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ABSTRACT

Controller design for a riderless bicycle is a difficult task due to its non-holonomic constraint and its complex dynamic. The problem will be more complex, when path tracking and stabilizing the roll angle of the bicycle are considered simultaneously. This paper proposes an analytical approach to stabilize the roll angle of an unmanned bicycle, in the meanwhile a desirable path is tracked by the bicycle. These two objectives are achieved due to the existing relation between the roll angle and the steering variable of the bicycle. In this paper, a multi-loop control structure is proposed to track a predetermined path. In the inner loop, the roll angle tracks a time-varying reference signal using the back-stepping method. This reference signal is manipulated in the outer loops to track the desired path. Moreover, the robustness of the system with respect to external disturbances is guaranteed by the Lyapunov redesign method. Finally, the efficiency of the proposed method is illustrated through computer simulations.

1. Introduction

Modelling and control of unmanned bicycles as clean transportation vehicles are still of interest for researchers (Arora, Diba, & Esmailzadeh, 2017; Chu & Chen, 2018). The goal is to track a predetermined path by manipulating the steering angle, pedal speed as well as the roll angle of the bicycle. The complexity of bicycle's dynamics and the lack of bicyclist's weight for stabilization have caused less attention to this topic in the literature. For simplicity, in some references, only control of the roll angle and balancing the bicycle have been considered. In this regard, feedback linearization methods and a variety of sliding mode and back-stepping approaches have been implemented to balance bicycles (Dao & Chen, 2011; Guo, Wei, & Huang, 2010; He, Zhao, & Stasinopoulos, 2015; Hwang, Wu, & Shih, 2011; Kawamura & Murakami, 2012; Yu & Yeh, 2011). The main problem of these methods is that after stabilizing the bicycle, there is no way to control its path; since control variables of the system (especially steering angle) have been used for stabilization and no effective control variable is left for tracking the path.

In some papers, the roll angle of the bicycle is used to make it to track a desirable path. For example, in Getz and Marsden (1995) with emphasis on the complexity of the controller design with non-holonomic constraints, a linear PID controller has been designed such that the bicycle tracks a specified path. Also in Tanaka and Murakami

(2009), with the assumption that the value of roll angle is small, a simple model has been derived and a linear PID controller has been designed such that the bicycle could track a predetermined path. Since linear controllers have many limitations and are not robust, in Mathieu and Hedrick (2010), path tracking of bicycles has been studied with two methods namely feedback linearization and sliding mode. Moreover, it has been shown that the sliding mode controller has a better performance and is more robust relative to feedback linearization. Also in Tagne, Talj, and Charara (2015), the control of intelligent vehicles has been done. However, in Mathieu and Hedrick (2010) and Tagne et al. (2015), roll angle and stability are not considered, and it is assumed that roll angle is always zero. As said before, ignoring the stability of roll angle may cause the bicycle to fall down before tracking the desired path. In some references such as Lee and Ham (2002), the unmanned bicycle has been stabilized using an extra weight instead of bicyclist's weight. However, this approach leads to mechanical complexities and an expensive product.

Proposed nonlinear controllers to control the roll angle and path tracking of the bicycle, simultaneously, could be categorized as intelligent and classic methods. Generally speaking, intelligent controllers use non-analytical approaches for this simultaneous control (Chen & Dao, 2005, 2007). Although non-analytical approaches may

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work well in the simulations, the performance of the control system could not be guaranteed for all situations. In Defoort and Murakami (2009), as the only paper that covers simultaneous nonlinear control of roll angle and path tracking, first using a second-order sliding mode controller, bicycles are controlled such that besides keeping the balance, the roll angle and longitudinal speed of the bicycle follow desired values. Then, by manipulating these values, tracking of a predetermined path is performed. However, in its control structure, two dynamics have been added to the closed-loop system, one is related to the roll angle estimator with a complicated equation and the other is a disturbance observer. Adding these two dynamics increases the order of the closed-loop system and consequently enhances the complexity of the controller and reduces the reliability of the system.

In this paper, an analytical approach is proposed to stabilize the roll angle of an unmanned bicycle and in the meanwhile to track a desirable path using a comprehensive model which considers the uncertainties. First, using the back-stepping approach the roll angle of the bicycle is forced to track a desired time-varying reference. Using this internal loop and the simulated data, a relationship between the desirable roll angle and the steering variable is estimated. The resulting relation is a polynomial which is a static equation and can be implemented easily. In the final stage, the path tracking controller is designed. To make the proposed method robust against external disturbances, the Lyapunov redesign method is used to avoid implementation of a disturbance observer. Computer simulations show the efficiency of the proposed method.

The rest of this paper is organized as follows: in Section 2, the considered model of the bicycle is reviewed. In Section 3, the proposed control method will be presented and finally, in Section 4, simulation results will show the performance of the proposed controller.

2. Bicycle model

The model of a bicycle can be divided into two parts: kinematics and dynamics. In this section, first the parameters of a bicycle are introduced and then kinematic and dynamic models of a bicycle are presented.

2.1. Parameters of an unmanned bicycle

A bicycle consists of three parts: front wheel, back wheel and the body. In analytical methods, a bicycle is considered as a point mass m that is located at its centre of mass. Bicycle moves forward by moving its pedal with speed equal to v and changes its path by rotating its steering
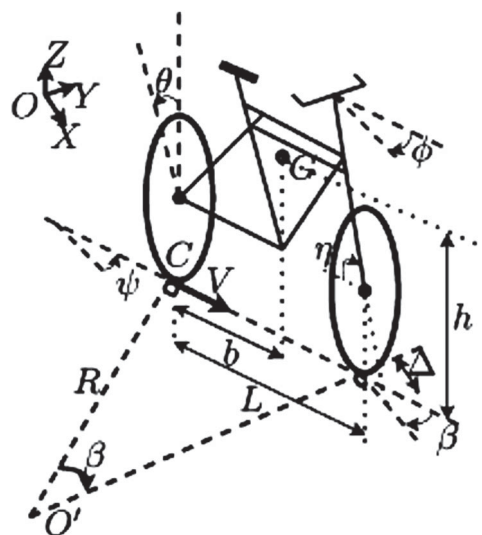

Figure 1. Side view of a bicycle and its parameters (Defoort & Murakami, 2009).

handlebar to right or left. Figure 1 shows a bicycle view and its parameters.

In Figure 1, G is the bicycle's centre of mass, H is the distance between the ground and the centre of mass, b is the distance between the centre of mass and the back wheel, and the variables $L, \eta, \Delta, \varphi, \theta$ and β are respectively the wheel base, castle angle, bicycle trail, steering angle, roll angle and the angle of front wheel rotation, and also ψ is the angle of the bicycle relative to the horizontal axis.

Castle angle (η) and bicycle trail (Δ) are the manipulating variables for the geometric stability of the system (Figure 1). These parameters introduce the effect of the steering angle at low speeds into the system geometry (Limebeer & Sharp, 2006). If the direction of steering handlebars changes the value of φ, front wheel will change the value of β accordingly (Figure 1). Equation (1) shows the relationship between these two angles.

$$\tan \beta \cos \theta = \tan \varphi \sin \eta. \tag{1}$$

Castle angle (η) is usually constant. For simplifying the dynamic model of the bicycle, this angle is set to 90° and φ is considered almost equal to β. This assumption induces an uncertainty into the model of the system.

Bicycle tyres are assumed to have no width. When the bicycle is tracking an inclined path, the tangential force between the tyre and road exposes the torque T_δ on the steering axis. This torque causes the bicycle body to move sidewise. In this case, the deviation angle of the body, which is named as δ, is estimated as (Astrom, Klein, & Lennartsson, 2005)

$$\delta = \frac{\Delta \sin \eta}{L} \beta. \tag{2}$$

As a result, T_δ is obtained by Equation (3)

$$T_\delta = -b\delta \, mg \, \cos \theta, \tag{3}$$

where g is the acceleration of gravity and δ is calculated from (2).

Besides that, in rotational movements inertia moments should be evaluated. Inertia moments of the bicycle are considered as I_h and I_x.

2.2. Kinematic model

The kinematic model of a bicycle includes non-holonomic constraint of the system that should be considered in path tracking. This constraint causes that the bicycle can't have a sidewise movement. Regarding this constraint kinematic model of the bicycle is obtained as follows:

$$\begin{cases} \dot{x} = v \, \cos \, \psi, \\ \dot{y} = v \, \sin \, \psi, \\ \dot{\psi} = v\sigma, \end{cases} \tag{4}$$

where σ is the steering variable and is defined as $\sigma = (\tan \beta / L)$.

2.3. Dynamic model

The dynamic model consists of two parts: steering and roll dynamics. In order to be more realistic, both dynamic models are considered in this paper.

2.3.1. Roll angle dynamic model

The dynamic model of the roll angle channel is derived from Lagrange equations. This model brings about the most complexity in the bicycles model. Linear equations of this model have been presented in Sharp (1971). Based on these equations, Getz has proposed a model for the bicycle using the definition of generalized coordinates of the bicycle (Getz, 1995). This model has been called 'Getz model'. In Defoort and Murakami (2009), a more complete model has been presented, by taking into account the uncertainties that exist in a bicycle and solving Lagrange equations. Equation (5) shows this model:

$$\ddot{\theta} = A \left(g \, \sin \, \theta - (1 - h\sigma \, \sin \, \theta)\sigma v^2 \, \cos \, \theta \right.$$
$$\left. + B \, T_\delta - bv\sigma\dot{\theta} \, \sin \, \theta - \frac{Dv}{\cos^2 \varphi}\dot{\varphi} \right), \tag{5}$$

where

$$A = \frac{mh}{I_x + mh^2}, B = \frac{1}{mh}, D = \frac{b \, \sin \, \eta}{L}.$$

By considering that $\varphi \in (-\frac{\pi}{2}, \frac{\pi}{2})$, therefore in (5), $\cos^2\varphi \neq 0$.

2.3.2. Steering angle dynamic model

Many physical phenomena like wind, unevenness of road surface, and gyroscopic forces, affect the bicycle. To consider these effects, the model of steering dynamics is considered as

$$I_h\ddot{\varphi} = \tau_{\text{motor}} - \tau_{\text{dist}}, \tag{6}$$

where τ_{motor} is the resulting torque of the motor to control the steering angle of unmanned bicycle and τ_{dist} is the torque due to disturbing forces.

3. Controller design

In this section, first the roll angle is controlled using the back-stepping approach. Then, the path tracking problem will be studied. Finally, the designed controller is made robust by applying the Lyapunov redesign method.

3.1. Control of roll angle

In the first stage, the derivative of the roll angle is considered as a virtual input of the system and by choosing an appropriate Lyapunov function, this virtual input is designed such that as well as ensuring the stability of the system and roll angle, a proper time-varying reference signal to be tracked. In the second stage, $\ddot{\varphi}$ is designed such that the designed derivative of steering angle obtained in the previous stage is followed. After the second stage, the control law (with the assumption of no disturbance) is designed.

Now, according to (5) and (6) and considering the state variable as $X = (\theta, \dot{\theta}, \varphi, \dot{\varphi})$, the control law is designed as shown below.

In the first stage, it is assumed that $\dot{\varphi}$ in (5) is the virtual input of the system. If the signal $\theta_d(t)$ is assumed as the desirable value of the roll angle, an appropriate Lyapunov function may be considered as follows:

$$V_1 = 0.5 \left((\dot{\theta} - \dot{\theta}_d)^2 + k \, (\theta - \theta_d)^2\right), \tag{7}$$

where k is an arbitrary positive constant. This form of Lyapunov function has been chosen to converge the variable $\theta(t)$ to the signal $\theta_d(t)$. Indeed, when V_1 is guaranteed to converge to zero, the terms θ and $\dot{\theta}$ will converge to θ_d and $\dot{\theta}_d$, respectively.

Then, by differentiating V_1 along the system (5), one has

$$\dot{V}_1 = \left(A \left(g \, \sin \, \theta - (1 - h\sigma \, \sin \, \theta)\sigma v^2 \, \cos \, \theta \right.\right.$$
$$\left. + BT_\sigma - bv\sigma\dot{\theta} \, \sin \, \theta - \frac{Dv}{\cos^2 \varphi}\dot{\varphi} \right)$$
$$\left. - \ddot{\theta}_d + k(\theta - \theta_d) \right) (\dot{\theta} - \dot{\theta}_d). \tag{8}$$

The selected virtual input in (8) must be designed to ensure negative definiteness of the derivative of the Lyapunov function. The virtual input is designed as

$$\dot{\varphi} = \frac{\cos^2 \varphi}{DAv}(A(g \sin \theta - (1 - h\sigma \sin \theta)\sigma v^2$$
$$\times \cos \theta + BT_\delta - bv\sigma\dot{\theta} \sin \theta)$$
$$- \ddot{\theta}_d + k(\theta - \theta_d) + k_1(\dot{\theta} - \dot{\theta}_d)), \qquad (9)$$

where k_1 is an arbitrary positive constant too. Then,

$$\dot{V}_1 = -k_1(\dot{\theta} - \dot{\theta}_d)^2 \leq 0. \qquad (10)$$

It is evident that the derivative of the Lyapunov function is negative semi-definite, thus only the stability of the closed-loop system with the designed control input can be guaranteed. However, Equation (10) leads to inequality of $V_1(t) \leq V_1(0)$ at $t \geq 0$ and consequently V_1 is bounded. The boundedness of V_1 leads to the boundedness of its expressions $((\theta - \theta_d)$ and $(\dot{\theta} - \dot{\theta}_d))$. According to Barbalat's lemma (Khalil, 2002) and differentiating Equation (10) again along the system (5) with the control law (9), one has

$$\ddot{V}_1 = 2k_1(\dot{\theta} - \dot{\theta}_d)[k(\theta - \theta_d) + k_1(\dot{\theta} - \dot{\theta}_d)]. \qquad (11)$$

Since $(\theta - \theta_d)$ and $(\dot{\theta} - \dot{\theta}_d)$ are bounded, for $\ddot{V}_1$ it is also the case. Boundedness of $\ddot{V}_1$ ensures the uniform continuity of $\dot{V}_1$. Thus the derivative of Lyapunov function tends to zero at infinity ($\dot{V}_1 \to 0$ if $t \to \infty$). Therefore, $(\dot{\theta} - \dot{\theta}_d)$ converges to zero.

Now, if the error variable z_1 is defined as $z_1 = \dot{\theta} - \dot{\theta}_d$, the dynamic of this variable according to (5), (9) will be as

$$\dot{z}_1 = -k(\theta - \theta_d) - k_1 z_1. \qquad (12)$$

Differentiating Equation (12) leads to

$$\ddot{z}_1 = (-k + k_1^2)(\dot{\theta} - \dot{\theta}_d) + k_1 k(\theta - \theta_d). \qquad (13)$$

The boundedness of $(\theta - \theta_d)$ and $(\dot{\theta} - \dot{\theta}_d)$, according to (13), leads to the boundedness of the second derivative of z_1. Thus $\dot{z}_1$ is uniformly continuous and according to Barbalat's lemma, $\dot{z}_1$ tends to zero at infinity. Thus, according to (12), in order that the derivative of error variable tends to zero at infinity, $(\theta - \theta_d)$ must tend to zero as well. In this way, asymptotic stability of the system is shown.

The designed control law (9) cannot be directly applied to the variable $\dot{\varphi}$, since it is a virtual input. Thus the designed control law (9) is considered as the desirable value of $\dot{\varphi}$ (i.e. $\dot{\varphi}_d$), then, the error variable z_2, is defined as $z_2 = \dot{\varphi} - \dot{\varphi}_d$. In the second stage, a new Lyapunov

function is considered as

$$V_2 = V_1 + 0.5 z_2^2. \qquad (14)$$

The derivation of V_2 along the system (6) with the substitution of $\dot{\varphi} = z_2 + \dot{\varphi}_d$ will be as follows:

$$\dot{V}_2 = -k_1(\dot{\theta} - \dot{\theta}_d)^2 - \frac{ADv(\dot{\theta} - \dot{\theta}_d)^2}{\cos^2 \varphi} z_2 + z_2(\ddot{\varphi} - \ddot{\varphi}_d). \qquad (15)$$

The virtual input in the second stage is considered as $\ddot{\varphi}$, which can be designed as

$$\ddot{\varphi} = \frac{ADv(\dot{\theta} - \dot{\theta}_d)^2}{\cos^2 \varphi} + \ddot{\varphi}_d - k_2 z_2. \qquad (16)$$

In this case, the derivative of the Lyapunov function turns out to be as

$$\dot{V}_2 = -k_1(\dot{\theta} - \dot{\theta}_d)^2 - k_2 z_2^2 \leq 0. \qquad (17)$$

Now, similar to the previous case, using the Barbalat's lemma, asymptotic stability of the system will be assured. But the physical input of the unmanned bicycle is the torque of the motor and, without considering input disturbances, one has

$$\ddot{\varphi} = \frac{\tau_{motor}}{I_h}. \qquad (18)$$

Thus the actual control input is given by

$$\tau_{motor} = I_h(\frac{ADv(\dot{\theta} - \dot{\theta}_d)^2}{\cos^2 \varphi} + \ddot{\varphi}_d - k_2 z_2). \qquad (19)$$

This control law is related to the nominal case (without considering disturbances). In the following section, the control law is developed to make the closed-loop system robust against τ_{dis}.

3.2. Robust controller

According to (6), disturbing input affects the steering variable. In this section, to make the designed control law (9) robust against external disturbances, Lyapunov redesign method is utilized.

As shown in the previous stage, the Lyapunov function (20) makes the nominal system asymptotically stable:

$$V = 0.5 ((\dot{\theta} - \dot{\theta}_d)^2 + k (\theta - \theta_d)^2 + (\dot{\varphi} - \dot{\varphi}_d)^2). \qquad (20)$$

A new control input to overcome the external disturbances is defined as $u' = \tau_{motor} + \upsilon$ (τ_{motor} is the designed control law without considering disturbances).

On the other hand, the unmanned bicycle has the following form:

$$\dot{x} = f(t, x) + G(t, x)[u' + \delta(t, x)]. \qquad (21)$$

The instantaneous value of additive disturbances is unknown, however, it is assumed that the upper bound of its norm is available as

$$||\delta|| = ||\tau_{\text{dist}}|| \leq 0.5. \qquad (22)$$

Now, if the controlled input u' is applied to the system (21), the closed-loop system will be as

$$\dot{x} = f(t, x) + G(t, x)\tau_{\text{motor}} + G(t, x)[\upsilon + \delta(t, x, u')]. \qquad (23)$$

Then, the derivative of a Lyapunov function V along the system (23) will be as follows:

$$\dot{V} = \frac{\partial V}{\partial x}(f + G\tau_{\text{motor}}) + \frac{\partial V}{\partial x}G(\upsilon + \delta). \qquad (24)$$

As it is seen, $\dot{V}$ consists of two expressions. The first one is due to the nominal system and the designed controller u, and the second expression is the result of disturbance δ and the added term υ. The first expression was shown to be negative in the previous design. Now, we are going to examine the second expression. By calling $\omega^{\mathsf{T}} = (\partial V/\partial x)G$, the virtual controller υ using the Lyapunov redesign control approach can be written as (Khalil, 2002)

$$\upsilon = \begin{cases} -\eta(t, x) \left(\dfrac{\varpi}{||\varpi||_2} \right) & \eta(t, x)||\varpi||_2 \geq \varepsilon, \\ -\eta^2(t, x) \left(\dfrac{\varpi}{\varepsilon} \right) & \eta(t, x)||\varpi||_2 < \varepsilon, \end{cases} \qquad (25)$$

where ε is the small positive value and $\eta(t, x)$ may be any function greater than the upper bound of δ (i.e. 0.5).

Thus the virtual control input to eliminate the effect of τ_{dis} is as

$$\upsilon = \begin{cases} -0.6 \dfrac{\dot{\varphi} - \dot{\varphi}_d}{|\dot{\varphi} - \dot{\varphi}_d|} & 0.6|\dot{\varphi} - \dot{\varphi}_d| \geq l_h \varepsilon, \\ -0.36(\dot{\varphi} - \dot{\varphi}_d)/(l_h \varepsilon) & 0.6|\dot{\varphi} - \dot{\varphi}_d| < l_h \varepsilon \end{cases} \qquad (26)$$

and the robust control law is as follows:

$$u' = l_h \left(\frac{ADv(\dot{\theta} - \dot{\theta}_d)^2}{\cos^2 \varphi} + \ddot{\varphi}_d - k_2 z_2 \right) + \upsilon. \qquad (27)$$

3.3. Path tracking

When the bicycle is going to change its path, the steering handlebar should be rotated. However, in the previous section, the steering input was used to stabilize the roll angle of the bicycle. This input has been designed such that the roll angle follows a desirable value. Since
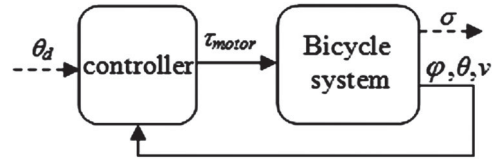

Figure 2. Block diagram of the internal loop (control of the roll angle).

the steering input changes the roll angle, the bicycle can be forced to follow a desirable predetermined path, by adjusting the proper roll angle. However, there is no strict relation between the track path and the roll angle. To obtain an appropriate relation, the internal loop shown in Figure 2 has been simulated for various values of θ_d, and in each case, the resulting steady-state value of the steering variable will be recorded. Then, an appropriate relation (a polynomial) can be fitted on the resulting data (between the variables θ_d and σ).

In this paper, the relation between desirable roll angle and the steering variable has been fitted with a fifth-order polynomial as

$$\theta_d = a_5 \sigma_d^5 + a_4 \sigma_d^4 + a_3 \sigma_d^3 + a_2 \sigma_d^2 + a_1 \sigma_d. \qquad (28)$$

However, it is observed that the coefficients of the derived equations intensely depend on the velocity of the bicycle. Therefore, the fitting process will be repeated for various velocities of the bicycle and the coefficients of Equation (28) will be velocity dependent as below:

$$a_1 = 0.0981v^2 + 0.025v,$$

$$a_2 = -0.0004v^4 - 0.0005v^3 - 0.0033v^2 + 0.0027v,$$

$$a_3 = -0.036v^4 + 0.1512v^3 - 0.291v^2 + 0.1693v,$$

$$a_4 = 0.0225v^5 - 0.154v^4 + 0.4531v^3$$
$$\quad - 0.5731v^2 + 0.2383v,$$

$$a_5 = 0.0258v^5 + 0.2177v^4 - 0.6854v^3$$
$$\quad + 0.8959v^2 - 0.381v.$$

Equation (28) only shows the relation between steering variable and the roll angle. However, to track a predetermined path the relation between favourable steering variable and the desirable path must be calculated. To do this, first it is assumed that the bicycle is oriented in the direction shown in Figure 3 (a vector with the angle of ψ) and tends to follow a predetermined angle (α). According to the third term of (4), if the value of the desirable steering variable is selected as

$$\sigma_d = -k_3 \frac{(\psi - \alpha)}{v}, \qquad (29)$$
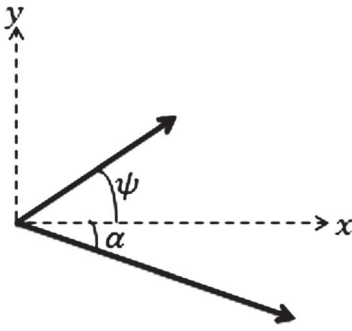
where k_3 may be any positive constant.

Figure 3. Real (ψ) and desired (α) directions of the bicycle.

Then, the desirable derivative of ψ angle will be as follows:

$$\dot{\psi}_d = -k_3(\psi - \alpha). \qquad (30)$$

By choosing the Lyapunov function as $V = 0.5(\psi - \alpha)^2$, it can be proved that a proper value of $k_3 > 0$ causes that the angle ψ tends to the angle α. Therefore, suitable steering variable for changing tracking path angle can be calculated from Equation (29). In order that bicycle can track a time-varying path, this proper angle should vary proportional to tracking path at each moment. Thus, according to Figure 4, if at each time instant the bicycle which is located at a position A is forced to go to point A_r on the desirable path, it should move with α angle to follow the reference path. In this case, the
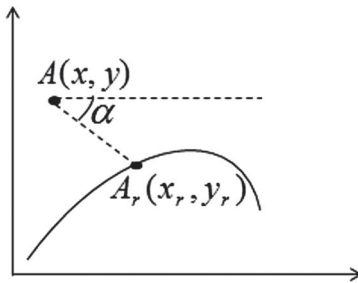

Figure 4. The position of the bicycle and the desirable path.

angle α at each moment is equal to

$$\alpha = \tan^{-1}\left(-\frac{y - y_r}{x_r - x}\right) = \tan^{-1}\left(\frac{y_r - y}{x_r - x}\right), \qquad (31)$$

where (x, y) is the position of the bicycle and (x_r, y_r) is the position of the virtual reference bicycle producing the reference path. By substituting Equation (31) in (29), the equation for desirable steering angle is derived, i.e.

$$\sigma_d = -k_3 \frac{\left(\psi - \tan^{-1}\left(\frac{y_r - y}{x_r - x}\right)\right)}{v}. \qquad (32)$$

When these two points (A_r and A) get close to each other, the argument of $\tan^{-1}$ in (31) tends toward 0/0 therefore the precision may be decreased. In order to reduce error, the following coefficient may be used:

$$k_3 = KR, \qquad (33)$$

where R is the distance between the two points and is defined as $R = \sqrt{(x_r - x)^2 + (y_r - y)^2}$ and K is a positive constant.

Thus by calculating the desirable steering variable to track the specified path (σ_d) and considering Equation (28), an appropriate roll angle (θ_d) will be applied to the internal loop. Figure 5 shows the corresponding block diagram.

Remark 1: The parameters which should be tuned in the proposed method are k, k_1, k_2 and K. Theoretically, these parameters only should be positive; however, they affect the transient response of the closed-loop system. For instance, increasing the values of k, k_1, leads to the fast convergence of θ to θ_d, while decreasing the value of k leads to a smoother response. Moreover, the value of k_2 affects on the convergence of the error variable z_2. Finally, increasing the value of K leads to a tight (but very sensitive) tracking.

4. Simulation

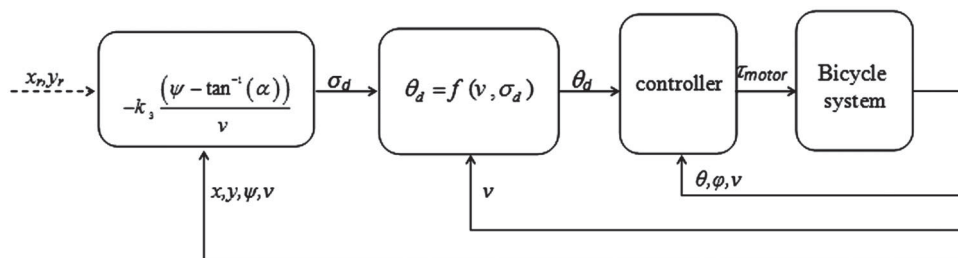In order to check the designed controller, the obtained results have been simulated in Matlab software. Table

Figure 5. The proposed block diagram to track a reference path.

Table 1. Physical parameters of the simulated unmanned bicycle.

m(kg)	31.4
g(m/s^2)	9.8
L(m)	1.2
b(m)	0.45
h(m)	0.85
Δ(m)	0.07
η(rad)	1.22
l_h(kgm^2)	0.46
l_x(kgm^2)	2

1 shows the values of nominal parameters of the unmanned bicycle that were used in the simulation.

To generate a desired path, a reference virtual bicycle has been used to generate the reference path. Equations of this reference bicycle are as follows:

$$\begin{cases} \dot{x}_r &= v_r \cos \psi_r, \\ \dot{y}_r &= v_r \sin \psi_r, \\ \dot{\psi}_r &= v_r \sigma_r. \end{cases} \quad (34)$$

In the simulation, it is assumed that the reference path is going to be tracked with an arbitrary but constant speed. Table 2 shows the initial values used in the simulation at a constant speed of 2 metres per second.

Table 2. Initial conditions of the unmanned and reference bicycles.

θ_0(rad)	−0.2
φ_0(rad)	0
x_0(m)	−0.5
y_0(m)	1
ψ_0(rad)	−0.39
x_{r0}(m)	0
y_{r0}(m)	0
ψ_{r0}(rad)	1.57

Table 3. Coefficients of the controller in the simulation.

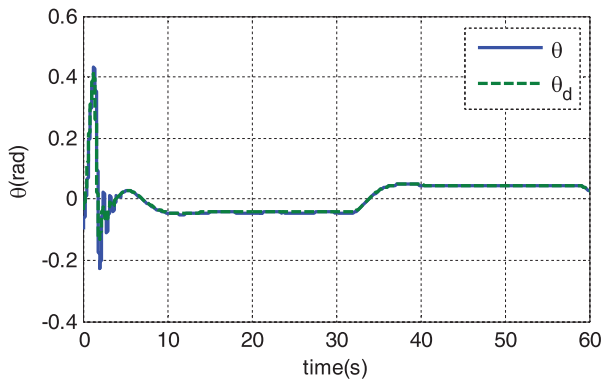| k	k_1	k_2	K	ε
85	5	15	0.2	0.001

Figure 6. Desirable and real values of the roll angle at $v = 2$ m/s.

Table 3 shows the applied control parameters in the simulation.

In Figure 6, the desirable roll angle and its real value are compared, and also Figure 7 shows the value of control signal.

As is evident, the roll angle has reached its desirable value in less than 8 seconds. In Figure 8, the desirable path and the path tracked by the bicycle are shown.

In Figure 9, the resulting steering angle to track the corresponding path by the bicycle is shown.
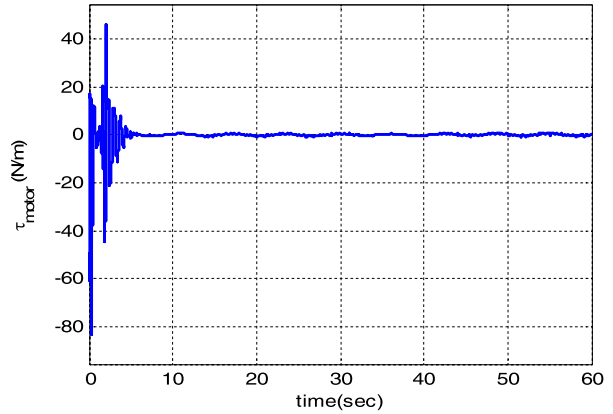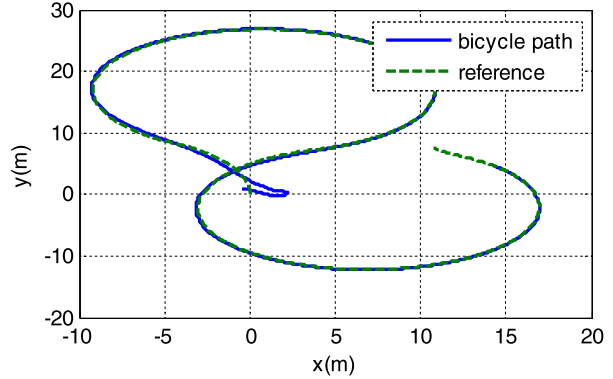

Figure 7. Time history of the control signal.

Figure 8. Desirable and real paths tracked by the bicycle in the coordinate plane.
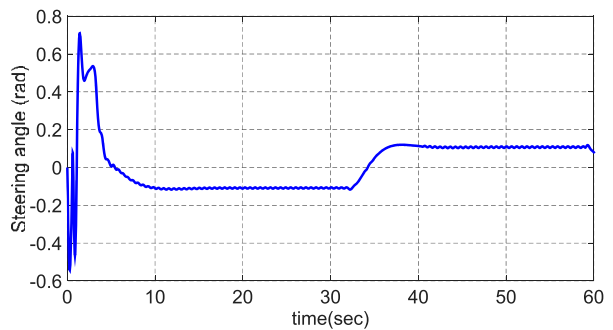

Figure 9. The resulting steering angle of the bicycle.

Simulation results show that the proposed algorithm is able to track a predetermined path as well as keeping the balance of the bicycle.

5. Conclusion

In this paper, control and path tracking of an unmanned bicycle using a back-stepping approach were studied. The designed controller was made the roll angle to follow an arbitrary time-varying signal with guaranteed asymptotic stability. In the model of bicycle, the effects of disturbing forces were modelled as an additive term in the channel of the torque of the motor. The designed controller was made robust relative to external disturbances, by applying the Lyapunov redesign method. First, it was shown that the roll angle of the bicycle could track the arbitrary time-varying signal by manipulating the steering variable (this formed an internal loop). Moreover, using the resulting closed-loop system (the internal loop) a nonlinear relation between desirable roll angle and the steering variable was extracted using a fifth-degree polynomial with speed-dependent coefficients. Finally, it was shown that a successful path tracking could be done using the set point of the internal loop. The presented simulation results showed that, by applying the proposed approach, the unmanned bicycle could track a predetermined path as well as keeping its balance properly.

Disclosure statement

No potential conflict of interest was reported by the authors.

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