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The Influence Of Discourse And Journal Writing On Second Graders' Acquisition Of Multidigit Addition Concepts

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THE INFLUENCE OF DISCOURSE AND JOURNAL WRITING ON SECOND GRADERS' ACQUISITION OF MULTIDIGIT ADDITION CONCEPTS

by

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ABSTRACT

The purpose of this study was to examine how second graders use writing and language when they are learning to add multidigit numbers in mathematics class. Second grade students were taught addition conceptually with a focus on sharing their strategies and thought processes with each other during the mathematics lesson. Two social norms were established with students so that sharing information and asking for clarity when they did not understand would be natural and expected. Students kept a daily mathematics journal to answer the class's Problems of the Day. Patterns found in student journals indicated three stages of multidigit learning. In Stage One, students used little or no words to explain their solution, illustrations show students using counting by ones strategies. Stage Two represents students using appropriate mathematics strategies and vocabulary to explain their solutions in detail. Lastly, Stage Three consists of students solving multidigit problems with little or no word explaining their solution process and illustrations are few. Results of the study indicated that students' oral explanations of solutions to addition problems included more detail compared to students' written justification of similar problems.

I dedicate this thesis to my mom and dad for their support and encouragement in all my adventures, I love you.

"There isn't anything that you can't find at Hensley's, and whatever that may be, it will always be the very thing."

Something about Hensley's, Patricia Polacco

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I speak for the trees.
Unless someone like you
Cares a whole awful lot,
Nothing is going to get better.
It's not.
The Lorax, Dr. Seuss

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CHAPTER ONE: INTRODUCTION

"Nothing is more rewarding than the confident feeling that ideas make sense; and nothing is more frustrating than not understanding" (Lambdin, 2004)

My heart is in teaching. In my years working with young children I have found that not only do I teach them but they are continuously teaching me. Over the years I've noticed some students were very good at mathematics and did not have to work at it too hard but others struggled. I wanted to seek out ways I could teach my young struggling mathematicians and strengthen the skills of the students who did not need the extra help. What I found through school workshops and masters' level courses was the need for me to reflect on my teaching practice. During reflection I challenged myself to look deeper into students' thinking and learning related to mathematics, specifically in how they add multidigit numbers. I wondered if my students were making meaning of the instruction or were they simply memorizing the taught lesson? Multidigit addition is one of the first skills taught in second grade and it tended to be one they struggled with the most. Was the instruction I provided enough for students to form their own thinking or did they need to interact and share ideas with one another to fully understand the concept? While reviewing literature related to my research questions, I found that it was necessary for me to look at how I incorporate proper mathematics vocabulary, provide student to student interaction and discussion in our mathematics lesson, and offer independent journal writing related to mathematics.

I grew up near the ocean sailing on my family's sailboat. It was common to be on the water with the wind blowing strong and for my dad to yell, "Tack." I did not know any other children that said they needed to use the 'head' or wanted to cook something in the 'galley.' It

was not until I got to college and met other boat people that told me they also had the job of putting up the jib or pumping the bilge when we got back into port.

I grew up in the world of boat people, people that spoke the language of the sea. My little mind sometimes thought that my dad was making up those words for the parts of the boat, since I had heard them no where else in my life. When I was grown up I automatically fit into a boat conversation because of my knowledge and experience. I soon felt sophisticated and worldly instead of alone. Not only do humans need to communicate their needs and wants in life but also their ideas. Using words that define an action give students the ability to share their thoughts.

I work hard teaching my students mathematics and as I reflected on my instruction I began to see how the conversation was centered on me. Therefore, I saw a need to learn to listen to the students and make sense of their ideas even if they were not from my instruction. The literature I read related to children's understanding of mathematics involves more than the relevant mathematical concepts from an adults' point of view but also knowing how children develop the concepts (Yackel, 2003). I understood that to mean that I need to really "listen" to my students' thinking; verbally and in their journals. I needed to create a classroom environment where I could listen to the students interact with one another. Journaling would provide insight on the thinking process of students who did not chose to voluntarily share. Modifying the social norms in the classroom would help take the center of instruction off of me. By creating a classroom where social norms foster children's listening and questioning, and by cultivating class discussion I would be setting the tone for students to learn mathematics differently than they may have learned it before.

Writing and mathematics did not always go together. When I was a learner in elementary school, writing sentences was only done during writing time and mathematics was associated strictly with numbers. Many elementary teachers defined mathematical knowledge in terms of memorizing facts and procedures (Gill & Thompson, 1995). When I look at today's textbooks and standardized tests they ask students to write their justifications. One way to understand the child's conceptual thinking is to ask them how they got their solution. Another way students learn mathematics strategies is adopting procedures that are demonstrated by other students (Hiebert & Wearne, 1996). Giving the solution is not enough for a student to show they understand the concept. I want to prepare my students for further learning in mathematics. I could help them master a concept by asking them to share their thinking and creating an environment for them to listen to each other's ideas. In addition to verbal communication journal writing is a personal form of expression and one that works easily with the many students and short time we have together in the classroom.

Purpose

The purpose of this study was to examine how second graders use writing and language when they are learning to add multidigit numbers in mathematics class. Using data collected during the study on my practice I sought to identify ways to improve my teaching so that I could guide my students in learning multidigit addition. Multidigit addition is an important skill learned in the early part of second grade and I wanted my students to have a strong understanding of the concept before moving on to other mathematics topics.

Research Questions

I changed my teaching style and wanted to formally evaluate my practice. I looked at two specific questions to help guide my study.

Question #1 In what ways is writing in mathematics useful when second graders are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established?

Question #2 In what ways is language useful when second grade students are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established?

Rationale

This is my second year using a particular mathematics series for second grade. I really struggled last year when I taught mathematics. I taught directly from the textbook because I did not know any other way to teach. I taught the way I was taught mathematics at a young age. I saw from my direct, traditional teaching method that my students and I did not take a strong interest in mathematics. There was no enthusiasm from the students in the classroom during mathematics class. My voice was primarily heard during the lesson. Academically, my students performed low on tests. I felt frustrated because I did not know how to help the students to 'get' mathematics.

I started taking master's level courses in mathematics education and reading professional books and journals during the year leading up to this study. The classes and readings opened my eyes to the possibility of teaching mathematics in a different way. I learned how to listen to my

students' thinking by asking open-ended questions such as, "How did you know you needed to add?" I saw how I could get more students to participate in sharing their ideas and strategies with the whole class. The class began to work with their peers to solve problems and I became the facilitator of the discussion instead of the lead. Students began to raise their hands to ask questions to other students to clarify their thinking. I started to see a new enthusiasm emerge when it came time to do mathematics everyday. My students became more engaged as they worked productively in small groups and started to share their own ideas with one another. Students started to look forward to mathematics because I gave them ownership in their own learning when they created methods for solving problems and tried to make sense of others' thinking. Test scores increased and parents were sharing with me that mathematics homework was not a struggle anymore. Research supports the evidence that students will rely on their own computational strategies when in an environment that fosters discussion and sharing ideas (Cobb et al., 1991). By creating an environment where students can be free to create and share their thinking, mathematics will be more meaningful.

The idea for this study emerged as a result of the positive experience I had with changing my teaching style during the spring of the previous year. I wanted to systematically study the impact on student learning in multidigit addition if I taught using a new teaching style in conjunction with changing the social norms during mathematics class.

Traditional learning goals for students must be expanded and reshaped to include a deep understanding of mathematics (National Research Council, 2001). Therefore I have sought out means to reshape my old ways of thinking to better meet the mathematical needs of my students.

Limitations

It is important to note that there were some limitations in my action research. A limitation of the study involved the time of day during which mathematics instruction was scheduled. I taught mathematics after lunch, towards the end of the day. The first 90 minutes of our school day was mandated for reading by the county. The first available uninterrupted time I had to teach was after lunch in the afternoon when students may have been more tired.

Another limitation of this study was the transient community in which my students come from. We had new students enter the classroom and lost some students during the data collection process.

Summary

The purpose of this study was to examine how second grade students use writing and language when learning multidigit addition. I had changed my teaching style and I wanted to see how that change affected my students when they learned multidigit addition. I specifically focused on how mathematics language was used in the classroom. Language was the bridge that would cultivate student interaction and discussion in our mathematics classroom. I thought the classroom conversations would lead me to some insight on how my students understood the addition concepts. One way for me to 'see' all of my students thinking would be through a daily mathematics journal. Armed with my ideas to focus on mathematics vocabulary, student conversations, and daily mathematics journals I found research that supported my methods. Literature related to conceptual learning strategies is shared in chapter two. Themes identified in the review were journaling, journals as assessments, conceptual teaching, multidigit addition,

and social norms. Chapter three describes the methods I used to collect the data for this study. Chapter four details the results of the study and chapter five concludes the thesis and provides recommendations for further research involving students' understanding of multidigit addition.

CHAPTER TWO: LITERATURE REVIEW

My study focused on my teaching practice and how second grade students learn multidigit addition in our classroom. The purpose of this literature review is to explore the different aspects of teaching mathematics and the subject of multidigit addition.

Researchers have identified different ways that children can add multidigit numbers. The way students find their sum is of interest to researchers, especially, if students are able to explain their reasoning (Carpenter *et al.*, 1998; Fuson & et al., 1997). Educators have approached the idea of understanding how students think through an addition problem in many different ways. Students and teachers have used pictures, symbols, numbers, and manipulatives to demonstrate solutions to a mathematical problem (Stix, 1994). Students can explain their thinking orally among their peers or with their teacher. Students can also communicate their thinking by writing an explanation (Liebars, 1997).

The National Council of Teachers of Mathematics (NCTM) identified mathematical literacy as a societal need arising from increasing mathematical and technological influences that require quantitative understanding (Pugalee, 1999). Current state standards suggest that conceptually-based instruction helps students to justify their solutions and that we can gain insight into student thinking by examining, questioning, and interpreting their representations (NCTM, 2000). How can we teach students to justify their answers if we are not modeling and practicing the 'how to'? Using a daily mathematics journal so that students can record their thinking will be one way I can make mathematics more observable in our students and really understanding their thinking and solutions. The purpose of this study was to examine how

second graders use writing and language when learning multidigit addition. Several themes were examined from a review of literature: journaling, journals as assessments, conceptual teaching, multidigit addition, and social norms.

Journaling

Student journaling can provide more than just insight on how a student can perform a task; it allows them to communicate using words to mark their intentions. Journal writing is a personal method of learning and one that could accommodate any student at any level of functioning (Burnaford *et al.*, 2000). Journaling allows students to shape his or her learning because it provides them with the freedom to outline, expand, and express their ideas in a safe environment. "Teachers can analyze students' representations in students' journals to gain insights on how students grasp the mathematical concept by questioning and interpreting their representations" (NCTM, 2000, p.136). Mathematical journals can provide children with regular experiences of recording information in writing. Students' journal entries provide authentic evidence for the teacher of the children's writing skills, the organization of their thinking about certain concepts, and their growth over time (McNair *et al.*, 1998). Writing in mathematics can be powerful. Writing can help students think through specific problems and clarify their questions. Communicating mathematical ideas is one of the goals of NCTM, writing about your solution can slow-down the thought processes and allow one to reflect and clarify his/her own thinking (Liebars, 1997).

Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas (NCTM, 2000). According to NCTM, teachers should help students learn how to talk about mathematics, to

explain their answers, and to describe their strategies. Working in pairs or small groups enables students to hear different ways of thinking and refine the way in which they explain their ideas (NCTM, 2000). Teachers can encourage students to reflect on class conversations and talk about talking about mathematics (Wood *et al.*, 1993). These kinds of interactions are a model for students and will give them the tools to be able to talk and write about their thinking later. Students can also construct meaning by adopting procedures that are demonstrated and shared by others (Hiebert & Wearne, 1996). In a study conducted by Jurdak and Zein (1998), there was evidence indicating journal writing in mathematics instruction had cognitive benefits to students, ages 11-13, in terms of increasing procedural knowledge, conceptual understanding, and mathematical communication.

Young students are not naturally inclined to write about mathematics because they are not formally taught mathematics vocabulary. Mathematics uses an entirely different vocabulary than what we use when we write in other fields. Writing rituals, such as daily journals, are a framework for tracking the development and use of mathematics vocabulary. At the same time, writing is a platform for showing how vocabulary growth is entwined with the growth of conceptual understanding (Murray, 2004). When students are writing about how they solve problems they will naturally use their mathematics vocabulary. Because mathematical language isn't used in a home environment often mathematical language needs to be directly taught (Murray, 2004).

Explicitly teaching mathematics vocabulary during instruction related to number concepts and operations is important because children must construct named-value and positional base-ten conceptual structures for the words and the marks and relate these conceptual

structures to each other and to the words and the marks (Fuson & Briars, 1990). Without a strong understanding of mathematics vocabulary, academic growth, communication, and mathematical issues can arise (Schwarz, 1999). Researchers realize that students may have a weak mathematical vocabulary and steps towards building up their vocabulary are necessary for academic growth.

Constructivist classrooms support conceptual understanding of addition. Giving students the opportunity to write and talk, rather than relying solely on tests, can signal to the teacher whether learning has occurred. Hinchey (1998) explains that the student who can explain in her own words what "add" means and who can invent and complete three or four addition problems demonstrates a thorough understanding of the concept of "addition."

Journaling allows students to reflect on their thinking because they have to put their thoughts into words. Frequently teachers look only for the answer and not the student's approach to solving the problem. Because journaling requires students to put their thoughts into words they are able to self correct their common mistakes as well as explain their reasoning with their peers. Reflections through journaling give students an outlet to analyze their own methods of thinking which will lead them to transferring knowledge to other similar situations (Grbavac *et al.*, 2003).

"It is during the process of writing that students' thoughts are clarified, thus revealing their levels of conceptual understanding" (Hopkins, 1997, pg. 6). Often it is difficult in the time constraints of a classroom to interview all the students. Journaling can provide the teacher with almost the same information as he/she would gain from a diagnostic interview, but in much less time (Hopkins, 1997).

When teachers review their students' writing and representations how will they assess them?

Journals as Assessment

Traditional assessments rarely assess how children are developing mathematics concepts (McNair et al., 1998). There are authentic and meaningful ways to assess these processes. Journals along with traditional standardized tests give teachers another piece of a child's understanding. "Multiple assessment windows ensure a richer, more balanced picture of a child" (McAfee & Leong, 1997, p. 77). Children's journal entries provide authentic evidence for the teacher of the children's thinking about concepts and it shows their growth over time. In fact, the ideas and thoughts the children are writing in their journals are sometimes too complex to show on a traditional assessment. "Writing forces a slow-down of one's thought processes, thereby allowing one to reflect and clarify their own thinking. It gives confidence to students who may not be mathematically inclined" (Liebars, 1997, p. 5). According to McNair, Thompson, and Williams (1998), children's journals provide teachers with an idea of their thinking and organization and especially growth over time.

There are various explanations on how to solve a problem and that there does not have to be one correct method to solve a problem (Ma, 1999). Pugalee (1998) finds that writing can be used to identify how students are growing as mathematically literate individuals because it creates an environment that supports mathematical reasoning. Evidence of representing, manipulating, reasoning, and problem solving, which are key factors that detect how literate students are in mathematics can be found in journal entries (Pugalee, 1999). It certainly would be challenging for a teacher to record those verbal interactions, writing provides concrete evidence

of ones' thinking. Pugalee's (1998) study shows writing has more implications for learning mathematics; it is a way for teachers to find evidence that students are using metacognitive behaviors.

If journal writing will be used as a way to assess students' understanding then how will students know what or how to write? Some students have never used writing or proper mathematics vocabulary as part of their mathematics lesson. Students will need to be taught in way that will prepare them for writing in a mathematics context.

Conceptual Teaching

We use the terms procedural knowledge and conceptual knowledge to denote a distinction often made between two forms of mathematical knowing (Hiebert & Lefevre, 1996). Procedural knowledge refers to mastery of computational skills and knowledge of procedures to solve problems (Eisenhart, 1993). Conceptual knowledge refers to knowledge of the underlying structure of mathematics-the relationships and interconnections of ideas that explain and give meaning to mathematical procedures (Eisenhart, 1993). For learning to occur in the way NCTM suggests, students need to be exposed to teaching which would facilitate them to make meaning out of their solutions. Learning opportunities that suggest students share their thinking with others is powerful because students are building on one another's ideas to develop their own ways of solving problems. In a classroom of second graders, a teacher used sharing and discussion strategies to solve a two digit subtraction problem. After the problem had been discussed and solved a girl from the group announced, "I love to talk about my thinking. When I share with the class, I can usually figure out any mistakes I made, and I think my ideas get much clearer!" (Scott, 2003). If we give students the opportunity to share their thinking the teacher

must be listening. As children are explaining, the teacher needs to be listening for what the children do say (Yackel, 2003).

One research article that supports conceptual teaching for second graders that use multiple strategies to solve problems with three addends was a study conducted using the *Thinking Mathematics* program (Gill & Thompson, 1995). The teacher in the research was trained in the *Thinking Mathematics* program designed by Lauren Resnick and Gaea Leinhardt that incorporates, The Ten Principles of Mathematics, the principles were developed from the work of cognitive and mathematics researchers that support the NCTM standards. The teacher encouraged students to find many different ways of solving problems while they also built their number sense using manipulatives in an environment that supported students using their prior knowledge to solve the problem. The researchers found that students solved problems in ways they found comfortable. The teacher bridged the meaningful thinking into their class work by allowing students to draw and write how they solved a three two-digit number addition problem. Student examples revealed the multiple ways that students solved the same problem. The teacher acted as a facilitator and supported the students as they explained their reasoning. Teachers can help guide their students, but understanding occurs as the students reflect upon their own thinking, either by discussion with another student or journal writing. Ma (1999) finds that teachers in the U.S. often display a procedural knowledge of multidigit subtraction whereas Chinese teachers presented a conceptual knowledge. Ma studied the American teachers' procedural knowledge of algorithms and found that it confined their expectations of student learning as well as their capacity to promote conceptual learning in the classroom. Therefore, the students taught by teachers with a procedural knowledge of an algorithm were not able to gain a

deeper understanding of mathematics."Conceptual learning not only promotes learning in the present, but also prepares students to relate their present learning to future learning" (p. 24). Teachers that use procedural knowledge of the topic exclusively are limiting their students' ability to think that there is only one correct way to solve a problem. Ma's research found that a striking difference was present between the U.S. and Chinese teachers' knowledge of elementary mathematics topics. The study showed most U.S. teachers tended to be procedurally focused. The Chinese teachers that were interviewed in this study used an old saying to introduce further discussion of an algorithm: "Know how, and also know why." "Exploring the "why" underlying the "how" leads step by step to the basic ideas at the core of mathematics" (p. 109). With the framework of how material will be presented let us look at how multidigit addition will be taught.

Multidigit Addition

NCTM recommends that educators need to increase the emphasis on conceptual understanding and decrease the emphasis on practicing routine skills (NCTM, 2000). Shifting from teaching routine and memorization skills to a conceptually-based classroom can be challenging for teachers not familiar with conceptual understanding. Educators need an idea of how conceptual understanding looks like. "One example of conceptual understanding in the area of multidigit arithmetic would be constructed by relating representations of quantities grouped by tens (e.g. physical and written representations), actions of decomposing the quantities into alternative groupings and recombining them, and facts of composition of quantities, such as two units plus three units are five units (whether the units are 1s, 10s, or other groupings)"(Hiebert & Wearne, 1996, p.253). Students that demonstrate conceptual understanding will be more likely to

develop appropriate new procedures and to adapt learned procedures for new tasks. NCTM suggests that students develop a solid understanding of the base-ten numeration system and place-value concepts by the end of grade two. An environment where students can share different strategies to approach a mathematics problem will help them to build their number sense and understanding further. Students do not necessarily need to wait for complete place-value understanding before being given the opportunity to solve problems with two and three digit numbers. Students also develop understanding of place value through the strategies they invent to compute (Fuson & et al., 1997). Fuson classified three invented strategies students use to solve 38 + 26; sequential, combining units separately, and compensating. Each different strategy illustrates a method that children have constructed to add without using manipulatives. Carpentar & et al.,(1998) focused their study on those three strategies and referred to them as *invented strategies.* Invented strategies are based on children's understanding of base-ten concepts, therefore, they need to understand those concepts before they can invent their own procedures for adding and subtracting multidigit numbers (Carpenter et al., 1998).

Fuson noted that children find it difficult to shift from 10 ones to 1 ten and back and its critical for the teacher to provide classroom activities that move students from unitary conceptions to count-by-tens and place-value-tens. These activities are crucial for understanding place value and multidigit methods. Multidigit addition and subtraction require student to have skills for both count-by-tens and place-value-tens knowledge (Fuson, 1998). Students in the primary grades can benefit from the use of manipulatives, visual aids, mental imagery, and discussion to solve problems (Nicol *et al.*, 2004). Observing children using those strategies can give the teacher a portrait of the students' thinking and understanding of numbers.

"According to Piaget's theory of cognitive development, one of the characteristics of operational functioning is the ability to create different efficient strategies" (Lemoyne & Favreau, 1981, p. 256). Lemoyne and Favreau studied students at both the preoperational and operational arithmetic level. They found that the operational functioning children showed a better knowledge of addition and subtraction tables and made more efficient use of them than preoperational children. The operational group understood the ordinal and cardinal aspects of numbers and the reversibility of operations. Therefore, strong place value and number sense topics are important prerequisite for teaching multidigit addition.

Social Norms

"Social norms refer to the expectations that the teacher and student have for one another regarding their ways of acting and communicating in the classroom" (Stephan & Whitenack, 2003, p. 153). During the first weeks of school teachers are establishing the norms in the classroom and setting the examples of how they want the students to interact with one another and it is key to creating an environment where problem solving can occur. To encourage students to contribute to discussions, the teacher should use strategies such as asking students to use their own words to explain their classmates' ideas that were shared (Stephan & Whitenack, 2003).

"Classrooms that focus on students' methods for constructing solutions, not simply answering the problem lead to genuine problem solving situations" (Stephan & Whitenack, 2003, p. 153). Teachers that create a classroom environment where students can share problem solutions openly are establishing sociomathematical norms. Sociomathematical norms are social norms that are specific to mathematical activity (Yackel & Cobb, 1996).

When elementary-grades students are asked to explain their thinking, typically they provide procedurally-oriented justifications. When the teacher tries to initiate a shift to conceptually oriented explanations, helping students make this shift can be quite challenging (Stephan & Whitenack, 2003).

> "The understanding that students are expected to explain their solutions and their way of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm." (Stephan & Whitenack, 2003, p. 154)

For example, when discussing a problem students will offer different solutions which will influence the learning opportunities for the students and teacher in the mathematics classroom. Cobb and Yackel (1997) described these normative understandings as continually regenerated and modified by the students and the teacher through their ongoing interactions. Teachers play a central role in establishing the mathematical quality of the classroom environment and in the process, the level of discourse and individual children's learning advanced (Cobb & et al., 1997). Yackel and Cobb's (1996) research in a second grade classroom shared ways that the teacher created new sociomathematical norms for her students. In their study, the teacher asked the students if anyone solved the mathematical problem differently than one student had solved it. The students were not accustomed to hearing the teacher ask the students if anyone solved the problem differently. The teacher in this study was creating a sociomathematical norm in her classroom on how sharing and explaining different solution methods to the same problem would occur. In my study, I established how solutions would be shared in the classroom similar to how

Yackel and Cobb (1997) researched a classroom where the teacher was changing the norm of sharing information in the classroom.

Summary

When students can choose computational strategies to solve problems, understand and explain their methods, and produce accurate answers efficiently it is called fluency (NCTM 2000). In my study, I examine the strategies my students used to solve multidigit addition problems by looking at their journals and listening to their discussions with one another. As teachers we usually teach the way we were taught in school (Huinker *et al.*, 2003). "Most elementary teachers have been immersed throughout their personal and professional lives in a culture that defined mathematical knowledge in terms of memorized formulas and efficient calculation" (Gill & Thompson, 1995, p. 349). I was taught in school to memorize the procedures in order to solve problems. I realize through the literature I reviewed that I need to take a close look at my own teaching practice and sociomathematical norms in my classroom in order for my students to have conceptual understanding of numbers.

In this study, I will be looking at how instruction and classroom norms affect the students' abilities to justify their solutions on paper. The purpose of the math journals is so that I can see my students' thinking. Mathematical knowledge and mathematics vocabulary go hand in hand; therefore, I will weave mathematical vocabulary words into my lessons so that students will be immersed in the language. Research supports that mathematical communication requires more than mastery of numbers and symbols (Murray, 2004). Mathematical communication means that students are in a position to acquire new procedures to solving problems such as

creating or adapting known procedures or using information learned from others to solve problems (Hiebert & Wearne, 1996).

While reviewing the literature related to my thesis question, I found research to support establishing two social norms in the classroom, specifically in mathematics instruction that would support my instruction style in this study (Yackel & Cobb, 1997; Stephan & Whitenack, 2003). In transitioning from a traditional teacher directed instruction to a whole group participation style, I used the research to guide my classroom culture (Gill & Thompson, 1995; Pugalee, 1999). I learned that student journals would be an important step in allowing the students to share their ideas in a personal form and also an alternative way to assess their understanding (McNair et al., 1998). Journaling would also be a way for me to listen to all my students' thinking where time constraints could pose a challenge. Another focus of my teaching style would be how I would have less of a directed role in the lesson. Research supports students sharing their problem solving ideas with one another not only for the teacher to assess them but for their peers to learn new strategies (Gill & Thompson, 1995). With the increase of dialogue in the mathematics classroom, the spoken mathematics language would give meaning to strategies and the solution process and would support clearer journal entries (Whitin & Whitin, 2000).

In the next three chapters I will explain the methodology, data analysis, and discuss conclusions of my study. I will explore my two research questions in greater detail in the following chapters: In what ways is writing in mathematics useful when second graders are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established? And, in what ways is language useful when second grade

students are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established?

CHAPTER THREE: METHODOLOGY

The purpose of this study was to examine how students use writing and language when they are learning multidigit addition. I wanted to find out in what ways is writing helpful when second graders are learning multidigit addition and in what ways is language used when students are learning multidigit addition? Qualitative methods were used to obtain data in this study. The data were collected using student and teacher journals, focus groups, and teacher field notes. Pre and post tests were also used to collect data on students' prior knowledge and level of ability after the study. This chapter chronicled the design, setting, procedure, data collection and analysis of the study.

Design of Study

The study was designed to help me understand how second graders learn multidigit addition. Data were collected based on my current understanding of how children learn multidigit addition. Qualitative research methods were used in this study. Mills (2003) defines qualitative research as "using narrative, descriptive approaches to data collection to understand the way things are and what it means from the perspectives of the research participant" (p. 4). Practical action research gives the teacher researcher the ability to choose her own focus, determine her data collection techniques, analyze and interpret her data, and develop action plans based on her findings (Mills, 2003).

School Setting

The school in this study is in a fast growing, but very transient community. In this school there is a mobility rate of 40%. The school is a Title One school with 72% of students on free or reduced lunch. Students represent many different ethnic groups, with the two highest percentages being Hispanic 43% and White 39.3%. A large percentage of children come from single-parent homes, live in mobile housing and move often. The state mandates both norm and criterion referenced testing. We follow a curriculum which meets the state and national academic standards.

Classroom Setting

This action research was conducted in a second grade classroom comprised of 11 boys and 9 girls between the ages of seven and nine. Students were assigned to me by my principal during the summer. All students were given consent and participated in my action research study. During the study several students moved and new students came into the study. Throughout the study, the class consisted of 6 Caucasian students (4 males/2 females), 10 Hispanic students (6 males/4 females), 3 multiracial (1 male/ 2 females) and one African American girl. Eighteen of the 20 students receive free or reduced lunch. Four students lived in a motel. Two students attend Exceptional Student Education (ESE) classes with one student attending for speech therapy twice a week for thirty minutes, and the other student attending daily for Language Arts for ninety minutes. Three students have been retained in second grade.

Procedure

I received Institutional Review Board (IRB) approval and principal approval for this study (Appendix B). This research study began in September and lasted for seven weeks concluding in mid October. During "Meet the Teacher" night, I distributed parent consent forms (Appendix A) and answered any questions parents or students had regarding the study. A signed parental consent letter was required before any data were collected. Parents not attending the teacher night received the parent consent form in their child's agenda on the first day of school. Students that entered my classroom after the first day of school received their consent forms in their agenda on the first day they attended class in my room. A pretest was used to determine what previous experience students had with adding multidigit numbers, as well as an explanation of their answer. I administered the pretest (Appendix C) to the class on the first day of data collection. The pretest was comprised of two and three digit addition problems, with and without regrouping. The pretest and posttest (Appendix D) contained a work space where student could add their written explanation. I provided the pretest and a pencil for each child. Students worked by themselves to complete the answers. I transferred the students' responses to a records sheet. The pretest was used later with the posttest to show growth in students' ability to correctly add multidigit numbers and use appropriate explanations to solve their problem.

I taught 60 minutes of mathematics instruction daily using conceptually based learning strategies in conjunction with the county adopted mathematics textbook. To monitor each student's addition and explanation progress, students kept a daily math journal. In the journal, students wrote their explanation of the "Problems of the Day," which took 5-10 minutes at the beginning of class instruction. The Problems of the Day were viewed by the class on the

overhead projector. At their seats, students would answer the questions. When all students were finished, we went over the answers together so that they could check their work. The Problem of the Day consisted of a word problem that was read to them aloud and 2-4 computation problems, along with one computation or problem solving question in the Florida Comprehensive Assessment Test (FCAT) format. The FCAT is administered to students in grades 3-11, and is used to assess students' subject knowledge in mathematics, reading, science, and writing. The journals were collected daily so that I could review all student responses. I followed our county pacing guide when I planned our daily objectives to meet county requirements (Table 1).

Table 1 Mathematical Content Addressed During the Study

Sequence of Instruction	Mathematical Content
Week One	Understand Place Value; Different Ways to Show Numbers; Count Groups of Ten and Identify their Corresponding Number of Ones, Recognizing Patterns
Week Two	Counting on a Number Line; Hundred Chart and Skip-Counting Patterns to 100; Even and Odd, Understand Place Value,
Week Three	Comparing Numbers; Equivalent forms of the Same Number, Recognizing Patterns; Ordinal Positions; Problem Solving , Counting On, Doubles and Doubles Plus One: Make a Ten
Week Four	Using a Data Table; Making a Concrete, Pictograph, and Tally Table; Problem Solving; Organizing Data, addition
Week Five	Multidigit addition, Problem Solving Strategies
Week Six	Multidigit addition; Problem Solving Strategies
Week Seven	Fact Families; Missing Addends; Mental Math

After each daily lesson was completed, I kept a teacher journal that described the interactions between me and the students. I also kept a pad of paper nearby during instruction in order to record observations during the lesson for minimal distractions.

Throughout the seven week data collection time, focus groups were conducted from a random sample of students once a week. In the focus groups, a random group of students were encouraged to discuss with one another their two digit addition solutions and share new ways of finding solutions. Research from Krueger (1998) suggested I use the pre-determined questions to keep the students focused in their conversation and I took field notes during the session (Appendix E). I established social norms for the focus group by encouraging conversation with myself and the other participates in the group. Just like in our daily mathematics lesson, students were able to share ideas, clarify steps, and question approaches to solving the problems with each other. I did not validate anyone's strategy or solution rather I observed while students worked in groups.

Finally, students took a post test comprised of four, two and three digit addition problems with an open space for students to supply their justification at the end of the study. I compared student responses on the pretest and posttest to determine student gains.

Student Journal Writing

In the beginning of the school year I established a procedure of journal writing before our daily math lesson. Students would get out their math journals and copy the Problems of the Day into their journals. In the first weeks of journal writing, I modeled how to copy the problems into their journal and shared out loud my thinking as I wrote. I kept a journal along with the students so as they were writing so was I. The Problems of the Day were shown on the overhead projector

and under each problem there was a sentence that said, 'Tell me how you got your answer." My students knew that they could use words or drawings to illustrate their solution method because I modeled it that way in my journal and I accepted all entries attempted.

Daily Math Instruction

As part of my study, the students kept a math journal. After journaling was completed daily, I collected the journals to review their work, and then our formal instruction began. Place value was taught at the beginning of the study. I began each lesson by passing out base ten blocks to each pair of students. I would circulate the classroom asking my students to manipulate the blocks in some way using open ended questions. I was looking for them to "discover" their answers through questioning. For example, when studying place value I would ask the students "Show me 2 tens and 11 ones." "What can we do so that we do not have as many ones?" Students were also encouraged to talk to one another at their table rather than just respond to me as was my practice prior to this shift in teaching strategy. I connected the conversation so that it would not just be the teacher and the student conversing, I wanted the class to know that I was looking for their ideas also so I would ask students to explain other students' explanations. I gave students time to interact with their table partners and I would circulate the classroom. While I was circulating, I listened as students made discoveries and shared their thinking with each other at their table. I would ask students to share with the whole class as a way of establishing the social norm of how we would share information in our mathematics class. By asking the class if someone did not understand someone else's thinking I was establishing the norm to ask for clarification when you do not understand. After the instruction portion of the lesson students would complete a guided practice page from their textbook related to the instruction

independently or with their desk partner. After all students had completed their guided practice page, the students would share their answers and solution strategies. I chose students randomly from a shuffled group of cards that had each student's name on them. Our lesson would conclude with passing out the homework assignment that would be due the following day. The homework would consist of questions similar to those in the guided practice assignment.

Analysis

The data collected from the pre and post test were used to examine any changes in the students' ability to write explanations for their solutions to multidigit addition problems and demonstrated the students' ability to add multidigit numbers with and without regrouping. Additionally, I collected work from the students' daily journal as an assessment tool. I used the journals to help me prepare for the next day of instruction based on what work students were able to correctly complete in their journal. I coded the information that I found when I went through the students' journals. I was specifically looking for students' ability to correctly solve the problem and noted what strategy they used to solve the problem. I used the teacher field notes and teacher journal to track the dialogue used during the lesson. I noted when students used correct mathematics vocabulary in their speaking. Focus groups allowed me to further answer my research questions. I observed the sociomathematical norms, students' ability to solve multidigit problems, and listened how students used mathematics vocabulary in their conversation. I saw how students used sociomathematical norms to help them solve their problems and used my teacher journal to record observations during focus groups.

Summary

The qualitative action research model was chosen because it could best be used in my particular classroom setting. Using a variety of data such as, pre and post tests, daily student and teacher journals, teacher field notes, and focus groups, I was able to collect the information to answer my research questions. The results of my study are reported in the next chapter.

CHAPTER FOUR: DATA ANALYSIS

The purpose of this study was to examine how second graders use writing and language when learning multidigit addition. Along with collecting daily mathematics journals, I observed interactions during classroom discussions that helped me understand how students were learning multidigit addtion. The journal helped me assess my students' ability to add multidigit numbers on a daily basis. I also changed the flow of the lesson delivery. Instead of teaching mathematics directly to the students, I implemented changes in the social norms of the classroom so that students would have a way of sharing their solutions. Pre and post tests were also used to show the students' ability to add multidigit whole numbers. I have observed that multidigit addition is a challenging topic for young learners and by giving them a way to communicate with one another and myself I was able to help teach to their needs.

I used the literature reported in Chapter 2 to guide my study. Themes were identified from student journals, focus groups, and conversations. Daily, I would write in my own journal about the interactions that took place during the mathematics lesson. I used my observations and students' journals to assess their understanding of addition. In this chapter I share the results of these implementations in my action research with my second grade classroom as I interpreted them.

Establishing New Social Norms in my Classroom

Traditionally, a teacher would stand up and deliver the lesson of the day in a whole group setting at the chalkboard to her students. Students would then work individually to complete a guided practice worksheet and homework would be assigned. Students would raise their hand to

ask the teacher a question. The class would be led by the teacher and the students would only ask questions of their teacher if they did not understand. Children in this type of traditional learning setting would sometimes lack involvement in the lesson and act disengaged. Typically, the same students would raise their hands while some looked bored or fell off task. I used to teach in a traditional classroom setup where the students sat at their seat and I stood by the overhead projector similar to those traditional methods explained above. In order for my students to change their sharing behaviors I needed to look at how I could change the role of the teacher (Egendoerfer, 2006).

Although the physical classroom arrangement was traditional with individual chairs and desks I wanted my students to share with the whole class comfortably. I grouped students into two tables of six and one table of eight. I wanted my students' mathematics experience to be meaningful to them. Research indicated that students learn from each other in the mathematics classroom (Stephan & Whitenack, 2003; Yackel & Cobb, 1996) so I established social norms regarding how students would share mathematics strategies. I wanted to take the role of the speaker off of me and give my students the resources to interact amongst each other. I specifically worked on how we would share information within the group so that I would not be the only one speaking. I established social norms by modeling the behavior that would be appropriate to use in class. The first social norm that I worked toward changing was how answers would be shared. For example, when a student shared an answer with me, I would not immediately tell them if they were correct or not, I used the opportunity to ask the child how he/she got his/her answer and then opened the conversation up to the class so that others could share their answers and strategies. I did not want to validate their answer with my own words

because that would put a stop in all further conversation. I modeled by directing the answer to the entire class. One day during an explanation of the solution to 4 + 5, Alison shared with the class that she got 9. I further inquired, "How did you get 9?" She explained her reasoning that if she added 1 to 4 she'd get 5. She knew that 5 + 5 is 10 because we had just studied double facts. Since she added 1 to 4 she took away the 1 from 10 and got 9. The steps to her solution were complex for many of my students so I asked the class if anyone did not understand Alison's method of solving the problem. The second social norm that I needed to model for the students was what to do when someone did not understand what was being said. Again, directing the question back to the class instead of validating it myself kept the students alert to see what would happen next. Mark raised his hand and said he did not understand what she meant. I asked Alison to share her thinking again; this time I used the overhead projector to give a visual while Alison explained. In the past, I observed that students stopped raising their hand to ask for help when they continuously do not understand. I found that having an open and risk free environment where students could dialog with one another became helpful because they did not feel singled out when they did not understand. Most importantly I saw the students become active members of the group. I used my field notes and teacher journal as a way of observing the participation in the classroom. According to my observations in my field notes, as students caught on to the norm of sharing their answers and clarifying misunderstanding, the number of participants in the discussion grew. I could also assess Alison's addition knowledge better because she was referring back to previously learned strategies and applying them to a new situation. After my students got used to the social norms of sharing and explaining to each other I noticed the transfer of these norms into instructional time with other subject areas. Here is one example of

how social norms were used throughout our school day. One day, Jake was at the board pretending to be the teacher during a spelling lesson. His job at the board was to help the class put the spelling words in order. As I watched the interaction between him and the students I noticed his language.

Jake: "How did you know that tie comes before try?"

Linda: "Since both of the first letters are the same, I compared the second letters. The letter i comes before R in the alphabet and that's how I know."

This example of how students interacted amongst each other outside of the mathematics lesson indicated to me that they were able to use the new social norm in a different context. By listening and observing the students converse this way I found that I could also assess their understanding. Because the conversation included an explanation as to 'why' the answer was solved that way helped reassure me that students truly understood the lesson objective not to mention that all students hear the reasoning and can make it more meaningful to them.

I also observed that because these social norms were established students became more active in their own learning even outside of our mathematics lesson. Building social norms in my classroom gave students the tools to interact as a group. Sharing their reasoning was another important part of understanding my students' needs. I was able to assess my students' mathematics understanding by listening to their interactions and reading their journal entries.

Place Value

Before the students begin learning addition in my second grade class they are taught place value. I did not incorporate the norms of sharing answers and explaining reasoning during place value instruction. It was not until we moved on to addition that I noticed through the

student journals the students did not truly understand place value. I found that I needed to go back and spend extra time on identifying tens and ones once I started the unit on addition. Chris's journal entry is an example of many students' lack of understanding of place value. I observed this particular journal entry as I circulated the room during Problems of the Day.

One of the Problems of the Day was 15 + 15 and students were busy solving the problem in their journals. I stopped at Chris's desk because he surprised me by copying down the problem horizontally instead of vertically like I did on the overhead projector. I studied his entry and asked him to explain his thinking to me. In talking with Chris it was obvious to me that he did not understand place value. I was surprised to see him still struggle with the concept since we had just finished a place value unit. His original explanation looked like this:

1 + 1= 2 5 + 5= 10

I asked Chris where did he get the ones from and he pointed to the 10's in the number 15. Therefore it was obvious that he did not understand the 1's really stood for 10's so I asked him to draw the number 15 using towers and cubes (base 10 blocks). He correctly added one tower and 5 cubes so I asked him where was the one? He said, "Oh, the one is a ten." He added a zero to the end of each of the ones making it look like this:

10 + 10 = 2

I asked him if 10 + 10 equals 2 and he said, "no, it is twenty" and then added a zero to the 2. I felt our one-on-one conversation moved too fast for him so then I asked him to tell me his steps again. Successfully he identified the 1 as a 10. "Now I want you to tell me using the paper and a pencil how you solved 15 + 15." Since I was assessing his understanding, I wanted him to have the experience of writing the correct explanation in his own words. For Chris, journal

writing was something he was more comfortable with than adding to a class discussion. This observation with one student had me start looking closer at the other students' entries to see if I found others still struggling with place value.

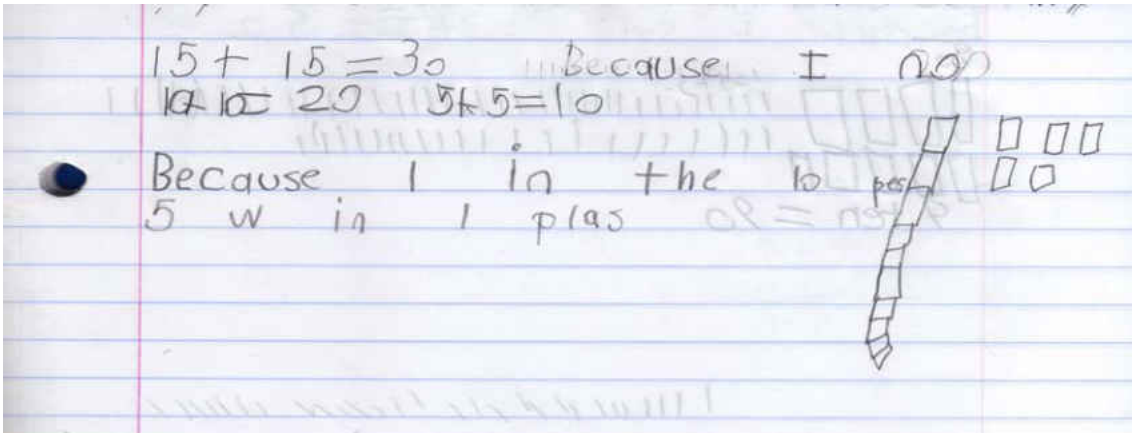

Figure 1 Chris's journal after our discussion

Chris's journal entry (see Figure 1) is representative of students who do not understand place value. Because journal writing is part of our daily routine, I was able to identify the group of students who struggle with this concept. In each case, the influence of place value on performance when adding multidigit numbers was clear.

I was surprised that as I went through more journals I could see the students lacked place value sense and that was why they were getting their addition problems incorrect. In the past, I did not ask for students' justification of their answers. My literature review convinced me to ask students for an explanation for their answers. I feel that if I had not implemented journal writing and modeled how to express mathematical thinking with words than I might have just seen these children as not understanding addition when really they needed help understanding place value.

Transition from Oral Language to Written Language

I noticed that students did not like to write sentences or words in their journals. I thought by giving them a vocabulary word wall they would find the words came to them easier when they were accessible to them. My field notes and daily classroom observations indicated that the students talked more than they wrote. It seemed that transitioning from oral language to written expression on paper was a challenge for them. I might not have gotten the true sense of their understanding if I just looked at their entries for assessment in their written justification.

A word problem is always included on weekly mathematics tests. Word problems have been a struggle for my students. We did a word problem that consisted of writing a justification during Problems of the Day in order to give them practice. I saw a pattern when we completed our daily word problem together in a discussion format; students gained insight into the word problem because of their interactions within the group whereas the test was strictly completed with no help from others. As another way of getting the students to talk to one another and share ideas students were grouped with a buddy to complete a word problem during practice. I observed students using their known strategies and manipulative counters to solve the problem. All student groups were able to get the correct answer and as I circulated the classroom I noticed that students were talking with one another and me sharing their solution ideas. It was clear that their oral descriptions were more detailed and lengthy than their writing in terms of a justification. Matt and Kayla's group told me they used a ten frame to count on but never explained that as their strategy on paper with they worked together to solve

8 + __ = 13.

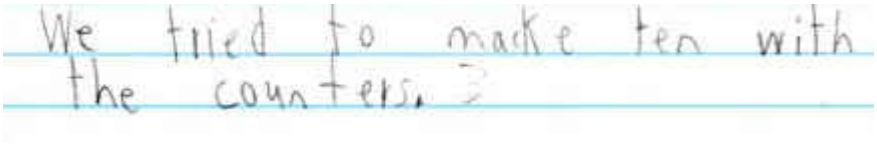

Figure 2 Matt and Kayla's work

Transitioning from oral language and expression to paper was definitely a challenge for the students. It is important to note that when they shared their answers with the class they were correct and able to use more words to explain their thinking processes. The short sentences that they wrote on their paper were not true representations of how they got their answer. Looking at Ann and Scott's work in Figure 3, they stated the fact of how they got their answer but never told the reader the process they used to get their answer. The problem was 8 + __= 13. I can see from their illustrations that they used counters on a ten frame to solve the problem correctly.

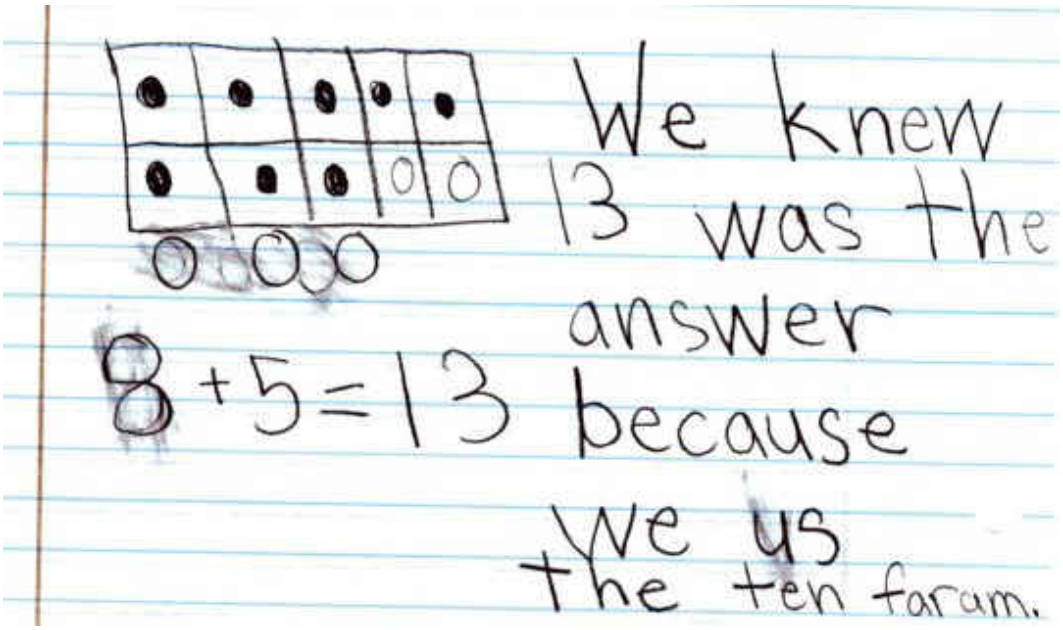

Figure 3 Ann and Scott's work

Figure 3 represents Ann and Scott's statement of how they solved the same problem. I can tell by the illustration that they started with eight counters and kept adding until they reached thirteen. That is also how they explained it to the group. The writing they turned in did not reflect

the complexity of their thinking. When it came time to share they were able to talk with more detail and justification orally than what was shown on their paper. At the start of our journal writing students' writings were short, misspelled, or incorrect. Sometimes I thought the reason they did not write more was that they did not have the vocabulary to express their thoughts. Based on their responses I knew I needed to give them extra instruction on vocabulary. They were able to converse using mathematics vocabulary but I did not see a transfer to their writings. I started a mathematics word wall in a pocket card display in our classroom. The mathematics word wall was unique because there were so many words that the chart could not display every word. I had to set up the word wall so that students had access to all the words even if I did not have much space. The chart had a designated spot for each letter of the alphabet. Behind each letter were lots of cards that displayed different vocabulary words and definitions with the same letter. Students were able to get out of their seat and manipulate the cards within the chart to give them words for the ideas in their heads. After the introduction of the math vocabulary chart I observed that their writings were clearer and their spellings were correct. The word wall was a resource to activate students' thinking and to support correct spelling.

Using Mathematics Vocabulary in Context

All businesses and careers come with their own language or dialect. Looking at the textbook curriculum for second grade mathematics I found that not a lot of attention is paid to the vocabulary and meaning of mathematics. My prior experience with teaching mathematics to second graders also interested me. I noted that it is hard to understand what someone is saying if they do not have a mathematics vocabulary. For example, when describing place value it is common for a student to use directional words such as 'over there, move it there, put it under the

four.' Listening to those commands I wondered if students understood place value since they were not using it correctly in context.

In my experience, students looked for key words that identified the operation in a problem. They used those key words to guide their problem solving with no deep understanding of what the question was asking them to solve. I wanted my students to not only define a mathematics word but to understand it in context. Throughout our mathematics lesson I referred back to the "How did you know you needed to add?" question so that my students would look again at what they were reading and try to use the correct problem solving strategy.

I noticed that after the social norms were changed in the classroom students were using more mathematics vocabulary in order for others to understand them. I noted in my journal that students were identifying numbers by place value. I listened in on students conversing with one another and using mathematics words such as addends and sum.

Andy: My sum is 23, how did you get 24?

Ann: Let's see, I used 10 + 13. They were both my addends.

Although the addition problem that Ann was working on did not give her the correct sum their conversation was easy to follow for both of them and for the onlooker.

Midway through my data collection I observed my students speak using mathematics language. I read their journal entries and saw their writing become more sophisticated and thoughtful as they incorporated mathematics vocabulary. I observed in focus groups students were able to understand each other's thinking and ideas because they all understood the same vocabulary. Specifically it was clearer to understand their intentions. Figure 4 is representative of students' journal entry where they correctly used the mathematics vocabulary tens and ones.

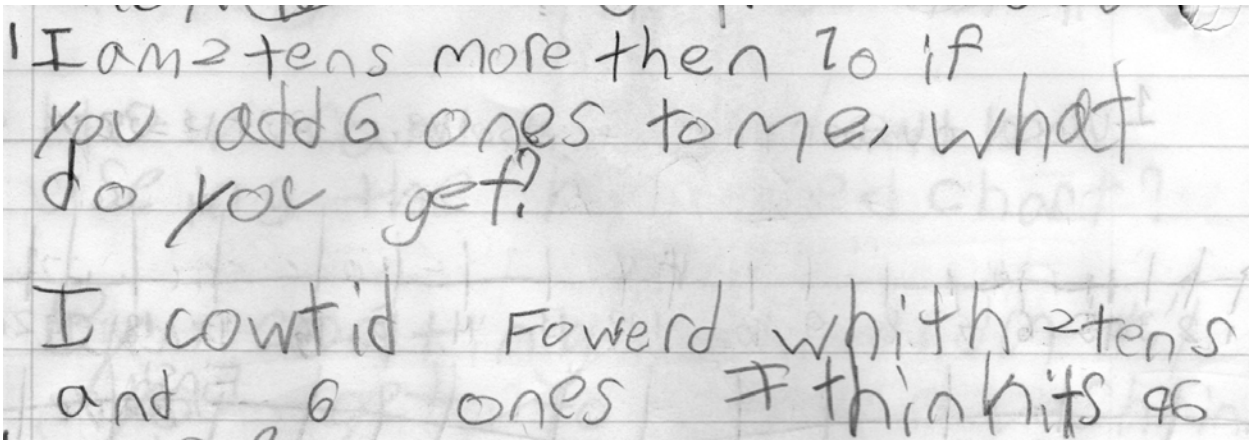

Figure 4 Using mathematics vocabulary

In Figure 4, it is evident that this child has an understanding of number sense because she stated she added 2 tens to the existing 70. She shows place value understanding when she added 6 ones to 90. If this student had not identified the digits according to their place value the justification would be incorrect. By writing her thoughts on paper, this student was able to show me that she could count on using tens and ones. Once vocabulary was taught using the word wall

(Figure 5), I found students used it often when they wrote and when they spoke.

Figure 5 Student using the mathematics word wall

Stages of Learning Multidigit Addition

Research indicates that students learn not only from the teacher but from each other (Hiebert & Wearne, 1996; Schwarz, 1999). I hoped that creating an environment where the students shared ideas amongst each other would help them to create strategies for finding answers. Students were able to pick up on our new social norms quickly. They seemed to enjoy talking with the class but I did not see evidence of strategies or mathematics vocabulary in their dialogue in the beginning of data collection. The early journal entries were consistent with the same phrases; I counted on, I used my head, or I just knew it. A few students were able to share

new strategies with the class. I taught strategies as they came up in the instruction because students were relying on the same ones and did not apply them to context. They used few sentences and illustrations to solve their problems. Figure 6 shows evidence that the student can solve the problem to get the correct answer but his sentence does not show convincing evidence that he knows the value of 1 in the number 10.

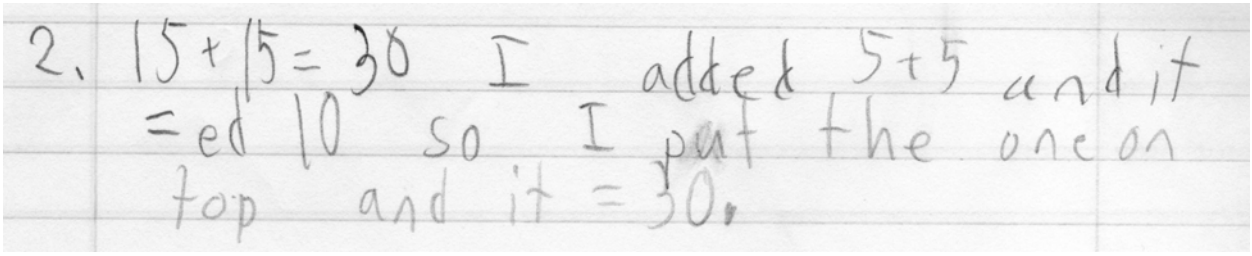

Figure 6 No convincing evidence that this student identifies place value

As shown in Figure 6, the student understood the procedural knowledge that he had to move a digit over to the tens place. He did not use the appropriate place value when he regrouped. By telling me he added a one on top of something is not specific nor is it correct since the 1 is really considered a 10. Journaling was a useful tool for individual assessment because it allowed me to see all my students' thinking whereas not every student would voluntarily share during the lesson. There were also time constraints in a classroom and not all students who wanted to share had the time. I was able to get a picture of the students' thinking when I read their journals; therefore, I was able to create lessons that would be meaningful and appropriate for them.

In the initial data collection stages most students were only able to use basic strategies for solving problems. As instruction continued, I could see their justification skills improve. In the middle of data collection students were writing and drawing using mathematics language and

labels for their drawings. Towards the end of instruction, I could tell students felt more comfortable with the addition skills and it became more procedural for them to solve the problems.

Stages of Learning

I classified students' understanding of multidigit addition into three stages. In Stage One students used counting by ones or 'counting on' strategies to solve their problems. The conversation vocabulary was minimal; that, this, move, and need more. Students' justifications were short, two or three word sentences that describe their thinking. In their journals I saw lots of drawings that represented counting by ones such as circles, see Figure 7.

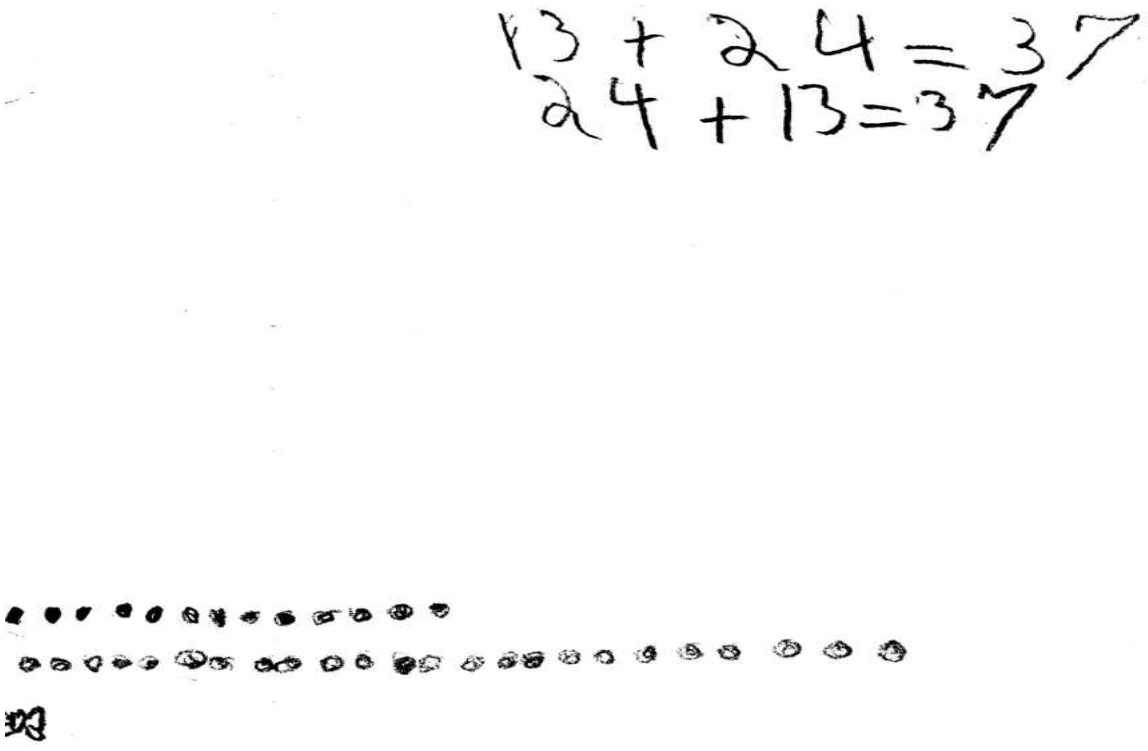

Figure 7 Counting by ones

In Figure 7, this student used counting by ones to answer her addition problem. Based on the fact she made one row of thirteen and one row of twenty-four circles with no words classifies her ability to see numbers as being worth more than just ones is minimal.

I could also tell that a group of students had already understood the basic ideas of adding because they used language such as; I practiced it at home, I used my mind, or I just know. In Figure 8 the student stated the answer was found, “I use my head to count the number.” The statement did not represent how the student solved the problem; 14 + 16 = 30

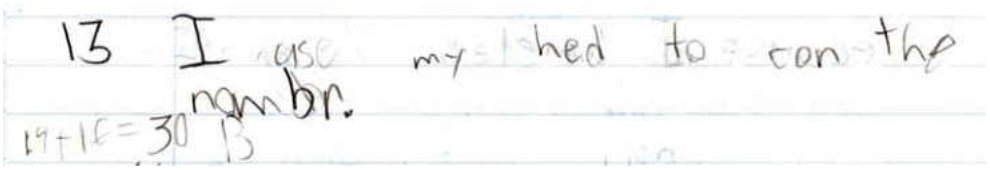

Figure 8 Stage One

Figure 8 is representative of Stage One of learning addition because the student uses a short sentence that does not correctly state his understanding. He may already understand the concept of adding two digits together but does not share vocabulary or process in his writing.

Around weeks three and four I began to see what I call Stage Two emerge. While reading their student journals I noticed the addition of mathematics vocabulary to justify and explain their thinking. Students were able to share different strategies for solving the same problem in the focus group setting. Students at this point had learned many different strategies to solve the addition problems and it is evident that they each had a favorite strategy to solve addition problems. In focus groups, where students solved the same problems, they used different strategies to solve the problem and they were able to explain in detail each step of their process. Students dialogued together to clarify and pose questions for each other. The students gave me

feedback after working in focus groups. It was unanimous; they loved working in such small groups because they could get attention and clarify word problems together. I noted students seemed to enjoy working together and dialoged more than writing individually on paper. Our daily journals did not seem to be the students' favorite part of mathematics; instead, they really enjoyed talking and sharing ideas with others at this stage. Figure 8 is an example of how students wrote out their thinking in a sentence format. This figure also shows their understanding of place value.

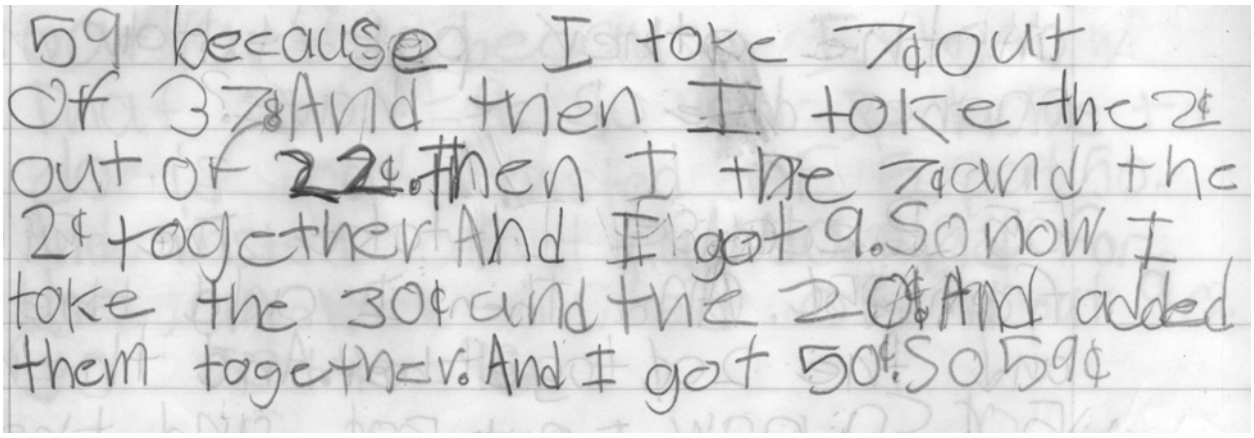

Figure 9 Stage Two

Figure 9 is a relevant example of Stage Two because it shows the student's own strategy to solving the addition problem. Also, use of place value is identified when the student refers to the digits in their proper place, tens and ones. The last addition step of adding 50 cents and 9 cents was left out of this illustration. I noted that most students in Stage Two were able to illustrate or write about their problem however, most students left out a part of the process especially when they had already written a lot, see Figure 10.

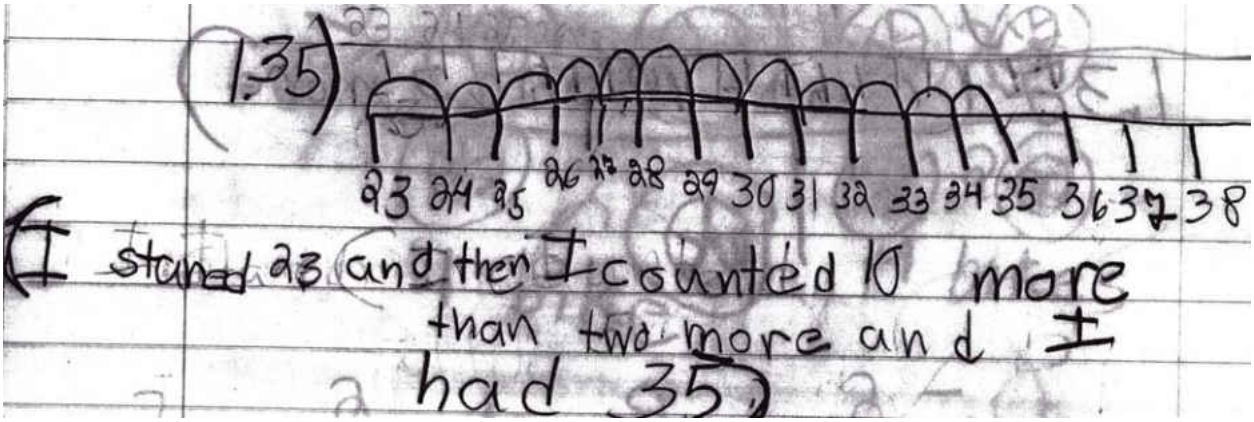

Figure 10 Stage Two

Figure 10 represents how a student started to use a number line to add numbers. The student clearly stated that he wanted to count by tens and then by ones however, his illustration does not represent the unitizing by tens. This is another example of students working in Stage Two. Stage Two represents trying out new strategies and using words and illustrations to map out their thinking. Not all students in Stage Two were getting the correct solutions at this time. I have noted from looking at their journals that not all answers are correct even if students write out their processes. It is also noted that students in Stage Two are making attempts to illustrate and write about their thinking. Figure 11 is an example of how students illustratesd their thinking without words defining their action.

Figure 11 Stage Two

In Figure 11 you can see how this child understands the reason she needed to regroup, she had enough ones to make a group of ten. Looking through student journals I noticed that students would rather illustrate their steps to you themselves rather than write it using words. Stage Two is the only stage that I identified my students attempting to write about their thinking as a process and not as a statement like in Stage One.

The final stage which I call Stage Three was surprising to me. Students went from writing in their journals with detailed explanations or illustrations to simply showing the procedural portion of their solution. The pages that were once covered with illustrations, diagrams, and sentences were now bare. Most students explained their thinking with short sentences, like Stage One, however, using different words. For example, Gabby wrote, "I used mental math." Evidence on paper showed me she regrouped properly but without any 'helping' strategies such as a tens and ones column like I had seen in earlier journal entries. It was evident to me that students were feeling confident that their answers were correct and did not see the need to explain their steps as they once did. The talking began to die down also. Students that were once excited to share their solution process and share without being prompted now just raised their

hand to give their final answer without any explanations, see Figure 12 for an example of Stage Three.

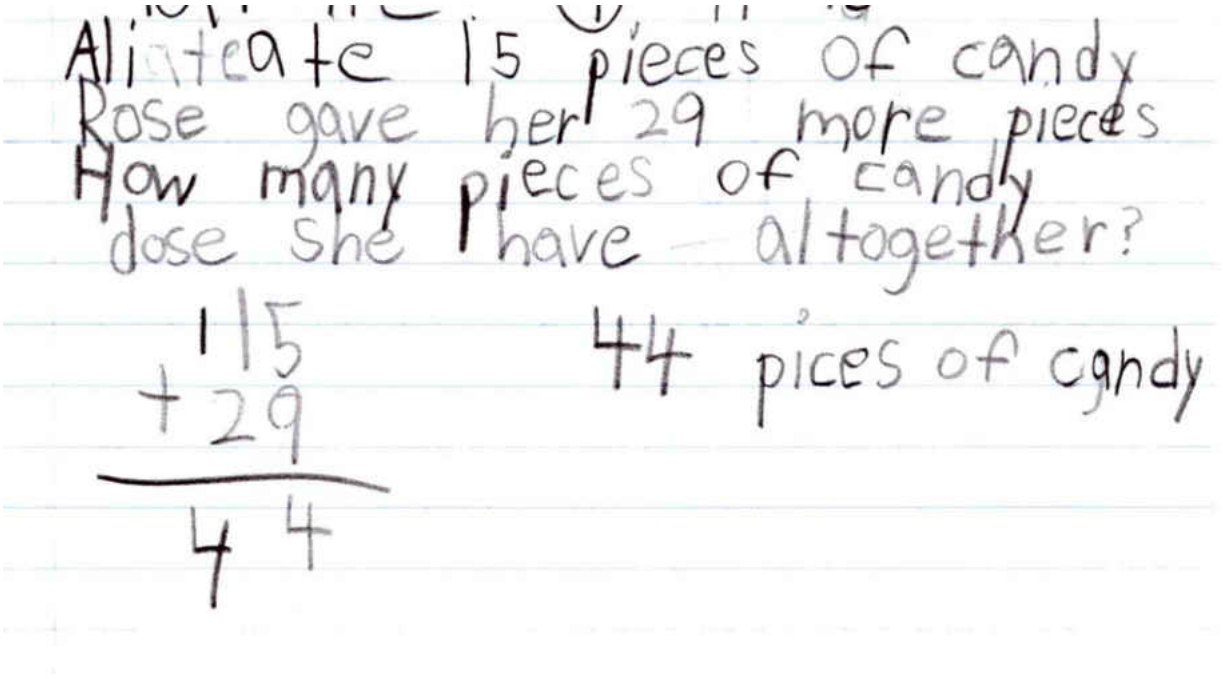

Figure 12 Stage Three

Figure 12 represents how most students answer addition problems in Stage Three. They are confident that they do not need an illustration to help them solve the problem. I also observed that students did not give a written explanation of the steps they followed to answer the problem. The use of different strategies is not evident in Stage Three. There was a time when students chose different strategies to complete a problem. At this time, most students in Stage Three set up their problem the same way, vertically, and begin to answer the problem starting in the ones place.

Summary

Data for this action research were collected from teacher and student journals, pre and post tests, focus groups, and teacher field notes. Students were able to write or illustrate their thinking in journals and during focus groups. Students also shared their thinking with others in the classroom. Results from the pretest showed most students did attempt to justify their solution. On the pretest, most students solved the multidigit addition problems incorrectly.

I thought that since my students could explain their thinking orally that writing would be evident in their journal entries. Since I did not see a lot of 'words' used to describe their thinking process I began to question why that was. Creating a mathematics word wall was one way that I thought students could get ideas for words that would help them clarify their thinking on paper. I did see students referring to the word wall and I felt that having a word wall was important and useful for the students to continue growing academically.

At the end of the seven week data collection period, I reread their journals and looked for patterns in their responses. Thumbing through the pages, I could clearly see that student entries could be placed into three stages of the learning addition unit. In Stage One, which was in the first two weeks of data collection, entries consisted of short sentences such as; "I knew it" or "I counted on." There usually were no illustrations or diagrams that accompanied the problem.

In weeks three through five I began to see students applying strategies to solve their problems. Some strategies were taught by me, other strategies students came up with on their own. Some strategies students learned from others when they shared. Student used the mathematics word wall for spelling and for words they needed to help them explain their thinking. My teacher journal and field notes indicated that the classroom was nosier; there was a

lot of talking and sharing going on. During this time, students asked to sit with partners to complete work together, they would work on the illustrations and writing together.

Stage Three occurred in the last two weeks of data collection. At this time students were getting solutions correct, I could tell they were confident when they gave their answer. Student journal entries contained less illustrations and words. There was evidence that students were solving the problems procedurally. When students explained their thinking they sometimes did not use proper place value language but the answers were correct. Most students in this stage set up their problem the same way, vertically. In Stage Three, students did not share their thinking process with others. Most students in this stage stated their answer. When asked how they got their answer, most explanations were procedural. For example, 24 + 58;

> "I added 4 + 8, I got 12. Twelve is bigger than 9 so I put down the 2 and carried the 1 and put it on top of the 2. One plus 2 is 3 plus 5 more is 8."

This example lacks the place value language that they once shared. When questioned if the 1 that was carried is a one, they would correct themselves and say, no I mean 10.

Post test results were similar to Stage Three responses. Most post test problems were answered correctly and it was the justifications that stood out. Like Stage Three, students were able to solve the problem correctly but lacked illustrations or words that let you know how they got their answer. Most students who told the reader how they got their answers on the post test stated their methods factually; "I regrouped, I counted on, or I changed the numbers so that I could solve the problem vertically." Few students illustrated what they meant when they regrouped or counted on. Very few students showed evidence of using their own strategies to solve the problem.

I questioned how mathematics vocabulary played a part in students' understanding and learning of place value and multidigit addition. At the beginning of data collection I observed discussions that students were using unspecific and unclear statements to describe their thinking. For example, a student would say; "I moved the zero here and placed the one there." When asked specifically what that meant the student was able to explain it with word choices that made mathematical sense. Therefore, I saw that I needed to provide students with a mathematics word wall to help them make word choices that were appropriate to mathematics. I was conscience in my own word choices in the classroom. I wanted students to hear what it sounded like to use those mathematics words in context.

The data collected offered me insight into how students learn to add. Results and future implications to a study like this are discussed in chapter 5.

CHAPTER FIVE: CONCLUSION

Introduction

I went into this study with a large amount of enthusiasm. Based upon what I had studied in my coursework and through journals I thought I could better help my students learn multidigit addition. I was excited to change from my traditional teaching methods into a group discussion style where my students could be more active in the conversation and learning. In this action research, I was able to answer my questions, "In what ways is writing in mathematics useful when second graders are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established? And, In what ways is language useful when second grade students are learning multidigit addition in a classroom where conceptual-based implementation and social norms are consistently established? In this chapter I will review the results of my study and offer recommendations for future study.

Results

In this action research I found that students seemed to enjoy sharing their mathematical thinking with others in the classroom. More students were engaged in the lesson than in previous years. Journal writing in mathematics was new to most of my students and they took to the new procedure easily.

Journals provided me an insight into my students' thinking. Looking through their entries on a daily basis gave me the opportunity to plan appropriately for the next day's instruction. In the very beginning of journal writing students did not write their thinking process down. I

noticed that most journal entries were consistent with the same phrases; I knew in my head, I counted on my fingers, or I used my mind. I labeled the beginning two weeks of instruction as Stage One. As instruction progressed, I observed students using new learned strategies to solve problems, I called this Stage Two. Most importantly, their answer justifications became more thoughtful and interesting to read during this time. I was able to assess students on their ability to correctly add multidigit numbers because their writings and illustrations were clear. Students shared their strategies verbally with one another. Stage Three occurred towards the end of data collection. Students at this stage were more confident in their answers when they shared. Entries contained fewer illustrations and words. Sometimes students' written explanations were procedural until they were questioned about specific steps. When asked students could identify place value and explain why they solved the problem the way they did.

Social norms in our mathematics classroom changed. Instead of direct instruction, students began to share their own thinking with one another. Often their ideas were different than the ones taught. Students were making meaning of addition by working with their peers to share information. Students also used our new social norm to ask for help when they did not understand a problem. It was common to hear students ask one another, "Can you explain your thinking to me again?" instead of saying, "No, that's wrong" as they once did.

The Impact of Outside Influences on Instruction

I had changed the way instruction and learning would take place in our classroom. Things did not change at home. When students needed help at home parents explained mathematics procedurally to the students, whereas they were getting conceptual and shared instruction at

school. In some of the student work, especially work done at home, the student reverted back to memorizing procedures. Their explanations during conversations in the classroom confirmed that the way it was taught at home was different than the way it was taught at school; sometimes students were confused. If I were to do this study again, I would organize a mathematics night in our classroom. Mathematics night would be an opportunity to bring parents and caregivers into our classroom to gain insight into our daily mathematics routine. I envision center activities set up throughout the room. The centers would be designed so the students could teach and share the mathematics topics they were learning in class. Most importantly, parents and caregivers would have an idea of the materials and sharing methods we use in the classroom to help solve our problems.

Recommendations

My students were motivated because they had new norms governing how they could share information with one another. The results of my study are encouraging to those students who are learning how to add. In the future, students should have a strong understanding for place value and number sense before beginning multidigit addition. Addition is not meaningful to students who do not understand numbers and their values.

I wanted to change social norms in my classroom so that I would not be the only presenter of information or the one who validates answers. If I were to do this study again I would want to do further research on other norms that would compliment the conceptual mathematics instruction. Specifically, taking the leader role off the teacher and giving more time for students to discover and explore their own strategies.

In this study, I relied on student journals to provide me an insight into their thinking process. In a class with so many children I wanted to be able to 'hear' what all my students were thinking, especially the ones who were shy about speaking in front of their peers. I would encourage mathematics teachers to use journals as a way of listening to their students' thinking. I also used teacher journals and field notes to record interactions, dialogues, and the overall feel of the class. I found that my journal and field notes provided me with something to reflect upon after the day was over. It also helped me 'see' what occurred during the lesson so that I could prepare for the next day. Sometimes conversations happened so fast that it was hard to get everything down on paper and I wished I would have recorded classroom interactions.

In an effort to get students to share with one another I grouped them in tables. I took the classroom size and the size of their desks into consideration when I chose groups of 6 and 8. Within the group, the students were paired off so that they always had a buddy. During group work the classroom would get loud but that did not seem to bother the students or the neighbors. The group size worked well in this setting because it allowed for enough talkers and listeners to be beneficial to all the students sitting at the table.

My students did not like the idea of writing in their journals. I found that my students loved to share verbally in small groups. If I were to do further research I would like to look at how students transfer their oral language into the written words and illustrations.

Summary

I discovered during this action research the passion I have about meeting the needs of my students. I have learned that the traditional textbook lessons and standardized assessments do not give the complete picture of the students' abilities in mathematics. Reflecting upon my practice

has given me the opportunity to seek out ways to be a better teacher. I found that writing was useful when second graders are learning mathematics because it offers a window into their thinking. Reading my students' writing opened my eyes to different solution possibilities. As a teacher, I thought I knew a lot about mathematics. I found as I was collecting data for my action research there are many strategies to use to solve mathematics problems that I had not ever used. I observed as they wrote and shared their thinking and it impressed me. Reading their journals also helped me to guide instruction to meet their specific needs. In my opinion, the most important thought I learned from this action research was the importance of listening to the students.

In my study I also looked for how mathematics vocabulary was useful when students were learning place value and multidigit addition. Before the mathematics word wall was set up, students often were unclear when they shared their thinking verbally or on paper. Often times I needed to ask for clarifications from the students, such as, "You told us that you moved the four over there, Where is over there?" I saw a pattern of students using unclear language in the beginning of this study, therefore, I added the mathematics word wall to our classroom so that students could see word choices that may define their thinking clearly.

Finding research that supported my thinking was inspiring. Becoming a better teacher for my students was important to me. Often times abandoning the traditional ways can be lonely and met with resistance. I encourage teacher researchers to continuously reflect on their practice.

APPENDIX A: PARENT CONSENT LETTER

Dear Parent or Guardian,
My name is Elizabeth Hensley and I am your child's teacher this year at Reedy Creek Elementary. During this academic year, your son/daughter will be asked to keep a daily math journal as a way of recording their mathematical thinking. I am presently working towards a Masters of Education degree at the University of Central Florida and I am working on my research thesis. The purpose of my research is to understand how children learn to add. In addition to normal math instruction, I will be using student math journals, interviews, and focus groups to gather data for my research. I plan on using an audio tape to record the students' interviews and focus group dialogue as well as note taking during class instruction on students' addition understanding.
I will need your permission for your child to take a pre-test and post-test, participate in focus groups related to math problems, and answer interview questions related to their mathematical thinking. The information obtained through these assessments will be kept confidential. There will be no identification of the students. Your child will be able to withdraw from the project at any given time. All data, including audio will be kept under lock and key, the audio will be destroyed at the end of the study. There will be no penalties for choosing not to participate in the research. Data will be collected during regular math classes. Students who choose not to participate will be engaged in the same classroom activities but no data will be gathered about them.

In order for your student to participate, I need a written release from the parent or guardian. Please fill out the form below and indicate whether you are giving permission for your child to participate or if you are denying permission.

Thank you for your help. If you have any questions, please call me between 8-8:15 a.m., (407) 935-3580.

Sincerely,

Elizabeth Hensley
Second Grade Teacher

Contact Information- UCF Faculty Advisor
Juli Dixon, Ph.D.
Associate Professor, Teaching and Learning Principles
University of Central Florida
4000 Central Florida Blvd.
Orlando, FL 32816-1250
407-821-4140
E-mail: jkdixon@mail.ucf.edu

Student's Name: _____

I have read the procedures described above. I voluntarily agree to participate in the procedure, and I have received a copy of this description.

I, _____ have read and understood the letter for participation in Ms. Hensley's research study on the way students learn to add.

____ Yes, _____has permission to participate.

____ No, _____does not have permission to participate.

_____ _____
Parent/Guardian Signature Date

Information regarding your rights as a research participant may be obtained from:
Barbara Ward, CIM
Institutional Review Board (IRB)
University of Central Florida (UCF)
12201 Research Pkwy, Ste 501
Orlando, Florida 32816-3246
Telephone: (407) 823-2901

APPENDIX B: IRB APPROVAL LETTER

University of Central Florida

Office of Research & Commercialization

June 7, 2006

Elizabeth Hensley
12813 Madison Pointe Circle 101
Orlando, FL 32821

Dear Ms. Hensley:

With reference to your protocol #06-3537 entitled, **"How does conceptually based multidigit addition instruction affect students' written justification of their solution?"** I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. **This study was approved on 6/5/06. The expiration date will be 6/4/07.** Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator. **Please notify the IRB office when you have completed this research study.**

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Joanne Muratori

Joanne Muratori
UCF IRB Coordinator
(FWA00000351 Exp. 5/13/07, IRB00001138)

Copies: IRB File
Juli Dixon, Ph.D.

JM: jm

12201 Research Parkway • Suite 501 • Orlando, FL 32826-3246 • 407-823-3778 • Fax 407-823-3299

APPENDIX C: PRETEST

Name:_____ Date:_____ Pretest

Answer each addition problem. Next to the problem, tell me how you solved it.

1.

$$\begin{array}{r} 45 \\ +17 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 32 \\ +\ 17 \\ \hline \end{array}$$

3. $55 + 32 =$

4.

$$\begin{array}{r} 258 \\ +\ 148 \\ \hline \end{array}$$

APPENDIX D: POST TEST

Name: _____ Date:_____ Posttest

Answer each addition problem. Next to the problem, tell me how you solved it.

1.

$$\begin{array}{r} 47 \\ +\ 18 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 33 \\ +\ 19 \\ \hline \end{array}$$

3. 56 + 66 =

3.

$$\begin{array}{r} 235 \\ +\ 127 \\ \hline \end{array}$$

APPENDIX E: SAMPLE FOCUS GROUP QUESTIONS

Sample Focus Group Questions Student Name:_____Date:_____

1. How would you solve this problem?
(Go around the group and record different responses)

$$\begin{array}{r} 28 \\ +34 \\ \hline \end{array}$$

2. Does it help to hear other students tell you how they got their answer?

3. Why do you like hearing other ways to solve problems?

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