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The Distribution of Individual Stock Returns in a Modified Black-scholes Option Pricing Model

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(Under the Direction of Patrica Humphrey)

ABSTRACT

There have been many attempts to find a model that can accurately price options. These models are built on many assumptions, including which probability distribution stock returns follow. In this paper, we test several distributions to see which best fit the log returns of 20 different companies over a period between November 1, 2006 to October 31, 2011. If a "best" distribution is found, a modified Black-Scholes model will be defined by modifying the Weiner process. We use Monte Carlo simulations to generate estimated prices under specified parameters, and compare these prices to those simulated by the model using the Weiner process. It was found the Student-t distribution did a better job at modeling the larger time intervals and the 3-parameter lognormal did a better job at modeling the smaller time intervals. We were not able to make any definite conclusion due to the cost of purchasing historical option data.

Key Words: Black-Scholes, Weiner process, mathematical finance

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CHAPTER 1

INTRODUCTION

In finance, options are one of the most important instruments used because of their application to both hedging and speculation. They are considered assets and are a type of derivative security because their prices are derived from the price of another asset. There are two types of options: calls and puts. Brodie, Kane, and Marcus define a call option as a contract that gives rights to the holder to *purchase* an asset for a specified price, called the exercise or strike price, on or before some specified expiration date.[Bodie et al., 2009] They define put options as giving the right to the holder to *sell* an asset for a specified strike price on or before some expiration date.[Bodie et al., 2009] Options can be further classified into different types - European, American, Long-Term, Exotic, etc. - depending upon the parameters of the option, such as time until expiration, the average price of the underlying asset during the life of the option, whether the underlying asset is a dividend-paying stock, whether a condition is satisfied by the price of the underlying asset, etc. With the many different ways that option contracts can be set up, sophisticated institutional traders can execute extremely complex strategies. For instance, large pension funds and investment banking firms trade options in conjunction with stock and bond portfolios to control risk and capture additional profits. Corporations use options to execute their financing strategies and hedge unwanted risks that they could not avoid in any other way.

The first historical account of options being used occurred in ancient Greece. In *Politics*, Aristotle described how the philosopher Thales earned a fortune using option contracts to speculate on the olive harvest. The next account occurred in Amsterdam during the 1630's: Investors used option contracts to speculate on the price of tulip bulbs. As prices for the bulbs increased, dealers began using what are now known as call options to secure a purchase price for the bulbs from the growers. This caused a buying frenzy. Eventually, the ordinary investor turned to options and began investing everything they had. As with all bubbles, the price of the asset could only increase for so long until it reached a saturation point when the price was so high no investor could afford to buy the asset. The bubble burst and a selling frenzy began. The price of the bulbs plummeted and all of the options expired worthless. Investors, dealers, and growers were wiped out, tarnishing the reputation of options for a long time.[Kairys and III, 1997, Poitras, 2009]

Eventually an option market was established in London, England during the mid-1690's but corruption ran rampant and in 1711, the English government concocted a plan to convince investors to buy the British government's debt accumulated from wars. The government teamed with the South Sea Company and granted them a monopoly to trade in Spain's South America. Holders of government debt would exchange the debt for shares in the company and the government placed tariffs on all goods coming from South America so it could pay the South Sea Company a perpetual annuity at six percent annually. For the government, this interest rate was much less than the rate on the short-term war debt. Investors believed that the company was making money by exploring South America, but in actuality the British government was trying to lower its interest rate instead of making money. When speculators got wind that a company had monopoly rights to trade the stock price began to soar. Management of the company took advantage of the situation and began to issue more stock. In 1720, South Sea Company management saw the stock price soar to £1,000 per share, but with earnings dismal, management began selling their own shares. Soon after, other investors began to exit. Many had invested in options and all they could do was to sit back and watch their life savings disappear as the options expired worthless.[Dale et al., 2007, Shea, 2007] In 1734, London eventually banned options trading by passing the Barnard's Act, which made brokers a principal in speculative transactions, requiring them to complete a transaction in the event of a default by a client.[Poitras, 2009] This eventually lead to the creation of the Option Clearing Corporation (OCC).

Options trading has been around for many years in America, although not on any public market just over the counter (OTC). It wasn't till 1872 that an investor by the name of Russell Sage developed the idea that is similar to modern day call and put options. He purpose to standardized option contracts which allowed them to become more liquid. These contracts had a set number of shares of stock for each option, specified the expiration date, and outlined the stock price pegged to each specific option. His idea of establishing an exchange to trade options was never formalized because it was hard for options to change hands past the initial buyer and seller due to the lack of standardized terminology.

The Chicago Board of Trade (CBOT) was established in 1848 to bring order to the chaotic commodities market. This was accomplished by giving buyers and sellers a place to meet in order to negotiate and settle contracts. This led to contracts being formalized and called forwards. It was not until 1973 that the CBOT decided to allow the trading of options. CBOT set up a separate facility, called the Chicago Board of Options Exchange (CBOE), in order to insulate themselves from the inherent risk in options trading and to ensure that the obligations associated with options contracts are fulfilled in a reliable and timely manner. They also standardized the price, expiration, and contract size for all listed options.[Thomsett, 2009] Options traded before 1970 - approximately 1.1 million - were typically basic call options. In 1977, the market had grown to over 39 million contracts traded because they also allowed put options. In 1983, the CBOE introduced options on broad-based stock indexes such as the S&P 100 Index (OEX) and the S&P 500 Index (SPX). In 1984, option contracts were being written for commodities, which were previously only futures contracts. This later led to options being written on futures, which is a form of leverage. In 1990, the long-term equity anticipation securities options, now referred to as LEAPS, introduced options that have much longer lifespan - as long as 8 to 30 months. This gave investors more flexibility in using options in their portfolios. With all the new developments and many improvements to how options are traded, this led to over 1.2 billion contracts being traded in 2008, the busiest year in the CBOE's 35 year history.[CBOE, 2011]

Pricing of option contracts has to be the most important aspects of trading options. One of the most important financial breakthroughs over the last century, came by Louis Bachelier in 1900. He was able to develop an option-pricing model based on the assumption that stock prices followed an arithmetic Brownian motion with zero drift.[Merton, 1973] The next major development came from Fisher Black and Myron Scholes in 1973; they were able to derive the Black-Scholes option-pricing model which allowed investors to approximate a price for a European style option. Previous models by Ayers [Ayres, 1963], Boness [Boness, 1964], Sprenkle [Sprenkle, 1961], and others had expressed the value of options in terms of warrants. Warrants are basically call options issued by a firm. The most important difference between warrants and call options is that when warrants are exercised the firm is required to issue new shares of stock. When call options are exercised the number of shares of the firm stays fixed.[Bodie et al., 2009] The formulas found by each showed similarities and each of these models had at least one arbitrary parameter. But it was the developers of the Black-Scholes model that were able to find a solution to their stochastic partial differential equation that was not dependent on any unknown variables. In order to obtain this solution they had to compromise and make some underlying assumptions. The most important of these states, "The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of a finite interval is lognormal. The variance rate of the return on the stock is constant."[Black and Scholes, 1973]

The key to the assumption is the fact that possible returns (daily price difference divided by yesterday's price) of stock prices follow a certain distribution, in particular, the lognormal. Before Black and Scholes published their paper, research was suggesting returns followed a Cauchy distribution or a Student-t distribution. These findings came from Mandelbrot [Mandelbrot, 1963], Fama [Fama, 1965], and Blattberg and Gonedes [Blattberg and Gonedes, 1974]. Their reasoning was that, due to the smaller tails of the normal distribution, the model tends to overvalue far out-of-the-money options and undervalue deep in-the-money, according to the explanation given by Robert Jarrow and Andrew Rudd in 1982.[Jarrow and Rudd, 1982] Even with the work of Mandelbrot and Fama in the sixties, Black and Scholes chose to use a lognormal distribution because the model does price options reasonably well and it leads to a realistic depiction. If returns are lognormally distributed, then the distribution of returns are "positively skewed," thus the lowest possible return is -100% and allows for a maximum return well beyond 100% in any given period.[Black and Scholes, 1973] This assumption seems to work when there is not much volatility in the market. But following the financial crisis of the late 2000's, there appears to be a sustained higher level of volatility in the markets. Thus one could hypothesize that currently returns should follow a Cauchy or Student-t distribution.

It has been this type of reasoning which has lead to researchers over the years to take a different look at the Black-Scholes option-pricing model by relaxing some of its assumptions. In 1973, Robert Merton was able to modify the model to account for dividends and variable interest rates. He was able to account for variable interest rates by letting σ^2, which represents the variance of the stock, also account for the variance in the value of a discounted bond where the value of the discount represents the interest rate.[Merton, 1973] In 1976, Jonathan Ingersoll was able to relax the assumption that there are no taxes or transaction costs.[Ingersoll, 1976] Also in 1976, John Cox and Stephen Ross showed that prices do not have to move continuously, but may instead jump from one price to the next.[Cox and Ross, 1976] In 1979, they collaborated with Mark Rubinstein and developed a new option pricing model that uses this idea, which is now called the Binomial Option pricing model. This model assumes that there are two (and only two) possible prices for the underlying asset on the next time period. The stock can either increase by a factor of u% (an uptick) or decrease by a factor of d% (a downtick). This is a "discrete-time" model and watching the model expand through time, one can see the model grow like branches on a tree forming a complex lattice network. If one were to divide the model into an infinity number of periods, instead of just N discrete periods,one would obtain the Black-Scholes model.[Cox et al., 1979] It must be mentioned that William Sharpe was the first to suggest a model that follows the binomial approach in 1978.[Sharpe et al., 1999] As mentioned earlier, there now seems to be a sustained higher volatility in the returns of individual stocks thus leading to doubts in the normality assumption of the Black-Scholes model. Thus in 2010, Daniel Cassidy, Michael Hamp, and Rachid Ouyed re-derived the Black-Scholes model using the Student-t distribution. They chose the Student-t because of research previously done by Blattberg and Gonedes.[Cassidy et al., 2010]

The purpose of this paper is to determine which family of distributions best fits the most current data better than a normal distribution and also to determine if there is a best time interval to refer to when determining how far in the past one must look. The daily closing prices on 20 different companies from Google finance were recorded for the dates of November 1, 2006 through October 31, 2011. The data were then segmented into one-month intervals, three-month intervals, six-month intervals, one-year intervals, and the entire five-year interval. Quantile-quantile plots provided a rough estimation as to whether the data followed a normal distribution and whether the data followed one of the alternative distributions. The Anderson-Darling test was then used to strengthen the argument against normality. Note, a visual comparison method was used in conjunction with the Anderson-Darling test because the Anderson-Darling test is very sensitive to deviations if the sample size n is too large, there are changes in sigma, or both. If it is determined that one of the alternative distributions is best, simulations of the new model determined most accurate will be compared to the Black-Scholes model.

The plan for the rest of the paper is as follows: the probability distributions will be introduced, along with certain statistical theorems used, and a derivation of the original Black-Scholes partial differential equation will be presented in Chapter 2 showing where the equations used to run the numerical simulations in Chapter 5 come from. In Chapter 3, we graph the probability plots and test the data to see which family of distributions best fits the data by visual inspection and the Anderson-Darling test. Then we discuss our results. In Chapter 4, we derive a new or modified Black-Scholes model using the best fitting distribution(s) and compare the results. Section 5, will deal with a final discussion of the results and give guidance to further research.

CHAPTER 2

CANDIDATE MODELS FOR DAILY STOCK RETURNS, DERIVATION OF THE BLACK-SCHOLES MODEL, AND USEFUL THEOREMS

In this chapter, we introduce the candidate distributions that will be used to model the daily stock returns, briefly review the properties of the candidate models, provide a derivation of the Black-Scholes Model, and theorems and definitions from statistics that will be used. We will be considering the following distributions: normal, Student-t, Cauchy, Weibull, and 3-parameter lognormal. Note that due to time constraints we were not able to test all distributions and chose these specific distributions from the literature because they have the necessary characteristics needed to model the data. The probability density functions for the distributions used below were obtained from Minitab and are the ones used by the software.

2.1 Normal Distribution

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\Big(\frac{x-\mu}{\sigma}\Big)^2 \right)$$

for $-\infty < x < \infty$, where $-\infty < \mu < \infty$ and $0 < \sigma < \infty$, denoted by $X \sim N(\mu, \sigma^2)$. Here μ is the location parameter also referred to as the mean, σ is the scale parameter (standard deviation), and σ^2 is the variance. The normal distribution is also referred to as the Gaussian distribution. Observe that all moments exist and the kurtosis for the normal family is 0. Note that the software Minitab used in Chapter 3 assumes that the kurtosis is 3. Stock returns were first recognized to follow a normal distribution by Louis Bachelier.[Shao et al., 2001] It was later confirmed by M. F. M. Osborne.[Osborne, 1959] The normality assumption was contested by Blattberg and Gonedes, Clark, Kon, and Nederhoffer and Osborne. They found that daily stock returns exhibit fatter tails and greater kurtosis than the normal distribution. Our hypothesis is in line with their reasoning, but we chose it as a competing model in order to test it against the most recent data.

2.2 Student's t Distribution

The probability density function of the Student's t distribution is given by

$$g(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} \Big(1 + \frac{x^2}{\nu}\Big)^{-\frac{\nu+1}{2}}$$

with ν degrees of freedom and $\nu \geq 0$, denoted by $X \sim t(\nu)$. The mean is 0 and the variance is $\frac{\nu}{\nu-2}$. It is known that as ν tends to infinity, the Student's t distribution tends to a standard normal probability density function, which has a variance of one. Blattberg and Gonedes were the first to propose that stock returns could be modelled by this distribution.[Blattberg and Gonedes, 1974] Platen and Sidorowicz later reaffirmed these findings.[Platen and Rendek, 2007] Finally, Cassidy, Hamp, and Ouyed used these findings to derive the Gosset formula, which is the Student-t version of the Black-Scholes model.[Cassidy et al., 2010] They found that $\nu = 2.65$ provides the best fit when looking at the past 100 years of returns. They realized that as markets become more turbulent, the degrees of freedom should be adjusted to a smaller value.[Cassidy et al., 2010]

2.3 Cauchy Distribution

The probability density function of the Cauchy distribution is given by

$$f(x) = \frac{1}{\pi\theta(1 + \big(\frac{x-\eta}{\theta}\big))}$$

where η is the location parameter and θ is the scale parameter, for $-\infty < x < \infty$. and is denoted by $X \sim CAU(\theta, \eta)$. This model is similar to the normal distribution in that it is symmetric about zero, but the tails are fatter. This would mean that the probability of an extreme event occurring lies far out in the distributions tail. Using a crude example, if the normal distribution gave a probability of an extreme event occurring of 0.05% and the "best case" scenario of this event occurring 300 years, then using the Cauchy distribution one would find that the probability of occurring would be around 5% and now the "best case" scenario might have been reduced to only 63 years. Thus giving extreme events more of a likelihood of occurring. The mean, variance, and higher order moments are not defined (they are infinite); this implies that η and θ cannot be related to a mean and standard deviation. The Cauchy distribution is related to the Student's t distribution $T \sim CAU(1, 0)$ when $\nu = 1$. In 1963, Benoit Mandelbrot was the first to suggest that stock returns follow a stable distribution, in particular, the Cauchy distribution.[Mandelbrot, 1963] His work was validated by Eugene Fama in 1965.[Fama, 1965] Recent research by Nassim Taleb came to the same conclusion as Mandelbrot, saying that stock returns follow a Cauchy distribution, as reported in his New York Times best-seller book "The Black Swan".[Taleb, 2010]

2.4 Weibull Distribution

The probability density function of the Weibull distribution is given by

$$f(x) = \frac{\beta}{\theta^\beta} x^{\beta-1} \exp\Big(-\frac{x}{\theta}\Big)^\beta$$

where $x \geq 0$, $\theta \geq 0$, and $\beta \geq 0$, denoted by $X \sim WEI(\theta, \beta)$. The mean is given by $\theta\Gamma\Big(1 + \frac{1}{\beta}\Big)$ and the variance is given by $\theta^2\Big[\Gamma(1 + \frac{2}{\beta}) - \Gamma^2(1 + \frac{1}{\beta})\Big]$. Due to the restriction on x, the returns for individual stocks were calculated as $ln\big(\frac{P(t)}{P(t-1)}\big)$. This distribution is a special case of an extreme value distribution and the generalized gamma distribution.[Chen et al., 2008] Returns were first suggested to follow a Weibull distribution by Mittnik and Rachev in 1993, when they looked at the S&P 500 daily index returns.[Mittnik and Rachev, 1989] Other authors have employed it as an error distribution in range data modeling and trading duration models.[Chen et al., 2008, Engel and Russell, 1998]

2.5 3-Parameter Lognormal Distribution

The probability density function of the 3-parameter lognormal distribution is given by

$$f(x) = \frac{1}{\sigma(x-\theta)\sqrt{2\pi}} \exp\left(-\frac{[\ln(x-\theta)-\zeta]^2}{2\sigma^2} \right)$$

where $x \geq 0$ and $\sigma \geq 0$. The location parameter is σ, the scale parameter is ζ, and the threshold parameter is θ. It was felt that the 3-parameter lognormal distribution was a logical choice to test because of the ability to shift the distribution and shape it in different ways in order to give it a slightly skewed right appearance.

2.6 Derivation of the Black-Scholes Model

In the remainder of this paper, we focus on the original Black-Scholes model proposed by Fisher Black and Myron Scholes in 1973. The Black-Scholes model is used to assess the market value of options at any given point in time and is referred to as "Newton's Law" or the "Schrödinger equation" of the whole field of financial engineering that makes these markets operate, according to Jeremy Bernstein.[Bernstein, 2004] Their great insight came from the fact that an investor can create a riskless portfolio by dynamically hedging a long (short) position in the underlying asset with a short (long) position in a European call option. Since the expected return on the portfolio is equal to the riskless rate of interest, then there is no arbitrage opportunity and the underlying asset is considered to be "risk neutral." This implies that in a risk neutral economy, the option written against that asset will trade for the same price as if it were traded in a risk-loving or risk-adverse economy. Therefore, the price of the option is not based upon the investor's preference of risk.[Garven, 2012] The derivation of the model is rather lengthy and complex and there are several assumptions that need to be made, to which we turn to our attention to the original paper. As Black and Scholes state in [Black and Scholes, 1973] the assumptions are:

1. The short-term interest rate is known and is constant through time.
2. The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is lognormal distribution. The variance rate of the return on the stock is constant.
3. The stock pays no dividends or other distributions.
4. The option is "European," that is, it can only be exercised at maturity.
5. There are no transaction costs in buying or selling the stock or the option.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
7. There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an equal amount to the price of the security on that date.

Most of the assumptions are self-explanatory. Special attention needs to be given to the second assumption, which will be the focus of this paper. Black and Scholes assumed that stock prices followed a random walk, which says that price changes should be random and unpredictable.[Black and Scholes, 1973] In particular, S_t follows a stochastic process governed by the stochastic differential equation:

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{2.1}$$

where μ is called drift and measures the average growth rate of the asset price, σ represents the volatility of the stock returns, and dW represents the infinitesimal change in Brownian motion.[Black and Scholes, 1973] A Brownian motion process is defined in

Definition A.0.2. Let it be noted that $W(t)$ is a Wiener process, which is a continuous-time stochastic process. It generates a random variable that is normally distributed with mean 0 and variance t, $\phi(0,t)$. The Wiener process is defined in Definition A.0.3.

Black and Scholes' idea to use geometric Brownian motion (GBM) stemmed from the work by Paul Samuelson. Samuelson realized that GBM differs from Brownian motion in the sense that if $\{W^x(t)\}_{t\geq 0}$ is a Brownian motion started at $t > 0$, then eventually the path W(t) will drift below 0.[Lin, 2006, Paul and Baschnagel, 1999] This is an unrealistic feature of the model because share prices of stocks cannot drop below 0.

To begin the derivation of the Black-Scholes model, we start with the second assumption given by Black and Scholes, which is given by (2.1). This equation can be rewritten as:

$$dS = \mu S dt + \sigma S dW$$

Next we need to use Ito's Lemma stated in Theorem A.0.1. If we let $\alpha(t,X) = \mu S(t)$, $\sigma(t,X) = \sigma S(t)$, and $F(t) = \phi(t,S) = \ln(S(t))$. Then we obtain:

$$dF(t) = \left[\phi_t + \mu S(t)\phi_S + \frac{1}{2}\sigma^2 S(t)^2 \phi_{SS}\right] dt + \sigma S(t)\phi_S dW$$

We can also see that: $\phi_S = \frac{1}{S(t)}$, $\phi_{SS} = -\frac{1}{S(t)^2}$, and $\phi_t = 0$.
This implies that:

$$\begin{aligned} dF(t) &= \left[0 + \mu S(t)\frac{1}{S(t)} - \frac{1}{2}\sigma^2 S(t)^2 \frac{1}{S(t)^2}\right] dt + \sigma S(t)\frac{1}{S(t)} dW \\ &= \mu dt - \frac{1}{2}\sigma^2 dt + \sigma dW(t) \end{aligned}$$

Now integrate both sides, using the fact that $\int_0^t \sigma dW(u) = \sigma(W(t) - W(0))$:

$$\int_0^t dF(t) = \int_0^t \Big(\mu dt - \frac{1}{2}\sigma^2 dt + \sigma dW(t)\Big)$$
$$F(t) - F(0) = \int_0^t \Big(\mu - \frac{1}{2}\sigma^2\Big) dt + \int_0^t \sigma dW(t)$$
$$F(t) = F(0) + \Big(\mu - \frac{1}{2}\sigma^2\Big)(t - 0) + \sigma(W(t) - W(0))$$

It is known from the assumptions of the Wiener process that $W(0) = 0$, then:

$$F(t) = F(0) + \Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma W(t)$$

Next substitute back in, $F(t) = ln(S(t))$:

$$ln(S(t)) = ln(S(0)) + \Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma W(t)$$

Take the exponential of both sides:

$$e^{ln(S(t))} = e^{ln(S(0)+\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)}$$
$$S(t) = e^{ln(S(0))} * e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)}$$

Therefore:

$$S(t) = S(0)e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)} \tag{2.2}$$

The equation above is the solution to the SDE (2.1) and since $W(t)$ is normally distributed, it follows that S(t) is lognormally distributed. This equation shows that stock prices evolve over time. It is known that the value of a call option (C) depends on the value of the underlying asset; i.e. $C = C(S, t)$. Since the price of call options depends directly on the stock price, Ito's lemma justifies the use of a Taylor-series expansion for the differential dC:

$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \text{higher order terms}$$

This is a first order approximation of the call option price. Note we let $S = S(t)$, $dS^2 = S^2\sigma^2 dt$, and dropped the higher order terms because any term with higher order than the order of dt is small enough to ignore.[Lin, 2006] Then we have:

$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}dt \tag{2.3}$$

The goal is to create a perfectly hedged portfolio: a portfolio that has no risk. We start by constructing a portfolio that has a short position in some quantity $\Delta(t)$ of the underlying asset worth S per share and of one call option position worth $C(S,t)$. Here $\Delta(t)$ is the hedge ratio and is a function of t because the portfolio will be dynamically hedged; i.e., $\Delta(t)$ will change as the price of the stock changes through time. We know that the value of this hedged portfolio is $\Pi = C(S,t) - \Delta(t)S(t)$, which implies that the portfolio changes as $d\Pi = dV - \Delta(t)dS$, then plug in:

$$\begin{aligned} d\Pi &= \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}dt - \Delta(t)dS \\ &= \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right)dt + \left(\frac{\partial C}{\partial S} - \Delta(t)\right)dS \end{aligned}$$

Because this is a risk-neutral economy, we can set $\Delta(t) = \frac{\partial C}{\partial S}$. This will give a perfectly hedged portfolio:

$$d\Pi = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right)dt \tag{2.4}$$

To make sure there is no arbitrage opportunity, let

$$d\Pi = r\Pi dt \tag{2.5}$$

This implies that the hedged portfolio must earn the riskless rate of interest r. Because we let $\Delta(t) = \frac{\partial C}{\partial S}$, then $V = C(S,t) - \frac{\partial C}{\partial S}S$. Substituting this into the right-hand side of equation (2.5) and equating the result with the right-hand side of equation (2.4), we obtain:

$$r\Pi dt = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right)dt \tag{2.6}$$

Divide both sides by dt and rearrange the terms to obtain:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{2.7}$$

This equation along with the boundary conditions, $C(S,t)) = max[S - K, 0]$ and $C(S,t) = 0$, represents the famous Black-Scholes partial differential equation. One can see that dynamic hedging causes the valuation relationship between a call option and its underlying asset to be risk neutral. It can also be observed that the price of a call option depends on four parameters: the exercise price X, the current stock price S, the time to expiration t, and the volatility of the underlying asset σ. Note, we could take the Black-Scholes pde and find an analytical solution but for the purpose of this paper we will not continue down that route.

CHAPTER 3

TEST AND RESULTS FOR NORMALITY

This chapter is dedicated to examining the returns in order to determine which family of distributions fits best. Probability plots are used to visually assess the distributions and descriptive statistics are provided in order to give an initial value for some of the parameters needed to test the models.

3.1 Data and Software

The goal was to find a distribution family that could model the daily returns of individual companies. It was felt that in order for a distribution to model stock returns accurately, one should look at a time frame where stock returns had gone through a wide range of volatility. It was felt that an option pricing model must be able to accurately price options even under the most volatile of times. The financial crisis of the late 2000's provided a recent time frame to focus on. We only focus on a five-year time interval because it seems to this point everyone that looks at this research area looks at many decades of past data when running their analysis. But we felt that when pricing options today the most relevant data would only be the most recent data, especially when the stock market becomes very volatile. It was decided that the five-year time frame between the dates of November 1, 2006 to October 31, 2011 would capture a period of normal growth $(11/06 - 01/08)$, a violent down-swing $(11/06 - 05/09)$, and then a violent up-swing $(05/09 - 05/11)$. With so much variability in the data, the distributions that were used to model the data needed to have the characteristic of fatter tails than a normal distribution. The following distributions have this characteristic: Student-t, Weibull, 3-parameter lognormal, and the Cauchy distribution.

For many of these distributions, we chose to use log returns for a couple reasons.

Let's say that at time i the price of the asset is p_i and if we let $j \equiv (i-1)$, then the return of the asset from period j to i is $r_i = \frac{p_i - p_j}{p_j} = \frac{p_i}{p_j} - 1$. The first property of interest is lognormality. Assuming prices are lognormally distributed, then using the fact that $1 + r_i = \frac{p_i}{p_j} = exp^{log(\frac{p_i}{p_j})}$ this implies that $log(1 + r_i)$ is normally distributed. The second property is approximate raw-log equality. If returns are small, $r \ll 1$, then the log returns are approximately the raw returns, $log(1 + r) \approx r$. Thus, the log returns were calculated by $r(t) = \ln\left(\frac{P(t+1)}{P(t)}\right)$ for the normal, Student-t, and Cauchy distributions. But there lies an inherent problem due to the restrictions for some of the distributions that variable values must be greater than zero. It was decided to use the gross returns of the form $r(t) = \frac{P(t)}{P(t-1)}$ for the Weibull and 3-parameter lognormal distribution.

The companies of interest are listed in table B1. These companies span the major indices and are spread through multiple industrial sectors. For the sake of being thorough, we tested the distributions on a couple of indices as well. This was determined to be important since options are not just for stocks but for indices as well. So, if there is a best distribution, it would need to accurately model returns of individual companies as well as indices. The indices that were looked at are the S&P 500 (SP) and the Wilshire 5000 (WIL). Their ticker symbols are located in table B1.

The software of choice was Minitab. It had a built-in function to find the descriptive statistics and many of the distributions were built-in, which made creating the probability plots rather effortless. Matlab was used to find the estimate of the scale parameter of the Cauchy distribution. Excel was used to calculate the log returns and gross returns using the daily closing prices of the companies and indices. The data was obtained from Google Finance (www.google.com/finance).

3.2 Decriptive Statistics

Due to the large number of plots that needed to be produced in order to carry out the analysis, it was decided to limit the number of companies presented in this paper. The companies of choice were determined from the normal probability plots, which we will discuss in more detail in Section 3.3.1. We narrowed the 20 companies down to three companies in order to provide the reader with a summary of what we felt was a true representation of all the companies. The normal probability plots were created for the entire five-year interval and over one month, three month, six month, one year intervals for all 20 companies. The results were based on how well the individual companies fit the normal distribution over all the intervals. From the plots, Bank of America (BAC) was one of the worst fitting companies, The Coca-Cola Company (KO) was one of the best, and AT&T (T) was average (for some intervals it was really good and others it was really bad). Thus, the statistics were calculated from the log-transformed data for BAC, KO, and T. The descriptive statistics were calculated in Minitab. The statistics concentrated on are the mean, standard deviation, skewness and kurtosis. The returns for each company were sectioned into different intervals: the full five-year, one-year, six-month, three-month, and one-month. This sectioning allowed us to observe how each of the statistics changed between the three companies and over the different time intervals. From the tables in Section B, it can be seen that for each interval for all the companies, the mean is approximately 0 and the standard deviation (SD), which is directly linked to the volatility, is not constant.

3.3 Procedures

The first objective is to provide evidence that the normality assumption does not hold for the distribution of stock returns. The second objective is to test a variety of other distributions to see which, if any, can model the returns. The five-years worth of returns will be tested over different intervals. This should provide evidence to practitioners showing them how far to look back in order to accurately price options. It was decided to test this assumption by using a combination of visual inspection of the quantile-quantile plots and the Anderson-Darling test.

The Anderson-Darling test is a goodness of fit test and belongs to the sub-class called distance tests. Minitab calculates the Anderson-Darling statistic using the weighted squared distance between the fitted line of the probability plot (based on the chosen distribution and using either maximum likelihood or least squares estimates) and the nonparametric cumulative distribution step function. This statistic is a squared distance that is weighted more heavily in the tails of the distribution. Here, a smaller test statistic indicates that the distribution fits the data better. Minitab calculates the test statistics as follow: $A^2 = -n - S$ where

$$S = \sum_{k=1}^{n} \frac{2k-1}{n}[ln(F(Y_k)) + ln(1 - F(Y_{n+1-k}))]$$

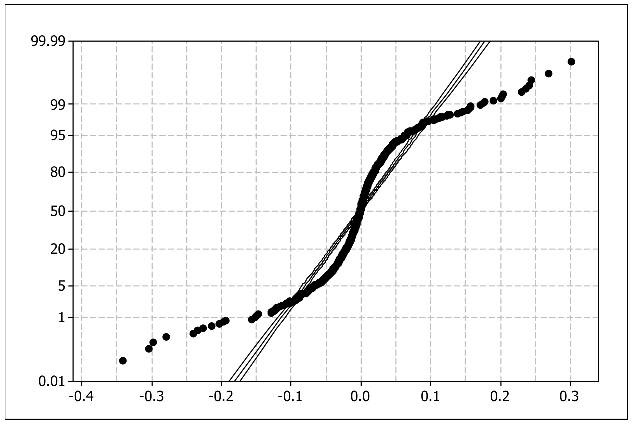
Y_i are the ordered observations, n is the total number of observations, and F is the cumulative distribution function of the specified distribution. The Anderson-Darling test is defined as:

$$H_0 = \text{The data follows the specified distribution}$$

$$H_a = \text{The data does not follow the specified distribution}$$

The analysis was started by importing the log returns into the software in its various intervals. Then the built-in function was used to create the normal probability plots. Minitab gives the researcher the option to enter a predetermined estimate for the mean and standard deviation but it was decided to let the software estimate those parameters. As part of the built-in function, the software calculates the Anderson-Darling test statistic. A sample of the output can be seen at the top of the next page with the data from BAC over the entire five-year interval. All of the Anderson-Darling test statistics generated can be found in Appendix B Table B.7.

Figure 3.1: BAC over 5-year interval



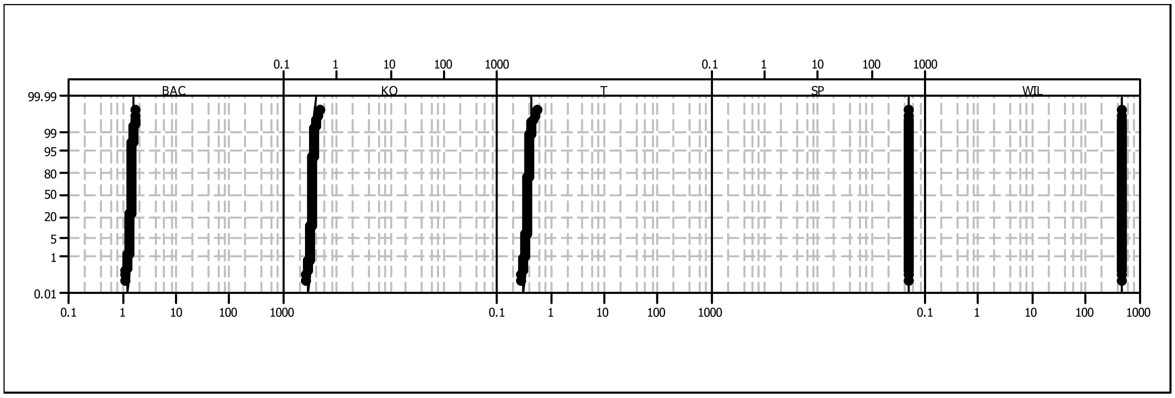
For the Student-t probability plots, Minitab does not have a built-in function. Thus, the plots will have to be constructed. Continuing with the log-transformed data, Minitab was used to order the data and to calculate the corresponding inverse cumulative probability values. The software allowed the user to enter a non-centrality parameter and the number of degrees to freedom to use. It was decided to let the non-centrality parameter equal zero. This was determined to be the correct assumption after calculating the descriptive statistics in Section 3.2, the mean value was approximately zero. For the degrees of freedom we turn to Section 2.2. There we referred to the paper by Cassidy, Hamp, and Ouyed, through their simulations they determined that setting the degrees of freedom to 2.65 would allow the Student-t distribution to properly model the data.[Cassidy et al., 2010] Next, we used the built-in function to create a time series plot where the ordered data values are on the x-axis and the calculated inverse values on the y-axis.

There is no built-in function to create the Cauchy probability plots, a similar procedure to the one used to create the Student-t probability plots will be used. But this time the log-transformed data will be imported into Matlab. Using Matlab's built-in function, *cauchyfit*, we can find the corresponding scale parameter for each interval of the data. With the ordered log-transformed data in Minitab, the inverse cumulative probabilities can be calculated for the Cauchy distribution using the scale parameter and again setting the location parameter to zero. Now, we used the built-in function to create a time series plot where the ordered data values are on the x-axis and the calculated inverse values on the y-axis.

As previously stated, there are some inherent problems with using log-transformed data to create the Weibull probability plots. When creating the inverse cumulative probabilities, the data must be greater than zero. The solution was to use the gross returns defined in 3.1. The returns are centered about one instead of zero. This would ensure that the data would not be less than or equal to zero. After importing the data into Minitab, the built in function was used to create the Weibull probability plots. It was decided to let the software estimate the shape and scale parameters.

Finally, we looked to see if the returns followed a 3-parameter lognormal distribution. Following the motif of the Weibull probability plots, we used the gross returns to perform this analysis. Using the built-in function for the 3-parameter lognormal probability plots we chose to set the location parameter to zero and to let the software estimate the scale. We set the threshold parameter to .00005 because the software has the requirement that this value must be less than the data minimum. One could also let the software estimate this parameter and then after creating the graph, uncheck the box that says, "adjust scale for threshold if distribution has this parameter." If it is left checked, the scales from graph to graph differ widely as can be seen in Figure 3.2 at the top of the next page. Note: due to the large number of plots produced, specific time intervals were chosen and the graphs for all the distributions are presented in the appendix C and are divided into different time intervals.

Figure 3.2: 3-parameter lognormal probability plot of BAC,KO, T, S&P, Wilshire over 5-year interval



3.4 Results of Normality

The analysis looked at how each distribution modeled the returns of the chosen companies and indices over a given time interval. The data was sectioned into intervals of 1 year, 6 months, 3 months, and individual months. In the appendix, abbreviations were used to designate which time interval we are looking at. For instance, if the plot is labeled BAC-Y3H2 then we are referring to Bank of America in the second half of the third year (this is referring to the interval: May 1, 2008 to October 31, 2011). In general this will mean H stands for half year and Q means quarterly. The corresponding graphs are located in Appendix C.2-C.6. The analysis will be broken up by time interval and the discussion will begin with the normal distribution. Then the discussion will turn to the Student-t and Cauchy distribution. They are examined this way because both of these distributions used log-transformed data and a built-in function was not available to create the plots. Thus, only visual inspection was used to analyze them. Finally, we will discuss the Weibull and 3-parameter lognormal distributions.

3.4.1 Five-year Interval

The analysis will begin by looking at the the normal probability plots on the five-year data set and the corresponding plots are located in Figure C.4. It is quite noticeable the plot for BAC has a definite S shape to it, implying that returns for BAC do not follow a normal distribution. The S-shape of the graph, where the values on the left hand side are above the line and the values on the right hand side are below the line, suggests that the normal distribution is underestimating the tails or that the "true" distribution should have heavier tails. Looking at the plot for KO, it can be seen that the normal distribution does a much better job of fitting the data but still with noticeable tails. This behavior leaves room for improvement. The plots for T, SP, and WIL each have a slight S-shape to them but they still have heavy tails. The plots over the five-year interval seem to make logical sense because if we look at Table B.2, we can see that the volatility for BAC is approximately four times as great leading to an emphasis of the S-shape. To confirm our results we turn our attention to the corresponding Anderson-Darling (AD) test statistics as can be seen in Table B.7. The AD values are many times greater than the AD critical value and according to the test we reject the null hypothesis. This suggest that the data does not follow the normal distribution.

The next set of plots looked at were for the Student-t and the Cauchy distributions. The plots are located in Figures C.5 and C.6, respectively. It seems that the Student-t distribution models the five-year returns rather well. There is some over estimating in the tails, especially for BAC. For the Cauchy distribution it is quite obvious that the distribution is overestimating the tails. One can see that the plots seem to be very symmetric.

Turning our attention to the Weibull and 3-parameter lognormal probability plots located in Figures C.7 and C.8, we can see that the Weibull distribution does a horrible job of fitting the data. The Weibull distribution with the parameters estimated from this data is left skewed. While the data is actually less left skewed than the distribution. This leads to the hanging tails we see in the plots. It seems that there are many observations on the lower end and this implies that the data is skewed. The AD test statistics,in Table B.7, for the Weibull distribution agree with the visual inspection. The values are many times greater than the critical value leading to the conclusion that the data does not follow a Weibull distribution. The 3-parameter lognormal distribution is very similar to the normal distribution. BAC has a definite S-shape, while T, SP, and WIL have a slight S-shape to them. The shape has the same implications as before. The 3-parameter lognormal distribution fits KO rather well, except for the tails. The AD test statistics for this distribution are similar to those of the normal distribution. We arrive at the same conclusion: the data does not follow a 3-parameter lognormal distribution. It seems that the Student-t distribution fits the five-year interval best.

3.4.2 One-Year Intervals

Next, let's look at the one-year intervals in Section C.3. The first set of plots that we will look at are the normal probability plots located in Figure C.9. We can see that the normal probability distribution does a decent job of fitting the returns in year 1. The AD values are close to the critical value and for T we fail to reject the null hypothesis. This implies that there is statistical evidence that the data for T in year 1 follows a normal distribution. As we move to years 2 and 3, one can see that the tails became fatter. This is logical because in Table B.3 it can be seen for each company that the standard deviation rises dramatically. Note, this rise corresponds with the financial crisis. As the standard deviation increases the fit of the data starts to weaken and outliers start to appear giving rise to the S-shape. Also, one could say this led to an increase in the AD values over this time interval. Through years 4 and 5, visually the fit was rather good for KO, T, SP, and WIL but the lower tails of BAC, SP, and WIL grew slightly. Referring to the corresponding AD values one can see that these values grew with the exception of T. But we still reject the null hypothesis for the AD test on all intervals. Overall, the normal distribution was a decent fit but not good enough.

Turning our attention to the Student-t and Cauchy probability plots in Figures C.10 and C.11, we can see that the Student-t distribution does a good job in the first year. As the volatility increases in year 2 and 3, a few outliers start to creep in and tails start to form. The tails are indicating, as before, the distribution is overestimating the outliers. But when compared to the other distributions it does a very good job. The Cauchy distribution overestimates the data on just about every account when looking at the data in one-year intervals. The Cauchy distribution does not provide a good fit to the data.

The Weibull and 3-parameter lognormal probability plots for the one-year intervals are located in Figures C.12 and C.13. One can see that the Weibull distribution fit the data rather well in years 1, 4, and 5. When the standard deviation increases in years 2 and 3, the fit broke down and we see that there are many observations at the lower end leading once again to the conclusion that the data are less skewed than the distribution. On closer inspection, we reject the hypothesis on all intervals. The AD test statistics can be found in Table B.7. The 3-parameter lognormal probability plots again have a similar outcome to the normal probability plots. They begin by fitting the data well and as the one looks into years 2 and 3, a tail and S-shape crop up. Looking at the AD values one can see that the 3-parameter distribution does not fit the data that well. There were only 2 intervals that fail to reject the null hypothesis AD and they were T in year 1 and year 5.

3.4.3 Six-Month Intervals

The next set of intervals to analyze are the six month intervals located in Section C.4 The figures of interest are C.14 and C.15. Here the normal distribution begins to fit the data quite well by visual inspection. Some of the intervals, such as BAC-Y1H1, BAC-Y1H2, WIL-Y2H1, have a slight curve or bend in them but does not seem to be too bad. It is quite obvious that as the second half of year two to comes around, there is more volatility introduced into the data and more outliers started to appear for all test subjects. The further through time one looks, the more the normal distribution does a good job of fitting the returns. Outliers occasionally pop up but mainly for BAC because on average it tends to be a more volatile stock. The AD test statistics tells us that the normal distribution was the appropriate model for 29 out of 50 intervals.

Let's turn our attention to the Student-t and Cauchy probability plots in Figures C.16 - C.19. By visual inspection, it can be seen that the model fits the data well for Y1H1 but then a noticeable bend appears. The bend is persistent throughout time and suggests that the returns in the tails are being overestimated. There are time where there are outliers that are being under estimated as in KO-Y2H2 and T-Y2H2. This seems to be the worst job of fitting that the Student-t distribution has done. Looking at the Cauchy probability plots we can see that the Cauchy distribution almost fits one time interval for KO, the second year, second half interval. But there are definite outliers and a slight bend. The conclusion can be made that the Cauchy overestimates the data and is not an appropriate distribution for this time interval.

The Weibull and 3-parameter lognormal probability plots are located in Figures C.20-23. The Weibull distribution does a mediocre job of modeling the 6-month interval returns. We can see that the Weibull distribution does a good job until year 2. Then we see that many of the observations tend to be on the lower end leading to the long hanging tail. If we look at the AD values one would reject the null hypothesis for all intervals. Implying that none of the intervals were appropriately modeled by the Weibull distribution. The AD test suggests that is not an appropriate distribution for this time interval. For the 3-parameter lognormal distribution, we can see that the fit falls apart starting in Year 2 half 2 for all test subjects. Applying the AD test, it was determined that one can fail to reject the null hypothesis on 24 out of 50 intervals. Though less than fifty percent of the intervals are appropriately modeled by the 3-parameter lognormal distribution, it is much better than most of the other distributions.

3.4.4 Three-Month Intervals

Let's continue our analysis by looking at the three month intervals located in Section C.5. As stated before, due to the large number of graphs the area of interest has been narrowed down to Y2Q3 - Y3Q4, Y4Q3 and Y4Q4. We start with the normal Probability plots that are located in Figure 24. Using visual inspection, we can see that BAC fails for Y2Q3 and the other subjects seem to pass for that interval. When looking over the other intervals, we can see that the outliers in some of the plots are being underestimated. For instance: SP-Y2Q4, KO-Y3Q2, and BAC-Y4Q2. This is similar to the 5-year interval for the subjects back in Section 3.4.1. Looking at the AD values determines that 36 of the 40 intervals fail to reject the null hypothesis. This is quite a success when compared to how the normal distribution fared over the 6-month, 1-year and 5-year intervals.

The next set of plots to look at will be for the Student-t and Cauchy distributions. We can see from Figure C.25 that the Student-t distribution does a fair job at fitting the returns for KO, T, SP and WIL. As time moves from Y2Q3 to Y2Q4, we can see that outliers appear for KO, T, SP and WIL. But there should be no cause for alarm since it looks like less than 5% of the returns are outliers. From Y3Q1 to Y4Q3, a noticeable curve appears and seems to persist in the plots. The curve suggests that the returns in the tails, are underestimated. It's interesting to note that all the plots seem to be very symmetric even though some quarters saw extreme loses and others saw extreme gains. Let's move on to the Cauchy distribution located in Figure C.26. With the initial look one can come to a conclusion as with the other intervals. It's obvious that every interval has about this interval noticeable tails. This implies that the returns in the tails are overestimated. Note that the width of the values on the Cauchy probability plot are directly dependent on the volatility of the underlying data. As one can see from the figures, it is obvious to spot difference in volatility from quarter to quarter. The plots seem to be implying that the volatility is not constant when viewed from a quarterly prospective.

The quarterly intervals for Weibull and 3-parameter lognormal probability plots are located in Figures C.27 and C.28. From an initial inspection, the Weibull distribution seems to fit KO, T, SP, WIL on Y2Q3 and Y3Q3; then add in BAC and the distribution fits the subjects over Y3Q4 - Y4Q4. For the quarters not mentioned, it can be seen that the long lower tails exist which leads to the same conclusion as previously stated in Section 3.4.3. The AD values state otherwise. According to the AD values, they suggests that 9 out of 40 of the quarterly intervals could be model by the Weibull distribution. This is a pass rate of only 22.5%. Let's see how the 3-parameter lognormal distribution performed. Looking at Figure C.28, it seems that the 3-parameter has done a good job with the exception of a few outliers on BAC-Y2Q3, BAC-Y2Q4, KO-Y2Q4 and a few others. Applying the AD test, it was found that 36 of the 40 quarterly intervals were appropriately fitted by the 3-parameter lognormal distribution.

3.4.5 One-Month Intervals

We finish our analysis by discussing the one-month intervals located in Section C.6. Note, we cut down the number of plots presented and our attention is now focused on May 2008 - October 2009 and we will be referencing each interval by the first 3 letters of the month. The corresponding normal probability plots are located in Figures C.29 and C.30. Looking at May 08 and Jun 08 everything is good. For BAC-Jul 08, T-OCT 08, and a few others, one can see that the plots do have the curve to them causing the data points on the ends to be below the straight line. This suggests that the data is positively skewed. All in all, the normal distribution looks to be a good fit, but there are a few places where there is an obvious outlier (ex. BA-Oct 08, SP-Dec 08, BAC-Apr 09). Reviewing the AD values, we can determine that the normal distribution fits the intervals 90 out of 90 times.

The next set of plots to look at will be for the Student-t and Cauchy distributions. The Student-t plots are located in Figures C.31-C.33. The data values seem to fall on a straight line. There are a few exceptions such as BAC-Jul 08, BAC-Aug 08, and T-Aug 08. Looking at the Nov 08 intervals, the curve appears and lasts for several months. Then the data straightens back out and becomes very symmetrical. This lasted until the September 08 intervals. Turning our attention to the Cauchy distribution located in Figures C.34-36. There is still a curve in the data plots but there seem to be some intervals that are somewhat straight (for example: SP-Jun 08, T-Jul 08, BAC-Apr 09, and SP-Aug 09). It can still be seen that there is much volatility in the data and it is not constant.

Finally, let's evaluate the Weibull and 3-parameter lognormal distributions. The Weibull probability plots are in Figures C.37 and C.38. One can see that most of the Weibull probability plots for BAC have the elongated tail hanging down, again indicating the data is less left skewed that the distribution. This also occurs for KO, T and SP on October 2008. This behavior seems to reoccur on March 2009 and April 2009. Reviewing the AD values, we can determine that the Weibull distribution fits 71 of the 90 intervals. This was a decent fit but other distributions were perfect to nearly perfect for the 1 month intervals. Let's see how the 3-parameter lognormal distribution performed. The plots are located in Figures C.39 and C.40 and they look quite linear with the exception of BAC starting in July 08 and ending in June 09. There were a few outliers (ex. KO-Oct 08, T-Oct 08, KO-Feb 09) but nothing to seem concerned about. Applying the AD test, it was found that 90 of the 90 monthly intervals were appropriately fitted by the 3-parameter lognormal distribution.

3.4.6 Conclusion of Normality Examination

The reason for analyzing so many different intervals of the same data set was to see if any conclusion could be drawn when deciding which length of time one should look back in order to know you have enough information to properly price options. This requires the knowledge of a distribution that models the returns. The goal was to find a statistical distribution that could model any set of returns no matter how volatile the returns may be. If the distribution is not normal, then we would modify the Black-Sholes model and test to see if the prices are more accurate when compared to the true value at option expiration. We found that when assessing all the intervals one notices that the normal and 3-parameter lognormal distributions did an effective job of fitting the data with the exception of BAC. The Cauchy and Weibull distributions were not nearly as effective. The success of the distributions just mentioned occurred with small sample sizes. Looking at how the distributions model the data with a large sample size, the Student-t distribution did the best job of fitting the data. And when one considers the examined distributions, it appears that the Student-t and 3-parameter lognormal distributions should be considered for further examination.

CHAPTER 4

SIMULATIONS

In this chapter, we look at what happens to option prices when the normality assumption is changed in the Black-Scholes model. Monte Carlo simulations were used to estimate the call price at some time t in the future. Using the results from Chapter 3 we will be examining the Black-Scholes model with the Student-t and 3-parameter lognormal distributions.

4.1 The Models

Looking back at Section 2.6, a derivation of the Black-Scholes model was presented and it finished with the famous Black-Scholes pde, Equation 2.7. As stated before, one could take the initial conditions along with the boundary conditions and finish deriving the Black-Scholes pde to arrive at an analytical solution but we will take a different approach which does not rely on solving the Black-Scholes pde. Instead we will try and model the behavior of the underlying asset itself, from which we will obtain estimates for the corresponding call option values. To allow us to do this, we make an assumption that all investors are "risk-neutral", that is they do not require a premium to encourage them to take risks. According to Higham, the phrase "risk neutral" comes from the phrase "risk-neutral investor" and he states the case as "an unlikely person who regards an investment with guaranteed rate of return r and a risky investment with expected rate of return r as equally attractive... we see that a risk-neutral investor would have no preferences between investing in a bank and in any asset."[Higham, 2004] A consequence of this assumption is that the average return on assets (μ) must be equal to the risk free interest rate (r). Thus, in Equation 2.2, we may replace μ (the expected return of the asset) with r (the risk free rate of returns on short-term Treasury bonds) and one finds an equation that will allow us to model the value of some asset through time. The new equation looks like:

$$S(t) = S(0)e^{\left(r-\frac{1}{2}\sigma^2\right)t+\sigma W(t)}. \tag{4.1}$$

. This is basically saying that for the purposes of pricing securities, one pretends that the asset price process is a geometric Brownian motion, $GBM(r, \sigma)$, instead of $GBM(\mu, \sigma)$.

A new question arises: how to find an equation to simulate the call option prices? We already know that the price of a call option at expiration time T is determined by the equation, $C_T = max(S_T - K, 0)$ where S_T is the asset price at expiration, T, and K is the option's strike price. At an earlier date t, the call option value will be the expected present value of the call option price and we obtain, $C_t = E[PV(max(S_T - K, 0))]$. This represents the expected payoff at discount rate r. Thus the price of a call option at time T is given by the resulting equation:

$$C_t = e^{-rT}E^Q[(max(S_T - K, 0))]$$

Note that E^Q makes it clear that we are taking the expectation in the risk-neutral world (the expectation in the Q-measure). Therefore the above equation can be rewritten for practical purposes as:

$$C_T = e^{-rT}E[(max(S_T - K, 0))] \tag{4.2}$$

Knowing what exactly the Q-measure is is not relevant to this paper and would take a while to explain; thus, it will be skipped. If one would like to read more about the topic, I refer you to the book by Lin.[Lin, 2006] The risk-neutral world is not reality. Investors would be unintelligent to think that the drift rate of risky investments is r. One would rather buy risk-free bonds in this case. Nevertheless, this concept is important and allows one to use Monte Carlo simulations to estimate the call option price.

With knowledge of Formulas 4.1 and 4.2 one can estimate the Black-Scholes call option price. The next idea to look at is what models should be examined to make reasonable comparisons to the Black-Scholes model. This will come down to altering one of the assumptions made by Black and Scholes, the assumption under consideration is number 2 in Section 2.6. As previously discussed, their assumption was that the underlying asset price moves as a random walk and in particular the asset prices process is governed by GBM. We talked about was the fact that GBM works using a Weiner process. Here lies the premise of our argument. Our hypothesis for the paper is that stock returns do not follow a lognormal distribution and must follow some other distribution, as discussed in Chapter 3. In order to modify the model to use a different distribution, we shall use a modified Weiner process. Typically, the Weiner process generates a random variable that is normally distributed but we are going to modify it to generate a random variable that has a Student-t distribution and then a 3-parameter lognormal distribution. This will in turn give us a modified GBM and cause the underlying stock and call prices to have the distribution we want.

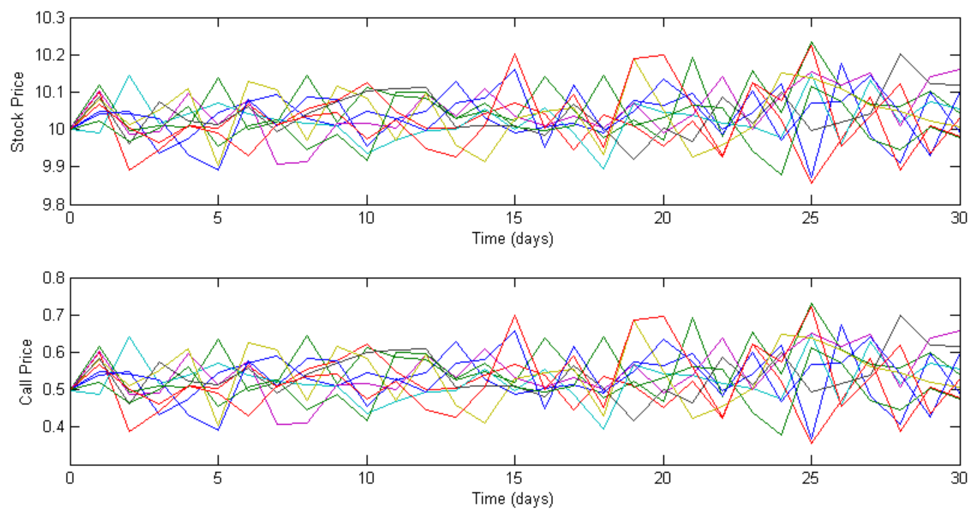
4.2 Simulations and Analysis

The goal was to determine if there are any price discrepancies when comparing the different models and if there are, then how large are they? Hopefully, this will lead us to a point where we are confident enough to say that the simulated prices are overvalued, undervalued, or priced correctly when compared to the baseline (the model using the normal distribution). Depending on the situation, a statement like this will give us a result that implies using one distribution is better than the others. Remember, the main strategy that seems to be implied by the Black-Scholes model is that of buying undervalued options (or selling overvalued ones) and holding them to expiration even in the face of any and all apparent setbacks in the position, trusting that the stock's historical volatility characteristic will be fulfilled over the longer term to raise the option's market value to equal the "fair value" that you calculated before you bought it. Hence, the goal would be to take advantage of a temporary market inefficiency to purchase the stock's volatility at a bargain rate.

To start the analysis, we need to determine the baseline. We are assuming that the model using the normal distribution (the Black-Scholes version) will be the baseline. This seems logical because as of now, that is the one a practitioner would use. It has been tested thoroughly and has been shown that, due to the smaller tails of the normal distribution, the model tends to overvalue far out-of-the-money (FOTM) options and undervalue deep in-the-money (DITM) options. FOTM call options are ones where the strike price is way higher than the current stock price and DITM options are the opposite. It seems that the better distribution would be give a lower price for FOTM options showing that they are not so overpriced (hopefully equal) when compared to the actual cost of the option and a higher price for DITM options showing that they are closer to the actual value of the the option. Herein lies a huge dilemma. We were not able to obtain historical options price data due to enormous cost. Thus, we will be able to determine if the prices of the modified Black-Scholes model are overvalued or undervalued when compared to the Monte Carlo simulated Black-Scholes prices but we won't be able to see exactly how they relate to the actual option price.

The idea behind the Monte Carlo simulation relies on the fact that the distribution of asset values at option expiry is determined by the process that generates future movements in the value of the asset. A simulation model can be viewed as progressing in three steps: 1) generate n random paths of the underlying variables; 2) compute the corresponding n discounted option payouts; and 3) average the last results to estimate the expected value of an options price at expiration. An example is displayed at the top of the next page. The top plot shows 10 simulated paths of the stock price and the bottom plot shows the resulting 10 paths of the call option price using the normal distribution and a 30-day to expiration option.

Figure 4.1: 30-day option using normal distribution w/ 10 paths



The simulations were run with Matlab R2011b on a computer with an Intel Core i5-2430M CPU at 2.40GHz and with 16.0 GB of memory on Windows 7 64-bit operating system. The corresponding Matlab code is located in Appendix D and was used to generate the baseline for comparison. The results can be seen in Tables B.8 to B.12. We chose to compare the options prices by distributions looking at how they change from deep in-the-money (DITM) all the way to far out-of-the-money (FOTM) options. Theory says that the deeper in the money an option is the more one will have to pay in order to purchase that option because there is a very high probability that the option will expire in the money. For FOTM options, the probability of the option price making it back to in the money is so low that the option is practically worthless. We should see that as we simulate the prices from DITM to FOTM options, the price should drop dramatically. Also, we will compare the option prices by looking at the corresponding volatility. Let's say we are going to model a 30 day option. The first simulation will use the one-month standard deviation and then the second simulation will use the five-year standard deviation. After the simulations we will compare and hopefully extract some useful information. Note we used Table 4.1 on the top of the next page to determine where to set S_0 and K for the corresponding ITM, ATM, etc.

Table 4.1: Table for S_0 and K

Type of Option	S_0	K
DITM	10	6
ITM	10	9.5
ATM	10	10
OTM	10	10.5
FOTM	10	14

The first set of simulations was for a 30-day option. The sigma used was from BAC-Oct 08 and had a value of 0.0962. We set n, representing the number of paths, to 1000. Note we only use 1000 because going higher would mean creating a unrealistically narrow confidence intervals. In general, the output will give Cmean which is the expected payoff (the price of the call option), the upper and lower bounds of a 95% confidence intervals and the standard deviation of the call price represented by Cstd. Looking at Table B.8 we can see the output for these simulations. One might notice that looking at the DITM option the normal and Student-t distribution are quite similar. When examining the standard deviations, we notice that the Student-t is significantly larger than the normal. This increase could be due in part to the fatter tails of the distribution. The 3-parameter lognormal is saying that the fair price of a 30-day option is lower than the price estimated by the normal and Student-t. As we move toward ATM and FOTM options, the prices estimated by the normal and Student-t drop to approximately zero as they should, while the standard deviation for the Student-t is noticeably higher than the corresponding value for the normal. Looking at the values for the 3-parameter lognormal and comparing it to the other two it appears that something is not right. The price estimated by the 3-parameter distribution for the FOTM options is outrageous. No rational investor would ever pay $1.94 for that option that has a probability approximately 0 of getting back into the money by expiration. As for the other two distributions, it seems that the Student-t would slightly overvalue a FOTM option when compare to the normal. As previously stated, studies have shown that the normal overvalues FOTM options. If we take that to be true, then the Student-t should noticeably overvalue FOTM options. The same seems to be true for DITM options because if the normal distribution is undervaluing DITM options then our estimated price in Table B.8 is too low and the true value of the option is worth more. We can see that the Student-t distribution does give a slightly higher price. This might suggest that the Student-t is closer to the true option value.

The results for the second set of simulations are contained in Table B.9. For this set of simulations, we have decided to use the volatility for BAC 5-year which has the value of .04832. Let's start off with the DITM options. One can see that the normal distribution has estimated a price that is slightly higher than that of the Student-t. This might be a moot point because the standard deviation is quite bigger for the Student-t. We see these attributes continue through the ITM and ATM options. Note, we see that the 3-parameter lognormal is estimating a price that is much lower than that given by the other two distributions for the DITM options. It would seem that it is deeply underestimating what the value of the call should be worth. For ITM options the 3-parameter distribution is vastly overestimating the fair value price. And this overestimating continues for the other types of options. Returning to the discussion on the normal and Student-t, it's not until OTM options that we see a noticeably gap between their simulated prices.

The third set of simulations was for a thirty day option and their results are contained in Table B.10. Here we used the one-month volatility for T Oct 08 which has a value of .0586. We can see that for the DITM options the normal and Student-t distributions have an approximately the same fair value price. The 3-parameter lognormal distribution has an estimated price that is significantly below the other two and with a standard deviation that is at least three times greater than the others. Moving down the table toward the FOTM options the prices drop as they should. For the ITM, ATM, OTM options, the Student-t distribution is giving an estimated fair value price that is higher than the one given by the normal distribution. The standard deviation for the Student-t was typically twice that of the standard deviation for the normal distribution with the exception for OTM options. Here the standard deviation was 4 times as great. The FOTM options have a similar outcome to the DITM options.

In the fourth set of simulations, we decided to use the one-month volatility for KO which has a value of .00886. The results for the fourth set of simulations are contained in Table B.11. For DITM options, we can see that the normal distribution is estimating a price that is slightly higher than the price from the Student-t distribution. This suggests that the Student-t distribution is producing an option's fair value that is slightly more undervalued than the normal distribution which is not something we want to see. Note the standard deviation is twice as high for the Student-t than for the normal. As we move down the table, the simulated prices for the normal distribution are quite similar to the simulated prices for the Student-t distribution. As usual, the estimated standard deviation was typically twice as high for the Student-t distribution when compared to the one produced by the normal distribution with the exception for ATM options. Here the standard deviation was 3 times as high.

In the fifth and final set of simulations, we decided to look at an index. Our choice was a 30 day option and then use the S&P 500's one-month volatility for Oct 08 which has a value of 0.05036. Since we are focusing on an index having the underlying asset price start at 10 is not logical. It does not make sense to use Table 4.1 as a guideline, more realistic numbers that will be used can be found in Table 4.2. Looking at the results in Table B.12 we can see what happens when the initial stock price is increased to a rather large number. For the DITM options the Student-t distribution has a simulated price that is noticeably higher than the one for the normal distribution. This agrees with the theory that if the Student-t is to be the best it would need to have a greater estimated fair value than the normal distribution hopefully not undervaluing the options price. As we move down the table one can see that the estimated price for the Student-t distribution remains significantly higher than the one for the normal distribution. This is not good because when we observe the FOTM options a better model would have a simulated fair value price that is lower than the normal distribution estimates. We can observe in the Table B.12 that this is not the case. Note that the same phenomena with the 3-parameter lognormal is occurring with extreme overvaluing and the volatility for the Student-t distribution is much higher for all the different types of options.

Table 4.2: Table for S_0 and K for the S&P 500 Index

Type of Option	S_0	K
DITM	1360	1330
ITM	1360	1355
ATM	1360	1360
OTM	1360	1365
FOTM	1360	1390

Comparing simulations in Tables B.8, B.9, and B.11 one can see that the only initial parameter to change was the value of volatility. We decided to pick intervals with a high level of volatility (BAC Oct 08), an average level of volatility (BAC 5 yr), and one on with a lower level of volatility (KO Dec 07). We can see that for DITM, ITM, and FOTM options in all three tables that the simulated call prices for the normal and Student-t distributions are quite similar for their being such a large difference in the standard deviations between the tables. The simulated prices seem to gradually change when going from ITM options to FOTM options. The change in the volatility parameter can be seen in the simulated call price. Looking at each type of option from Tables B.8, B.9, and B.11 we can see that the simulated call prices either stays the same or drops in value as they should. The less variable stock returns are the less the option will be worth. One can see that the simulated call price standard deviation decreases from Table B.8, B.9, and B.11, this makes sense because the volatility parameter is decreasing as well.

Looking at the simulations in Tables B.10 and B.12 one can see that when the initial stock price changes we see dramatic differences in the simulated call price. Note the volatility parameter for each set of simulations was approximately the same. The initial stock price was change from 10 to 1360 which would better represent the value of an index. We observe that from Table B.10 to B.12 the call price has risen dramatically. Table B.10 shows that there is only a slight if any difference between the estimated call prices from the normal to the Student-t distribution but we can see in Table B.12 that there is a noticeable difference between the normal and Student-t simulated call prices. The call price standard deviation is larger in Table B.12 than in Table B.10 but this is to be expected. Note that the standard deviation for the Student-t distribution is twice that of the standard deviation for the normal distribution in Table B.12 and this is similar to what we found in Table B.10.

CHAPTER 5

RESULTS

From Chapter 3, we feel that it is a safe conclusion to say that returns for individual companies and even indices, do not follow a normal distribution. In other words, we could say that there are other distributions that fit the data better. Further analysis needs to be done in order to determine if there is an overall distribution that best fits the data. It seemed that for the entire interval the Student-t distribution was the best fit but when analyzing the smaller intervals the 3-parameter lognormal distribution was the best, especially for the 1-month intervals.

In Chapter 4 it is hard to discern tangible results without having the historical options data. Using Monte Carlo simulations the results can change rather dramatically from one set to the next. There is something noticeably wrong with the 3-parameter lognormal distribution results. We believe it has to do with the scaling of the data. We see that the model with the Student-t distribution is allowing more movement in the simulated fair value price than the model using the normal distribution. This is a good sign because even though we cannot say with certainty that the Student-t distribution is best it is heading in the right direction. We saw that using the Student-t distribution does allow for more variability, or in other words volatility, in the option prices but it was interesting to see that Student-t distribution gave a tighter 95% confidence interval.

Future research directions include instead of assuming the normal distribution is "correct," taking the simulated prices and comparing them to the historical option prices using the historical parameter data. This will show if the normal and Student-t distributions are overvaluing FOTM options or undervaluing DITM options and by how much then a true comparison can be made. One can also try to model stock returns using other distributions, ARMA models, or GARCH models. Also, it would make sense to try to apply changepoint analysis to option pricing and see if this will lead to a better option pricing model. This will allow a model to start pricing options using, say, a normal distribution when times are less volatile and when the volatility estimates exceed some parameter the model switches over to another distribution.

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Appendix A

THEOREMS AND DEFINITIONS

Theorem A.0.1. (One Dimensional Ito's Lemma) *Let $\{S(t)\}$ be a solution of the stochastic differential equation $dX = \alpha(t, X)dt + \sigma(t, X)dW$ and $\phi(t, x)$ a deterministic function which is continuously differentiable in t and twice differentiable in x. Then the stochastic process $\phi(t, X(t))$ is a solution of the following SDE:*

$$\begin{aligned} d\phi(t, X) &= \left[\phi_t(t, X) + \alpha(t, X)\phi_x(t, x) + \frac{1}{2}\sigma^2(t, X)\phi_{xx}(t, X)\right]dt \\ &\quad + \sigma(t, X)\phi_x(t, X)dW \end{aligned}$$

Proof. A proof of Theorem A.0.1 is given by Lin.[Lin, 2006] □

Definition A.0.2. (Brownian Motion) *A random process B_t, $t \in [0, T]$, is a (standard) Brownian motion if:*

1. *The process begins at zero, $B_0 = 0$.*
2. *B_t has stationary, independent increments.*
3. *B_t is continuous in t.*
4. *The increments $B_t - B_s$, have a normal distribution with mean zero and variance $|t - s|$:*

$$(B_t - B_s) \sim N(0, |t - s|)$$

Definition A.0.3. (Weiner Process) *A Weiner process, $W(t)$, satisfies three properties:*

1. *$W(0) = 0$.*
2. *$W(t) - W(s)$ has a normal distribution with mean 0 and variance $\sigma^2(t - s)$ for $s \leq t$.*
3. *$W(t_2) - W(t_1), W(t_3) - W(t_2), ..., W(t_n) - W(t_(n - 1))$ are independent for $t_1 \leq t_2 \leq ... \leq t_n$.*

Appendix B

TABLES

Table B.1: Company and Index Names with Ticker Symbols

Company and Index Names	**Ticker Symbol**
3-M Company	MMM
Alcoa	AA
AT&T Inc.	T
Bank of America	BAC
Caterpillar	CAT
The Coca-Cola Company	KO
E L Du Ponte Nemours and Co.	DD
Eastman Kodak Company	EK
Exxon Mobil Corp.	XOM
General Electric	GE
Goodyear Tire	GT
International Business Machines	IBM
Intel Corp.	INTC
International Paper	IP
Johnson & Johnson	JNJ
Microsoft Corp.	MSFT
Owens-Illinois, Inc.	OI
The Procter & Gamble	PG
Sears Holdings Corp.	SHLD
S&P 500	.INX
Wal-Mart Stores, Inc.	WMT
Wilshire 5000	W5000FLT

Table B.2: Descriptive Statistics for Logged Data on Entire Interval

Ticker	Mean	SD	Skewness	Kurtosis
	Over 5 years			
BAC	-0.00162	0.04832	-0.19	11.83
KO	0.000299	0.013876	0.63	11.6
T	-0.000122	0.01742	0.56	8.87

Table B.3: Descriptive Statistics for Logged Data on 1-Year Intervals

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	1^{st} Year				2^{nd} Year			
BAC	-0.000436	0.010252	0.02	2.27	-0.00273	0.05378	-0.33	6.96
KO	0.001112	0.007880	0.13	1.50	-0.00133	0.01987	1.00	11.20
T	0.000793	0.012347	-0.01	0.34	-0.00176	0.02572	0.95	7.00
	3^{rd} Year				4^{th} Year			
BAC	-0.00198	0.08349	-0.02	3.36	-0.00093	0.02235	-0.43	1.59
KO	0.00075	0.01721	0.51	2.39	0.000538	0.009839	-0.40	1.62
T	-0.000160	0.022080	0.20	1.73	0.000405	0.009799	-0.17	0.62
	5^{th} Year							
BAC	-0.002040	0.035200	-0.63	8.46				
KO	0.000427	0.010641	-0.35	2.12				
T	0.000108	0.011112	-0.36	1.53				

Table B.4: Descriptive Statistics for Logged Data on 6-Month Intervals

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	Year 1 Half 1				Year 1 Half 2			
BAC	-0.000550	0.008879	-0.77	3.66	-0.00041	0.01141	0.37	1.49
KO	0.000907	0.006647	-0.05	4.84	0.00131	0.00891	0.17	0.14
T	0.001005	0.010972	-0.10	1.37	0.000590	0.01356	0.05	-0.23
	Year 2 Half 1				Year 2 Half 2			
BAC	-0.002030	0.027290	0.87	1.20	-0.00341	0.07055	-0.31	3.62
KO	-0.000390	0.011720	-0.08	0.03	-0.00225	0.02536	1.05	7.74
T	-0.000620	0.020080	-0.14	0.40	-0.00286	0.03022	1.26	7.08
	Year 3 Half 1				Year 3 Half 2			
BAC	-0.007840	0.110830	0.09	1.05	0.00383	0.04106	0.71	3.38
KO	-0.000180	0.021470	0.50	1.17	0.00167	0.01154	0.87	2.03
T	-0.000350	0.028150	0.25	0.45	0.00002	0.01373	-0.31	0.60
	Year 4 Half 1				Year 4 Half 2			
BAC	0.001550	0.019710	-0.48	0.96	-0.00341	0.02454	-0.31	1.66
KO	0.000020	0.009672	-0.59	1.57	0.00106	0.01002	-0.24	1.70
T	0.000116	0.009312	-0.23	0.81	0.00069	0.01029	-0.14	0.49
	Year 5 Half 1				Year 5 Half 2			
BAC	0.000560	0.001720	0.59	0.57	-0.00458	0.00403	-0.48	4.93
KO	0.000763	0.008065	0.01	0.60	0.00010	0.01269	-0.37	1.48
T	0.000698	0.009489	-0.10	0.24	-0.00047	0.01251	-0.41	1.55

Table B.5: Descriptive Statistics for Logged Data on 3-Month Intervals

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	Year 1 Quarter 1				Year 1 Quarter 2			
BAC	-0.000397	0.006640	-0.05	0.71	-0.00053	0.01078	-0.88	2.87
KO	0.000402	0.005080	1.87	7.53	0.00792	0.00792	-0.67	3.63
T	0.001540	0.01111	0.08	1.08	0.000470	0.01089	-0.29	1.84

Table B.5: (continued)

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	Year 1 Quarter 3				Year 1 Quarter 4			
BAC	-0.001110	0.008000	-0.12	0.99	0.00028	0.01401	0.33	0.53
KO	-0.000020	0.008080	-0.18	-0.54	0.00261	0.00955	0.27	0.21
T	0.000180	0.01284	-0.04	-0.84	0.001000	0.01432	0.09	0.15
	Year 2 Quarter 1				Year 2 Quarter 2			
BAC	-0.001440	0.026660	0.45	0.57	-0.00262	0.02811	1.25	1.96
KO	-0.000740	0.013280	-0.05	-0.32	-0.00004	0.01003	-0.05	0.39
T	-0.001330	0.02152	0.05	0.32	0.000090	0.01867	-0.39	0.62
	Year 2 Quarter 3				Year 2 Quarter 4			
BAC	-0.002060	0.052700	1.74	4.31	-0.00470	0.08500	-0.74	2.36
KO	-0.002090	0.014310	0.20	0.91	-0.00240	0.03294	0.97	4.70
T	-0.003570	0.01582	0.20	0.24	-0.002160	0.03973	1.07	3.81
	Year 3 Quarter 1				Year 3 Quarter 2			
BAC	-0.020300	0.098800	-0.06	2.27	0.00480	0.12130	0.06	0.39
KO	-0.000480	0.022990	0.56	0.75	0.00012	0.02000	0.43	1.98
T	-0.001310	0.03228	0.33	0.22	0.000630	0.02344	0.15	0.24
	Year 3 Quarter 3				Year 3 Quarter 4			
BAC	0.007760	0.050030	0.68	2.27	-0.00023	0.02894	-0.24	0.25
KO	0.002250	0.013670	0.99	1.42	0.00107	0.00889	-0.04	0.41
T	0.000360	0.01516	-0.36	0.53	-0.000340	0.01218	-0.24	0.49
	Year 4 Quarter 1				Year 4 Quarter 2			
BAC	0.000620	0.020890	-0.27	0.55	0.00248	0.01857	-0.74	1.79
KO	0.000270	0.009140	-0.47	0.57	-0.00023	0.01024	-0.67	2.25
T	-0.000190	0.00994	0.07	0.89	0.000420	0.0087	-0.64	0.82
	Year 4 Quarter 3				Year 4 Quarter 4			
BAC	-0.003680	0.028130	-0.46	1.52	-0.00314	0.02056	0.15	0.74
KO	0.000470	0.012090	-0.37	0.72	0.00164	0.00744	0.77	2.13
T	-0.000070	0.01147	0.08	0.19	0.001460	0.00898	-0.43	1.02
	Year 5 Quarter 1				Year 5 Quarter 2			
BAC	0.002880	0.022040	0.40	-0.04	-0.00180	0.01576	0.65	1.35
KO	0.000390	0.007950	0.24	1.15	0.00114	0.00822	-0.21	0.33
T	-0.000570	0.00934	-0.55	0.84	0.001980	0.00954	0.30	-0.65
	Year 5 Quarter 3				Year 5 Quarter 4			
BAC	-0.003730	0.017110	0.12	0.12	-0.00541	0.06204	-0.35	1.69
KO	0.000130	0.007960	0.73	3.56	0.00007	0.01606	-0.45	0.15
T	-0.000980	0.00817	-0.26	0.20	0.000030	0.01566	-0.45	0.57

Table B.6: Descriptive Statistics for Logged Data on 1-Month Intervals

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	Nov-06				Dec-06			
BAC	-0.000020	0.005940	0.50	0.63	-0.000430	0.007450	-0.22	1.58
KO	0.000112	0.004339	0.37	0.30	0.001490	0.006820	2.12	6.50
T	-0.000480	0.012600	0.04	0.28	0.002640	0.006870	-0.43	0.14
	Jan-07				Feb-07			
BAC	-0.000760	0.006800	-0.17	0.07	-0.001780	0.010650	-2.43	7.91
KO	-0.000385	0.003620	0.19	-0.12	-0.001340	0.008010	-2.10	6.82
T	0.002560	0.013000	0.36	1.14	-0.001170	0.011730	-1.35	3.87
	Mar-07				Apr-07			
BAC	0.000170	0.012870	-0.38	1.26	-0.000120	0.008640	-0.24	-0.25
KO	0.001270	0.007850	-0.70	1.02	0.004180	0.007320	0.89	3.19
T	0.003140	0.012200	0.21	0.23	-0.000910	0.008180	-0.19	-0.35
	May-07				Jun-07			
BAC	-0.000170	0.005160	-1.74	4.58	-0.001740	0.007950	0.23	0.49
KO	0.000690	0.008090	-0.12	-0.54	-0.000620	0.006520	0.00	-0.90
T	0.002980	0.009210	-0.18	-0.61	0.000180	0.015300	-0.18	-1.21
	Jul-07				Aug-07			
BAC	-0.001450	0.010430	0.09	0.26	0.002890	0.016930	0.17	-0.17
KO	-0.000180	0.009670	-0.31	-0.71	0.001370	0.010780	0.72	0.12
T	-0.002760	0.013360	0.60	-0.42	0.000780	0.018800	0.04	-0.48
	Sep-07				Oct-07			
BAC	-0.000430	0.012080	0.93	2.68	-0.001760	0.012360	-0.29	0.35
KO	0.003490	0.010030	-0.34	0.86	0.003130	0.008000	0.45	0.95
T	0.003130	0.013820	-0.18	-0.43	-0.000540	0.008960	0.69	0.04
	Nov-07				Dec-07			
BAC	-0.002170	0.029390	-0.04	-0.63	-0.005580	0.016630	-0.17	0.36
KO	0.000260	0.012350	0.47	-0.75	-0.000590	0.008860	-0.55	0.23
T	-0.004260	0.020080	-0.52	-0.23	0.004200	0.020240	1.05	0.87
	Jan-08				Feb-08			
BAC	0.003220	0.031640	0.63	0.39	-0.005260	0.020320	-0.48	-1.06
KO	-0.001880	0.017550	-0.05	-1.05	-0.000460	0.010730	-0.47	-1.07
T	-0.003650	0.023980	-0.09	-0.15	-0.005000	0.023360	-0.60	-0.45
	Mar-08				Apr-08			
BAC	-0.002360	0.037360	1.39	1.10	-0.000450	0.025450	1.33	2.47
KO	0.002020	0.011140	0.25	1.80	-0.001520	0.008310	-0.42	-0.63
T	0.004750	0.018860	0.32	-0.61	0.000480	0.012300	0.17	0.87

Table B.6: (continued)

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	May-08				Jun-08			
BAC	-0.004700	0.019530	1.02	1.08	-0.016860	0.025590	-0.06	-0.56
KO	-0.001320	0.010860	0.48	1.83	-0.004610	0.015800	0.59	1.24
T	0.001440	0.014040	-0.17	0.14	-0.008060	0.016150	0.69	0.28
	Jul-08				Aug-08			
BAC	0.014600	0.082600	0.82	-0.19	-0.002620	0.040380	-0.40	-1.02
KO	-0.000420	0.015940	-0.16	0.88	0.000520	0.015010	0.32	-0.47
T	-0.004060	0.016410	0.25	1.89	0.001790	0.016770	0.53	-1.02
	Sep-08				Oct-08			
BAC	0.005600	0.105000	-0.56	0.69	-0.016100	0.096200	-0.90	2.48
KO	0.000740	0.018730	-0.29	0.19	-0.007900	0.050800	1.14	1.78
T	-0.006480	0.030840	-0.53	0.18	-0.001800	0.058600	1.11	1.31
	Nov-08				Dec-08			
BAC	-0.019900	0.087600	1.07	3.09	-0.006200	0.080000	-0.61	2.24
KO	0.003090	0.030120	0.71	-0.45	-0.001510	0.022410	-0.08	0.99
T	0.003240	0.043740	0.14	-0.61	-0.000090	0.028490	0.13	0.14
	Jan-09				Feb-09			
BAC	-0.036200	0.126300	-0.01	1.79	-0.026900	0.135900	0.09	-0.16
KO	-0.002760	0.015200	-0.30	-1.16	-0.002360	0.026760	1.15	2.07
T	-0.006970	0.022740	0.12	-0.59	-0.001850	0.024750	-0.58	-0.46
	Mar-09				Apr-09			
BAC	0.024800	0.123200	0.31	-0.72	0.012300	0.105100	0.00	4.22
KO	0.003320	0.020600	-0.76	0.74	-0.000940	0.011190	-0.23	0.75
T	0.002660	0.027610	0.73	0.03	0.000750	0.018030	-0.47	0.07
	May-09				Jun-09			
BAC	0.011600	0.073400	0.70	0.49	0.007190	0.039520	-0.73	1.65
KO	0.006640	0.017200	0.89	0.33	-0.001090	0.011750	1.10	2.10
T	-0.001650	0.019310	-0.24	0.46	0.000090	0.011120	0.28	-0.53
	Jul-09				Aug-09			
BAC	0.004940	0.033380	0.68	0.48	0.008670	0.029390	-0.08	0.42
KO	0.001640	0.011280	0.14	0.88	-0.001090	0.008820	-0.34	0.13
T	0.002370	0.014860	-0.61	-0.13	-0.000340	0.009080	-0.43	0.67
	Sep-09				Oct-09			
BAC	-0.001850	0.023910	-0.63	1.38	-0.006770	0.031980	-0.21	-0.29
KO	0.004590	0.009170	0.25	0.45	-0.000330	0.008000	-0.47	0.12
T	0.001720	0.013540	0.19	-0.24	-0.002310	0.013420	-0.65	0.40

Table B.6: (continued)

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	Nov-09				Dec-09			
BAC	0.003980	0.019860	0.47	-0.28	-0.002220	0.016440	0.06	0.00
KO	0.003350	0.008370	-0.41	0.48	-0.000150	0.008650	-0.96	2.16
T	0.002300	0.009020	1.56	2.62	0.001720	0.008030	-0.79	0.01
	Jan-10				Feb-10			
BAC	0.000380	0.026140	-0.76	0.46	0.004650	0.022430	-0.71	1.37
KO	-0.002350	0.009860	-0.07	0.40	-0.001430	0.014120	-0.71	1.60
T	-0.004770	0.011460	0.33	0.58	-0.001100	0.008180	-0.41	0.82
	Mar-10				Apr-10			
BAC	0.003000	0.011640	-0.24	-0.54	-0.000050	0.021040	-0.82	1.44
KO	0.001840	0.007160	-0.02	-0.18	-0.001300	0.008860	0.07	-0.77
T	0.001770	0.007440	-1.47	3.19	0.000390	0.010390	-0.43	0.46
	May-10				Jun-10			
BAC	-0.005940	0.033780	0.13	0.47	-0.004140	0.022090	0.22	-0.70
KO	-0.001860	0.013460	-0.05	0.62	-0.001150	0.013810	-0.31	0.12
T	-0.003330	0.012780	0.37	-0.05	-0.000210	0.012780	0.22	0.18
	Jul-10				Aug-10			
BAC	-0.001060	0.028700	-1.64	4.94	-0.005430	0.014620	0.30	0.03
KO	0.004310	0.007650	0.89	0.25	0.000580	0.008870	0.68	1.49
T	0.003170	0.007770	0.62	1.37	0.001870	0.010930	-0.21	-0.13
	Sep-10				Oct-10			
BAC	0.002280	0.020740	0.97	1.08	-0.006410	0.025030	-0.34	-0.53
KO	0.002150	0.007630	1.46	3.84	0.002230	0.005610	0.21	0.68
T	0.002570	0.005740	0.63	0.40	-0.000130	0.009690	-0.76	1.39
	Nov-10				Dec-10			
BAC	-0.002130	0.021900	0.57	0.04	0.008970	0.020470	0.27	-0.55
KO	0.001420	0.008340	0.26	0.63	0.001830	0.008000	0.55	2.10
T	-0.001230	0.008410	0.49	-0.46	0.002530	0.006620	1.13	1.53
	Jan-11				Feb-11			
BAC	0.001440	0.023330	0.60	1.19	0.002100	0.017230	0.03	1.68
KO	-0.002270	0.007180	-0.45	1.12	0.000890	0.006390	0.52	2.25
T	-0.003270	0.011930	-0.77	-0.53	0.001620	0.007220	-0.25	-0.73
	Mar-11				Apr-11			
BAC	-0.003020	0.017230	1.38	2.05	-0.004100	0.012240	-0.43	0.12
KO	0.001620	0.010390	-0.57	-0.13	0.000840	0.007270	0.30	-0.20
T	0.003290	0.012030	0.10	-1.19	0.000830	0.008470	0.62	-0.49

Table B.6: (continued)

Ticker	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
	May-11				Jun-11			
BAC	-0.002100	0.010960	0.47	0.10	-0.003160	0.020150	-0.28	-0.22
KO	-0.000460	0.006150	0.11	0.82	0.000330	0.008250	-0.87	0.81
T	0.000670	0.008650	-0.04	0.65	-0.000220	0.007970	-0.29	-1.27
	Jul-11				Aug-11			
BAC	-0.006050	0.019260	0.73	0.19	-0.007500	0.082900	-0.43	1.14
KO	0.000530	0.009560	1.97	5.75	0.001530	0.020860	-0.56	-0.25
T	-0.003550	0.007630	-0.92	1.17	-0.001170	0.021680	-0.39	-0.45
	Sep-11				Oct-11			
BAC	-0.013760	0.041140	0.26	-0.49	0.005200	0.053600	-0.21	-0.63
KO	-0.001990	0.014520	-0.10	-1.10	0.000530	0.011360	-1.08	2.16
T	0.000070	0.011820	-0.25	-1.40	0.001300	0.011170	0.28	-0.17

Table B.7: Anderson-Darling Values (Note: * represents AD values with pvalues less than .05 and ** represents AD vales with pvalues less than .01)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
5-Year	Normal	66.753**	22.600**	18.784**	30.880**	29.290**
	Weibull	154.312**	152.290**	135.129**	105.655**	97.414**
	3-Parameter	66.696**	22.558**	18.744**	30.838**	29.249**
Year 1	Normal	3.271**	1.584**	0.352	4.625**	4.155**
	Weibull	11.015**	8.642**	4.189**	7.443**	6.207**
	3-Parameter	3.243**	1.562**	0.345	4.596**	4.128**
Year 2	Normal	6.503**	5.205**	3.474**	5.512**	5.314**
	Weibull	17.538**	28.620**	22.626**	21.245**	20.202**
	3-Parameter	6.458**	5.156**	3.435**	5.466**	5.270**
Year 3	Normal	6.491**	2.286**	1.927**	2.667**	2.490**
	Weibull	15.966**	12.933**	9.815**	7.412**	6.870**
	3-Parameter	6.453**	2.260**	1.906**	2.643**	2.467**

Table B.7: (continued)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
Year 4	Normal	1.218**	2.693**	1.184**	3.160**	2.958**
	Weibull	4.109**	6.510**	4.579**	8.540**	8.168**
	3-Parameter	1.201**	2.671**	1.172**	3.135**	2.933**
Year 5	Normal	6.295**	2.049**	0.662	4.722**	4.402**
	Weibull	16.661**	6.691**	3.848**	9.574**	9.633**
	3-Parameter	6.250**	2.024**	0.648	4.688**	4.295**
Y1H1	Normal	1.683**	1.767**	0.655	2.221**	2.185**
	Weibull	3.286**	6.821**	3.03**	2.768**	2.368**
	3-Parameter	1.651**	1.728**	0.638	2.181**	2.148**
Y1H2	Normal	1.623**	0.524	0.206	1.71**	1.465**
	Weibull	5.894**	2.908**	1.824**	2.574**	2.034**
	3-Parameter	1.601**	0.52	0.209	1.694*8	1.452**
Y2H1	Normal	1.188**	0.285	0.368	0.320	0.300
	Weibull	5.982**	1.51**	1.873**	2.674**	2.456**
	3-Parameter	1.175**	0.285	0.360	0.315	0.297
Y2H2	Normal	2.166**	3.015**	3.338**	3.917**	3.920**
	Weibull	5.895**	12.953**	13.027**	10.416**	10.012**
	3-Parameter	2.131**	2.967**	3.294**	3.874**	3.879**
Y3H1	Normal	1.167**	0.643	0.528	0.344	0.321
	Weibull	4.372**	4.573**	3.129**	1.760**	1.571**
	3-Parameter	1.148**	0.63	0.522	0.337	0.316
Y3H2	Normal	1.351**	1.300**	0.236	0.694	0.665
	Weibull	7.808**	6.900**	1.338**	1.917**	1.887**
	3-Parameter	1.321**	1.280**	0.231	0.687	0.658

Table B.7: (continued)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
Y4H1	Normal	0.947*	0.881*	0.474	1.911**	1.754**
	Weibull	1.707**	1.559**	1.955**	1.481**	1.295**
	3-Parameter	0.933*	0.865*	0.463	1.896**	1.740**
Y4H2	Normal	0.765*	2.338**	0.965*	1.554**	1.438**
	Weibull	2.850**	5.150**	2.752**	4.503**	4.254**
	3-Parameter	0.746*	2.314**	0.954*	1.534**	1.418**
Y5H1	Normal	0.708	0.344	0.222	1.634**	1.416**
	Weibull	4.263**	2.526**	1.871**	2.990**	2.558**
	3-Parameter	0.702	0.334	0.217	1.615**	1.399**
Y5H2	Normal	2.572**	1.221**	0.669	1.107**	1.04**
	Weibull	6.289**	2.792**	2.031**	2.542**	2.645**
	3-Parameter	2.536**	1.201**	0.654	1.084**	1.016*
Y2Q3	Normal	2.666**	0.520	0.241	0.515	0.392
	Weibull	6.591**	2.070**	1.430**	0.784*	0.718
	3-Parameter	2.636**	0.507	0.237	0.517	0.393
Y2Q4	Normal	0.723	1.384**	1.402**	1.105**	1.085**
	Weibull	1.083**	5.332**	5.057**	3.294**	3.100**
	3-Parameter	0.698	1.343**	1.370**	1.079**	1.061**
Y3Q1	Normal	0.884*	0.536	0.407	0.288	0.294
	Weibull	2.642**	2.330**	1.642**	0.788*	0.678
	3-Parameter	0.856*	0.530	0.407	0.290	0.296
Y3Q2	Normal	0.457	0.401	0.263	0.296	0.268
	Weibull	1.672**	2.560**	1.344**	1.360**	1.296**
	3-Parameter	0.449	0.384	0.258	0.289	0.263

Table B.7: (continued)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
Y3Q3	Normal	0.839*	1.151**	0.195	0.735	0.741
	Weibull	3.574**	3.710**	0.545	1.594**	1.644**
	3-Parameter	0.813*	1.140**	0.194	0.733	0.738
Y3Q4	Normal	0.283	0.254	0.218	0.471	0.514
	Weibull	0.725	1.036**	0.896*	0.325	0.343
	3-Parameter	0.278	0.246	0.211	0.472	0.517
Y4Q3	Normal	0.517	1.017*	0.542	0.474	0.443
	Weibull	1.175**	1.708**	1.758**	1.711**	1.637**
	3-Parameter	0.499	1.006*	0.536	0.465	0.434
Y4Q4	Normal	0.366	1.092**	0.933*	0.804*	0.725
	Weibull	1.710**	3.888**	1.344**	2.522**	2.333**
	3-Parameter	0.356	1.068**	0.920*	0.783*	0.706
May 08	Normal	0.647	0.351	0.300	0.560	0.567
	Weibull	1.197**	1.039**	0.398	0.325	0.289
	3-Parameter	0.658	0.334	0.306	0.572	0.578
Jun 08	Normal	0.260	0.333	0.388	0.566	0.543
	Weibull	0.413	0.935	0.968	0.522	0.595
	3-Parameter	0.279	0.329	0.394	0.564	0.540
Jul 08	Normal	0.616	0.211	0.453	0.269	0.217
	Weibull	1.105**	0.472	1.077**	0.394	0.356
	3-Parameter	0.638	0.202	0.431	0.296	0.242
Aug 08	Normal	0.420	0.224	0.748*	0.307	0.298
	Weibull	0.277	0.530	1.047**	0.598	0.490
	3-Parameter	0.454	0.239	0.790*	0.322	0.318

Table B.7: (continued)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
Sep 08	Normal	0.297	0.219	0.499	0.382	0.390
	Weibull	0.248	0.341	0.432	0.285	0.348
	3-Parameter	0.291	0.218	0.508	0.385	0.396
Oct 08	Normal	0.381	0.830*	0.662	0.472	0.401
	Weibull	0.386	1.725**	1.534**	1.245**	1.121**
	3-Parameter	0.363	0.823*	0.658	0.468	0.397
Nov 08	Normal	0.538	0.574	0.188	0.349	0.337
	Weibull	1.481**	0.886*	0.349	0.492	0.473
	3-Parameter	0.520	0.604	0.207	0.387	0.374
Dec 08	Normal	0.472	0.309	0.382	0.383	0.439
	Weibull	0.652	0.677	0.736*	0.336	0.275
	3-Parameter	0.453	0.299	0.386	0.366	0.348
Jan 09	Normal	0.439	0.519	0.331	0.358	0.434
	Weibull	0.901*	0.455	0.657	0.556	0.553
	3-Parameter	0.420	0.560	0.349	0.373	0.454
Feb 09	Normal	0.159	0.445	0.446	0.268	0.277
	Weibull	0.413	1.175**	0.287	0.402	0.454
	3-Parameter	0.167	0.439	0.470	0.269	0.276
Mar 09	Normal	0.402	0.579	0.516	0.171	0.161
	Weibull	0.786*	0.441	1.149**	0.627	0.589
	3-Parameter	0.426	0.581	0.528	0.175	0.167
Apr 09	Normal	0.965*	0.290	0.243	0.308	0.254
	Weibull	1.738**	0.571	0.295	0.153	0.125
	3-Parameter	0.927*	0.283	0.247	0.306	0.251

Table B.7: (continued)

Interval	Distribution	BAC	KO	T	S&P	Wilshire
May 09	Normal	0.335	0.490	0.179	0.236	0.271
	Weibull	0.948*	1.041**	0.357	0.419	0.505
	3-Parameter	0.335	0.500	0.178	0.264	0.297
Jun 09	Normal	0.301	0.444	0.312	0.511	0.538
	Weibull	0.321	1.312**	0.636	0.612	0.648
	3-Parameter	0.287	0.435	0.329	0.501	0.526
Jul 09	Normal	0.297	0.425	0.330	0.875*	0.781*
	Weibull	0.895*	0.990*	0.161	1.226**	1.097**
	3-Parameter	0.299	0.416	0.344	0.862	0.769
Aug 09	Normal	0.408	0.251	0.282	0.188	0.178
	Weibull	0.697	0.283	0.310	0.205	0.232
	3-Parameter	0.407	0.255	0.277	0.187	0.181
Sep 09	Normal	0.298	0.240	0.502	0.283	0.269
	Weibull	0.414	0.613	0.861*	0.305	0.268
	3-Parameter	0.289	0.748	0.521	0.288	0.273
Oct 09	Normal	0.157	0.780*	0.256	0.195	0.237
	Weibull	0.316	0.708	0.215	0.159	0.175
	3-Parameter	0.165	0.792*	0.262	0.208	0.251

Table B.8: Option Estimates for 30-day option using volatility from BAC Oct 08 with 95% Confidence Interval and Standard Deviation

Type of Option	Distribution	Cmean	Confidence Interval		Cstd
			Lower	Upper	
DITM	Normal	4.0272	3.8615	4.1929	0.4707
	Student-t	4.0894	4.0027	4.1762	1.3993
	3-Para	3.5705	3.5581	3.5828	2.8159
ITM	Normal	0.5697	0.4196	0.7198	0.4263
	Student-t	0.6384	0.5968	0.6801	0.6722
	3-Para	2.7827	2.7706	2.7948	2.7631
ATM	Normal	0.2114	0.1065	0.3162	0.2978
	Student-t	0.3145	0.2811	0.3479	0.5394
	3-Para	2.6734	2.6613	2.6854	2.7520
OTM	Normal	0.0447	-0.0033	0.0927	0.1364
	Student-t	0.1852	0.1298	0.2407	0.8947
	3-Para	2.5838	2.5717	2.5960	2.7633
FOTM	Normal	0.0000	0.0000	0.0000	0.0000
	Student-t	0.0071	-0.0069	0.0211	0.2259
	3-Para	1.9354	1.9240	1.9468	2.5938

Table B.9: Option Estimates for 30-day option using volatility from BAC 5 year with 95% Confidence Interval and Standard Deviation

Type of Option	Distribution	Cmean	Confidence Interval		Cstd
			Lower	Upper	
DITM	Normal	4.0201	3.9379	4.1024	0.2336
	Student-t	4.0178	3.9949	4.0407	0.3699
	3-Para	3.0919	3.0155	3.1682	1.2318
ITM	Normal	0.5471	0.4643	0.6298	0.2350
	Student-t	0.5540	0.5324	0.5757	0.3489
	3-Para	2.2178	2.1396	2.2960	1.2611
ATM	Normal	0.1295	0.0726	0.1864	0.1616
	Student-t	0.1625	0.1422	0.1828	0.3277
	3-Para	2.0955	2.0220	2.1690	1.1863
OTM	Normal	0.0022	-0.0038	0.0083	0.0173
	Student-t	0.0333	0.0227	0.0439	0.1712
	3-Para	1.9929	1.9174	2.0684	1.2182
FOTM	Normal	0.0000	0.0000	0.0000	0.0000
	Student-t	0.0000	0.0000	0.0000	0.0000
	3-Para	1.1726	1.1039	1.2413	1.1088

Table B.10: Option Estimates for 30-day option using volatility from T Oct 08 with 95% Confidence Interval and Standard Deviation

Type of Option	Distribution	Cmean	Confidence Interval		Cstd
			Lower	Upper	
DITM	Normal	4.0235	3.9215	4.1254	0.2895
	Student-t	4.0292	3.9982	4.0602	0.5004
	3-Para	3.1461	3.0530	3.2391	1.5017
ITM	Normal	0.5488	0.4499	0.6476	0.2808
	Student-t	0.6065	0.5761	0.6369	0.4907
	3-Para	2.2420	2.1514	2.3327	1.4632
ATM	Normal	0.1387	0.0742	0.2032	0.1832
	Student-t	0.1870	0.1668	0.2073	0.3270
	3-Para	2.1963	2.1048	2.2878	1.4764
OTM	Normal	0.0079	-0.0061	0.0218	0.0396
	Student-t	0.0380	0.0266	0.0495	0.1847
	3-Para	2.0410	1.9520	2.1300	1.4355
FOTM	Normal	0.0000	0.0000	0.0000	0.0000
	Student-t	0.0000	0.0000	0.0000	0.0000
	3-Para	1.3729	1.2858	1.4600	1.4053

Table B.11: Option Estimates for 30-day option using volatility from KO Dec 07 with 95% Confidence Interval and Standard Deviation

Type of Option	Distribution	Cmean	Confidence Interval		Cstd
			Lower	Upper	
DITM	Normal	4.0311	4.0158	4.0464	0.0435
	Student-t	4.0233	4.0182	4.0285	0.0833
	3-Para	2.9249	2.9123	2.9375	0.2031
ITM	Normal	0.5451	0.5305	0.5597	0.0416
	Student-t	0.5484	0.5433	0.5534	0.0818
	3-Para	2.0921	2.0795	2.1047	0.2032
ATM	Normal	0.0511	0.0377	0.0646	0.0382
	Student-t	0.0592	0.0528	0.0656	0.1027
	3-Para	1.9675	1.9547	1.9803	0.2064
OTM	Normal	0.0000	0.0000	0.0000	0.0000
	Student-t	0.0001	-0.0001	0.0002	0.0019
	3-Para	1.8653	1.8520	1.8787	0.2150
FOTM	Normal	0.0000	0.0000	0.0000	0.0000
	Student-t	0.0000	0.0000	0.0000	0.0000
	3-Para	1.0263	1.0136	1.0390	0.2051

Table B.12: Option Estimates for 30-day option using volatility from SP Oct 08 with 95% Confidence Interval and Standard Deviation

Type of Option	Distribution	Cmean	Confidence Interval		Cstd
			Lower	Upper	
DITM	Normal	39.2424	28.4796	50.0053	30.5740
	Student-t	43.5767	40.499	46.6544	49.6561
	3-Para	297.4209	287.0188	307.8230	167.8307
ITM	Normal	19.7909	11.7650	27.8168	22.7990
	Student-t	26.8870	23.9547	29.8193	47.3110
	3-Para	292.0421	281.6131	302.4711	168.2657
ATM	Normal	17.8427	9.9476	25.7378	22.4276
	Student-t	22.8106	20.4002	25.2209	38.8901
	3-Para	293.5280	282.5561	304.4999	177.0251
OTM	Normal	13.6840	6.7589	20.6092	19.6722
	Student-t	20.8551	18.4381	23.2721	38.9965
	3-Para	292.7601	281.8656	303.6546	175.7761
FOTM	Normal	4.9415	0.6679	9.2150	12.1399
	Student-t	9.0587	7.1062	11.0112	31.5024
	3-Para	283.3558	272.5533	294.1583	174.2912

Appendix C

GRAPHS

C.1 Daily Closing Prices of Indexes Over 5-Year Interval

Figure C.1: DJIA over 5-year interval

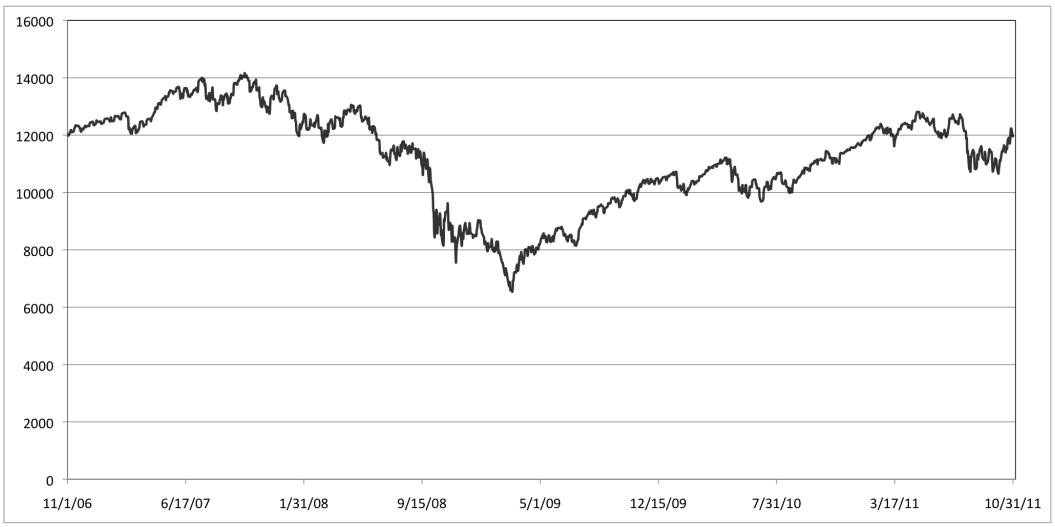


Figure C.2: Nasdaq over 5-year interval

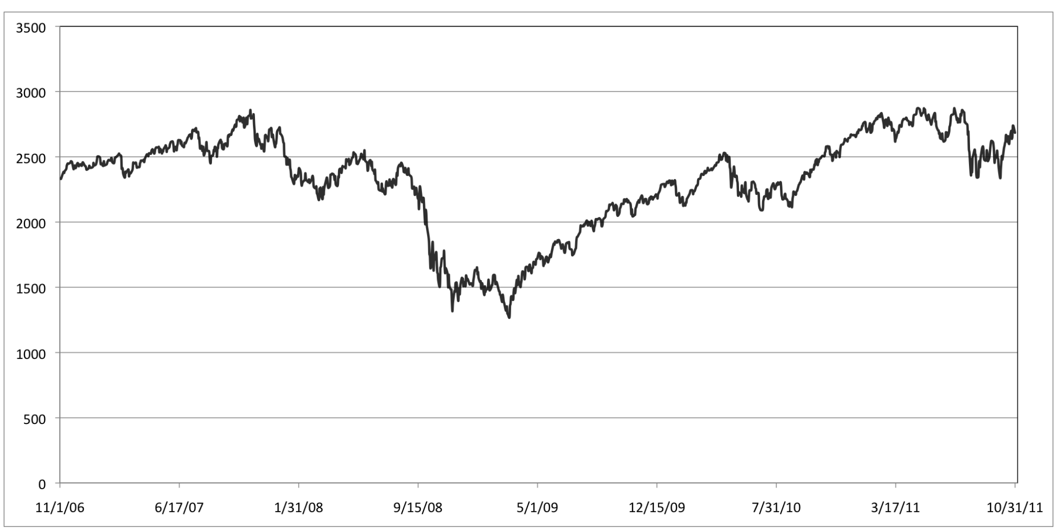
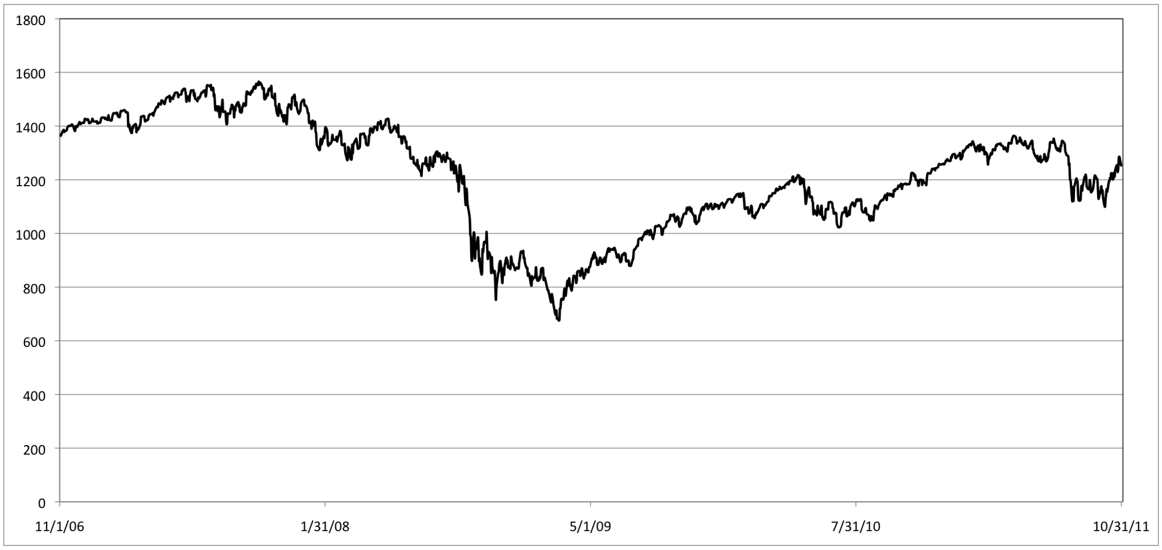


Figure C.3: S&P 500 over 5-year interval



C.2 5-Year Interval

Figure C.4: Normal probability plots over 5-year interval with a 95% confidence interval

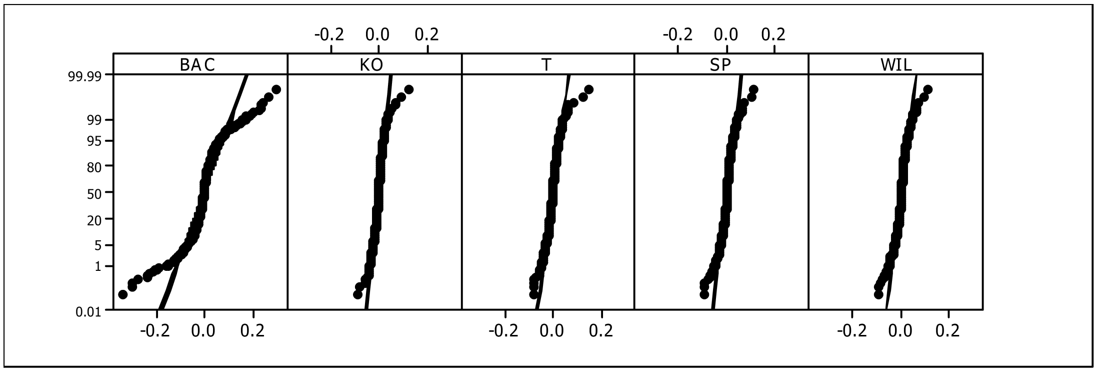


Figure C.5: Student-t probability plots over 5-year interval

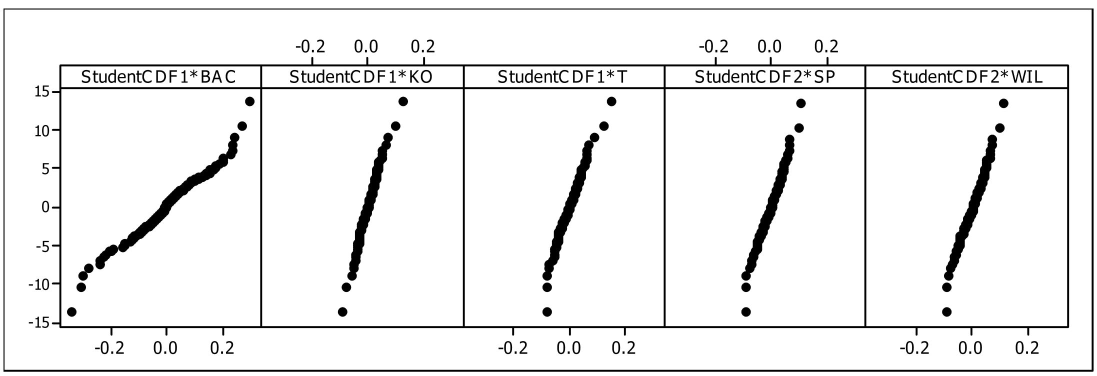


Figure C.6: Cauchy probability plots over 5-year interval

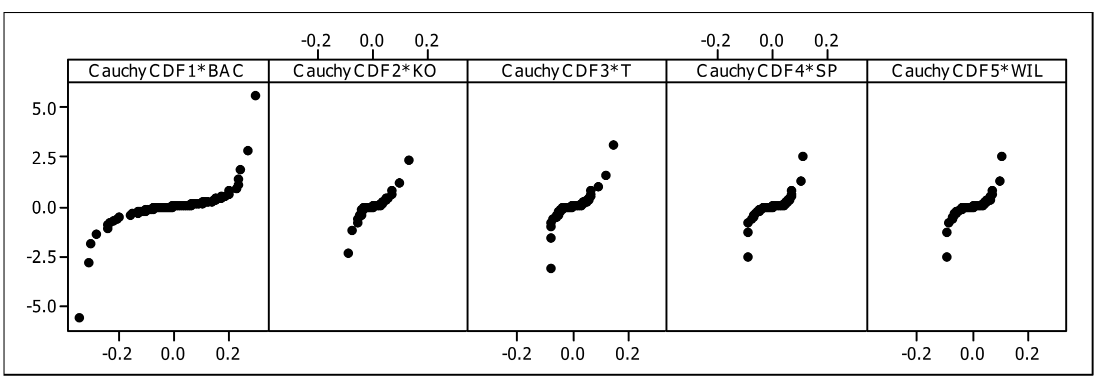


Figure C.7: Weibull probability plots over 5-year interval with a 95% confidence interval

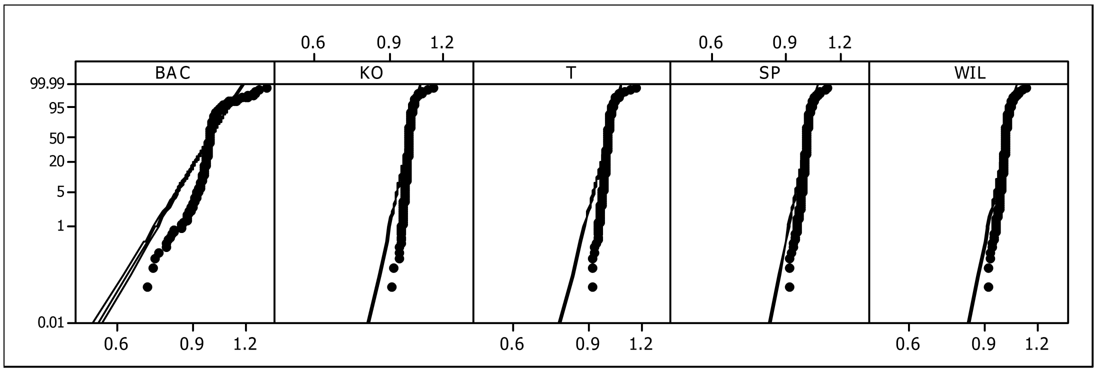
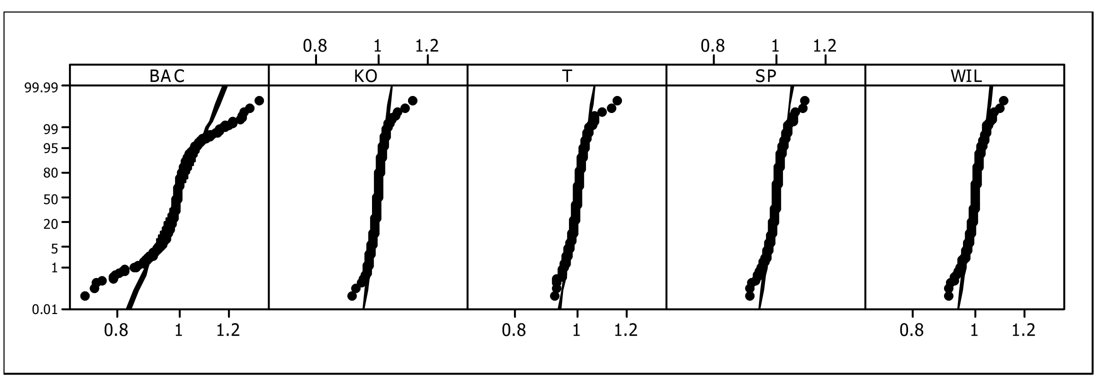


Figure C.8: 3-parameter lognormal probability plots over 5-year interval with a 95% confidence interval



C.3 1-Year Interval

Figure C.9: Normal probability plots over 1-year intervals with a 95% confidence interval

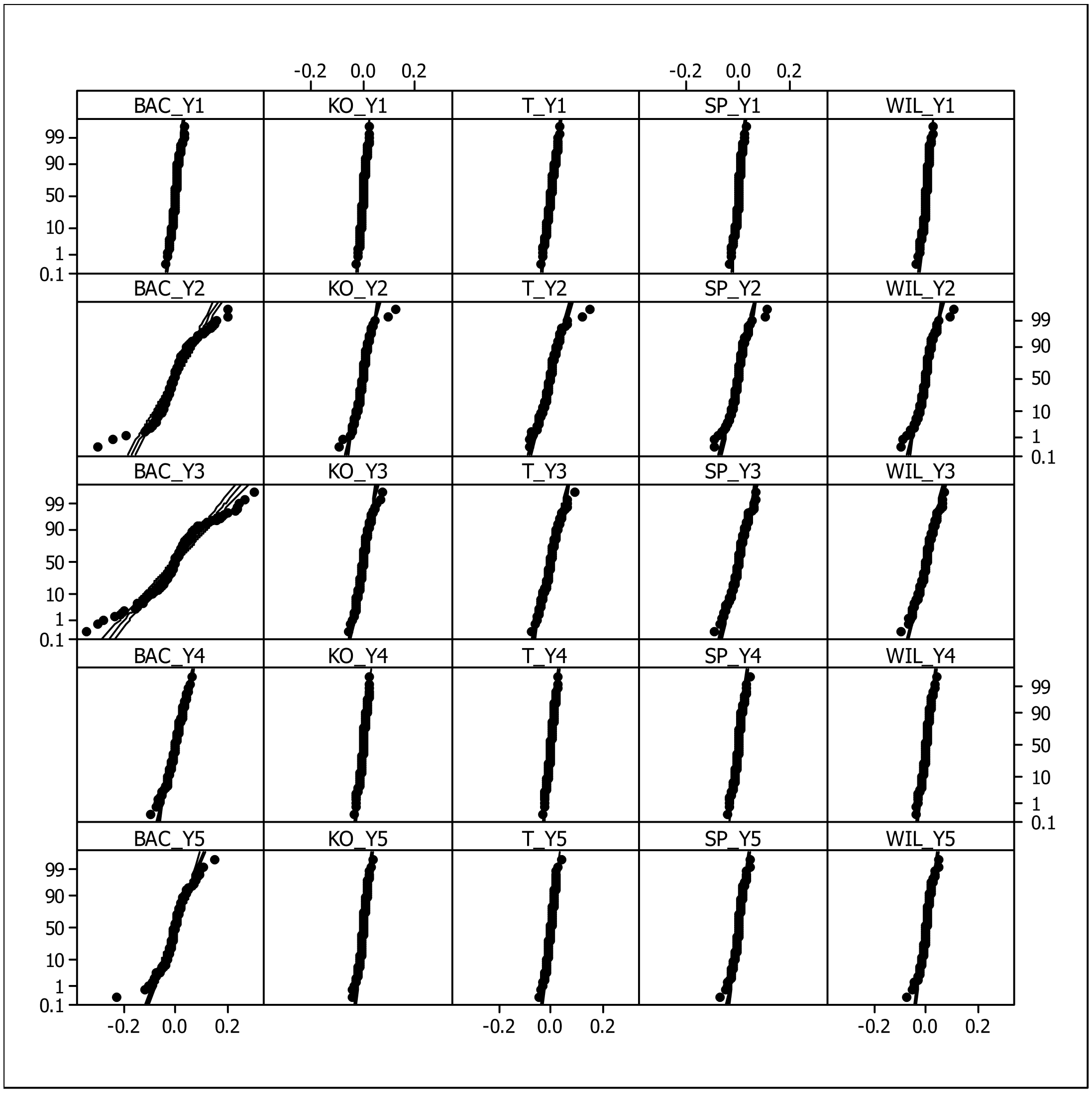


Figure C.10: Student-t probability plot over 1-year intervals

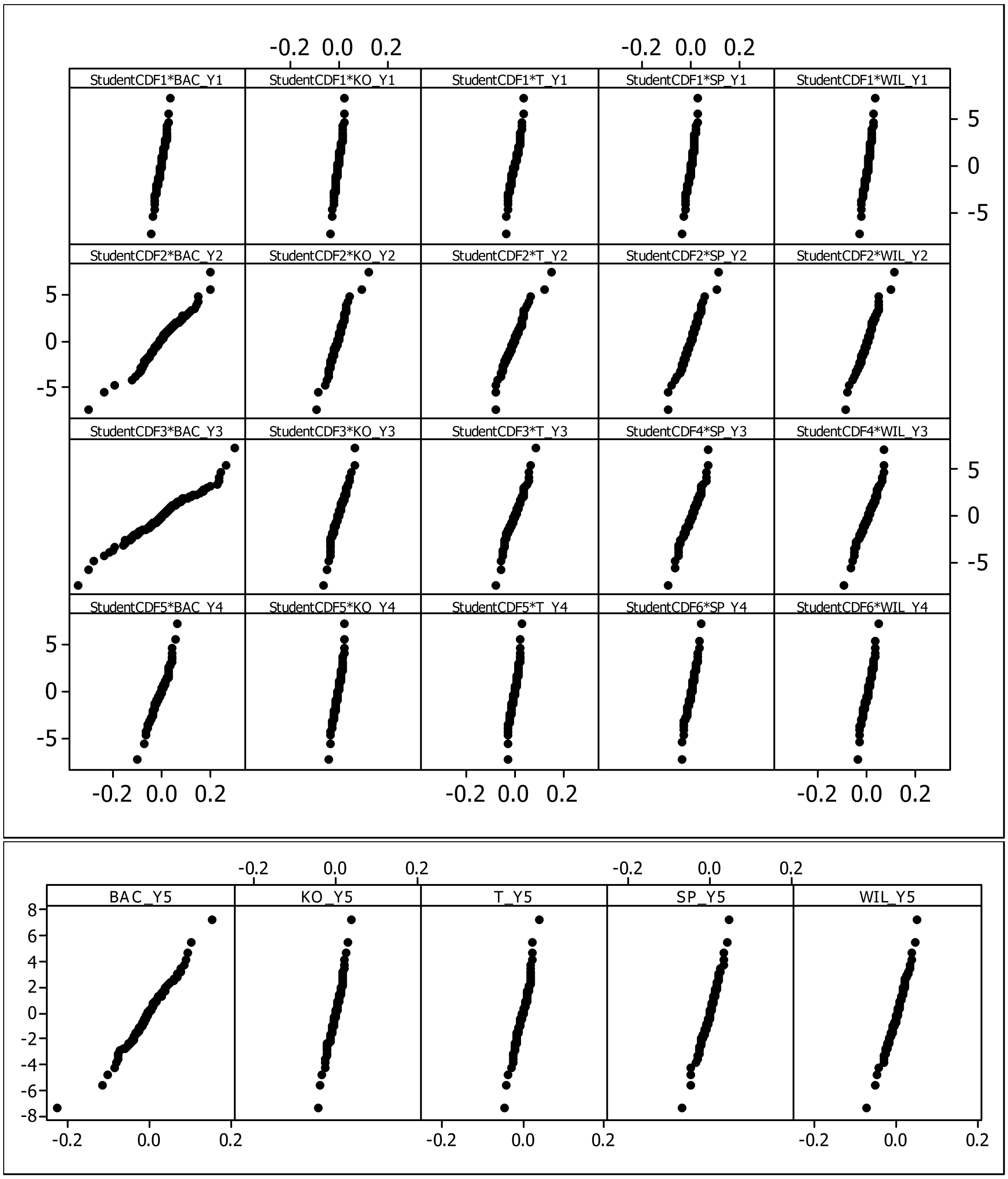


Figure C.11: Cauchy probability plot over 1-year intervals

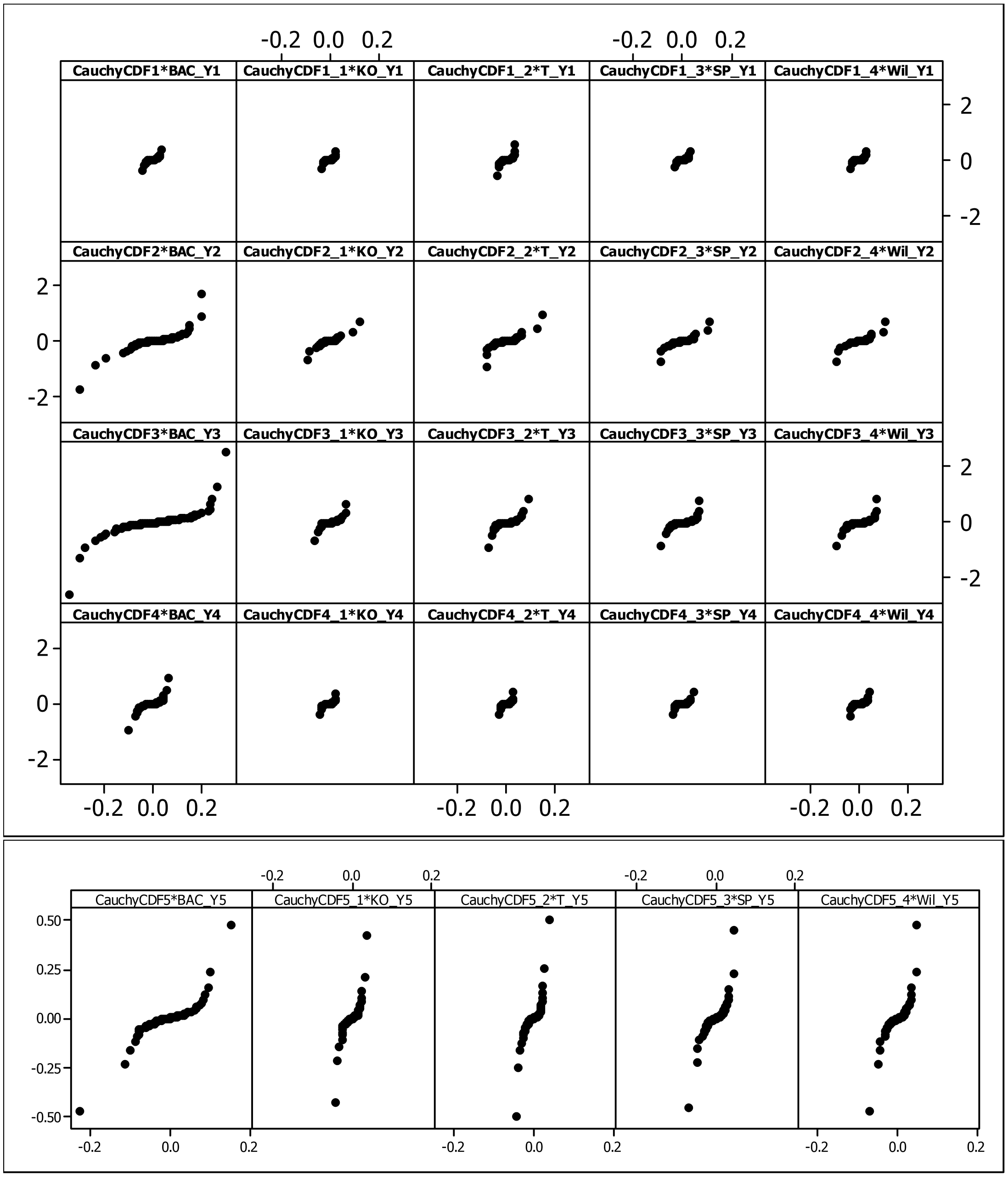


Figure C.12: Weibull probability plots over 1-year intervals with a 95% confidence interval

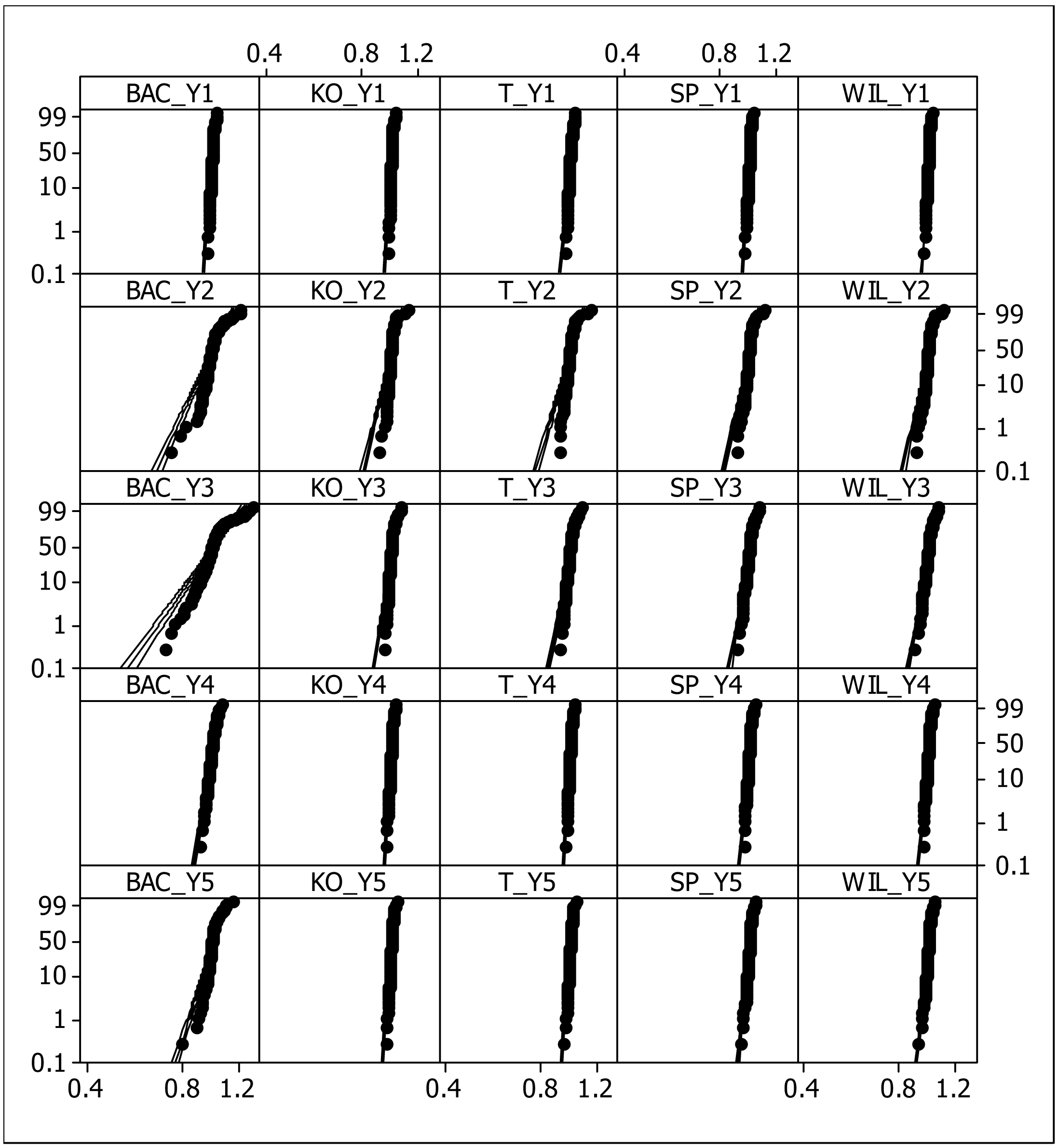
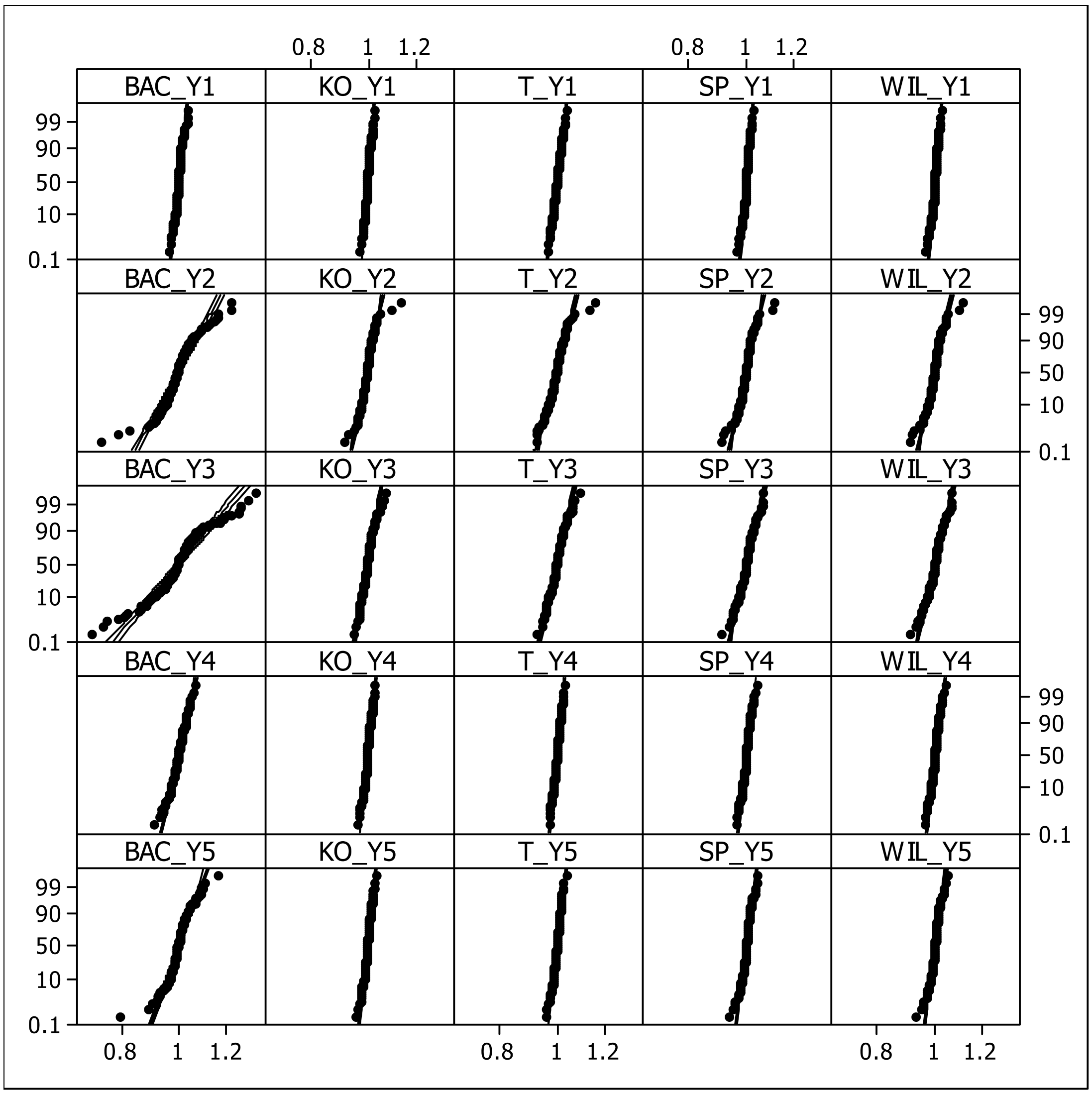


Figure C.13: 3-Parameter Lognormal probability plots over 1-year intervals with a 95% confidence interval



C.4 6-Month Intervals

Figure C.14: Normal probability plots over 6-month intervals with a 95% confidence interval

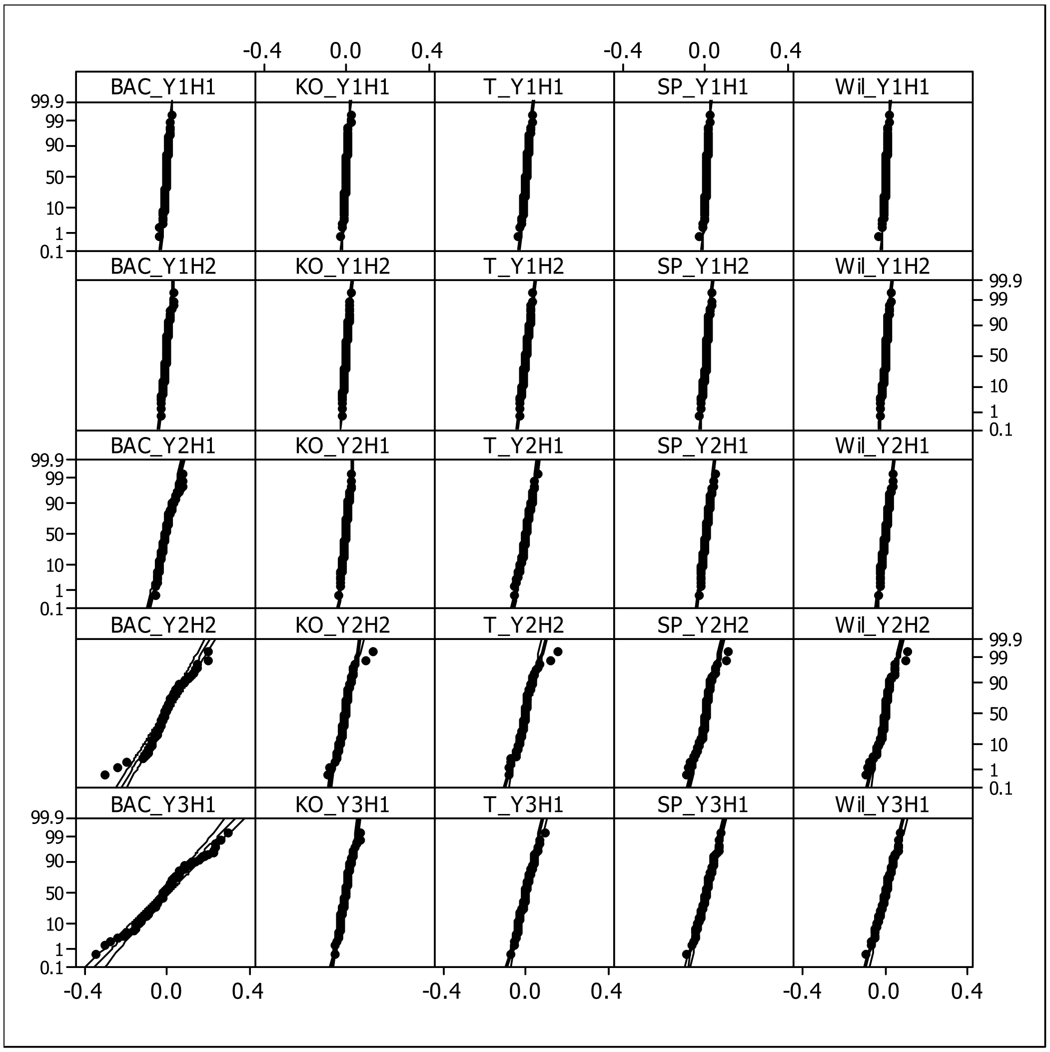


Figure C.15: Normal probability plots over 6-month intervals with a 95% confidence interval (Cont'd)

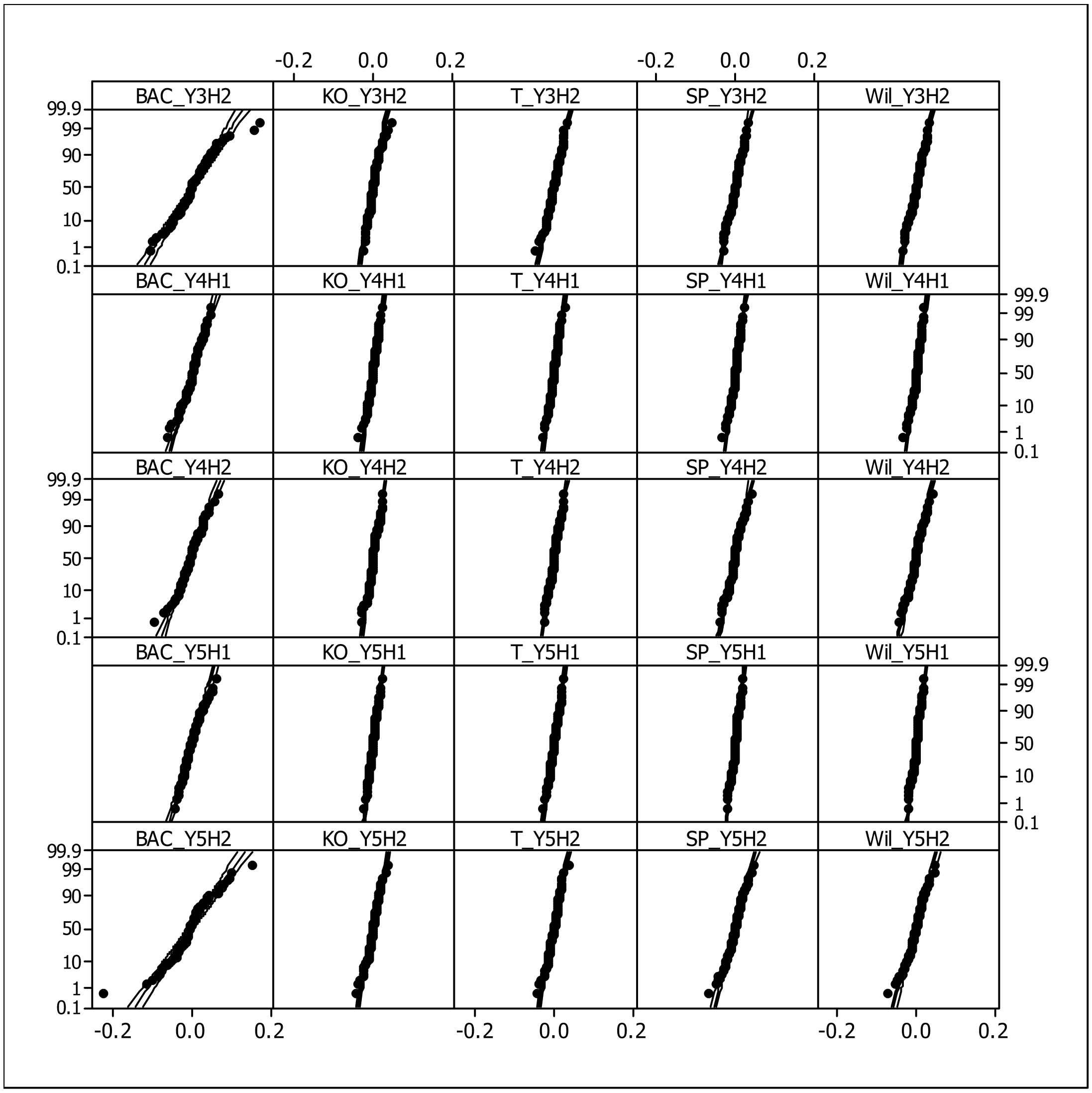
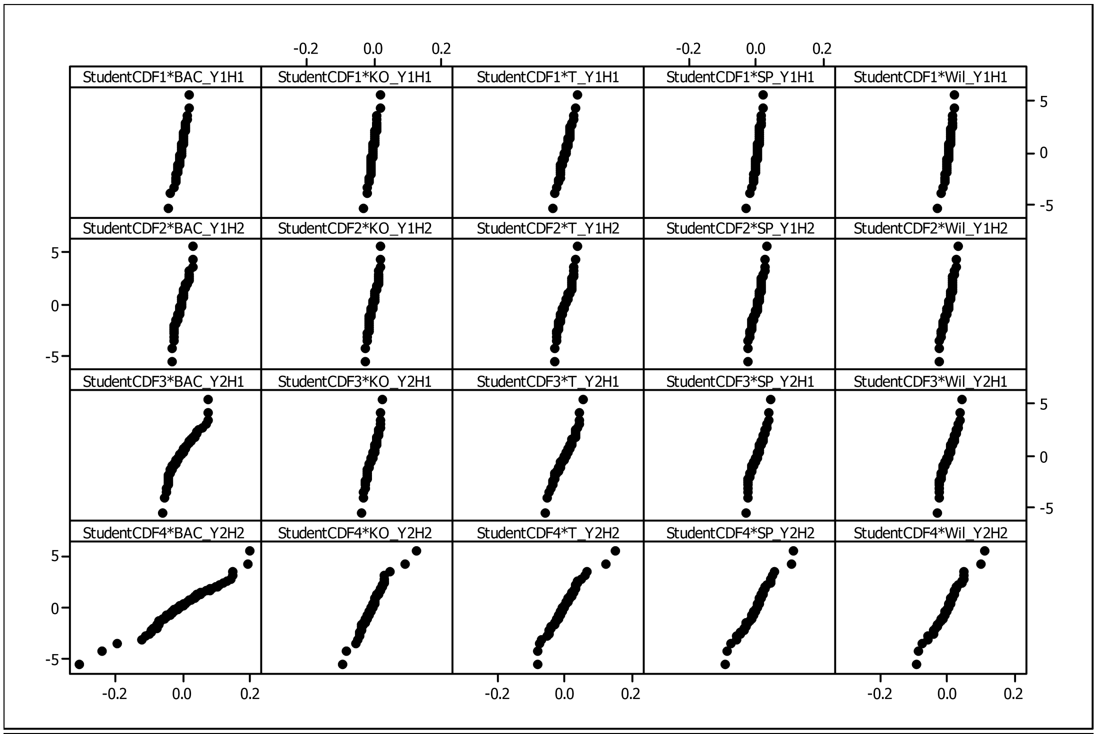


Figure C.16: Student-t probability plots over 6-month intervals with a 95% confidence interval



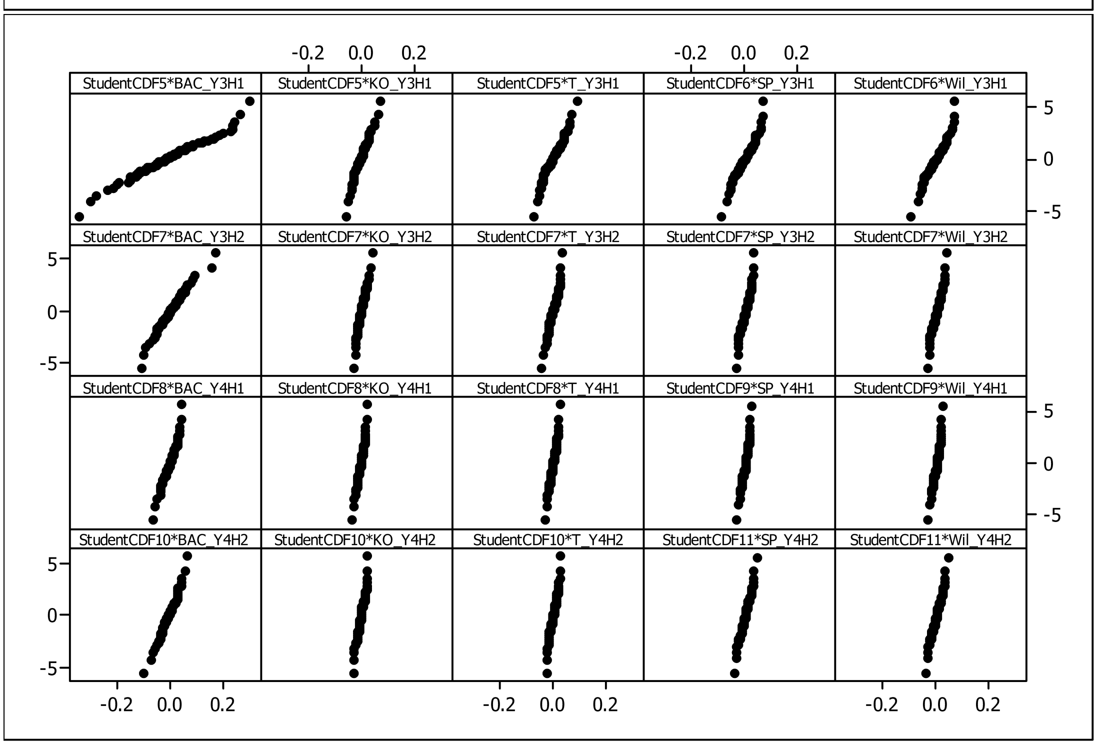


Figure C.17: Student-t probability plots over 6-month intervals

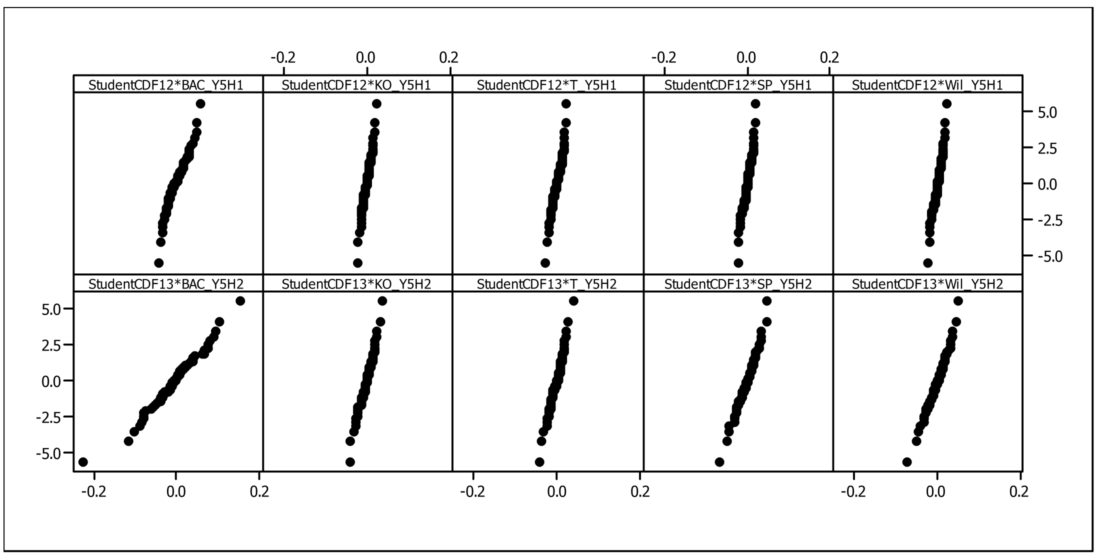


Figure C.18: Cauchy probability plots over 6-month intervals

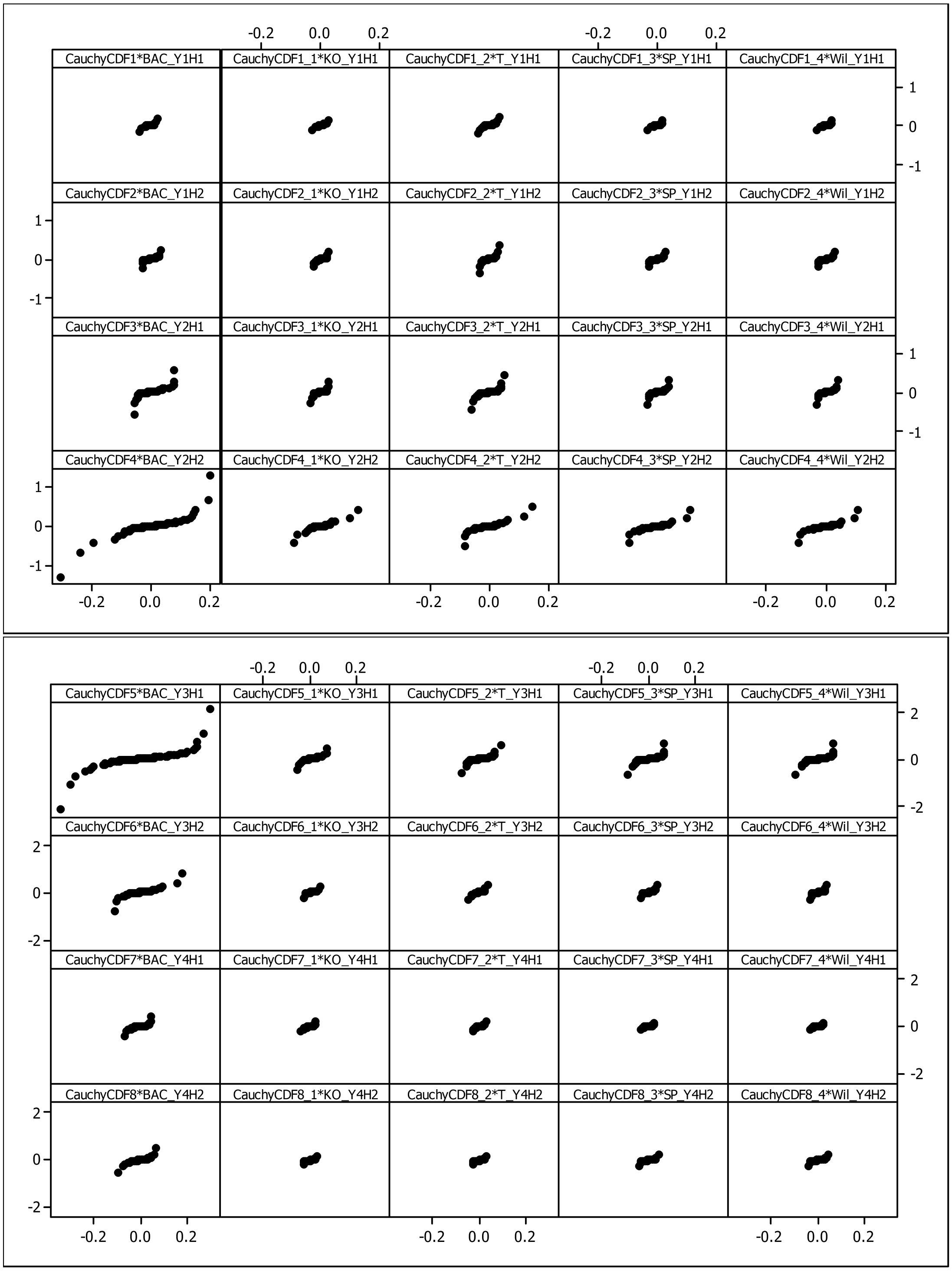


Figure C.19: Cauchy probability plots over 6-month intervals (Cont'd)

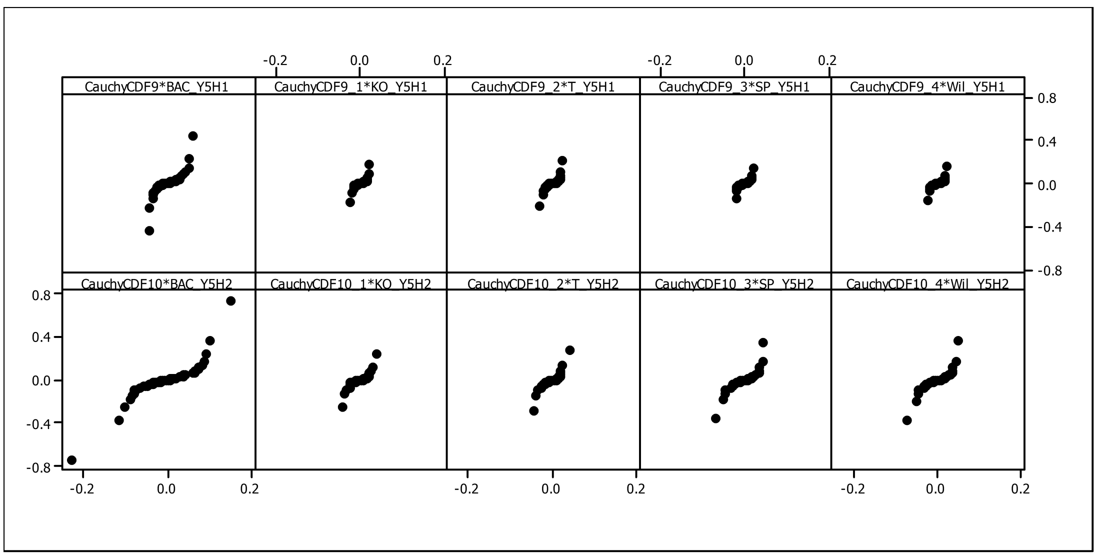


Figure C.20: Weibull probability plots over 6-month intervals with a 95% confidence interval

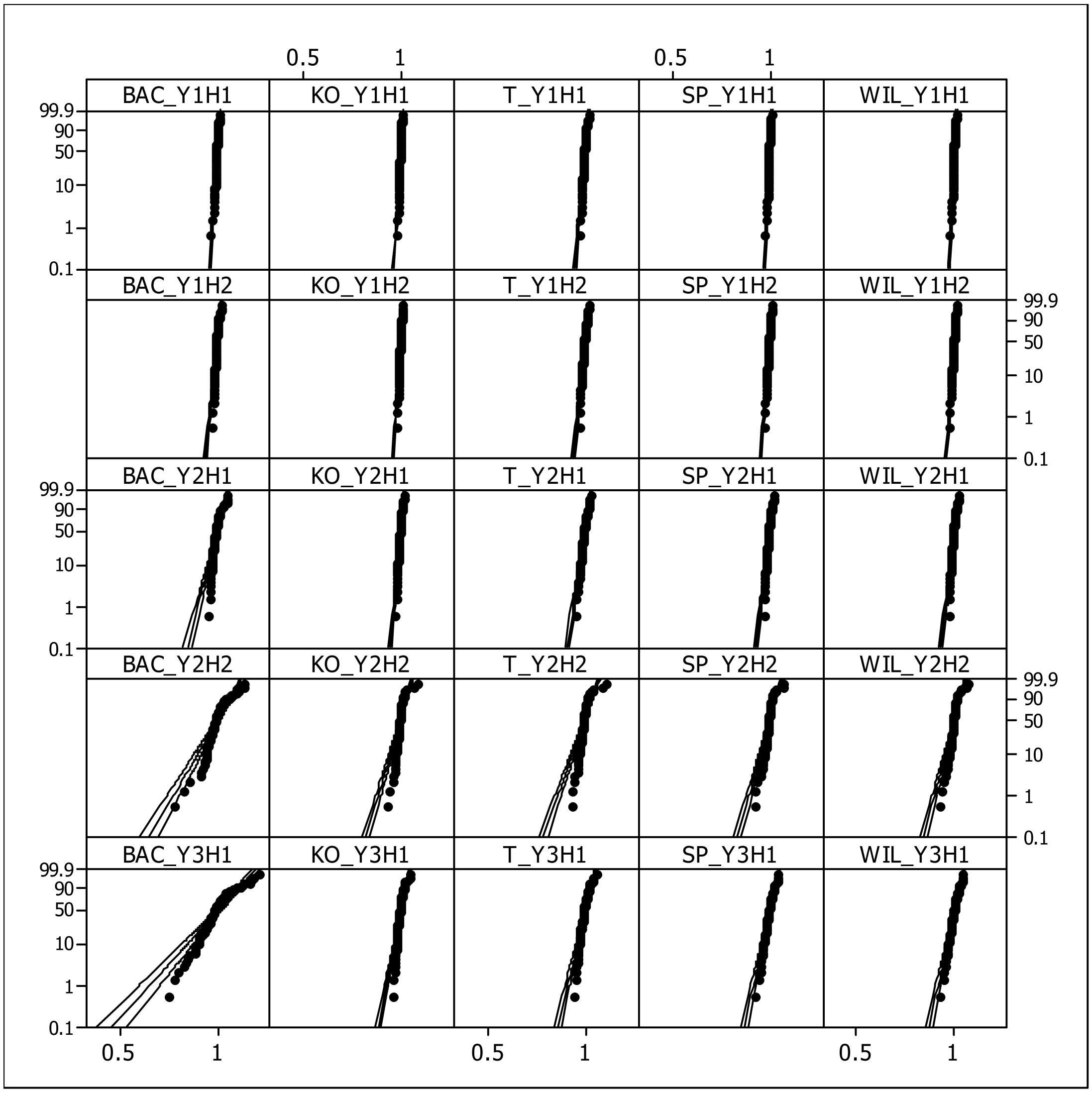


Figure C.21: Weibull probability plots over 6-month intervals with a 95% confidence interval (Cont'd)

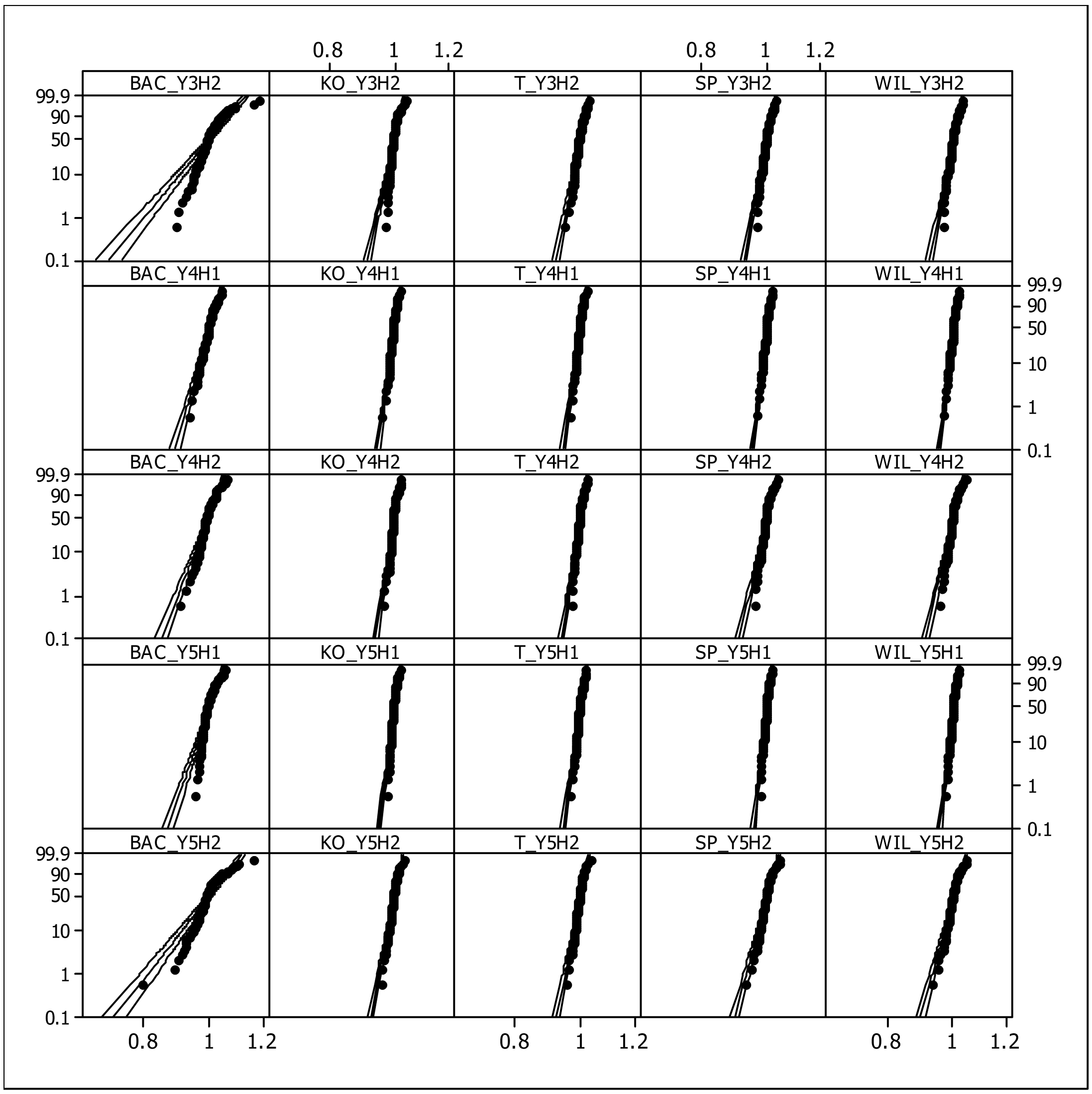


Figure C.22: 3-parameter lognormal probability plots over 6-month intervals with a 95% confidence interval

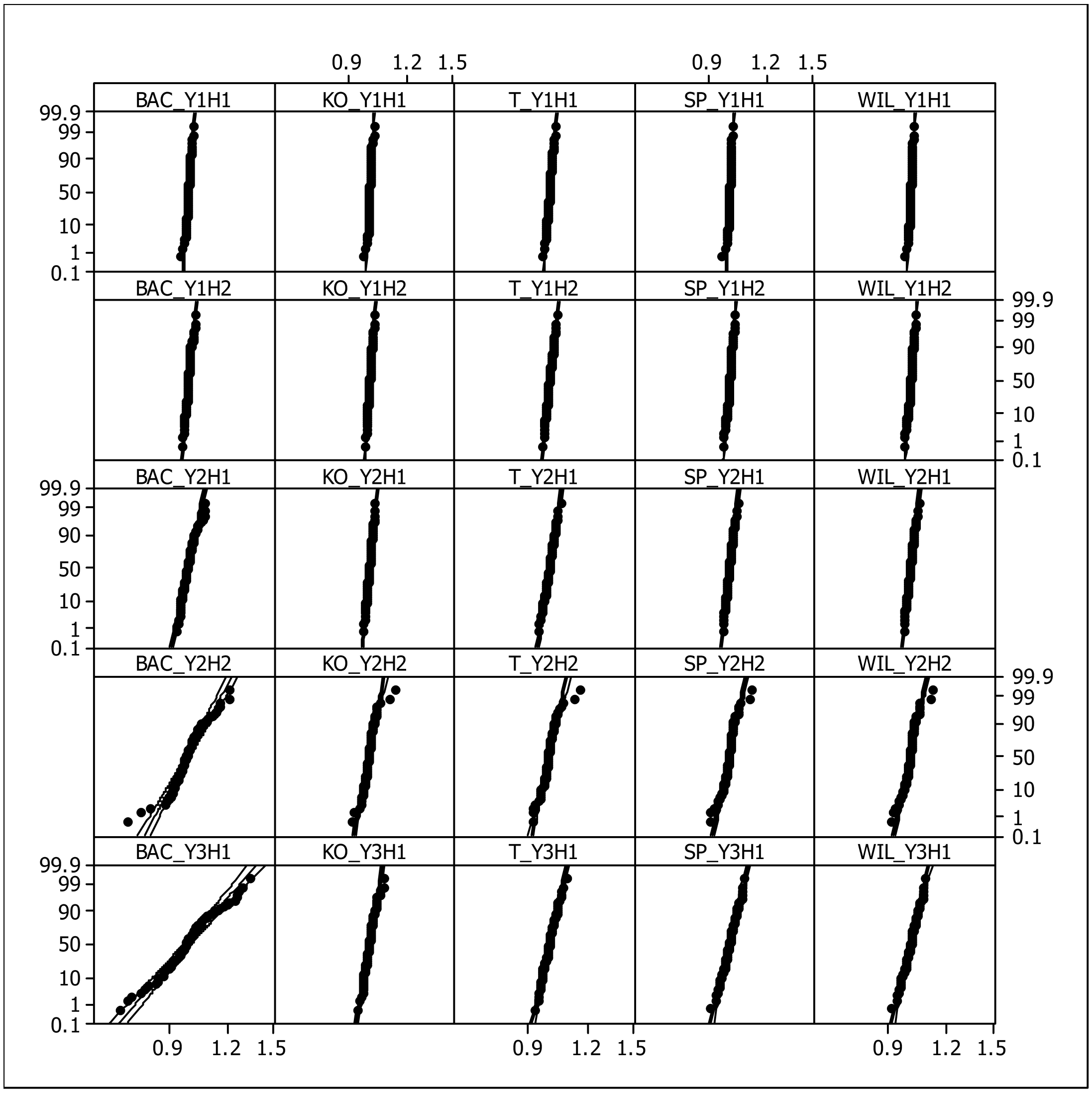
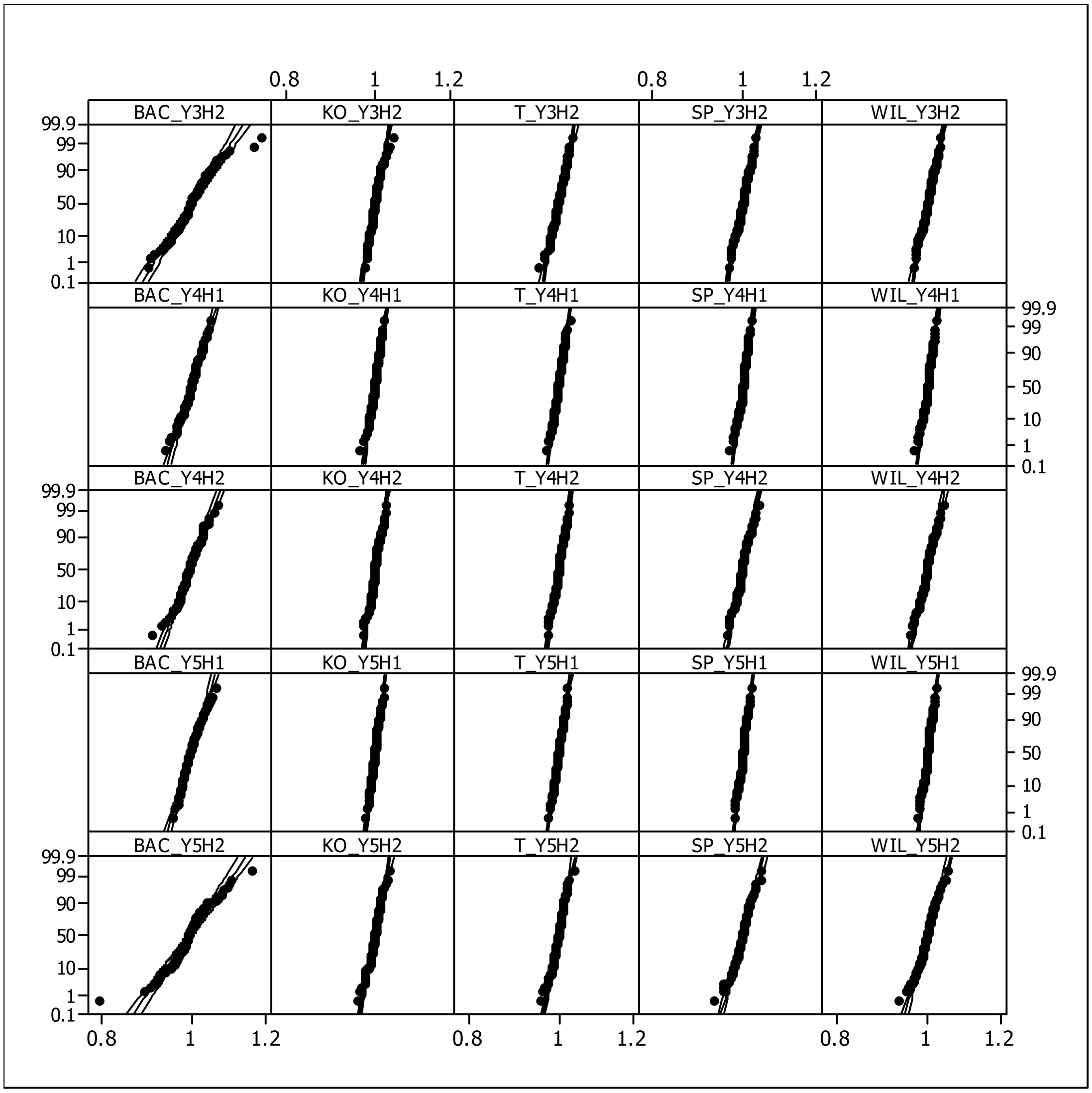


Figure C.23: 3-parameter lognormal probability plots over 6-month intervals with a 95% confidence interval (Cont'd)



C.5 3-Month Intervals

Figure C.24: Normal probability plots over 3-month intervals with a 95% confidence interval

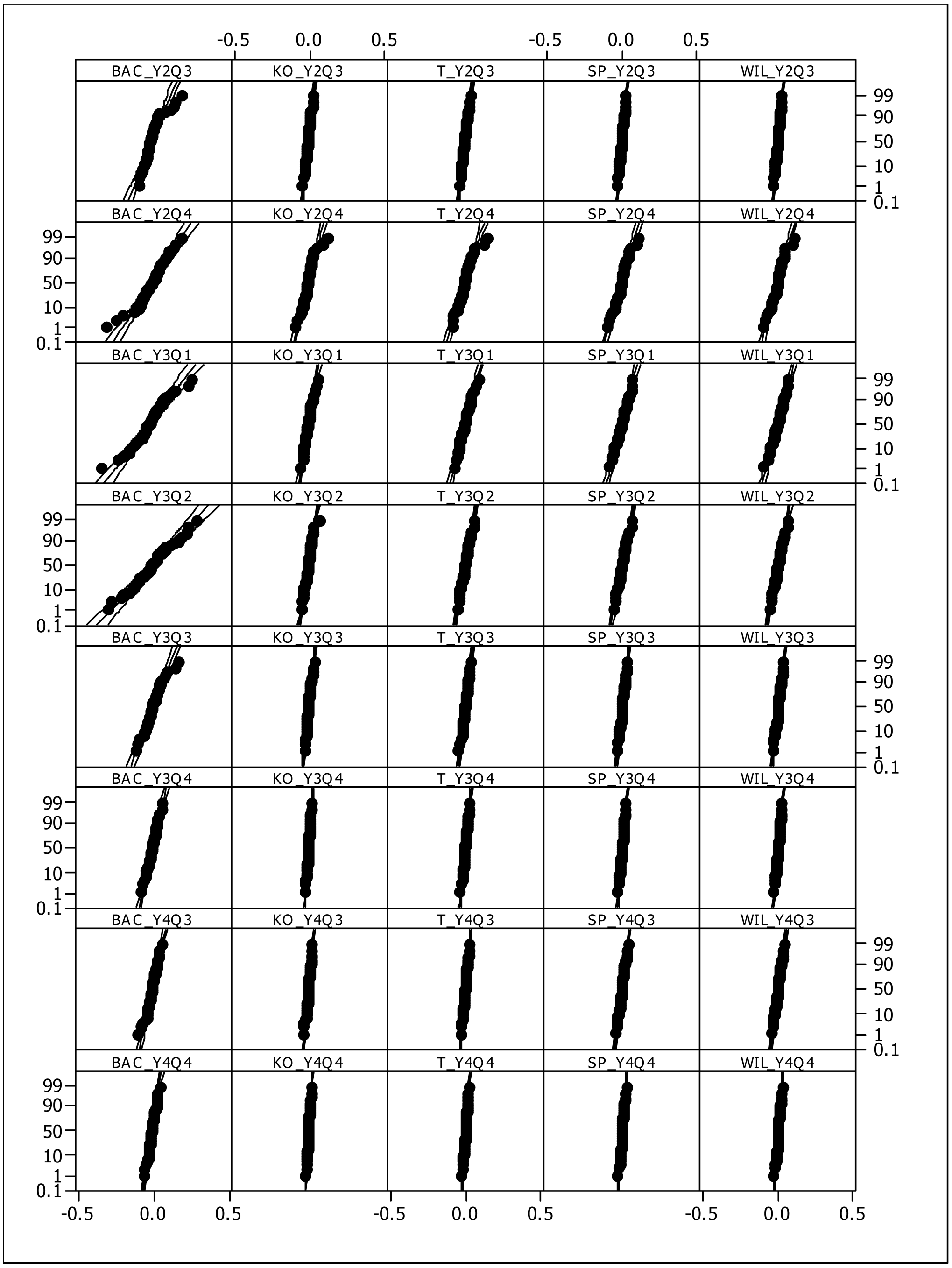


Figure C.25: Student-t probability plots over 3-month intervals

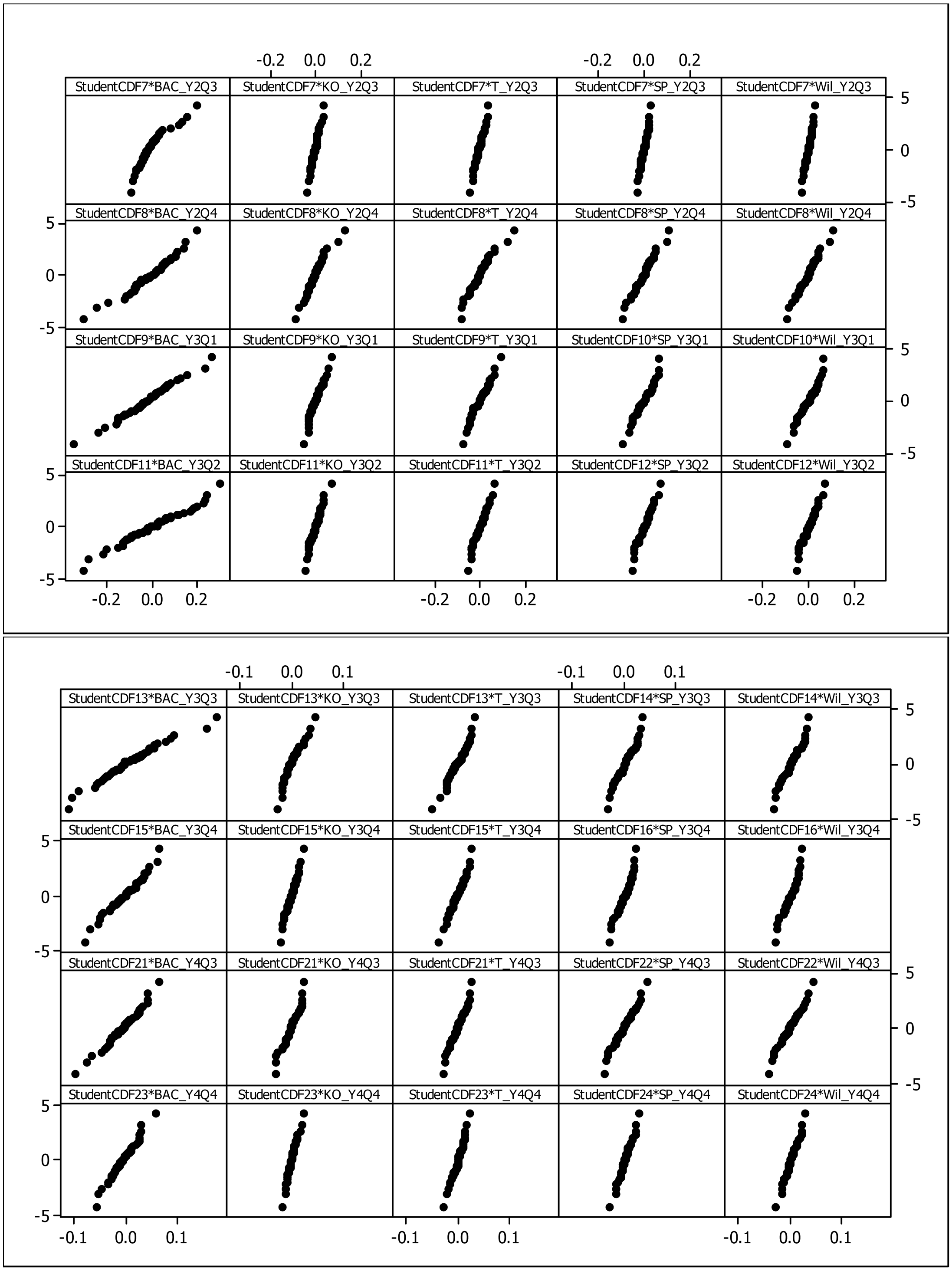


Figure C.26: Cauchy probability plots over 3-month intervals

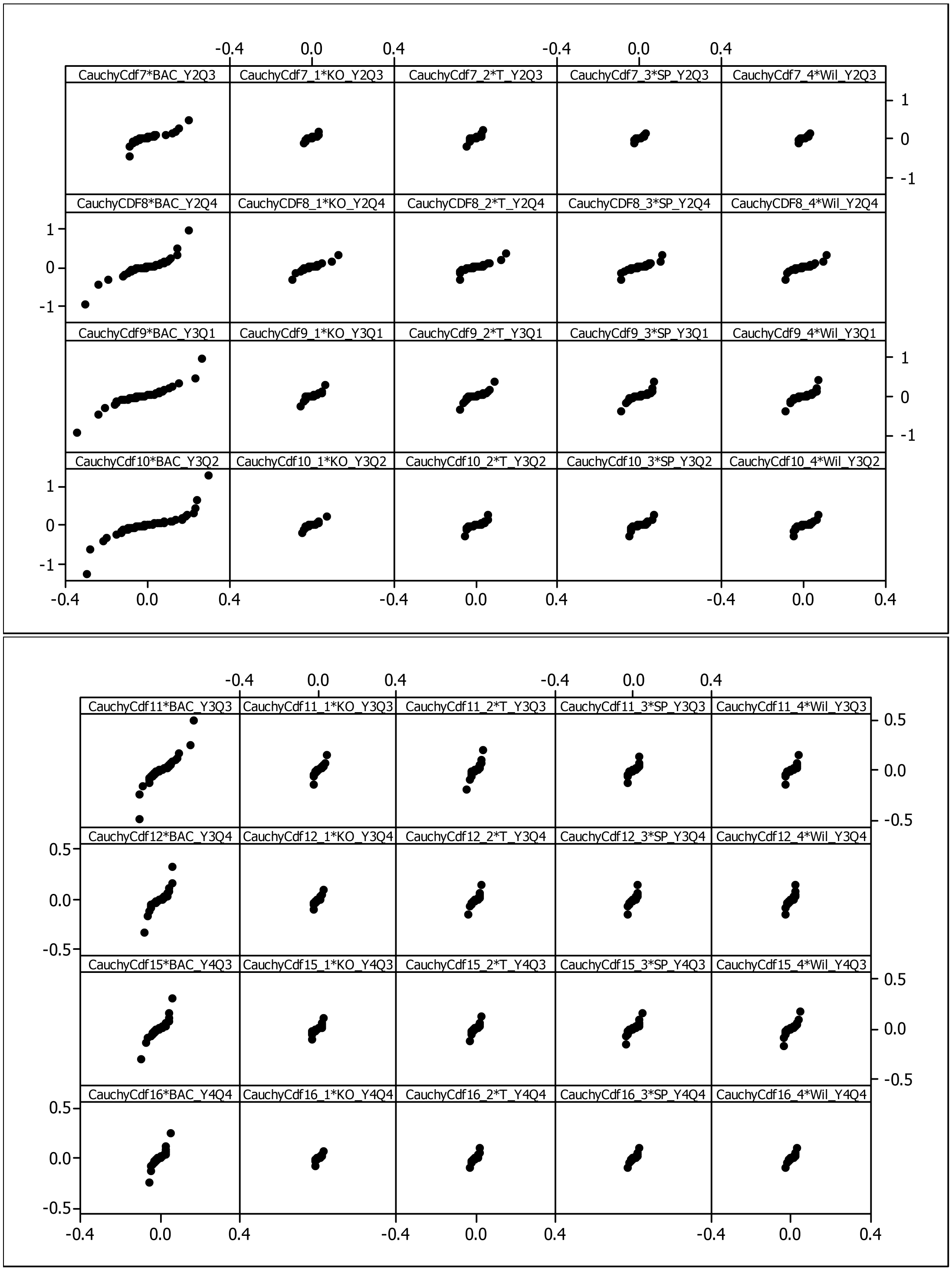


Figure C.27: Weibull probability plots over 3-month intervals with a 95% confidence interval

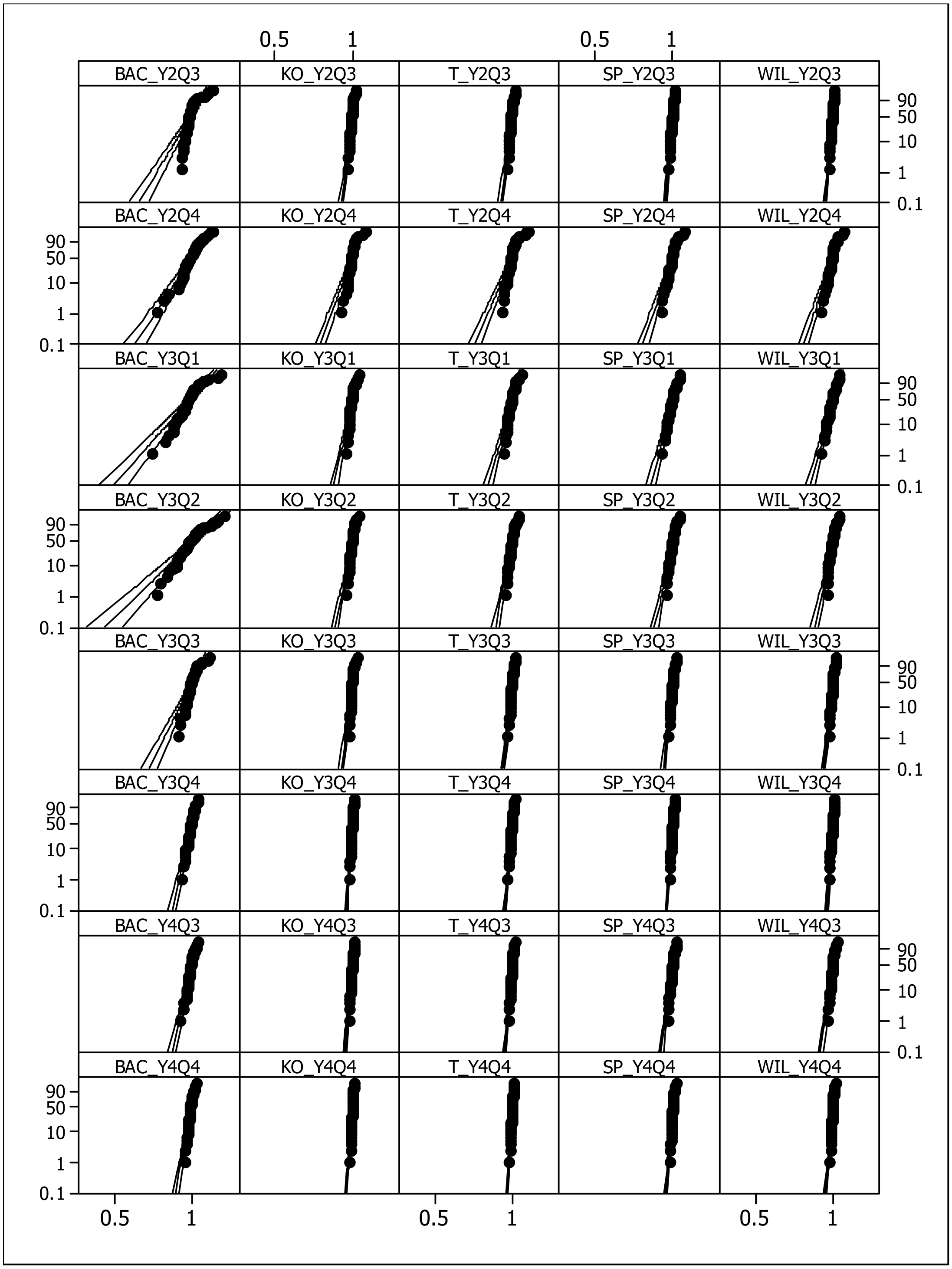
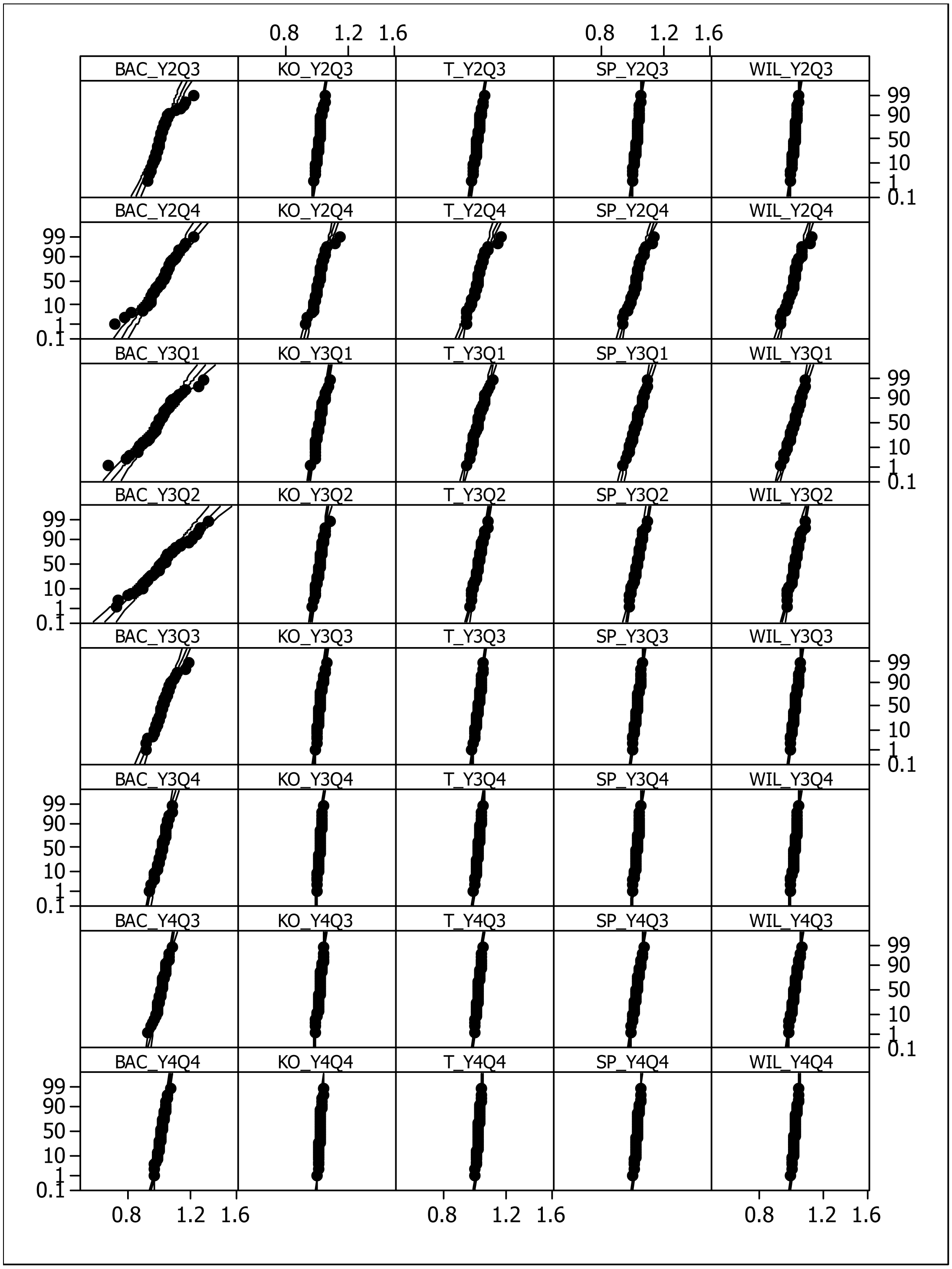


Figure C.28: 3-parameter lognormal probability plots over 3-month intervals with a 95% confidence interval



C.6 1-Month Intervals

Figure C.29: Normal probability plots over 1-month intervals with a 95% confidence interval

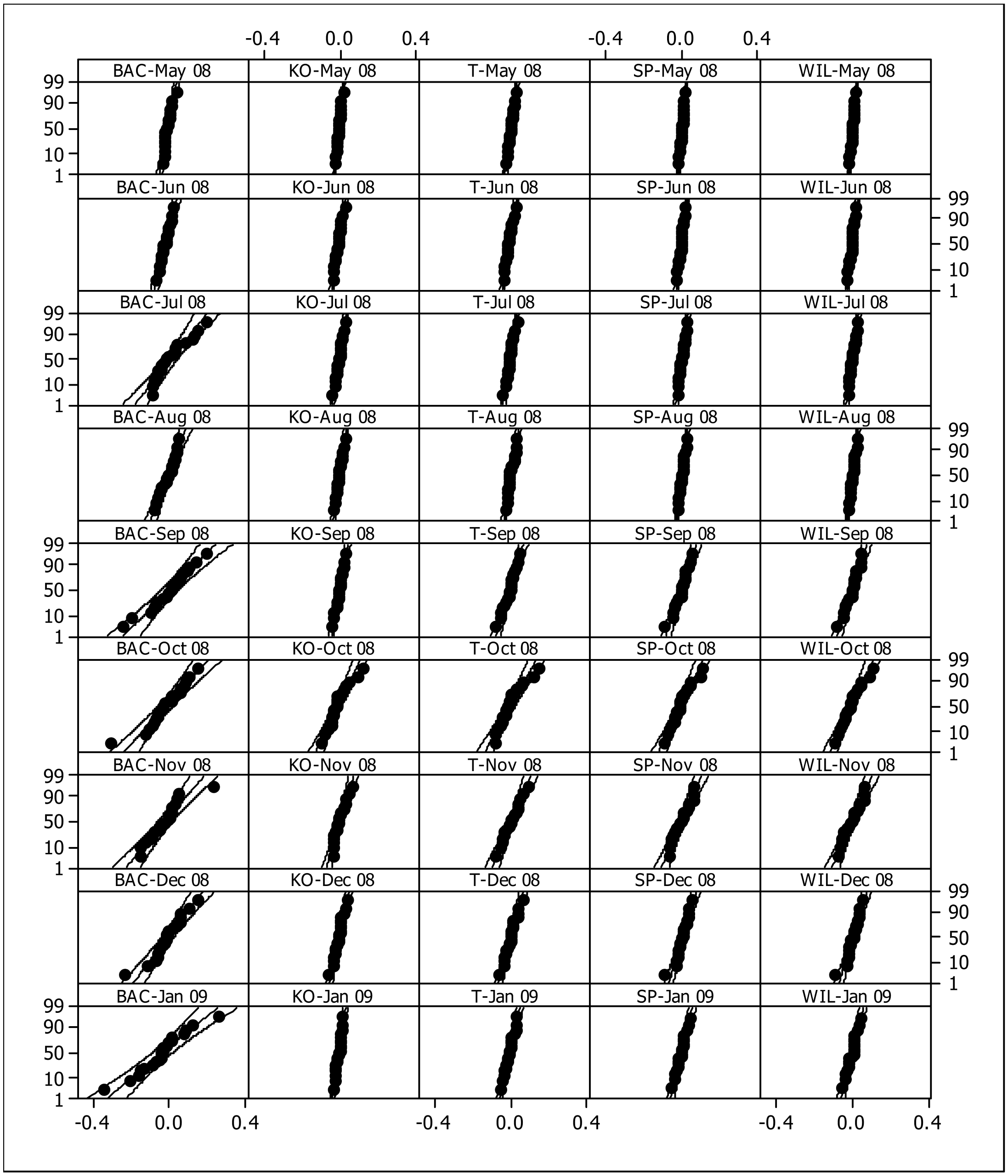


Figure C.30: Normal probability plots over 1-month intervals with a 95% confidence interval (cont'd)

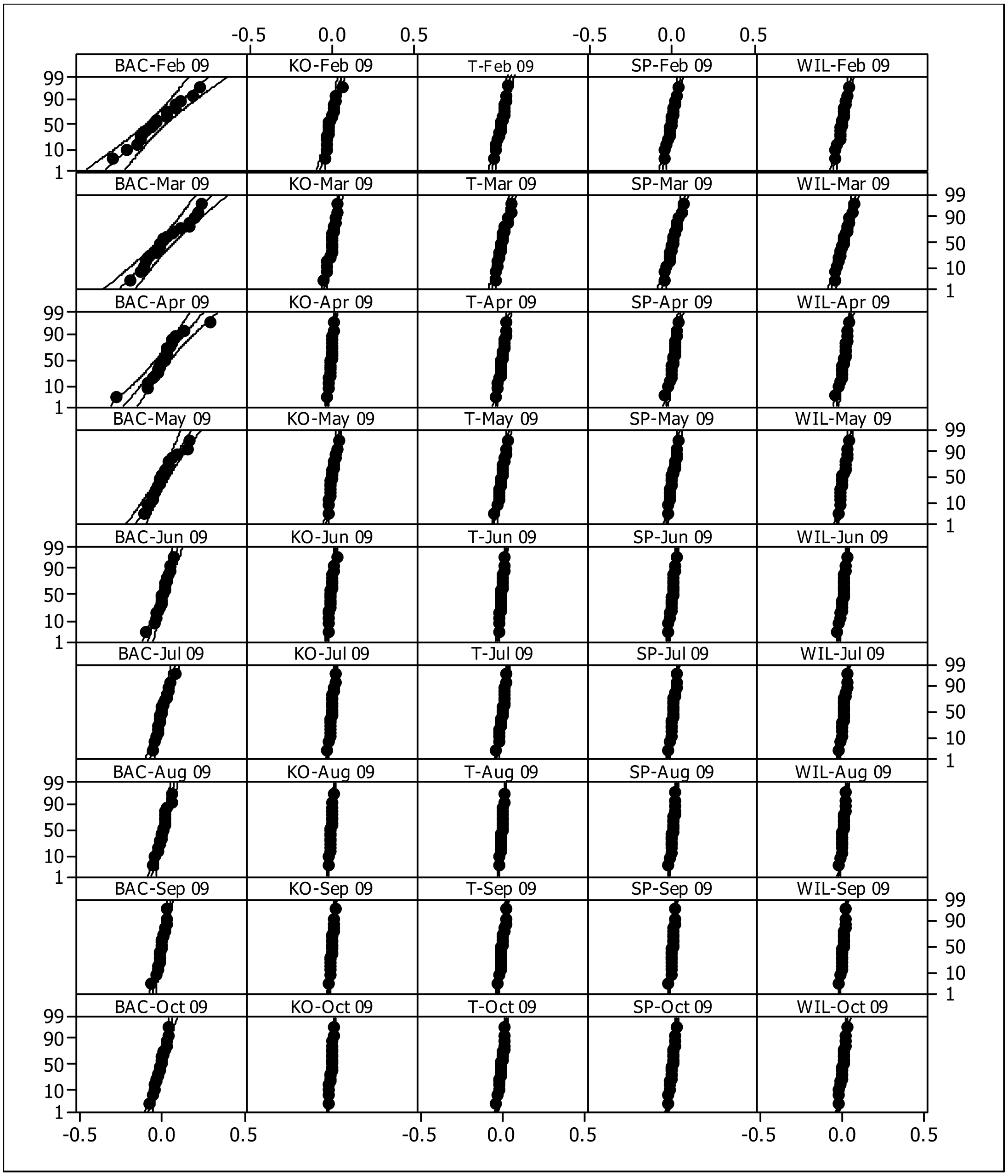


Figure C.31: Student-t probability plots over 1-month intervals

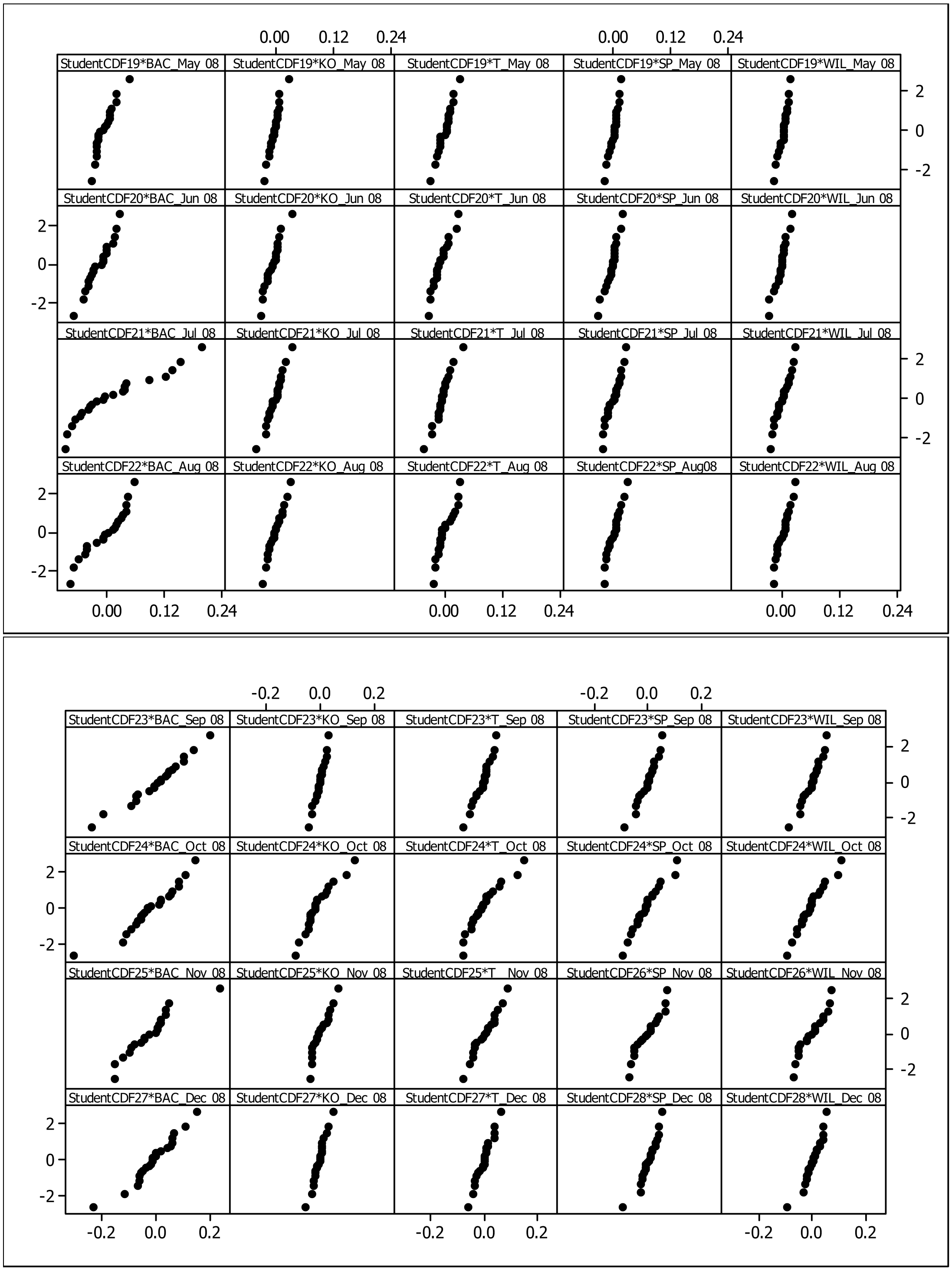


Figure C.32: Student-t probability plots over 1-month intervals (cont'd)

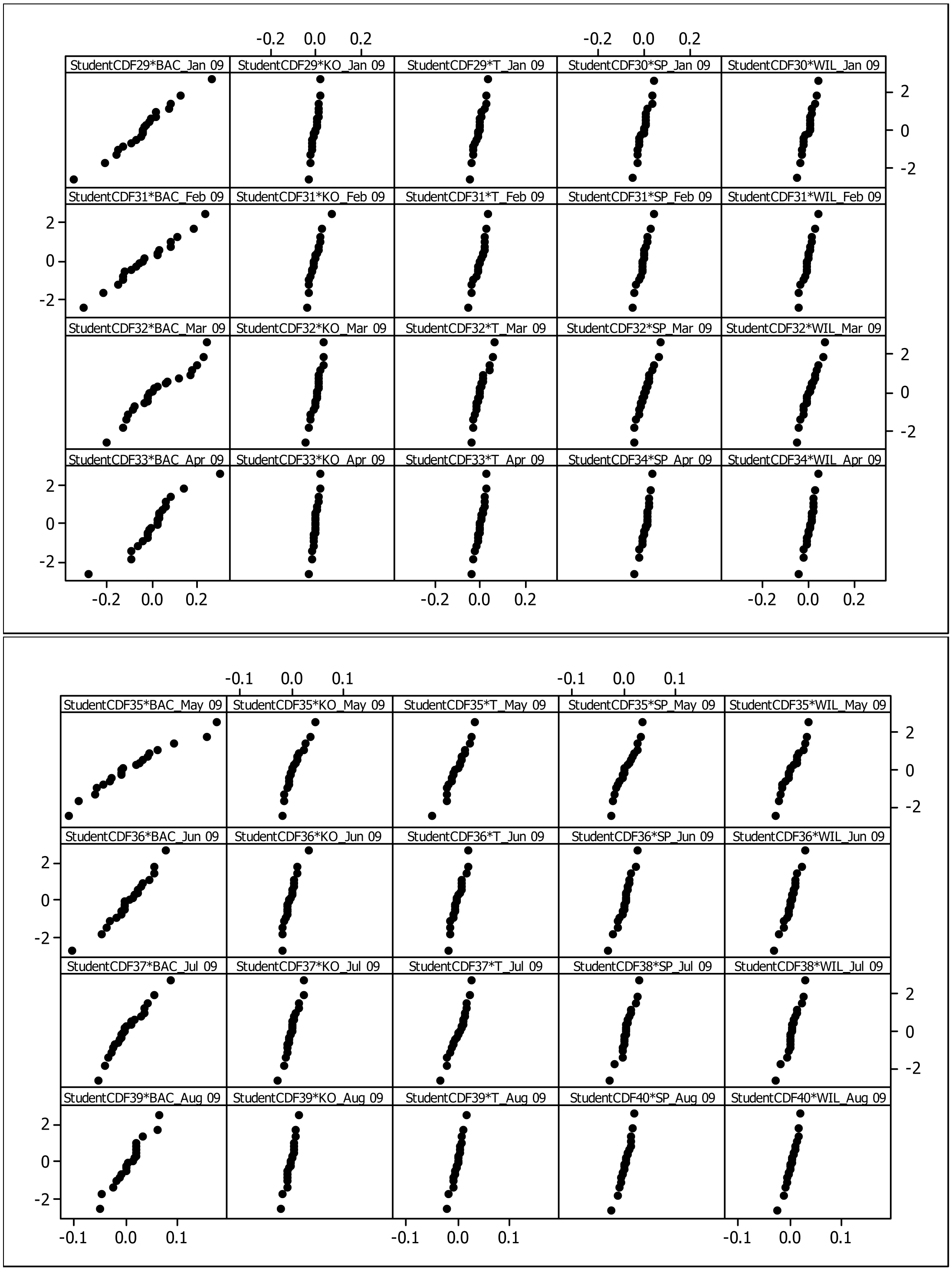


Figure C.33: Student-t probability plots over 1-month intervals (cont'd)

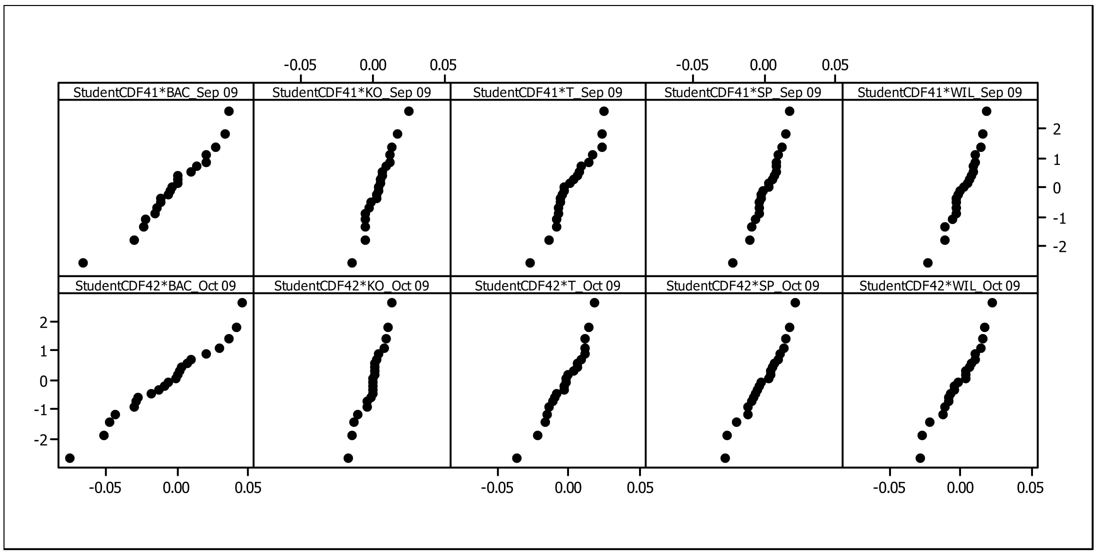


Figure C.34: Cauchy probability plots over 1-month intervals

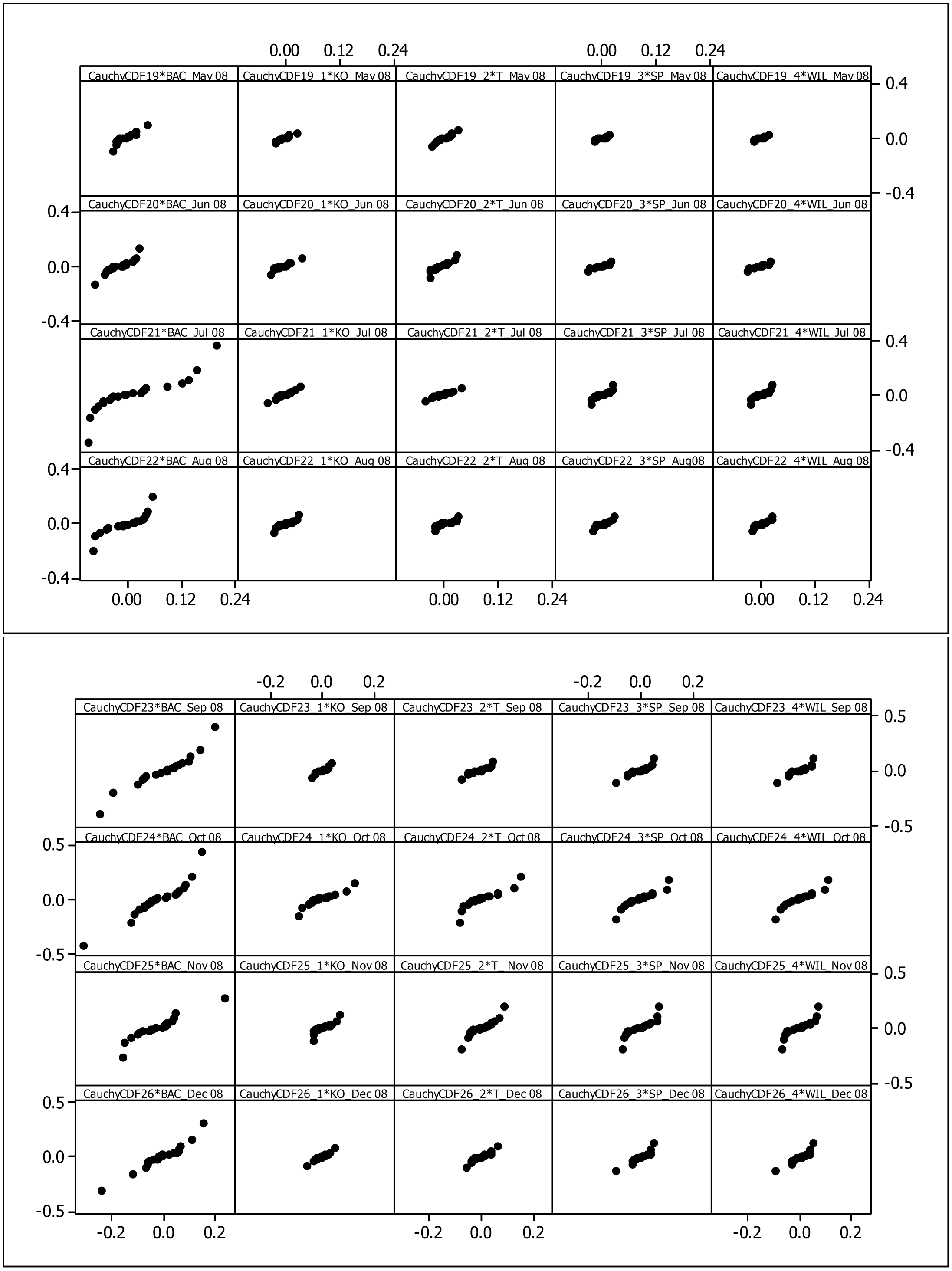
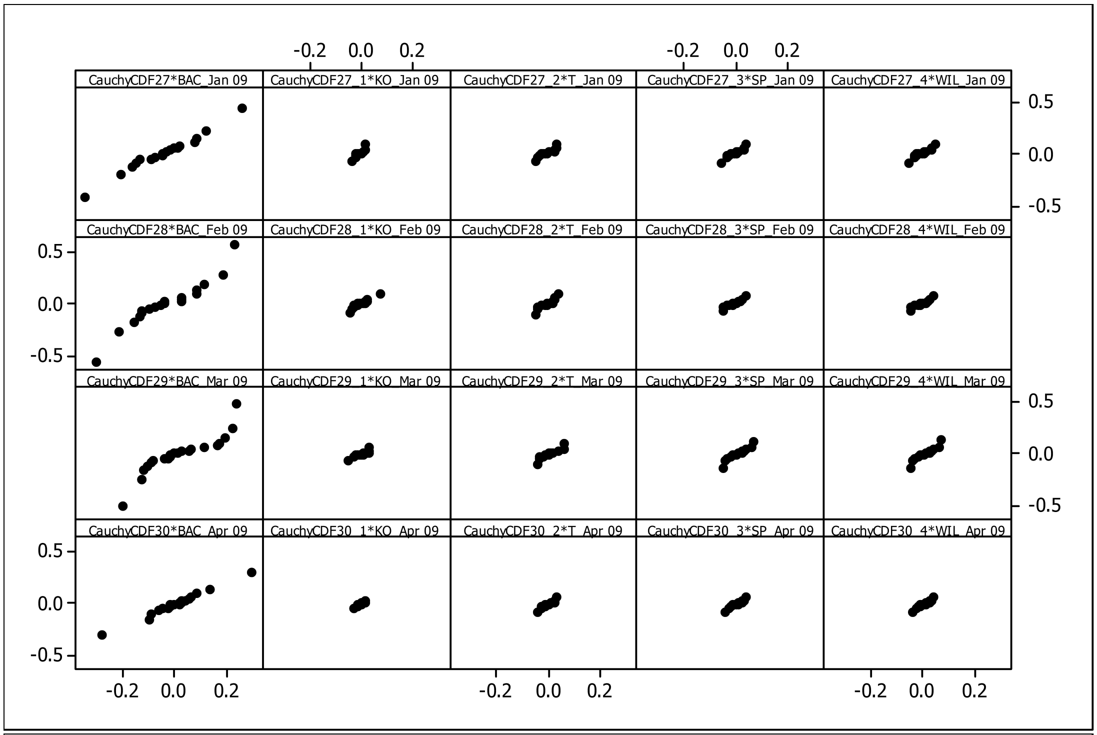


Figure C.35: Cauchy probability plots over 1-month intervals (cont'd)



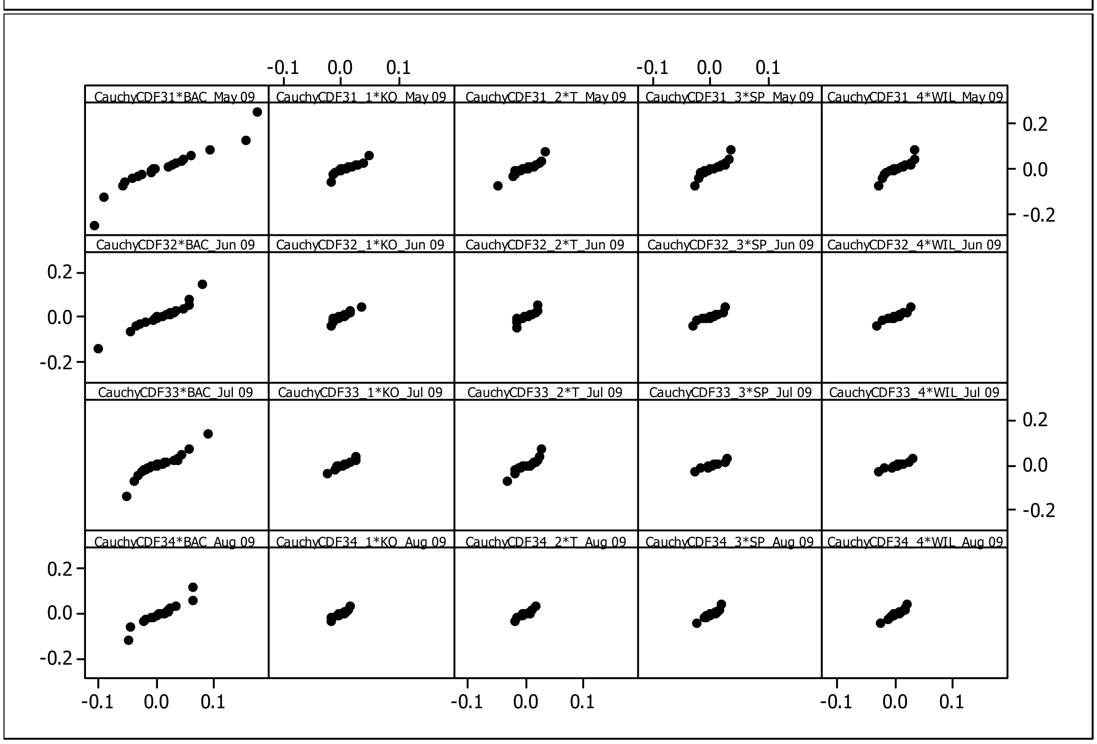


Figure C.36: Cauchy probability plots over 1-month intervals (cont'd)

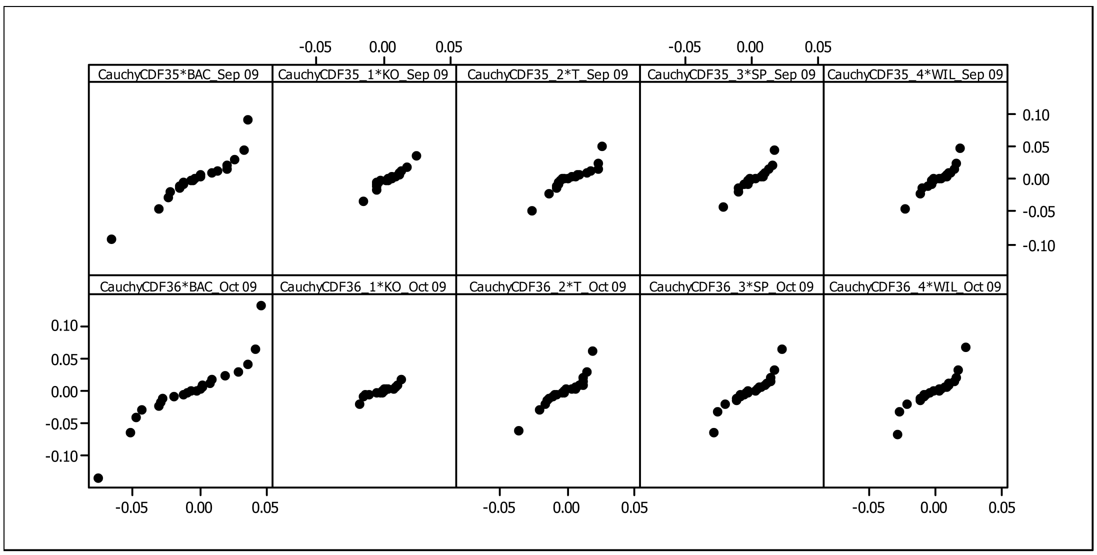


Figure C.37: Weibull probability plots over 1-month intervals with a 95% confidence interval

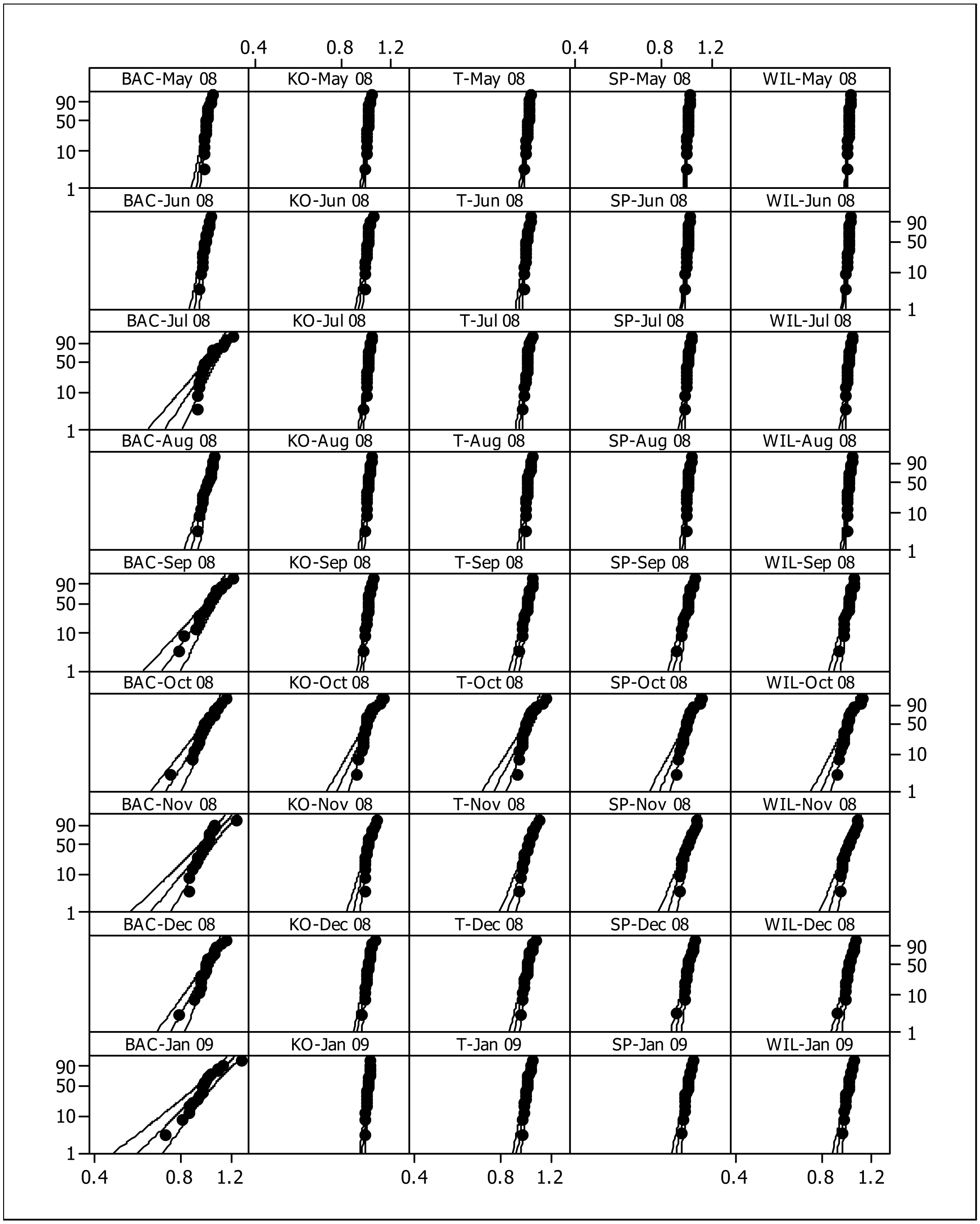


Figure C.38: Weibull probability plots over 1-month intervals with a 95% confidence interval (cont'd)

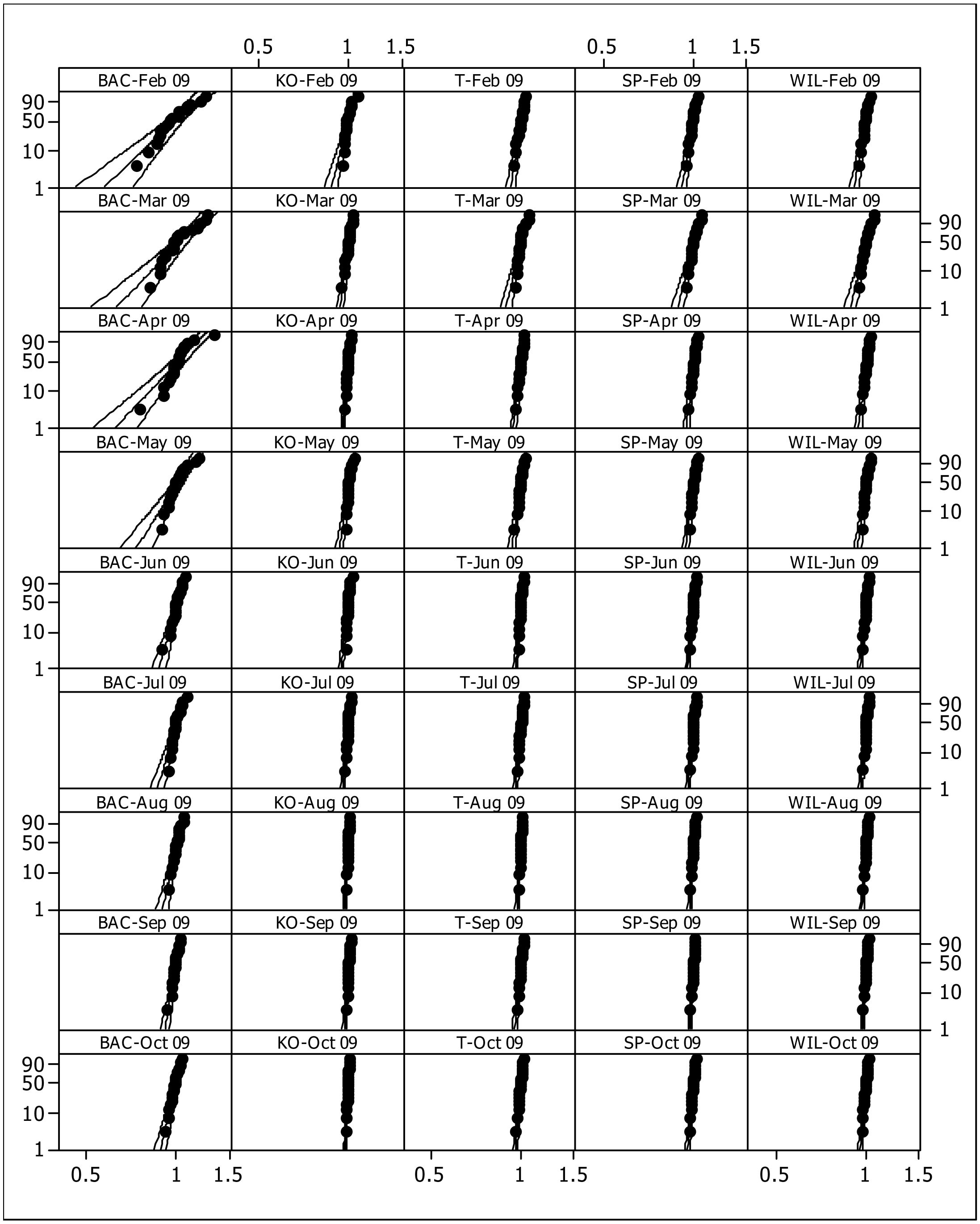


Figure C.39: 3-parameter lognormal probability plots over 1-month intervals with a 95% confidence interval

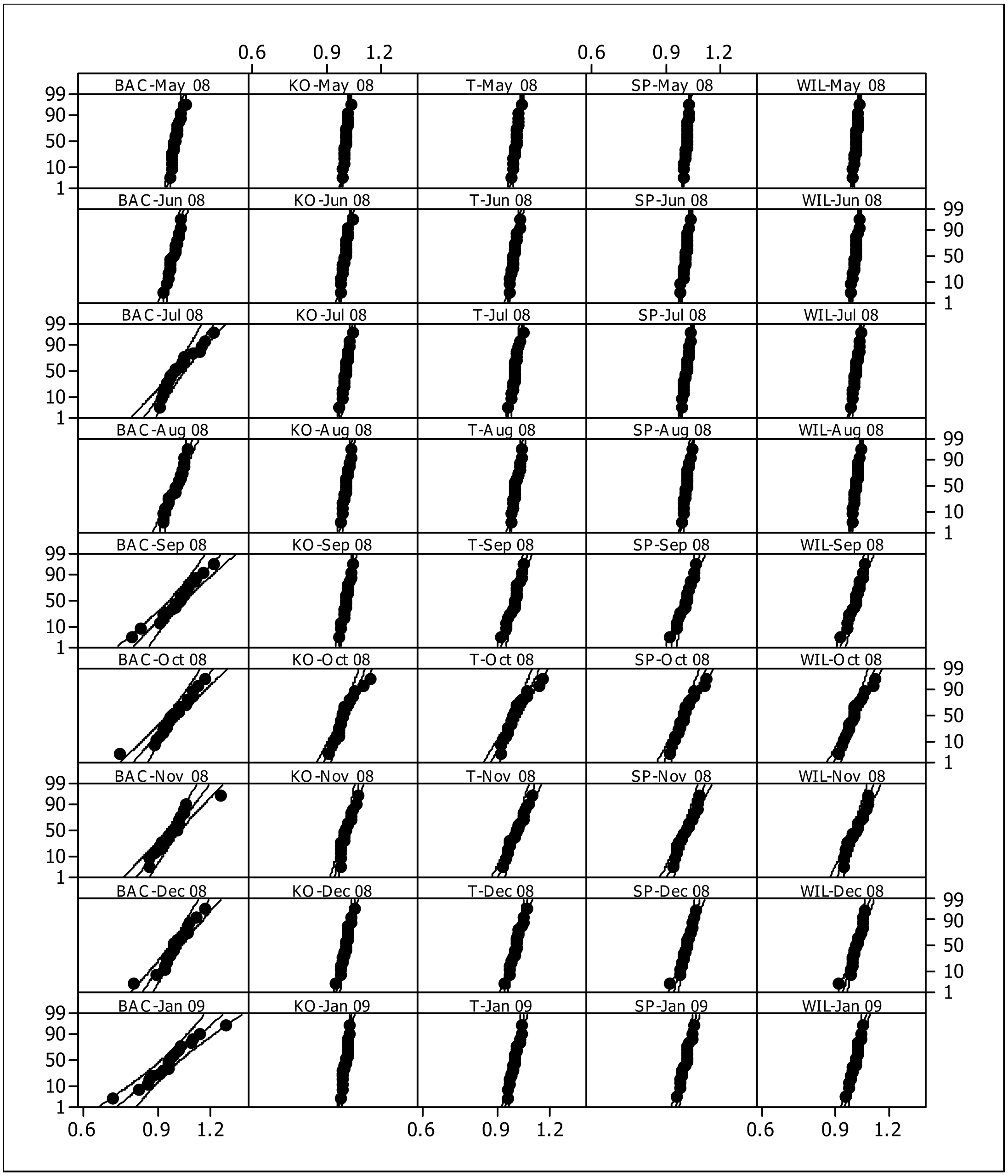
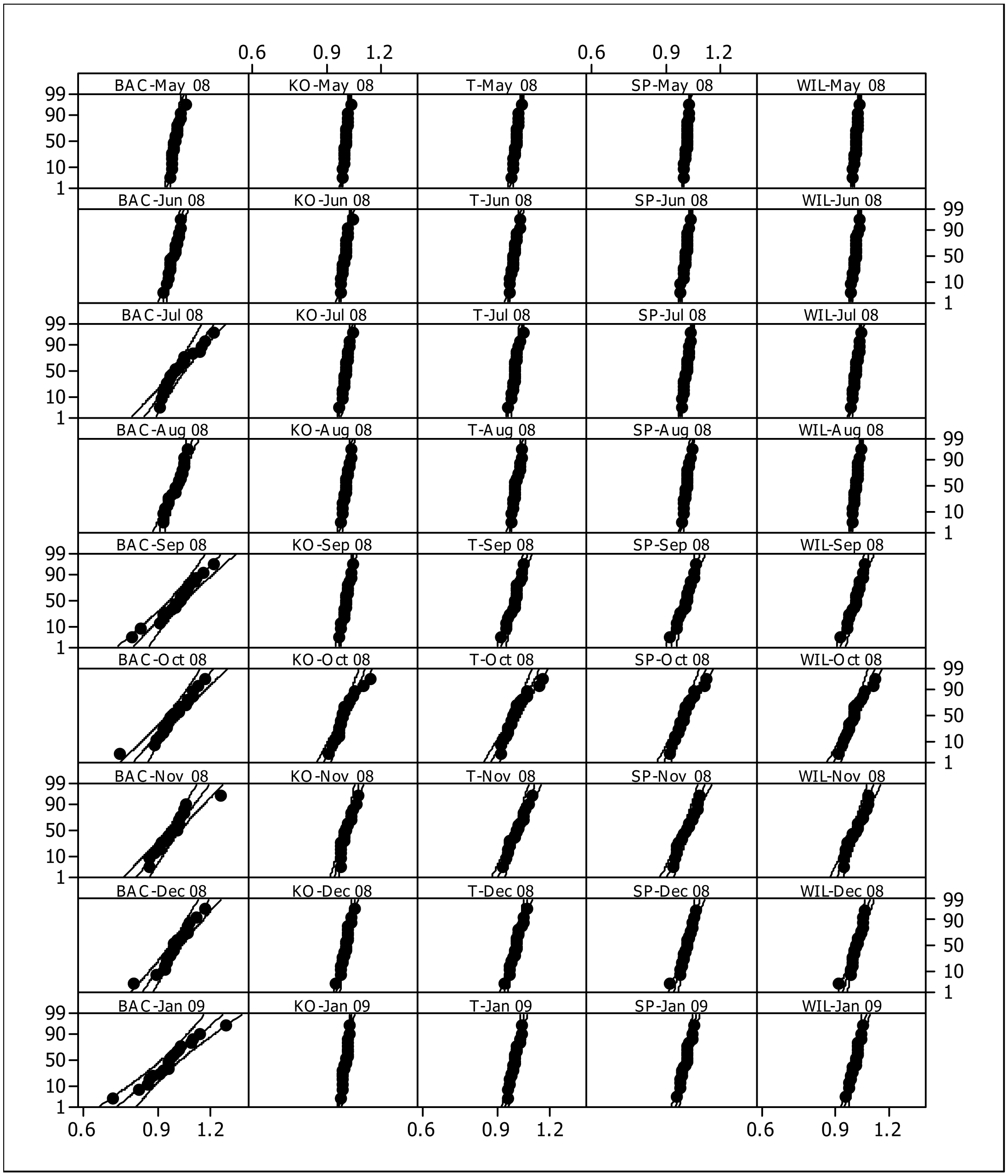


Figure C.40: 3-parameter lognormal probability plots over 1-month intervals with a 95% confidence interval (cont'd)



Appendix D

MATLAB CODE

```
% Run this code for normal distribution
days = 30;
S0 = 1360;              %Inital Stock Price
K = 1330;                %Strike Price
r = .02;             %Treasury bond rate correspondging to option life
sig = 0.05036;            %Standard Deviation
m = days+1;
n = 1000;              %Number of simulated paths
T = days/252;        %Time to maturity (in days)
t = 1/252;
S = zeros(m,n);
C = zeros(m,n);
q = normrnd(0,1,m,n);
for j = 1:n
   S(1,j)=S0;
    for i = 2:m
       S(i,j) = S0*exp((r-0.5*sig^2)*(T+(i-1)*t) + sig*sqrt(T+(i-1)*t)*q(i,j));
    end
    for i = 1:m
        C(i,j) = exp(-r*(T+(i-1)*t))*max(S(i,j)-K,0);
    end
end
Cmean = mean(C(m,:))                          % Payoff Mean
width = 1.96*std(C(m,:))/sqrt(m);
Conf = [Cmean - width, Cmean + width]         % Confidence Interval
Cstd = std(C(m,:))
x = [1:m];
subplot(2,1,1);
plot(x-1,S)
xlabel('Time (days)')
ylabel('Stock Price')
subplot(2,1,2);
plot(x-1,C)
xlabel('Time (days)')
ylabel('Call Price')
```

```
% Run this code for Student-t distribution
days = 30;
S0 = 1360;           % Inital Stock Price
K = 1390;             % Strike Price
r = .02;           % Treasury bond rate correspondging to option life
sig = 0.05036;          % Standard Deviation
m = days+1;
n = 1000;           % Number of simulated paths
T = days/252;      % Time to maturity (in days)
t = 1/252;
S = zeros(m,n);
C = zeros(m,n);
q = trnd(2.65,m,n);
for j = 1:n
   S(1,j)=S0;
    for i = 2:m
       S(i,j) = S0*exp((r-0.5*sig^2)*(T+(i-1)*t) + sig*sqrt(T+(i-1)*t)*q(i,j));
    end
    for i = 1:m
        C(i,j) = exp(-r*(T+(i-1)*t))*max(S(i,j)-K,0);
    end
end
Cmean = mean(C(m,:))
 width = t_confidence_interval(C(m,:));
 Conf = [Cmean - width, Cmean + width]
 Cstd = std(C(m,:))
x = [1:m];
subplot(2,1,1);
plot(x-1,S)
xlabel('Time (days)')
ylabel('Stock Price')
subplot(2,1,2);
plot(x-1,C)
xlabel('Time (days)')
ylabel('Call Price')
```

```
% Run this code for 3-parameter lognormal distribution
days = 30;
S0 = 1360;            % Inital Stock Price
K = 1390;              % Strike Price
r = .02;            % Treasury bond rate correspondging to option life
sig = 0.05036;           % Standard Deviation
m = days+1;
n = 1000;             % Number of simulated paths
T = days/252;       % Time to maturity (in days)
t = 1/252;
S = zeros(m,n);
C = zeros(m,n);
 q = lognrnd(r,sig,m,n) + .00005;
for j = 1:n
    S(1,j)=S0;
    for i = 2:m
        S(i,j) = S(i-1,j)*(q(i,j));
    end
    for i = 1:m
        C(i,j) = ((T+(i-1)*t))*max(S(i,j)-K,0);
    end
end
Cmean = mean(C(m,:))
width = t_confidence_interval(C(m,:));
Conf = [Cmean - width, Cmean + width]
Cstd = std(C(m,:))
x = [1:m];
subplot(2,1,1);
plot(x-1,S)
xlabel('Time (days)')
ylabel('Stock Price')
subplot(2,1,2);
plot(x-1,C)
xlabel('Time (days)')
ylabel('Call Price')
```