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Supporting Middle School Students with Emotional or Behavioral Disorders in Blended Learning: A Fraction Intervention using Virtual Manipulatives

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SUPPORTING MIDDLE SCHOOL STUDENTS WITH EMOTIONAL OR BEHAVIORAL DISORDERS IN BLENDED LEARNING: A FRACTION INTERVENTION USING VIRTUAL MANIPULATIVES

by

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ABSTRACT

Students with emotional or behavioral disorders (EBD) are more likely than other students with disabilities to drop out of school (Blackorby & Wagner, 1996) and suffer societal consequences that include higher probabilities and rates of incarceration, poverty, drug abuse, homelessness, low wages, and unemployment (Bradley, Doolittle, & Bartolotta, 2008; Wagner, 1995). High school graduation is a critical factor to improve post-secondary outcomes for students with EBD; yet it is often mathematics, specifically algebra, that stands in the way of graduation (Blackorby & Wagner). Students with EBD often enter middle school lacking foundational mathematics skills, such as fractions, which sets them up to struggle with pre-algebra and ultimately fail algebra (U.S. Department of Education, 2008).

The purpose of this study was to improve the ability of middle school students with EBD to access online grade-level mathematics content by providing fraction remediation to improve conceptual fraction knowledge and procedural fluency. The intervention consisted of *Initial Fraction Idea*s, an intervention curriculum from the Rational Number Project (RNP; Cramer, Behr, Post, & Lesh, 2009), in conjunction with three online virtual manipulatives (VM).

The unique blended learning environment of this study provided the opportunity to evaluate the use of an evidence-based fraction intervention, in conjunction with VMs, in a single subject, multiple baseline across subjects design. Fluency data was gathered from daily fraction addition assessments (FAA) administered after each intervention session. A second component of the study featured a non-experimental repeated measures design that assessed student conceptual understanding of fraction equivalency through the administration of pre, post, and delayed-post Equivalent Fraction Tests (EFT).

The intervention was effective in producing increases in median group fluency with high effect sizes, across three replications, establishing a functional relationship between the intervention and the dependent variable for these groups of students with EBD. In addition, all groups posted mean gains in equivalent fraction knowledge from pre to post-EFT, and maintained those gains for at least 15 days after the intervention concluded.

For you Mom…You said I could do anything.

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There are countless students and families that have fueled my passion and motivation to learn more so I could do more. The names I often forget, but the faces I will remember forever. Justin and Cheryl, Wilson and Anthony, Solomon and Jonathon, Debra and Tabitha, Nicholas and Amber, Chell and Laniya, Zeke and Caitlyn, Jacob, Mason, Luna, Ralph, Madison, Abby, Zofia, and baby Jake…you represent all who spark the passion in my purpose.

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(Dr. Seuss, *Oh, The Places You'll Go!*).

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CHAPTER ONE: INTRODUCTION

Overview

The doors of public school opened to all children, regardless of their income, culture native language, ability, disability, or propensity for learning when Massachusetts passed the nation's first compulsory education law in 1852 (Bellamy, 1891). Despite the intention of this educational law, it took less than 50 years for many students, some with emotional or behavioral disorders (EBD), to be removed from school and placed into classrooms for "special instruction" (Boston City Council, 1857; Gardiner-Chase, 1904). This removal of students was intended to restore order and instruction to the classroom, mollify teachers, and refocus attention on the students who were eager to learn. Nevertheless, for the next 70 years, separate settings for students with disabilities became the norm in public education, especially those with EBD (Osgood, 2008).

This trend of segregation began to shift in the 1970's, in response to a host of factors including the tenure of two presidents with personal disability experiences. Exposés by the authors of *Willowbrook* (Rivera, 1972) and the photographic essay, *Christmas in Purgatory* (Blatt & Kaplan, 1966), revealed the merciless conditions inside asylums and institutions, raising the voices of parents and advocates in protest, as well as civil rights litigation and legislation (Osgood, 2008). These events led up to the passage of the *Education for all Handicapped Children Act of 1975,* later to be renamed the *Individuals with Disabilities Education Act* (*IDEA;* 1997), that guaranteed every child and adolescent the right to a free and appropriate public education (FAPE) in the least restrictive environment (LRE). With the passage of this legislative initiative, many students with disabilities were headed back into the regular classroom. The debate about educating students with disabilities was no longer "should we or shouldn't we" but instead "how much and in what ways" (Osgood, 2008, p. 117).

In the four decades since the passage of IDEA, the challenge of educational placement for students with EBD has continued (Osgood, 2005, 2008). Mental health professionals estimate that as many as 6% to 9% of all children and adolescents have EBD, but within the school system less than 2% are identified and receiving services (Kauffman & Landrum, 2013). Students with EBD present a complicated set of behaviors and unique challenges for general education teachers to manage in an inclusive classroom (Temple-Harvey & Vannest, 2010). The overwhelming majority of students with EBD are males. This population of students often exhibits disruptive maladaptive behaviors and can be verbally aggressive, abusive, and noncompliant. In the classroom, students with EBD often have difficulty establishing and maintaining social relationships, arousing negative feelings in others, and alienating classmates and teachers by contributing to negative school outcomes (Kauffman & Landrum, 2013). Academically, when compared to students with other high-incidence disabilities, these students have lower test scores and graduation rates as well as higher rates of retention, course failure, and overall school failure (U. S. Department of Education, 2012).

Educational Placement of Students with EBD

Students with EBD are placed in separate classrooms or schools more than students in any other disability category (Landrum, Katsiyannis, & Archwamety, 2004). Decisions of the U.S. Court of Appeals, over three years (1994-1997) in three separate cases and in three different circuits, framed a set of principles that schools must use in determining LRE. The four themes that guide that framework are "individualization", "a presumptive right to an integrated education", "appropriateness", and "options" (Yell, 1998, p. 73). Despite the courts' rulings for students with disabilities, the discussion continues on what constitutes LRE for students with EBD.

Proponents on both sides of this issue have strong arguments for their positions. Some educators support segregation (e.g., Carpenter & Bovair, 1996; Crockett & Kauffman, 1999; Milloy, 2001; Yell, 1998). They believe that certain students with EBD, or other disabilities, are best served in a separate special education setting where their unique academic, social-emotional, and behavioral needs can be met. Others strongly support the full inclusion of all students with disabilities, citing the legal mandate in IDEA (1975). Those who hold this inclusive view (e.g., Lipsky & Gartner, 1987; Stainback & Stainback, 1991; Wang, Reynolds, & Walberg, 1988) believe that the LRE must be the general education classroom; that separate classrooms or schools, even pull out programs or resource rooms, are unfair or illegal simply because they are separate. Despite opposing viewpoints regarding placement for students with EBD, general education may be less than what is required because of this population's unique behavioral or emotional needs.

The behavioral and educational needs of some students with EBD often make them more likely to be separated from the general education population in order to receive the intensive services and support they require (Jackson & Neel, 2006; Maccini et al., 2008; Wagner et al., 2006). When students with EBD transition to middle school, the separate classroom can present a challenging placement dilemma for special education staffing specialists. The NCLB mandate to have highly qualified, content certified teachers (*20 U.S.C. § 9101*, 2002), coupled with a chronic shortage of mathematics and special educators, make it difficult for schools and districts to appropriately staff these classrooms (Boe & Cook, 2006; McLeskey, Tyler, & Flippin, 2004). The proliferation of online learning options in many states has presented a timely solution to the placement dilemma making standards based, high quality instruction by content certified teachers available to students anywhere (Watson, Murin, Vashaw, Gemin, & Rapp, 2013). Inclusive online instruction, delivered by content certified teachers, to students working in self-contained classrooms, under the supervision of special education teachers, is a form of blended learning that may have the potential to meet all the requirements of IDEA, NCLB and the LRE directive, as well as meeting the needs of students with EBD.

Potentially a range of problems that adversely affect academic and behavioral performance of students with EBD could be addressed in a blended learning environment. A separate physical setting staffed with special educators often provides the needed support for internalizing and externalizing behaviors associated with students with EBD as well as the necessary academic supports and interventions required to improve performance and minimize negative experiences in school; negative school experiences lead to increased drop-out rates for this population of students (Reid, Gonzalez, Nordness, Trout, & Epstein, 2004). Dropout is the most common method of school exit for students with EBD (Landrum et al., 2004). Students with disabilities in general dropout at twice the rate of their peers (U. S. Department of Education, 2011), with the highest rates among all disability categories attributed to students with EBD (Blackorby & Wagner, 1996). Unacceptable high school dropout rates result in a population of adolescents with EBD who have the highest rates of arrest, lowest wages, and least access to post-secondary education. Overall, these students are most likely to never earn a high school diploma or GED, which severely limits their post-secondary options and long-term outcomes.

Researchers have established a correlation between the successful completion of algebra and high school graduation (Impecoven-Lind & Foegen, 2010; Reid et al., 2004). There is evidence that the risk of algebra failure can be predicted in elementary school by identifying students with deficits in division and fraction concepts (Siegler et al., 2012). The early identification of these students may present an opportunity to correct deficits and alter future mathematics outcomes. Mathematics proficiency is critical for students with EBD in the elementary and middle grades because it prepares them for algebra, a graduation gatekeeper (Impecoven-Lind & Foegen, 2010) and graduation requirement in 45 states (Steele & Steele, 2003). Failure to successfully complete algebra increases the probability that students with EBD will drop out of high school and be at risk for a lifetime of low wages, unemployment, and incarceration with little access to post-secondary education (Blackorby & Wagner, 1996; Cavanaugh et al., 2005; Watson & Gemin, 2008).

Mathematics

Academically, mathematics difficulties are a significant part of the reason that many students with EBD fail to thrive in school (Blackorby & Wagner, 1996; Nelson, Benner, Lane, & Smith, 2004; Reid et al., 2004). Those difficulties begin early, are often unrecognized, and create not only academic concerns but behavioral and dispositional ones as well (Hodge, Riccomini, Buford, & Herbst, 2006; Templeton, Neel, & Blood, 2008). Currently compounding the existing mathematics deficits and difficulties for students with EBD is a nationwide effort to increase rigor in mathematics through the Common Core State Standards (CCSS) and their central role (Tienken, 2011) in the federal government's efforts to engage the states in a *Race to the Top*.

In the CCSS, algebra concepts are a critical focus of the early grades mathematics standards with the goal of preparing students to take Algebra in 8th grade (NGA Center/CCSSO, 2012c). With the successful completion of algebra serving as a key predictor of student graduation (Silver, Saunders, & Zarate, 2008), this content area is critical for the overall academic success of students with EBD. Common Core State Standards and increasingly more rigorous graduation requirements, that include at least algebra and its associated end of course (EOC) exam, have become a central feature of education policy in 45 states (NGA Center/CCSSO, 2012b). State leaders continue to look at ways to increase academic standards for graduation, with 22 states now requiring four credits or four Carnegie units in mathematics to earn a standard high school diploma (Zinth, 2012).

In Florida last spring, 81% of 9th grade students with EBD failed to pass the Algebra I EOC exam (Florida Department of Education, 2013b). That failing percentage increases dramatically for students taking Algebra I in 10th grade or later (Florida Department of Education, 2012). Stumbling at this critical milestone in high school is particularly problematic as algebra is considered a gateway to higher mathematics, all further learning, and post-secondary success for all students, including students with EBD (Blackorby & Wagner, 1996; Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2005). Success in algebra, as well as secondary mathematics, is contingent on adequate preparation in elementary school (Wu, 2009).

The primary goal of elementary mathematics is to prepare students with a strong foundation for higher mathematics, including algebra (Wu, 2009). The integrated theory of number development (Siegler, Thompson, & Schneider, 2011) states that conceptual understanding of numeration requires an awareness that all real numbers have magnitudes that can be ordered on a number line. This theory promotes the conceptual understanding of fractions to a critical prerequisite for advanced mathematics concepts (Siegler, Thompson, & Schneider).

Elementary teachers, as well as secondary mathematics instructors, report the difficulties they have teaching fraction concepts (Siegler et al., 2011); some of those difficulties may be the result of their own deficits in conceptual fraction understanding (Ball, Hill, & Bass, 2005; Hill, Schilling, & Ball, 2004; Ma, 1999; Siegler et al., 2011). Teaching mathematics, particularly abstract concepts like fractions and algebra, require depth of conceptual knowledge and insight into student errors and misconceptions (Hill et al., 2004). Fractions are the first abstract concepts introduced in elementary mathematics that require students to construct mental models to compensate for the absence of physical units (like fingers) that can be touched and counted. The abstract nature of fractions and the importance of understanding this sub-concept of rational numbers make it a critical prerequisite for algebra (Wu, 2009).

Evidence Based Practices

The amount of research on effective mathematics instruction, interventions, or strategies for students with disabilities is limited. Even more limited are studies that target the unique needs of students with EBD (Hodge et al., 2006). Empirical research utilizing technology in mathematics for students with disabilities is equally sparse. Researchers who have focused on online learning in general are just beginning to proliferate, but research related to students with disabilities in online environments is still in its infancy (Cavanaugh, Barbour, & Clark, 2009; Coy, Marino, & Serianni, in review; Straub, 2012; Vasquez & Straub, 2012).

A review of literature by Vasquez and Straub (2012) was conducted to identify empirically validated online instructional practices designed to support students with disabilities. Of the seven identified studies, none were related to mathematics and only one involved students with EBD as participants. A more recent literature review conducted in 2012 by Coy and colleagues, found 15 articles related to K-12 students with learning disabilities participating in some form of online learning. Eight of the 15 studies involved middle school students but none of those students were identified with EBD (Coy et al., in review).

A dearth of research exists related to online instruction in general, but particularly for students at risk; also needed is research on students with EBD learning in online environments (Black, Ferdig, & DiPietro, 2008; Cavanaugh et al., 2009; Vasquez & Straub, 2012). Researchers in online environments need to evaluate if and how face-to-face evidence based practices transition to online and blended learning environments for students with disabilities. Research in this area may contribute to the development of an inventory of evidence based practices for use by teachers and teacher preparation programs especially for the critical content area of mathematics and the often overlooked population of students with EBD (Cavanaugh et al., 2009; Vasquez & Straub, 2012).

By middle school, the large majority of students with EBD have multiple procedural and conceptual learning gaps in mathematics (Lane, Wehby, Little, & Cooley, 2005; Reid et al., 2004). Grade-level content assumes that students have prerequisite knowledge and skills making it difficult for students with EBD to successfully learn new grade-level content (Hodge et al., 2006). That challenge is particularly evident when students are engaged in online learning and requires teachers in blended classrooms to be prepared to design supplemental instruction and interventions to provide deficit skill remediation concurrently with grade-level online instruction.

Virtual Manipulatives

Researchers have documented the use of virtual manipulatives (VM) for over 20 years as an effective practice in mathematics instruction for students, including students with disabilities (Bouck & Flanagan, 2009; Durmus & Karakırık, 2006; Moyer, Niezgoda, & Stanley, 2005; Moyer, Salkind, & Bolyard, 2008; Moyer-Packenham & Suh, 2012; Moyer-Packenham, Westenskow, & Salkind, 2013; Ozgun-Koca & Edwards, 2011; Suh, Moyer, & Heo, 2005). When compared with other instructional interventions, the use of VM produced moderate effects in the domains of numbers and operations, fractions, geometry, measurement, and integers (Moyer-Packenham, Westenskow, et al., 2013). Virtual manipulatives are tools that have the potential to support students with EBD taking online mathematics in blended settings who need conceptual and procedural remediation to successfully interact with grade-level content.

The Problem

Mathematics performance data for students with EBD are not available because the National Assessment of Educational Progress (NAEP) data are not disaggregated by disability category (2013). However authors of the *2013 Nation's Report Card* reveal that 65% of all 8th grade students did not achieve proficiency in mathematics on the 2012 NAEP additionally, 92%

of students with disabilities failed to reach proficiency. The lack of evidence based practices for teaching mathematics to secondary students with EBD in any type of classroom (brick and mortar, fully online, blended, or special education) is a pressing issue in education (Cavanaugh et al., 2009; Coy et al., in review; Gunter & Denny, 1998). Researchers and practitioners note that students with EBD are being placed into online instructional environments with little support and without adequate preparation for students or teachers (COLSD, 2012). Providing teachers with evidence based instructional strategies and tools, including digital solutions, has the potential to support mathematics learning for students with EBD in blended learning environments (COLSD, 2012; Watson & Gemin, 2008).

Purpose and Significance

The primary purpose of this research study was to investigate the effects of a remedial, conceptual fraction intervention, using lessons from the Rational Number Project (RNP) and VM, on fraction addition fluency and equivalent fraction knowledge for middle school students with EBD learning online in self-contained special education classrooms.

Research Questions

The researcher in this study evaluated the effect of an intervention package designed to improve conceptual knowledge in fractions comprised of the RNP's *Initial Fraction Idea* lessons (Cramer et al., 2009) and fraction virtual manipulatives (RNP/VM). The researcher sought to answer these questions:

1. To what extent does RNP/VM increase group median fluency on a five-minute fraction addition assessment for students with EBD?
2. Does RNP/VM increase equivalent fraction knowledge as evidenced by the results of a pre, post, and delayed post-test administration of the EFT?
    a. To what extent do mean scores increase from pre to post administration of the 20-item EFT, for groups of students with EBD?
    b. Given an increase in mean group scores from 2.a., to what extent do those gains persist after the end of the intervention?
3. How do the students with EBD and their respective teacher and paraprofessional perceive the goals, procedures, and outcomes of RNP/VM?

Research Design

The researcher utilized an intervention package, RNP/VM, and gathered data from assessments for this two-part study. Part one of the study utilized a single subject design with multiple baselines across participants clustered in small groups. The single subject design allowed the researcher to collect data on the intervention delivered to three groups of participants selected from three separate classes (Red, n = 3; White, n = 3; Blue, n = 3). The researcher evaluated group median computational fluency from a daily Fraction Addition Assessment (FAA) across the baseline and intervention phases of the study. Part two of the study employed a non-experimental repeated measures design that allowed the researcher to evaluate mean Equivalent Fraction Test (EFT) scores for the three groups of participants (n =3) as well as the whole group (N = 9). Data were gathered from the pre, post, and delayed post-test administration of the EFT. To add context to the quantitative analyses the researcher maintained anecdotal records of observations of student and teacher behaviors as well as verbal communications. Additionally, participants' perceptions of intervention goals, instruction, VMs, and their own performance were gathered through informal questions and a Social Validity survey constructed and delivered in Qualtrics.

Assumptions

The assumptions of the research in this single subject design included independence across the legs and a similarity in the way legs are related to the independent variable.

Independent Variables

The conceptual fraction intervention utilized in this study consisted of the RNP *Initial Fraction Ideas* (IFI; Cramer et al., 2009), lessons 1-6, along with three VMs developed through RNP funding (http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html). Throughout this manuscript the intervention package will be referred to as RNP/VM.

Dependent Measures

Group median fluency was the primary dependent variable for the single subject portion of this study. Fluency, for the purposes of this study, was operationally defined as the number of correct responses on the FAA divided by time. Data were gathered from the daily administration of the 5-minute, 20-question FAA during the baseline and intervention phases of the study. The maximum possible raw score on the FAA was 20, representing a maximum fluency of 4.0 problems/minute (p/m; fluency = # correct/5 minutes).

The group mean score on the EFT was the dependent variable for the non-experimental repeated measures portion of this study. The EFT contained 20 questions, four with two-part answers and one with a three-part answer, with a maximum possible raw score of 25 points.

Participants

A purposive sample was used to select participants from three classes of middle school students with EBD who were physically located in self-contained special education classrooms and simultaneously enrolled in inclusive online courses. Students rotated into the self-contained classroom daily for a 90-minute math and science block. The classroom was staffed with one special education teacher and one paraprofessional who taught and supported all three of the classes that were a part of this study. All the students were enrolled in an online mathematics course appropriate for their grade level and accessed their course from computers located in the special education classroom. The special education teacher was highly qualified in mathematics, by district standards, and served as teacher of record in the Flex model of blended learning (Staker & Horn, 2012) for her students enrolled in online mathematics.

Data Analysis

A multiple baseline design was used to evaluate the effect of RNP/VM on the dependent measure across groups of participants (Gast, 2010). Data were collected and used to calculate median fluency for each group of participants. Daily medians were graphed to enable the researcher to perform a systematic visual analysis across the phases of the study. Results of visual analysis including medians, range, stability, trend, percentage of non-overlapping data points (PND; Scruggs, Mastropieri, & Casto, 1987), and Tau-U (Parker, Vannest, Davis, & Sauber, 2011) are discussed.

Definitions

The following definitions are provided to provide clarity as they relate to this study.

Blended Learning

Blended learning is "a formal education program in which a student learns at least in part through online delivery of content and instruction with some element of student control over time, place, path, and/or pace and at least in part at a supervised brick-and-mortar location away from home" (Staker & Horn, 2012, p. 3).

Emotional or Behavioral Disorders (EBD)

Emotional disturbance is defined in the IDEA (1997) regulations as:

> A condition exhibiting one or more of the following characteristics over a long period of time and to a marked degree that adversely affects a child's educational performance: (a) an inability to learn that cannot be explained by intellectual, sensory, or health factors; (b) an inability to build or maintain satisfactory interpersonal relationship with peers and teachers; (c) inappropriate types of behavior or feelings under normal circumstances; (d) a pervasive mood of unhappiness or depression; (e) a tendency to develop physical symptoms or fears associated with personal or school problems. The term includes

> schizophrenia. The term does not apply to children who are socially maladjusted, unless it is determined that they have an emotional disturbance (*CFR* §300.7(a)(9)).

The National Special Education and Mental Health Coalition adopted the term emotional or behavioral disorder in 1987 to recognize that students labeled with emotional disturbance may have disorders of emotion or behavior, or both (Kauffman & Landrum, 2013). As a result EBD has become a commonly used term and will be the one used throughout this paper.

Least Restrictive Environment (LRE)

Identified in IDEA 2004, LRE requires that:

> "To the maximum extent appropriate, children with disabilities, including children in public or private institutions or other care facilities, are educated with children who are not disabled, and special classes, separate schooling, or other removal of children with disabilities from the regular educational environment occurs only when the nature or severity of the disability of a child is such that education in regular classes with the use of supplementary aids and services cannot be achieved satisfactorily" (Sec. 612[a][5]).

Online Learning

Online learning is "education in which instruction and content are delivered primarily over the Internet" (International Association for K-12 Online Learning, 2011, p. 7).

Rational Number Project (RNP)

RNP is a collaborative multi-university research project, funded by the National Science Foundation. Investigators in the project conducted research on how children learn fractions, decimals, ratios and proportionality since 1979.

(http://www.cehd.umn.edu/ci/rationalnumberproject/)

RNP/VM

The pseudonym for the intervention package under investigation in this research study is RNP/VM. The intervention package consists of *Initial Fraction Idea* lessons from the Rational Number Project along with three web-based virtual manipulatives developed through project funding. Details on the lessons and links for the virtual manipulatives are available on the RNP website (http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html).

Virtual Manipulatives

Virtual manipulatives are "interactive, web-based visual representations of dynamic objects that present opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373).

CHAPTER TWO: LITERATURE REVIEW

Overview

Students with emotional or behavioral disorders (EBD) have provided unique challenges to classroom teachers and special educators since compulsory education laws were first enacted in Massachusetts giving every child an opportunity to attend public school (*Mass. Gen. Laws ch. 240 § 1-5*, 1852). Despite the potential challenges for this population, by 1918 every state had passed legislation ensuring all children could attend school (Katz, 1976). These compulsory education laws brought a diverse group of students with varying experiences, cultures, and abilities into schools and classrooms (Cubberley, 1924; Gardiner-Chase, 1904). Although the intention of legislators regarding compulsory education in each state was to serve all students, students with EBD were often not expected to attend school, were quickly expelled or in some instances tolerated, and received limited educational opportunities (Groszmann, 1922).

Despite the intent of that early legislation, it was not until 1975 that students considered behaviorally or emotionally challenged were given the right to be educated in public schools (*EHA,* 1975). Since the passage of the *Education for All Handicapped Children Act of 1975* (EHA), the identification, placement, and provision of special education services for students with EBD has been the focus of collaborative efforts across the disciplines of education, medicine, and psychology (Becker et al., 2010; Bradley et al., 2008; Wagner et al., 2006). The path of services and research continues to be one of challenges to equal access, opportunity, and positive educational outcomes for this population of students (Bradley, Henderson, & Monfore, 2004); a promise by legislators with the passage and continuing revisions of the *Individuals with Disabilities Education Act* (IDEA; 1975, 1997, 1990, 2004).

Equal access and educational outcomes for students with EBD are the foci of this literature review. The researcher provides a synopsis of current and historical literature related to the legislation and educational placement, including the potential for online or blending learning, to support the academic and behavioral needs of students with EBD. The content of the research and literature in this chapter is organized around a conceptual framework that connects high school graduation and post-secondary outcomes to success in algebra and a conceptual understanding of fractions. The researcher provides an analysis of the current research related to conceptual and procedural understanding of fractions noting an absence of studies that address students with EBD. The literature review culminates with a synopsis of the Rational Number Project (RNP), the project's book of *Initial Fraction Ideas* (IFI), the use of virtual manipulatives (VM) in the concrete-representational-abstract (CRA) sequence of instruction, and the potential of these tools for students with EBD to support their learning in a blended environment.

Educational Placement of Students with EBD

History

Educational support services for students with disabilities were unnecessary until students who had difficulty learning or behaving began to enter school. Compulsory education laws added a sizeable number of students who could not learn to the "one size fits all" public school structure of the 19th century (Mann, 1849). In the larger cities, these laws compelled immigrants with limited English proficiency, along with children who had mental, physical, and psychological disabilities into public school classrooms. Teachers "who received no training" struggled to provide instruction, "some were good and some were not" (Gardiner-Chase, 1904, para. 35). The laws worked as intended in the larger cities, moving the "truant" and "incorrigible" from the streets and into public school classrooms (Cubberley, 1924; Gardiner-Chase, 1904). Boston, a city that was taking the lead in creating structure and standardizing practices in public education (Osgood, 2008), created the first segregated settings (schools for special instruction) that would address the individual needs of students (Boston City Council, 1857). Over the next half-century, Boston established separate schools and classrooms for students with a variety of special needs including programs for students who were hearing impaired or deaf, "mentally deficient," vision impaired, and those who had speech disorders, chronic illnesses, giftedness, or were disciplinary problems (Ellis, 1933; Osgood). Other major cities quickly followed Boston's lead creating their own special classrooms for students who were unable or unwilling to learn in the standard public school setting (Gardiner-Chase, 1904; Osgood).

The National Education Association (NEA), acknowledging the growing trend toward dedicated separate classrooms for students with disabilities, formed a Department of Special Education and introduced the term *special education* at their 1902 annual meeting (Bell, 1902). During this early era of separate settings, special education services primarily came in the form of a special teacher charged with the task of educating a large number of students with a wide variety of needs (Farrell, 1914; Wallin, 1917). Efforts by public school officials to move the problem of disruptive students, unable or unwilling to learn, from regular to special classes resulted in rooms and schools that were described by critics as "dumping grounds for all kinds of non-conforming pupils" (Groszmann, 1922, p. 102). The challenges faced by the special teachers in these ungraded classrooms illuminated the need for better teacher preparation and disability-specific separation of students to better meet the diverse needs (Gardiner-Chase, 1904; Groszmann).

During the next several decades, the public schools sharpened their identification and placement methods through the use of tools to identify students who struggled academically or behaviorally. The advent of standardized intelligence tests like the Stanford-Binet provided, what was thought of at the time, an objective measurement to identify disabilities (Terman, 1916). While ungraded special classrooms were still the norm, schools were moving to further segregate these populations both by ability and disability. Results of this segregation were still mixed for both students and teachers in the special classes. However, the overall impact to the school system was perceived to be so positive that the model spread nationwide (Farrell, 1914). By the 1920's establishing special programs for students with a variety of special needs was the norm, further segregating students by their specific disabilities (Ellis, 1933). By the 1930's, special education in segregated settings was part of the national public school model and was having an effect on the lives of hundreds of thousands of students with disabilities (Osgood, 2008; Stullken, 1940; Wallin, 1924). The special classroom became the primary educational setting for children with mental disabilities and emotional disorders and remained that way until the 1980's (Osgood, 2008).

The tenure of two U.S. Presidents, Franklin Roosevelt and John Kennedy, did much to advance the cause of individuals with disabilities through their efforts to raise public awareness and support social and educational initiatives (Osgood, 2005). While Kennedy's days in the White House were abruptly terminated by his assassination, the legislative ideas for the education and rights of individuals with disabilities, were conceived during his short tenure

(Osgood, 2008). “A child with disabilities born in the United Sates in 1960 came into a nation that was finally claiming to have a significant interest in her or his welfare” (Osgood, 2008, p.99).

Emerging from Kennedy’s ideas were a dozen pieces of legislation enacted between 1965 and 2004 that brought special education from the sidelines to center stage as a civil rights issue protected by federal law. Through the *Elementary and Secondary Education Act of 1965 (ESEA)*, the federal coffers were opened to begin the flow of funds to support the education of students with disabilities. In 1970, reformers began the push for an expansion of services for students with disabilities, including a call by Madeline Will, Assistant Secretary for the Office of Special Education Services, to educate this group of students alongside their same-age peers to the greatest extent possible when it was for their benefit (Will, 1986) .

The passage of the *Education for All Handicapped Children Act of 1975* mandated a free and appropriate public education (FAPE) in the least restrictive environment (LRE), for all students with disabilities. The original legislation, along with its related amendments and reauthorizations, including its renaming in 1990 as the *Individuals with Disabilities Education Act* (IDEA), set the stage for the inclusion of students with disabilities, including EBD, in general education classrooms around the nation (1975, 1990, 1997, 2004). The 1975 legislation provided a legal, but vague (Kavale et al., 2005) definition of emotional disturbance and omitted a definition for LRE, suggesting that the term was intended as a guiding principle rather than a location. This lack of specificity left special educators in disagreement as to how to apply the principle of LRE (Crockett & Kauffman, 2013; Kauffman, Bantz, & McCullough, 2002). The final protection afforded to students with EBD by the IDEA legislation was the right to receive instruction from highly qualified, content certified teachers, using evidence-based instructional practices to ensure that they made adequate progress in the general education curriculum (*Individuals with Disabilities Education Improvement Act*, 2004). Positive behavioral interventions and supports were added in the 1997 amendments to IDEA. The latest revision and amendments to IDEA in 2004 emphasized functional assessment and positive interventions in an attempt to prevent problem behaviors. Today the statutes of IDEA continue to define how students with EBD are educated in U. S. classrooms.

Current Climate

Despite the purported protections of IDEA, students with EBD continue to be the population of students most often staffed into self-contained special education classrooms in an attempt to provide the behavioral, social/emotional, and academic supports they need. The Office of Special Education Programs (OSEP) reported statistics on the percentage of time students with EBD spent in regular classes in their 2009 annual report to Congress. The graphic in Figure 1 provides a pictorial representation of those percentages (OSEP, 2009). Most students with EBD spent some time in regular classes, which included time spent in lunch, recess, and study periods.

Teachers who are not prepared to work with students with EBD in an array of settings can unwittingly contribute to the intensification of behaviors, resulting in significant behavioral consequences and an increase in academic deficits (Espin & Yell, 1994; Kauffman & Landrum, 2013; Oliver & Reschly, 2010; Regan, 2009). Since this population of students often presents a complicated set of behaviors, teachers in the general education classroom may be challenged by those behaviors without adequate preparation (Oliver & Reschly, 2010; Reid et al., 2004; Vannest, Temple-Harvey, & Mason, 2009).

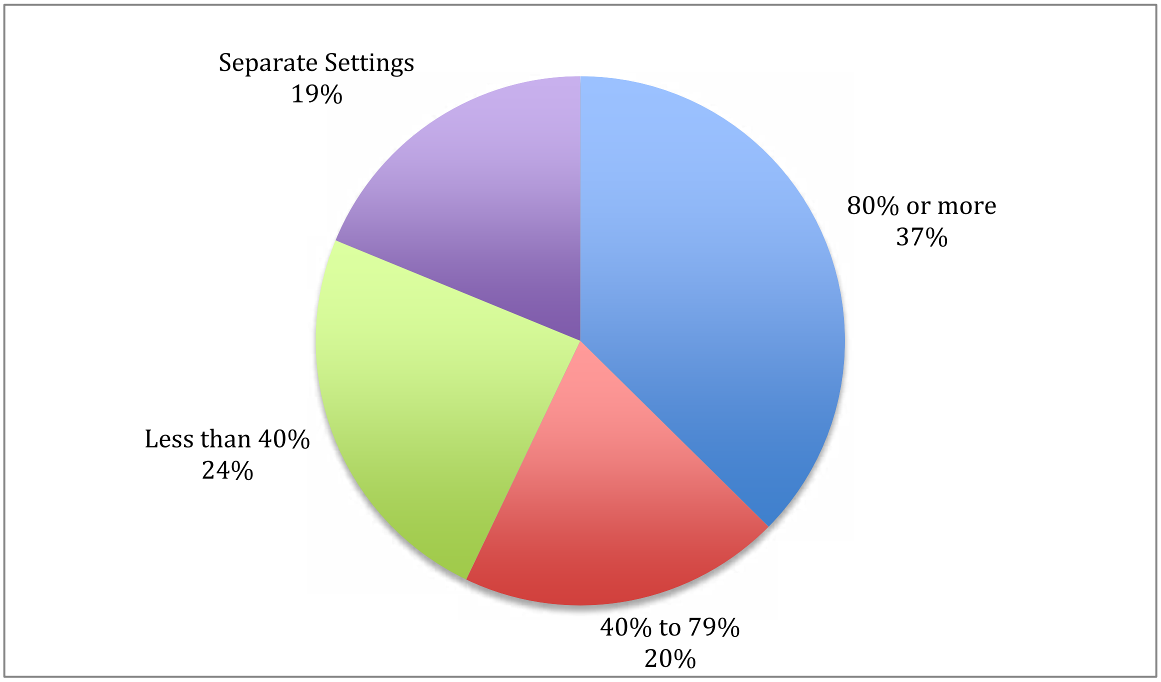

Figure 1: Time spent in regular classes by students with EBD, Fall 2007.

To gain a national perspective on teacher preparation and academic programs for students with EBD, Wagner and colleagues (2006) analyzed the Special Education Elementary Longitudinal Study (SEELS) and the National Longitudinal Transition Study-2 (NLTS2) related to outcomes for this population. The researchers documented that general education teachers, who reported teaching students with disabilities, averaged over ten years of experience, few reported that they had received inservice training regarding the needs of students with EBD (17% of elementary teachers, 21% of middle school teachers, 6% of high school teachers). In addition, the researchers reported that only 23% of elementary school students and 30% of middle school students with EBD had teachers who "strongly agreed" that they were sufficiently prepared to teach students with disabilities (Wagner et al., 2006).

With regard to academic programs, students with EBD are being served most often in separate settings (Landrum et al., 2004) to receive behavioral and instructional supports and interventions designed to improve academic performance such as (a) content enhancement in a structured learning environment (Boudah, Lenz, Bulgren, Schumaker, & Deshler, 2000), (b) independent learning strategies (Deshler & Schumaker, 1986), and (c) teachers with a strong repertoire of behavior-management skills (Landrum, Tankersley, & Kauffman, 2003; Walker et al., 1998). The self-contained special education classroom seems to be the choice that many schools make in an effort to deliver persistent, structured, and intensive instruction with precision, while monitoring and adapting to meet the needs of students with EBD (Kauffman, Bantz, & McCullough, 2002; Landrum, Tankersley, & Kauffman, 2003).

Intensive instruction and intervention delivered in the separate setting is intended to support academic and behavioral success for students with EBD; but researchers' analyses of data from studies conducted in this setting have not revealed improved academic or behavioral outcomes for this population of students (Lane et al., 2005; Wehby, Lane, & Falk, 2003). For example, Lane and colleagues (2005) conducted a study to determine if students with EBD receiving services in a self-contained school or classrooms were benefiting from their placement in a more restrictive setting. Their sample students (n=60) included 26 in self-contained classrooms and 24 in self-contained schools. Reported demographics of the students included: gender (M-41, F-19), ethnicity (Caucasian-13, African American-45, Hispanic-2), primary disability label (ED-42, LD- 8, OHI/ADHD-6, OHI-2, MMR-1, L-1), grade level (elementary-

33, secondary-27), age (*M*=10.87, *SD*=1.76), IQ (*M*=81.08, *SD*=22.40). The researchers reported little academic gain with no significant difference in the academic progress of students in the two settings (with the exception of a decrease in writing scores for students in the self-contained school as compared to the students in the self-contained classrooms). The reported results revealed no academic benefit, which is an indication that the requirements for LRE had not been met in the separate setting (Lane et al.). Despite efforts toward inclusion, students with EBD continue to be separated from the general education population at a higher rate than students in all other categories of disabilities (Landrum et al., 2004).

This separate setting for students with EBD presents a unique challenge at the secondary level. NCLB requires that special education teachers be highly qualified, but they often lack content certification in core subject areas like mathematics (Simpson, Lacava, & Graner, 2004). Combining secondary special education teachers without content certification with students who are unable to receive their instruction in the general education setting often results in students receiving mathematics instruction from non-certified teachers (Simpson et al.). Jackson and Neel (2006) found students in self-contained special education classrooms are not likely to receive high quality, standards-based instruction in mathematics.

Students with EBD require and deserve academic supports and interventions as well as accommodations from highly qualified teachers (Hodge, Riccomini, Buford, & Herbst, 2006; Maccini et al., 2008; Templeton, Neel, & Blood, 2008). In the NLTS-2 Wave 2 Teacher Survey (2003), teachers reported that they received no in-service training on the needs of their students with EBD. However, teachers reported receiving the following supports to meet the needs of their students with EBD: co-teaching 17%, consultation 53%, aides 22%, reduced class size 10%, and information about student needs 64%.

Just as teachers need support, so do students. The National Longitudinal Transition Survey-Wave 2 (NLTS-2) Student School Program Survey (2003) provided information on the level of support received by students with EBD in all content areas. Supports for students with EBD included extra time for tests (62%), test questions read aloud (21%), modified tests (35%), alternative tests (28%), modified grading standards (22%), slower-paced instruction (25%), more time for assignments (57%), modified or alternative assignments (19%), and more frequent feedback (48%). The researchers found that more than half of students with EBD placed in general education classes accessed the general education curriculum with some level of modification. More than half of students with EBD in separate special education classes also used subject-area general education curriculum, without modification (7%), with some modification (35%), and with substantial modification (17%). A specialized individualized curriculum (as required by IDEA legislation) was used by 24% of students with EBD in separate classes, but 17% of the students in that setting there was no subject-area curriculum at all. Figure 2 depicts the use of general education curriculum (GEC) and individualized curriculum for students with EBD in separate special education classes.

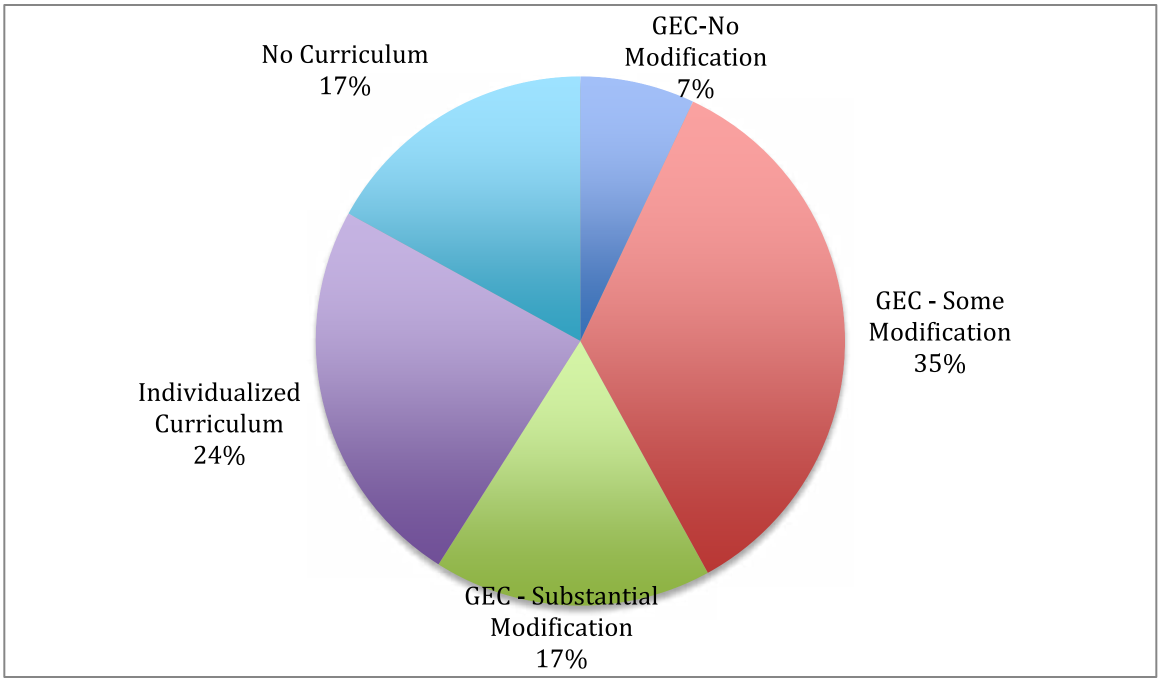

Figure 2: Curriculum use by students with EBD in separate special education classes.

Conceptual Framework: Connecting Mathematics to Post-Secondary Outcomes

"Knowledge of mathematics is crucial to educational and financial success in contemporary society" (Siegler et al., 2012, p. 1). Society often measures financial success by earning potential, which has a high correlation with successful high school completion (Blackorby & Wagner, 1996). Conversely, students who drop out of high school experience a number of negative post-secondary outcomes and societal consequences that impact their quality of life such as unemployment, poverty, incarceration, substance abuse, and homelessness (Blackorby & Wagner).

For students with EBD, this societal balancing act often hinges upon mastery of mathematics, specifically algebra (Reid et al., 2004). It is algebra that researchers have identified as the tipping point for high school graduation (Impecoven-Lind & Foegen, 2010).

Siegler and colleagues (2012) established that success in algebra could be predicted by students' knowledge of fractions and division, identifying those concepts as tipping points for success in algebra and all high school mathematics. The conceptual framework depicted in Figure 3 represents the delicate balance of these tipping points, the overarching educational factors that can lead to positive post school experiences and financial independence for all students, including students with EBD. These critical factors should be the focus of the instructional planning and support for students with EBD.

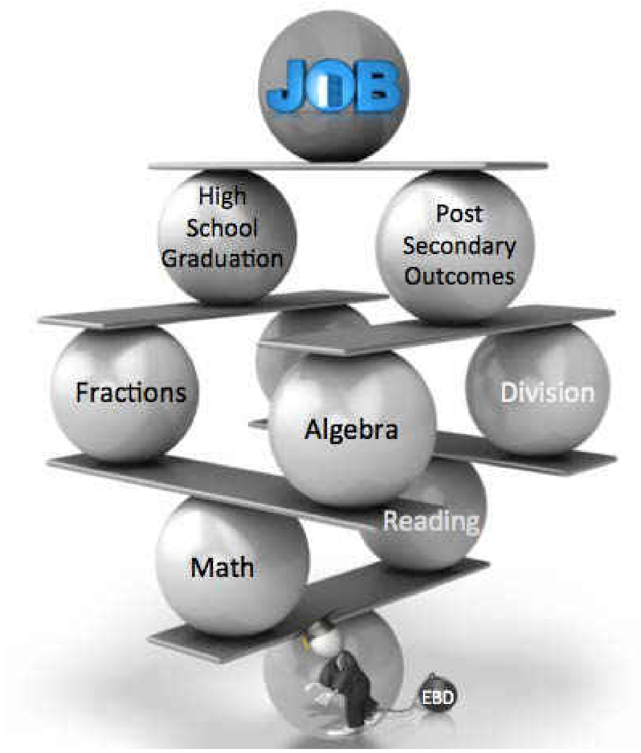

Figure *3:* Academic tipping points that influence outcomes for students with EBD.

The image in Figure 3 depicts a conceptual framework based on the work of Chan, Leu, and Chen (2007) that illustrates the tenuous balance between the fundamental skills of mathematics and reading and the critical abstract concepts of fractions and algebra (Chan et al., 2007; Deshler & Schumaker, 1986; Deshler, 2005; Deshler et al., 2001; Mong & Mong, 2011; Rakes, Valentine, McGatha, & Ronau, 2010; Siegler et al., 2012, 2011; Suh et al., 2005; Suh & Moyer-Packenham, 2007; Wu, 2009). The graphic also highlights the delicate equilibrium of high school graduation and post-secondary outcomes, such as employment and career opportunities, which ultimately result in financial independence for students with EBD. The image and factors reflected in this tipping point model provide a structure for this review of literature and justification for this proposed research study.

Improving Post-secondary Outcomes

Achievement in mathematics is crucial to students' well-balanced K-12 education, participation as a productive community member, attainment of personal goals, and contribution to the nation's economy and competitiveness (Cavanaugh et al., 2005). The aggregate loss to the U.S. economy resulting from failure to complete high school is estimated at 154 billion dollars a year (Alliance for Excellent Education, 2011). Each year over three million students drop out of high school, almost half are students with EBD (Blackorby & Wagner, 1996) Improving post-secondary outcomes, college and career readiness, rose to the top of the national education agenda in 2002 with the enactment of NCLB and continues to be the focus of Obama's education agenda as evidenced by the *Race to the Top* competition (GAO, 2011).

In an effort to improve high school graduation outcomes, mathematics has been targeted as a critical content area for all students, particularly for students with EBD. The push for higher standards, better performance outcomes, and mastery of mathematics are key components of both the NCLB legislation and the Common Core State Standards (CCSS) initiative.

Common Core State Standards

At the time of this study CCSS has been adopted by forty-five states requiring all students to meet the more rigorous mathematics standards of the CCSS framework. The framework provides a continuum of grade-level standards for mathematical content that requires and integrates fluency in foundational skills such as numeration, operations, and algorithms. Foundational skills vary based on grade-level and standard, but these same skills are revisited as stepping-stones to higher-level mathematics. The progressive complexity of the standards makes recall and fluency of the foundational skills critical to allow students to address more complex problems, to advance through the standards, and to demonstrate the required competencies on CCSS grade-level assessments. Students with EBD need to have access to the general education mathematics curriculum with supports for their unique academic, behavioral, and emotional needs to ensure they do not fail to master critical concepts and move on with gaps in the CCSS sequence of instruction (NGA Center/CCSSO, 2013; Powell, Fuchs, & Fuchs, 2013).

In addition to rigorous annual grade-level mathematics standards that must be mastered by students, the CCSS also requires teachers to support students in mastering eight *Standards for Mathematical Practice.* These standards required teachers to push for coordinated, sequenced, and linked mastery of standards across grade levels. These standards reveal the potential difficulties students with EBD may have in mathematics if they are not engaged in learning that will promote skill mastery through effective instruction from high quality teachers. The *Standards for Mathematical Practice* include:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reason of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (NGA Center/CCSSO, 2012, para. 2-9).

The shift in expectations for both students and teachers as a result of CCSS is important to both understand and monitor. Success for students is dependent on continuity of instruction and building mathematics concepts across grade-levels (Powell et al., 2013). For students with EBD, long term success in mathematics will require consist exposure across all grades to the general education grade-level content aligned to CCSS along with instruction, grounded in evidence based practices, delivered by highly qualified mathematics teachers who are prepared to address the unique behavioral needs of this population of students (NGA Center/CCSSO, 2013; Powell et al.). Interruption of the progressive development of critical mathematics skills by removing students with EBD from general education classrooms to separate settings can exacerbate the mathematics difficulties of these students and contribute to their failure in school (Gagnon & Bottge, 2006; K. L. Lane et al., 2005).

CCSS Assessments

Mastery of mathematical concepts is not the only potential challenge with the CCSS for students with EBD; an equal challenge is assessing this population's level of learning. The new sets of assessments for students in grades 3-12 will be computer-based and administered for the first time in 2014-2015. A major advantage of the consortia created assessments for students with disabilities is in how the tests are being designed (Edyburn, 2013; Frizzell, 2013). Both the Partnership for Readiness for College and Careers (PARCC) and Smarter Balanced Assessment Consortium (SBAC) assessments are being created using Universal Design for Learning (UDL) principles to address visual, auditory, and physical barriers (http://www.udlcenter.org/aboutudl/whatisudl/3principle) and will provide a full range of accommodations that allow for flexibility in presentation, response, timing and scheduling, and settings (PARCC Assessment Consortia, 2013; SBAC, 2013). A number of accessibility features will be made available to all students. Students, teachers, parents, or test administrators should to be able to select accessibility features to individualize the assessment based on student needs and preferences. At the same time, planned accommodations for students with disabilities should be available in accordance with students' IEPs and include accommodations for presentation of questions, student response, as well as timing, scheduling, and setting considerations (PARCC Assessment Consortia, 2013; SBAC, 2013). How these tools may support students with EBD in mathematics is not yet discussed in the literature and will eventually emerge over time.

Concerns for Students with Disabilities

Despite efforts to build fully accessible and adaptive assessments, it is not yet known how these tools will impact the performance of students with disabilities, including students with EBD. The new tests will include more tasks that require students to use higher-level skills such as the ability to analyze, critique, evaluate, and apply knowledge with increasing cognitive expectations. In contrast, the testing conducted by most states for accountability under NCLB has consisted almost entirely of multiple-choice and short answer questions that primarily require only recall and recognition (Darling-Hammond & Adamson, 2013). The transition to the new standards and assessments will likely see an initial decrease in performance for all students as teachers and students acclimate to the new academic expectations (Darling-Hammond & Adamson).

It will be important to conduct research related to the implementation and application of CCSS in classrooms, curricula, and assessments for all students, and particularly on how the implementation of CCSS impacts the overall academic progress of students with EBD. This population of students, with their unique behavioral and learning needs, could potentially experience negative outcomes from the increased rigor of CCSS or suffer from unintended consequences of the implementation of the new standards. These revised standards require teachers to integrate algebra concepts and algebraic thinking beginning in kindergarten, highlighting the need to address deficits in skills, conceptual understanding, and mathematics reasoning early and continuously (Powell et al., 2013).

At this time there is no research or discussion in the literature on potential behavioral complications from increased rigor in content standards and assessments for students with EBD as a result of CCSS. The potential negative impact to graduation and drop out rates for students with EBD as a result of the alignment of state mandated high stakes tests and EOC exams to the CCSS is yet to be realized. The impact of these reform efforts on mathematics performance will eventually be seen in the data gathered through the National Assessment of Educational Progress (NAEP).

National Assessment of Educational Progress

The NAEP is the largest uniform national assessment of student academic progress in the areas of mathematics, reading, science, writing, the arts, civics, and economics. Results are based on representative samples of students in 4th, 8th and 12th grades and provide current levels of knowledge as well as long term trends by student grade, age, race/ethnicity, gender, status as English learner, national school lunch program eligibility, disability status, and content area (National Center for Education Statistics, 2011).

Despite decades of education and mathematics reform, including over 30 years of standardized testing, the mathematics performance of high school students has remained virtually unchanged; scores for students with disabilities, over the eight years they have been disaggregated, also have been stagnant (NAEP Data Explorer, 2013a; U.S. Department of Education, 2008). Data gathered from the national administration of state standardized tests for compliance with NCLB along with NAEP provides a picture of outcomes and trends for students with disabilities that include students with EBD.

Mathematics achievement-level results for NAEP can be disaggregated by disability status, but no detailed information is available for specific disability categories such as students with EBD. Achievement levels are determined by scale score cut offs (range 0-500) and are identified for eighth-grade mathematics related to skills in five content areas (number properties and operations, measurement, geometry, data analysis and probability, and algebra) as:

- Below Basic (< 262)
- Basic (262) - Evidence of conceptual and procedural understanding in the five content areas. This level indicates an understanding of arithmetic operations on whole numbers, decimals, fractions, and percents.
- Proficient (299) - Consistent application of mathematical concepts and procedures to complex problems in the five content areas.
- Advanced (333) - Evidence of the ability to generalize and synthesize concepts and principles in the five content areas (National Center for Education Statistics, 2006).

An extraction of NAEP results by disability status reveal that in 2013, 65% of students with disabilities scored below basic in mathematics, 27% achieved basic, 7% were proficient, and only 1% were advanced. For students without disabilities during the same year 21% scored below basic, 40% achieved basic, 29% were proficient, and 10% were advanced. The 65% of eighth-grade students with disabilities below basic did not demonstrate an understanding of arithmetic operations on whole numbers, decimals, fractions, and percents (National Center for Education Statistics, 2006). A visual display of comparison data for both groups that can be seen in Figure 4 shows students without disabilities making greater progress towards proficiency than students with disabilities.

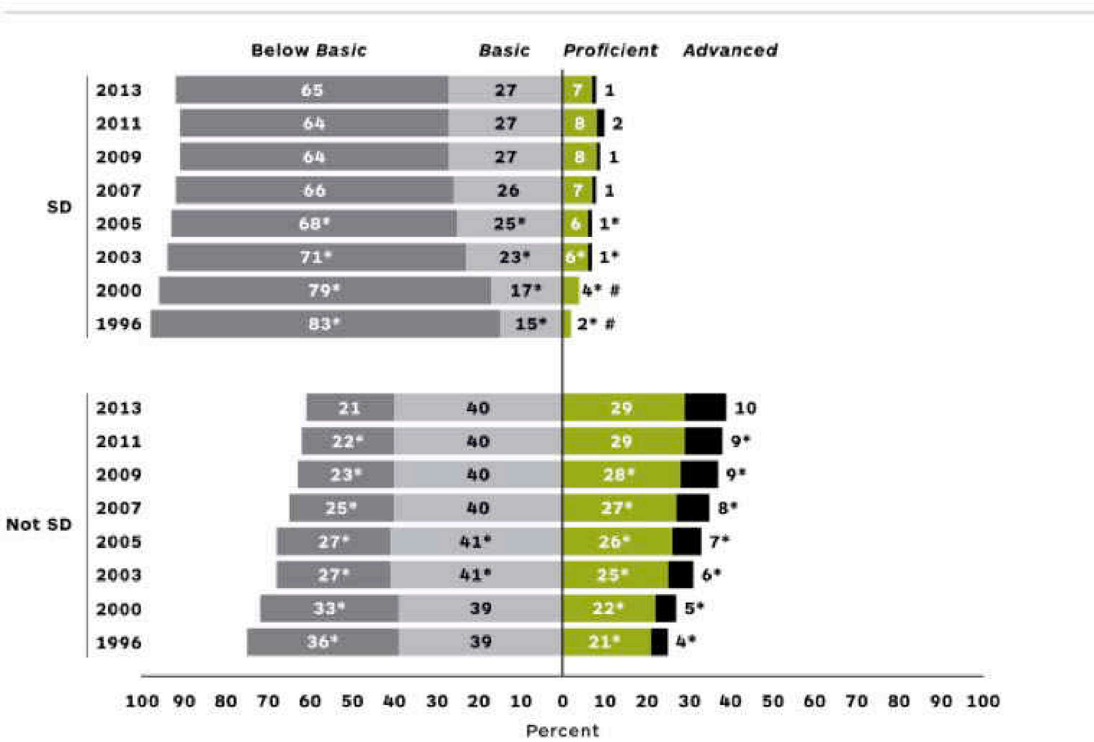

Figure 4: Trends in eighth-grade NAEP mathematics results by disability status for 2013.

Connecting Data from the NAEP and NTLS-2

Despite the lack of disaggregated NAEP performance results for students with EBD, their national academic performance can be discussed by correlating information from other national sources. The *Student School Program Survey* from the NLTS-2 reports that 58.7 % of students with EBD receive their mathematics instruction in the general education classroom. In addition, OSEP's *Annual Report to Congress* indicates that students with EBD comprise 7.3% of all students with disabilities (OCEP, 2009). By combining data from these two sources, it can be postulated that students with disabilities, including students with EBD, do score below basic on the NAEP and have skill deficits that puts them at risk for failure in algebra (Cavanaugh et al., 2005; Stein, Kaufman, Sherman, & Hillen, 2011) potentially requiring an increase in the amount of support and remediation that will be necessary in order for students with EDB to be successful in grade level, standards-based mathematics curricula (Archambault et al., 2010; Müller, 2009; 2010) like the Common Core. Special educators have raised concerns with CCSS and assessments as they are related to students' annual progress and passing end of course (EOC) exams required by most states.

Impact on EOC Exams and High School Graduation

Currently 26 states require a high school comprehensive exit exam or selected EOCs to qualify for high school graduation, 22 of those states require an EOC in algebra. Reports released from the Florida Department of Education show the overall passing rate in 2013 for the Algebra 1 EOC exam was 50% for all students, with only 15% of students with EBD receiving a passing score. As of 2013-2014, algebra is a graduation requirement in 45 states (NGA Center/CCSSO, 2012b). The potential impact of this requirement for students with EBD is yet to be realized and is a significant concern in light of historical Florida algebra EOC results for this population of students.

The Florida Example

Standards mandated by Florida officials require students to have a passing score on the algebra EOC exam to earn high school credit, and qualify for graduation. The additional rigor of CCSS and the new assessments tied to those standards create a potential problem for students who are already struggling to complete coursework that is required for graduation (Blackorby, Edgar, & Kortering, 1991; Bradley et al., 2008). In 2013, failure rates reported by Florida Department of Education on the Algebra 1 EOC exam; disaggregated to account for all students, students with disabilities (SWD), students with EBD; as well as students' grade levels at the time of the exam were: 9th grade (SWD-71%, EBD-81%), 10th grade (SWD-86%, EBD-87%), 11th grade (SWD-87%, EBD-90%), 12th grade (SWD-86%, EBD-83%; Florida Department of Education, 2013a). The graphic displayed in Figure 5 provides a visual representation of how failure rates for all students compares to rates for SWD and students with EBD, and illuminates the challenge that EOC Algebra 1 exam presents for all Florida students, particularly those with disabilities (Florida Department of Education, 2013a). For students with EBD, this critical milestone in high school is problematic as algebra is considered a gateway to higher mathematics and all further learning (Blackorby & Wagner, 1996; Cavanaugh et al., 2005)

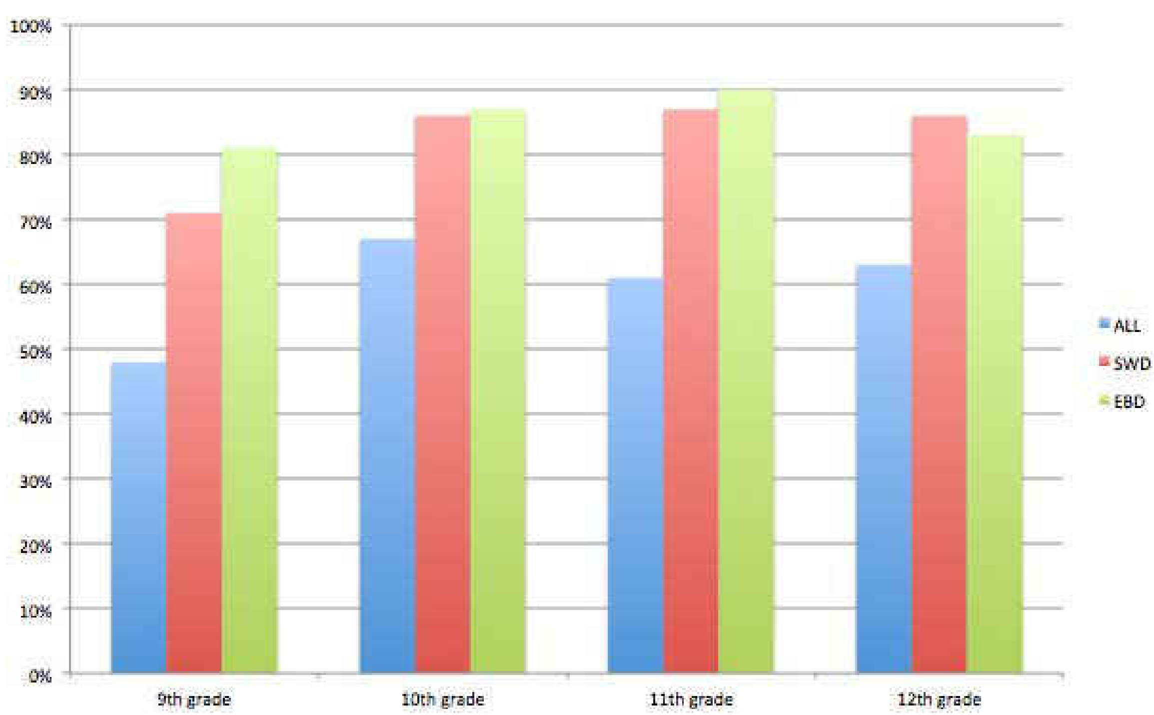

Figure 5: *2013 Algebra 1 EOC exam failure rates.*

Failure of students with EBD to successfully complete algebra is positively correlated with dropping out of high school and increases the likelihood of a lifetime of low wages, unemployment, and incarceration with little access to post-secondary education (Blackorby & Wagner, 1996; Cavanaugh et al., 2005; Watson & Gemin, 2008). Students with EBD have the highest dropout rates of all disability categories (Blackorby & Wagner, 1996) and any future changes in the education system could exacerbate their already dismal post-secondary picture. The failure to graduate from high school puts significant limits on post-secondary options and long-term career success for this population of students (Bradley et al., 2008; Goodman et al., 2011; Reid et al., 2004) and passing algebra is a key element to addressing this issue (Blackorby & Wagner, 1996; Cavanaugh et al., 2005; Watson & Gemin, 2008).

Mathematics: Outcomes for Students with EBD

So what is the potential solution? For students with EBD to be successful in algebra, their difficulties in mathematics must be recognized and addressed through early intervention and monitored throughout their educational experience (Powell et al., 2013; Siegler et al., 2012). The unique social and emotional needs of this population of students must be considered to ensure they do not interfere with students' access to the depth of knowledge and escalating learning expectations of general education mathematics. The CCSS concepts and skills found in the elementary grade levels build the foundational knowledge required for learning more advanced mathematics concepts in middle and high school where the ability to use elementary numeration skills with automaticity and fluency is critical (Hutchinson, 1993; Mayer, 1998). These prerequisite skills include conceptual and procedural understanding of whole numbers, operations, geometry, measurement, and fractions (U.S. Department of Education, 2008). Among this group of skills, fractions are cited as a major source of difficulty and anxiety for many students (Wu, 2009) and the most poorly developed of all the foundational skills (U.S. Department of Education). In their final report, *The National Mathematics Advisory Panel* stated that "fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected" (U.S. Department of Education, p. 18). Evidence of that proclamation can be seen in the falling percentages of students who are proficient on the NAEP mathematics assessment as they age and move to higher grade levels (National Center for Education Statistics, 2012).

Florida: A State-Level Example

This need for revision and an increased focus on fractions is not germane to national data, but is also found in the mathematics outcome data for the State of Florida. High failure rates on Algebra 1 EOC exams by students with EBD in Florida, present an opportunity to look more closely at the performance statistics for younger students on statewide assessments conducted to meet NCLB requirements. For example, the 2013 Florida Comprehensive Assessment Test (FCAT) was used to assess 174,360 eighth-grade students across the state, including 1,835 students identified with EBD. The mean developmental scale score, aggregated for all students on 2013 FCAT was 240, with 51% of all students reaching proficiency or above and 49% of all students scoring below proficient. By comparison on the same assessment, students with EBD earned a mean developmental scale score of 219, with only 16% of students with EBD reaching proficiency or higher and 84% scoring below proficient. The mean score for all students with disabilities was slightly higher than the mean score for students with EBD, with the aggregated mean developmental scale score for all students with disabilities reported as 224, with 22% of all students with disabilities reaching proficiency or better and 78% of all students with disabilities scoring below proficient.

Students with EBD also had the lowest mean points earned across mathematics reporting categories (with the exception of students who are intellectually disabled) with a mean of 4 of 12 possible points for number operations, problems, and statistics; a mean of 8 of 19 possible points for expressions, equations, and functions; and a mean of 4 of 17 possible points for geometry and measurement (Florida Department of Education, 2013c). More students with EBD failed to reach proficiency, across all grade levels, than students in any other disability category (Figure 6).

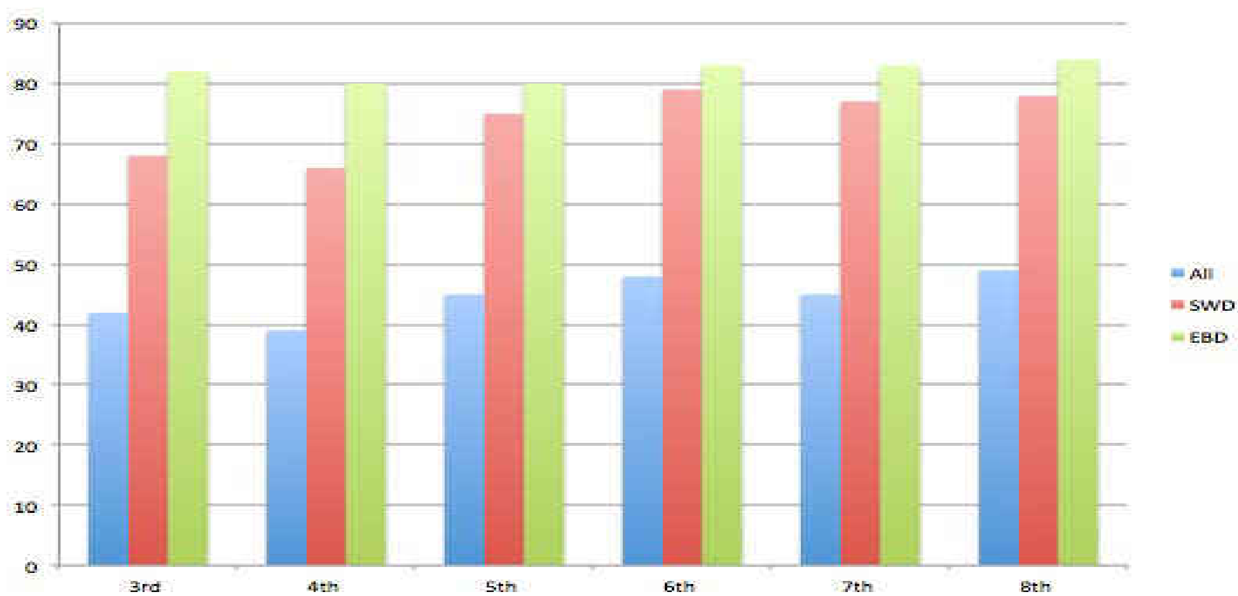

Figure 6: Percentage of non-proficient students, 2013 FCAT, by grade level and disability.

Fractions: A Key to Success in Algebra

Ongoing assessments nationally, and in states like Florida, provide evidence that students with EBD struggle academically. Documented algebra failure rates, specifically for students with EBD, and the correlation of failing algebra with high school dropout statistics substantiates researchers' conclusions that success in algebra is critical to positive post-secondary outcomes for this population of students (Blackorby & Wagner, 1996; Cavanaugh et al., 2005; Watson & Gemin, 2008). Higher-level mathematics is often difficult for adolescents with EBD who have difficulty focusing on complex problems (Carr & Punzo, 1993; Gunter & Denny, 1998; Hogan & Prater, 1993). Students with learning disabilities (LD) and EBD have been found to struggle with high-level problem solving (Hutchinson, 1993; Maccini, McNaughton, & Ruhl, 1999), deconstructing word problems (Montague, Bos, & Doucette, 1991), finding solutions (Algozzine, O'Shea, Crews, & Stoddard, 1987), and self-monitoring their problem-solving performance (Brown & Palincsar, 1982). Kameenui and Carnine (1998) note that students with LD and EBD have common deficits in short and long-term memory, poor attention, lack of metacognition, and difficulty storing and retrieving verbal information. Without appropriate levels of support and remediation for missing or weak prerequisite skills, students with LD and EBD are not likely to acquire the prerequisite skills necessary to successfully complete algebra or demonstrate proficiency on any state mandated mathematics assessment (Impecoven-Lind & Foegen, 2010; Maccini et al., 1999).

In order to be successful in algebra, students need to effectively utilize a variety of prerequisite numeration skills (Hutchinson, 1993; Mayer, 1998; U.S. Department of Education, 2008), but those necessary skills go far beyond what is required in elementary mathematics. A strong understanding of arithmetic principles and skills, along with the fluent application of those skills in problem solving activities, is necessary to successfully progress from middle grades mathematics to algebra. For example, a student with a strong conceptual understanding of abstract representations may struggle with algebraic proficiency as a result of poor arithmetic fluency (Witzel, Mercer, & Miller, 2003) in the same way that poor reading fluency effects reading comprehension. Fluency is defined as accuracy with speed and is considered a demonstration of true mastery (Binder, Haughton, & Bateman, 2002).

Algebra has a number of components that present challenges to students with LD and EBD who often have similar learner characteristics and skill deficits. One such component is the use of symbolic language to represent values and relationships along with properties and principles that define those relationships (Maccini et al., 1999). The complex series of tasks required in solving algebra problems places an increasing cognitive load on students with LD or EBD who may not have mastered basic mathematical operations. In addition to recognizing and using basic mathematical terms and symbols and performing basic computations, students must represent problems using variables for unknown quantities, identify appropriate strategies, perform calculations, apply relationship principles, as well as monitor the problem solving process (Hutchinson, 1993; Impecoven-Lind & Foegen, 2010; Steele, 2006; Witzel, Mercer, & Miller, 2003).

Missing conceptual understanding, deficits in procedural knowledge, and lack of fluency of the application of those competencies contribute to the difficulty that many students with EBD have in mathematics in the middle grades regardless of whether there is a comorbid LD or other cause of mathematics difficulty (Cullinan & Epstein, 2001; Fuchs & Fuchs, 2002). Mathematics difficulties generally begin early and persist throughout middle and high school (Powell et al., 2013). When difficulties persist they can result in issues such as decreased motivation and mathematics anxiety, key factors in the development of the negative attitudes towards mathematics that often appear in upper elementary and middle school students with EBD (Hensel & Stephens, 1997).

Elementary and secondary teachers of mathematics have historically reported difficulties associated with teaching fraction concepts and skills to students; those difficulties are often the result of poor conceptual understanding of fractions (Siegler et al., 2011). The concept of fractions is difficult for students to understand because it violates many of the properties of whole numbers: they are countable, have a single successor, are expressed as a single symbol, increase with addition and multiplication, decrease with subtraction and division, and have a finite number of symbols in a given interval. Fractions however, get larger when you divide, smaller when you multiply, have an infinite number of successors, and are not countable (Siegler et al., 2011). The introduction of fractions is the first departure from concrete whole number reasoning (where students can count and add on their fingers) to something that is abstract and requires a mental model. The abstract nature of fractions, and the connection of fraction understanding to overall rational number understanding, makes this a critical concept all students must master to be well prepared for algebra (Wu, 2009).

The CCSS requires teachers to introduce fraction concepts in third grade. Using five single digit denominators (2, 3, 4, 6, 8), third grade students are expected to understand a fraction as part of a whole, how to represent a fraction on the number line, and how to explain equivalence with visual models and use reasoning to compare fractions. In fourth grade, the concepts of equivalence and ordering are extended to include fractions with nine denominators (2, 3, 4, 5, 6, 8, 10, 12, 100). In addition, fourth grade students are expected to understand how to build fractions from unit fractions using part/whole relationships, decompose fractions into a sum of fractions with the same denominator, add and subtract mixed numbers with like denominators, and solve word problems involving addition and subtraction of fractions with like denominators by using visual models and equations.

Evidence Based Practices in Fractions

Although some instructional practices and interventions have been documented in the research literature on mathematics interventions as useful in the instruction of students with EBD, research is scarce (Hodge et al., 2006; Templeton et al., 2008). A meta-analysis of mathematics intervention research from 1976 to 2006 conducted by Templeton and colleagues (2008) included studies with the following characteristics: (a) mathematics performance was a dependent variable, (b) participants included students with EBD having no comorbid disabilities, and (c) an intervention was identified as the independent variable. In the final selection analysis 15 single-subject studies were identified for inclusion and provided information for analysis on 43 participants, 16 interventions, and 106 effect sizes. Nine of the identified interventions involved students in elementary school (≤11 years old), four involved students in secondary schools (>11 years old), and three used mixed age groups of students (9-12 years old). In the studies that reported gender, 91.7% were male and 8.3% were female. Only 50% of the studies identified school settings with six interventions in self-contained classrooms, one in an "adjustment class," and a final one in a resource room. Percentage of non-overlapping data points (PDN) was used to evaluate intervention effectiveness. Six interventions with math facts as the focus (*M* PND = 89.70) and ten interventions with computational focus (*M* PND = 80.89) produced mean PNDs that were effective by Scruggs & Mastropieri standards (2001). In their conclusion, Templeton and colleagues (2008) stressed the paucity of research in the area of mathematics for students with EBD and highlighted the critical absence of recent studies and studies on explicit mathematics strategies.

Hodge and colleagues also completed a review of literature on instructional interventions in mathematics from 1985 through 2005. Initial criteria for identification and selection of articles included: (a) quantitative or single-subject design; (b) participants were all students with EBD or in studies with diverse participants, outcomes were disaggregated for students with EBD; (c) use of a mathematics instructional intervention; and (d) mathematics achievement as dependent measures. Thirteen studies were identified, 12 addressed basic computation skills and one addressed both computation and problem solving. Single subject designs were utilized by 12 of the 13 studies and one used a post-test only group design. A total of 152 students participated in the 13 studies. Demographic information was inconsistently reported. Studies reporting student age described participants ranging in age from 9 to 16 years. Eleven studies reported gender (55 males, 5 females), three reported student race (6 African-American, 4 Caucasian) and the largest study, a group design, did not report gender or race. Hodge and colleagues (2006) noted a dearth of research evidence on improving academic performance in mathematics for students with EBD, and concluded that little is known about the effectiveness of mathematics interventions for this population of students.

Using the same search criteria from the Hodge and colleagues' study, this researcher extended the years of that analysis to include articles from January 2006 through November 2013. The initial search of Psych Info and ERIC located 27 articles that were not duplicates, but only eight studies that met the original qualifications for inclusion. The addition of these studies raised the participant count to 203 students in 21 intervention studies on mathematics instruction for students with EBD spanning 28 years. These additional studies used primarily single-subject design with only one additional study utilizing a group design. Among these 21 studies, all produced positive if not significant results for the students in the intervention conditions, and none replicated or expanded prior studies that built an evidence base for specific instructional practices for this population of students. The researchers of the studies reported that students with EBD responded positively to supports and interventions as supplements to the standard mathematics curriculum with reported effects on learning gains numerically positive if not significant. Only one of the 21 studies identified involved fraction computations, which was an intervention using peer tutoring that reported improved academic outcomes for participants (Franca, Kerr, Reitz, & Lambert, 1990). None of the studies in the meta-analysis, or this researcher's extension, pertained to the use of manipulatives in a concrete-to-representational-to-abstract sequence of instruction in an empirical study or the use of VM for this population of students. A synthesis of key factors from the 13 studies identified by Hodge and colleagues and the eight identified by this researcher can be seen in Appendix B.

Concrete-Representational-Abstract Sequence of Instruction

The potential of VM to support all students in learning mathematical concepts, such as fractions, is well documented in the literature (Moyer-Packenham et al., 2013), with much of that research coming from extensions of earlier research that involved the use of physical manipulatives in a concrete-to-representational-to-abstract (CRA) sequence of instruction. Physical manipulatives, not virtual manipulatives, were the basis for this historical definition, "Manipulative materials are concrete models that incorporate mathematical concepts, appeal to several senses, and can be touched and moved around by students" (Hynes, 1986, p. 11). Swan and Marshall (2010) expanded upon this definition by adding that "A mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (p. 14). The manipulatives themselves are not an instructional strategy but a tool to be utilized in conjunction with an effective teaching strategy to maximize their effective use (Ball, 1992; Moyer & Jones, 2004). One such strategy is the concrete-representational-abstract (CRA) sequence of instruction.

Instruction that uses CRA teaches skills and concepts to students through the sequential use of concrete objects, pictorial representations, and abstract numerals. This three-stage process has been shown to be successful in teaching students difficult mathematics concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, & Jackson, 2003; Maccini & Hughes, 2000; Witzel, 2005). Much of this research has been conducted in the elementary grades for arithmetic instruction because of its concrete nature. It has been harder to develop concrete representations of abstract concepts, like those found in algebra, but increasingly researchers are documenting success in this area (Borenson, 2011; Sherman & Bisanz, 2009;

Witzel et al., 2003; Witzel, 2005). In addition, the use of physical manipulatives in the CRA sequence of instruction has been documented as effective in improving conceptual and procedural knowledge for mathematics skills and concepts for all students (Carbonneau, Marley, & Selig, 2013; Rakes et al., 2010).

Virtual Manipulatives

In recent years virtual manipulatives (VM) have begun to supplement or replace physical manipulatives, particularly for secondary students who may be reluctant users of concrete materials (Moyer et al., 2002). The proliferation of digital media and its accessibility via the Internet has created an environment for the creation and use of VM. The Center for Implementing Technology in Education (CITEd) describes VM as "digital 'objects' that resemble physical objects and can be manipulated, usually with a mouse, in the same ways as their authentic counterparts" (CITEd, n.d.). Moyer and colleagues defines a VM as "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). The interactive nature of VM is what sets them apart from other digital representations.

"Interactive tools provide experiences that help students discover and verify the relationships among symbols and representations of algebraic operations" (Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2008, p. 68) by allowing students to move or manipulate objects. These manipulatives differ from static visual representations which also can be web-based, resemble physical manipulatives, and are similar to pictures in books but cannot be manipulated by users. When virtual tools are interactive they allow users to handle representations with input devices (e.g. mouse, joystick, keyboard, trackpad) in order to investigate concepts, build understanding, and construct meaning (Moyer, Bolyard, & Spikell, 2002). The availability of VM is growing rapidly and research results on the effectiveness of these interactive tools are promising (Moyer et al., 2005; Ozgun-Koca & Edwards, 2011; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2007).

Cavanaugh and colleagues conducted a study on the use of one type of VM in a large state virtual school (Cavanaugh et al., 2008). The study was conducted in an algebra course on the use of an online tool for graphing linear equations. Students were randomly assigned to the control group (no online tool; $n = 14$) or the experimental group (online tool; $n = 33$). No demographic or disability information was provided by the researchers. For the purposes of this study, two equivalent course models were constructed and verified as equivalent, to provide otherwise similar experiences for the students in the two groups. Each participant took a pre and post-test on the concept of graphing linear equations and both groups showed learning gains, with the mean increase (pre to post) for the control group 1.71 and 3.07 for the experimental group. The variance in gains between the two groups did not represent a significant difference. There was a moderately high correlation ($r = .73$) between the pre and post-test scores with no significant interaction.

Cavanaugh noted the limitations of the study due to small sample sizes and observation inconsistencies. In addition she commented on the unique challenges of virtual classrooms in conducting research (bimodal age range of students in the same course, varying rates of course progress). She went on to conclude that the use of online or VM is effective for at least some students, and as a result teachers should make these types of tools available to students (Cavanaugh et al., 2008).

Reimer and Moyer (2005) looked at the effect of the use of VM by 19 third-grade students during a unit on fractions. The participant group was culturally and linguistically diverse (10 Caucasian, 2 Hispanic, 1 African-American, 3 Asian, and 3 Middle-Eastern) including three English learners, three students with LD and four students identified as gifted and talented. The researchers provided no other demographic information. These students used several web-based VM in a computer lab as a part of their mathematics instruction and had this tool available to use on the post-test. Data were collected through pre and post-tests on conceptual knowledge, procedural computation skills, and a student attitude survey. The conceptual knowledge post-test mean (M = 11.0, SD = 3.61) was significantly higher, $t(18)$ = 2.05, $p < .05$), than the pretest mean (M = 9.58, SD = 4.53), supporting the researcher's hypothesis that the use of fraction VM would improve conceptual knowledge. While there were numerical gains in the tests of procedural knowledge, no significant differences were seen, possibly in part because pretest scores averaged 90% leaving little room for improvement. One limitation of this study was the high pretest scores by all students creating a ceiling effect. Even with those high pretest scores, 74% of the participants had post-test scores that were at least as high as their pretest scores. Student responses to the survey questions were 59% positive, 23% neutral, and 18% negative. While the majority of the students had positive attitudes about VM, they did not perceive them as a tool to assist them in answering test questions. Student comments indicated that they appreciated the immediate feedback of the VM and liked the ease of use and the way the use increased the speed at which they could work problems (Reimer & Moyer).

Moyer-Packenham and Suh (2012) conducted a study to examine the impact of VM on the learning gains for different achievement groups. The study included a group with students that were low achieving (LA; $n = 12$), two groups with students that were average achieving (AA; $n = 12$, $n = 12$), and a group of students that were high achieving (HA; $n = 21$). Standardized test scores were used to place students into achievement level groups. The researchers provided no other demographic data. One of the groups of AA students was randomly selected to be a control group that would use PM. Data were gathered through fraction pre and post-tests on fraction equivalence and fraction addition as well as videotapes of classroom activities. Although all groups showed a numerical gain from pre to post-test, the group with students who were LA had the largest and only significant increase ($p < .01$), indicating that they benefited the most from the use of VM. The group of students who were LA had a post-test mean ($M = 81.31$, $SD = 12.34$) that was 11.16 points higher than the group pretest mean ($M = 70.15$, $SD = 21.44$). No significant difference in the mean pre to post-test gains were found when comparing the group of AA students that used VM and the group that was AA that used PM.

The videotapes of classroom activities, along with interviews with students, revealed that students in each of the ability groups had different experiences using the VM. Students in the group that was HA identified multiples and factors, used mental math strategies, recognized equivalence and proportions, and focused on symbolic features to work fraction problems. Students in the group that was AA also used mental math strategies and equivalence relationships to a lesser degree, used VM later in the instructional unit, used a procedural method to identify multiples and common denominators, used counting strategies in place of proportional relationships, and relied on pictorial and symbolic features to complete tasks. The group of students who were LA used procedural methods to find multiples and common denominators, were dependent on pictorial models in the applets, used counting strategies rather than proportional relationships but did not recognize equivalence, were confused by common denominators, and used trial and error to complete tasks. The researchers noted that VM provided five interrelated affordances that stimulated learning in mathematics that varied in their impact on student learning based on the achievement level of the learner (Moyer-Packenham & Suh).

Online Availability of Virtual Manipulatives

Moyer and colleagues attempted to demystify VM in a 2002 article by listing various web-based tools for the reader to explore. More than 12 years ago, the sites provided by Moyer et al. represented state of the art technology. Some of the recommended sites have evolved and are still available today, but many are no longer active. A recent search of the Internet for VM by this researcher revealed more than 30 active sites that either host or provide links to online mathematics VMs including research supported free tools, tools provided by curriculum publishers, sites that catalogue tools and direct users to external sites with content specific VM, sites containing information on VM for mobile devices and tablets, and sites containing strategies for incorporating VM into instruction. Table 1 contains a selection of the most comprehensive sites that host VM or provide related resources.

Table 1

Online Virtual Mathematics Manipulatives and Resources

Name	Website	Level
*National Library of Virtual Manipulatives	http://nlvm.usu.edu/en/nav/vlibrary.html	All Levels
Shodor: InterActiviate	http://www.shodor.org/interactivate/	6-12
*NTCM: Illuminations	http://illuminations.nctm.org/Search.aspx?view=search&type=ac	All Levels
Glencoe Online	http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html	P-8
InThinking: Teach Mathematics	http://www.teachmathematics.net/	6-12
Foundations of Algebra	http://www.foundationsofalgebra.com/manipulative-mathematics/	6-12
McGraw Hill	http://highered.mcgraw-hill.com/sites/0073519456/student_view0/virtual_manipulative_kit.html	K-5
*Manipula Math	http://www.ies-math.com/math/java/	6-12
Drexel: Math Tools	http://mathforum.org/mathtools/index.html	All Levels
GeoGebra	http://www.geogebratube.org/	All Levels

Note: * On Moyer and colleagues original list.

Virtual manipulatives are one of many digital tools, when made easily available via the Internet, have the potential to allow students with EBD to investigate and build conceptual understanding in mathematics, practice procedural tasks, and support their own learning (Miller, Brown, & Robinson, 2002). There is clear evidence in the literature regarding the impact of VM on learning outcomes for students in mathematics (Moyer-Packenham, Westenskow, et al., 2013).

In a conceptual analysis of literature on the use of VM conducted by Moyer-Packenham and colleagues (2013), the researchers identified five interrelated affordances that benefit students: focused constraint, creative variation, simultaneous linking, efficient precision, and motivation. Those affordances have produced positive outcomes and learning gains in geometry concepts (Olkun, 2003; Steen, Brooks, & Lyon, 2006), algebra concepts (Cavanaugh et al., 2008; Suh & Moyer-Packenham, 2007), integer addition and subtraction (Bolyard & Moyer-Packenham, 2006), and fractions (Moyer-Packenham, Ulmer, & Anderson, 2012; Reimer & Moyer, 2005; Suh et al., 2005; Westenskow, 2012).

The Rational Number Project

Despite the potential value of VM, these tools do not provide a specific curriculum that may address potential barriers to fraction learning for students with EBD. However, a curriculum that has been shown to directly impact student learning in fractions is the Rational Number Project (RNP) *Initial Fraction Ideas,* which was developed to be used with physical manipulatives (PM). The Rational Number Project was a collaborative research project that included faculty from eight major universities from the U.S. and one partnership from Germany. The National Science Foundation (NSF) continuously funded this project from 1979 to 2009 with the exception of 1983-1984. By 2013 the project had contributed over 100 publications and tools to the field of mathematics education including journal articles, books, book chapters, curriculum, professional development, a website, and three web-based VM. The University of

Minnesota, College of Education and Human Development (UM/CEHD; 2013) currently hosts and maintains the RNP website under the guidance of Kathleen Cramer, one of the project's principal investigators.

Much of the research conducted by project investigators, through the NSF funding, led to the development and validation of a book of fraction lessons originally published in 1997 as *Fraction Lessons for the Middle Grades Level 1*. Those lessons were revised in 2009, and renamed *Initial Fraction Ideas* (IFI)*;* a title that the authors believed better reflected the content. Twenty-three IFI lessons assist teachers in helping students to develop conceptual understanding of fractions and promote (a) the development of fraction meaning by using a part-whole model, (b) the construction of informal ordering strategies using mental representations of fractions, and (c) the creation of meaning for fraction equivalence and conceptual understanding of fraction addition and subtraction through the use of concrete models.

The RNP researchers and others have documented the use of PM to build conceptual understanding in mathematics, a critical foundation for procedural proficiency (Cass et al., 2003; Cramer & Henry, 2002; Cramer, Post, & delMas, 2002; Cramer, Wyberg, & Leavitt, 2008; Moyer et al., 2005; Ozgun-Koca & Edwards, 2011; Post & Cramer, 1989; Sherman & Bisanz, 2009; Sowell, 1989; Witzel, 2005; Witzel, Riccomini, & Schneider, 2008). These researchers and others have expanded the literature to include VM, as those tools became more accessible and available (Bolyard & Moyer-Packenham, 2006; Cavanaugh et al., 2008; McLeod, Vasinda, & Dondlinger, 2012; Moyer et al., 2005; Moyer-Packenham & Suh, 2012; Moyer-Packenham, Westenskow, et al., 2013; Olkun, 2003; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2007; Westenskow, 2012).

Application of *Initial Fraction Ideas*

Westenskow (2012) conducted a mixed methods study that provides a framework for further research on the use of VM in fraction interventions. As a result, her study is examined in detail as part of this literature review. Westenskow evaluated the effectiveness of a fraction intervention curriculum used with three groups of students one group that used PM only, one that used VM only, and one that used a combination of PM and VM. Forty-three fifth grade students with mathematics difficulties participated in the study and were assigned to one of three intervention groups by using a stratified selection process. The purpose of Westenskow's study was to identify variations in students' conceptual understanding of fraction equivalence as a result of the intervention and relate variations to the type of manipulative used.

Westenskow was first interested in the overall learning gains related to conceptual understanding of fraction equivalence when using the IFI lessons (modified for the use of VM) in an intervention, setting in combinations with the use of PM and VM in three conditions: (a) PM only, (b) VM only, and (c) a combination of PM and VM. To assess overall learning gains, an equivalent fraction test (EFT) was created and validated in her study. All students completed the EFT pretest to provide pretest data and determine eligibility for participation in the study.

Scores, reported as percentage correct, ranged from 5% to 100% with a mean of 51.1%. Criteria for participant selection included (a) identification by a teacher as having math learning difficulties and (b) pretest score was below 40%. An exception was made, at the teachers' requests, to include eight students who had a history of mathematics difficulty but whose pretest scores (54%-57%) were above the cut-off. Forty-two fifth-grade students from four schools, who had not participated in fifth-grade fraction instruction, consented to participate in the research. Westenskow provided no detailed demographic information, except a concluding statement that the groups were primarily white, middle class students. Because the study was designed to target Tier II intervention, students with disabilities were excluded from the three phase project.

The pre-intervention phase of Westenskow's study was used to develop and validate the EFT. The assessment was intended as the dependent variable for a repeated measures analysis and would be administered three times during the study, pre-intervention, post-intervention, and a third time as a delayed post-test. The test contained three types of questions: multiple choice, short answer, and open response. The 20 questions consisted of four questions from each of the five fraction equivalence sub concepts: modeling, evaluating, building groups, solving sentences, and simplifying fractions. Westenskow's validation of the EFT was conducted using a three-step process.

1. The researcher developed a pool of 60 potential questions, based on research literature.
2. A team of experts was convened to evaluate each question's content validity. During this step, internal validity was also assessed by administering the questions one at a time to three students with different achievement levels (one high, one medium, one low). As a student answered each question, the researcher asked the student to explain their reasoning process. Questions that did not initiate equivalent fraction thinking were refined.
3. Three pilot tests (A, B, C) were created from the resulting questions from the pretest, post-test, delayed post-test and delivered to 81 students who provided an average of 55

responses to each question. Reliability and item difficulty was determined by an item response analysis. Analysis results are summarized in Table 2.

Table 2

Reliability and Item Difficulty for Equivalent Fraction Tests

Variable	Pretest	Post-test	Delayed Post-test
Reliability	0.74	0.760	0.740
Mean item difficulty	-0.10	-0.002	-0.005

(Westenskow, 2012, p. 66)

The three forms of the EFT were administered before, immediately after, and three weeks after the intervention. Student responses on the 20-item test were scored as five points for a correct response and zero for an incorrect response, making the maximum score 100. Table 3 contains a summary of question types. Summative results of the pre, post, and delayed post-test administration of the EFT were analyzed using paired sample *t* tests. The analyses showed significant learning gains in all treatment conditions, the largest gains in the PM group, followed by the VM group with learning gains on the EFT, from pretest (M = 25.07, SD = 8.72) to post-test (M = 59.79, SD = 22.57), significant at, $t(13) = 6.66$, $p < .001$ with a large effect size (d = 2.03) reported.

The mean pretest score (M = 51.1) of all 5th grade students (N =183) and the mean pretest score (M = 51.1) of all the intervention groups (n =183) were calculated to provide a basis for comparison to the overall population sample. Students who participated in the study showed a mean gain of 36.9 points between the pretest and post-test with 69.8% of study participants scoring higher than the pretest mean for all 5th grade students and 46.3% scoring 75% or higher.

Westenskow's analyses demonstrated that all three interventions increased the students' understanding of equivalent fractions.

Table 3

Pre/Post/Delayed Test Question Types

Content	Representation Level	Type	Questions
Modeling equivalence	4 pictorial	2 multiple choice	1,2
		2 open response	16,18
Evaluating equivalence	3 pictorial	2 multiple choice	4,5
		1 matching	3
		1 open response	17
	2 symbolic only	1 multiple choice	6,8
Building equivalent groups	3 pictorial	1 multiple choice	7,10
		1 open response	19
	1 symbolic only	1 open response	13
Completing equivalent sentences	1 pictorial	1 multiple choice	9
	1 symbolic only	1 open response	14
Simplifying fractions	3 pictorial	2 multiple choice	11,12
		1 open response	20
	1 symbolic only	1 short answer	15

(Westenskow, 2012)

A final paired sample *t* test was conducted comparing the post-test results with the delayed post-test. No significant difference in scores were found for any intervention group indicating that students retained their achievement gains three weeks after the intervention. Small effect sizes were reported ($d \leq 0.12$) for all interventions. In Westenskow's study students used VM from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/) and NCTM Illuminations (http://illuminations.nctm.org/) as either a replacement for or supplemental to PM. Virtual manipulatives have similar characteristics to PM, but digital technology gives VM the capacity to provide students with additional supports. Four of the six VM selected for Westenskow's study provided support designed to guide students in the completion of a system-generated task. All PM allowed students to work on IFI activities as written, by freely using the PM to explore specific fraction equivalency tasks; four of the six selected VMs did not. When using the IFI lessons for the VM groups, Westenskow found it necessary to modify practice activities and pre-plan the use of alternate VMs to parallel the PM activities and achieve the same learning goals. Her study was a foundational study to identify variations in learning related to the types of manipulatives, to pilot methods of fraction intervention for use in future research, and to pilot the use of two types of learning trajectories.

Virtual Manipulatives from the Rational Number Project

Since the conclusion of Westenskow's study, the RNP research team has expanded their work to the use of VM. In 2013, RNP contracted with Twin Cities Public Television to create a set of three virtual manipulatives that closely resemble the PM used in IFI lessons. Applets that mimicked fraction circles, chips, and paper folding were developed and are available online for teachers to use in teaching fractions (http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html). Each of these VM was created to imitate the manipulation of PM, with the added features of immediate feedback and the ability to have the applet display fraction names. This set of manipulatives will allow future researchers to utilize the IFI lessons as originally designed by RNP, substituting VM for PM in the structured activities. Figures 7, 8, and 9 contain screen shots of each of the virtual manipulatives.

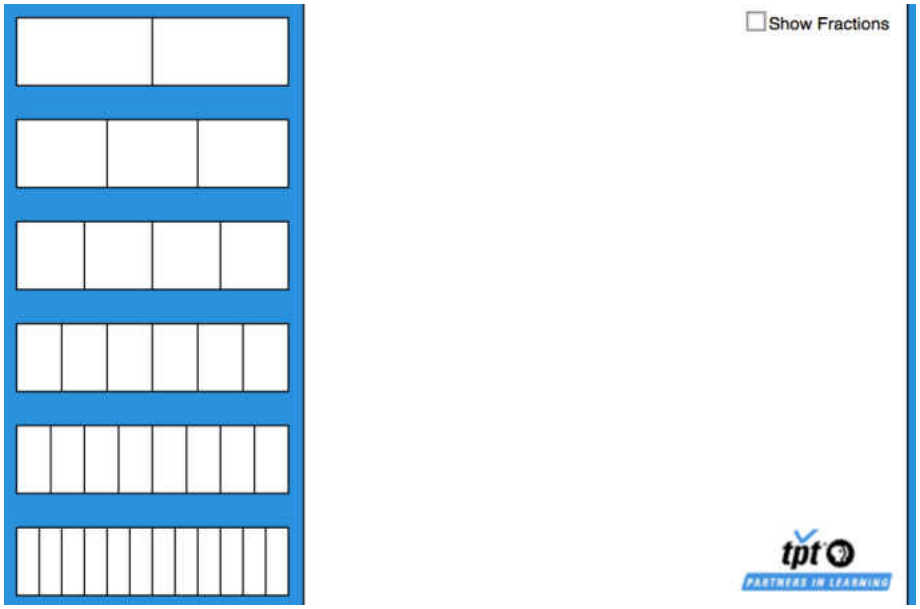

Figure 7: *Paper folding virtual manipulative.*

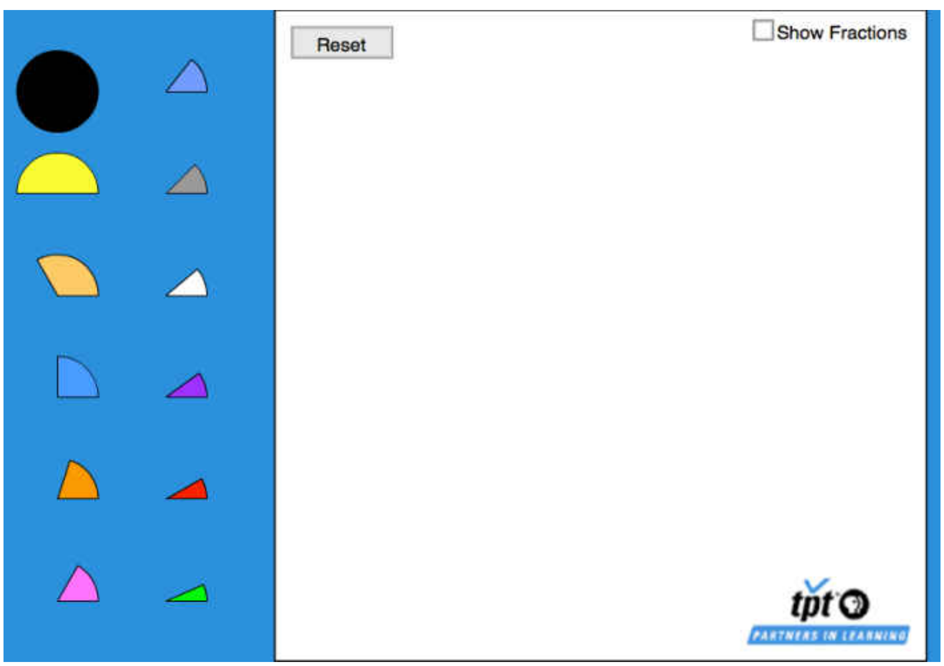

Figure 8: Fraction circle virtual manipulative.

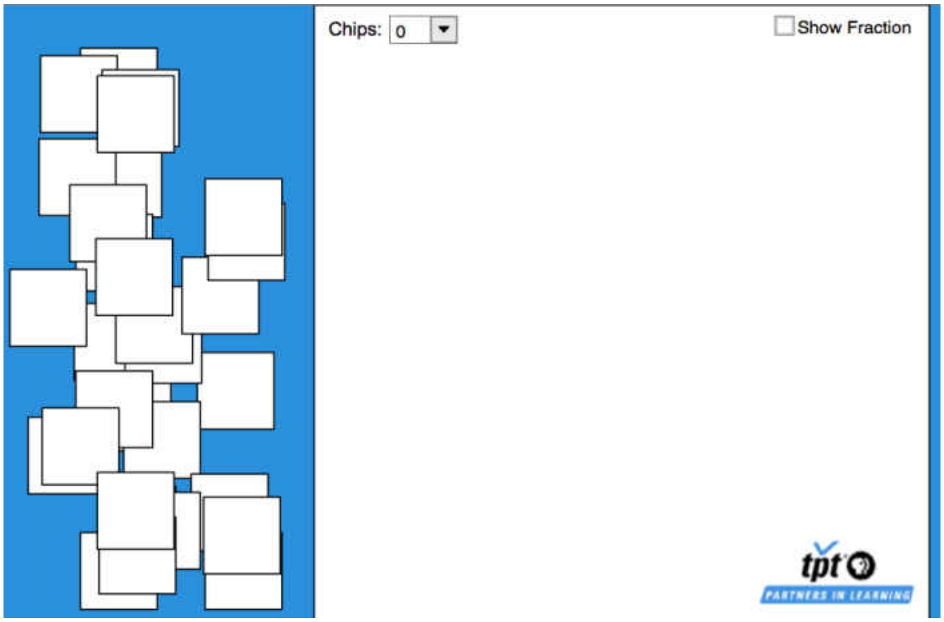

Figure 9: Chips virtual manipulative.

Using Virtual Manipulatives and the Rational Number Project with Students with EBD

Researchers have documented the potential for student use of manipulatives in mathematics instruction for students with disabilities (Witzel et al., 2008) and the use of VM for students who are low achieving and have mathematics difficulties (Moyer-Packenham & Suh, 2012). Research on the use of VM in conjunction with or as a replacement for PM has shown positive effects on students' mathematics achievement (Moyer-Packenham, Westenskow, et al., 2013; Olkun, 2003). "*The Rational Number Project* was the longest lasting cooperative multi-university research project in the history of mathematics education" (Post, 2002, para. 1) and produced volumes of evidence to document the validity of the scripted sequential instruction that comprises *Initial Fraction Ideas*. The legacy of this project includes a significant body of literature (see Cramer & Henry, 2002; Cramer et al., 2002), curricula for providing intervention instruction for fraction and decimal concepts, and a set of three virtual manipulatives for use in understanding the concepts of part-whole and equivalency.

Westenskow's study on equivalent fraction learning extended the use of the *Initial Fraction Idea* lessons to the use of VM in a study that compared groups of students with mathematics difficulties who used VM only, PM only, or a combination of VM and PM. Manipulative use in all three groups showed significant gains in equivalent fraction knowledge in a pre to post-test analysis with the VM condition producing the largest effect size.

Moyer-Packenham and Suh (2012) suggested that VM may have multiple affordances as compared to PM, and those affordances may be more beneficial to students who are lower achieving. Westenskow indicated that an extension of her research should include a more diverse sample of students and potentially focus on motivation and attitude related to the type of manipulative. To date, no studies have been identified that used manipulatives (PM or VM) in mathematics instruction for students with disabilities, especially students with EBD. Many students in this population are performing poorly on state high stakes tests (Bradley et al., 2008; Nelson et al., 2004), are failing to pass algebra (Florida Department of Education, 2013a; Nelson et al., 2004; Zhang, Katsiyannis, & Kortering, 2007), and are dropping out of high school at twice the rate of all students with disabilities (Landrum et al., 2004). The use of VM in building the foundational skills necessary for success in algebra should be considered for this population of students.

This research extended the use of RNP lessons, Westenskow's EFT, and piloted the use of the RNP funded VMs. The study setting was a unique blended learning environment where students with EBD, who were physically separated in special education classrooms, were enrolled in inclusive online courses. The teacher indicated that students were struggling to meet the academic and time constraints of the online coursework in this hybrid academic environment. The researcher addressed a specific problem of practice, that was identified as a critical concern by the teacher, that would support students with remedial instruction on fractions while potentially allowing them to keep pace with their online, CCSS aligned, mathematics courses. The short-term goal for this intervention study was to meet the needs in this classroom, remediation to provide background knowledge, improve learning outcomes and course completion rates, was the short-term goal for this intervention study. The long-term goal was the potential impact of a strong conceptual understanding of fractions on students' future success in algebra and other more advanced mathematics courses as they enter high school and prepare for their post-secondary education and careers.

Online and Blended Learning

Accessibility to highly qualified teachers and individualized curriculum in key content areas like mathematics are among the challenges that exist today for students with EBD. NCLB requires that all students receive core content instruction from highly qualified teachers. Online learning may provide an opportunity to provide this type of instruction without exacerbating behavioral challenges of students with EBD. With a continued shortage of highly qualified special education teachers, particularly teachers dually certified in special education and targeted content areas, like mathematics or science, administrators struggle to staff secondary special education classrooms (Boe & Cook, 2006; McLeskey et al., 2004). A potential solution to the conundrum of providing highly qualified teachers and standards-based curriculum in critical shortage areas like mathematics and science may be found in blended or fully online learning (Hassel & Terrell, 2004; Rose & Blomeyer, 2007; Staker & Horn, 2012; Vasquez & Serianni, 2012).

Staker and Horner (2012) identify four models of blended learning. In the Rotation model students rotate on a fixed (teacher directed) schedule between online and teacher led instruction. Station rotation, lab rotation, individual rotation, and flipped classrooms are four commonly used Rotation models. The Flex model provides online instruction and an on-site teacher-of-record. Individual student needs dictate their movement between online instruction and various types of face-to-face support within the classroom. In the Self-Blended model students choose to take one or more classes online in addition to their traditional face-to-face classes. In the Self-Blended model the online teacher is the teacher of record for online courses. The Enriched Virtual model is a fulltime online program where students spend some of their time in brick-and-mortar classroom away from home. These four models provide a framework for face-to-face support for online instruction.

The advent of online and blended learning presents viable alternatives to connect students with EBD, whose LRE is a self-contained classroom, with standards-based mathematics content taught by highly qualified teachers (Archambault et al., 2010; Müller, 2009; 2010; Watson et al., 2012). Providing high quality mathematics instruction, regardless of placement or mode of delivery, is critical to the outcomes of students with EBD (Gagnon & Bottge, 2006). Researchers have expressed concerns that without the proper supports in the online environment, outcomes for students with disabilities could remain unchanged (Deshler, East, Rose, & Greer, 2012).

CHAPTER THREE: METHODS

Overview

Students with emotional or behavioral disorders (EBD) need promising practices and evidence-based strategies, tools, and interventions to succeed in mathematics whether they are enrolled in face-to-face or online courses. The inability of middle school students with EBD to acquire both conceptual and procedural knowledge in mathematics as well as fluency in problem solving tasks is an ongoing barrier to their success in this content area (Poncy, Skinner, & Axtell, 2010; Shih, Speer, & Babbitt, 2011). The researcher in this study investigated the effects of a remedial mathematics intervention on middle school students with EBD. Using a combination of RNP *IFI* lessons and VMs (RNP/VM) the researcher examined the impact of these combined tools on computational fluency in adding fractions and conceptual understanding of equivalent fractions.

To assist in understanding the overall components of the study the researcher provides the research questions, the independent and dependent variables, a description of the participants and setting, and an in depth explanation of the research design. The chapter concludes with a discussion of the reliability and validity of the study.

Research Questions

The researcher in this study evaluated the effect of RNP/VM, an intervention package designed to improve conceptual knowledge in fractions on middle school students with EBD in a blended learning environment that consisted of physical placement in a separate special education setting and enrollment in an online grade-level mathematics course. The researcher investigated the conceptual (EFT) and procedural (FAA) performance of three groups of students in this setting before, during, and after the intervention. The first component of the research study evaluated students' procedural performance as a result of teacher delivery of the intervention package and a daily fraction addition assessment (FAA). The data gathered from the FAAs were used to evaluate to what extent the intervention increased median fraction addition fluency for these groups of students. The second component of the study evaluated changes in conceptual understand of equivalent fractions. Students were administered a validated Equivalent Fraction Test (EFT; Westenskow, 2012) as a non-experimental pre, post, and delayed post-test measure to evaluate changes and retention of equivalent fraction knowledge. The researcher sought to answer these questions:

1. To what extent does RNP/VM increase group median fluency on a five-minute fraction addition assessment for students with EBD?
2. Does RNP/VM increase equivalent fraction knowledge as evidenced by the results of a pre, post, and delayed post-test administration of the EFT?
    a. To what extent do mean scores increase from pre to post administration of the 20-item EFT, for groups of students with EBD?
    b. Given an increase in mean group scores from 2.a., to what extent do those gains persist after the end of the intervention?
3. How do the students with EBD and their respective teacher and paraprofessional perceive the goals, procedures, and outcomes of RNP/VM?

Participants and Setting

Students

The population of students targeted for this study included 24 students in 6th, 7th, and 8th grade who were identified with EBD and assigned to a separate classroom for 100% of their school day. The study took place in a single Central Florida middle school in a district that had mandated the preceding year that students with EBD in this setting be enrolled in inclusive online courses to receive grade-level content instruction, which included mathematics. The primary exceptionality for all 24 students in this setting was EBD. All students received their mathematics instruction in this unique blended learning environment from the same special education teacher who was dually certified in mathematics and special education.

Participants selected for this study scored below scored below 50% on baseline fraction addition assessments, had at least an 80% attendance rate, scored 80% or less on the equivalent fraction pretest, and consented to participate in the study. The pre-study and baseline periods were used to obtain student consent, assess students' fraction knowledge, observe attendance, and determine eligibility for the study. Students who did not meet selection criteria participated in the study instruction, used VMs, and took assessments along with their peers, but their information was not included in the study. The teacher provided an incentive of "extra credit" for all students who actively participated in the intervention instruction and assessments regardless of their inclusion in data collection. The level of participation varied depending on individual attendance, student disposition, and on-task behavior; all typical factors for this population of students. Responses from non-participating students were excluded from data collection and analyses. All of the students in each of the three classrooms participated to some extent in some or all of the intervention activities. Nine students, three in each class, were selected as participants, 16 of the 24 students were excluded from the study for the following reasons: Four declined to participate; 10 had intermittent attendance below the 80% study threshold, confirmed by historical attendance records or observed during the three-week the pre-study period; one student was out of school during the study on home health care; and one student demonstrated proficiency in fraction addition as demonstrated by two FAA scores above 75% during baseline.

Selected Participants

Nine students remained after applying pre-determined exclusion criteria. Three students in each class made up the three groups identified by the teacher as the Red group, the White group, and the Blue group. These nine students participated in the pre, post, and delayed-post assessments of equivalent fraction knowledge, the intervention (RNP/VM), daily fraction addition assessments, and the social validity portion of this study. The multiple baseline component of the study and post-test EFT concluded just before the school's spring break. When the nine students returned 11 days later, two students had exited the program and as a result were not available to participate in the delayed post-test EFT or the social validity survey. Table 4 provides selected demographic information on the student participants arranged by group. All nine students participated in the intervention and took the daily fraction assessments when present. When absences occurred, returning students received one-on-one instruction for the missed lesson and then participated in the group lesson for the day.

Table 4

Selected Demographic and Academic Information on Participants

Characteristic	Red Group	White Group	Blue Group
Age			
11	0	0	1
12	1	2	0
13	1	1	1
14	1	0	1
Grade Level			
6th	1	1	1
7th	2	1	1
8th	0	1	1
Gender			
Male	3	2	1
Female	0	1	2
Race			
Caucasian	3	1	3
Black	0	2	0
Disability Status			
EBD	3	3	3
SLD	0	1	0
OHI/ADHD	1	2	2
ASD	1	0	0
Accommodations			
Reading	1	2	1
Calculator	3	3	3
Additional Time	3	3	3
ESOL	0	0	0
State Annual Assessment Reading			
No score	1	1	0
1	2	2	1
2	0	0	2
3	0	0	0
State Annual Assessment Math			
No score	1	1	0
1	2	1	2
2	0	1	1
3	0	0	0
Attendance During Study			
Days absent w missed assessment	1	4	2
Days absent w make-up assessment	0	0	0

Setting

The setting for this study was a self-contained special education classroom for middle school students identified with EBD located in a Central Florida public school. This particular school district had previously mandated online courses for all students in this educational placement. The EBD unit consisted of three distinct content classrooms housed in separate portables: English Language Arts, social studies/ reading, and mathematics/science (mathematics). A special education teacher staffed each of these content classrooms and all students rotated among the classrooms and received their instruction or accessed online courses for each content area daily. This study was confined to the mathematics class and included one portable classroom, one teacher, and included students in grades 6-8.

The Flex model defined by Staker and Horn (2012) best describes the blended learning model planned for this classroom. Students were enrolled in online mathematics, and their special education mathematics teacher served as the teacher-of-record. The diagram in Figure 10 depicts the design features of this classroom that are similar to those in Staker and Horn's report of the Flex model.

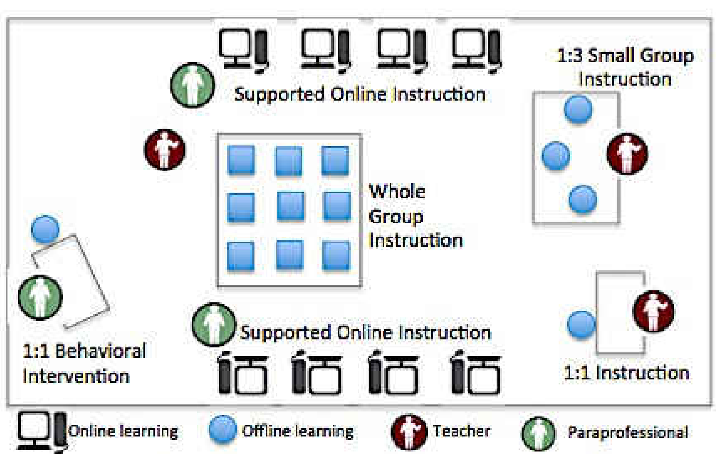

Figure 10: Flex model of blended learning used in study setting.

Using the Flex model the teacher planned to provide a variety of face-to-face supports for the online content in this blended environment. The following vignette typifies a day in this classroom prior to intervention.

Students enter the mathematics/science classroom targeted for this study and make their way to an assigned computer to access their online courses. They are expected to enter, sign into their science or mathematics course, and stay engaged in the online content for the duration of the 90-minute block. The transition time (that begins upon entering the classroom from breakfast, lunch, or another classroom) varies widely by student and often takes as much as 20 minutes before all students are seated and in their online course. Once online, the students are expected to independently read, listen, and answer questions as part of the online content. Most students call out loud to the teacher or paraprofessional for help in understanding the content or answering practice or assessment questions.

In response to student requests for assistance, the teacher and/or paraprofessional move to each individual student who has asked for help to provide one-on-one assistance. All academic support is provided at the student's computer station. Many questions involve reading support or reminding students of math concepts and procedures they should already know. If a student is unable to make the connection and move forward, the teacher provides on the spot remediation or support to help them work problems, understand concepts, and move forward. Each student works in a grade-level mathematics course and is generally at unique place in the curriculum. Despite the diversity of progress in the curriculum, the teacher and paraprofessional repeatedly address the same types of help questions; most involve arithmetic skills first taught in elementary school. The frequency of student requests for help make it necessary for students to wait for help, increasing time off task, disruptions, and other behavioral issues.

An announcement is made 10 minutes prior to the end-of-block transition to allow students to complete a section or test before logging out of their course. Many students quickly end their online sessions as soon as the announcement is made and spend most of that time engaged in off task behaviors.

The structure and challenges of the setting are clearly depicted in a typical day in the classroom setting. Due to the unique nature of this setting, the researcher had to consider several variables related to both selecting participants and introducing interventions. Dispositions and behaviors of this group of students with EBD made it difficult to single out a student or small group of students for a prescribed intervention. In order to avoid negative reactions, outbursts, refusals, and discord in the classroom during this study the researcher used the natural grouping of students by class to define treatment groups. This grouping allowed all students in the same class to simultaneously enter the various phases of the study and experience the same instruction and use of VM. The groups drawn from the classes were designated as Red, White, and Blue groups.

All students were enrolled in their own grade-level appropriate online mathematics course (6^{th}, 7^{th}, or 8^{th}) and rotated daily into a 90-minute block of time for mathematics instruction. The master schedule of class times across the three content area classrooms rotated weekly to mitigate the behavioral challenges of the after lunch block. The rotating schedule meant that each week a different content area class was scheduled after lunch for each group of students.

The groups were all served in the same mathematics classroom that had eight computers, one for each student's individual use. Computers were positioned around the perimeter of the room and students sat adjacent to each other, four on one side of the room and four on the opposite side of the room. The teacher used a Dell laptop computer with Windows XP that accessed the Internet through the school network. Student computers varied, but were typically older PC's of various brands with Widows XP that accessed the Internet through the school network. A ceiling mounted projector allowed the teacher to share her laptop screen, providing easy viewing for students. The following describes a typical day in the classroom during the study intervention.

On a typical day the students enter the mathematics classroom and move to their computer stations for group instruction. The teacher quickly addresses students who are slow in entering, and they move promptly to their assigned computers. The teacher introduces the lesson and tells students which manipulatives will be used during the lesson. Students use bookmarks in Internet Explorer to go to the virtual manipulative pages they need to access.

The teacher begins the semi-scripted lesson and demonstrates how to use the VM to perform the instructional tasks. Students mimic the teacher's manipulations of the VM and duplicate the tasks. The teacher goes through each step of the instruction prompting students to perform tasks and answer questions with the VM. Each student performs the tasks on their own computer and responds aloud to teacher questions. This portion of the intervention takes approximately 12 minutes and involves all students.

After the group instruction, the daily practice activities are distributed to each student, and they are expected to work independently to perform the VM tasks and answer questions on the worksheets. The teacher circulates around the room prompting students to be on task and provides support to individual students as needed.

After the 8-10 minute independent practice time, worksheets are collected and the daily FAA is distributed facedown. Once all students have their FAA, the teacher begins the 5-minute timed countdown for the assessment. At the end of 5-minutes all assessments are collected. Students are instructed to sign on to their online course and attend to the instruction and assessments. Students work on their online course with teacher and paraprofessional support until the end of the period.

The primary change in classroom routine in this setting was the addition of a block of time devoted to face-to-face instruction by the teacher which resulted in a corresponding reduction of time available for online instruction. In addition, the 10-minute warning bell near the end of the block was eliminated to minimize the reduction of time available for online coursework. Due to the unique nature of this setting and the behavioral considerations of these students, changes were implemented during the week prior to the initiation of baseline.

Teacher and Paraprofessional

The dually certified special education mathematics teacher assigned to all three groups of students was a highly qualified dually certified teacher qualified by district standards. The teacher had all three groups of students during sequential 90-minute blocks. Three groups of mixed grade students rotated into her classroom each day to access their online course.

The teacher reported that she has been teaching mathematics to students with EBD in a segregated setting at this school since she began teaching 12 years ago. She held a Florida

Professional Teaching Certificate with certification in 6-12 English and K-12 special education (EH and ASD) with reading and ESOL endorsements. In addition she meets the requirements of a highly qualified mathematics teacher through the High Objective Uniform State Standard of Evaluation (HOUSSE) plan that is a part of NCLB. Since receiving her initial certification, the teacher has participated in graduate courses and has obtained a Master's degree in Special Education and enough additional hours for a specialist certificate in curriculum and instruction.

All three classes had the same full time instructional paraprofessional. This paraprofessional's first day on the job coincided with the first day of baseline data collection. She had been an instructional assistant in an elementary classroom for students with ASD for 1½ years prior to this position and a one-on-one high school aid for a student in a mild/moderate disabilities classroom. This setting was her first assignment in a middle school classroom for students with EBD. The paraprofessional held a B.S. in Agriculture and was a stay at home mother prior to her employment as a paraprofessional.

Because the paraprofessional was new to the class, she took a passive role in the classroom during the weeks of the study. For the five weeks of the study her activities were primarily observational; she recorded behavior points and watched students travel to and from the classroom as necessary. The paraprofessional did not interact with the students related to their instruction or independent learning activities during the course of this study.

Interobserver

The Interobserver selected to evaluate the instructional fidelity of the intervention was a master teacher with 19 years of teaching experience, dually certified in special education and middle school mathematics. The Interobserver agreed to participant in a review of the intervention and practice observations in preparation for evaluating the intervention instruction by video recording.

Independent Variable

The conceptual fraction intervention selected for this study utilized the RNP *Initial Fraction Ideas* (IFI) lessons 1-15 (Cramer et al., 2009) along with three VM (fraction circles, chips, and paper folding) developed by RNP. The lessons used in the study can be found on the Rational Number Project website (http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html).

IFI Lessons

The IFI lessons were designed to build basic fraction conceptual knowledge based on the National Council of Teachers of Mathematics (NCTM) *Principals and Standards for School Mathematics* (2000). They are consistent with the Common Core State Standards (CCSS) in the areas of part/whole (CCSS.Math.Content.3.NF.A.1), equivalence (CCSS.Math.Content.3.NF. A.3, 4.NF.A.1) and ordering (CCSS.Math.Content.4.NF.A.2). The IFI curriculum consists of 23 lessons with the following components: an overview of the mathematical idea being developed (learning goal), a materials list, and loosely scripted teacher actions. Each lesson contains the following components: a 5-10 minute warm up activity that is a review of the previous lesson, a teacher led large group instruction session targeting concept exploration through the guided use of manipulatives, independent tasks that allow students to explore concepts through lesson activity sheets and manipulatives, and a wrap up activity designed to bring the class back together as a group to discuss independent activities or work a final problem. Instructional time varies for each lesson and generally ranges from 30-45 minutes. Mimicking Westenskow's 2012 use of the intervention, Lesson 1 and Lesson 2 were combined to create a more comprehensive initial lesson.

Virtual Manipulatives

The IFI lessons were designed for use with physical manipulatives and had not been modified at the time of this study to reflect minor changes in lesson directions that were necessary when using VM in place of their concrete counterparts. The researchers who created RNP added a set of three VM that are described in Chapter Two and pictured in Figures 7, 8, and 9. Each of these VM was designed to mimic the manipulation of a physical manipulative with the added features of immediate feedback (in the fraction circle manipulative) and the ability to display fraction values.

Validity

Validity of the components of RNP/VM was established through numerous research studies. The IFI lessons were developed and validated as a part of the extensive research provided to the RNP by NSF (see Cramer & Henry, 2002; Cramer et al., 2002). The use of VM as a substitute for physical manipulatives has been validated in studies unrelated to the IFI lessons (Moyer-Packenham, Baker, et al., 2013; Olkun, 2003; Suh & Moyer-Packenham, 2007) as well as a study that utilized IFI lessons (Westenskow, 2012). While there is no prior research evidence that validates the suite of VMs recently developed through RNP funding, this researcher piloted their use in a content analysis of the IFI lessons.

Validation of RNP VMs through Content Analysis

In order to provide evidence of the VMs validity this researcher utilized them as part of a content analysis of the IFI lessons. The RNP VMs used in this study were online applets that provided students the opportunity to work with virtual fraction circles, chips, and fraction strips that were designed to mimic the use of physical manipulatives written into the original IFI lessons. To conduct this analysis the researcher first reviewed the lessons for language that was not applicable for the use of VMs. Next she clarified instructions and directions to guide the teacher and student in the use of VM as a part of IFI lesson activities.

The content analysis revealed the need for only minor changes in the semi-scripted instructions to students and task directions for the teacher related to the use of the fraction strip manipulative in lieu of a physical paper strip. Modifications were identified for lessons 4, 7, 10, and 12. These changes in language did not change the intent or content of the lessons; they were simply adaptations for use of VM in lieu of physical manipulatives. For example: a lesson instruction to students stated, "fold a paper strip in half to make two equal lengths " which required a modification in verbal instructions for using the VM to state, "select a fraction strip that has two equal parts." A table of changes to lessons, a sample lesson from the RNP series, and a modified version of the same lesson can be found in Appendix C.

Finally in a series of simulated lessons with the researcher in the role of teacher, she delivered the revised IFI instruction calling for the use of VMs to a graduate student, unfamiliar with VMs and uncomfortable with fractions, serving as a surrogate student. The intent of this simulation was to (a) confirm that the virtual form of the physical manipulative found in each IFI lesson would allow students to complete the practice activities as designed by the IFI authors, (b) that the use of the VM in those activities would develop equivalent fraction knowledge in the same manner as use of the physical manipulative, and (c) to validate the clarity and accuracy of the modified language in the semi-scripted instructions.

The researcher, in conducting the simulated lesson activity, identified a need for an informal session to introduce the use of each of the VMs and demonstrate how each could be used to explore fraction concepts. This introductory session was necessary to demonstrate how to access the tools, to allow students to practice a response to applet's automated feedback, and familiarize students with how to turn the fraction labeling option on and off. Practice sessions for each tool were scheduled for the first day that each VM appeared in a lesson. Practices sessions were scheduled for fraction circles prior to Lesson 1, for paper folding prior to Lesson 4, and for chips prior to Lesson 12.

Research Design

Gast and Hammond (2010) note that all single subject experimental design studies use baseline logic as the foundation for establishing experimental control. Participants serve as their own control when measures are taken on target behaviors during a baseline period and then compared to measures in an adjacent intervention condition. A number of single subject models exist and all incorporate baseline logic by comparing adjacent conditions and replicating those initial results by withdrawal or reversal of intervention; or replication across participants, conditions, or behaviors. In this study the researcher used a multiple baseline across participants design with groups selected from three mathematics classes for the purposes of inter-group replication (Sidman, 1960). In total nine students qualified for participation in the study and formed groups that were designated as Red (*n*=3), White (*n*=3) and Blue (*n*=3). Data were reported using group medians and means as well as the individual effects for each participant, exposing variability among participants and creating a simultaneous replication design (Gast & Ledford, 2010; Kelly, 1980).

Dependent Measure

Group median fluency was the primary dependent variable for the single subject study. Fluency was defined as the number of correct responses divided by time.

Instruments, Data Collection and Analysis

Data were collected through a series of five-minute FAAs that were randomly generated to contain 20 fraction addition problems with numerators and denominators ranging from 1-5 and excluding mixed numbers and improper fractions. The decision to limit the range of numerators and denominators and exclude mixed numbers and improper fractions was made in an effort to minimize frustration and improve engagement. The online worksheet generator used in this study can be found at The Teachers Corner (http://worksheets.theteacherscorner.net/make-your-own/math-worksheets/basic-math/fractions-equations.php) and contains options to create randomly generated problem worksheets with most of those specifications. A sample assessment can be found in Appendix D. To control for variations in difficulty, assessments generated by the worksheet website were reviewed by the researcher and discarded if they contained less than two or more than three common denominator problems.

The daily FAA was delivered to the entire class at the close of the RNP/VM activities each day. The teacher distributed the assessment to each student face down on their workspace. The teacher started the assessment by setting a timer for five minutes and saying, “start”, indicating that the students could begin working. Students with calculator accommodations were allowed to use a calculator on the assessment. Students answered as many questions as possible in any order they chose, skipping problems at will during the five-minute time period. The teacher gave a one-minute warning, by saying “one more minute” at the four-minute mark, indicating to students that time was coming to an end. When the timer sounded, the teacher called out “stop, put your pencils down.” The teacher and researcher observed student compliance with the stop command and marked late responses with “time expired” as assessments were collected from all students. Any student responses recorded after time expired were not included in the number correct during scoring.

The teacher scored the FAAs using a solution key. A correct response was defined as a fraction solution in any form, lowest terms or any equivalent fraction. Each correct response counted as one point, making 20 points the total possible score on the FAA. All assessments were scored a second time by the researcher to insure grading reliability. Point-by-point IOA required an item-to-item comparison of the two scores, no scoring discrepancies were identified during the study. As a result, point-by-point IOA was calculated to be 100% and confirmed fidelity of scoring. Individual fluency scores were calculated to the nearest tenth by dividing the number of correct responses by five minutes. The result was a fluency value ranging from 0.0 to

4.0, representing problems per minute (p/m), which was used in the median calculations for each group. Appendix D contains a sample FAA with a scoring key and a daily score recording sheet with the median calculation framework that was used to arrive at group median scores.

Procedures Used to Avoid a Ceiling Effect

Based on consultations with three mathematics experts, the study mathematics teacher, an assistant professor of special education with mathematics expertise at a western university, and a professor of mathematics education at the University of Central Florida, it was anticipated that few if any participants would complete all the problems within the five-minute time frame at any point in this study. Twenty problems were selected as a number that was adequate to allow for variances in ability and allow growth, but low enough to prevent frustration or anxiety from the perceived magnitude of the task.

As additional protection against a ceiling effect, baseline data were charted for individual students to identify any that had the potential of completing all 20 problems in the five-minute time frame with 85% accuracy. Number correct and number attempted were charted for each student during baseline and compared to the priori threshold of concern (85% accuracy on at least 12 attempted problems). The researcher conducted a visual analysis and identified one student who had exceeded the threshold of concern on two assessments during baseline. That student was excluded from data collection and analysis during the study.

Anecdotal Records

The researcher maintained a daily log to capture observations related to the classroom and school environment as well as student and teacher dispositions and verbal expressions during the study. The goal of this data collection was to provide context that could prove helpful in analyzing results as well as identifying and dismissing rival explanations. Anecdotal notes were made during researcher observations that aligned with the following questions: (1) Who is present? (2) What are students engaged in doing? (3) How are students responding to the intervention? (4) What are students saying about the intervention? (5) What factors may be interfering with student engagement? (6) How are students responding to instructions from the teacher? (7) How are the teacher and paraprofessional responding to disruptions or off task behavior? (8) Were there any other factors that impacted today's instruction?

Procedures

This two-component research study had four distinct phases. The pre-study phase was used to prepare the students for a change in classroom routine, prepare the teacher to deliver the intervention instruction, identify participants for intervention groups, gather data on students' current level of performance along with demographic and historical information from school records, and administer the pretest EFT. The baseline phase commenced once all pre-study tasks were completed. Once stability of baseline had been established and maintained over five data points, the intervention phase began. A final post-intervention phase allowed for an analysis of student retention and began immediately after the final administration of the EFT for each group and concluded for all groups after the administration of the delayed post-test to all groups one day after students returned from an 11-day spring break vacation.

Phase I: Pre-Study

Ten tasks were identified as necessary before baseline data collection could begin. A list of those tasks can be found in Appendix E. The pre-test EFT was administered to all students prior to the commencement of baseline data collection.

Phase II: Baseline

Baseline began when the pre-study tasks were complete and the pretest EFT had been administered. During baseline, the teacher intentionally changed the normal classroom routine to include a short whole group instructional session at the beginning of each math block, which was followed by delivery of the FAA. This routine continued daily for five consecutive days at which time all three groups had met the baseline stability criteria required to enter intervention.

In order to determine which group would enter the intervention phase first, the researcher used factors such as student compliance, anticipated attendance, teacher input, and scheduled mathematics times to evaluate the best intervention rotation. The Red group was selected to receive the intervention first while the White group and Blue group continued in an extended baseline taking FAAs two times each week including the day before moving into the intervention phase.

Phase III: Intervention

During intervention, the teacher delivered the IFI daily lesson while students performed the instructional tasks using the specified VM on their own computers. Appendix F contains a table of the scope and sequence of lessons that were presented to students. The teacher assisted students as needed in following instructional directives and demonstrating fraction concepts with VMs. The RNP practice activities were originally designed for small groups and it was the intention of the researcher that students work together to complete the activities. However, on the first day of intervention students in the Red group could not find their partners and remain on task during the transition to small groups, consuming so much time that the teacher decided to have each student work independently on the activities. As activities moved from teacher directed to independent, the teacher circulated around the room to guide, support, clarify, reteach and provide positive reinforcement to students as they completed fraction tasks. The daily intervention time concluded with a five-minute FAA.

All students present when intervention instruction was delivered to their class participated in the intervention activities. Five absences by participants required that they make up the missed instruction in addition to participating in the current day’s intervention activities. No student missed more than one day. Students made up lesson content when they returned from an absence but did not take the missed FAA.

Phase IV: Post-Intervention

Spring break began for these students immediately after the third group (Blue group), completed the intervention phase of the study. Following the final post-EFT, school was closed for a spring break that spanned 11 calendar days. After the break, only seven of the nine study participants returned to the classroom; both had returned to the general education setting. As a result only seven students took the delayed post-EFT and the social validity survey, which were the culminating events of the study.

Implementation with Fidelity

A number of factors were considered to insure the fidelity of implementation of the various features of this research. The delivery of the intervention (instructional fidelity) and the scoring of student assessments were both identified and scored by an Interobserver.

Instructional Fidelity

Three factors were considered in the effort to ensure that the intervention instruction was delivered with fidelity. First, a rubric was developed to evaluate the fidelity of the teacher's instructional delivery. The rubric contained seven points of competency with rubric component scores ranging from 0 to 2. A total of 14 points could be earned. For purposes of verifying instructional fidelity, proficiency was established at 12 of 14 rubric points, which equated to the 85% criteria. Second, the teacher was provided with an orientation including a rationale for the intervention and an opportunity to practice the delivery of instruction with feedback. Finally the Interobserver was prepared in a similar fashion and also participated with the researcher in a detailed collaborative review of the evaluation rubric components and standards. Competency for instructional fidelity was established at 85% with required IOA also 85%.

Both the teacher and Interobserver were provided with materials designed to prepare them for the delivery (or evaluation) of the instructional lessons. Those materials and activities included a brief history and sampling of research evidence on the effectiveness of RNP; an overview of the structure of the research study; a copy of the RNP curriculum; the evaluation rubric; opportunities to explore the use of the VM; a review of RNP goals, scope, and sequence; and separate guided practice (or evaluation) time under the tutelage of the researcher.

Pre-study Fidelity Check

At the conclusion of the self-study and guided practice and prior to the baseline phase, the teacher's competency in delivering the intervention instruction was assessed by the researcher and Interobserver in two consecutive demonstration lessons. The teacher delivered the RNP lessons to a surrogate who assumed the role of a student. The researcher scored the instructional delivery live on the rubric while the Interobserver watched a video recording of instruction later the same day and scored the teacher on the same rubric. The teacher's score was determined by dividing the earned rubric score by the total possible rubric points resulting in a percentage score. Interobserver agreement was determined by a point-by-point comparison of the seven rubric components to identify the number of components that were scored identically by the two observers. The total number of observable components divided by the number of components with identical scores for the two sessions resulted in a reportable IOA percentage for the pre-study period.

Ongoing Fidelity Checks

Once the intervention phase commenced, the researcher assessed the teacher's instructional delivery every two days and one of those sessions each week was video recorded to allow remote scoring by the Interobserver. During the intervention phase, the researcher evaluated a total of 15 sessions, eight of which were recorded for IOA purposes. Point-by-point IOA was used to determine the number of rubric components scored identically by the two observers. The IOA percentage was calculated by dividing the number of components with identical scores by the total number of rubric components from the eight sessions.

Fidelity of Scoring

Point-by-point IOA was used to ensure accuracy of scoring. All assessments were scored first by the teacher and a second time by the researcher (see Appendix G). The criterion for IOA was established at 100% due to the sensitive nature of the reported score to scoring errors as a result of the low number of problems (20) on the assessment and small group size.

Data Analysis

The researcher utilized visual analysis to evaluate the effect of the intervention on the dependent variable. The data gathered during the multiple baseline study provided group median fluency rates from daily FAAs that were graphed for visual analysis. Between-condition and within-condition analyses were conducted. Two measures of effect size were calculated. The effects of the intervention on student performance on the EFT were calculated by comparing the pre, post, and delayed post-test group means for the three groups.

Visual Analysis of Multiple Baseline Data

A systematic visual analysis is generally used to analyze data displayed in graphs that result from single subject studies. Data for this multiple baseline design were gathered from the administration of the FAA. The FAA was administered during the first five consecutive days of baseline and twice a week for each group that was waiting to enter the intervention phase of the study. In addition, the FAA was administered to each group during their respective intervention phases, delivered daily at the conclusion of the intervention instruction. As a result, the Red group took five FAAs during baseline and five FAAs during intervention, the White group took seven FAAs during baseline and five FAAs during intervention, and the Blue group took nine FAAs during baseline and five FAAs during the intervention phase of the study.

All data were graphed on a series of vertically adjacent y- axes, one for each tier of the study. Each y-axis represents group median fluency and the x-axis represents the 20 days of the study from the first day of baseline to the last day of intervention for the Blue Group, the last to enter the intervention phase of the study. The data were displayed in graphs of adjacent conditions that allowed for a visual analysis of the effects of the intervention on group fluency. These analyses were conducted on the level, trend, and stability of the data points (Gast & Ledford, 2010). Gast describes these terms in detail as (a) trend – progress over time, (b) level – magnitude of the data, and (c) stability – variability or bounce of the data (Gast, 2005). These factors provided the foundation for visual analysis (Lane & Gast, 2013).

In addition to group means, the simultaneous replication design described by Kelly (1980) required that data for each participant be graphed in addition to group data, providing inter-subject replication. Individual fluency data for all participants was also graphed for each group with a scale and ratio for the visual display that allowed a clear interpretation (Gast & Ledford, 2010).

Within-Condition Analysis

A within-condition analysis was conducted to evaluate data patterns within the baseline and intervention phases of this study (Gast & Spriggs, 2010). In the baseline condition, the researcher looked for stability of data across five data points before qualifying a group to enter the intervention phase (Horner et al., 2005). In the intervention phase the researcher looked for stability of data across five consecutive data points at which point data collection was terminated. Lane and Gast (2013) present seven steps for conducting a within-condition analysis of graphed data:

Step 1. An upper case letter was used to identify and label conditions. The baseline conditions were identified with the letter A and the intervention conditions were identified with the letter B.

Step 2. Sessions in each condition were counted and numbered.

Step 3. Stability of data was established in each condition. Two measures of central tendency and the range of data for each session were calculated for each group by using the scores of the session participants. Adding the scores of session participants and dividing the resulting sum by the total number of participants present for that session produced the group mean. Listing the scores of participants in numerical order and identifying the middle value (averaging the two middle values if there are an even number of scores) calculated the median for each session. The range for each session was designated by noting the least and greatest values in the session. A stability envelope was established by applying the criteria of 80% of data points contained within ± 25% of the median. The final step in establishing stability was to calculate the percentage of data points that fell on or within the stability envelope in each condition.

Step 4. (a) The relative level change in a condition was calculated. In each condition the data points were split in two and the median value identified for each half using the median procedure described. Since there were an odd number of data points, the middle value was excluded. Next the smaller median value was subtracted from the larger, and trends were identified as improving (a positive value) or deteriorating (a negative value).

(b) The absolute level change in each condition was calculated. The researcher identified the value of the first and last data points, subtracted the smaller of the median values from the larger and indicated if the change was improving (a positive value) or deteriorating (a negative value).

Step 5. The trend in each condition was estimated. The midpoint of the sessions in a condition was marked with a dashed vertical line. A solid vertical line marked the mid-date (middle session) for each half. Since there were an odd number of sessions, the middle session was excluded. The median value for each half was found on the ordinate and marked with a short solid horizontal line at the midpoint for each half to identify the mid-rate. A line was drawn to connect the two points at which the mid-date and mid-rate intersected in each half to create a trend line. The trend line was moved vertically until an equal number of data points were positioned above and below the trend line.

Step 6. Trend stability in each condition was calculated. The stability envelope that was established in Step 3 was applied to the trend line created in Step 5 to create a trend line that was paralleled by a stability envelope. The final step was to calculate the percentage of data points that fell on or within the stability envelope in each condition.

Step 7. The data paths in each condition were evaluated. The original graphic display of data, along with the calculations and information generated when required in Steps 1-6, was used to evaluate the trend in each condition and allow the researcher to describe trend direction (improving, deteriorating, or unchanged), stability (stable or variable), and presence of multiple data paths (yes or no) for each condition.

Between-Condition Analysis

The researcher conducted a between-condition analysis to evaluate the data across adjacent conditions (baseline and intervention) to look for an immediate change in level and trend. Trend is considered the more important factor to the researcher when conducting the visual analysis (Gast & Spriggs, 2010). In addition to level and trend, the researcher considered the stability or variability of the data through the observation of overlapping data points across conditions. Lane and Gast (2013) present five steps for conducting a between-condition analysis of graphed data. Those steps were applied to this study as listed and described below.

Step 1. The number of variables that changed between conditions was noted. Only one independent variable, RNP/VM, was introduced between baseline and treatment.

Step 2. Any changes in direction of trend between conditions were categorized. Changes in trend direction were described as decelerating or accelerating. Changes in trend between conditions with respect to the anticipated therapeutic direction were described as improving or deteriorating.

Step 3. Stability across conditions was summarized for each condition.

Step 4. Level changes between conditions were noted.

(a) The relative level change was calculated by subtracting the median value from the second half of the baseline condition from the median value from the first half of the intervention phase. Level was characterized as improving or deteriorating.

(b) The absolute level change was calculated by subtracting the last value of the baseline condition from the first value in the intervention phase. Level was characterized as improving or deteriorating.

(c) The median level change was calculated by subtracting the median value of the baseline condition from the median value of the intervention condition. Level was characterized as improving or deteriorating.

(d) The mean level change was calculated by subtracting the mean value of the baseline condition from the mean value of the intervention condition. Level was characterized as improving or deteriorating.

Step 5. Overlap of data points between conditions was noted.

(a) The percentage of non overlapping data points (PND) was calculated by marking the highest value in baseline and counting the number of data points that were above that value in the intervention condition. That number was divided by the total number of sessions in the intervention phase and then multiplied by 100%.

Effect Size

Two effect size analyses were conducted to measure the effects of the intervention by using the graphic display of data. The PND, a commonly used estimate of effect size in single subject studies, was calculated by counting the number of data points in the intervention condition that were above the highest value in the baseline condition (scores were expected to increase) and dividing that number by the total number of data points in the intervention condition then multiplying the result by 100% (Scruggs et al., 1987). The resulting values were characterized as >90%, a large effect; 70% to 90%, a medium effect; and 50% to 70%, a small effect.

The Tau-U is "an index of between and within-phase trend" (Parker, Vannest, Davis, & Sauber, 2011, p. 8) and combines non-overlapping data and trends. This non-parametric measure is useful for small data sets, calculated by the formula $\frac{(C-D)}{\frac{n}{2}}$, with C representing the number of concordant pairs and D representing the number of non-concordant pairs. The Tau-U Calculator found at http://www.singlecaseresearch.org/calculators/tau-u was used to conduct the Tau-U analyses for each group.

Analysis of Pre, Post, and Delayed Post-test EFT

Three forms of the EFT were created by Westenskow (2012) to serve as a pretest, post-test and a delayed post-test. The pretest was administered on the last school day before baseline (day 0), the post-test was administered the school day following the end of the intervention phase for each group (day 8, 15, or 22), and the delayed post-test was administered to all participants on day 44, the second day after students returned from spring break. As a result, the number of days between the post-test and the delayed post-test varied for the groups. The delay between the post and delayed post-EFT was 29 calendar days for the Red group, 22 calendar days for the White group, and 15 calendar days for Blue group.

To evaluate the impact of the intervention on conceptual understanding of equivalent fractions, the following group statistics were calculated and reported for each test administration and comparison: mean, range, and standard deviation. Comparisons were made between the pretest and post-test group means to evaluate any change in equivalent fraction knowledge. The comparison between the post-test and delayed post-test means required an adjustment to the post-test means for both the White and Blue groups to eliminate the post-test scores of the two students (one from each group) who did not have a delayed post-test score for comparison. These adjusted group means for the post-test were compared to the delayed post-test means to evaluate retention of equivalent fraction knowledge. In an effort to expose variability of the reported group results, individual raw scores were reported and evaluated for net change in performance over the three administrations of the EFT.

Social Validity

The social validity of RNP/VM was evaluated using the data collected through the survey questions found in Appendix H. Student participants, the teacher, and the paraprofessional completed a Qualtrics survey online to validate the social importance of the goals, procedures, and outcomes of RNP/VM (Wolf, 1978). After spring break and the administration of the delayed post-test EFT, student participants were asked to sit at one designated computer to complete the online survey. Only six of the seven students that took the delayed post-test also completed the survey. One student left school early, before the survey was administered.

The classroom teacher or paraprofessional sat with each student as they took the survey to read or clarify questions as needed. The teacher and paraprofessional received the link to their survey immediately after the completion of the delayed post-test. Both individuals completed the survey within 24 hours. Participants recorded their responses by selecting their level of agreement from a five-point Likert scale with the number one correlated to strongly agree and the number five correlated to strongly disagree.

Results of the Qualtrics survey were exported to an Excel spreadsheet and individual questions were organized and labeled by themes related to the survey goals: Theme A – perceptions about the use of RNP/VM, Theme B – level of agreement with goals of the intervention, and Theme C – level of agreement with academic and behavioral outcomes for students and teachers. Additionally, Theme C for the student survey was further disaggregated by whether the question was related to academic problem solving (AP), academic conceptual knowledge (AC), or behavior (B). The teacher survey utilized the same major themes with Theme C further disaggregated by whether the question was related to student academics (A), student behavior (B), or teacher behavior (T). The researcher exported the Qualtrics data into Excel, coded questions with themes and sub-themes, and created summary statistics. To ensure the accuracy of the researcher's reported results the Interobserver duplicated the researcher's process of exporting, coding, and recreated 100% of the data tables and statistics.

CHAPTER FOUR: RESULTS

The purpose of this study was to investigate the effects of a remedial, conceptual fraction intervention, a combination of lessons from the Rational Number Project and virtual fraction manipulatives (RNP/VM), on procedural fluency (fraction addition) and conceptual understanding of equivalent fractions in working with students labeled EBD. Three groups of middle school students with EBD, who were already assigned to self-contained special education classrooms and enrolled in online mathematics, were participants in the study. The researcher sought to answer the following questions:

1. To what extent does RNP/VM increase group median fluency on a five-minute fraction addition assessment for students with EBD?
2. Does RNP/VM increase equivalent fraction knowledge as evidenced by the results of a pre, post, and delayed post-test administration of the EFT?
    a. To what extent do mean scores increase from pre to post administration of the 20-item EFT, for groups of students with EBD?
    b. Given an increase in mean group scores from 2.a., to what extent do those gains persist after the end of the intervention?
3. How do the students with EBD and their respective teacher and paraprofessional perceive the goals, procedures, and outcomes of RNP/VM?

These research questions were answered using a multiple baseline design across three groups of participants who all had the same mathematics teacher and attended a single public middle school located in the Central Florida area. Using graphic displays, the researcher reports the results of the study including median fluency for each group on a daily assessment of fraction addition fluency. Student numbers are used to replace student names for purposes of anonymity. Student groups were identified as the Red, White, and Blue groups.

Research Question One

Research question one was addressed by the researcher using a single subject, multiple baseline across groups design and sought to answer the question: To what extent does RNP/VM increase group median fluency on a five-minute fraction addition assessment for students with EBD? The multiple baseline component of the research study was structured to address this question through the use of the RNP/VM intervention package and the FAA.

Multiple Baseline Across Groups of Participants

Baseline

To confirm stability in baseline the researcher calculated mean, median, and range for each group; all results are reported in fluency, problems per minute (p/m). During the initial baseline the following results were recorded. The Red group demonstrated a stable baseline; all five data points graphed were zeros (mean 0.0, range 0.0); no participants were absent during the initial baseline period. The White group demonstrated a stable baseline with four of five (80.0%) data points on or within the stability envelope (mean 0.08, range 0.0 to 0.4); no participants were absent during the initial baseline period. The Blue group demonstrated a stable baseline, all five data points graphed for the initial baseline period were zeros (mean 0.0, range 0.0); Student 7 was absent on Day 2 and Student 8 was absent on Day 9 resulting in the calculation of a group median for Day 7 and Day 9 from two rather than three FAA scores. All three groups met the 80% stability criteria during the initial five days of baseline, qualifying each group to move into the intervention phase of the study.

With three groups qualified to begin intervention, a decision had to be made about which group to move to intervention first. At the teacher's suggestion, it was decided to order the groups based on the expected class rotation schedule to insure that each group would enter treatment when the mathematics block was scheduled in the morning. As a result the Red group was selected to enter intervention first.

By day five of baseline, all participants reported they were tired of taking what they perceived as the same FAA every day. They reported frustration at "failing" the test every day and stated that it was "pointless" to continue. After consulting with the teacher, the researcher decided to discontinue daily FAAs during the extended baseline period for the White group and Blue group, and only probe for baseline fluency twice a week until intervention began, to prevent a potential behavioral confound in a "real" classroom setting. Probes were planned for Tuesdays and Fridays, with that schedule adjusted to ensure that a final baseline probe would be administered the day before a group would enter intervention. The White group waited an additional nine days after the initial baseline period to enter the intervention phase and took two additional FAAs. The Blue group waited an additional sixteen days after the initial baseline period to enter the intervention phase and took four additional FAAs.

Extended Baseline

The Red group entered the intervention phase first and as a result did not require an extended baseline. The total baseline phase for the White group spanned two weeks (Days 1-14), included both the initial and extended baseline periods, and produced seven baseline data points. Stability continued across the total baseline with six of seven (85.7%) data points on or within the stability envelope (mean 0.1, range 0.0 to 0.4), no participants were absent during the extended baseline period. The total baseline phase for the Blue group spanned three weeks (days 1-21), included both the initial and extended baseline periods, and produced nine baseline data points. Stability continued across the total baseline for the Blue group with all nine data points (100%) on or within the stability envelop, all nine data points graphed for the total baseline period were zeros (mean 0.0, range 0.0). In addition to the initial baseline period absences reported above for the Blue group, there was one absence during the extended baseline period. Student-8 was absent for Session 9, resulting in the calculation of the group median for session from two rather than three FAA scores. The assessment results during the extended baseline period were consistent with the results from the initial baseline period indicating that baseline stability was maintained for both the White and Blue groups.

The multiple baseline component of the study began with the administration of the FAA to all three groups of participants on the first day of baseline (Session 1). Group median fluency was calculated and graphed to allow for visual analysis of the data during the baseline and intervention phases of the study. A graph of the multiple baseline results can be found in Figure 11.

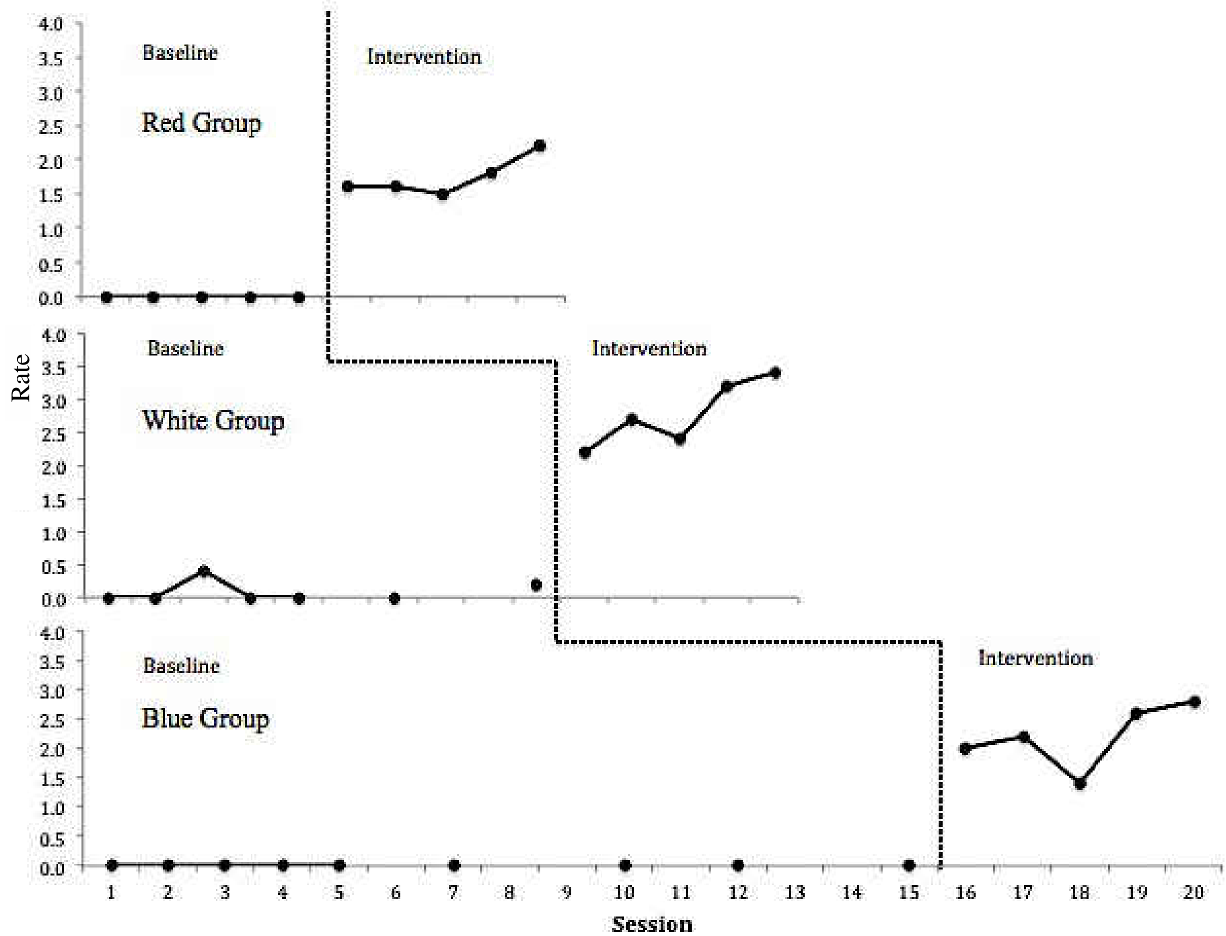

Figure 11: Multiple baseline across three groups of participants.

<u>Red Group Intervention</u>

Intervention began for the Red group on Day-8 (Session 6) of the study. All three participants were in attendance and expressed being happy at the prospect of doing something different. The researcher observed the students attending to the teacher's introduction to the intervention and each used his or her own computer to mimic the teacher's manipulation of the fraction circle VM in response to verbal instructions. All students in the Red group received the intervention instruction and participated in the independent activities and assessment.

The intervention phase for the Red group continued for five sessions (6-10) on consecutive days and terminated on Day-12 of the study. One data point was gathered for each session of intervention and graphed for visual analysis. During the intervention phase, each of the three participants was absent for one session; no more than one participant was absent for any single session. Each participant made up the missed intervention instruction the following day, but the missed FAA was not administered. Daily medians were calculated with the scores of only those participants who were present on the day the lessons were originally delivered; three of the five medians calculated represented the performance of two of the three participants. The median group fluency on the FAA for the five-day intervention phase (Sessions 6-10) was 1.6 p/m (range 0.7, mean 1.74), an increase of 1.6 p/m from the baseline median of zero. Visual analysis of intervention data confirmed an upward (improving) trend and stability (100% of the data points located within or on the stability envelope).

White Group Intervention

Intervention began for the White group on Day-15 of the study (Session 11). All three participants were in attendance on Day-15 and the researcher recorded their mixed opinions about the change in routine. Two participants, Students 4 and 5 stated they were concerned that the intervention would cause them to fall farther behind in their online math course. Student-6 told the researcher that he was very interested in learning to use the VMs. All three participants were observed cooperating with the teacher and participating in all intervention activities without further comments or complaints. The researcher observed the participants attending to the teacher's introduction to the intervention and each used their own computer to mimic the teacher's manipulation of the fraction circle VM in response to verbal instructions. Two of the students explored other uses of the VM in between activities. The teacher provided four verbal prompts to Student-5 during the practice activity to encourage him to complete the independent practice assignment. All students in the White group received the intervention instruction and participated in the independent activities and assessment.

The intervention phase for the White group spanned five sessions (11-15) on consecutive days and terminated on Day 19 of the study. One data point was gathered for each session of intervention and graphed for visual analysis. During the intervention phase, Student-4 was absent for Session 12 and made up the instruction the following day; the missed FAA for Student-4 was not administered. As a result the daily median for Session12 was calculated with the scores of only the two participants who were present for that session. The median group fluency on the FAA for the five-day intervention phase (Sessions 11-15) was 2.7 p/m (range 1.2, mean 2.8), an increase of 2.7 p/m from the baseline median of zero. Visual analysis of intervention data confirmed an upward (improving) trend and stability (100% of the data points located within or on the stability envelope).

<u>Blue Group Intervention</u>

Intervention began for the Blue group on Day-22. All three participants were in attendance on Day-22 and verbalized strong objections to what they perceived as a loss of time due to the intervention instruction. In response to their objections, the teacher explained how the skills they would learn would help them on the upcoming FCAT. The explanation seemed to allay their concerns because each of the three turned their attention to readying their computers to access the VM. Student-8 did not attempt to answer any problems on the FAA during baseline, when the assessment was placed on her desk she wrote her name and turned it over without looking at the problems. On Day-1 she had stated that she did not know how to add fractions.

Beginning with Session 16 (Day 22), when the VM was introduced, Student-8 attended to the instruction, mimicked the teacher examples, and worked every practice activity using the VM. She was observed twice asking the teacher for clarification on a task. Latency (time from teacher prompt to student action) varied (immediate to > 3 minutes) for each of the Blue group participants. All eventually engaged in the instruction, practice activities using VMs, and the daily assessment. Students 7 and 8 were given four prompts during the independent activity to stay on task and complete the activity. The teacher used proximity control (stood near students who were off task) to encourage them to attend to the assignment. All students in the Blue group received the intervention instruction and participated in the independent activities and assessment.

The intervention phase for the Blue group spanned five sessions (16-20) on consecutive days and terminated on Day 26 of the study. One data point for each day of intervention was gathered and graphed for visual analysis. During the intervention phase, Student-8 was absent for Session-17 and made up the instruction the following day; the missed FAA for Student-8 was not administered. As a result the daily median for Session 17 was calculated with only the two scores of the participants who were present. The median group fluency on the FAA for the five-day intervention phase (Sessions 16-20) was 2.2 p/m (range 1.4, mean 2.2), an increase of 2.2 p/m from the baseline median of zero. Visual analysis of intervention data confirmed an upward (improving) trend and stability (80% of the data points located within or on the stability envelope). Figure 12 displays a calendar that highlights critical study days.

Mon	Tues	Wed	Thurs	Fri	Sat	Sun
		Pre Study Days		**Day 0** All Groups **Pre EFT**		
Day 1 ***Session 1*** Red - Baseline White - Baseline Blue - Baseline	**Day 2** ***Session 2*** Red - Baseline White - Baseline Blue - Baseline	**Day 3** ***Session 3*** Red - Baseline White - Baseline Blue - Baseline	**Day 4** ***Session 4*** Red - Baseline White - Baseline Blue - Baseline	**Day 5** ***Session*** 5 Red - Baseline White - Baseline Blue - Baseline	Day 6	Day 7
Day 8 ***Session 6*** Red Intervention White – Ex BL Blue – Ex BL	**Day 9** ***Session 7*** Red Intervention White – Ex BL Blue – Ex BL	**Day 10** ***Session 8*** Red Intervention White – Ex BL Blue – Ex BL	**Day 11** ***Session*** 9 Red Intervention White – Ex BL Blue – Ex BL	**Day 12** ***Session 10*** Red Intervention White – Ex BL Blue – Ex BL	Day 13	Day 14
Day 15 ***Session 11*** **Red Post EFT** White Intervention Blue – Ex BL	**Day 16** ***Session 12*** Red – Post Int White Intervention Blue – Ex BL	**Day 17** ***Session 13*** Red – Post Int White Intervention Blue – Ex BL	**Day 18** ***Session 14*** Red – Post Int White Intervention Blue – Ex BL	**Day** 19 ***Session 15*** Red – Post Int White Intervention Blue – Ex BL	Day 20	Day 21
Day 22 ***Session 16*** Red – Post Int **White Post EFT** Blue Intervention	**Day 23** ***Session 17*** Red – Post Int White – Post Int Blue Intervention	**Day 24** ***Session 18*** Red – Post Int White – Post Int Blue Intervention	**Day 25** ***Session 19*** Red – Post Int White – Post Int Blue Intervention	**Day 26** ***Session 20*** Red – Post Int White – Post Int Blue Intervention	Day 27	Day 28
Day 29 **Blue Post EFT** Post Intervention All Groups	**Day 30** Post Intervention All Groups	Day 31 Post Intervention All Groups Spring Break	Day 32 Post Intervention All Groups Spring Break	Day 33 Post Intervention All Groups Spring Break	Day 34	Day 35
Day 36 Post Intervention All Groups Spring Break	Day 37 Post Intervention All Groups Spring Break	Day 38 Post Intervention All Groups Spring Break	Day 39 Post Intervention All Groups Spring Break	Day 40 Post Intervention All Groups Spring Break	Day 41	Day 42
Day 43 Post Intervention All Groups School Day	**Day 44** Post Intervention All Groups **Post EFT**					

Figure 12: Study Calendar

Summary

All groups achieved baseline stability in the initial five-day baseline period (Sessions 1-5) and qualified to enter the intervention phase of the study. The Red group was chosen to enter the intervention phase first. The White group followed the Red group and the Blue group followed the White group into intervention once the former group exited intervention and the twice-weekly baseline probes confirmed continued baseline stability. Each group required five days of intervention to demonstrate a consistent change in level of fluency and trend stability. In Table 5 the researcher has provided a summary of the within-condition analyses conducted to qualify groups to enter and exit study phases along with Figure 13, a visual display of the data.

Table 5

Within-Condition Analysis: Baseline and Intervention

	Red Group		White Group		Blue Group	
Conditions	Baseline	Intervention	Baseline	Intervention	Baseline	Intervention
Condition Label	A	B	A	B	A	B
Number of Sessions	5	5	7	5	9	5
Condition Median	0	1.6	0	2.7	0	2.2
Condition Mean	0	1.74	.09	2.8	0	2.2
Condition Range	0	.7	.4	1.2	0	1.4
± 25%	0	.44	.02	.56	0	.55
Relative Level Change	Stable	Improving	Improving	Improving	Stable	Improving
Absolute Level Change	Stable	Improving	Improving	Improving	Stable	Improving
% Data points in SE	100%	100%	85.7%	100%	100%	80%
Trend Direction	Unchanged	Improving	Unchanged	Improving	Unchanged	Improving
Trend Stability	Stable	Stable	Stable	Stable	Stable	Stable
Multiple Data Paths	No	No	No	No	No	No

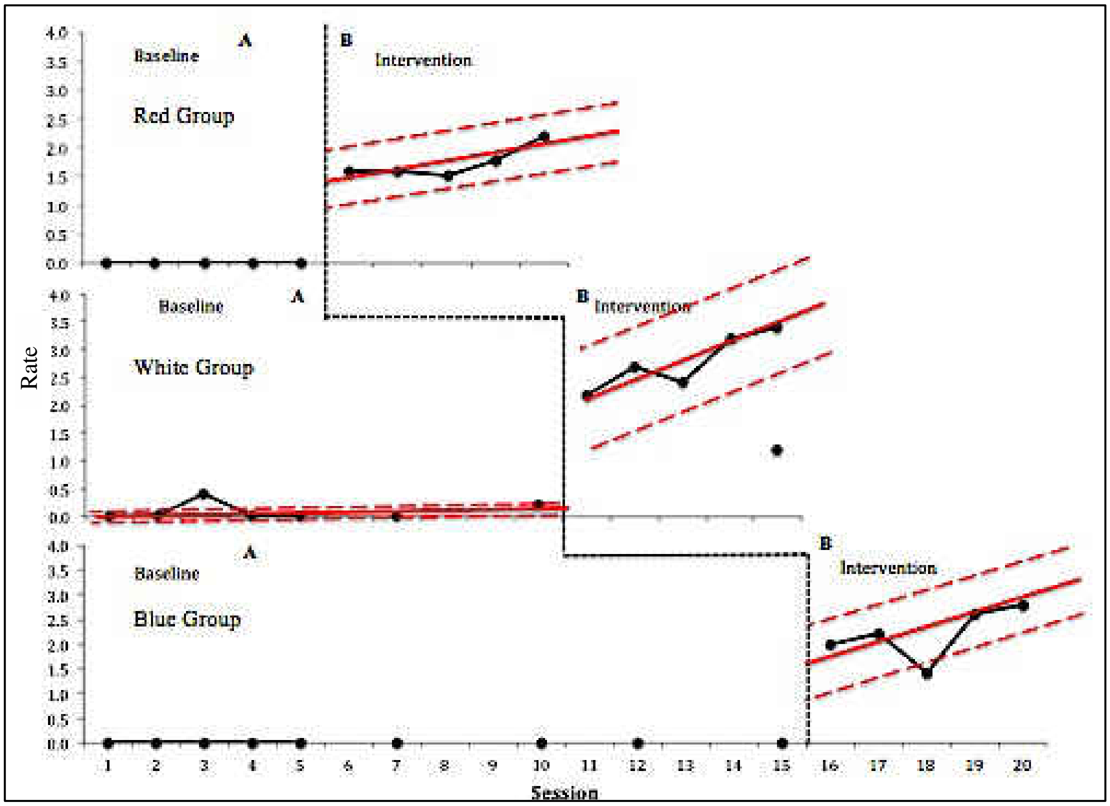

Figure 13: Within-condition stability.

Effect Size

Both PND and Tau-U were calculated in a between-condition analysis to measure the effects of the intervention on each group. Visual analysis of graphed data across conditions provided the information to calculate PND for each group. All intervention data points were higher than the highest baseline data point for each group. As a result, PND for the Red, White, and Blue group was calculated at 100%, a large effect size. The calculated Tau-U for the Red group and Blue groups displayed by the Tau-U calculator was 1.0, a large effect size. Table 6 contains a summary of the data resulting from the between-condition analyses. After the Tau-U calculator corrected for a baseline trend for the White group, Tau-U was calculated at 0.9143 for the White group, also a large effect size. Table 7 reports the complete results from the Tau-U calculator.

Table 6

Between-Condition Analysis

Conditions	Red Group	White Group	Blue Group
# Changed Variables	1	1	1
Direction of Trend	Accelerating	Accelerating	Accelerating
Relative Level Change	+1.6	+2.5	+2.1
Absolute Level Change	+1.6	+2.0	+2.0
Median Level Change	+1.6	+2.7	+2.2
Mean Level Change	+1.7	+1.9	+2.2
PND	100%	100%	100%
Tau-U	1.0	.9143	1.0

Note: + indicates improving

Table 7

Tau-U Analysis of Results for Research Question One

ID	Label	S	PAIRS	TAU	TAUb	VARs	SD	SD Tau	Z	P Value	CI 85%	CI 90%
Trend:												
0	RB vs RB	0	10	0	0	16.6667	4.0825	0.4082	0	1	-0.588<>0.588	-0.672<>0.672
1	WB vs WB	3	21	0.1429	0.1875	44.3333	0.3171	0.3171	0.4506	0.6523	-0.314<>0.599	-0.379<>0.664
2	BB vs BB	0	36	0	0	92	0.2664	0.2664	0	1	-0.384<>0.384	-0.438<>0.438
4	WB vs WB	3	21	0.1429	0.1875	44.3333	0.3171	0.3171	0.4506	0.6523	-0.314<>0.599	-0.379<>0.664
Phase:												
3	RB vs RI	25	25	1	1	91.6667	0.3830	0.3830	2.6112	0.0090	0.449<>1.551	0.370<>1.630
6	BB vs BI	45	45	1	1	225	0.3333	0.3333	3	0.0027	0.520<>1.480	0.452<>1.548
Corrected Baseline:												
5	WB vs WI	32	35	0.9143	0.9143	151.6667	0.3519	0.3519	2.5984	0.0094	0.408<>1.421	0.335<>1.493
Combined:												
	-	-	-	-	-	-	-	-	-	-	-	-

Weighted Average:

Label	TAU	Var-TAU	Z	P Value	CI 85%	CI 90%	CI 95%
#3+#6+#5	0.9712	0.2059	4.7166	0.0000	0.6747<>1.2677	0.6325<>1.3099	0.5676<>1.3748

Simultaneous Replication Design

The researcher graphed the results of individual participants as a demonstration of inter-subject repetition or simultaneous replication design (Gast & Ledford, 2010; Kelly, 1980). Each group (Red, White, and Blue) had three participants whose individual fluency scores were used to determine group medians reported in Table 5. The researcher evaluated individual student performance by calculating and plotting mean and median fluency for both conditions as well as the standard deviation and variance for the mean values.

Figure 14 displays the individual fluency performance of the three Red group participants designated as Student-1, Student-2, and Student-3. Student-1 demonstrated an increase in mean fluency of 1.95 p/m from baseline (mean 0.0, median 0.0, range 0.0) to intervention (mean 1.95, median 2.2, range 1.0 to 2.4). Student-1 was absent for Session 7. The student's return the following day was the only demonstration of a drop in fluency (-1.2 p/m). Student-2 demonstrated an increase in mean fluency of 1.4 p/m from baseline (mean 0.0, median 0.0, range 0.0) to intervention (mean 1.4, median 1.3, range 1.2 to 1.8). Student-2 was absent for Session 8. The student returned to school the following day and demonstrated the same fluency (1.2 p/m) as Session 7, prior to the absence. Student-3 demonstrated an increase in mean fluency of 2.29 p/m from baseline (mean 0.36, median 0.0, range 0.0 to 0.8) to intervention (mean 2.65, median 2.5, range 1.6 to 4.0). Student-3 was absent for Session 9. The student returned the following day and demonstrated an increase in fluency (+1.0 p/m) over the rate demonstrated for Session 8, prior to the absence. All participants in the Red group demonstrated increases in mean fluency that contributed to the overall mean fluency for the Red group, an increase of 1.88 p/m from baseline ($\bar{x}$ = 0.12, s = 0.21, s^2 = 0.04) to intervention ($\bar{x}$ = 2.0, s = 0.63, s^2 = 0.39).

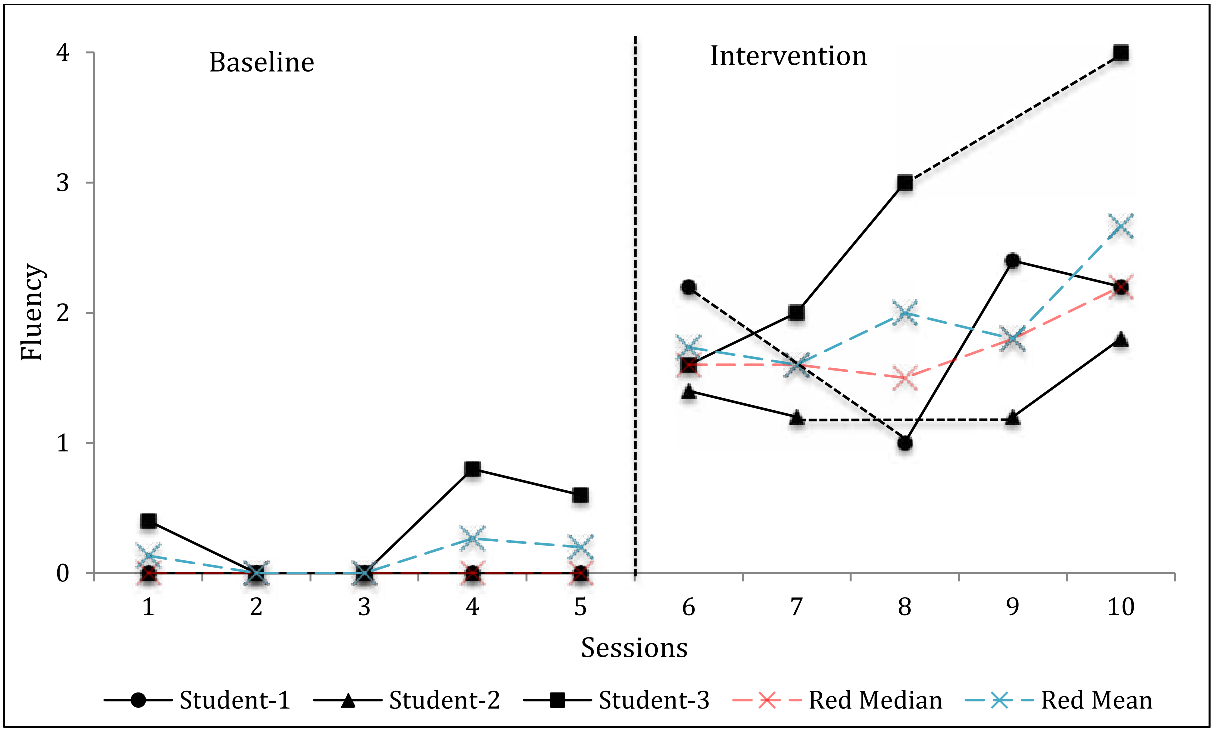

Figure 14: Participant data - Red group. Dashed black lines indicate a missing data point.

The individual fluency performance of the three White group participants designated as Student-4, Student-5, and Student-6 is displayed in Figure 14. Student-4 demonstrated an increase in mean fluency of 2.26 p/m from baseline (mean 0.14, median 0.2, range 0.0 to 0.4) to intervention (mean 2.4, median 2.4, range 1.8 to 2.6). Student-4 was absent for Session 12. When the student returned the following day, there was a demonstrated increase in fluency (+0.8) compared to the fluency score from Session 11. Student-5 demonstrated an increase in mean fluency of 2.7 p/m from baseline (mean 0.14, median 0.0, range 0.0 to 0.4) to intervention (mean 2.84, median 2.8, range 2.4 to 3.4). Student-5 was not absent during the intervention. Student-6 demonstrated an increase in mean fluency of 2.80 p/m from baseline (mean 0.0, median 0.0, range 0.0) to intervention (mean 2.80, median 2.6, range 2.0 to 3.6). Student-6 was not absent during intervention. All participants in the White group demonstrated increases in mean fluency contributing to the overall mean fluency for the White group, an increase of 2.59 p/m from baseline ($\bar{x}$ = 0.09, s = .08, s^2 = .01, c_v = .89) to intervention ($\bar{x}$ = 2.69, s = .24, s^2 = .06, c_v = .09).

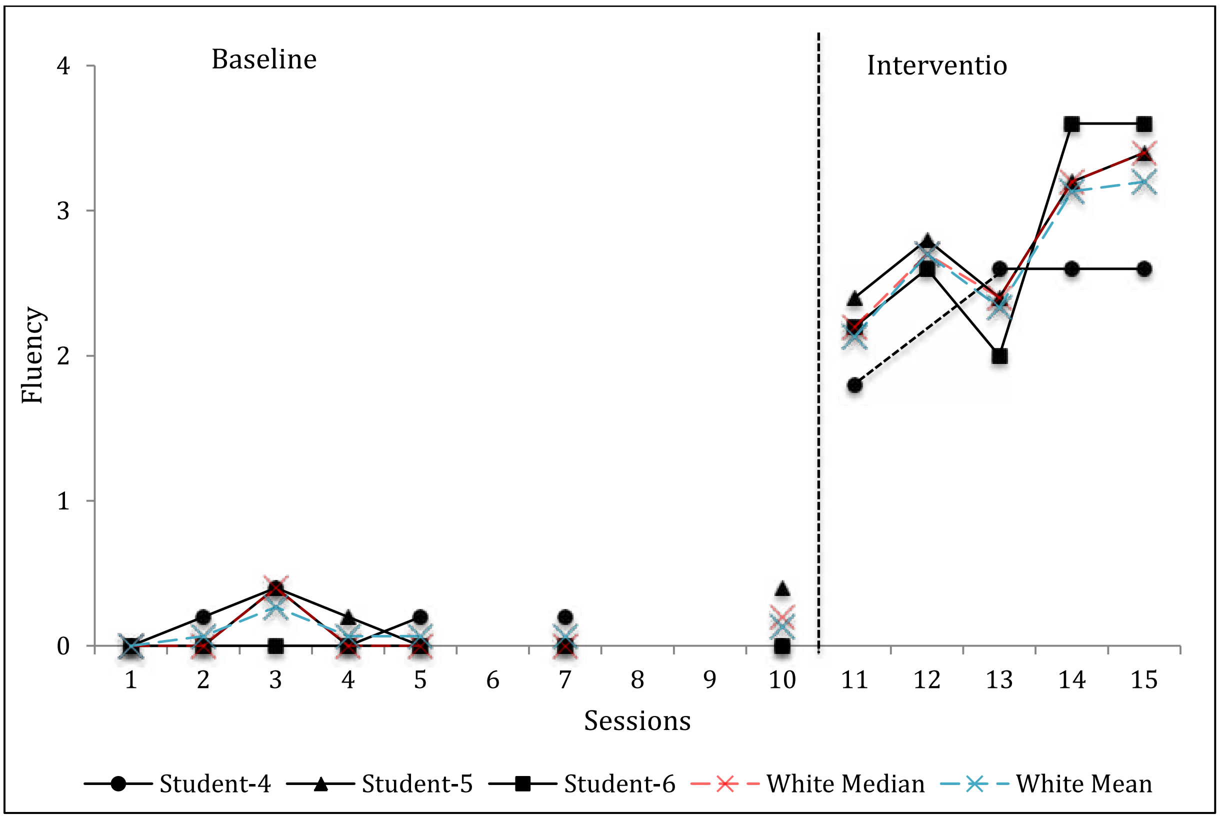

Figure 15: Participant data - White group. Dashed black lines indicate a missing data point.

Figure 16 displays the individual fluency performance of the three Blue group participants designated as Student-7, Student-8, and Student-9. Student-7 demonstrated an increase in mean fluency of 2.73 p/m from baseline (mean 0.23, median 0.0, range 0.0 to 0.8) to intervention (mean 2.96, median 3.4, range 2.2 to 3.6). Student-7 was not absent during intervention. Student-8 demonstrated an increase in mean fluency of 1.5 p/m from baseline (mean 0.0, median 0.0, range 0.0) to intervention (mean 1.5, median 1.6, range 0.2 to 2.6). Student-8 was absent for Session 17 and demonstrated a one-day decline in fluency (-0.8) the following day. Student-9 demonstrated an increase in mean fluency of 1.75 p/m from baseline (mean 0.9, median 0.0, range 0.0 to 0.4) to intervention (mean 1.84, median 1.7, range 0.8 to 2.8). Student-9 was not absent during intervention. Overall the Blue group demonstrated an increase in mean fluency of 2.16 p/m from baseline ($\bar{x}$ = 0.10, s = .12, s^2 = .02, c_v = 1.2) to intervention ($\bar{x}$ = 2.26, s = .60, s^2 = .36, c_v = .27). Appendix J contains a table summarizing the statistics used to evaluate individual student performance for each group.

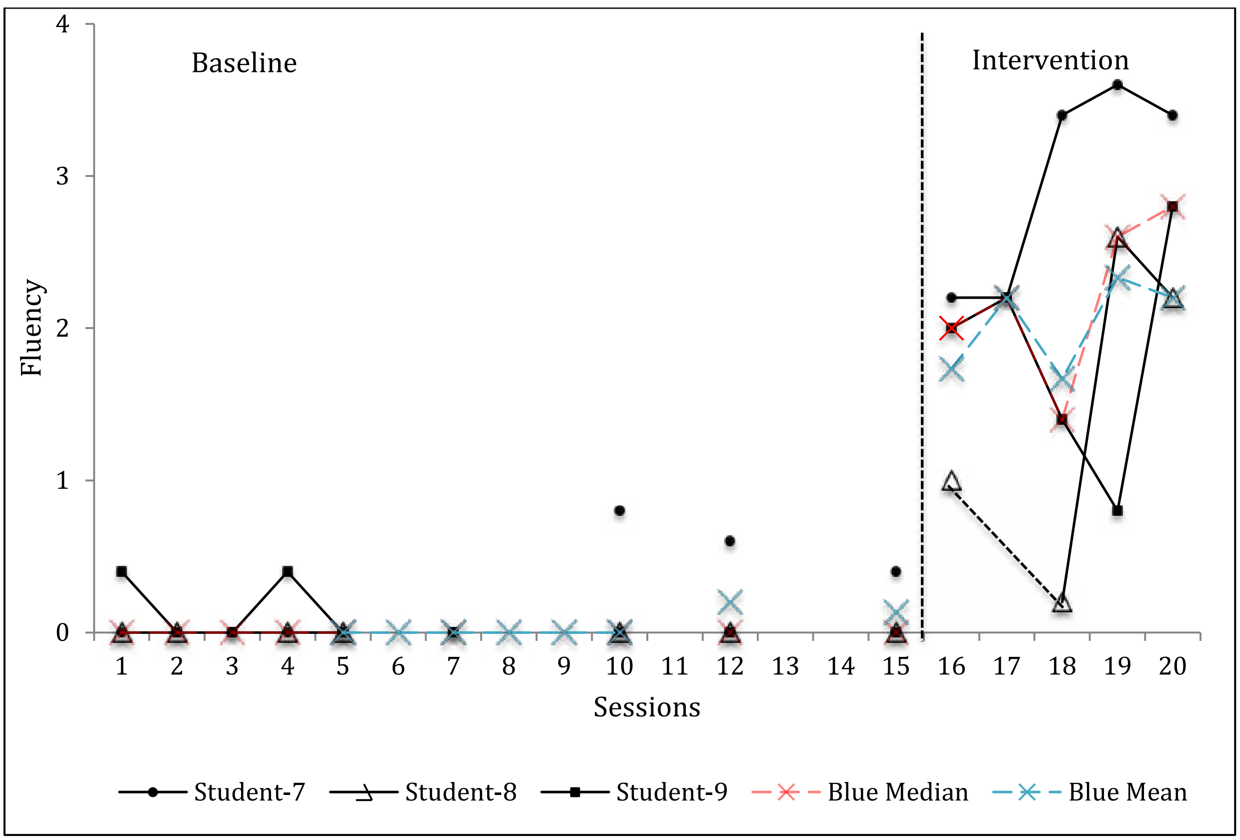

Figure 16: Participant data - Blue group. Dashed black lines indicate a missing data point.

Research Question Two

Research question two was addressed through a non-experimental pre, post, and delayed post-test design using the equivalent fraction test (EFT) as the repeated measure of equivalent fraction knowledge to answer the following question: Does RNP/VM increase equivalent fraction knowledge as evidenced by the results of a pre, post, and delayed post-test administration of the EFT? Specifically the researcher was looking to see if the intervention produced an increase in mean scores from pre to post administration of the EFT and given an increase, did that increase persist 15 or more days after the intervention.

Non-experimental Pre, Post, and Delayed Post Test

The Pre EFT was administered on Day-0, immediately before baseline commenced. The post-test was delivered to each group the day following the conclusion of their intervention (Day 15 - Red group, Day 22 - White group, and Day 29 - Blue group. The delayed post-test was administered to all groups on Day-44. Student-5 from the White group and Student-9 from the Blue group did not return to the classroom after spring break necessitating an adjustment to post-test means (removing scores for Student-5 and Student-6 from post group mean calculations) for purposes of comparison to the delayed post-test.

All nine participants took the pretest and post-test version of the EFT. Mean scores for the pretests included: Red group ($\bar{x}$ = 8.0, range 0 to 20, s = 10.6), White group ($\bar{x}$ = 6.0, range 2 to 18, s = 8.7), and Blue group ($\bar{x}$ = 4.0, range 0 to 11, s = 6.1). The corresponding post-test mean scores included: Red group ($\bar{x}$ = 12.7, range 4 to 23, s = 9.6), White group ($\bar{x}$ = 14.0, range 8 to 19, s = 5.6), and Blue group ($\bar{x}$ = 8.7, range 2 to 18, s = 8.3). All three groups demonstrated an increase in mean EFT scores from pre to post with the largest gain attributed to the White group (+8.0) followed by both the Red and the Blue groups with the same increase (+4.7). A summary of pre to post increases demonstrated by the three groups can be found in Table 8.

Table 8

Pre to Post Change in Mean EFT Scores by Group

	Pre	Post	Change	% Change
Red Group	8.0	12.7	+ 4.7	
White Group	6.0	14.0	+ 8.0	
Blue Group	4.0	8.7	+ 4.7	
Overall Mean	6.0	11.8	5.8	96.7%

Note: + indicates improving.

The mean score for individual participants on the EFT was 6 (range 0 to 20, *s* = 6.87) on the pretest and 11.8 (range 2 to 23, *s* = 7.34) on the post-test, an increase of 5.8 (97%). Seven students had increases in scores from pre to post-test with a mean increase of 8.1 points. One student had no change in score and one student had a score decrease of 5 points from pre to post-test. Table 9 summarizes the individual pre to post comparison of EFT scores.

Seven students took the delayed post-test version of the EFT allowing a comparison of the post-test and delayed post-test scores. Two students, one from the White group and the other from the Blue group, did not take the delayed post-test EFT, requiring an adjustment to the post-test mean calculations to make an accurate comparison. The adjusted post-test mean for the White group was 11.5 (range 8 to 15, *s* = 4.9) and the adjusted post-test mean for the Blue group was 4.0 (range 2 to 6, *s* = 2.8). The Red group post-test mean remained unchanged at 12.7. Delayed post-test scores included the White group mean of 17.5 (range 15 to 20, *s* = 3.5), the Blue group mean of 2.5 (range 1 to 4, *s* = 2.1), and the Red group mean of 14.7 (range 9 to 24, *s*

= 8.1). Two groups demonstrated increases from post to delayed post, the largest posted by the White group (+6.0) and the smallest by the Red group (+2.0). The Blue group posted a decrease (-1.5). Table 10 summarizes the post to delayed post comparison of EFT scores.

Table 9

Pre to Post Change in EFT Scores by Student

	Pre	Post	Change
Red Group			
Student-1	4	4	+ 0
Student-2	0	11	+ 11
Student-3	20	23	+ 3
White Group			
Student-4	12	15	+ 3
Student-5	2	19	+ 17
Student-6	4	8	+ 4
Blue Group			
Student-7	11	6	- 5
Student-8	0	2	+ 2
Student-9	1	18	+ 17
Overall Mean	6.0	11.8	+ 5.8

Note: + indicates improving

Table 10

Post to Delayed Post Change in Mean EFT Scores by Group

	Post	Delayed Post	Change	% Change
Red Group	12.7	14.7	+ 2.0	
White Group	*11.5	17.5	+ 6.0	
Blue Group	*4.0	2.5	- 1.5	
Overall Mean	9.4	11.6	2.2	23.4%

Note: * Adjusted post-test mean.

The seven participants that took both the post and delayed post-test EFT had a mean score of 9.9 (range 2 to 23, *s* = 7.2) on the post-test and a mean score of 11.6 (range 1 to 24, *s* = 8.3) on the delayed post-test, an increase of 2.2 points (22%). Five students demonstrated increases in scores from post to delayed post-test with a mean increase of 4.0 points. One student had no change in score and one student had a score decrease of 5 points. There was no apparent correlation between the number of days of delay or the number of days that a student was present for intervention sessions and change in scores from post to delayed post EFT. Table 11 summarizes individual student results.

Table 11

Post to Delayed Post Change in EFT Scores by Student

	Post	Delayed Post	Change	# Days of Delay	# Intervention Days Present
Student-1	4	9	5	29	4
Student-2	11	11	0	29	4
Student-3	23	24	1	29	4
Student-4	15	20	5	22	4
Student-6	8	15	7	22	5
Student-7	6	1	-5	15	5
Student-8	2	4	2	15	4
Overall Mean	9.9	12.0	+ 2.2		

Research Question Three

Research question three was addressed through two online surveys, one for the student participants and the other for the teacher and paraprofessional. These surveys were constructed with the purpose of gaining an understanding of the participants' perceptions related to the goals, procedures, and outcomes of RNP/VM.

Social Validity Questionnaire Results

An online questionnaire for students with items rated on a Likert scale (5 "strongly agree", 4 "agree", 3 "neither agree nor disagree", 2 "disagree", and 1 "strongly disagree") was used to assess the following: a) the perceptions about the use of RNP/VM procedure, (b) the level of agreement with the goals of the intervention, and (c) the level of agreement with academic and behavioral outcomes resulting from the intervention. Overall 100% of the students that took the survey agreed with the intervention goals as assessed by four survey questions (22 question responses were agree/strongly agree, 2 question responses were disagree). The students also agreed that the use of RNP/VM helped them to better understand fractions as assessed by three survey questions (18 responses were agree or strongly agree). Questions related to the academic and behavioral outcomes of the intervention were divided into two groups: student effort and attention was assessed by four questions and academic outcomes were assessed by four others, two related to conceptual knowledge and two to problem solving ability. The 75% of student responses to the two questions related to their level of equivalent fraction knowledge were positive (9 responses were agree or strongly agree, 2 were neutral, and 1 disagreed). Problem solving responses were mixed with responses to the two questions split evenly between agree/strongly agree (6) and disagree/strongly disagree (6). Appendix K contains a table of the student survey responses.

An online questionnaire for the teacher and paraprofessional with items rated on a Likert scale (5 "strongly agree", 4 "agree", 3 "neither agree nor disagree", 2 "disagree", and 1 "strongly disagree") was used to assess the following: (a) the perceptions about the use of RNP/VM procedure, (b) the level of agreement with the goals of the intervention, and (c) the level of agreement with academic and behavioral outcomes for teachers and students resulting from the intervention. Overall the teacher and paraprofessional agreed with the intervention goals as assessed by four survey questions (8 question responses were agree/strongly agree). The teacher and paraprofessional also agreed that the use of RNP/VM helped students to better understand fractions as assessed by five survey questions (8 question responses were agree/strongly agree, 1 response was neutral, and 1 disagreed). Questions related to the academic and behavioral outcomes of the intervention were divided into three groups: student effort and attention was assessed by two questions, academic outcomes were assessed by two questions, and teacher intentions for future use of VMs was assessed with four questions. The teacher and paraprofessional agreed that the students were able to easily and independently use the VMs to learn about fractions (4 responses were agree/strongly agree). They also agreed that the intervention helped the students to understand fractions (4 responses were agree/strongly agree). Last, the teacher and paraprofessional reported mixed responses that were generally positive that they would use and/or explore the use of other VMs for teaching fractions or other mathematics concepts (5 responses were agree/strongly agree, 2 responses were neutral, 1 disagreed). Appendix K contains a table of the teacher and paraprofessional survey responses.

Reliability

Dependent Measures

Reliability of the dependent measures (FAA and EFT) used to evaluate research questions one and two was confirmed by double scoring all assessments, first by the teacher and a second time by the researcher. The researcher decided to score all assessments twice due to the low number of questions and the small number of participants in each group, which would cause the group median sensitive to scoring errors. This point-by-point Interobserver agreement uncovered zero scoring differences, resulting in 100% IOA for both the FAA and the EFT.

Fidelity of Instruction

The Fidelity of Instruction Rubric was used to assess the procedural fidelity of the teacher's instruction. Initially, the teacher read the intervention materials and conducted one practice session with two instances of corrective feedback from the researcher. Next, both the researcher and the Interobserver watched the teacher deliver instruction in two separate practice sessions rating the instructional delivery on the rubric; the researcher observed the live instruction and the Interobserver watched a video recording of the instruction. The teacher earned a mean score of 13 (range 12-14) or 93% across both observers with 100% IOA on the two consecutive demonstration sessions delivered during the pre-study phase, which met the criteria to deliver the RNP/VM instruction with fidelity during the intervention phase of the study.

Once intervention began, the researcher continuously assessed fidelity through evaluations of teacher instruction on alternating days and a second weekly evaluation of one of those sessions (recorded) by the Interobserver. In total, the researcher evaluated the teacher's instruction 15 times during intervention and the Interobserver evaluated eight of those lessons a second time with a mean rubric score of 13.7 (range 12-14) or 98.25% with IOA on 53 of 56 observable behaviors (94.6%) during intervention. Over the course of the study including pre-study and intervention phase observations there were a total of 70 observable instructional behaviors. The two observers agreed on 67 of the 70 observation components resulting in a reportable IOA for instructional fidelity of 98.6%. All video recordings were destroyed after the Interobserver completed the evaluations. Appendix I contains a summary of all rubric scores and IOA calculations.

Interobserver agreement was utilized to ensure that summary statistics and tables from the Social Validity Surveys were an accurate representation of the Qualtrics survey results. The researcher exported the Qualtrics data into Excel, coded questions with themes and sub-themes, and created summary statistics. To ensure the accuracy of the researcher's reported results, the Interobserver duplicated the researcher's process of exporting, coding, and creating the data tables and statistics comparing the researcher's results at each stage of the process. The Interobserver reported zero errors (100% IOA), confirming the reliability of the researcher's data analysis.

CHAPTER FIVE: DISCUSSION

Students with EBD are served in self-contained settings more than students in any other disability category (Landrum et al., 2004). At the same time, there is a national trend to include this population of students in classrooms with their non-disabled peers (Bradley et al., 2008; Landrum et al., 2004). Despite the 1975 legislation requiring that all students with disabilities receive their instruction in the least restrictive environment (LRE; *Education for All Handicapped Children Act of 1975, 20 U.S.C. § 1400 et seq*), educational services and settings vary widely for students with EBD. The inconsistency in district application of LRE for this population of students is due in part to the unpredictable and at times volatile nature of their behaviors and the lack of a formal definition in the law (Becker et al., 2010; Crockett & Kauffman, 2013; Kauffman et al., 2002). The proliferation of online and blended learning presents an opportunity for students with EBD to be served in more inclusive virtual classroom settings despite their wide range of behavioral needs.

Two of Staker and Horn's blended learning models (2012) best describe the instructional design of the separate special education setting at the school chosen for this study. The primary instructional design of the EBD unit aligns with The Rotation model. Students placed in a separate physical setting with a special education teacher were enrolled in online inclusive courses where the online teacher served as the teacher-of-record. The design in the mathematics classroom differed because of the unique qualifications of the special education teacher. She was also certified in mathematics. As a result, she was qualified to serve as the mathematics teacher-of-record, a basic requirement of the Flex model of blended learning (Staker & Horn, 2012). This model provides a variety of teacher-directed, individualized support for students within the classroom. This hybrid environment was the setting for this study.

Students with EBD who participated in this research study on fraction remediation with virtual manipulatives (VM) were receiving their mathematics instruction in a Flex model of blended learning. The flexibility of this instructional design provided a unique opportunity to evaluate the effectiveness of an evidence-based fraction intervention, *Initial Fraction Ideas,* delivered face-to-face in concert with online VMs (RNP/VM) to support the students' ability to access online grade-level mathematics content. To contextualize a discussion of this study, Chapter Five is constructed with four foci: (a) a summary of the unique structure of this setting, (b) a summary presentation of the study results; (c) a brief discussion linking study outcomes and researcher reflections with existing literature on students with EBD; and (d) a conclusion that highlights the strengths and limitations identified by the researcher along with implications for practitioners and future investigators.

The school district that provided the setting for this study had first mandated the use of online courses for students with EBD in separate settings at the beginning of the 2012-2013 school year. At the time of this study, the students were in their second consecutive year of online grade-level mathematics instruction. The sudden shift to grade-level content the preceding year triggered challenges related to student reading levels, deficits in prerequisite mathematics skills, and problem behaviors; all factors that had an impact on the 2012-2013 course completion rates. Those challenges remained unresolved as students returned for the 2013-2014 school year. The special education teacher voiced a number of concerns about the impact of the online curriculum on her students' frustrations with school, off task behaviors, and academic progress in mathematics.

The teacher's concerns were related to the challenges that her students experienced the previous year, difficulties that continued to frustrate students at the time of the study. Her primary worry was the students' need for remediation of prerequisite skills, instruction that would consume valuable class time and impact their ability to maintain acceptable rates of progress in the online courses. Without remediation, her students also were unlikely to make adequate progress; lack of prerequisite skills could result in failed assessments and requirements to repeat course modules. The teacher's concerns regarding these skill deficits paralleled the concerns expressed by Cavanaugh and colleagues (2005) and Stein and colleagues (2011) and put students at risk of algebra failure.

The teacher and students reported other difficulties with the online coursework. The teacher was concerned about the reading level of the courses. She noted that the school policy for general education students allowed only those who were proficient (Level 3) in reading on the FCAT to enroll in online courses; her students however, were required to take the online courses, despite having less than proficient (Level 1 or Level 2) reading scores on the state assessment. Students expressed a dislike for online work and believed that it was difficult and boring. The teacher reported that it was hard to motivate the students to engage in the course, and that the reinforcers that historically served to motivate students had been cut from the school budget. She noted that there had been an escalation in disruptive behaviors and office referrals since the implementation of the online curriculum. The teacher also reported that when the students fell behind they were unwilling to work after school or at home to catch up. Students reported that they could not work on mathematics at home because they did not have access to a computer or their responsibilities after school did not allow time for homework. This combination of factors made it critically important for the teacher to maximize student productivity during class time.

Even if this group of students with EBD were to be productive during class time, preliminary indications from researchers suggest that students with disabilities may experience little if any success in online learning (Deshler et al., 2012). Existing literature on the effectiveness of online instruction for K-12 students is scarce and does not include academic outcomes for students with disabilities or any study related to supporting mathematics instruction for students with EBD in online or blended learning environments (Black, Ferdig, & DiPietro, 2008; Cavanaugh et al., 2009; Vasquez & Straub, 2012).

Summary of Results

The conceptual framework described in Chapter Two and depicted in Figure 3 represents the delicate balance of academic tipping points that can lead to mathematics achievement, high school graduation, and positive post school experiences for students with EBD. Despite the identification of these critical concepts and their connection to high school graduation and positive post-secondary outcomes, little progress has been made in improving academic or societal outcomes for this population of students (Bradley et al., 2008). This lack of progress can be seen in the NAEP results published by the National Center for Education Statistics. The mathematics performance of high school students with and without disabilities has remained virtually unchanged despite reform and research efforts to improve mathematics outcomes for all students (NAEP Data Explorer, 2013a; U.S. Department of Education, 2008). For students with EBD, their unique behavioral and academic needs make targeting gains in mathematics achievement, in inclusive or separate settings, a challenge for both teachers and researchers (Temple-Harvey & Vannest, 2010).

Three primary research questions framed the work of the researcher in this investigation of the effects of *Initial Fraction Ideas*, a Rational Number Project (RNP) intervention proven effective for middle school students with mathematics difficulties (Cramer et al., 2009; Westenskow, 2012) and the use of VMs (Moyer-Packenham, Westenskow, et al., 2013; Westenskow, 2012) on the procedural fluency and conceptual understanding of students with EBD. Data were collected through the daily administration of a Fraction Addition Assessment (FAA) during the baseline and intervention phases of the multiple baseline study; an Equivalent Fraction Test (EFT), as a repeated measure, delivered pre, post, and delayed post intervention to assess equivalent fraction knowledge; and an online survey provided to teachers and students to assess social validity. These data were gathered to answer the following questions:

1. To what extent does RNP/VM increase group median fluency on a five-minute fraction addition assessment for students with EBD?
2. Does RNP/VM increase mean equivalent fraction knowledge as evidenced by the results of a pre, post, and delayed post-test administration of the EFT?
    a. Is there an increase in mean scores, from pre to post administration of a 20-item EFT, for groups of students identified with EBD?
    b. Given an increase in mean group scores from 2.a., do those gains persist after the end of the intervention?

3. How do the students identified with EBD and their respective teacher and paraprofessional perceive the goals, procedures, and outcomes of RNP/VM?

Summary of Results from Multiple Baseline

The researcher utilized a single subject design with a multiple baseline across groups of participants to examine the effects of the intervention and address research question one. This methodology provided the experimental control and replication required to establish a functional relationship between the independent and dependent variables. The analysis of individual student results served to further strengthen the findings of the group analysis providing inter-subject repetition or simultaneous replication design (Gast & Ledford, 2010; Kelly, 1980).

The researcher's visual analysis of FAA median group scores indicated that each of the three groups experienced immediate changes in level of fluency and an accelerating (improving) trend as a result of the intervention. Effect sizes were large for all groups as demonstrated by both PND and Tau-U (Table 6). To further collaborate those results, an in depth analysis of mean performance by and within groups was conducted. All three groups demonstrated an immediate change in level of mean fluency along with varied and accelerating trends; all individual participants experienced gains in fluency, trends were generally positive with more variability within groups than between groups (Figures 12-14). The gains in fluency rates demonstrated across the three groups, along with similar gains by individual participants demonstrated by the inter-subject analysis, provides evidence of a functional relationship between the RNP/VM intervention and fraction addition fluency for this group of students with EBD.

The effectiveness of students' use of VMs to build fraction knowledge has been documented in a number of studies (Bolyard & Moyer-Packenham, 2006; Moyer-Packenham & Suh, 2012; Moyer-Packenham, Westenskow, et al., 2013; Moyer-Packenham, Baker, et al., 2013; Westenskow, 2012). The gains in fraction addition fluency posted by the participants affirm the results of earlier research and extends the use of VMs as a part of a fraction intervention to students with EBD. Rational Number Project researchers worked for two decades to develop the manipulative activities that could build part-whole understanding through fraction modeling, comparisons, equivalence, and using fraction circles to practice adding and subtracting fractions without using symbols. Researchers documented how the use of fraction circles built conceptual foundations necessary for ordering, adding, and subtracting fractions (Cramer & Henry, 2002; Cramer et al., 2002, 2008). The improved fraction addition fluency demonstrated in this study validated the results of the RNP researchers and affirmed the results of other studies that validated the effectiveness of the concrete-to-representational-to-abstract (CRA) sequence of instruction (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, & Jackson, 2003; Maccini & Hughes, 2000; Witzel, 2005).

Summary of Results from Non-experimental Repeated EFT

Research question two was addressed by the repeated administration of the EFT (pretest, post-test, delayed post-test). All groups experienced an increase in mean EFT scores from pre to post-test, with an overall mean increase of 96.7% (see Table 8), consistent with the pre to post test gains (+138%) reported by Westenskow for the VM condition in her 2012 study. A second group comparison was conducted to evaluate retention of conceptual understanding through mean group scores on the EFT from post-test to delayed post-test. The Red and White groups experienced an increase in mean EFT scores and the Blue group experienced a decrease. The overall mean change was an increase of 23.0% for the three groups. Westenskow reported an overall decrease of 4.2% in EFT scores from post-test to delayed post-test for the VM condition. Possible explanations for the disparity in post to delayed post means between the two studies include: (a) missing delayed post-test scores for two participants in the current study who did not return to the classroom after spring break; (b) the sensitivity of mean values to variability and missing data, particularly in small samples (Stavig & Gibbons, 1977); and (c) timing differences in the administration of the delayed post-tests (21 to 28 days after intervention for Westenskow's participants and 15-29 days for participants in the current study).

This non-experimental analysis of EFT results is not sufficient evidence to confirm a functional relationship between the intervention and conceptual knowledge. It does however suggest a correlational relationship between RNP/VM and performance on the EFT. The results of this study are consistent with the conclusions of researchers from RNP, Westenskow (2012), and others (Bolyard & Moyer-Packenham, 2006; Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2008; McLeod, Vasinda, & Dondlinger, 2012; Moyer, Niezgoda, & Stanley, 2005; Moyer-Packenham & Suh, 2012; Moyer-Packenham, Westenskow et al., 2013; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2007); the use of virtual and physical manipulatives as a part of fraction instruction builds and maintains fraction conceptual understanding including equivalent fraction knowledge. The findings of this study demonstrate a similar correlation between RNP/VM and improved equivalent fraction knowledge for students with EBD.

Summary of Results from Student and Teacher Surveys

The student and teacher surveys were designed to assess: (a) the perceptions about the use of RNP/VM, (b) level of agreement with goals of the intervention, and (c) level of agreement with academic and behavioral outcomes for students and teachers. Overall all of the students as well as the paraprofessional and teacher agreed with the intervention goals and the use of RNP/VM. Responses to questions about academic and behavioral outcomes had more variability. Seventy-five percent of the student responses reported improved equivalent fraction knowledge and 50% of student response reported improved problem solving ability as a result of the intervention. The teacher and paraprofessional agreed that RNP/VM was easy for students to use and improved student understanding about fractions (see Appendix K). Overall the teacher, paraprofessional, and students' perceptions supported the goals and outcomes of the study. They expressed that using VMs was fun, easy, and helpful for understanding fractions, which is consistent with the literature on the use of VMs in fraction teaching (Ozgun-Koca & Edwards, 2011; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2007).

Relating the Research to Existing Literature

The researcher in this study conducted a multifarious extension of four areas of existing literature: (1) the extension of general education mathematics interventions to students with EBD, (2) the use of virtual rather than physical manipulatives as a part of a fraction intervention, (3) the effects of a conceptual fraction intervention on procedural fluency, and (4) the application of an existing face-to-face evidence based practice to a unique blended learning environment. The students posted procedural as well as conceptual learning gains as a result of the intervention and reported positive perceptions, of the intervention instruction and use of the VMs. The teacher and paraprofessional also reported positive perceptions of the intervention instruction and use of VMs.

Rational Number Project researchers, experienced positive results using the *Initial Fraction Ideas* curriculum with over 1600 fourth and fifth grade students in general education settings (Cramer et al., 2009). Project researchers have made this evidence-based fraction instruction widely available for use through a website maintained by the University of Minnesota (http://www.cehd.umn.edu/ci/rationalnumberproject/). Westenskow (2012) extended the use of the RNP intervention by limiting participants to general education students with mathematics difficulties along with incorporating the use of virtual as well as physical manipulatives in the scripted instruction. Westenskow's analysis of EFT scores for participants in the physical manipulative condition affirmed the positive outcomes of the RNP studies that used only physical manipulatives. This current study extended the use of the RNP intervention to students with EBD and affirmed the positive results of the RNP and Westenskow studies.

Studies comparing the use of virtual and physical manipulatives have validated VMs as an effective substitute for their physical counterparts in over 30 studies (Moyer-Packenham et al., 2013). Westenskow's 2012 study extended that research by applying the RNP intervention curriculum with the goal of identifying variations in students' conceptual understanding of fraction equivalence and identified variations to the types of manipulative used (virtual only, physical only, and a combination of virtual and physical). Her study results affirmed, through pre and post-test comparisons of the EFT, that the use of VMs as part of a fraction intervention could produce significant learning gains, a mean increase in scores of 138% with a large effect size (*d* = 2.03), for students with mathematics difficulties. This present study utilized Westenskow's EFT for the non-experimental repeated measure. A comparison of the same pre and post-test scores showed students had learning gains on the EFT that were similar to the participants in Westenskow's study, a 97% mean increase from pre to post-test. Retention, evaluated through a post to delayed post-test comparison of EFT scores, also produced similar results. Westenskow's participants in the VM condition demonstrated significant levels of retention with only a minor drop in mean EFT scores from post to delayed-post test (-4%) whereas the participants in this present study had a mean increase from post to delayed post-test of 23%.

Researchers have identified a correlation between conceptual understanding and procedural knowledge in mathematics (Hill et al., 2004; Siegler et al., 2012, 2011; Wu, 2009). That correlation is particularly noteworthy in the area of fractions (Siegler et al., 2011). Prior to this present study, the existing researchers utilizing the RNP fraction intervention assessed only changes in conceptual knowledge. The researcher in this study extended that literature to include the evaluation of the effect of a conceptual fraction intervention on fraction procedural fluency (addition). The researcher's analyses of results affirmed previous studies by documenting improvements in conceptual knowledge as well as gains in procedural fluency. To evaluate fluency, participants took a 5-minute daily assessment consisting of 20 fraction addition problems. Fluency was calculated by dividing the number of correct answers by five minutes. Group medians for each group were calculated daily from the individual participant scores during the baseline and intervention phases. Daily medians were graphed to allow for visual analysis of the data. All three groups, in the multiple baseline design, posted an increase in fluency after the first day of intervention and an improving trend over the 5-day intervention periods.

Research related to students with disabilities in online environments is still in its infancy (Cavanaugh, Barbour, & Clark, 2009; Coy, Marino, & Serianni, in review; Vasquez & Straub, 2012). Empirical research on online learning is just beginning to emerge in current peer-reviewed publications. At the time of this study, the published literature did not yet included any studies validating existing evidence-based practice in mathematics for students with disabilities in online or blended settings (Coy et al., in review; Deshler et al., 2012; Vasquez & Straub, 2012). This current study was an effort to validate the effectiveness of an existing face-to-face evidence-based fraction intervention in a blended learning environment for students with EBD. The classroom teacher delivered the RNP instruction to the groups of students face-to-face in a whole-class setting, combining intervention activities with the use of online VMs as a remedial supplement to the online mathematics curriculum. The gains in procedural fluency, the large effect sizes for each group, and the numerically positive improvement in equivalent fraction knowledge all demonstrated the effectiveness of RNP/VM for this group of students with EBD.

Implications

Although some instructional practices and interventions have been documented in the research literature on mathematics interventions as useful in the instruction of students with EBD, research is scarce (Hodge et al., 2006; Templeton et al., 2008). Twenty-one experimental studies were conducted over the last 28 years that involved 203 students with EBD. These studies were identified through the Hodge and Templeton meta-analyses along with an extension conducted by this researcher. Together they provide data on interventions that should be considered promising practices. These researchers confirmed that students with EBD can and do respond with positive learning gains when provided with supports and interventions that supplement the standard mathematics curriculum (Hodge et al.; Templeton et al.). Despite a positive trend in the research, the diverse and infrequent single subject studies conducted in special education fail to meet the standards of documentation described by Horner and colleagues (2005) for establishing a practice as evidence based. Those standards include: (a) at least five published (peer reviewed) single-subject studies that have demonstrated experimental control and meet minimum methodological standards, (b) studies must have been conducted by at least three different researchers in at least three different geographic locations, and (c) the studies (five or more) must have included 20 or more participants (Horner et al.).

With those standards as a guide, the field's ability to document evidence-based practices depends on the collaborative efforts of multiple researchers. The field of special education could benefit from a collaborative and systematic effort by researchers to document the effectiveness of existing general education evidence-based practices for students with disabilities, especially students with EBD. In addition, much work is needed to test those evidence-based practices for students with EBD in online environments. This type of collaborative effort has the potential to produce a toolbox of resources that could potentially impact the mathematics learning and overall academic success of students with EBD in both face-to-face and virtual learning environments.

This researcher investigated the effects of a remedial, conceptual fraction intervention, consisting of a combination of lessons from the RNP and VMs, on procedural fluency (fraction addition) and conceptual understanding of equivalent fractions. The researcher's intention was to conduct a partial operational replication (Klein, 2004) of the VM condition tested in Westenskow's 2012 study to build upon the evidence from that study and extend the intervention's use to students with EBD. Overall, the researcher demonstrated that the fidelitous delivery (98.25% IOA) of RNP/VM improved both conceptual understanding of fractions (equivalence) and procedural fluency (addition) for this group of students with EBD. The implications of this study stretch across the fields of mathematics, special education, and online learning.

Implications for Special Education

Researchers who have reviewed the body of special education literature reveal that students with EBD often struggle in mathematics (Lane, Wehby, Little, & Cooley, 2005; Reid et al., 2004), yet research studies designed to validate effective instructional practices in mathematics are limited (Hodge et al., 2006; Templeton et al., 2008). A correlation between success in mathematics, high school graduation, and positive post-secondary outcomes for students with EBD is apparent (Blackorby & Wagner, 1996). High school dropouts (42% with EBD) are a high cost to the U.S. economy with an estimated ten year impact of 1.5 trillion dollars in lost wages with billions more related to incarceration, rehabilitation, and social services (Alliance for Excellent Education, 2011). In light of these facts, researchers in special education need to prioritize research on effective mathematics practices for students with EBD.

Implications for Online and Blended Learning

The proliferation of online learning options in states like Florida has presented what may be a timely and potentially controversial twist to the conversation that surrounds the least restrictive environment (LRE) directive; physically place students with EBD in separate educational settings and provide them with standards-based, high quality instruction by content certified teachers through fully-inclusive online courses (Vasquez & Serianni, 2012; Watson et al., 2013). Once again a lack of specificity and no formal definition for LRE in the law (Kavale et al., 2005) may rekindle the decades old disagreement among special educators on how to apply the LRE directive (Crockett & Kauffman, 2013; Kauffman, Bantz, & McCullough, 2002) and spark new debates on the appropriateness of online and blended learning for students with EBD. While this improved availability of content certified teachers may provide scheduling and financial benefits to schools and districts, researchers must evaluate the social and behavioral ramifications of further isolating students with EBD by placing them in online courses even when the overarching instructional design is a blended learning model supported by researchers (Staker & Horn, 2012).

Blended Learning in Separate Settings

Students with EBD continue to be excluded from the general education setting despite national trends towards inclusion for all categories of students with disabilities (Bradley et al., 2008; Landrum et al., 2004). The availability of inclusive online learning for this population of students, that can be accessed while they remain physically and socially isolated from their peers, has the potential to misrepresent the placement of these students as fully included in local, state and national reports. While the potential of blended learning to provide high quality grade-level instruction by content certified teachers is appealing in light of the high numbers of students with EBD who currently have limited access (Jackson & Neel, 2006; Simpson et al., 2004), educators must be aware of the potential overuse that this convenience might permit.

The use a blended learning model such as the Rotation or Flex models (Staker & Horn, 2012) may "solve" a number of district LRE dilemmas related to the placement of students with EBD (Hassel & Terrell, 2004; Rose & Blomeyer, 2007; Vasquez & Serianni, 2012) without providing students the behavioral and social/emotional services and supports they need to successfully return to the general education setting. The human contact, teacher-student relationships, peer-to-peer interactions, and critical opportunities to practice appropriate behaviors and social skills could be lost in the online environment. How will students with EBD learn to engage and respond appropriately if they never come face-to-face with their peers or the world around them because they are "fully included," yet they are not? What incentive will remain for teacher preparation programs and school districts to prepare general education teachers for the challenges of students with EBD in their classrooms (*NTLS-2: Student school program survey*, 2003) if those students slowly disappear into "more appropriate" placements in "inclusive" but separate blended learning environments? How will these students cope with the transition to the real world if they are actually successful in completing online coursework and graduating from high school? These questions and more need to be answered as more forms of blended learning become a part of instructional design for students with EBD served in brick and mortar K-12 schools.

Research Initiatives

The Office of Special Education Programs (OSEP), an agency of the U.S. Department of Education, has recognized the critical need for research in this area. In January 2012, OSEP funded the Center on Online Learning for Students with Disabilities, a collaborative project that includes the Center for Research on Learning at the University of Kansas, the Center for Applied Special Education Technology (CAST), and the National Association of State Directors of Special Education (NASDSE). The Center began its work by surveying the field, generating a research agenda, and publishing on the history of online learning for students with disabilities along with perceptions from the field. Now, in its third year, the Center is managing a variety of research efforts designed to evaluate existing practices in virtual environments and the accessibility of online courses. The critical need for solutions dictates that researchers begin with what is proven, test existing evidenced-based practices in online and blended environments.

While the Center faculty are focusing on evaluating systems and instructional practices others, like Vasquez (2009) and Straub (2012), are attempting to adapt face-to-face evidence based practices to the online environment. The researcher in this study began the task of adapting and applying existing evidence based practices, the RNP curriculum and VMs, to support mathematics learning for students with EBD in a blended learning environment. The next steps for this research is (a) replication to document evidence-based practices, (b) delivery of the intervention online, synchronously and asynchronously, and (c) measuring the effect of the intervention on the related mathematics performance of students in their online course.

Limitations

Single subject experimental designs using multiple baselines across participants or groups may have both internal and external threats to validity. This study is not without limitations, despite the large effect size and reported improvements in procedural fluency and equivalent fraction knowledge across all groups.

This multiple baseline design across groups utilized purposive sampling to identify participants for each of the three groups. This type of sampling has an inherent weakness because participants were selected according to a predetermined set of criteria. In this study all nine participants selected were students with EBD who were confined to self-contained special education classrooms for 100% of the school day. Within that narrow sample of students, the researcher attempted to minimize the variability of student participants by using a priori criteria to create groups that were as similar as possible. However after applying exclusion criteria, the small numbers of students eligible for participation precluded any attempt to closely match other participant characteristics across groups. As a result, a limitation of this study is the potential academic and behavioral variability among participants in the three groups. The small sample size and purposive sampling prevents the researcher from being able to generalize the results to the larger population of students with EBD (Gast & Ledford, 2010; Gast, 2010; Horner et al., 2005).

The FAA was selected as a dependent measure to remove reading ability as a confounding variable during this study. The assessment required no reading; all 20 problems were algorithms. The choice of median as an appropriate measure of central tendency was based on the researcher's desire to minimize the variability of the small samples (Stavig & Gibbons, 1977). The group median score from the fraction addition assessment was used to conduct the visual analysis of data. A limitation of this study may be that the groups were not closely matched and group scores using medians may mask the variability of students within groups (Gast & Ledford, 2010). This limitation was mitigated by an intra-subject analysis of individual students in addition to the inter-group analysis creating a simultaneous replication design (Kelly, 1980).

The online generation and manual screening of the FAA did not take into account the number of problems with fractions that represent whole number equivalents (e.g. 5/5, 3/3). Because this variance was not considered in setting the criteria for manual screening, the frequency of whole number equivalent fractions on each FAA ranged from 4 to 11. The wide range of problems that contained whole number equivalents may have introduced a variation in difficulty across the multiple forms of the FAA and as a result should be considered a weakness of this study.

Another potential limitation may have been missing data due to absences. While absent participants made up the intervention instruction and activities, they did not make up FAAs. As a result, some days when the median was calculated for each group only two participants were present. Since this median score was used as the measure of central tendency because it is less sensitive to missing data than mean or mode (Stavig & Gibbons, 1977), the missing data could limit the overall results of this study.

Lack of external validity may be a limitation of this study due the unique characteristics of this population of students, the limited number of groups (three), and the small number of student participants ($n = 9$). Subsequent replications of this study with similar groups of students may strengthen the findings of this study (Odom et al., 2005).

This study was conducted in a blended learning environment; students accessed their online course from a self-contained classroom led by a special education teacher certified in mathematics. The teacher's educational background and certification made her uniquely qualified to administer the intervention, which may be a factor in future attempts to replicate this study in other settings.

In the present study the researcher chose to focus on the effects of a two-part intervention package, RNP/VM. The researcher did not seek to isolate the effects of the component parts of the intervention package; data were not collected on frequency, duration, or appropriate use of the VMs. Future studies should be designed to gather data on the use and effectiveness of VMs and to further look at separate as well as blended approaches to learning.

Additionally, the researcher did not seek to answer the question of how the intervention and observed improvements in fraction knowledge affected students' performance on fraction related questions in their online mathematics courses or end of year high stakes testing. Studies by future researchers should attempt to address those questions and drill down to assess student performance on individual online problems where the use of those skills may be required.

Conclusion

Several authors and researchers provide ample evidence to underscore the critical connection between the conceptual understanding of fractions and success in algebra (Hutchinson, 1993; Mayer, 1998; Siegler et al., 2012; Witzel et al., 2003; Wu, 2009) as well as algebra's direct connection to high school graduation and positive post-secondary outcomes, especially for students with EBD (Blackorby & Wagner, 1996; Bradley et al., 2008; Cavanaugh et al., 2005; Watson & Gemin, 2008). Despite the documented connection, literature supporting effective evidence-based practices for instruction or intervention in fractions for this population of students is scant as is research measuring mathematics learning gains as a result of instructional practices and strategies (Hodge et al., 2006; Templeton et al., 2008). This paucity of research led both Hodge and Templeton, along with their colleagues, to conclude that little is known about effective mathematics instruction for this population of students. The two meta-analyses uncovered only one study that involved fraction knowledge (Franca et al., 1990), which leads this researcher to suggest that nothing is known about effective fraction instruction for students with EBD.

Despite the limited evidence for mathematics interventions, one theme is clear in each of the studies published in the mathematics literature for students with EBD; interventions work. The investigators who conducted the 21 diverse studies that included 203 students with EBD over the 28 years reviewed in two meta-analyses (Hodge et al., 2006; Templeton et al., 2008) plus the extension of the Hodge study conducted by this researcher, all reported positive if not significant mathematics learning gains for students who participated in the interventions. A correlation between mathematics interventions and learning gains for students with EBD was apparent; this population of students responded positively to all mathematics interventions.

This population of students responds positively to intervention. Students with EBD provide outstanding opportunities for researchers to test the effectiveness of existing evidence based practices in mathematics on students whose post-secondary outcomes depend on educators identifying and using effective instructional practices and interventions (Blackorby et al., 1991; Bradley et al., 2008).

Online learning is changing the face of education (Watson et al., 2013) and has attracted the interest of school districts struggling with limited financial resources and shortages of content certified special education teachers. The emergence of district mandated online instruction for students with EBD in Florida, a state that is leading the nation in online learning options (Watson et al.), increases the likelihood that other states and districts may follow Florida's lead. A dearth of research exists on effective online instructional practices for students with EBD and on the impact of those practices on the social/emotional and behavioral goals of this population of students (Black, Ferdig, & DiPietro, 2008; Cavanaugh et al., 2009; Vasquez & Straub, 2012). Despite a lack of research evidence on the effectiveness of online instruction, the staffing and financial benefits of this mode of instruction may increase the likelihood that more schools and districts will choose online instructional delivery for students in self-contained settings, particularly for students with EBD.

Mandated online learning, as in the case of the school district in this study, is a pivotal departure from student choice, one of the founding principles of online learning. It was the appeal of choice – "any time, any place, any path, any pace" – touted first by the nation's oldest and largest virtual school that sparked the various applications and exponential growth of online and blended learning.

Call to Action

Without replication, this study will be just another disjointed attempt at demonstrating the effectiveness of an intervention for students with EBD; the study count for this population simply moves from 21 to 22. Students with EBD have a documented academic need for mathematics intervention and researchers have found this population to be responsive to a variety of mathematics interventions and strategies (Hodge et al., 2006; Templeton et al., 2008). A collaborative effort to establish a research agenda related to students with EBD along with research studies that align with that agenda would not only intentionally replicate studies that have identified promising practices but also generate new studies in special education. This effort could potentially produce a catalogue of evidence based practices for this population of students that align with the standards outlined by Horner and colleagues (2005). A similar collaborative effort is indicated for research related to online instruction for students with EBD (Coy et al., in review).

Researchers need to evaluate not only the academic outcomes of online learning for students with EBD, but also question the impact of many of the latent factors of the online environment. What are the effects of the physical and social isolation inherent in online learning for students with EBD on developing social skills and peer relationships? In pure online learning environments, what is the impact of no human contact or touch on the social and behavioral goals for students with EBD? To what extent does online or blended learning improve academic persistence or time on task for students with EBD? What types of reinforcers are valuable to students with EBD learning in virtual environments? What strategies can be used to motivate students with EBD to persist in the successful completion of online courses? What types of online environments foster the best blends of technology and human interaction to meet the academic, social/emotional, and behavioral needs of students with EBD? Countless research questions and opportunities lie ahead as online learning expands to serve more students with EBD.

The only barriers to testing and identifying effective instructional practices for students with EBD in face-to-face, online, and blended environments are researchers and resources. It is time to escalate research efforts related to effective mathematics instruction and remediation for students with EBD. The continued proliferation of online and blended learning and the ability of those environments to mitigate staffing and financial crises in special education create an opportunity for mandated online instruction to spread rapidly if touted as an effective solution to school and district special education dilemmas for students with EBD. This potential shift from choice to mandate provides the perfect time to advance a research agenda related to mathematics learning in online environments.

APPENDIX A: INSTITUTIONAL REVIEW BOARD LETTER

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Barbara A. Serianni**

Date: **February 10, 2014**

Dear Researcher:

On 2/10/2014, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	Exempt Determination
Project Title:	Supporting Middle School Students with Emotional or Behavioral Disorders in Online Mathematics: A Fraction Intervention Using Virtual Manipulatives
Investigator:	Barbara A Serianni
IRB Number:	SBE-14-10054
Funding Agency:	
Grant Title:	
Research ID:	n/a

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Signature applied by Joanne Muratori on 02/10/2014 12:25:39 PM EST

IRB Coordinator

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APPENDIX B:
RESULTS AND EXTENSION OF META-ANALYSIS

Results of Hodge et al., 2006

Summary of Mathematics Intervention Studies for Students With EBD

Authors	Subjects	Intervention	Setting	Design	Dependent Measure	Results
Cade & Gunter, 2002	3 males with EBD 2 AA, 1 Cau Ages: 11, 12,14 years	Mnemonic strategy to solve basic division facts	Special day school	Multiple baseline across subjects with multiple probes	Accuracy on division by 7 worksheets	Mnemonic strategy increased students accuracy on division by 7 math facts
Carr & Punzo, 1993	3 AA males with EBD Ages: 13–15 years	Self-monitoring during independent work	Middle school self-contained class	Multiple baseline across skills	Accuracy, and productivity on basic operation math worksheets	Increased accuracy and productivity across subjects and settings
Davis & Hajicek, 1985	7 students with EBD Ages: 9–15 years	Strategy training and self-instructional training for problem solving	Psycho- educational center	Multiple baseline across treatments	Rate of accuracy & attention to the multiplication problems with decimals	Self-instruction condition led to greater improvements in accuracy and attention compared to strategy training alone
Franca, Kerr, Reitz, & Lambert, 1990	8 males with EBD Ages: 13–16 years	4-step peer tutoring	Self-contained classroom in a private school	Multiple Baseline across subjects	Social interactions and error rates for fraction problems	Both tutor and tutee improved social interactions and math performance
Jolivette, Wehby, & Hirsch, 1999	3 males with EBD Ages: 9, 10, 10 years, 4th grade	Preassessment, strategy instruction, preferred strategy replication	Summer school program	Rapid reversal design	Subtraction worksheet	Increased accuracy for all students
Landeen & Adams, 1988	10 males with EBD, Ages: 8–10 years	Paper and pencil & computer-assisted drill and practice	Self-contained special education class	Multi-element baseline design	Numbers of correct and incorrect subtraction facts	Increased ratio of correct to incorrect for both conditions
Lazarus, 1993	18 students with EBD: 14 males, 4 females Ages: 11–13 years	Teacher directed instruction & self-management	Middle school self-contained special education class	Multiple baseline across subjects	Accuracy on 20 math problems	Increased math performance
Lee, Sugai, & Horner, 1999	2 males with EBD Age: 9 years 3rd grade	Individualized direct instruction	Self-contained special education classroom	Within-subject alternating treatment design	Accuracy on the Math Skills Assessment (MSA)	Increased accuracy on math skills assessment and decreased off-task behavior
Levendoski & Cartledge, 2000	4 males with EBD: 3 Cau, 1 AA Ages: 9–11 years	Self-monitoring with visual and auditory cues	Self-contained classroom	Withdrawal design	Percentage correct math problems	Increase in on-task behavior and math productivity

Scruggs, Mastropieri, & Tolfa-Veit, 1986	85 total: 63 males, 22 females 41 EBD 44 LD 4–6th grade	Test taking strategies	Self-contained classroom	Group comparison posttest-only	Stanford Achievement Test	Significant effects for word study skills and math concepts; no difference between groups
Skinner, Bamberg, Smith, & Powell, 1993	3 males with EBD Ages: 9, 12, 12 years	Cover, copy, and compare practice strategy	Private school self-contained elementary class	Multiple baseline across tasks	Rate of accurate responding to division problems	Increased rate of accurate responses
Skinner, Turco, Beatty, & Rasavage, 1989	4 students with EBD: 3 males, 1 female two 4th grade and two10th grade	Look, cover, write answer, evaluate	Special school for students EBD	Multiple baseline across subjects	Multiplication facts	Increased rates and accuracy; maintained overtime
Skinner, Ford & Yunker, 1991	2 males with EBD Ages: 9, 11 years	Verbal and written cover, copy, and compare	Residential school	Alternating treatments design	Number of digits correct/ minute on multiplication facts	Verbal cover, copy, and compare more effective than written, cover, copy, and compare

Note. EBD = emotional/behavioral disorders; AA = African-American; Cau = Caucasian.

Extension of Hodge et al., 2006

Authors	Subjects	Intervention	Setting	Design	Dependent Measure	Results
Lee, D. L., Lylo, B., Vostal, B., & Hua, Y. (2012).	3 males, 14, 16,18 with EBD	Use of high preference problems	Alternative school for students with EBD	SSD Multiple Baseline across participants	Mean latency	Decreased latency to initiate low-pref math problems
Rafferty, L. A., & Raimondi, S. L. (2009).	1 female 3^{rd} grader w EBD 2 male 3^{rd} graders w EBD 2 males 1 2^{nd} 1 3^{rd} grade comparison students	Comparison of Self monitoring of attention vs self monitoring of academic performance	2 Self contained classrooms in a public elementary school VE class size 15	SSD alternating treatment design	On task behavior Academic productivity Academic accuracy	Increased accuracy, productivity, and on task behavior with SMP
Alter, P., Brown, E., & Pyle, J. (2011).	2 4^{th} grade males w EBD 1 1^{st} grade female w EBD	Teaching strategy delivered by teacher who was trained by workshop on conceptual understanding	Self contained classroom for students with EBD	SSD Multiple Baseline across participants	% of word problems solved correctly AND % of time on task	Mean accuracy and time on task increased compared to baseline
Haydon, T., Hawkins, R., Denune, H., Kimener, L., McCoy, D., & Basham, J. (2012	1 11^{th} grade female wEBD 1 10^{th} grade male w EBD 1 11^{th} grade male w EBD	Comparison of iPads vs worksheet for independent mathematics practice	Alternative school for students with EBD	SSD alternating treatment design	Active engagement Correct responses per minute	iPad condition produced higher rates of active engagement and increase in correct responses per minute
Billingsley, G., Scheuermann, B., & Webber, J. (2009)	10 HS students gr 9-11 w EBD 9 male 1 female	Comparison of 3 instructional methods, direct teach, computer-assisted instruction and a combination of both	Self contained special education classroom in public high school	SSD alternating treatment design	Math learning from a teacher created Curriculum Based Measurement	Mixed results across participants, no condition was clearly superior for all participants
Mulcahy, C. A., & Krezmien, M. P. (2009)	4 middle school students w EBD, 2 female 2 male	Contextualized instruction package for geometry	Self contained classroom for students with EBD	SSD Multiple Baseline across participants	Accuracy on 3 probes: domain, objective and transfer/maintenance	Performance for perimeter and area was improved, but transfer & maintenance results were mixed
Ramsey, M. L., Jolivette, K., Patterson, D., & Kennedy, C. (2010).	3 female w EBD 13, 14,15 2 male w EBD 15,16	Choice/no choice conditions	Residential setting for children with ED	SSD ABAB withdrawal design	Time on task Task completion Accuracy	Functional relationship shown for all three DVs
Glago, K., Mastropieri, M. A., & Scruggs, T. E. (2009)	21 4^{th} & 5^{th} grade students with LD and EBD	5 step self determination strategy	General education classroom, suburban elementary	Quasi-experimental pre/post randomized	Pre/post problem solving Pre/post strategy application	Both measures showed a sig difference favoring the experimental group

APPENDIX C:
RESULTS OF CONTENT ANALYSIS

Information Gathered in Content Analysis using Virtual Manipulatives

IFI Lesson	Student VM	Teacher VM	Comments
Lesson 1	FC	FC	1- Virtual FCs are used easily with lesson. 2- Students need an introduction to all virtual manipulatives. 3- No changes to script required.
Lesson 2	FC	FC	1- Virtual FCs are used easily with lesson. 2 -No changes to script required.
Lesson 3	FC	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 4	PF	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 5	FC	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 6	FC	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 7	PF	PF	1- Instructional language needs to be modified substituting "select the paper strip" for folding instructions.
Lesson 8	FC	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 9	FC		1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 10	FC, PF		1- Instructional language needs to be modified substituting "select the paper strip" for folding instructions. 2- Virtual FCs are used easily with lesson.
Lesson 11	FC	FC	1- Virtual FCs are used easily with lesson. 2- No changes to script required.
Lesson 12	PF, C	C	1-Virtual Cs are used easily with this lesson. 2- Minor modifications to lesson instructions needed for virtual PF.
Lesson 13	C	C	1- Virtual Cs are used easily with lesson. 2- No changes to script required.
Lesson 14	C		1- Virtual Cs are used easily with lesson. 2- No changes to script required.
Lesson 15	C	C	1- Virtual Cs are used easily with lesson. 2- No changes to script required.

Note: Fraction circles (FC), Chips (C), Paper Folding (PF).

Table of Changes to RNP Initial Fraction Idea Lesson 4

#	Language to be Changed	New Language
1	Prior to using paper strips to model fractions it is necessary to practice folding strips into 2, 3, 4, 6, 8, and 12 equal parts. Ask students to follow along with you as you model how to fold paper strips. Fold paper strip into two equal parts:	Prior to using paper folding to model fractions it is necessary to practice selecting strips with 2, 3, 4, 6, 8, and 12 equal parts. Ask students to follow along with you as you model how to select fraction strips. Select strip with two equal parts:
2	Keep it folded. Now fold it again into two equal parts, unfold.	Keep your first fraction strip. Now select one that divides each half into two equal parts. Ask: how many equal parts do you think we have?
3	Ask students to verbalize how to fold paper strips to form four equal parts	Ask students to verbalize how to select fraction strips that have four equal parts.
4	Model folding into three equal parts. Form the letter "S" with a paper strip to get close to 3 equal parts. Press down on paper.	Model selecting a strip with three equal parts.
5	Model sixths. Fold paper strip into thirds and then fold into two equal parts. Have students do this and guess, before unfolding, the number of equal parts that they expect.	Model selecting a strip divided into sixths. Place the strip of 6ths directly under the strip of 3^{rds}. Have students model this and describe how a strip of 3rds can become a strip of 6ths
6	Ask students if they could have obtained sixths by folding first in halves and then in thirds? Try it.	Ask students if they could have obtained sixths by talking a strip with halves and cutting each half in thirds? Demonstrate.
7.	Ask students to think of strategies for folding 8ths and 12ths. Encourage trial and error strategy. Have them verbalize successful ways. For 12ths reinforce multiple ways.	Ask students to think of strategies for getting 8ths and 12ths. Encourage trial and error strategy using paper folding. Have them verbalize successful ways. For 12ths reinforce multiple ways.
9	Ask: How can you show me one-fourth with a paper strip? Have students fold into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth.	Ask: How can you show me one-fourth with a virtual fraction strip? Have students select strip divided into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth
Student Pages E & F	You'll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction you have shown.	You'll need to use the paper folding VM. For any four of the figures shown below, select paper strips to model the fraction that the figure represents. After you have selected the strip and shaded the part, write the name of the fraction you have modeled (use words, not symbols).

Table of Changes to RNP Initial Fraction Idea Lessons

Lesson 7	Language to be Changed	New Language
1	Ask children to fold a strip of paper into 4 equal parts. Using the same strip of paper ask them how they can increase the number of equal parts to 8. Have them do so, but before they open up the strip of paper to show eighths ask: Before you open up the strip, can you tell me if the size of the equal parts will be larger or smaller than fourths? Why?	Ask children to select a fraction strip with 4 equal parts. Ask them how they can increase the number of equal parts to 8. Have them demonstrate that by selecting the appropriate paper strip. Ask students, can you tell me if the size of the equal parts will be larger or smaller than fourths? Why?
3	Now ask students to fold, shade, and label these fractions with paper folding.	Now ask students select a paper strip, shade it , and label these fractions using paper folding:

Lesson 10	Language to be Changed	New Language
1	Ask students to fold strips of paper into thirds. Shade 2/3 of the area. Write the symbol for that amount on the strip	Ask students to select paper strip divided into thirds. Shade 2/3 of the area. Write the symbol for that amount on the strip here.
2	Now ask students to fold, shade, and label these fractions with paper folding.	Now ask students select a paper strip, shade it , and label these fractions using paper folding:
3	Now have students select a second strip that shows 6 equal parts. Ask them to guess the number of parts they will shade.	Now have students fold the same strip to show 6 equal parts. Before they actually open up the folded paper, ask them to guess the number of shaded parts.
4	Open up the amount and record on the paper strip	Record student guesses on the board. Have students record their guess on a piece of paper.

Lesson 12	Language to be Changed	New Language
2	Can you partition your paper strip into 3 equal pieces?	Can you select a paper strip with 3 equal pieces?

Rational Number Project

Initial Fraction Ideas
Lesson 4: Overview

Students use paper folding to model and name unit and non-unit fractions. Students compare the paper-folding model to fraction circles. Students record fractions in words: one-fourth, two-thirds.

Materials
- ∞ Paper strips for folding for students
- ∞ Fraction Circles for teacher
- ∞ Student Pages A-L

Teaching Actions

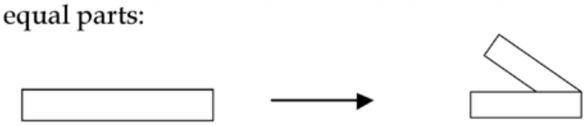
Warm Up

Name the red piece in three different ways by changing the unit. What different units did you use?

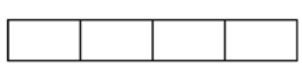
Large Group Introduction

1. Prior to using paper strips to model fractions it is necessary to practice folding strips into 2, 3, 4, 6, 8, and 12 equal parts.

   Ask students to follow along with you as you model how to fold paper strips. Fold paper strip into two equal parts:



2. Keep it folded. Now fold it again into two equal parts. Ask: how many equal parts do you think we have? Unfold:



3. Ask students to verbalize how to fold paper strips to form four equal parts.

Comments

This lesson may take two class periods. Students are still recording fractional amounts using word names; symbols are introduced in lesson 5.

Cut paper strips from 8.5″ by 11″ sheets of paper about 1 inch wide and 8.5″ long.

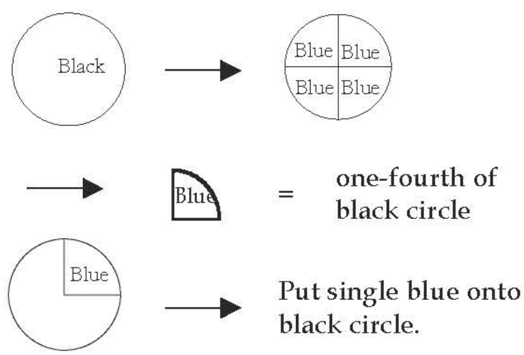
Teaching Actions

5. Model sixths. Fold paper strip into thirds and then fold into two equal parts. Have students do this and guess, before unfolding, the number of equal parts they expect.

6. Ask students if they could have obtained sixths by folding first in halves and then in thirds? Try it.

7. Ask students to think of strategies for folding 8ths and 12ths. Encourage trial and error strategy. Have them verbalize successful ways. For 12ths reinforce multiple ways.

8. Students can shade equal parts of paper strips to show fractions. Using fraction circles, show one-fourth using a black circle as the unit.



Say: To show one-fourth of a black circle I divided it into four equal parts. Pick up one of the parts to show one-fourth.

9. Ask: How can you show me one-fourth with a paper strip? Have students fold into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth.

10. Discuss how the two displays for one-fourth are alike and different.

11. Repeat for 1-third, 1-eighth, 1-twelfth.

Comments

Students often will expect 5 equal parts (3+2). They are more apt to think additively than multiplicatively.

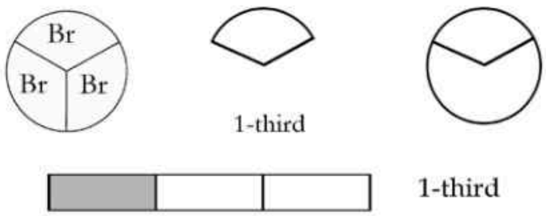
<u>To get 12ths</u>
Halves → halves → thirds
Thirds → halves → halves
Halves → thirds → halves

The similarity between the two displays is what's important. A unit is divided into <u>equal</u> parts and one or more equal parts are highlighted in some way. This is a *manipulative to manipulative* translation.

Teaching Actions

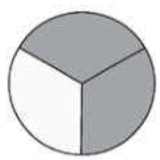
Comments

12. Look at two displays for one-third:



13. Shade in another third on the paper strip.

Ask: how many thirds are shown now? How can I show two-thirds with circles? (Pick up two browns and say these are two-thirds of black.) State that 2-thirds is 1-third and 1-third more:



Comment: Non-unit fractions are introduced as sums of unit fractions: 2-fourths is 1-fourth and 1-fourth.

14. Now draw a picture of a square. Divide it into 4 equal parts and shade 3 of 4 parts. Ask students to fold paper to show the same fraction that you drew. Record fractions as 3-fourths: 1-fourth + 1-fourth + 1-fourth.

Comment: Students now have seen two models for fractions. Practice pages that follow this lesson give students a chance to apply their new learning to pictures of units in different shapes.

15. Return to fraction circles. Model problems as in lesson 3, this time with non-unit fractions.

<u>Examples</u>:

- ∞ The black circle = 1. What is the value of 1 blue; 3 blues; 1 brown; 2 browns; 3 reds.
- ∞ The yellow piece = 1. What is the value of 1 blue; 2 reds; 3 grays; 2 pinks.

Teaching Actions

Small Group/Partner Work

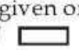
16. There are several student pages in this lesson. **Select the most appropriate ones for your students.** Students may need some assistance to do some of the pages. See Comments for clarification.

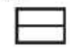
Wrap Up

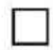
17. Go over problems 6 and 7 from Student Page B. Have students share their solutions. Pick and choose other problems for students to share.

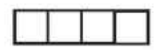
Comments

Teacher Notes for Student Pages:

B: Problems 6 and 7 provide some problem solving. Students reconstruct the unit given one part. For example if  equals 1-half, then the whole must be two of those parts:

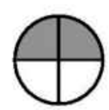


If  equals 1-fourth, then the whole must be four of those parts:



G: Clarify with students that a picture may need to be modified to determine if 2-fourths are shaded in. For example:

Is 2-fourths shaded?



2-fourths can easily be seen once the picture is completed by drawing in the needed lines.

Translations

- Manipulative to verbal
- Manipulative to manipulative to verbal
- Manipulative to verbal to written symbols (word names)

Modified Sample Lesson
(*Only Page 1 And 2 Required Modification*)

Rational Number Project

Initial Fraction Ideas
Lesson 4: Overview

Students use paper folding to model and name unit and non-unit fractions. Students compare the paper-folding model to fraction circles. Students record fractions in words: one-fourth, two-thirds.

Materials

- ☐ Paper folding VM for folding for students
- ☐ Fraction Circle VM for teacher
- ☐ Student Pages A-L

Teaching Actions

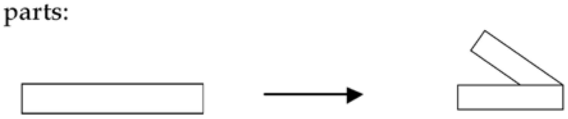
Warm Up

Name the red piece in three different ways by changing the unit. What different units did you use?

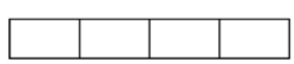
Large Group Introduction

1. Prior to using paper folding to model fractions it is necessary to practice selecting strips with 2, 3, 4, 6, 8, and 12 equal parts.

   Ask students to follow along with you as you model how to select paper strips. Select strip with two equal parts:



2. Keep your first paper strip. Now select one that divides each half into two equal parts. Ask: how many equal parts do you think we have?



3. Ask students to verbalize how to select paper strips that have four equal parts.

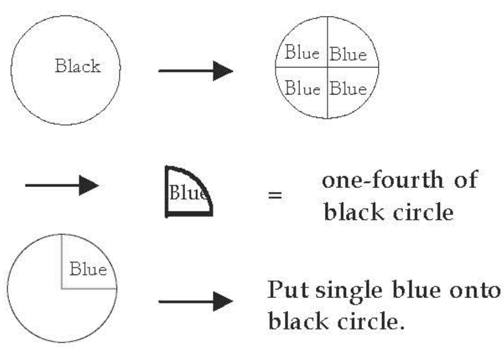
4. Model selecting a strip with three equal parts.

Comments

This lesson may take two class periods. Students are still recording fractional amounts using word names; symbols are introduced in lesson 5.

Cut paper strips from 8.5″ by 11″ sheets of paper about 1 inch wide and 8.5″ long.

Teaching Actions	Comments
5. Model selecting a strip divided into sixths. Place the strip of 6ths directly under the strip of 3rds. Have students model this and describe how a strip of 3rds can become a strip of 6ths.	Students often will expect 5 equal parts (3+2). They are more apt to think additively than multiplicatively.
6. Ask students if they could have obtained sixths by talking a strip with halves and cutting each half in thirds? Demonstrate.	
7. Ask students to think of strategies for getting 8ths and 12ths. Encourage trial and error strategy using paper folding. Have them verbalize successful ways. For 12ths reinforce multiple ways.	<u>To get 12ths</u> Halves → halves → thirds Thirds → halves → halves Halves → thirds → halves
8. Students can shade equal parts of paper strips to show fractions. Using fraction circles, show one-fourth using a black circle as the unit.	



Teaching Actions	Comments
Say: To show one-fourth of a black circle I divided it into four equal parts. Pick up one of the parts to show one-fourth.	
9. Ask: How can you show me one-fourth with a paper strip? Have students select strip divided into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth.	
10. Discuss how the two displays for one-fourth are alike and different.	The similarity between the two displays is what's important. A unit is divided into <u>equal</u> parts and one or more equal parts are highlighted in some way. This is a *manipulative to manipulative* translation.
11. Repeat for 1-third, 1-eighth, 1-twelfth.	

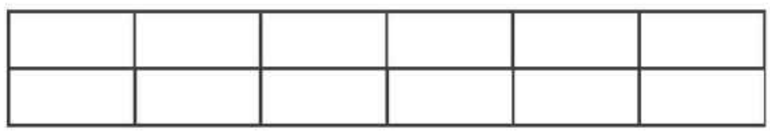
Sample Lesson Activity Sheets

Name________________________________ Lesson 4 / Student Page A

1. Here is a picture of a candy bar.

Draw to show the candy bar divided into 5 equal-sized pieces.

2. Here is a picture of a pan of brownies.

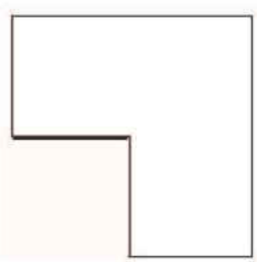


The pan of brownies is cut into _____________ equal-sized parts.

Each piece is _____________ of the whole pan.

3. O-So-Good candy bars come in the shape of a square. After Janis ate one pieces of an O-So-Good candy bar, it looked like the shape below.

   The piece that Janis ate is ______________ of the whole candy bar.



4. Hamdi's garden is a rectangle. Draw a picture of Hamdi's garden. Show on your drawing that the garden is in 9 equal-sized parts.

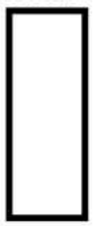
5. Devan's garden is in the shape of square. Draw a picture of Devan's garden.

Draw on Devan's garden to show it divided into 3 equal-sized parts.

Each part is ____________ of Devan's patio.

6. One-half of a coffee cake was left after a party was over.

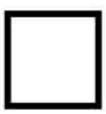
The half looked like this:



Draw a picture of the whole cake.

Explain to your classmates how you solved the problem.

7. Willis, Vang, Ellen, and Marta shared part of a candy bar equally. Marta's share looked like this:



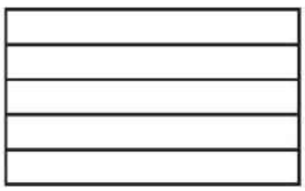
Draw a picture to show the whole candy bar.

Explain to your classmates how you solved the problem.

Name________________________________

For each exercise, look at the figure and then answer the question and write the word name for each fraction.

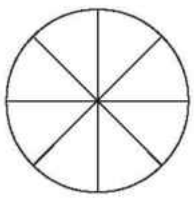
1.



________ equal-sized parts.

Each part is ______ ____________ of the whole.

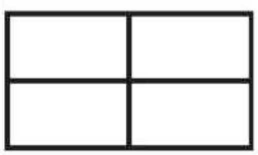
2.



________ equal-sized parts.

Each part is ______ ____________ of the whole.

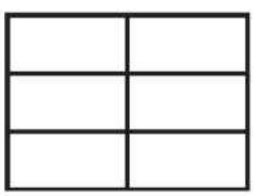
3.



________ equal-sized parts.

Each part is ______ ____________ of the whole.

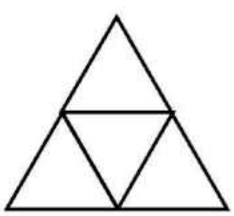
4.



________ equal-sized parts.

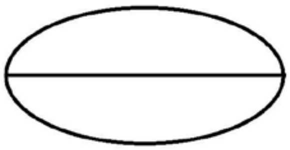
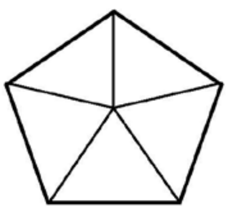
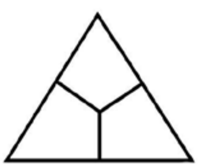
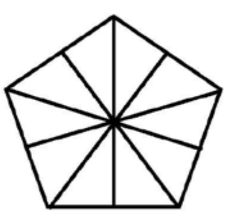
Each part is ______ ____________ of the whole.

5.



________ equal-sized parts.

Each part is ______ ____________ of the whole.

6. 	________ equal-sized parts. Each part is ______ ___________ of the whole.
7. 	________ equal-sized parts. Each part is ______ ___________ of the whole.
8. 	________ equal-sized parts. Each part is ______ ___________ of the whole.
9. 	________ equal-sized parts. Each part is ______ ___________ of the whole.
10. 	________ equal-sized parts. Each part is ______ ___________ of the whole.

APPENDIX D:
ASSESSMENT INSTRUMENTS

Name: ______________________________

Fraction Addition Assessment (sample)

Complete the following and reduce answers to lowest terms. You may skip problems or work problems in any order.

1. $\frac{7}{9} + \frac{5}{8} =$

2. $\frac{7}{7} + \frac{7}{7} =$

3. $\frac{6}{7} + \frac{5}{8} =$

4. $\frac{4}{8} + \frac{1}{5} =$

5. $\frac{2}{3} + \frac{4}{5} =$

6. $\frac{3}{4} + \frac{6}{7} =$

7. $\frac{3}{9} + \frac{1}{4} =$

8. $\frac{5}{7} + \frac{8}{9} =$

9. $\frac{1}{5} + \frac{8}{9} =$

10. $\frac{5}{7} + \frac{7}{7} =$

11. $\frac{6}{6} + \frac{7}{10} =$

12. $\frac{1}{6} + \frac{3}{4} =$

13. $\frac{8}{9} + \frac{7}{8} =$

14. $\frac{1}{4} + \frac{3}{8} =$

15. $\frac{3}{4} + \frac{1}{10} =$

16. $\frac{5}{8} + \frac{3}{5} =$

17. $\frac{1}{7} + \frac{3}{9} =$

18. $\frac{4}{7} + \frac{7}{10} =$

19. $\frac{3}{7} + \frac{3}{9} =$

20. $\frac{5}{6} + \frac{9}{10} =$

Name: ______________________________

Fraction Addition Assessment (sample) -KEY

Complete the following and reduce answers to lowest terms. You may skip problems or work problems in any order.

1. $\frac{7}{9} + \frac{5}{8} =$ 101/72 = **1 29/72**

2. $\frac{7}{7} + \frac{7}{7} =$ 2/1 = **2**

3. $\frac{6}{7} + \frac{5}{8} =$ 83/56 = **1 27/56**

4. $\frac{4}{8} + \frac{1}{5} =$ 7/10 = **7/10**

5. $\frac{2}{3} + \frac{4}{5} =$ 22/15 = **1 7/15**

6. $\frac{3}{4} + \frac{6}{7} =$ 45/28 = **1 17/28**

7. $\frac{3}{9} + \frac{1}{4} =$ 7/12 = **7/12**

8. $\frac{5}{7} + \frac{8}{9} =$ 101/63 = **1 38/63**

9. $\frac{1}{5} + \frac{8}{9} =$ 49/45 = **1 4/45**

10. $\frac{5}{7} + \frac{7}{7} =$ 12/7 = **1 5/7**

11. $\frac{6}{6} + \frac{7}{10} =$ 17/10 = **1 7/10**

12. $\frac{1}{6} + \frac{3}{4} =$ 11/12 = **11/12**

13. $\frac{8}{9} + \frac{7}{8} =$ 127/72 = **1 55/72**

14. $\frac{1}{4} + \frac{3}{8} =$ 5/8 = **5/8**

15. $\frac{3}{4} + \frac{1}{10} =$ 17/20 = **17/20**

16. $\frac{5}{8} + \frac{3}{5} =$ 49/40 = **1 9/40**

17. $\frac{1}{7} + \frac{3}{9} =$ 10/21 = **10/21**

18. $\frac{4}{7} + \frac{7}{10} =$ 89/70 = **1 19/70**

19. $\frac{3}{7} + \frac{3}{9} =$ 16/21 = **16/21**

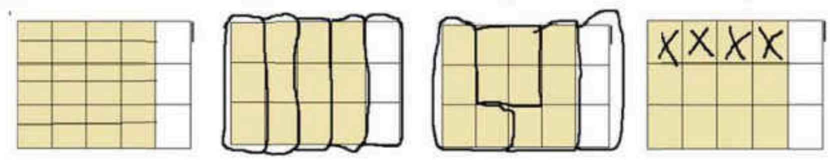
20. $\frac{5}{6} + \frac{9}{10} =$ 26/15 = **1 11/15**

Equivalent Fraction Pretest

Name____

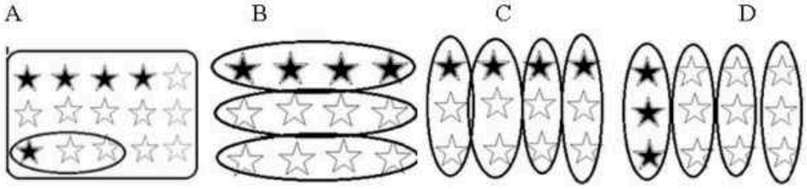
1. Maria wants to show that $\frac{4}{5}$ equivalent to $\frac{12}{15}$. Circle the drawing that shows $\frac{4}{5}$ is equivalent to $\frac{12}{15}$
.

A B C D



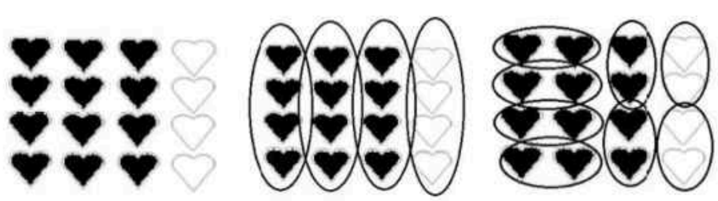
2. Sammy wants to show that $\frac{4}{12}$ is equivalent to $\frac{1}{3}$. Which drawing shows that $\frac{4}{12} = \frac{1}{3}$.

A B C D

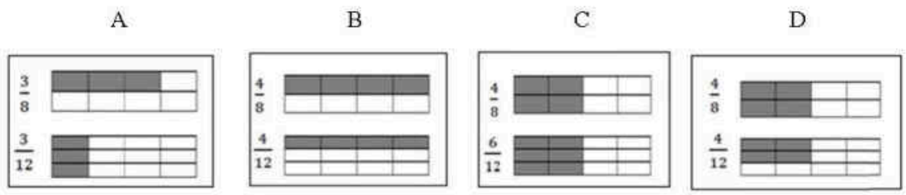


3. Draw lines to match the pictures with the correct fractions.

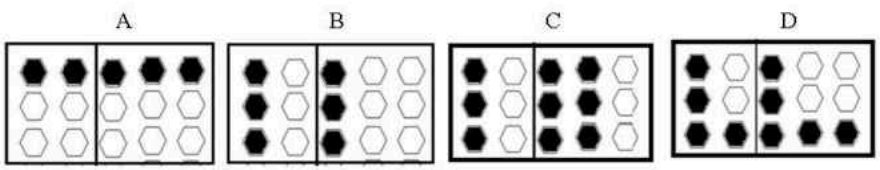
$\frac{3}{4}$ $\frac{6}{8}$ $\frac{12}{16}$



4. Circle the pair of drawings which show two equivalent fractions.



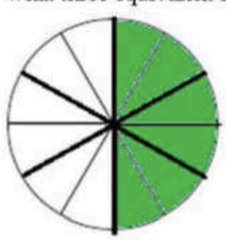
5. Circle the pair of sets which shows that the two fractions are equivalent.



6. Circle the statement that is correct,

A. $\frac{2}{3}=\frac{3}{2}$ B. $\frac{5}{6}=\frac{10}{12}$ C. $\frac{7}{9}=\frac{8}{9}$ D. $\frac{5}{7}=\frac{5}{8}$

7. What three equivalent fractions are shown in the circle?

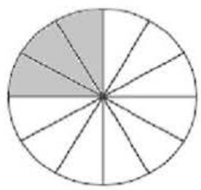


A. $\frac{1}{2}\ \frac{3}{4}\ \frac{6}{12}$ B. $\frac{1}{2}\ \frac{3}{2}\ \frac{6}{2}$ C. $\frac{1}{2}\ \frac{2}{2}\ \frac{3}{2}$ D. $\frac{1}{2}\ \frac{2}{4}\ \frac{3}{6}$

8. Which of the following groups show three equivalent fractions?

A. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}$ B. $\frac{1}{3}, \frac{2}{6}, \frac{4}{9}$ C. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}$ D. $\frac{1}{3}, \frac{3}{1}, \frac{3}{3}$

9. This picture shows that $\frac{1}{4} = \frac{3}{\square}$. What number can be put in the box to make the sentence true?



A. 9 B. 3 C. 4 D. 12

10. This picture shows that $\frac{6}{9} = \frac{\square}{3}$. What number can be put in the box to make the sentence true?

A. 3 B. 2 C. 1 D. 9

11. The box below shows that $\frac{10}{12}$ of the rectangle is shaded.

Which fraction is the simplified form of $\frac{10}{12}$?

A. $\frac{2}{6}$ B. $\frac{1}{4}$ C. $\frac{5}{12}$ D. $\frac{5}{6}$

12. $\frac{6}{10}$ of Liz's stars are black. Rename the fraction in its simplest form.

A. $\frac{3}{5}$ B. $\frac{4}{10}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

13. Write three fractions that are equivalent to $\frac{3}{5}$.

$\frac{3}{5}=$ ______ $\frac{3}{5}=$ ______ $\frac{3}{5}=$ ______

14. Fill in the missing numerator

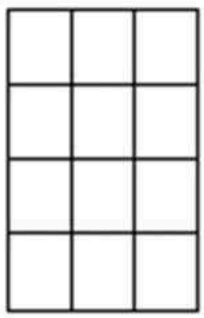
$$\frac{3}{4}=\frac{\square}{8}$$

15. Write $\frac{10}{15}$ in simplest form.

$$\frac{10}{15}=\frac{\quad}{\quad}$$

16. Using the box below show that $\frac{2}{3}$ is equivalent to $\frac{8}{12}$. Shade $\frac{8}{12}$ and then circle boxes to show $\frac{2}{3}$. Explain in words how your model shows that the two fractions are equivalent.

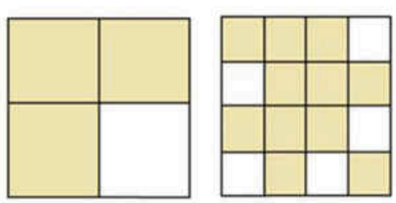
$\frac{8}{12} = \frac{2}{3}$



17. Sam said that the two squares below have the same fraction of shaded area.

Is Sam right or wrong?______________________

Write the fractions.



Fraction :_______ Fraction: _______

Explain why you think Sam is right or wrong.

18. This box shows that $\frac{1}{2} = \frac{6}{12}$

$\frac{1}{2} = \frac{6}{12}$

In the next two boxes show two <u>other</u> fractions which are also equivalent to ½.

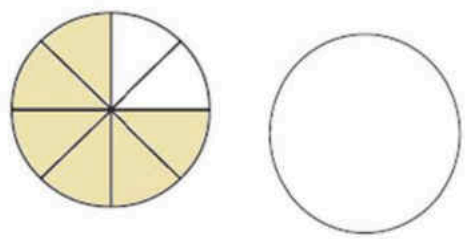
$\frac{1}{2} =$ ________

$\frac{1}{2} =$ __________

19. Nancy has 10 eggs. She colored $\frac{1}{5}$ of them blue. Draw a picture to show how many eggs Nancy colored blue.

20. The first circle shows $\frac{6}{8}$.
What is $\frac{6}{8}$ in simplest form? ___________

On the second circle draw and label a picture of the most simplified form of $\frac{6}{8}$.

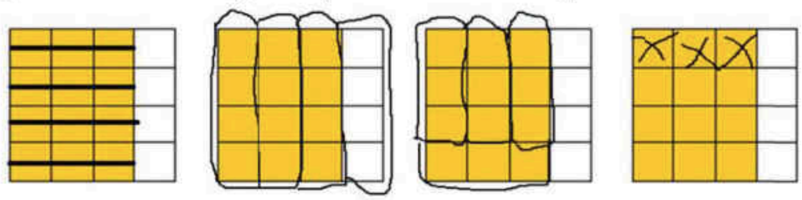


Equivalent Fraction Posttest 1

Name___________________________

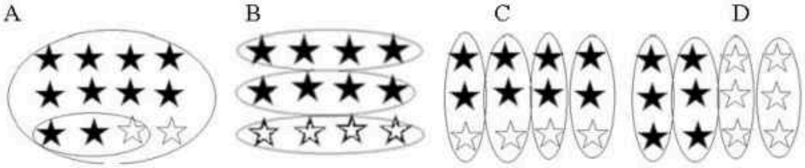
1. Maria wants to show that $\frac{3}{4}$ is equivalent to $\frac{12}{16}$. Circle the drawing that shows $\frac{3}{4}$ is equivalent to $\frac{12}{16}$

A B C D



2. Sammy wants to show that $\frac{8}{12}$ is equivalent to $\frac{2}{3}$. Which drawing shows that $\frac{8}{12}=\frac{2}{3}$?

A B C D

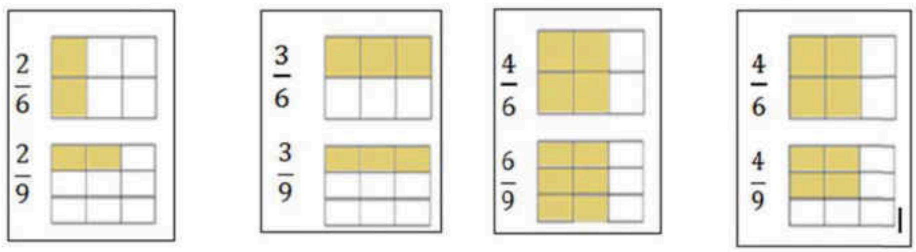


3. Draw lines to match the pictures with the fractions.

$\frac{2}{3}$ $\frac{4}{6}$ $\frac{8}{12}$

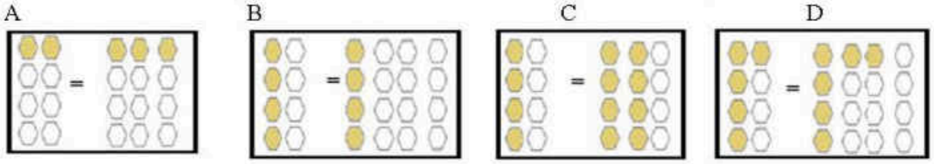
4. Circle the box with a pair of drawings which shows that the fractions are equivalent.

A B C D



5. Circle the box with a pair of sets which shows that the fractions are equivalent.

A B C D



6. Circle the statement that is correct.

A. $\frac{3}{4} = \frac{4}{3}$ B. $\frac{4}{5} = \frac{8}{10}$ C. $\frac{5}{7} = \frac{3}{7}$ D. $\frac{4}{7} = \frac{4}{9}$

7. What three equivalent fractions are shown in the circle?

A. $\frac{1}{4}\ \frac{2}{6}\ \frac{4}{16}$ B. $\frac{1}{4}\ \frac{2}{4}\ \frac{4}{4}$ C. $\frac{1}{4}\ \frac{2}{6}\ \frac{3}{16}$ D. $\frac{1}{4}\ \frac{2}{8}\ \frac{4}{16}$

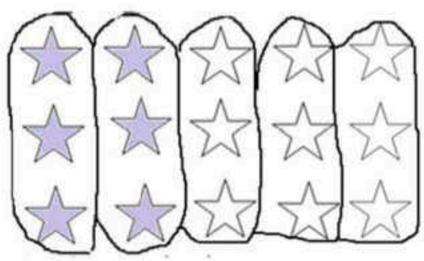
8. Which of the following groups show three equivalent fractions?

A. $\frac{1}{5}\ \frac{2}{6}\ \frac{3}{7}$ B. $\frac{1}{5}\ \frac{3}{10}\ \frac{6}{20}$ C. $\frac{1}{5}\ \frac{2}{10}\ \frac{3}{15}$ D. $\frac{1}{5}\ \frac{5}{1}\ \frac{5}{5}$

9. This picture shows that $\frac{3}{4} = \frac{9}{\ }$. What number can be put in the box to make the sentence true?

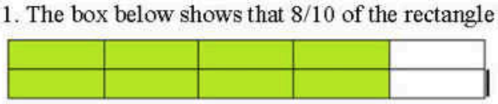
A. 9 B. 3 C. 4 D. 12

10. This picture shows that $\frac{6}{15} = \frac{\ }{5}$. What number can be put in the box to make the sentence true?



A. 6 B. 2 C. 3 D. 15

11. The box below shows that 8/10 of the rectangle is shaded.



Which fraction is the simplified form of 8/10?

B. $\frac{4}{7}$ B. $\frac{1}{3}$ C. $\frac{4}{10}$ D. $\frac{4}{5}$

12. $\frac{6}{8}$ of Ty's stars are black. Rename the fraction in its simplest form.

A. $\frac{3}{4}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$

13. Write three fractions that are equivalent to $\frac{4}{5}$.

$\frac{4}{5} = \frac{\quad}{\quad}$ $\frac{4}{5} = \frac{\quad}{\quad}$ $\frac{4}{5} = \frac{\quad}{\quad}$

14. Fill in the missing numerator.

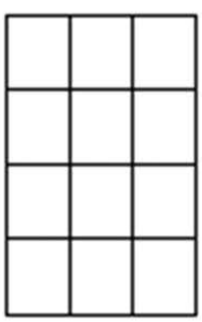
$$\frac{5}{6} = \frac{\quad}{12}$$

15. Write $\frac{4}{10}$ in simplest form.

$$\frac{4}{10} = \frac{\quad}{\quad}$$

16. Using the box below show that $\frac{3}{4}$ is equivalent to $\frac{9}{12}$. Shade $\frac{9}{12}$ and then circle the boxes to show $\frac{3}{4}$. Explain in words how your model shows that the two fractions are equivalent.

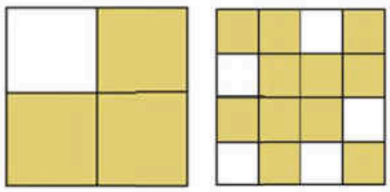
$\frac{3}{4} = \frac{9}{12}$



17. Sam said that the two squares below have the same fraction of shaded area.

Is Sam right or wrong? ___________________________

Write the fractions.



Fraction :_______ Fraction: _______

Explain why you think Sam is right or wrong.

____

____

18. This box shows that $\frac{1}{3} = \frac{4}{12}$

$\frac{1}{3} = \frac{4}{12}$

In the next two boxes make two <u>other</u> fractions which are also equivalent to ½.

$\frac{1}{3} = -$

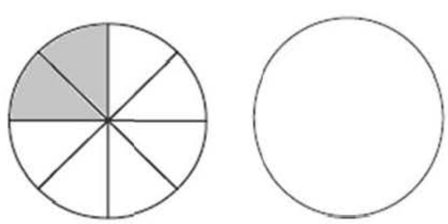
$\frac{1}{3} = -$

19. Nancy has 8 cup cakes. 1/4 of them are chocolate. Draw a picture to show how many cup cakes are chocolate.

20. The first circle shows $\frac{2}{8}$

What is $\frac{2}{8}$ in simplest form? ___________

On the second circle draw and label the simplified fraction for $\frac{2}{8}$.

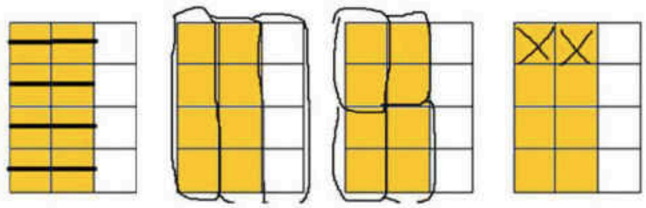


Equivalent Fraction Delayed Posttest

Name____________________

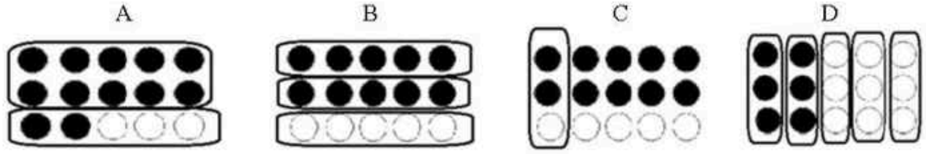
1. Maria wants to show that $\frac{2}{3}$ is equivalent to $\frac{8}{12}$. Circle the drawing shows $\frac{2}{3}$ is equivalent to $\frac{8}{12}$.

A B C D



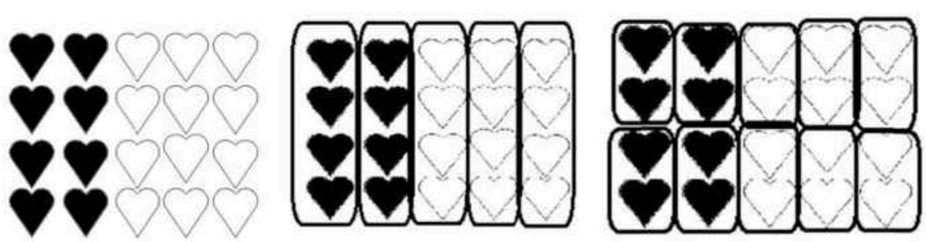
2. Sammy wants to show that $\frac{10}{15}$ is equivalent to $\frac{2}{3}$. Circle the drawing that shows $\frac{10}{15} = \frac{2}{3}$?

A B C D



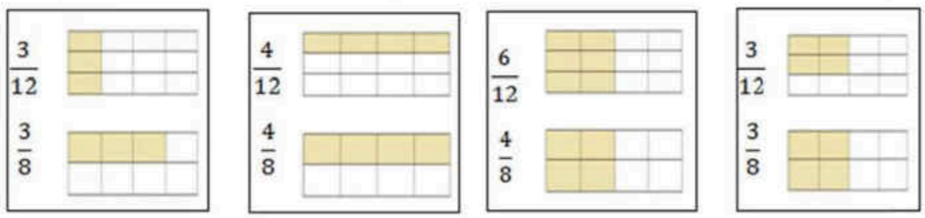
3. Draw a line to match each fraction with the correct picture.

$\frac{2}{5}$ $\frac{4}{10}$ $\frac{8}{20}$



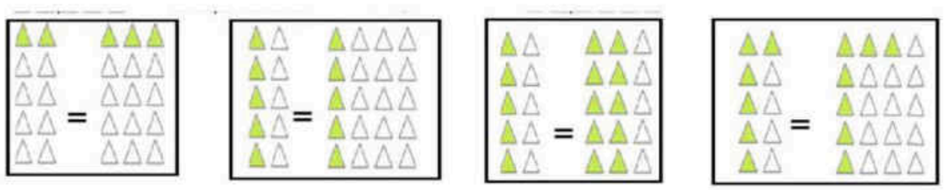
4. Circle the pair of drawings that shows that the fractions are equivalent.

A B C D



5. Circle the pair of sets that shows two equivalent fractions

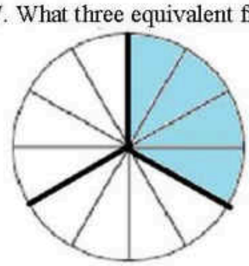
A B C D



6. Circle the statement that is correct?

A. $\frac{3}{5}=\frac{5}{3}$ B. $\frac{2}{3}=\frac{8}{12}$ C. $\frac{3}{9}=\frac{4}{9}$ D. $\frac{2}{3}=\frac{2}{5}$

7. What three equivalent fractions are shown in the circle?

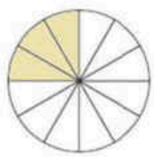


a. $\frac{1}{3}\ \frac{4}{4}\ \frac{4}{12}$ b $\frac{1}{3}\ \frac{2}{5}\ \frac{4}{7}$ c $\frac{1}{3}\ \frac{2}{3}\ \frac{3}{3}$ d $\frac{1}{3}\ \frac{2}{6}\ \frac{3}{9}$

8. Which of the following groups show three equivalent fractions?

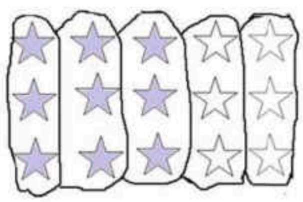
B. $\frac{1}{4}\ \frac{2}{5}\ \frac{3}{6}$ B. $\frac{1}{4}\ \frac{3}{8}\ \frac{9}{16}$ C. $\frac{1}{4}\ \frac{2}{8}\ \frac{3}{12}$ D. $\frac{1}{4}\ \frac{4}{1}\ \frac{4}{4}$

9. This picture shows that $\frac{1}{4} = \frac{\ }{12}$. What number can be put in the box to make the sentence true?



A. 1 B. 12 C. 4 D. 3

10. This picture shows that $\frac{9}{15} = \frac{\ }{5}$. What number can be put in the box to make the sentence true?



A. 9 B. 3 C. 2 D. 1

11. The box below shows that $\frac{6}{8}$ of the rectangle is shaded.

Which fraction is the simplified form of $\frac{6}{8}$?

C. $\frac{3}{5}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{3}{4}$

12. 8/10 of Liz's stars are black. Rename the fraction in its simplest from.

A. $\frac{4}{5}$ B. $\frac{1}{2}$ C. $\frac{4}{10}$ D. $\frac{1}{3}$

13. Write three fractions that are equivalent to $\frac{2}{3}$.

$\frac{2}{3} = \frac{}{}$ $\frac{2}{3} = \frac{}{}$ $\frac{2}{3} = \frac{}{}$

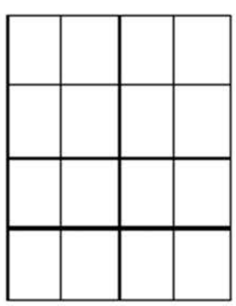
14. Fill in the missing denominator

$$\frac{3}{4} = \frac{9}{}$$

15. Write $\frac{8}{12}$ in simplest form.

$\frac{8}{12} = \frac{}{}$

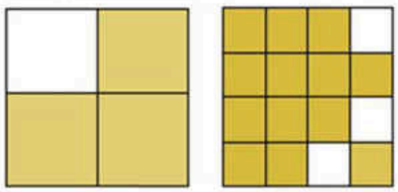
16. Using the box below show that $\frac{3}{4}$ is equivalent to $\frac{12}{16}$. Shade $\frac{12}{16}$ and then circle boxes to show $\frac{3}{4}$. Explain in words how your model shows that the two fractions are equivalent.



17. Sam said that the two squares below have the same fraction of shaded area.

Is Sam right or wrong?____________________

Write the fractions.



Fraction :_______ Fraction: _______

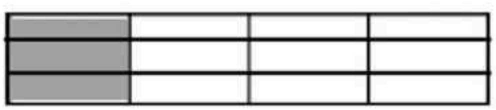
Explain why you think Sam is right or wrong.

____

____

18. This box shows that $\frac{1}{4} = \frac{3}{12}$

$\frac{1}{4} = \frac{3}{12}$



In the next two boxes make two <u>other</u> fractions which are also equivalent to ½.

$\frac{1}{4} = \frac{}{}$

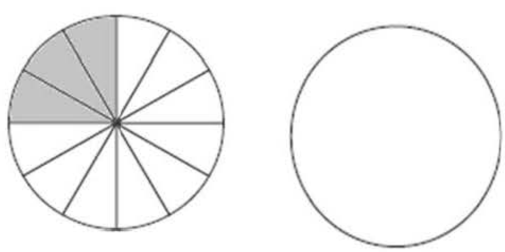
$\frac{1}{4} = \frac{}{}$

19. Nancy has 10 cup cakes. $\frac{1}{5}$ of them are chocolate. Draw a picture to show how many cup cakes are chocolate.

20. The first circle shows $\frac{3}{12}$

What is $\frac{3}{12}$ in simplest form? ___________

On the second circle draw and label the simplified fraction for $\frac{3}{12}$.



Equivalent Fraction Test Scoring Summary Form Group # _____

Record scores from two independent scorers for each student's EFTs, ☐OA, reconcile any differences per collaborative conference.

	Pretest (Date:_______)			Post-Test: (Date:______)			Delayed Post (Date: ______)		
Student #	**Scorer 1 (100 pts)**	**Scorer 2 (100 pts)**	☐	**Scorer 1 (100 pts)**	**Scorer 2 (100 pts)**	☐	**Scorer 1 (100 pts)**	**Scorer 2 (100 pts)**	☐
Student 1									
	Notes:			Notes:			Notes:		
Student 2									
	Notes:			Notes:			Notes:		
Student 3									
	Notes:			Notes:			Notes:		
Student 4									
	Notes:			Notes:			Notes:		

APPENDIX E:
PRE-STUDY TASK LIST

Pre-Study Task List

The following tasks should be completed prior to the commencement of the baseline phase:

1. Obtain IRB and district approval for the study,
2. Meet with district level special education support staff and school administration to inform all parties of study timing (these individuals have already consented to the study),
3. Recruit, select, and train graduate research assistants,
4. Complete teacher training and fidelity assessment,
5. Familiarize the paraprofessional with the intervention and study design,
6. Distribute and collect consent forms,
7. Gather student academic and demographic information,
8. Retrieve pre-assessment data from most recent district progress monitoring assessment,
9. Administer pretest version of EFT to all students ($N = 24$), and
10. Make final decisions about including/excluding individual students in groups.

APPENDIX F:
LESSON SCOPE AND SEQUENCE

Initial Fraction Idea Lessons: Scope and Sequence

Lesson	Manipulative & Activity	Topic
1	Explore Fraction Circles	Exploration with the circles.
2	Apply Fraction Circles	Model and verbally name: 1-half, 1- third, 1-fourth.
3	Apply Fraction Circles	Model and verbally name unit fractions with denominators greater than 4.
3	Apply Fraction Circles	Model and verbally name unit fractions with denominators greater than 4.
4	Explore Paper Folding	Compare paper folding to fraction circles. Model and name (verbally and with written words) unit and non-unit fractions.
5	Apply Fraction Circles	Model fractions and record with symbols a/b.
6	Apply Fraction Circles	Model the concept that the greater the number of parts a unit is divided into, the smaller each
7	Apply/Practice Paper Folding	Reinforce the concept that the greater the number of parts a unit is divided into, the smaller each part is.
8	Apply Fraction Circles	Fraction Equivalence
9	Apply Fraction Circles, Pictures	Fraction Equivalence
10	Apply Paper Folding	Fraction Equivalence.
11	Apply/Practice Fraction Circles	Order fractions by comparing to 1- half.
12	Explore Chips	Introduce new model for fractions less than one by comparing to a familiar model.

Lesson	Manipulative & Activity	Topic
13	Apply Chips	Model fractions using several units for the same fraction
14	Apply Chips	Model fractions using chips; determine fractions that can be shown given a set of chips.
15	Apply/Practice Chips	Fraction Equivalence.
15	Apply/Practice Chips	Fraction Equivalence.
16	Apply/Practice Fraction Circles	Reconstruct the unit given the fraction part.
17	Apply/Practice Fraction Circles	Model fractions greater than one sing mixed and improper fraction notation.
18	Apply/Practice Fraction Circles	Fraction equivalence for 1-half based on a number pattern.
19	Apply/Practice Fraction Circles	Estimate sum of two fractions within story contexts.
20	Apply/Practice Fraction Circles	Find the sum of two fractions using fraction circles.
21	Apply/Practice Fraction Circles	Estimate and solve concretely fraction subtraction using “take- away” and “difference” contexts.
22	Apply/Practice Fraction Circles	Estimate and solve fraction subtraction using “difference” and “how many more” contexts.
23		Summary activities to tie together students’ number sense and addition and subtraction.

(Cramer et al., 2009; Westenskow, 2012)

APPENDIX G:
SCORE RECORDING SHEET
WITH MEDIAN CALCULATION

Daily Score Recording for Fraction Assessment with Median Calculation

Day/Date/Time of Assessment
Scorer # 1 _____ Scorer #2 _____

Group #	Student #	Scorer 1	Scorer 2	☐	Comments
	1				
	2				
	3				
	4				

Record the score from each scorer in the appropriate column for each student. Use ☐column to mark agreement between scorer 1 and scorer 2. Note in comments if student is absent or tardy or other relevant information. Document IOA disagreement and how difference was resolved.

# Scores	Score 1	Score 2	Score 3	Score 4	Score 5	Score 6	Score 7	Green Boxes are Medians
1 score								
2 scores								
3 scores								
4 scores								
5 scores								
6 scores								
7 scores								

Daily Median Calculation

Choose ONE of the lines below that corresponds with the number of scores collected today. List scores in order from least to greatest and read median from green box for odd number of scores OR compute mean for two scores in yellow boxes and record the mean in the green box at the end of the road. Green boxes contain median scores when procedure is followed correctly.

APPENDIX H:
SURVEY

Student Survey

Student Survey Questions

Rank your level of agreement with the following statements related to learning about and using fractions:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
It is important to understand about fractions.	☐	☐	☐	☐	☐
It is important to be able to work problems using fractions.	☐	☐	☐	☐	☐
Understanding fractions is important for my future.	☐	☐	☐	☐	☐
It is important to be able to add fractions QUICKLY.	☐	☐	☐	☐	☐
I like to work problems that have fractions in them.	☐	☐	☐	☐	☐
It is HARD to add fractions that don't have the same denominator.	☐	☐	☐	☐	☐
I am good at working problems with fractions.	☐	☐	☐	☐	☐

Rank your level of agreement with the following statements related to the fraction lessons and using virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
Lessons with manipulatives help me to understand fractions.	☐	☐	☐	☐	☐
Lessons about fractions are boring.	☐	☐	☐	☐	☐
I HAVE used physical manipulatives to learn about fractions.	☐	☐	☐	☐	☐
I have NEVER used virtual manipulatives to learn about fractions.	☐	☐	☐	☐	☐
Using manipulatives to learn about fractions is hard.	☐	☐	☐	☐	☐
I like using virtual manipulatives better than physical manipulatives.	☐	☐	☐	☐	☐

Rank your level of agreement with the following statements about doing the fraction lessons and using virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
I can add fractions that have common denominators.					
I can add fractions that have different denominators.					
I can add fractions with the SAME denominators fast.					
I can add fractions with DIFFERENT denominators fast.					
I CAN reduce a fraction to its lowest terms.					
I CAN write at least two equivalent fractions for 1/3.					
I can make equivalent fractions by using virtual manipulatives.					
I can use virtual manipulatives to add fractions that have different denominators.					

Rank your level of agreement with the following statements about your behavior during the fraction lessons using virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
I completed the worksheets almost every day.					
I used the virtual manipulative to do lesson activities.					
I finished early most of the time.					
I almost never had enough time to complete the activities.					
When I worked with a partner we completed all our work carefully.					
My partner and I fooled around a lot instead of doing our fraction worksheets.					
I helped my classmates complete the lesson activities.					
I tried hard on the daily quiz to add fractions accurately and quickly to improve my score.					

Teacher Survey

Teacher/Paraprofessional Survey

Student Survey Questions

Rank your level of agreement with the following statements related to the need for students to learn about and use fractions:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
It is important for students to understand about fractions.	☐	☐	☐	☐	☐
It is important for students to be able to work problems using fractions.	☐	☐	☐	☐	☐
Understanding fractions is important for my students' future.	☐	☐	☐	☐	☐
It is important for students to be able to add fractions QUICKLY.	☐	☐	☐	☐	☐
Most students like to work problems that have fractions.	☐	☐	☐	☐	☐
Most students find it HARD to add fractions that don't have the same denominator.	☐	☐	☐	☐	☐
Most students are good at solving computation problems that incorporate fractions.	☐	☐	☐	☐	☐
Fraction equivalence is a difficult concept for my students to understand.	☐	☐	☐	☐	☐

Rank your level of agreement with the following statements related to the fraction lessons and using virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
Lessons with manipulatives help students to understand fractions.	☐	☐	☐	☐	☐
Students had fun using the virtual manipulatives to learn about fractions.	☐	☐	☐	☐	☐
I HAVE used physical manipulatives to help students learn about fractions.	☐	☐	☐	☐	☐
I have NEVER used virtual manipulatives to help students learn about fractions.	☐	☐	☐	☐	☐
Students find it hard to use manipulatives to learn about fractions.	☐	☐	☐	☐	☐
Students like using virtual manipulatives better than physical manipulatives.	☐	☐	☐	☐	☐
Most students found it easy to use the virtual manipulatives.	☐	☐	☐	☐	☐

Rank your level of agreement with the following statements about teaching the fraction lessons with virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
I think the RNP lessons were well structured and easy to use for teaching fraction concepts to students.	○	○	○	○	○
The virtual manipulatives were easy to learn how to use.	○	○	○	○	○
It was easy to demonstrate lesson concepts with virtual manipulatives.	○	○	○	○	○
Students were engaged in using the virtual manipulatives during the whole group activities.	○	○	○	○	○
Students were engaged in using the virtual manipulatives complete worksheet tasks during the small group activities.	○	○	○	○	○
I understand how to use virtual manipulatives to demonstrate equivalent fractions.	○	○	○	○	○
I can demonstrate fraction addition using virtual manipulatives.	○	○	○	○	○
I believe systematic instruction coupled with the use of virtual manipulatives helps students understand fraction concepts.	○	○	○	○	○
I will use the lessons and virtual manipulatives again to help students understand fraction concepts.	○	○	○	○	○
I not use this intervention (lessons + virtual manipulatives) again.	○	○	○	○	○
I will use virtual manipulatives to teach fractions again.	○	○	○	○	○
I will explore other types of virtual manipulatives to help my students understand other mathematics concepts.	○	○	○	○	○
Students were able to independently use the virtual manipulatives to complete worksheet activities.	○	○	○	○	○
I had to provide only minor assistance to students when they were performing independent activities.	○	○	○	○	○
Click to write Statement 15	○	○	○	○	○

Rank your level of agreement with the following statements about student behavior during the fraction lessons using virtual manipulatives:

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
Mos of the students completed the worksheets almost every day.	○	○	○	○	○
Most of the students used the virtual manipulative to do lesson activities.	○	○	○	○	○
Most of the students finished early most of the time.	○	○	○	○	○
Most of the students did not have enough time to complete the activities.	○	○	○	○	○
Students were highly engaged in learning activities using the virtual manipulatives.	○	○	○	○	○
Students' perceptions of using the virtual manipulates were generally positive.	○	○	○	○	○
Students' perceptions of completing the worksheet tasks were generally positive.	○	○	○	○	○
Students felt like they were learning about fractions.	○	○	○	○	○
Students felt that the lessons helped them to add fractions more accurately and more quickly.	○	○	○	○	○
Students were often disengaged from the group instructional activities.	○	○	○	○	○
Students were often disengaged during the small group/partner activities.	○	○	○	○	○

APPENDIX I:
FIDELITY OF INSTRUCTION

Fidelity of Instruction and Interobserver Agreement

	Pre-Study				Intervention Sessions																						
Component	A	A	B	B	6	6	7	8	8	9	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	20
1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
4	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
5	1	1	2	2	1	1	2	2	2	2	2	2	2	1	1	2	2	2	2	2	2	2	2	2	2	2	2
6	1	1	2	2	1	1	2	1	2	1	2	2	2	2	2	2	1	2	1	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Total	12	12	14	14	12	12	14	13	14	13	14	14	14	13	13	14	13	14	13	14	14	14	14	14	14	14	14
IOA Mean		12		14		12			13.5			14			13			13.5			14			14			14
% IOA		100%		100%		100%			93%			100%			100%			93%			100%			100%			100%

Note: Shaded columns represent Interobserver data.

APPENDIX J:
ANALYSIS OF STUDENT PERFORMANCE WITHIN GROUPS

Analysis of Mean Fluencies Across Conditions, Groups, and Individuals

Participants	Baseline Mean Fluency	Intervention Mean Fluency	Increase
Red Group			
Student-1	0.00	1.95	1.95
Student-2	0.00	1.40	1.40
Student-3	0.36	2.65	2.29
Red Mean	**0.12**	**2.00**	**1.88**
SD	0.21	0.63	0.45
VAR	0.04	0.39	0.20
Range	0.0 to 0.36	1.40 to 2.65	1.40 to 2.29
White Group			
Student-4	0.14	2.40	2.26
Student-5	0.14	2.84	2.70
Student-6	0.00	2.80	2.80
White Mean	**0.09**	**2.68**	**2.59**
SD	0.08	0.24	0.29
VAR	0.01	0.06	0.08
Range	0.0 to 0.14	2.40 to 2.84	2.26 to 2.80
Blue Group			
Student-7	0.20	2.96	2.76
Student-8	0.00	1.50	1.50
Student-9	0.09	1.84	1.75
Blue Mean	**0.10**	**2.10**	**2.00**
SD	0.10	0.76	0.67
VAR	0.01	0.58	0.30
Range	0.0 to 0.2	1.50 to 2.96	1.50 to 2.76
Overall			
Mean of Means	.103	2.26	2.16
SD	0.02	0.37	0.38
VAR	0.00	0.13	0.14

APPENDIX K:
SOCIAL VALIDITY SURVEY RESULTS

Student Responses to Social Validity Survey

Survey Question	Theme	5	4	3	2	1
These lessons with virtual manipulatives helped me to understand fractions better.	A	2	4	0	0	0
I liked using virtual manipulatives to learn about fractions.	A	4	2	0	0	0
I would like to try other virtual manipulatives to learn about other math concepts.	A	5	1	0	0	0
It is important to be able to do calculations with fractions quickly	B	2	3	0	1	0
It is important to be able to work problems using fractions.	B	4	2	0	0	0
It is important to understand about fractions.	B	3	3	0	0	0
Understanding fractions is important for my future.	B	4	1	0	1	0
I am better at working problems with fractions than before the intervention.	C-AP	1	2	0	2	1
CAN reduce a fraction to its lowest terms.	C-AP	1	2	0	3	0
I CAN write at least two equivalent fractions for 1/3.	C-AC	3	2	1	0	0
I CAN make equivalent fractions with virtual manipulatives.	C-AC	3	1	1	1	0
I completed the worksheets almost every day.	C-B	4	2	0	0	0
I used the virtual manipulatives to do lesson activities.	C-B	6	0	0	0	0
I finished early most of the time.	C-B	3	2	1	0	0
I tried hard on the daily quiz to add fractions accurately and quickly to improve my score.	C-B	6	0	0	0	0

Notes: Themes A: C theme divided by A (academic; P-problem solving, C-conceptual) and B (behavioral).
Likert scale: 5-strongly agree, 4 agree, 3-neither agree nor disagree, 2-disagree, 1-strongly disagree.

Teacher/Paraprofessional Responses to Social Validity Survey

Survey Question	Theme	5	4	3	2	1
Students had fun using the virtual manipulatives to learn about fractions.	A	1	0	0	1	0
Students are more willing to use VM than physical manipulatives for learning.	A	0	1	1	0	0
I think the RNP lessons were well structured and easy to use to teach fraction concepts.	A	0	2	0	0	0
The virtual manipulatives were easy to learn how to use.	A	1	1	0	0	0
It was easy to demonstrate lesson concepts with virtual manipulatives.	A	0	2	0	0	0
It is important for students to be able to do calculations with fractions QUICKLY.	B	0	2	0	0	0
It is important for students to be able to work problems using fractions.	B	2	0	0	0	0
It is important for students to understand about fractions.	B	2	0	0	0	0
Understanding fractions is important for my students' futures.	B	2	0	0	0	0
Most students found it easy to use the virtual manipulatives.	C-B	0	2	0	0	0
Most students were able to independently use the virtual manipulatives to complete worksheet activities.	C-B	1	1	0	0	0
I understand how to use virtual manipulatives to demonstrate equivalent fractions.	C-T	0	1	0	1	0
I will use the lessons and virtual manipulatives again to help students understand fraction concepts.	C-T	1	0	1	0	0
I will use virtual manipulatives to teach fractions or other math concepts in the future.	C-T	1	1	0	0	0
I will explore other types of virtual manipulatives to help my students understand other mathematics concepts.	C-T	1	0	1	0	0
Lessons with virtual manipulatives help students to understand fractions.	C-A	0	2	0	0	0
I believe systematic instruction coupled with the use of virtual manipulatives helps students understand fraction concepts.	C-A	2	0	0	0	0

Notes: Themes - C theme divided by A (academic), B (student behavior), T (teacher behavior).
Likert scale: 5-strongly agree, 4 agree, 3-neither agree nor disagree, 2-disagree, 1-strongly disagree.

APPENDIX L:
PERMISSIONS

Kathleen Cramer December 26, 2013 11:42 AM
To: Barbara Serianni <Barbara.Serianni@ucf.edu>
Re: inquiry RNP Inbox - UCF 2

Barb

You have my permission to use the RNP lessons for your research.

The online professional workshop was created several years ago and isn't available.

In terms of staff development, make use of the teacher's guide in the lessons. Ask teachers to pay close attention to the notes section of the lessons as they communicate information on student thinking.
Teachers may want to read some of the articles in the NCTM journals and yearbook. When I am back in the office next week I will send you some suggestions. Feel free to send me a reminder to do this.

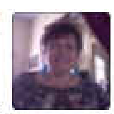
Let me know things work out.

Kathy Cramer

See More from Barbara Serianni

--
Kathleen Cramer
Associate Professor Mathematics Education
Co-Executive Director, STEM Education Center
College of Education and Human Development
University of Minnesota

Barbara Serianni <barbara.serianni@ucf.edu> December 23, 2013 8:40 AM
To: Kathleen Cramer
inquiry RNP Sent - UCF 1



Dr. Cramer,

I am a doctoral candidate in special education at the University of Central Florida in the midst of writing my dissertation proposal which will feature the RNP Initial Fraction Ideas with virtual manipulatives as a remedial intervention for middle school students with severe emotional disturbance (SED) in special education classrooms learning mathematics via an online curriculum. This group of students struggle with basic elementary skill deficits the most challenging (according to their teacher) is fractions.

I would like to secure permission to use the RNP materials and virtual manipulatives as a part of my study which will begin in schools February 1, 2014. In addition I noticed on the website a reference to an **online professional development workshop** to prepare teachers to use the material, but I can't seem to find it online. I was wondering if I could have access to that workshop to assist me in training the participating teachers to use the materials.

I also have read Arla Westenskow's dissertation and plan to ask her for permission to incorporate some of the modifications she made to the lessons for the use of virtual manipulatives in my study.

I would appreciate any information, permissions, or direction you may have for me. If you feel it would be beneficial to speak by phone I would love to try to schedule a time to meet with you.

I look forward to hearing from you soon.

Hoping this holiday season brings joy and peace to you and yours.

Barb Serianni

Kathleen Cramer January 21, 2014 10:09 AM
To: Barbara Serianni <Barbara.Serianni@ucf.edu>
permission to use RNP materials for your doctoral study

Barbara

You have my permission to use for your dissertation the RNP Initial Fraction Idea lessons including activity sheets and the teacher's guide as well as the virtual manipulatives developed in conjunction with Twin Cities Public Television accessible from
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html

Kathleen Cramer

--
Kathleen Cramer
Associate Professor Mathematics Education
Co-Executive Director, STEM Education Center
College of Education and Human Development
University of Minnesota

From: **Kathleen Cramer** crame013@umn.edu
Subject: Re: inquiry RNP
Date: December 26, 2013 at 11:42 AM
To: **Barbara Serianni** Barbara.Serianni@ucf.edu

Barb

You have my permission to use the RNP lessons for your research.

The online professional workshop was created several years ago and isn't available.

In terms of staff development, make use of the teacher's guide in the lessons. Ask teachers to pay close attention to the notes section of the lessons as they communicate information on student thinking.
Teachers may want to read some of the articles in the NCTM journals and yearbook. When I am back in the office next week I will send you some suggestions. Feel free to send me a reminder to do this.

Let me know things work out.

Kathy Cramer

See More from Barbara Serianni

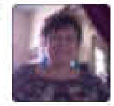
--
Kathleen Cramer
Associate Professor Mathematics Education
Co-Executive Director, STEM Education Center
College of Education and Human Development
University of Minnesota

arlawestenskow@gmail.com December 23, 2013 12:55 PM
To: Barbara Serianni <Barbara.Serianni@ucf.edu>
Re: Dissertation permissions Inbox - UCF 4

You have my permission to use any of my dissertation materials and to make any modifications needed for your study.
Good luck. It was nice to talk with you.
Arla Westenskow

Barbara Serianni <barbara.serianni@ucf.edu>
To: arla.westenskow@aggiemail.usu.edu
Dissertation permissions

December 23, 2013 8:48 AM
Sent - UCF 1



Dr. Westenskow,

I am a doctoral candidate in special education at the University of Central Florida. I am in the midst of writing my research proposal in which I incorporate the use of RNP materials and virtual manipulatives as a remedial intervention for middle school students with severe emotional disturbance (SED) taking online mathematics in self-contained special education classrooms. I noticed in your dissertation that you made some modifications to the RNP materials of the use of virtual manipulatives and I would like to secure permission to use your modified materials in my project.

If you agree to this I would appreciate access to any materials that you believe would be of value in my study that may not be a part of your published dissertation. I would love to chat with you about your dissertation research and glean any insights you gained from conducting your study.

If you are willing, please email me permission to use your work in my study.

If you are willing to speak with me, suggest a couple of days and times that might work for you.

Thanks so much for your time and attention.

Barb Serianni

Barbara Serianni
Doctoral Candidate
University of Central Florida
Exceptional Education
Provost Fellow
AACTE Fellow
Project LEAD Scholar
407-222-4452

Seth Leavitt
To: Barbara Serianni <Barbara.Serianni@ucf.edu> Cc: Dr. Kathleen Cramer
Re: RNP virtual manipulatives

December 30, 2013 10:14 PM
Inbox - UCF 2

Hello Barb.

Thank you for your interest in the Rational Number Project materials.

Dr. Cramer cc'd me her response to your initial inquiry about the virtual manipulatives. Unfortunately, I don't have any additional information. The virtual manipulatives were created by an outside programmer for the project that Dr. Cramer and I were doing with Twin Cities Public Television (TPT). The project compared the student outcomes from two groups of teachers. The first group participated in a one week, face-to-face workshop during the summer. The second group participated in an analogous online workshop in the Fall and Winter. TPT contracted for the manipulatives to be created but we never used them. I have not heard of any use of the virtual manipulatives since I posted them on our web site. I posted them with the intention that teachers use them. I think it will be interesting to hear your results.

I'm sorry I can't be of any more help. Feel free to contact me if you do have any other questions.

Seth Leavitt
Mathematics
Field Middle School
Minneapolis Public Schools
Minneapolis, Minnesota

LIST OF REFERENCES

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