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Finite-time H_∞ control for a chaotic finance system via delayed feedback

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ABSTRACT

This paper is concerned with the finite-time H_∞ control problem for a chaotic finance system subject to energy-bounded external disturbance via delayed feedback controller. Based on an augmented Lyapunov-like functional, the Wirtinger-based inequality and the novel finite-time boundedness analysis approach, a delay-dependent sufficient condition is proposed in terms of linear matrix inequalities such that the closed-loop error system is finite-time bounded with a prescribed H_∞ performance level. By simple modification, the sufficient condition based on the standard H_∞ control design is also obtained. Finally, simulation results are given to demonstrate the effectiveness and advantages of our proposed results.

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1. Introduction

Over the past several decades, dynamical behaviours for economic/financial systems have received considerable attention (Cesare & Sportelli, 2005; Chen, 2008a; Chen, Liu, & Xu, 2014; Chen & Ma, 2001a, 2001b; Gao & Ma, 2009; Stutzer, 1980; Yu, Cai, & Li, 2012). In particular, it has been identified that the chaotic behaviour may be encountered in an economic/financial system. Note that the financial crisis is essentially a kind of chaotic phenomenon. Moreover, the existence of the chaotic phenomenon in a real economic/financial system will bring the inherent indefiniteness into the macroeconomic operation. Therefore, various kinds of control schemes have been proposed to control the chaotic behaviour of economic/financial systems over the past decade (e.g. Chen, 2008b; Chen & Chen, 2007; Dadras & Momeni, 2010; Salarieh & Alasty, 2008; Son & Park, 2011; Tacha et al., 2016; Wang, 2016; Wang, Huang, & Shen, 2012; Xin & Zhang, 2015). For example, the delayed feedback approach and the adaptive algorithm have been used in Chen and Chen (2007) to control the chaos in the Cournot-Puu model. In Dadras and Momeni (2010) and Wang et al. (2012), the sliding mode control schemes have been proposed to stabilize fractional-order economic systems and, in Wang (2016), adaptive fuzzy scheme has been used to address the control problem for a class of uncertain fractional-order finance systems subject to input saturations.

In the above-mentioned literature, it is noted that the external disturbances have been ignored. In reality, finance systems should be inevitably influenced by external disturbances stemmed from environmental interference (Zhao & Wang, 2014). More importantly, the occurrence of external disturbances could lead to difficulties in achieving desirable control performances and, in the worst case, destabilize the closed-loop system. Therefore, it is more imperative to address the control problem of chaotic economic/financial systems in the presence of external disturbances. Recently, the global H_∞ control problem has been studied in Zhao and Wang (2014) for a class of chaotic finance systems with external disturbances. The main objective of the H_∞ control discussed in Zhao and Wang (2014) is to ensure that the L_2 gain from external disturbance to the error between the system state and the desirable orbit is less than a prescribed positive scalar. However, it should be pointed out that the global design in Zhao and Wang (2014) is concerned with all unstable fixed points, which has the more rigorous requirement on the finance system. Therefore, the global control scheme might be ineffective or cannot achieve the desirable performances. In addition, it is worth mentioning that the analysis approach used in Zhao and Wang (2014) is conservative, which might lead to the failure in designing the delayed feedback controller.

On the other hand, it is noted that most existing results are concerned with the behaviour of economic/financial

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systems over an infinite-time horizon. However, in many practical applications, our main attention may be the finite-time behaviour of a control system. Based on such a requirement, the concept of finite-time stability/boundedness has been proposed in Amato, Ariola, and Dorato (2001). The finite-time boundedness introduced in Amato et al. (2001) means that once we fix a time interval, the system state does not exceed a certain bound within the prescribed time interval for given initial conditions and external disturbances. Over the past years, the finite-time control and state estimation problems have been widely investigated for various kinds of control systems (e.g. Ali, Saravanan, & Arik, 2016; Amato, Ariola, & Dorato, 2005; Ma, Wang, & Lu, 2012; Niamsupa & Phat, 2016; Shen, Park, & Wu, 2014; Shen, Park, Wu, & Zhang, 2015; Song & He, 2015; Wang, Wang, Wei, & Alsaadi, 2018; Zhao, Shen, Li, & Wang, 2013). For example, in Zhao et al. (2013), the finite-time H_∞ control problem has been addressed for a class of delayed Markovian jump systems with input constraints and, in Ali et al. (2016), the problem of finite-time H_∞ state estimation has been investigated for a class of delayed switched neural networks.

Up to now, despite the significant advances made on the finite-time analysis and control problems, it is worth mentioning that the existing results cannot be applied to nonlinear finance systems. The main reason is that the existing results cannot deal with the nonlinear characteristic occurred in finance systems effectively. In addition, it should be pointed out that the existing finite-time boundedness analysis approaches in Ali et al. (2016), Ma et al. (2012), Niamsupa and Phat (2016), Song and He (2015) and Zhao et al. (2013) might be conservative for systems with time delays to some extent, since the information of time delays has been not sufficiently explored.

Motivated by the above discussions, the main objective of this work is to investigate the finite-time H_∞ control problem for a class of finance systems via delayed feedback controller. Using an augmented Lyapunov-like functional, the Wirtinger-based inequality and the novel finite-time boundedness analysis approach, a sufficient condition is obtained by means of linear matrix inequalities (LMIs), which can guarantee that the closed-loop system is finite-time bounded with a prescribed H_∞ performance level. As the by-product, the condition of H_∞ control design over the infinite-time horizon is also proposed. Finally, simulations results show that our proposed conditions are not only effective but also less conservative.

The main contributions of this paper are summarized as follows: (1) the finite-time H_∞ control problem is considered, for the first time, for a chaotic finance system

and corresponding condition is established; (2) by incorporating the time-delay information sufficiently, a novel finite-time boundedness analysis approach is proposed for time-delay systems; and (3) an improved condition of designing standard H_∞ controller is also obtained for a chaotic finance system.

Notation. The superscript ' T ' denotes the transpose of a matrix. $L_2[0, \infty)$ is the space of square integrable vector functions over an interval $[0, \infty)$. $P > 0$ means that P is a real symmetric and positive definite matrix. $\|\cdot\|$ denote the 2-norm of a vector. $\lambda(\cdot)_M$ and $\lambda(\cdot)_m$ refer to the minimum and the maximum eigenvalue value of a matrix, respectively. I denotes an identity matrix with proper dimension. The symmetric terms in a symmetric matrix are denoted by $*$.

2. Problem formulation

Let us consider the dynamical model of financial system proposed in Chen and Ma (2001a, 2001b). The model consists of four sub-blocks, i.e. production, money, stock and labour force, and can be described by the following ordinary differential equations:

$$\begin{aligned}\dot{x}_1(t) &= x_3(t) + (x_2(t) - a)x_1(t), \\ \dot{x}_2(t) &= 1 - bx_2(t) - x_1^2(t), \\ \dot{x}_3(t) &= -x_1(t) - cx_3(t)\end{aligned}\quad (1)$$

where the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ denote the interest rate, the investment demand and the price index, respectively; the scalars $a > 0$, $b > 0$ and $c > 0$ represent the saving amount, the cost per investment and the demand elasticity of commercial markets, respectively.

For the model (1), it can be verified that, if the condition $c - b - abc \triangleq \Delta < 0$ holds, there exists a unique fixed point $(0, 1/b, 0)$. However, under the condition $c - b - abc \triangleq \Delta > 0$, the system (1) has the following three fixed points:

$$(0, 1/b, 0), \quad (\pm\sqrt{\Delta/c}, (ac + 1)/c, \mp\sqrt{\Delta/c^3}). \quad (2)$$

In addition, it has been shown that the dynamic behaviours of the system (1) are seriously affected by the parameters a , b and c . For many cases, the system (1) cannot maintain its stability at the fixed points (Chen, 2008b; Zhao & Wang, 2014). For example, when parameters are chosen as $a=3$, $b=0.1$ and $c=1$ or $a=2.5$, $b=0.2$, $c=1.2$, the system (1) will demonstrate the chaotic behaviour. Therefore, how to control the finance system (1) has become an interesting research topic during the past decade.

On the other hand, it should be pointed out that the finance systems may be inevitably influenced by external disturbances stemmed from environmental interference (Zhao & Wang, 2014). Therefore, the external disturbances should be incorporated in the finance system (1) to reflect the real commercial market.

Denoting that $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$,

$$A = \begin{bmatrix} -a & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{bmatrix}, \quad f(x(t)) = \begin{bmatrix} x_1(t)x_2(t) \\ 1 - x_1^2(t) \\ 0 \end{bmatrix}.$$

and adding both the disturbance term and the control term to (1), the finance system (1) can be modified as follows:

$$\dot{x}(t) = Ax(t) + f(x(t)) + B\omega(t) + u(t) \quad (3)$$

where $\omega(t)$ denote the disturbance belonging to $L_2[0, \infty)$ and B is a matrix with compatible dimension.

In this paper, we adopt the following delayed controller:

$$u(t) = K_1(x(t) - x^*) + K_2(x(t - \tau) - x^*) \quad (4)$$

where K_1 and K_2 are controller gains, $\tau \geq 0$ denote the time delay and x^* is an unstable fixed point.

Remark 2.1: The delayed controller (4) has been used in most existing literature (Zhao et al., 2013; Zhao & Wang, 2014). If we set $K_1 = -K_2 \triangleq K$, the feedback controller (4) become the case $u(t) = K(x(t) - x(t - \tau))$ that has been recognized to be effective in controlling the chaotic finance system (Chen, 2008b; Chen & Chen, 2007; Son & Park, 2011). If one sets $K_1 = 0$, the feedback controller (4) become the completely delayed case $u(t) = K_2(x(t - \tau) - x^*)$.

For the fixed point $x^* = [x_1^* \ x_2^* \ x_3^*]^T$, it is clear that

$$Ax^* + f(x^*) = 0. \quad (5)$$

Letting $r(t) \triangleq x(t) - x^*$ and applying the Equations (3)-(5), we have the following closed-loop error system:

$$\begin{aligned} \dot{r}(t) &= (A + K_1)r(t) + K_2r(t - \tau) \\ &\quad + f(x(t)) - f(x^*) + B\omega(t). \end{aligned} \quad (6)$$

Using Taylor expansion, it is seen that

$$f(x(t)) = f(x^*) + Fr(t) + [H.O.T.] \quad (7)$$

where $[H.O.T.]$ is the higher order term in $r(t)$ and

$$F \triangleq \left[\frac{\partial f(x)}{\partial x} \right]_{x=x^*} = \begin{bmatrix} x_2^* & x_1^* & 0 \\ -2x_1^* & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, we can obtain the following linearized model:

$$\dot{r}(t) = (A + F + K_1)r(t) + K_2r(t - \tau) + B\omega(t). \quad (8)$$

We now introduce the following lemma and definitions, which are indispensable in obtaining the results of this paper.

Lemma 2.1 (Seuret & Gouaisbaut, 2013; Sun, Liu, & Chen, 2009): Let an $n \times n$ symmetric matrix $Z > 0$, two scalars a and b satisfying $b > a$ and a vector function $\rho(t) \in \mathbb{R}^n$ be given. If the integrations concerned are well defined, then the following three integral inequalities hold:

- (1) $(b - a) \int_a^b \rho^T(s)Z\rho(s) ds \geq \left(\int_a^b \rho(s) ds \right)^T Z \left(\int_a^b \rho(s) ds \right) + 3\Omega^T Z \Omega,$
- (2) $(b - a) \int_a^b \rho^T(s)Z\rho(s) ds \geq \left(\int_a^b \rho(s) ds \right)^T Z \left(\int_a^b \rho(s) ds \right),$
- (3) $\frac{(b^2 - a^2)}{2} \int_{-b}^{-a} \int_{t+\theta}^t \rho^T(s)Z\rho(s) ds d\theta \geq \left(\int_{-b}^{-a} \int_{t+\theta}^t \rho(s) ds d\theta \right)^T Z \times \left(\int_{-b}^{-a} \int_{t+\theta}^t \rho(s) ds d\theta \right) \quad (b > a \geq 0)$

where

$$\Omega = \int_a^b \rho(s) ds - \frac{2}{b-a} \int_a^b \int_\theta^b \rho(s) ds d\theta.$$

Remark 2.2: In Lemma 2.1, the inequality (1) is referred to as the Wirtinger-based inequality and the inequality (2) is the Jensen inequality. Noting that $3\Omega^T Z \Omega \geq 0$, it is clear that the Wirtinger-based inequality provides a more accurate estimate of the term $(b - a) \int_a^b \rho^T(s)Z\rho(s) ds$ than the Jensen inequality. Of course, when using the Wirtinger-based inequality, the selected Lyapunov functional should contain the augmented term $\int_a^b \rho(s) ds$ (Seuret & Gouaisbaut, 2013). Otherwise, the Wirtinger-based inequality cannot demonstrate its superiority.

Definition 2.2 (Amato et al., 2001; Zhao et al., 2013):

Let the scalars $c_2 > c_1 \geq 0$, $\delta > 0$ and $T > 0$ be given. The error system (8) is said to be finite-time bounded with respect to (c_1, c_2, δ, T) if, for any $\omega(t)$ satisfying $\int_0^T \omega^T(s)\omega(s) ds \leq \delta$, the following relationship holds:

$$\sup_{-\tau \leq s \leq 0} \{\|r(s)\|, \|\dot{r}(s)\|\} \leq c_1 \Rightarrow \|r(t)\| \leq c_2, \quad \forall t \in [0, T].$$

Definition 2.3 (Zhao et al., 2013): The error system (8) is said to be finite-time bounded with an H_∞ performance level $\gamma > 0$ if the error system (8) is finite-time bounded with respect to (c_1, c_2, δ, T) and, under the zero-initial condition $(r(t) = 0, t \in [-\tau, 0])$, the error system (8) satisfies the following constraint:

$$\int_0^T r^T(s)Sr(s) ds < \gamma^2 \int_0^T \omega^T(s)\omega(s) ds$$

for all nonzero $\omega(t) \in L_2[0, \infty)$, where $S > 0$.

In this paper, our main objective is to design the time-delayed feedback controller (4) such that the error system (8) is finite-time bounded with an H_∞ performance level γ .

3. Main results

Theorem 3.1: Let the scalars $c_2 > c_1 \geq 0, \delta > 0, T > 0, \tau > 0, \alpha > 0, \gamma > 0$ and $\epsilon \neq 0$ be given. The error system (8) is finite-time bounded with an H_∞ performance level $\gamma^* = e^{\alpha T/2}\gamma$ if, there exist matrices $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0, Q > 0, Z > 0, R > 0, J_1 > 0, J_2 > 0, X, Y_1, Y_2$, and scalars $\lambda_i > 0$ ($i = 1, 2, \dots, 5$) such that the following LMIs hold:

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & XB & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & 0 & \epsilon Y_2^T \\ * & * & \Omega_{33} & 0 & P_{12}^T \\ * & * & * & -I & \epsilon B^T X^T \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0, \quad (9)$$

$$\begin{bmatrix} \Pi_{11} & P_{12} - 2Z \\ * & \Pi_{22} \end{bmatrix} \geq 0, \quad (10)$$

$$P \leq \text{diag}\{J_1, J_2\}, \quad (11)$$

$$J_1 \leq \lambda_1 I, \quad J_2 \leq \lambda_2 I, \quad Q \leq \lambda_3 I, \quad Z \leq \lambda_4 I, \quad R \geq \lambda_5 I, \quad (12)$$

$$[(\lambda_1 + \tau^2 \lambda_2 + \tau e^{\alpha \tau} \lambda_3 + 0.5 \tau^3 \lambda_4) c_1^2 + \delta] e^{\alpha T} \leq \lambda_5 c_2^2 \quad (13)$$

where

$$\begin{aligned} \Omega_{11} &= X(A + F) + (A + F)^T X^T + Y_1 + Y_1^T \\ &\quad + P_{12} + P_{12}^T + Q + S/\gamma^2 - 4Z - \alpha P_{11}, \\ \Omega_{12} &= Y_2 - P_{12} - 2Z, \quad \Omega_{13} = P_{22} + 6Z/\tau - \alpha P_{12}, \\ \Omega_{15} &= -X + \epsilon(A + F)^T X^T + \epsilon Y_1^T + P_{11}, \\ \Omega_{22} &= -e^{\alpha \tau} Q - 4Z, \quad \Omega_{23} = -P_{22} + 6Z/\tau, \\ \Omega_{33} &= -12Z/\tau^2 - \alpha P_{22}, \quad \Omega_{55} = \tau^2 Z - \epsilon(X + X)^T, \\ \Pi_{11} &= P_{11} + 2\tau Z - R, \quad \Pi_{22} = P_{22} + Q/\tau + 2Z/\tau. \end{aligned}$$

Moreover, if feasible solutions exist, then the controller gain matrices are given by $K_1 = X^{-1}Y_1$ and $K_2 = X^{-1}Y_2$.

Proof: Let a positive scalar $\alpha > 0$ be given and choose the following augmented Lyapunov-like functional:

$$\begin{aligned} V(t) &= \eta^T(t)P\eta(t) + \int_{t-\tau}^t e^{\alpha(t-s)} r^T(s)Qr(s) ds \\ &\quad + \tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{r}^T(s)Z\dot{r}(s) ds d\theta \end{aligned} \quad (14)$$

where $\eta(t) = [r^T(t) \int_{t-\tau}^t r^T(s) ds]^T$ and $P > 0, Q > 0, Z > 0$. By direct calculations, it follows that

$$\begin{aligned} \dot{V}(t) &\leq 2\eta^T(t)P\dot{\eta}(t) + r^T(t)Qr(t) + \tau^2 \dot{r}^T(t)Z\dot{r}(t) \\ &\quad - e^{\alpha \tau} r^T(t - \tau)Qr(t - \tau) - \tau \int_{t-\tau}^t \dot{r}^T(s)Z\dot{r}(s) ds \\ &\quad + \alpha \int_{t-\tau}^t e^{\alpha(t-s)} r^T(s)Qr(s) ds. \end{aligned} \quad (15)$$

Using the first inequality of Lemma 1, we have

$$\tau \int_{t-\tau}^t \dot{r}^T(s)Z\dot{r}(s) ds \geq \begin{bmatrix} \vartheta_1(t) \\ \vartheta_2(t) \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ 0 & 3Z \end{bmatrix} \begin{bmatrix} \vartheta_1(t) \\ \vartheta_2(t) \end{bmatrix} \quad (16)$$

where

$$\begin{aligned} \vartheta_1(t) &= r(t) - r(t - \tau), \\ \vartheta_2(t) &= r(t) + r(t - \tau) - \frac{2}{\tau} \int_{t-\tau}^t r(s) ds. \end{aligned}$$

For a given non-zero scalar ϵ and any matrix X , it is seen from the system (8) that the following equation is ensured:

$$\begin{aligned} 2[r^T(t)X + \epsilon \dot{r}^T(t)X][(A + F + K_1)r(t) + K_2r(t - \tau) \\ + B\omega(t) - \dot{r}(t)] = 0. \end{aligned} \quad (17)$$

Adding the left side of (17) to $\dot{V}(t)$ and using (16), we have

$$\begin{aligned} \dot{V}(t) &\leq \xi^T(t)\Omega\xi(t) + \alpha V(t) \\ &\quad - r^T(t)(S/\gamma^2)r(t) + \omega^T(t)\omega(t) \end{aligned} \quad (18)$$

where

$$\begin{aligned} \xi(t) &= \left[r^T(t)r^T(t - \tau) \int_{t-\tau}^t r^T(s) ds \omega^T(t) \dot{r}^T(t) \right]^T, \\ \Omega &= \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & XB & \tilde{\Omega}_{15} \\ * & \Omega_{22} & \Omega_{23} & 0 & \epsilon K_2^T X^T \\ * & * & \Omega_{33} & 0 & P_{12}^T \\ * & * & * & -I & \epsilon B^T X^T \\ * & * & * & * & \Omega_{55} \end{bmatrix}, \end{aligned}$$

and $\Omega_{13}, \Omega_{22}, \Omega_{23}, \Omega_{33}, \Omega_{55}$ are denoted in Theorem 1 and

$$\begin{aligned} \tilde{\Omega}_{11} &= X(A + F + K_1) + (A + F + K_1)^T X^T \\ &\quad + P_{12} + P_{12}^T + Q + S/\gamma^2 - 4Z - \alpha P_{11}, \\ \tilde{\Omega}_{12} &= XK_2 - P_{12} - 2Z, \\ \tilde{\Omega}_{15} &= -X + \epsilon(A + F + K_1)^T X^T + P_{11}. \end{aligned}$$

Denote that $Y_1 = XK_1$ and $Y_2 = XK_2$. Clearly, if LMI (9) holds, we obtain $\Omega < 0$. Moreover, we have from (18) that

$$\dot{V}(t) \leq \alpha V(t) - r^T(t)(S/\gamma^2)r(t) + \omega^T(t)\omega(t). \quad (19)$$

Integrating both sides of (19) from 0 to t yields that

$$V(t) \leq e^{\alpha t}V(0) + \int_0^t e^{\alpha(t-s)}\omega^T(s)\omega(s)ds - \frac{1}{\gamma^2} \int_0^t e^{\alpha(t-s)}r^T(s)Sr(s) ds. \quad (20)$$

Under the zero-initial condition, it is seen from (20) that

$$\int_0^T r^T(s)Sr(s) ds < (\gamma^*)^2 \int_0^T \omega^T(s)\omega(s) ds \quad (21)$$

where $\gamma^* = e^{\alpha T/2}\gamma$, which means that the system (8) satisfies the performance requirement for all nonzero $\omega(t) \in L_2[0, \infty)$.

Next, it remains to prove that the closed-loop error system (8) is finite-time bounded with respect to (c_1, c_2, δ, T) .

Using the inequalities (2)–(3) of Lemma 1, one obtains

$$V(t) \geq \eta^T(t)P\eta(t) + \frac{1}{\tau} \left(\int_{t-\tau}^t r(s) ds \right)^T Q \times \left(\int_{t-\tau}^t r(s) ds \right) + \frac{2}{\tau} \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{r}(s) ds d\theta \right)^T \times Z \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{r}(s) ds d\theta \right) = \eta(t)\Pi\eta(t) \quad (22)$$

where

$$\Pi = \begin{bmatrix} P_{11} + 2\tau Z & P_{12} - 2Z \\ * & P_{22} + Q/\tau + 2Z/\tau \end{bmatrix}.$$

From (10) and (22), it follows that

$$V(t) \geq r^T(t)Rr(t) \geq \lambda_m(R)\|r(t)\|^2. \quad (23)$$

On the other hand, from (11), (12) and (20), we have

$$V(t) \leq \left\{ [\lambda_M(J_1) + \tau^2\lambda_M(J_2) + \tau e^{\alpha\tau}\lambda_M(Q)] \sup_{-\tau \leq s \leq 0} \|r(s)\| + 0.5\tau^3\lambda_M(Z) \sup_{-\tau \leq s \leq 0} \|\dot{r}(s)\| + \delta \right\} e^{\alpha T} \quad (\forall t \leq T) \leq [(\lambda_1 + \tau^2\lambda_2 + \tau e^{\alpha\tau}\lambda_3 + 0.5\tau^3\lambda_4)c_1^2 + \delta] e^{\alpha T}. \quad (24)$$

Noting that $R \geq \lambda_5 I$ in (12) and (13), it is clear from (23) and (24) that the relationship $\|r(t)\| \leq c_2$ ($\forall t \in [0, T]$) is ensured for all initial conditions $r(t)$ ($-\tau \leq t \leq 0$) satisfying $\sup_{-\tau \leq s \leq 0} \{\|r(s)\|, \|\dot{r}(s)\|\} \leq c_1$. This completes the proof. ■

Remark 3.1: In Theorem 3.1, the parameters $\delta, \tau, c_1, c_2, T$ and γ play a vital role in the design of finite-time H_∞ controller. Generally speaking, we can optimize one of such parameters by fixing the values of other parameters in designing the controller. For example, we can search the maximum admissible c_1 by fixing the values of δ, τ, c_2, T and γ . Of course, we can also minimize c_2 for given δ, τ, c_1, T and γ .

Corollary 3.2: For given scalars $\tau > 0, \gamma > 0$ and $\epsilon \neq 0$, the system (8) is asymptotically stable with an H_∞ performance level γ if, there exist matrices $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0, Q > 0, Z > 0, X, Y_1$ and Y_2 such that the following LMIs hold:

$$\begin{bmatrix} \check{\Omega}_{11} & \Omega_{12} & \check{\Omega}_{13} & XB & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & 0 & \epsilon Y_2^T \\ * & * & \check{\Omega}_{33} & 0 & P_{12}^T \\ * & * & * & -I & \epsilon B^T X^T \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0 \quad (25)$$

where $\Omega_{12}, \Omega_{15}, \Omega_{22}, \Omega_{23}, \Omega_{55}$ are denoted in Theorem 1 and

$$\check{\Omega}_{11} = X(A + F) + (A + F)^T X^T + Y_1 + Y_1^T + P_{12} + P_{12}^T + Q + S/\gamma^2 - 4Z, \check{\Omega}_{13} = P_{22} + 6Z/\tau, \check{\Omega}_{33} = -12Z/\tau^2.$$

Moreover, if feasible solutions exist, then the controller gain matrices are given by $K_1 = X^{-1}Y_1$ and $K_2 = X^{-1}Y_2$.

Remark 3.2: Recently, the global H_∞ control problem has been studied in Zhao and Wang (2014) for the chaotic finance system (3) via the delayed feedback controller (4). It is noted that the global design is concerned with all unstable fixed points. Thus, such a global scheme might be ineffective in designing the delayed controller. Different from the approach in Zhao and Wang (2014), our proposed results are local and each fixed points can have its own controller. Moreover, the advanced Wirtinger-based inequality is used in this paper to deal with the time delay. Therefore, our obtained conditions will be more effective than the result proposed in Zhao and Wang (2014).

Remark 3.3: Over the past a decade, the finite-time control/state estimation problems have been widely investigated for various kinds of dynamical systems with time delays. However, it is worth pointing out that the existing finite-time boundedness analysis approaches in Ali et al. (2016), Ma et al. (2012), Niamsupa and Phat (2016), Song and He (2015) and Zhao et al. (2013) are conservative for time-delay systems, since the time-delay information has been partly overlooked. Different from the existing approaches, the useful terms $\int_{t-\tau}^t e^{\alpha(t-s)}r^T(s)Qr(s) ds$

and $\tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{r}^T(s)Z\dot{r}(s) ds d\theta$ are specifically taken into account in this paper when performing the finite-time boundedness analysis (see (22)) and then the delay-dependent constraint condition (10) is induced. Noting that the Lyapunov-like functionals $V(t)$ in Ali et al. (2016), Ma et al. (2012), Niamsupa and Phat (2016), Song and He (2015) and Zhao et al. (2013) are simply bounded by a delay-independent term of the form $r^T(t)Pr(t)$, it is obvious that our proposed finite-time boundedness analysis approach utilize the time-delay information sufficiently, which allows us to establish the less conservative design conditions.

If the terms $\int_{t-\tau}^t e^{\alpha(t-s)}r^T(s)Qr(s) ds$ and $\tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{r}^T(s)Z\dot{r}(s) ds d\theta$ are ignored in finite-time boundedness analysis, then LMI (10) is degraded as follows:

$$\begin{bmatrix} P_{11} - R & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0. \tag{26}$$

4. Numerical simulation

In this section, we will show the effectiveness and benefits of our proposed results by numerical simulation. In the simulation, the parameters are selected as $a = 2.5, b = 0.2, c = 1.2, S = I, B = [0.2 \ 0.2 \ 0.2]^T$ and $\omega(t) = 1/(1 + 2t)$. In this case, it can be verified that the finance system (1) has three unstable fixed points, which are represented as follows:

$$(\kappa_1, \kappa_2, -\kappa_3) \triangleq \mathbb{P}_1^*, \quad (-\kappa_1, \kappa_2, \kappa_3) \triangleq \mathbb{P}_2^*, \quad (0, 5, 0) \triangleq \mathbb{P}_3^*$$

where $\kappa_1 = 0.5774, \kappa_2 = 3.3333$ and $\kappa_3 = 0.4811$.

First, we consider the H_∞ control problem by the delayed feedback $u(t) = K(x(t) - x(t - \tau))$. By simple verifications, it is found that the condition in Zhao and Wang (2014) is infeasible for any γ and τ . For the fixed point \mathbb{P}_3^* , our proposed LMI (25) is also infeasible. However, letting $\tau = 1, \epsilon = 0.7$ and solving LMI (25) with $Y_1 = -Y_2 \triangleq Y$ in this paper, one can obtain the minimum H_∞ performance levels $\gamma = 1.12$ and $\gamma = 0.88$, respectively, for the points \mathbb{P}_1^* and \mathbb{P}_2^* . The corresponding controller gains $K_1 = -K_2 \triangleq K$ are obtained as follows:

$$K = \begin{bmatrix} -0.5462 & 0.7042 & 0.2062 \\ -0.5538 & -0.2991 & 0.0803 \\ -0.1078 & 0.0220 & -0.9713 \end{bmatrix} \tag{\mathbb{P}_1^*},$$

$$K = \begin{bmatrix} 0.0055 & -0.7986 & 0.7417 \\ 0.7523 & -0.6346 & 0.0924 \\ -0.1655 & 0.3078 & -1.2487 \end{bmatrix} \tag{\mathbb{P}_2^*}.$$

If a larger γ is selected in solving LMI (25), e.g. $\gamma = 3$, we have the following controller gain matrices:

$$K = \begin{bmatrix} -0.8621 & 0.5919 & 0.2198 \\ -0.4349 & -0.8928 & -0.1930 \\ 0.0836 & -0.1669 & -0.2246 \end{bmatrix} \tag{\mathbb{P}_1^*},$$

$$K = \begin{bmatrix} -0.8186 & -0.5928 & 0.1874 \\ 0.3871 & -0.9055 & 0.1359 \\ 0.0956 & 0.1236 & -0.2363 \end{bmatrix} \tag{\mathbb{P}_2^*}.$$

Using the above controller gains, state responses of the error system (6) are plotted in Figures 1–4. It is seen from Figures 1 and 2 that our proposed control scheme is indeed effective in controlling the unstable fixed points \mathbb{P}_1^* and \mathbb{P}_2^* . In addition, it can be seen from Figures 3–4 that the control design based on a larger level γ means the worse closed-loop performance.

Next, we will address the H_∞ control problem by the delayed feedback $u(t) = K_2(x(t - \tau) - x^*)$. For this problem, it is checked that the global design condition in Zhao and Wang (2014) is infeasible. Letting $\gamma = 2$ and solving LMI (25) with $Y_1 = 0$ in this paper, we can obtain the maximum admissible time delays $\tau = 1.39$ ($\epsilon = 11$), $\tau = 1.54$ ($\epsilon = 1.5$) and $\tau = 0.36$ ($\epsilon = 1.5$), respectively, for the fixed points $\mathbb{P}_1^*, \mathbb{P}_2^*$ and \mathbb{P}_3^* . The corresponding controller gains K_2 are obtained as follows:

$$K_2 = \begin{bmatrix} -0.6067 & -0.2869 & -0.5521 \\ 0.9195 & -0.1878 & 0.8320 \\ 0.5096 & 0.0880 & 0.4151 \end{bmatrix} \tag{\mathbb{P}_1^*},$$

$$K_2 = \begin{bmatrix} -0.4200 & 0.3094 & -0.2747 \\ -0.8401 & -0.0617 & -0.5395 \\ 0.4434 & -0.1581 & 0.2204 \end{bmatrix} \tag{\mathbb{P}_2^*},$$

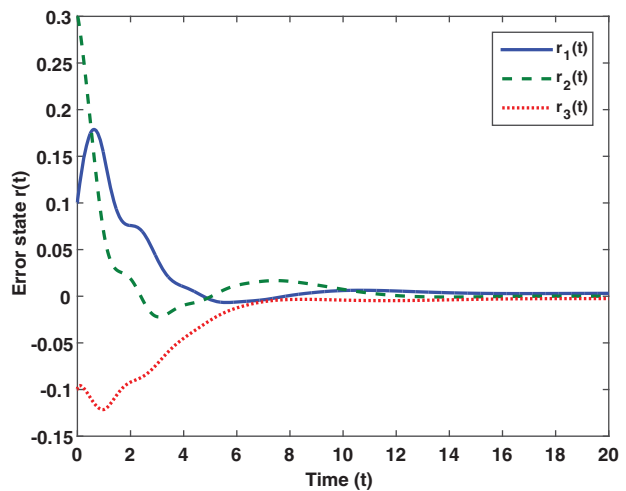


Figure 1. State responses of the error system (6) involving \mathbb{P}_1^* ($\gamma = 1.12$).

$$K_2 = \begin{bmatrix} -2.7160 & -0.1202 & -0.7065 \\ -0.0436 & -0.7533 & -0.2815 \\ 0.6867 & 0.0270 & -0.3279 \end{bmatrix} \quad (\mathbb{P}_3^*).$$

Based on the above controller gains, we plot the state responses of the error system (6) in Figures 5–10. It is clear from Figures 5–7 that our proposed controllers behave well. However, it can be seen from Figures 8–10 that the stability cannot be guaranteed for somewhat larger time delays.

Finally, we are concerned with the finite-time H_∞ control problem. Here, we adopt the feedback controller $u(t) = K_2(x(t - \tau) - x^*)$. Note that $\int_0^{+\infty} \omega^T(s)\omega(s) ds = 0.5$. Choose $c_1 = 0.25, \delta = 0.5, T = 5, \alpha = 0.05$ and $\gamma = 2$, and solve LMIs (9)–(13) with $Y_1 = 0$, we obtain the minimum $c_2 = 1.54$ ($\tau = 1.39, \epsilon = 1.5$), $c_2 = 1.56$ ($\tau = 1.54, \epsilon = 1.5$) and $c_2 = 1.84$ ($\tau = 0.36, \epsilon = 1$), respectively, for

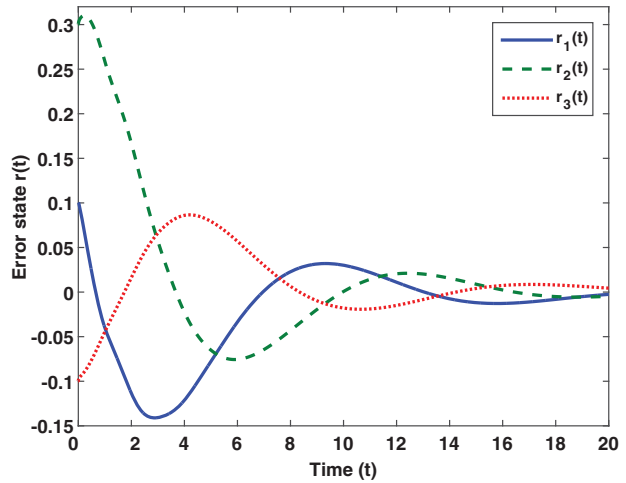


Figure 4. State responses of the error system (6) involving \mathbb{P}_2^* ($\gamma = 3$).

the fixed points $\mathbb{P}_1^*, \mathbb{P}_2^*$ and \mathbb{P}_3^* . The corresponding controller gains K_2 are obtained as follows:

$$K_2 = \begin{bmatrix} -0.5397 & -0.4390 & -0.2821 \\ 0.7886 & -0.0579 & 0.5392 \\ 0.4288 & 0.1686 & 0.2561 \end{bmatrix} \quad (\mathbb{P}_1^*),$$

$$K_2 = \begin{bmatrix} -0.4284 & 0.3986 & -0.1456 \\ -0.7891 & 0.0024 & -0.5219 \\ 0.4039 & -0.1788 & 0.2081 \end{bmatrix} \quad (\mathbb{P}_2^*),$$

$$K_2 = \begin{bmatrix} -2.7629 & 0.0116 & -0.6888 \\ 0.0467 & -0.9307 & 0.0415 \\ 0.8045 & -0.0776 & 0.0472 \end{bmatrix} \quad (\mathbb{P}_3^*).$$

In Figure 11–13, we plot the the state responses of the error system (6) based on the above controller gains. It is seen from Figure 11–13 that the state of the error system (6) satisfies the corresponding constraint $\|r(t)\| \leq c_2$

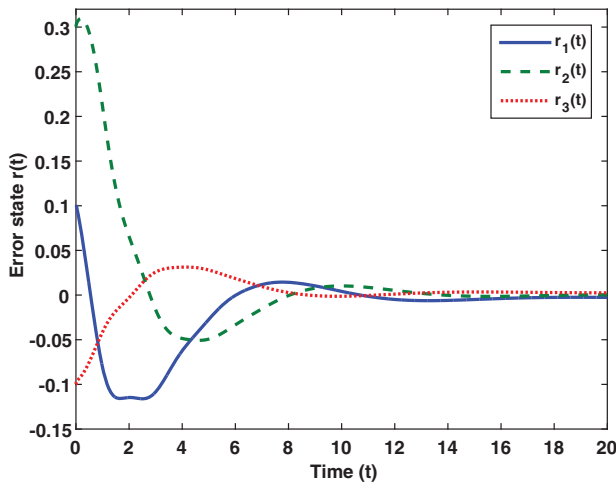


Figure 2. State responses of the error system (6) involving \mathbb{P}_2^* ($\gamma = 0.88$).

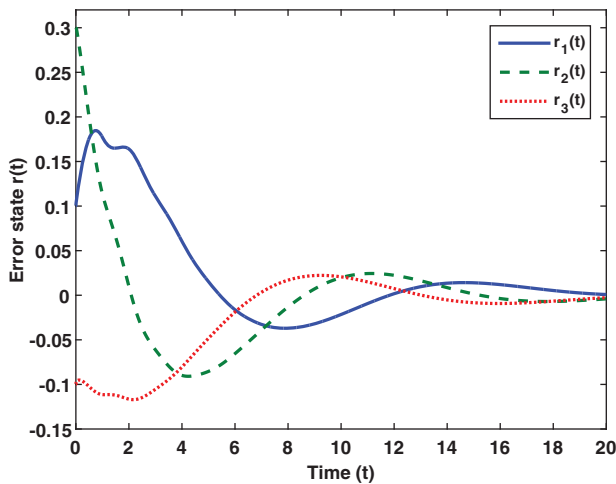


Figure 3. State responses of the error system (6) involving \mathbb{P}_1^* ($\gamma = 3$).

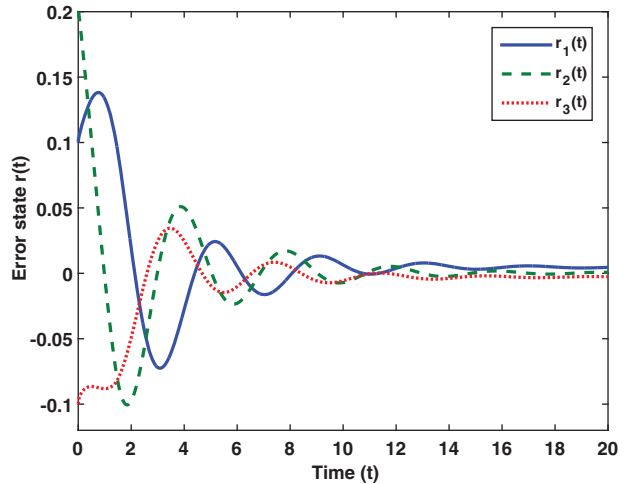


Figure 5. State responses of the error system (6) involving \mathbb{P}_1^* ($\tau = 1.39$).

($\forall t \in [0, 5]$), which shows the effectiveness of our proposed control scheme.

On the other hand, it is worth mentioning that, if LMI (10) is replaced by LMI (26) in minimizing the values of c_2 , the larger $c_2 = 1.84$, $c_2 = 1.84$ and $c_2 = 1.91$ can be achieved, respectively, for \mathbb{P}_1^* , \mathbb{P}_2^* and \mathbb{P}_3^* , which means that our proposed finite-time boundedness analysis approach is specifically effective in reducing the conservatism.

Remark 4.1: To reduce the possible conservatism, the adjusting parameter $\epsilon \neq 0$ is introduced in this paper. Generally speaking, different choosing of the scalar ϵ might result in different system performance, which can be shown by simulation. For example, let us consider the H_∞ control design for the fixed point \mathbb{P}_1^* under the delayed feedback $u(t) = K(x(t) - x(t - 1))$. Selecting the

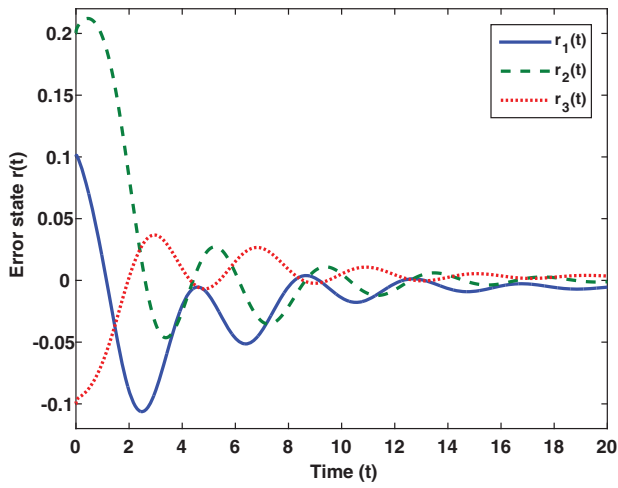


Figure 6. State responses of the error system (6) involving \mathbb{P}_1^* ($\tau = 1.54$).

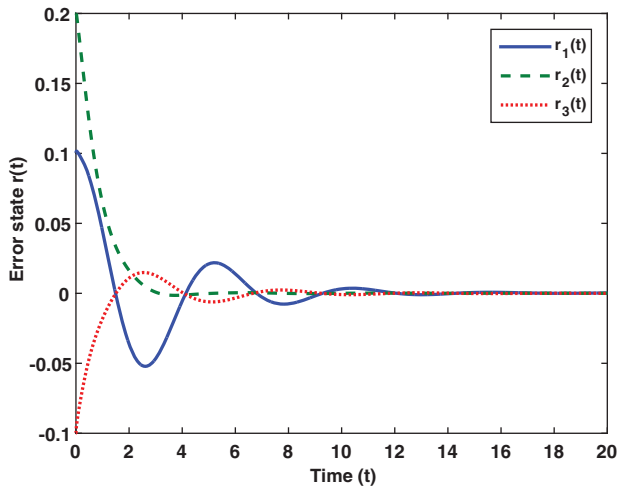


Figure 7. State responses of the error system (6) involving \mathbb{P}_2^* ($\tau = 0.36$).

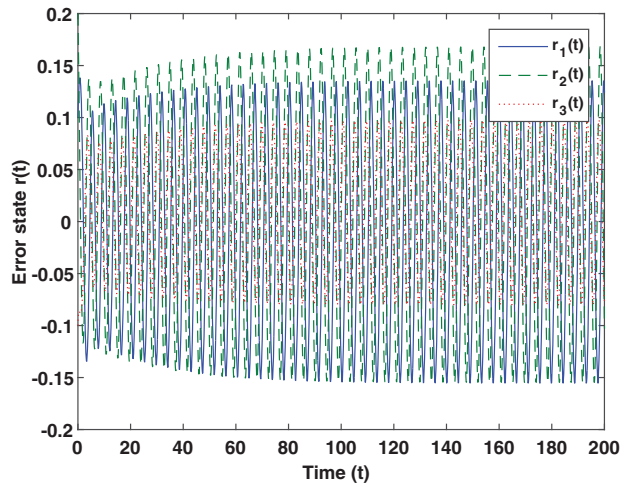


Figure 8. State responses of the error system (6) involving \mathbb{P}_1^* ($\tau = 1.78$).

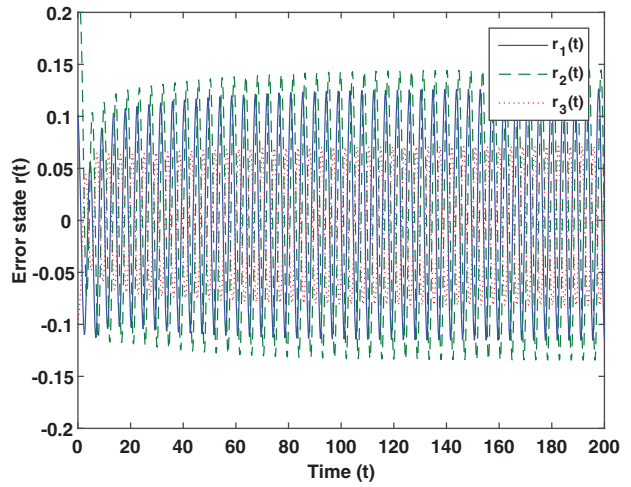


Figure 9. State responses of the error system (6) involving \mathbb{P}_2^* ($\tau = 1.89$).

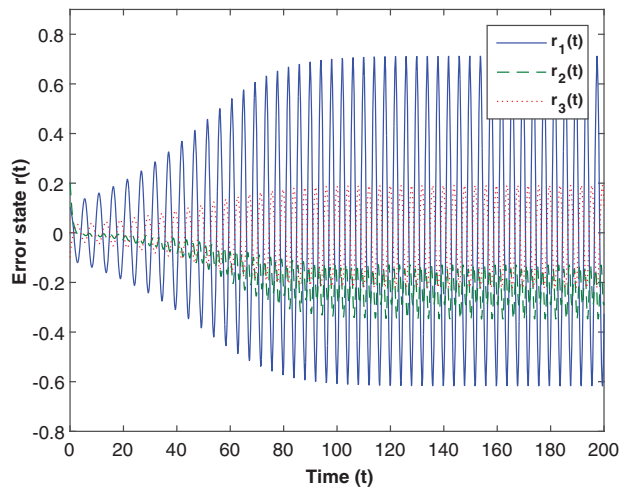


Figure 10. State responses of the error system (6) involving \mathbb{P}_3^* ($\tau = 0.42$).

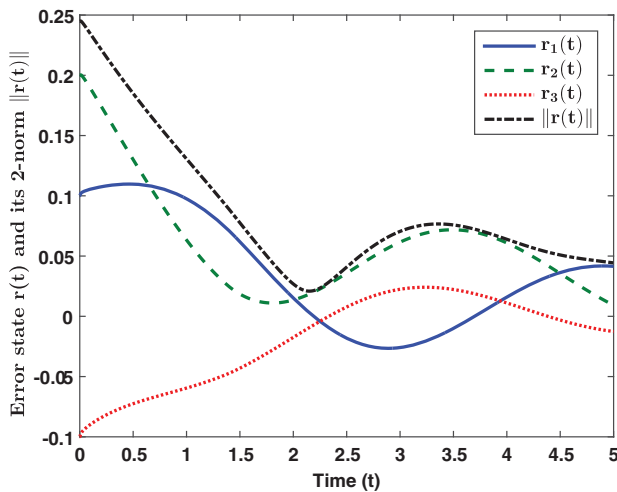


Figure 11. The error state $r(t)$ and its 2-norm $\|r(t)\|$ involving \mathbb{P}_1^* .

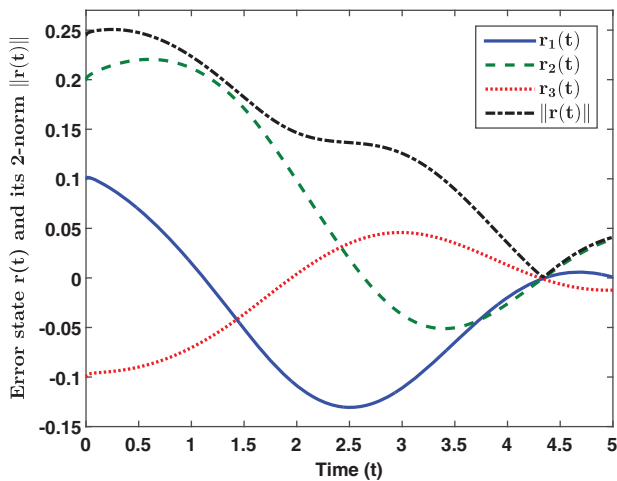


Figure 12. The error state $r(t)$ and its 2-norm $\|r(t)\|$ involving \mathbb{P}_2^* .

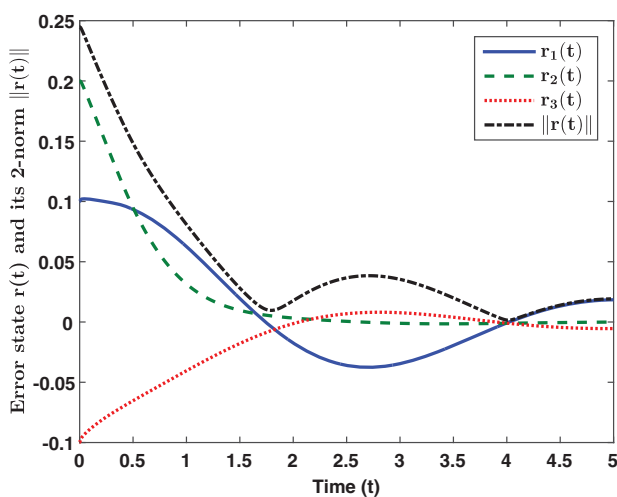


Figure 13. The error state $r(t)$ and its 2-norm $\|r(t)\|$ involving \mathbb{P}_3^* .

parameter ϵ as 0.1, 0.5, 0.7, 0.9 and 2.0, and solving LMI (25) with $Y_1 = -Y_2 \triangleq Y$, the achieved minimum H_∞ performance levels γ are, respectively, 4.33, 1.14, 1.12, 1.15 and 1.54. In addition, it is worth pointing out that the system performance is highly related to the time delay τ , which can also be illustrated by simulation. For example, let us address the H_∞ control problem for \mathbb{P}_1^* under the feedback controller $u(t) = K(x(t) - x(t - 1.2))$. Letting $\epsilon = 0.7$ and using Corollary 3.2, we have the minimum H_∞ performance level $\gamma = 1.26 > 1.12$, which implies that the system performance become worse as τ increases.

5. Conclusions

In this paper, we have addressed the finite-time H_∞ control problem for a class of chaotic finance system via delayed feedback controller. By using some less conservative techniques, a sufficient condition has been developed, which can guarantee that the closed-loop error system is finite-time bounded with a prescribed H_∞ performance level. As the by-product, a sufficient condition based on the standard H_∞ control scheme has also been established. The effectiveness and advantages of our obtained results have been sufficiently demonstrated by numerical simulation. The proposed analysis approach in this paper can be extended to some other kinds of finance systems.

On the other hand, it should be pointed out our proposed results are based on the simple linearization technique and, thus, the application range of our results is restricted. As the further research, we would like to utilize some advanced techniques, such as the piecewise-linear approach, the T-S fuzzy approach and neural networks to improve the results of this work. In addition, it is also interesting to investigate the synchronization control issue (Chen, Wang, Shen, & Dong, 2018; Wang, Wang, Han, & Wei, 2018) and the saturated control problem (Chen, Fei, & Li, 2017; Ma, Wang, Liu, & Alsaadi, 2017) for chaotic finance systems without or with stochastic disturbances (Ma, Wang, & Lam, 2017; Ma, Wang, Liu, et al., 2017).

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