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A Descriptive Framework For The Problem-solving Experiences Of Physics Students

Sherry Savrda
University of Central Florida

A DESCRIPTIVE FRAMEWORK FOR THE PROBLEM-SOLVING EXPERIENCES OF PHYSICS STUDENTS

by

SHERRY L. SAVRDA
B.S. University of Central Florida, 1986
M.S. University of Central Florida, 1991

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Major Professor: David N. Boote

ABSTRACT

This study investigated the applicability of a general model of physics students' problem-solving experiences that suggests there are four dynamic factors affecting the problem-solving process: categorization, goal interpretation, resource relevance, and complexity. Furthermore, it suggests an overarching control factor called stabilization, which describes the inter-relatedness of the other factors over the problem-solving process. Think-aloud protocols of problem-solving experiences were used to investigate the model. Results of the study showed that conceptual resources had a significant impact on the success of problem-solving attempts. Participants who exhibited a lack of understanding of physics concepts were less likely to check their work, use diagrams effectively, set subgoals, or to use geometric or trigonometric resources, and were more likely to use a formula-driven search for a solution than those who exhibited evidence of conceptual understanding. However, conceptual understanding did guarantee problem-solving success. Mathematical and procedural knowledge was also seen as important.

While many of the specific observations were consistent with the existing literature, the model provides an alternative framework with which to understand and synthesize those observations. The model was shown to be partially successful in describing participants' problem-solving experiences. Categorization, resource relevance and goal interpretation were supported to varying degrees; however, there was less evidence to support the construct of complexity. Determination of evidence for stabilization was guided by a working definition based on the participants' search for a stable understanding of the problem. Implications of these results for research and practice were noted.

To Steve: Thank you for bringing joy to my life.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE: INTRODUCTION

The development of competence in problem solving has traditionally been a primary goal of physics instruction. Physics instructors would argue that conceptual understanding is a necessary prerequisite for success in problem solving, yet research has shown that students generally do not to call on conceptual knowledge as they attempt to solve unfamiliar physics problems. Instead, students tend to resort to trial and error equation-matching, looking for equations that contain the unknown quantity and attempting to fit the equation to the problem at hand (Heyworth, 1999; Savelsbergh, Jong, & Ferguson-Hessler, 2002; VanLehn, 1998). It has also been shown that students frequently refer to examples while solving problems, but that they do not consider the differences between the examples and new problem situations, and often apply equations from the examples incorrectly. (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi & VanLehn, 1991). Furthermore, although students in general have sufficient mathematical resources learned in their calculus and/or algebra-trigonometry courses, they often fail to recognize those resources as being relevant to the solution of physics problems (Bassok, 1990; Bassok & Holyoak, 1989; Cui, Rebello, & Bennett, 2005; Ozimek, Engelhardt, Bennett, & Rebello, 2004; Tuminaro & Redish, 2003).

While there has been a considerable amount of research done in the area of physics problem solving, most of this research has focused on the identification of various processes and strategies students use in the solution of problems (Chi, Glaser, & Rees, 1982; Dhillon, 1998; Larkin, McDermott, Simon, & Simon, 1980a; Savelsbergh et al., 2002), or on the differences between expert and novice problem solvers (Chi, Feltovich, & Glaser, 1981; Finegold & Mass, 1985; Larkin et al., 1980a). Studies such as these are descriptive in nature, seeking to identify the cognitive processes students undertake as they solve problems. Other studies have focused on the development of computer models that emulate the problem-solving process (Larkin, McDermott, Simon, & Simon, 1980b; Ross & Bolton, 2002; VanLehn, 1998). The models developed to date, particularly the computer models, tend to focus on one aspect of the problem-solving process, while de-emphasizing other aspects. Few of the models discussed in the physics education literature to date have considered the motivational or attitudinal aspects of problem solving. While work has been done in these areas (Elby, 1999, 2001; Hammer, 1994, 2000; Redish, Saul, & Steinberg, 1998), it is only recently that attitudes and personal perspectives have been directly investigated in relation to problem solving (Cummings & Lockwood, 2003). What is needed is a general model of students' problem-solving experiences, one which includes not only the strategies students apply but also the resources and personal perspectives they bring to the process.

Further evidence on the need for additional investigation into physics problem solving comes from a resource letter published in the *American Journal of Physics* (Hsu, Brewe, Foster, & Harper, 2004). This letter, published as a "guide to the literature on research in problem solving, especially in physics" (p. 1147), lists 109 references on problem solving. However, only 17 of those references were published in 2000 or later, and of those, only 9 were physics-specific. Additionally, only 4 of those 9 were reports on research; the remainder were reports on instructional strategies, computer applications or tutorials, or student workbooks. While some research has appeared in the literature since the publication of the resource letter, much of the work has been focused on specific aspects of problem solving, such as transfer (Cui et al., 2005;

Ozimek et al., 2004), the use of multiple representations (Leone & Gire, 2005; Rosengrant, Heuvelen, & Etkina, 2004, 2005), problem context (Park & Lee, 2004), or in the use of specific instructional strategies (Grossman, 2005). There is a need for further investigations into general problem-solving models, and especially into the applicability of alternate models of problem solving.

McGinn & Boote (2003) have proposed a model of mathematical problem solving that might be useful for representing students' experiences in solving physics problems. Their model identifies four primary factors that influence the problem-solving process: categorization, goal interpretation, resource relevance, and complexity. The attractiveness of the McGinn and Boote model lies in the fact that the factors that make up the model have, for the most part, been identified in the literature as being significant to the problem-solving process. The four primary factors are brought together with a fifth factor, stabilization, which describes the inter-relatedness of the other four factors over the problem-solving process. Stabilization is a dynamic process in which the problem-solver seeks to reach a balance, or congruence, among the other factors, until a stable understanding of the problem is achieved. Unlike linear models of problem solving, the stabilization model attempts to represent the continually shifting evaluations of factors undertaken by the problem solver.

The purpose of this study was to investigate the applicability of the McGinn and Boote (2003) model of problem solving to the experiences of students enrolled in introductory, calculus-based college physics courses. The primary research question was "To what extent does the stabilization model describe physics students' problem-solving experiences?" Investigation of this question required exploration of the secondary questions "What are the basic processes that physics students undertake as they attempt to solve physics problems?" and "What resources do students bring to the problem-solving process?" The goal was to develop a clearer understanding of where and why students experience difficulties when solving physics problems. Understanding the actions taken by students in their problem-solving processes is a critical component in the development of models to describe those processes. In the sections that follow, literature relevant to this study will be reviewed, and case studies will be presented. Analysis of the protocols from the study will outline the problem-solving processes observed. The results of the application of the stabilization model will be presented, and the strengths and deficiencies of the model will be discussed.

CHAPTER TWO: LITERATURE REVIEW

The development of problem-solving skills is an important goal of introductory physics instruction. As a result, a considerable body of research has been undertaken in an effort to understand the problem-solving process. Because of the highly mathematical nature of most physics problems, much of the research in physics problem solving has common historical roots with mathematical problem-solving research. In the past twenty years, however, physics education research has matured into a legitimate specialized field, with research programs focused on the unique combination of conceptual, intuitive and mathematical skills that comprise problem solving in physics.

Research in physics problem solving can generally be divided into three broad categories: expert-novice studies, which attempt to characterize the differences between expert and novice problem-solving processes; pedagogical studies, which focus on instructional strategies for improving problem-solving skills; and computer-based studies, which attempt to model the problem-solving process through computer algorithms. There is, of course, considerable overlap among these areas. More recently, in recognition of the role of conceptual understanding in successful problem solving, researchers have begun looking at the relationships between concepts and problem solutions.

This review will begin with a synopsis of both historical and recent work in the area of mathematical problem solving that has particular relevance in the domain of physics. It will then move to studies that specifically targeted physics problem solving. This discussion will include various aspects of both expert-novice studies and problem-solving models. Next, more recent work investigating the role of conceptual understanding will be connected to the previous studies. The review will include a discussion of the limitations of the methodologies typically used to investigate problem-solving processes. Finally, the McGinn and Boote (2003) stabilization model will be described, with a discussion of how their model relates to, as well as differs from, the models used in earlier studies.

Mathematical Problem Solving

Research into problem solving, as well as the emphasis on problem solving in the disciplines, can be traced back to the classic work *How to Solve It* (Pólya, 1945). Pólya, through a self-study, identified a number of heuristics, or "mental operations useful for the solution of problems" (p. 2). Additionally, he identified a four-step problem-solving process general enough to apply to any discipline, with the use of heuristic strategies integrated throughout the problem-solving process. His model suggested that if one is to be successful in solving a problem, one must first understand the problem. Strategies such as drawing figures, the use of appropriate notation or symbolism, and separating the conditions of the problem are used at this point. Second, the problem solver should find the connection(s) between the data given in the problem and the unknown, eventually obtaining a plan for solution of the problem. It is in this step that heuristics play the most prominent role. Pólya suggested such strategies as finding related problems, breaking the problem into smaller steps, choosing a different unknown to solve for, or temporarily changing the conditions of the problem (for example, looking at extremes). Third, the problem solver carries out the plan developed in step two. Finally, the solution is examined for accuracy, possible alternative solutions, and possible applicability to other problems.

The promise of Pólya's work was that if particular strategies used by experts within a discipline could be identified, those strategies could be explicitly taught to students. This would conceivably reduce the time needed for students to become proficient problem solvers. Unfortunately, the promise of general heuristics was not fulfilled. Despite many attempts, general heuristics as a means to improve problem-solving performance have not been found to be easily teachable or transferable. According to Schoenfeld (1985a), there are several reasons for the lack of success. Primary among these is the fact that heuristics alone cannot assure problem-solving proficiency. Learners must have an adequate domain-specific knowledge base, as well as a certain level of sophistication in the use of that knowledge, before heuristics become a useful tool. In other words, a problem solver must become reasonably proficient within a domain before he or she can make effective use of heuristics. In addition, the learner must possess adequate control mechanisms to guide the problem-solving process (Sweller, 1983). These are the metacognitive skills that provide the problem solver with the ability to recognize when to abandon a technique that is proving fruitless, and when to continue with a process that is burdensome but which may well lead to a solution (Dhillon, 1998). An additional factor contributing to the difficulty of teaching general heuristics is that heuristics are complex processes. The various heuristics as used by proficient problem solvers are not characterized in sufficient detail to make them directly useful to students (Schoenfeld, 1985a).

It should be noted that the statement concerning the limited success in teaching heuristics does not imply complete failure. For example, Schoenfeld (1985a) carried out a series of studies on the teaching of heuristics to mathematics students. However, he framed his studies to take into account the limitations he identified for the teaching of heuristics. That is, he taught the techniques to adult learners with mathematical backgrounds that included at least introductory calculus, and who were working on problems that were within their grasp given their mathematical background. Using a limited selection of five heuristic strategies, he found that the direct teaching of *domain-specific* strategies did result in modest but significant increases in problem-solving ability. Likewise, Oladunni (1998) looked at the teaching of both heuristics and metacognitive strategies in a comparison study. He found that while metacognitive strategies provided the largest improvement in problem-solving skills, the teaching of heuristics did result in modest gains. One interesting result from Oladunni's study is the fact that high-ability students, as measured by a pre-test, realized the greatest gains in problem-solving skill. This supports Schoenfeld's assertion that one must already be moderately proficient within a domain to make effective use of heuristics.

Analogical techniques have also been an important area of research in mathematical problem solving (Cummins, 1992; Gick & Holyoak, 1980; Schoenfeld & Herrmann, 1982). Analogical problem solving involves either comparing a new problem to other problems one already knows how to solve, or recognizing a new problem as a member of a category of problems that one is familiar with. For example, in algebra, a problem solver might recognize a problem about the time needed to fill a vat with liquid as being analogous to a problem about the time to travel to a given destination: Both problems belong to the category of rate problems. Recognizing the category to which a problem belongs enables the problem solver to recall a general solution strategy, or schema, which can be applied to the new problem. The ability of students to identify the category to which a problem belonged has been shown to be dependent on the level of processing students are asked to undertake while reading the problems prior to categorization (Cummins, 1992). Students who were asked to make comparisons between problems were generally able to sort new problems according to problem structure, and to recognize a new problem as belonging to a particular category. In contrast, students who were only asked to analyze individual problems, or were given no instructions other than to read the problems, tended to sort problems according to surface features such as whether the problems involved moving objects or money. Students who made appropriate categorizations were also able to successfully solve the problems at a higher rate. Specific training in heuristic techniques and in identifying problem categories also seems to improve the ability of students to make appropriate categorizations (Schoenfeld & Herrmann, 1982). These results suggest that the use of analogy is a powerful strategy in problem solving. The problems used in the cited studies of analogy in mathematics are typical of those seen in an introductory algebra or geometry course. As a result, these studies do not address the issue of analogical comparison and transfer of similar solution strategies across domains.

The use of analogical examples is a strategy frequently seen in the problem solving attempts of physics students (Chi et al., 1989; Chi & VanLehn, 1991; Cummins, 1992; VanLehn, 1998). Students taking introductory physics courses, particularly at the college level, are also expected to bring with them a number of mathematical skills necessary for the solution of problems in physics. These two observations make the question of whether the skills and strategies learned in mathematics courses readily transfer to physics problems of particular interest (Bassok, 1990; Bassok & Holyoak, 1989; Cui et al., 2005; Ozimek et al., 2004). Bassok and Holyoak studied students who learned problem categories either through algebra or physics; for example, arithmetic sequences or series. They found a high degree of transfer to physics problems when students learned problem categories and solutions through algebra instruction, but very little transfer when the problem categories were learned in the context of physics problems. The likely reason for this unidirectional transfer, according to Bassok and Holyoak, is the result of content-specific derivation of learned solution techniques in a physics context. The difference between the types of variables used in physics problems and those used in algebra problems is also believed to affect transfer. Many of the variables encountered in introductory algebra are extensive, involving only one measured entity. Extensive variables, such dollars, meters, or time, are conceptually less difficult than intensive variables, which involve more than one entity combined as a single unit. While intensive variables are certainly used in other domains, they are typically explicitly stated as rates, such as in miles per gallon or dollars per year, rather than given specialized names such as velocity or acceleration as they are in physics. As a result, problems involving intensive variables are perceived to be more difficult than those involving extensive variables, even when the solution technique is identical. Transfer of solution techniques between problem categories learned with an extensive variable to a similar problem involving intensive variables is lower than that between problems with the same type of variable.

One possible explanation for the lack of transfer between physics and algebra that was not noted by Bassok and Holyoak (1989) and Bassok (1990) lies in the way in which the problems used in the studies were stated. The authors provide several examples of the physics problems used in their studies. As an example, consider the following arithmetic sequence problem in a physics context:

"What is the acceleration (increase in speed each second) of a train, if its speed increased uniformly from 15 m/s at the beginning of the 1^{st} second, to 45 m/s at the end of the 12^{th} second?" (Bassok, p. 529)

This problem is stated much like a sequence problem would be stated in an algebra text. It is not, however, in the format that a typical physics textbook would use. This same problem might be stated in a physics text as follows:

What is the acceleration of a train that starts from a speed of 15 m/s and reaches a speed of 45 m/s after 12 s?

It could be argued that the format of the former problem informs students who had learned solution techniques in an algebra context of the fact that the problem, although in an unfamiliar context, was a sequence problem. The parenthetical comment "increase in speed each second" in particular could be interpreted as a hint to students to use a sequence technique. In fact, hints have been found by other researchers to have a strong positive influence on the rate of transfer between analogical problems, even between different contexts (Anolli, Antonietti, Crisafulli, & Cantoia, 2001; Gick & Holyoak, 1980; Perfetto, Bransford, & Franks, 1983). Physics students, familiar with problems stated in the form of the latter example, would be less likely to recognize a sequence problem stated in typical algebraic format:

"Jaunita went to work as a teller in a bank at a salary of $12,400 per year and received constant yearly increases coming up with a $16,000 salary during her 13^{th} year of work. What was her yearly salary increase?" (Bassok, 1990, p. 529)

Here, the format of the problem is so unlike the analogous physics problem previously stated that is seems unlikely students who learned sequences only within the domain of physics would find a cue within the problem to inform them that it is equivalent in form to an acceleration problem. The context of the learned solution technique, in combination with the differences in problem statement format, suggests that transfer between problems would be unlikely. It would be interesting to see if problem statement format had a direct effect on the level of interdomain transfer of solution techniques, if the high rates of transfer noted by Bassok would be seen if the physics problems used were stated in forms more typical of problems in physics texts, or if hints to physics students resulted in higher transfer rates to problems outside the domain of physics; however, no such studies were found in the literature.

More recent studies have looked at transfer between trigonometry (Ozimek et al., 2004) or calculus (Cui et al., 2005) and physics. In a study of second-semester engineering physics students, Cui et al. found that the students in their study were able to solve calculus problems without difficulty, but were unable to solve isomorphic physics problems. The researchers noted that the students' primary difficulties were in setting up the problems, such as in choosing limits or appropriate variables of integration, rather than with the calculus per se. They also noted that students were unsure about what criteria they should apply to determine when calculus should be used in a physics problem. Ozimek et al. found similar results in transfer from trigonometry. In their study, students were found in general to have retained their trigonometry knowledge, but were unable to apply it to isomorphic problems in physics. However, Ozimek et al. also looked at transfer from two "contemporary" perspectives; one which viewed transfer in light of the students' ability to apply prior knowledge to learn to solve problems in a new context; and the other which viewed transfer as an ability to recognize similarity relations between the prior knowledge and the new context. When applying these perspectives of transfer, the researchers found that there was evidence of transfer in the areas of geometrical and functional applications of trigonometry. Both of these studies suggest that students need considerable support and scaffolding in order to successfully apply knowledge from their mathematics courses to physics courses. Just as earlier studies showed that hints can greatly increase the transfer of knowledge between analogical problems across different contexts (Gick & Holyoak, 1980; Perfetto et al., 1983), scaffolding during instruction of problem-solving instruction can act as the "hint" to increase transfer of knowledge from mathematics to physics.

Physics Problem Solving

Research in physics problem solving began in earnest in the early 1980s. At that time, much of the work focused on expert-novice differences (Chi et al., 1981; Finegold & Mass, 1985; Larkin et al., 1980a). These studies were not designed to develop a model for problem solving, but rather to identify those factors that distinguished expert problem solvers from novices. Among the most robust and oft-repeated results of the expert-novice studies is recognition of the tendency of novices to categorize physics problems according to surface features, first noted in the seminal work of Chi, Feltovich and Glaser (1981). When asked to group a selection of problems according to similarity of solution, novices tend to sort the problems according to the visual similarities of tangible objects in the problem statement. Focusing on surface features resulted in categories such as "pulley problems" or "incline problems." These results have been replicated in various forms by a number of researchers (Cummins, 1992; Jong & Ferguson-Hessler, 1986; Savelsbergh et al., 2002; Snyder, 2000).

When asked how they might approach the solution to a problem, novices respond only in vague, general statements, and tend to list equations they might try to apply to the problem (Chi et al., 1981). This suggests that the novice's focus is on the unknown quantity and the equations that will provide a value for it (Savelsbergh et al., 2002). Further evidence to support this view of novice problem solving comes from analysis of problem-solving protocols. Novices' attempts at problem solutions are guided by search processes, as they work backward from the unknown solution to the known quantities. They often attempt a one-step solution, selecting an equation because it contains the unknown quantity. Other unknowns within the chosen equation become new target quantities, with new equations chosen to find those values. This process of equation selection and solution continues until a solution to the problem is found (Chi et al., 1982; Dhillon, 1998; Heyworth, 1999; Larkin et al., 1980a; Savelsbergh et al., 2002). At each step after an equation is chosen, the novice must reevaluate and decide how to proceed. This equation-driven solution process often takes place bereft of any understanding of the physical principles underlying the problem situation. Novices' problem-solving schemata are thus surface-feature-oriented and equation-driven. To the novice, the equations *are* the knowledge (Larkin et al., 1980a). This is in contrast to the expert's view, in which the principles are the knowledge, and equations are simply a way to represent that knowledge.

Knowledge of principles acts in other ways to guide the experts' problem-solving processes. Experts were found to categorize physics problems according to the underlying physics principles that would used to solve the problem, resulting in categories such as "Newton's second law" or "conservation of energy" (Chi et al., 1981; Chi et al., 1982). Categorization of problems according to physics principles is indicative of the central importance that principles play in the problem-solving processes of experts (Savelsbergh et al., 2002). Problem solving begins with a qualitative analysis of the states and conditions implicit in the problem statement, a step that novices rarely take (Chi et al., 1982). These conditions act as cues, suggesting the principle applicable to the problem situation. Most importantly, *how* to solve the problem is encoded with categorization, so that recall of the principle calls up the entire procedure for obtaining the solution (Chi et al., 1981; Larkin et al., 1980a; Savelsbergh et al., 2002). The automated sequences and principle-indexed knowledge form a schema that guides the problem-solving process. Once initiated, the solution process proceeds with a forward-working strategy, moving from known quantities to the unknown solution. Experts select equations not because they contain the unknown quantity, but rather because all quantities but one are known. Knowing that the intermediate quantity is attainable, they proceed as if it is already known, repeating the process until the desired quantity is reached (Chi et al., 1982; Dhillon, 1998; Larkin et al., 1980a).

Conceptual Understanding and Problem Solving

The results of expert-novice studies have provided a general description of the characteristics of both experts and novices. In particular, the studies showed the importance of understanding the physical principles underlying a problem. Recognition of the importance of principles has led to investigations of the relationships between conceptual understanding and problem solving.

The importance of conceptual knowledge is clearly seen in the results of those studies targeting the differences between good and poor novice problem solvers. Researchers have suggested that there are three general areas that distinguish these two groups. First, good and poor novice problem solvers differ in the strategies they use. Poor problem solvers tend to use a formula-driven, working-backwards approach, much like the novices in expert-novice studies (Heyworth, 1999; VanLehn, 1998). Second, good problem solvers, like experts, use a qualitative description of the problem to guide the solution process (Chi et al., 1989; Finegold & Mass, 1985; Heyworth, 1999; Jong & Ferguson-Hessler, 1986; Robertson, 1990). Finally, novices who are better problem solvers have much better conceptual understanding than do poor problem solvers (Heyworth, 1999; Jong & Ferguson-Hessler, 1986; Kim & Pak, 2002; Robertson, 1990; VanLehn, 1998). According to Schoenfeld (1985a), problem solvers who possess poorly structured concept systems will interpret a problem situation to fit their inaccurate conceptions, focusing on obvious surface features while ignoring important facts and/or properties related to the problem situation. Clearly, conceptual understanding is a critical component in successful problem solving. However, increased conceptual understanding does not automatically translate into increased problem-solving ability (Hoellwarth, Moelter, & Knight, 2005; Hung & Jonassen, 2006). A study carried out at California Polytechnic State University found that students in an active-learning-mode class that emphasized concepts had far higher scores on a standard measure of conceptual understanding than their counterparts in a traditional lecture-lab course, but scored on average at or below the level of the students in the traditional course on quantitative problem-solving exams (Hoellwarth et al., 2005). While it could be argued that the difference in emphasis for the two courses was in large part the determining factor in these results, the results also suggest, as noted by Hoellwarth et al., that for maximum problem-solving proficiency students should be taught both concepts and explicit problem-solving skills. It should also be noted that it is possible for novices to memorize algorithms and equations with little conceptual understanding and be reasonably successful problem solvers, a fact that is evident in the results of several studies (Heyworth, 1999; Kim & Pak, 2002; Robertson, 1990; VanLehn, 1998). However, novices who rely on algorithms for problem solving have been found to be limited in their ability to solve transfer problems which are structurally different yet conceptually similar to problems they were previously successful in solving (Chi et al., 1989; Robertson, 1990; VanLehn, 1998).

Novices with incomplete or inaccurate integrated knowledge structures can often make conceptually correct statements about a problem, even as far as selecting the correct physics principle to apply to a given problem. During the problem-solving process, novices will then rely on incorrect conceptions or everyday intuition to solve the problem, rather than thinking through the implications of the application of the chosen principle (Robertson, 1990; Schoenfeld, 1985a). There are two possible explanations for this behavior. One is simply that the conceptual knowledge is incomplete, leading to incorrect application of the chosen principle (Hammer, 2000; Kim & Pak, 2002; Robertson, 1990). This explanation supports what Maloney and Siegler (1993) have called *conceptual competition*, in which a novice possesses more than one alternative understanding of physical phenomena at a time (see also Mildenhall & Williams, 2001). These alternative conceptions coexist and compete with each other in the cognitive structure of the learner. One conception or another might be used in a given problem, depending on the problem solver's extent of understanding and cues within the problem statement. The other possible explanation is that novices possess sufficient conceptual understanding, but that understanding is not supported by adequate procedural knowledge. There is some evidence that a significant portion of the difficulties that arise in problem solving are a result of deficiencies in applying knowledge already possessed by the problem solver (Chi et al., 1989; Finegold & Mass, 1985; Hammer, 2000; Kim & Pak, 2002; Maloney, 1994). The presence of knowledge in memory does not mean that one can access that knowledge and use it to solve a problem (Perfetto et al., 1983). Declarative knowledge about a domain must be coupled with adequate procedural knowledge to facilitate successful problem solving.

Researchers investigating expert-novice differences have suggested that students generally have the declarative knowledge needed to solve introductory physics problems (Chi et al., 1989; Chi et al., 1981; Savelsbergh et al., 2002; Sherin, 2001). They can, for example, state Newton's second law and can make inferences about related physics concepts (Chi et al., 1989), and might be able to make some basic statements about general conditions for applicability of the law. What novices are lacking is a deep understanding of the principles, represented by a knowledge structure rich in connections between related information that makes the principle applicable to the solution of a problem. A problem solver must know more than just the statement of a principle; he or she must also know how that principle relates to other concepts. In short, he or she must know the structure of the discipline (Dufresne, Gerace, Hardiman, & Mestre, 1992). Dhillon (1998) called this aspect of domain knowledge structural knowledge. Just as important as structural knowledge is situational or procedural knowledge (Chi et al., 1989; Chi et al., 1982; Dhillon, 1998; Jong & Ferguson-Hessler, 1986; Larkin & Simon, 1995; Robertson, 1990; Savelsbergh et al., 2002). This procedural knowledge allows the problem solver to make productive use of the features of the problem, knowing which physics principles are valid, conditions of applicability of those principles, how to connect mathematics to the concept and the problem at hand, and how to detect errors. Procedural knowledge is highly domain-specific, although it may be integrated with general mathematical procedures. It is this integrated declarative and procedural knowledge that novices frequently lack.

The approach taken to solve a problem depends in large part on the extent of one's structural and procedural knowledge. The procedural knowledge the problem solver has available is based on previous experiences in similar problems (Savelsbergh et al., 2002). Since previous problem solutions form the basis of the existing procedural knowledge, the novice is placed in a situation in which the more successful past problem-solving attempts have been, the more likely that the problem solver will possess the procedural knowledge needed to solve a new problem. Furthermore, good problem solvers have the metacognitive skills to recognize when they don't understand, and are likely to follow that admission of lack of understanding with an appropriate search for understanding (Chi et al., 1989). Novices often fail to recognize when they don't understand, either because they lack the metacognitive skills to monitor their own understanding, or because they do not recognize the need for conceptual understanding in the problem-solving process (Elby, 1999; Hammer, 1989). Unfortunately, the same skills that are needed to recognize success (or failure) in problem solving are the same skills needed to produce a correct problem solution. If a problem solver lacks the ability to correctly solve a problem, he or she is also unlikely to recognize when a solution is incorrect (Dunning, Johnson, Ehrlinger, & Kruger, 2003).

Part of the procedural knowledge required for successful problem solving in physics is knowledge of the use of multiple representations of physical situations, including but not limited to verbal, mathematical, graphical, and pictorial representations. Typical introductory courses in physics include instruction in the use of these representations as an aid to conceptual understanding and problem solving. Domain-specific representations such as free-body diagrams, energy bar charts, and state diagrams are of particular interest in understanding how physics students solve problems. Recent studies suggest that students who make appropriate use of diagrammatical representations have a greater likelihood of success in problem solving (Leone & Gire, 2005; Rosengrant et al., 2005), but that use of a physically and/or conceptually incorrect diagram results in a greater chance of error than no use of diagrams at all (Rosengrant et al., 2004). If students do not recognize the relevance of the diagram to the physical situation in the problem statement, they are likely to consider the diagram as nothing more than a short-hand method of keeping track of known information rather than a tool for connecting the verbal problem statement to the mathematics needed for the solution. The problem solver must have the appropriate procedural skills to make use of multiple representations of physical situations, including knowledge of how to construct a good diagram, confidence in computational processes, and the processes needed to connect the various representations to physics principles (Larkin & Simon, 1995).

The mathematics and physics literature that has been discussed to this point has provided a wealth of knowledge about problem-solving processes. The differences between expert and novice problem solvers have been delineated, as have some of the differences between good and poor novices. The importance of conceptual understanding has been emphasized and linked to the procedural knowledge needed for success in problem solving. Despite the fact that these studies have provided a wealth of information on problem solving, the studies have shortcomings that limit the applicability of the information provided. While the same statement could be made about virtually all investigations of human behavior, it is important to recognize those limitations. In the next section, some of the strengths and shortcomings of the methods used in previous studies will be addressed.

Methodological Limitations

Although the problem-solving research that has taken place to date has provided us with important information about problem-solving processes, there are limitations to the applicability of the studies, primarily as a result of the methodologies used. Because of natural variability in human behavior, no conclusions drawn from problem-solving studies will have universal applicability. Understanding the limitations of the various methods used to study problem-solving behavior will help assure that the information obtained from problem-solving research can be used to the greatest possible benefit.

Early studies in problem solving involved the use of introspective techniques, in which the researcher investigated his or her own thought processes. The primary limitation of self-study is that one can never be sure that one's problem-solving behavior is typical. It is also not possible to know for sure that one is appropriately tracing the important steps in the thought process. According to modern information-processing theory, the only thoughts available for processing are those thoughts that are attended to in working memory, under the immediate attention of the thinker (Ericsson & Simon, 1993). In problem solving in particular, many processes are automated, and take place without access through working memory. This is the type of automation, for example, that causes an experienced driver to go through the motions of driving without conscious attention to the myriad small actions that make up the process of operating a vehicle (Bereiter & Scardamalia, 1993). These automated processes limit the problem-solving processes that can be related to others, either verbally or through the written word. A second limitation of self-study methods is that the data to be analyzed are processes that take place in the consciousness of a single observer, and are thus not available for verification or replication by other observers (Ericsson & Simon, 1993; Someren, Barnard, & Sandberg, 1994). In relation to problem solving, self-studies tend to be undertaken by experts, and as a result the processes described by self-study are the processes undertaken by experts. Pólya (1945), for example, was an expert mathematician who developed his four-step problem-solving model through the process of studying his own expert problem-solving processes. The result is that while his model might describe expert behavior very well, it would not be expected to be applicable to novice mathematics problem solvers.

Another related limitation of Polya's (1945) study is that the model he developed describes problem solving as a linear, step-by-step process. This is a general limitation that is seen in many models, which assume that problem solving can be described in terms of a linear or cyclical process. A prime example of a linear model in the domain of physics is the Minnesota problem-solving model (Heller & Heller, 2000; Heller, Keith, & Anderson, 1992), which has been taught as a problem-solving strategy in introductory physics courses. Linear models such as Polya's and the Minnesota model tend to be based on idealized, expert behavior in situations that are familiar to the problem solver. Research into the behavior of novices, or of experts solving unfamiliar or particularly difficult problems, suggests that such step-wise problem-solving behavior occurs only in problem situations that are familiar to the problem solver. While it can certainly be argued that a primary goal of problem-solving instruction in physics is to get novices to solve problems in the same manner as experts, it is important to recognize that such linear models are essentially models of exercise solving (Bereiter & Scardamalia, 1993; Schoenfeld & Herrmann, 1982; Singh, 2002).

Schoenfeld and Herrmann (1982) noted this limitation in relation to the expert-novice studies that were prevalent in the early 1980s. Apart from possessing superior domain-specific knowledge and considerably more experience, experts also tend to be older than the novices used in the studies, and more likely to possess a greater aptitude for the domain. In the domain of physics, the problems used in expert-novice studies tended to be typical practice problems such as might be seen in introductory physics texts. While textbook problems generally *are* problems to novices, they tend to be perceived as exercises by experts (Bereiter & Scardamalia, 1993). There have been a few attempts to address this shortcoming by looking at the problem-solving behavior of experts on more difficult or unfamiliar problems (Singh, 2002). In these cases it was found that experts resort to more novice-like behavior, relying on means-ends analysis or other "weak" problem-solving techniques.

Another significant limitation of expert-novice studies is that they tend to treat both experts and novices as homogeneous groups. There are qualitative differences in both levels and styles of expertise, just as there are for novices. Treating each group as a uniform whole necessarily loses some of these distinctions. There have also been only limited attempts to delineate between true expert problem solving and that of experienced novices, who may be proficient problem solvers but who lack the integrated knowledge structure of a discipline possessed by experts. There have been some attempts to bring out some of the distinctions between good and poor novice problem solvers (Chi et al., 1989; Finegold & Mass, 1985; Heyworth, 1999; Jong & Ferguson-Hessler, 1986; Joshua & Dupin, 1991; Snyder, 2000). Clarifying the differences between successful and unsuccessful novices is of particular pedagogical importance. Knowledge of the characteristics of successful novices can inform instruction, allowing physics educators to make techniques and strategies used by successful novices an explicit component of introductory physics courses.

Given that so many studies over a period of over two decades have provided very similar qualitative results, one could argue that the particular limitations of expert-novice studies have not hindered the understanding of the general characteristics of novice or expert problem solving in physics. However, with some exceptions, the majority of these studies have been descriptive in nature. Current models of problem solving in physics tend to focus on the series of steps taken and equations generated, while ignoring aspects of the problem-solving process such as qualitative analysis, metacognitive strategies, and conceptual understandings (Chi et al., 1982). Focusing on steps and procedures limits the applicability of the results of these studies to the improvement of problem-solving instruction in physics. Although a multifaceted description is a critical first step to developing understanding of problem-solving behaviors, physics education researchers must look beyond description towards informing those instructional practices that will assist in the development of problem-solving skills. To do this, a more complete understanding of problem-solving processes must be developed, one which takes into account those aspects of problem solving that simple descriptions are not able to account for. One research methodology that has been developed to address the limitations of descriptive studies is the think-aloud method. Although several different versions of think-aloud studies have been discussed in the literature, the main assumption of all versions is the same: that it is possible to instruct subjects to think out loud during the process of problem solving in a way that does not alter the problem-solving process, and that analysis of the resulting protocols can provide valuable insight into the various aspects of problem solving that are not addressed in purely descriptive models (Ericsson & Simon, 1993; Someren et al., 1994; Taylor & Dionne, 2000). In the next section, the think-aloud methodology will be discussed, and arguments provided for its use as the method of choice for investigating problem-solving processes.

Think-Aloud Methods

If one is to understand problem solving, one must observe the problem-solving process. In addition, one must have some way of inferring the cognitive activities taking place in the mind of the problem-solver as he or she works towards a solution. This philosophy has led to the use of the think-aloud protocol as the method of choice for investigating problem-solving processes. A number of the studies discussed in previous sections have utilized think-aloud methods for all or part of their data (see for example Bassok, 1990; Chi et al., 1982; Dhillon, 1998; Schoenfeld, 1985a). During think-aloud sessions, participants are asked to concurrently verbalize their thought processes as they attempt to solve problems, which allows for observation of solution strategies in real time. The problem-solving session is audio and/or video recorded, and the recorded protocols are analyzed using one of several qualitative techniques. By focusing on the actions taken and cognitive processes verbalized, it is hoped that a characterization of the problem-solving process can be obtained. It is also possible to collect quantitative data, such as the number of errors per solution or time to solution. However, because of the additional cognitive load that verbalization of the problem-solving process can place on the solver, it has been suggested that quantitative protocol data is not particularly reliable (Chi et al., 1982).

Early use of verbalization techniques raised concerns about the validity of the technique (Nisbett & Wilson, 1977). As problem-solving research matured as a field of study, the use of verbal data has also matured. Through the application of the information-processing model and verbalization theory, think-aloud methodology has gained respectability as a valid, reliable technique for gathering data about cognitive processes (Ericsson & Simon, 1993; Schoenfeld, 1985b; Someren et al., 1994; Taylor & Dionne, 2000). Verbal think-aloud methods avoid some of the limitations of introspective techniques by treating the verbal data objectively, since the protocols are not interpreted by the subject of study and are available for review by anyone. However, in order for think-aloud techniques to produce data which accurately reflect the cognitive processes of the subject, there are several conditions that must be met, both in the collection of the verbalizations and in the subsequent analysis.

The first requirement for obtaining protocols which represent a direct correspondence between the thoughts of the subject and the verbalizations produced is that the thoughts be verbalized concurrently with the problem-solving process. According to the information-processing model of human cognition, the only information that is available for verbalization is information which is in working memory, under the immediate attention of the problem solver

(Ericsson & Simon, 1993). Instructions to the subject to verbalize information not normally heeded during problem solving will result in additional information brought into working memory, which can in turn result in changes in the course and structure of the problem-solving process (Schoenfeld, 1985b). The primary reason for these changes is an increase in the cognitive load. Studies comparing silent problem solving with think-aloud protocols have shown that if instructions to the problem solver emphasize only to think out loud, and not to explain or analyze his or her actions, there is little additional cognitive load and the course of the problem solution is not changed (Ericsson & Simon, 1980).

Concurrent verbalization has been shown to provide the direct correspondence to cognitive processing needed for analysis of problem solving, but there are situations in which retrospective protocols can also provide valuable information. If only retrospective techniques are used, the reliability of the verbalizations produced is found to decrease as the time between the task and the report increases (Ericsson & Simon, 1993). However, if retrospective reports are taken immediately following the task completion, valuable information can still be obtained. Combining retrospective reports with other records of the problem-solving process, such as the written solution and/or videotape, can help assure the validity of the information obtained (Ericsson & Simon, 1980; Taylor & Dionne, 2000).

The use of retrospective reports, or debriefings, alone generally results in insufficient information from which to deduce the problem solver's cognitive processes. In particular, the problem solver might have difficulty remembering all that he or she did, or might present his or her thought processes as more coherent than they actually were. The problem solver's memory of the processes used is guided by knowledge of the result obtained, which could cause the problem solver to report more generalized strategies than were actually used (Someren et al., 1994). However, when retrospective debriefing is used in conjunction with concurrent think-aloud protocols, the data from each source can act as a means of validation for the other. The think-aloud protocol provides directly observable behavior against which the validity of retrospective debriefing statements can be compared, while the retrospective debriefing data can be used to verify and clarify the researcher's interpretations of the think-aloud protocols in situations in which the researcher must draw inferences from the data (Schoenfeld, 1985b; Taylor & Dionne, 2000). This is particularly important in light of the fact that even in the best of situations, think-aloud protocols will not be complete. During the course of any problem-solving situation, there will be certain cognitive processes that are automated, which do not enter working memory and are therefore not available for verbalization. Likewise, there are thoughts that will not be verbalized simply because of the time required for verbalization (Ericsson & Simon, 1993; Someren et al., 1994). Estimates suggest that under the very best conditions information can be obtained every second for verbal data, while the information processes that are taking place may only be in the range of a few tens to a few hundreds of milliseconds long (Larkin et al., 1980a). Thus there will always be situations in which the researcher must infer, on the theoretical basis of the coding and the actions/verbalizations prior to and following, what cognitive processes the problem solver undertook during a pause in the verbalization. Triangulation between the think-aloud protocols and retrospective debriefings makes the inferences less problematic (Taylor & Dionne, 2000).

The second requirement for meaningful think-aloud protocols is related to the task undertaken by the participants. As noted previously, there are certain processes that are automated in any problem solution. As a general example, suppose one step in a problem required the addition of 2 and 5. The problem solver might verbalize “2 and 5 is 7,” but if one was to ask the problem solver how they knew that 2 and 5 was 7, the subject would not be able to verbalize that process. Likely responses might be that he or she “just knew,” or that it “was memorized.” Single-digit addition is so automated that it cannot be verbalized; the answer is retrieved directly from long term memory without intervening processing in working memory. This factor in the collection of think-aloud protocols is particularly relevant in relation to expert problem solvers, whose basic problem-solving processes may be chunked and automated (Chi et al., 1982; Larkin et al., 1980a; Savelsbergh et al., 2002). To avoid this source of incomplete think-aloud protocols, it is important for the task provided to participants to be difficult enough so that its solution is not automated, yet at the same time be solvable by at least some portion of the participants, and not so difficult as to discourage verbalization (Ericsson & Simon, 1993; Someren et al., 1994; Taylor & Dionne, 2000).

The final requirements for obtaining useful think-aloud protocols are related to the actual protocol collection procedures. Participants should be provided with clear verbal and written instructions that emphasize the need for accurate, continual verbalization without explanation or interpretation (Schoenfeld, 1985b; Someren et al., 1994). The researcher should not interrupt the protocol unless absolutely necessary, and then only with neutral reminders to “Keep talking.” In addition, participants should be provided with adequate time to practice the thinking-aloud process until they are comfortable with it. Thinking aloud is natural to most people, and so introduces little or no additional cognitive load (Ericsson & Simon, 1993); however, it is important to provide practice problems to put participants at ease and to assure that the instructions were interpreted in the way the researcher intended (Schoenfeld, 1985b).

One significant benefit of the use of the think-aloud method is that it allows the various aspects of problem solving identified in other studies to be investigated in the real-time context of problem solving. For example categorization, typically investigated using problem-sorting tasks (Chi et al., 1981; Savelsbergh et al., 2002), can be studied during actual problem-solving activities. In addition, they provide access to knowledge about cognitive activities that would not be apparent from simple observation of completed written solutions. This focus on the actions taken, coupled with the verbalizations of cognitive processes, has provided a more complete view of problem solving. For example, think-aloud protocols of problem solvers have resulted in a number of computer models designed to emulate the problem-solving behavior of both experts and novices (Larkin et al., 1980b; Plotzner, 1994; Ross & Bolton, 2002). However, even with think-aloud protocol data, there is still much to be learned about problem solving.

The studies and methods discussed so far have provided a rich view of problem solving in physics. The differences between expert and novice problem solvers have been delineated, as have some of the differences between good and poor novices. The importance of conceptual understanding has been emphasized and linked to the procedural knowledge needed for success in problem solving. A more complete view of the problem-solving process must take these known factors into account, integrating the results of expert-novice studies with conceptual understanding, the use of cognitive resources, and procedural knowledge. It should reflect the dynamic nature of the problem-solving process while integrating the robust results of previous problem-solving studies. One possible candidate for such a model is that proposed by McGinn and Boote (2003).

The McGinn and Boote Model

The McGinn and Boote (2003) model of problem solving was developed as a result of self-study research in mathematical problem solving. As noted previously, there are limitations to the applicability of self-study techniques. However, the use of introspection to help understand mathematical problem solving is not without precedent. To provide one notable example, the basis of Pólya's (1945) historic four-step problem-solving model outlining the use of heuristics was the result of an examination of his own problem-solving processes.

The need for an alternative model of problem solving arises from recognition of the fact that purely cognitive models focus on problem solving as a series of discrete steps, divorced from the social, material, motivational and emotional aspects of the process. These aspects of problem solving, and of learning in general, have been acknowledged in the physics education literature (for example, see Redish et al., 1998), but have only recently been investigated in direct relation to physics problem solving (Cummings & Lockwood, 2003).

McGinn and Boote (2003) set out on their self-study as a result of a desire "to understand the processes of problem solving and what could be learned by solving problems" (pp. 88 - 89). In particular, they wanted to understand how various social, material and conceptual resources were recognized and used to support problem-solving efforts. As a result of their study, they learned that even highly structured and well-defined problem statements often led to problem solving efforts that were not necessarily highly structured or well defined. While the mathematical backgrounds of the authors preclude them from being categorized as true novices, they found that frequently, when faced with a difficult problem, their problem-solving activities became far more novice-like. Similar results were found in the domain of physics, when experts were faced with particularly difficult problems (Singh, 2002). Experiences in struggling with difficult problems led McGinn and Boote to recognize that problem difficulty, and their personal perceptions of difficulty, were a major factor in understanding their problem-solving experiences. This realization caused them to set out to determine what caused a problem to be perceived as difficult.

Their experiences in attempting to solve problems led McGinn and Boote (2003) to identify a continuum of problem difficulties. This continuum ranges from trivial exercises with automated solutions to ill-defined difficulties, or predicaments. Automated activities are those things which are done essentially without active consideration of the processes involved, such as single-digit addition. Exercises are defined as problems that have been categorized and are solvable by automated solution procedures. It is in the case of exercises that step-wise solution processes are most often observed. Difficulties occur when appropriate resources are not recognized, the problem cannot be categorized, the complexity of the problem is high, and/or the goal of the problem is not recognized or appropriately interpreted. In this case, a solution is not possible. According to McGinn and Boote, the two extremes of this continuum represent situations in which no problem solving takes place. If the solution is automated, there is no problem. Likewise, if the problem is ill-defined and the problem solver has no conceptual or procedural understandings upon which to base the solution, no problem solving takes place. It is the area between those two extremes that is of most interest to problem-solving research.

Analysis of their problem-solving experiences led McGinn and Boote (2003) to identify two levels of factors that influenced problem difficulty. At the first level are four factors: categorization, goal interpretation, resource relevance and complexity. At the second level is one factor, stabilization, which describes the shifting inter-relationships of the other four factors over time during the problem-solving process. The second-level factor is needed because, according to this model, a problem solver's perception of problem difficulty is related not just to the four previously identified factors, but also to how the relationships between those factors change over time. Understanding of the possible relationships that might arise during the problem-solving process requires an understanding of the individual first-level factors.

The first factor, categorization, has been fully documented in both the mathematics and physics problem-solving literature. Many of the early expert-novice studies focused on differences in problem categorization (Chi et al., 1981; Chi et al., 1982; Cummins, 1992; Larkin et al., 1980a; Medin, 1989). A problem that is properly categorized has been recognized as belonging to a certain genre of problems, for example, a conservation of energy problem. Categorization of a problem often allows for recall of a general problem-solving procedure, so that the solution becomes automated and is, in effect, no longer a problem but rather an exercise.

Closely related to categorization is goal interpretation, which is defined essentially as an understanding of how a solution should look. If the goal of a problem is adequately interpreted, the problem solver understands the general pattern that the solution will follow. Sweller (1983) noted that clearly specified goals that are properly interpreted by the problem solver serve two purposes. First, they act as a feedback mechanism to guide the problem solver to a solution, and to assist in the choice of strategies selected to apply to the problem (Vollmeyer, Burns, & Holyoak, 1996). This is particularly important when means-ends analysis is being employed in the search for a solution. A second, related function is as a control mechanism. If a chosen path is not proving to be fruitful, knowledge of the goal directs the solver to search for alternate solution paths. For experts, the categorization of a problem is in large part tied to the interpretation of the problem goal, and solution patterns are tied to the categorization. Together, categorization and goal interpretation affect what, if anything, will be learned from the solution process. If a problem is categorized in such a way that the solution pattern is automated and the solution itself is trivial, the solver learns little. When the solution process is not automatic, the solver is cognitively engaged with the process, and the possibility for learning is high. In this regard, it is important to note that the answer is only one part of an acceptable solution (McGinn & Boote, 2003). In contrast, students often approach problem solving as if the final answer is the only important part of the solution. This attitude was reported by Good (1984), who found that experts view problem solving as a process, while novices view it as a recall task (see also Singh, 2003); and by research suggesting that novices' solution attempts are unknown- and equation-driven (Chi et al., 1981; Larkin et al., 1980a; Savelsbergh et al., 2002). However, it is not only during automated solutions that the possibility of no learning occurs. Sweller notes that equation-driven solutions guided by means-ends analysis can lead to situations in which nothing is learned from the solution process, which in turn results in little or no transfer to new situations.

Of particular importance in physics problem solving is resource relevance. This factor refers to the various conceptual, material and social resources that the problem solver brings to the solution attempt. There are two closely related aspects to this factor. First, the solver must have available and recognize existing resources as relevant to the solution process. The second aspect is related to the idea of transfer; it refers to the recognition of domain knowledge which can be applied to the problem at hand, even if the new situation is in a domain outside that of the existing knowledge. Schoenfeld (1985a; see also Schoenfeld & Herrmann, 1982) looked at resource recognition in mathematical problem solving and found that novices frequently fail to use the conceptual resources available to them. Recently this factor has received renewed attention in the physics education research literature. Physics educators often claim that many of their students do not have the mathematical background to become successful problem solvers in physics. However, recent studies in physics parallel the results reported by Schoenfeld, showing that students in physics in general possess adequate knowledge within the domain of mathematics, but do not recognize its relevance within the domain of physics (Cui et al., 2005; Ozimek et al., 2004; Tuminaro & Redish, 2003). As noted earlier, similar results have been reported in relation to conceptual resources (Chi et al., 1989; Hammer, 2000; Kim & Pak, 2002; Savelsbergh et al., 2002; Sherin, 2001).

The final first-level factor in the McGinn and Boote (2003) model is complexity, which is simply an indication of the number of steps or operations needed to reach a solution. Regardless of the level of mathematical sophistication required for a problem solution, the problem is perceived as difficult if it requires a large number of discrete steps to attain the solution, an observation that has also been noted in the domain of physics (Foster, 2007; Reif & Heller, 1982). A problem with one step, or at most a few steps, is viewed as simple, and in fact might be perceived as an exercise rather than a problem.

The four primary factors discussed so far work together to affect the solver's perception of problem difficulty. For a problem to be perceived as solvable, the solver must believe the problem is appropriately categorized, the goal is correctly interpreted, relevant resources are available, and that the level of complexity is reasonable. If a problem is improperly categorized, or resources are unavailable or unrecognized, the problem might be perceived as unsolvable. At the other extreme, if all four factors are rated as low, the solution might be automated, and the problem is perceived as a mere exercise. Thus the perception of problem difficulty is determined by the inter-relatedness of the four factors over time. If perceptions of the factors are inconsistent with each other, such as if the recognized resources do not support the categorization of the problem, the solver is forced to re-evaluate his or her perception of the four factors (McGinn & Boote, 2003).

The shifting of relationships between the four primary factors over time during the solution attempt is called stabilization, the second-level factor in the McGinn and Boote (2003) model. According to the model, a stable representation of the problem must be attained in order for a solution to be reached; that is, there must be a stable relationship among the four factors. Thus problem solving can be considered to be an active search for stability in the problem, represented by the relationships between the factors of categorization, goal interpretation, resource relevance and complexity. If all factors are stabilized, all that remains is computation.

McGinn and Boote (2003) originally established their description of problem solving as a means of identifying the factors that affect problem difficulty. Other researchers have looked at problem difficulty, including aspects of the recognition of goals (Larkin et al., 1980a; Sweller, 1983; Vollmeyer et al., 1996), categorization (Chi et al., 1981; Snyder, 2000), context (Chi et al., 1989; Larkin et al., 1980a), the use of diagrams (Larkin & Simon, 1995; Leone & Gire, 2005; Rosengrant et al., 2004, 2005), and mathematical complexity (Bassok & Holyoak, 1989; Reif & Heller, 1982). Recent work by Foster (2007) provides a framework for determining problem difficulty based on twenty-one traits grouped into three major categories: Approach to the Problem, Analysis of the Problem, and Mathematical Solution. Approach to the Problem includes those traits of a problem that affect the problem solver's choice of concepts and principles to apply to the problem, but not the actual choice itself. This includes such things as goal interpretation, context, level of abstraction, and the number of principles required for the solution. This category as described by Foster has considerable overlap with aspects of categorization as described by McGinn and Boote, as well as with goal interpretation and complexity. Analysis of the Problem includes those traits that affect the translation of the problem into a physics representation, including the actual categorization, use of diagrams, and generation of equations. This category also overlaps with McGinn and Boote's use of categorization as a factor affecting problem difficulty, as well as with complexity, although Foster's definition of complexity as a factor is much broader that that defined by McGinn and Boote. Foster's final category, Mathematical Solution, relates primarily to the level of mathematics utilized in the problem, but also includes aspects of complexity as defined by McGinn and Boote.

Despite the considerable amount of research into the factors that affect problem difficulty, none of the researchers have extended their interpretation to the point where it would be applicable to the entire problem-solving process. McGinn and Boote's (2003) understanding of problem difficulty seems to go beyond simply identifying factors; rather, they seem to suggest actions that take place during the process of solving a problem. It does not require a significant extension of McGinn and Boote's descriptions of factors to formulate a framework for the problem-solving process. For example, the contention that the categorization of a problem would affect the problem solver's perception of the difficulty of the problem suggests that categorization is an action that the problem solver must carry out at some point during the process of solving the problem. Likewise, if recognition of resource relevance affects problem difficulty, then it seems clear that the problem solver must recognize the relevance of their available resources to the solution of the problem. That is, recognition of applicable resources is an action that the problem solver must carry out. Of particular interest in this interpretation of McGinn and Boote's work is the superordinate factor, stabilization. This factor is defined in terms of the shifting relationships between the other four factors over time. However, it can also be interpreted as the use of the other four factors in the process of a search for a stable understanding of the problem. In order for a problem to be solvable, the problem solver must reach an understanding of the problem in terms of interpreting the goals, accessing relevant resources, making appropriate categorizations, and navigating the perceived or actual complexity of the problem. That is, stabilization is an action that the problem solver must undertake in order to reach a solution to the problem.

Viewed simultaneously as a way to describe both problem difficulty and the problem-solving process, McGinn and Boote's (2003) model could be considered a hybrid: In assessing those factors that determine how difficult a problem is perceived to be, we are determining the steps that the problem solver must take in order to reach a stable understanding of the problem and, ultimately, a solution. As a model, it suggests that problem solving is the process of searching for a stable understanding of the problem at hand, a situation that may not be reached until a solution is found. While attempting to stabilize understanding, the problem solver utilizes relevant resources, attempts to categorize the problem, and interprets the goal of the problem, all while navigating the complexity of the problem. Up to the point where the problem is solved, there is always the possibility that destabilization might occur. The problem solver, previously believing that the problem is fully understood, may reach an impasse in the solution process, forcing a reassessment of the problem situation.

While the identified factors can be considered as actions in a problem solving process, it is important that they not be interpreted as a sequence of steps. As noted by McGinn and Boote (2003), as well as other researchers (Bereiter & Scardamalia, 1993; Schoenfeld & Herrmann, 1982; Singh, 2002), step-wise problem solving only takes place when the problem solver is working on problems that are familiar. That is, step-by-step problem solving only takes place when the problem solver is working on an exercise. In real problem solving, the problem solver follows no discernable steps or pattern, but might instead wander between categorization, utilizing resources, reinterpreting goals and subgoals, all while struggling with the perceived complexity of the problem. It is this manner that the proposed McGinn and Boote model differs from traditional linear or cyclical models of problem solving.

There are several other differences between this proposed model and traditional cognitive models of problem solving. As noted, the primary difference is that there are no discrete steps or stages in the problem-solving process, as suggested by Pólya (1945) and others (Schoenfeld, 1985a; Singh, 2003). According to the McGinn and Boote (2003) model, the problem will be solved in a step-wise fashion only if the factors are rated low and stabilized, at which point it is an exercise rather than a problem. Second, there is an on-going assessment of the problem as perceptions of the four factors change over the course of the solution attempt. Rather than proceeding in linear or cyclical fashion, the solution process may proceed in an apparent random fashion, what McGinn and Boote call "flailing about" (2003, p. 103). Finally, the model arises from the interplay among social, material and cognitive aspects of problem solving. Neither cognitive nor situative aspects of problem solving alone are adequate to describe the problem-solving process proposed by this model.

The McGinn and Boote (2003) stabilization model was developed through an investigation of problem solving in mathematics. Problem solving in physics is also highly mathematical, suggesting that the model could be applicable to that domain as well. This study investigated the applicability of the stabilization model to novice problem solvers in the domain of physics. The model is particularly attractive because, as discussed earlier, the four first-level factors have already been identified by other researchers as being important indicators of success in physics problem solving. The findings of Singh (2002) with regards to the problem-solving behavior of experts attempting particularly difficult problems also raised important questions about what it means when we say we trying to model problem solving. The problem-solving processes of novice physics problem-solvers have been well-documented by other researchers (Chi et al., 1981; Finegold & Mass, 1985; Jong & Ferguson-Hessler, 1986), and models have been developed to describe those processes (Larkin et al., 1980b; VanLehn, 1998). The characteristics of experts solving similar problems have also been documented (Chi et al., 1982; Heyworth, 1999; Larkin et al., 1980b). When a novice exhibits problem-solving processes that show higher levels of automation, appropriate categorization, and a forward-working strategy, we say that the problem solver is exhibiting more expert-like behavior. But at that point, the problem is no longer a problem; it is an exercise. Likewise, Singh demonstrated that when faced with a difficult problem, experts behave more like novices. In this respect, it could be argued that the models that describe expert behavior are models of exercise solving. Ideally, a model of problem-solving should be general enough that it can describe the behavior of a wide range of experience, from novices to experts. The problem-solving behavior of experts solving familiar types of problems is easily characterized by step-wise models; it is the highly variable problem-solving behavior of novices that is more challenging.

CHAPTER THREE: DESIGN AND METHODOLOGY

The purpose of this study was to investigate the applicability of the McGinn and Boote (2003) model of problem solving to the experiences of students enrolled in introductory, calculus-based college physics courses. The primary research question was "To what extent does the stabilization model describe physics students' problem-solving experiences?" Investigation of this question required exploration of the secondary questions "What are the basic processes that physics students undertake as they attempt to solve physics problems?" and "What resources do students bring to the problem-solving process?" The challenge for any model of problem solving is in describing the problem-solving experiences of novices, who possess incomplete conceptual and procedural knowledge and a limited number of pattern-induced schemata. Expert-novice studies have done an excellent job of helping us understand what experts do, and the differences between the general characteristics of expert problem solving and those of novices. What is needed is a model that will enable a deeper understanding of the specific processes undertaken by novices as they attempt to solve problems. Even more powerful would be the demonstration of a general model that would be valid for both experts and novices. This study investigated the applicability of a model of problem solving that has a level of generality that might make it applicable to both experts and novices. The applicability to novice problem solvers is more problematic, given that previous researchers have demonstrated that novices tend to carry out problem solutions in a less organized fashion than experts. For this reason, this investigation of the McGinn and Boote (2003) model focused on novices.

Procedure

The primary data for the study was collected using an integrated approach, combining concurrent think-aloud protocols with retrospective debriefing. This methodology was chosen in order to investigate the various factors associated with the stabilization model within the context of actual problem solving. Participants were asked to verbalize their thought processes during the solution of physics problems typical of those found in introductory physics textbooks (see Appendix A). Each participant was provided with both verbal and written instructions (see Appendix B), and was provided with practice problems to use for becoming comfortable with thinking aloud while problem solving. Because the McGinn and Boote model (2003) notes the importance of material resources, participants were invited to bring a physics text, calculator, and/or other material resources with them to the problem session. The problem sessions were videotaped for later transcription and analysis. Practice problems were also videotaped with the intent of putting the participants at ease with the presence of the video equipment while they were solving problems.

The researcher conducted semi-structured, retrospective debriefing interviews immediately following the problem-solving sessions, during which the participant and researcher reviewed the videotape of the session. The debriefing session consisted of the researcher asking questions directed at clarifying the steps taken during the problem solution and determining the participants' perceived points of difficulty in that process. While some of the categories of questions could be anticipated in advance, the details of the questions used could not, as they were based on the specific processes and procedures used by individual participants. In addition, more general questions related to problem solving in general were asked. The retrospective session was audiotaped for later transcription and analysis. Complete problem sessions, including the debriefing, ranged in length from approximately one hour to just over two hours.

Following transcription, several layers of analysis took place, first to characterize the actions taken by individual participants. Participants were grouped according to error indicators, and group characteristics were determined. Finally, the protocols were reviewed according to the framework of the stabilization model to ascertain the extent to which the model applied to the problem-solving experiences of the participants. At each point parallel analysis was carried out by an independent reviewer as means of validating the interpretations of the researcher. To assure that the factors that make up the stabilization model were operationalized in a manner consistent with the original study (McGinn & Boote, 2003), one of the authors of that study (Boote) also served as a reviewer.

Participants

Participants were obtained from a pool of volunteers solicited from introductory, calculus-based physics courses. The researcher visited six physics classes, three at a research university and three at a community college. During those visits the researcher explained the purpose of the study and solicited volunteers to participate in the study. In addition, the researcher provided written text of the same information to a colleague at a second university, which was read aloud in one elective laboratory class to solicit for additional volunteers. At the time of solicitation, volunteers were asked to provide information concerning mathematical preparation, SAT/ACT scores, and current grade in their physics course (see Appendix C). This process resulted in a pool of 47 potential participants, which included both men and women of various ethnicities. All volunteers were contacted to schedule a problem-solving session; of those nineteen responded and made appointments. Three volunteers did not keep their scheduled appointments and did not respond to follow-up contacts. The remaining 16 participants completed problem-solving sessions. However, there were four participants whose resulting protocols were not used in the analysis, for reasons to be discussed in Chapter 4. The remaining 12 participants included seven university and five community college students, with three females and nine males. The age of the participants ranged from 19 to 22, with one exception who was 43. The majority (10) was Caucasian; two were Hispanic. All participants had completed at least one semester of calculus-based physics; several had completed two or three semesters. The majority of participants reported declared majors of physics or engineering. A summary of participant demographics can be seen in Table 1.

Table 1: Participant Demographics

Participant	Sex	Age	SAT[#]	School Type[†]	Last or Current Physics Course[‡]	Last Math Course*	Major*
Alex	M	43	NR	CC	Physics I	Calculus I	Computer Engineering
Andrew	M	19	1010	CC	Physics I	Calculus I	Engineering
Arnold	M	20	1190	U	Physics III	Linear Algebra	Physics
Art	M	22	1180	U	Physics II	Linear Algebra	Physics
Ben	M	21	1200	U	Physics II	Calculus I	Physics
Beth	F	20	1280	U	Physics I Honors	Diff. Equations	Mathematics
Betty	F	19	1340	U	Physics I Honors	AP Calculus BC	Psychology
Bob	M	21	NR	U	Physics III	Diff. Equations	Physics
Brittany	F	19	NR	U	Physics III	Calculus II	Physics
Carl	M	19	NR	CC	Physics I	Calculus I	Pre-med
Chuck	M	20	1230	CC	Physics II	Calculus I	Engineering
Cory	M	19	1000	CC	Physics I	Calculus I	Engineering

[#] NR: Not reported
[†] CC: Community college; U: University
[‡] Generic titles
*As self-reported

Problems

The problems used in this study were chosen from a selection of introductory-level physics textbooks, or from problems written by the researcher. Care was taken to ensure that the problems selected were of a suitable level of difficulty. If the selected problems were too easy, participants might have perceived the problems as exercises, resulting in automated problem-solving processes that would contribute little to the investigation (Ericsson & Simon, 1993; Someren et al., 1994; Taylor & Dionne, 2000). On the other hand, if the selected problems were too difficult, few participants would have been able to solve them, again leading to protocols that provided only minimal information. The problems selected were required to be difficult enough to be perceived as problems to the majority of the participants in the study, to assure that protocols of the solution process provided rich data. In order to assess the level of difficulty, a group of problems selected by the researcher was reviewed by a physics instructor who was not otherwise associated with this research. The instructor was asked to judge the difficulty of the sample problems, and to provide an estimate of the percentage of students in his classes who would find the problems particularly challenging. Based on the feedback from the instructor, a small group of problems was selected for use in the study. Through the course of collecting the earliest protocols, the researcher ultimately settled on three problems which appeared to satisfy the research criterion for appropriate level of difficulty. These problems were characterized by the fact that they required multiple steps and concepts for successful completion of the problem, and could be solved with alternate methods. The problems used in the study can be seen in Appendix A.

Data Analysis

Videotaped problem-solving sessions were transcribed verbatim, using a line-numbered format without punctuation. Natural pauses, obvious implied punctuations, and obvious changes in the nature of the activity carried out by the participants were used for the initial separation of lines. Transcriptions also included non-verbal actions taken by the participants, such as drawing diagrams, referring to references, using a calculator, or referring to problem statements. All identifying information was removed from the transcripts, which were labeled with the participants' chosen pseudonyms to help assure confidentiality.

The 12 protocols that resulted from the data collection process were first reviewed to ascertain the level of success the participants had reached with the problem. In order to facilitate comparisons between participants, only one problem was used for the analysis, even if the individual attempted two or three. All of the participants to whom this applied had completed Problem 1 (see Appendix A); since that is the problem that the majority of all participants had completed, it was the problem selected from their protocols for inclusion in the analysis.

The next step was to carry out an initial careful reading of the protocols while referencing the written work of the participants. The goal was to ascertain where students made mistakes, a task which was not unlike an instructor grading an exam. Once an error was noted, a determination was made as to the nature of that error, with errors categorized as primarily procedural, mathematical or conceptual. Because this step required a certain level of judgment and inference on the part of the researcher, three protocols were provided to a physics instructor not otherwise associated with this research for verification of the error categorization. In all three cases, the judgments of the reviewer were in agreement with those of the researcher. The protocols were then divided into three groups: Group A, whose members made primarily procedural/mathematical errors; Group B, whose members made primarily conceptual errors; and Group C, whose members made no errors.

Once the nature of the errors was determined, the protocols were reviewed again, and the basic activity being carried out by each individual was noted. The goal of this analysis was to determine if the members of each of the groups had any problem-solving characteristics in common. At this level, no attempt was made to interpret the actions of the participants. It was literally a listing of steps taken, including such actions as reading the problem statement, drawing a diagram, writing an equation, and so on. Because the majority of the steps noted were universally carried out by all participants, a second pass was made through the protocols to note the context and manner in which the actions took place. After completing the step analysis for each participant, a list of the characteristics of each protocol was created. Included in this list were such items such as whether or not the participant checked his or her work, stated goals or unknowns, referred to formula sheets or books, and so on.

An initial coding scheme based on the McGinn and Boote (2003) model of problem solving was then used to code the transcribed protocols. Initial coding categories included the four first-level factors identified by the model: categorization, goal interpretation, resource relevance, and complexity. The second-level factor, stabilization, was used as a fifth category. An additional category for actions and verbalizations not related to problem solving was included. Each of the categories was clarified by use of conjectured prototypical statements that would be coded within that category (see Appendix D). The coding scheme was tested on three protocols selected at random from the pool of transcribed data, to determine if the identified categories were sufficient to fully code the protocols. To test the reliability of the coding scheme, a second coder was asked to code the same three randomly selected protocols, and the inter-coder reliability factor, Kappa, was determined.

When coding was complete, the coded protocols were analyzed within the framework of the stabilization model. Think-aloud methodology can be applied from two perspectives: theory verification or theory building. This investigation had aspects of both perspectives. Because the stabilization model is new, there was an aspect of using the data to build and modify the theory. Yet at the same time, most aspects of the model have been previously identified in the research literature as being important to the problem-solving process, so the analysis also included an aspect of verification, assuring that those previously identified factors fit within the framework of the stabilization model. The goal of this portion of the analysis was to determine if the majority of protocol fragments were found to be coded within the context of the model. If so, the implication is that the model is applicable to the problem-solving experiences of the participants. If, on the other hand, a large portion of the protocol fragments were not coded, the results would suggest that the model is inadequate. Segments of the protocol that could not be coded correspond to cognitive processes or sub-processes that are not explained by the model, thus suggesting that a modification of the model is required. According to Someren et al. (1994), there are three possible reasons for deviations between the coded protocols and the model to which they are compared. First, the protocols might show processes that are not predicted by the model, suggesting that it is incomplete. Second, the model might predict processes that are not seen in the protocols, suggesting that the model overspecifies the process it is intended to describe or that an aspect of the model is invalid. Third, the protocols may show processes in a different order than predicted by the model. This third possibility is not applicable to the stabilization model, as it specifically excludes step-by-step processes except in the case of exercises. The analysis of the protocols in this study was carried out with the understanding that there were two possible explanations for a small number of deviations from the model. The first possibility is that the model, while basically applicable, needs minor adaptation. This possibility corresponds to a generalization of the first two interpretations by Someren et al. The second possible explanation is that the minor deviations are a result of individual differences within the problem-solving process rather than a global failure of the model. Ideally one would like such individual differences to fit within the framework of the model, and realistically it is probably not possible to distinguish between the two alternate explanations.

Limitations and Assumptions

The primary assumptions for this study were the assumptions associated with the use of verbal protocol data. First, it was assumed that verbal behavior is recordable behavior that can be observed and analyzed like any other behavior. Second, it was assumed that the participants in the study were actually verbalizing the cognitive processes that they were attending to during the problem-solving process; in other words, we trusted the verbal reports. This is essentially an empirical issue, in that the verbal reports could be validated with other recorded information, such as the videotapes and written solutions. Overriding both of these assumptions is information-processing theory, which places constraints on the cognitive processes that are verbalized. Fundamental among these is that cognitive processes can be described in terms of the sequences of information processes that the participant attends to during the course of the problem solution. Other assumptions included that the information verbalized was the information that was under immediate attention in working memory, and that information retrieved directly from long term memory without intermediate processing was not available for verbalization (Ericsson & Simon, 1993). Processes that were automated would fall into this category. It can also be expected that not every thought was recorded, primarily because the process of verbalizing thoughts takes longer than simply thinking those thoughts. Estimates suggest that under the very best conditions information can be obtained every second for verbal data, while the information processes that are taking place may only be in the range of a few tens to a few hundreds of milliseconds long (Larkin et al., 1980a). This means that some level of inference was required as the verbal protocols were reviewed. The use of videotaping and post-session interviews helped reduce the level of inference required, and allowed for validation with the participant when necessary (see, for example, Taylor & Dionne, 2000).

The study also had limitations. As with any verbal protocol study, the large time commitment necessary to transcribe, code and analyze the protocols limited the amount of data that were used. In addition, many interpersonal differences in the cognitive processes undertaken by the participants were expected to be, and were, observed. While it was expected that all participants in the study would be novices in the area of physics problem solving, it was also expected that there would be considerable variation in ability. Each of these factors limits the extent to which the results of the study can be generalized.

An additional limitation of the study is that it did not test the social resource aspect of the model. Examples of social resources would be the interactions between students as they work on problems together, or the interactions between a student and a professor. This study focused on the actions and strategies carried out by single individuals; to investigate social interactions as they relate to problem solving would require the observation of dyads or triads as they work together to solve a problem, or of in-field interactions between professors and students. Such observations add a level of complexity to the analysis that is beyond the scope of this study. This is not to say these interactions should not be studied, as socially-situated cognitive theory suggests that social interactions are a critical aspect of learning. Likewise, the study did not address motivational, emotional, or epistemological factors related to problem solving. In the domain of physics, investigations of physical phenomena are often carried out moving from the simple to the complex. By understanding the basic underlying principles, we have a foundation upon which to build understanding of the nuances and perturbations that occur in more complex systems. The same argument can be applied to building understanding of problem solving in physics. If an initial test of the stabilization model shows that the model is applicable to individual students' problem-solving experiences in relation to their primarily cognitive actions, the groundwork would be laid for future investigations of applicability of the model to situations in which problem solving involves social resources. Additional studies could then investigate emotional, motivational and epistemological issues, which are perhaps best investigated outside of actual problem-solving activities.

The level of expertise of the participants used in the study is an additional limitation. Ultimately the goal is to find a model of problem solving that is applicable to all levels of expertise. The challenge for any model is not to describe the organized, highly automated behavior of experts, but rather to describe the less organized, highly variable behavior of novices. If a general model of problem solving can be found to apply to the problem-solving behavior of novices, the next logical step would be to test the model on experts. Clues to the applicability of the model to more experienced novices could be inferred from the results of this study, as it is expected that any group of novices will contain persons with varying degrees of problem-solving ability. If the model applies to the problem-solving experiences of more expert-like novices as well as to those who struggle with problems, it would suggest that the model could be extended into the realm of expert behavior. This is not, however, to suggest that the results of this study could be generalized to expert behavior; rather, it suggests the possibility of further investigations involving expert problem solvers.

Confirmation Bias

Confirmation bias is the tendency to search for, notice, or place more weight on evidence that supports one's hypotheses, while at the same time ignoring or giving less credence to evidence that is counter to those hypotheses (Nickerson, 1998). It is a form of selection bias which leads one to collect only such evidence that supports the desired outcome of a study. In effect, confirmation bias results in one seeing what one is looking for and what one expects to find. Confirmation bias appears to be a normal human tendency, and as such care must be taken to establish conditions to avoid it. The primary way that this is done is by application of the standard rules of the scientific method; that is, researchers should seek evidence to attempt to disprove their hypotheses rather that to confirm them.

Nickerson (1998) suggests that the tendency to find what one is seeking is particularly strong when an investigation involves taxonomies, or categories. If a group of categories exists, the tendency is to view information obtained in an observation in terms of those categories. Given that the factors that make up the stabilization model are in effect categories of actions, special care had to be taken in this study to avoid confirmation bias. Nickerson provides several examples of cases within the domain of science in which repeated presentation of evidence contrary to the hypothesis induced change, even when the researcher held strong belief in his or her hypothesis. Thus, to reduce the possibility of confirmation bias in this study, the analysis was carried out with an active search for data that did not support the applicability of the stabilization model. The focus of the analysis on clearly identifying the actions taken by the participants prior to attempting to apply the stabilization model was helpful in that respect. The use of an independent reviewer to carry out analysis and coding on several randomly selected protocols also served to reduce the possibility of confirmation bias.

CHAPTER FOUR: PRELIMINARY ANALYSIS AND CASE STUDIES

The primary research question for this study was "To what extent does the stabilization model describe physics students' problem-solving experiences?" In order to adequately address this question, we must first understand the actions taken by physics students as they attempt to solve problems. Several layers of analysis were required to attain a level of understanding which would allow testing of the model at a reasonable level of confidence. In this chapter the results of the preliminary analysis of the protocols will be described in a stepwise fashion. This preliminary analysis was focused on obtaining descriptions of the actions taken by the participants, with no attempt to interpret those actions in light of the stabilization model. Excerpts from analyzed protocols will be provided, so that the reader can more fully understand the steps taken. After the actions taken by the participants are fully characterized, those actions will be interpreted in a way that integrates the retrospective debriefing interviews with the protocol data. Finally, case studies will be provided to illustrate the primary findings in relation to the actions taken by the participants.

Initial Analysis

Sixteen participants completed problem-solving sessions, using five different problems. Three of the participants attempted two problems, and one attempted three. It became apparent early in the process of gathering data that two of the original problems were not appropriate for the study. The first problem involved only one concept for its solution, and was quickly completed by the two participants who attempted it. According to the requirements for the collection of verbal protocols, a problem that is too easy results in protocols that give little insight into the problem-solving process (Ericsson & Simon, 1993; Someren et al., 1994; Taylor & Dionne, 2000). Because the problem appeared to not fit the criterion of appropriate level of difficulty, it was eliminated from the pool of potential problems. The participants who had solved the deleted problems had also completed an alternate problem, so the deletion of the problem did not result in deletion of the participants. The second problem was used only once, and was found upon review to require the use of a concept not generally covered in a typical introductory level course. It too was deleted from the pool of questions. The remaining three problems can be seen in Appendix A. Twelve participants attempted Problem 1, three attempted Problem 2, and two attempted Problem 3.

The protocols of four of the participants who completed problem sessions were not included in the final analysis. The requirements for the collection of verbal protocol data include the stipulation that the collection of the data put little or no additional cognitive load on the participant beyond that required for the problem-solving attempt (Ericsson & Simon, 1980). Two of the participants explicitly expressed considerable difficulty in translating thoughts from their native language to English for verbalization. The process of translation was clearly placing additional cognitive load on the individuals during the problem-solving process. Because of this, it was determined that the conditions under which their protocols were collected did not meet the requirements for valid verbal data, and their protocols were not included in the analysis. The third individual attempted two problems during his problem session during the course of forty minutes, and was unable to move beyond writing down basic given information. The expressions of self-criticism that made up his protocol would make an interesting psychological study, but provided no insight into the problem-solving process. Because the individual did no problem solving, his protocol was not included in the analysis. The protocol of the fourth individual was not included in the analysis because of technical difficulties during the collection of the protocol.

The remaining twelve protocols were first reviewed to ascertain the level of success that the participants had reached with the problem. This first review resulted in three groups: one which was characterized by the fact that they ended the problem session before reaching a solution; a second group which contained participants who completed the attempted problem but with errors; and a final group which included those individuals who successfully completed the problem with the correct answer. The second group was by far the largest, containing nine individuals. One participant did not complete his solution, and two completed their problems with a correct solution.

The next step was to carry out an initial careful reading of the protocols while referencing the written work of the participants and their comments during the post-session interviews. Taylor and Dionne (2000) have suggested that the use of retrospective debriefing data can be used to verify and clarify the researcher's interpretations of the think-aloud protocols in situations in which the researcher must draw inferences from the data. In order to take maximum advantage of the information gathered in the debriefings and to facilitate further analysis of the protocols, the statements made by participants during the interviews were tabulated next to the sections of the protocols to which they applied. Tabulation of post-session interview questions and answers next to the sections of the protocols to which they referred allowed for cross-referencing during the second level of analysis in situations where additional information was needed in order to interpret the actions taken by the participant. The primary result of this second reading was to find that there were two broad sub-categories within the group who completed the problem with errors: those participants whose errors were a result of conceptual difficulties, and those who made primarily procedural and/or mathematical errors. This observation parallels that made by Chi, et al. (1989), who found that errors occurred either as a result of math errors or of what they called inference errors, which were essentially references to physics concepts applicable to the problem. In most cases, determination of type of error was inferred easily from the actions and verbalizations of the participants. For example, after carrying out some initial calculations, Betty realized she had forgotten to take into account the force of friction:

> "but I have to account for friction… friction not going to count on this [referring to diagram] so it's going to oppose the motion so the force the motion is like this [indicating direction with pencil] this will have… so the force backwards… the *unintelligible* energy… is going *unintelligible* up to a point… how does friction come in… so kinetic energy minus the force of friction is going equal to the potential energy… it should…"

Betty proceeds to write the equation she verbalized, subtracting the force of friction from the kinetic energy, and completes the problem. Her conjecture that it is the *force* of friction that is subtracted from the kinetic energy rather than the *work done* by friction has the appearance of a conceptual error. In comparison, Arnold also applies energy concepts to the solution of his problem, but to a section of the problem with no friction:

> "[writing an equation] one half m v squared… equals m g h… cause that's where its coming from… all right… that cancels that [canceling mass terms]… v equals the square root of two g h… all right h in this case is going to equal… sohcahtoa …we have opposite over hypotenuse sine three hundred sine five degrees… so the change in elevation is twenty six point… one meters… so the change in velocity… square root of two… two times nine point eight times twenty one point six… change in velocity is going to be twenty two point six meters per second so final velocity is the initial plus the

> change which is going to be forty four point six meters per second before it goes up the incline…"

Arnold verbalizes that he is calculating a velocity, takes the square root of that velocity, and adds it to the initial velocity. In other words, he is treating the velocity in his equation as a Δv, and in fact later refers to it as a change in velocity and writes: $\Delta v = 22.6 \, {}^{m}\!/_{s}$. However, the correct expression is not $\frac{1}{2}mv^2$ as he has written, but rather $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$. Arnold appears to understand the physical principles he needs to solve the problem, but makes an error when he applies those principles. One might argue that it was a conceptual error in his first statement that caused his difficulties, since it ignores the initial kinetic energy that should be with the potential energy. However, Arnold appears to understand the transfer of potential energy into kinetic energy – "cause that's where it's coming from" – and he both verbalizes and writes Δv, suggesting that his original intent was to calculate the change in velocity. As a result, Arnold's error was categorized as primarily procedural/mathematical.

The example of Arnold's error illustrates one of the difficulties of carrying out this analysis. Occasionally it was not clear whether an error was conceptual or procedural/mathematical. Four of the five participants who exhibited conceptual difficulties appeared to have partial conceptual understanding, suggesting a transitional stage in developing understanding (Maloney & Siegler, 1993; Mildenhall & Williams, 2001). For those participants, it was often difficult to determine the difference between a procedural difficulty and a conceptual error. Sometimes those differences were clarified in the post-session interviews, such as when Beth described why she multiplied an expression for gravitational potential energy by the coefficient of friction ("because that's where the friction is"). Other times the determination had to be made from the context of the protocol itself. Taken out of context, Arnold's first statement might be categorized as a conceptual error. Similarly, participants would occasionally make both conceptual and mathematical errors simultaneously at a given point in the solution process. This required the researcher to take the error made in the context of the statements and actions taken by a participant, and to make a judgment as to the primary difficulty experienced by the participant. The type of error inferred by the verbalizations and actions carried out could often be supported by statements made in the retrospective debriefings. Because of the difficulty of judging the type of error, three random protocols in which the participant made errors were provided to a second physics instructor who was not otherwise associated with this research. The classifications made by the independent reviewer were in all cases in agreement with the judgment of the researcher.

Because the type of errors made seemed to be an important difference in the protocols of the members of the second group, the group was divided into two subgroups: Group A, whose members made primarily procedural and/or mathematical errors, and Group B, containing those who made primarily conceptual errors. It was also determined that the participant who did not complete his problem had considerable conceptual difficulties. As a result, that participant was included in Group B. Finally, it was noted that one member of the original group of participants with errors had completed the problem correctly down to the next to the last line, at which point he incorrectly copied a number from his calculator. Because his solution would have been correct if not for that minor error, he was moved to Group C with the other participants who had completed the problems correctly. This inclusion of a participant with a minor error in the group with no errors is in agreement with steps taken by other researchers in the analysis of student problem solving (Leone & Gire, 2005). The final composition of the groups used for further analysis is seen in Table 2, where it is seen that for easy reference each participant has been assigned a pseudonym which corresponds alphabetically to the group to which they are assigned.

Table 2: Group Definitions and Membership

Group	Defining Characteristic	Members
A	Mathematical/Procedural Errors	Alex, Andrew, Arnold, Art
B	Conceptual Errors	Ben, Beth, Betty, Bob, Brittany
C	Problem completed correctly	Carl , Chuck, Cory

The next step was to see if the members of the three groups had any problem-solving characteristics in common. The protocols were carefully reviewed, and the basic activity being carried out by the individual at each step was noted. No attempt was made to analyze or interpret the actions of the participants at this point. Instead, the listing of steps included such things as reading the problem statement, drawing a diagram, writing an equation, and so on. This process is illustrated by the protocol of Beth, which can be seen in Appendix E. To verify an accurate identification of the steps taken by each problem solver, the step analysis was also carried out by an independent reviewer on a randomly selected protocol from each group. The steps listed by the reviewer were largely in agreement with those listed by the researcher, even when different terminology was used. For example, the researcher listed the following steps taken by Alex at the beginning of his protocol: refers to problem statement, gathers info, and categorizes. For the same segments, the reviewer listed reads problem, underlines givens, and decides on momentum.

The step analysis of the protocols revealed that there were a number of actions that were universally carried out by all the participants in this study. These steps included reading the problem statement, drawing diagrams, writing equations, and using a calculator. Other steps were not universal, but were carried out by the majority of participants, such as stating the goal or unknown, checking work, and stating the type of problem. Because there were several steps that were almost universally carried out by the participants, the important part of the analysis was not that they occurred, but rather the manner in which they occurred. Was a diagram used for the construction of equations that helped support the solution to the problem? Or was the diagram simply used as a method of illustrating the quantities given in the problem statement? It is the answers to questions such as these that characterize the three groups of protocols.

Group Characteristics

After completing the step analysis for each participant, a list of the characteristics of each protocol was created. Included in this list were such items such as whether or not the participant checked his or her work, stated goals or unknowns, referred to formula sheets or books, and so on. We continue with the example of Beth by showing the characteristics of her protocol in Table 3.

Table 3: Characteristics of Beth's Protocol

Implicit categorization of problem
Checks work
No explicit statement of goal(s)
Makes primarily forward progress
Does not refer to formula sheet, book or problem statement during solution
Writes both symbolic and numeric equations; solves only numerically
Makes occasional reference to her diagrams
Diagram is not fully used to construct the equations used in the solution
Pauses at several points to consider her approach to the problem

Several of the characteristics shown in Beth's protocol require further explanation. Categorization has been shown to be an important step in problem solving (Chi et al., 1981; Jong & Ferguson-Hessler, 1986; Savelsbergh et al., 2002; Snyder, 2000). At this level of analysis, no attempt was made to determine the basis from which categorization took place; instead, the analysis focused simply on determining whether or not some form of categorization took place. Participants in this study who categorized the problem they attempted did so in one of two ways. The first was an explicit statement of the type of problem, as exhibited in the protocol of Chuck:

> "all right… so maybe we should do this from an energy perspective I think we should say that… this part where Jill goes into the jumps in the sled could say that… momentum is conserved so…"

Chuck has clearly and explicitly stated how he will approach the problem, and he carries through on the categorization in his solution. The second way in which categorization was exhibited was through the process of selecting a solution method. Participants did not explicitly state the type of problem, but started solving the problem with a chosen approach and continued with that approach throughout their solution. This form of implicit categorization is seen in Cory's protocol, when he says "OK… so first we have the truck… which is going down slope it has the weight force… do the free body diagram of the truck…" and then follows up a short time later with "f net on the x axis equals the mass times the acceleration." Although Cory does not explicitly state "this is a Newton's second law problem," he draws a free body diagram, applies the mathematical statement of the second law, and carries through on this approach to the end of the problem. This inference is supported in the post-session interview, when he states that he "started drawing the free body diagram" and that "the force way of looking at it" would be the best approach.

All the participants in this study drew diagrams as part of their solution process, but how they used those diagrams varied considerably. Some participants drew a diagram at the start of their solution, but did not refer to it at all during the solution process. Other participants referred to the diagram to show directions of motion, to indicate components of displacement, or to determine angles. At the highest level of use, the diagrams were used to help construct the equations that were used to solve the problem. This was evidenced by direct correspondence between the quantities labeled on the diagram and the quantities included in the equations, by the participant directly referencing the diagram as the equations were written, and was supported by the verbalizations of the participants. Use of diagrams to construct equations was found to be a universal characteristic of the Group C participants, and by the Group A participants, although not as obviously so. Similar observations were noted by Rosengrant et al. (2005), who found that the most successful problem solvers were able to use their diagrams to help construct the mathematical representations (equations) used to solve the problem.

Another characteristic of most of the protocols, including Beth's, is one or more pauses in the solution during which the participant would consider his or her approach to the problem. The interlude during which the participant considers the approach is not a pause in the strictest sense of the word, since there appears to be active processing taking place. Perhaps the best way to describe this pause would be a search for, or verification of, an understanding of the problem. It is a rather complex characteristic, involving a number of activities. During a pause, the participant might consider the type of problem, what equations might be used, or whether an approach other than the one currently being used might be better. Sometimes the search would lead a participant to refer to a book to consider an example or look for an equation, while at other times he or she might review what has already been done to see if the physical situation warrants the approach taken. In the case of Beth, a significant amount of time is spent in the search for an approach to the problem:

> "ok so I know its initial kinetic energy is that going to help… no it's not it doesn't matter I know its initial velocity then it slides down the slope for this given period of time ok so [writing an equation] velocity equals distance times time I don't know time… I do know d I do know v I can find t ok… traveling down the slope does that matter… no… it shouldn't then it goes up then it's going back up it's going back up the ramp... does that matter kinetic ener... should I use kinetic energy I already know v [referring to diagram]… I already know v the velocity's changing I have to find how far along if I know the vel… the velocity that's not going to help me…he applies the brakes wait he applied the brakes velocity's changing… (sighs) applying the brakes doesn't do… because his truck starts sliding he doesn't actually slow down so velocity is still the same…"

During this interlude Beth appears to be trying to make sense of the problem as she refers to her diagram, states her unknown quantity ("I have to find how far along…"), and refers to her own embodied knowledge, all while questioning herself about which solution method she should use ("should I use kinetic energy…"). It is interesting to note that the members of Group C, who solved the problem correctly, spent only minimal, if any, time considering the approach they would take. In contrast, the members of Group B, who exhibited conceptual errors, in general spent a great deal of time searching for an understanding that would lead to an approach to the problem. Observations such as this led to the justification for determining the characteristics of each participant's actions.

The reason for determining the primary characteristics of each participant's actions was to determine whether or not there was a common list of characteristics for each of the groups of protocols. While there was considerable overlap in the actions taken by the participants regardless of their success with the problems, there were certain features that seemed to be more characteristic of each of the groups. These group characteristics are seen in Table 4.

Table 4: Group Problem-Solving Characteristics

Group	Characteristics
A	Frequent checks of work Steady forward progress Diagrams used to construct the equations used References to formula sheet or book for checking equations Occasional to frequent pauses to consider approach
B	Infrequent or no checks of work Occasional backward progress (starting new approach or 'going back') Diagrams not used consistently to construct the equations used Frequent references to formula sheet or book for solution approach Frequent pauses to consider approach
C	Frequent checks of work Continual forward progress Diagrams used extensively to construct the equations used Few if any references to formula sheet or book Few if any pauses to consider approach
All groups	Explicit or implicit categorization of problem Explicit or implicit statement of goal(s) – Group C all explicit Use of both symbolic and numerical equations

Group A: Errors in solution; concepts ok
Group B: Errors in solution; concept errors
Group C: Solution correct

Although the primary purpose of this study was to test the applicability of the stability model of problem solving, a secondary purpose was to determine at what points in the problem-solving process students have difficulties. If students making similar categories of errors have similar characteristics, that information could be used to inform the development of a framework for physics problem-solving instruction. It is important to note, however, that within the groups there was still variation in the actions undertaken by the participants. The group characteristics should be considered as broad descriptions within a continuum of problem-solving behaviors.

Not every Group A participant, for example, exhibited exactly the behaviors listed as characteristic of Group A. However, each Group A participant had more in common with other Group A participants than he or she did with members of the other two groups.

Several features of the group characteristics are worthy of further discussion. First, there were several characteristics that were found in the protocols of all participants. All participants in this study drew one or more diagrams as part of their solution. This is not surprising given that the use of diagrams, including sketches and more domain-specific diagrams such as free body diagrams, are an integral part of physics instruction. Recent work in the area of multiple representations has raised questions about the efficacy of diagrams in helping students become better problem solvers. Rosengrant et al. (2005) found that while use of a free body diagram or other diagrammatical representation was correlated with a higher probability of reaching a successful solution, use of a diagram did not guarantee success. In fact, they found that in many cases an incorrect diagram was worse that not using a diagram at all. As noted by Larkin and Simon (1995), it is not enough to draw a diagram; the diagram must be correct, and one must have the procedural knowledge required to connect that diagram to both relevant physical principles and the mathematics required for the solution.

Another characteristic found in the protocols of all participants was the explicit or implicit statement of the goals or subgoals of the problem. This is an interesting feature, given that the problems used in the study were in the format of typical textbook problems with clearly stated goals; that is, they included statements such as “how far” or “how fast” as part of the problem statement. However, all of the problems required more than one step for their solution, so that there were subgoals that must be attained in order to reach a final correct answer. The recognition of a goal or subgoal has been found to be an important factor in problem-solving success (Sweller, 1983). When a goal is recognized, it acts as a control mechanism for the problem-solving process, guiding the problem solver in the choice of solution method. This would be especially important for novice problem solvers, who have been found to frequently use means-ends analysis or search procedures to solve unfamiliar problems (Chi et al., 1982; Dhillon, 1998; Heyworth, 1999; Larkin et al., 1980a; Savelsbergh et al., 2002).

Although all participants made some sort of statement of goal or subgoal, the manner in which the statements were made varied according to group. Group C participants, for example, all made at least one explicit statement of a goal during their problem statement, as seen in the protocol of Cory:

> "Now we're gonna try to find the final velocity… ca…because this velocity here [referring to diagram] but we're trying to find the final velocity at the bottom…"

In this case Cory is referring not to the final goal of the problem, but rather a subgoal that he must reach in order to continue with his solution. In contrast, most of the goal-related statements made by the members of Groups A and B tended to be embedded in other verbalizations, if they were made at all. This type of implicit statement is illustrated by Brittany as she tries to determine what solution method she will use:

> "he's traveling down the mountain so gravity's pulling down… how far does the truck go… before coming to a stop… maybe I want to use kinematics on this…"

Brittany appears to be making a statement of the ultimate goal of the problem, but it seems to be made almost as an aside, as she is reviewing the physical characteristics of the problem and considering possible approaches to the problem.

An unexpected feature of the protocols was that nearly all participants made either explicit or implicit categorizations of the problem they attempted. There is a large body of literature suggesting that novice problem solvers are generally unable to make an appropriate categorization, or that they make categorizations solely on the basis of the surface features of the problem (Chi et al., 1981; Chi et al., 1982; Cummins, 1992; Larkin et al., 1980a; Medin, 1989). Despite the many studies showing the difficulties that novices have with categorization, there is some evidence suggesting that novices *can* make appropriate categorizations and statements about applicable physical principles (Chi et al., 1989; Savelsbergh et al., 2002). This does not imply that they know how to proceed once they make an appropriate categorization, since categorization must be coupled with the necessary structural and procedural knowledge to complete the problem. The actions of the participants of this study seem to support this notion. However, caution should be applied in interpreting the protocols in this way. Only seven of the 12 participants made explicit statements of categorization, such as "collision that means I want to use momentum to solve this" or "so then we probably need to use Newton's second law in order find… ah the acceleration." One participant provided no indication that the problem was categorized, while for the remaining four participants categorization was implied by the actions taken during the solution process. This would include, for example, statements such as "do the free body diagram of the truck" followed by calculations of net force, or by observing that the participant started to solve the problem from an energy perspective and carried that choice through to the solution of the problem. It is unclear whether this implicit categorization would satisfy the criteria required as indication of categorization as referenced in earlier studies, or if the participants, when asked to state a category or type of problem, would have been able to do so. There is also only minimal evidence as to the basis used by the participants to make their categorizations.

Case Studies

The characteristics of each group are best understood in the context of the protocols themselves. In order to provide the best possible understanding of the problem-solving experiences of the participants, this section will present an exemplary protocol from each of the three identified groups. In addition, an exceptional case from Group B will be provided as a counterexample. The protocols will be integrated with explanations provided by the participants in the retrospective debriefings, and with commentary including description and analysis. While reviewing the exemplary cases it is important to recognize the interpersonal differences in the actions taken by the participants, and to consider the general characteristics of each group. No single participant can epitomize every feature of the general characteristics of each group, but when taken as a whole it becomes clear that the members of a given group have more in common with each other than with the members of other groups. The full partitioned protocols for each of the exemplars, including their written work, can be found in Appendix F.

The case studies were chosen by carefully reviewing the protocols, and selecting a participant whose actions best exemplified the group characteristics identified in the initial analysis. As previously noted, individual differences in problem-solving approaches preclude any single participant as having exactly the characteristics of the group; however, care was taken to select the individual that was most representative. The cases which follow include large segments of the verbalizations of the participants, interspersed with commentary to orient the reader to the features and evidence used to characterize the actions of the problem-solvers. Segments of the post-session interviews are also included when the comments made by the participant in the interview serve to further clarify the actions taken, or to support the inferences made in relation to the actions. Each case study is followed by a brief analysis in which the general features of the protocol are discussed.

Group A: Alex

Alex, who works on Problem 2 (see Appendix A), begins by reading the problem statement out loud, underlining the given values as he encounters them. As he reads, it appears that he is also interpreting the problem. This assumption is supported by the fact that he makes a categorization during this first read-through of the problem:

> "mmm let's see… Jack and Jill are sledding uh-oh angle… on a snow covered hill ok that's important… let's see friction constant… kinetic friction ok it's sledding down a snow covered hill that is inclined here's where I get my angle at an angle of twenty degrees to the horizontal what's that mean… I guess that means the x axis… Jill mass ok we got mass runs at four meters per se… meters per second oh that's a velocity across the top of the hill landing on a five kilogram sled which is at rest ooo… collision that means I want to use momentum to solve this…which at rest at the very edge of the hill her brother Jack stands at rest a distance of fourteen point six meters down the slope as Jill pa Jill passes Jack thirty meters he jumps onto the back of the sled oop two problems… eh two collision problems looks like it…"

As Alex reads the problem statement, he categorizes the situation as first one, then two collisions, and notes that he will use momentum to solve the problem. He also appears to be making an initial assignment of a coordinate system. However, his comment about the x axis foreshadows the procedural difficulties he will have later in his solution. Alex proceeds by listing the quantities given in the problem statement. Although Alex has verbalized an initial categorization of the problem, it does not appear that he has attained a complete understanding of the problem situation or of how he should proceed with his solution. He seems to struggle with the details of the procedures he is utilizing, and expresses doubt about his approach:

> "let's go ahead… hmmm… what do I need to do… I need to figure out the x right this is going to be a two dimensions it's going to be an x dimension and a y dimension now what have I got here what what do I have here I have an angle here… twenty degrees and the radius total distance is forty meters ok… so what I need to do I need to figure out… I need to figure out I don't know what why I'm figuring this out but I think that it's going

to be important I need to figure out… what the actual distance are in the x and the y… ah dimensions so…let's make a ninety degree angle… ninety degrees… degrees… so y is gonna be the sine… and x is gonna be the cosine… so we know ok we take the radius and the radius is forty meters…"

Here we see the beginnings of Alex's procedural difficulties. He knows the problem involves momentum, and states this knowledge several times during the solution process. But his approach suggests that he is unsure of how to handle the fact that the collision is taking place on an incline. The motion takes place entirely along a line, but the presence of the slope prompts Alex to treat the problem as two-dimensional. Even after starting this process, he is unsure about whether or not it is correct. In the post-session interview his uncertainty about this approach becomes even more evident. When asked about his comment "I don't know what why I'm figuring this out but I think that it's going to be important" he states:

> "I was thinking I realized you know what I'm probably gonna need this in a free body diagram somewhere because it's in the free body diagram that I dissect the x and the y… and I knew with an angle as soon as I see an angle I go I hate angles because I have to think I have to sit there and I have to use trigonometry to figure out what the x and the y value is for the forces and I don't like to do that so but… you know I said well you know what I might as well get that started I might as well figure this out and I'll worry about it later…"

Alex has at this point completed his initial diagram and listing of the quantities given in the problem, as seen in Figure 1.

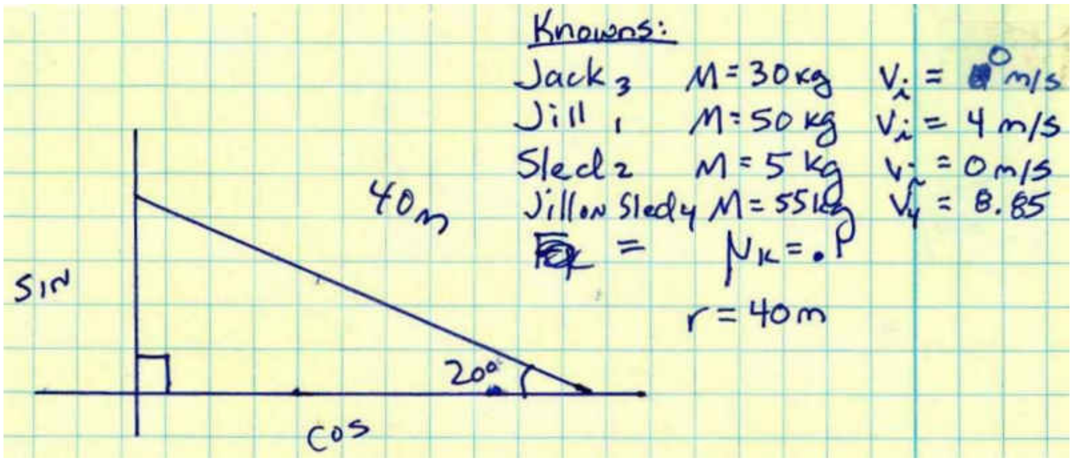


Figure 1: Alex's First Sketch

After completing his diagram, Alex goes on to calculate the components of the displacement down the incline. He evaluates his answer for reasonableness, and then refers to his book for momentum formulas:

> "we know that the momentum's going to be conserved so I know… [writing equation] momentum of Jill plus momentum… the sled… is going to equal the momentum of Jill on the sled ok what's momentum mass times velocity of Jill… so mass mass Jill velocity of Jill times plus mass of the sled times the velocity of sled equals the… mass of Jill and the sled so it's I guess that's gonna be four [referring to the subscripts on the symbols] Jill on sled it's gonna be four the mass of that's gonna be fifty five… and… what's gonna be the velocity… well let's figure that out oh this is gonna be simple…"

At this point in the solution it appears that Alex has settled on an approach to the problem. He seems to believe that from this point the solution will proceed quickly, as evidenced by his comment about the ease of solution. He has not used the components he calculated for the positions, but apparently does not think that he needs them. This interpretation is further supported by his comments during the post-session interview, during which he states that he was very confident that this was going to be a simple problem:

"I thought it was just gonna be a simple at that point I thought it was going to be a simple momentum problem oh I already know most everything I don't have to think I'll just throw in some numbers into the formula and churn away and I'll be done…"

After writing his equation for momentum conservation, Alex proceeds to calculate the final velocity of the sled and Jill, but then stops to consider the friction.

"did I forget… oh I'm forgetting about the coefficient of friction uh-oh… I should have known this is this not as easy as I thought it was… oh hold on wait a second no no no no no no no friction hasn't come into play yet…how far down the hill… is Jack… stands at rest at a distance of fourteen point ah ok… fourteen point six meters down the slope… all right that's another problem now I have to figure out… how fast the sled is going when it gets to Jack…"

After considering the friction, Alex realizes that he has another problem to solve. He recalculates the x and y components of the displacement down this slope, this time using the distance between Jack and Jill as the displacement, and moves on to constructing a free body diagram, seen in Figure 2.

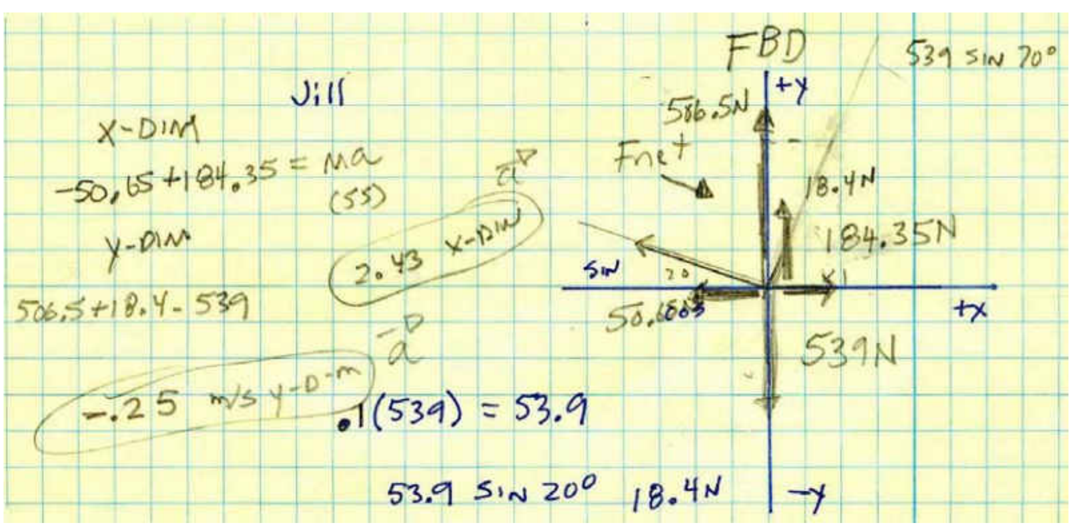


Figure 2: Alex's Free-Body Diagram

It is not clear from his comments why he has decided to do a free body diagram. In the post-session interview Alex provides information suggesting that the diagram was an attempt to find a solution process:

> "I started doing this before I truly realized what I was doing it for why I was doing it but it was just something that ah it's it's like if I get into trouble I run back to the free body diagram and the picture rep representation… so anyway that's ah so I so I ah I think for awhile I was confused I was swimming I was just doing calculations and I didn't know why… and I think it's even at this point I don't know what I'm doing it isn't until I make reference to the formulas… it wasn't until then that I started to feel comfortable with the problem at this point I'm just treading water..."

After calculating the combined weight of Jill and the sled, he completes the free body diagram. It is at this point that Alex makes a major procedural error. The primary difficulty is that he draws his coordinate system in a standard vertical-horizontal orientation, instead of with the x axis corresponding to the slope. This choice, while not physically incorrect, dramatically increases the complexity of the problem:

> "we have… force of gravity five hundred and thirty nine newtons downward and we know…ok this is twenty degrees this way [drawing slope in quadrant II of coordinate system]… and ninety degree angle… that's twenty degrees… and this angle must be… seventy degrees [drawing normal vector]… and we know… that… what do we know… we know that that's gonna equal to the five hundred thirty nine but how much in the x how much in the y this is where I always this is where I always mess up so let's not try to mess up ok let's think about this now… we know that the radius is gonna equal five hundred and no… five hundred and thirty nine newtons ok…"

This approach gives Alex the correct magnitude of the normal force (506.5 N), but his interpretation is that it is the component of the normal *perpendicular to horizontal ground*, with an *additional* horizontal component of 184.4 N. Further evidence of his belief that the normal force is equal in magnitude to the weight is seen when he calculates the friction force using the weight of 539 N:

> “ok I need the total amount ok it’s the coefficient… friction times the the normal force so that coefficient what was the coefficient point one… equals point one… so then point one times five thirty nine newtons… ok.”

Alex proceeds to calculate the x and y components of the friction in his coordinate system. After completing the free body diagram, he seems to lose track of where he is going with the problem:

> “all right now I’ve got my free body diagram forces… what do I care… what do why did I do all that what’s the point… I’m lost what am I doing… ok… time to do a pictorial all right pictorial may save me we’ve got a hill… it’s going down twenty degrees… here to here… here’s the sled… and Jill… that was a momentum problem we solved that… now we have to get to Jack… ok all we have to do to get to is get to Jack once I get to Jack then Jack becomes a momentum problem… and then get to the bottom of the hill and that’ll be oh my god it’s four problems… oh… this is hard… ok let me think about this…”

At this point Alex is realizing the complexity of the task before him. During the post-session interview he states “I’m obviously I’m panicking… I’m realizing that I now have four problems to solve…” He recovers fairly quickly, however, and moves to find an approach to the solution:

> “well if you find out ah… I need the acceleration once I know the acceleration then I can figure out what the velocity is when they get to Jack all right that’s what the free body diagram is for second law calculations and of course there’s an angle so I have to figure it out in the x and the y directions…”

He proceeds to calculate the x and y components of the acceleration. After completing these calculations, he briefly considers “putting them back together… to get the acceleration in the direction of the twenty degrees…” but instead calculates the components of the velocity in the x and y directions of his coordinate system. He still seems unsure of where he is going with the problem, and once again verbalizes an attempt to find an appropriate approach to the problem:

> “did I figure this out right… let me look at the kine you know what let me look at the kinematic equations… that’s what that’s what I’m messing up I just wasted all this time well maybe not let me think ok… we know that… ok here’s what we need to do… ok

what am I figuring out what do I need ok it's a collision problem once we get so we're gonna need the mass and the velocity that's what I need once I get to Jack how do I figure out the velocity… ok I know what the acceleration is… I know what the positions are… I know what the… need the final velocity and I know what the initial velocities are ok all right… [referring to his calculated values] this is the acceleration in the x dimension this is the acceleration in the y direction these are accelerations ok… these are velocities in x and y directions all right let me think about this now… where is… ok here are the distances these are what I need for the position ok all right here we go we'll use this one…the formula I'm gonna use velocity final squared equals velocity initial squared plus two times acceleration of the… x final plus the x initial ah oh ook I need… I I need on my pictorial diagram I need a coordinate system…"

Alex draws a sketch, seen in Figure 3, showing the slope and its horizontal and vertical distances above his original slope diagram, then uses his chosen equation to calculate the x and y components of the velocity of the sled just prior to the second collision.

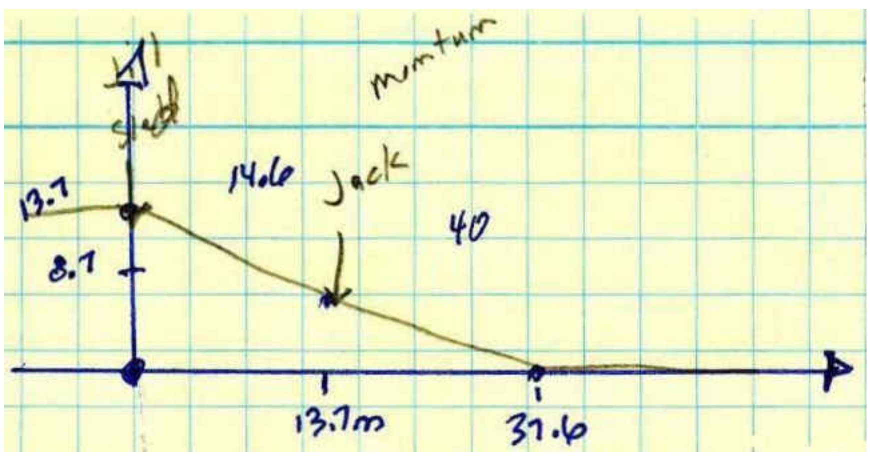


Figure 3: Alex's Second Sketch

Alex appears to run into difficulty for a short time when he calculates the y component of the velocity:

"all right take the squ oh no that's gonna be… an imaginary number… what have I done something wrong… acceleration is that a scalar or a vector… it's a vector… what have I done wrong I've messed up on the signs again oh… that kills me... signs kill me… is it important yes… [long pause]… what am I doing what should I do now I need to I need to go and I need to walk through this and I need to figure out… why it's not a negative… ok let me think about this now… how did I get… point wh how did I get the acceleration… I got the acceleration from my free body diagram and the second law pairs… and that makes sense because it's accelerating down right… ok so that's correct all right now… the velocity the velocity should be a negative… ok where's the velocity how'd I figure out the velocity velocity came from here [referring to calculations]… ok… ok… velocity I figured out the velocity… I should have used… three hundred forty degrees instead of twenty degrees… what would have happened how would that would have looked… if I had done that… ok three point six five times the cosine three forty degrees is gonna equal… right but… the cosine… what's going on use the sine sine for the y… negative ah ok… that's where I made my mistake cause it's going downhill all right now what did I use that for what did I use that for… so velocity… it doesn't matter… it doesn't matter… ok… five here… we have this… well… maybe it's not important I know it is but… I just I gotta keep going ok let me think about this so…"

What Alex actually has done is reversed the initial and final velocities in his equation; however, he does not find this mistake until later in the solution. He proceeds with his solution despite the fact that he has not resolved his sign problem. After referring to the book to see how to put the components "back together," he calculates the magnitude of the final velocity. After a quick evaluation of the magnitude of the answer, Alex proceeds with the solution. In comparison to the first part of the problem, the remainder of the solution goes fairly quickly. He begins by considering the collision between Jack and the sled:

"ok now the second part of the problem the third no this is the third part of the problem momentum… ok we've got Jill on sled… that's eight point… eight point four right… ok we don't need we it's a momentum problem I don't nee I don't care about the angles…"

Despite the fact that he feels two dimensions are needed for the velocity and acceleration portions of the problem, Alex is not concerned about the angle when working with momentum conservation. During the post-session interview he states:

"at that point I'm thinking hey I never did the angles in the first problem… and then I realized that that was one of the nice things about the momentum you don't care about

> the angles… I don't know why but it just popped in my head that I didn't need to know… what what the angle was because oh I know because I'm… it's it's it's just a one dimensional problem…"

It is not clear, either from the protocol or the interview, why Alex believed the angles were important for velocity and acceleration but not for momentum. He also provides no insight into why he considered the motion down the hill to be two-dimensional at the same time he considered the collision to be one-dimensional. This uncertainty seems to be at the root of his procedural difficulties. With an appropriate choice of coordinate system for his free body diagram and the kinematics calculations, the entire problem would have been one-dimensional, and the complexity of the problem would have been reduced dramatically.

From this point Alex's solution parallels that of the first half of the problem. He calculates the x and y components of the velocity, draws a free body diagram, and calculates the acceleration in the x and y directions. It is while calculating the final velocities in the x and y directions that Alex realizes his earlier mistake with the sign of the velocity:

> "all right focus back to the formula… initial velocity here is the initial velocity x direction the initial velocity is five point three five squared plus two times two point four three ah here we go initial position is going to be… hold on it's final position final position first…"

He corrects his mistake, and then goes back to his earlier work:

> "my calculations for the second problem may be wrong let's go back to that… those silly signs… oh no it's on this page… signs… this is reversed [referring to the initial and final positions in the calculation of velocity] this should be here this should be here… that makes sense ok… all right… I feel so much better now that I know what's going on with the signs… I inverted the positions… all right… ok no problem… whew… it's not going to cascade through the work all right everything I've got is fine so far is ok…"

After checking that the error did not propagate through his solution, he calculates the magnitude of the final velocity, reviews the problem statement to make sure he has answered the question, and declares that he has the answer.

Like many of the other participants in this study, Alex evaluated his work at various points. His evaluations frequently support the interpretation that his conceptual knowledge was fairly well-developed. Evidence for this lies in the fact that he assessed not only the magnitude of the numbers, but the signs and physical interpretations as well. For example, when calculating the acceleration in the y direction, he obtains a negative answer:

> "does that make sense… yea cause we're going downhill… ok so that would be the negative y direction ok so does this makes sense yes…"

Another example that suggests a reasonable level of conceptual understanding comes after Alex calculates the velocity of the sled after Jack jumps on:

> "my god… that slowed 'em down quite a bit… does it make sense… the mass increased… so the velocity had to shrink… it had to… ok…"

The apparent contradiction between Alex's verbalized conceptual understanding and the convoluted path he takes to reach a solution to the problem is in agreement the results reported by other researchers concerning the conceptual knowledge of students. In particular, it has been noted that students frequently have sufficient declarative conceptual knowledge, but lack the procedural knowledge needed to successfully solve problems (Chi et al., 1989; Chi et al., 1981; Savelsbergh et al., 2002). As noted by Hoellwarth et al. (2005), conceptual understanding does not necessarily equate to effective problem solving. Alex appears to be an exemplary example of this assertion.

It is interesting to note that despite the unnecessary complexity that Alex injects into the problem by his choice of coordinate system, he gets an answer that is numerically correct. Indeed, if no mistakes had been made he should have obtained the correct answer, since the coordinate system was not physically incorrect, just unnecessarily complex. Alex makes an error in assuming that the normal is equal to the weight in his calculation of friction, which is primarily a conceptual error. However, because the coefficient of friction is so small the error is correspondingly small. Later in the problem, in the calculation of the final velocity in the y direction, he makes a math error, but once again the magnitude of the error is small. The overall result is that Alex's answer is correct to the first decimal point, which is within the limits of the significant figures appropriate to the problem.

Alex's protocol in general exhibits all the characteristic features of Group A participants. Although he makes one conceptual error in understanding the normal force, his primary difficulties appear to be procedural. He makes steady, although slow, progress in his solution. The mathematics of Alex's solution are fully supported by the diagram he draws, even though his choice of coordinate system increases the complexity of his solution. Alex makes frequent checks of his work, checking the magnitudes, signs, and physical sense of calculated quantities. He also uses his formula sheet to check equations, but not as a search for a solution process. Finally, he pauses frequently to consider his approach to the problem.

Group B: Brittany

Brittany, who works on Problem 1 (see Appendix A), begins by reading the problem statement and writing down the given information as she encounters it. She then draws a sketch showing the truck and the two slopes, which she labels with the given information. At this point she refers to her textbook:

> "ok… mmm… ahm… I'm trying to find a formula that fits… the uhm problem I'm given mass velocity… and I got angles… so I need to find… a uhm… sort of a… a collision… problem… that's two dimensional collisions… I like to look at examples they help me the most… mmm… I can't exactly find what I'm looking for… [pause]… I'm confused… the problem is there's these problems [referring to examples in the book] don't involve friction and it's based on a frictionless surface and this one involves… friction and I have to find a formula that involves that ah there we go… ha found it…"

Brittany does not seem to understand what type of problem she is dealing with, and refers to her book for help. Apparently cued by the given information of mass and velocity, she starts by looking in the chapter on collisions. She states that she is confused, because the examples she refers to do not include friction. She turns to the section on friction and states that she has found what she is looking for. In the post-session interview Brittany states that she was looking for "a formula that involves friction or an example that I could look… and compare." She also indicates that she likes use examples. When asked during the interview how she knows if she has the right example, she says that she "[writes] down what is given… compare to what they have and I kind of…work around it." In reference to this problem, she also states that she was looking for how to do a free body diagram, which is the next step she takes in her solution:

> "ok…let's do the diagram… truck is traveling down a mountain road when it hits a thick patch of ice…in a panic the driver hits the brakes which…without friction down the slope… and it's a five degree slope… that's what it's traveling on… it's without friction… it's five degrees… so you have a normal force… and we have m g sine theta…

> that's theta… and this is m g… and there's no friction back so… mmm… m g cosine theta… and… since there's no friction we just solve for m g sine theta…"

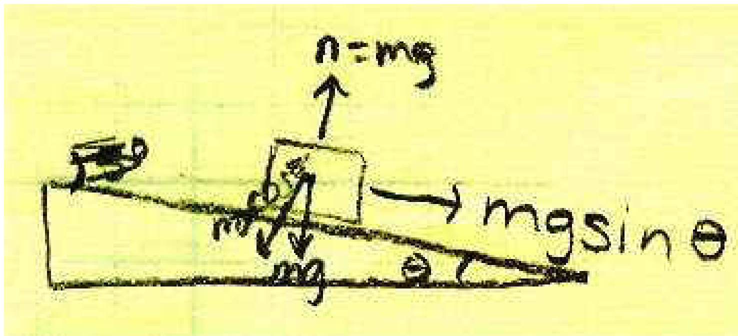


Figure 4: Brittany's Free-Body Diagram

Brittany's sketch (see Figure 4), which was copied from an example in her book, is technically not a free body diagram, but it does show the forces acting on the truck. However, it appears that she has indicated the total weight force rather than the cosine component as directly opposing the normal force. She determines that the force down the incline is mgsinθ, although she has obtained this information from the example she is referring to in the text as she writes the equation, not from the diagram she has drawn. She writes equations for the sum of the forces in the x and y directions which are symbolically correct. The value of the force in the x direction is calculated correctly, but she makes her first error in calculating the y component of the force:

> "and the normal force is m g… and the total forces in the y direction… are… uhm [referring to example]… n minus m g cosine theta… so fifteen hundred kilograms times ten meters per second minus fifteen hundred kilograms times ten meters per second times cosine of five degrees…fifteen hundred times ten… one five zero zero zero minus… times ten times cosine five… oops… which equals four nine four two point nine two… minus one five… fourteen point… equals fifty seven point oh eight…. and that's equal to m a… m a y equals zero… ok"

Here it is seems that Brittany is following the example in her book without considering its relevance to the problem she is solving. She states that "the normal force is mg" and proceeds to calculate a value for the net force using the normal as mgcosθ, and then writes:

$$\sum F_y = 57.08 = ma_y = 0.$$

She apparently does not understand why mass times acceleration in the y direction is zero, and does not recognize the mathematical contradiction in her written equation.

At this point Brittany refers to the problem statement again and draws a new diagram (seen in Figure 5) showing the two slopes and the given information.

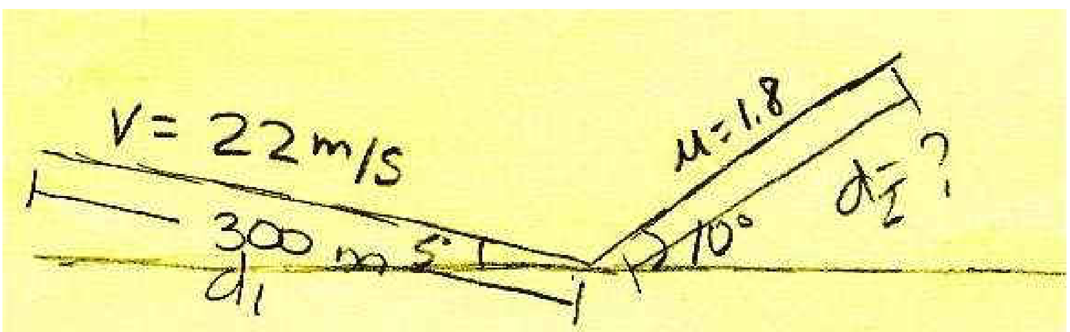


Figure 5: Brittany's Sketch

Brittany then enters into a lengthy verbalization which appears to be an attempt to make sense of the problem and to find a way to solve it:

> "f equals m a [writing equation]… in the x direction… huh… I just now realized I have a distance [erases f = ma]… and I need to find distance… hmm [referring to an example in the text of an object on a slope]…oh that could *unintelligible* that doesn't help… he's traveling down the mountain so gravity's pulling down… how far does the truck go… before coming to a stop… maybe I want to use kinematics on this… I think they have one of those… nice tables… ah there found them [referring to table of kinematics formulas in textbook]… finally… uhm… and I have distance… x i we're not really given position so that doesn't work… [writes d_2 = ? on diagram] distance two is distance one… I got the

> two mixed up its initial velocity equals twenty two meters per second… its final velocity will equal zero meters per second… its initial distance… three hundred me oops meters… and it wants the final distance... ok … uhm… I'm gonna assume… ok… yea… now I'm just trying to figure out which formula I need… now that I finally realize what I have… uhm… the first slope angle one five degrees angle two is ten degrees… uh-huh and we have mu one point eight on the second slope… don't need time… could find … no… no… don't have acceleration don't have time… acceleration's… the key [referring to textbook]... velocity… acceleration (*unintelligible*) constant acceleration… acceleration… do they not have… force of gravity is m g… x direction… equals that [circles answer for sum of forces in x direction] uh-huh… have that… huh… now I need the distance distance… I've got all that… I need to find distance…"

Brittany appears driven by her attempts to find the correct formula, an interpretation supported by her comments during the problem session and the post-session interview. She repeatedly states that she is looking for a formula, refers to equations in her book, and when asked about the use of formulas in the retrospective debriefing states that she "think[s] what I have that can just fits and see if it works… so… I mean I just look at my symbols and see if it matches up with their symbols."

After referring to an example showing an object on a slope in the section on kinematics, and to the table of equations, Brittany indicates that she has found the formula she wants:

> "ah-ha found it [writing equation] distance equals one half of a x t squared… and a x is given… g sine theta… ok…. uhm… I'm gonna use the equation [writing equation]… v x of that squared equals two a x d… so that one's gonna be zero equals two x d… and that a x is gonna be ten meters second times sine of ten degrees ten times sine ten degrees… one point seven… four… um-hmm… so two times one point seven four distance…"

At this point Brittany's conceptual and mathematical difficulties become apparent. The example she refers to shows an object sliding from rest down an incline. The only information from the example that is relevant to the current problem is the acceleration formula ($a_x = g \sin\theta$), which would give the acceleration of the truck down the first incline. Brittany uses this formula and correctly calculates the acceleration, but then applies it to the upward incline. She states that the final velocity of the truck is zero, and carries out the following sequence of steps:

$$
\begin{aligned}
v_{x_f}^2 &= 2a_x d \\
0^2 &= 2a_x d \\
&= 2(1.74)d \\
d &= 3.473
\end{aligned}
$$

There are two major errors here. First, Brittany has applied the acceleration appropriate to the downward slope to the upward slope. Second, she writes an invalid mathematical expression. One might interpret this as a careless mistake if not for the steps she takes next:

> "that didn't take that's including the friction though… I left that part out… so… ok… scratch that [referring to example with object on level surface]… a x equals negative coeffriction [sic] g ok… ah… hmm… ok… that's the movement [indicating direction on slope]… so that's gonna equal mu of one point eight times ten meters per second… and that equals negative one point eight times ten negative eighteen… and uhm *unintelligible*… v of x final equals two a x d that'd be zero squared equals two times… negative eighteen and that's times d negative thirty six… d equals thirty six… oops finally…"

Brittany refers to another example in the book, this time one which shows an object on a level surface being acted on by friction. She takes a formula from the example $(a_x = -\mu g)$ which is inappropriate for an object on an incline, and applies it to her problem. When she uses that expression to calculate the final position of the truck, she makes exactly the same mathematical error as in her previous attempt at the solution:

$$
\begin{aligned}
0 &= 2(-18\, {}^{m}\!/_{s})d \\
-36 &= d \\
36\,m &= d
\end{aligned}
$$

Not only does Brittany fail to recognize the mathematical impossibility of her equation, she does not seem at all bothered by the negative sign. She simply does not include it in her final answer.

Brittany appears to be hindered by both conceptual and mathematical difficulties. She relies heavily on formulas and examples to help her solve problems, but as a result of her conceptual difficulties she does not recognize when an example or formula is inappropriate for the problem she is attempting. Her use of examples is characteristic of the analogical problem-solving behavior noted by other researchers (Chi et al., 1989; Chi & VanLehn, 1991; VanLehn, 1998). She apparently recognizes her lack of conceptual understanding. In reference to a question about examples, Brittany states that in her introductory physics course she "pretty much used the book if you have the book you're fine… sometimes without the use of a book… it's just not very…very you know step by step which I like…" She also states that on "hypothetical stuff I'm stuck."

Brittany's work is characteristic of the Group B protocols. She makes multiple conceptual and mathematical errors, and on one occasion changes her approach and goes back to change a quantity she has already calculated. She takes only two pauses to assess her work; once when she checks that her calculator is in the correct mode, and a second time when she realizes she has forgotten to take friction into account. While her free body diagram appears superficially correct, her actions suggest that is was copied directly from an example referred to in the text. Later, when she refers to examples for formulas for the acceleration caused by friction, Brittany does not recognize that the examples she chooses provide equations that are inappropriate for the problem at hand. She pauses frequently to search for way to solve the problem, and seems to characterize the search for a one-step solution noted in a number of expert-novice studies (Chi et al., 1982; Dhillon, 1998; Heyworth, 1999; Larkin et al., 1980a; Savelsbergh et al., 2002). Brittany refers to her book repeatedly in a search for "a formula that involves friction or an example that [she] could look… and compare." She seems to be an example of the typical novice, using a formula-driven search for the solution to the problem. For Brittany, as for many novices, the equations are the knowledge (Larkin et al., 1980a).

Group C: Cory

Cory, who solves Problem 1 (see Appendix A), starts the problem-solving process by reading the problem statement out loud and writing down given information as he encounters it. He then states that it would be helpful to him to read the problem silently, which he does. Immediately after reading the problem silently, he starts drawing two sketches showing the two slopes labeled with the given information. He moves on to discussing the solution while drawing a free body diagram (seen in Figure 6):

> "OK… so first we have the truck… which is going down slope it has the weight force… do the free body diagram of the truck x axis… the y axis… mm…. mm… five degree slope… have the weight force… the normal force… there's no friction force it's just just sliding…"

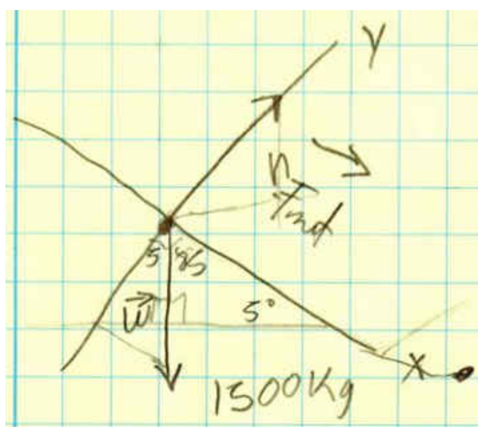


Figure 6: Cory's First Free-Body Diagram

Cory does not explicitly categorize the problem, but his immediate launching into a free body diagram can be interpreted as implicit categorization of the problem as a second law situation. This interpretation is supported by two observations. First, Cory does not deviate from his chosen approach, carrying it through to the completion of the problem. Second, during the post-session interview he notes that he briefly considered work and energy, but then decided that "cause when I knew you had to find the ah acceleration… I started drawing the free body diagram" and that "the force way of looking at it" would be a better approach.

Cory's activities from this point on suggest that he knew where he was going in the problem from the start of his solution. The remainder of the problem-solving session was devoted to alternately calculating quantities, assessing his work, and referring to the problem statement to verify information. This problem appeared not to be a problem for Cory; it was an exercise. He calculates the weight of the truck, finds the components of the weight, and moves on to applying Newton's second law:

> "f net…f net equals…f net is on that in this direction [indicating down the slope]… f net on the x axis equals the mass times the acceleration… which equals… mm… cosine of five degrees… cause it's the opposite… cosine of five degree times fourteen… seven… seven hundred newtons… so cosine whoa… opposite sine… sine of five degrees… sine of five degrees times fourteen seven hundred… equals… twelve eighty one point one eight nine newtons… which equals the f net…twelve… eighty one point one eight nine newtons…. the mass equals…ok talking about the mass not the weight ok … minus fifteen zero zero… kilograms… which equals… times the acceleration equals… twelve eighty one point one eight nine newtons… acceleration equals this over this… twelve eighty one point one eight nine divided by fifteen zero zero… acceleration whoa equals twelve eighty one… twelve eighty one point one eight… fifteen equals… the acceleration equals eight point five four meters per second square… now we're gonna try to find the final velocity… ca…because this velocity here [referring to diagram] but we're trying to find the final velocity at the bottom…"

This segment illustrates the main features that characterize Cory's solution. First, he uses his free body diagram in conjunction with Newton's second law to develop the equations he will solve. As noted in the recent work by Rosengrant et al. (2005), the use of diagrams to construct the equations used to solve a problem is suggestive of well-developed conceptual and procedural resources. Second, Cory carries out on-going evaluation of his work, such as when he notes that he should be using sine rather than cosine for the component of the weight down the slope. Finally, he pauses to verbalize a goal for the next step in his solution: "we're trying to find the final velocity at the bottom…"

The actions Cory takes throughout his solution seem to suggest that he has solid conceptual understanding and well-developed metacognitive skills. After noting that he must find the velocity at the bottom of the first slope, he selects an equation, substitutes in the known values, and calculates the velocity. He then stops to consider the answer:

> "I can tell this answer makes sense because if you're going down the hill and there's no friction you're gonna go from… twenty two meters per second to thirty one meters per second so your velocity increase… that makes sense…"

This type of ongoing analysis of both the physical situation and the numbers he has calculated takes place throughout Cory's solution, even as he is outlining the next steps he will take. After calculating the velocity at the base of the first slope, Cory draws a new free body diagram (seen in Figure 7), calculates the components of the forces and the friction, and then the acceleration.

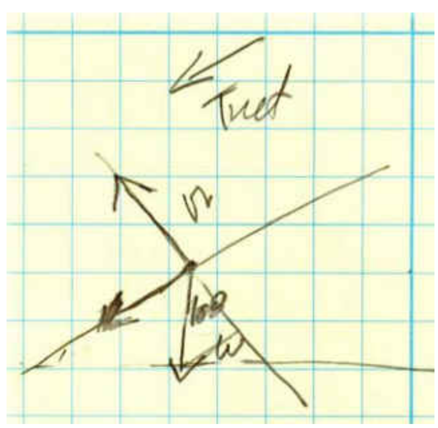


Figure 7: Cory's Second Free-Body Diagram

He pauses to consider his goal and the physical conditions that will allow him to calculate it:

> "all right… we wanna find the final position it say how far along the truck ramp does the truck go before coming to a stop… so if we're trying to find the final position we can use this equation … final velocity square equals initial velocity square plus two acceleration final position minus initial position… final velocity is gonna be zero cause the cars gonna go all the way up and come back down… but the maximum position is gonna comes when the velocity's zero… zero equals the initial velocity… the initial velocity is thirty one point five six six… squared plus two the acceleration… acceleration is negative nineteen point zero seven… initial position is gon the final position is what we're looking for and initial position is zero…"

Cory seems to know what he has to find, how the physical situation relates to the unknown quantity, and how he is going to reach that goal. He quickly finishes the exercise.

In his final evaluation of the answer, Cory refers to the problem statement and briefly considers whether another approach (momentum and impulse) might have worked as well. He rapidly dismisses the idea ("whoa") and declares the problem solved. Cory is the only participant who considered possible alternative approaches, a step which Pólya (1945) identified as a feature of effective problem solving. In the post-session interview Cory notes that he knew he was done with the problem, but wondered whether another approach would have been easier.

Cory's protocol illustrates all of the features of the Group C characteristics. He makes continual progress in his solution, and does not pause to consider whether his chosen approach is correct. His actions and verbalizations seem to suggest that he knew after reading the problem statement what approach he should take to solve the problem. Cory uses diagrams effectively to help generate the equations he uses to solve the problem, and makes no reference to a formula sheet or his book. Finally, he carries out frequent evaluation of his work, considering the magnitudes, signs, and physical sense of the quantities he has calculated. Cory appears to exhibit more expert-like behavior than many of the participants.

Group B: Ben – an Exception

Ben's protocol is included in the cases not because he was an exemplar of his group, but rather because he was an exception. In fact, his actions during the problem solution attempt were so unlike any other participant that the possibility of making him a group unto himself was considered. In the end it was the apparent conceptual inadequacies exhibited in his protocol that resulted in his being included as a member of Group B. Here we consider his actions as he attempts Problem 1 (see Appendix A).

Like all of the participants, Ben starts by reading the problem statement. The first difference noted in what Ben does as compared to the other participants is that he starts a sketch before he reads the entire problem statement:

> "all right a truck with a mass of fifteen hundred kilograms is traveling down a mountain road at twenty two meters per second when it hits a thick patch of ice… all right you need to start by drawing a truck traveling [drawing diagram]… with a velocity of twenty meters… so let's draw a box… truck it's got a mass of fifteen hundred kilograms um is traveling… down a mountain road so we'll just… use a y axis for down [drawing vertical line downward from sketch]…"

The premature start on the diagram results in Ben having to alter his diagram, seen in Figure 8, after he reads a little further in the problem statement:

> "in a panic the driver hits the brakes which fail causing the truck to slide essentially without friction down the five degree slope ok that changes that [erases vertical line, draws slope]… five… equals… five degrees… theta equals five degrees… after traveling down the slope for a distance of three hundred meters…the driver manages to get the truck onto a runaway ramp… so he goes three hundred meters on that axis [labels 300 m on slope]…"

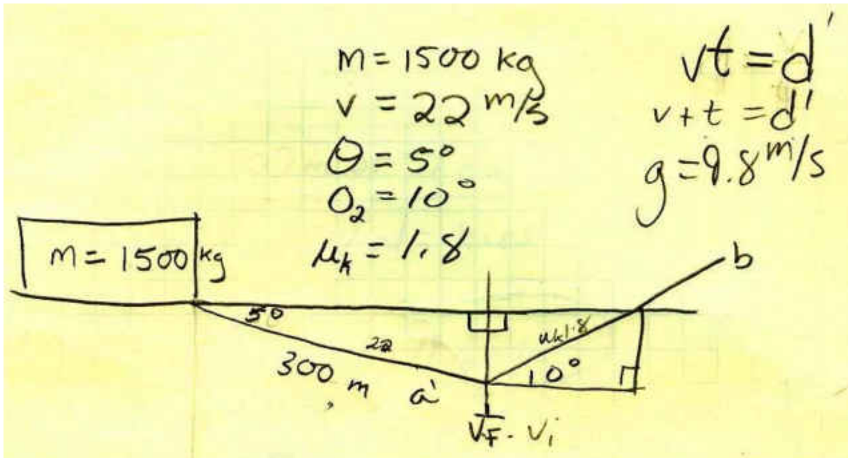


Figure 8: Ben's Sketch

From this point Ben continues reading the problem statement and completes the diagram, which accurately represents the information in the problem statement. He then refers to his notes, and considers the approach he will take with the problem:

> "coefficient of frictions [verbalizing topics as he looks through notes]… force and motion mass… total forces… distance… motion… that's work… kinetic energy… collisions… velocity… harmonic motion… I don't know what happened to… what section was that… friction… there's no friction I want to find… [writing an equation] distance is velocity over time… no… distance over time is velocity… so velocity times time is equal to distance… we've got… two triangles [referring to diagram]… and… given that's ninety degrees and that's ninety degrees and that's a right triangle need to get… the velocity… after he slides to here [indicating common base of slopes]… and then… using this coefficient of friction and the velocity…"

At this point it seems as if Ben has a general plan for the problem, as he gestures at the base of the slopes and states that he will "need to get… the velocity… after he slides to here… and then… using this coefficient of friction and the velocity…" The apparent solution process that this verbalization suggests is valid, but he seems not to know how to carry out the process he has described. He has also written an equation for average velocity which would not be applicable to this problem situation. Ben refers to his book, and appears to be once again considering possible approaches:

> "motion…motion… all right… force equals mass times acceleration… all right g equals nine point eight meters per second… and the velocity… after… force of friction… its kinetic so its going to be…kinetic friction sliding… is when an object is in motion… ok [referring to example in book] force of kinetic friction…is equal to the coefficient of kinetic friction times n where n…I presume is…mass times gravity…in the opposite direction of gravity… n…two…so you're looking at… the force is going to equal mass times gravity times the sine of theta mass g sine theta is equal to force… is going to lead to my change… in distance… so I need fifteen hundred times nine point eight times sine of… five degrees…"

In this segment Ben has found an example in his book that shows an object sliding down an incline, which is an appropriate analogical example for this problem. However, the example does not show the calculation of the normal force, and he states that the normal force is "mass times gravity… in the opposite direction of gravity," a statement that would be correct only if the object was on level ground. He does obtain from the example the correct expression for the force in the direction of the incline, and correctly infers that it is that component of the force that is relevant to the motion down the incline. Ben proceeds to calculate the force, and then the acceleration:

> "that's… sine of five… so that's negative point nine five nine for the sine of five times nine point eight… times… mass which is fifteen hundred… gives you a number which is equal to force divided by the mass which we just multiplied by… fourteen hundred and *unintelligible* divided by the mass is equal to the acceleration… if that's a negative [referring to force and acceleration] that's gonna be a negative..."

So far Ben appears to be doing fine. He carries out the calculation correctly, but seems bothered by value of -9.40 m/s^2 that he gets for the answer:

> "that can't be right…force is equal to mass…the distance… derivative… plus t all right… the derivative of the distance is gonna be v plus t equals d prime… ah… both derivatives are gonna equal one that's using the product rule… so… d prime… still I'm

> not in the right direction ah…looking for… how far along the truck ramp does the truck go before coming to a stop…"

What Ben has done, but does not notice, is put his calculator into radian mode instead of degree mode. This mistake led to the unreasonable answer for the acceleration, which clearly troubled him. In the post-session interview he states that "the number itself was wrong and I could look at it and tell… it's just too big." It is unclear from either the protocol or the interview what Ben meant by his reference to the derivatives.

From this point Ben seems to get caught up in the defining equations for physical quantities that he finds in the book. Most of the remainder of the time he spends working on the problem is spent looking through the book:

> "so its going… so it [sighs]… d prime… that's a prime [referring to the acceleration he calculated]… velocity acceleration…. instantaneous acceleration… velocity… relative acceleration… velocity over time… motion in two dimensions and we're traveling that way [gesturing diagonally]… velocity…"

At the point where Ben mentions two dimensions he has referred to an example in the book on relative velocity, with a boat on a river. He gestures between his work and the example, then returns to his work:

> "velocity in the I'm gonna call this side a this side b velocity b direction… er velocity… of b v… we know velocity… this part's velocity… constant acceleration… it should be all constant acceleration constants of revolution… I guess I'll go back to this"

After apparently trying to fit his problem to the relative velocity example, he returns to his notebook, and finds another example with an object on a slope:

> "acceleration along the x axis… is going to be… acceleration… is going to be nine point eight gravity… times sine of theta one… (pause) ok so velocity… I have to find velocity final… acceleration deceleration… lets work with change in time velocity equals derivative of x in relation to change in time or… change in distance over change in time… t in reference… so [writing equation] change in v… is equal… delta v over delta t

> equals a… and we want a prime…we want yea a prime to get v final for right here [point between two slopes] we've got v initial to be in the equation for that problem the b side of it"

Ben verbalizes the same formula for the acceleration down the slope that he used previously, but does not calculate the value again. He then starts referring to defining equations, but his verbalizations and written work do not provide any indication of how they will apply to his problem. Yet at the same time, it seems that he still has a general idea of what he needs to do: "get v final for right here we've got v initial to be in the equation for that problem the b side of it." In other words, he knows that if he finds the velocity of the truck at the bottom of the first slope, it will be the initial velocity for the second slope. Ben refers again to his notebook, and writes an equation:

> "change in… average v times… a [writing equation]… delta x… delta x is equal to… average v… delta x equals v final velocity final squared minus velocity initial squared over 2… but a equals… average speed instantaneous velocity… velocity on the x axis… instantaneous velocity… in the x direction…equals the limiting value of the ratio delta x over delta t as delta t approaches zero…displacement delta x also approaches zero as delta t approaches zero… velocity one of x…equals…"

Ben refers to his book and his notebook throughout this segment. He writes an equation that could be useful, but ultimately does not use it. He verbalizes a list of topics from the section of the book he is referring to, and then reads the formal definition of instantaneous velocity. When asked during the retrospective debriefing about the equation he wrote, he states that at this point he was confused about where he was going with the problem, and did not use the equation because "I wasn't finding the…number [acceleration] that I was looking for and I couldn't really conceptualize the equation to get that number."

Ben seems to be unable to make any further progress in his solution attempt. As he had stated several times previously, he knows he wants to find the velocity, but does not appear to have any idea how to go about it. He continues to refer to his book and his notes, gestures over his diagram as he considers the motion of the truck, and verbalizes several comments without any apparent connection to the problem:

> "I still want v (sighs)… d prime… ok we've got d… d prime… is how fast that's changing… he's just going… that's v… v prime… one point eight velocity here [point between slopes]… sine no… sine of five degrees… times… velocity's moving that way… that's down… a vector quantity… it's going to be slowing it down… times the velo what… oh… so the sine of five times twenty twenty two meters… sine of twenty two... that's not right… sine of five… actually it'd be sine of… one fifty five… times… my friction… and its three hundred meters… [writing equation] x squared plus y squared equals three hundred… what would you get… twenty two (sighs)… rate of change multiplied out with rate of change… is gonna be… sine is opposite… cosine is adjacent so its gonna be cosine… of five… ok…multiply…distance…good night to this…"

At this point Ben states that he is stuck and ends the problem session.

It is difficult to characterize what Ben's primary difficulties are. He seems to know what he has to do ("need to get the velocity… after he slides to here"), but appears to lack the procedural skills to accomplish the task, and is never able to get to the point where he can begin to think about how he will handle the second slope. It also appears that Ben is deficient in conceptual understanding. Several aspects of his problem attempt appear to support this assertion. He draws a sketch of the physical situation, but does not start trying to solve the problem until he has referred to examples in the book and his notes. In the retrospective debriefing, he indicates that he was looking for "how sine of theta and gravity equated into the acceleration." He goes on to clarify:

> "I was gonna look for the static I mean the kinetic friction and how it equated in…and…also…how the sine or cosine of theta equated in I thought I had that kind

> of…figured out…and…I needed to figure out how acceleration…how the change in acceleration affected the final velocity at the bottom."

He was asked what he meant by change in acceleration, to which he replied, "what the acceleration was to get the velocity final so from velocity initial to velocity final using the acceleration." Again, it is not clear what his conceptualization of acceleration and its relation to velocity is. He states "change in acceleration," but then correctly indicates that there would be a change in velocity. His statements suggest the velocity-acceleration confusion that is commonly seen in students in introductory physics courses.

As Ben proceeds with his solution attempt, he verbalizes a mathematical statement of Newton's second law, but does not draw a free body diagram. Instead, he copies a formula for the force down the slope from an example. The answer he obtained as a result of having his calculator in the wrong mode appears to have set Ben off track in his solution, and he never recovers. The remainder of his solution attempt is spent referring to the book and his notes. He spends a considerable amount of time reviewing irrelevant examples and reading defining equations. We will never know how Ben might have proceeded if his calculator had been in the correct mode, which would have given him the correct value of the acceleration. But it appears from his actions that he lacked the control mechanisms, as well as the conceptual and procedural skills, needed for success.

Summary of Participant Actions

The layers of analysis carried out on the protocols to this point focused on the actions carried out by the participants. This analysis first suggested that the participants could be divided into three broad groups: those who completed their problems successfully, those who completed the problem with primarily only mathematical and/or procedural errors, and those who exhibited significant conceptual errors. Further analysis showed that these groups were relevant, in that the members of the groups had certain problem-solving characteristics in common. Group A participants, who completed their problem with mathematical or procedural errors, appeared to have reasonably good understanding of underlying physics concepts. They made steady, if slow, forward progress in their solutions, making high-level use of diagrams to connect to the mathematics of the solution, and making frequent checks of their work. If they used texts or formula sheets at all, it was as a means of checking that they had remembered the form of an equation correctly. Group B participants exhibited significant conceptual inadequacies, which limited their ability to successfully complete their problems. They were seen to make frequent use of texts or formula sheets to search for an approach, to make only lower-level use of diagrams in their solution, and to make few if any checks of their own work. Group C participants, who completed their problems correctly, were found to carry out their solutions in a more or less step-wise fashion, with high-level utilization of diagrams to connect to the mathematics of the solution, while making frequent checks of their own work. They also made few, if any, references to texts or formula sheets.

This preliminary analysis was undertaken with the goal of describing as completely as possible the actions taken by the participants as they attempted to solve the problems they were given. Only minimal interpretation of those actions took place, limited to what was required to develop the description adequately. The purpose of this stage of the analysis was to reach an understanding of the actions of the participants at level that would serve to strengthen the next step in the analysis: the attempt at application of the stabilization model.

CHAPTER FIVE: RESULTS

Once the actions taken by the participants were identified and the group characteristics were determined, the initial coding scheme informed by the McGinn and Boote (2003) stabilization model (see Appendix D) could be tested against the protocols. However, the first attempt at applying the stabilization model to the partitioned protocols suggested that there were features of the protocols that were not able to be coded with the original five factors. In particular, it was noted that most participants spent a considerable amount of time verbalizing the calculation of various quantities during their solutions. While calculation was not one of the factors identified by McGinn and Boote (2003), it is nonetheless an action taken during the course of problem solving and must be taken into account in any description of the problem-solving process. In fact, several participants in this study made errors during calculations; not coding specifically for calculations would eliminate the context in which those errors took place. For this reason an additional coding category was added to the coding scheme, not as an additional factor in the model to be tested, but as a way to account for those periods of time in which the participant was actively carrying out a calculation. That the independent reviewer, who was initially provided only the original coding scheme, also noted the need for a category to account for calculation further supports its inclusion. After some consideration, it was decided that an algebraic manipulation of a symbolic relationship, such as solving for a particular variable, would also count as a calculation. It should be noted that McGinn and Boote did mention calculation in their original work, but only to note that once a problem was stabilized, with a known solution path, "all that remained was computation" (p. 100).

The second feature that could not be accounted for by the original coding scheme was the ongoing evaluation of work carried out by the majority of the participants. McGinn and Boote (2003) discuss ongoing assessment in their original formulation of the stabilization model. In their discussion of the factors and actions that relate to problem difficulty, they note that there was ongoing assessment in relation to the four identified factors of categorization, goal interpretation, resource relevance and complexity as they evaluated and reevaluated their approach to a given problem. It was this ongoing assessment in search of a stable understanding of the problem that McGinn and Boote associated with the process of stabilization. There was a considerable body of evidence of this type of assessment activity in the protocols of the participants of this study as they attempted to reach an understanding of the physical situations described in their problems and searched for an approach to the problem, particularly in Group B. But there was an additional form of assessment that took place which is not mentioned in the original study by McGinn and Boote. The majority of participants of this study paused at one or more points to evaluate the work they had already completed. This evaluation consisted, for example, of considering the magnitude or sign of a calculated number, assessing the accuracy of a written equation, or checking whether the correct angle or vector component had been used. Because these self-checks of work were carried out by the majority of participants, it was decided that an additional code to account for those episodes was needed. The independent reviewer also noted that there appeared to be two levels of assessment taking place, although in his first analysis he did not add an additional code to distinguish between the two.

With additional codes to account for calculation and evaluation in place, an attempt at application of the stabilization model to the partitioned protocols could take place. According to

McGinn and Boote (2003), the factors of categorization, complexity, resource relevance, and goal interpretation are primary to the problem-solving process, while stabilization is an overarching factor describing the inter-relatedness among the four primary factors. That is, the four primary factors form the foundation of the problem-solving process, while stabilization governs the manner in which those factors interact with each other. The superordinate status of stabilization suggests that it should be evidenced as an activity that overlays, or encompasses, the other four factors. If stabilization is a valid factor, we should see the problem solver addressing issues of resources, complexity, categorization and goals as sub-actions within the process of stabilization, as part of the search for a stable understanding of the problem. This interpretation informed the decision to code the protocols for the four primary factors first, and then to review the coded protocols to look for evidence of stabilization.

In the sections that follow, each of the primary factors will be reviewed in relation to the McGinn and Boote (2003) model and to evidence of the factors suggested by other researchers. These factors then will be discussed in relation to findings from the protocols generated in this study. In particular, explanations of the decisions and assumptions made with regards to determination of evidence of the factors will be explained. Finally, aspects of the protocols that were inconsistent with each of the factors in the stabilization model will be described, as well as those features that seem to validate them.

Categorization

Categorization has long been recognized as an important factor in the problem-solving process. The results of early expert-novice studies in the domain of physics suggested that novices are unable to adequately categorize physics problems, focusing on surface features rather than underlying physics principles (Chi et al., 1981; Chi et al., 1982). More accomplished problem solvers categorize according to underlying principles that are linked with a solution process, or schema, so that recall of the principle calls up the entire procedure for obtaining the solution (Chi et al., 1981; Larkin et al., 1980a; Savelsbergh et al., 2002). As noted by McGinn and Boote (2003), appropriate categorization of a problem results in the problem being perceived as easier, even if all the details of the solution are not recalled.

Categorization can affect problem complexity in other ways, particularly within the domain of physics. Many problems can be solved in more than one way, with one method typically resulting in a more complex solution process. This was the case for the problems used in this study. Each of the problems had portions that could be solved either with Newton's second law or work-energy relationships. In all three problems, application of work and energy resulted in a less complex solution process. The category decision made by the participants thus had a direct impact on the length and complexity of the solution. For the participants of this study, there was no clear connection between problem-solving proficiency and choice between alternate solution processes.

Although early expert-novice studies suggested that novices were unable to categorize physics problems appropriately, more recent studies provide evidence that introductory physics students frequently do have sufficient conceptual knowledge to appropriately categorize problems and select appropriate physics principles (Hammer, 1996; Kim & Pak, 2002; Robertson, 1990). However, they are unable to carry the categorization through to the successful solution of a problem, either because their knowledge is incomplete, or because they lack the procedural knowledge required for applying the chosen principle.

The categorization studies of Chi et al (1981) and others asked participants to simply group sample problems on cards by type or category. No study was found in the domain of physics that looked explicitly at the phenomenon of categorization within the context of think-aloud protocols. As a result, a decision had to be made regarding those verbalizations that would be considered as evidence that categorization had taken place. Examination of the actions taken by the participants in this study showed that explicit statements of categorization were made by seven of the twelve participants. For example, immediately after reading his problem statement, Chuck states "all right… so maybe we should do this from an energy perspective I think we should say that… this part where Jill goes into the jumps in the sled could say that… momentum is conserved so…" Likewise, Beth spends a few minutes considering possible approaches to her problem, then states "ok so I do know the energy kinetic energy the other kind of energy's going to be potential energy got it." Although her statement is not of the form 'this is an energy problem,' it is clear that she has decided what approach she should take in the problem, and she carries through with that approach to the end of the problem. In all but one case the explicit statements of categorization made by the participants in this study were correct.

The other way in which categorization was evident was implicitly, through the choice of solution process. Because no actual statement of problem type was made, the researcher had to make a judgment as to whether or not the actions taken by the participant represented actual categorization. This type of categorization was described earlier in the case of Cory, who drew a free body diagram and applied the mathematical statement of Newton's second law to his problem. Although Cory never stated "this is a second law problem," his actions seem to suggest that he had made a categorization. Because determination of categorization in these cases required inference from the actions of the participants, a more rigorous test was applied to cases of implicit categorization. In order for the actions taken by a participant to be considered as evidence of implicit categorization, the participant's actions must show that he or she chose a solution process based on a particular physics principle (i.e. Newton's second law), and that process was followed to the end of the solution attempt. Four participants in this study were found to have taken actions that met these criteria for implicit categorization. One participant provided no clear indication of categorization of any kind. A summary of indications of categorization is seen in Table 5.

Table 5: Indications of Categorization

Group	Participant	Explicit	Implicit	Apparent Conceptual Basis	Category
A	Alex	√		√	Momentum/Second law
	Andrew		√		Second law
	Arnold	√		√	Work-Energy
	Art		√		Second law
B	Ben				None
	Beth	√		√	Energy
	Betty	√		√	Work-Energy
	Bob		√		Second law
	Brittany	√			Collision*
C	Carl	√			Second Law
	Chuck	√		√	Momentum/Work-Energy
	Cory		√	√	Second Law

*Invalid categorization; eventually completes attempt without clear categorization

Based on the evidence collected in the protocols, the majority of the participants in this study appeared to make appropriate categorizations of the problems they attempted, although the basis of those categorizations was sometimes questionable. This is in agreement with the results cited by other researchers, which indicate that students in general have the declarative knowledge needed to solve physics problems (Chi et al., 1989; Hoellwarth et al., 2005; Savelsbergh et al., 2002). The results of this study are also seem to support the assertion that the declarative knowledge needed to adequately categorize a problem does not necessarily imply complete conceptual or procedural knowledge. Despite the fact that the majority of participants were able to make appropriate categorizations, only three participants were able to carry that categorization through to the successful completion of the problem. The largest group in the study, Group B, showed significant conceptual gaps in their knowledge, as evidenced by both the verbalizations and the post-session interviews. For those participants in Group A whose conceptual understanding appeared more robust, there were still deficiencies in the procedural knowledge needed for successful application of the principles they selected.

The two exceptions in this discussion of categorization were among the weaker problem solvers. Brittany initially explicitly categorized her problem as a collision problem, apparently cued by the fact that the problem statement listed mass, velocity and angles. She did not carry through on this inappropriate categorization, and eventually reached an answer based on application of force principles. However, she never clearly categorized the problem, basing her work instead on formulas culled from examples she referenced in her textbook. Ben, on the other hand, never gives any indication of categorization, and ends his problem-solving attempt before reaching a solution.

The choice of categorization was also observed to affect problem complexity, as reported by McGinn and Boote (2003). Participants who selected work-energy relationships over Newton's second law in general completed solutions that were shorter than the solutions of those participants who used the second law to solve the same problem. Alex's protocol is a prime example of how problem categorization can affect solution complexity. He correctly categorized the first part of Problem 2 (see Appendix A) as a momentum problem, and then chooses Newton's second law to apply to the second part of the problem. This was an appropriate choice, but one which resulted in more lengthy solution than one based on work-energy principles would have been. In addition, Alex categorized the motion constrained to an incline as a two-dimensional problem, which additionally increased the complexity of the problem.

In short, the protocols from this study seem to support the validity of problem categorization as an important factor in the problem-solving process in introductory physics. This is not surprising given the large body of literature related to categorization both in physics and mathematics problem solving. What is notable about these results is that nearly all the participants, even those whose problem-solving sessions provided evidence of significant conceptual difficulties, were able to make appropriate categorizations *within the context of problem solving*. However, the basis upon which those categorizations were made is questionable. Only six of the participants had any apparent conceptual basis for their categorizations, and of those, in only three of those cases did the conceptual understanding appear robust and not at least partially based on surface features of the problem. The results of this study also highlight the relationship between categorization and complexity, the next factor to be discussed.

Complexity

McGinn and Boote (2003) note that the number of operations needed to complete a problem solution influences the perceived difficulty of the problem. An increase in the number of discrete steps needed to reach a solution makes it more difficult for the problem solver to see how they will get from the initial state to the final goal state. This would have particular relevance to the experiences of novices, who frequently rely on means-ends analysis to solve unfamiliar problems. The measure of the number of steps to reach a solution is defined by McGinn and Boote as complexity.

In this study, only one participant made explicit reference to the complexity of the problem attempted as defined in the stabilization model. Alex, when he realizes that the sled-on-a-hill problem has four distinct parts, makes the comment “and then get to the bottom of the hill and that’ll be oh my god it’s four problems… oh… this is hard…” In the post-session interview, Alex indicates that when he realized there were four parts to the problem he was “panicking” because he saw the task as far more difficult than he originally perceived it to be. In fact, earlier in the problem, at a point when he believed the problem to involve only the application of conservation of momentum, he stated “this is gonna be simple…” An increase in the perceived number of steps needed to reach the solution resulted in Alex modifying his assessment of the difficulty of the problem.

Although Alex’s statements concerning the difficulty of the problem he attempted appear to support the stabilization model’s definition of problem complexity, he was an exception. No other participant made an explicit statement concerning problem complexity or difficulty. There were a few isolated statements that might be considered as statements of complexity; however, given the context in which they took place it is difficult to ascertain whether they were statements of complexity or simply statements about the next steps that must be taken in the solution process. The statements that come closest to meeting the definition of complexity as outlined in the stabilization model are those made by Ben and Andrew. Early in the problem, as he is considering the approach he will take, Ben states that he will “need to get the velocity after he slides to here… and then using this coefficient of friction and the velocity…” Likewise, about midway through his solution, Andrew notes “I have the time that it takes me to get right here… how far along the truck so I can find the final velocity now that I know the acceleration here the final velocity at which I can get this here will be the initial for this ramp… if I'm not mistaken…" Ben and Andrew have alluded to the number of steps, even though they do not specifically mention them. Later in the problem Andrew notes "I know this is three hundred I'm trying to find this [x_2] so three hundred this is given [velocity] then this is given [acceleration] I have two unknowns I have this [x_2] I have the time…" Again, no specific mention of the number of steps is made, but he notes that there are two unknowns to deal with. A similar statement concerning two unknowns was made by Carl during the course of his solution.

Other than the four participants already mentioned, there were no apparent statements related to problem complexity. All that can be determined from these results is that the verbalizations of these participants do not appear to support the contention that problem solvers *consciously* consider the complexity of a problem as part of their solution process. The results of the study must be considered as inconclusive with regards to the factor of complexity.

Goal Interpretation

The stabilization model defines goal interpretation as knowledge about how the solution should look (McGinn & Boote, 2003). Goal interpretation is closely tied to categorization, since the categorization of a problem is often linked to recall of a complete solution pattern, or schema, at least for more experienced problem solvers. To the expert, the complete solution is a way in which information about the problem situation can be communicated. To better understand the concept of a solution pattern communicating information, and to illustrate the relationship between categorization and goal interpretation, consider again the example of Alex. He categorized the non-momentum conservation portions of his problem in terms of Newton's second law, and further categorized the problem as two-dimensional, despite the fact that the motion was constrained to a line. His categorization automatically set a solution pattern based on components of the vector quantities involved in the problem. In other words, his categorization determined what the solution would look like. Alex's solution then communicates information about components of forces and accelerations that would not have been communicated if he had categorized the situation as a work-energy problem.

As noted by McGinn and Boote (2003), the answer is also part of the overall goal. Goal interpretation takes into account the fact that the answer is only one part of a complete solution, even though for many novices the answer might be considered as the only *important* part of the solution. Providing only the answer to a problem does not communicate information about the assumptions and/or approximations that might have been made during the course of the solution, nor about the principles applied for that solution. Knowledge of the final goal, or answer, does not necessarily imply knowledge of how to reach that goal, a situation that is well-understood by every student who has ever looked in the back of a textbook for an answer. The problems used in this study were well-defined, in that the final goal was stated in the problem statement. However, all of the problems required multiple steps to reach the final goal, so that there were unstated subgoals that must be reached for a successful solution. The number of subgoals within a solution was dependant upon the problem attempted and the approach chosen by the participant. That is, although the overall goal was stated in the problem description, the subgoals were related to the categorization of the problem.

Once an understanding of goal interpretation was reached, a decision had to be made as to the criterion to be used as evidence of goal interpretation within the protocols. The basis of the definition of goal interpretation is the solution pattern as established in large part by the subgoals within the problem. Informed by this definition based on subgoals, this study used a criterion that sought evidence of goal interpretation in the verbalizations of the subgoals that defined the solution pattern. The justification for this decision was that if a participant was able to verbalize a goal or subgoal, he or she had some knowledge of what the solution pattern should look like. To strengthen the evidence, the subgoals verbalized by each participant were compared with the actual solution pattern produced. Final determination of whether or not goal interpretation was evident was made on the basis of the level of agreement between the verbalized subgoals and the actual solution path.

In the first pass through the protocols, any goals and subgoals actually verbalized by the participants were noted. Table 6 summarizes the results of this review, which showed that every participant made at least one statement of the final problem goal beyond the reading of the problem statement. In addition, all but two participants made at least two statements of subgoals. In these results, repeated statements of the same subgoal were treated as a single statement. Problem goals and subgoals were often verbalized several times, so that the total verbalizations made by the participants varied from one to eight goal statements, and from zero to 11 subgoal statements.

Table 6: Verbalizations of Goals and Subgoals

Group	Participant	Verbalized Final Goal	Verbalized Subgoal(s)*	Subgoals define solution pattern	Solution pattern matches expert
A	Alex	√	√ (7)	√	
	Andrew	√	√ (6)	√	√
	Arnold	√			√
	Art	√	√ (4)	√	√
B	Ben	√	√ (3)		
	Beth	√			
	Betty	√	√ (3)	√	
	Bob	√	√ (2)		
	Brittany	√	√ (3)	√	
C	Carl	√	√ (5)	√	√
	Chuck	√	√ (5)	√	√
	Cory	√	√ (4)	√	√

*Repeated statements of same subgoal grouped as one statement

Comparison of the verbalized subgoals with the actual solution steps showed that in seven of the 12 protocols, the subgoals defined the solution path; that is, the pattern of steps indicated in the verbalizations matched the pattern of steps carried out in the solution. Two participants verbalized no subgoals to compare to, and one participant (Ben) verbalized subgoals that represented a valid solution pattern but was unable to proceduralize those subgoals. In several cases, the actual solution included more steps than were verbalized, but these were cases when an additional calculation was required to reach the stated subgoal. For example, Alex verbalizes that he needs to calculate the friction force, and then calculates the normal force first. Four of the protocols fell in this category.

Two of the participants expressed subgoals that at first did not appear to match the solution they actually carried out. Bob verbalizes two subgoals: find the acceleration and find the final position. In his solution, he uses kinematics equations in an attempt to generate a symbolic relationship for the final position in one step, calculates an (incorrect) acceleration which he then substitutes into his symbolic relationship. Although in his solution Bob does eventually calculate acceleration and final position, he carries out a number of steps prior to those calculations which are not verbalized. For this reason, his solution was determined not to match the verbalized subgoals. Likewise, Cory carries out a number of steps in his solution that are not verbalized. Cory is an interesting case in relation to goal interpretation, as he is one of the individuals who reached a correct solution. He verbalizes subgoals only for the second half of his solution, and for that portion the subgoals match the solution steps. For the entire first half of the solution, he carries out steps which parallel the calculations he performs for the second half of the solution, but without verbalizing the subgoals of those calculations. Although it appears that Cory knows what the solution pattern should look like, he does not verbalize that pattern. However, because the verbalized subgoals for the second half of the problem mirror the steps taken in the first half, his protocol was categorized as matching the verbalization of subgoals.

It is important to note that the condition of matching verbalized subgoals and solution does not indicate a valid solution process. As noted earlier, the protocols of all Group B participants showed significant conceptual errors in application of physics principles. However, as seen in Table 6, Group B participants were also less likely to provide verbalized subgoals that matched actual solution steps they took. Although two of the five Group B members provided verbalized subgoals that matched their actual solution steps, both of the solutions contained conceptual errors. Likewise, both of the verbalized paths represented possible valid solutions, but the participants lacked the conceptual and procedural knowledge needed to carry out the solutions correctly.

Another aspect of goal interpretation that is inherent in its definition in McGinn and Boote (2003) lies in knowledge of the magnitude, direction and units of the answer. For a good problem solver, this sense of how the answer should look begins to be developed early in the solution of the problem. Understanding of how the answer should look, both for the overall goal of the problem and the subgoals needed to attain it, provides cues to the problem solver as to whether or not he or she is making progress in the solution (see also Sweller, 1983). This aspect of goal interpretation is illustrated in the protocol of Andrew. Early in his solution, Andrew draws a diagram (Figure 9) and indicates the horizontal plane at the base of the incline as the distance that he is looking for.

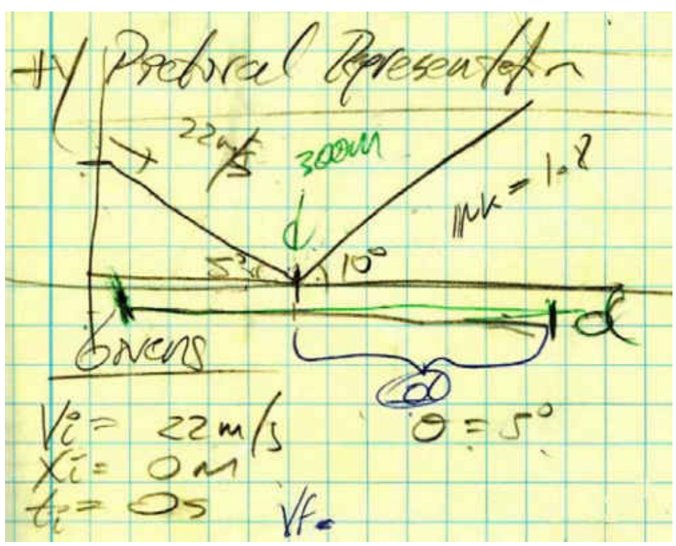


Figure 9: Andrew's Sketch

In other words, he has interpreted the question "How far along the truck ramp does the truck go before coming to a stop?" as asking for the horizontal distance. Later in the problem he reevaluates that interpretation:

> "and this is how far along the truck… on the truck ramp does the truck go before coming to stop… so… earlier I was explaining this distance I thought it was going this distance [indicating horizontal distance] no this is the three hundred there so… what is that distance [indicating distance along upward slope]…"

Andrew has reinterpreted the goal of the problem based on the work he has carried out up to that point. Unlike Andrew, problem solvers without a clear sense of what the final answer should look like would not have the cues needed to alert them to the presence of errors or misinterpretations of the goals of the problem.

Understanding these results in relation to goal interpretation was difficult. In order facilitate further understanding, the steps that would be taken by an expert in solving the problems were listed. Because each of the problems could be solved either with work/energy principles or by application of Newton's second law, the expert steps for each method were listed, and were verified by an independent physics professor. The steps taken by the participants were then compared to the expert's steps. This comparison, which is also seen in Table 6, was far more enlightening than looking at the participants' protocols in isolation. Comparison to an expert solution path showed that each of the Group C participants verbalized a solution pattern based on subgoals that matched the expert pattern. This was true even in the case of Cory, who did not verbalize the first half of his solution. The majority of the solution paths of the Group A participants also matched those of the expert, further supporting the original categorization of this group as exhibiting mathematical or procedural errors only. The only exception was Alex, who incorrectly categorized his problem as two-dimensional.

The analysis of the protocols with respect to goal interpretation, defined as knowledge of the solution pattern, suggests that in order to reach a successful solution, the problem solver must be able to verbalize the necessary subgoals leading to that solution. Additionally, the more expert-like the verbalized pattern is, the more likely it is that a correct solution will be reached. However, these results are not conclusive. Arnold does not verbalize any subgoals in his solution, yet his solution pattern matches that of the expert. Likewise, Cory verbalizes subgoals for only half of his solution, yet he also produces a solution which matches the pattern of that of an expert. Based on the evidence of their protocols, it seems likely that both of these individuals knew what the subgoals and solution pattern were, but simply did not express them. What appears clear from these results is that conceptual understanding is important for successfully solving unfamiliar problems, although it is not sufficient. None of the Group B participants were able to verbalize an expert-like solution pattern, and none were able to correctly solve the problems they attempted. This is not surprising, given the conceptual difficulties evident in their protocols. But Group A participants, who seemed to have a reasonable level of conceptual understanding and in general produced expert-like solution patterns, were also not able to reach successful solutions. These results parallel those reported by Hoellwarth et al. (2005), who found that increased conceptual understanding did not necessarily translate into increased problem-solving proficiency. Understanding why conceptual understanding is necessary but not sufficient in physics problem solving leads one to consider the resources that problem solvers bring to their solution attempts.

Resource Relevance

Resource relevance is another factor which has been well-documented in the literature. Schoenfeld (1985a) noted that students' inability to recognize the relevance of their existing domain knowledge was a significant barrier to success in mathematical problem solving. The highly mathematical nature of physics problem solving suggests that the same statement could be made about physics students. This connection between physics problem solving and mathematical problem solving has been investigated by other researchers (Bassok, 1990; Bassok & Holyoak, 1989; Cui et al., 2005; Ozimek et al., 2004). Their results in general suggest that novices within the domain of physics have difficulties in recognizing their mathematical knowledge as relevant to problem solving in physics.

Other aspects of resource relevance are related to the ways in which students use the resources they do recognize as relevant. Many physics students use examples as a resource in solving unfamiliar problems. The manner in which those examples are used is related to the problem solver's conceptual understanding and ability to recognize an example as relevant to the problem at hand (Chi et al., 1989; Chi & VanLehn, 1991; VanLehn, 1998). In particular, problem solvers who lack sufficient conceptual understanding are more likely to copy information verbatim from an example, without considering its relevance to the problem they are attempting to solve.

The investigation of resource relevance in this study began with a simple tabulation of the resources used by the participants. The majority of the listed resources, seen in Table 7, were identified by McGinn and Boote (2003), but several were identified solely from the work of the participants in this study. This list does not include all of the resources listed by McGinn and Boote, primarily because this study did not consider social resources. The tabulated information shows that prior knowledge was the most frequently used resource, a finding which is not surprising. As noted by Bruner (1996), the most important thing a student brings to the classroom is his or her prior knowledge.

Table 7: Resources Explicitly Noted in Sessions

Resources*		Group A				Group B					Group C		
		Alex	Andrew	Arnold	Art	Ben	Beth	Betty	Bob	Brittany	Carl	Chuck	Cory
Material	Ruler	√											
	Colored Pens		√										
Textual Resources	Textbook Formulas	√				√			√	√			
	Textbook Examples					√				√			
	Formula Sheet	√									√		√
	Lecture Notes		√		√	√		√					
Prior Knowledge	Geometry/ Trig.	√	√	√	√	√	√	√			√	√	√
	Physics Principles	√	√	√	√		√	√	√		√	√	√
	Physics Concepts	√	√	√	√	√		√			√	√	√
	Memorized Equations	√	√	√	√	√	√	√	√		√	√	√
	Units	√			√		√						
Other	Mnemonics			√									√
	Memory of Lecture	√	√		√								
	Own Earlier Work	√	√				√	√	√	√	√	√	√

* All participants used paper/pencil, calculator, algebra knowledge, problem statement and self-drawn diagrams.

The first information to come from the review of utilized resources was that all participants utilized algebraic knowledge in their attempted solutions, and in all but one case they used those resources correctly. The exception was Brittany, who made two significant mathematical concept errors. First, while summing the forces using an equation she copied from an example in her textbook, she writes:

$$\sum F_y = 57.08 = ma_y = 0.$$

She looks at the equation and states "ok," apparently not recognizing the contradiction. Later in the solution she writes:

$$\begin{aligned} v_{x_f}^2 &= 2a_x d \\ 0^2 &= 2a_x d \\ &= 2(1.74)d \\ d &= 3.473 \end{aligned}$$

She again does not notice the mathematical impossibility of what she has written. This same mistake is made again at a later point in the solution. Brittany appears to have mathematical conceptual inadequacies, and apparently recognizes her deficiencies, as she states in her post-session interview: "my weakest subject's math… and I'm like well where did that negative sign come from I didn't see it there so you know."

The other resource that was used by all the participants was self-drawn diagrams, as noted earlier in the analysis. The current resource tabulation does not take into account the manner in which the participants used the diagrams they drew, an aspect of the protocols that will be discussed later. However, it was found that use of diagrams to generate the equations used to solve the problem was a universal characteristic of Group C. Use of diagrams in this way was also seen in Group A, though not as obviously so. Group B, the group with conceptual errors in their solutions, used their diagrams primarily as a means of recording the information in the problem statement or for determining angles and/or components.

The most significant finding from the tabulation of resources used was the indication that Group B participants were lacking in their use of physics principles and conceptual understandings, which is not surprising given the evidence of conceptual errors in their protocols. For the purpose of this analysis, *physics principles* were taken to mean formal principles within the domain of physics, such as Newton's second law, conservation of energy, or conservation of momentum. *Conceptual understandings* was taken to refer to knowledge of basic conditions within physical situations, such as the signs of velocities and accelerations, the fact that velocity is zero at the highest point in a motion, and that kinetic friction opposes the motion of an object. Clearly these two ideas are related, but for the purpose of understanding the actions taken by the participants it was useful to separate them. As illustrated by the protocols of Betty and Ben, it is possible to refer to basic physical concepts without reference to the underlying physical principles.

All of the members of Groups A and C referred to fundamental physics principles in the solution of their problems, often making categorizing statements related to those principles. Likewise, they all made use of physical concepts, using appropriate signs for physical quantities, and taking note of such things as points where velocities were zero. This was not the case for the Group B participants. Two of the five Group B members made no reference to physics principles, and three of the five made no reference to basic concepts. Only one participant, Betty, made use of both, although her understanding of the principles and concepts was flawed.

It was also noted that all Group A and Group C participants made use of geometrical and/or trigonometric knowledge, while only three of the five Group B participants did so. The reduced use of geometrical and trigonometric knowledge by Group B participants could be related to the fact that many physics concepts, such as net force, are based on underlying geometrical concepts.

Only two participants made reference to textbook examples during the course of their solution attempts, both of whom were members of Group B. Coincidently, they both are exemplary models of students applying the results of examples inappropriately. Both of these participants were attempting Problem 1 (see Appendix A). During the course of his solution Ben refers to a relative motion example, and then tries to fit his problem to the inappropriate example. He even refers to the notation used in the book for the example, stating which of his variables correspond to the variables in the example. Brittany, on the other hand, refers to a more appropriate example at first, but does not know what to do with the information it provides her. Later, when she realizes that the example did not include friction, she refers to a second example which does include friction, but which is on a level surface instead of an incline as her problem is. Despite the fact that the example is not applicable to her problem, she uses the equation for acceleration from that example for her solution. Both Brittany and Ben exhibit the inappropriate use of examples documented by other researchers (Chi et al., 1989; Chi & VanLehn, 1991; VanLehn, 1998).

We can summarize these findings by noting that the participants in this study bring significant mathematical and conceptual resources with them to the problem-solving process, and for the most part are able to make appropriate use of those resources. Participants who exhibited conceptual difficulties in their solutions were also less likely to have relevant resources to call upon, an observation which supports the presumption that conceptual understanding is a necessary component of successful problem solving. That it is not a sufficient component is evident in the protocols of Group A. All of those participants made use of relevant mathematical and conceptual resources, and yet were unable to reach a correct solution. It is important to note that this summary of use of resources does not consider whether those resources were applied correctly, or whether or not the conceptual resources used were accurate. To connect the use of resources to the accuracy and correct use of those resources requires looking back at the initial analysis, where the determination of type of error took place. It is also not apparent from a simple tabulation whether or not a conscious consideration of the appropriateness of a resource took place. These considerations lead to the conclusion that while resource relevance is an important factor in problem solving, it is not evident from this study that *conscious* consideration of those resources took place on the part of the problem solver.

Stabilization

What makes a problem a problem, as opposed to an exercise? This is essentially the question that McGinn and Boote (2003) were trying to answer through their introspective study of problem solving. As noted earlier, the problem situations they encountered ranged from automated activities and exercises to solvable problems to unsolvable "difficulties" (p. 98). It is the activities that qualify as problems that were of interest to McGinn and Boote, and to this study. Problems are situations in which the problem solver does not have ready access to an automated means of solution (Schoenfeld, 1985a), and the four previously identified factors of categorization, resource relevance, goal interpretation and complexity must be evaluated, balanced, and reevaluated to find a workable solution.

McGinn and Boote (2003) suggest a model of problem solving that considers the changing relationships between the four previously identified factors as the problem solver attempts to find a solution. The shifting of relationships between the four factors, moderated by ongoing assessment of the problem situation, is what McGinn and Boote called stabilization. They define it as a "shifting salience of primary factors… superordinate to the primary factors" (p. 99). They note that when the problem solver's perceptions of the factors are inconsistent with each other, reevaluation of the primary factors will result in actions taken to attempt to change the situation. In other words, the process of problem solving is a search for a stable relationship between the four factors, corresponding to a stable understanding of the problem situation. The search for stability requires an ongoing assessment of the problem situation, considering each of the factors in relation to the others. When a problem is stabilized quickly and appropriately, all that remains for its solution is calculation. On the other hand, if a problem is stabilized inappropriately, ongoing assessment may result in destabilization of the situation. The need for destabilization may be cued by an impasse in the solution, or by the solution exceeding some conscious or unconscious threshold, such as if the problem is taking longer that expected to solve.

Stabilization can be perhaps better understood in the context of metacognition, which is in effect the process of thinking about one's own cognitive processes (Kuhn, 2000). In relation to problem solving, metacognition acts as a control mechanism, informing the problem solver about whether or not progress is being made towards a solution. The control process governs selection and pursuit of appropriate solution processes, the selection of goals and subgoals (see also Sweller, 1983), recovery from inappropriate choices, resource allocation and general monitoring of the problem-solving process (Schoenfeld, 1985a). Lesh (as cited in Schoenfeld) noted that in the process of working a problem, a novice might realize that a completely different conceptualization of the problem is needed than the one that he or she had to that time been working under, and in fact may go through several reconceptualizations of the problem before a solution is reached. It is this monitoring process that is governed by metacognition, and is the type of activity that McGinn and Boote (2003) discuss in relation to stabilization, which is described in terms of the metacognitive ongoing assessment of whether or not progress is being made towards a solution. Lesh also argues that weaknesses in novices' conceptual and procedural systems result in problems with that control, leading to situations in which the problem solver may neglect important properties related to the problem, focus on surface features while neglecting underlying principles (see also Chi et al., 1981), lose track of overall goals while focusing on individual steps, or carry out individual steps incorrectly while remaining focused on the overall goals. In this respect, it is not the attainment of a stable understanding that is important, but rather the metacognitive awareness to realize when progress was not being made; that is, the awareness of when destabilization is necessary. Stabilization is thus suggestive of flexibility in problem solving, which is defined as the ability to change the direction of a problem-solving process in such a way that the fit between the problem situational constraints and the problem solver's solution process can be optimized (Frensch & Sternberg, 1989).

In order to evaluate the protocols in this study for evidence of stabilization, a working definition of stabilization had to be developed that would lend itself to the search. In other words, what actions taken by the participants would qualify as evidence of stabilization? McGinn and Boote (2003) discuss an ongoing evaluation of the relationships among the four factors; that is, evaluation of problem category, relevant resources, appropriate interpretation of the goal, and identification of sufficient operations to reach the solution. This definition was taken to mean that the problem solver is searching for a stable understanding of the problem that will lead to a solution. This appears to be consistent with the McGinn and Boote definition, in that a thorough understanding of the problem will be reached through the processes of categorization and application of relevant resources, combined with knowledge of the goal state and any necessary subgoals, all taking place while navigating a reasonable number of operations. The alternate definition as the *search for understanding that will lead to a solution* has the advantage of providing something more concrete to look for in the protocols. If the participant was considering solution options, verbalizing a search for understanding, but was not actively engaged in carrying out a solution, it was considered to be evidence of stabilization. On the other hand, if the participant was actively engaged in carrying out a solution and then stopped to reconsider their approach, it was considered evidence of destabilization. This assessment of approach to the problem excludes pauses to simply check work, such as when the participant is checking a calculation, evaluating the sign of an answer, or verifying the form of an equation. Assessment that qualifies as destabilization was considered to be an active consideration of the validity of the approach being taken. Destabilization may be temporary, in that it does not change the course of the problem solution, or it may result in a significant change in approach.

Analysis of the protocols to determine whether or not there was evidence of stabilization or destabilization necessarily took place after the protocols were reviewed for the four primary factors, since stabilization is defined as an overarching factor governing the relationship among the other four. That is, the process of stabilization or destabilization may involve a change in categorization, the use of resources, and consideration of the goal state and problem complexity.

Table 8: Indications of Stabilization

Group	Participant	Stabilization Episodes as Verbalization of Search for Understanding	Destabilization Episodes as Verbalization of Search for New Understanding	Cues for Destabilization
A	Andrew	Continual; no explicit categorization	Two; one resulting in change of course and one resulting in modification of approach	Consideration of given information; impasse in solution
	Alex	Continual with uncertainty evident throughout; explicit categorization	Three; all resulting in change of course of solution	Consideration of friction; impasse in solution (two times)
	Art	Beginning; no explicit categorization	One; result is affirmation of the approach being used	Uncertainty regarding equations
	Arnold	None apparent; rapid categorization; step-wise solution	None	NA
B	Beth	Beginning; explicit categorization	One; result is modification of approach	Consideration of friction
	Bob	Beginning with uncertainty evident throughout; no explicit categorization; equation-driven solution	Two; one resulting change of course and one resulting modification of approach	Consideration of given information; impasse in solution
	Brittany	Continual; no explicit categorization; example- and equation-driven solution	Two; both result in change in course of solution	Impasse in solution; consideration of friction
	Betty	Beginning; explicit categorization	One; result is modification of approach	Consideration of friction
	Ben	Continual; no explicit categorization; equation-driven solution	One; understanding never attained; no solution reached	Magnitude of calculated quantity
C	Chuck	None apparent; explicit categorization; step-wise solution	None	NA
	Cory	Beginning; no explicit categorization; stepwise solution	None	NA
	Carl	Beginning; no explicit categorization; stepwise solution	One; result is affirmation of the approach being used	NA

The review of the protocol data showed that nine of the 12 participants exhibited verbal evidence of thinking about how they should solve the problem (see Table 8). Three of the participants, Chuck, Cory, and Arnold, showed no evidence of a search for an approach to their problems. In each of these cases, the participant read the problem statement, immediately categorized the problem, and proceeded to engage in a more-or-less stepwise solution of the problem. Cory and Chuck both obtained correct solutions for the problems they attempted, and their solutions matched the solution process of the expert. Likewise, although Arnold made a minor error near the end of his problem, his basic solution process matched that of the expert. These observations are consistent with those made be McGinn and Boote (2003), who noted that when a problem is readily categorized, relatively automated solution procedures are activated. At this point the problem is, in effect, an exercise.

The remainder of the participants all verbalized an active interpretation process indicative of a search for an understanding of the problem situation and identification of a solution approach at the beginning of their solution attempt. In addition, four of those participants exhibited almost continual searches for understanding, frequently expressing questions about their approach to the problem or of their interpretation of the problem. The searches carried out often made use of resources such as self-drawn diagrams, formula sheets, the problem statement, or textbooks, but did not involve any calculations or other progress towards a solution. The distinction, as outlined in the criterion for evidence of stabilization, is that the participant was *considering* a solution, not actually *carrying out* a solution. Beth provides an excellent example of the type of search for solution that was considered as a likely candidate for stabilization activities:

> “ok so I know its initial kinetic energy is that going to help… no it’s not it doesn’t matter I know its initial velocity then it slides down the slope for this given period of time ok so velocity equals distance times time I don’t know time… I do know d I do know v I can find t ok… traveling down the slope does that matter… no… it shouldn’t then it goes up… then it’s going back up it’s going back up the ramp... does that matter kinetic ener… should I use kinetic energy I already know v… I already know v the velocity’s changing I have to find how far along if I know the vel… the velocity that’s not going to help me… he applies the brakes wait he applied the brakes velocity’s changing… (sighs) applying the brakes doesn’t do… because his truck starts sliding he doesn’t actually slow down so velocity is still the same… ok so I do know the energy kinetic energy the other kind of energy’s going to be potential energy got it”

Here we see Beth considering the problem situation, using problem statement, diagram and prior knowledge resources, and ultimately categorizing the problem. After this interlude she engages in actively pursuing the solution using her chosen energy approach, which she carries through to the end of the problem. Similar actions were taken by Bob as he considers possible approaches to his problem:

> “ok we’ve got… got forces in the x direction forces in the y …and…I wonder if I could solve this with an energy concept hmm… actually yea I probably could that’s a good idea… let’s see… so if they want distance and if we use kinetic energy to find the velocity it was going… uhm… that won’t tell us… actually yea it would help… so… no they gave us the velocity already… all right ok never mind… so kinetic energy won’t help us there… good ok… brain fart there… so… soo… we need another basic uhh let’s see what one would work here… hmm… let’s see v final equals v naught plus… what is it… one-half a t… ah let’s look that up…”

After considering forces, then energy, Bob settles on kinematics to solve his problem.

For two of the participants (Art and Carl) there was only minimal evidence of an apparent search for understanding. Carl starts his solution as if he already had a plan in mind. He draws a free body diagram, and then a second diagram listing the known quantities from the problem statement. He then pauses to consider what to do next:

> “we have in ah initial position a initial time initial velocity and then we’ll have a final distance… final position a final time and a final velocity at that point the final position

> will be zero meters the I mean the initial position the initial time will be zero and the initial velocity was given to be twenty-two meters per second… and then you we have the final position being five three hundred meters and the final time being unknown and the final velocity is what we're calculating… so for the what we're finding… the ramp is covered… how far along the truck ramp ramp does the truck go before coming to a stop hmm… in order to solve the problem we probably need the final velocity at a later time so yes that that is what we are solving for so then I look at my formula sheet and see what I can do with that… I know I know the final position I know the initial I know the ini ah initial velocity I do not know the time… acceleration… [refers to diagram] there would be… yes there would be an… because of the weight force of the truck there would be an acceleration so the truck is accelerating… so then we probably need to use Newton's second law in order find the acc… ah the acceleration along the horizontal… the x…"

This is the only pause during which Carl considers his approach to the problem. Once an understanding was reached, his ultimate decision was to utilize the resources he had already made available (the free body diagram and kinematics information) to solve the problem.

At the other end of the continuum were Brittany and Ben, who engaged in nearly continuous searches for a stable understanding of their problem. Both of these participants were highly equation-driven, searching their books for equations and/or examples that would help them solve the problem. This was particularly true of Brittany, who made several comments during her protocol indicating that she was "trying to find a formula that fits… the uhm problem…" Although Brittany's search for understanding took place throughout the protocol, her search was punctuated by pauses during which she would attempt various approaches. In contrast, Ben appears to have decided on a solution process shortly after reading the problem statement. He draws diagrams illustrating the problem situation, lists the known quantities, and then states: "need to get… the velocity… after he slides to here [point between the two slopes] and then… using this coefficient of friction and the velocity…" After outlining this basic solution process, Ben refers to his book, but appears to be looking for specific formulas related to friction. He starts a solution based on his outline, but hesitates after he gets an answer for the acceleration that he doesn't think is correct. He states "that can't be right…" and then starts again considering his approach to the problem. The remainder of his protocol is a nearly continuous verbalization of formulas from the textbook, conditions in the problem, definitions, and reviews of the problem statement. Ben never returns to an active attempt at the solution. It appears from his statements that he is never able to balance his available resources (text, problem statement and prior knowledge) with the conditions of the problem. In other words, Ben is unable to stabilize his understanding of the problem in a way that will lead to a solution.

As seen in Table 8, there were eight participants who verbalized significant reevaluations of their solution process. These episodes are particularly interesting, because they represent points in the solution process where the participant is questioning their initial understanding of the problem. There were several apparent cues that led the participants into episodes of destabilization, which were interpreted as searches for a new understanding of the problem. In five cases, destabilization was brought about by an impasse in the solution. That is, when the participant realized that they were at a point where they could go no further, or where they realized they had too many unknown quantities, they were forced to reevaluate their approach to the problem. Curiously, in four cases the reevaluation was cued by the participant realizing that friction, which they had previously ignored, played an important role in the problem situation. Other cues for destabilization included features of the problem statement, uncertainty regarding an equation, and the magnitude of a calculated quantity.

The destabilization episodes carried out by the participants had three possible outcomes. The first, which was seen in only two participants, was an affirmation of the solution process already being undertaken. Art, who attempted Problem 1 (see Appendix A), calculates a velocity for the midpoint of the problem, and then stops to reconsider his approach:

> "ok, I'm trying to re… remember equ… ah it how to… if I need to… what I what I'm trying to solve for I'm trying to figure out what I'm trying to solve here and I am trying to solve for x ok… let me try to visualize it again make sense of the equations all right so time independent that's fine oh wait a minute that's right it's gonna go to a stop so dig… final velocity's going to be zero what am I thinking all right (laughs) all right so that's fine yea same equation same equation…"

Art's reevaluation of his understanding seems to be cued by uncertainty about the equations he is using and how they relate to the goals of the problem. Once he reestablishes his goal and relates the equations to that goal, he determines that his initial approach was appropriate.

The second result of destabilization was to reach a new understanding of the problem that caused a significant change in the course of the solution, a result seen in the protocols of six participants. The change in the course of the problem would either be in response to an impasse in the solution with the currently utilized method, or by recognition of a new section of the solution that required a new approach. For example, Andrew starts his problem by using kinematics, but reaches an impasse in the solution when he realizes that the problem solution requires knowledge of acceleration. He reviews his equations, but notes that he has three unknown quantities: time, velocity and acceleration:

> "ok, but what about this velocity here…mmm I wonder if I could get it from the beginning right to the end… I know this (indicating drawing) some acceleration (indicating equation) mmm… see if I can find some equations (refers to notes)… kinda wake up… my physics bulb… let's see position acceleration initial oh I have this one x f equals x 1 plus v I the change in time time here time initial is zero seconds time the final time I don't know… final time I don't know plus one half a t squared…"

This impasse causes Andrew to reconsider the problem situation by reviewing the problem statement and his diagrams, and reviewing the work he had already completed. He draws a new diagram, reviews the given information, and then notes the friction. Consideration of the friction causes him to refocus his attention on the forces involved. Andrew draws a free-body diagram, and continues with his solution by applying Newton's second law. In the post-session interview, Andrew states that he started trying to solve the problem with just kinematics, but that he got to a point where he felt he could not go any further:

> "because I wasn't getting anywhere with just that (referring to kinematics) so I figure out… hey I have and I saw that there's a frictional force there so I figured that had to help me somehow seeing that frictional force pulling it down everything's going up a ramp you know with gravity affecting it so I just basically had run into a wall right there…"

Bob verbalizes a similar reassessment of his approach to his problem. Bob starts his problem with kinematics. Later in the problem he briefly reconsiders his approach when he realizes he doesn't have the acceleration he needs:

> "we need the acceleration and that can be ah… that can be found with uhm… hmm let's look at some other stuff here (refers to book)… let's see here… *unintelligible* you don't have time… hmm let's see I know what I'm trying to find I need the acceleration… ya got… we got the now let's see we have the velocity we got distance traveled we don't have time but let's see… ah ok let's… let's start by going through here and… listing all the stuff… the the ah listing all the forces that the truck is undergoing…"

This interlude represents a pause during the solution process during which Bob is no longer actively working on the solution, but rather is considering where to go next. Once he decides that looking at the forces will work, he carries his solution forward to completion. Again, this verbalization seems to fit the criterion of search for a new understanding of the problem.

The final observed result of destabilization was simply a modification of approach. In this case, the participant would apparently note that the basic approach they were applying was appropriate, but that it needed to be modified to take into account new or just-realized information. For example, Beth decides to approach her problem from an energy standpoint. She calculates the distance requested in the problem statement, then pauses to evaluate her work:

> "ok does that make sense potential energy *unintelligible* I know the final kinetic energy is zero how far up does the truck go before coming to a stop therefore v final is zero ok potential energy though first it went down so I called that negative cause it started at zero so its going down and that had negative potential energy that makes sense also then since this is L d sine alpha (refers to sketch) I need to find d I know alpha… I never used the friction force… I never used the friction force shoot…"

At this point Beth does not change her solution; she simply modifies the equation she has already written to account for friction.

The examples provided here appear to provide evidence of problem solvers engaging in active searches for a stable understanding of the problem they attempted to solve. However, it is important to note that stabilization does not necessarily imply a "correct" understanding of the problem; it implies only that the problem solver has reached his or her understanding of the problem. The working definition of stabilization as *search for a stable understanding* used in this study appears to be consistent with McGinn and Boote's (2003) definition as "shifting salience of primary factors" (p. 99). If it is, then the data from the protocols, taken together with the evidence of the four primary factors, suggests that the stabilization model, while perhaps incomplete, might provide a framework for an alternate way of looking at problem solving in physics. In order to provide for validation of the interpretations of the researcher, one protocol was selected at random from each of the groups and provided to a reviewer not otherwise associated with this research. The goal was to determine if the coding scheme developed from the stabilization model was sufficiently developed, and to see if another physics professor analyzing the protocols within the framework of the coding scheme would reach conclusions that were similar to those reached by this researcher. In the next section the instructions provided to the independent reviewer will be described, and the results of his analysis reviewed.

Results from the Independent Review

Confirmation bias is the natural human tendency to seek out, or place more weight on, evidence that supports one's hypothesis (Nickerson, 1998), which can lead researchers to collect only that evidence that supports the desired outcomes of a study. In effect, confirmation bias results in one seeing what one is looking for and what one expects to find. To reduce the possibility of confirmation bias affecting the conclusions of this study, the coding scheme and one protocol selected at random from each of the three groups of participants was provided to a reviewer who was not otherwise associated with this research. This individual was not familiar with the McGinn and Boote (2003) study, nor with the research associated with this study; however, he was an experienced professor familiar with the teaching of problem solving in physics.

The reviewer was provided with the coding grid and given the opportunity to ask questions to clarify the working definitions of each of the factors in the model. He was then asked to review the protocols provided to him, with the purpose of assigning codes to the various verbalizations and actions taken by the participants, with special attention to those sections of code that did not appear to fit within the coding scheme. He was also asked to indicate if he felt that additional codes were necessary to fully characterize the actions of the participants as they attempted to solve their problems.

After reviewing the protocols, the reviewer noted that a great deal of time during the problem-solving sessions was dedicated to the process of calculation. As a result, he created a category for noting those segments of the protocols. That an independent reviewer also noted the need for a code for calculation further supports the contention of this researcher that the problem-solving activities carried out by the participants could not be fully described without accounting for periods of verbalized calculations.

The reviewer was then asked to comment on any problems or points of confusion noted during the process of coding the protocols. His comments indicate that there were two factors that gave him difficulty as he was coding the protocols: complexity and stabilization. In relation to complexity, he noted that the complexity of a mathematical operation consisted of more than a simple count of steps, suggesting, for example, that a five-step calculus problem was inherently more complex than a five-step algebra problem, simply because of the level of abstraction of the mathematics involved. Therefore he felt that it was necessary as he was coding the protocols to expand the definition of complexity to take into account such activities as the participant verbalizing multiple unknown quantities, or even just a general expression of confusion. In relation to stabilization, he commented on some difficulty recognizing when stabilization was taking place, stating that the definition in the coding scheme was "rather vague." He noted that at times assessment seemed to take place on two levels: a consideration of strategy, and a checking of the mechanics of the work. These comments are again in agreement with the conclusions of this researcher in relation to stabilization.

As a result of the comments made by the reviewer, he was provided with the description of the factors from the original McGinn and Boote (2003) paper. He was also given the sections on the working definition of stabilization and the need for an additional category for checking work from a draft of this paper. He was then asked to review his coding of the protocols to see if he wanted to change any of his codings based on the information provided. His review resulted in 34 changes in the original 203 segments coded, as well as 13 additions and five deletions. Of the changes made, 13 were of an original coding of stabilization being changed to assessment, and six were of an original coding of calculation being changed to resource relevance in situations where an equation recalled from memory was written without being immediately utilized in a calculation. The additions and deletions were a result of, in his words, "being more careful the second time through."

After the reviewer's coding was complete, it was compared to the coding done by this researcher, using a quantified intercoder reliability measure known as Kappa, which is based on the measure of the percent of codes which correspond, corrected for marginal frequencies to take into account those codes which are used more or less frequently by one or the other of the coders. If intercoder reliability is low, it suggests that the coding scheme is ambiguous. In general, a Kappa of 0.70 is considered the minimum for an acceptable reliability (Someren et al., 1994).

To determine Kappa, a cross-table (seen in Table 9) was constructed which showed the correspondence between the codes assigned by the two coders. The diagonal of the table represents those segments for which the coders were in agreement; that is, they assigned the same codes to the same segments. All entries off-diagonal represent segments of disagreement. The differences in the marginal frequencies show the relative frequency with which each coder used a particular code. For example, in Table 9 it can be seen that Coder 1 (the researcher) coded for assessment (31 times) far more frequently than did Coder 2 (the reviewer; 18 times). The calculation for Kappa takes into account these differences by correcting the level of agreement for the expected proportion of codes corresponding based on the marginal frequencies. This gives a conservative estimate of intercoder reliability, because "similar marginal frequencies will make Kappa low where one could argue that similar marginal frequencies themselves indicate intercoder reliability" (Someren et al., 1994, p. 130). On the other hand, the proportion of codes in correspondence provides an optimistic estimate of reliability. As a result, both proportion correspondence and Kappa are generally reported. The full calculation for Kappa can be seen in Appendix G.

Table 9: Cross-Table for Intercoder Reliability (Kappa)

	Coder 2: Reviewer									
	Code	RES	CAT	GOL	COM	CAL	ASE	NRP	STA	Total
Coder 1: Researcher	RES	49	7	7	4	9	1	0	7	84
	CAT	2	16	1	0	0	0	0	1	20
	GOL	7	1	13	0	0	0	0	2	23
	COM	0	0	0	2	1	0	0	0	3
	CAL	4	0	0	0	34	1	0	1	40
	ASE	2	0	0	4	6	16	0	3	31
	NRP	0	0	0	2	0	0	2	0	4
	STA	0	0	0	0	0	0	0	9	9
	Total	64	24	21	12	50	18	2	23	214

RES: Resource Relevance
CAT: Categorization
GOL: Goal Interpretation
COM: Complexity

The proportion correspondence for the coded protocols was found to be 0.66, with a Kappa of 0.57. This level falls below the level considered acceptable for intercoder reliability, although not excessively so. There are several possible reasons for this. First, as noted in earlier discussions, the original definition of stabilization was somewhat vague. This would make it difficult to ascertain when a stabilization activity was actually taking place. Second, the reviewer's coding showed some inconsistencies, such as a segment coded as calculation in one protocol, and an almost identical segment in another protocol being coded as resource relevance. This could possibly be a result of the conceptualization of the factors evolving in the mind of the reviewer as the coding took place. Third, the interview with the reviewer following his coding revealed several conceptualizations of the factors that differed from those used by the researcher. The reviewer noted that he teaches that the choice of an equation automatically indicates the physics principles being applied to a problem. As a result, the verbalization of the writing of an equation was often coded as categorization by the reviewer, rather than resource relevance as was done by the researcher. Additionally, the reviewer indicated that an expression of confusion about the problem on the part of the participant was coded as an indication of complexity, while similar expressions typically indicated part of a destabilization process to the researcher.

Stabilization seemed to be the most problematic for the reviewer to conceptualize, and was also the factor that limits the extent to which the quantification of intercoder reliability can be interpreted. One of the primary differences in the two sets of coded protocols was that the reviewer only rarely coded for stabilization in conjunction with other factors, while the researcher viewed stabilization as a factor that could encompass other activities. As a result, all but one of the reviewer's segments coded for stabilization were in isolation; that is, the segment was indicated as stabilization only. In contrast, the researcher coded for the four primary factors first, and then reviewed the activities in relation to the others to look for evidence of stabilization. As a result, the researcher's codings for stabilization frequently encompassed two or more segments, which were additionally coded for the four primary factors. The diagonal value for stabilization indicated in the cross-table is thus only indicative of those segments where both the reviewer and the researcher had noted stabilization. Those cases where the reviewer noted stabilization and the researcher did not are seen in the off-diagonal entries. Cases where the researcher indicated stabilization as encompassing other factors are not seen in the table, as the determination of Kappa cannot account for varying levels of factors. There were a total of 52 segments that fell in this category, falling into 10 individual episodes.

According to Someran et al. (1994), there is no real solution to the problem of determining intercoder reliability when there are multiple levels of components involved. Their recommended solution is to report the different levels separately. This was done for the same protocols discussed above. For the primary factors only, the proportion correspondence is 0.69, with a Kappa of 0.60. This result suggests that there was some ambiguity in the factors of the coding scheme other than stabilization.

The determination of the correspondence of the coding of stabilization is more problematic, given that it is the only factor in its level. The proportion corresponding, indicating those segments for which the coders were in agreement, was 0.73, but Kappa was found to be only 0.14. The value of the proportion corresponding is inflated by the fact that there were 147 segments which were not coded by either coder as stabilization; that is, both coders are in agreement that the majority of the protocol segments do not represent stabilization activities.

There were a total of 67 segments that were coded by either the researcher or the reviewer (or both) as being representative of stabilization. When only those segments coded as stabilization by either or both coders are considered, we find that the proportion corresponding is only 0.13 and Kappa is essentially zero. These results suggest that the coders were in close agreement in determining what activities were *not* stabilization, but were apparently not in agreement as to what activities actually *did* represent stabilization. It should also be noted that all of the segments coded as stabilization by the researcher were also assigned codes corresponding to other, primary factors.

In this chapter the results of applying the stabilization model to the protocols were discussed, and the points of agreement and disagreement with the model were pointed out. The results of the independent review were presented, with the primary conclusion being that the reviewer was in agreement with the assertions made earlier in the chapter regarding the need for additional codes to account for calculation and checking of work. The review also appears to support the statements regarding the need for a more concrete working definition for stabilization. In the next chapter, the results of this analysis will be summarized, and the strengths and shortcomings of the model will be described. In addition, the implications of the model for future research and the ways in which the results of this study might be used to inform physics teaching will be discussed.

CHAPTER SIX: SUMMARY AND CONCLUSIONS

Problem solving is a highly complex activity, involving interplay between various conceptual, procedural, motivational, metacognitive, social factors and epistemological beliefs. It has been said that "problem solving is the most written about, but possibly the least understood, topic in the mathematics curriculum" (Lester, 1994, p. 661). The same could be said for physics problem solving. The majority of participants in this study exhibited similar levels of declarative knowledge and mathematical ability and carried out similar types of activities, such as drawing diagrams, listing known quantities, and stating physical principles. In addition, the participants were mainly the same age and had completed similar levels of mathematics course work. Yet despite these similarities, they showed varying procedural and overall problem-solving abilities. The question remains: What accounts for the variations in problem-solving abilities among problem solvers of similar educational backgrounds? The answer most likely lies in the fact that problem solving in physics, as in mathematics, involves far more than just the simple recall of facts and procedures. This study does not presume to answer the question of the absolute causes of variations in problem-solving proficiency, if indeed an answer is even attainable. It does, however, provide some important observations about those variations. In the sections that follow, the evidence provided by the participants will first be reviewed in light of the two secondary research questions: "What are the basic processes that physics students undertake as they attempt to solve physics problems?" and "What resources do students bring to the problem-solving process?" Following the summary of processes and resources, the primary research question, "To what extent does the stabilization model describe physics students' problem-solving experiences?" will be addressed. The implications of the results of this study for the instruction of physics problem solving will be discussed, and possible directions for future research suggested.

Problem-Solving Processes

Although variations in problem-solving abilities were observed, the specific processes undertaken by the participants as the attempted their solutions actually exhibited little variation. The majority of the participants listed known quantities, made use of diagrams, made accurate problem categorizations, exhibited sufficient mathematical ability, and referenced goals and subgoals within the problem solution. What differed was the success attained as these processes were carried out. In this section the primary observations will be discussed in relation to the three groups of participants identified in the initial analysis of the protocols and the level of success each group reached.

Observation One: Students Can Categorize

This is perhaps one of the more surprising results of this study. Although a few studies have suggested that students *can* make accurate categorizations of physics problems and select appropriate physics principles (Hammer, 1996; Kim & Pak, 2002; Robertson, 1990), none of these studies were undertaken in the context of actual problem-solving sessions. The closest was the study by Robertson; however, all the problems in that study were Newton's second law problems, and the focus of the study was to look for evidence of one particular physics concept:

the system concept. Thus, the results of this study can be considered as unique, in that categorization of problem situations took place within the context of actual problem-solving.

Analysis of the protocols in this study suggests that students can make accurate assessments of categorization of typical textbook physics problems. In the context of physics instruction, students have cues as to what principles they should be applying to a given problem. If the topic of study is work and energy, and the problem to be solved is in the chapter on work and energy, the student knows that work and energy are the appropriate principles to apply to the problem at hand. Similarly, Schoenfeld (1985a) noted that calculus students were highly effective at applying techniques of integration when they knew what technique they were supposed to use, such as when they were working exercises at the end of a particular chapter. But when faced with a problem where they had to decide what technique to use, they were far less effective. There were no explicit cues in the problem statements used in this study to inform participants as to what techniques they should use to solve the problem. As a result, any categorization made by the participants must be a result of prior understandings learned in their physics classes. In every case but one, the participants in this study made appropriate categorizations of the problems they attempted. Even in the case of the inaccurate categorization, the participant quickly abandoned the initial categorization and launched into a search for an alternate solution process. While at the surface these results appear to be in direct conflict with the assertion that students do not appropriately categorize problems, it is possible that the categorizations are a result of students utilizing key words or tangible objects represented in the problem statements to make their categorizations. This would be in agreement with the results found by other researchers who found that even accurate categorizations by students were not based on physics principles (Chi et al., 1981; Jong & Ferguson-Hessler, 1986; Savelsbergh et al., 2002; Snyder, 2000). Nonetheless, the fact that appropriate categorizations were made even by participants who exhibited significant conceptual misunderstandings suggests that physics instruction is effective in helping students learn to recognize the cues in problem statements that indicate appropriate solution methods. However, recognition of the problem category does not imply knowledge of the underlying concepts, nor of appropriate procedures to reach the correct solution. The majority of the participants in this study did not obtain the correct answer to the problem they attempted, despite the fact that they were able to correctly categorize the problem. This result emphasizes the need for the structural and procedural knowledge noted by Dhillon (1998) and others (Chi et al., 1989; Jong & Ferguson-Hessler, 1986; Larkin & Simon, 1995; Robertson, 1990; Savelsbergh et al., 2002).

Despite the apparent clear categorizations made by the participants in this study, some caution must be applied to the interpretation of these results. Although the participants were able make appropriate categorizations, there is little evidence in the protocols or in the post-session interviews as to the basis the participants used to make those categorizations. Two participants provided clues as to one possible explanation for the observed categorization effects. For example, Art explained his choice of a Newton's second law approach to his problem: "always think of an inclined block on a plane type thing you know always gotta set up the ah… force diagram first…" Likewise, Brittany states "I guess in Newton's law when an object's in motion down an incline… that's what I first thought of…" The seminal work of Chi et al. (1981; see also Chi et al., 1982; Jong & Ferguson-Hessler, 1986) suggested that novices rely on the surface features of problem situations rather than underlying physics principles to make categorizations.

Based on the comments of Art and Brittany, it appears that they did just that, using the cue of 'incline' to categorize the problem as a Newton's second law problem. This is a particularly relevant observation in the case of Brittany, whose protocol suggested that she had significant gaps in her conceptual understandings. Even when the categorizations appeared to be made on the basis of conceptual understanding, there is evidence that the understanding was not necessarily robust. Beth, for example, appears to make her choice of the use of energy on the basis of understanding of potential energy, but later makes a conceptual error in using potential energy in relation to the work-energy relationship. While six participants appeared to make their categorizations on the basis of at least a limited conceptual understanding, in only three cases did that understanding appear robust enough to carry the problem solver through to the end of the solution process.

Observation Two: Conceptual Understanding Affects the Use of Available Resources

A review of the resources used by the participants in this study supports this observation. All of the participants in this study, with one exception, exhibited algebraic understanding at a level sufficient for the solution of the problems they attempted. In addition, all of the participants of Groups A and C used geometrical and trigonometric principles, memorized equations, and physics concepts and principles in the solutions of their problems. In comparison, only three of five participants in Group B used geometrical and trigonometric principles or physics principles, while only two of five referred to specific physics concepts. The reduced level of use of physics concepts and principles is not surprising, given that membership in Group

B was based on evidence of conceptual misunderstandings in the protocols. However, that conceptual errors in physics should be related to a reduced level of use of geometrical and trigonometric concepts is not as clear. One possible explanation lies in the fact that many physics concepts and procedures are based on geometrical and/or trigonometric understandings. The methodology of this study did not allow for interpretation of why this connection is present; however, the fact that only participants who had conceptual difficulties exhibited a lack of application of geometrical and trigonometric concepts is suggestive of a connection worthy of further study.

At the same time, there is no evidence that conceptual difficulties are associated with an increased likelihood of the use of textual references for the purposes of obtaining formulas. While Group B participants were more likely to use a textbook to look up formulas, when the use of a formula sheet was included in the category of formula references, there was no evidence of increased use according to group membership. Two of four Group A members, three of five Group B members, and two of three Group C members referred to either the text or a formula sheet during the course of their problem solution. However, the manner in which the references were used *was* dependent upon group membership. Group A and Group C members used their textual references primarily to verify formulas; that is, they knew the formula they were looking for and used the reference to assure that they had remembered its form correctly. Group B members, on the other hand, were more likely to use their textual references as part of their search for understanding leading to a solution process. In this case, they did not know for sure what formula they were looking for, but used the reference in a search for a formula that would fit the information given in the problem statement.

The other way in which textbooks or notes were used in the problem solutions was as a resource of examples. Only three participants in this study referred to example problems during the course of their problem solutions; two of those three were members of Group B who exhibited problem-solving processes that were equation- and example-driven, with little if any reference to specific physics principles. The third was a member of Group A, who briefly referred to an example in his class notes to verify that the approach he was taking was correct.

All participants in this study used diagrams as part of their solution process. However, the manner in which those diagrams were used appeared to be dependent upon their exhibited level of conceptual understanding. Group A and Group C participants, who were characterized by apparent sufficient understanding of physics concepts, tended to use diagrams as a means of generating the mathematical relationships used in their solutions. This is in agreement with the results of Rosengrant et al. (2005), who found that successful problem solvers in physics used their diagrams to help construct the mathematical representations used in their problem solutions. In contrast, the members of Group B, who showed evidence of lack of understanding of physics concepts, used their diagrams primarily as a means of recording information provided in the problem statement or for visualization of the problem situation.

Observation Three: Students Evaluate Their Work

Pólya (1945) listed evaluation of work as an important part of the problem-solving process. In his four-part problem-solving model, he indicated that the final step in a problem solution was evaluation of accuracy, consideration of alternate approaches, and an examination of possible applicability to other problem situations. More modern work in the area of problem solving groups the process of evaluation of accuracy in the realm of metacognitive monitoring, the self-evaluation of the problem-solving process. There is some evidence to suggest that novices do not carry out ongoing evaluations of their work. Dhillon (1998), for example, noted that while experts carried out evaluation of their work throughout their problem solutions, checking both the sense and mathematics of their work, novices tended to evaluate only at the end of their problem, and then primarily only to check the mechanics of the solution. Schoenfeld (1985a) grouped checking work under the category of control, along with such actions as selecting reasonable solution methods and evaluating the effectiveness of an approach. He emphasized the importance of verification of answers in his teaching of strategies for improving problem solving, but found that students were less likely than more experienced problem solvers to check their work at any time during their solution process. However, Chi et al. (1982) did see evidence of evaluation processes in novice problem solvers in their seminal work on expertise in physics problem solving. They noted the qualitative analysis of the problem situation often occurred throughout the problem-solving process, not just at the beginning.

In general, the participants in this study did make ongoing checks of their work, particularly the members of Groups A and C. The number of checks of work during the problem solution ranged from one to 18. This ongoing evaluation looked at not only the mechanics of the work, such as if an equation had been solved correctly or the appropriate vector component was used, but also the magnitudes, units and signs of the quantities involved. The analysis of the signs of quantities calculated is of particular interest, since in physics the signs of quantities are indicators of direction, and thus are tied directly to the physics concepts themselves.

All participants in this study made at least one check of their work during their solution attempt; however, the level of evaluation varied according to group membership. Members of Group B, whose protocols showed evidence of conceptual misunderstandings, were far less likely to evaluate their work, with the number of checks made ranging from one to four times. In addition, very few of the evaluations were conceptually substantive. That is, few of the checks involved such things as units, signs or magnitudes of the quantities calculated. Most of the evaluations made by Group B members were to check that an equation was written correctly or that the calculator was in the correct mode (angle measure as opposed to radian measure).

In contrast, members of Groups A and C made many more evaluations of their work, ranging from five to eighteen checks, with one exception who made only two checks. There were no noticeable differences between the two groups with respect to number or substance of the evaluations made. Additionally, the majority of the checks were conceptually substantive. For example, Alex calculates a velocity using conservation of momentum, then pauses to consider the number he obtained: "my god… that slowed 'em down quite a bit… does it make sense… the mass increased… so the velocity had to shrink… it had to… ok…" Here we see Alex not just checking his math, but checking the physical sense of his answer, using his knowledge of physics concepts to reference his check. Members of Groups A and C also made more non-conceptual checks, such as checking equations, substitution of values into equations, and accuracy of math, than did the members of Group B. As was noted in Observation One, conceptual understanding appears to affect far more than just the use of concepts within the solution process.

Observation Four: Students Use Goals and Subgoals to Guide Their Solutions

Schoenfeld (1985a) emphasized the establishment of goals and subgoals as an effective problem-solving heuristic, but noted that teaching the use of subgoals is a difficult task. The difficulty is primarily a result of the fact that defining subgoals for a problem requires an extensive set of other skills, many of which are domain-specific and based on conceptual understanding. If goals and subgoals are utilized, they act as feedback for the solution process. If a subgoal is successfully attained, it indicates to the problem solver that the previous steps were correct, so that work can proceed on the next step in the problem (Sweller, 1983).

Ten of the 12 participants in this study verbalized subgoals during the course of their solution attempt, with the number of stated subgoals varying from two to eleven. Of those participants who verbalized subgoals, eight were able to utilize the stated goals to define their solution process, and five of those processes matched the solution that would be carried out by an expert. There is no way of knowing whether or not the technique of setting subgoals was an explicit part of the mathematics and/or physics instruction experienced by the participants, but apparently it is a technique that they have learned to employ.

As with the other observations noted in this study, it is illuminating to look at the differences in the use of subgoals among the three groups. All three members of Group C used subgoals to guide their solution process, and their solution patterns matched those of an expert. Likewise, three of the four Group A members utilized subgoals; however, the one member who did not verbalize subgoals nonetheless used them in his solution, which matched the pattern of the expert. The only member of Group A whose solution did not match that of the expert was Alex, who had incorrectly categorized his incline problem as a two-dimensional problem.

It is in Group B that differences in the use of subgoals become apparent. Only two of the five members of Group B verbalized subgoals. None of the members of this group generated solutions that matched the solution pattern of the expert, and one member ended the problem attempt prior to reaching a solution. As was the case with the other observations of problem-solving processes, we see those participants whose protocols suggest conceptual misunderstandings exhibiting significant differences in problem-solving behaviors from the other participants in the study.

Conclusion Regarding Processes: Concepts Matter

The initial analysis of the protocols simply grouped participants according to the types of errors, if any, made in the problem solutions. Subsequent analysis showed that this grouping was consistent with other characteristics of the problem solvers that were not directly related to type of error. Group A participants, who exhibited reasonable levels of conceptual understanding but who made mathematical and/or procedural errors, had more in common with Group C participants who obtained correct solutions than they did with Group B participants who made conceptual errors. The similarities ranged from how frequently the participants made checks of their work to the ways in which they used diagrams in their solutions. Participants who made concept errors were less likely to make substantive checks of their work, use diagrams to build mathematical relationships, have sufficient available resources, set subgoals, or to make continual forward progress on their problems. These participants were also more likely make frequent pauses to consider their approach, and to make use of a formula-driven search for a solution.

None of the participants whose protocols showed evidence of conceptual errors were able to reach a successful solution to their problem, yet all but one of the participants with conceptual errors carried out a solution to the point of reaching an answer. This supports the assertion that novice problem solvers are unable to monitor their own conceptual understandings (Chi et al., 1989). However, not all participants who exhibited evidence of conceptual understanding reached correct answers to their problems. The fact that the presence of conceptual errors seemed to be related to other difficulties encountered by the participants does not imply that conceptual understanding is the only determining factor in the success of the problem solver. As evidenced by Group A, mathematics and domain-specific procedures are also important. This is in agreement with statements made by Chi et al. (1982), who noted that it is procedural knowledge that ultimately determines problem-solving success (see also Dhillon, 1998; Jong & Ferguson-Hessler, 1986; Savelsbergh et al., 2002). Likewise, a recently reported study by Hoellwarth et al. (2005) indicated that instructional emphasis on concepts did not improve problem-solving ability. Anecdotally, physics instructors often complain about students who claim that they understand the concepts but just can't solve the problems. The protocols of the Group A participants seem to suggest that there may be a basis for the students' comments. Group A participants made statements both in their protocols and during the post-session interviews that implied they had a reasonable grasp of the concepts needed to solve the problems they attempted. Yet because of mathematical or procedural difficulties, they were unable to reach a correct solution. As noted by Hoellwarth et al., conceptual understanding must be coupled with adequate procedural knowledge.

Without further research, one can only conjecture about the reasons why participants with sufficient mathematical resources and conceptual understanding are unable to reach a successful solution. However, the observations in this study suggest one possible reason. In all but one case, participants reached the point in their solution where they got an answer, and in only one case did a participant question the validity of the answer obtained. In other words, the participants apparently did not recognize that they had made an error. This suggests that the mathematical and/or conceptual difficulties exhibited in the protocols do not, in general, cause the problem solver to become stuck, unable to proceed with the solution. Rather, it suggests a lack of sufficient metacognitive monitoring skills necessary to realize when one has made a mistake. This conclusion is supported by the fact that Group B participants made few checks of their work. The evidence is less conclusive for Group A participants, who in general made as many checks of their work as the Group C participants. However, the general observations related to monitoring of work is in agreement with the results of a study by Dunning et al. (2003), who compared the results of psychology students' exams with the students' perceptions of their success. They found that the less competent the student was, the more likely he or she was to overestimate their level of success. The reason, according to Dunning et al., is that the same knowledge needed for success is the same knowledge needed to recognize when one has made an error. In relation to physics problem solving, this implies that the same procedural and conceptual skills needed to attain the correct solution to a problem are the same skills needed to recognize those conceptual and/or procedural errors when they occur. Blissfully unaware that a mistake has been made, the problem solver continues on to an answer.

Application of the Stabilization Model

The primary research question for this study was "To what extent does the stabilization model describe physics students' problem-solving experiences?" Addressing the secondary questions, "What are the basic processes that physics students undertake as they attempt to solve physics problems?" and "What resources do students bring to the problem-solving process?" resulted in a level of attention to and scrutiny of the protocols to allow the attempted application of the model to take place with a reasonable level of confidence. In the previous section, the answers to the secondary research questions were addressed, with the overall result being recognition of the importance of conceptual understanding to the problem-solving process. However, it was also noted that conceptual understanding did not guarantee problem-solving success, unless it was accompanied by mathematical and procedural proficiency. In this section the primary research question will be addressed. The results of the study will be reviewed in light of the four primary factors, followed by a description of the attempt to apply the stabilization model to the protocols. The section will conclude with an overall summary of the applicability and limitations of the stabilization model.

Factor One: Categorization

The results of this study suggest that the participants were able to make appropriate categorizations of the problems they attempted. Seven of the 12 participants made explicit statements of categorization, and all but one of those categorizations was correct. Another four participants made implicit categorizations, a determination based on observation of a choice of solution method that was carried through to the end of the solution attempt. Only one participant made no categorization of the attempted problem in any form.

McGinn and Boote (2003) suggest that problem categorization is fundamental to the problem-solving process, affecting not only the solution schema that will be applied to the problem, but also complexity, resource allocation, and goal interpretation. Evidence from this study supports this interpretation. The problems used in this study could be solved by application of either Newton's second law or work-energy principles. The categorization choice made by the participant affected the knowledge and resources the problem solver had to bring to the solution, the subgoals that the problem solver had to meet to reach the final goal state, and the length of the solution. In addition, the categorization affected the information that could be obtained from the solution. The application of work-energy principles, for example, resulted in information about acceleration to be absent from the solution.

That categorization should be supported by the results of this study as a valid factor in a problem-solving model is not surprising, given the volume of literature on the subject of categorization. That all participants were able to make appropriate categorizations *within the context of problem solving* was unexpected. The majority of categorization studies reported in the literature relate to problem sorting exercises (Chi et al., 1981; Jong & Ferguson-Hessler, 1986; Schoenfeld & Herrmann, 1982), and none were found that addressed categorization in the process of think-aloud protocols. In addition, the majority of those studies suggested that individuals with limited conceptual understanding were unable to make appropriate categorizations based on underlying physics concepts. The results of this study are consistent with the results of earlier studies in that regard. However, the results of this study could be considered as unique with regard to categorization within the context of problem solving.

As noted earlier, recognition of the fact that the participants made appropriate categorizations should not be interpreted as necessarily implying an underlying conceptual understanding. Most of the participants appeared to be making their categorizations on the basis of surface features or cues in the problem statement. Regardless of the basis for their categorizations, the participants did make appropriate categorizations of the problems that, in most cases, guided their solution attempts. These results support categorization as a valid factor within the stabilization model.

Factor Two: Complexity

Problem complexity is defined by McGinn and Boote (2003) as the number of operations needed to complete the problem solution. As noted in Chapter Five, only one participant in this study made explicit reference to the number of steps needed to complete his solution. Three others made mention of the steps they needed to undertake in relation to subgoals of the problem, or of the fact that they had two unknown quantities to deal with, but it is not clear from their verbalizations whether those statements met the definition of complexity as laid out by McGinn and Boote. Other than these four participants, there were no apparent statements related to problem complexity. The lack of verbalized statements of complexity is not to say that the participants did not consider their attempted problems as difficult or complex. Some of the solutions carried out by the participants did have numerous steps, particularly those solutions that involved application of Newton's second law. It is also quite likely that in some cases, a lack of mathematical ability hampered the participant's ability to solve the problem. This was certainly the case with Brittany, who not only had conceptual difficulties but significant mathematical inadequacies as well. One possible reason for the success of the better problem solvers in this study is that their comfort with the level of mathematics involved enabled them to automate some of the solution steps, so that despite the length of the solution the problem did not appear difficult to them. Chuck, for example, carried out the solution for Problem 2 (see Appendix A) in a step-wise fashion despite the large number of steps involved. In contrast, Alex had considerable difficulty navigating the solution of the same problem. He often lost track of where he was, because the steps he was carrying out were not automatic, as evidenced by the fact that on several occasions he found it necessary to refer to his book for a formula or for the method for dealing with the magnitude of a vector. Without a clear understanding of the goals and subgoals of the problem, the mathematics involved, and/or the conceptual understanding of underlying principles, problem solvers like Alex lack the resources to recognize the difference between a process is lengthy but correct and one which is lengthy because it is not correct (Dhillon, 1998).

Part of the difficulty in interpreting the results of this study in relation to complexity lies in the narrow definition provided by the original researchers (McGinn & Boote, 2003). While a greater number of steps to complete would make a problem more difficult, the level of mathematical sophistication of those steps would also clearly be important. For example, one would expect that the completion of five calculus steps would be more difficult than the completion of five arithmetic steps.

There have been a number of studies that have investigated, either directly or indirectly, factors other than the length of a solution that affect problem difficulty. Park and Lee (2004), for example, found that the situational context of a problem had a significant impact on the perceived difficulty of a problem (see also Foster, 2007; Reusser, 1988). This was particularly true if the problem required the application of scientific concepts to everyday situations, where the problem solver's common sense knowledge might interfere with the application of appropriate scientific knowledge. Likewise, the observable or tangible objects in a problem situation are more concrete than the concepts and principles that must be used to solve it (Savelsbergh et al., 2002), making it more likely that the less experienced problem solver will focus on the surface features of the problem (Chi et al., 1981; Jong & Ferguson-Hessler, 1986; Snyder, 2000). The focus on tangibles can also increase the level of perceived difficulty, since the problem solver will not be accessing the principles necessary for the solution of the problem. Finally, the quantities referred to in a typical physics problem are more complex than the quantities typically experienced in everyday life. Many of the quantities are rates, but unlike rates in everyday life (such as miles per gallon), the rates in physics are given specialized names like velocity or acceleration that mask the fact that the quantity is a rate. These intensive variables (Bassok, 1990) also act to increase the perception of problem difficulty, as does the application of particularly abstract scientific ideas or principles such as Gauss' law, magnetic flux or a varying angular momentum vector (Foster, 2007).

The studies described above have focused on the inherent features of the problem that affect problem difficulty. McGinn and Boote (2003) state that a given situation becomes a problem when the problem solver's standard practices break down. The problem solver's perceptions of problem difficulty result from perceptions of whether or not he or she believes there are relevant resources available, the goals and subgoals of the problem are identified, the problem is appropriately categorized, and operations needed for solution can be implemented. However, one must remember that these perceptions of problem difficulty are primarily functions of the problem solver: Good problem solvers more know, and know what they know differently, than do poor problem solvers. This implies that a five-step calculus-based problem could seem trivial to one problem solver while a five-step algebra problem seems overwhelming to another. Thus the perceived difficulty of a problem is highly dependent upon the experience of the problem solver with a particular class of problem (Singh, 2002). In this respect, problem difficulty is primarily a function of the characteristics of the problem solver, which may include factors such as disposition, motivation, intuition and interest (Lester, 1994). This study focused on the more cognitive aspects of problem solving; as a result, these personal factors were not able to be accounted for.

What can be determined from these results is that the verbalizations of these participants do not appear to support the contention that problem solvers *consciously* consider the complexity of a problem as part of their solution process. However, there is limited evidence that some participants do take note of the number of steps they will have to go through to reach a solution, and of the difficulty that some participants had navigating the number of steps required for their solutions. Because of this, the results of the study must be considered as suggestive yet inconclusive with regards to the factor of complexity. It is suggested that a broader definition of complexity would assist in developing a deeper understanding of this construct and how it affects problem solving processes.

Factor Three: Goal Interpretation

Goal interpretation is defined as knowledge about how the solution should look (McGinn & Boote, 2003). Investigation of this factor led to questions concerning what features of the protocols should be considered as evidence of goal interpretation, primarily because the definition is somewhat ambiguous. Informed by the work of Schoenfeld (1985a), it was determined that the goals and subgoals of a problem determine what the ultimate solution will look like. As a result, evidence of goal interpretation was sought in the verbalizations of subgoals that led to the solution. The evidence was then interpreted based on the level of agreement between the verbalized subgoals and the actual solution pattern. This operational definition was verified by one of the authors (Boote) of the stabilization model as being consistent with the original definition.

All but one of the participants in this study expressed two or more subgoals during the course of their solutions. Upon review, it was noted that eight of the 12 participants expressed subgoals that matched their solution pattern. That is, based on statements made prior to reaching each subgoal, it appeared that the participant knew what their solutions should look like. In addition, four of the verbalized solution patterns matched the solution pattern of an expert solving the problem. This further supports the definition of goal interpretation based on subgoals.

However, the results are not as clear as for the categorization factor. Three participants verbalized subgoals that matched their solution, but which did not match the solution pattern of an expert. In other words, they verbalized *their* solution, but apparently did not know what the *correct* solution should look like. Also, one participant did not verbalize any subgoals, but nonetheless produced a solution which matched the solution pattern of an expert. This latter example may be the result of activation of an automatic solution schema, or of the limitations of think-aloud protocol data, in that not all thoughts are verbalized. Based on the evidence that eight of 12 participants verbalized subgoals that appeared to support their solutions, we conclude that the evidence is suggestive of goal interpretation as a valid factor in problem solving. However, based on the questions raised by the fact that all but one participant stated subgoals, while only eight produced solutions that matched the pattern of their stated subgoals, we must conclude that overall, while the evidence is suggestive of goal interpretation as a valid factor, it is not conclusive.

Factor Four: Resource Relevance

As was discussed in Chapter Five, the analysis of the protocols suggested that the participants in this study brought significant mathematical and conceptual resources into the problem-solving process. The majority of the participants utilized sufficient algebraic resources, in ways that were appropriate to the problem. There were two exceptions: Brittany, who demonstrated significant algebraic difficulties, and Ben, who did not complete enough calculations from which to judge his algebraic resources. In addition, most participants demonstrated the geometric and/or trigonometric resources sufficient for application to their problems. In this case the exceptions were two of the five Group B participants, who did not utilize any geometrical/trigonometric resources despite the fact that the correct solution of the problem they attempted would have required their use.

Another resource used by all the participants was self-drawn diagrams. Most physics courses emphasize the use of sketches and domain-specific diagrams such as free body and vector diagrams, so the use of diagrams was a feature that was expected to be evident in the protocols. The types of diagrams drawn and the manner in which the diagrams were used, however, varied across the groups. Members of Group B tended to used diagrams primarily as a means of recording information given in the problem statement, or for visualization of the problem situation. Members of Groups A and C, however, tended to use the diagrams not only for keeping track of information, but as conceptual tools for generating the mathematical representations used to solve the problem. This is consistent with the results reported by Hoellwarth et al. (2005), who noted that students with higher levels of conceptual understanding were more likely to connect diagrams to the mathematics of a solution.

It was in the use of physics principles and concepts that the three groups of participants were most different. This is to be expected, given that the groups were defined in terms of the types of errors made during the course of the problem solutions. All of the members of Groups A and C made explicit reference to physics principles and concepts in their solutions. However, only three of five Group B members made reference to physics principles, and only two of five referred to specific concepts. Only one Group B member made reference to both. In addition, it appeared that members of Group B had fewer physics conceptual resources at their disposal than did the members of Groups A and C. Even when statements of physics principles and/or concepts were verbalized by the Group B participants, those statements frequently showed evidence of conceptual misunderstandings.

The review of the protocols suggests that the majority of participants in this study had sufficient mathematical resources with which to complete their problem solutions. And while some participants' protocols suggested conceptual misunderstandings in the domain of physics, there were still seven of the 12 who appeared to possess sufficient declarative physics knowledge with which to reach a successful solution to their problem. As noted in Chapter Five, it should be noted that the tabulation of resources used does not indicate whether the resources were applied correctly or whether the conceptual understanding expressed in the protocols was fully developed in the mind of the participant. In fact, the protocols of Group B participants seem to support what Maloney and Siegler (1993) called *conceptual competition,* in which alternate conceptions exist in the mind of the novice problem solver (see also Mildenhall & Williams, 2001). It is also not apparent from the protocols or the post-session interviews whether or not a conscious consideration of the appropriateness of a resource took place. It is quite likely in the case of some resources, such as algebraic knowledge, that the use of the resource is so automated that there was no need for consideration of its relevance. McGinn and Boote (2003) note that identification of relevant resources is not difficult when a problem has been appropriately categorized. A lack of evidence on the part of the Group C participants of a conscious consideration of resource relevance may be a result of their ease of categorization and their partially automated solution processes. The lack of verbalized reference to resource choices on the part of the members of Groups A and B is more problematic, since their solution processes were clearly not as automated as those of Group C.

The observations described here and in Chapter Five suggest that there were two ways in which the use of resources was evident. The first, simpler way is to support an already stable problem situation. The problem solver already knows how he or she will solve the problem, and uses a formula sheet to verify the form of and equation, or creates a free body diagram to help visualize the forces acting on an object and to assist in the summation of those forces. Cognitive resources also fall in this category, such as when the problem solver recalls a memorized formula or the procedure for solving a system of equations, or accesses conceptual knowledge about the relationship between friction and motion. Far more complex is the use of resources in an attempt to reach an understanding of the problem. In this case, the problem solver is attempting to reach a balance between the four primary factors, and the use of resources is directed to assist in attaining that balance. The problem solver may in this case search for an example that might cue an appropriate categorization, which would in turn cue the correct equations to use. This may in turn help the problem solver to recognize the level of complexity involved in the problem.

This analysis suggests that the participants of this study made use of relevant mathematical and conceptual resources in ways that were in general appropriate to support their problem-solving processes. These observations support the assertion that resource relevance is an important factor for consideration in a model of problem solving. However, there is no evidence from the protocols that those resources were determined to be relevant through *conscious* processes on the part of the problem solver.

The Superordinate Factor: Stabilization

McGinn and Boote (2003) define stabilization as a "shifting salience of primary factors… superordinate to the primary factors" (p. 99). This rather vague definition was the first challenge to be met in attempting to apply the stabilization model to the protocols in this study. The description of a problem given by McGinn and Boote provided the clues needed to develop a working definition that could be used to search for evidence of stabilization within the protocols. They state that a problem is a situation in which the four primary factors of categorization, resource relevance, goal interpretation and complexity must be evaluated and balanced in order to find a workable understanding of the problem. They go on to note that when a problem is stabilized, all that remains is calculation; that is, the solution process is known. It was also noted that a critical part of success in problem solving was related to destabilization, that is, knowing when a current understanding of a problem situation is inadequate and is not leading to a solution. These descriptions led to the working definition of stabilization for this study: *the search for an understanding of the problem that will lead to a solution*. If a participant was noted to be considering a solution, not actually carrying out a solution, it was considered as evidence of stabilization. This working definition of stabilization was verified by one of the authors (Boote) of the stabilization model as being consistent with the original definition.

Several examples of protocol segments that were deemed to be evidence of stabilization were provided in Chapter Five; they will not be repeated here. However, the results of that analysis will be summarized. First, it was noted that three participants exhibited no evidence of stabilization activities. These participants were all members of Groups A or C, who almost immediately categorized their problems and launched into apparently automated solution processes. The solution process for these individuals seemed to be characteristic of the step-wise processes of competent problem solvers solving familiar categories of problems (Bereiter & Scardamalia, 1993; Schoenfeld & Herrmann, 1982; Singh, 2002). For these participants, the problems were exercises. This is consistent with statements made by McGinn and Boote (2003), who noted that when a problem is categorized appropriately and resources are readily available, the problem solution is highly automated and all that remains is calculation. Two of the participants carried out activities that appeared to be searches for understanding early in their protocols, but decided on a solution process very quickly. One of these participants was a member of Group C, who quickly carried out his solution with only a minor copying error at the end. The other was a member of Group B, who determined a solution method, which was then carried out incorrectly and with conceptual errors. Of the remaining participants, three carried out searches for understanding only near the beginning of their protocols. Three others paused throughout their solutions to consider or reconsider their solution process. One participant decided quickly on a solution method without any evidence of a search, but then stopped midway through his rather lengthy solution to reconsider if what he was doing was correct.

There was also evidence of destabilization, but the monitoring that is part of the destabilization process was not always seen to be successful. All participants in the study, with one exception, were able to reach an answer to the problem they attempted, despite evidence of conceptual and procedural difficulties. The majority of the participants had episodes in their problem-solving processes that could be interpreted as searches for a new understanding of the problem, and most of those episodes resulted in at least a modification of approach if not an actual change in the course of the solution. Despite this, the monitoring process failed in most of those cases, as evidenced by the fact that only three participants reached a correct solution to their problem. Despite destabilization, the metacognitive monitoring carried out as the participants searched for a stable understanding of the problem was not sufficient to catch the conceptual or procedural errors present in the solutions. Chi and her colleagues (1989) noted that novice problem solvers were likely to detect and admit their lack of mathematical or procedural understanding, but were far less likely to detect gaps in their conceptual understanding. This appears to be the case for the Group B participants, who proceeded to an answer despite significant conceptual errors. However, why the participants in Group A were able to carry out destabilization activities and still proceed to an answer even with procedural or mathematical errors is less clear. While some studies have suggested that novice problem solvers do not make ongoing checks of their work (Dhillon, 1998; Schoenfeld, 1985a), that was not the case in this study. Despite ongoing checks, mathematical and procedural errors were still made. Destabilization occurred, but the monitoring of work that takes place with it was not always successful.

The results of this review of the protocols suggest that the participants in this study did carry out active searches for an understanding of the problem that would lead to a solution. During these apparent searches, the participants made use of their available resources, considered the goals and subgoals of the problem, and attempted categorization. In addition, there is also evidence in the protocols that certain cues within the problem-solving process, such as reaching an impasse or the magnitude of a calculated quantity, can trigger destabilization: a reevaluation of the problem situation leading to a renewed search for understanding. The evidence is also supportive of McGinn and Boote's assertion that stabilization is not needed when a problem is perceived as an exercise, as in the cases of Chuck, Cory, Arnold, and Carl. At this point, the tentative conclusion is that stabilization is a valid construct as related to problem solving, in that it describes the problem solver's active and conscious search for understanding and a valid solution process.

Putting It Together: The Stabilization Model

The previous section has summarized the extent to which the various components of the stabilization model were evident in the problem-solving protocols generated by the participants in this study. Although the results were mixed, the question remains: When taken as a whole, is the stabilization model a valid descriptor for the problem-solving experiences of the participants? Or, to restate the primary research question: "To what extent does the stabilization model describe physics students' problem-solving experiences?" To address this question, the strengths and weaknesses of the model and the evidence supporting and refuting it must be evaluated.

The strength of the model lies in the fact that three of the four primary factors that form the backbone of the model have already been identified through research in physics and/or mathematical problem solving as important factors in problem-solving success. Categorization (Chi et al., 1981; Jong & Ferguson-Hessler, 1986; Savelsbergh et al., 2002; Snyder, 2000) and resource relevance (Cui et al., 2005; Ozimek et al., 2004; Schoenfeld, 1985a; Schoenfeld & Herrmann, 1982; Tuminaro & Redish, 2003) both have a long history in the literature and are well-documented. The importance of goals and subgoals in problem solving has also been documented (Sweller, 1983), although not so extensively as categorization and the use of resources. While there has been some mention of complexity was found in the literature (Foster, 2007; Lester, 1994; Reif & Heller, 1982; Singh, 2002), the definitions used by other researchers appear to differ somewhat from that used by McGinn and Boote (2003). Nevertheless, it would seem reasonable that the complexity of a problem would have a direct impact on problem-solving success, particularly for the novice. Of course, reasonableness does not guarantee validity. Consideration of the evidence supporting each of these factors *as a group* must be taken into account in order to make a determination as to the applicability of the model.

The evidence regarding two of the primary factors is particularly strong. With one exception, all of the participants in this study were able to make appropriate categorizations of the problems they attempted. Although it could not in general be determined on the basis of the protocols or the post-session interviews whether or not those categorizations were grounded in understanding of underlying physics principles or based on surface features, it appeared that participants were able to use their categorizations successfully in the search for a valid solution process based on their understanding of the problem. That the basis of the categorizations could not be determined is not particularly problematic in relation to support of the stabilization model, as the model makes no mention of the basis of categorization. Likewise, the protocols provide strong evidence of the use of relevant resources in the problem-solving process, albeit without evidence of conscious consideration of those resources.

The evidence on goal interpretation is suggestive, but not conclusive. Part of the difficulty in interpreting the results in light of this factor lies in the definition provided by McGinn and Boote (2003) as *knowledge of how the solution should look*. The research design of this study did not include methodology that would easily determine if the participant would know the form of the solution prior to actually carrying out that solution. An operational definition based on stated subgoals, informed by earlier work by Schoenfeld (1985a) and verified by one of the authors (Boote) of the original stabilization model, was developed in order to look for evidence of goal interpretation. Based on an assumption of agreement between the definition used in this work and that originally proposed by McGinn and Boote, the overall conclusion is that the data is suggestive of goal interpretation as a valid factor. The data is not considered conclusive because although all but one participant stated subgoals as part of their solution process, only eight protocols showed solution paths that matched the verbalized subgoals. That is, although the remainder of the participants stated subgoals, there was no evidence that those stated goals indicated knowledge of the solution process.

Complexity was the only primary factor for which there was little direct supporting evidence in the protocols. Only one participant explicitly stated any indication of knowledge of the complexity of the problem attempted, although three others made brief mention of the steps that needed to be taken or of the presence of two unknown quantities. Therefore, it must be stated that the concept of complexity as a factor in problem solving as defined by McGinn and Boote (2003) is only minimally supported by the evidence exhibited in the protocols. However, there is some evidence that some of the participants had difficulty navigating the complexity of their problems, suggesting that an expanded definition of complexity may be needed.

Finally, the overarching factor of stabilization must be considered. It is in stabilization that the limited definitions provided by McGinn and Boote (2003) prove to be most problematic. Because their definition was so vague, a new working definition informed by McGinn and

Boote's descriptions of the stabilization process had to be developed. If the working definition of stabilization as the coordination of a *search for an understanding of the problem that will lead to a solution* is valid, then there is strong evidence that the participants of this study did engage in active searches for an understanding of the problems they attempted to solve. There was also evidence of destabilization, whereby situations arising within the solution process cued the problem solver to reevaluate their solution and engage in a renewed search for understanding. In addition, the initial and renewed search processes involved the use of relevant resources, interpretation of goals, and attempts at categorization, which supports the assertion of stabilization as superordinate to the primary factors.

The aspects of the problem-solving processes that were not addressed by the model must also be taken into consideration. First, there is no mention of calculation or other mathematical manipulation in the description of the stabilization model. This is potentially problematic. It was noted during the review of the protocols that errors were often made not in the setup of the problem, but rather in the process of calculations. Those errors then affected the portions of the solution that followed. If a model is to fully describe problem-solving processes, it must take into account all aspects of that process. Not accounting for calculations risks removing the context in which errors, which might affect the course of the solution, occur. Second, it was noted that all participants evaluated their work during the course of their solutions. The level of evaluation varied from superficial to extensive, but even so, it was a universal feature of the protocols in this study. The same argument that was made in relation to calculations can be made for evaluation. If an action is an important part of the problem-solving process, it should be integrated into any model describing that process. That evaluation should be a part of a problem-solving model is supported by the literature (Dhillon, 1998; Pólya, 1945; Schoenfeld, 1985a).

The previous discussion leads to the conclusion that while the evidence supporting at least one aspect of the stabilization model is inconclusive, the model does hold some promise for applicability to physics problem solving. This conclusion is based on the operational definitions used in this study and verified by Boote (personal communication, October 11, 2006). This is the primary shortcoming of the model as originally described. Without sufficiently rigorous definitions of the factors, the model is left open to interpretation, much like that which was required in this study. While care was taken to develop working definitions that were consistent with those originally proposed, and those definitions were verified by Boote, it must be recognized that if the model is described in terms that are too vague, there is danger of misinterpretation.

Although stabilization, with appropriately rigorous definitions, could be considered as a model of problem solving, it might be more productive to consider it not as a model, but rather a descriptive framework. The terminology of model implies some ideal, while descriptive framework is more suggestive of an aid to developing understanding. If, on the other hand, further refinement of the framework including more rigorous definitions of its primary constructs shows that the framework is effective in describing both novice and expert behavior, then perhaps the status of model would be more appropriate.

The model as described by McGinn and Boote (2003) does appear to predict the behavior observed by the novice problem solvers in this study. For example, the model predicts that in the case of a problem that is quickly categorized and stabilized, the solution path is known and the solution will proceed in an automated, step-wise fashion. This is precisely the behavior exhibited by Chuck, whose problem solution matched that of the expert. It also predicts that if the categorization is not valid, the problem is perceived as more difficult. Alex's protocol was an exemplary case of this situation. Likewise, the model predicts that if categorization is not possible, the search for a solution can lead to what McGinn and Boote called "flailing about" (p. 103), behavior that was observed in Ben. Further support for the applicability of the model comes from the post-session interviews. When asked to describe how he usually goes about solving a problem, Arnold stated:

> "the first think I like to do is just write down my knowns ahm and then draw a picture and somehow connect the knowns to the picture…and then figure out what kind of a law if not I already know figure out what kind of problem it is… uhm and then write down maybe a few relevant equations that might be helpful… I don't usually like to calculate numbers until the end I like to end up with one final equation… and then just plug into all of it which I kind of did toward the end"

Arnold's description of his problem solving process indicates that he attempts to make sense of the problem (stabilizes the situation) by categorizing, gathering information and formulas (finds relevant resources), working to get a single equation (navigates the complexity), and working towards the goal (plugging in at the end). Arnold describes a problem-solving process that sounds very much like that described by McGinn and Boote.

That the framework as described was able to provide general descriptors of the behavior observed in the novices in this study suggests that the framework is solid. That there were features of the protocols that were not fully supportive of or described by the framework suggests that it is incomplete. Like any framework, in needs to be filled in with appropriately detailed structure. The operational definitions used in this study could form the basis of a more rigorous definition of the constructs that make up the model. More concrete definitions or additional verification of the current working definitions would be required before further investigations into the applicability of the model take place. Adding additional structure to the framework, such as an accounting of evaluation and calculation, would provide a more complete descriptive framework. Interpreted as a descriptive framework, stabilization would not replace existing models of problem solving, but rather could be considered as an alternative way of viewing the process of problem solving. Much like physics has alternate conceptions of light, all of which have validity in their areas of application, stabilization could be could be considered as an alternate conception of problem solving, providing another tool for developing understanding what is unquestionably a highly complex process.

Limitations to Applicability and Generality

This study addressed several of the limitations of previous studies discussed in Chapter Two. McGinn and Boote's (2003) original study was carried out utilizing introspective techniques. In addition, neither of the original researchers could be categorized as novices in the field of mathematics. As a result, one cannot be sure that the observations they made in developing the stabilization model were general enough to be applied beyond their own problem-solving experiences, or whether they were unique to their own experiences as quasi-experts. This study addressed that shortcoming by looking specifically at novices, utilizing think-aloud protocol techniques which allowed for verification of McGinn and Boote's observations in other problem solvers. This study did not assume that problem-solving was a linear process, an assumption that was the basis of a number of studies and strategies reported in the literature (Heller & Heller, 2000; Pólya, 1945; Schoenfeld, 1985a). It also did not consider the novice problem solvers as a homogenous group; in fact, the preliminary analysis of the protocols emphasized that fact that the group was quite diverse in their problem-solving abilities. An additional strength of the study was in that it investigated conceptual understanding in the context of the actual problem-solving process, an approach that has only recently begun to see attention in the literature (Hoellwarth et al., 2005; Hung & Jonassen, 2006).

As in any study of human behavior, there are limitations to the applicability of the results of this study, and to the extent to which they can be generalized. Most of the limitations are a result of the research methodology of the study, as discussed in Chapter Three. Although the general limits of the study were outlined at that point, it would be prudent to review those limitations in light of the analysis undertaken and the results obtained. There were also additional limitations that were noted as a result of the attempt to apply the stabilization model to the protocol data obtained.

The primary assumptions outlined in Chapter Three are still considered valid. These assumptions are those associated with the use of verbal protocol data, informed by information-processing theory. In short, we assume that verbal behavior can be recorded and analyzed and that participants verbalized the cognitive processes they actually attended to during their problem-solving sessions. Information-processing theory holds that these verbalizations are of information under the immediate attention of the problem solver within their working memory. As a means of validating the data obtained, the sessions were videotaped, which allowed for the researcher to review the session with the participant, to clarify actions, and to elaborate on any periods of silence in the protocols.

It can be expected that not every thought will be recorded, primarily because the process of verbalizing thoughts takes longer than simply thinking those thoughts. As noted earlier, estimates suggest that under the very best conditions information can be obtained every second for verbal data, while the information processes that are taking place may only be in the range of a few tens to a few hundreds of milliseconds long (Larkin et al., 1980a). This means that a certain amount of inference was necessary as the verbal protocols were reviewed. The use of videotaping and post-session interviews with the participants greatly reduced the amount of inference, and allowed for validation with the participant when inference was necessary (see, for example, Taylor & Dionne, 2000). Even though videotaping allowed for validation and cross-checking that would not have otherwise been possible, there were still questions that arose during the process of analysis. A passing comment or action taken by the participant may not have been noted during the problem-solving session, but was seen during the transcription or analysis phase. Although there were few of these situations, in those cases that did arise inferences had to be made in light of the context of the action and, whenever possible, supported by statements in the post-session interviews.

A second limitation of this study is the small number of participants. The large time commitment required to record, transcribe, code and analyze the protocols necessarily limits the amount of data that can be utilized in think-aloud protocol studies. Because of this, extreme caution should be exercised in trying to generalize the results beyond the population from which the participants were drawn; that is, beyond similar groups of introductory calculus-based college physics students. However, it should be noted that there was variation in the problem-solving abilities of the participants, from a level at which the participant could not finish the attempted problem to one in which the participant produced an expert-like solution pattern. This suggests the possibility of extension to problem solvers of other abilities, although more study would be required.

A significant limitation of the study was a result of the somewhat ambiguous definitions of the factors in the stabilization model as provided by the original authors (McGinn & Boote, 2003). There was no question about the meaning of categorization or resource relevance, as these were previously identified factors that are well documented in the literature. Goal interpretation, while present in the literature, appeared to take on a slightly different meaning in the stabilization model than that traditionally used in the literature, which required the researcher to consider the ways in which goal interpretation might be evidenced in the protocols in this study. The other two factors, complexity and stabilization, were constructs developed by McGinn and Boote through their original research and as such have only limited historical background in the literature from which to draw from. This required the researcher to develop a personal understanding of those constructs, and then determine working definitions to use in analyzing the protocols. Although great care was taken to assure that the working definitions were consistent with those used by McGinn and Boote, and were verified by consultation with one of the original authors (Boote) any interpretation of the results of this study must take into consideration that the working definitions that guided the analysis were this researcher's interpretations of those constructs.

Finally, the original work by McGinn and Boote (2003) included discussions of the importance of social interactions in the problem-solving process. According to the original model, social resources such as student-student and student-professor interactions must be included if a full understanding of problem solving is to be developed. The importance of social interactions in the development of expertise within a domain has been noted by other researchers (Heller et al., 1992; Redish et al., 1998; Tobias, 1990). Likewise, the importance of consideration of motivational, emotional, and epistemological factors was noted (Cummings & Lockwood, 2003; Hammer, 1994). Cognitive activity is at the heart of problem solving. If a model of problem solving cannot describe or explain the cognitive activities undertaken by the problem solver, then it would be pointless to see how it applies to the other factors, some of which are perhaps best studied outside the context of actual problem solving. Thus we start with the heart of problem solving, verify its applicability there, and then move on to other factors. For these reasons, although the relevance of those other factors is acknowledged, this study focused on the cognitive aspects of problem solving only.

Implications for Instruction

A number of research studies have resulted in recommendations for instructional practice. Schoenfeld (1985a), for example, used Pólya's (1945) four-step problem-solving model to develop instruction in the use of heuristics for mathematics. Likewise, the step-wise problem-solving processes observed in experts has been used as a guide for teaching physics problem solving (Heller & Heller, 2000). Some of these practices have attained a moderate level of success (Heller et al., 1992), but in general instruction in physics problem solving has not made significant advances despite the increased understanding of both expert and novice behavior that has been the result of the past 25 years of research. A large part of the reason is what we have learned: Expertise in physics problem solving, as in any domain, is the result of extended, deliberate practice over a long period of time (Bereiter & Scardamalia, 1993; Lester, 1994). Nonetheless, recommendations for good practice in problem solving instruction are still needed, with the goal of assisting students in the attainment of that goal.

The most substantial outcome of the review of the participants' behavior in this study was the observation that concepts matter. While this might sound like stating the obvious, it is the level to which conceptual understanding was observed to be linked to the behavior of the participants that is of particular importance. Participants who demonstrated a lack of understanding of basic physics concepts and principles were also less likely to use their geometrical/trigonometric resources, use diagrams to support their solutions, use examples or equations appropriately, check their work, or to generate subgoals to guide their problem-solving attempts. This is not to imply that there is a causal relationship between lack of conceptual understanding and these other behaviors; however, the data do suggest a relationship. The implication is that the increased emphasis on conceptual understanding that has been the focus of physics education reforms over the past two decades is addressing an important factor in helping students to develop effective problem-solving techniques. Physics educators should continue that emphasis. Some studies suggest that students believe that studying concepts will not help them get good grades, and that only by focusing on formulas and problem solving can they be successful in physics class (Elby, 1999). Rather than emphasizing formulas and their algebraic manipulation only, instructors should include emphasis on concepts, and incorporate the physical concepts into problem-solving instruction explicitly (Hammer, 1989).

The results of this study also suggest that conceptual understanding is not the only factor in successful problem solving; procedural knowledge is also important. Successful problem solving requires that conceptual and procedural knowledge be linked, so that recognition of particular physics principles within a problem calls up appropriate procedures for solving the problem. It is this linked knowledge that a number of the participants in this study appeared to be lacking. Problem solving skill is a traditional goal of physics instruction, but how to attain that goal is still a subject of debate. The traditional approach is to assign students lots of problems; however, there is evidence to suggest that approach is not effective for the majority of students (Joshua & Dupin, 1991; Kim & Pak, 2002). Similarly, the emphasis on conceptual understanding in recent years has not been shown to increase problem-solving ability (Hoellwarth et al., 2005; Hung & Jonassen, 2006). Clearly an approach that finds a balance between procedural and conceptual knowledge is needed.

At the same time, there were some features of effective problem solving that were exhibited by all participants, such as appropriate categorization and the use of diagrams. The fact that these behaviors were seen even in unsuccessful problem solvers suggests that these features could be used as scaffolding to help support the development of appropriately linked conceptual and procedural knowledge through the course of problem-solving instruction. An instructional process that started with the student's surface-feature referenced categorization as a basis for linking to the underlying physics principles could be followed by instruction in generalized procedures for applying those principles to problem solving. Instruction should explicitly verbalize and model the connections between concepts and procedures. The possible effectiveness of carrying through on this suggestion is supported by statements made by several participants during the post-session interviews, in which they mention that they utilize the techniques demonstrated by professors as they attempt to solve unfamiliar problems. Alex, for example, mentioned trying to emulate his professors, Art mentioned learning to work in symbols because his professor encouraged him to do so, and Andrew stated that he typically calculates everything possible in case he needs it later because that's what his professor taught him to do. The use of diagrams as connections to the mathematics of a solution should also be demonstrated and emphasized. The idea is to link what students are already able to do – categorize problems and draw diagrams – to what they are not able to do – call up appropriate principles and procedures. Explicit instruction in the use of subgoals to outline the problem-solving process is also encouraged.

Suggestions for Future Research

In the original paper describing the stabilization model, McGinn and Boote (2003) discuss the importance of non-cognitive factors to the problem-solving process. These motivational, social, attitudinal, epistemological and emotional factors were not addressed by this study. Other researchers in the domain of physics education have begun to look at issues such as expectations and epistemology (Elby, 2001; Hammer, 1994, 2000; Redish et al., 1998), attitudes (Cummings & Lockwood, 2003) and metacognition (Koch, 2001; Oladunni, 1998). In her now-classic *They're Not Dumb, They're Different: Stalking the Second Tier,* Tobias (1990) reported on the importance of the social structure and interactions within the physics classroom and department to the teaching and learning of physics. If a modern model of problem solving is to address the many facets that make up effective problem solving, and how students learn to be effective problem solvers, testing of that model must investigate those factors. This suggests several directions for future research. Problem-solving sessions with dyads and triads would be a good place to start, given that six of the 12 participants in this study noted that they typically do homework problems with others. In-field observations of interactions between students and professors and/or teaching assistants would also be relevant.

The motivational aspects of physics problem solving are another area which has received only limited attention in the literature (Cummings & Lockwood, 2003; Elby, 1999; Redish, Saul, & Steinberg, 1998). The actions taken by the problem solvers in this study suggest that this aspect is worth of further study. Investigations in why students such as Ben end a problem-solving attempt after only 10-15 minutes without reaching a solution, while other problem solvers such as Alex or Chuck continue on through a lengthy solution of 30-60 minutes might help us better understand the role of motivation in the development of competence in problem solving.

The results of the analysis in this study suggest several other possible directions for future investigations. Categorization in particular bears further study. All but two participants were able to appropriately categorize the problem attempted, but there is little indication of the basis used to make those categorizations. A closer look into categorization in the context of problem solving, with particular attention to determining whether surface features or underlying principles were used, could provide information on how best to scaffold to physics principles for those students who focus primarily on surface features. Another possible avenue for future research is suggested by the connections noted in this study between conceptual understanding and other aspects of the problem solving process. A study connecting the results of commonly utilized tests of conceptual understanding, such as the Force Concept Inventory (Hestenes & Halloun, 1995; Hestenes, Wells, & Swackhamer, 1992), and think-aloud protocols of problem solving could further investigate those connections.

As previously mentioned, the definitions of the some of the constructs that make up the stabilization model are somewhat vague. If more robust definitions for complexity, goal interpretation and stabilization were to be established, further testing would be needed to establish the validity of the model. For example, in relation to goal interpretation, it is not clear from the results of this study whether or not problem solvers truly know what the solution would look like. Investigation of goal interpretation, perhaps in the context of identifying subgoals prior to the solution of the problem, could help answer that question. In relation to complexity, it was noted that there are numerous other factors that appear to affect problem difficulty besides the number of steps needed to solve the problem. One factor that was seen in this study but not addressed directly was the choice of solution methods. In this study, all of the problems used had sections that could be solved either by application of Newton's second law or by work-energy principles, with the Newton's law approach resulting in more steps needed to reach a solution. Foster (2007) noted that a problem with more than one possible solution method was in general perceived as more difficult by students. No other study was found in the literature that specifically addressed the choice of solution techniques, suggesting that it is an area in need of further investigation.

Finally, it was proposed in the introduction to this study that the establishment of a problem-solving model that was equally valid for both experts and novices would be a worthy goal. Investigation of the applicability of the stabilization model to the problem-solving behavior of experts would be the next logical step in that process.

The results of the testing of the stabilization model in this study were mixed. Some aspects of the model, in particular categorization, resource relevance and goal interpretation, were strongly supported by the data. This was expected, given the volume of research reported in the literature on these topics. The construct of complexity, however, was not strongly supported. Whether this was a result of problem solvers not consciously considering problem complexity or of a shortcoming of the methodology or of the working definition could not be determined. Stabilization as a construct appeared to be supported by the data, but even that must be considered as inconclusive, given the vague definition provided by the original report. However, overall there seems to be enough evidence to support the model to warrant further study. If the shortcomings in the definitions of model constructs can be addressed, the framework filled in with sufficient detail, and it is found to describe the problem-solving experiences of experts and novices alike, then the stabilization model may be able to attain the status of an alternate problem-solving model.

APPENDIX A: PROBLEMS USED IN THE STUDY

The problems selected for use in the study were gathered from introductory physics texts or were written by the Researcher. The criteria for the problems were that a) they be of a level of difficulty such that they would be perceived as problems by the Participants, not exercises, b) they require the use of multiple concepts and steps in order to reach a successful solution, and c) they can be solved by more than one method. From a pool of ten questions, the following three were ultimately selected for use in the study.

> Problem 1: A truck with a mass of 1500 kg is traveling down a mountain road at 22 m/s when it hits a thick patch of ice. In a panic, the driver hits the brakes, which fail, causing the truck to slide essentially without friction down the 5^o slope. After traveling down the slope for a distance of 300 m, the driver manages to get the truck onto a runaway truck ramp, which is inclined at an angle of 10^o *upwards* from the horizontal. The ramp is covered with a soft material, which results in a coefficient of friction of 1.8. How far along the truck ramp does the truck go before coming to a stop? (Adapted from Young & Freedman, 2004, p. 279, problem 7.66)

Problem 2: Jack and Jill are sledding on a snow-covered hill ($\mu_k = 0.1$) that is inclined at an angle of 20^o to the horizontal. Jill (m = 50 kg) runs at 4 m/s across the top of the hill, landing on the 5 kg sled which is at rest at the very edge of the hill. He brother Jack stands at rest a distance of 14.6 m down the slope. As Jill passes Jack (m = 30 kg), he jumps onto the back of the sled. They continue together on down the slope, reaching the bottom after traveling a total distance of 40 m along the slope. How fast are they going when they reach the bottom of the hill? (Adapted from Serway & Beichner, 2000, p. 283, problem 20)

Problem 3: At the intersection of University Blvd. and Rouse Rd. a subcompact car with a mass of 950 kg traveling east collides with a truck of mass 1900 kg traveling north. The two vehicles become entangled as a result of the collision, and travel a distance of 15 m at an angle of 66^o north of east before coming to a stop. The officer arriving on the scene measures the coefficient of friction between the tires and the road and finds that it has an average value of 0.85. What was the speed of each vehicle just before the collision occurred? (Written by the researcher)

APPENDIX B: INSTRUCTIONS TO PARTICIPANTS

The purpose of this research is to gain a better understanding of how people solve physics problems. The best way to learn about how people solve physics problems is to ask them to think aloud as they solve a problem. We are not concerned with whether or not you ultimately get the correct solution to a problem; we are primarily interested in the processes you undertake as you attempt to solve a problem.

When you think aloud, the goal is to verbalize as much as possible of what you are thinking. Almost everything you say will be important for the researchers as they attempt to develop a better model of problem solving. You won't be able to verbalize everything you think, but the idea is to verbalize as much as is possible, to provide the best report of your thinking processes. To achieve this, it is important that you talk constantly throughout the solution process.

As you think aloud, don't try to plan what you are going to say. Your report does not have to be well structured or grammatically correct. What is most important is that you accurately reflect your thoughts, or even fragments of your thoughts. Ideally, you should verbalize directly what is going through your mind as you attempt to solve the problem, without paying too much attention to how it is coming out.

Adapted from Taylor & Dionne (2000).

APPENDIX C: PARTICIPANT INFORMATION SHEET

Your interest in participation in the Physics Problem-solving Study is appreciated. To help the researcher with the participant selection process, please provide the following information. After all student information has been tabulated, students chosen to participate in the study will be contacted to set up an appointment to meet with the researcher.

Your expressed interest in the study does not in any way obligate you to participate in the study if you are chosen. Likewise, if you participate in the study, you have the right to withdraw from the study at any time for any reason without penalty.

The information you provide on this form, or as a result of the study, will not be released to any person in any form that would enable you to be identified. It will be held in confidence by the researcher, and all identifying information will be removed from the data collected before any results of the study are made public.

Name ____ Sex M / F

Phone _______________________ Email ___________________________________

Major _______________________________________ Current physics grade ________

Last math class completed ________________________________ Grade earned _______

SAT score: Composite ______ Quantitative ______ Qualitative ______ ACT score ______

____ Which of the following best describes your reason for taking this class?

a. It is a prerequisite for other courses.
b. It is in my major field.
c. I am interested in the material.
d. I am taking it as an elective only.

____ Which of the following best describes how you *usually* solve physics problems?

a. Alone, as the material is presented in class.
b. Alone, at the last minute before it is due or before an exam.
c. With one other person.
d. With a study group.
e. By copying other people's problems.
f. By copying the solution manual.

____ Which of the following best describes what you hope to learn from this class?

a. Fundamental physics concepts.
b. Problem-solving strategies.
c. Both a and b.
d. Nothing that will apply to future classes; I'm only taking the course because I have to.
e. I'm not sure.

____ Which of the following best describes your perceptions of the purpose of problem solving in physics courses? (You may include more than one response if appropriate.)

a. A way to learn physics concepts.
b. A way to learn mathematical procedures.
c. Busy work with no real relevance to success in the course.
d. A way to learn how to apply physics concepts to real-world situations.
e. Useful for learning procedures, but having no real relevance to real-world situations.

APPENDIX D: CODING GRID

Table 10: Protocol Coding Grid

Category	Definition and Prototype Statements	Code
Categorization	Participant expresses that a problem belongs in a particular category "This is a work-energy problem." "I can apply the 2^{nd} law here."	CAT
Goal Interpretation	Participant expresses what the problem is asking for and what the solution should look like "They are asking me to find the final velocity of the ball." "It looks like I'm supposed to provide a proof."	GOL
Resource Relevance	Participant utilizes a particular material, conceptual, social or mathematical resource "I need the cosine component…" (participant reaches for calculator) "I don't remember that formula…it's in the book." "There's more than one unknown…I'll need a system of equations."	RES
Complexity	Participant expresses an assessment of the difficulty of the problem "This is going to take several steps." "Before I can find the final velocity, I need the acceleration. But all I know is the force." "This is simple. I just need to…"	COM
Stabilization	Participant expresses an attempt to stabilize the problem "This can't be right. There are too many unknowns." "Work-energy won't work here. There's not enough information." "Maybe I need to try momentum instead."	STA
Not Related to Problem Solution	Participant expresses verbalizations or carries out actions not directly related to the problem-solving process. "It's warm in here." "What was that noise?"	NRP

APPENDIX E: BETH'S STEP FORMATTING

Table 11: Beth's Step Coding

Verbalization	Actions
A truck with a mass of 1500 kg is traveling down a mountain road at 20 miles per hour when it hits a thick patch of ice lovely in a panic the driver hits the brakes which fail causing the truck to slide essentially without friction down a five degree slope after traveling down the slope for three hundred meters the driver manages to get the truck… onto a… runaway truck ramp which is inclined at ten degrees upward from the horizontal the ramp is covered with a soft material which results in a coefficient of friction of one point eight how far along… the truck ramp does the truck go before coming to a stop	Refers to problem statement
ok (refers to problem statement) the truck (starts writing) truck mass m is traveling down a mountain road at velocity v (draws arrow) when it hits a thick patch of ice ok in a panic it hits the brakes (writes a and draws arrow) point negative acceleration	Refers to problem statement Writes given information
truck slides without friction down the five degree slope (draws downward slope, labels angle) theta after traveling slowly down the ramp for a length of three hundred meters (labels slope L) the driver manages to get the truck onto a runaway ramp which is inclined ten degrees ten degrees up (draws slope, labels angle) use alpha I have theta alpha has a coefficient of friction of one point eight (labels, then sighs)	Draws diagram
ok so I know its initial kinetic energy is that going to help… (pause) no its not it doesn't matter I know its initial velocity then it slides down the slope for this given period of time ok	Refers to knowledge Refers to diagram
so velocity equals distance times time (writes equation) I don't know time… I do know d I do know v I can find t ok… traveling down the slope does that matter… no… it shouldn't then it goes up (pause)	Writes equation Refers to diagram
then it's going back up it's going back up the ramp... does that matter kinetic ener... should I use kinetic energy I already know v…	Refers to diagram
I already know v (starts writing kinetic energy equation) the velocity's changing I have to find how far along if I know the vel… the velocity that's not going to help me… he applies the brakes wait he applied the brakes velocity's changing… (sighs) applying the brakes doesn't do… because his truck starts sliding he doesn't actually slow down so velocity is still the same…	Writes equation States goal
ok so I do know the energy kinetic energy the other kind of energy's going to be potential energy got it (finishes writing kinetic energy equation)	Refers to knowledge Categorizes problem (implicit) Writes equation
the initial… the change in kinetic energy is velocity final minus velocity initial there is no final velocity because its going to stop once it is… up the ramp although I don't know how far… potential energy started at zero but then it went down it went down three hundred meters an angle of five degrees	Writes equation Refers to diagram
so times sine (starts drawing triangle) going down slope of five degrees (points to side of triangle) so sine… ok the first change is going down this far (indicates on sketch) the second change is going up… this I don't want I don't want to use d again (erases) I called this L I called this L we'll call that L call that d sin alpha…	Draws diagram Refers to diagram

ok the initial kinetic energy is go then gong to be one-half fifteen hundred times twenty meters per second… squared equals m g… uhm… (adds mg to potential energy statement already written) its fifteen hundred again so the fifteen hundreds are going to cancel fifteen hundred gravity and then that is ok I know L is three hundred times the sine of five I know the final is… d which I don't know and the sine of ten…	Writes equation
(reaches for calculator) the fifteens canceled twenty two is squared (writes answer) ninety six point eight equals g times that mess (writes other side of the equation) ok we need to divide by g… (writes answer of 9.9)	Calculates
and that's meters per second I have kilograms (gestures over previous line of work) two kilograms (indicating other side of equation) that's meters per second squared got meters per second yea meters per second… equals… plus three hundred meters sine...	Checks units
wait a minute is that right… (pause) I'm dropping kinetic energy to potential energy why aren't my units right… meters per second squared is all I have left over here I divided by kilograms on both sides… meters per second times meters… so I ha this should be nine point eight meters squared per second squared… then I divided by g yes I divided by meters per second so that is just seconds… so three hundred times five degrees equals… d… divide sine ten degrees… do I have meters I have meters yes… duh squared	Checks work Refers to work
(goes to calculator) nine point eight point nine plus three hundred times the sine of five…wait un-oh *unintelligible* calculator *unintelligible* nine point nine time plus three hundred times sine of five…is thirty six meters over sine of ten…which is point one seven…equals d which is equal to two hundred…two hundred twelve meters	Calculates
ok does that make sense potential energy *unintelligible* I know the final kinetic energy is zero how far up does the truck go before coming to a stop therefore v final is zero ok potential energy though first it went down so I called that negative cause it started at zero so its going down and that had negative potential energy that makes sense also then since this is L d sine alpha (gestures over sketch) I need to find d I know alpha… I never used the friction force… I never used the friction force shoot… (writes mu above d sine theta term in equation for potential energy)	Checks work States goal Refers to problem statement Refers to diagram Checks work
I multiply that… (pointing to same term in equation) I multiply that (pause) yes I multiply that by that by the sine d sine theta as well as that that's here that's here that's here and that's here (adding mu to various terms in the equation) that makes my answer a lot smaller…	Refers to work Writes equation
mu times one point eight times sine... that's the sine of ten… is point three one makes it divide by that is… one hundred fifteen meters…	Calculates
to me that makes sense… there's no friction over that one *unintelligible* it does make sense… ok I lost initial kinetic energy final potential energy… that canceled I got meters	Checks work Checks units
ok I'm done	Ends problem

APPENDIX F: GROUP EXEMPLAR PROTOCOLS

Group A Exemplar: Alex

(9:28:59) Researcher: When you're ready you may begin.

(9:29:01) All right this looks like a huge fortune cookie…

mmm let's see…Jack and Jill are sledding uh-oh angle…on a snow covered hill ok that's important… let's see friction constant (underlines mu k in problem statement)… kinetic friction ok it's sledding down a snow covered hill that is inclined here's where I get my angle at an angle of twenty degrees (underlines in problem statement) to the horizontal what's that mean… I guess that means the x axis… Jill mass ok we got mass (underlines) runs at four meters per se… meters per second oh that's a velocity (underlines) across the top of the hill landing on a five kilogram sled (underlines) which is at rest

ooo… collision (writes 'collision' at top of problem statement) that means I want to use momentum to solve this…

which at rest at the very edge of the hill her brother Jack stands at rest a distance of fourteen point six meters down the slope as Jill pa Jill passes Jack thirty meters (underlines 30 kg) he jumps onto the back of the sled

oop two problems… eh two collision problems looks like it

they continue together on down the slope reaching the bottom after traveling a total distance of forty meters (underlines) along the slope how fast are they going at the bottom of the hill

(9:30:45) all right let me think about this… (reviewing problem statement) ok… reaching the bottom after traveling a total distance of forty meters along the slope so that's the radius (writing r = on problem statement) the radius is going to equal (puts problem statement aside and starts on fresh sheet of paper)

(9:31:00) what do I know let's stop let's stop and let's focus let's focus… all right let's do what my knowns are what are my knowns (writing)… knowns ok… I've got Jack… I've got… Jill so two objects… I need to know what the mass and the velocity of each object are… Jack's initial velo… well the masses aren't gonna change… not yet…

ok so the mass of Jack equal to thirty kilograms… and Jill ooh Jill's a little bigger she's fifty kilograms… ok and… Jill's initial velocity… initial i equals four meters per second… no oh that's not it (wrote next to Jack – corrects) Jack's is zero… four meters per second for Jill did I get that right let me check (refers to problem statement) Jack's thirty kilograms zero cause he's standing still… Jill's fifty she's running at four meters per second…

(9:32:27) ok now… I think this is gonna be a momen… I think it's gonna be two momentum problems…

let's go ahead… hmmm (starts drawing diagram)… what do I need to do… I need to figure out the x right this is going to be a two dimensions it's going to be an x dimension and a y dimension now what have I got here (Researcher: could you speak up…) what what do I have here I have an angle here… twenty degrees and the radius total distance is forty meters ok…

so what I need to do I need to figure out… I need to figure out I don't know what why I'm figuring this out but I think that it's going to be important I need to figure out… what the actual distance are in the x and the y… ah dimensions so… let's make a ninety degree angle… ninety degrees… degrees… so y is gonna be the sine… and x is gonna be the cosine… so we know ok we take the radius and the radius is forty meters… that's (writing equation) forty meters times the sine… twenty degrees will give me my y component…

and that's gonna be (calculating) forty sine

oop hold on let me check my calculator… oop yea that's what I thought it's in radians because for some reason radians in calculus degrees in physics so I gotta remember that

twenty degrees… ok what's that hmm right rounding (writing) thirteen point seven meters in the y dimension all right now this is the x dimension forty meters cosine of twenty degrees equals (calculating) ok I don't want to do all that all over again so I'll just do that (working with calculator)… that that oh…

does that sound right thirty seven point six meters in the x dimension… hmm ehh… yea that looks… that's possible…

(9:35:07) all right back to the problem I need… ok I've got the initial mass and initial velocity for Jill… and then she collides… the sle… ah I forgot the sled…

the sled (refers to problem statement, adds info to list of knowns) has a mass of five kilograms and initial velocity… zero meters per second…

all right this is definitely a momentum…

(refers to formula sheet) where's my momentum stuff… huh got all my energy stuff all my kinematics stuff I don't have any momentum stuff… that's ok (refers to book)… momentum was chapter eight… go to the back of the chapter… and the summary should have what I need… huh was it chapter eight… no let's see the last chapters were ten and eleven… were energy energy light and energy with calories… so nine nine should be the chapter that I'm looking for… does this stuff look ah ok there's my impulse and there's my momentum…

(9:36:51) ok… so… we know that the momentum's going to be conserved so I know…

(writing equation) momentum of Jill plus momentum… the sled… is going to equal the momentum of Jill on the sled ok what's momentum (writing equation) mass times velocity of Jill hmm what am I going to do that how am I going to tell Jack Jill and the sled apart ok I'm going to make this one this two this three (adding subscripts to previously listed masses)… so mass mass Jill velocity of Jill times plus mass of the sled times the velocity of sled equals the… mass of Jill and the sled so it's I guess that's gonna be four (adding combined mass to knowns list) Jill on sled it's gonna be four the mass of that's gonna be fifty five… and… what's gonna be the velocity…

(9:38:20) well let's figure that out oh this is gonna be simple

ok mass four times the velocity of four… ok well we know that the initial velocity of the sled is zero so we don't have to worry about that ok let's figure this out

(writing equation) mass what that's fifty times four don't need a calculator for that it's two hundred… plus zero equals fifty five v four ok let's figure this out (calculating)… two hundred divided by fifty five equals ahh fractions hmm… hmm… I'm gonna go ahead and do an extra digit just because… so it's gonna be three point six five meters per second all right that's the first part of the problem ok that's the first…

(9:39:24) now… Jill… why do I need the *unintelligible*… *unintelligible* did I forget… oh I'm forgetting about the coefficient of friction uh-oh… I should have known this is this not as easy as I thought it was… oh hold on wait a second no no no no no no no friction hasn't come into play yet…

how far down the hill (refers to problem statement)… is Jack… stands at rest at a distance of fourteen point ah ok… fourteen point six meters down the slope…

all right that's another problem now I have to figure out… how fast the sled is going when it gets to Jack…

(9:40:24) all right… hmm… let's read through these numbers but this time we're gonna use fourteen point six (crossing through the 40 on the sine and cosine terms and writing in 14.6) fourteen point six fourteen point six that's what I need…

ah I'm gonna let's see (calculating) fourteen point six times the… sine of twenty degrees equals… five meters along the y axis… and let me see… cosine… thirteen point seven ok so he only actually travels thirteen point seven meters along the x axis…

(9:41:21) all right free body diagram time let's see (drawing)… this might be kinematics I think that's what this is…

ok… what is the force of gravity of Jill on sled that's gonna be (calculating) fifty five times nine point eight meters per second squared
ok we have (drawing)… force of gravity five hundred and thirty nine newtons downward and we know… ok this is twenty degrees this way (drawing)… and ninety degree angle… that's twenty degrees… and this angle must be… seventy degrees (labeling)… and we know… that… what do we know… we know that that's gonna equal to the five hundred thirty nine but how much in the x how much in the y this is where I always this is where I always mess up so let's not try to mess up ok let's think about this now…

we know that the radius is gonna equal (writing) five hundred and no (erasing)… five hundred and thirty nine newtons ok… in the y direction it's going to be five hundred and thirty nine times the sine of seventy degrees ok

(calculating) five thirty nine times the sine of seventy degrees is that right… yea… yea that's that's right ok so that means we have in the y direction the force of gravity… is going to be five hundred and six point five newtons and let's see what it is in the other direction (calculating)… ok… ooh… ok force of… ok let's get rid of that (erasing vector at seventy degrees) I don't need you anymore… in the x direction it's going to be one hundred and eighty four point three five newtons

ok so… we've got acceleration here... so… oh force of friction again (drawing vector on diagram) at an angle ok twenty degrees and…

(9:44:42) ok I need the total amount ok it's the coefficient… friction times the the normal force so that coefficient what was the coefficient

(refers to problem statement) point one up should have put that into my knowns (writes in list of knowns) ok f k equals now what's the

how he does it it's a backwards u with a k

equals point one… so then (writing equation) point one times five thirty nine newtons…

ok that should be easy I shouldn't ne really use (calculating) the calculator but I've made some really bonehead moves over the years so I'd better be safe that's fifty three point nine all right and that's the radius now we have to figure out the x and the y have to figure out the x and the y… sine cosine I hate trigonometry…

all right fifty three point nine sine twenty degrees what is that (calculating) fifty three point nine the sine of twenty degrees equals eighteen point four newtons of force… in the y direction ok all right so we've got something more pushing up on this thing… eighteen point four newtons ok and what is it in the x direction (calculating)… ok we have a fifty point six five newtons in that direction… ok…

(9:46:56) all right now I've got my free body diagram forces… what do I care… what do why did I do all that what's the point… I'm lost what am I doing… ok… time to do a pictorial all right pictorial may save me

we've got a hill (drawing)… it's going down twenty degrees… here to here… here's the sled… and Jill… that was a momentum problem we solved that… now we have to get to Jack… ok all we have to do to get to is get to Jack once I get to Jack then Jack becomes a momentum problem…

and then get to the bottom of the hill and that'll be oh my god it's four problems… oh… this is hard… ok let me think about this…

well if you find out ah… I need the acceleration once I know the acceleration then I can figure out what the velocity is when they get to Jack all right that's what the free body diagram is for second law calculations and of course there's an angle so I have to figure it out in the x and the y directions

(9:48:20) x dimension… what's the sum of the x direction all right we've got a negative (writing equation) fifty cause it's going in ah…

always do that (labeling on free body diagram) plus x plus y… that's why I lose points on my tests…

ok…. fifty point six five and that's a negative plus… plus one hundred and eighty four point three five equals mass times acceleration… right…

and this is going to happen so what's the f net f net is going to be it's not going to be in the x or the y it's gonna be both ok so it's going that's it f net is always in the direction of the acceleration… ok so what do we got…

oh we've got that it's fifty five oh simple math time let's see… negative fifty point six five plus one hundred and eighty four point three five equals divided by fifty five oh ok we have an acceleration of two point four three… in the x dimension what is it in the y dimension cause now we have to figure out what it is… in the y y we've got in the plus direction five zero six point five plus eighteen point four minus fifty three nine… ok… five (calculating) point six point five plus eighteen point four minus fifty three nine cosine negative twenty… and that's gonna be the mass divi now what's that divided by fifty five equals negative point two five meters per second…

(9:50:43) does that make sense… yea cause we're going downhill… ok so that would be the negative y direction ok so does this makes sense yes…

ok now… (sighs) now we have to put them back together… to get the acceleration in the direction of the twenty degrees so…

how do I do that… hmm… how do I do that… come on think… wait a second (pause)… maybe… what was the (refers to previous work)… here we got the three point six five… acceleration at twenty degrees… this needs to be broken into x and y pieces (pause)… am I doing this right… ok let's think about this… think about this don't worry about the time don't worry about the time… ok this needs to be broken up…

so this will be three point six five times the sine of twenty degrees… (calculating) three point six five times the sine twenty degrees equals… so we have one point two five in the y direction and… (calculating) cosine is … three point four three in the x direction all right ok now… from Jill to Jack…

ok…. did I figure this out right…

(9:53:09) (Researcher: keep talking) let me look at the kine (referring to equation sheet) you know what let me look at the kinematic equations… that's what that's what I'm messing up I just wasted all this time well maybe not let me think ok… we know that… ok here's what we need to do… ok what am I figuring out what do I need ok it's a collision problem once we get so we're gonna need the mass and the velocity that's what I need once I get to Jack how do I figure out the velocity… ok I know what the acceleration is… I know what the positions are… I know what the… need the final velocity and I know what the initial velocities are ok all right… this is (referring to previous work) the acceleration in the x dimension this is the acceleration in the y direction these are accelerations ok… these are velocities in x and y directions all right let me think about this now… where is… ok here are the distances these are what I need for the position ok all right here we go we'll use this one…

(9:55:00) ahm ah let me think… oh jeez I used up all my paper… let me go (Researcher: you may you may go to the next page if you want) ok… all right let's carry over what do I need to do (Researcher: if you need to you may remove this sheet and then we'll just clip it back in if that's helpful to you) I'll you know what I'm not going to do that I'm going to go I need I know what I need…

(9:55:24) I need… ok x dimension (writing)… y dimension… ok I need to know… the formula I'm gonna use (writing equation) velocity final squared equals velocity initial squared plus two times acceleration of the… x final plus the x initial ah oh ook I need…

I I need on my pictorial diagram I need a coordinate system (adding to previous diagram) yes *Instructor* is gonna yell at me for that… you're supposed to do that from the very beginning ok that's probably why I'm having such a tough time with this problem ok we will make this the origin ok… blink blink (marking points on diagram)… all right we got two things we need to know about… ok this… is thirteen point seven meters (labeling)… this… is thirty seven point six meters… ok… total distance here… this is going to be thirteen point seven meters… and ah… wait a second…

(9:57:02) (Researcher: Keep talking) I'm think oh um… this is what you would call pardon my language a brain fart I just stopped thinking I just stopped thinking ok well this is the problem I'm think I'm I'm looking at this five meters here in the in the y direction… and it doesn't seem it drops five it just it drops five or does it drop more… cause we've got thirteen so its thirteen point seven div minus five… does that sound right…

let's see what would that thirteen point seven minus five would be ah… thirteen (calculating) point seven minus five would be eight point seven so this is an eight point seven from this coordinate system… so I've got now it's gone thirteen point seven ok in the x direction and over here this is a total of fourteen point six that makes sense that makes sense ok overall it's forty right that makes sense ok all right now…

(9:58:15) x dimension ok final velocity that's what I'm working for let's do x dimension first… ok initial velocity in the x dimension was…

(writing equation) three point four three and that's gonna be squared and acceleration in the x direction was a positive… two point four three… initial x position was gonna be zero minus… no that's final position and it went… ah final position thirteen point seven minus zero and that's gonna equal final velocity squared… let's do this now

(calculating)… three point four three squared thank god for calculators… eleven point seven six plus… two point four two times two equals that times thirteen point seven equals… sixty six point five eight… add those together… eleven point seven six plus sixty six point five eight equals that oh uh square root… square root of that is gonna be eight point eight so final velocity in the x direction is eight point eight meters per second all right let's do it for y…

(writing equation) final velocity f plus ok… initial velocity in the y direction… is one point two five and it should be a negative… right… it doesn't matter because I'm squaring it anyway… plus… two times acceleration in that in the x direction which is a negative point two five and initial position is… aoh polo… do I have thirteen point seven twice… thirteen point seven… oh I do… oh that's confusing… ok… initial so it's thirteen point seven… for that minus the final position so eight point seven so oh that's five…

so (calculating) one point two five squared one point five six… and this is gonna be negative… ok… two times point two five equals point five times that's two point five… so one point five six minus two point five equals negative point nine four…

(10:01:54) all right take the squ oh no that's gonna be… an imaginary number… what have I done something wrong (referring to previous work)… acceleration is that a scalar or a vector… it's a vector… what have I done wrong I've messed up on the signs again oh… that kills me... signs kill me… is it important yes (referring to previous work)… (long pause)… what am I doing what should I do now I need to I need to go and I need to walk through this and I need to figure out… why it's not a negative… ok let me think about this now… how did I get… point wh how did I get the acceleration (referring to previous work)… I got the acceleration from my free body diagram and the second law pairs… and that makes sense because it's accelerating down right… ok so that's correct all right now… the velocity the velocity should be a negative… ok where's the velocity how'd I figure out the velocity velocity came from here… ok… ok… velocity I figured out the velocity… I should have used… three hundred forty degrees instead of twenty degrees… what would have happened how would that would have looked… if I had done that…

ok (calculating) three point six five times the cosine three forty degrees is gonna equal… right but… the cosine… what's going on use the sine sine for the y… negative ah ok… that's where I made my mistake cause it's going downhill all right now what did I use that for what did I use that for… so velocity…

(10:04:56) (refers to v^2 equation – slaps paper) it doesn't matter… it doesn't matter… ok… five here… we have this (referring to values in equation)… well… maybe it's not important I know it is but… I just I gotta keep going ok let me think about this so… I don't think I'm going to bother taking the point five… so… the final velocity in the y direction is gonna be point nine seven…

it's gonna be in the negative direction… cause it's going in the y direction ok…

so now I need to put ’em together… ok… how do I do that… how do I put them back together that’s in the very beginning (refers to book) that’s with vectors… vectors… vectors in the coordinate system… it’s gonna be chapter three page seventy eight… seventy eight I know I should know how to do this but… I don’t feel like thinking so let’s go to the book seventy eight seventy eight scalars and vectors… now let’s go just go right to the back it should be there… should be there in the summary… all right… oh of course duh ok… now all right ok so it’s x squared plus y squared equals r squared well root r… ok… all right here we go… ah x what’s x x is…

(calculating) eight point eight… squared plus… negative nine no negative point nine seven squared… so the final velocity is going to be… eight point eight five (sighs)…

(10:07:56) does that sound right… ok we started out what was the initial velocity of sled on Jill… three point six five… that was it that one’s it right there… three point six five you’ll never be neat… you’ll never be neat… ok does that make sense… oh man he’s definitely well yea… that’s oh… that’s that’s pretty fast… I guess…

(10:08:29) ok now the second part of the problem the third no this is the third part of the problem momentum… ok we’ve got Jill on sled… that’s eight point… eight point four right… ok we don’t need we it’s a momentum problem I don’t nee I don’t care about the angles…
it’s the mass four velocity four plus mass of Jack is three mass of three velocity three equals mass five velocity five this is so much easier than that kinematics stuff… all right Jack is thirty and he’s standing still so I don’t care… about him ok fifty five for that the velocity is… eight point eight five… ah the further along the messier I get oh well as long as the problem’s right what do I care eight five… and then what’s the final velocity…

here we go ok (calculating)… so it’s fifty five times eight point eight five equals blah blah blah divided by eighty five equals… ooh five point seven meters per second (refers to previous work)…

my god… that slowed ’em down quite a bit… does it make sense… the mass increased… so the velocity had to shrink… it had to… ok…

(10:10:41) now… back to a two dimensional kinematic problem… what do we got left here all right hey no problem… go to the formula go to the formula use the formulas remember the formulas will save you… formulas will save you… free body diagrams will save you… picture diagrams will save you… ok look at the problem that’s what I need (indicating v_f^2 equation) let’s do the x dimension x dimension what do we know… oh I have to go figure out what the velocity is in the x dimension that's (writing equation) five point seven times the cosine of three hundred and forty degrees… ok (calculating) five point seven times the cosine of three hundred and forty degrees equals right fine it's positive it's a positive number it's going in the positive x direction therefore that's cool five point seven sine three forty equals what I don't' know… negative negative one point nine five ok so *unintelligible* initial velocity in the x direction equals… five point three five and the velocity initial in the y direction equals a negative one point nine five don't forget the period ok got that… (chuckles)…

(10:12:23) acceleration… free body diagram time (drawing)… free body diagram free body dia hold on (scribbling through diagram) stop it stop it be neat… be neat… be neat (starting new diagram with ruler) stop panicking… stop worrying about the time… ok eighty five… what's eighty five

(calculating) eighty five times nine point eight equals eight hundred and thirty three newtons we've got a force over this way… seventy degrees it's gonna be eight thirty three… but what is it… along the y axis… we've got (calculating) sine of seventy degrees…

come on come on you stupid machine there you go
so it's gonna be seven hundred eighty two point seven this way… and what is it in the other way… it's gonna be something because it's gonna be acceleration has to be downhill cosine… stop it there we go… two eight five newtons in x direction y direction… ok friction… twenty degrees… (calculating) eight thirty three times point one equal ok so we got eighty three point three… eighty three point three sine twenty degrees equals… twen… ok so it's twenty eight point five… why are you doing this oh acceleration duh ok… *unintelligible* (calculating) cosine… seventy eight point three newtons…

(10:14:44) all right let's figure this out now what do I got… what are the forces let's think about this let's think about this… gravity duh… gravity… normal force… is going to be perpendicular to the surface… friction is gonna be in the opposite direction of the motion the motion is that way (indicating on diagram) that's the acceleration so that's f net… ok let me think… oh second law pair that's for the x dimension take a two dimensional problem and make it two one dimensional problems ok what do we got neg let's do the negatives first

(writing equation) negative seventy eight point three plus… two hundred and eighty five equals eighty five times a… all right that's simple we can do this one eight seven point three… that's a negative right yea plus two hundred and eighty five equals two oh six point seven divided by the mass… which is eighty five comes to two point four three acceleration in the x dimension… cool we're rollin' now y dimension y dimension what do we got here what are the negatives… do we care about the y dimension yea we do…

we're *unintelligible* biggest negative is ah eight thirty three (calculating) cause that's the negative y coordinate plus… twenty eight point five… plus seven eight two point seven equals negative number of course it's gonna be a negative number because it's going down… negative y direction so… acceleration of y is gonna be… negative twenty one point eight… ok… ok…

(10:17:58) so I've got the acceleration part and I've got all right because I used… a pictorial diagram I know the position stuff is so let's plug it in and this is it this is it ok first x do the x first what's x velocity of x oh my god you've lost it in all this now I know why you're supposed to be neat… there it is… what's that… that's the velocity (labels)…

what's the acceleration… wait a second… twenty one point eight… that's not right… what are you thinking you've gotta redo that… let me think about this… oh you nitwit you forgot to divide it (calculating) by the mass… there we go that's better point negative point two five… gee surprise surprise the acceleration in the y direction is the same because it's the gravity… so it should be the same in the x direction too is it… what was it last time (referring to previous work) ch yea… it's con accel of course it wouldn't change why are you doing all this work well at least you're on the right track ok… it doesn't matter it was it was it was practice it's you need the practice…

all right focus back to the formula… initial velocity here is the initial velocity x direction the initial velocity is (writing equation) five point three five squared plus two times two point four three ah here we go initial position is going to be… hold on it's final position final position first… thirty seven point six minus… thirteen point seven there's that that's ah that's confusing ok… equals final velocity in the x direction… of x… what's that is that a one or a seven ok…

(calculating) thirty seven point six minus thirteen point seven equals twenty three point nine five point three five to the second power is twenty eight point six twenty eight point six… two point four three times two equals four point eight six… times twenty three point nine equals that… plus twenty eight point six equals that… and the square root of that is gonna be twelve ok so what am I oh final velocity final velocity of x is gonna be twelve meters per second what is it gonna be for y…
(10:20:27) negative one point nine five… (calculating) three point three point eight… initial position is going to be… negative eight point seven… because the final position is zero… so I don't have to worry about that…

two times ah that's gonna be negative (drops pencil)… that's why I've ok that ok… they're timesing each other that one was a negative that one was a negative that's why it becomes positive ok all right… negative (writing) so it's negative five times no negative point five…

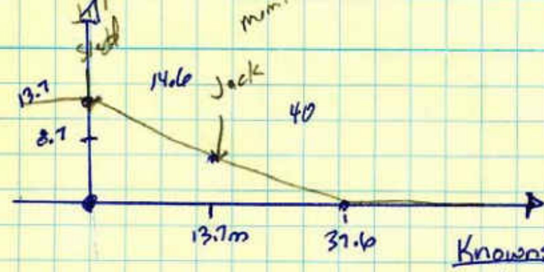
(calculating) let's just point five times eight point seven… equals four point three five… plus three point eight plus three point eight close that square root of that… it's two point eight five… final velocity in the y direction however…

(10:21:59) my calculations for the second problem may be wrong let's go back to that… those silly signs… oh no it's on this page… signs… (pause)… this is reversed (refers to positions in calculation of Jill's final velocity) this should be here this should be here… that makes sense ok… all right… I feel so much better now that I know what's going on with the signs… I inverted the positions… all right… ok no problem… whew… it's not going to cascade through the work all right everything I've got is fine so far is ok… go back… what was I… what was I doing…

(10:22:56) ok two point eight five is the final velocity so let's go and let's figure that out… blink (calculating) blink… it's gonna be… what is it gonna be… oh it's x… eight point eight is that it hold on… there it is right there… whew… (calculating) twelve squared plus two point eight five squared and that… twelve point three meters per second is the final velocity…

(10:23:46) you know what… how's the problem begin with… Jack and ok what do I want… Jack and Jill are sledding on a snow covered hill that is inclined an angle of twenty degrees to the horizontal blah blah blah blah blah… and Jill's overweight… she's running at four point meters per second… across… up on the hill… brother stands… Jill passes what do I want they came down what am I looking for how fast are they going at the bottom of the hill… that's it I got it I got it what was it… where is it I better right that down… it's twelve… answer right there… that's what I want

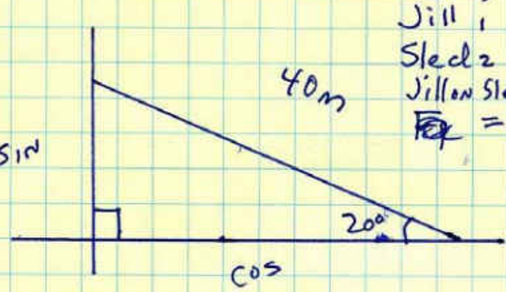
(10:24:32) I'm done



MV

Knowns:

Jack 3 M = 30 kg V_i = 0 m/s
Jill 1 M = 50 kg V_i = 4 m/s
Sled 2 M = 5 kg V_i = 0 m/s
Jill on Sled 4 M = 55 kg V_4 = 8.85
F = μ_k = .1
r = 40m



14.6 ~~40~~ sin 20° = 13.7 m   5 m y-axis
14.6 ~~40~~ cos 20° = 37.6 m   13.7 m x-axis

13.7 / 5

$M_o^{Jill} + M_o^{Sled} = M_o^{Jill\ on\ Sled}$

$(m_1 V_1) + (M_2 V_2)^{0} = M_4 V_4$

50 4
200 = 55 V_4

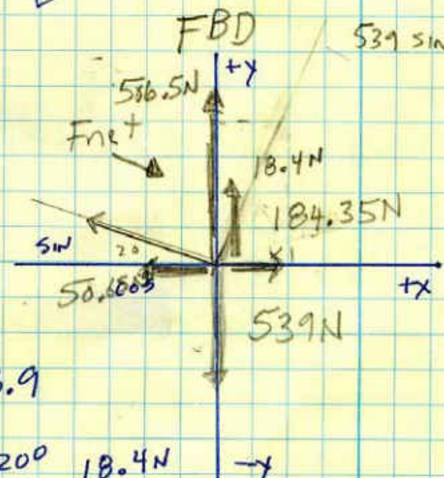
3.65 m/s

3.65 m/s 340°
3.65 sin 20° -1.25 y
3.43 x
r = 539 N

539 sin 70°

Jill

X-DIM
-50.65 + 184.35 = ma
(55)
$\vec{a}$
Y-DIM
2.43 X-DIM
506.5 + 18.4 - 539



$\vec{a}$
-.25 m/s y-dim

.1(539) = 53.9

53.9 sin 20° 18.4N

X-DIM

$v_f^2 = v_i^2 + 2\vec{a}(x_f - x_i)$

X-DIM   Y-DIM

$(v_f)^2 = (3.43)^2 + 2(2.43)[13.7 - 0]$

$11.76 + 66.58$

$V_{fx} = 12$ m/s

$[37.6 - 13.7] = V_{fx}$

23.9 $8.8\frac{m}{s}$ X-DIM

$(5.35)^2 + 2(2.43)[$

4.86

28.6

8.7 18.7

$(v_f)^2 = (-1.25)^2 + 2(-.25)[-]$

-5

$1.56 + (-2.5)(-.5)$

(-1.95)

12.3 m/s

$[-8.7]$ $V_f y = .97$

$\sqrt{x^2 + y^2} =$

$3.8 + 2f$

-5

$(4.35) + 3.8$

$2.85 \; V_{fy} =$

$V_f \; 8.85$

$M_4 V_4 + M_3 V_3 = M_5 V_5$

$55(8.85)$ $30 \; 0$ 85

V

5.35

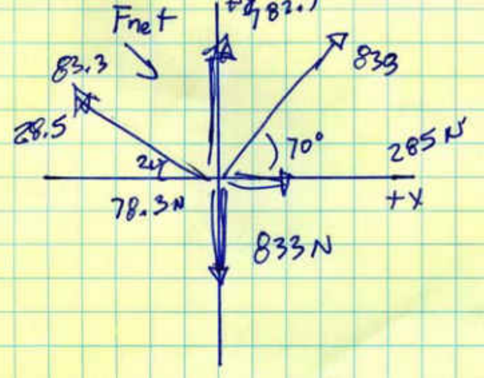
5.7 m/s

$5.7\cos(340°) =$

$5.7\sin(340°) = (-1.95)$

$V_{nx} = 5.35$

$V_{ny} = -1.95$



X-DIM

$-78.3 + 285 = 85 a$

2.43 a X-DIM

Y-DIM

$\vec{a}_y = -.25$

Group B Exemplar: Brittany

Researcher: Ready – you may begin

(4:21:20) (starts writing) I always like to write down what I'm given first I have a truck mass of fifteen hundred kilograms and it's traveling down a mountain road with a speed of twenty two meters per second when it hits a thick patch of ice in a panic the driver hits the brakes which fail causing the truck to slide essentially without friction … towards the five degree slope so slope with an angle of five degrees after traveling down the slope for a distance of three hundred meters the driver manages to get the truck onto a runaway truck ramp which is inclined at an angle…(adds "hill" to first angle written) ramp…ten degrees upwards…from the horizontal the ramp is covered with a soft material which results in a coefficient of friction of… so mu is one point eight how far along the truck ramp does the truck go before coming to a stop so that's…(writes d then erases) I'll do x…mmm

(4:22:46) mmm…it's traveling down a mountain road… when it hits a patch of ice (starts drawing rectangle, labels with mass and velocity)…mm twenty two meters…down a five degree slope…(*unintelligible* under breath – starts drawing incline) five degrees travels a distance of three meters (labels) three hundred meters (referring to problem statement) to get the truck…which is inclined…(draws second incline) that's ten degrees…coefficient of friction one point eight…

(4:23:53) (Researcher: be sure to speak up) ok…(opens textbook – turning pages) mmm…

(4:24:19) (Researcher: keep talking tell me what you're thinking) ahm…I'm trying to find a formula that fits… the uhm problem I'm given mass velocity…and I got angles…so I need to find… a uhm…sort of a…a collision…problem…(continues turning pages) that's two dimensional collisions…*(unintelligible*…I like to look at examples they help me the most…mmm…(turns to table of contents) I can't exactly find what I'm looking for…(pause)…(taps rhythm on book)…

(4:26:23) (turns pages to chapter on collisions) I'm confused…(turns back to table of contents, then back to collision chapter) the problem is there's these problems don't involve friction and it's based on a frictionless surface and this one involves…friction and I have to find a formula that involves that (turning back to table of contents) ah there we go….(turns to section on friction) ha found it…(pause – turning pages more slowly – looking at examples)

(4:27:39) (returns to work page) ok…let's do the diagram…(refers to problem statement) truck is traveling down a mountain road when it hits a thick patch of ice…in a panic the driver hits the brakes which…without friction down the slope (starts drawing) …and it's a five degree slope (erases, then draws again)…that's what it's traveling on…it's without friction…it's five degrees…so you have a normal force (starts drawing arrows on diagram) …and we have m g sine theta… that's theta (labels angle on diagram)…and this is m g…and there's no friction back so…mmm… m g cosine theta…and…since there's no friction we just solve for m g sine theta…

so the total forces in the x direction…m g *unintelligible* one thousand five hundred kilograms times (Researcher: be sure to speak up) does it matter if I used nine point eight or ten (Researcher: what you would normally do) I'll just make it easy…times sine of five degrees…

(calculating) fifteen hundred times ten times make sure I'm in degrees yep sine five and given the total force is one thirty zero seven point three four…

and the normal force is m g…and the total forces in the y direction… are…uhm… (starts writing equation) n minus m g cosine theta…so fifteen hundred kilograms times ten meters per second minus fifteen hundred kilograms times ten meters per second times cosine of five degrees…

(calculating) fifteen hundred times ten…one five zero zero zero minus…times ten times cosine five…oops…which equals four nine four two point nine two…minus one five…fourteen point…equals fifty seven point oh eight….and that's equal to m a…m a y equals zero…ok

(4:32:16) (refers to problem statement) after traveling down the slope for a distance of three hundred meters…

(starts drawing new diagram) travels down the slope distance (labeling) three hundred meters…five degree slope the driver manages the truck onto a runaway truck ramp which is inclined at an angle of ten degrees upwards…this is the trucker's slope… from the horizontal…this is the horizontal…that's ten degrees that's five degrees (labeling on diagram)…the ramp is covered with a soft material which results in a coeffriction of one point eight…(sighs) ok…

(4:33:22) (starts writing equation below diagram) f equals m a…in the x direction…huh…(Researcher: tell me what you're thinking) (erases equation) I just now realized I have a distance…(taps paper with pencil) and I need to find distance *unintelligible*

(4:33:56) (refers to book – turns pages – still in friction section) …hmm… (pause)… oh… that could (looking at example with object on slope) *unintelligible* …that doesn't help…

(4:34:48) (refers to problem statement) he's traveling down the mountain so gravity's pulling down…how far does the truck go…before coming to a stop… maybe I want to use kinematics on this… (turning pages)… I think they have one of those…nice tables (laughs)… ah I think I found it…(continues turning pages) …should be one *unintelligible* … (continues turning pages) … (period of silence while continuing to look through book)… ah there found them… finally (laughs) …(referring to kinematic equations in textbook)

(4:36:55) (Researcher: keep talking) …uhm… and I have distance… *unintelligible* ..x i (writing) we're not really given position (erasing) so that doesn't work…(writes velocity on diagram)… (reads problem statement under breath – gestures over diagram with pencil while reading – writes d_2 = ? on diagram) distance two is distance one…I got the two mixed up (starts writing known information under diagram) it's initial velocity equals twenty two meters per second…it's final velocity will equal zero meters per second… it's initial distance… three hundred me oops meters…

and it wants the final distance... ok …uhm…I'm gonna assume… ok… yea…

(4:39:07) (Researcher: tell me what you're thinking) now I'm just trying to figure out which formula I need…now that I finally realize what I have (laughs)… uhm…the first slope (starts writing) angle one five degrees angle two is ten degrees…uh-huh and we have mu one point eight on the second slope…don't need time…could find … no… no…don't have acceleration don't have time…acceleration's…the key...

(starts turning pages of book) velocity… acceleration (*unintelligible*) constant acceleration… acceleration… (continues turning pages of book) …do they not have…force of gravity is m g… (reviewing previous calculations on page) x direction… equals that (circles answer for sum of forces in x direction – 1304.34) uh-huh… have that…

(4:42:10) (Researcher: keep talking) huh… now I need the distance (phone rings in background) distance…perfect huh (chuckles) …I've got all that …

I need to find distance…ah-ha found it (referring to example with object on slope) distance equals one half of a x t squared… and a x is given… g sine theta…ok….uhm… I'm gonna use the equation… v x of that squared equals two a x d…so that one's gonna be zero equals two x d… and that a x is gonna be ten meters second times sine of ten degrees

(calculating) ten times sine ten degrees…one point seven… four… um-hmm…so two times one point seven four distance (calculating)…(writes answer)…

(4:44:17) that didn't take that's including the friction though (starts turning pages of book) …I left that part out…so…ok…scratch that…

(starts writing new equation) a x equals negative coeffriction g ok… ah…hmm…

(refers to book – looks at example on level surface with friction) ok… that's the movement… so that's gonna equal mu of one point eight times ten meters per second…and that equals negative one point eight times ten negative eighteen… and uhm…

uhm *unintelligible* (crosses out previous answer for d – starts writing new equation) v of x final equals two a x d that'd be zero squared equals two times…negative eighteen and that's times d

(calculates) negative thirty six…d equals thirty six…oops (erases – rewrites answer with units – boxes answer) finally… (drops pencil, puts cover on calculator)

(4:46:13) (Researcher: done) yes (Researcher: ok)

Given:

$m = 1500\ kg$

$v_i = 22\ m/s$

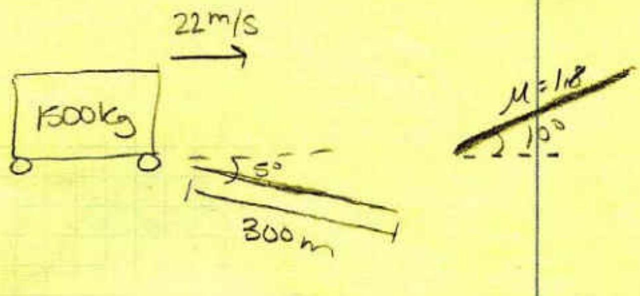
hill slope $= 5°$

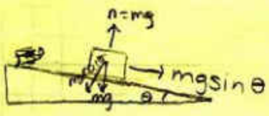
$d = 300\ m$

ramp slope $= 10°$ ↑

$\mu = 1.8$

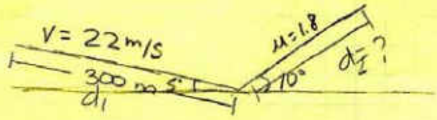
$x_f = ?$





$\Sigma F_x = mg\sin\theta = (1500kg)(10\ m/s)\sin(5°)$
$= 1307.34$

$\Sigma F_y = n - mg\cos\theta = (1500kg)(10m/s) - (1500kg)(10m/s)\cos(5°)$
$= 15000 - 14942.92$
$\Sigma F_y = 57.08 = ma_y = 0$



$v_i = 22\ m/s$

$v_f = 0\ m/s$

$d_1 = 300m$

$d_2 = ?$

$\theta_1 = 5°$

$\theta_2 = 10°$

2nd slope $\mu = 1.8$

$d = \frac{1}{2} a_x t^2$

$a_x = g\sin\theta$
$= 10\ m/s \cdot \sin(10°)$
$= 1.74\ m/s^2$

$v_xf^2 = 2a_x d$
$0 = 2a_x d$
~~$= 2(1.74)$~~ d
$d = 3.473$

$a_x = -\mu g$
$= -1.8 \cdot 10\ m/s$
$= -18\ m/s^2$

$v_xf^2 = 2a_x d$
$0^2 = 2(-18\ m/s)\ d$
$-36 = d$

36m = d

Group C Exemplar: Cory

Researcher: You may begin.

(5:43:42) (Participant reads problem out loud – begins writing information as reading) a truck with a mass of… fifteen hundred kg… (starts writing) so mass equals fifteen… hundred kilograms… is traveling down a mountain road… at twenty two meters per second (writes) the velocity equals twenty two meters per second… when it hits a thick patch of ice… (pause) in a panic the driver hits the brakes… which fail causing the truck to slide essentially without friction down… a five degree slope… after traveling down the slope for a distance of three hundred meters… the driver manage to get up the truck and onto a runaway truck ramp which is incline of ten degrees upward from the horizon… the ramp is covered with a soft material which is result in a coefficient of friction of one point eight how far along the truck ramp does the truck go before coming to stop (pause)

(5:45:13) usually it helps me when I read the problem not out loud…

Researcher: You, you do what you normally do when you solve a problem. (Participant pauses while reading silently)

(5:45:32) (begins drawing slope on paper) the initial velocity is twenty two meters per second when he hits a thick patch of ice… in a panic the driver hits the brake… which fail… causing the truck to slide essentially without friction down…the five degree slope… slope with five… the slope has five degrees (draws horizontal line at base of slope and labels angle)… five degree slope… and slides down (gestures with pencil down the slope drawn)… after traveling down the slope for a distance of three hundred meters… the driver manage to get the truck onto a runway… runaway truck ramp which is inclined 10 meters upward… (draws slope on paper) ten meters upward ten degrees upward… from the horizontal the ramp is covered with a soft material which is a result in a coefficient of friction of one point eight… how far along the truck ramp does the truck go before coming to stop…

(5:47:10) ok… so first we have the truck… which is going down slope (starts diagram on page) it has the weight force… do the free body diagram of the truck (erases first diagram, starts again) x axis… the y axis… mm… mm… five degree slope… have the weight force… the normal force… there's no friction force its just just sliding… truck the mass of fifteen hundred kilograms is traveling down the mountain… road… at twenty two meters per second when it hits a thick patch of ice… in a panic the driver hits the brake which fail… causing the truck to slide essentially without friction down the five degree slope…

(5:48:52) ok... so to here (marks spot on problem statement, gestures over free body diagram)… the weight force is gonna be... fifteen hundred kilograms… is the mass… weight equals mass times acceleration…

weight equals the mass… fifteen hundred times acceleration which is nine point eight (uses calculator)… times nine point eight… the weight equals fourteen hun..fourteen thousand seven hundred newtons… (pause) (draws triangle at midpoint of free body diagram)

(5:49:57) Researcher: Keep talking.

Oh yea… ok… (gestures with pencil on free body diagram) right here we have… the um… x component of the weight force and the y component of the weight force…

um… (writing equation) f net on the y component equals mass times acceleration there's no acceleration… so it equals zero… which equals… the normal force… minus… the y component of the weight force…

(5:50:46) ok… y component of the weight force equals… (writing mnemonic) soh-cah-toa… (draws triangle below mnemonic; moves to free body diagram) ok so we have (moving to free body diagram)… five degrees here… that means that… oh… I was just going to get that angle there… if that's five degrees that's ninety… it is ninety here…

this right here is gonna be… (uses calculator) ninety minus five… eighty five… and then five degrees is this angle cause this is a ninety degree angle too (erases triangle previously drawn under mneumonic)…

the triangle is like looks like this… mm… just like this… five degree angle… the hypotenuse is… fourteen seven hundred… ok the y component is gonna be the adjacent side which is the y... so cosine of five degrees equals adjacent over hypotenuse…

fourteen seven hundred… fourteen seven hundred… times cosine of five degrees equals… the y component of the weight force… the normal force… is gonna equal the same… cause there's no movement…

(5:53:03) ok… so… (uses calculator) fourteen seven hundred times cosine of five equals… fourteen six four four… (pause)

that's not right… ok… (moves pencil over to previously calculated weight) fourteen… ok its smaller than this… that's the y component of the weight which equals… the normal force… so we don't have any acceleration in the y axis… mm

(5:53:53) f net… f net equals… f net is on that in this direction (writes on free body diagram)… f net on the x axis equals the mass times the acceleration… which equals… mm… (pause) cosine of five degrees… cause it's the opposite…

cosine of five degree times fourteen… seven… seven hundred newtons… so cosine whoa… opposite sine… sine of five degrees…

sine of five degrees times fourteen seven hundred… equals… twelve eighty one point one eight nine newtons… which equals the f net…

(5:55:18) twelve… eighty one point one eight nine newtons… the mass equals…

ok talking about the mass not the weight

ok … minus fifteen zero zero… kilograms… which equals… times the acceleration equals… twelve eighty one point one eight nine newtons… acceleration equals this over this…

(uses calculator) twelve eighty one point one eight nine divided by fifteen zero zero… acceleration whoa equals twelve eighty one… (pause, goes to calculator) *unintelligible*… twelve eighty one point one eight… fifteen equals… the acceleration equals eight point five four meters per second square (takes up problem statement after writing answer)

(5:56:44) now we're gonna try to find the final velocity… ca… because this velocity here but we're trying to find the final velocity at the bottom (indicates points on diagram)…

(moves to equation sheet) ok we're gonna use this equation here (indicates $v^2 = v_o^2 + 2a(x - x_o)$ on equation sheet) cause we know the final velocity we know the initial velocity the acceleration but we don't know how much time this happens… so we're gonna have to use this equation here cause we know everything here…

the final velocity… that's what we're looking for

(starts writing equation) the final velocity square equals the initial velocity squared initial velocity is… (looking at previous work) initial velocity… twenty two meters per second (starts writing) twenty two meters… initial velocity is twenty three meters per second…and all this is squared… plus two acceleration equals point eight five four… the change in position initial position is zero (erases zero just written) the final position is three hundred and the initial position is zero..

ok so final velocity equals the square root of (goes to calculator) twenty two squared… squared plus two times point eight five four times three hundred… the square root of (writing answer) nine nine six point four… that equals (goes to calculator)… the square root of… answer… thirty one point five six five… point six meters per second…

I can tell this answer makes sense because if you're going down the hill and there's no friction you're gonna go from… twenty two meters per second to thirty one meters per second so your velocity increase… that makes sense…

(5:59:39) like… (turns page) final velocity equals (transferring information to second page)… thirty one point… five six six… meters per second… um all right…

(picks up problem statement) after traveling down the slope for a distance of three hundred meters... meters per second unh-k… the driver manage to get to the truck onto a runaway truck ramp which is inclined at an angle of ten degrees upward… from the horizontal…

ok so now we have (starts drawing)… free body diagram… we know f net is gonna be this way (draws arrow to indicate direction)… we have the weight force (draws arrow on diagram)… the friction force… if the truck is moving upwards (indicates with hand) then the frictions gonna be pointing this way (draws on diagram)… the friction with a coefficient… so (writing) mu is gonna equal one point eight…

ok… (picks up problem statement) through the… the driver manage to get to the runway run which is inclined at an angle of ten degrees upward… (starts drawing triangle on diagram) ten degrees upward… so then that means that's gonna be eighty (indicating on diagram) and that's gonna be ten degrees… ok (starts erasing part of diagram) I can erase this right here… ten degrees… the ramp is covered with a soft material which is a result in a coefficient of friction of one point eight how far along the truck ra… how far along the truck ramp does the truck go before coming to a stop… all right…

(6:02:03) Final velocity… all right… so we have the normal force (indicating on diagram) the weight force and the friction force… (labeling arrows on diagram) the weight force and the normal force… mm…

(writing) the summation of the y component of forces equals mass times acceleration acceleration is zero… so its zero… all right…

right here (indicating on diagram) we have then on the y component we have the normal force… minus the y component of the weight force… uhhuh… on…

ok so we need to find the normal force… the weight…

we find the weight back here (turning back page) to be… the weight is fourteen seven hundred newtons…

wi I'm doing this right now cause I need to find… the normal force to find the normal force times mu equals the…normal force times mu equals the force the kinetic friction force…

mm-k fourteen thousand is the weight force (indicating diagram) I'll have a triangle like this… (drawing new triangle below previous diagram) ten degrees… un… the weight force is the hypotenuse fourteen seven hundred… the y component and the x component… the y component is… (writing) soh-cah-toa… ok we're looking yea the y component is the adjacent…

cosine of ten degrees times fourteen seven hundred… equals the y component… which equals the normal force…

(uses calculator) cosine of ten degrees times fourteen… seven hundred… equals one four four seven six point six seven… newtons… that's gonna be the normal force

(6:05:17) (moves to diagram) ok I have the normal force… (very quietly reading problem) incline of upwards *unitelligible* the ramp is covered with a soft material which is… (louder) how far does the truck ramp does the truck go before coming to stop… ok… uh-huh…

(6:05:37) Ok so… now we're getting into the (starts writing) summations… on the y component… which equals mass times acceleration

we're trying to find the acceleration in this case… which equals…

we have n… no we don't have n (erasing)… so the only things on the x component is gonna be the friction force which is negative… minus… the x component of the weight… the x component of the weight is gonna equals to sine of ten degrees times fourteen seven hundred…

(uses calculator) so I just change this… to sine of ten degrees times fourteen seven hundred… the weight equals twenty five fifty two point six… three newtons… huh… ok… that's gonna be equal to… the weight component of the x….

now… f k equals n times mu… which equals n… equals… n equals fourteen four seven six times six seven… times mu mu equals one point eight…

(uses calculator) fourteen four seven six point seven six times one point eight… the friction force equals twenty six zero five eight point one six eight newtons… the summation of y component equals mass… mass times…

ok (erasing) we're trying to find acceleration so…

acceleration equals… minus f k minus the weight force over the mass… acceleration equals… minus kinetic friction which equals twenty six zero five… eight point one six eight… minus the weight force… the weight force is twenty five fifty two point six three… all divided by the mass the mass is… hmm…

(refers to problem statement) the mass of the cart is (writing) fifteen hundred kilograms ok…

(uses calculator) negative two six zero five eight minus two five five two point six three divided by fifteen zero zero… (writing) acceleration equals nineteen point zero seven meters per second squared

(6:09:39) All right… we wanna find the final position it say how far along the truck ramp does the truck go before coming to a stop… so if we're trying to find the final position we can use this equation… final velocity square equals initial velocity square plus two acceleration final position minus initial position… final velocity is gonna be zero cause the cars gonna go all the way up and come back down… but the maximum position is gonna comes when the velocity's zero… zero equals the initial velocity… the initial velocity is thirty one point five six six… squared plus two the acceleration… acceleration is negative nineteen point zero seven… initial position is gon the final position is what we're looking for and initial position is zero…

(uses calculator) ok… so I have this right here (indicating initial velocity term) is adding goes to subtracting minus thirty one point five six six… squared divided by… by this (indicating term with acceleration and change in position) its multiplying goes dividing two nineteen point zero seven… equals the final position… (uses calculator) negative thirty one point five six six square root of two divided by… two times negative nineteen point zero seven… (writing) equals negative twenty six but since its going up in this direction its gonna be twenty six point one… three meters… ok

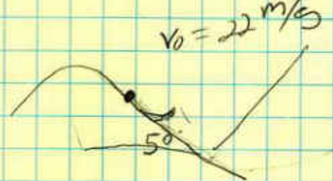
(6:12:18) (picks up problem statement) A truck with a mass of fifteen hundred kilograms… we can use momentum to solve this too… cause we know the initial mass initial velocity… mmm can we use momentum… no… ok if we want to use momentum… (starts writing) mass… momentum equals the ma… the mass times velocity… which equals mass times velocity (fading) momentum impulse… no (erasing)… *unintelligible… unintelligible…*impulse if we could find the impulse… whoa (louder) how far along the truck ramp does the truck go before coming to stop... twenty six point thirteen meters… ok…

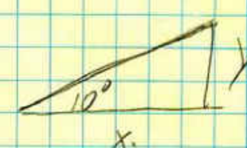
Researcher: That's it?

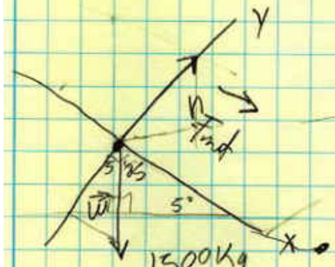
(6:13:32)That's it.

$m = 1500\ kg$

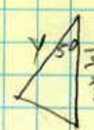
$v = 22\ m/s$







soh cah toa



$\cos(5°) = \frac{a}{14700}$

$w = m \cdot a$

$w = 1500 \cdot 9.8$

$w = 14700\ N$

$14700 \cos(5°) = W_y$

$14644\ W_y = n$

$F_{net_y} = m a^{0} = 0 = n - W_y$ $\quad \sin(5°)\ 14700\ N$

$F_{net_x} = ma = 1281.189\ N$ $\quad 1281.189\ N.$

$1500\ kg\ a = 1281.189\ N$

$a = .854\ m/s^2$

$v_f^2 = (22\ m/s)^2 + 2(.854)(300 - 0)$

$v_f = \sqrt{996.4} = 31.565\ m/s$

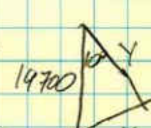
$V_f = 31.566 \ m/s$

$\Sigma_y = m a^0 = 0 = n - w_y \qquad U = 1.8$

14700 N

$m_i v_i = m_f v_f$

sdi cah toa



$\cos(10°) \cdot 14700 = n$

14476.67 N

$\sin(10°) \cdot 14700$

$\Sigma_y = m a = -f_k - w_x \qquad W_x = 2552.63 \ N$

$\Sigma_y = 1 \ a = \frac{-f_k - w_x}{m} \qquad f_k = nU = 14476.67 (1.8)$

$f_k = 26058.168 \ N.$

$a = \frac{(-26058.168 - 2552.63)}{1500 \ kg} = -19.07 \ m/s^2$

not originally present - added in retrospection

$V_f^2 = V_i^2 + 2a(x_f - x_i)$

$0 = (31.566)^2 + 2(-19.07)(x_f)$

$\frac{(-31.566)^2}{2(-19.07)} = 26.13 \ m$

Group B Exception: Ben

Researcher: You may begin.

(10:23:18) All right a truck with a mass of fifteen hundred kilograms is traveling down a mountain road at twenty two meters per second when it hits a thick patch of ice… all right you need to start by drawing a truck traveling… with a velocity of twenty meters…

so let's draw a box (starts drawing a box)… truck its got a mass of fifteen hundred kilograms (labels box) um is traveling… down a mountain road so we'll just… use a y axis for down (draws an arrow vertically downward from one end of box) velocity of twenty two meters per second he hits a thick patch of ice… so here equals fifteen hundred kilograms (writes at top of page) v equals twenty two meters per second (writes under mass) in a panic the driver hits the brakes which fail causing the truck to slide essentially without friction down the five degree slope ok that changes that (erases previously drawn vertical arrow)… (draws slope next to box, labels angle) five… equals… five degrees… theta equals five degrees (writes under velocity at top of page)… (pause)

(10:25:02) Researcher: keep talking

(10:25:05) after traveling down the slope for a distance of three hundred meters… the driver manages to get the truck onto a runaway ramp… so (starts labeling distance on slope) he goes three hundred meters on that axis… and… (reading under breath - *unintelligible*) in a panic (reading under breath - *unintelligible*) after traveling down the slope for a distance of three hundred meters the driver manages to get the truck onto a runaway truck ramp… which is inclined at an angle of ten degrees upwards from the horizontal… the ramp is covered with a soft material which results in a coefficient of friction how far along the truck ramp does the truck go before coming to a stop

(10:25:53) ok so we… know the coefficient of friction from the ice… essentially none… after traveling down the slope (reading under breath – *unintelligible)* manages to get the truck onto a runaway truck ramp so he goes down the road three hundred meters and then he hits the runaway truck ramp which has an inclination of… (draws second upward incline) ten degrees (labels slope)… this… so (writes at bottom of list of known quantities) theta two equals ten degrees… and the coefin of coefficient of friction… is going to be one point eight… (writes at bottom of list of unknown quantities) one point eight…

(10:26:49) (drops pencil and reaches for notebook, flips pages of notebook) coefficient of frictions… force and motion mass… total forces… distance… motion… that's work… kinetic energy… collisions… (continues flipping pages)… velocity… (reading from notes) harmonic motion… I don't know what happened to… (continues flipping pages)… what section was that… friction… there's no friction

(10:28:32) (lays notes aside and reaches for book) I want to find… (opens book but does not refer to it, picks up pencil)

distance is velocity over time (writes equation at top of page)… no… distance over time is velocity (erases equation)… so velocity times time is equal to distance…

we've got… (draws perpendicular line between two previously drown slopes) two triangles… and… given that's ninety degrees and that's ninety degrees and that's a right triangle (draws under upward slope) need to get… the velocity… after he slides to here (indicating the low point between the two slopes)… and then… using this coefficient of friction and the velocity…

(10:29:46) (reaches for book again, refers to table of contents) motion… motion… (pause – reading table of contents)

(10:30:33) (opens book to first page of chapter one) all right… force equals mass times acceleration… all right g equals nine point eight meters per second (writes under previous velocity equation)… and the velocity… after… (refers to book again – flipping pages) force of friction… (finding friction in text) its kinetic so its going to be… kinetic friction (reading in text) sliding… is when an object is in motion… (continues reading)

(10:32:06) (turns pages of book back to earlier pages; goes back to friction page)

(10:32:20) Researcher: keep talking

(10:32:22) ok force of kinetic friction… is equal to the coefficient of kinetic friction times n where n… I presume is… mass times gravity… in the opposite direction of gravity… (flips back one page and refers to an example with an object on a slope) n… two… so you're looking at (reading)… the force is going to equal mass times gravity times the sine of theta

(10:33:07) (goes back to paper and starts writing) mass g sine theta is equal to force… (gestures over diagram) is going to lead to my change… in distance… so I need (starts writing) fifteen hundred times nine point eight times sine of… five degrees…

that's… (reaches for calculator) sine of five… (speaks under breath while using calculator – *unintelligible*) so that's negative point nine five nine for the sine of five times nine point eight… times… mass which is fifteen hundred… gives you a number which is equal to force divided by the mass which we just multiplied by… (continues punching buttons on calculator – has not written any results at this point) fourteen hundred and *unintelligible* divided by the mass is equal to the acceleration…

(writes result) if that's a negative that's gonna be a negative (adds negative to answer written)...

(10:35:56) that can't be right… force is equal to mass… the distance… derivative… (speaking under breath – *unintelligible*) plus t

(10:35:10) Researcher: speak up please

(10:35:12) all right… the derivative of the distance is gonna be v plus t equals d prime… ah… both derivatives are gonna equal one that's using the product rule… so… d prime… (pause) still I'm not in the right direction (returns to book)… ah… looking for…

(looks to problem statement) how far along the truck ramp does the truck go before coming to a stop… so its going… so it… (sighs)… d prime… (pause)… that's a prime… that's a (writes check next to number on paper)… (mumbles under breath – *unintelligible*) velocity acceleration…

(refers to book) instantaneous acceleration… (continues flipping pages of book)… velocity… relative acceleration… velocity over time… (is referring to section in book on relative velocity – stops at an example showing boat crossing a river) motion in two dimensions and we're traveling that way (gestures in diagonal direction)… velocity…

(10:38:01) Researcher: be sure to speak up for me

(10:38:02) velocity in the I'm gonna call this side a this side b (indicating the two slopes previously drawn) velocity b direction… er velocity… of b v…

we know velocity… (starts turning pages of book) this part's velocity… constant acceleration… it should be all constant acceleration (continues to turn pages of book) constants of revolution…

I guess I'll go back to this (puts book aside, picks up notebook, starts turning pages) (*Unintelligible* as turns page – stops at an example with an object on an incline) acceleration along the x axis… is going to be… acceleration… is going to be nine point eight (refers to example in notebook – picks up calculator) gravity… times sine of theta one… (pause) ok so velocity… *(unintelligible* – under breath) I have to find velocity final… acceleration deceleration… (refers to notebook) lets work with change in time velocity equals derivative of x in relation to change in time or… change in distance over change in time… t in reference…

so change in v (starts writing)… is equal… delta v over delta t equals a… and we want a prime…

we want yea a prime to get v final for right here (indicating point between two slopes) we've got v initial to be in the equation for that problem the b side of it

(10:41:44) (refers to problem statement) the ramp is covered with a soft material which results in a coefficient of friction of one point eight… (pause)… change in… average v times… a…

(writing) delta x… delta x is equal to…

(pause – refers to notebook) average v… (pause) delta x equals v final velocity final squared minus velocity initial squared over 2… but a equals… (pause)…

(refers to notebook – mumbles – refers to textbook)… average speed instantaneous velocity… velocity on the x axis… instantaneous velocity… (reading aloud from text) in the x direction… equals the limiting value of the ratio delta x over delta t as delta t approaches zero… displacement delta x also approaches zero as delta t approaches zero… (long pause – reading text)…

(10:44:53) (writing) velocity one of x… equals… I still want v… (sighs)… (gestures over diagram)

(10:45:19) Researcher: keep talking

(10:45:22) d prime… ok we've got d… d prime… is how fast that's changing… he's just going… (pause) that's v… v prime… (long pause)… (draws right triangle indicator – mumbles – *unintelligible*) one point eight (writes on upward slope) velocity here (indicating point between two slopes)….

(10:46:39) (gets calculator) sine no… sine of five degrees… times…

velocity's moving that way (holds hands together, backs of fingers touching, to make two slopes)… that's down… a vector quantity… it's going to be slowing it down… times the velo what… oh… so the sine of five times twenty twenty two meters… sine of twenty two... that's not right… sine of five… actually it'd be sine of… one fifty five… (gestures over diagram)… times… my friction… and its three hundred meters… (long pause)…

(writing) x squared plus y squared equals three hundred… what would you get… twenty two… (sighs)… rate of change multiplied out with rate of change… is gonna be… sine is opposite… cosine is adjacent so its gonna be cosine (writes)… of five… (mumbles – *unintelligible*)

(10:49:43) ok… multiply… distance… good night to this… (pause)

(10:50:08) can I just say that I'm stuck

Researcher: you can

(10:50:15) ok cause…I'm pretty much stuck and…

$m = 1500\ kg$

$v = 22\ m/s$

$\theta = 5°$

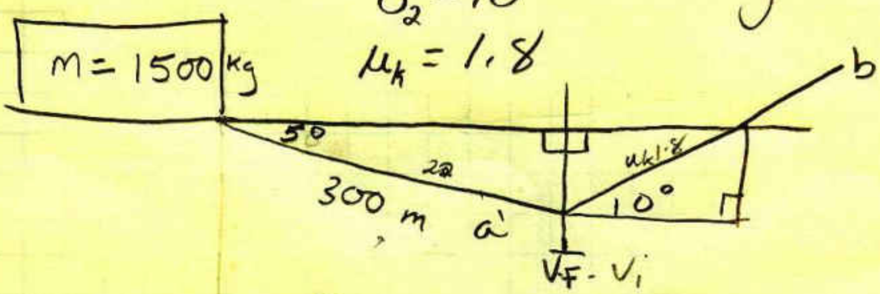
$\theta_2 = 10°$

$\mu_k = 1.8$

$vt = d'$

$v + t = d'$

$g = 9.8\ m/s$



cos5

$\frac{\Delta v}{\Delta t} = a$

22 m/s v

$mg \sin\theta = F$/

$1500 \cdot 9.8 \cdot \sin 5 =$

$x^2 + y^2 = 300$

$-9.40\ m/s$

a'

$v_x =$

a

$a \Delta x = \frac{V_F^2 - V_i^2}{2}$

APPENDIX G: CALCULATION OF KAPPA

For reference purposes, Table 9 from Chapter 5 is repeated here.

Table 9: Cross-table for Intercoder Reliability (Kappa)

	Code	Coder 2: Reviewer RES	CAT	GOL	COM	CAL	ASE	NRP	STA	Total
Coder 1: Researcher	RES	49	7	7	4	9	1	0	7	84
	CAT	2	16	1	0	0	0	0	1	20
	GOL	7	1	13	0	0	0	0	2	23
	COM	0	0	0	2	1	0	0	0	3
	CAL	4	0	0	0	34	1	0	1	40
	ASE	2	0	0	4	6	16	0	3	31
	NRP	0	0	0	2	0	0	2	0	4
	STA	0	0	0	0	0	0	0	9	9
	Total	64	24	21	12	50	18	2	23	214

RES: Resource Relevance
CAT: Categorization
GOL: Goal Interpretation
COM: Complexity

The proportion correspondence of the results of two independent coders is determined by adding the entries on the diagonal, which correspond to segments coded the same by both coders, and dividing by the total number of coded segments. For the data above, this is

$$\text{Proportion correspondence} = \frac{141}{214} = .66$$

This is an optimistic estimate of the reliability of the coding scheme, as it does not take into account the effect of one coder applying one or more codes at a higher or lower rate than the other coder. These effects are seen in the marginal values, such as in Table 9 where it can be seen that Coder 1 was more likely to use the code ASE (31 times) than Coder 2 (18 times). To take into account this variation in the proportions of the different coding categories used by the coders, the expected proportion corresponding is calculated. This is done by multiplying and adding marginal frequencies:

$$\frac{64}{214} \times \frac{84}{214} = .1174 \qquad \frac{24}{214} \times \frac{20}{214} = .0105$$

$$\frac{21}{214} \times \frac{23}{214} = .0106 \qquad \frac{12}{214} \times \frac{3}{214} = .0008$$

$$\frac{50}{214} \times \frac{40}{214} = .0437 \qquad \frac{18}{214} \times \frac{31}{214} = .0122$$

$$\frac{2}{214} \times \frac{4}{214} = .0002 \qquad \frac{23}{214} \times \frac{9}{214} = .0045$$

The sum of these marginal frequencies is the expected proportion corresponding, in this case 0.1999. The measure of the intercoder reliability, Kappa, is defined to be the proportion corresponding corrected for the marginal frequencies. It is determined by:

$$\text{Kappa} = \frac{(\text{proportion corresponding} - \text{expected proportion corresponding})}{(1 - \text{expected proportion corresponding})}$$

For this data,

$$\text{Kappa} = \frac{(.6588 - .1999)}{(1 - .1999)} = 0.57$$

Kappa can be considered as a conservative estimate of the reliability of the coding scheme. Because stabilization is an overarching factor which may encompass the use of other factors, this analysis was repeated with the row and column corresponding to stabilization (STA) removed. In this case, the proportion corresponding is 0.6911 and the sum of marginal frequencies is 0.2282, giving a Kappa of 0.60.

APPENDIX H: INTERNAL REVIEW BOARD LETTERS

University of Central Florida

Office of Research & Commercialization

September 22, 2005

Sherry L. Savrda
Seminole Community College
100 Weldon Blvd.
Sanford, FL 32776

Dear Ms. Savrda:

With reference to your protocol #05-2829 entitled, **"A dynamic model for the problem-solving experiences of physics students,"** I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. **This study was approved by the Chairman on 9/18/05. The expiration date for this study will be 9/17/06.** Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator. **Please notify the IRB when you have completed this study.**

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Barbara Ward

Barbara Ward, CIM
UCF IRB Coordinator
(FWA00000351, IRB00001138)

Copy: IRB file
David Boote, Ph.D.

BW:jm

12443 Research Parkway • Suite 302 • Orlando, FL 32826-3252 • 407-823-3778 • Fax 407-823-3299

University of Central Florida

Office of Research & Commercialization

August 22, 2006

Sherry Savrda
Seminole Community College
100 Weldon Blvd.
Sanford, FL32776

Dear Ms. Savrda:

With reference to your protocol #06-3711 entitled, "A dynamic model for the problem-solving experiences of physics students" I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. **This study was approved on 8/21/06. The expiration date for this study will be 8/20/2007.** Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Joanne Muratori

Joanne Muratori
UCF IRB Coordinator
(FWA00000351 Exp. 5/13/07, IRB00001138)

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David Boote, Ph.D.

JM:jm

12201 Research Parkway • Suite 501 • Orlando, FL 32826-3246 • 407-823-3778 • Fax 407-823-3299

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