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Credit risk control and management using limited diversification

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ABSTRACT

The diversified strategy can reduce the systematic risk efficiently, but may fail to account for emergent and default risk that many decision-makers usually face at large-scale level. Modern data-driven methodologies allow optimizing both systematic and non-systematic risks in a unified framework. In this article, we demonstrate an approach to analyze and compare partial-diversified portfolios of Credit Default Swap. We classify and investigate different metrics of credit risks and integrate them with limited diversification and other performance objectives. We test the developed approach in a study of hundreds of business contract investments over the recent financial crisis. The results indicate that the decisions using limited diversification are more robust in terms of allocation structure and out-of-sample downside risks reduction. Therefore, the partial-diversified optimization models provide alternatives to support a variety of problems involving unknown risks.

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Risk management; risk analysis; uncertainty; limited diversification; credit default swap

1. Introduction

The goal of distributing available resources into multi-projects is to diversify and reduce systematic and non-systematic risks. Diversification is a viable strategy to balance risk and reward and enhance the efficiency of resource allocation when investing candidates with homogeneous risks (Joshi and Lambert 2011). A company may incorporate with multi-partners when it is expanding the scope of its business activities into areas where it has little experience to reduce the uncertainty from asymmetric information. However, as the main category of the non-systematic risk, the total default loss cannot be fully hedged through diversification due to the endogenous counterparty relationship, see Maggio, Kermani, and Song (2017). This is especially true for a decision-maker who optimizes this type of loss by holding all associated insurance contracts, e.g. Credit Default Swap (CDS). Under an agreement, CDS issue or seller pay a recovery to the buyers due to the default and terminate the contract. Otherwise, he will receive periodic premiums over the maturity. This mechanism allows the market participants to estimate the credit risk about the health of the underlying. It turns out that CDS trading reduce the transaction cost, offers liquidity to the market, and affect the underlying significantly, see (Oehmke and Zawadowski 2015).

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Additionally, Leccadito, Tunaru, and Urga (2015) show that trading CDSs appropriately can efficiently enhance investment performance.

Although CDS is relatively new to other derivatives, options for example, it has become an important alternative for hedging uncertainties after its appearance in the 1990s. Starting from a total notional amount of US$180 billion in 1997, the size of the CDS market increased exponentially to approximately US$62.2 trillion by the end of 2007 and then declined to US$21 trillion by December of 2013 (Fung et al. 2008; Augustin et al. 2014). The changes reflect the strong demand and the success of the financial innovation. However, due to the herd behaviour, the derivative market would be counterproductive to the underlying market. For example, CDS negatively affect the market when the credit events trigged the subprime mortgage crisis in 2008 (Stulz 2010), as well as the Eurozone debt crisis in 2011 (Aizenman, Hutchison, and Jinjarak 2011). Evidence by Duffie, Scheicher, and Vuillemey (2015) showed that CDS market overreacts in both crisis, and result in the key regulatory reforms for the modern financial system. That is, main CDS trading should be cleared through central counterparties to improve the transparency of the credit market. Another reform in the EU is to ban the short selling of the naked position of sovereign CDS, partially because of the concerns about the liquidity risk from CDS trading, despite that the borrowing costs may increase significantly (Calice, Chen, and Williams 2013; Oehmke and Zawadowski 2015).

CDS is a powerful weapon for risk-averse investors to avoid the default losses of the underlying entities, it is also useful for risk-seeking investors to pursue high profits due to extremely high penalty value specified in the contract. Risk management for using CDS is crucial for trading participants. Compared with the stock market, CDS trading and market are more complex since the investment involves uncertainties derive from spread changing and default losses. Spread changing in the CDS market generates systematic risk, while credit events related to the contract drive the counterparty failure risk. Given a CDS, the default risk usually dominates the systematic risk in a depression environment. The investor may enter different positions for the contracts in order to diversify the portfolio risk. However, extreme portfolio losses still can occur for large-sized credit portfolios, see recent studies by Giesecke et al. (2015); Muhlbacher and Guhr (2017). When one or a few contracts violate, all correlated CDSs may be propagated and face high penalty values, and aggregated portfolio loss may be magnified compared with that from uncorrelated CDS set. This encourages us to limit the portfolio size to reduce potential default risk and management complexity. That is, avoid selecting bad contracts through intelligent model by setting a trade-off between the systematic and default risks, the return, and the size of the CDS portfolio.

An integrated approach is designed and proposed CDS portfolio construction in this paper. According to the special structure of the contract, we incorporate the cardinality and solvency constraints to the Mean-Variance and CVaR models, respectively. We show that the spread-changing risk and the credit risk can be identified and managed in our multidimensional trade-off framework. Therefore, we can improve the allocation structure and reduce the counterparty-risk. In addition, we propose a dynamic updating procedure to evaluate the robustness of the developed models. We study the comparison metrics include portfolio returns, variances, Sharpe ratios and other indicators. Risk analysis for the comparison results illustrates the effectiveness of the cardinality constraints for reducing the downside risk. In addition, out-of-sample testing indicates constantly better performances by the CVaR model with cardinality restriction than those of the mean-variance strategy.

The rest of the article is structured as follows. Section 2 briefly reviews the relevant literatures on the microstructure and macrostructure of the CDS market. Section 3 develops the CDS selection models and associated dynamic updating procedure. A comparison of the results is then discussed in section 4. Finally, we present the conclusions of our work in section 5.

2. Literature review

CDS based derivative and portfolio have been studied from different perspectives such as inner and outer structures. Hull and White (2000, 2004) value the price of single-name CDS using a two-step procedure, where they firstly estimating the risk-neutral default probability and then calculating the CDS spread by aggregating all claim amounts under various default circumstances. The authors show that the recovery rate is a key factor to determine the equilibrium of the CDS spread. Griffin (2014) investigates the important role of accounting information that reveals the counterparty risk. The author discusses seven contributions of a financial statement that can be used to affect CDS spread. The correlation of a CDS with other markets has been emphasized in terms of exposing credit risks. Typical factors such as excess yields to treasuries, the interest rate, and swap based specific information to explain the variability are used to study the credit risk in Feldhutter and Lando (2008). Other impacts from the country-specific factor, GDP-based ratios and financial market indices on the daily changing of sovereign CDS spreads are examined, see Fender, Hayo, and Neuenkirch (2012). The authors extend a GARCH model to explore the relevant default risk in an emerging market. Friewald, Wagner, and Zechner (2014) study the cross-section effect of spread and the stock returns, where they show that there is a strong positive correlation between the credit risk premia and stock returns.

Many researchers pay enough attention to the impact of the transparency of standardized over-the-counter (OTC) derivatives after the financial crisis. For example, Loon and Zhong (2014) showed that both systematic and counterparty risks can be improved by using central clearing for CDS trading since this operation allows us to estimate the potential default risk of the OTC market. Duffie, Scheicher, and Vuillemey (2015) carefully compared the difference between central clearing after financial reformation in 2013 and prior bilateral clearing mechanisms. The authors simulated the impacts of increasing collateral demand on different aspects of the central clearing parties. This indicates that collateral demand has become a significant factor in CDS investment. Giesecke et al. (2014) have taken account the collateral demand for CDS portfolio, where relevant practical constraints such as initial capital requirement, position limit, and solvency ability are integrated into an optimization model. The authors illustrated their approach by analyzing the performance of a group includes fifteen references. Indeed, the solvency of CDS trading is one of the main concerns for risk-averse investors. The credit risk measurement attracts other researchers from a practical perspective. Iscoe et al. (2012) structured the default loss by combing the non-systematic risk, e.g. credit events, and systematic risk factors. The authors approximated derivative portfolios' value at risk (VaR) and associated expected shortfall using CVaR approach. Despite the efficiency of large-scale computation, other types of restriction such as cardinality constraint have not been studied by their algorithm. In our work, we apply a return structure to express solvency since it is more intuitive and natural from a decision-maker's point of view. To the best of our knowledge, this work is the first attempt to investigate the impact of cardinality constraints on CDS portfolio construction.

3. Models

With a limited budget, people try to maximize profit through investing different financial instruments for a given tolerance of loss. A tradable financial instrument, a credit contract such as CDS, is a legal standard agreement with a monetary value between traders. One difficulty of investment under uncertainty is that parameter estimation errors could affect the optimal decision significantly when using incomplete market information. The process of determining and combining the weights of selected securities is called portfolio selection. A portfolio generally diversifies non-systematic risk, but may not be efficient for credit based asset. Due to the unique structure of the contract, holding a larger number of CDS positions may not efficient enough to hedge default risk. In addition, the portfolio may underperform using diversification. Thus, a trade-off that limits the portfolio size and improves the allocation structure is established in our work. In this section, we propose a CDS portfolio selection procedure to enhance the robustness of CDS portfolios under a dynamic environment. We first present different risk measurements and practical constraints; then, we develop CDS portfolio selection models that are used as key elements of an information updating process.

3.1. Portfolio risk measurement

Suppose that there are n risky asset to be selected. Let r_i be the random return of asset i, the expected return of asset i is μ_i, and the covariance between assets i and j is σ_{ij}, then for a given weight x the portfolio return $r_p = \sum_{i=1}^{n} r_i x_i$, the expected return of the portfolio $\mu_p = \sum_{i=1}^{n} \mu_i x_i$, and the portfolio variance measures the symmetrical risk (Markowitz (1952)) is expressed as:

$$\begin{aligned}\sigma_p^2 &= E\left[(r_p - \mu_p)^2\right] = E\left[\left(\sum_{i=1}^{n} r_i x_i - \sum_{i=1}^{n} \mu_i x_i\right)^2\right] = E\left[\left(\sum_{i=1}^{n} (r_i - \mu_i) x_i\right)\left(\sum_{j=1}^{n} (r_j - \mu_j) x_j\right)\right] \\ &= E\left[\sum_{i=1}^{n} \sum_{j=1}^{n} (r_i - \mu_i)(r_j - \mu_j) x_i x_j\right] = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j\end{aligned} \tag{1}$$

Portfolio variance in (1) captures the correlation information across the horizonal observations. However, normal distribution on the assets' return is not a valuable assumption in practice because many factors have significantly impact on the price and distort the bell curve of the distribution. For example, a black swan event may have an unexpected effect on investor's confidence and cause a sudden jump in a firms price in the market; indeed, such a phenomenon is ordinary in an uncertain environment. In order to take account of this downside risk, the VaR measurement was first introduced by JP Morgan Chase and then standardized by the Basel Committee on Banking in 1996. Subsequently, it has been broadly adopted as a measure of the risk exposure of a trading portfolio in the financial industry. Recently, Artzner et al. (1999) proposed CVaR to overcome the drawback associated with a lack of sub-additivity property in VaR; namely, maintaining diversification when taking more portfolios into account. Given a loss or default probability α, $CVaR_\alpha(x)$ is the average of the portfolio losses that are beyond the portfolio $VaR_\alpha(x)$. Mathematically,

$$\begin{aligned}CVaR_\alpha(x) &= \gamma + \frac{1}{1-\alpha} \int_{f(x,y) \geq \gamma} (f(x,y) - \gamma) p(y) dy \\ &= \gamma + \frac{1}{1-\alpha} \int \left(f(x,y) - \gamma\right)^+ p(y) dy\end{aligned} \tag{2}$$

where γ is the VaR exposure, x denotes the portfolio weights and y denotes the realization of uncertainty, $p(y)$ is the probability of the realization. CVaR reflects the magnitude of the losses right to portfolio VaR, which is more reasonable to represent the loss distribution.

In this study, we consider the two aforementioned risk objectives with regard to portfolio selection in the CDS market based on the following reasons. First, portfolio variance cannot be ignored because it captures aggregative correlations that represent the connection between the CDS market and other markets. This special structure allows us to simplify variance in terms of reducing computational complexity. Second, although both portfolio variance and CVaR are common measures of risk premium, the differences between these two approaches in the CDS market are unclear. From a risk point of view, a CDS portfolio may encounter the uncertainty of credit events and result in a default risk that are hard to be diversified. These considerations encourage us to handle both risks through the trade-off between risk measurements and constraints.

3.2. Cardinality and solvency constraints

From the Markowitz-type framework, we know that the optimal portfolio weight x^* is generated by minimizing the portfolio variance for a given portfolio return requirement. Basic Mean-Variance Optimization (MVO) enables one to incorporate different practical constraints in the selection procedure. We describe the constraints that used for CDS portfolio construction as follows.

• Solvency constraint. In the CDS market, the main concern is the solvency ability. Portfolio values must be no lower than the debt in the worst-case scenarios under either long or short positions for any asset, see Giesecke et al. (2014). For each dollar after enter into the position at initial point, we implement the constraints as follows:

$$\sum_{i=1}^{n}(1+\mu_i)x_i - \sum_{i=1}^{n}C_i\min(x_i,0) = -\sum_{i=1}^{n}(1-C_i)\min(x_i,0) + \sum_{i=1}^{n}S_i\max(x_i,0) \tag{3}$$

where $\sum_{i=1}^{n}(1+\mu_i)x_i$ is the expected portfolio value under weight x. We reduce this amount to $\sum_{i=1}^{n}x_i = 1$ by assuming no investment yield under conservative situation. C_i is the recovery ratio for one dollar investment after default of *ith* CDS occurred, S_i is total periodic discounted payment until the maturity for holding the CDS i. If an investor buys a CDS but he immediately realized there is no default risk, the total cost after any discount must be less than the total portfolio value at initial point. If an investor sells a CDS but default occurred at once, then, after receipt of the recovery money, the total portfolio value must cover the investment loss. This constraint connects the microstructure of the contract with CDS investors. The piecewise linear variables $\max(x_i,0)$ and $\min(x_i,0)$ can be linearized as $z_i^+ \geq x_i, z_i^+ \geq 0$ and $z_i^- \leq x_i, z_i^- \leq 0$, respectively.

• Cardinality constraint, which is used to control the portfolio size via introducing new binary variable y, is expressed as:

$$\begin{cases} lb_i y_i \leq x_i \leq ub_i y_i, & \forall i = 1,\ldots,n \\ \sum_{i=1}^{n} y_i = K & \\ y_i \in \{0,1\}, & \forall i = 1,\ldots,n \end{cases} \tag{4}$$

where K sets the portfolio size, lb and ub are the lower and upper bounds for the portfolio weights. The weight is force to 0 when the asset i is not included into the portfolio, i.e. $y_i = 0$. The second constraint then limits the number of assets of the portfolio. Cardinality constraint can help to reduce transaction costs, simplify the complexity of asset management, and save administrative overheads and costs. Thus, these constraints have been extensively studied for portfolio construction in stock market (Beasley, Meade, and Chang 2003; Lejeune and Samatl i-Paç 2013; Chavez-Bedoya and Birge 2014; Kwon and Wu 2017). When we select portfolios from the CDS market, we find that in some instances allocation just applies to a few assets; in other instances, allocation is across the whole asset set. Consequently, portfolio variance or return may be unsatisfactory for investors. In order to overcome this issue, we control portfolio size to centralize the allocation or diversify across more CDSs. Although this set of constraints increases the computational complexity, we shown that it can improve the CDS portfolio performance from a return/risk perspective in Section 4.

3.3. Cardinality constrained CVaR optimization

The MVO selection incorporates cardinality and solvency constraints into the following optimization model (5–13).

$$\min \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \mu_i x_i \geq R, \tag{6}$$

$$\sum_{i=1}^{n} x_i = 1, \tag{7}$$

$$1 - \sum_{i=1}^{n} C_i z_i^- = -\sum_{i=1}^{n} (1 - C_i) z_i^- + \sum_{i=1}^{n} S_i z_i^+, \tag{8}$$

$$z_i^+ \geq x_i, z_i^+ \geq 0, \forall i = 1, \ldots, n, \tag{9}$$

$$z_i^- \leq x_i, z_i^- \leq 0, \forall i = 1, \ldots, n, \tag{10}$$

$$lb_i \delta_i \leq x_i \leq ub_i \delta_i, \quad \forall i = 1, \ldots, n \tag{11}$$

$$\sum_{i=1}^{n} \delta_i = K, \tag{12}$$

$$\delta_i \in \{0, 1\}, \quad \forall i = 1, \ldots, n \tag{13}$$

where lb, ub are the lower and upper bounds of the proportion to asset i, K is designed portfolio size. We can adjust the lower bound lb to obtain long or short position, e.g. $lb \geq 0$ denotes the short selling is prohibited. The objective (5) minimizes the portfolio risk while constraint (6) satisfies the minimum return requirement R. Constraints (8–10) implement the solvency constraint after linearization and constraints (11–13) restrict the portfolio size. If one asset is not be selected, then $y = 0$ force the weight $x = 0$ in constraint (11), otherwise the weight lies in the boundary interval. In addition, this set of constraints can avoid small or large fraction investment in the portfolio by adjusting the values of lb_i and ub_i for asset i.

Next, we change the objective in (5) by minimizing the expected downside risk and formulate the CVaR selection model as follows:

$$\min \quad \gamma + \frac{1}{(1 - \alpha)S} \sum_{s=1}^{S} z_s \tag{14}$$

s.t. (6)–(13)

$$z_s \geq f(x, y_s) - \gamma, s = 1, \ldots, S \tag{15}$$

$$z_s \geq 0, s = 1, \ldots, S \tag{16}$$

where the expected portfolio loss in (3) is approximated by using the expected loss of the scenario set. $f(x, y_s)$ could be linear or non-linear loss function. For example, $f(x, y_s) = \sum_{i=1}^{n} \bar{r}_i x_i - r_{is}, \forall s = 1, \ldots, S$ where $\bar{r}_i$ is the expected returns of the scenario set. Compared with MVO that only includes the first two moments information, CVaR optimization (Rockafellar and Uryasev 2000) is more flexible since appropriate scenario set can capture higher order moments and across multiregimes, which can generate a more robust portfolio. Moreover, model (14–16) maintains the linear form that allows large scenario set used in practice.

3.4. Dynamic updating process

To test the robustness of the model performance, we describe the dynamic updating procedure as follows.

Step 1: Pick up in-sample spreads start at t.

Set the in-sample length $\triangle t$ and the out-of-sample size $\triangle t_1$.

Generate model parameters using spreads from t to $(t + \triangle t)$.

Step 2: Solve model (5) - (13) and model (14) - (16) simultaneously; then remove the cardinality constraints (11) - (13) and resolve the models.

With the optimal weights, calculate the associated statistics information for periods between $(t + \triangle t)$ and $(t + \triangle t + \triangle t_1)$.

Step 3: If t is less than the end time of the samples,

$t \leftarrow (t + \triangle t)$. Go to Step 1.

Otherwise, Stop.

The purpose of the rolling up testing process is that partial of the spread information are updated, which captures the uncertain feature of the CDS market. In Step 2, we can use periodic out-of-sample returns or prices to represent the dynamical movement of the market. More analysis based on the real data to illustrate this process are shown in next section.

4. Empirical analysis

CDS is an alternative tool to hedge against default loss and enhance the financial health. The growing CDS market reflects the success of this derivative innovation created by JP Morgan Chase in the 1990s. The notional amount of the global CDS market increased approximately 35 times to US 62.2 trillion by the end of 2007 from US 180 billion at the beginning of 1997, see Fung et al. (2008). This increase represents more than 300% annual growth on average. We focus on single-name CDS trading because of its fundamental role in trading activities.

We construct CDS portfolios through different weighted strategy using daily CDS spreads from the US market. These strategies include equal weighting, MVO with and without cardinality and CVaR with and without cardinality constraints respectively. We found that portfolios with moderate size generally have more robust performance in terms of portfolio returns and Sharpe ratios during out-of-sample periods. Since CVaR selection strategy takes higher moments into account (see larger skewness and kurtosis values in Table 1), the associated portfolios have potential advantages for immunizing default risk.

4.1. Data

All corporate CDS are collected from the CDS data service provider - Markit. We collect spreads written on senior debt from 65 firms whose CDS curves are priced on US dollar. Our reference entities cover all sectors that are used in the Standard & Poors (S&P) 500 index. Each firm has protection contracts maturing at different tenors; namely, 6 months, 1–5 years, 7 years, 10 years, 15 years, 20 years and 30 years, respectively. We expand the spreads of the reference entities across different tenors and remove a CDS where one-third of the trading data is missing; then we obtain 501 valid series of CDS spreads and use them to generate the model parameters. We also collect the recovery ratios and discount all periodic payments back to when the CDS portfolios were constructed.

Table 1. Statistics for data summary across sample returns and moment information.

	Min	Percentile (25%)	Median	Average	Percentile (75%)	Max
Daily return	−0.9959	−0.0106	0.0000	0.0352	0.0105	239.2413
Expected return	−0.0002	0.0051	0.0114	0.0352	0.0304	0.4075
Covariance	−0.6227	−0.0005	0.0004	0.0278	0.0024	53.3206
STD	0.0172	0.0911	0.1963	0.4943	0.4939	7.3021
Skewness	−4.6645	4.6635	9.9722	11.6294	17.4281	35.9747
Kurtosis	7.6071	61.3876	146.6847	254.2822	360.1551	1297.1055

The expected returns and covariance matrix are computed using equation $\mu_i = \frac{1}{T}\sum_{t=1}^{T} r_{it}$ and $\Sigma_{ij} = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \mu_i)(r_{jt} - \mu_j)$ separately. The daily return matrix r_{it} includes 1304 trading days across 501 CDSs which covered the periods from 3 January 2005 to 31 December 2009. We estimate the CDS returns directly from historical data because of the high correlations between the CDSs and the equities market. Table 1 summarizes the observations statistics. First, 75% of the daily returns do not exceed 105 basis points (BPS). Moveover, the maximal daily return could be up to 239.2413, indicating that the reward from the CDS investment is probably extremely large. Second, the median of the standard deviation (SD) is 19.63%, whereas the median of the expected return is only 1.14%. The contrast tells us that the symmetrical risk of entering a single CDS may be relatively high. Of the elements in the covariance matrix, 25% are negative, which shows that part of the symmetrical risk can be offset by combining different CDSs into one portfolio appropriately. Finally, it is clear to see that more than 75% of the CDS have large positive skewness ($\geq$ 4.6635) and kurtosis ($\geq$ 61.3876). The general asymmetric and heavy-tailed shape represents a boom in insurance and the concerns of default risk from the underlying entities. Table 1 shows the statistical information of the whole data set. Similar structures, namely large skewness and kurtosis values, exist in the selected sub-data sets during the rolling test procedure.

Two types of test are applied in our calculation. The first evaluates the portfolio performances of the models developed in section 3.3, while the second compares the robustness of the portfolios under uncertain market by the process described in section 3.4. We call the first one a model comparison and the second a rolling test. For the model comparison, we use daily spreads from December 29, 2006 to December 31, 2007 as initial in-samples for portfolio construction. We then collect daily portfolio out-of-sample performance without rebalance during the year of 2008, which was a critical period of financial crisis. For the rolling test, the in-samples are still set as one year's daily spreads. We update the in-samples every 3 months. The minimal portfolio returns during the out-of-sample periods are collected to compare the worst dynamic movements using different strategies.

In our calculation, we observed that covariance matrix generally cannot maintain a convex property when using large CDS sets. One method to overcome this issue is to approximate a nearest positive semi-definite covariance matrix by minimizing the distance between the two matrices in Frobenius space (Higham 1988). Numerical result shows that the conversion process enhances stability for solving the models.

In order to generate a meaningful scenario set for the CVaR model, we apply mvtrnd function in MATLAB to compute the probability of multivariate Student's t distribution, which captures the 'fat-tails' effect by dividing the multivariate normal distribution with a chi-square random value. Although we can apply large re-sampling, for example 1 million, to fit the statistical moments of the historical observation, this approach increases the memory consumption and complexity of solving the CVaR model. In practice, we find that the degree of freedom equals 5 and the scenario number sets as 10,000 matches the in-sample moments information well and stably. Here note that any other notable scenario generation approach that interprets risks better can also be used for the CVaR model. For example, for the given scenario, bootstrapping methods generate a new sample by scaling and unifying the mixed scenario set; then the process is repeated until the designed size is satisfied, see Guastaroba, Mansini, and Speranza (2009) together with other referenced techniques.

The expected portfolio return equals 25% of the maximal return of the CDS set. The size is set at $q = 15$. We allow the lower bound $lb = -0.03$ and the upper bound $ub = 0.25$ because a large trading position may concentrate allocation and increase variance. The loss probability $\alpha = 5\%$ in the CVaR model. Then, we solve MVO and CVaR models by considering cardinality constraints, or not, on an AMD Dual-Core laptop with 2GB of RAM with standard solver (Gurobi 2015). We set enough running time for the solver to guarantee the solution quality is good

Table 2. Statistic about CDS portfolio size and weight allocation under different strategies with and without cardinality constraints.

	MVO		MVO with cardi.		CVaR		CVaR with cardi.	
Weight range	# of CDS	Total Pct	# of CDS	Total Pct	# of CDS	Total Pct	# of CDS	Total Pct
−3%–−0.1%	367	−9.96	3	−0.07	239	−4.41	5	−0.15
0.1%–5%	50	1.03	5	0.19	216	3.75	4	0.12
5%–15%	60	5.21	6	0.63	19	1.41	1	0.05
15%–25%	22	4.72	1	0.25	1	0.25	5	0.97
Total	499	1.00	15	1.00	475	1.00	15	1.00

Pct: Percentage.

enough when using larger CDS sets; for example, the gap between lower and upper bounds should shrink to 5% or less. More analytical details are listed in next section.

4.2. Allocation and performance comparison

We investigate the model comparison with and without cardinality constraints and then conduct a dynamic rolling test using the data procedure described in Section 4.1. After obtaining the optimal weights under different strategies, we compare the portfolios under different performance metrics, including portfolio returns, SD or variance, Sharpe ratios, skewness, and kurtosis, during in-sample and out-of-sample periods. We use five selection strategies namely, equally weighted, MVO with and without cardinality constraint, and CVaR with and without cardinality constraint. The equally weighted strategy that diversifies risk across entire CDS set is used as a benchmark portfolio. Note that the MVO and CVaR models also include the solvency constraint that considers default risk under worst-case scenarios.

We first generate the portfolios using one year's daily spreads starting from December 29, 2006 and calculate daily returns during the year of 2008 with the fixed optimal weights. In order to look inside the portfolios, we count the number of selected CDS and associated total percentage of the budget in different weight ranges in Table 2.

As can be seen without the cardinality constraint, both models have tremendously large number of short positions and CDSs located in the range of [−3%, −0.1%], while the models with cardinality constraint reduce selling actions significantly. Specifically, the investor sells the protection contracts at a rate that is 9.96 times larger than the budget he has by using an MVO strategy. In contrast, only 7% of budget is used to short 3 CDSs if the investor restricts the portfolio size. Similarly, the CVaR strategy suggests short positions for nearly half of the CDSs, while with cardinality constraint only 5 CDSs, which account for 15% of the budget, are sold. In addition, cardinality constraint forces most of the budget to positive ranges; for example, MVO with cardinality constraint uses 63% of the budget to buy 6 CDSs in which each weight is at least larger than 5%. Moreover, we observe that the allocation differences are ordinary during our rolling testing.

Holding a large number of short positions is undesirable because this generates large transaction costs and possible default losses. First, it is clear to see from Table 2 that without cardinality constraint both strategies invest almost all CDS entities with different weightings; for example, 499 and 475 out of 501 CDSs by MVO and CVaR separately. This holding structure increases the complexity of portfolio management in terms of rebalance when taking the transaction costs into account for the rolling test. As aforementioned, CDS investors need to handle spread changing and default risks. Suppose one, or a few, default(s) of the entered short positions are realized in the future: A sudden jump of the spreads due to the default loss leads to poor performance. Controlling portfolio size is an efficient way to reduce a large proportion of negative holdings and thus improve the associated out-of-sample performance by using the trade-off mechanism.

Table 3. Summary statistics of CDS portfolios under different selection strategies.

Strategies under different periods		Mean (BPS)	SD (BPS)	Sharpe ratio	Skewness	Kurtosis	Min (BPS)	Max (BPS)
Equal weighed	In-sample	43.67	147.74	0.29	1.61	9.34	−526.82	788.83
	Out-of-sample	87.03	225.45	0.38	1.06	7.19	−601.61	1181.05
MVO	In-sample	90.05	68.56	1.30	−0.55	1.30	0.00	148.77
	Out-of-sample	−50.91	986.39	−0.05	−0.35	3.92	−3786.16	2954.33
MVO with cardi.	In-sample	90.05	200.73	0.44	1.30	4.50	−301.77	817.95
	Out-of-sample	95.32	375.78	0.25	0.90	4.62	−917.16	1468.91
CVaR	In-sample	90.05	188.26	0.47	2.71	15.27	−284.14	1434.72
	Out-of-sample	97.42	503.02	0.19	2.59	20.44	−1382.18	4235.21
CVaR with cardi.	In-sample	90.05	308.34	0.29	2.92	12.69	−520.28	1694.19
	Out-of-sample	98.91	494.60	0.20	5.06	51.28	−1060.16	5395.77

In-sample: 29 December 2006 – 31 December 2007; Out-of-sample: 1 January 2008 – 31 December 2008.

Then the optimal weights are fixed; namely, there is no rebalancing during out-of-sample testing periods. We calculate portfolio daily returns and list associated statistical properties in Table 3 for both in-sample and out-of-sample periods. Because of diversification and the prohibition of short selling, the equally weighted strategy can usually maintain a stable performance and is a suitable indicator for comparing the developed strategies in our study.

From Table 3, we know that all expected daily returns from optimization models for given in-sample period are 90.05 BPS. This result indicates the return requirement constraint (6) is satisfied and that the expected returns are generally larger than the equally weighted portfolio of 43.67 BPS for the same period. It is clear to see that the expected returns are diverse during the out-of-sample period because the market environment changed significantly in 2008. The MVO expected daily return during the out-of-samples decreases to –50.91 BPS while the CVaR out-of-sample return increases to 97.42 BPS. By incorporating cardinality constraint, the MVO model has been significantly improved to 95.32 BPS and the CVaR strategy has also been improved to 98.91 BPS daily. This improvement can be interpreted in terms of the cardinality constraint efficiently reducing the number of short positions and avoiding a potentially large loss in the future (see the allocation structure in Table 2).

The equally weighted portfolios have relative small SD for both periods because of the diversification effect. The MVO model has lowest SD (68.56 BPS) since it has least selection restriction; however, the SD in the out-of-sample period increases more than 10 times and becomes the largest SD (986.39 BPS). After considered cardinality constraint, the SD of MVO-typed CDS portfolio in the out-of-samples only increases 1.87 times to 375.78 BPS. Analogously, the SD of CVaR portfolio during the out-of-sample period increases 2.67 times to 503.02 BPS while the SD of the cardinality constrained CVaR model only raise 1.60 times to 494.60 BPS in the same period. Note that cardinality constrained CVaR portfolio has a relative larger SD (494.60 BPS) than that from the cardinality constrained MVO model (375.78 BPS). This can be interpreted in the context of the CVaR model holds 5 CDSs, which account for 97% of the budget in the range of [15%, 25%], while the MVO model only distributes 63% of the budget on 6 CDSs in the range of [5%, 15%], see Table 2. From the allocation structure, we know that CVaR models tries to short one group of sub CDS set and concentrate on another group of sub CDS set, which may lead to high-risk performance.

Taken portfolio return and SD together, we also examine the portfolio Sharpe ratios for different periods. The Sharpe ratio measures how much excess return a risky asset can achieve for each unit of volatility in a given period, see Sharpe (1994), which is calculated by $\frac{E(r_p)-r_f}{\sqrt{var(r_p)}}$ where r_f is the return of risk-free assets, e.g. 10 year US Treasury bond. From Table 3, we see that both MVO and CVaR models have high Sharpe ratio values in the in-sample period but sharply plunge during the out-of-sample period. However, the models have been improved after incorporating cardinality constraint, that is, MVO model decreases 103.85% but only 43.18% deduction with cardinality constraint, and CVaR portfolio falls 59.57% while CVaR strategy with cardinality constraint decreases 31.03% in a depression market. The equal weighted Sharpe ratio is more robust during the out-of-

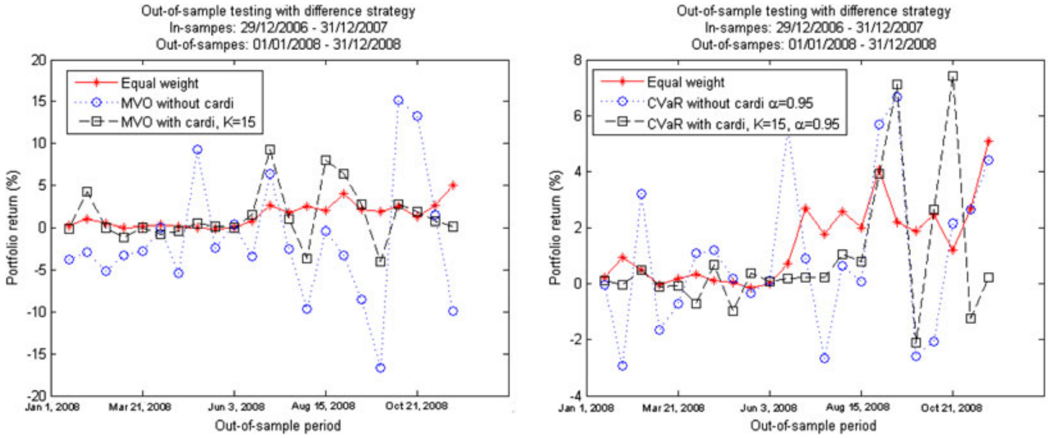

Figure 1. Out-of-sample testing with and without cardinality constraint.

sample period mainly because most of the spreads increased during the financial crisis and the aggregate returns drive the Sharpe ratio up. For example, 420 out of 501 CDSs in this study increase their spreads and 53% of CDSs double their spreads during the out-of-sample period. Therefore, a portfolio with short positions may has large variance and push the Sharpe ratio down.

It is also necessary to compare the higher-order statistics due to the skewed and heavy-detailed distribution of CDS portfolio returns. From Table 3, we see that MVO strategy has smallest skewness and kurtosis as it only considers symmetrical risk. The equal-weighted strategy has moderate values represents the arithmetic average performance of CDS set. The CVaR models, on the other hand, have relatively larger skewness and kurtosis values than those from other strategies, which indicate the CVaR portfolios capture more large returns benefited from the downside market. This situation can be verified by that both minimal and maximal returns of CVaR models are better than those value from MVO and equal-weighted strategies during the out-of-sample period. For example, without the cardinality constraints during the out-of-sample period, the minimal return of CVaR model is 1.74 times smaller than that from MVO portfolio and the maximal return of CVaR model is 1.43 times larger than maximal returns of MVO portfolio. The same trend can be verified using the model with cardinality constraint in out-of-samples.

We then sketch the evolution of portfolio out-of-sample returns in Figure 1. We collect the returns bi-monthly and check the robustness for the optimal weights. Figure 1 clearly shows that cardinality constrained portfolios have more stable performance. From the left-hand side of Figure 1, we see that MVO-type portfolio has more volatility than that from MVO model with cardinality constraint after mid-June, 2008. For example, the portfolio return from MVO model decreases 16.64% compared with only a 4.01% deduction by cardinality constrained MVO model from August to October in 2008, a main stage during financial crisis. Most instances of MVO model underperform the equal weighted portfolio at the same period, while the movement of cardinality constrained MVO model is closer to the path of the equally weighted strategy. A similar trend is shown on the right-hand side of Figure 1, although the volatility range is much smaller than that of the MVO models. The CVaR model faces a greater number of sharp slashes; for example, the periods in January, June and October of 2018, respectively. Meanwhile, the cardinality constrained CVaR model has fewer, and relatively smaller, jumps.

4.3. Robust rolling test

We then dynamically update the model parameters by replacing a portion of the in-samples and build the worst bound of return evolution for the rolling test. For example, starting from 3

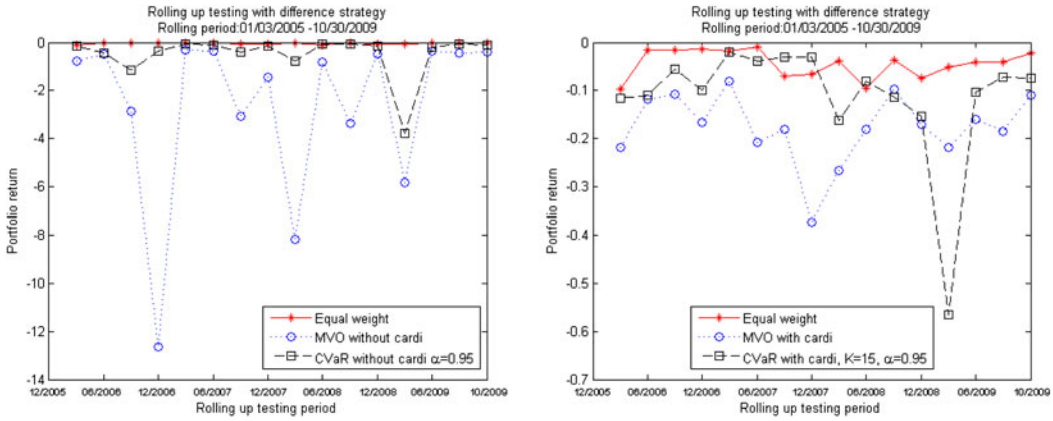

Figure 2. Rolling testing across MVO and CVaR selection.

January 2005, we use one year's daily spreads to obtain the model weightings and pick the worst portfolio return in the first quarter of 2006. Then, we use spreads between the second quarter of 2005 and the first quarter of 2006 as updated in-samples and calculate the worst portfolio return in the second quarter of the year 2006. We repeat the rolling process every 63 trading days, with 21 trading days per month, up to the end of 2009. We found that without the cardinality constraints, it is common that larger amount of short positions appear in the portfolio allocation when different rolling periods are tested. These short positions fluctuate and magnify the volatility significantly. However, the models with cardinality constraints force most of the budget onto long positions and avoid the bad CDS, which therefore reduce the default risk. To better understand the portfolio performance, we display the worst boundary of the movements in Figure 2, which compares the selection strategies under the same restriction.

Figure 2 depicts boundary comparison across different selection strategies. From left to right we see that the volatility of movement shrinks 20 times after incorporating the cardinality constraints. From the left-hand side of Figure 2, it is clear to see that the lower bounds of the rolling returns by the CVaR model are better than those of the MVO model; for example, –5.8389 vs –3.7768 under the worst-case scenario during March 2009. Despite cardinality constrained CVaR portfolio being 35% worse than the rolling return of MVO model in March 2009 from the right-hand side, we can still see that the fluctuation of the black-broken line to the red line is much smoother than that between the blue-dash line and the red line in other periods. Therefore, Figure 2 indicates that the CVaR models have better rolling lower boundary performance compared with associated MVO models.

Overall, the analysis exhibits insights about CDS portfolio performance driven by default risk. It turns out that cardinality constraints improve allocation structure by reducing the number of short positions. We also show that the portfolios using cardinality restriction have stable performance in terms of out-of-sample testing and dynamic rolling-up testing. Since more long tail information can be captured by the CVaR risk measurement, associated portfolio selection model is more suitable to be used in CDS market, which can be verified based on our comprehensive comparison results.

5. Conclusions

We propose a CDS-based portfolio selection model by integrating different risk measurements and solvency and cardinality constraints together in this article. The solvency constraints can isolate the default risk under worst-case scenarios. Cardinality constraints are used to limit the short positions in the allocation structure. Base on the comparison results generated by different strategies, we found that cardinality restriction is an efficient way for improving portfolio out-of-sample performance. In addition, CVaR-type portfolios generally have relatively better rolling up results than those from mean-variance and equally-weighted strategies. This work could be further studied by incorporating other practical consideration such as transaction costs constraints at a large scale level of a derivative set. Other risk measurements, e.g. tracking errors between the portfolios and associated CDS indices, could also be explored due to more transparent information available from CDS market.

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