, 2001). This refers back to changing the students’ view of their role in the classroom. Teachers should be helping students to become more self-sufficient and confident in mathematical explanations and justifications.

Another key part of discourse is discussion of the solution. According to Yackel and Cobb’s research (1996), the norm of identifying mathematically different solutions occurs when the teacher requests students to offer different solutions. The teacher can help this process by encouraging and calling attention to different solutions. As solutions are discussed, students learn to determine if a solution is actually mathematically different and make judgments about similarities and differences between solutions (Yackel & Cobb, 1996).

Right answers are not the only solutions that should be discussed in class. Incorrect solutions provide “opportunities to reconceptualize a problem, explore contradictions to a solution approach, and try out alternative strategies” (Kazemi & Stipek, 2001, p. 72). If students see that another classmate is incorrect in their method or their solution, they must be able to explain why, not simply state that it is wrong. Students should develop the ability to “listen to and attempt to make sense of each other’s interpretations of and solutions to problems, and to ask questions and raise challenges in situations of misunderstanding or disagreement”(Yackel, 2001, p.6). This questioning and explanation shows their understanding of the concept and their ability to analyze a solution. This should become a sociomathematical norm in the classroom (Yackel, 2001).

The teacher and students together work to create and negotiate the sociomathematical norms that develop in the classroom (Kazemi, 1998). Once these norms are established, they continue to grow and develop. This is key to causing and helping students to think more conceptually and supports higher level cognitive thinking (Yackel & Cobb, 1996). In turn, this will help student's own understanding of the mathematics improve. Kazemi (1998) identified four sociomathematical norms that relate to this and are evident throughout this discussion:

- Explanations consisted of mathematical arguments, not simply procedural summaries of the steps taken to solve the problem.
- Errors offered opportunities to reconceptualize a problem and explore contradictions and alternatives strategies.
- Mathematical thinking involved understanding relations among multiple strategies.
- Collaborative work involved individual accountability and reaching consensus through mathematical argumentation. (p.411)

Kazemi (Kazemi, 1998) determined that these four norms were key to the mathematics classroom and increasing conceptual understanding. Students need to go beyond procedure and explain the mathematics used. They also need to discuss and compare the various strategies that are used in solving problems and the solutions reached.

Conclusion

Based on the literature discussed, problem solving, writing, and discourse are all important in the mathematics classroom. Great value is placed on students being able to communicate their thinking both in writing and classroom discussions. Communication "should be a fundamental component in implementing a balanced and effective mathematics program" (Pugalee, 2001a, p. 297). By introducing and practicing these methods of instruction, students will possibly be affected in the same way as those students in Yackel's study where "It became routine for students to explain their thinking, to ask questions and raise challenges, and to elaborate their explanations and justifications spontaneously without prompting from the instructor" (2001, p. 11). Based on the literature read related to my research questions, I created a plan for collecting and analyzing data. Chapter 3 will explain in detail how I carried out my research.

CHAPTER 3: METHODOLOGY

Problem and Rationale

In the six years that I have been teaching, I have observed my students' performance and attitude towards problem solving and the steps involved. I have observed their disinterest when a problem-solving lesson is presented in their text. The dislike they have toward writing any more work than necessary to solve the problem is also evident. My students are very used to being taught the steps to a problem rather than finding a solution for themselves. Many of them prefer it that way. Therefore, part of the reason I believe they dislike, and have less confidence in their work during these lessons, is the fact that it requires them to think "outside of the box." They have to come up with a plan themselves. I conducted this study to determine the effects of heuristic instruction, journal writing, and discourse on student performance in problem solving. The purpose of this study was to reflect on my instruction of problem solving heuristically as well as my use of journal writing and discourse in mathematics to help improve my students' performance in problem solving.

Design of the Study

The study conducted was an action research study. By conducting action research, I was able to study and evaluate my own practices in the classroom. This study included both qualitative and quantitative methods of collecting data for the purpose of evaluating student performance in problem solving. The qualitative data included the students' reflective journal prompts and whole class discourse on problem solving. The quantitative data included the performance rubric used to assess student performance in problem solving.

Reflective journal writing prompts were used as a means of checking student understanding as well as connecting more with my students on math related topics. Students were also instructed to write in their journal to provide feedback on the journal writing and discourse related to problem solving. Samples of journal prompts can be found in Appendix A.

Audio recordings were taken of several of the beginning problem solving strategy lessons as well as of several uses of whole class discourse. Samples of the topics and questions discussed can be found in Appendix B. Classroom discourse related to their problem solving prompts was recorded to study student participation, understanding, and performance in problem solving. These recordings were based on problems already completed by the class. It was also recorded to study my own participation in the discourse in order to better my role in classroom discourse.

Problem-solving prompts were used to assess student performance in problem solving skills, explanation, and justification. Samples of the problem solving prompts can be found in Appendix C. Student performance was measured using a rubric that assessed the following specified criteria: identifying the important information in the problem, selecting and implementing an appropriate strategy, showing work and solving the problem, and a complete explanation and justification of how the problem was solved. The rubric was adapted from Krulik and Rudnick's (1998) *Assessing Reasoning and Problem Solving: A Sourcebook for Elementary School Teachers* and can be found in Appendix D.

Assumptions and Limitations

This study was conducted with the assumption that the instruction of problem solving lessons on specific strategies, journal writing including both writing and problem solving prompts, and whole class discourse would improve student performance in problem solving. This assumption is based on the review of literature conducted and on my graduate courses related to mathematics instruction.

One limitation of this study was the sample. Due to the requirements of the administration in certain classes, research was only conducted in advanced math classes. Therefore, there is no comparison of these practices between advanced and general math students. Another limitation was the length of my research period. The problem-solving unit was not taught at the start of the school year, which would have been preferred. Rather, instruction of the unit took place after the first quarter of the school year. The unit took three weeks to complete. During that time, students did no writing in their journals. Upon completion of the unit, data was collected from their journals for five more weeks without discourse, and then another five weeks with discourse. This time for journaling was also interrupted by Christmas break.

Setting

This study took place in a Central Florida parochial school that is in the downtown of mid-size city. The school is available to students from pre-kindergarten through eighth grade. It is nationally accredited school and undergoes ongoing improvement. Enrollment for the 2007 through 2008 school year was 194 students. There were 86 girls and 108 boys in the school. Of the 194 students, 6.7% were Hispanic, 1% was Asian, 5.7% were African American, and 86.6% were Caucasian. 4.6% of the students receive a government scholarship and 7.2% of the students receive internal scholarships provided by the school.

The study was conducted in a sixth grade advanced mathematics class. The class consisted of ten students, of which 7 were female and 3 were male. From this group 70% were Caucasian, 20% were black, and 10% were Hispanic.

Mathematics instruction took place one class period per day. The periods lasted forty-eight minutes four days of the week. Class periods on Wednesdays only lasted forty-three minutes. All ten students participated in the study from beginning to end. No students left these classes, nor were any added to the class throughout the study.

Prior to this research, my classroom was very traditional. My role as the teacher was to teach the students the ways and methods to solve problems. Students were expected to listen as well as participate in solving problems. They were encouraged to raise their hand before speaking. However, classroom discussions tend to move the class away from this social norm. I did not strictly enforce this norm either.

Work was mostly done individually. The students sit in rows facing the whiteboard at the front of the classroom. Classroom discussions were led by me with student participation whenever possible.

In order to carry out my research, some social norms were adjusted. Students were permitted to talk more freely without raising their hand. The students were also permitted to talk and work with each other on a problem to help each other. New sociomathematical norms were also introduced. Students were instructed to explain and justify their work and solution when solving a problem.

Data Collection Procedures

Before beginning this study, permission was sought from the school principal, president of the school board, and pastor. All three granted permission to conduct the study in my classroom using my students. Permission was also sought and obtained from the University of Central Florida Institutional Review Board (IRB). Upon receiving permission from the IRB, consent was then sought from the students as well as their parents. All ten students and their parents granted consent and assent for participation in this study. A copy of the IRB approval letter is found in Appendix F. The consent forms for my administrator, students, and their parents are found in Appendices G-I.

The qualitative data collected for this action research study included assignments and assessment from a problem-solving unit, student journal entries including both reflective and problem solving prompts, and audio recordings of whole class discourse. In order to respect the confidentiality of the students, pseudonym names were used throughout the study. The quantitative data consisted of a rubric for scoring student problem solving performance (see Appendix E). The rubric was adapted from Krulik and Rudnick's (1998) *Assessing Reasoning and Problem Solving: A Sourcebook for Elementary School Teachers*. The rubric, reflective writing prompts, and problem solving prompts used in this study are included in the appendices. These instruments were used to create opportunities for assessment and to assess student performance.

Problem Solving Unit

Before I began the unit, I introduced explanations and justifications to the students with a basic numeric problem (see Appendix J). Definitions and expectations of explanation and justification were also discussed. Eight different strategies were then taught heuristically to the students. The instruction focused on one strategy each day and one problem together in class. The strategies taught were guess and check, work backwards, write an equation, make a Venn Diagram, draw a picture, make a table or list, find a pattern, and logical reasoning. Students solved a problem as a whole class and discourse focused on how to solve the problem, the strategy used, and student explanations and justifications.

The students then solved two problems on their own as an assignment. These problems were to be solved using the strategy taught that day in class. This was to help students recognize problems where the strategy could be used. They began these problems in class and discussed the strategies and solutions with their classmates. What the students had not completed by the end of the period was taken home for homework. The students were asked to follow the four-step plan and explain and justify their work for each problem in writing. This work would later serve as a basis of comparison to problems in the journals to see how performance had been affected.

After four strategies were taught, the students received a review worksheet. The worksheet consisted of three problems where the solution could be found using any of the four strategies already learned. The students were not informed which strategy went with which problem. They were to determine this themselves. The purpose for this was for me to determine if they were able to recognize the types of problems that various strategies could be used with. The idea was to see if teaching heuristically had helped them with selecting an appropriate strategy. A second review sheet was also given after the last four strategies were taught.

At the conclusion of the problem-solving unit, after 3 weeks, instruction then returned to mathematically-specific lessons appropriate for the class. Data was then collected from the journals through prompts given throughout the next five weeks.

Math Journals

Each student had a spiral bound notebook to use as his or her math journal. The notebooks were kept in the classroom and identified by student. Students were instructed that they were to only write in them in class, so the notebooks would never leave my room. However, due to lack of time in class, the students were allowed to take them home occasionally to finish their journal entries. In those cases, students returned the journals the next day.

Students began the year with a few simple reflective writing prompts related to mathematics class. This was to help the students become more comfortable with writing in math class before we began more in depth reflective prompts and the problem-solving prompts. Several of these prompts were given the first weeks of school, with each prompt consisting of five minutes of writing time. These prompts were all completed in their journals. There was a break taken from writing in the journals as I taught the students the problem-solving unit. After the unit, reflective prompts then shifted to questions about mathematics instruction and lessons, as well as problem solving related to the topics at hand. As students completed their journal entries, comments and responses to journal entries were given to the students. Students were also given prompts that requested feedback on the problem-solving unit, journals, and the discourse of the classroom. I wanted to hear the students' ideas related to if and how they felt these practices were helping them. I also wanted to see if the students made any connection at all between these practices and their performance.

The student math journals were also used as a record of problem solving prompts given throughout the research period after the unit and continued to be used throughout the rest of the year. These prompts began the week after the problem-solving unit concluded. As the prompts were given, students were instructed to follow a version of Polya's four-step plan for problem solving, which included explore, plan, solve and check. Word problems were pre-typed on mailing labels, which each student placed in their notebook. These problems required the use of at least one strategy. The students were required to select the strategy that they thought was most appropriate. As part of the explore step, students were asked to underline or highlight the important information and the question. As the students solved the problem, they were required to show all of their work. Finally, the students wrote the solution next to their work. Upon completing the problem, students then needed to write out an explanation for how they solved the problem, step by step. This included stating which strategy they used. Their explanations were not to be simply procedural, but mathematical. They were also asked to justify what they did. The students needed to prove their work.

Discourse

After five weeks of solving problems in their math journals, I continued this process in with the class for another five weeks, but added discourse to my research. As stated previously, the students completed problems individually in their journals during class time. On occasion they were taken home to complete when there was not enough time in class. Following each problems completion, the journals were collected and the problem was scored using the rubric. Upon completion of the problem, the entire class discussed the strategies used and the solutions found. This discussion was held either before or after the student work was scored using the rubric. Several students were asked to show their work on the board and explain their process to the class. These students were selected in one of two ways. If the problem had been scored before discussion, I selected the students based on their use of strategy. If the problems were not scored first, I would ask for volunteers who had solved the problem using a different strategy, or who had found a different solution. The other students were encouraged to ask questions for clarification as well as to make comments in support of or against the work presented. As these comments were made by the other students, they were expected to explain why the student's work was either correct or incorrect. In viewing various students' work and strategy usage, discussion also included the comparing and contrasting of the various strategies used. These discussions were recorded for data related to discourse in the classroom.

Performance Rubric

As students solved various word problems in their journals, a performance rubric was used to score their work. The original rubric is found in Krulik and Rudnick's (1998) *Assessing Reasoning and Problem Solving: A Sourcebook for Elementary School Teachers*. The portion of the rubric used from Krulik and Rudnick included understanding the problem, selecting a plan, carrying out the plan, and communicating the solution. The rubric was altered slightly to better fit my needs. Students were provided with a copy of the rubric as a reference as to how their journal entries would be assessed. The problems solved by the students during the heuristic instruction were scored with this rubric. The students were allowed to look at how they were scored and any comments made. Math journals, primarily used after the problem-solving unit, were checked once to twice a week throughout the rest of the data collection period. The math journals included both reflective and problem solving prompts. Only the problem solving prompts were scored. As the problems and work were looked through and scored, comments or corrections were also written back to the students. This method was used in order to collect data on student performance in problem solving.

Data Analysis Procedures

Math Journals

Students were given one to two reflective writing prompts per week throughout the data collection period starting with the problem-solving unit. After the journals had been passed out to the students, the prompt for the day would be read aloud. The students were directed to write for up to five minutes on this topic. A reminder was given to write anything that came to mind related to the topic. The first several writing prompts were designed to be more opinion and definition related to help the students become more comfortable with writing in mathematics class. When the writing prompts were reflective, the student writing was not scored. I read the journals each week and wrote comments or questions back to each student. As the study progressed, the writing prompts changed and related more to the mathematics topics being taught.

Problem solving prompts were given one to two times per week. The stickers with the problem were passed out to each student and placed in their journal. I read the problem aloud as the students followed along. They were then instructed to underline or highlight the important information in the problem before they determined their plan and carried it out. Students were instructed to show all their work and then asked to explain and justify their solution and solution methods. While the students worked, I would walk around the classroom to help as needed. I would not offer solutions to their problems but ask questions to guide them toward the solutions themselves. Students were also permitted to talk with each other to work on the solution.

After completing the problem, the students turned in their journals. Whenever possible, I scored them that night and returned them the next day. Each of the requirements made in their problem solving were scored using the rubric. I also made any necessary additional comments to their writings.

After the journals were returned to students, the class would then discuss the solution to the problem and possible alternative solution strategies. Several students were called to the board to show their work and solution to the problem. They were also asked to explain how and why they solved the problem in this way. The students selected for this part of the instruction were based on their methods for solving as noted when the journals were scored. This would allow students to see the various ways that one problem could be solved. Discussion also centered on these variations for comparison.

Discourse

Audio recordings were taken of several of the discussions held after a problem-solving prompts. The recordings were transcribed and analyzed for student participation. In analyzing the discourse, changes in how the students participated, questioned each other, explained their work, and justified their work were examined. I also listened for my own participation and questioning in these discussions.

Performance Rubric

In the notebook that was provided for the students' math journal, students recorded their work and explanations to various problem-solving prompts. As stated previously, the rubric was from Krulik and Rudnick (1998). Permission to use this instrument was obtained from Rudnick (see Appendix F). A copy of the rubric was glued into the front of the students' math journals as a reference. This rubric was used to score the students' work on each problem solving prompt in their journal.

The performance rubric was broken down into the following characteristics: exploring the problem, planning a solution, solving the problem, and explanation of the work and solution. Students received a score of 0-3 in each of these areas based on their level of work. The total possible points students could receive for each problem was twelve.

Data Analysis Plan

In examining the impact of the heuristic problem-solving unit on student performance in problem solving, student work was examined before the problem-solving unit, during the unit, and after the unit. Math journal writing prompts, which were reflective but related to problem solving, were also used. Student work consisted of problem solving prompts only. Problem-solving prompts were given either as worksheets or as prompts for their journals. Worksheets were only given during the problem-solving unit as assignments. The problem solving prompts in the journals began after the unit was completed. All of the problems were analyzed using the rubric for problem solving. I focused on two parts of the rubric, the use of strategy and the explanation and justification.

Three students were selected for this focused analysis and to compare their work to the work of the class. The students selected were Laura, Thomas, and Karen. Based on their previous work Laura was a low-achieving student; Thomas a middle-achieving student; and Karen a high-achieving student. I selected these three students to provide data from a cross section of the class. Their work, as well as the work of the rest of the class, was analyzed for use of strategy and the written explanation and justification. The rubric was used to find their scores in each area. The first problem analyzed was given to the class to complete before instruction in any strategies began. Halfway through instruction of these strategies, a review worksheet was given. Problems on this sheet could be solved using any of the strategies taught up to that point. Students, however, were not told which strategy to use with which problems. The first problem from this review sheet was analyzed. Students were also assessed at the end of the entire problem-solving unit. The first problem from their assessment was analyzed. These scores for the pre-unit problem, the average score during the unit, and the first problem after the unit were then compared.

In comparing the individual work of these three students to the work of the rest of the class, I looked for similarities and differences in the strategy they selected and how they carried out the strategy. I also looked at the level of their explanations and justifications as compared to the level of the rest of the class. In this, I looked for improvements or weaknesses individually and how these improvements or weaknesses compared to those of the whole class. To make these comparisons, I used the scores of the individual three students and the average scores of the whole class, including these students.

Upon completion of this unit, students began completing problem-solving prompts in their journals. One to two problems were given per week. Two of the problems given to the class, the handshake and lawn problems, were analyzed. The first problem analyzed was given to the students upon completion of the problem-solving unit. The second problem analyzed was given about two weeks later. The three individual students' work was again analyzed and compared to the class's strategy use as well as their explanations and justifications. The scores found using the rubric were used for this comparison also.

After five weeks of solving problems in their journals, the students had Christmas vacation. When the students returned to classes, I began to focus more on discourse in the classroom as the students continued solving problems in their journals. Two problems given during this time were selected for analysis. These problems were the earnings and barbecue problem. The first problem analyzed was given almost one month after the problem solving unit. The second problem analyzed was after another two weeks of problem solving in their journals. These problems were then analyzed in two ways. The first was using the classroom averages found from the scores the students received on these two problems. Each of the three students whose individual work was analyzed earlier were again used to compare their strategy use, solutions, explanations, and their scores to the class averages.

In examining the effect of classroom discourse on student performance in problem solving, audio recordings of classroom discourse and student reflection were used in addition to the problem-solving prompts. The audio recordings were taken of the whole class discourse of each of the problem-solving prompts that had been given since the focus on discourse began. Upon completion of the data collection, I listened to all of the recordings where classroom discourse was a focus. However, I primarily focused on the first and last recording. I analyzed them to see if improvement took place in student participation, student questioning, and discussion on various strategies used in solving one problem. The three individual students selected were also analyzed more closely in these areas.

I also used the students' thoughts pertaining to their performance. The students were asked if and how they felt they had improved. The responses to these prompts were written in their journals and were not scored. These responses combined with the rubric scores helped me to identify any themes in the data.

Upon completion of the problem-solving unit, the students were given the first reflective prompt pertaining to their own view of their performance in problem solving. Student reflection was used again upon completion of several problems in the journals. After completion of data collection pertaining to classroom discourse, the students were asked to reflect again. They were asked if and how they felt their performance in problem solving had improved or not, and to state why or why not. The reflective journal prompts, along with student work and the rubric scores, were the used to identify common themes in the data throughout the collection period.

Conclusion

Data was collected from both reflective and problem-solving journal entries, classroom discourse, and the scoring rubric. It was then recorded, triangulated, and analyzed to show the effects of teaching problem solving heuristically, journal writing, and discourse on students' performance in problem solving. The data was analyzed for any changes in student performance over the course of thirteen weeks. For a complete timeline of events, see Appendix M. The findings from this reasearch will be discussed in the next chapter.

CHAPTER 4: DATA ANALYSIS

Introduction

As I began and completed the coursework that was part of my master's program, I learned and read a great deal on the importance of communication in the mathematics classroom. Discussions and literature focused on the value of students' ability to explain and justify their work. Value was also placed on the use of discourse in the classroom. This is what led me to the research questions of this thesis:

1. How does heuristic problem solving instruction affect my 6th grade advanced mathematics students' performance on problem solving tasks?
2. How does journaling affect my 6th grade advanced mathematics students' performance on problem solving tasks?
3. How does discourse affect my 6th grade advanced mathematics students' performance on problem solving tasks?
4. How did this sequence of instruction and practices affect my 6th grade advanced mathematics students' performance on problem solving tasks?

It was my goal to see how my use of heuristic instruction, journal writing, and discourse affected student performance in problem solving. It was my belief that all three will improve my students' performance.

Data was collected with information relating to the three research questions. It was analyzed to identify common themes.

Teaching Heuristically

In an effort to answer my first research question related to heuristic instruction, I taught a problem-solving unit. Instruction within the problem-solving unit focused on one strategy per day. One sample problem was completed as a whole group during the class period. Students were then given two problems covering the same strategy to complete on their own. The problems students solved on their own were used to determine themes related to the impact of heuristic instruction on problem solving ability.

The first set of problems given to the students was given before strategy instruction began. A version of Polya's 4-step problem solving plan, as well as discussion of what were effective explanations and justifications, had already been introduced. This assignment allowed me to see where the students were in terms of selecting a strategy and using it as well as how detailed their explanations and justifications were. I used a performance rubric to score the two problems given for each student. There were four areas for scoring. These were: exploring the problem, planning a solution, solving the problem, and explanation of the work and solution. Each category was on a three-point scale with a possible holistic score of 12.

Handshake Problem from the Problem-Solving Unit

For the purpose of analyzing the data, the first problem from each of the three chosen assignments was selected. The first problem given to the sixth grade class that was analyzed was:

Fifteen people are at a party. If each person shakes hands with everyone else (JUST ONCE), how many handshakes are there in all? Use the problem solving plan. Explain and justify your work.

The students were instructed to underline any important information and the question asked. This determined the number of points received for exploring the problem. As the problem was solved, all work was to be shown, making their use of strategy clear. This was part of planning the solution. Upon finding the solution, students were asked to identify it clearly as part of solving the problem. Finally, students were asked to explain and justify their work and solution.

Student Work Analysis

The work completed by the sixth grade class was scored. Class averages for this problem are identified in Table 1.

Table 1: 6th Grade Averages – Handshake Problem

Characteristics	Score Problem 1
Exploring the Problem (out of 3 points)	2.4
Planning a Solution (out of 3 points)	3
Solving the Problem (out of 3 points)	2.7
Explanation of Work and Solution (out of 3 points)	1.6
Total Score	9.9

The class did fairly well in exploring the problem. This was the first time they were asked to underline the important information, and only four out of the ten students in this class did not underline all of the important information. The class all scored a three on planning a solution. The reason for this is that the class decided as a whole to act the problem out together and solve a simpler problem. Therefore, they all selected an appropriate strategy for this problem. Eight out of the ten students scored a three on solving the problem, which means they showed all of their work and found the correct solution. The other two made small errors. As a whole, the class did not score well on the explanation and justification. The students had a fairly good understanding at the beginning of what a good explanation was but they did not all understand what was a good justification.

In an effort to provide a snapshot into the classroom at various points throughout the study, three students, Laura, Thomas, and Karen, were selected from the class to analyze and compare their work to the class as a whole. The students selected provided a cross-section with one low, one middle, and one high achieving student. Laura was a student who had difficulty understanding new concepts. Thomas was a bright student who made careless mistakes and did not always take time with his work. Karen was also a bright student who put forth effort into her work and caught on to new concepts easily. The scores that Laura, Thomas, and Karen received are in Table 2.

Table 2: Student Scores – Handshake Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	3
Planning a Solution	3	3	3
Solving the Problem	1	3	3
Explanation of Work and Solution	1	1	2
Total Score	9	10	11

Laura, Thomas, and Karen all scored above the class average on exploring the problem. As stated earlier, this was the first time the students were asked to underline important information. Even after being asked, the students still needed to remember to complete this. If they did remember, as these three students did, they underlined everything that was appropriate. In the class as a whole it seemed that students either identified all important information or did not identify any important information.

Thomas and Karen received full points for solving the problem. They both showed all of their work and found the correct solution. Laura however, did not show any work. She did have the right solution. Therefore, she lost points for not showing the work.

Both Laura and Thomas scored lower than the class average on the explanation and justification of the work and solution. Laura had a limited explanation with no real justification (see Figure 1).

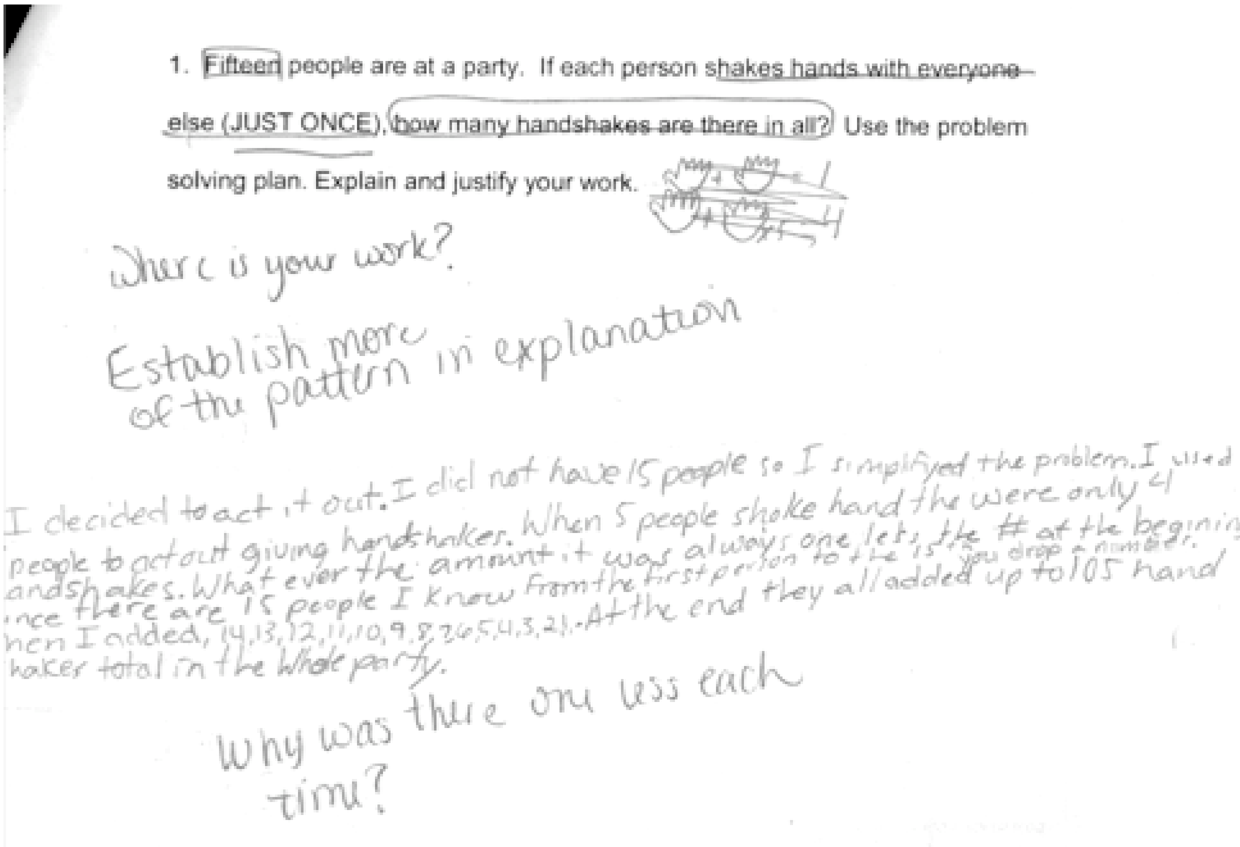


Figure 1: Laura's Work on the Handshake Problem

She begins to describe the pattern that develops in the number of handshakes when she points out that with five people, there are four handshakes. Laura does not fully describe or explain what continues to happen with the number of handshakes other than saying it "was always one less". She was on the right path of establishing a pattern but did not completely explain her thinking.

Thomas's writing was even weaker than Laura's. He showed his work; however in his written explanation, he made statements but had no support for where his claims came from and how he got them (see Figure 2).

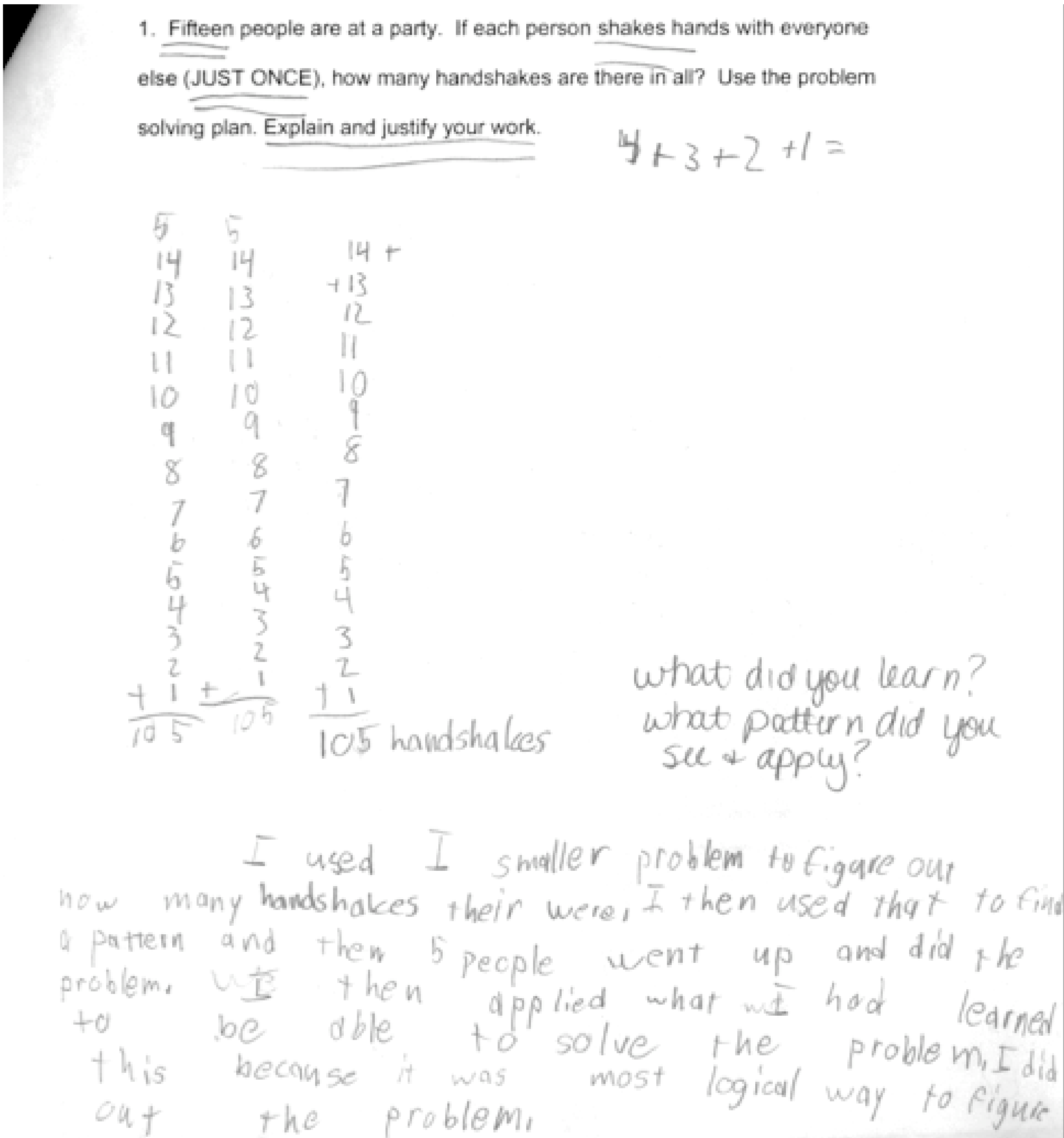
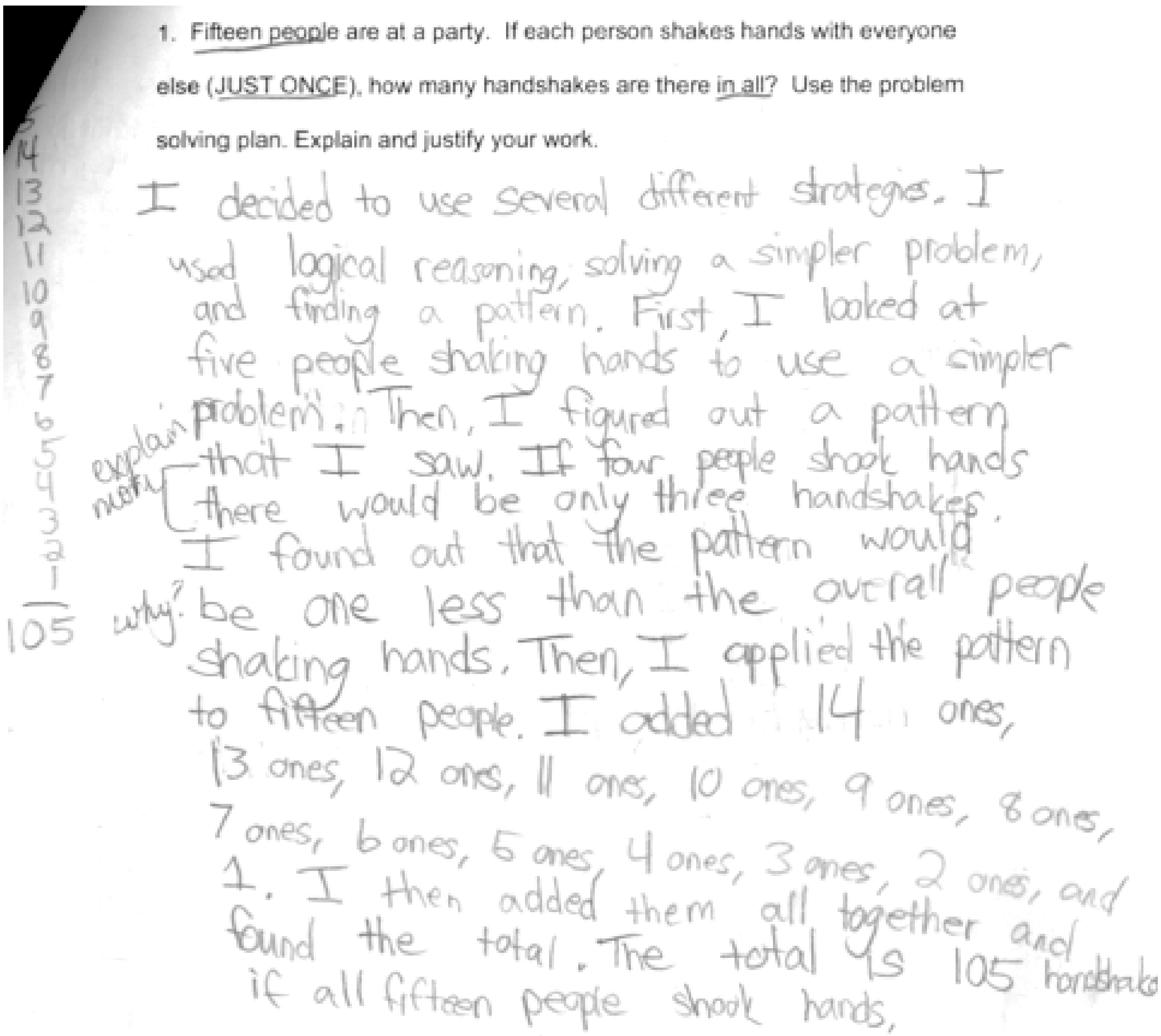


Figure 2: Thomas's Work on the Handshake Problem

As you can see, Thomas described nothing of what happened in the simpler problem of five people. But it is evident from his work that he did recognize and understand what happened and applied it to the problem with fifteen people. This understanding of process was not written in his explanation and justification. He had no explanation and therefore he had nothing to justify.

Karen's work and writing was better than the class average. She showed her work and her explanation was much more descriptive (see Figure 3).



1. Fifteen people are at a party. If each person shakes hands with everyone else (JUST ONCE), how many handshakes are there in all? Use the problem solving plan. Explain and justify your work.

15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
105

I decided to use several different strategies. I used logical reasoning, solving a simpler problem, and finding a pattern. First, I looked at five people shaking hands to use a simpler problem. Then, I figured out a pattern that I saw. If four people shook hands there would be only three handshakes. I found out that the pattern would be one less than the overall people shaking hands. Then, I applied the pattern to fifteen people. I added 14 ones, 13 ones, 12 ones, 11 ones, 10 ones, 9 ones, 8 ones, 7 ones, 6 ones, 5 ones, 4 ones, 3 ones, 2 ones, and 1. I then added them all together and found the total. The total is 105 handshakes if all fifteen people shook hands.

explain more

why?

Figure 3: Karen's Work on the Handshake Problem

Karen used the same strategies as the rest of the class, but also stated that she used logical reasoning as well. She, like Laura, began to describe the pattern found in the number of handshakes but did not complete it. She used this pattern to find the solution for the larger problem. Her explanation was almost complete; however, her writing was missing justification. She claimed that the handshakes would lessen by one, but did not fully justify why that would happen.

Lawn Problem from the Problem-Solving Unit

One strategy was discussed each day in class. As a whole class, and with instruction from me, the students solved a problem using the strategy of the day. We also wrote out an explanation and justification together. After four lessons from the problem-solving unit, I gave the students a review worksheet with three problems on it. These problems could involve any of the strategies that were taught up to this point. As stated earlier, the first problem from the review sheet was used for analyzing.

Work Analysis

The first problem which was given to the sixth grade class was:

> Morey mowed half of Mickey's lawn. Matty mowed ¼ as much as Morey did. Midge mowed twice as much as Matty. How much of Mickey's lawn has not been mowed?

The students were instructed to underline any important information and the question asked. I told them this as we began this unit, and continued to remind them throughout the problem-solving unit. This determined the number of points received for exploring the problem. As the problem was solved, all work was to be shown, making their use of strategy clear. This was part of planning the solution. Upon finding the solution, students were asked to identify it clearly as part of solving the problem. Finally, students were asked to explain and justify their work and solution. This was made clear in the instructions on the sheet. The class averages were scored using the rubric (see Table 3).

Table 3: 6th Grade Averages – Lawn Problem

Characteristics	Score Problem 1
Exploring the Problem	3
Planning a Solution	3
Solving the Problem	2.9
Explanation of Work and Solution	1.3
Total Score	10.2

As stated earlier, the class had completed four other problem-solving lessons by this point. Even though I was still reminding the whole class, the students had become used to the practice of exploring the problem by underlining all the important information and the question. The entire class received a score of three in this area of the rubric. This same score was given to all of the students in planning a solution. They all selected an appropriate strategy to solve the problem. For this problem, the drawing strategy was used by all the students. Nine out of the ten students solved the problem and found the correct solution, which created a class average of 2.9. The one student who had an incorrect solution explained one of the fractions incorrectly.

The weak area again was the explanation and justification, with both explanations and justifications being problematic. The reason for this seemed to be that the problem dealt with fractions. My students did not have enough conceptual understanding of fractions to create a good explanation and justification. The students are able to used fractions but are unable to verbalize their thinking and understanding of them. This will be evident in the student samples presented.

I again focused on the same three sixth grade students and compared their work to the class averages. The class averages were determined using the rubric (see Table 4).

Table 4: Student Scores – Lawn Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	3
Planning a Solution	3	3	3
Solving the Problem	3	3	3
Explanation of Work and Solution	1	1	3
Total Score	10	10	12

All three students had remembered to underline the important information in the problem. As a result, they all received full points in exploring the problem. Also, they all selected an appropriate strategy and found the correct solution. Variation in the individual student scores occurred in the explanation and justification.

Laura and Thomas both scored close to or the same as the class averages. Like the class as a whole, they had chosen an appropriate strategy of drawing out the picture. Their understanding of fractions was also strong enough for them to find the correct solution. Their low area, like the class as a whole, was the explanation and justification. Laura's work is found in Figure 4.

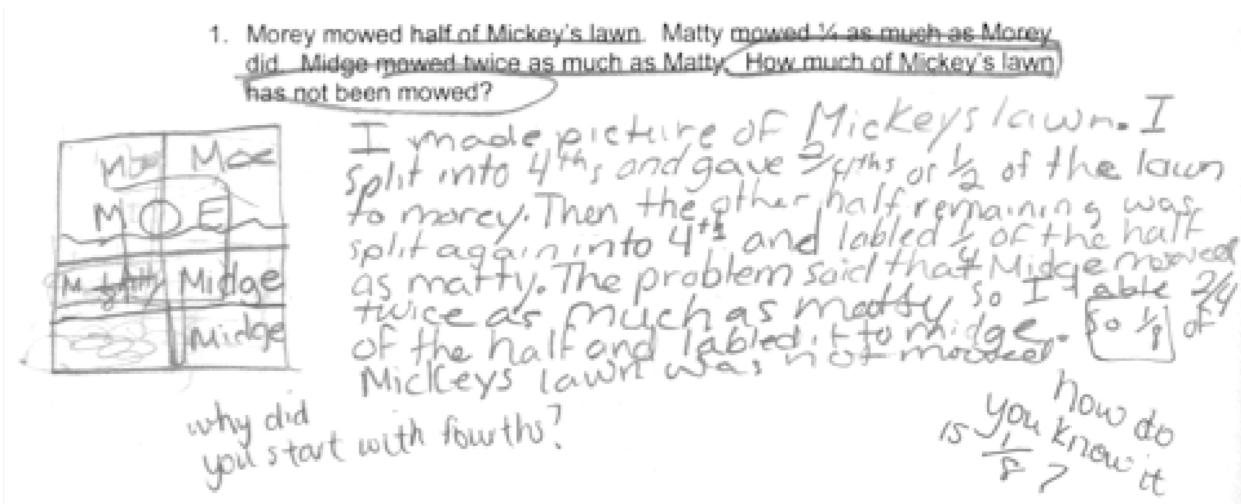


Figure 4: Laura's Work on the Lawn Problem

Laura used a rectangular representation for Mickey's lawn. She was able, through her work to find the correct solution. Laura's explanation was fairly strong. She described each step of her work. Her understanding of the whole was very apparent as she used terminology like "2/4 of the half". However, Laura lacked justification for her work and solution. She did not justify why she started her drawing with fourths, nor did she justify her solution of 1/8. Her work does support the use of fourths but does not support how she knew it was eighths for her solution.

Thomas also used a rectangle to represent the whole of the yard (see Figure 5). He did cut his differently than Laura. His work also supports the solution he found.

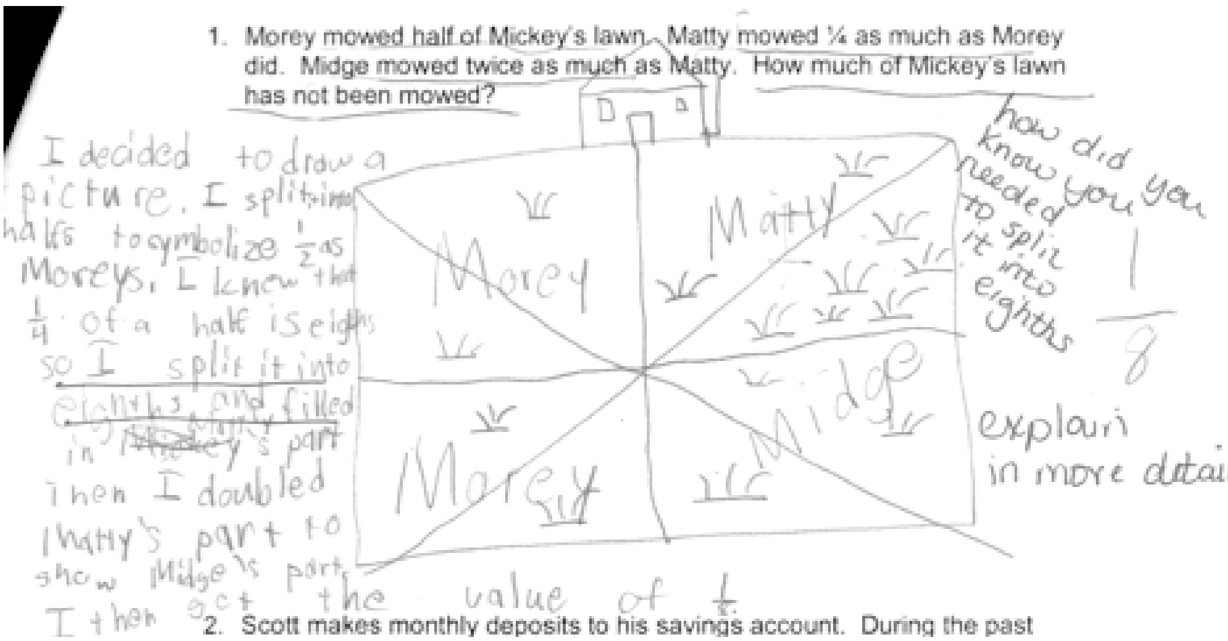


Figure 5: Thomas's Work on the Lawn Problem

Thomas's explanation and justification was lacking in both areas. His thought and step process was not easily followed because he was lacking details. Thomas also did not justify the eighths he drew in the pictures. He stated that he drew eighths because he "knew that ¼ of a half is eighths". This may be knowledge that a sixth grader has, but I was looking for their understanding of that knowledge. I wanted to know the reason that Thomas drew the eighths and how he knew he needed them.

Karen followed the class averages in the first three areas of the rubric. However, she scored above the class average on the explanation and justification of her work. She also used the rectangle to represent the whole yard. Her cuts to the yard were similar to Laura's (see Figure 6).

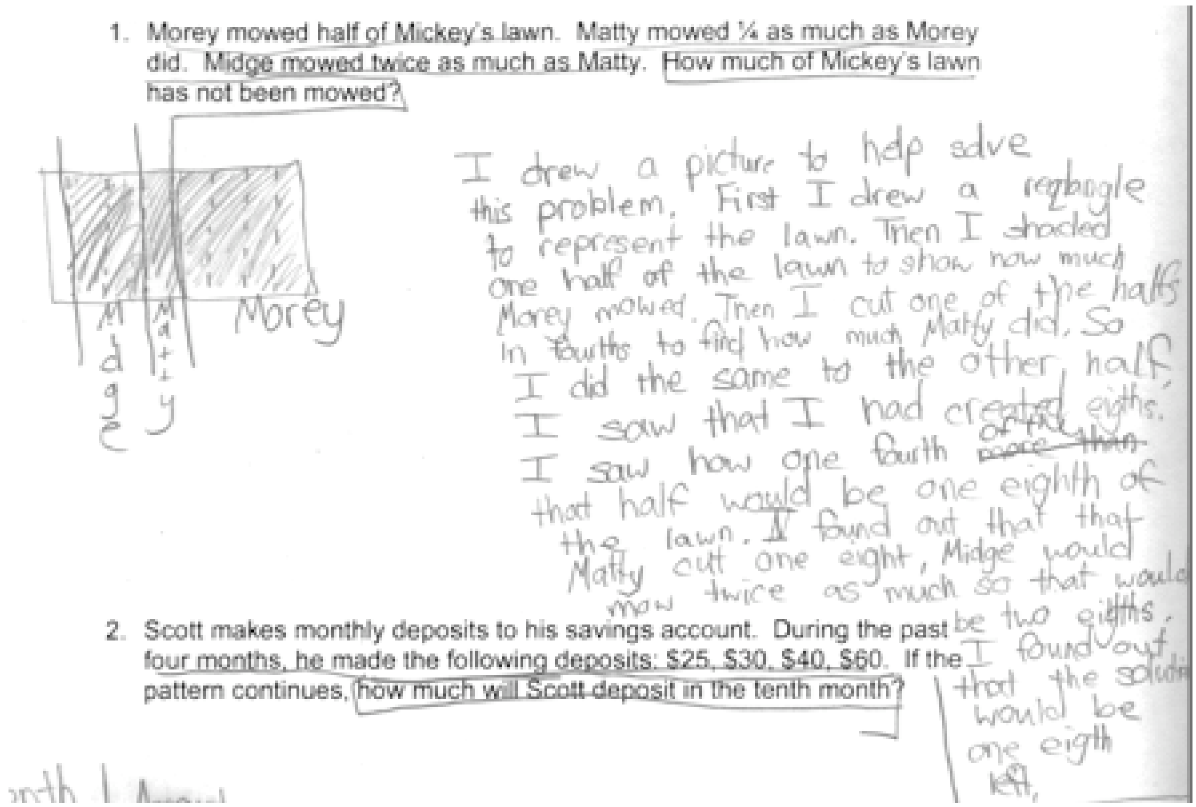


Figure 6: Karen's Work on the Lawn Problem

The steps Karen took in her drawing were detailed in her writing and therefore she had a clear explanation. Her justification was also strong. Karen justified why she ended up with eighths in her drawing. She worked through and explained all the steps she took. She, therefore, was able to justify why her solution was 1/8. Her only error was in some of her wording. Instead of saying that she saw one fourth of the half would be one eighth, she stated that one fourth more than that half would be one eighth of the lawn.

Travel Problem from the Problem Solving Unit Assessment

The final part of this unit was the assessment. The problem-solving test for the class consisted of three problems. Students were instructed to solve each problem using the most appropriate strategy or strategies. They were also instructed to explain and justify their solution. For the purpose of this paper, only the first problem from the assessment was selected to be analyzed.

Student Work Analysis

The first problem on the sixth grade assessment was:

> Mrs. Dixon and her family had traveled about 90 miles, or 2/5 of the way to the campsite. How much farther do they have to go?

The scoring rubric that was used for the lessons and work was also used for this assessment. Therefore, the rubric aided in my analysis of this data. I used the class averages and then focused again on several students' work to determine if there was growth in student performance in problem solving from the beginning to the end of this unit. The class averages for this problem were scored using the rubric (see Table 5).

Table 5: 6th Grade Averages – Travel Problem

Characteristics	Score Problem 1
Exploring the Problem	2.7
Planning a Solution	2.3
Solving the Problem	2.4
Explanation of Work and Solution	1.8
Total Score	9.2

The class average for exploring the problem was a 2.7 due to one student who did not remember to underline the important information. This student consistently did not remember this part of solving the problem. As the unit progressed, I would not always remind the students verbally to do this because it had become a norm for most of the students.

The class average for planning a solution was lower than it had been throughout the unit. Two of the ten students selected an appropriate strategy but did not implement it completely. They used a drawing to represent the fifths in the problem, but did not identify in their work the miles those represented. One student made use of a table, which would work, but the way she implemented it was not helpful to her for solving the problem because she did not fully understand what the numbers in her table meant. Another student, Laura, did not use any strategy. She resorted back to operations, which will be seen in her work. This was not typical for most of the students. There were occasional problems where the operations had to be used, but this was mostly required with guess and check problems.

The class average for solving the problem was also low. Four students out of the ten either had an incorrect solution or the right solution, but the work did not fully support it. This problem dealt with fractions, as did the first problem analyzed in the review sheet. Based on the student work and solutions to this problem, not all of the students seem to have a strong understanding of what the fractions in this problem represent. All of the students recognized that fifths were needed in this problem. The students were also able to determine the mile equivalence to each fifth. The problem for some students was that they totaled these values rather than finding the total mileage that Mrs. Dixon still needed to go.

The explanation and justification was also still low. Only one student out of this class wrote a clear explanation and justification. The rest of the class struggled with either one or both of these. As stated in earlier analysis, this difficulty again is associated with a mathematical concept, in this case, fractions. In problems that were not based on more difficult mathematical concepts the students seemed to write explanations that were slightly more acceptable. Students need to have a strong understanding in order to write an explanation and justification about them.

The same three students from earlier analysis were used again to compare to the class averages. The scores for these students were found using the rubric (see Table 6).

Table 6: Student Scores – Travel Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	3
Planning a Solution	1	3	3
Solving the Problem	3	3	3
Explanation of Work and Solution	1	2	2
Total Score	8	11	11

Laura was able to explore the problem and find the correct solution. Laura's difficulty was in finding a strategy and the explanation and justification of her work (see Figure 7).

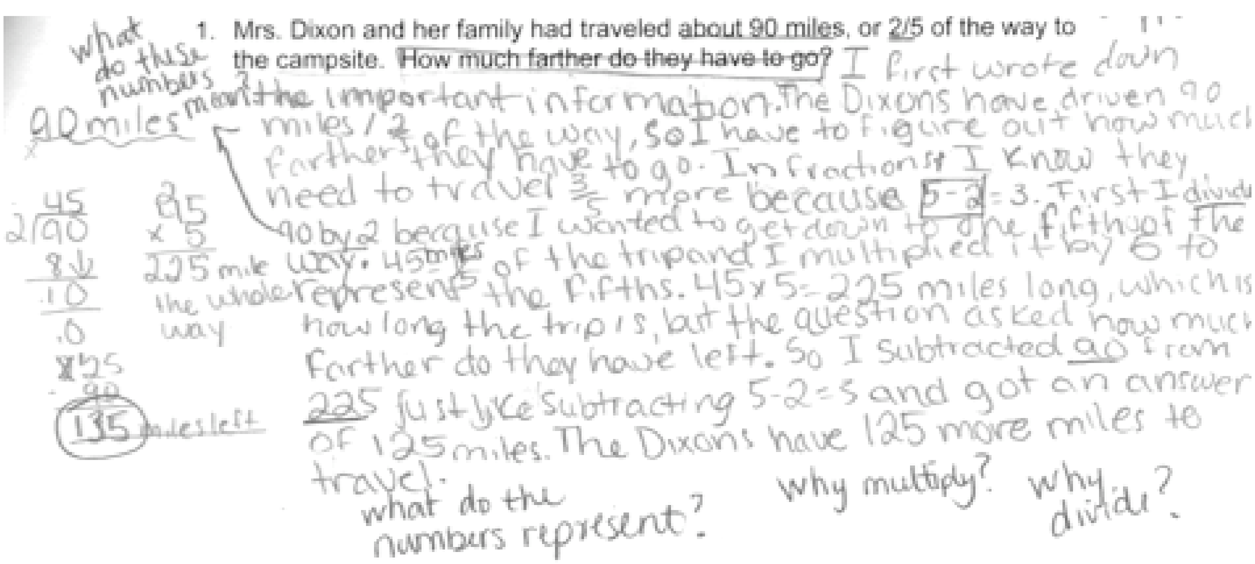


Figure 7: Laura's Work on the Travel Problem

Laura was the one student who resorted back to using only operations for this problem. There was no instruction of operations as a strategy. Instruction was focused on other strategies in order to demonstrate to students that there were other methods to solving problems rather than always looking for an algorithm.

Other students had used some operations to solve, but Laura relied fully on it. She did not select the drawing or list strategy that the others had. As a result, Laura struggled with explaining clearly what the numbers in the operations meant and justifying the operations she used. This was one point I emphasized throughout instruction to my students about the use of operations. If they used them, they needed to explain the numbers used as well as justify why they used the operation that they did.

Thomas scored well in exploring the problem, planning a solution, and solving the problem. He drew a picture to help solve the problem. Thomas used a rectangular model again as he did earlier with the lawn problem (see Figure 8).

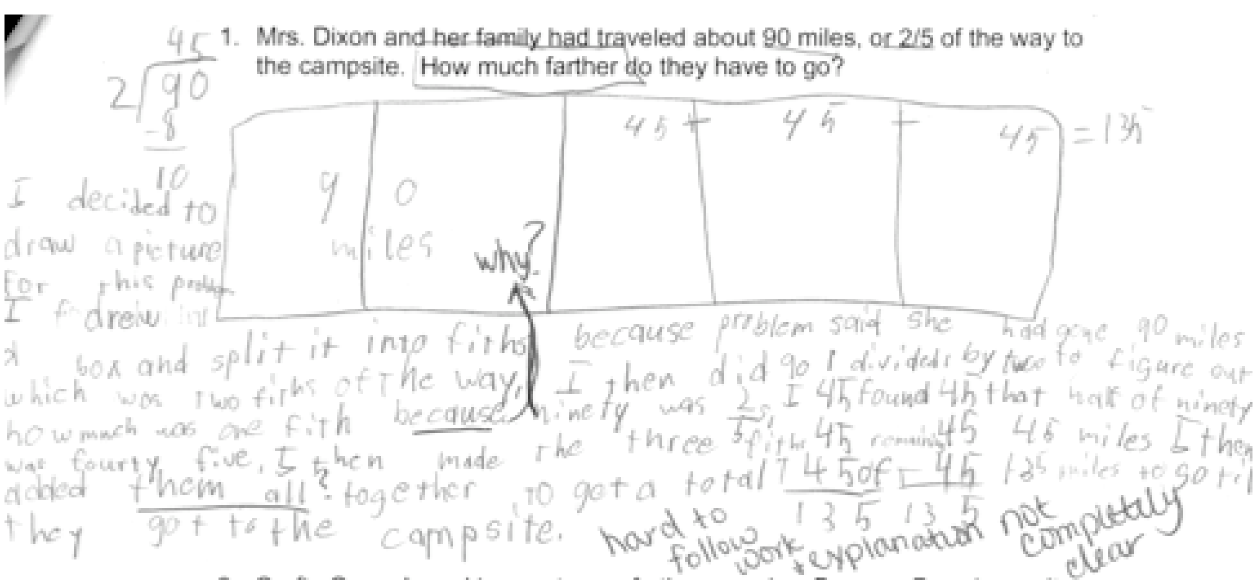


Figure 8: Thomas's Work on the Travel Problem

Thomas's writing showed more effort to me than it had in the past. He knew what he needed to write about, but he was still struggling with how to get it out in writing. Thomas had most of the details to his work; however, it was hard to follow. He did not fully identify the numbers he was using every time, but did only for part of the time. His justification was slightly lacking. He, like Laura, used some operations in his work but did not provide reasoning for them. He stated that he added three 45's together but did not say why he added just those three.

Karen also scored well in exploring the problem, planning a solution, and solving the problem. She also drew a picture to solve. She used a number line to represent the drive (see Figure 9 and 10).

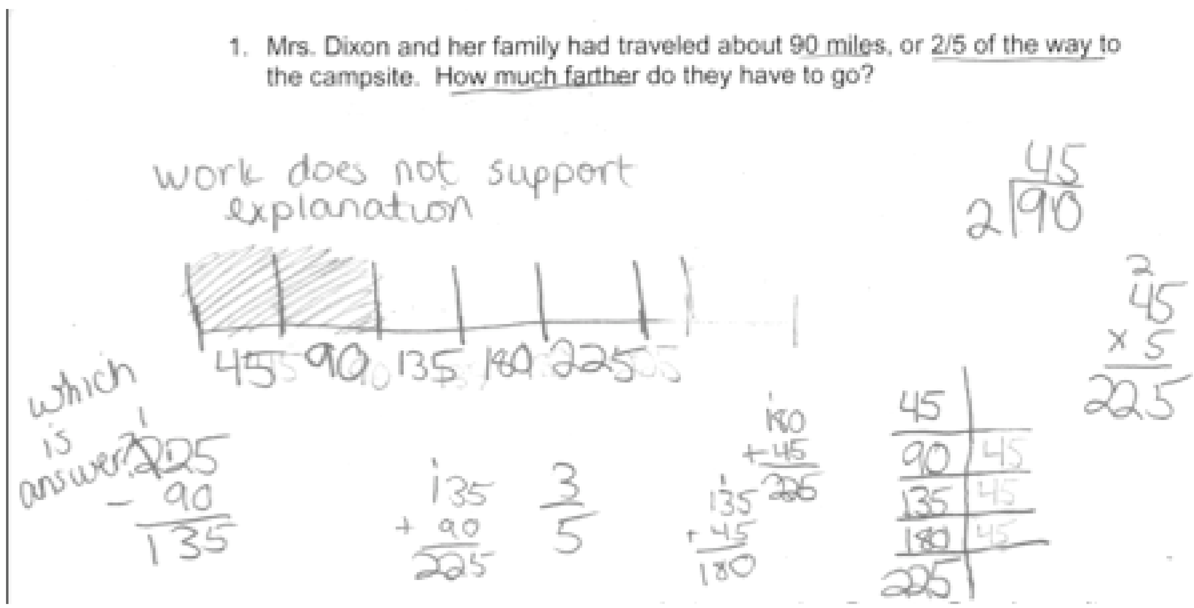
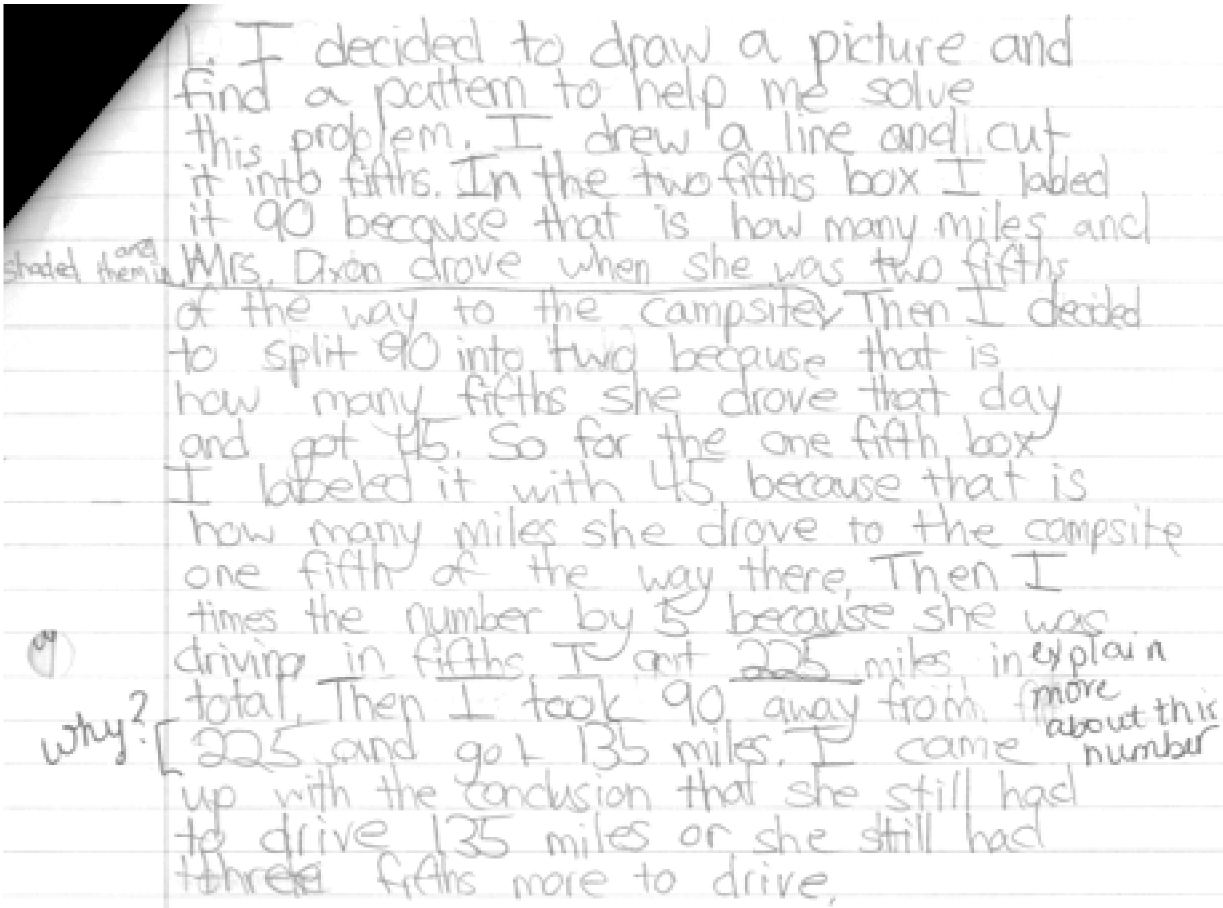


Figure 9: Karen's Work on the Travel Problem


1. I decided to draw a picture and find a pattern to help me solve this problem. I drew a line and cut it into fifths. In the two fifths box I labled it 90 because that is how many miles and Mrs. Dixon drove when she was two fifths of the way to the campsite. Then I decided to split 90 into two because that is how many fifths she drove that day and got 45. So for the one fifth box I labeled it with 45 because that is how many miles she drove to the campsite one fifth of the way there. Then I times the number by 5 because she was driving in fifths. I got 225 miles in total. Then I took 90 away from 225 and got 135 miles. I came up with the conclusion that she still had to drive 135 miles or she still had three fifths more to drive.

shaded them in and

why?

explain more about this number

Figure 10: Karen's Work on the Travel Problem

I thought it was interesting how Karen identified the fifths in her drawing. Most students identified each fifth as 45 miles. Karen used a running total instead, adding 45 miles each time. Karen had an understanding of what the numbers she wrote meant and was able to find the solution even while using the running total. Two other students (not Laura or Thomas) from the class labeled their work the same way but did not demonstrate that they fully understood it. Karen saw that the way she counted the fifths gave her the total distance from beginning to end and therefore knew to subtract the 90 miles from the 225 miles. The other two students did not recognize what the final mileage was and added the 135, 180, and 225 miles seen in Karen's work.

Karen also wrote a good explanation and justification. Only two things were left out. She did not fully explain the 225 miles in her explanation. Karen also did not justify her final step of taking 90 miles away from 225 miles. I know she understood this in order for her to do it, but she did not write about that understanding.

Problem Solving Unit Conclusion

At the beginning of the problem-solving unit, the students did have the ability to select a strategy that will work with a problem, even before they have received instruction on the individual strategies. This is evident from the first problem analyzed, the handshake problem. The class did discuss as a whole which strategy to use. The weak area was the explanation and justification. Some of the students were able to explain to an extent, as seen with Karen's work, but were not able yet to complete it. Very few of the students were able to provide a strong justification, which was evident in all three student samples of the first problem. This would hopefully improve as the students continued in the problem-solving unit and beyond.

The next problem analyzed was the lawn problem from the first review worksheet. The difference between this assignment and the individual lessons was that the students knew that the problems from the individual lessons were based on the strategy taught that day. They knew they would use that strategy to solve the problems. This review sheet, however, was not like the other assignment sheets. Students were informed that they could use any of the strategies already taught, but they did not know specifically which strategy went best with each problem. This was what they had to determine. As shown by the class averages and these individual students, the teaching of strategies heuristically helped these students to select appropriate strategies and receive the highest score possible in this area. The other two problems on the review sheet support this finding. The students all selected an appropriate strategy for the second problem. All but one of the students selected an appropriate strategy for the third problem. There were also two students who selected an appropriate strategy but did not implement it correctly.

Based on the student work discussed and the class averages at the end of this unit and based on the third problem analyzed, the sixth grade class seemed to grasp the process presented for problem solving and selected appropriate strategies; however, the class was still struggling with writing complete explanations and justifications. Some of the students were beginning to improve in this area as seen in Karen's work. However, this class's average dropped in comparing the problem from the first lesson and the one from the review lesson. This seemed to be due to the mathematical concept understanding of fractions that was necessary. For students to be able to justify fully, they must understand the mathematics behind it. This struggle could also be based on their ability to verbalize what they know about these concepts.

At this point, several themes had developed. First, the students were remembering to underline the important information in the problem. This was becoming a problem-solving norm for them. Second, the students were able to select and implement appropriate strategies. Finally, the students seemed to improve in their explanations, however the justifications were still lacking.

As for the explanation and justification, the sixth grade did improve in the class averages over all. They seemed to grasp the idea of a good explanation by the end of the unit, as long as they actually took the time and made the effort to do it. The class still did not understand what it meant to justify in their writing. Some of the students seemed to have an understanding of the difference between explaining and justifying. The students were asked at one point to write in their journals what they thought explanation and justification meant. Of the three students analyzed, all three seemed to have an understanding of what explanation means. Laura does not have a clear understanding of what justification is. This would explain why her justifications have been lacking. Laura describes these terms as "explaining and justifying your work is proving that its right. When answering a problem you are explaining the question and your work justifies your answer." Thomas has a better idea of what these two terms mean. Thomas describes it as "explaining and justifying your work means saying why you did it and how you did it. Justifying it means that you are saying why you think your way is right and that you cannot be proven wrong." Karen has tended to score higher in this area, but her understanding of the difference between the two is also weak. Karen's writing stated "explaining your work is saying how you got your answer. Justifying your work is saying how that your answer would be correct. They are alike because it helps explain your work." As the students continued with problem solving, it was my hope that they continue to improve in this area.

The students were also asked to write in their journal how they felt the problem-solving unit had helped their performance. Thomas stated " I didn't know anything about problem solving before we did this so this helped. I still don't like it, especially explanations. But this unit really helped a lot but I think I am better at problem solving now than I was before." Laura also agreed that the problem-solving unit helped to improve her skills. She stated " I was a little bit confused [about problem solving]. Now I feel a lot better about problem solving because I've grasped more on the entire process and its become very clear to me now." Karen wrote in more detail about her improvements. She stated, "After the unit I am way more confident about problem solving. I know different ways to approach the problem than just doing it the regular way. I know what justifying your work means and I know how to explain and justify my work."

The two primary areas focused on in the student performance at an individual and class level were planning a solution and the explanation of the work. The sixth grade class did well at selecting and implementing a strategy on the first two problems analyzed, which were the handshake and lawn problems. Class averages in this area were lower for the assessment. As explained in the analysis of the travel problem, this was due to the implementing of the strategy and not the selection and shows that teaching strategies heuristically can improve student performance in problem solving.

There was definite improvement over the course of this unit. The scores from the rubric and class averages demonstrate this improvement. The students also believe they have made improvements as a result of this unit. Some of the students believe they had improved more than the scores reflected.

Problem Solving in Math Journals

Upon completion of the problem-solving unit, students began solving problems into their math journals. The problems were typed on mailing labels for the students to stick in their journal. They were expected to do all of the same work as was required during the problem-solving unit. This included reading and underlining the important information in the problem, selecting a strategy and showing all work, finding a solution, and explaining and justifying their solution. After completing the problem, students would turn in their journal to be scored. The same assessment rubric was used. Students also had a copy of the scoring rubric inside the front cover of the journal for a reference to the scores they received. The problem would be discussed the next day in class. Several students were selected to show the various strategies that could be used to solve the problem. The primary focus in these class discussions was their strategy, work, and solutions.

Two problems were selected from those given to the sixth grade class for analyzing. The class scores were averaged. The same three students were again used for comparing and analyzing.

Earnings Problem from Math Journal

Student Work Analysis

The first problem selected from the sixth grade data was completed shortly after the completion of the problem-solving unit. The problem selected was:

> Jake earned $576 during the month of February. He was paid $6 per hour. He did not work more than five hours each day, nor did he work on Sunday. If he worked the same number of hours each day, how many hours per day did he work?

Student work was scored and averaged. The class averages for each category of the rubric are shown in Table 7.

Table 7: 6th Grade Averages - Earnings Problems

Characteristics	Score Problem 1
Exploring the Problem	3
Planning a Solution	2.7
Solving the Problem	2.9
Explanation of Work and Solution	1.9
Total Score	10.5

The whole class scored a three in exploring the problem. Students all seemed to remember to underline the important information and question without being asked. Planning a solution also continued to be a strong area for the class. For this problem, the students all chose to use guess and check with operations. However, two of the ten students did not identify that they used guess and check. Only one student found an incorrect solution. Explanation and justification was still low for this problem.

Three students; Laura, Thomas, and Karen, were again selected to compare their scores to the class averages and to analyze more closely. The scores that these three students received are found Table 8.

Table 8: Student Scores - Earnings Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	3
Planning a Solution	3	2	3
Solving the Problem	3	3	3
Explanation of Work and Solution	1	1	2
Total Score	10	9	11

Laura varied from the class averages in some areas. She chose a correct strategy to solve the problem like the rest of her classmates. She therefore received full points in planning a solution. From her strategy use and work, she was able to find the correct solution (see Figure 11).

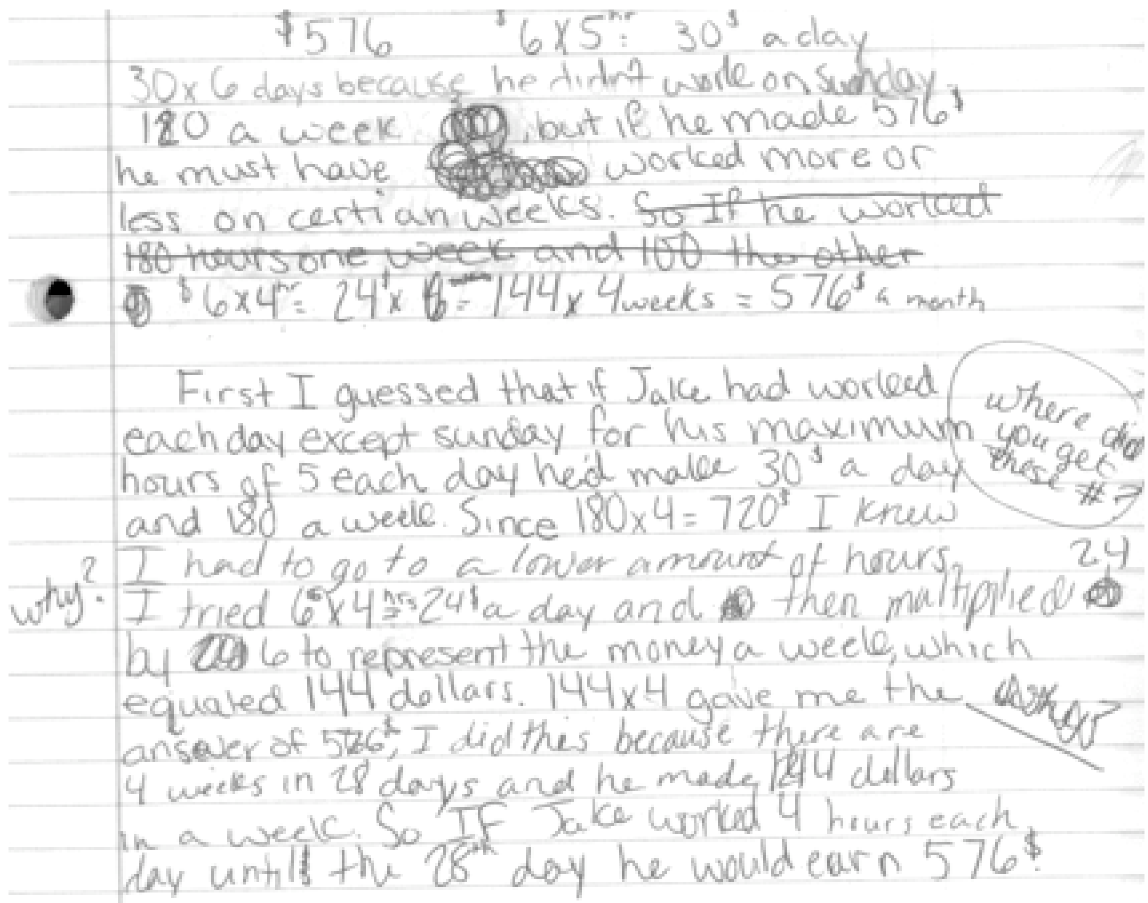


Figure 11: Laura's Work on the Earnings Problem

Laura used the guess and check strategy. She also demonstrated her use of estimation. The only work seen is two guesses, one of which is the solution she found to be correct. Laura's lowest score was in explaining and justifying her work. As shown in her work, she guessed two different possibilities for the solution and used operations to calculate both. Laura's explanation was detailed in the steps she took, but she did not always identify the meaning of the numbers she used in her work. She wrote several mathematical expressions in the problem. In one of them, you can see she identified what the numbers represented with symbols. She did not do this with the other. Laura's work was also mixed in with her explanation.

Thomas's scores also varied from the class averages.   He used guess and check, but did not identify this in his writing. This is why he received a two in this category. Thomas was able to find the correct solution, but his work does not show every step in getting this solution. His work also tends to be more disorganized, making it difficult to follow the order in which he worked (see Figure 12).

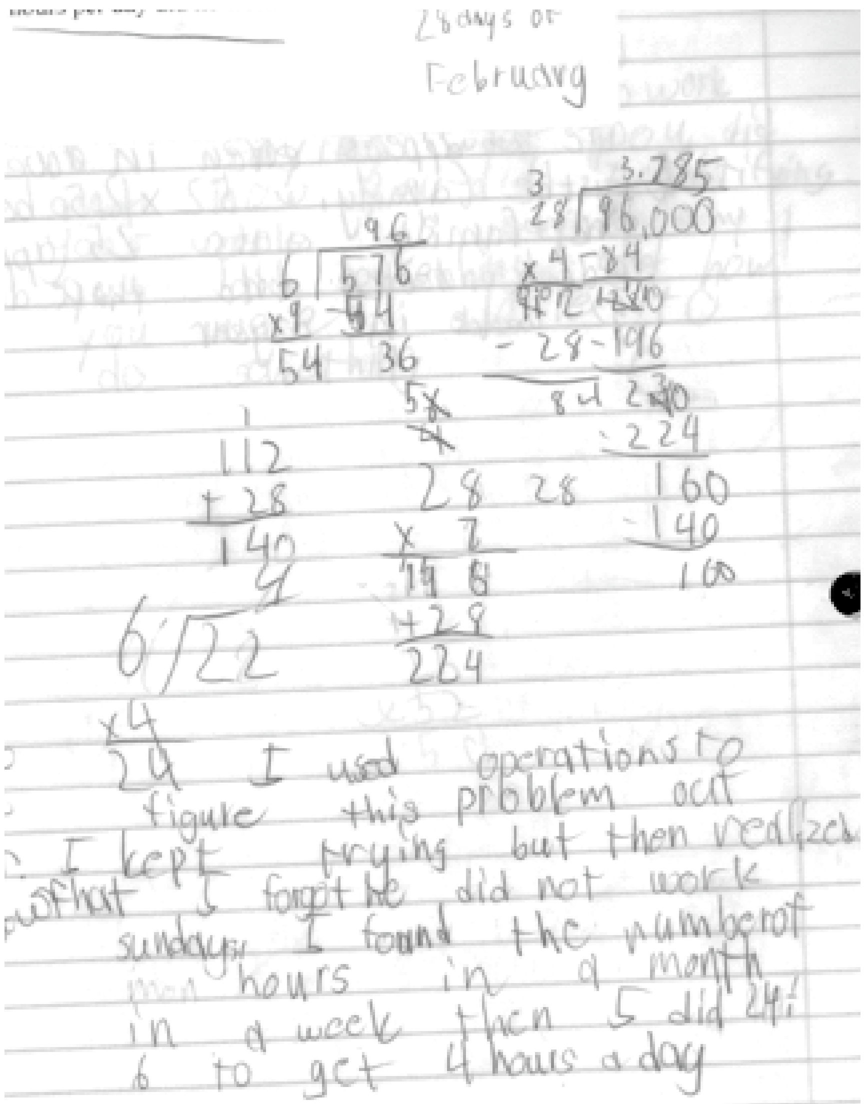


Figure 12: Thomas's Work on the Earnings Problem

Thomas's explanation and justification also received a low score. His explanation is very limited. He only described three things: his use of operations, the mistake he made, and that he did 24 divided by 6 to get the answer of 4 hours a day. His justification is completely lacking.

Karen had scores equal to or above the class averages. Karen, like her classmates, used the guess and check strategy with operations. She properly identified this and received the full three points for this category. Karen also found the correct solution (see Figure 13).

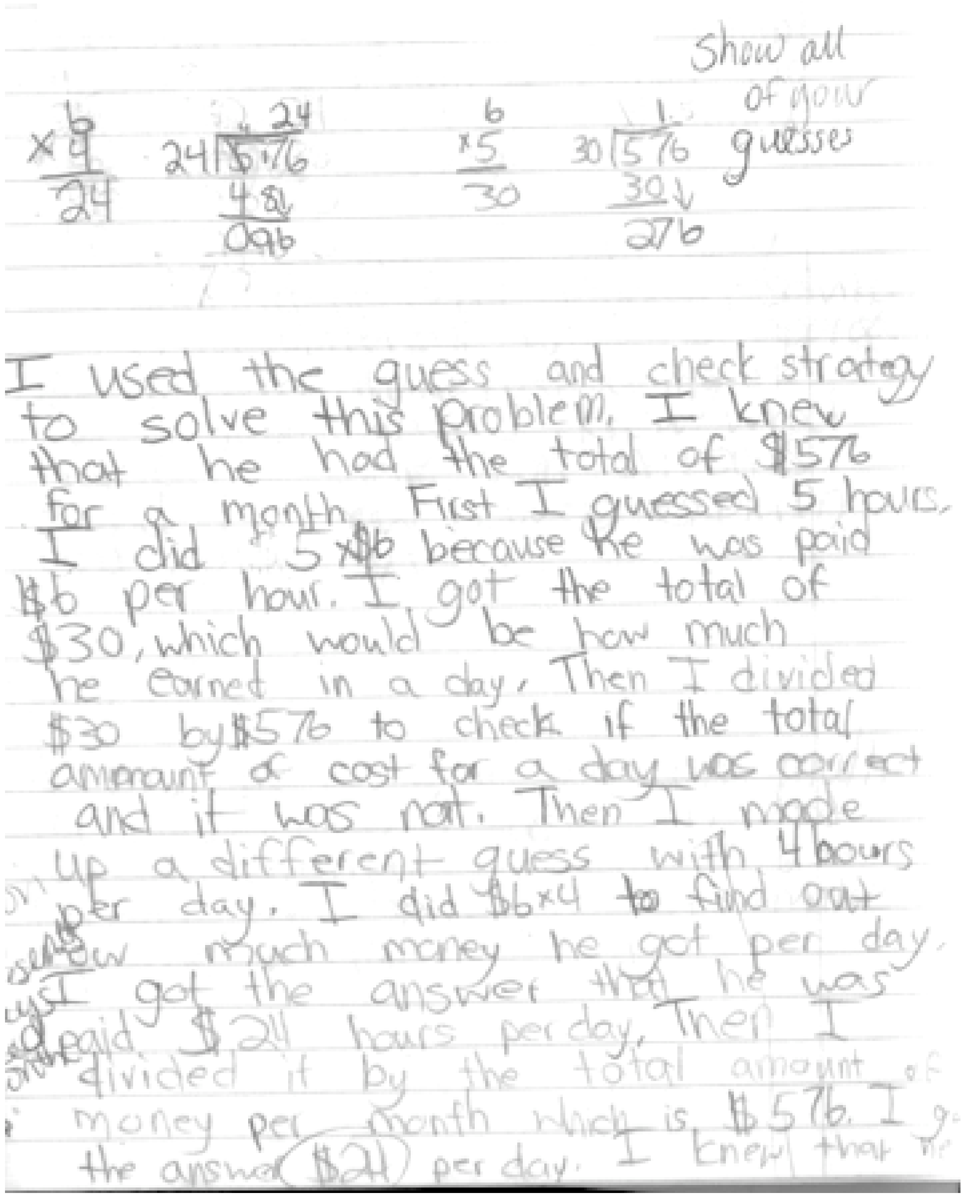


Figure 13: Karen's Work on the Earnings Problem

Only two of Karen's guesses are seen. There were eraser marks on her paper indicating that she had guessed other numbers first. Karen wrote a strong explanation and justification. She properly identified the numbers she used and the reasons for the operations chosen and did this for each step she took. She only made two errors. The first was a misidentification of one of the 24's as dollars instead of the number of days worked per month. She also wrote the wrong number as her solution in the explanation.

Barbeque Problem from Math Journal

The second problem chosen for analysis from the math journal was given to the students two weeks after the last problem analyzed. It was the last problem they solved immediately before Christmas break. This was also the last problem given before I added more discourse to problem solving. Up to this point, the class had been discussing the problem as a class upon completion, but I had not focused greatly on my leading and student participation in the classroom discourse.

Student Work Analysis

The problem given to the class was:

> Hannah sold \$65 worth of barbecue tickets. Adult tickets cost \$4 each and children's tickets cost \$3 each. How many adult tickets could Hannah have sold? Is there more than one possible solution to this problem?

The class was given this problem to solve in their math journals. Very little instruction was given at this point other than reading through the problem. This is because the students have solved a variety of problems with the same expectations for each so that it has become routine in the class.

The class scores were collected and averaged for analysis and are found in Table 9.

Table 9: 6th Grade Averages - Barbecue Problem

Characteristics	Score Problem 1
Exploring the Problem	2.7
Planning a Solution	3
Solving the Problem	2.8
Explanation of Work and Solution	1.5
Total Score	10

Even though much of what was expected of the students in problem solving had become routine, occasionally students forgot to underline the important information. One student out of the ten forgot to do this for this problem, lowering the average to 2.7. The students were expected to underline the important information as part of exploring the problem. The students were all successful at selecting an appropriate strategy. All ten students again used guess and check. The students all implemented the strategy correctly as well.

Some of the students had begun in the problem-solving unit to not clearly identify the solution they found. As a result, the average score for solving the problem was 2.8. By this point in our work on problem solving I had determined that the students were still struggling with justifying their work. Their explanations seemed to be good, but their writing was lacking the reasons for their work. Only a few students were still struggling with the explanation part. This is why the class average was 1.5.

Laura, Thomas and Karen's work were again used for analyzing. Their scores were compared to the class averages. The scores for these students can be found in Table 10.

Table 10: Student Scores - Barbeque Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	0
Planning a Solution	3	3	3
Solving the Problem	3	2	3
Explanation of Work and Solution	1	1	2
Total Score	10	9	8

Laura, like the rest of the class, used the guess and check strategy to solve this problem. As you can see from her work, some of her guesses were scribbled out as she realized they were incorrect (see Figure 14).

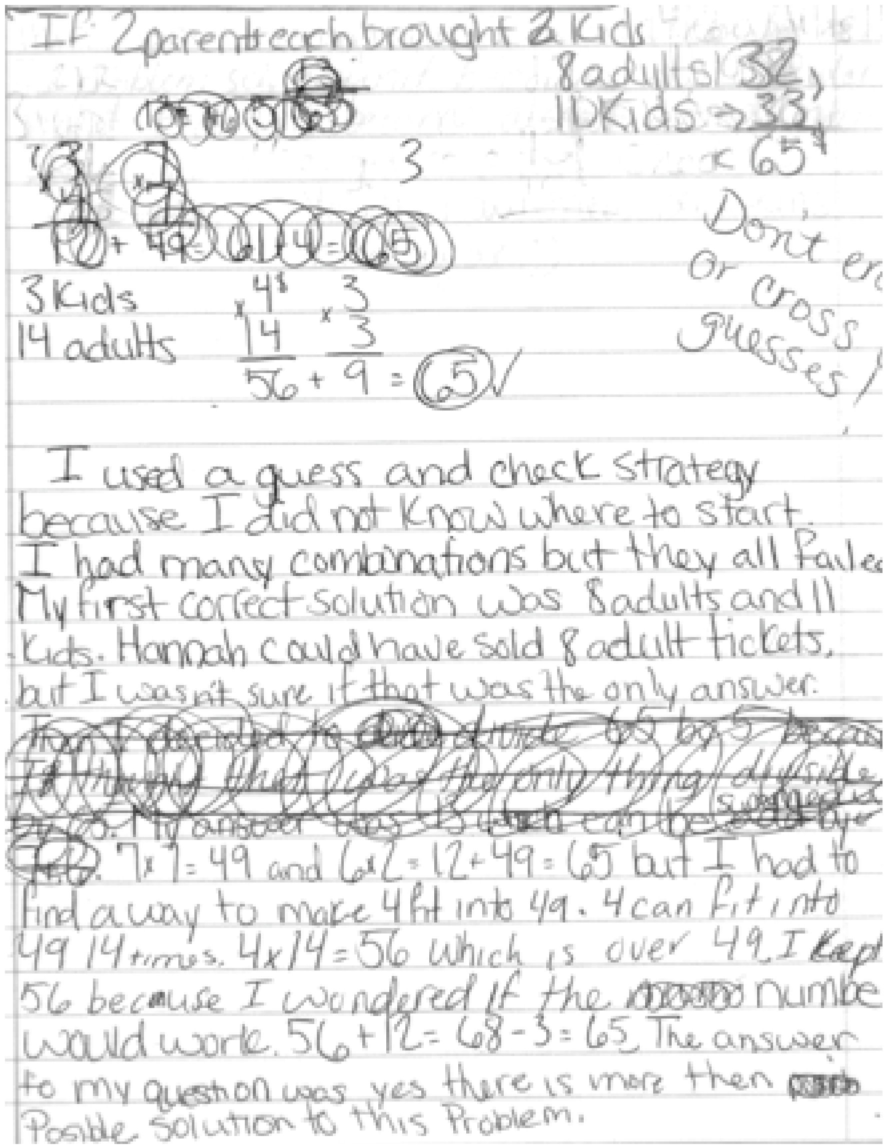


Figure 14: Laura's Work on the Barbeque Problem

The students had been asked not to cross anything out, so I made a note in her journal concerning this. She did find the correct solution like all of her classmates. Laura was one of the students who struggled with explaining as well as justifying. She basically listed out the operations she undertook but gave no reasoning for them. This has been evident in the last two problems Laura has solved. She writes expressions in her work as an explanation for what she did rather than using words. In answering the second question of the problem, Laura had found another solution, but it was not clearly justified either.

Thomas also used the guess and check strategy. When he solves problems, his work tends to be all over the page, which makes it difficult to follow and find the solution (see Figure 15 - 16).

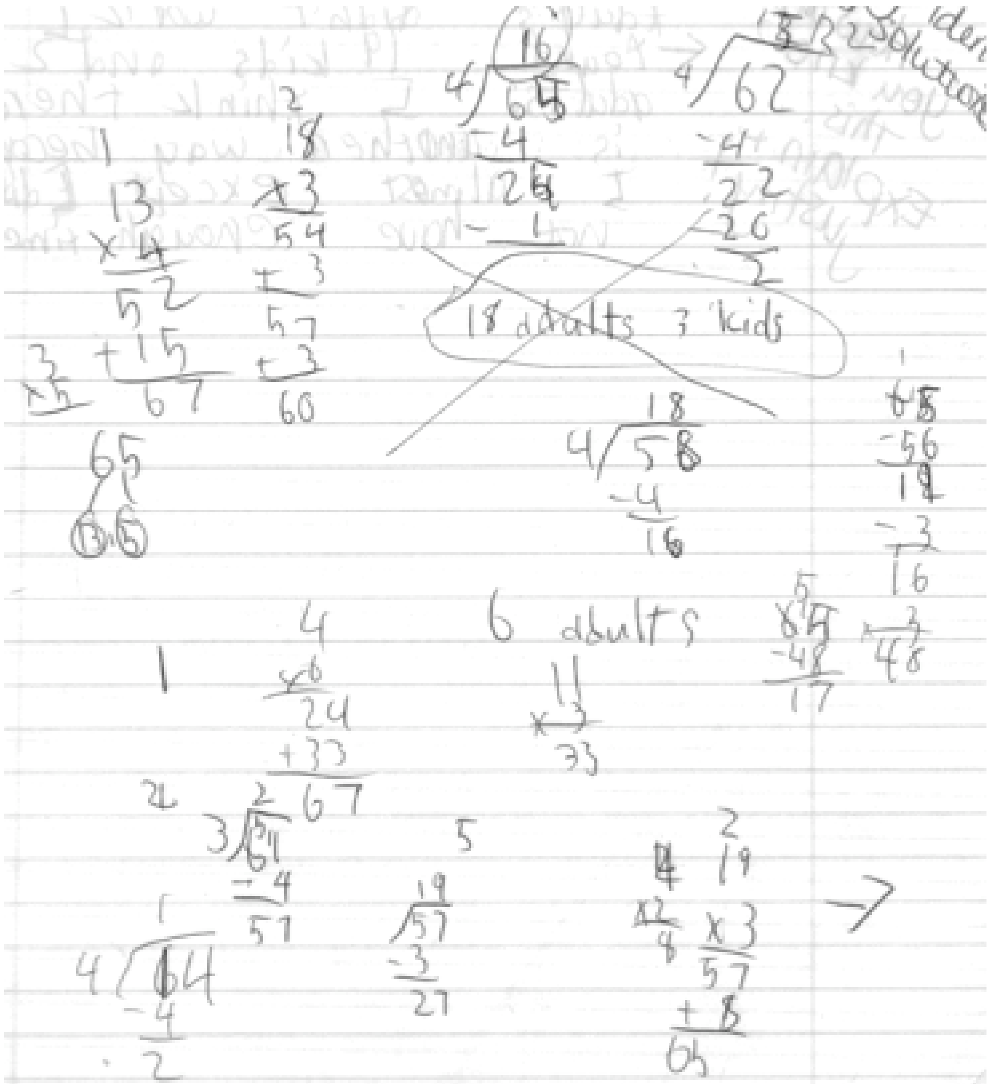


Figure 15: Thomas's Work on the Barbeque Problem

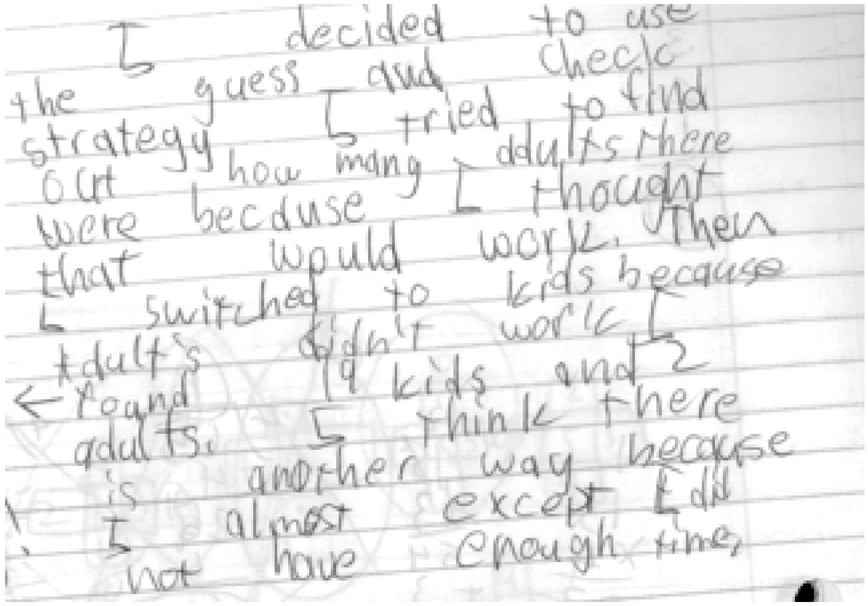
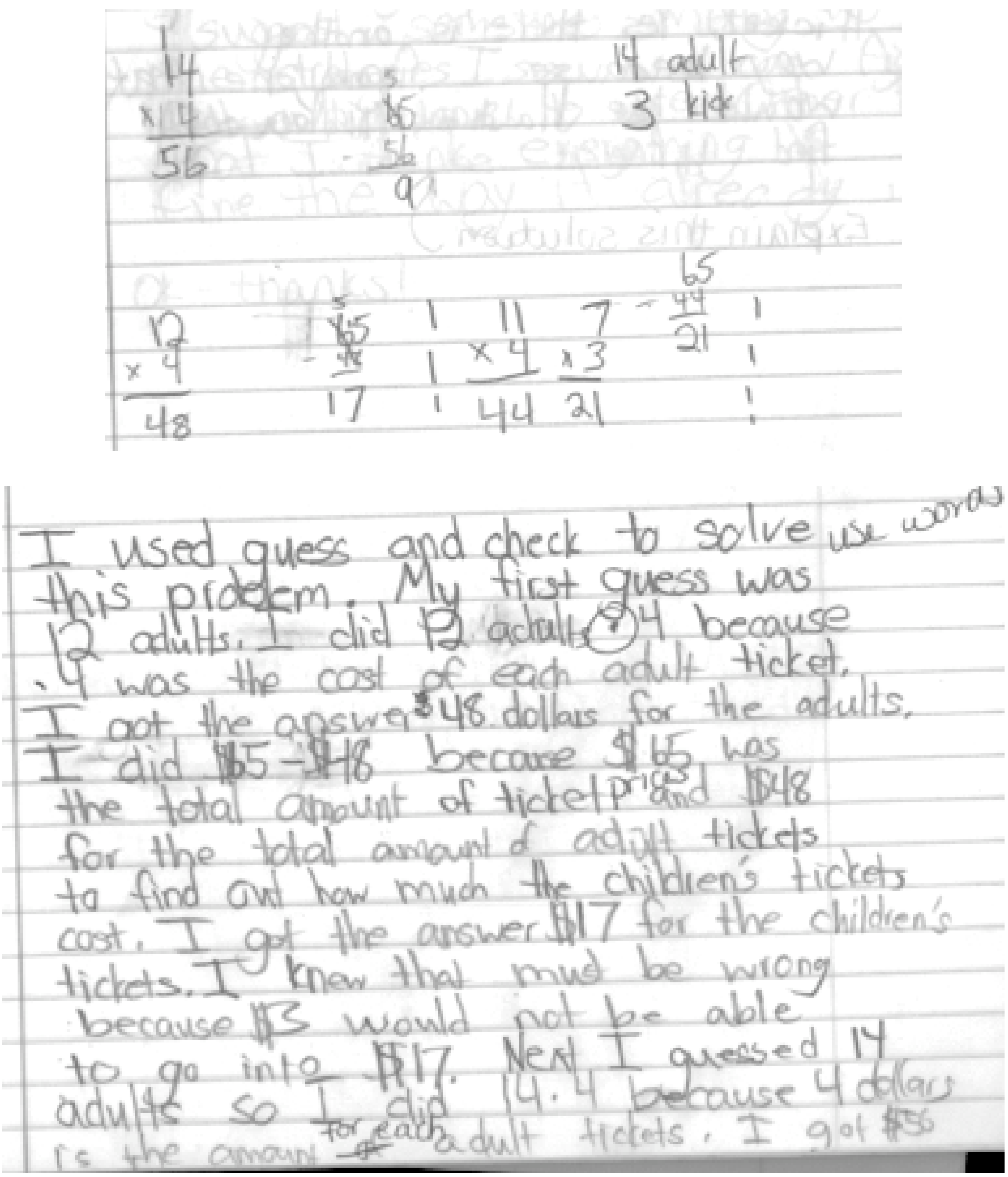

I decided to use
the guess and check
strategy I tried to find
out how many adults there
were because I thought
that would work. Then
I switched to kids because
adult's didn't work I
found 19 kids and 2
adults. I think there
is another way because
I almost except I did
not have enough time.

Figure 16: Thomas's Work on the Barbeque Problem

Thomas explained how he approached the problem at first and then how he changed it because it "didn't work". From this he jumped to stating his solution. There was no explanation of what operations he used or steps to get the solution and therefore did not justify either. Thomas seems to be consistently struggling with this portion of the problem solving. He did run out of time and was unable to find a reason for the answer to the second question to be yes. But I do not think that is this is the reason his writing is so poor. For some reason, he does not seem to grasp what to write even though it was discussed after each problem and I made notes on all of his work.

Karen was the one student who forgot to underline the important information. This happens very rarely for this class because it has become routine for them. Karen happened to forget on this day. She also used the guess and check strategy and found two solutions in order to answer both questions. Karen's work and explanation are in Figure 17 and 18.



I used guess and check to solve this problem. My first guess was 12 adults. I did 12 adults · 4 because 4 was the cost of each adult ticket. I got the answer $48 dollars for the adults. I did $65 − $48 because $65 was the total amount of ticket prices and $48 for the total amount of adult tickets to find out how much the children's tickets cost. I got the answer $17 for the children's tickets. I knew that must be wrong because $3 would not be able to go into $17. Next I guessed 14 adults so I did 14 · 4 because 4 dollars is the amount for each adult tickets. I got $56

Figure 17: Karen's Work on the Barbeque Problem

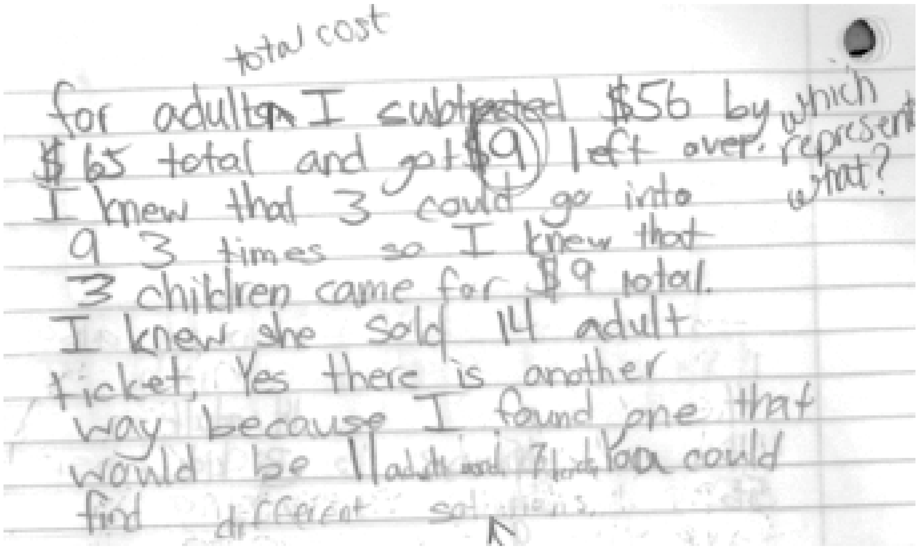


Figure 18: Karen's Work on the Barbeque Problem

It is evident from Karen's work that she did some of the work in her head. In the first solution she found, she demonstrated that after 14 adults, there was $9 left. The price of child tickets was $3 so she assumed that meant three children went. She did not show any work to demonstrate this step.

Karen had a fair explanation and justification, but was missing a few things. There were some minor mistakes in her word choices, but what she was trying to explain was clear that she understood. She also did not write what each number meant in her work, which she has done in the past. She also neglected to explain the second solution she found in order to answer the second question.

Conclusion

All three of these students, like the rest of the class, were able to use an appropriate strategy for solving the earnings problem. Thomas, for some reason, did not identify that he had used this strategy. This was a problem for only one other student. As a whole, the class was still able to identify appropriate strategies without reminders or hints. The explanation and justification still seemed to be a difficulty, even after completing the unit. For the most part, the class seemed to explain well, except for Thomas on this problem. The difficulty for many of them is identifying what the numbers mean in their work. They also struggled with justifying operations. Karen was one of the students in this class that was most successful. Laura had also shown improvements her explanations and justifications.

All three students, as well as the class, selected and implemented correctly an appropriate strategy for the barbecue problem as well. This again goes back to the instruction they received during the unit. Selecting a strategy is not difficult for them.

The students still are consistently struggling with the explanation, justification, or both. There was a slight improvement from the assessment problem to the first problem in the journal. However the score dropped again for the second problem studied from their journals. These problems both required the use of operations to solve. The students demonstrate that they know which operations to use in solving the problem. They do not, however, have the ability to verbalize the reasoning for their usage or what the numbers in the problem mean.

Upon completion of this section of the data collection, the students were asked to write in their journals about how they think problem solving in their journals had helped them. They all said that it had helped them in some way to improve their skills. Laura stated, " I still need to work on the writing." She felt that she had improved in everything but still needed to work on the writing portion of problem solving. Karen recognized that she was learning how to use different strategies and was improving in her performance. She also recognized that the rubric helped her to see "my score and notes I can find out what I need to work on." Thomas's was interesting. The only improvement he saw was in exploring and planning the problem. Thomas stated, "Now I go through look at the problem pick out the important information and select a strategy to use." I would agree that these are his strengths as well. What these students have written supports what their scores and work have shown as far as their performance.

Discourse in Problem Solving

As the last two sections have shown, the students' problem solving skills in exploring the problem, planning a solution, and finding a solution either improved or continued to be a strong area. This indicates that heuristic instruction and problem solving in the journals helps their problem solving skills. Due to a lack of verbal emphasis on explanation and justification, this area was still a struggle for the class as a whole. As a result, I began to work more on the discourse and verbalization in problem solving to see if that would impact their performance in explaining and justifying.

As the class started solving problems in their journals after the problem-solving unit, the students discussed the problem as a class. Very little discussion took place though. The focus was primarily on the strategy used. Students were selected to show how they solved the problem on the board and to verbally explain to the class what they did. Other students were selected if they had chosen a different way to solve the problem. Discussion focused on the similarities and differences in those strategies. There was little emphasis placed on explanation and justification.

As a result of continuing low scores in this area, I realized that I needed to make improvements in the discourse. I began by having students read their explanations and justification aloud. The rest of the class was instructed to listen. They were then asked, upon listening, if the explanation and justification was clear and strong enough. Conversation would then follow on what was good and ways to improve. I eventually also added the demonstration of work back into the discourse along with the verbal reading of the explanation. Discussion would follow on the strengths and weaknesses of the explanation and justification. This discussion would also include similarities and differences in the various strategies used as well as how those individual students had looked at the problem.

Recordings of various class discussion of a problem were analyzed for whole class involvement as well as the involvement of the three students whose individual work has been analyzed. My goal was to have less instruction and questioning by me, and more participation and leading of the discussion by the students. This in turn would hopefully lead to improvement in their explanation and justification skills. Therefore, one problem at the end of several weeks of discourse was again analyzed based on the scores the students received using the rubric.

Discourse Analysis

As discourse was emphasized more in the problem solving, the focus was first on the strategy usage. Upon completion of a problem, students were asked to show their work on the board. Student selection was based on the use of a different strategy. The first problem covered at this point allowed for variety. The problem was:

> From the bottom of a thirty-foot hole, a frog can climb up four feet each day, but slips back two feet each night. In how many days does the frog escape from the hole?

Three students demonstrated their work on the board. One used operations, another used a list, and the third made a number line. The students each explained verbally how they solved the problem, but did not read the explanation from their journals.

Student participation in the discourse was relatively little to the amount that I talked. Several students did ask questions and participate in discussion. Questions that were asked included:

1. “How did you find 15 days?”
2. “Why did you do it like that with your chart and all?”
3. “Why did you do it?”

This was the students’ first attempt at creating discourse in the classroom. The questions tended to be more about choices than about the process or the understanding.

My participation in the discourse was much greater than the students. I asked questions, explained student work, and clarified or restated statements made by students. I was trying to demonstrate for the students what type of questions they could ask. My questions also focused on creating deeper understanding. I asked about the similarities and differences in the strategies selected. The class, as a whole, answered questions.

As we continued on with solving other problems, students began to participate more and ask questions themselves. One problem in particular involved fractions. One student, who had gotten the solution wrong, asked a question of another student’s solution. He stated, “I don’t understand, like when Marcus has ¼ why wouldn’t it be 1/3 because there was, um…, um, well if he got half of it, then there was like a half left, and then Jan would have had the other piece in the middle, so why wouldn’t it be 1/3?” This student had changed the whole to what was left each time rather than leaving the whole as the original pizza. The student at the board responded. She restated that the half was taken, leaving half behind and then she gave a half of that half away. Unfortunately, the student still did not understand. I restated at that point to help. I also reemphasized the half of the half, but I added to this statement that this was of the original whole pizza. This meant there was ¼ of the pizza left of the whole, rather than 1/3 or one of three pieces.

The questions had become more about understanding than about strategy choice. My questions continued, but they continued to improve in terms of looking for or leading to greater understanding.

After several problems, I recognized that the students were still struggling with explanation and justification. I decided to alter the discourse for a while. The students seemed to demonstrate the ability to select an appropriate strategy, so we did not focus on this in discourse for several weeks. Instead, I began to have students read their explanation and justifications aloud to the class.

The idea was for students to begin to recognize good explanations and justifications. In order to do this, I would ask for a student volunteer to read their explanation aloud. Upon reading, I would ask the class two questions.

1. Do you think this was a good explanation?
2. Do you think there was good justification?

The questions would be worded differently at times. It was also difficult for the students to listen to an entire explanation at one time and then answer these questions. I would usually have the student go back and read parts and then ask the questions for each section. If the answer to either of these questions was no, the class would then discuss what improvements could be made.

After focusing discourse on the explanation and justification for several problems, I then combined the two focuses of discourse together. Students were selected to show work on the board and read their explanation and justification aloud. On occasion, the students would begin the discussion by stating that they did not understand something shown in the work or written in the explanation. This occurred in the last recording done for research. The student at the board would then reply. If the students did not start the conversation, I would ask if any students had questions. If not, we would move on to discuss the explanation and justification as we had in the past. The final questions related to the similarities and differences in the strategies used and how the student viewed the problem.

I had added one more aspect to improve their explanations and justifications. One problem the students had consistently was that they never read over the work themselves to catch any mistakes or lack of explanation and justification. As a result, I began having the students exchange their journals to be looked over before the journals were scored and before we talked about them as a class.

Student Involvement in Discourse

There were ten students participating in these discussions on problems. All of the students participated at some point due to being called on to participate. There were eight students who participated regularly on their own accord. One student participated less on his own and the last students only participated when called upon.

There were three students whose work has been analyzed throughout my research. Their participation in discourse was also tracked throughout the discussions. Laura participated on a daily basis. She would volunteer to read her explanation and justification, she would ask questions, and she would participate in the discussions on how to improve explanations. Karen participated just as much as Laura and in the same aspects. These two students were willing to participate and read even if they knew their work was not perfect. They had an interest in improving. Thomas did not participate as regularly. Thomas tended to offer solutions when he knew he was right. He never volunteered to read his explanation and justification aloud. Thomas has had difficulty writing explanations and justifications since the beginning, which could explain his reluctance to participate.

Student Work Analysis During Discourse

Upon completion of data collection from the student journals, I had recognized some common themes in the student work. The students were able to select an appropriate strategy and use it to find the solution. There was, however, a continued weakness in their explanations and justifications. It was at this point that discourse became part of my research focus. I wanted to see if it would help to improve my students' performance in problem solving.

The last problem given before discourse began was used in previous analysis. The scores for this problem, the barbeque problem, are listed again in Table 11. Included in this table are the average scores from the last problem given as part of the research on discourse. I used the scores from these problems for analysis on the improvement or lack of improvement in problem solving.

Table 11: 6th Grade Averages - Journal Problems Before and After Discourse (Barbeque and Clock Problem)

Characteristics	Before Discourse	After Discourse
Exploring the Problem	2.7	2.7
Planning a Solution	3	2.8
Solving the Problem	2.8	2.8
Explanation of Work and Solution	1.5	2.4
Total Score	10	10.7

The last problem given to the students as part of my research was:

> A clockmaker must wind his clocks on a regular schedule. He winds part of his clocks every two days, part of his clocks every three days, and part of his clocks every five days. How often must he wind all of his clocks on the same day?

The class, except for one student, consistently did well with exploring the problem. Cathy was the student in the class who forgot to underline the important information and put little effort into her work. She rushed to get things done and did not remember to do things that are expected of her. Cathy needs constant verbal reminders. Even though underlining the important information had become a norm for the rest of the class, it had not for her.

The scores for planning a solution had dropped a little due to one student as well. This student had listed logical reasoning, draw a picture, and make a list as strategies used. In looking at her work though, the list was the only strategy she used. Therefore, she lost a point for selecting an appropriate strategy.

All but one of the students in this class had used some form of a list to solve the problem. The lists all appeared in vertical columns except for one student who listed the days clocks were wound horizontally. The one student who did not use a list drew a picture. Her picture was based more on the days and what clocks were wound. The day she drew all three clocks was the solution to the problem. All the students who made the lists looked at the clocks individually and had to find the day they all matched. This difference was part of the class discussion on the problem.

All of the students had identified the correct solution. However, one of the students had not found the solution based on work. Cathy is a student who has repeatedly put little effort into solving a problem (see Figure 19).

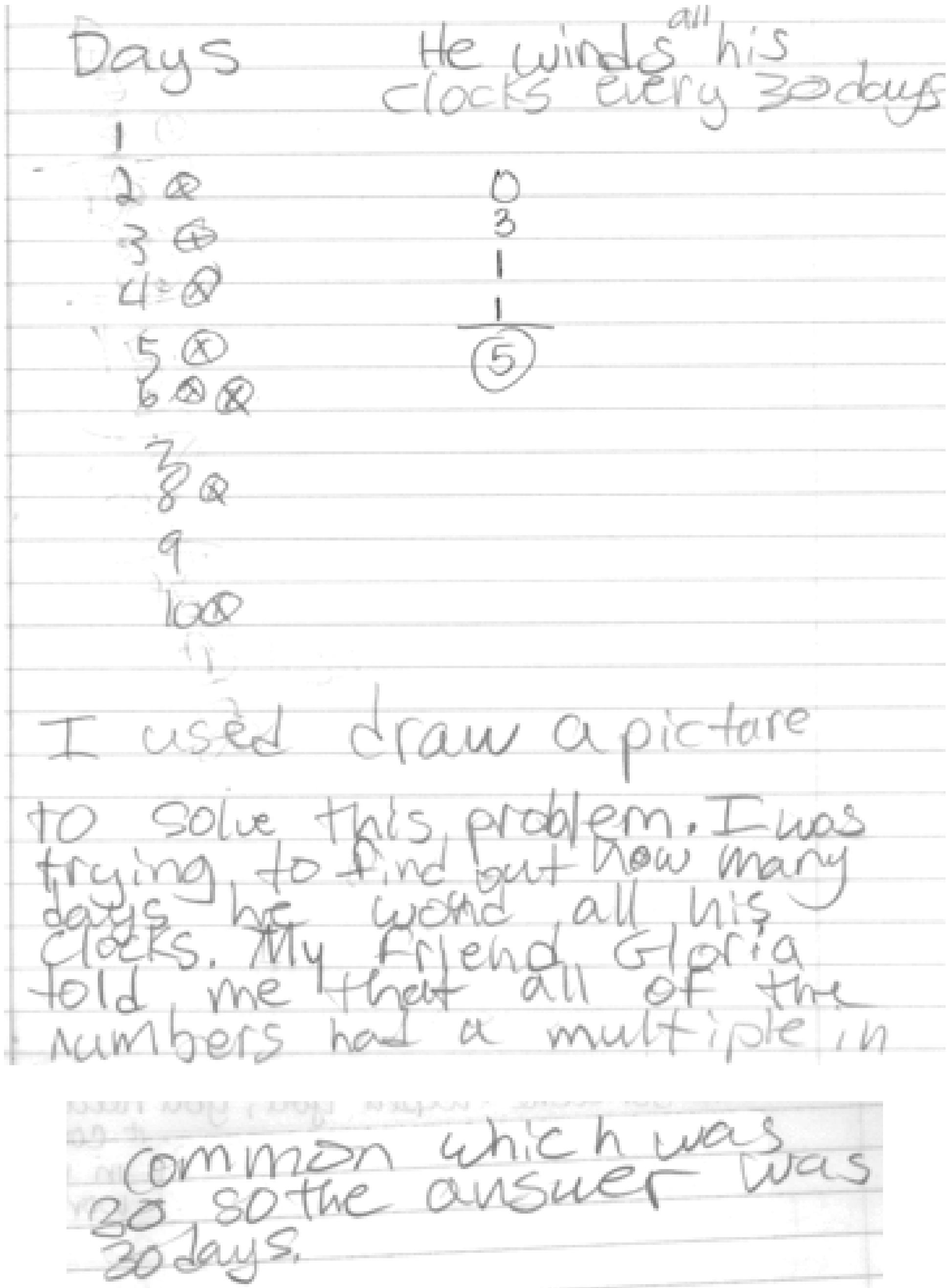


Figure 19: Cathy's Work on the Clock Problem

Cathy's lack of effort was the reason the average score was lower. Her work was not complete. She did not take the time to think about this problem and as a result got a hint from one of her classmates. She then did not finish the work but figured the solution out in her head.

Explanations and justifications for the class had improved since the beginning of discourse. All but one of the students had written a fairly good explanation and justification. This student was again Cathy. Her explanation and justification are included in Figure 19.

Little effort was put into understanding this problem. She started to use a strategy that would have helped her find the solution, but she did not complete it. Her explanation was based on the hint she had received from a classmate but it was not justified in her work or writing.

As stated earlier, Cathy's scores had a great impact on the class averages for this problem. The class averages are shown below in Table 12 with and without Cathy's scores.

Table 12: 6th Grade Averages - Clock Problem With and Without Cathy's Score

Characteristics	With Cathy's Score	Without Cathy's Score
Exploring the Problem	2.7	3
Planning a Solution	2.8	2.8
Solving the Problem	2.8	3
Explanation of Work and Solution	2.4	2.8
Total Score	10.7	11.6

Cathy's score had caused the averages in exploring the problem, solving the problem, and explanation to be lower. The class overall, therefore demonstrated more improvement in their problem solving skills than was evident with her work included.

Individual Student Work Analysis

After analyzing the class as a whole, the individual work of three students was again analyzed. The work of Laura, Thomas, and Karen were again used for this. Scores they received on this problem are found in Table 13.

Table 13: Student Scores – Clock Problem

Characteristics	Laura	Thomas	Karen
Exploring the Problem	3	3	3
Planning a Solution	3	3	3
Solving the Problem	3	3	3
Explanation of Work and Solution	3	3	3
Total Score	12	12	12

Thomas, Laura, and Karen scored at or above the class averages. Laura had used the list strategy to help solve this problem (see Figure 20).

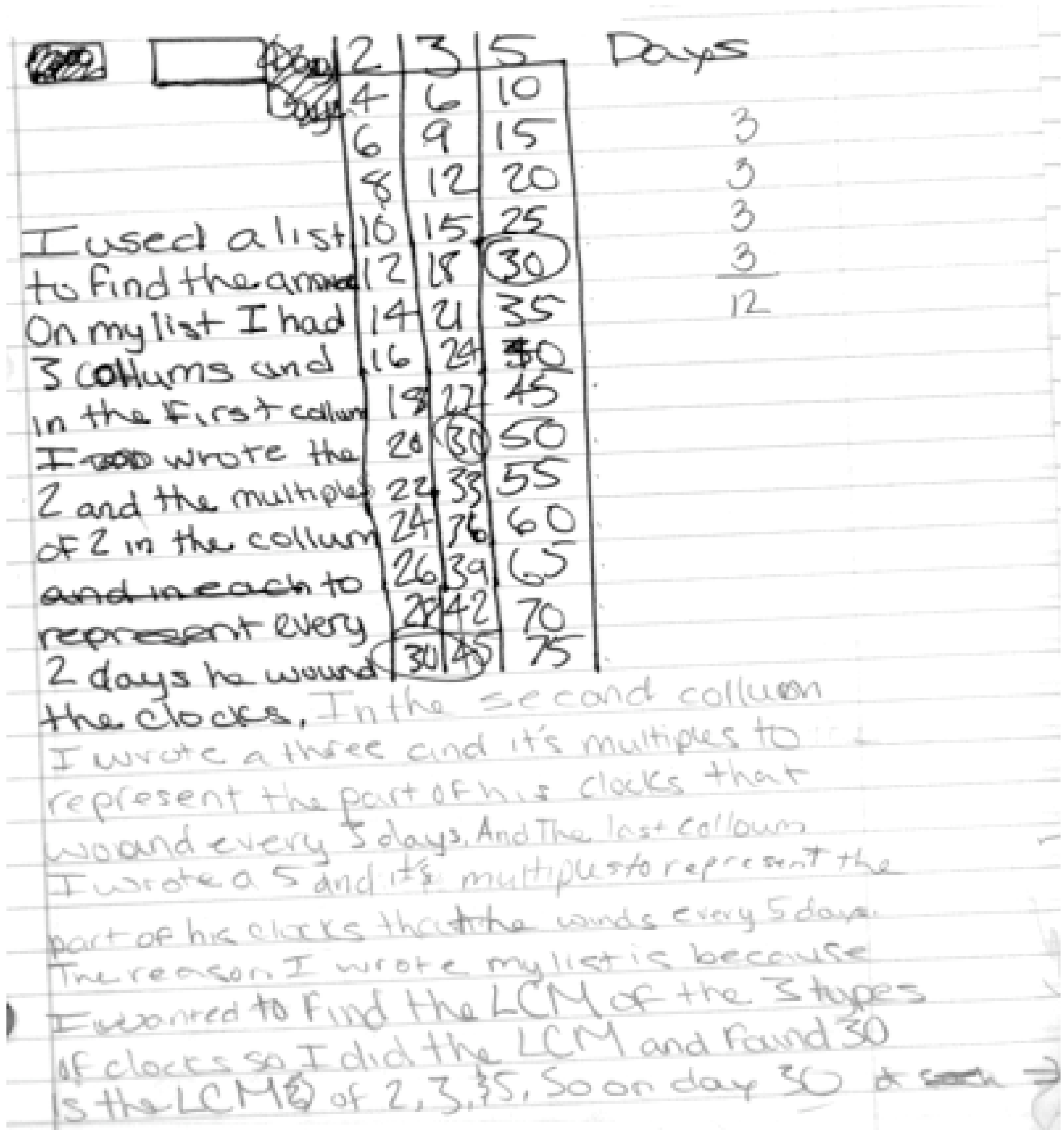
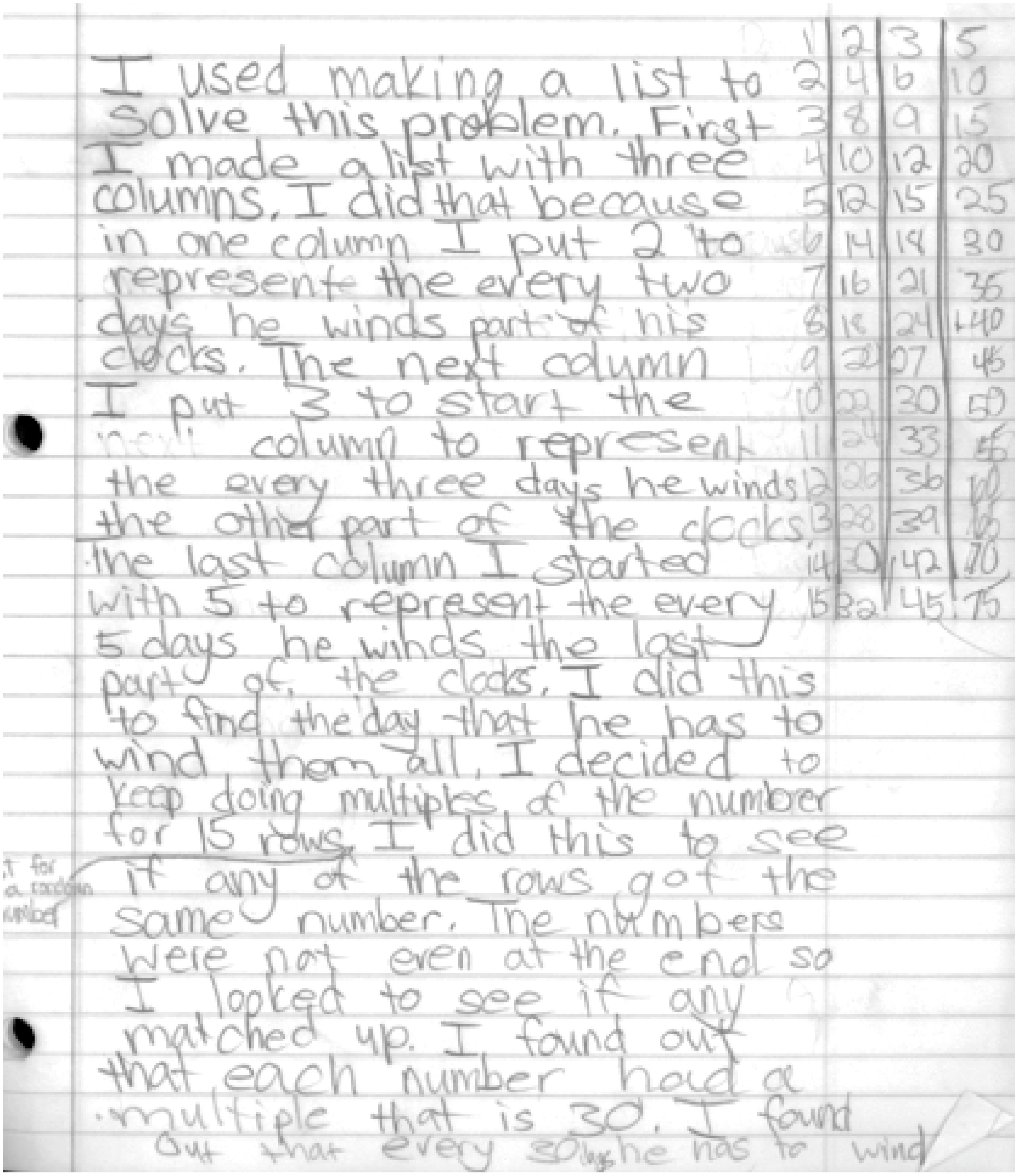


Figure 20: Laura's Work on the Clock Problem

She made a column for each of the different clocks and listed the multiples of the days each was wound. She found the common day was 30. Her explanation and justification were also very clear. She even used the term multiples to explain and justify what she did.

Karen also used the list strategy. Her work however had one additional column (see Figure 21).



I used making a list to solve this problem. First I made a list with three columns. I did that because in one column I put 2 to represent the every two days he winds part of his clocks. The next column I put 3 to start the next column to represent the every three days he winds the other part of the clocks. the last column I started with 5 to represent the every 5 days he winds the last part of the clocks. I did this to find the day that he has to wind them all. I decided to keep doing multiples of the number for 15 rows. I did this to see if any of the rows got the same number. The numbers were not even at the end so I looked to see if any matched up. I found out that each number had a multiple that is 30. I found out that every 30 days he has to wind

1	2	3	5
2	4	6	10
3	8	9	15
4	10	12	20
5	12	15	25
6	14	18	30
7	16	21	35
8	18	24	40
9	20	27	45
10	22	30	50
11	24	33	55
12	26	36	60
13	28	39	65
14	30	42	70
15	32	45	75

Figure 21: Karen's Work on the Clock Problem

Karen was one of the students who volunteered to show her work on the board to the class. In explaining her work to the class, she had thought originally that the number would be the same in an entire row. She realized as she solved the problem that the numbers did not need to be in the same row to be the same day. She stated that her first column was actually not important to the solution. Karen's explanation and justification was also very clear and well written. She also used the term multiple in her writing.

Thomas struggled a little at first with this problem. The first strategy he selected was to draw a picture. He then realized that his picture was not helping him to find a solution. At this point he asked me for some help. I asked him what other strategy might be useful and he selected the list. He then created lists for each of the different clocks knowing that he was looking for the day they had in common (see Figure 22).

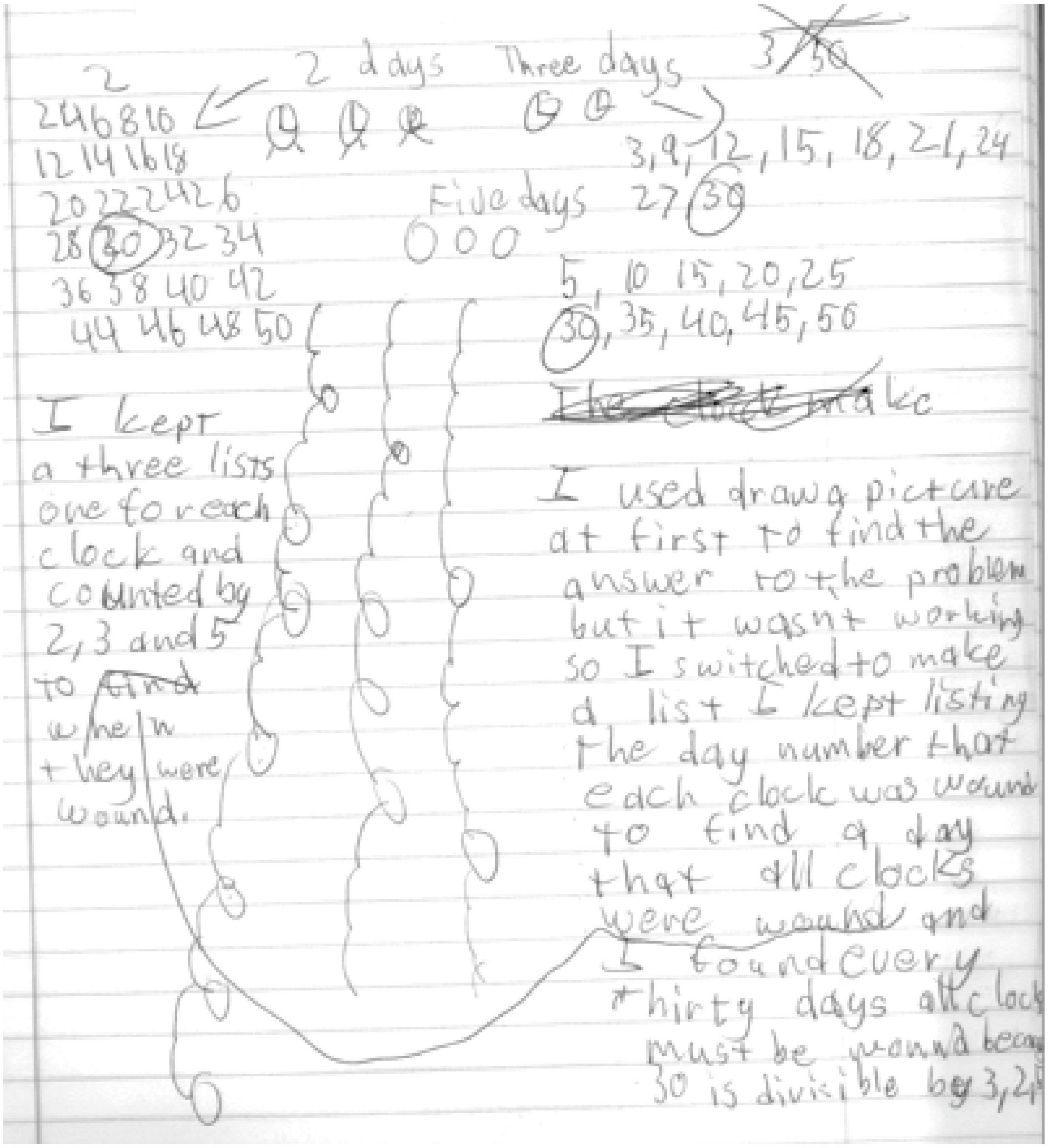


Figure 22: Thomas's Work on the Clock Problem

Thomas's explanation and justifcation were also much stronger than he had really written before. He had made more of an effort in this problem. It also helped that the class had exchanged journals to read each other's explanations. Thomas had written a fairly good and clear explanation and justification to begin with. His classmate had made one recommendation which led to the added statement on the left side of the page.

Conclusion

Based on the holistic average for the class before and after discourse there was improvement in their explanations and justifications. There would have been even more improvement if Cathy had put more effort into problem solving. The class averages without her scores demonstrated the level of improvement of the rest of the class. There was a definite improvement in the explanations and justifications. This was related to the classroom discussions on strategy usage, verbal readings of explanations and justifications, and the exchanging of student writings before discussions.

The students also reflected on how discourse had helped their explanations and justifications. Laura and Karen both wrote responses that said discourse had helped them. Thomas was absent during this reflection and therefore, there is no comment from him. Laura stated, " Talking in class about justification and explanation has helped me a lot because I now see better ways to justify something." Karen also said the verbal discussion helped. She focused also on the comments of her classmates. She stated that these comments "… helped me. I learned how I can explain and justify my work better."

This has completed my analysis of students work from the heuristic instruction of problem solving to problem solving of journals and the use of discourse. The data demonstrated that the students' performance in problem solving had improved. The scores of the individual students and the class averages demonstrated growth and improvement from beginning to end. These scores were taken not only from the problem-solving unit, but also from problems solved in the journals. There was a definite difficulty in teaching students to justify their explanations. The students will continue to develop their skills in this as they continue to get more practice.

The students' reflective journal prompts also supported that the students had improved in their skills. They themselves stated how they had improved. Finally, discourse was also key to the improvement in their problem solving skills. It took verbal discussions of strategies, verbal readings of explanations and justifications, as well as reading each other's explanations in order for the class to continue to improve in their performance.

In the next chapter I will discuss the data as a whole. I will identify common themes throughout the data collection as well as report on any improvements made by the students in their problem solving skills.

CHAPTER 5: CONCLUSION

Introduction

NCTM has set four key standards for instruction in problem solving. These four standards were related to building new mathematical knowledge, solving problems, applying a variety of various strategies, and be able to reflect on the process of problem solving. In reference to this last standard, communication is also very important (NCTM, 2000). In accepting the importance of problem solving instruction, the purpose of this study was to observe and examine the way that I instructed my students in problem solving and the impact of that instruction on their performance in problem solving. The focus of student performance included strategy usage, finding the solution, and providing appropriate explanations and justifications. Throughout the research period, data was collected to assess student performance in problem solving through the use of heuristic instruction with journaling and discourse. A problem-solving unit was first taught to the class to introduce Polya's four problem solving steps (Polya, 1957) and to teach various problem solving strategies heuristically. Journals were then used to continue instruction and practice in problem solving. Problem solving prompts were given to the students regularly to solve in their journals. These journals were also used as a means for assessment, my own communication with the students regarding their work, and for reflection. Finally, discourse was emphasized in the class discussions of problem solving in order to hopefully continue improvement in performance. Student responses, a problem-solving rubric, and recorded classroom discussions were analyzed to provide understanding and insights into students' performance in problem solving.

According to Johnson (2008), action research "has the potential to change education, to keep our teaching practices evolving"(p.214). Action research can help us to improve our level of teaching in the classroom. With this in mind, I conducted action research in my 6th – 8th grade advanced math classes. However, I only analyzed data from the sixth grade advanced mathematics class. My action research included adding problem solving, journal writing, and discourse as part of a regular routine during class time. By starting with the problem-solving unit, I was able to introduce the importance and meaning of problem solving. I also introduced some of the various strategies that could be used in problem solving. This unit also served as the first introduction that all my students had to writing, explaining, and justifying within mathematics. This unit provided an opportunity for me to model problem solving for my students and for them to learn through imitation and practice (Polya, 1957). Instruction focused on one strategy in each lesson. One example problem was worked together as a whole class. We also completed the explanation and justification as a class as a model from which they could learn. Assignments focused on individual strategies with the intent of students learning to recognize problems for which the strategy could be used. These assignments, review sheets, and the assessment from this unit were used for my analysis. I was able to use the rubric to evaluate student work and determine any growth in their skills. Through reflection of student work throughout this unit, I was able to recognize strengths and weaknesses that had developed. This allowed me to then find ways to improve on their weaknesses. These changes were then made during the time spent completing problem solving prompts in the journals. The same rubric was used, but discussion was now included on the various strategies used. Instruction was still given on the expectations for the explanations and justifications. As I continued reflecting throughout this research, discourse was added, students began reading explanations and justifications aloud, and students read each other's work for improvement. This constant reflection and focus on improving my students' performance in problem solving has improved my own teaching performance by causing me to reflect in all areas of my instruction, not just on this one topic.

Problem Solving Unit

The main purposes of the problem-solving unit was to instruct students heuristically in problem solving strategies and to introduce them to explanation and justification. Practice in both areas was also the intent. Higgins (1997) had stated that teaching strategies heuristically had its positives and negatives. I found this to be true in my research as well. By teaching problem solving strategies heuristically, students did learn to recognize to an extent the types of problems in which various strategies could be used . This was evident in the scores the class received throughout the unit and journal writing. It was very rare for one of my students to select an inappropriate strategy or implement it wrongly. However, I also saw the negatives to teaching heuristically. Several students still looked first for an operation that could solve the problem. There was also difficulty if the class as a whole did not immediately know the best strategy to use. I believe they had learned and accepted that the solution would not be immediately known; however, they had replaced this with the expectation that they should know immediately how to go about solving it. One of my students would struggle and get upset if he could not figure out the strategy right away. However, he would eventually determine a strategy to use.

The result of teaching in this method also led to a lack of creativity in the use of the strategies. This is similar to Higgins findings in her research (1997). The students rarely used a strategy other than those they received instruction on. However, as DeCorte (1995) and Hohn and Frey (2002), and even Higgins (1997), found in their research, there were more benefits from this. I believe this is evident in the fairly consistent scores my students received in this area of the problem solving.

Writing

Greater value and importance has been placed on writing in mathematics (NCTM, 2000; Pugalee, 2004; Rose, 1989). Writing instruction began in my research at the beginning of the problem-solving unit and continues in my class currently. The students had great difficulty in the beginning when writing was introduced. Many of them had never before been asked to write during a mathematics lesson or instruction. The primary focus of their writing was on explanation and justification but students were also asked occasionally to reflect, describe, or define in their writings.

Explanations and Justifications

The most difficult area for my students to learn and improve in throughout the research was in explanations and justifications. As the instruction and modeling of explanations and justifications began, my students struggled in both areas. Their writings tended to be more procedural and very limited. This supports research conducted by Yackel and Cobb (1995). They argue that students have difficulty with this because mathematics lessons are taught very procedurally. This very idea was discussed with my class as we talked about their continuing difficulties with explaining and justifying. They have been continuously taught procedures and algorithms but rarely instructed as to why they do them or what the procedures may mean. My students demonstrated their skills in exploring, planning, and finding a solution. However, they struggled in explaining and justifying their solutions due to their lack of experience in writing and the procedural instruction they had received previously.

Throughout the problem-solving unit, I modeled explanations and justifications as the research has suggested (Martinez, 2006; Polya, 1957). By the end of the problem-solving unit, the class had seemed to have a better understanding of the expectations for the explanations. The students were writing out their process completely except for one or two who continued to demonstrate a lack of effort in their work. However, as they began solving problems in their journals, the students were still having difficulty with providing strong justifications, or their reasoning, behind each step. Modeling did not seem to be enough. This indicated that heuristic instruction with journal writing and without discourse does not help students in their explanations and justifications, only in their problem solving skills.

Eventually, discourse was included more in the hope of improving their writing, but this will be discussed more in the next section. By the end of the research I had noticed a trend in their writing. The students were able to write strong explanations and justifications if fewer operations were involved. If the problem could be solved with a strategy and a simple operation, they were better able to justify. However, if the problem involved more difficult mathematical concepts, such as fractions, the class did not perform as well in their justifications. This refers back to Yackel and Cobb's (1995) research, that math is taught procedurally. The students lack enough conceptual understanding and that directly affected their ability to explain and justify.

But, again, lack of experience could also be part of the problem. These both provide reasons that modeling explanations and justifications was not enough for the students.

Writing also served as a means of assessment not only in problem solving but in student understanding (Baxter et al., 2005; Burns & Silbey, 2001; Doerr, 2006). This research is supported by my own research in identifying my students' weaknesses in conceptual understanding. I was also able to use student reflections and writings for assessment. For example, the sixth grade class was asked to explain the difference between a factor and a multiple. My students had great difficulty with this. The last problem used as part of my research demonstrated that several of my students had developed a greater understanding of what these terms meant when they used the term multiples in their explanation. I was able to assess their understanding of these concepts through their writing.

Discourse

Discourse plays an important role in the development of conceptual understanding (Pugalee, 1999). This practice was also important to improving student performance in problem solving. It allowed for students to discuss various strategies and solutions. It can also help in improving explanations and justifications.

When I began discourse, I was very worried about implementing it correctly. The reason for this was as Breyfogle (2005) found in his research, few teachers have received instruction on how to implement this in the classroom. I had received very little and was somewhat unsure of myself. We began by discussing various strategies used for the problems the students solved. I also encouraged the students to begin asking questions. The sixth grade class began this immediately, however, their questions were very procedural and not conceptual, which supports research conducted by Williams and Baxter (1996). There was little focus on the explanations and justifications at first, but as I became more comfortable, I included this in our discourse. Students read their explanation and justification aloud and the rest of the class was encouraged to identify whether or not the writing was strong and clear. They were also encouraged to suggest to their classmates possible improvements. Students had also begun asking questions for clarification and understanding. By the end of my research, we had included both of these topics in our discussions. I think the discourse helped them to improve their problem solving skills in terms of their writing. As seen at the end of my research, my students had improved in this area overall.

I have become more comfortable with discourse in my classroom and attempt to implement it in other areas besides problem solving. However, I know this is a practice I have not perfected and will continue to use it. Hopefully, as I become more comfortable with it, my students will as well and will continue to re-develop their own role in the classroom.

Norms

The most important aspect of this research has been in the area of social and sociomathematical norms. The establishment and development of norms is key to helping students think more conceptually and supports higher levels of thinking (Yackel & Cobb, 1996). There were several practices related to problem solving that have became norms in my classroom. The first was that students were expected to identify all information in the problems before they began solving them. The students were instructed to underline the information.

Most of my students remembered this each time they solved a problem. Occasionally, one or two students forgot because they were not given a verbal reminder.

Another norm that was established in problem solving was writing acceptable explanations and justifications. The students worked on this a great deal and knew that it was expected each time they solved a word problem. As this norm continued and developed, the student comfort level and performance also improved.

Discourse has also been established as a norm. The students knew that after a problem was solved the class would discuss the various strategies implemented, the solutions found, and identify acceptable explanations and justifications. I realize that these norms will continue to be established and negotiated (Yackel, 2001; Yackel & Cobb, 1995, 1996).

Recommendations

There were several limits to this research. First, the sample size was small. It would be beneficial to conduct this research on a larger sample and to see the results. This research was also conducted in a small parochial school. It would also be beneficial to conduct this research in the public school system.

Another limit to my research included the problem-solving unit itself. The students received two weeks of instruction on problem solving without any breaks. I believe this caused some students to look more negatively on problem solving and this might have impacted the data. As I begin instruction in this next year, I plan to teach one problem solving strategy per week. This will mean the unit takes longer to get through, but I feel student attitude towards it would improve.

There were also various factors that had an impact on my data. I felt some of the problems chosen for the problem-solving unit and the prompts may have impacted my data. The most appropriate strategy to use was very evident in some problems. This led to the entire class solving the problem the same way on several occasions, which then affected the classroom discourse. However, this indicates that students were able to recognize typical examples of each problem solving strategy without difficulty. This indicates that heuristic instruction did have a positive impact. As I continue teaching, I will continue collecting problems that can be used so that I have a greater selection to pull from. Obviously the students recognizing which strategy to use is a positive to the instruction, but selecting problems for which various strategies could be used would improve their skills even more.

One final factor that had an impact on my research was Cathy. Cathy rarely put forth any effort in solving the problems as well as writing explanations. As seen in the last problem analyzed, this had a great impact on my data. Every class will have at least one student similar to Cathy. I would recommend identifying these students early and begin working with them individually. The personal attention in this might help them to realize that, as the teacher, my requirements will be met and they will learn early to make the effort.

Another recommendation I have pertaining to this action research is more time. I am interested to see where my students will be at in their performance by the end of this year, especially since much of what was expected of them was very new. I also would recommend starting immediately at the beginning of the school year.

One final recommendation that I have pertains to the use of discourse. I focused on discourse after the students had completed a problem-solving unit and moved on to solving problems in their journals. This did provide information as to how discourse affects explanations and justifications. I would recommend a strong focus on discourse from the beginning of instruction. As I continue my instruction in this area and am now more comfortable with it, I will include discourse from the beginning.

Discussion

Problem solving, writing, and discourse are all very important aspects of the mathematics classroom. Literature describes their importance not only in the mathematics classroom but outside of it as well. With greater value being placed on these, more and more research is being conducted; however, there is still a need for further research on these three practices at the middle and high school level.

Through my research, I was able to study the affects of these practices on my students' performance in problem solving. The problem-solving unit improved my students' performance in selecting and implementing appropriate strategies. This unit and word problem prompts in the journals, as well as teacher modeling, improved my students' performance in writing acceptable explanations. Discourse was the practice that helped to improve the justifications in their writings. The sequence in which I conducted this research had an impact on my student performance in problem solving tasks. Each focus built off of the previous. In this, my students skills and performance improved as each focus was added to the research.

I plan to continue this practice in future years of teaching. As I continue this, my own comfort level in writing in math and leading discourse will improve. The students and I will continue to develop our new roles in the classroom. I also hope that as I continue this instruction in problem solving, the students will continue to improve in their performance, as most of them have instruction from me for all three years of middle school mathematics. As stated earlier, most of these practices were very new for them. The continuity of teaching in this manner will continue to develop their own conceptual understandings of mathematics and lead them to becoming better problem solvers both in and out of the classroom.

APPENDIX A: SAMPLE JOURNAL PROMPTS

1. What do you like about math?
2. What do you dislike about math?
3. What does problem solving mean to you?
4. Do you think calculators should be used in math class? Why or why not?
5. What do you think about showing work for math?
6. What did you learn today?
7. What did you not understand about today's lesson?

APPENDIX B: SAMPLE DISCOURSE QUESTIONS

1. Was that a good explanation?
2. Was that a good justification?
3. Do you understand what was said?
4. Do you agree?
5. Do you disagree?
6. Reasons for agree/disagree.
7. How are the two strategies used similar or different?
8. Why does the strategy (not) work?
9. Explain what you did.
10. Why did you choose this method?
11. Can you solve it in a different way?
12. Can someone restate what __________ said?

APPENDIX C: SAMPLE PROBLEM SOLVING PROMPTS

1. A toy shop makes tricycles and four-wheel wagons. Seven customers ordered six items each. Every order was different. How many wheels were needed for each customer?
2. Hannah sold $65 worth of barbecue tickets. Adult tickets cost $4 each and children's tickets cost $3 each. How many adult tickets could Hannah have sold? Is there more than one possible solution to this problem?
3. Jan sat down to eat a whole pizza. Barry asked for some, so Jan gave Barry half. Marcus also wanted pizza, so Jan gave Marcus half of what was left. Then Nina asked for pizza too, so Jan gave Nina half of what was left. Next Demetrius asked for pizza, so Jan gave him half the remaining pizza. How much pizza did each person get?
4. A jigsaw puzzle has 50 borer pieces and other non-border pieces. If each piece is one unit in length, how many units wide and how many units long could the puzzle be? Is there more than one possible answer? Explain.
5. A zookeeper is ordering food for the zebras. She knows that three zebras eat 25 pounds of hay every three days. How much hay should she order for 12 zebras to have enough hay for 30 days?

APPENDIX D: ANALYTIC PROBLEM SOLVING RUBRIC

Analytic Problem Solving Rubric

Characteristics	Score	Criteria
Exploring the Problem	3	Identifies the necessary information and question to be answered AND illustrates the problem when necessary
	2	Identifies most of the necessary information and question to be answered
	1	Only identifies the necessary information OR the question to be answered
	0	Does not identify information or the question
Planning a Solution	3	Selects and implements an appropriate strategy
	2	Selects an appropriate strategy but does not implement correctly OR selects incorrect strategy but implements it
	1	Wrong strategy selected
	0	No attempt made
Solving the Problem	3	Work shown and correct solution
	2	Work shown with minor computation error OR not enough work shown
	1	Work shown but incorrect solution OR gives solution but no work
	0	No work or solution
Explanation of Work and Solution	3	Gives solution with complete explanation of work, AND work is neatly presented
	2	Gives solution with limited explanation, AND work is neatly presented
	1	Gives solution with limited explanation
	0	Gives solution with no explanation
Total Score		

APPENDIX E: PERFORMANCE RUBRIC APPROVAL LETTER

Melissa Wittcop

From:	skrulik@temple.edu
Sent:	Tuesday, October 23, 2007 2:59 PM
To:	Melissa Wittcop
Subject:	Re: permission request

Dear Ms Wittcop:

Please feel free to use the rubric you wih from our book. They were put there for people to use.

Good luck on your dissertation.

S. Krulik

1

APPENDIX F: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL LETTER

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901, 407-882-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Notice of Expedited Initial Review and Approval

From : **UCF Institutional Review Board**
FWA00000351, Exp. 5/07/10, IRB00001138

To : **Melissa Wittcop**

Date : **August 16, 2007**

IRB Number: **SBE-07-05088**

Study Title: **The Effects of Journal Writing and Discourse on Student Performance in Problem Solving**

Dear Researcher:

Your research protocol noted above was approved by **expedited** review by the UCF IRB Vice-chair on 8/14/2007. **The expiration date is 8/13/2008.** Your study was determined to be minimal risk for human subjects and expeditable per federal regulations, 45 CFR 46.110. The category for which this study qualifies as expeditable research is as follows:

7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

The IRB has approved a **consent procedure which requires participants to sign consent forms.** Use of the approved, stamped consent document(s) is required. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

To continue this research beyond the expiration date, a Continuing Review Form must be submitted 2 – 4 weeks prior to the expiration date. Advise the IRB if you receive a subpoena for the release of this information, or if a breach of confidentiality occurs. Also report any unanticipated problems or serious adverse events (within 5 working days). Do not make changes to the protocol methodology or consent form before obtaining IRB approval. Changes can be submitted for IRB review using the Addendum/Modification Request Form. An Addendum/Modification Request Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at http://iris.research.ucf.edu .

Failure to provide a continuing review report could lead to study suspension, a loss of funding and/or publication possibilities, or reporting of noncompliance to sponsors or funding agencies. The IRB maintains the authority under 45 CFR 46.110(e) to observe or have a third party observe the consent process and the research.

On behalf of Tracy Dietz, Ph.D., UCF IRB Chair, this letter is signed by:

Signature applied by Joanne Muratori on 08/16/2007 10:22:17 AM EDT

Joanne Muratori

APPENDIX G: PARENT CONSENT LETTER

August 2007

Dear Parent or Guardian,

Welcome back to another school year! As you know, I will be teaching your child in math class this year. I have spent time this summer planning for some new activities for class this year.

In addition to teaching at Trinity this last year, I also began coursework for a Master's Degree in K-8 Math and Science at the University of Central Florida. I began researching and writing my thesis over the summer. This thesis is based on an action research study. This means that I have to study one or more of my instructional practices in my classroom. I am planning to do this with all of my math students in grades 6 through 8 during the course of the 2007-2008 school year.

The purpose of my research is to study the effects of using math journaling, which will include problem solving and reflection, and discussion in the classroom on student performance in problem solving. I am interested in seeing if journaling and discussion will have an effect on your child's ability in problem solving.

I do need your permission to have your child participate in the study. Data collected from your child's math journal. This includes solving one or two problems and reflecting once or twice a week on their math class experiences. I will also audiotape class discussions and focus group interviews. In addition to these, your child will also be assigned homework and class work that can be used for data. Participation in this study is completely voluntary and consent may be withdrawn at any time. There is no compensation for participating, nor will there be a negative affect if you do not give permission. Please realize though, that even if your child does not participate, he/she will still be required to complete the work assigned. This work then will not be used as data.

Your child's identity will be kept confidential during this study. Audio recordings will also be destroyed upon completion of my thesis. The purpose of this study is really to analyze my teaching practices and not your child's math abilities. I do not anticipate and risks to your student as a result of participating in this study. There is the potential benefit of identifying an effective teaching and learning strategy for middle school mathematics. Upon completion of the study and thesis, any data collected from your child and his/her class will be destroyed.

If you have any questions regarding this study, you may contact me at any time. You may also contact either of my advisors: Dr. Enrique Ortiz at 407-823-5222 or Dr. Janet Andreasen at 407-823-1378. If you have questions or concerns about participant's rights, you may contact the UCF IRB Office of Research and Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826. The hours of operation are Monday through Friday, 8:00 am – 5:00 pm. The phone number is 407-823-2901.

Thank you so much for your help in this process!

Sincerely,
Melissa Witcop

____

I have read the procedure above and understand what is being asked of my child as a participant of this research study. I voluntarily agree to allow my child to participate in the study and to be audiotaped during class and interview sessions. I will also allow his/her journals and work to be used. I have received a copy of this form.

____
Name of Child (Printed)

☐ I give voluntarily give consent for my child to participate in Miss Witcop's study.

☐ I give permission for my child to be audio taped during class discussions.

☐ I would like more information about this study.

☐ I do not give consent for my child to participate in Miss Witcop's study.

__________________________ __________________________ ________________
Name of Parent/Guardian (printed) Name of Parent/Guardian (signed) Date

UCF University of Central Florida IRB
IRB NUMBER: SBE-07-05068
IRB APPROVAL DATE: 8/14/2007
IRB EXPIRATION DATE: 8/13/2008

APPENDIX H: PRINCIPAL LETTER OF CONSENT

Informed Consent to Conduct Research
Principal Consent

June 28, 2007

Dear Dr. Shaffer,

This coming school year, I will be conducting an action research project in my classroom on the effects of journal writing and discourse on student performance in problem solving. This study is a part of my Master's program in K-8 Math and Science Education at the University of Central Florida. You are being asked to give your consent to allow me to conduct my research with my class for the 2007-2008 school year.

During my study, I will be audio taping whole class discussion and small focus group interviews. I will also use the students' journals, which will include problem solving and reflection. In addition, homework and class work will be used as data for analysis. All of this data will be collected from the various math classes that I teach to the middle school.

I will be obtaining informed consent from my students' parents allowing their children to participate in the study and to be audio taped and for my use of their journals and work. The letter of consent will also explain the purpose of the study and my expectations of the students. Participation in this study is not mandatory. They will then be informed that if they do not give permission, the students will still have to do the same work, it just will not be used in my study. Math grades of those students not participating in the study will not be affected.

I do not anticipate any risks to my students during this study other than the possibility of a break in confidentiality. Any data that I collect will be destroyed upon completion of the study. Student names will not be used in my thesis. The key identifying the students will also be destroyed upon completion of my study and thesis.

If you have any concerns or questions about this study, you may also contact one of my faculty supervisors: Dr. Enrique Ortiz at 407-823-5222 or Dr. Janet Andreasen at 407-823-1378. Thank you for your support in this process.

Sincerely,

Melissa Wittcop

My signature below indicates that I have read and understand the details of this study and that I give consent for Miss Melissa Wittcop to conduct research with her 6th - 8th grade math students during the 2007-2008 school year.

Robert L. Shaffer

Dr. Robert Shaffer
Principal, Trinity Lutheran School

Date

APPENDIX I: STUDENT LETTER OF CONSENT

August 2007

Dear Student,

Welcome to another year of math class! I am very excited about this year. As some of you know, I am a graduate student at the University of Central Florida. I am working to receive a Master's Degree in K-8 Math and Science Education. One of the requirements for this program is that I write a thesis. For me to do this, I need to conduct research about the way I teach math. For me to study this, I need your help. This year you are going to use a math journal. I would like permission to use the writing and work you put in this. I would also like permission to use you homework and class work. I am also planning to audio record some of the discussions we will have as a whole class and interviews that I will conduct with small groups. Your journal will include problem solving and reflections about math class. The interviews will include questions about class as well.

You do not have to participate in this study if you do not want to. My goal in this study is to look at certain ways that I teach math. You and your work will also remain confidential during my study and in my paper. That means that your name will not actually appear in my writing. The only people who will look at your work and listen to your recordings will be my supervisor and I.

If you decide not to participate, you will still need to do this work in class. I just will not be able to use any of your work as data. There will be no extra credit given for participation, nor will your math grade be affected in any way.

Thank you!

Miss Wittcop

By signing below, I am saying that I understand my role in the study to be conducted by my teacher. I have asked any questions that I may have had and they have been answered so that I understand what is expected of me. By signing, I am saying that I am willing and would like to participate in this study.

____

Student Name – printed

______________________________ ____________________

Student Name – signed Date

APPENDIX J: NUMERIC PROBLEM

Jessica has ½ of a candy bar and Ryan has 2/3 of a candy bar. How much do they have together?

Use the problem solving plan to solve. Explain and justify your answer.

APPENDIX K: PROBLEM SOLVING REVIEW WORKSHEET

6th Grade Review Sheet Name__________________________

Solve each problem using the 4-step plan. Choose the best strategy for each problem. Remember to show your work and write an explanation and justification for your solution.

1. Morey mowed half of Mickey's lawn. Matty mowed ¼ as much as Morey did. Midge mowed twice as much as Matty. How much of Mickey's lawn has not been mowed?

2. Scott makes monthly deposits to his savings account. During the past four months, he made the following deposits: $25, $30, $40, $60. If the pattern continues, how much will Scott deposit in the tenth month?

3. Holly, Carlyle, Sarah Jane, and Bryan are competing in the Fourth Annual One-Legged Race! They're now on the last leg of the race. How many different ways could they finish?

APPENDIX L: PROBLEM SOLVING ASSESSMENT

6th Grade Name______________________

Problem Solving Test

Solve each problem using the most appropriate strategy or strategies. Be sure to explain and justify your solution.

1. Mrs. Dixon and her family had traveled about 90 miles, or 2/5 of the way to the campsite. How much farther do they have to go?

2. Crafty Corey is making costumes for the new play, Bugs on Broadway. It stars the same number of 8-legged spiders as it does 100-legged centipedes. Corey's costumes have a total of 10,800 legs. How many spiders are in the show?

3. Carter Middle School has 487 fiction books and 675 nonfiction books. Of the nonfiction books, 84 are biographies. How many books are not biographies?

APPENDIX M: RESEARCH TIMELINE

RESEARCH TIMELINE

Research Timeline	Dates
Heuristic Problem Solving Unit (3 weeks)	Oct. 18 – Nov. 9
Handshake Problem	Oct. 18
Lawn Problem	Oct. 29
Travel Problem	Nov. 8
Problem Solving Prompts Solved in Journals (5 weeks)	Nov.12 – Dec. 14
Earnings Problem	Dec. 4
Barbeque Problem	Dec. 14
Problem Solving Prompts Solved in Journals with Discourse Added (5 weeks)	Jan. 7 – Feb. 15 (no problems given during exam week)
Clock Problem	Feb. 14

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