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## Modelling endurance and resumption times for repetitive one-hand pushing

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### ABSTRACT

This study's objective was to develop models of endurance time (ET), as a function of load level (LL), and of resumption time (RT) after loading as a function of both LL and loading time (LT) for repeated loadings. Ten male participants with experience in construction work each performed 15 different one-handed repeated pushing tasks at shoulder height with varied exerted force and duration. These data were used to create regression models predicting ET and RT. It is concluded that power law relationships are most appropriate to use when modelling ET and RT. While the data the equations are based on are limited regarding number of participants, gender, postures, magnitude and type of exerted force, the paper suggests how this kind of modelling can be used in job design and in further research.

**Practitioner Summary:** Adequate muscular recovery during work-shifts is important to create sustainable jobs. This paper describes mathematical modelling and presents models for endurance times and resumption times (an aspect of recovery need), based on data from an empirical study. The models can be used to help manage fatigue levels in job design.

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Modelling; endurance;  
recovery; work-rest;  
repetitive load

## 1. Introduction

Understanding how to design jobs that lead to high system performance, sustainable work conditions and well-being for the employees is becoming increasingly important for companies active in the global market. This is also in line with the two main objectives in ergonomics (IEA 2014). Time aspects in loading and recovery play an important role in the development of musculoskeletal disorders (MSDs) and are thus important in creating sustainable jobs (Putz-Anderson 1988; Wells et al. 2007; El ahrache and Imbeau 2009). When jobs are designed, there is a need for predictive tools to design appropriate work-recovery job patterns. For this, models on how the recovery need varies with loading amplitude and duration conditions are needed.

Over the years several models for estimating time aspects, such as endurance time, ET, and recovery need, have been presented (e.g. Rohmert 1960a; El ahrache and Imbeau 2009; Frey Law and Avin 2010; Ma et al. 2010; Sonne and Potvin 2015). Several models have been reported to have an inability to simulate the recovery

process adequately, stimulating research on how the recovery process varies across different loading conditions (e.g. Rashedi and Nussbaum 2015a; Sonne and Potvin 2015). Reported insufficiencies of existing endurance and recovery models (e.g. El ahrache, Imbeau, and Farbos 2006; Perez et al. 2014; Rose et al. 2014; Rashedi and Nussbaum 2015b, 2017) include: (i) Models use the time to regain maximum force generating capacity in defining the recovery time (e.g. Frey Law and Avin 2010), although no tentative relationship between the maximum force generating capacity and MSDs has been supported by any study results. (ii) Models are based on and consider only one task repetition despite the repetitive nature of most work (e.g. Rose, Ericson, and Örtengren 2000). (iii) Some models suggest that at low load level (LL) no recovery is needed, expressed with an asymptote in the model which is not substantiated by more recent research (e.g. Rohmert 1960b; Frey-Law, Looft, and Heitsman 2012). (iv) Models are based on static loads, despite the dynamic nature of many work tasks (e.g. Mathiassen and Åhsberg 1999). (v) Models are based on several other studies, and models with inconsistent prerequisites in different studies (e.g. El

ahrache, Imbeau, and Farbos 2006), which can be questioned from theoretical and modelling perspectives. (vi) Models do not consider task dependency or initial conditions which can affect fatigue recovery processes (e.g. Ma et al. 2011). The models presented in this paper try to meet the first three of these insufficiencies.

In this paper the same experience-based definition of ET and resumption time (RT) is used, as by Rose et al. (2014). The instructions to the participants were: 'The endurance time is when you would stop working if this was your work task, given that you strongly want to continue the task to accomplish an amount of work over the day and get the job done, but when, due to very strong discomfort, pain or fatigue (almost max), you choose to take a pause.' RT is operationally defined as the time until the participants were willing to resume work, if the task had been their job. This RT definition, also used by Rose et al. (1992a, 1992b, 2000, 2001), focuses on the participants' subjectively determined recovery need, rather than objective phenomena such as force generating capacity (Frey Law et al. 2010), or mean power frequency of electromyographic signals (Glimskär, Höglund, and Örtengren 1987).

The overall objective of the work presented in this paper was to derive mathematical relationships for perceived RT based on an empirical data-set. While a detailed description of the data collection has been previously published (Rose et al. 2014), here brief summary follows. Ten male participants, who had professional construction work experience (average 5.7 years, SD 9 years), were an average age of 23 years (SD 11), 181 (SD 6) cm height, and 80 (SD 11) kg weight, engaged in one-handed, individually normalised, pushing tasks at shoulder-height. After a training session, followed by a session to determine maximum voluntary contraction (MVC) levels, each participant carried out the series of 15 sessions. Each session consisted of two subsequent trials, with each trial consisting of a predetermined loading time, LT, during which a handle was pushed at a certain load level, LL, followed by a resting period of subjective duration which established the resumption time, RT. LL varied between 10, 30 and 50% of each participant's MVC, denoted as LL10, LL30 and LL50, respectively. LT varied between 10, 30, 50, 70 and 100% of the subjectively determined ET in the first trial. The second trials in the sessions were included to capture effects of repeated loading, such as possible changes in the RT. In total, 300 trials were performed and data from all trials were used in the modelling described in this paper. To meet the objective, the modelling work aimed to resolve three specific research questions:

*Research question 1:* What model form best describes the relationship between endurance time and load level for two repeated trials?

*Research question 2:* What model form best describes the relationship between resumption time, load level, and loading time for two repeated trials?

*Research question 3:* What model forms can describe the effect of multiple repetitions on endurance time and resumption time?

## 2. Methods

ET and RT were mathematically modelled by regression analysis using Octave 3.6.2. Akaike's Information Criterion corrected,  $AIC_c$ , was calculated, in order to compare models with different numbers of parameters, since in such cases  $AIC_c$  should be used rather than the sum of squared residuals, SSR (see Appendix 1).

For ET and the first research question, linear regressions were performed. The SSR were computed for deviations from  $\log_{10}(ET)$ , since the variability in the ET was found to be logarithmically distributed (see Section 3.1).

For RT and the second research question, all equations with two parameters<sup>1</sup> in the equations (A, B) in Table 1 were linearised. The linear regressions and the SSR were computed for deviations from  $\log_{10}(RT)$  rather than RT, since the variability in RT was found to be logarithmically distributed (see Section 3.2.2). The equations for RT with three parameters<sup>2</sup> (A, B and C) in Table 2 and the mathematical equations derived for RT (Equation 2) were fitted by seeking the minimum SSR for  $\log_{10}(RT)$  using the Nelder–Mead method and the function 'nelder\_mead\_min', part of the 'optim' package in Octave. Since this function does not report the standard deviation for the best-fit parameters and these can be difficult to compute, the standard deviation of parameters for these equations were omitted.

To determine whether the exponential or power law provided the best relationship between the ET and the LL, the goodness of the fits in terms of the SSR were compared. These values were determined by performing a linear regression of  $\log_{10}(ET)$  versus either LL (exponential) or  $\log_{10}(LL)$  (power law). The two equations were linearised as described in Appendix 2.

From the  $\pm$  SD determined for the SSRs and  $AIC_c$ s, the  $p$ -value when comparing a pair of models was computed. First this measure was computed and reported in comparing the exponential vs. power law expression for ET vs. LL in Table 1, then in comparing the power law in expression 1 with a shared exponent for both trials to the one in Table 1. In Table 2, the  $p$ -value provides the significance of the difference in  $AIC_c$  between the best model (lowest  $AIC_c$ ) on the first row of Table 2 and each one of the other models. In Equations (2) and (3), the fixed-exponent expressions to the best-fit expression (first row) of Table 2 was compared.

Kernel density estimation (KDE) was used to estimate the unknown probability distribution of a variable by estimating the probability density function of that variable, using a sample of data points from that distribution. KDE

was used to compute the fraction of participants willing to resume work right away,  $f_{\text{RESUME}}$ :

$$f_{\text{resume}}(x) = \frac{\sum_{i=1}^{N_{\text{resume}}} \exp\left\{-\left([\log_{10}(x) - \log_{10}(RT_i)]^2\right)/2\sigma^2\right\}}{\sum_{j=1}^{N_{\text{total}}} \exp\left\{-\left([\log_{10}(x) - \log_{10}(RT_j)]^2\right)/2\sigma^2\right\}}$$

where  $N_{\text{RESUME}}$  is the number of participants that were willing to resume work right away (i.e.  $RT = 0$ ) and  $RT_i$  is the theoretical, computed non-zero RT for the  $i$ th such participant based on Equation (2),  $N_{\text{TOTAL}}$  is the total number of sessions (150 for each trial) and  $RT_j$  is the computed RT for the  $j$ th such session based on Equation (4). For  $\sigma$  in the above equation, the Gaussian approximation (Silverman 1998), was used:

$$\sigma = \left(\frac{4}{3N_{\text{total}}}\right)^{\frac{1}{5}} \cdot \sqrt{\left(\sum_{j=1}^{N_{\text{total}}} [\log_{10}(RT_j) - \overline{\log_{10}(RT)}]^2\right) / (N_{\text{total}} - 1)} = 0.09659$$

where  $N_{\text{TOTAL}}$  is the total number of values (300) over both trials (150 sessions and 2 trials) and  $\log_{10}(RT)$  is the average of  $\log_{10}(RT)$  for all sessions and for both trials. Fitting of an exponential decay function,  $f_{\text{RESUME}}(RT) = \exp(-A(RT-B))$ , to the KDE of  $f_{\text{RESUME}}(x)$  was done using the theoretical, non-zero RT values of the  $N_{\text{TOTAL}} = 150$  for each trial as input data points for the KDE  $f_{\text{RESUME}}(x)$ , and using the octave function `nelder_mead_min` to minimise the SSR for  $f_{\text{RESUME}}$ . This allows the two trials to share their value for the exponential parameter  $B$ , but not for  $A$ .

### 3. Results

The main results of this study are the presented models for RT. In this section, results are presented in the order of the research questions. The results of the modelling of RT are presented in three different settings: (i) equations fitted to empirical data where participants stated a need for rest before resuming work (Section 3.2.2), (ii) equations fitted to empirical data for all cases, both when participants stated a need for a rest and when they were willing to resume work immediately (Section 3.2.3), and (iii) based on data for all cases with two repetitions, presenting a prediction model for more than two repetitive loadings (Section 3.3).

#### 3.1. Modelling ET for two repeated loadings

In order to answer the first research question and determine how best to represent the relationship between ET

and LL, the data of ET versus LL were plotted. This was done since visual inspection often provides important information about the nature of the relationship between two variables. In Figure 1, ET versus LL has been plotted using different axis scales.

In contrast to when plotting ET on a linear scale (Figure 1(a)), where a wider range of ET values is seen for lower LLs, when plotting ET on a logarithmic scale (Figure 1(b) and (c)) the range is rather the same for all three LLs. This suggests that the variability of ET is distributed logarithmically. Further, visual inspection reveals a linear relationship when plotting  $\log_{10}(\text{ET})$  versus  $\log_{10}(\text{LL})$  (Figure 1(c)), which suggests a power law relationship between ET and LL (whereas a linear relationship when plotting  $\log_{10}(\text{ET})$  versus LL (Figure 1(b)) would suggest an exponential relationship instead, see Appendix 2). In addition, Table 1 shows the comparison of the fits of the exponential and power law relationships with data from trials 1 and 2. The power law offers a better fit with the data (lower SSR) than the exponential function. Hence, based on the data used in this study, ET is best described by a power law relationship.

**Table 1.** Comparison of the relationships for ET vs. LL for trials 1 and 2.<sup>†</sup>

ET=	Trial	A ± SD (s)	B ± SD	SSR ± SD	p-value
$Ae^{(B \cdot LL)}$	1	568 ± 80	-4.43 ± 0.41	1.53 ± 0.24	0.2
	2	418 ± 59	-4.54 ± 0.41		
$A(LL)^B$	1	31.1 ± 4.4	-1.13 ± 0.09	1.24 ± 0.23	1
	2	22.7 ± 3.2	-1.11 ± 0.09		

<sup>†</sup>ET: Endurance time. LL: Load level. A: coefficient parameter. B: exponent parameter. SD: Standard deviation. SSR: sum of squared residuals. p-value: statistical significance of the SSR difference between the exponential and power relationships.

The power law equations for the two trials are similar in their exponent parameter ( $\Delta B = 0.08 \pm 0.13$ ), but differ in their coefficient parameter ( $\Delta A = 8.4 \pm 5.4$  s). By repeating the fit of the data, holding the exponent parameter,  $B$ , fixed for both trials, allowing only the coefficient parameter,  $A$ , to vary between the trials, resulted in the simpler equation for ET:

$$\begin{aligned} \text{Trial 1: ET} &= 32 \text{ s (LL)}^{-10/9} \\ &\quad \text{[SSR} = 1.24; p\text{-value} = 1] \quad (1) \\ \text{Trial 2: ET} &= 23 \text{ s (LL)}^{-10/9} \end{aligned}$$

In Figure 2, Equation (1) is plotted alongside the experimental data.

There is good agreement between Equation (1) for ET as a function of LL and the data from trial 1 (Figure 2(a)) and trial 2 (Figure 2(b)). Further, the decrease in the ETs for trials 1 and 2 is clearly visible.

### 3.2. Modelling RT for two repeated loadings

#### 3.2.1. Immediate work resumption or non-zero RTs

The second research question aims at modelling resumption times. This was complicated by the fact that, in some cases participants were willing to resume work immediately ( $RT = 0$ ), whereas in other cases participants required a resting period,  $RT$  before resuming their task ( $RT > 0$ ). Figure 3 illustrates the percentage of participants willing to resume work immediately as both a function of the LL, and as a function of a relative loading time, RLT. The main reason to choose to express the LT relative to the ET (i.e.  $LT/ET$ ) rather than simply using the LT alone is because while the LT and ET differ for different participants, their ratio differs less, providing a more predictive, and less individual, unitless quantity. Figure 3 shows that participants were less willing to resume work immediately in trial 2 than in trial 1.

#### 3.2.2. Modelling RT for cases with non-zero RTs

Results in this section are based on data where participants stated the need for a rest period and reported an  $RT > 0$ , before resuming the task. In order to determine how the RT relates to the effort, LL and LT of the activity, possible relationships between RT and various combinations of absolute and relative factors were explored. The variability in RT was found to be distributed logarithmically. Therefore, only RTs which are non-zero were considered, because  $\log_{10}(RT = 0) = -\infty$ ,

which makes fitting and plotting these measures impossible. Table 2 presents different possible relationships for RT, as power law and exponential functions of ET, LL and LT.

Since some of the models differ in the number of parameters, the SSR alone is not sufficient to determine the most appropriate model. A model with more parameters typically agrees better with the data. However, the deviation of the data from the relationship curve might be due to noise (inaccuracies) in the data, and if this is the case, allowing the data to deviate from the curve is more appropriate than to seek a perfect fit. It is important to weigh goodness of fit against the risk of over-parametrizing.

Akaike's Information Criterion AIC (see Appendix 1), was used with the model with the lowest AIC or  $AIC_c$  providing the best description of the relationship based on the available data. In Table 2, of the relationships explored, the one which offers the lowest  $AIC_c$  is  $RT = A(LT)^B \cdot (LL)^C$ . Here the coefficient parameter  $A$  varies most between the two trials, whereas the exponent parameters,  $B$  and  $C$ , respectively, are more similar between the two trials. Thus, it was decided that the two trials could share the parameters  $B$  and  $C$  in the modelling. Since LT is expressed in units of seconds, the coefficient parameter  $A$  in this equation in Table 2 has an awkward unit. By dividing LT by a constant with the unit of time,  $A$  can be expressed in units of seconds. These steps led to the simpler expression for RT:

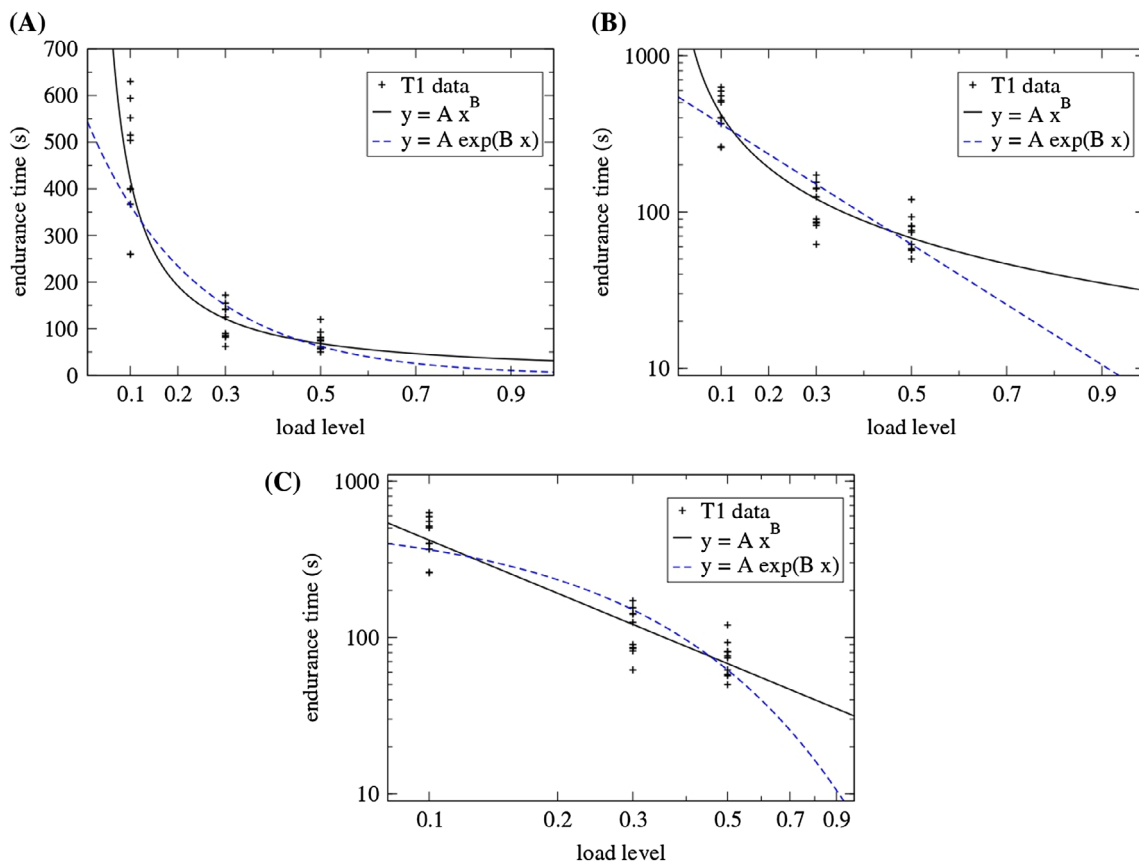
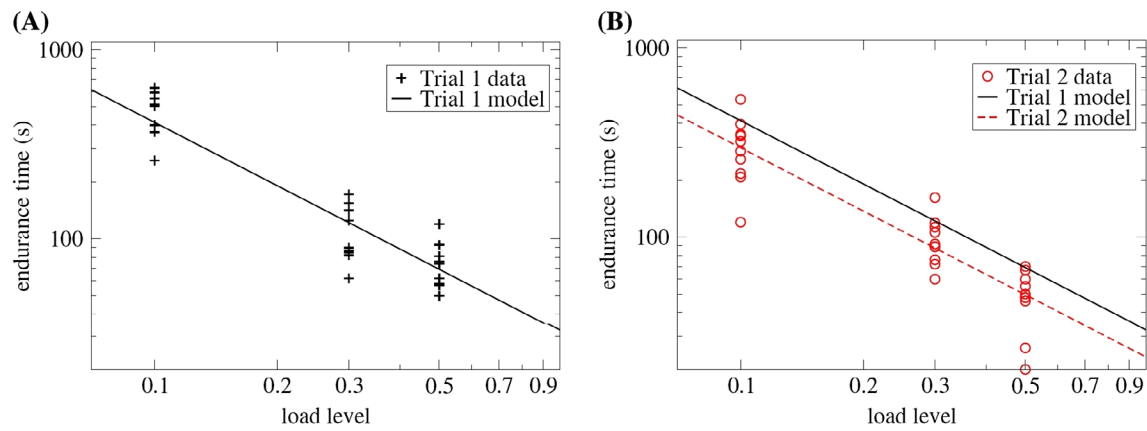


Figure 1. Relationship between endurance time, ET, and load level, LL, on a (a) linear-linear, (b) log-linear, and (c) log-log plot.





**Figure 2.** Model for endurance time, ET, as a function of load level, LL, for (a) trial 1 data and trial 1 model, and (b) trial 2 data, trial 2 model and trial 1 model. (Measured values, calculated lines).

**Table 2.** Comparison of relationships for RT for trial 1 and trial 2. Based on  $RT \neq 0$  cases only.<sup>†</sup>

RT=	Trial	A ± SD	B ± SD	C ± SD	SSR	AIC <sub>C</sub>	p-value
$A(LT)^B \cdot (LL)^C$	1	6.84 ± 1.7	0.729 ± 0.07	0.595 ± 0.099	12.3 ± 1	-702 ± 21	1
	2	10.4 ± 2.4	0.67 ± 0.07	0.525 ± 0.097			
$A(LT \cdot LL)^B$	1	8.28 ± 1.9	0.73 ± 0.071		12.8 ± 1.1	-698 ± 21	0.4
	2	12.8 ± 2.7	0.67 ± 0.071				
$A(LT/ET_1)^B \cdot (ET)^C$	1	38.2 ± 13	0.747 ± 0.073	0.227 ± 0.061	12.7 ± 1.1	-696 ± 22	0.4
	2	64.2 ± 19	0.666 ± 0.073	0.174 ± 0.057			
$A(LT/ET_1)^B \cdot (LL)^C$	1	89.0 ± 11	0.729 ± 0.073	-0.211 ± 0.072	12.9 ± 1.1	-691 ± 22	0.4
	2	108 ± 13	0.666 ± 0.073	-0.216 ± 0.069			
$A(LT/ET_1)^B$	1	119 ± 8.7	0.708 ± 0.075		13.9 ± 1.2	-678 ± 22	0.2
	2	144 ± 11	0.641 ± 0.075				
$Ae^{(B \cdot LT/ET_1)}$	1	27.6 ± 3.3	1.62 ± 0.18		14.3 ± 1.3	-671 ± 23	0.1
	2	36.3 ± 4.3	1.64 ± 0.2				
$A(LT)^B$	1	10.7 ± 2.8	0.434 ± 0.056		15.7 ± 1.4	-648 ± 21	0.03
	2	15.7 ± 3.9	0.396 ± 0.054				
$Ae^{(B \cdot LT)}$	1	51.2 ± 4.2	0.00266 ± 0.00042		17.6 ± 1.8	-620 ± 25	0.005
	2	61.2 ± 4.8	0.00299 ± 0.0005				
$A(LT/ET_1 \cdot LL)^B$	1	126 ± 19	0.254 ± 0.062		20.7 ± 2	-581 ± 23	<10 <sup>-4</sup>
	2	134 ± 20	0.201 ± 0.06				
$A(LT/ET_1)^B (LL - 0.15)^C$	1	116 ± 29	0.761 ± 0.099	0.042 ± 0.15	8.29 ± 0.9	-423 ± 17	<10 <sup>-24</sup>
	2	157 ± 39	0.671 ± 0.099	0.131 ± 0.14			
$A(LT)^B (LL - 0.15)^C$	1	7.1 ± 3	0.752 ± 1	0.476 ± 0.17	8.4 ± 0.86	-421 ± 16	<10 <sup>-25</sup>
	2	12.1 ± 4.7	0.690 ± 1	0.503 ± 0.16			

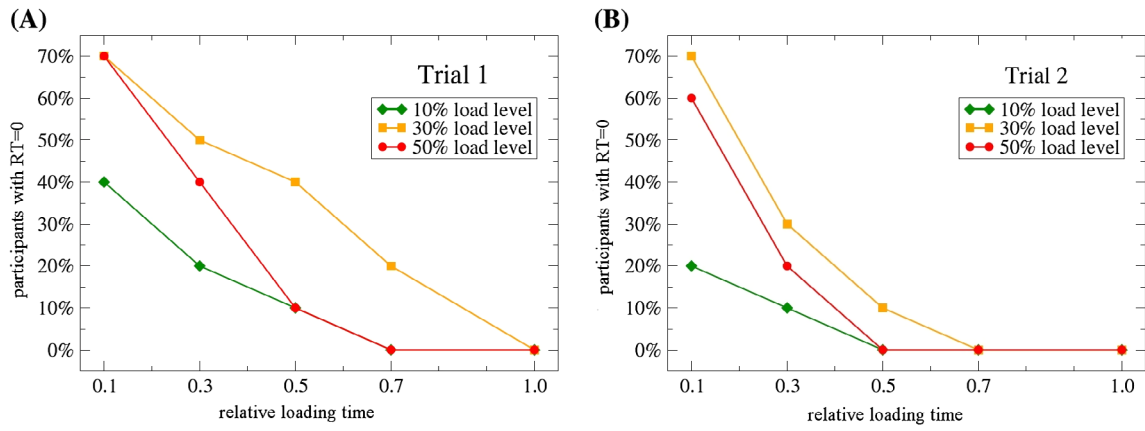
<sup>†</sup>RT: Resumption time. ET<sub>1</sub>: Endurance time in trial 1. LL: Load level. LT: Loading time. A: coefficient parameter. B and C: exponent parameters. SD: Standard deviation. SSR: sum of squared residuals. AIC<sub>C</sub>: Akaike's Information Criteria corrected. The p-value indicates the statistical significance of AIC<sub>C</sub> differences between the model on the first row and each of the other models.

$$\text{Trial 1: } RT = 80 \text{ s } (LT/32 \text{ s})^{3/4} (LL)^{5/9} \quad [SSR = 12.5; p\text{-value} = 0.4] \quad (2)$$

$$\text{Trial 2: } RT = 80 \text{ s } (LT/23 \text{ s})^{3/4} (LL)^{5/9}$$

where the time coefficient parameter,  $A = 80 \text{ s}$ , is the same in both trials. Equation (2) predicts that RT increases with increased LL as well as with increased LT.

In other models where recovery time, not RT, has been modelled (e.g. Rohmert 1960b), the relative recovery time, i.e. the recovery time relative to LT, is considered. While this has been explored here ( $SSR = 12.5$ ,  $p\text{-value} = 0.4$ ), a somewhat better fit is found instead by expressing the RT relative to the ET time



**Figure 3.** Percentage of participants willing to resume work immediately after completing their task (a) in trial 1 and (b) in trial 2.

$$\text{Trial 1: } RT/ET_1 = 80/32 \left[ 32 \text{ s LT} / ET_1^2 \right]^{3/4}$$

$$\text{Trial 2: } RT/ET_1 = 80/32 \left[ (32 \text{ s})^2 / (23 \text{ s}) \text{ LT} / ET_1^2 \right]^{3/4}$$

$$\left[ \text{SSR} = 12.4; p\text{-value} = 0.5 \right] \quad (3)$$

In Equation (3), LL has been replaced by ET at that LL in trial 1 ( $ET_1$ ) by using Equation (1). This transformation improves the SSR.

### 3.2.3. Modelling RT for all cases – immediate work resumption and non-zero RTs

The RT equations derived in 3.2.2 for a given LT and LL are based on data where participants needed a rest period. As Figure 3 illustrates, for some values of LT and LL, some participants did not require a rest period. Therefore, using

$$\begin{aligned} \text{Trial 1: } f_{\text{RESUME}}(RT) &= 1 \text{ (or 100\%)} \\ &= \exp\left(-\frac{RT - 9.5 \text{ s}}{32 \text{ s}}\right) \end{aligned}$$

$$\begin{aligned} \text{Trial 2: } f_{\text{RESUME}}(RT) &= 1 \text{ (or 100\%)} \\ &= \exp\left(-\frac{RT - 9.5 \text{ s}}{23 \text{ s}}\right) \end{aligned}$$

as RT increases since the number of participants willing to resume work right away will decrease as the theoretical, non-zero, RT increases.

It is possible to compute a theoretical, non-zero RT value for each participant in the study, even those who did not require rest, by using Equation (2). KDE was used to compute the fraction of participants willing to resume work right away,  $f_{\text{RESUME}}$  and to construct a function for  $f_{\text{RESUME}}(RT)$ . The function  $f_{\text{RESUME}}(RT)$  is well estimated by an exponential decay function Equation (5):

$$\begin{aligned} &\text{for } RT \leq 9.5 \text{ s} \\ &\text{for } RT > 9.5 \text{ s} \\ &\left[ \text{SSR} = 0.900, R^2 = 0.971 \right] \end{aligned} \quad (5)$$

$$\begin{aligned} &\text{for } RT \leq 9.5 \text{ s} \\ &\text{for } RT > 9.5 \text{ s} \end{aligned}$$

Equation (2) will result in the computed RT being longer than the average RT observed. A more accurate average RT,  $RT_{\text{AVG}}$ , can be expressed as:

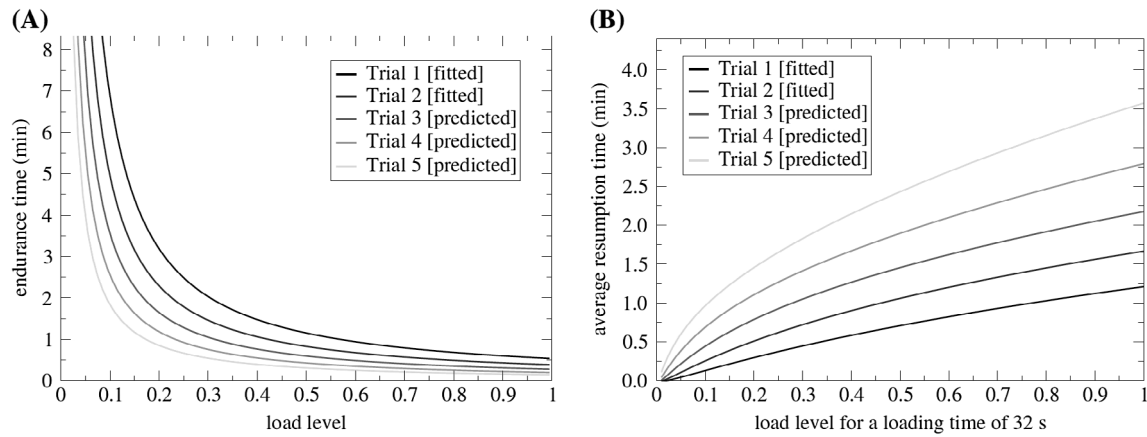
$$RT_{\text{AVG}} = (1 - f_{\text{RESUME}}) RT + f_{\text{RESUME}} 0 = (1 - f_{\text{RESUME}}) RT \quad (4)$$

where  $f_{\text{RESUME}}$  is the fraction of participants willing to resume work right away and RT is derived from Equation (2). In Equation (4), both  $f_{\text{RESUME}}$  and RT depend on LL and LT. While we have a theoretical equation for RT that depends on these two variables, we have no such equation for  $f_{\text{RESUME}}$ . We assumed that  $f_{\text{RESUME}}$  will decrease monotonically

Combining Equation (5) with the equation for the  $RT_{\text{AVG}}$  introduced in Equation (4), the following equation for  $RT_{\text{AVG}}$  is obtained, which also answers the second research question:

$$\begin{aligned} \text{Trial 1: } RT_{\text{AVG}} &= 0 \text{ (no resumption time)} && \text{for } RT \leq 9.5 \text{ s} \\ &= \left[ 1 - \exp\left(-\frac{RT - 9.5 \text{ s}}{32 \text{ s}}\right) \right] \cdot RT && \text{for } RT > 9.5 \text{ s} \\ \text{Trial 2: } RT_{\text{AVG}} &= 0 \text{ (no resumption time)} && \text{for } RT \leq 9.5 \text{ s} \\ &= \left[ 1 - \exp\left(-\frac{RT - 9.5 \text{ s}}{23 \text{ s}}\right) \right] \cdot RT && \text{for } RT > 9.5 \text{ s} \end{aligned} \quad (6)$$

where the theoretical RT is computed using Equation (2). Equation (6) incorporates all participants, both those who did and those who did not require a rest period.



**Figure 4.** (a) Predicted endurance time, ET, and (b) predicted average resumption time,  $RT_{AVG}$ , where the curves for the first two trials are based on Equation (1) and (6), respectively, and the curves for trials 3–5 are based on Equation (7) and Equations (8–9), respectively.

### 3.3. Modelling ET and RTs for repeated loadings

In this section the third research question is addressed. A visual presentation of the shape of the relationships obtained above is of interest, as is what they might imply for additional trials. Figure 4(a) illustrates the predicted decrease in ET for increasing numbers of trials, where the curves for the first two trials are based on Equation (1) and the curves for the predicted trials 3–5 are based on Equation (7), produced by assuming that the coefficient in the equation for ET changes by the same percentage with each subsequent trial. ET for repetitive loadings can be expressed as:

$$ET = (23/32)^{n-1} [32 s(LL)^{-10/9}] \quad (7)$$

where  $n$  is the trial number such that at the maximum LL ( $LL = 1$ ) and ET would be 32 s for the first trial ( $n = 1$ ), 23 s for the second trial, 17 s for the third, 12 s for the fourth, 8.5 s for the fifth, et cetera.

Figure 4(b) illustrates the predicted increase in the  $RT_{AVG}$  after successive trials, where the curves for the first two trials are based on Equation (6) and the curves for the predicted trials 3–5 are based on Equations (8) and (9), produced by assuming that the divider for LT and the exponential decay in  $f_{RESUME}$  change by the same percentage with each subsequent trial:

$$RT_{AVG} = \begin{cases} 0 \text{ (no resumption time)} & \text{for } RT \leq 9.5 \text{ s} \\ \left[ 1 - \exp\left(-\frac{RT(n) - 9.5 \text{ s}}{32 \text{ s}} \cdot \left\{\frac{32}{23}\right\}^{n-1}\right)\right] \cdot RT(n) & \text{for } RT > 9.5 \text{ s} \end{cases} \quad (8)$$

where

$$RT(n) = \left[\frac{32}{23}\right]^{\frac{3}{4}(n-1)} \cdot 80 \text{ s} \cdot \left(\frac{LT}{32 \text{ s}}\right)^{\frac{3}{4}} \cdot (LL)^{\frac{5}{9}} \quad (9)$$

where  $n$  is the trial number, with  $n = 1$  corresponding to trial 1. Equations (7)–(9) answer the third research question.

## 4. Discussion

The main contribution of this study is that it expands the available pool of empirically based endurance and recovery models. Uniquely, this study included two repetitions of the loading trial, providing some insight into the impacts of repetition on endurance and resumption response, also for low LLs. Methodologically, we demonstrate how these kind of data can be adequately analysed and modelled. The models here avoid the need to use the time to regain maximum force generating capacity for defining the recovery time and includes attention to recovery needs at low LL – problems noted with other models.

### 4.1. Discussion of study results

#### 4.1.1. On modelling ET for two repeated loadings

As described in 3.1, the analysis of the ET data showed that it is most appropriate to use  $\log_{10}(ET)$  or  $\ln(ET)$  rather than ET when performing a regression with ET. Further, the analysis (visual inspection as well as comparing SSR between possible mathematical relationships) showed that the power law offers a better fit to the experimental data used in this study than the exponential function. The analysis approach used here, with visual inspection of the variables in linear and logarithmic scales, and distribution shape analysis exploring power and exponential functions is rarely, if ever, presented in modelling papers. Such analysis may reveal that it would be appropriate to redevelop some of the existing models. This paper contributes, with descriptions, on how such analysis can be performed.



The suggested Equation (1) for ET (with  $SSR = 1.24$ ) models how participants could exert maximum force ( $LL = 1$ ) for 32 s in the first trial, and for 23 s in the second trial. At  $LL = 1$  the ET for the first trial is somewhat longer than in some other models (e.g. Sato et al. 1984; Frey Law and Avin 2010). At low LLs ET is similar to the Sato et al. model (1984), but shorter than the Rohmert model (1960a). It should be noted that ET in this study is subjectively determined by participants with professional experience of work which was simulated in the study – not based on force generating capacity as used in other studies.

#### 4.1.2. On modelling RT for two repeated loadings

The experimental data show that, in some cases, some participants were willing to resume the task immediately after completing their task (see Figure 3), which made the modelling of RT more complicated. However, the results do not indicate an LL asymptote, at which no fatigue would occur, for example as the model by Rohmert (1960b) suggests. The willingness to resume the task decreases as a function of the RLT. The percentage was lower in trial 2 than in trial 1 for all points, except at LL30 (30% MVC) for 10% RLT, where it was equal to the percentage in trial 1. The percentage did not decrease monotonically as the LL decreased. Participants were least likely to want to resume work at LL10, and were more likely to want to resume work at LL30 than at LL50. One explanation may be that the absolute LT was considerably longer for LL10 than for the other two LLs (the mean values of  $ET_1$  was 448 s for LL10, 114 s for LL30 and 75.2 s for LL50), which may have resulted in the onset of other fatigue mechanisms, such as central fatigue (e.g. Bigland-Ritchie 1981; Boyas and Guével 2011), in the cases with long LTs at LL10 compared to the other cases. These results suggest the possible existence of an optimal LL between 10 and 50% of MVC where participants are most willing to resume work immediately, indicating less fatigue. To our knowledge, this has not been reported in the fatigue research literature previously and further investigation is warranted.

Equation (5) states that for activities with a critical theoretical RT of less than 9.5 s, all participants will be willing to resume work immediately. The theoretical RT at a given LL and LT (see Equation 3) is 28% longer in trial 2 than in trial 1. Fewer participants are willing to resume work right away in trial 2 than in trial 1. The larger exponential decay rate in trial 2 compared to trial 1 also means that the fraction of participants willing to resume work right away drops off even faster in trial 2 than in trial 1 on any given loading condition. The decreases observed in RT and ET in the second repetition suggests that, although participants were allowed to set their own RT and decide how much rest they required before resuming work, they have

not allowed sufficient time to ‘fully recover’. This is a novel finding regarding RT, that could not be observed in studies with a single repetition (e.g. Rose, Ericson, and Örtengren 2000). While it is well known that different variables used to describe different fatigue aspects recover at different rates (e.g. Jonsson 1984), further research is needed to understand this tendency in repetitive loading scenarios.

#### 4.1.3. On modelling ET and RT for repeated loadings

The modelling of ET and RT for repeated loadings, when  $n > 2$ , was based only on two repeated loadings. In Section 3.3 the assumptions used in this modelling are described. However, alternatives to the above assumptions could also have been chosen, e.g. that the coefficient in the equation for ET changes with different percentages with increased repetitions, or different LLs. Gaining data from more than two repetitions is necessary to be able to form more adequate assumptions for the predictive models when  $n > 2$ .

## 4.2. Discussion of methods

The models demonstrated here have several limitations. One weakness of the study is the limited data-set with a small sample size of only 10 participants. Only data from male participants could be used, despite the ambitions to also include female participants, mainly because there are very few females among construction workers and its subgroup plumbers. Further, the task specificity is also a limitation as different motor tasks, with different postures and angles of force, may affect the response of ET and RT. The study design was adapted to the resources, given the time-intensive methodological approach used.

The experimental data that the models are based on are in the range between 10 and 50% MVC levels. It is suggested that the models initially are used for assessment of tasks up to 50% MVC. While 50% MVC is very high, it is not outside of the range of possible operational range of workloads, which was the range in focus for the modelling work. Not having data for higher MVC levels likely affects the accuracy of the models for very high load tasks. From a modelling perspective, gathering data for higher LLs and sampling ET at high LL values in future would be preferable. Since different fatigue processes and mechanisms are considered to be at play at different types of loading patterns (e.g. Liu, Brown, and Yue 2002) it is conceivable that different curve forms might apply at different parts of the force-fatigue relationship. However, we suggest a single form here across the force range examined for the sake of parsimony in future applications. Further research is needed to gain better resolution, in both load level and time domains, in order to improve the accuracy of these kinds of models.

### 4.3. General discussion

There are differences between existing endurance and recovery need models. While one of the older models uses a polynomial relationship between ET and LL, (Rohmert 1960a) most of the others use either power law, (e.g. Sato et al. 1984; Sjøgaard 1986; Frey Law and Avin 2010), or exponential relationships (e.g. Manenica 1986; Rose et al. 1992a, 1992b, 2000; Mathiassen and Åhsberg 1999). Differences between existing models can be explained by differences in the mathematical forms applied, different definitions of the included variables (Rose et al. 2014), different study designs (El ahrache, Imbeau, and Farbos 2006), different tasks, and whether the models are general or based on loading of specific muscle groups or body parts (Frey Law and Avin 2010) or on individual specific (Ma et al. 2015). Possible explanations of the differences in existing models also include whether an endurance limit (a % MVC below which force is considered to be possible to exert with 'infinite duration' without fatigue) is used or not, (El ahrache, Imbeau, and Farbos 2006).

Recovery need expressed as RT, which is based on data from experienced, healthy workers, in studies where they have carried out tasks similar to their normal work tasks and where they subjectively have determined when they would resume the task has been modelled in several studies (e.g. Rose et al. 1992b, 2014; Rose, Örtengren, and Ericson 2001). It is hypothesised that this may be more adequate than determining the recovery need as the time needed to recover maximum force generating capacity in healthy, but in the experimental task setting, untrained participants. These two variables, RT and recovery time, reflect two different aspects of the fatigue and recovery processes at play. The experience of the RT model approach in industry (Combs, personal communication, 2016), indicates that this approach may be at least as adequate for industrial applications as the more traditional recovery time model approach.

In this study static loading tasks were studied. However, many jobs have both static and dynamic components. Bakke et al. (1996) found that signs of fatigue in dynamic work appear much later than in static work and Perez et al. (2014) found the best fit with a correction factor of 1/6 in recovery time to cover dynamic work with static models. It would be of interest to evaluate how valid the equations presented here are for dynamic work, and to investigate what kind of correction factors might be needed to evaluate more repetitive, dynamic work (cf. Perez et al. 2014).

Applicability of any new model is always a concern and becomes increasingly problematic as the context of application moves further away from the context of the model creation. We recommend caution when making inferences based on these models in different working circumstances without evaluating the models' validity. Despite the

limitations discussed here, we believe this study provides interesting and potentially useful data on the relationship between load amplitude, load duration, and recovery.

## 5. Conclusions

This study adds to knowledge in the ergonomics domain in two ways: by describing how certain characteristics of empirical data can be accounted for in mathematical modelling, and by presenting models for ET and RT derived from a mathematical methodology which is unusual in the ergonomics literature.

The variability for ET and RT was found to be logarithmically distributed. Thus it is concluded that when performing regression for these two variables, logarithmic presentation of ET and RT should be used rather than a linear presentation, based on the current data-set. It is also concluded that power law relationships are most appropriate to use when modelling ET and RT with data distributed as in the experimental study, with two subsequent loading trials. Among the presented equations, Equations (7)–(9) are suggested for use for repeated loadings, although evaluation of the models' validity is recommended before implementation in industry. An unexpected inverse 'U' relationship between load level and recovery times was observed – a phenomenon worthy of further investigation.

## Notes

1. *A* and *B* are parameters for the first and second terms, respectively, in the equations.
2. *A*, *B* and *C* are parameters for the first, second and third terms (if a third exists), respectively, in the equations.

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## Appendix 1. Akaike's Information Criterion corrected

The AIC corrected for small samples,  $AIC_c$ , is computed as

$$AIC_c = (n_1 + n_2) \ln \left( \frac{SSR_1 + SSR_2}{n_1 + n_2} \right) + \frac{2(N_{\text{pars}} + 1)(n_1 + n_2)}{n_1 + n_2 - N_{\text{pars}} - 2}$$

where  $N_{\text{pars}}$  is the total number of parameters (the sum of those in the equations for trials 1 and 2), and the other symbols are as defined in Section 2 (Burnham and Anderson 2002).

## Appendix 2. Linear regression of $\log_{10}(\text{ET})$

If the relationship between ET and LL is exponential, then  $\log_{10}(\text{ET})$  versus LL yields

$$\text{ET} = A e^{B \text{LL}}$$

$$\underbrace{\log_{10}(\text{ET})}_{y} = \underbrace{\log_{10}(A)}_{y\text{-intercept}} + \underbrace{(B/\ln(10))}_{\text{slope}} \cdot \underbrace{\text{LL}}_x$$

Once the slope ( $m \pm \Delta m$ ) and y-intercept ( $b \pm \Delta b$ ) are obtained from the linear regression, the value of  $A$ ,  $B$ , and their standard deviation ( $\Delta A$  and  $\Delta B$ ) is simply given by

$$A \pm \Delta A = 10^b \pm [10^b \cdot \ln(10) \cdot \Delta b]$$

$$B \pm \Delta B = \ln(10) \cdot [m \pm \Delta m]$$

If the relationship between ET and LL is a power law, then  $\log_{10}(\text{ET})$  versus  $\log_{10}(\text{LL})$  yields

$$\text{ET} = A(\text{LL})^B$$

$$\underbrace{\log_{10}(\text{ET})}_{y} = \underbrace{\log_{10}(A)}_{y\text{-intercept}} + \underbrace{B}_{\text{slope}} \cdot \underbrace{\log_{10}(\text{LL})}_x$$

such that the value of  $A$ ,  $B$ , and their standard deviation is now given by

$$A \pm \Delta A = 10^b \pm [10^b \cdot \ln(10) \cdot \Delta b]$$

$$B \pm \Delta B = m \pm \Delta m$$