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The Impact Of Using A Computer Algebra System In High School Calculus On High Performing Students' Conceptual And Procedural Understanding

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THE IMPACT OF USING A COMPUTER ALGEBRA SYSTEM IN HIGH SCHOOL CALCULUS ON HIGH PERFORMING STUDENTS' CONCEPTUAL AND PROCEDURAL UNDERSTANDING

by

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ABSTRACT

Recently, there has been an increasing interest in high school mathematics education, especially in the teaching and learning of calculus. For example, studies conducted by Bressoud (2010); Judson and Nishimori (2005); Koh and Divaharan (2011); and St. Jarre (2008) all looked at how to improve the understanding of calculus students and what roles the educator must take to ensure that their students are successful. The purpose of this study was to determine if there was a significant difference between instruction using computer algebra system (CAS) compared to instruction using the graphing calculator in high school calculus on students' conceptual and procedural understanding. This study explored and compared two different types of instruction based on the use of two different types of technology, CAS and graphing calculator.

The total population for this study consisted of 333 students. There were 187 students classified as using the graphing calculator and 146 students classified as using CAS. The data for this study were collected from four Advanced Placement (AP) calculus AB courses from high schools in Florida. The study used observations and two sets of calculus tasks in order to gather data. The research questions for this study looked at comparing the grades of students categorized based on the type of instruction received during the learning of calculus. The statistical procedure that was used was a simple one-way analysis of variance (ANOVA). The results indicated that there was no significant difference between the two types of instruction on the students' procedural knowledge,

however, there was statistical significance on the students' conceptual understanding in favor of the CAS students.

The study introduces a framework on how to obtain information about the effects of different types of instruction on students' understanding of calculus. The results of this study contribute in assisting teachers and future researchers on how to analyze student work in order to obtain information about the students' conceptual and procedural understanding of first semester calculus.

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LIST OF ACRONYMS/ABREVIATIONS

AP	Advanced Placement
ANOVA	Analysis of Variance
CAI	Computer Assisted Instruction
CAS	Computer Algebra System
DGS	Dynamic Geometry Software
FCAT	Florida Comprehensive Assessment Test
GC	Graphing Calculator
ICT	Information and Communication Technology
NCTM	National Council of Teachers of Mathematics
NSF	National Science Foundation
NPS	Non-participating Schools
OECD	Organization for Economic Co-operation and Development
PISA	Programme for International Student Assessment
SMG	Symbolic Math Guide
STEM	Science, Technology, Engineering, and Mathematics
TRS	Technology Ranking Scale

CHAPTER ONE: INTRODUCTION

Rationale

According to the Organization for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA), 2009 results, the United States ranked at 32 out of 65 countries in mathematics performance. The highest percentage of United States students were at level three, which is categorized as students having the ability to execute, clearly describe procedures, and select and apply simple problem-solving strategies (PISA, 2010). United States students' have been ranked as low achieving since the 1950's (Robinson, 2003).

Students that wish to pursue careers in science, technology, engineering, and mathematics (STEM) will have calculus as their entry-point and pre-requisite to undergraduate mathematics (Robinson, 2003). Due to the importance in STEM disciplines and the size of the initial enrollment into those degree programs, there have been many research studies (e.g., Ferrini-Mundy & Graham, 1991; Heid, 1988; White & Mitchelmore, 1996) examining students' understanding of calculus. According to Chappell (2003), the students' failure to develop a conceptual understanding of calculus is the fault of the traditional calculus curriculum. As a result, argue Cadena, Travis, and Norman (2003), the path to reform calculus was paved.

The review of literature discusses studies that have investigated the effects on students' understanding and retention of secondary school calculus on performance in first and second semester calculus in college (Ferrini-Mundy & Gaudard, 1992; St. Jarre, 2008; Wu, 1997; Bressoud, 2010; Bressoud, 2004). Ferrini-Mundy and Gaudard found that students who complete high school calculus struggled in second semester calculus. Bressoud (2010) reports that calculus two enrollments declined by more than 30 percent between 1995 and 2005. As a result, there is also a steep decline in students pursuing STEM careers.

Since the early 1990s, there has been much change in the teaching of calculus in secondary school and college calculus. According to Garner and Garner (2001), the advances in technology have been challenging the teaching of the traditional curriculum in calculus. In addition, researchers (Bressoud, 2010; Chappell, 2003; Ferrini-Mundy & Gaudard, 1992; Garner & Garner, 2001; St. Jarre, 2008) have shown that students who take traditional taught calculus courses lack retention of facts and skills as the students take second semester calculus courses. Garner and Garner suggest that this finding is due to the lack of conceptual understanding that the students obtain from a traditional curriculum. Due to the high failure and withdrawal rates from traditional courses and researchers (Edwards & Heid, 2001; Heid, 1988; , Judson, 1988) findings that using ideas of the reform calculus movement promotes a higher student success rate, reform calculus has become popular. Recent research (Scheja & Petterson, 2010; Tarmizi, Ayub, Baker, & Yunus, 2010; Zerr, 2010) has investigated the use of technology in calculus instruction to improve students' conceptual understanding.

The National Council of Teachers of Mathematics (NCTM) (2000) emphasized the importance of the use of technology in mathematics education, stating that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p.11). There are different types of technology that are and can be used within instruction. The graphing calculator, Dynamic Geometry Software (DGS), computer algebra systems (CAS) and the interactive white board are just a few of the technologies used today in classrooms. Research has shown that students can benefit from technology integration because it allows students of different mathematical skills and levels ways to use visualization and exploration in multimedia environments, which can foster understanding in new ways (Hohenwarter, Hohenwarter, & Lavicza, 2008).

The NCTM (2000) also mentions that teachers can benefit from the integration of technology because it can provide creative opportunities for supporting students' learning and fostering the gain of mathematical knowledge and skills. Students "may focus more intently on computer tasks and may benefit from the constraints imposed by a computer environment" (p. 24). With the knowledge of the uses of technology, studies have shown that the teachers' perspectives on the use of technology during instruction were positive (Haciomeroglu, Bu, & Haciomeroglu, 2010).

In a study by Ozgun-Koca (2010), the researcher found that pre-service secondary teachers were able learn and understand algebra through the use of advance calculators with CAS in algebra instruction. Ozgun-Koca used an open-ended questionnaire and group interviews in order to reveal the pre-service teachers beliefs of when and why to use three possible versions of CAS; black box, white box, or Symbolic Math Guide (SMG). The study suggests that white box and SMG were preferred over the black box method. In addition, the black box method could be effective if students mastered the basic skills needed to preform simple computations by hand.

Related Research

Computer Algebra System (CAS)

In the spring of 2011, the researcher audited an AP calculus AB course. This was the first time the researcher had seen the use of CAS. DERIVE, which is the type of CAS that was used, was built into the TI N-spire; the calculator set the class used. To promote conceptual understanding, Hughes-Hallett et al. (2000) argue, teachers need to consider the "rule of four" when teaching mathematical concepts in calculus. The "rule of four" refers to concepts being viewed algebraically, graphically, numerically, and verbally. In order for teachers to use CAS effectively and be able to explain the system to students, argues Nunes-Harwitt (2005), they must understand the mechanisms behind the software. When teachers improve their knowledge of CAS, they can better apply the "rule of four."

According to Heid and Edwards (2001), CAS has been available in graphing calculators since the mid 1990s. The use of CAS in secondary calculus courses, mention Heid and Edward, support the development of deeper conceptual understanding by allowing linked multiple representation. According to King (1997), when CAS was used in the classroom, the students' overall achievement was greatly improved. Barton (2001) conducted a review of 52 studies on the effects of using CAS in a calculus environment and found that that those students who used CAS in the classroom had a better overall achievement of conceptual understanding, while there was no significant difference in procedural understanding.

Pierce and Stacey (2001) collected data on college calculus students using DERIVE through surveys, observations, and assessments. The study found that students were able to relate algebraic and graphical representations of functions better than the students that did not use DERIVE. According to Pierce and Stacey, the use of CAS had positive effects on using various representations and encouraging student discussions. In a similar study looking at CAS usage, Ball and Stacey (2004) found that students who used CAS during an exam illustrated their answers using more symbols and mathematical notations rather than words.

Palmiter (1991) investigated the difference between students taught using CAS in calculus versus students using paper and pencil methods and their knowledge of procedures. According to Palmiter, there was a significant difference in scores, illustrating those students in the CAS group scored higher on both the conceptual and computational parts of the final exam. Monaghan, Sun, and Tall (1994) found that when students used CAS to carry out the process of differentiation, they responded by explaining the keystrokes that were necessary to get the result rather than illustrating comprehension of the topic. Monaghan et al. concluded that students were replacing one procedure, which had little conceptual meaning, with another. According to Gaulke (1998), there were insignificant differences found between students using CAS and students who did not in terms of conceptual understanding. Gaulke did conclude that using CAS did not result in any loss of computational ability.

Conceptual Understanding and Procedural Knowledge

Mathematical knowledge, Hiebert and Lefevre (1986) suggest, can be thought of as being either, conceptual or procedural. According to Chappell and Killpatrick (2003), based on years of research, there is an important distinction that has been made between the two types of knowledge. Skemp (1976) began the idea of the distinction by discussing two types of understanding, instrumental and relational. Instrumental, argues Skemp, means having procedural knowledge, which he described as the knowledge of performing without the understanding of the why. Skemp, referred to relational understanding, as the understanding of the "how" and the "why". Hiebert and Lefevre began using the terminology procedural and conceptual to represent instrumental and relational, respectfully. According to Hiebert and Lefevre, procedural knowledge refers to an understanding of the rules for completing mathematical tasks with the inclusion of symbolic language. Conceptual understanding, on the other hand, is knowledge of mathematical ideas, with the understanding of why algorithms, those are being used, work and when they should be applied. According to Slesnick (1982) students' use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Furthermore, knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Calculus Reform

In the 1960s, the first idea of "New Math" was being implemented in United States classrooms and covered various areas of mathematics that included elementary algebra, geometry, trigonometry, and calculus (Bosse, 1995). The idea was to encourage mathematical fluency, support answers with proofs, and to recognize mathematical concepts from a concrete or abstract situation (Wu, 1997). As a result, in 1987, "Calculus for a New Century," was a reform movement that was supported by the National Science Foundation (NSF) (Hurley, Koehn, & Ganter, 1999). Cadena et al. (2003) argue that the major concern that led to the 1987 reform was that half of the students that enrolled in a first semester calculus course would fail or withdraw from the course.

Studies on comparing reform-taught calculus versus traditionally taught calculus and the results on student learning and understanding illustrates different results. One side makes a strong argument that those students who completed reform calculus courses are not able to keep up with traditionally taught students due to their lack of basic skills (Chappell, 2003). On the other side, instructors found that the majority of students grasped concepts more effectively, with a higher level of conceptual understanding than the traditionally taught students, as well as being more motivated to learn calculus than the traditionally taught students (Cadena et al., 2003). The arguments come down to procedural understanding versus conceptual understanding, based on the major factor of the reform, which was the introduction of using technology to assist in teaching calculus (Garner & Garner, 2001). When given a closer look, the technology that was introduced was the use of computer labs and a computer algebra system (CAS) empowered calculator (Hurley, Koehn, & Ganter, 1999).

In two studies, Chappell (2003) and Cadena et al. (2003), both showed that students in the reform calculus course earned more A's and B's than the traditionally taught calculus students. In addition, there was no significant difference in grades in second semester calculus students. However, Chappell noticed that students who completed the reform taught calculus course had a higher withdrawal rate than the traditionally taught calculus course in second semester calculus. According to Chappell, there were three issues observed from the reform taught calculus students that withdrew from second semester calculus.

1) They were no longer content to work through procedural problems without a conceptual understanding of what they were doing.

2) The effort required to be successful increased tremendously in second semester calculus than their reform taught calculus course.
3) In second semester calculus they relied primarily on memorization when taking exams rather than the conceptual instruction that helped them in the reform taught calculus course.

Ferrini-Mundy and Gaudard (1992) addressed the relationship of experience of secondary school calculus to performance in first-year college calculus and second-year calculus. The study focused on the retention of students' coming in with previous knowledge of calculus and if taking secondary school calculus actually improves overall performance in college calculus. Ferrini-Mundy and Gaudard investigated if there were performance differences based on procedural skills and conceptual understanding. The findings of Ferrini-Mundy and Gaudard's study showed that students that studied secondary school calculus were more successful in the college first semester calculus course. The major difference in performance, argued Ferrini-Mundy and Gaudard, was due to procedural proficiency illustrated by students who completed a year of secondary school calculus.

In 2005, Judson and Nishimori conducted a study that compared the best calculus students from the United States with calculus students from Japan. They looked at the impact of using a reformed calculus course that allowed the use of technology compared to the traditional calculus curriculum whose focus was on hand computations. Judson and Nishimori found little difference in the conceptual understanding of calculus between the Japanese and United States high school calculus students. Even though the Japanese students had no experience with the calculus reform problems, they were just as successful as the United States students. Both the

Japanese and United States students illustrated a strong grasp of the mechanics of calculus and understood the derivative as a rate of change and how it can be used to sketch the graphs of functions. However, the United States students did have a weaker understanding of the meaning of the Fundamental Theorem of Calculus, and understanding of functions. The main advantage that the Japanese students had was that they showed a higher understanding of procedural skills than the United States students.

Purpose of Study

According to the NCTM (2000), technology can support students' conceptual understanding of various areas of mathematics such as geometry, trigonometry, and calculus. Saha, Ayub, and Tarmizi (2010) argue, advances in computers have brought great innovations and teachers need to be up to date on the advances in order to maximize the uses in teaching and learning. The present study attempted to address the relationship of the secondary school (high school) calculus experience, with and without, the use of CAS empowered graphing calculators, on students' conceptual and procedural understanding of calculus concepts. The goals of the study were to:

- determine if taking secondary school calculus using CAS will improve overall performance in calculus; and
- determine if using CAS will affect conceptual and procedural performance.

Research Questions

This study examined the following research questions.

1) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of procedural calculus tasks?
2) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of conceptual calculus tasks?
3) Is there a significant difference in performance between high performing students that have had instruction with computer algebra system (CAS) and high performing students that have had instruction with the graphing calculator (GC) on calculus tasks?

Potential Contribution

This study attempted to build on Judson and Nishimori's (2005) findings by incorporating the CAS environment into the methodology. An item that was lacking in the literature was looking at how using a CAS in secondary school calculus instruction can affect the performance of students' understanding of calculus concepts and how it can impact the students' conceptual understanding and procedural knowledge. This study contributes to the literature relative to students' understanding of calculus and promoting higher enrollments in STEM majors.

CHAPTER TWO: LITERATURE REVIEW

Introduction

The literature review for this study includes five parts: (a) history of computer's in mathematics education; (b) technology in mathematics education; (c) debate on conceptual understanding versus procedural fluency; (d) the calculus reform movement; and (e) retention of skills and knowledge of mathematics. Discussing these topics will contribute to the understanding of the changes that occurred in the past sixty years in mathematics education, with focus on how the teaching of calculus has evolved. With change comes argument, which in turn creates research studies as well as expert opinionated discussions. I will begin with the history of computers and technology in and the influence it has had on the teaching and learning of calculus. The review of literature will focus on the debate related to conceptual understanding and procedural knowledge and discuss how this debate has influenced the reform movement in calculus. Finally, I will complete the review of the literature with the comparison of significant studies that have helped shape my own study.

Computers in Mathematics Education

Researchers argue that the computer can be used as a tool to complement mathematical thinking. However, the same researchers also argue that to be a successful tool in education, the computer should be used for conceptual purposes rather than for symbolic manipulation (Dubinsky & Tall, 1991; Kaput, 1992; Papert, 1980). When first introduced into the classroom, the computer primarily was used as a provider of drill-and-practice examples in addition to those already included with the textbooks being used (Kaput, 1992). However, as time progressed, computer software began to focus on the conceptual teaching of mathematics and has influenced both the study of mathematics and the curriculum to support conceptual understanding (Rubin, 1999).

History

Computers began being used in education in the early sixties following the "Sputnik" launch of 1957. The U.S.S.R. launched the satellite and the United States, according to Bybee (1997), realized that a change in the curriculum, especially in mathematics and science, was eminent. The introduction of computers into education, argues Kaput (1992), had many obstacles to overcome. There were few computers available in schools and those that were available had very limited computational and display features. Software was scarce and the software that computers did have had been commercially produced computer-assisted instruction (CAI) software. Early CAI software was primitive and programmed to a drill-and-practice style of the learning of computation skills.

Aydin (2005) describes CAI programs, as course content in the form of drill-and-practice, tutorial, and simulations. CAI provides demonstrations, testing, information, and simple communications to the user. Researchers, (Hatfield, 1985; Kaput, 1992; Papert, 1980) argue that the main issue with CIA programs is they are mainly drill-and-practice based. As a result, CIA programs could not improve students' conceptual or relational understanding (Skemp, 1976). As described by Dubinsky and Tall (1991), to achieve success, computers in education should be used for explicit conceptual purposes. If computers are added to the curriculum without specific aims in mind, failure is inevitable.

In addition, during the 1960s and 1970s, teachers were not prepared for a computers presence in the classroom environment (Kaput, 1992). The computers were too difficult for teachers to implement into a constantly changing curriculum and the teachers had little to no training on how to use a computer. The results of a study conducted by Russell and Bradley (1997) showed that 71% of the 248 teachers used in their study indicated that they had no computer related subjects while attending Teacher Colleges or Universities. Following Russell and Bradley's study, researchers (Koh & Divaharan, 2011; Mishra & Koehler, 2006; Thompson & Mishra, 2007), conducted studies on Information and Communication Technology (ICT) in education because it articulated the different types of knowledge needed by teachers for technology integration. This lack of training for pre-service teachers increases the anxiety of using a computer in the classroom and supports Cuban's (1986, as cited by Kaput, 1992) statement; "Educators appear to have a deep set skepticism towards anything that plugs into the wall" (p. 2).

Unfortunately for teachers, however, fortunate for education, Hatfield (1985) found a correlation that there was a strong connection between thinking processes of learners during the writing of their own computer programs and aspects of mathematical thought. Dubinsky and Tall (1991) concur that computers provide a much-needed meaning to mathematic concepts and ideas that are agreed to be easier to understand when seen more concretely rather than as abstract.

Early uses of symbolic manipulators in mathematics education, according to Hodgson (1987, as cited by Dubinsky & Tall, 1991), had little effect on the teaching of mathematics in the classroom. In addition, students felt the software was too difficult due to its lack of user-friendliness to accept the program. Since the early 1980s, symbolic manipulators, explains Heid and Edwards (2001), performed complex computations that were not only time consuming when using paper and pencil, but in some cases, impossible. However, when performing the difficult computations, argues Char (1986, as cited by Dubinsky & Tall, 1991), the early versions of symbolic manipulators, prevent the students from understanding the steps to perform the computation and the meaning of what the concept represents. Students would give the right answers for the wrong reasons and give the wrong answers with a rational argument (Dubinsky & Tall, 1991).

As a result, in the 1990s, Heid and Edwards (2001) write, symbolic manipulators were incorporated into CAS that included graphical and numerical systems in addition to the symbolic utilities. Thus, Dubinsky and Tall (1991) recommended that the symbolic manipulator should be used in an appropriate conceptual environment, which might broaden the students' understanding. Heid and Edwards also mention that, unlike basic symbolic manipulators, CAS allowed students the opportunity to explore real world situations, find approximate answers using the graphical features, and continue to find exact answers with the symbolic algebra tools. Aydin (2005) concurs in that learning occurs when the students are given the opportunity to construct their own knowledge. CAS provides what researchers (Aydin, 2005; Dubinsky & Tall, 1991; Heid & Edwards, 2001) agree to be one of the most important aspects of learning mathematics, making abstract ideas more concrete. And when that is accomplished, argues Aydin, students will be able to shift from concrete to abstract.

Computer Algebra Systems (CAS)

CAS are computer programs that have existed for over 35 years and have been used in college calculus courses since the early 1990s, (Arnold, 2004; Nunes-Harwitt, 2005), to perform algebraic manipulation and computation. According to Nunes-Harwitt (2005), a CAS is incorporated in curricula designed for advanced courses, such as calculus and linear algebra. Heid and Edwards (2001) introduce four possible roles for using CAS in mathematics education. They were (a) using CAS to perform computations in order for the user to focus on the concepts, (b) using CAS to generate many examples to assist students on noticing symbolic patterns, (c) using CAS to generate results for problems posed in abstract form, and (d) using CAS as a pedagogical tool which creates and generates symbolic procedures to assist with the achievement of conceptual understanding in students. Researchers (Arnold; McAndrew; Nunes-Harwitt; Ozgun-Koca) have found that one of the great benefits of a CAS on student learning is that it offers a variety of options for students to experiment and explore the mathematic concepts, while removing the long and time wasting algebraic manipulations. An example of using CAS can be seen in Figure 1.

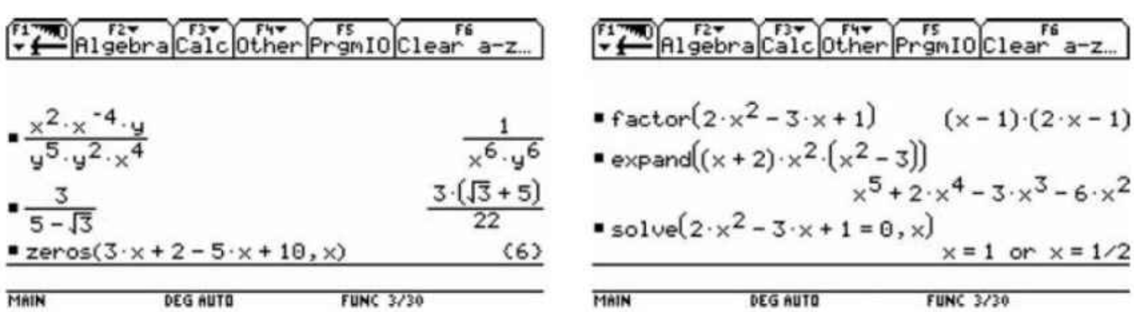


Figure 1: Example of CAS being used to solve equations and simplify expressions (Ozgun-Koca, 2010).

CAS in High School

When the idea of introducing CAS into secondary school (high school) mathematics courses emerged, teachers felt uncomfortable using CAS in the classroom for three reasons (Arnold, 2004). First, the CAS would completely take over the teaching aspect of the subject and leave little room for teacher involvement. Second, the students will never learn how to do the mathematics because, with the use of the CAS, the students' algebra skill will weaken. Finally, teachers had great concerns on what type of questions they would be able to ask on exams. Wilf (1982) questioned the possible effects of technology in the classroom, stating "teach more concepts and less of mechanics. What happens when computers can solve differential equations, do multiple integrals, vector analysis, etc.?" (p. 8).

Black Box

The first two criticisms and concerns stated above are addressed in a study by Ozgun-Koca (2010). In studying prospective teachers', Ozgun-Koca, offered three versions of CAS, which he labeled as "black box," "white box," and "Symbolic Math Guide (SMG)." Using "black box" meant that the CAS would provide the user with an answer, keeping the intermediate steps hidden. For example, if a student inputted an expression and ask for the first derivative the result would show without any work being displayed on the arrival of the answer. The "black box" version of CAS could be used to discuss Heid and Edwards (2001) (b) and (c) roles stated before. An example of the "black box" version of CAS can be seen in Figure 2. Arnold (2004) argues that this type of system supports the claims made by the teachers in his study. Drijvers (2004) argues that when the "black box" version of CAS performs the complex procedures, it is not transparent to the students. Drijvers believes that students need to approach problems using the traditional paper-and-pencil method in order to overcome, not avoid, the long process, to encourage relational understanding (Skemp, 1976) through procedural fluency. According to Drijvers, the "black box" version of CAS inhibits instrumentation.

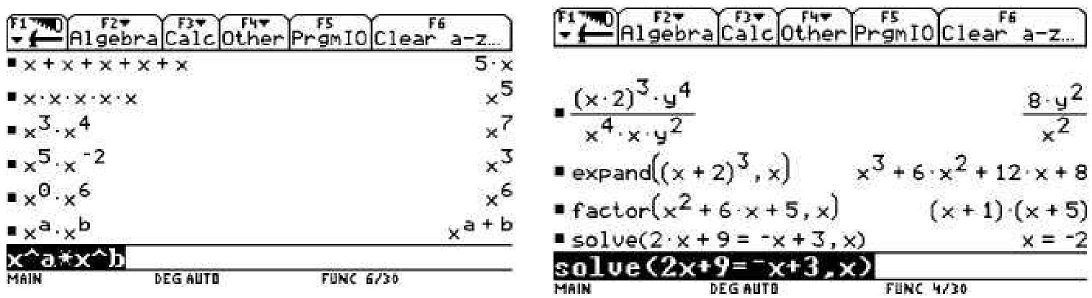


Figure 2: Example of the "Black Box" version of CAS (Ozgun-Koca, 2010).

However, McAndrew (2010) disagrees and mentions that using a system like the "black box," will provide time for students to be in an environment to experiment and explore, by removing time consuming and tedious algebraic manipulations. Edwards (2003) agrees with McAndrew, in that, using a CAS like that of "black box," could be used for quick and accurate algebraic computation, leaving room for, as well as, support of more conceptual understanding and development of problem-solving skills. Lumb, Monaghan, and Mulligan (2000) argue teachers might have to refocus their instruction on pattern recognition rather than memorization to gain understanding of the procedures using the "black box" version of CAS. Otherwise, as Drijvers (2004) argues, students' might not gain the conceptual insight to the mathematics being taught.

White Box

With the "white box" version of CAS, the student would input a question, and attempt a step to observe the outcome. The students' will then input the answer that was provided by the CAS and again attempt the next step and the output will show another completed step. The process would continue until the final answer is reached. Using the "white box" version, argues Drijvers (2004), algorithms are studied thoroughly since the steps are not hidden as in the "black box" version. The "white box" version supports Heid and Edward's (2001) fourth role for using CAS in mathematics education, (d). The intermediate steps are shown to the students and the students are free from the algebraic and arithmetic computations. The "white box" version of CAS, Ozgun-Koca (2010) suggests, will allow students to apply learned procedures without the worry of making mistakes during the arithmetic computations. An example of the "white box" version of CAS is illustrated in Figure 3.

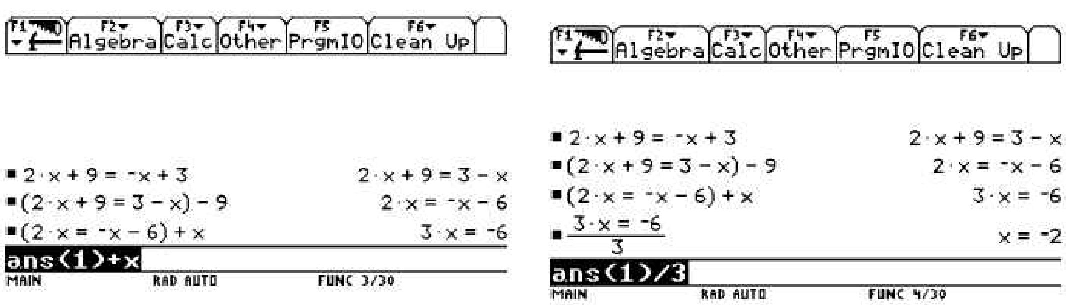


Figure 3: Example of the "White Box" version of CAS (Ozgun-Koca, 2010).

Edwards (2003), suggests, using the "white box" version of CAS before using the "black box" version, could increase the conceptual level of understanding for secondary school mathematics students. Drijvers (2004) agrees with Edwards, in that, by using the "black box" version without first experiencing the "white box" version, students could be unable to interpret and assess the answers produced by the CAS. However, Drijvers (2000) did mention in a previous study, that students who use on the "black box" version elicit curiosity, which has the potential for exploration, which in turn, might lead to interesting discoveries. Eventually, argues Drijvers, discoveries can lead to the explanation phase where students can provide explanations and develop new concepts. However, McAndrew (2010) found that the "white box" version of CAS allowed students more opportunities to investigate findings while making stronger connections with the contexts. McAndrew writes, "One of the greatest benefits of a CAS in a teaching environment is that it allows students to experiment and explore, while at the same time removing the cognitive load associated with messy algebraic manipulations" (p. 601).

Symbolic Math Guide (SMG)

Ozgun-Koca (2010) discusses a third use of CAS in mathematics classrooms. SMG is an application created by Texas Instruments and aims at helping students with symbolic and algebraic transformations. Brown, Stillman, and Herbert (2010) describe SMG as a CAS that assist students' in solving equations by offering a finite list of items dedicated to the particular form of the algebra at that point of the solution. Ozgun-Koca agrees and describes SMG as an advanced "white box" version of CAS. In a SMG environment, students' enter an equation into the CAS and the CAS offers all possible transformations from which to choose. Students make a choice and the CAS illustrates what the outcome would be for that choice before the equation is simplified. Ozgun-Koca found that SMG received approval by teachers as a pedagogical tool because of its capabilities of creating an effective teaching environment as well as guiding students to different methods without hinting to which method is more appropriate. An example of the SMG version of CAS is illustrated in Figure 4.

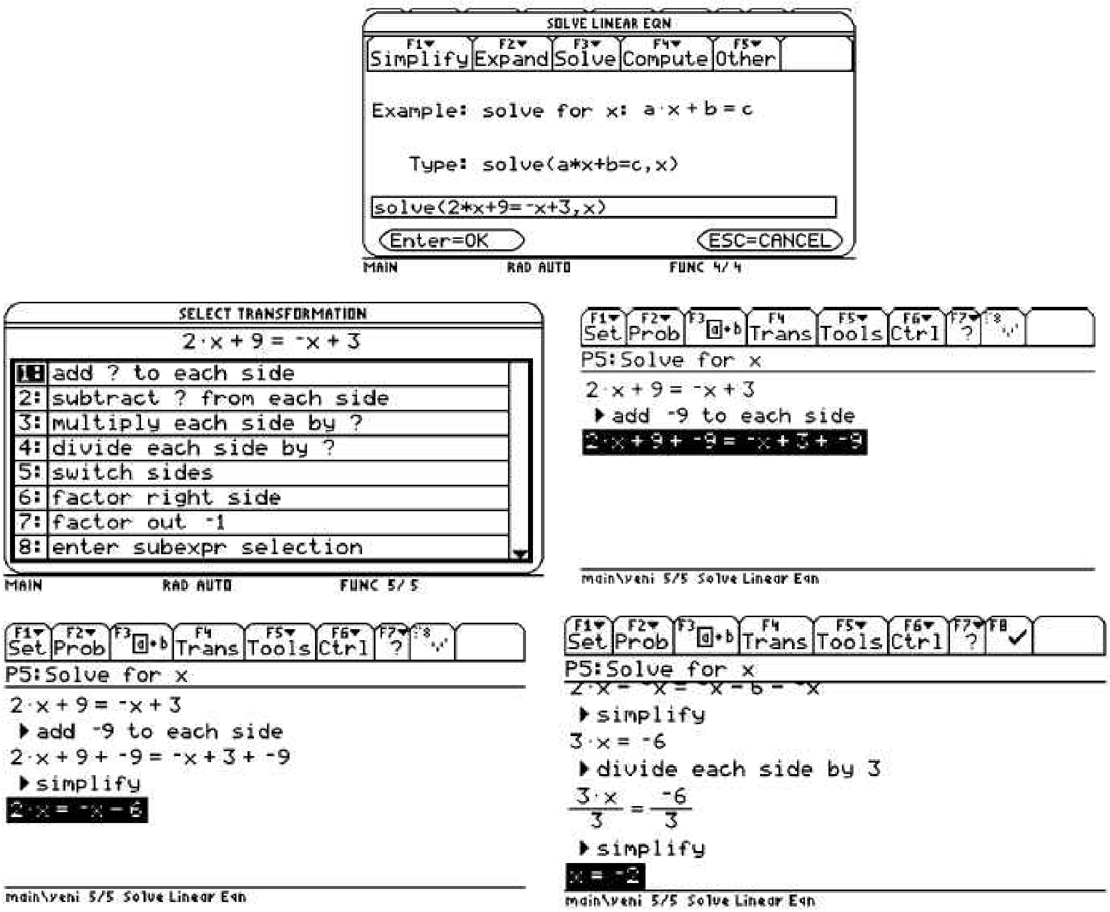


Figure 4: Example of the "SMG" version of CAS (Ozgun-Koca, 2010).

The third teacher concern that Arnold (2004) mentions, was the challenge of what type of exam questions could be asked when students have access to a CAS. Nunes-Harwitt (2005) addresses this question by arguing, CAS could simplify instructors' task with exam construction. Since CAS are programmable, discusses Nunes-Harwitt, they allow instructors to construct questions by writing problem-generating procedures. Pierce and Stacey (2001) mention, a major change in examination questions, when using CAS, was the extension of assessment from students' recall facts and pencil-and-paper skills to questions that require interpretation and explanation. In their study, Pierce and Stacey argue, students' were required to go beyond routine mathematical manipulations or translating mathematical words into CAS commands, to explaining and justifying reasoning, as well as interpreting results. The students were required to verbalize their mathematical thinking.

Technology

Using technology in the mathematics classroom, argues the National Council of Teachers of Mathematics (NCTM) (2000), has the potential to enhance student learning. According to the *Principles and Standards of School Mathematics* (NCTM, 2000), the "Technology Principle" is one of the six principles that help promote high quality mathematics education. The principle states, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). Habre and Grundmeier (2007) mention, from the early eighties to the end of the twentieth century, there have been great advancements of technological aids in mathematics courses. Graphing calculators and CAS are, considered by researchers, the most influential form of technology that paved the way for radical change in the way mathematics is taught (Cooley, 1997; Dubinsky & Tall, 1991; Ellington, 2006; Gaulke, 1998; Habre & Grundmeier, 2007; Heid, 1988; Koh & Divaharan, 2011; Porzio, 1995). According to Heid (1997), when the calculator was first introduced into the classroom, educators were concerned that students would develop dependency on the calculators and interfere with the learning of mathematical skill.

Calculators in Mathematics Education

Ellington (2006) mentions, due to the evolution of technology, the graphing calculator became the everyday tool in mathematics classrooms. In a study by Laughbaum (1999), 85% of the mathematics departments of 600 colleges and universities reported that the faculty utilizes the graphing calculator in the teaching of mathematics in at least one course. NCTM (1989) supports the use of the graphing calculator and notes "the emergence of a new classroom dynamic in which teachers and students become natural partners in developing mathematical ideas and solving mathematical problems" (p. 128). The question is, are graphing calculators helping or hindering students' development of the skills necessary to do well in their mathematics courses? According to Ellington, educators have been researching this question for the last thirty-five years.

NCTM (2000; 1989) has a strong stance that the graphing calculator, as an educational tool, enhances developmental skills in mathematics. The focus here will be on calculus courses and the use of the graphing calculator during calculus instruction. In a study by Porzio (1995), students that used the graphing calculator were proficient at using graphical representation; however, the students had some trouble using symbolic representations even though the use of the same representations were stressed during the course. Hembree and Dessart (1992) found in their statistical evaluation of the effects of calculators on students' achievement based on eighty-eight research reports, that the graphing calculator had no meaningful effect on students' understanding of mathematical concepts. Ellington (2006) argues that it is not as simple as to say the graphing calculator helps or hinders the students' understanding of calculus. Ellington found that when the graphing calculators were included in instruction but were not allowed in the testing process, the students did not develop the skills necessary to apply mathematical formulas and procedures. However, Ellington suggests, when the graphing calculators are included in both, instruction and testing, the students benefitted in their development of mathematical skills in procedural and conceptual achievement.

Porzio (1997) found that students perceived the focus of the calculus I course to be on learning about and using graphical representations of calculus concepts, and not symbolic representations. Day (1996) mentions, when students concentrate on the calculator to produce answers, they have trouble seeing and understanding the underlying mathematics. Dubinsky (1995) and Andrew (1999) share the concern with Day, arguing that when the graphing calculator is used to achieve answers by pressing buttons that students have memorized, without understanding the underlying concepts. This produces poor skills in arithmetic, algebra, and trigonometry.

Hall and Martin (1999) suggest that instructors lack the training of how to use the calculator as a tool in the classroom and therefore, blame should not be placed on the effects of the graphing calculator. According to Laborde (2001), teachers must decide the best way to incorporate the use of the calculator into the classroom and what tasks to assign students to promote understanding. Doerr and Zangor (2000) argue teachers' are poorly prepared on the knowledge of the calculator's capabilities, as well as other technology, and its potential to enhance student learning. According to Schwartz (1999), when technology is used correctly, it provides a setting for conjecture and creativity for the students. Healy and Hoyles (2001) warn technology might present problems for teachers that are accustomed to routine instruction, leaving little room for discovery.

According to Smith (1996), students that used graphing calculators for problem solving, computation and conceptual understanding had a significantly higher achievement than students that did not use graphing calculators. Kulik (2003) conducted a review of 12 research studies on the effectiveness of using graphing calculators in calculus. Kulik found that when students used the graphing calculator, they had higher test scores than those students that did not use the calculator. Simonsen and Dick (1997) found that the graphing calculator equipped students with the ability to visualize problems using multiple representations.

CAS in Calculators

According to Heid and Edwards (2001), CAS has been available in graphing calculators since the mid 1990s. The use of CAS in secondary calculus courses, mention Heid and Edward, support the development of deeper conceptual understanding by allowing linked multiple representation. According to King (1997), when CAS was used in the classroom, the students' overall achievement was greatly improved. Barton (2001) conducted a review of 52 studies on the effects of using CAS in a calculus environment and found that that those that used CAS in the classroom had a better overall conceptual understanding, while there was no significant difference in procedural understanding.

Monaghan, Sun, and Tall (1994) found that when students used CAS to carry out the process of differentiation, they responded by explaining the keystrokes that were necessary to get the result rather than illustrating comprehension of the topic. Monaghan et al. conclude that students were replacing one procedure, which has little conceptual meaning, with another. According to Gaulke (1998), there were insignificant differences found between students using

CAS and students that did not in terms of conceptual understanding. Gaulke did conclude that using CAS did not result in any loss of computational ability.

Drijvers (2000) suggests, students using graphing calculators with CAS capabilities without proper guidance from teachers lack the knowledge of the "why" and focus only on the "how." Drijvers argues, to effectively use graphing calculators with CAS capabilities, students need to go through many phases before becoming proficient users. The most important phase, according to Drijvers is to perform the pencil-and-paper method before using the calculator.

Palmiter (1991) conducted a study comparing two groups of calculus students' and the study of integration that disagrees with Drijvers (2000) argument. In the study, one group used CAS enabled graphing calculator and the other did not use any calculator. After both groups took the same conceptual examination and computational examination, the results showed that the CAS group achieved a higher overall average in both examinations. Palmiter argues that based on his study, it is clear that students can be more successful in conceptual and computational tasks working with CAS, than students working in a traditional environment.

Porzio (1995) suggests CAS students were better able to use and to recognize and make connections between different representations compared to the students who did not use CAS. Porzio concludes from his study, that traditional students were the least proficient at using graphical representations and had the most difficulty recognizing and making connections between different representations. Cooley (1997) reports students in the technology based calculus class that used CAS enabled calculators, scored significantly higher in three of the six conceptual areas: limit, derivatives, and curve sketching. The non-technology group did not score higher in any of the conceptual areas.

Conceptual Understanding versus Procedural Skills

One of the reasons researchers (Aspinwall & Miller, 1997; Heid, 1988; Mahir, 2009; Star, 2005) believe students' difficulties in learning calculus is due to the instructors' insufficient focus on the conceptual understanding of the material. According to Aspinwall and Miller (1997), students end their study of calculus with little conceptual understanding because students' feel computation is most essential. Studies (Mahir, 2009; Star, 2005) suggest that students' use the process of memorization, to perform routine procedures. Therefore, there is a misconception of using memorization and having conceptual understanding of the concepts. However, Mahir (2009) finds that for students' to be successful in calculus there must be a balance between procedural and conceptual knowledge by putting emphasis on concepts and relations rather than problem-solving techniques.

Mathematical knowledge, Hiebert and Lefevre (1986) suggest, can be thought of as being either, conceptual or procedural. According to Chappell and Killpatrick (2003), based on years of research, there is an important distinction that has been made between the two types of knowledge. Skemp (1976) began the idea of the distinction by discussing two types of understanding, instrumental and relational. Instrumental, argues Skemp, means having procedural knowledge of which he described as the knowledge of performing without the understanding of the why. Skemp, referred to relational understanding, as the understanding of the "how" and the "why". Hiebert and Lefevre began using the terminology procedural and conceptual to represent instrumental and relational, respectfully. According to Hiebert and Lefevre, procedural knowledge refers to an understanding of the rules for completing mathematical tasks with the inclusion of symbolic language. Conceptual understanding, on the other hand, is knowledge of mathematical ideas, with the understanding of why algorithms, those are being used, work and when they should be applied. According to Slesnick (1982) students' use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Slesnick (1982) believes that conceptual understanding is superior to memorized algorithms for solving math problems. According to Slesnick, conceptual understanding refers to an integrated and functional grasp of mathematical ideas and students' that have a conceptual understanding know more than isolated facts and methods. Students' understand why a mathematical idea is important and the kinds of contexts in which it is useful. In addition, Slesnick suggests, students' have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Slesnick also mentions that although teachers often look for evidence of conceptual understanding in students' ability to verbalize connections among concepts and representations, conceptual understanding need not be explicit and students often understand before they can verbalize that understanding. Plato (2000) suggests that to solve mathematics problems there needs to be a connection between mathematics concepts and everyday life and connections among the different mathematics concepts in a discipline. Someone who is good at solving problems, argues Plato, does not randomly connect concepts, which might occur when using memorized algorithms to solve problems (Star, 2005), but rather integrates the concepts into a well-structured knowledge base. According to Plato, to attain computational proficiency, students need to be able to communicate clearly using mathematical symbols and apply algorithms when needed. To attain conceptual understanding, adds Plato, students' must understand the relevance of what they are doing when they use their computational skills. For example, students should understand that the technique of substitution works because it replaces the unknown with a calculation of obtaining the solution. Plato argues that educators need to know that developing computational proficiency and conceptual understanding is the primary goals to ensure students succeed in mathematics.

Burns (1998) believes that procedural skills are more important to the life of a person than conceptual understanding. "Arithmetic skills are the necessary life tools that children must learn and continue to review … far more important than anything else" (p. 2). Burns argues that as adults, students' will use arithmetic daily and a person who cannot do arithmetic is handicapped in many situations. Arithmetic prepares children for real-world mathematics and almost all of the daily situations that require arithmetic call for more than merely computing. Burns says that, a person needs to problem solve in order to decide on the numbers to use or which operation to choose. "Learning to compute mentally is an essential skill that can only be achieved with 'skill and drill" practice' (p. 3). Burns believes that to calculate efficiently, it is important to know basic facts and students' should have sufficient procedural skills before graduating from high school. Grundmeier, Hansen, and Sousa (2006) define procedural fluency in calculus as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms, which are the systematic routines needed to perform arithmetic operations.

Star (2005) argues that using the terms conceptual and procedural confounds knowledge types with knowledge quantities. Star found that when assessing knowledge of concepts, it is often found verbally and through a variety of tasks. This method suggests that conceptual knowledge is complex and procedural knowledge is one-dimensional. However, Star argues that "procedures should not be assumed to be rote but rather as objects with several different sources of training" (p. 6). Star agrees that procedural knowledge is the knowledge of rules to complete mathematical tasks (Hiebert & Lefevre, 1986), however, he suggests conceptual knowledge is better understood in terms of knowledge of concepts or principles, rather than the understanding of the "why" alone. According to Star, conceptual knowledge is superficial and weakly connected. Being well connected, argues Star, is obtained when there is a deep procedural knowledge. In addition, Star says, although the word procedural may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics.

Baroody, Feil, and Johnson (2007) concur with Star (2005), by suggesting that procedural and conceptual knowledge represents a mutually supportive aspect of learning mathematics. According to Baroody et al., the development of knowledge and understanding can be described along a continuum. At one end, understanding is unconnected and coupled with superficial procedural knowledge and a lack of conceptual knowledge. At the opposite side of the continuum, there is a strong connection between procedural and conceptual knowledge. Hiebert and Carpenter (1992) found both types of learning are important for students' to be proficient in mathematics. When making a decision on which of the two types, conceptual understanding and procedural fluency, should come first, researchers and educators offer different points of view.

Hiebert and Carpenter recommend that focus should be on concepts before procedures. Oaks's (1988) found when students preferred to memorize procedural rules and algorithms and were not aware of the concepts underlying the procedures, the production of conceptual understanding illustrated limited success.

Devlin (2006) suggests that the issue of the relationship between procedural skills and conceptual understanding is one of the most frequently raised and difficult subjects to deal with in mathematics education. According to Devlin, few educators argue that either is irrelevant, however, generations of mathematicians and educational researchers found, and continue to find that it is difficult to explain how these two important components can be balanced. Wu (1999) discusses the difficulty in finding a balance between these components with respect to mathematics education and links the difficulty to the fact that researchers still lack a convincing universally accepted account of the distinction in the first place. Wu argues that the distinction between procedural skills and conceptual understanding is a theoretical fiction. He suggests that there cannot be good mathematics teaching without considerable emphasis on standard basic skills, that is, formal standard symbolic technique. According to Wu, computational proficiency and conceptual understanding must be synthesized. Without the ability to compute, argues Wu, students' might be unable to effectively solve problems and communicate their solutions and without conceptual understanding students' might be lost in a world of meaningless rote and mathematics that is extremely repetitive which doesn't require any thinking.

Chappell and Killpatrick (2003) also found that when students learned in a conceptual environment, their procedural skills and abilities became more efficient. In their study, the students exposed to the concept-based learning environment scored significantly higher that the students in the procedure-based environment on questions that measured conceptual understanding and procedural skills. Byrnes and Wasik (1991) found that conceptual knowledge and procedural knowledge actually enhance one another. Whether instruction is procedural or conceptual, understanding of both, Byrnes and Wasik suggest, will be obtained. Therefore, Baki (as cited in Mahir, 2009) argues, conceptual knowledge includes procedural knowledge and vice versa.

Schafersman (1991) describes problem solving as the ability to formulate, represent, and solve mathematical problems and problems generally fall into three types: one-step problems, multi-step problems, and process problems. Most problems that students will encounter in the real world, suggests Schafersman, are multi-step or process problems. The solutions of these problems involve the integration of conceptual understanding and procedural knowledge. Schafersman believes that students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution (Heid, 1988). Therefore, Schafersman feels mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself, argues Plato (2000), is not useful in problem solving unless students recognize when and where to use the procedure, as well as when and where it does not apply. As a result, according to Schafersman, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

The debate over conceptual versus procedural knowledge, has been the vigor of the calculus reform movement (Roddick, 2001). Researchers (Hurley, Koehn, & Ganter, 1999; Katz,

1993) mention that the calculus reform movement involved introducing technology to improve conceptual understanding. As a result, argues Katz, the reform idea hindered the procedural skills that the students would need to be successful in calculus-dependent courses. Davis, Porta, and Uhl (1994) found when using technology, together with instruction that focused on procedural fluency, students' achieved a stronger conceptual understanding. Engelbrecht, Harding, and Potgieter (2005) concur and found a significant difference in terms of confidence and performance of procedural and conceptual problems between students that took a calculus course emphasizing understanding and interpretation than students in the traditional course. NCTM (2000) agrees that competency in the area of calculus requires both procedural and conceptual fluency.

Reform

According to Wu (1997), the "New Math" movement failed to deliver the strong case of how students should learn to be effective mathematics students'. As a result, argue Hurley et al. (1999), with the support of the National Science Foundation (NSF), the reform idea of "Calculus of a New Century" was introduced in 1987. This reform came, as described by Cadena, Travis, and Norman (2003), as a result of research showing that approximately 50% of the students that enrolled in calculus I in United States universities, failed the course. According to Chappell and Killpatrick (2003), more than 500 post-secondary institutions in the United States are engaging in calculus reform strategies. Heid (1988) describes the reform curriculum as one that challenges held beliefs that students could not adequately understand concepts without prior mastery of basic skills.

During the 1980s, mathematicians, according to Douglas (1986), were concerned with the quality of student learning in calculus. This concern led to a Calculus Reform movement in the United States that suggested the integration of technology to make the subject more meaningful to a more diverse group of students (Tall, Smith, & Piez, 2008). In 1986, the Tulane Conference held in New Orleans, Louisiana, focused on five concerns about traditional calculus courses and according to Mittag and Collins (2000), they were

- a very low pass rate for calculus students;
- the students had little understanding implementing symbolic algorithms and little facility at using calculus in subsequent mathematics courses;
- there was elevated frustration in faculty due to the need to work harder to help poorly prepared and poorly motivated students in advance courses based on material that was supposed to be learned in calculus;
- some disciplines were using calculus as a required unmotivated and unnecessary filter; and
- mathematics was behind other disciplines in the use of technology.

Roberts (as cited in Tall et al., 2008) mentions there were desirable characteristics for a calculus reform course that were summarized from the Mathematical Association of America's Subcommittee on Calculus Reform and the First Two Years. These four characteristics were

- students that complete calculus should have a sense of the contributions calculus has made to modern world views, its place in the realm of intellectual and science history, and the role it continues to play;
- the applications should be found by the students and instructors to be real and

compelling;

- the instructors should have high expectations from their students and employ pedagogical strategies, such as, cooperative learning and laboratory experiences, in order to engage students' interest and enhance success; and
- all students should learn to read and write reasoned arguments at appropriate levels for their stage of development.

According to Mittag and Collins (2000), the conference's 25 participants included faculty from two-year and four-year universities, and mathematics education researchers. Together, the participants proposed three suggestions for calculus reform, which Mittag and Collins suggest, have been the influential guide for the reform effort. The suggestions were,

- there needed to be a focus on a conceptual understanding that used a variety of intuitive graphical and numerical approaches and were geared to the needs of average students;
- emphasis needed to be placed on the importance of changing the modes of instruction and the use of technology to develop active learners.
- there needed to be a promotion on the inclusive spirit in the reform ideas and emphasis to be placed on the importance of cooperation and broad spreading at every stage.

As a result of the conference, in 1987, "Calculus for a New Century" began to take place. According to Hurley et al. (1999), the NSF started the initiative by funding 19 various grants. After the publication of the *Curriculum and Evaluation Standards for School Mathematics* (1989), Mittag and Collins mentioned that high schools began their involvement in the calculus reform effort. The NSF began funding grants that help train high school calculus teachers on how to use the graphing calculator and as a result, according to Mittag and Collins, the Advanced

Placement (AP) program began the requirement of the graphing calculator on the AP examinations.

Supporters of the calculus reform (Chappell & Killpatrick, 2003; Heid, 1988; Hiebert & Lefevre, 1986; Hurley et al., 1999; Porzio, 1995) argued that being able to obtained correct solutions to procedural questions without the understanding of how and why the algorithms work, or without being able to correctly interpret methods or answers in problem contexts, is not essential. In addition, individuals will increase their use of technology to perform mathematical calculations. Therefore, focus should be placed on a more conceptual teaching while encouraging the use of technology in order to be used appropriately and effectively. Those that oppose the calculus reform (Wu, 1997; Rochowicz, 1996; Wilson, 1997; Ratay (1993); Friedman, 1993) argue that students cannot fully understand concepts until they gain proficiency in skills. The opposition believes that the most important aspect of mathematics is to know the algorithms and being able to use them.

The curriculum design revolved around the debate of procedural skills versus conceptual understanding (Wilson, 1997). Wilson criticizes the calculus reform movement and argues that in an effort to enhance the conceptual understanding of students, the secondary and post-secondary calculus course now requires a minimal amount of knowledge compared to the traditional calculus course. Students in the reform calculus course, argues Wilson, focus on only superficial use of skills. Wu (1997) further argues that the reform calculus course illustrated no significant improvement in conceptual understanding and lessened the use of computational techniques. Rochowicz (1996) supports Wilson and Wu's claim, in a study he conducted where he found the students who benefited from the reform course were students who illustrated weak mathematical skills coming into the course. The students with a stronger mathematical premise gained little improvement. Ratay (1993) also found that only the weaker students benefited from the reform calculus courses.

Heid (1988) investigated the effect of re-sequencing skills and concepts. In her experiment, she placed emphasis on concepts, and let the computer perform the algorithms for 12 weeks out of a 15-week calculus course. During the last three weeks, she placed emphasis on computation skills, with students performing algorithms by hand. Heid found that the students in the experimental calculus reform course, showed better conceptual understandings than the traditional class. Students from the reform calculus group were able to speak about the concepts with more detail, clarity, and flexibility, than the students from the traditional class. The students from the reform course applied concepts more appropriately and were better able to answer conceptually oriented questions. According to Heid, the new curriculum emphasized concepts, applications, and problem solving and using technology as the primary method of algorithm execution. As a result, evidence showed that the students in the reform course understood concepts better than students in the traditional calculus course.

Similar to Heid's (1988) findings, in a study looking at using a reform calculus course and how it effects students' conceptual understanding and procedural knowledge of calculus, Chappell and Killpatrick (2003) found, when devoting significant class time to the development of conceptual understanding, there would not be any damaging results to the students' procedural skills. After conducting the study twice, Chappell and Killpatrick also found, students in the reformed based learning environment scored significantly higher than students enrolled in the traditional environment on an exam that contained both procedural questions and conceptual understanding questions. Based on their findings, Chappell and Killpatrick suggest conceptual understanding aids the gaining of procedural skills.

Porzio (1995) found that students in a reform environment were better able to use and make connections between different representations. However, Porzio also suggests that the addition of technology into the calculus curriculum does not necessarily improve students' understanding of calculus. For the reform courses to be effective, argues Porzio, teachers need to be prepared to help students make connections between different representations of the same concepts. In addition, instead of showing the students the connection, teachers need to work on the problems with the students in order to encourage exploration, establishment, and reinforcement of the connections. If the reform calculus course is to be effective, Porzio suggests, the curriculum, including revisions to the calculus textbooks, should be done in a way that promotes multiple representations, recognition of connections between representations, and using technology appropriately.

Similar to Heid's (1988) study, Judson (1988) also showed students did not need to learn calculus procedures before they can understand calculus concepts. Like Heid, Judson's study focused on the re-sequencing of skills and concepts, where concepts and applications were taught prior to algorithms for computation. However, unlike Heid, Judson did not find a better conceptual understanding in the students from the reform group. Judson's findings support Byrnes and Wasik's (1991) claim that the relationship between conceptual knowledge and procedural knowledge does not rely on which comes first. However, Hiebert and Lefevre (1986) note that one of the central goals of curricular reforms in calculus is the emphasis on procedures with deeper conceptual understanding.

Garner and Garner (2001) found no statistically significant difference between the students from a reform calculus course and students from a traditional calculus course. However, using interviews with students, Garner and Garner, showed that students from the reform calculus course seemed to be more conceptually oriented than the traditional students. In addition, the reform group was able to provide explanations with more confidence, discuss applications more clearly, and used graphical explanations more than the traditional group. Garner and Garner suggest that the later was due to the fact that the traditional group did not spend as much time on graphical applications as the reform group; however, the traditional group had stronger analytical knowledge.

Judson and Nishimori (2005) conducted a cross-cultural study between United States calculus students in a reform calculus environment and Japanese calculus students taking the traditional calculus course. The goal of the study was to find differences between the two calculus courses and the students' conceptual thinking. Judson and Nishimori found little difference in the conceptual understanding of calculus between the two groups. The Japanese students performed just as well as the United States students, even though the Japanese students had no previous experience with calculus reform type problems. Both the United States students and the Japanese students had a solid grasp of the mechanics of calculus and how to sketch the graphs of functions. Both groups, however, argue Judson and Nishimori, lacked the skills and experience to link several concepts to solve difficult problems. Opposite to the findings of Heid (1988), Judson and Nishimori found the students in the reform group lacked fluency in manipulating algebraic expressions, while the traditional group was able to illustrate a strong understanding. Judson and Nishimori concluded that students in the reform group relied on the use of the calculator as a problem-solving tool, which supports Wu's (1997) and Wilson's (1997) concerns.

Ganter (1997) argues that since studies on the differences between reform calculus and traditional calculus have not shown great significance, educators seem to discard reform efforts. Together with the increase in workload brought from reform, creates argues Ganter, an environment of uncertainty that might lead to withdrawal of support for the calculus reform movement. However, researchers (Cadena et al., 2003; Chappell, 2003; Ferrini-Mundy & Gaudard, 1992; Garner & Garner, 2001; Kwon, Allen, & Rasmussen, 2005; St. Jarre, 2008) have looked beyond which calculus curriculum produces more understanding; they studied which group retains the knowledge longer.

Retention

Sousa (2000) defines retention as a measure of how well a student remembers the material over time and successfully retrieves information from long-term memory. According to Sousa, successful retrieval of information relies on making an association with exiting knowledge that can facilitate future retrieval. Retention of knowledge, according to Kwon et al. (2005), has been studied to address questions such as,

- does retention of conceptual understandings differ from that of procedural understandings?;
- what are some ways that educators can facilitate students' retention of mathematical understandings and skills for longer periods of time?;

- what is the relationship between instructional approaches and retention of mathematical knowledge?

Researchers (Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986) argue that both conceptual and procedural types of learning, are important to proficiency in mathematics and if the two were linked, a creation of a network of information would be better retained. Heid (1988) and Judson (1988) investigated the effects of re-sequencing skills and concepts. Emphasis in both studies was on concepts, with the assistance of technology performing the algorithms. However, Kwon, et al. argue that studies, like Heid and Judson's, evaluate student understanding at the end of the instructional intervention, and fewer studies have examined retention of the mathematical knowledge weeks or months later.

Semb and Ellis (1994) found that when instructional differences were quantitative, there were no differential effects on retention. They claim what is taught in the classrooms is soon forgotten. However, the types of differences varied depending on the instructional treatments. According to Semb and Ellis, students from an active learning environment, forgot less than those in a passive learning environment, where active refers to a reformed curriculum extensively using technology while a passive environment refers to a traditional curriculum (Kwon, et al., 2005). Anderson, Austin, Barnard, and Jagger (1998) agree with Semb and Ellis, in that minimal retention was observed in their study. Anderson et al. report that from an assessment to study retention, only 20% of responses were correct and 50% of the responses had little to no connection with the questions.

Garner and Garner (2001) suggest, comparing reform curriculum to traditional curriculum should not be an important question or focus. Instead, argue Garner and Garner, studies ought to focus on the retention of important concepts and skills of students that completed a reform calculus course versus students that completed a traditional calculus course. Garner and Garner investigated the retention of students after completing calculus seven months prior and found both groups had forgotten most of what they had learned. However, the reform group performed significantly better on the conceptual questions compared to the students from the traditional curriculum. On the other hand, Garner and Garner found that students from the traditional calculus course performed significantly better than the reform students on items that tested procedural knowledge. As a result, Garner and Garner suggest, that when choosing the type of instruction, educators should consider the goals of the students. If the goal is to ensure that students gain a conceptual understanding, Garner and Garner recommend the reform calculus curriculum. However, if the goal is to enhance and strengthen procedural skills, a more traditional curriculum is recommended.

Engelbrecht, Harding, and Preez (2007) looked at retention of basic knowledge and skills two years after completing calculus. Following students into their third year of college, the study consisted of a mastering test in the first year of study and then a similar exam in the students' third year. As reported by Engelbrecht et al., only 42 of the original sample group of 103 were available to continue in the study. The exam used consisted of five topics. They were: properties of logarithms; sketching of graphs using transformations; limits; differentiation; and integration. The study reported that although much effort was made to reinforce basic knowledge and rote skills during the first year, retention was minimal. During the first year, explains Engelbrecht et al., students were exposed to repeated exercises to help acquire knowledge. The instruction did not focus on any methods that researchers (Hiebert & Lefevre, 1986; Skemp, 1976) have argued promotes conceptual understanding. The findings of Engelbrecht et al. support Garner and Garner's (2001) argument, that an instruction that focuses on promoting conceptual understanding will support retention.

In a similar study Kwon et al. (2005) researched if retention of conceptual understandings differs from retention of procedural understandings. They found similar results as Garner and Garner in that students' retention of conceptual understandings were stronger from the reform calculus group; however, they found no difference between the reform group and traditional groups' retention of procedural understandings. In addition, students from the reform group relied on graphical visualization as they explored and sought to understand any new mathematical concepts in class. As a result, Kwon et al. argue, retention of visual images may be longer term than retention of semantic knowledge, such as procedures and algorithms.

Significant Studies

Ferrini-Mundy and Gaudard (1992) addressed the relationship of experience of secondary school calculus to performance in first-year college calculus and second-year calculus. The study focused on the retention of students' coming in with previous knowledge of calculus and if taking secondary school calculus actually improves overall performance in college calculus. Ferrini-Mundy and Gaudard investigated if there were performance differences based on procedural skills and conceptual understanding. The study was done during one academic year in a school that offered a single calculus track. The college used the Swokowski (1988) text, which is similar to the Thomas and Finney (1988) textbook, in that it offers more applied problem sets. These textbooks differ from reform textbooks such as those written by Hughes-

Hallett et al. (2000) and Finney, Demana, Waits, and Kennedy's (2007) textbooks, which focuses less on the algebraic and takes a more intuitive approach to learning calculus.

The study began with a 41-item pretest in algebra and trigonometry to provide information about students' characteristics. There were similar methods used by Chappell (2003), Cooley (1997), and Serhan (2006) to measure the basic understandings of the students' before taking the college level one calculus course. Each following exam was generated through faculty collaboration and consisted of questions requiring open-ended responses. However, the final exam was solely a multiple-choice exam. The use of calculators was not permitted on the exams. Ferrini-Mundy and Gaudard (1992), developed the final exam following the discussion of Hiebert and Lefevre (1986), and then had the mathematics education faculty classify which questions would be considered procedural and which were conceptual. The findings of Ferrini-Mundy and Gaudard's study showed that students that studied secondary school calculus were more successful in the college first semester calculus course. The major difference in performance, argue Ferrini-Mundy and Gaudard, was due to procedural proficiency illustrated by students that completed a year of secondary school calculus.

Findings in the second semester calculus course differed from those in the first semester calculus course. Ferrini-Mundy and Gaudard found that the procedural facility of the students that took the one-year secondary calculus course to be less of a significant difference in performance. Ferrini-Mundy and Gaudard suggest that procedural, technique-oriented secondary school courses in calculus may predispose students to attend more to the procedural aspects of the college course. Students might believe that procedures are the core to solving calculus problems and when moving to the theoretical aspects of the course in second semester calculus, students' experiences with calculus might be less receptive to these aspects (Ferrini-Mundy & Gaudard, 1992).

In a study following Ferrini-Mundy and Gaudard (1992), Chappell (2003) assess the performance of students enrolled in second semester calculus that took either reform first semester calculus or traditional first semester calculus. Her study consisted of 185 students enrolled in a large state university classified as a Carnegie I research institution. The study took place during the fall of 1998 and spring of 1999. In the fall, 49 students were enrolled in the reform calculus course and 136 students were enrolled in the traditional calculus course. In the spring, all 185 participants enrolled in the traditionally taught second semester calculus course. All 185 participants took the same exams in first semester calculus. The exams were prepared by the faculty and assessed two dimensions of students' knowledge, procedural skills and conceptual understanding. Both groups used the same textbook, Stewart (1999). Stewart's textbook used the "Rule of Three" to implement the goal of teaching conceptually. According to Stewart, topics should be presented geometrically, numerically, and algebraically, in order for students to gain conceptual reasoning. Stewart (2008) expanded his "Rule of Three" to the "Rule of Four," by emphasizing the verbal and descriptive point of view. Stewart argues that the reform elements in his textbook remain in the context of a traditional curriculum.

Chappell found that students enrolled in the reform calculus course had significantly higher course grades than those students enrolled in the traditional course. However, the results of the second semester calculus course showed very little significance between the two groups. However, Chappell also found that students from the reform calculus group had a higher withdrawal rate in second semester calculus than those students enrolled in traditional calculus.

Chappell concluded three reasons for why students from the reform calculus course could not succeed in a traditional second semester calculus course. According to Chappell,

- after students were placed in a traditional environment, they were no longer content to work through procedural problems without having a conceptual understanding of what they were learning,
- the effort needed to be successful in second semester calculus was much greater than first semester calculus, and
- the conceptual instruction the students became used to in first semester calculus helped in taking exams because of the thoroughness of the material. In second semester calculus, the students relied on memorization when taking exams.

Chappell argues that the results of her study make a powerful case for the importance of reform calculus instruction because it assists students that often have difficulty approaching and solving unfamiliar problems.

CHAPTER THREE: METHODOLOGY

Introduction

The review of literature has produced many studies that investigate what factors and methods can be used to promote success in calculus and higher-level mathematics. The literature discusses the debate on how computers and technology either enhance or hinder student understanding of calculus. The argument on whether or not the calculus reform movement has improved students' conceptual understanding and procedural knowledge has been investigated since the reform movement began in 1987. Different methods and instruments were used in the studies. This chapter will discuss the procedures and methods that were used in this study. The methodology section will include, (a) the purpose of the study; (b) the research questions; (c) population and setting; (d) sample and sampling; (e) reliability and validity; (f) description of measures and data collection; (g) data analysis methods; and (h) limitations.

Purpose of the study

Students that want to pursue careers in science, technology, engineering, and mathematics (STEM), will have calculus as their entry-point and pre-requisite to undergraduate mathematics (Robinson, 2003). According to the OECD Programme for International Student Assessment (PISA), 2009 results, the United States ranked at 32 out of 65 countries in mathematics performance. The highest percentage of United States students were at level three, which is categorized as students having the ability to execute, clearly describe procedures and select and apply simple problem-solving strategies (PISA, 2010). United States students have been ranked as low achieving since the 1950's (Robinson, 2003). The review of literature discussed studies that have investigated the effects on students' understanding and retention of secondary school calculus on performance in first and second semester calculus in college (Ferrini-Mundy & Gaudard, 1992; St. Jarre, 2008; Wu, 1997; Bressoud, 2010; Bressoud, 2004). Ferrini-Mundy and Gaudard found that students that completed high school calculus struggled in second semester calculus. Bressoud (2010) reports that calculus II enrollments declined by more than 30 percent between 1995 and 2005. As a result, there is also a steep decline in students pursuing STEM careers.

An item that is lacking in the literature is looking at how using a computer algebra system in secondary school calculus instruction can impact the performance of students' understanding, both conceptual and procedural, of calculus concepts. The purpose of this study is to contribute to the literature on examining the effects of instruction on calculus performance.

Research Questions

1) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of procedural calculus tasks?
2) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of conceptual calculus tasks?
3) Is there a significant difference in performance between high performing students that

have had instruction with computer algebra system (CAS) and high performing students that have had instruction with the graphing calculator (GC) on calculus tasks?

Population and Settings

The target sample for this study was determined based on a population of students from two different categories, CAS and GC classroom environments. In order to identify which environments were CAS or GC, AP calculus teachers were contacted by the researcher from high schools in two counties in Florida. Based on the statistics gathered at the end of the 2010/2011 school year, one of the county high schools consisted of 12,526 students that took the grade 10 Florida Comprehensive Assessment Test (FCAT) in mathematics. There were 6,298 females and 6,228 males. The breakdown for the ethnicity percentages is listed in Table 1. The percentage breakdown illustrates white males and females consisted of approximately 34%. African American, non-Hispanic males and females consisted of approximately 27% and Hispanics/Latinos made up approximately 32%. The rest of the ethnicities make up approximately 7% of the population. Table 1 also illustrates the passing percentage for each ethnicity group.

Table 1
Ethnicity percentages of FCAT scores: Grade 10, County One

Ethnicity	% Of population (n = 12,526)	% Passing (n = 12,526)
White, Non-Hispanic	33.6	90.0
African American, Non-Hispanic	26.8	64.0
Hispanic/Latino	32.3	74.0
Asian, Non-Hispanic	4.9	92.0
Native American/Alaska Native	0.4	85.0
Native Hawaiian/other Pacific Islander	0.0	0.0
Multi-Racial, Non-Hispanic	2.0	88.0
Unknown Race/Ethnicity	0.0	0.0
Total	100.0	78.0

Note: Data for Table 1 obtained from the Florida Department of Education website.

Based on the statistics gathered at the end of the 2010/2011 school year, the high schools in the second county consisted of 5,260 students that took the grade 10 FCAT in mathematics. There were 2,658 females and 2,602 males. The breakdown for the ethnicity percentages is listed in Table 2. The percentage breakdown illustrates that the majority of the students were white males and females consisting of approximately 67%. African American, non-Hispanic males and females consisted of approximately 13% and Hispanics/Latinos made up approximately 12%. The rest of the ethnicities make up approximately 8% of the population. Table 2 also illustrates the passing percentage for each ethnicity group.

Table 2
Ethnicity percentages of FCAT scores: Grade 10, County Two

Ethnicity	% Of population (n = 5,260)	% Passing (n = 5,260)
White, Non-Hispanic	66.9	94.0
African American, Non-Hispanic	13.0	72.0
Hispanic/Latino	11.9	87.0
Asian, Non-Hispanic	2.1	94.0
Native American/Alaska Native	0.2	90.0
Native Hawaiian/other Pacific Islander	0.0	0.0
Multi-Racial, Non-Hispanic	5.9	90.0
Unknown Race/Ethnicity	0.0	0.0
Total	100.0	90.0

Note: Data for Table 2 obtained from the Florida Department of Education website.

The study was conducted during the 2011/2012 school year. The population size was dependent on the number of schools visited. The researcher made contact with four AP calculus teachers in one county. During brief discussions, the researcher determined that the teachers were graphing calculator users and had a total of 280 AP calculus AB students among them. The researcher then made contact with the second county's AP calculus teachers and found three teachers that stated they used CAS empowered graphing calculators and related most of their instruction to CAS environments. Among those teachers, there are 188 AP calculus AB students. The AP calculus AB course focuses on the study of analytic geometry, discussing limits, continuity, differentiation of algebraic and trigonometric functions, applications of derivatives, integration and the fundamental theorem of calculus, and the applications of definite integrals.

Sample and Sampling

Following the determination of which schools were willing to participate for the study, the researcher gathered participants to represent the sample of the population of the students. The researcher performed a priori power analysis to determine the number of students in each group the study would need for 80% power, which is typically considered sufficient power (Cohen, 1988). The researcher used .05 for alpha and .05 for the effect size, which is considered acceptable for a moderate to large difference according to Cohen. As a result, the study required a sample size of 64 participants per group. Based on the teachers' records of the students, the researcher grouped students from different schools that shared common grades in their AP calculus course. The researcher asked teachers to share the names of their students that were maintaining a grade of A or B (high performing) and which students were maintaining a grade of C or D (low performing).

The researcher chose to compare the high performing students for this study because the study used different schools. In order to increase the support and validity of the findings, the researcher used high performing students from the four schools. If the sample consisted of a mix between high performers and low performers, then the chance that one group outperformed another could be due to one group consisting of more low performing students. This method is similar to the sample selection used in studies conducted by Cooley (1995), Ferrini-Mundy and Gaudard (1992), and Garner and Garner (2001).

Cooley (1995), Ferrini-Mundy and Gaudard (1992), and Garner and Garner (2001), used mathematical background questionnaires to verify that the students used in their studies shared common basic mathematical knowledge in different areas of mathematics. Since this study was conducted during the second half of the AP calculus curriculum, the students were grouped as high performing and low performing based on the teachers' records. The goal was to gather 64 students per group and that goal was achieved. The researcher obtained more than the suggested sample number in case some students were absent during the data collection portion of the study or decided to stop participating or withdraw during the course of the study. The researcher ran data analyses using 64 students from each group.

A random sample population was drawn. According to Nesbary (2000), the larger the sample size, the greater probability the sample will reflect the general population. Patten (2004) argues that obtaining an unbiased sample is the main criterion when evaluating the adequacy of a sample. To help support Patten's argument, in that every member of the population will have an equal opportunity of being selected, this study used random sampling to help ensure an unbiased sample population. Patten also discusses the issue of sampling errors. The researcher used stratified random sampling in order to reduce sampling error. Kott and Vogel (1995) describe stratified sampling as taking a population and partitioning it into groups prior to selecting the sample and then randomly choosing a sample within each group. According to Patten, the same percentage of participants, not the same number of participants, are drawn from the stratum.

Reliability and Validity

Reliability is defined as the consistency of the results obtained from a measurement

(Patten, 2004). The focus of reliability should be on the extent to which the results remain consistent over time. Patten argues an instrument is valid if it measures what it is intended to measure and achieves the purpose for which it was designed. According to Patten, validity is a matter of degree and discussion should focus on how valid a test is and not whether it is valid or not. There is not a test instrument that is completely valid and Patten suggests a researcher needs some kind of assurance that the instrument being used will result in correct conclusions. Patten discusses three principles to help improve content validity, (a) the researcher should use a broad sample of content rather than a narrow one, (b) the researcher should emphasize on important material, and (c) the researcher should write questions to measure the appropriate skill. The researcher took precautions so that the data collection instruments for this study abided by the three principles in order to help improve content validity. The calculus tasks selected cover graph-analyzing capabilities, integration, derivatives, and limits. Researchers (Chappell, 2003; Ferrini-Mundy & Gaudard, 1992; Garner & Garner, 2001) emphasize that these topics are the focus of any first semester calculus course. The tasks have been shown to measure the appropriate skills of these concepts by various studies (Eisenberg & Dreyfus, 1991; Garner & Garner, 2001).

Description of Measures and Data Collection

Observation Rubric and Technology Ranking Scale

The sample selected for this study was determined based on a population of students from two different categories, CAS and GC classroom instruction environments. In order to identify which environments were CAS or GC, various high schools were visited throughout the two Florida counties. An observation rubric (see APPENDIX A) and the technology survey with the Technology Ranking Scale (TRS) (see APPENDIX B) was developed and used by the researcher in order to classify instruction. The rubric consisted of two sections. The first section was used to identify the demographics of the school. The section included the name of the school, the county the school was located in, and the date and time the observation took place. In addition, the researcher made note of the name of the teacher and background of the teacher, including the experience based on the number of years teaching the calculus course. Data also included the number of male and female students.

The second section of the rubric was used to gather information of the classroom environment. The amount of technology available to the students is one of the major differences between traditional and reform calculus (Hurley, Koehn, & Ganter, 1999; Katz, 1993). In addition, group work compared to individualized instruction, and discussion versus lecture, are the other key components of reform calculus (Chappell, 2003; Ferrini-Mundy & Gaudard, 1992; Judson & Nishimori, 2005).

The technology survey and the TRS was used in order to decide if the schools met the criteria of the study in order to be considered as part of the population setting. While observing the class instruction, the researcher used the TRS to evaluate the level of technology used for different concepts and areas. The ranking system ranges from level one to level six. The following is a key to the ranking scale system developed by the researcher:

1. Using calculator for computational purposes only
2. Using built-in functions of calculator or "Black Box" version.

3. Using calculator for graphical illustrations only.
4. Using built-in functions of calculator with reference to graphical illustrations.
5. Using graphical representations and symbolic manipulation functions.
6. Using graphical representations, symbolic manipulation, and animation functions.

According to Mittag and Collins (2000), using a calculator for computational purposes only, will assist with helping students gain a conceptual understanding of calculus; however, this approach will fall short as it will not offer any visual perspective of the material. Wilson (1997) found that students that used the calculator to perform the computations and created graphical illustrations were able to gain a deeper understanding of the concepts than those that used the calculator for computational purposes only.

Rank two is noticed when the instructor or students use the calculator functions to perform tedious pencil-and-paper tasks, without observing the steps which can be used to promote conceptual understanding (Ozgun-Koca, 2010). When using the calculator as avoidance to the pencil-and-paper method, researchers (Heid & Edwards, 2001; Ozgun-Koca, 2010) labeled this idea as "black box." Drijvers (2004) argues that when the "black box" version of using calculators performs the complex procedures, it is not transparent to the students. Drijvers believes that students need to approach problems using the traditional paper-and-pencil method in order to overcome, not avoid, the long process, to encourage conceptual understanding (or relational, Skemp, 1976) through procedural fluency. According to Drijvers, the "black box" version inhibits instrumentation.

Researchers (Andrew, 1999; Day, 1996; Dubinsky, 1995; Porzio, 1997) argue that when using the graphing calculator to graph only, students perceived the focus of the course to be on learning about and using graphical representations of calculus, and not symbolic representations. Day (1996) found students had trouble seeing and understanding the underlying mathematics. Dubinsky (1995) and Andrew (1999) agree with Day, and found students that did not understand the underlying concepts, produced poor skills in arithmetic, algebra, and trigonometry. If the researcher notices rank four, which incorporates the graphical aspects and built-in functions of the graphing calculator, then the researcher can conclude that the use of the calculator is supporting the development of the students' conceptual understanding of calculus (NCTM, 2000).

NCTM (1989; 2000) has a strong stance that the graphing calculator, as an educational tool, enhances developmental skills in mathematics. According to Smith (1996), students' conceptual understanding improved when they used graphing calculators for problem solving, computation, and graphical representations. In addition, Smith found that the students had a significantly higher mathematical achievement than students that did not use graphing calculators. Kulik (2003) conducted a review of 12 research studies on the effectiveness of using graphing calculators in calculus. Kulik found that when students used the graphing calculator, for both graphing and "black box" computations, they had higher test scores than those students that did not use the calculator. Simonsen and Dick (1997) found that the graphing calculator equipped students with the ability to visualize problems using multiple representations and as a result, promoted a higher conceptual understanding.

Instruction that received a majority of the ranking at level four was considered to qualify as the GC instruction environments. These schools showed advanced use of the graphing calculator. Even though a school used a CAS empowered calculator, the researcher was only able to conclude that it was a GC school since it did not rank higher than four in the areas observed during instruction. Levels one through four illustrate use with a GC graphing calculator. The functions are available in both types of technology; however, using the CAS features, which are at level five or six, illustrate actual usage of CAS.

CAS empowered calculators are a fully interactive mathematical software system that includes various types of functions the help it stand apart from graphing calculators. These built-in functions can simplify, factor, expand algebraic expressions, perform differentiation and integration, solve differential equations, and perform matrix calculation among other capabilities. The visualization functions are capable of producing 2- and 3-dimensional graphs and create animations. The symbolic manipulation tools and animation features are two aspects of utilizing CAS in a calculus classroom (Nunes-Harwitt, 2005). Heid and Edwards (2001) found that when CAS is used as a pedagogical tool, which creates and generates symbolic procedures, it will assist with the achievement of conceptual understanding in students. Researchers (Arnold, 2004; McAndrew, 2010; Nunes-Harwitt, 2005; Ozgun-Koca, 2010) found that one of the great benefits of CAS for student learning is that it offers a variety of options for students to experiment and explore the mathematic concepts while removing the long and time wasted algebraic manipulations. Rank five in the scaling system that the researcher developed looked for the use of the symbolic manipulation of CAS or the Symbolic Manipulation Guide (SMG). Ozgun-Koca (2010) discusses symbolic manipulations as an advanced version of "white box" and a more useful pedagogical tool than "black box" as it aims to assist students with symbolic and algebraic transformations. When used with graphical representations, Brown, Stillman, and Herbert (2010) found the use of SMG as a pedagogical tool because of its capabilities of creating an effective teaching environment as well as guiding students to different methods without hinting to which method is more appropriate.

Symbolic manipulation with graphing capabilities is what helps distinguish CAS use apart from using a graphing calculator. Symbolic manipulation is often used to generate the first and second derivatives of functions. The researcher chose rank six on the scale system if there was evidence during instruction that supported the use of animation. According to Lipeikiene and Lipeika (2006), visualization is important in general cognitive skill acquisition and problem solving processes. Animation is considered a more advanced visualization form, which offers visualization of complex mathematical concepts and provides convincing demonstrations of ideas and helps generate and encourage exploration. Using animation allows students to explore, experiment, and visualize mathematics as a dynamic process.

For instructional strategies of a teacher to be considered a CAS environment, the researcher needed to observe at least the use of symbolic manipulation in the classroom. More advanced instructors or those experimenting would illustrate the use of animation and were considered as advanced CAS users. Instruction was considered a CAS setting if it involved the use of technology at the five or six level. The times and days that observations by the researcher took place were chosen by the instructors. The researcher made use of the observation rubric during each session and made a decision on the environment after a level was reached.

Two of the four graphing calculator school environments were visited three times each. During those times, the researcher observed the technology used be each instructor and according to the ranking system, the schools were ranked as levels one and two . This meant that the schools did not use technology as a tool of instruction, rather as a tool for computational purposes. The other two schools that were classified as graphing calculator schools were also visited three times each. These two schools ranked at a level four which illustrated use of the built-in functions in order to analyze graphical representations. The researcher then visited two schools that claimed to be using CAS graphing calculators during instruction. The first school was visited five times in order to for the researcher to be sure that the school ranked at level six, which illustrates the use of the symbolic manipulation and animation features of CAS. The second school was visited three times as the instructor continually illustrated the advance uses of CAS during each lesson.

Based on the observation conducted by the researcher, the population for two groups was decided and it categorized the students' type of instruction. The groups were: (a) students that used the graphing calculator with CAS capabilities (CAS) and (b) students that used the graphing calculator (GC).

Calculus Tasks

The calculus students were given two sets of calculus tasks that contained both conceptual understanding and procedural knowledge questions based on material covered during the integral half of the calculus curriculum. The tasks covered five topics addressed in first semester calculus courses. They are: (a) description and analysis of graphs of functions, (b) interpretation of the definite integral, (c) interpretation of the derivative, (d) computing the definite integral, (e) analyzing limits and (f) analyzing slopes tangent to functions and maximum slopes. The tasks came from two different studies, Eisenberg and Dreyfus (1991) and Garner and Garner (2001). The tasks by Garner and Garner were designed so that they would be solvable for any type of calculus student. The questions were piloted twice prior to the study with mathematics students that completed a calculus course and content validity was established by a panel of mathematicians and mathematics educators from all across the United States. The tasks by Eisenberg and Dreyfus were given to calculus courses and were used by researchers Selden, Selden, Hauk, and Mason (1999), and Tufte (1988). Each of the tasks rested firmly upon a concept, which has a visual interpretation. According to Eisenberg and Dreyfus (1991), thinking visually makes higher cognitive demands than thinking algorithmically; therefore students have issues with visual thinking.

The set of tasks in APPENDIX C was considered procedural, as they required the students to recall knowledge to perform symbol manipulation without knowing the "why" (Hiebert & Carpenter, 1992; Skemp, 1976). Procedural tasks require mastery of algebraic procedures and manipulation. Task One on the procedural set of tasks involved symbol manipulation and recall of facts pertaining to the characteristics of an integral. Based on the definition of instrumental understanding (Skemp, 1976), this question was considered a procedural question. The students did not need to explain the "why" for their answers; they only needed to show the "how." Task Two asked students to illustrate what they understand about limits. Heid (1988) found students show difficulty in the understanding of limits. In the study, Heid found students that were taught using graphical and symbol manipulation computer programs were able to identify the notion of a limit with a process rather than a number. Task Three required students to analyze a function and then produce a graph by following methods of integration. This process was considered instrumental (Skemp, 1976). Task Four focused on Heid's findings that students' confusion of limits permeated their explanation of derivatives as they described derivatives as approximations for slopes of tangent lines rather than as being equal to the slope.

The set of tasks in APPENDIX D was considered conceptual because the tasks required explanation and understanding of principles (Hiebert & Carpenter, 1992; Skemp, 1976). Task One on the conceptual set of tasks was used to assess the graph-analyzing capabilities of the students. This type of understanding, according to Garner and Garner (2001), would be considered conceptual. Task Two will be used to analyze the students understanding of the visual concept of an integral. Task Three asked students to provide a meaning rather than an actual value for the question (Garner & Garner). Task Four involved both symbol manipulation and interpretation of results, and therefore, was labeled conceptual (Garner & Garner). Task Five would be considered conceptual since it pertains to analyzing an integral and asks to explain the meaning and recall the properties of integration (Garner & Garner).

The researcher administered the tasks to the students from each of the four selected schools. The only motivation the researcher used to have the students make a strong effort in completing the tasks was telling them that the tasks can help find weaknesses in their understanding in order for them to better prepare themselves for the AP exam. The procedural tasks were distributed first and the researcher told the students that they were allowed to use the calculator of their choice. In addition, the instructor asked the students to show as much work as possible and if all the work was done on the technology, to write out the steps taken to solve the question. After the students completed the procedural tasks, the researcher either allowed the class to take the conceptual tasks immediately after, if time permitted, or came back the next day.

For the conceptual task, the researcher informed the students that they cannot use any type of calculator and were only allowed to use pencil and paper.

After collecting all the exams, the researcher graded each paper. If the students achieved the correct answer, they earned 10 points for each question. If the students illustrated the correct understanding of how to solve the task, they earned five points. If there was no evidence of understanding, the students earn zero points. After the grading, the researcher used the names provided by the instructors to sort the exams into four groups. The first group consisted of the low performing GC students. The second group consisted of the high performing GC students. The third group consisted of the low performing CAS students. And lastly, the fourth group consisted of the high performing CAS students. In order to select a sample the researcher selected from the two high performing groups.

Data Analysis

The purpose of the background information, especially the data on the experience of the instructor assisted the researcher in grouping instructors that share similarities. Similar to the study conducted by Porzio (1997), where three different instructors were used and the only relation they shared was the number of years experience they had teaching calculus. Ferrini-Mundy and Gaudard (1992) and Chappell (2003) conducted studies in similar conditions, where the type of instruction was classified into two groups, traditional and reform; however, the instructors teaching the courses were irrelevant to the study. Instead, the studies focused on the class environment as a whole.

The research questions for this study looked at comparing the grades of students categorized based on the type of instruction received during the learning of calculus. The statistical procedure that was used was a simple one-way analysis of variance (ANOVA). This is the procedure of choice when there is only one grouping dimension. The scores on the set of tasks were entered into SPSS and a Levene's Test of Equality of Error of Variances, Descriptive Statistics, and One Way ANOVA was performed at the probability of 0.05. The researcher also ran a Post Hoc Test (Tukey or Scheffe dependent on if the groups are equal or not) to see the difference in the groups (Triola, 2011).

The analyses include multiple tables illustrating the results of the data collected. These tables provide descriptive statistics for the calculus students. The tables were grouped into three categories. The first category provides the demographics of the population and sample used in the study. The second category shows analyses on the procedural set of tasks. The third category shows the analyses on the conceptual set of tasks.

Limitations and Ethical Issues

McNamara (1994) discusses five ethical concerns to be considered when conducting research studies. The researcher must deal with voluntary participation, take precautions to avoid harm to the respondents, anonymity and confidentiality, identifying purpose and sponsor, and analysis and reporting. The researcher made sure the participants were aware that involvement in the study was completely voluntary. The researcher took precautions to avoid any embarrassment or feelings of discomfort towards the participants. The identities of the participants were kept confidential and all data were stored on a locked computer. The purpose of the study was explained to the participants and any procedures that warrant details were disclosed. Finally, the researcher reported all methods and results to professional colleagues for feedback and suggestions.

CHAPTER FOUR: DATA ANALYSIS

Introduction

The purpose of the current study was to investigate if using computer algebra system (CAS) empowered graphing calculators compared to graphing calculators (GC) without CAS capabilities, influenced student performance during Advanced Placement (AP) calculus AB instruction. In addition, this study ventured to determine if CAS and GC students differed in their conceptual and procedural understanding during calculus instruction. In order to compare the two groups, classroom observations were conducted. Following the observations, the students completed two sets of tasks consisting of questions evaluating their conceptual understanding and procedural skills.

The sample for this study consisted of 128 high school AP calculus AB students enrolled in the 2011-2012 academic year at four public high schools. All the schools were located in the state of Florida and spanned over two counties. After observing teachers to classify the level of CAS and GC use during their instruction, four schools were selected in order to participate in the study. The four schools were selected based on the results of the observation rubric and the TRS. After observing each teacher three to five times, the researcher administered two sets of calculus tasks to the students. The two sets of calculus tasks were used to measure the students' procedural and conceptual understanding of calculus. The first set of tasks consisted of four procedural questions. During the completion of these tasks the students were allowed to use the preferred calculator used during their instruction. Depending on the group, the students used either CAS or GC. In addition, the students from both groups had a choice to complete the task using only paper and pencil procedures. The second set of tasks consisted of five conceptual questions and the students were not allowed to use any form of technology and were to rely on paper and pencil only.

This chapter reports the results of the study on the effect of CAS instruction on the students' conceptual and procedural understanding of the second half of an AP calculus AB course that covers topics of integration. This chapter presents the results of the statistical analyses of the data collected from the two sets of tasks that were used to investigate the research questions. The first part discusses the results of the data collected during the researchers observations used to gather details about the amount, type, and extent of the technology that was used during the calculus instruction. In addition, data on the demographics of each school are discussed. The second part reports the descriptive statistics and an independent samples *t*-test for the procedural set of tasks. Each of the four procedural questions was analyzed and an interpretation is offered in order to discuss the findings. The results for each of the four schools are reported. Comparisons include results on the procedural set of tasks for each of the four schools individually followed by comparisons of the GC students and CAS students. In addition, there was an analyses looking at the high performing students (students with course grades of A's and B's) and the low performing students (students with course grades of C's and D's).

The third part reports the descriptive statistics and an independent samples *t*-test for the conceptual knowledge set of tasks. Each of the five tasks was analyzed and an interpretation is offered in order to discuss the findings. The results for each of the four schools are reported. As with the procedural set of tasks analysis, comparisons include the findings on the conceptual set of tasks for each of the four schools individually followed by comparisons of the GC students and CAS students. In addition, there were analyses looking at the high performing students and the low performing students. The fourth part reports the descriptive statistics for the overall results from both sets of tasks that are need to answer the research questions. This part also reports the overall results of the AP calculus AB performance between the two groups.

The following are the three research questions and their corresponding hypotheses addressed in this study, reiterated for ready reference, in order to serve as the organizing framework for this chapter.

1) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of procedural calculus tasks?
2) Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of conceptual calculus tasks?
3) Is there a significant difference in performance between high performing students that have had instruction with computer algebra system (CAS) and high performing students that have had instruction with the graphing calculator (GC) on calculus tasks?

Demographics of Participants

Graphing Calculator Students

In the remainder of the chapter the terms “CAS” and “GC” will be used to describe instructions on learning environments with computer algebra system (CAS) and graphing calculator (GS) respectively. During this study a total of seven schools were visited. Four of the schools used GC and three of the schools used CAS. In order to determine which schools extensively used technology during the instruction, the researcher visited each school three to five times in order to observe the extent in which the technology of choice was used. The researcher did classroom observations during regular scheduled classes during classroom instruction, for both the GC environments and CAS environments. The observation rubric (see Appendix A) was used to rate the amount of technology used and the TRS (see Appendix B) was used to rate the level of the technology used during the calculus instruction and also to record various information about the class and instructor. Since this study gathered data from various schools, instructors and instruction methods, the researcher wanted to find as many similarities as possible in order to have a comparable population. The demographics of the seating arrangements and the amount of technology used during each class session assisted in identifying common features between the different schools. The features included organization of the lesson, when to use the technology, and the overall presentation of the concepts being learned.

There were four GC schools visited that were located in the same county. Each school had one instructor that the researcher observed and obtained information about. Each school offered a mathematics background entrance exam to the students in order to determine their eligibility for taking AP calculus AB. However, one of the schools (non-participating school (NPS)_1) used a college algebra entrance exam in order to determine which students could enroll into college algebra at a local community college. If the students were unsuccessful in passing the college algebra entrance exam, they would then be placed into AP calculus AB. As a result, as per the teacher, the students enrolled in school NPS_1 lacked a strong foundation to be successful in the course. All four of the schools arranged their classrooms using traditional rows and group work consisted of working in pairs with one's neighbor. Two of the schools (labeled GC_1 and GC_2 for this study) had one graphing calculator per student each owned by the students. The third school (NPS_2) had a class set with less than 50% of the students owning their own, and NPS_2, allowed any type of calculator as it was used for computational purposes only.

During instruction, schools GC_1 and GC_2 reinforced each lesson while using the graphing calculator. Based on the rating system used in the observation rubric, the schools used the advanced features of the graphing calculator. Each school received a majority rating of four for parts that were applicable during the lessons. Table 3 illustrates the TRS used by the researcher in order to rank the use of technology for the schools.

Table 3
Technology Survey and Technology Ranking Scale (TRS)

To what extent is the calculator used during each of the following:

Demonstrations during teaching of concepts	1	2	3	4	5	6	N/A
When analyzing functions	1	2	3	4	5	6	N/A
When analyzing graphs of functions	1	2	3	4	5	6	N/A
When modeling questions dealing with graphs	1	2	3	4	5	6	N/A
While solving real world word problems	1	2	3	4	5	6	N/A
While solving equations	1	2	3	4	5	6	N/A
When performing integration	1	2	3	4	5	6	N/A
When analyzing what the integral means	1	2	3	4	5	6	N/A
When estimating with finite sums	1	2	3	4	5	6	N/A
When analyzing or finding maximum area	1	2	3	4	5	6	N/A
When working on anti-derivatives	1	2	3	4	5	6	N/A
When finding the area under curves	1	2	3	4	5	6	N/A
When working with definite integrals	1	2	3	4	5	6	N/A
When evaluating sums and limits of integrals	1	2	3	4	5	6	N/A
When solving integrals with trigonometric functions	1	2	3	4	5	6	N/A

The following is a key to the ranking survey:

1. Using calculator for computational purposes only
2. Using built-in functions of calculator or "Black Box" version.
3. Using calculator for graphical illustrations only.
4. Using built-in functions of calculator with reference to graphical illustrations.
5. Using graphical representations and symbolic manipulation functions.
6. Using graphical representations, symbolic manipulation, and animation functions.

Table 4 shows the summary of the ranking of each school.

Table 4
Percentages of Ranking System for the use of Technology in GC Schools

School	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank N/A
NPS_1	73%	0%	0%	0%	0%	0%	27%
NPS_2	0%	60%	20%	7%	0%	0%	13%
GC_1	0%	13%	20%	60%	0%	0%	7%
GC_2	0%	6%	20%	67%	0%	0%	7%

A rating of four constitutes using built-in functions of the graphing calculator with reference to graphical illustrations. According to Reznichenko (2007), using the built-in functions and graphical illustrations of the graphing calculator constitutes using the most advanced features of the calculator. The built-in functions should relate to the subject in discussion. For example, when studying calculus, integration and differentiation functions are often used.

During integration, the function keys dealing with integration were utilized. In addition, graphing was used to enhance student understanding by providing an image that would relate to the situation discussed during the lesson. Students were encouraged to use the calculator to assist in answering problems as well as provide graphical illustrations when applicable. The instructors from GC_1 and GC_2 displayed the calculator abilities on the board via a projector attached to a computer coherent to the lesson being delivered. NPS_1 did not use a calculator to demonstrate graphs. Instead, the instructor used other means of producing graphs. It is important to note that the graphs drawn by hand were just as clear as the ones produced by the graphing calculator. However, not all the students could copy an accurate illustration on their own and therefore, may not have been able to produce vivid graphs to help in their studies and enhance their understanding. NPS_2 did not use graphing calculators on a regular basis, and did all computations by hand and also produced graphs, when applicable, by hand.

The students from GC_1 and GC_2 seemed very comfortable with the graphing calculator. The students used the graphing calculator efficiently to complete their tasks during class. For example, when evaluating an integral, the instructor would illustrate the expansion on the board and then ask the students to evaluate the integral using their graphing calculators. During one lesson in GC_1 the researcher observed the following question being answered.

$$\int_{0}^{\frac{\pi}{6}} \sin(3x)$$

Figure 5: Sample problem during graphing calculator instruction.

The students quickly went to their graphing calculators and entered the integral into the technology in order for the graphing calculator to perform the evaluation. In the calculator the students would find the "fint" function. Then they would enter the function, sin (3x), followed by x, 0, π/6. When the students then pressed the enter key, the result of 1/3 would appear on the screen.

The instructors from both the GC_1 and GC_2 schools acted more like facilitators rather than lecturers. Students were actively engaged when assigned problems to complete on the graphing calculator. With the help of the graphing calculator, the students engaged through discovery methods and visual representations that the graphing calculator would produce. The classes exhibited a more reform calculus environment where the instructors presented a concept and activity and the students discovered the main ideas at the end of the activity, as opposed to a traditional calculus environment.

Based on the results from the observations of the instruction with the graphing calculator, (NPS_1, NPS_2, GC_1, and GC_2), the researcher chose students from GC_1 and GC_2 to participate in the current study. In total there were 187 students. From the observation rubric the researcher determined there were 99 male students and 88 female students. Therefore, 53% of the GC students were male and 47% were female. For this study, the researcher grouped the students as high performing and low performing. The instructors from each school provided the researcher with the names of the students that were maintaining a grade of an A or a B in the course. These students were considered high performers. The students that were maintaining a grade of a C or a D in the course were considered low performers. From the information provided by the instructors, the researcher determined that 92 students were considered high performers and 95 students were considered low performers. This means that approximately 49% of the students were high performers from the GC schools used in the present study. Table 5 summarizes the demographics of the GC students.

Table 5
Demographics of the Students from GC_1 and GC_2

	GC_1	GC_2	Total	Percentage
Male	61	38	99	53
Female	42	46	88	47
High Performers	59	33	92	49
Low Performers	44	51	95	51

CAS Students

The researcher visited three schools in which the AP calculus AB instructors acknowledged, via phone, that they were using CAS during instruction. All three schools were from the same county. The first school observed could not be used in the present study because the students completed the AP curriculum prior to the winter break and the students were only reviewing for the AP calculus exam. The researcher was unable to obtain any information about the type of instruction that was used during the teaching of the calculus concepts. The second and third schools visited (CAS_1 and CAS_2) illustrated many similarities with the type of instruction used. Both schools organized their students in traditional rows. Each student owned their own CAS empowered graphing calculator and were required to bring them to each class. Based on the rating system, both schools were rated a six for the majority of the areas covered whenever applicable to the observation rubric. A rating of six indicates that the instructor used graphical representations, symbolic manipulation, and animation functions when applicable during instruction. The difference between rating 4, which was the most advance use of the graphing calculator and rating 6 is the animation function capabilities that the CAS offers. According to Pierce and Stacey (2002), the most advance uses of CAS are when symbolic manipulation and animation are utilized. Table 6 illustrates the percentages for the results of the ranking system used in the observation rubric.

Table 6
Percentages of Ranking System for the use of Technology in CAS Schools

School	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank N/A
CAS_1	0%	0%	0%	10%	40%	37%	13%
CAS_2	0%	0%	0%	7%	47%	33%	13%

The instructors from both CAS schools reinforced each lesson with calculator usage. Instructors from both schools had CAS software installed on the classroom computer and projected the calculator on the white board for demonstration and reference purposes during the instruction. When solving problems the instructors from both schools expected their students to provide graphical support for their answers and use the symbolic manipulation features of the calculators to help work out the problems. The major difference between the schools was that the instructor from CAS_1 would show the pencil-and-paper computation prior to illustrating how to perform the tasks on the calculator. The instructor from CAS_2 would provide details on how to perform the tasks via calculator and did not focus on the pencil-and-paper computations.

One example of the use of CAS during instruction was during a lesson on definite integrals. The question shown in Figure 6 below was worked out using the advanced capabilities of CAS.

Find the area of the region bounded by $f(x) = x^3$ *and* $g(x) = e^{x^2}$

Figure 6: Example of a question used during CAS instruction.

In order to solve the problem the instructor provided the formula shown in Figure 7.

$$A = \int_a^b (f(x) - g(x))\ dx$$

Figure 7: Formula provided to solve area question.

The students were left to work on their own for a few minutes. As the students were working the instructor suggested they first graph the problem by using the graphing capabilities of the CAS in order to visualize the problem. When the CAS produced the graph the students were able to find the upper and lower limits by using the symbolic manipulation function "root finder." Then the students were able to use the computational features to find the area.

Similar to the GC instructors, the instructors from both the CAS_1 and CAS_2 schools acted as facilitators. The lessons always began with an idea and led to a problem to solve. Students were actively learning when assigned problems to complete on the CAS. The CAS provided help on graphing and symbolic manipulation in order to make computational tasks simpler. The students learned through discovery and collaboration. As with the GC classes, the CAS classes exhibited a more reform calculus environment where the instructors presented a concept and activity and the students discovered the main ideas at the end of the activity, as opposed to a traditional calculus environment.

Based on the results of the observations of the CAS instruction, the researcher chose to use both schools in the present study. In total, there were 146 students. From the observation rubric the researcher determined there were 83 male students and 63 female students. Therefore, 57% of the CAS students were male and 43% were female. As with the GC students, the researcher grouped the CAS students as high performers and low performers based on the information provided by the instructors. From the information provided by the instructors, the researcher determined that 76 students were considered high performers and 70 students were considered low performers. This means that approximately 52% of the students were high performers from the CAS schools used in the current study. Table 7 summarizes the demographics of the CAS students.

Table 7
Demographics of the CAS Students

	CAS_1	CAS_2	Total	Percentage
Male	49	34	83	57
Female	32	31	63	43
High Performers	48	28	76	52
Low Performers	33	37	70	48

Data Analysis

The data analysis part of the study contains analyses of the results obtained on the two sets of tasks given to the students to complete. The first set of tasks contained four procedural tasks used to measure the procedural skills attained by the students. The second set of tasks contained five conceptually oriented problems used to measure the conceptual understanding attained by the students. The maximum score for the procedural skills set of tasks was 40 points and for the conceptual knowledge set of tasks was 50 points.

For the procedural skills set of tasks, each question was worth 10 points. If the student made a fair attempt they received five points. A fair attempt was illustrated if the student was able show some understanding of the concept depicted by the question asked. If the error was computational and illustrated understanding, five points were awarded. Zero points were given if the question was left blank or there was evidence of misunderstanding. For example, if the work provided had no relation to the question being asked and/or there was a conceptual mistake illustrated. The researcher graded each problem and recorded the results.

The total population for this study consisted of 333 students. There were 187 students classified as GC and 146 students classified as CAS. GC_1 consisted of 103 students and GC_2 consisted of 84 students. CAS_1 consisted of 81 students and CAS_2 consisted of 65 students.

Analysis of Procedural Set of Tasks

The set of procedural tasks given to the students consisted of four tasks that were used in order to examine the students' procedural skills. For the GC students, they were allowed to use their graphing calculator and for the CAS students, they were allowed to use their CAS empowered graphing calculator. The first task, shown in Figure 8, was used to determine if students could evaluate two definite integrals. In order to accomplish this task, the students would have used symbol manipulation and recalled facts pertaining to the characteristics of an integral. In order receive the full ten points; the students would need to obtain the correct answer. The researcher instructed students to show all work and explain the steps used when using the calculator. Figure 9 shows an example of a solution that would receive full credit.

Compute the following definite integrals:

a. $$\int_1^2 4x^3\,dx$$

b. $$\int_0^{\frac{1}{2}} e^{2x}\,dx$$

Figure 8: Procedural Task One (Garner & Garner, 2001).

a. $$\int_1^2 4x^3\,dx = \left[x^4\right]_1^2 = 16 - 1 = 15$$

b. $$\int_0^{\frac{1}{2}} e^{2x}\,dx = \quad Let\, u = 2x,\ then \quad du = 2dx$$

$$therefore,\ dx = \frac{1}{2}du.$$

$$\int_0^{\frac{1}{2}} e^{u}\,du = \left[\frac{1}{2}e^{2x}\right]_0^{\frac{1}{2}} = \frac{1}{2}e^{0} = \frac{1}{2}e - \frac{1}{2}.$$

Figure 9: Sample solution to procedural Task One.

This solution illustrates the use of symbolic manipulation in order to evaluate the integrals. In total, approximately 73% of the population received full credit on this question.

The second task, shown in Figure 10, was used to determine what the students understood about limits. Heid (1988) found students show difficulty in the understanding of limits. In the study, Heid found students that were taught using graphical and symbol manipulation computer programs were able to identify the notion of a limit with a process rather than a number. Task Two consisted of two parts. The first part asked the students to find the limit of a piecewise function as it approached one from the positive side. Many students had little to no trouble with this part of the question. The second part of the task asked students to find the derivative of the function from the left at $x = 1$. Here students illustrated the most trouble. Approximately 92% of the students were able to answer the first part of Task Two correctly. However, 32% were able to answer the second part correctly. The most common error, made by 45% of the students that received the first part correctly, was evaluating the derivative at $x = 1$. The function was not continuous at $x = 1$, and the derivative of a function does only exist if the function is continuous. Figure 11 illustrates a correct answer for Task Two.

Consider: $f(x) = \begin{Bmatrix} x^2, & x \leq 1 \\ x+3, & x > 1 \end{Bmatrix}$

a. *Find* $\lim_{x \to 1^+} f(x)$

b. *What is the derivative of f(x) from the left at x = 1?*

Figure 10: Procedural Task Two (Eisenberg & Dreyfus, 1991).

a. $\lim_{x \to 1^+} x+3=1+3=4$

b. $f(x)=x^2$

$f'(x)=2x=2(1)=2$

This answer is incorrect because the function is not continous therefore the derivative does not exist.

Figure 11: Sample solution to procedural Task Two.

The third task, shown in Figure 12, on the procedural skills set of tasks required students to understand how to analyze an anti-derivative in the calculator. Then the students were asked to produce the graph of the original function using methods of integration. This question was the most challenging out of the four procedural tasks. Approximately 35% of the population was able to answer this question fully. The students illustrated two common errors. The first error noticed was the students graphed the derivative rather than the original function. The second error was the students had the right function but seemed to make an error typing the function into the calculators. Figure 13 illustrates a student working the problem out by hand and then drawing the graph produced by the calculator.

Using the graph of $\frac{dy}{dx}=f'(x)=(x-1)(x-2)^2(x-3)^3$, *sketch a graph of* $y=f(x)$.

Figure 12: Procedural Task Three (Eisenberg & Dreyfus, 1991).

$$\begin{aligned} f'(x) &= (x-1)(x^2-4x+4)(x^3-9x^2+27x-27) \\ &= (x^3-5x^2+8x-4)(x^3-9x^2+27x-27) \\ &= x^6-14x^5+80x^4-238x^3+387x^2-324x+108 \end{aligned}$$

$$\int f(x) = \frac{x^7}{7} - \frac{7x^6}{3} + 16x^5 - \frac{119x^4}{2} + 129x^3 - 162x^2 + 108x + C$$

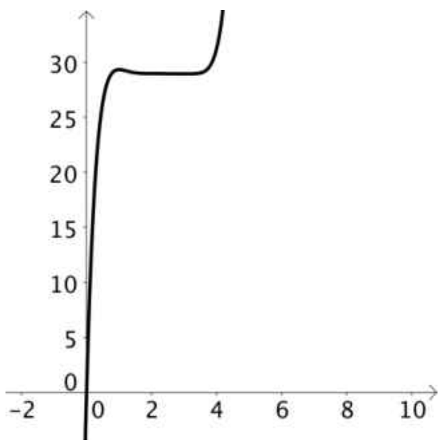


Figure 13: Sample solution to procedural Task Three.

The fourth task, shown in Figure 14, on the procedural skills set of tasks asked students to find the maximum slope of a graph. According to Heid (1988), students that are confused about limits permeated their explanation of derivatives as they describe derivatives as approximations for slopes of tangent lines rather than as being equal to the slope. During the observations done by the researcher, each class discussed how to find the maximum of a function. The idea is taking the first derivative. In addition, the researcher observed students learning that the maximum slope is found using the second derivative. For Task Four the students were asked to find the maximum slope. Approximately 42% of the students answered task four correctly. The most common error, which was made by 35% of the students, was using the first derivative rather than the second to find the maximum slope. This may have occurred from the students mistakenly assuming that maximum slope and slope are interchangeable terms. Figure 15 illustrates a correct solution that would receive full credit.

Find the maximum slope of the graph of $y = -x^3 + 3x^2 + 9x - 27.$

Figure 14: Procedural Task Four (Eisenberg & Dreyfus, 1991).

$$
\begin{aligned}
&f'(x) \quad = -3x^2 + 6x + 9 \\
&f''(x) \quad = -6x + 6 \\
&\textit{Set } f''(x) = 0, \qquad -6x + 6 = 0, \quad \textit{then}, \ x = 1 \\
&f'(1) = -3(1)^1 + 6(1) + 9 \\
&\qquad = 12
\end{aligned}
$$

Figure 15: Sample solution to procedural Task Four.

The next part of this analysis section will provide the results of an independent *t*-test in order to provide descriptive statistics for the tasks completed by the students on the procedural set of tasks. The independent samples *t*-test will help determine if there were any statistically significant differences among the students. Table 6 shows the descriptive statistics for the procedural set of tasks for the GC instruction group and Table 8 shows the descriptive statistics for the procedural set of tasks for the CAS instruction group.

Table 8

Descriptive Statistics for Tasks on set of Procedural Tasks (GC group)

Variable	GC_1		GC_2	
	M	*SD*	*M*	*SD*
Task 1	8.35	3.16	8.10	2.68
Task 2	7.01	3.38	7.15	2.34
Task 3	5.29	3.69	5.36	3.36
Task 4	5.87	3.86	6.07	3.48
Procedural Overall	66.31	22.23	66.70	16.95

Note: *M* = Mean; *SD* = Standard Deviation.

Table 9

Descriptive Statistics for Tasks on set of Procedural Tasks (CAS group)

Variable	CAS_1		CAS_2	
	M	*SD*	*M*	*SD*
Task 1	9.01	2.00	8.85	2.46
Task 2	7.37	2.94	7.40	3.07
Task 3	6.23	3.49	6.54	3.74
Task 4	6.05	4.01	5.69	4.50
Procedural Overall	71.67	18.63	71.19	18.07

Note: *M* = Mean; *SD* = Standard Deviation.

As Table 8 shows, GC_1 (M = 66.31, SD = 22.33) and GC_2 (M = 66.70, SD = 16.95) students did not differ in their overall procedural scores. There was no statistically significant difference between the two groups on individual items of the set of tasks. The Levene's test of equality of error variances was not significant for the data between the two groups (F = 9.51, p = 0.90). Table 9 illustrates that the CAS_1 students (M= 71.67, SD = 18.63) and CAS_2 students

(M = 71.19, SD = 18.07) did not differ in their overall procedural scores. The Levene's test of equality of error variances was not significant for the data between the two groups (F = 0.00, p = 0.88).

In addition, Table 8 shows that there was no significant difference in the mean scores on individual tasks between GC_1 students and GC_2 students. Table 9 shows there was also no significant difference in the mean scores on individual tasks between CAS_1 students and CAS_2 students. Overall, when comparing Table 8 and Table 9, they show that the CAS students had higher mean scores than GC students. Table 10 shows the mean scores between GC students and CAS students when all 187 GC students are grouped together and when all 146 CAS students are grouped together. The independent *t*-test was used in order to offer an analysis on the mean scores as well as provide insight on any significant differences between the two groups. A 95% confidence interval was set in order to interpret if there was statistical significance ($p > 0.05$) or if there was no statistical significance ($p \leq 0.05$) between the mean scores.

Table 10
Descriptive Statistics for GC Students versus CAS Students on Procedural Tasks

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Task 1	8.24	2.95	8.94	2.21	-2.40	0.02
Task 2	7.05	2.95	7.38	2.99	-0.94	0.35
Task 3	5.32	3.54	6.37	3.60	-2.66	0.01
Task 4	5.96	3.69	5.89	4.22	0.17	0.87
Procedural Overall	66.48	19.98	71.46	18.32	-2.34	0.02

Note: degree of freedom (df) = 331.

As Table 10 shows, the CAS students had a higher mean score (M = 8.94, SD = 2.21) than the GC students (M = 8.24, SD = 2.95) on Task 1. Task 1 asked students to evaluate definite integrals. A mean score of 82% and 89% illustrate that both groups were able to use the technology adequately in order to obtain the correct answer. Navigating through both types of calculators is important in order to perform tedious calculations and requires strong procedural skills in order to properly input the correct keystrokes to obtain the requested output. Although both groups performed well on this task, there was a statistically significant difference in the mean scores between the two groups with $t(331) = -2.40$ and $p < 0.05$. A factor that can offer some insight on this result is the CAS calculator offers a more visual interpretation when inputting integrals that require evaluating than the graphing calculator. When asked to evaluate integrals, the CAS shows the actual integral symbol and offers input boxes so the students can easily enter the problem given without having to remember order. Unlike the GC which offers less animation and requires students to remember the order of inputs or the answer will return either "error" or the incorrect response. Figure 16 shows an example of an integral being evaluated on a CAS calculator and Figure 17 shows an example of an integral being evaluated on graphing calculator.

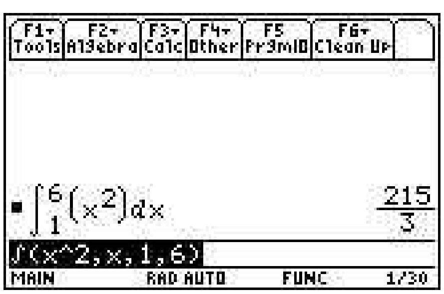


Figure 16: An example of evaluating integrals using CAS.

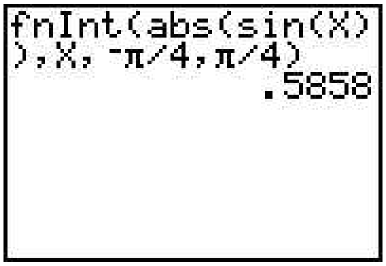


Figure 17: An example of evaluating integrals on a graphing calculator.

Task Two is designed to measure the students' knowledge on key concepts of limits and their properties. In particular, the students' knowledge about limits from left and right, numerical representation of limit, function continuity, interval continuity, and derivative continuity was the focus. Table 10 shows the result that the differences in the mean scores between GC students and CAS students were not significant. In addition, both groups had mean scores of approximately 70%, this offers some insight on the misunderstandings of both groups of students. Students seemed to have difficulty in understanding that in order for the limit to exist, the limit from the left must approach the limit from the right. Students seemed to show difficulty in relating the limit and continuity concepts. Although in class, as observed by the researcher, the students learned how to interpret limits when approaching from the left and the right, the issue seemed to arise when the function was a piecewise.

According to Table 10, there was a statistically significant difference between the mean scores of the GC students ($M = 5.32$, $SD = 3.54$) and the CAS students ($M = 6.37$, $SD = 3.60$) on Task 3, $t(331) = -2.66$ and $p < 0.05$. Both groups scored low on this task as the students seemed to have difficulty using an appropriate scale when graphing a function on the calculators in order to view the behaviors. The CAS students scored slightly higher with approximately a 64% than the GC students with approximately 53%. Again, this might be due to the animation features available on the CAS technology that is not available on the graphing calculator. In addition, the ease of entering functions into the CAS provides some assurance of less input errors than using the GC which requires multiple uses of parenthesis in order to group terms together. As mentioned earlier, only 35% of the overall population was able to fully answer this question. Many students received partial credit for being able to provide the correct function, but showed difficulty graphing the function after it was obtained.

The results on Task 4 show similar results to Task 3. As Table 10 shows, the GC students ($M = 5.96$, $SD = 3.69$) mean score and the CAS students ($M = 5.89$, $SD = 4.22$) mean score were low. Both groups had mean scores less than 60% although there was no statistically significant difference with $t(331) = 0.17$ and $p < 0.05$, between them. The most common error illustrated by both groups was entering the wrong function into the corresponding calculator and asking for the slope. In both calculators, there exists a built in function asking to evaluate the derivative of a function. However, the common error was finding the slope at the first derivative rather than the second derivative, which is where one would find the maximum slope. As mentioned in chapter three, this question could be used to analyze both procedural skills as well as conceptual knowledge. The common error shown by the students illustrates a lack of conceptual knowledge rather than a lack of procedural skills.

In conclusion, the CAS students of the population achieved an overall higher score on the procedural set of tasks. As Table 10 shows, there was a statistically significant difference between the mean scores of the GC students ($M = 66.84$, $SD = 19.98$) and the CAS students (M

= 71.46, SD = 18.32) with t(331) = -2.34 and $p < 0.05$. The CAS students seemed to appreciate the animation features of the CAS as it offered a somewhat easier functionality over the GC. However, the overall population is not the focus of this study. As mention in chapter three and in the introduction, a sample of the high performing students is what will be used to address the research questions for this study.

In order to select a sample from each group, two further analyses have been conducted. The first will compare the high performing students with the low performing respectively from each group. This analysis will be used in order to decide if there exists a significant difference in order to further separate the group. The second analysis will select the sample from the high performing students and compare the high performing CAS students with the high performing GC students in order to address research question one.

High and Low Performing Students' Performance on Procedural Tasks

According to the information provided by the teachers that participated in this study, 49% of the GC students were considered high performing with grades of A's and B's, and 51% of the GC students were considered low performing with grades of C's and D's. The CAS teachers provided similar information and as a result, 52% of the CAS students were classified as high performing and 48% were classified low performing. Table 11 shows the results of an independent samples *t*-test that was used to analyze the differences in the mean scores between the high performing GC students and low performing GC students. The focus is on the overall mean score as the factor needed in order to be able to select a sample. The analyses of the individual tasks illustrated and helped support the claim that the high performing students scored higher on the tasks than the low performing students.

Table 11
Descriptive Statistics for GC High Performing versus Low Performing Students on Procedural Tasks

Variable	High Performing *M*	High Performing *SD*	Low Performing *M*	Low Performing *SD*	*t-value*	*p (2-tailed)*
Task 1	8.91	2.44	7.58	3.25	-	-
Task 2	7.42	2.61	6.74	3.23	-	-
Task 3	5.71	3.44	4.95	3.61	-	-
Task 4	6.09	3.91	5.84	3.47	-	-
Procedural Overall	70.33	18.88	62.76	20.40	0.23	0.01

Note: degree of freedom (df) = 185.

As shown in Table 11, the high performing GC students had higher mean scores on all four tasks. In addition, there overall mean score was significantly higher and there was a statistically significant difference in the mean scores on all four procedural tasks between the GC high performing students (M = 70.33, SD = 18.88) and the GC low performing students (M = 62.76, SD = 20.40) with t(185) = 0.23 and $p < 0.05$. As a result, a sample of 64 students from the GC high performing group will be selected to compare with the CAS sample selection.

In order to select a sample of CAS students, an analysis comparing the CAS high performing students and the CAS low performing students was performed. Table 12 shows the results of the independent samples *t*-test that was performed on the CAS group.

Table 12
Descriptive Statistics for CAS High Performing versus Low Performing Students on Procedural Tasks

Variable	High Performing *M*	High Performing *SD*	Low Performing *M*	Low Performing *SD*	*t-value*	*p (2-tailed)*
Task 1	9.41	1.63	8.42	2.63	-	-
Task 2	7.63	2.32	7.11	3.57	-	-
Task 3	6.78	3.34	5.93	3.83	-	-
Task 4	6.05	4.19	5.71	4.28	-	-
Procedural Overall	74.67	14.23	67.96	21.48	2.24	0.03

Note: degree of freedom (df) = 144.

As shown in Table 12, the high performing CAS students had higher mean scores on all four tasks. Task One and Task Three show approximately a 10% difference in mean scores. This may suggest that the high performing students were more comfortable with the CAS as those two tasks, as describe earlier, require a stronger understanding with regards to the calculator's capabilities. In addition, the high performing students overall mean score was significantly higher and there was a statistically significant difference in the mean scores on all four procedural tasks between the CAS high performing students ($M = 74.67$, $SD = 14.23$) and the CAS low performing students ($M = 67.96$, $SD = 21.48$) with $t(144) = 2.24$ and $p < 0.05$.

Analysis of Conceptual Set of Tasks

The conceptual set of tasks given to the students consisted of five tasks that were used in order to examine the students' conceptual knowledge of integration from an AP calculus AB course. While solving the tasks, students were not allowed to use their calculators. In actuality, the calculators would have been useless as the tasks consisted of short answer and/or open-ended questions that did not require computation or any type of arithmetic evaluation. The first task, shown in Figure 18, was used in order to assess students' ability to analyze graphs. This type of understanding, according to Garner and Garner (2001), would be considered conceptual. The students would receive full credit if they were able to correctly state the intervals and identify the minimum and maximum points. Partial credit was given if only one of the two parts was answered correctly. Figure 19 illustrates an example of a solution that received full credit.

Given below is the graph of a function.

a. On what intervals is f(x) increasing? Decreasing?

b. Estimate any points at which a local maximum or a local minimum occurs.

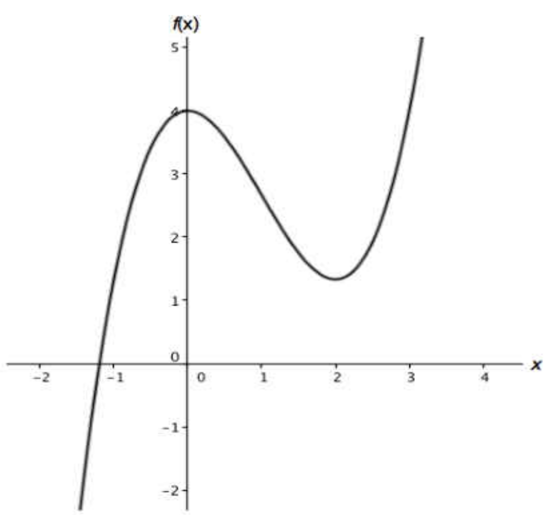


Figure 18: Conceptual Task One (Garner & Garner, 2001).

Increasing: $(-\infty,0]\cup[2,\infty)$
Decreasing: $[0,2]$
Local Maximum: (0, 4)
Local Minimum: (2, 1.5)

Figure 19: Sample solution to conceptual Task One.

In total, approximately 52% of the population received full credit for this question and approximately 45% of the population received partial credit. The most common error was in part a. The students labeled the dependent or *y* values as the interval.

Task Two, shown in Figure 20, on the conceptual knowledge set of tasks would be considered conceptual since it pertains to analyzing an integral and asks to explain the meaning and recall the properties of integration (Garner & Garner, 2001). This question produced low scores with approximately 34% of the students from the whole population able to answer the question correctly. This task stands out because the students were able to evaluate integrals on Task One from the procedural set of tasks; however, the low number of responses on the conceptual portion indicates a low conceptual understanding of the properties of integrals from the students. Figure 21 shows an example of a student's correct answer.

Why is the following computation obviously wrong?

$$\int_{-1}^{1}\frac{dx}{x^2}=\int_{-1}^{1}x^{-2}dx=\left.\frac{x^{-1}}{-1}\right|_{-1}^{1}=\left.\frac{-1}{x}\right|_{-1}^{1}=\frac{-1}{1}-\frac{-1}{-1}=-2$$

Figure 20: Conceptual Task Two (Eisenberg & Dreyfus, 1991).

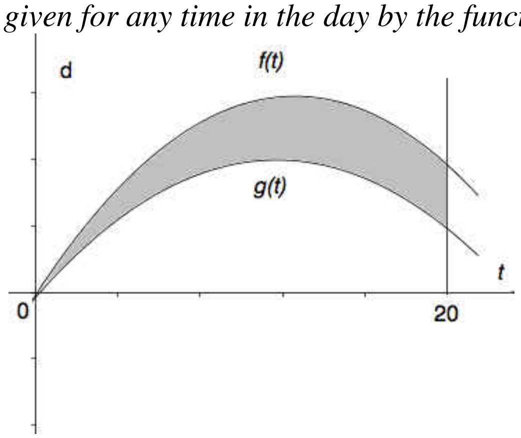
The following integral is discontinuous at x=0 and therefore cannot be evaluated.

Figure 21: Sample solution to conceptual Task Two.

The most common wrong answer to this question, given by 59% of the population, was "It is not wrong." The researcher accepts some responsibility for such a low percentage of correct responses. The use of the word "obviously" might have given the assumption that the question was not wrong and the students' thought it was a trick question. As a result of this outlier, during the analyses of the data, an additional portion was analyzed with Task Two omitted to determine if the overall mean score comparisons was affected.

Task Three, shown in Figure 22, on the conceptual set of tasks assessed if students know when it is appropriate to use integrals. The students illustrated knowledge of how to evaluate integrals using the technology. In this task the students needed to illustrate knowledge of when it is appropriate to use integrals and the appropriate way to use integration. This task consisted of three parts without providing any reference to integrals. However, 97% of the students were able to explain that integrals were needed to analyze the given graph in the problem. Approximately 64% of the population was able to answer the question completely correctly and 33% received partial credit for using integrals but incorrectly. One incorrect method was when computing the shaded region; students reversed the functions, which would result in a negative area. Ferrini-Mundy (1987) found that calculus students' errors related more to setting up integrals rather than solving them. In this case students did not try to make sense of the fact that negative areas do not exist on a real plane. Another common error was some students did not understand what the shaded region represented. Many different reasons could explain this confusion. Bremigan (2005) argues that problems traditionally included in integration draw upon students' abilities to interpret a diagram representing some situation geometrically, with the ideas of calculus concepts occurring later in the problem. Garner and Garner (2001) suggest that this confusion arises from a lack of real world applications being introduced during instruction. Figure 23 illustrates a full correct answer to Task Three.

Suppose that on a typical day the rate at which the electric company accumulates revenue (in dollars per minute) is given for any time in the day by the function f(t). The rate at which the company's operating costs accumulate (in dollars per minute) is given for any time in the day by the function g(t). These functions are graphed below.



a. *How could you use calculus to find the company's revenue and cost functions?*
b. *How could you use calculus to compute the shaded area?*
c. *What does the shaded area represent economically?*

Figure 22: Conceptual Task Three (Garner & Garner, 2001).

a. $\int f(t)dt = Revenue\ Function$

$\int g(t)dt = Cost\ Function$

b. $\int_0^{20} (f(t) - g(t))dt$

c. *Profit: (Revenue – Cost)*

Figure 23: Sample Solution to conceptual Task Three.

Task Four, shown in Figure 24, involves both symbol manipulation and interpretation of results, and therefore, would be labeled as conceptual (Garner & Garner, 2001). Similar to Task Three, the students were required to interpret a situation that involves a real world situation. In addition, this would be considered a common question dealing with related rates. In order to answer the question correctly, students had to understand that even though values can be obtained from functions, the values, when given a situation, have restrictions. Approximately 58% of the population was able to correctly answer both parts of the question and approximately 39% was able to answer one part correctly. The most common mistake was students' accepted negative values for the unit of miles, which cannot be the case. White and Mitchelmore (1996) found that students are often unsuccessful in translating verbal problem statements to abstract symbols due to a limited conceptual understanding of the concepts needed to solve such problems. Figure 25 shows a correct response to Task Four.

A compact car in motion on the freeway uses f(x) gallons of gas in driving x miles.

a. *For what values of x, if any, is f'(x) positive? For what values of x, if any, is f'(x) negative?*

b. *A motor home uses g(x) gallons of gas in driving x miles. How would you expect the graphs of f(x) and g(x) to compare?*

Figure 24: Conceptual Task Four (Garner & Garner, 2001).

a. *All values of x from (0, ∞) will be positive for f'(x). f'(x) cannot be negative.*

b. *The graph of g(x) will increase at a more rapid rate that the graph of f(x) because a motor home would use up gas faster than a compact car.*

Figure 25: Sample solution to conceptual Task Four.

Task Five, shown in Figure 26, was used to analyze the students understanding of the visual concept of an integral. According to Eisenberg and Dreyfus (1991), thinking visually makes higher cognitive demands than thinking algorithmically; and as a result, students show difficulty with visual thinking. The students from the current study's population showed some evidence of understanding integral properties. In this specific task, students were required to recall the property of an odd function and that an odd function that is integrated from –*a* to *a*, where *a* is any number, will equal zero. Approximately 59% of the students illustrated knowledge of the property however; only 27% of the students were able to illustrate understanding of how to evaluate the integral in its general form. Figure 27 shows the response to Task Five that would receive full credit.

$$\textit{If f is an odd function on [-a, a], evaluate .} \int_{-a}^{a} b + (f(x))\,dx$$

Figure 26: Conceptual Task Five (Eisenberg & Dreyfus, 1991).

$$\begin{aligned}\int_{-a}^{a} b + (f(x))dx &= \int_{-a}^{a} b\,dx + \int_{-a}^{a} f(x)dx \\ &= bx\int_{-a}^{a} \quad + 0 \\ &= ba - (-ba) \\ &= 2ab\end{aligned}$$

Figure 27: Sample solution to conceptual Task Five.

The next part of this analysis section provides the results of an independent samples *t*-test in order to provide descriptive statistics for the tasks completed by the students on the conceptual understanding set of tasks. The *t*-test helps determine if there were any statistically significant differences among the groups of students. Table 13 shows the descriptive statistics for the tasks on the conceptual set of tasks.

Table 13
Descriptive Statistics for Tasks on Conceptual Set of Tasks (GC group)

	GC_1		GC_2	
Variable	*M*	*SD*	*M*	*SD*
Task 1	7.04	2.92	6.79	2.97
Task 2	3.01	4.61	3.10	4.65
Task 3	7.86	2.94	7.86	2.49
Task 4	7.67	2.51	7.45	2.60
Task 5	4.12	3.86	3.75	4.17
Conceptual Overall	59.42	17.70	57.88	15.92

Note: M = Mean; *SD* = Standard Deviation.

Table 14
Descriptive Statistics for Tasks on Conceptual Set of Tasks (CAS group)

	CAS_1		CAS_2	
Variable	*M*	*SD*	*M*	*SD*
Task 1	8.02	2.58	8.08	2.45
Task 2	3.82	4.89	3.85	4.90
Task 3	8.27	2.64	8.31	3.10
Task 4	8.09	2.80	7.77	3.31
Task 5	4.81	3.75	4.62	4.53
Conceptual Overall	66.05	16.86	65.23	22.16

Note: M = Mean; *SD* = Standard Deviation.

Table 13 illustrates that GC_1 students (M = 59.42, SD = 17.70) and GC_2 students (M = 57.88, SD = 15.92) did not differ statistically on their overall scores on the conceptual understanding set of tasks. In addition, there was no statistically significant difference between the GC_1 and GC_2 on individual items of the conceptual set of tasks. The Levene's test of equality of error variances was not significant for the data between the two groups (F = 0.42, p = 0.54). Furthermore, Table 13 shows that there was no significant difference in the mean scores on individual tasks between GC_1 students and GC_2 students.

Table 14 shows that the scores for the CAS_1 students (M= 66.05, SD = 16.86) and CAS_2 students (M = 65.23, SD = 22.16) did not differ statistically on the overall conceptual set of tasks. The Levene's test of equality of error variances was not significant for the data between the two groups (F = 9.04, p = 0.80). In addition, Table 14 illustrates that there was no significant difference in the mean scores on individual tasks between CAS_1 students and CAS_2 students. The data from Table 13 and Table 14 shows that the CAS students had higher mean scores than GC students on every task.

Table 15 shows the mean scores between GC students and CAS students when all 187 GC students are grouped together and when all 146 CAS students are grouped together. The independent samples *t*-test was used in order to offer an analysis on the mean scores as well as provide insight on any significant differences between the two groups. A 95% confidence interval was set in order to interpret if there was statistical significance ($p \leq 0.05$) or if there was no statistical significance ($p > 0.05$) between the mean scores.

Table 15
Descriptive Statistics for GC Students versus CAS Students on Conceptual Tasks

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Task 1	6.93	2.94	8.05	2.52	-3.68	0.00
Task 2	3.05	4.62	3.84	4.88	-1.51	0.13
Task 3	7.86	2.74	8.29	2.84	-1.39	0.17
Task 4	7.57	2.54	7.95	3.03	-1.22	0.22
Task 5	3.96	4.00	4.73	4.10	-1.72	0.10
Conceptual Overall	58.73	16.90	65.68	19.33	-3.50	0.00

Note: degree of freedom (df) = 331.

As Table 15 shows, the CAS students had a higher overall mean score (M = 8.05, SD = 2.94) than the GC students (M = 6.93, SD = 2.94) on Task 1. Task 1 asked students to analyze a graph and to state the intervals of areas that are increasing and decreasing. In addition, the students were asked to find the maximum and minimum points. Although both groups did fairly well, the mean score of the GC students was approximately 70% and CAS students approximately 80%, there was a statistically significant difference in the mean scores between the two groups with $t(331) = -3.68$ and $p < 0.05$. The researcher noticed that more of the GC students misinterpreted the intervals by providing the *y*-axis values rather than the *x*-axis values.

Task 2 was designed to assess the students' knowledge on properties of integrals as well as facts about real numbers. As the analysis in Table 15 shows, the students from both groups performed poorly on this task. Although the CAS students (M = 3.84, SD = 4.62) had a higher mean score than the GC students (M = 3.05, SD = 4.62), both groups seemed more occupied by finding an error in the computation rather than an error from a law or property. The researcher's attempt in using the word "obviously" to give students a hint to avoid the computation did not prevail. Students seemed to have difficulty identifying asymptotic behavior. Very few students from both groups mentioned asymptotic behavior as the reason the computation was wrong. An analysis conducted with the omission of Task 2 will follow.

According to the results illustrated in Table 15, both the GC students and the CAS students did fairly well on Task 3, with a mean score of 79% and 83% respectively. The goal of Task 3 was to interpret if students could appropriately use integrals and when integrals can be used in real world situations. The CAS students had an overall higher mean score (M = 8.29, SD = 2.84) than the GC students (M = 7.86, SD = 2.74), however there was no statistically significant difference.

Task 4 was used in order to continue to measure and analyze students' understanding about real world situations and again the students from both groups did fairly well. There was no statistical significance between the CAS students (M = 7.95, SD = 3.03) and the GC students (M = 7.57, SD = 2.54. Both groups illustrated that miles cannot be negative when driving and a motor home could most likely burn more gas than a compact car. Based on the results of Task 3 and Task 4, there is no sufficient evidence that the type of technology used during instruction hindered the students' conceptual understanding nor is there statistically significant evidence one technology impacts the conceptual understanding of real world applications more than the other.

The final task on the conceptual understanding set of tasks given to the AP calculus AB students was designed in order to analyze the students' understanding of the visual concept of an integral. The students were required to recall two ideas about integrals. The first was to recall the property of an odd function. When an odd function is on $[-a, a]$, the integral will equal zero.

As mentioned earlier, 59% of the students were able to remember this concept. However, the second idea that the students needed to illustrate was how to take the integral of a constant on [-*a*, *a*], which only 29% of the students were able to do. Although the mean scores between the GC students (M = 3.96, SD = 4.00) were lower than the CAS students (M = 4.73, SD = 4.10), there was no statistical significance. In addition, as the results on this task were also low, the researcher will provide an additional analysis to see if the overall mean scores are significant when omitting Task 2 and Task 5.

The final parts of Table 15 show that the CAS students of the population achieved an overall higher score on the conceptual set of tasks. As Table 15 shows, there was a statistically significant difference between the mean scores of the GC students (M = 58.73, SD = 16.90) and the CAS students (M = 65.68, SD = 19.33) with t(331) = -3.50 and $p < 0.05$. The CAS students seemed to have a better understanding of how to analyze graphs, and apply the idea of integrals on real world situations. In order to confirm that the CAS students performed statistically better on the conceptual set of tasks, an additional analysis was conducted in which Task Two and Task Five were omitted. This was only done in order to continue analyzing the students' scores as a whole throughout the rest of the analysis portion of the study. As the means for the tasks would not differ, only the overall conceptual mean scores will be illustrated in Table 16.

Table 16
Descriptive Statistics for Conceptual Set of Tasks without Tasks 2 and 5

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Conceptual Overall	74.52	16.86	80.94	18.98	-3.26	0.00

Note: degree of freedom (df) = 331.

As Table 16 shows, the CAS students as a whole still outperformed the GC students on the conceptual set of tasks and there was a statically significant difference in the mean scores between the two groups with $t(331) = -3.26$ and $p < 0.05$. However, the overall population is not the focus of this study. As mention in chapter three and in the introduction, a sample of the high performing students is what will be used to address the research questions for this study. In order to select a sample from each group, two further analyses have been conducted. The first will compare the high performing students with the low performing respectively from each group. This analysis will be used in order to decide if there exists a significant difference between two groups in order to separate the groups further. The second analysis will select the sample from the high performing students and compare the high performing CAS students with the high performing GC students in order to address research question three.

High and Low Performing Students' Performance on Conceptual Tasks

Table 17 shows the results of an independent samples *t*-test that was used to analyze the differences in the mean scores between the high performing GC students and low performing GC students. The focus is on the overall mean score as the factor needed in order to be able to select a sample. The analyses of the individual tasks will illustrate and help support the claim that the high performing students scored higher on the tasks than the low performing students.

Table 17

Descriptive Statistics for GC High Performing vs. Low Performing Students on Conceptual Tasks

Variable	High Performing *M*	High Performing *SD*	Low Performing *M*	Low Performing *SD*	*t-value*	*p (2-tailed)*
Task 1	7.34	2.62	6.56	3.19	-	-
Task 2	3.48	4.79	2.60	4.41	-	-
Task 3	8.34	2.68	7.34	2.71	-	-
Task 4	8.22	2.49	6.98	2.46	-	-
Task 5	4.73	3.58	3.28	4.28		
Conceptual Overall	64.26	15.49	53.54	16.54	4.58	0.00

Note: degree of freedom (df) = 185.

As shown in Table 17, the high performing GC students had higher mean scores on all five tasks. In addition, there overall mean score was significantly higher and there was a statistically significant difference in the mean scores between the GC high performing students (M = 64.26, SD =15.49) and the GC low performing students (M = 53.54, SD = 16.54) with $t(185) = 4.58$ and $p < 0.05$. As a result, a sample of 64 students from the GC high performing group will be selected to compare with the CAS sample selection.

In order to select a sample of CAS students, an analysis comparing the CAS high performing students and the CAS low performing students was performed. Table 18 shows the results of the independent samples *t*-test that was performed on the CAS group.

Table 18
Descriptive Statistics for CAS High Performing vs. Low Performing Students on Conceptual Tasks

Variable	High Performing *M*	High Performing *SD*	Low Performing *M*	Low Performing *SD*	*t-value*	*p (2-tailed)*
Task 1	8.49	2.31	7.57	2.66	-	-
Task 2	4.47	5.01	3.14	4.68	-	-
Task 3	8.68	2.22	7.86	3.36	-	-
Task 4	8.49	2.31	7.36	3.58	-	-
Task 5	5.26	3.73	4.14	4.42		
Conceptual Overall	70.79	13.93	60.14	22.68	3.45	0.00

Note: degree of freedom (df) = 144.

As shown in Table 18, the high performing CAS students had higher mean scores on all four tasks. In addition, the high performing students' overall mean score was significantly higher and there was a statistically significant difference in the mean scores between the CAS high performing students (M = 70.79, SD = 13.93) and the CAS low performing students (M = 60.14, SD = 22.68) with $t(144) = 3.45$ and $p < 0.05$. As a result, a sample of 64 students from the CAS high performing group will be selected to compare with the GC sample selection.

Summary for Population

Within groups, GC_1 and GC_2 students did not differ on the overall procedural tasks. In addition, both groups had similar mean scores on the individual procedural tasks. CAS_1 students and CAS_2 students showed no difference in either the overall procedural skills task or on the individual skills tasks. CAS students as a whole had higher scores with significance on

Tasks One and Three. However, although CAS students had higher scores on Tasks Two and Four, they were not significant. On the overall procedural tasks, CAS students did perform better than the GC students. When looking at the high performing GC students and the low performing GC students, the high performing performed better on the individual tasks, as well as the overall set of tasks. Similarly, the CAS high performing students had higher scores on both individual tasks and the overall set of tasks than the low performing CAS students.

On the conceptual skills tasks portion of the study, there was no difference between GC_1 students and GC_2 students on either the individual tasks or the overall set of tasks. Similarly, CAS_1 students and CAS_2 students showed little differences between the scores on the overall set of tasks not the tasks individually. Overall, the CAS students had higher scores on the individual tasks with a significant difference on Tasks One and Five. In addition, the CAS students scored higher on the overall set of conceptual tasks. When comparing the high performing GC students and the low performing GC students, the high performing students had higher overall scores on both the individual tasks and overall tasks and the scores were significantly higher. Similarly, the CAS high performing students illustrated significantly higher overall scores on the individual tasks as well as the overall set of tasks than the low performing CAS students. Table 19 illustrates the percentages of the students that answered each procedural task completely correct. Table 20 illustrates the percentages of the students that answered each conceptual task completely correct.

Table 19
Percentages for Procedural Tasks completed by population

Variable	GC	CAS
Task 1	68%	80%
Task 2	31%	41%
Task 3	28%	43%
Task 4	39%	46%

Note: correct answers only.

Table 20
Percentages for Conceptual Tasks completed by population

Variable	GC	CAS
Task 1	44%	62%
Task 2	30%	38%
Task 3	58%	71%
Task 4	52%	65%
Task 5	24%	31%

Note: correct answers only.

Analysis on Sample of High Performing Students

Comparison CAS and GC Sample on Procedural Tasks

The procedural set of tasks was used to measure students' procedural skills in the integration portion of AP calculus AB. In order to address research question two, a sample of the high performing students was selected from the GC students and from the CAS students. A sample size of 128 students was selected, 64 from each group. Table 21 shows the results of the independent samples *t*-test used to analyze the overall mean score of the selected sample. The independent samples *t*-test confirmed that there was a statistically significant difference between the GC students (M = 69.30, SD = 15.90) and the CAS students (M = 76.97, SD = 13.10).

Table 21
Descriptive Statistics for GC High Performing versus CAS High Performing Students on Procedural Tasks

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Task 1	8.98	2.21	9.39	1.64	-	-
Task 2	7.56	2.33	7.83	2.12	-	-
Task 3	5.39	3.25	6.89	3.37	-	-
Task 4	5.78	4.01	6.67	3.95	-	-
Procedural Overall	69.30	15.90	77.10	13.28	-3.01	0.00

Note: degree of freedom (df) = 126.

The Levene's test of equality of variances was significant for the mean score data between the two groups (F = 1.95, p = 0.00). This implies that the effect of CAS embedded calculators was significant for the procedural skills tasks for the integration portion of AP calculus AB.

Comparison CAS and GC Sample on Conceptual Tasks

The conceptual set of tasks was used to measure students' conceptual understanding in the integration portion of AP calculus AB. In order to address research question three, a sample from the high performing students was selected from the GC students and from the CAS students.

A sample size of 128 students was selected, 64 from each group. Table 22 shows the results of the independent samples *t*-test used to analyze the overall mean score of the selected sample. The *t*-test confirmed that there was a statistically significant difference between the GC students (M = 64.56, SD = 15.21) and the CAS students (M = 73.28, SD = 15.74).

Table 22
Descriptive Statistics for GC High Performing versus CAS High Performing Students on Conceptual Tasks

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Task 1	7.27	2.51	8.52	2.30	-	-
Task 2	3.44	4.79	4.84	5.04	-	-
Task 3	8.24	2.77	8.52	2.30	-	-
Task 4	8.22	2.53	8.67	2.23	-	-
Task 5	4.84	3.33	6.09	3.39		
Conceptual Overall	64.56	15.21	73.28	15.74	-3.19	0.00

Note: degree of freedom (df) = 126.

The Levene's test of equality of variances was significant for the mean score data between the two groups (F = 0.22, p = 0.00). This implies that the effect of CAS embedded calculators during instruction was significant for the conceptual understanding tasks during the integration portion of AP calculus AB.

Data Analysis of Overall Performance

The researcher also performed an analysis on the overall score of all tasks combined, in order to determine if there was any significant difference between the GC students' scores and the CAS students' scores. This analysis was based on the combining of the students' scores on each task into one set of tasks using the same sample of 128 students. The results in Table 19 will offer evidence in order to address research question one. Table 23 shows that the CAS students had a higher mean score than the GC students.

Table 23
Mean Scores Overall Performance for High Performers

	GC		CAS			
Variable	*M*	*SD*	*M*	*SD*	*t-value*	*p (2-tailed)*
Overall Performance	66.58	12.72	72.70	10.20	-3.00	0.00

Note: degree of freedom (df) = 126.

This analysis showed that there was a statistically significant difference in mean scores in favor of the CAS student (M = 72.70, SD = 10.20) with p = 0.00 on the overall tasks. Overall, 62% of the CAS students achieved an overall grade of 70% or greater and 42% of the GC students achieved an overall grade of 70% or greater on both sets of tasks combined. This analysis also provides evidence that the effect of CAS instruction during AP calculus AB was significant for the students' performance in the calculus course.

CHAPTER FIVE: DISCUSSION, LIMITATIONS, AND RECOMMENDATIONS

Introduction

According to NCTM (2000), technology can support students' conceptual understanding of calculus. Saha, Ayub, and Tarmizi (2010) argue, advances in computers have brought great innovations and teachers as well as schools need to keep current on the advances in technology in order to maximize its use in the teaching and learning of calculus. In addition, students that wish to pursue careers in science, technology, engineering, and mathematics (STEM), will have calculus as their entry-point and pre-requisite to undergraduate mathematics (Robinson, 2003). According to the OECD Programme for International Student Assessment (PISA), 2009 results, the United States ranked at 32 out of 65 countries in mathematics performance. The highest percentage of United States students were at level three, which is categorized as students having the ability to execute, clearly describe procedures and select and apply simple problem-solving strategies (PISA, 2010). United States students' have been ranked as low performing since the 1950's (Robinson, 2003). In chapter two, the review of literature discusses studies that have investigated the effects on students' understanding of calculus on performance in first and second semester calculus in college (Ferrini-Mundy & Gaudard, 1992; St. Jarre, 2008; Wu, 1997; Bressoud, 2010; Bressoud, 2004). Ferrini-Mundy and Gaudard found that students that complete high school calculus struggled in second semester calculus. Bressoud (2010) reports that calculus II enrollments declined by more than 30 percent between 1995 and 2005. As a result, there is also a steep decline in students pursuing STEM careers.

An item that was lacking in the literature was looking at how using computer algebra systems (CAS) during high school calculus instruction can impact the performance of students' understanding, both conceptually and procedurally, of calculus concepts. The purpose of this study was to contribute to the literature by evaluating high school courses using CAS to see if these supported students' understanding of calculus and attempt to find a way to promote higher enrollments in STEM career majors.

This study examined the impact of using CAS during instruction on students' understanding of calculus compared to using a graphing calculator. The general research question this study purported to answer was, "Does the use of CAS during instruction in high school calculus, affect conceptual and procedural performance in AP calculus AB?" Details of the procedures and statistical results are offered in chapter four. The results from chapter four were used to address the research questions for this study. The following chapter includes a discussion of the results of the study, conclusions about the study, and recommendations for future studies.

Performance on Procedural Tasks

The first research question asked; *Is there a significant difference between high performing students that have had computer algebra system (CAS) instruction and high performing students that have had graphing calculator (GC) instruction on performance of procedural calculus tasks?* In order to address this question an independent sample *t*-test was conducted on each of the four tasks from the procedural set of tasks and on the overall mean scores on the procedural set of tasks. The details of *t*-test outputs are in chapter four. According to the analysis, the CAS students achieved higher mean scores on all four tasks, with significance on Task One and Task Three. Task One, shown in Figure 28, asked students to evaluate two definite integrals. In order to accomplish this task, the students would have to have used symbol manipulation and recalled facts pertaining to the characteristics of an integral. Approximately 80% of the CAS students solved the task correctly on this task and approximately 68% of the GC students answered this task correctly.\

Compute the following definite integrals:

a. $$\int_{1}^{2} 4x^3 \, dx$$

b. $$\int_{0}^{\frac{1}{2}} e^{2x} \, dx$$

Figure 28: Task One on procedural set of tasks (Garner & Garner, 2001).

One reason for the difference between the two groups might be that the CAS offers a more user-friendlier visual when inputting integrals that require evaluating than the graphing calculator. While observing the CAS classes evaluate integrals, the researcher noticed that the instructors would go over the step-by-step process of entering the problem into the calculator. Emphasis was constantly placed on accessing the default integral equation that students saw during problems and they were able to continue from there. In the graphing calculator classes, the steps are more tiresome and lengthy. Students have to memorize which menu to visit to find the function key as well as the order of entering the function or they will get an error message. This might be frustrating to some students as the error messages are not specific and do not lead students to the correct input procedure. Porzio (1995) supports this result and suggests CAS students were better able to use and make connections with the technology, due to the more advanced animation features found in the CAS that are not available on the graphing calculator. Task Two, shown in Figure 29, assessed the students' knowledge on key concepts of limits and their properties. In particular, the students' knowledge about limits from left and right, numerical representation of limit, function continuity, interval continuity, and derivative continuity was the focus. According to Cooley (1997), CAS students score significantly higher on the topic of limits than GC students. There was no statistically significant difference between the two groups and therefore this study does not show similar findings as with Cooley. In addition, approximately 59% of the CAS students and approximately 69% of the GC students were not able to solve the task correctly. During the observations, the researcher noticed the students were comfortable in evaluating limits. In the GC instruction environments, students evaluated limits however; the properties and theorems were not discussed while the researcher was present. If the properties of discontinuity are not commonly discussed, students might focus on the procedural concepts and loose insight on the underlining conceptual aspects of the material (Hiebert & Lefevre, 1986). These findings support those found by Gaulke (1998) in that CAS students and GC students did not differ in procedural skills.

Consider: $f(x) = \begin{cases} x^2, & x \le 1 \\ x+3, & x > 1 \end{cases}$

a. *Find* $\lim_{x \to 1^+} f(x)$

b. *What is the derivative of f(x) from the left at x = 1?*

Figure 29: Task Two on procedural set of tasks (Eisenberg & Dreyfus, 1991).

The third task, shown in Figure 30, from the procedural skills set of tasks required students to understand how to analyze an antiderivative using a calculator. Then the students were asked to produce the graph of the original function using methods of integration. This question showed to be the most challenging out of the four procedural tasks. The results from chapter four show that there was a statistically significant difference between the CAS students and GC students on task three. However, both groups scored low on this task as the students seemed to show difficulty using an appropriate scale when graphing the function. The researcher noticed difficulties illustrated by students during the observations, particular when challenging graphs were discussed during instruction.

Using the graph of $\frac{dy}{dx} = f'(x) = (x-1)(x-2)^2(x-3)^3$, *sketch a graph of*

$y = f(x)$.

Figure 30: Task Three on the procedural set of tasks (Eisenberg & Dreyfus, 1991)

During the observations of a CAS instruction, the use of the technology was shared with the whole class. After students were given time to work on the problem assigned, the educator would project the CAS on the front board and begin with a visual to start a discussion with the students. The CAS, unlike the graphing calculator, offered more animation and symbolic manipulation possibilities. However, during the observation, the researcher noticed that some of the output provided by CAS confused the students. For example, the graph windows were not labeled and piece-wise functions were confusing to the students because the CAS showed vertical lines where there was discontinuity. When the teacher switched to the algebra window, the CAS showed both complex and real solutions. Students that previously used graphing calculators probably had trouble understanding the complex notation. As a result, students might misinterpret the outputs that the CAS provides. This can explain some of the low scores on the individual tasks.

Forty-three percent of the CAS students answered this task correctly compared to 28% of the GC students. Again, this might be due to the animation features available on the CAS technology that Porzio (1995) suggests are more user-friendly and are not available on the graphing calculator. Monaghan, Sun, and Tall (1994) suggest that when students used CAS, the keystrokes are easier to remember as the animation guides the students; however, students illustrated a lack of comprehension and therefore, the CAS could not support the concepts the teachers were providing for their students nor expand on any valid meaning to the students. As illustrated by the students' work, most students graphed the given function rather than the antiderivative as requested by the task. Approximately 65% of the students that answered this question incorrectly had graphs illustrating this error. The researcher believes the students graphed each and could only see a graph produced from the first derivative function and assumed that it was the correct answer.

Task Four, shown in Figure 31, asked students to find the maximum slope of a function. Thirty-five percent of the students that answered this task incorrectly found the slope of the function rather than the maximum slope. An assumption made by the researcher is that students make reference to maximum using the first derivative and setting the first derivative to zero in order to find the slope. A gap in the understanding might be when using the second derivative to inquire whether the function has a local minimum or maximum is its only function.

Find the maximum slope of the graph of $y = -x^3 + 3x^2 + 9x - 27.$

Figure 31: Procedural Task Four (Eisenberg & Dreyfus, 1991).

Overall, there was a statically significant difference between the CAS students and GC students on the procedural set of tasks. This finding supports Pierce and Stacey's (2001) findings that students that used CAS made less procedural errors. In a similar study, Pierce and Stacey (2002) found that when using CAS, the students became more sophisticated with the technology, and as a result were able to perform more advanced procedures that cannot be done on the graphing calculator.

Performance on Conceptual Tasks

The second research question asked, *Is there a significant difference between high performing students that have had computer algebra (CAS) instruction and high performing* *students that have graphing calculator (GC) instruction on performance of conceptual calculus tasks?* In order to address this research question an independent sample *t*-test was conducted on each of the five tasks from the conceptual knowledge set of tasks as well as on the overall set of tasks. The details of the *t*-test outputs can be seen in chapter four. According to the analysis, the CAS students achieved higher mean scores on all five tasks; however, there was only statistical significance on Task One, shown in Figure 32. Task One was used in order to assess the students' ability to analyze graphs. Approximately 62% of the CAS students and approximately 44% of the GC students were able to correctly solve all parts of Task One. These findings support Heid and Edwards (2001) argument that students are able to comprehend graphical representations more effectively due to the user-friendliness of multiple representations offered by the CAS. During one of the observations in the CAS environment, the instructor illustrated to students the different ways of analyzing a graph produced by the technology. When accessing the "algebra" menu while on the graph screen, the different calculations that the calculator can do appear in full description. This allows students a more friendly approach to finding maximums, minimums, etc. The graphing calculator uses more abbreviations that students would have to become more accustomed to in order to analyze properties of the graphs produced. Cooley's (1997) findings also support this study's claim and reports that students that used CAS, scored significantly higher on topics about graphs due to the friendlier environment.

Given below is the graph of a function.

a. On what intervals is f(x) increasing? Decreasing?

b. Estimate any points at which a local maximum or a local minimum occurs.

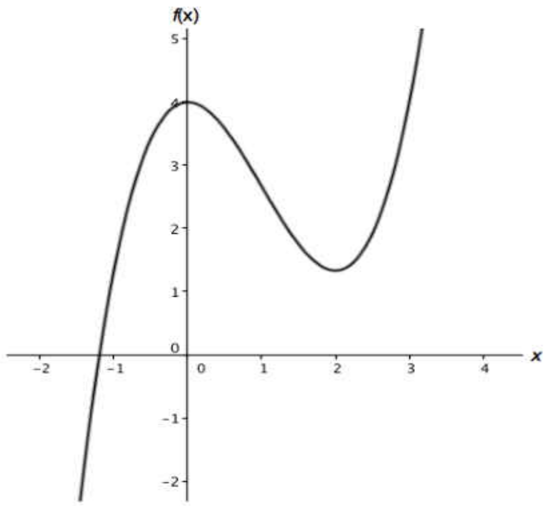


Figure 32: Task One on conceptual set of tasks (Garner & Garner, 2001).

Task Two shown in Figure 33, on the conceptual set of tasks received the most incorrect answers. Approximately 30% of the GC students and 38% of the CAS students were successful in recognizing that the integral cannot be computed because it was not continuous on the given interval. Although the students from both groups were able to compute integrals, as evidence from Task One on the procedural set of tasks has shown, they illustrate a lack of understanding about the properties of integrals. Again, the researcher believes that this might be due to the lack of focus on properties during instruction from both types of environments. Therefore, students from both groups were able to evaluate integrals, indicating a strong procedural understanding (Hiebert & Lefevre, 1986). However, the low success percentage of Task Two on the conceptual set of tasks indicates a lack of conceptual understanding of integral (Hiebert & Lefevre, 1986).

Grundmeier, Hansen, and Sousa (2006) support this claim with their finding that there is a gap between the conceptual and procedural understanding of integrals. In the current study, the gap might be that the students focused more on the evaluating of integrals and were less concerned about the properties and uses of integration. Grundmeier, et al. suggests that students are unable to draw connections or see relationships between ideas of integration and evaluating integrals. Furthermore, Grundmeier, et al. argue that the instructors often mistaken the student's ability to conduct routine procedures as an indication of conceptual understanding. During the analysis process, the researcher omitted this question to see if the overall mean scores between the groups would vary. The result, as shown in chapter four, indicates that the CAS students still achieved higher mean scores when this question was not included. As a result this question remained for the remainder of the data analyses.

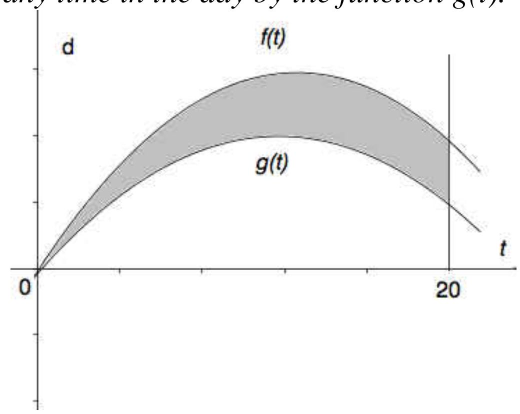
Why is the following computation obviously wrong?

$$\int_{-1}^{1} \frac{dx}{x^2} = \int_{-1}^{1} x^{-2} dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{1} = \left.\frac{-1}{x}\right|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$$

Figure 33: Task Three on conceptual set of tasks (Eisenberg & Dreyfus, 1991).

The third task, shown in Figure 34, illustrates a traditional problem included in integral calculus and attempts to draw upon the students' abilities to interpret a diagram representing a situation geometrically (Bremigan, 2005). Approximately 97% of the students were able to illustrate that integrals would be needed to solve the problem in Task Three. However, approximately 58% of the GC students were able to correctly explain how integrals could be used. In addition, approximately 71% of the CAS students were able to correctly explain how to use integrals and therefore the CAS students achieved a higher overall score on Task Three. According to Bremigan, students may encounter obstacles connected to their own limited understanding of calculus concepts, procedures, and/or symbols. In addition, Bremigan found that students have difficulty in interpreting diagrams due to the students being faced with unrecognized challenges. Although both the CAS and GC students experienced questions with diagrams, they might not have had enough experience with abstract examples. The most common error was the students incorrectly illustrated the use of integrals. White and Mitchelmore (1996) found that students had difficulties translating verbal problems statements to the abstract symbols due to the students' limited conceptual understanding of the concept of variable, which is a key component in the process of constructing an equation from a diagram.

Suppose that on a typical day the rate at which the electric company accumulates revenue (in dollars per minute) is given for any time in the day by the function f(t). The rate at which the company's operating costs accumulate (in dollars per minute) is given for any time in the day by the function g(t). These functions are graphed below.



a. *How could you use calculus to find the company's revenue and cost functions?*
b. *How could you use calculus to compute the shaded area?*
c. *What does the shaded area represent economically?*

Figure 34: Task Three on the conceptual set of tasks (Garner & Garner, 2001).

Task Four, shown in Figure 35, on the conceptual set of tasks showed lower results than Task Three. Task Four is a common question that relates to rate problems implicit on the variable of time. Approximately 52% of the GC students and 65% of the CAS students were able to correctly answer Task Four. The students' responses show that they might not have a clear connection on how integrals can be used. Many students drew various diagrams to help make sense of the tasks and relate them back to the word problem. Bremigan (2005) found that students would normally construct diagrams to represent the problem situation and this is one strategy that can be useful in solving rate problems. The students illustrated that they could evaluate integrals in Task One from the procedural set of tasks, however, similar to the findings by Ferrini-Mundy (1987), calculus students' errors are related to setting up the integral more often than solving the integrals themselves. This can offer some explanation to the low correct answer rate to Tasks Three and Four. Orton (1983) supports the researcher's claim and concludes that the main difficulty for students was their inability to make connections between the definitions and process of evaluation. Orton argues that students would have a better understanding if teachers would introduce the topic of integration by focusing on the basic concept of determining area under the curve. For this study the researcher observed integration being taught through examples first, then through application. This method of instruction aligns with the Orton's findings and as a result may explain the gap in the students' connection between integrals and their applications.

A compact car in motion on the freeway uses f(x) gallons of gas in driving x miles.

a. *For what values of x, if any, is f'(x) positive? For what values of x, if any, is f'(x) negative?*

b. *A motor home uses g(x) gallons of gas in driving x miles. How would you expect the graphs of f(x) and g(x) to compare?*

Figure 35: Task Four on the conceptual set of tasks (Garner & Garner, 2001).

Task Five, shown in Figure 36, was used to analyze the students understanding of the visual concept of an integral. According to Eisenberg and Dreyfus (1991), thinking visually makes higher cognitive demands than thinking algorithmically. The current study supports with Eisenberg and Dreyfus's findings as approximately 56% of the GC students and approximately 64% of the CAS students were able to recall the properties of odd functions in reference to integrals and correctly answered Task Five. Although the CAS students achieved a higher mean score on Task Five, both groups performed poorly on this task. The students that answered the question correctly drew an image to help understand the situation better. Tall (1991) argues that students have difficulties relating representations, but more difficulty expressing situations with an image. As a result, the students had trouble integrating a constant without providing a visual representation to see the simplicity of the task. The findings from Task Five's results are similar to the findings of Heid (1988). Heid found that CAS students performed better on visual representation problems but that there were no statistically significant differences from students that did not use CAS. In addition, Palmiter (1991) achieved similar results and found that the CAS students achieved higher mean scores on integral problems however they too were not statically significant.

If f is an odd function on [-a, a], evaluate. $\int_{-a}^{a} b + (f(x))\, dx$

Figure 36: Task Five on the conceptual set of tasks (Eisenberg & Dreyfus, 1991).

Overall, the CAS students had higher overall scores on the conceptual set of tasks, and the overall scores were statistically significant. These findings support Barton's (2001) conclusions that CAS students performed better on conceptual questions than students that did not use CAS. In addition, Barton finds that students that used CAS had a higher overall conceptual understanding of calculus tasks than classrooms that did not use CAS.

Overall Performance on Calculus Tasks

The third research question asked; *Is there a significant difference in performance between high performing students that have had instruction with computer algebra system (CAS) and high performing students that have had instruction with the graphing calculator (GC) on calculus tasks?* In order to address this question an independent samples *t*-test was conducted on the overall mean scores of the sample of the high performing students from both AP calculus AB groups. There was a statistically significant difference in favor of the CAS students ($F = 2.73$, $p = 0.00$) on the overall mean scores on the calculus tasks. In addition, 62% of the CAS students achieved an overall grade of 70% or greater and 42% of the GC students achieved an overall grade of 70% or greater on both set of tasks combined. The analysis offers some insight that the effect of CAS instruction during AP calculus AB was significant for the students' performance in the calculus course.

The analysis showed that the CAS students had higher mean scores than the GC students. However, since the scores were low from both groups, we cannot state that the use of CAS during AP calculus AB instruction increases students' overall understanding of calculus concepts during the integral half of the calculus curriculum.

The higher scores produced by the CAS group illustrate similar findings as in King's (1997) study when CAS was used in the classroom; the students' overall achievement was greatly improved. In another study, Palmiter (1991) found that when CAS enabled graphing calculators were used during calculus instruction; the CAS group achieved a higher overall mean score on both, the conceptual examination and computational examination. The group outperformed the group that did not use CAS during calculus instruction. Palmiter argues that based on his study, it is clear that students can be more successful in conceptual and computational tasks working with CAS, than students not working with CAS.

The CAS students' achieving higher overall scores can account for many different reasons including:

- Access to more realistic problems,
- Deeper exploration of mathematical concepts,
- Increased opportunities to develop connections among mathematical ideas
- A wider range of examples, and
- More abstraction.

According to Pierce and Stacey (2001), using CAS generated results that demanded interpretation, something that the GC lacked. In addition, the integration of CAS in calculus allows students to focus on applications and concepts and changes the role of the teacher. With the use of CAS in the classroom, the interactions between the teacher, student, and technology become more complex (Heid, 1996). Although CAS and graphing calculators have graphical functions linked to multiple representations, students failed to use them to solve these tasks. In most cases, they could have visualized and/or sketched the graphs without using the technology to solve the tasks. Polya (1945) and Schonenfield (1985) encourage the strategy of drawing a diagram as an important component in problem solving.

The scores on Tasks Two and Five were low for both groups. Although students attempted to draw a diagram for Task Five, it was rare to find two identical diagrams for the students that answered the question incorrectly. This finding is similar to those of Bremigan (2005), who found that calculus students' errors often related to the diagrams the students drew or manipulated in the problem. Reasoning with visuals, suggests Bremigan, is important to learning the theories and applications of calculus. The 27% of the students that answered Task Five from the conceptual set of tasks correctly; drew a correct diagram to accompany the work they produced. On Task Two, the students that received partial credit illustrated a correct set up, sometimes with a diagram, but neglected to see that the calculation is impossible because of the discontinuous characteristic of the integral.

One of the main purposes of this study was to investigate whether graphing calculator instruction and CAS instruction produced different calculus performance. The CAS students achieved higher scores on all the tasks than the GC students, and this study supports the claim that CAS instruction produces deeper conceptual and procedural understanding; in addition, students did illustrate that they developed superior understanding of concepts, which agrees with the findings of Heid and Edwards (2001).

Implications

The results of this study agree with those studies that found CAS instruction to be more effective on student learning (Arnold, 2004; McAndrew, 2010; Nunes-Harwitt; 2005; Ozgun-Koca, 2010), using CAS in the classroom has potential benefits. The studies listed make it clear that the presence of CAS in the classroom does not guarantee those potential benefits will be realized. Students must become comfortable and learn how to operate the technology effectively and be able to integrate the uses of the technology during the learning of the mathematics. The role of the teacher is to find more productive ways to integrate the use of CAS into the curriculum. During instruction, teachers can hold discussions about the meaning of the functions used while solving a problem via CAS. While navigating through the menus on the CAS, the teacher can educate the students on the different options in each menu as well as identify what other areas with which the menu item can be used in the calculus course. Prior to selecting a menu item, the teacher can prepare a list of important concepts that the menu item can be referred to throughout the calculus course. For example, when evaluating integrals, the teacher would illustrate the step-by-step procedure of how to solve the problem using CAS. In addition, rather than focusing on the final answer, discuss the steps CAS took in order to achieve the answer. CAS provides sample work on how to achieve the final answer and students can use this animation feature in order to understand the methods as well as obtain the final solution.

In addition, teachers should integrate the use of CAS as a lesson supporter. If the lesson, for example, is finding the area under a curve, display the animation features from the CAS in order to project to the front screen and revolve the lesson around what is seen by the technology. Use the manipulation tools to show students how the area can change as the minimum or maximum point is increased or decreased. Students seem to relate to the information more effectively when they are seeing interactive changes rather than having the teacher point and verbal explain changes. This important task requires full involvement by the teacher.

In assessing the students' understanding, the tasks should be selected with caution. The wording used for the questions should make sense to the students participating in the study. The researcher feared that the wording used in asking the questions influenced the students' abilities to answer the tasks. For example, Task Two from the conceptual set of tasks used the word "obviously." Some of the students might have believed that the question was a trick question and this may have affected their judgment. In addition, the time spent during observations should be longer in order to better prepare tasks that the students would be more comfortable with. Although, the tasks used in this study are tasks that have been proven to be answerable by any calculus student (Garner & Garner, 2001), an observer's opinion can contribute to more effective questions.

In a future study, new tasks should be piloted. The tasks used for this study came from Garner and Garner (2001) and Eisenberg and Dreyfus (1991). The tasks were 11 years and 21 years old respectively. Since some of the questions were meant to represent real world problems, a new set of updated real world scenarios might relate to the students of today more.

Limitations of the Study

This study had several limitations that must be noted when interpreting the study's results and conclusions. The study used participants from four different schools in which there were four different instructors. Some of the studies that showed significant differences and that were used as a guide for this study (Ferrini-Mundy & Gaudard, 1992; Heid & Edwards, 2001, Pierce & Stacey, 2001) used one teacher teaching two different ways, one with CAS and the other without CAS. In addition, a total of 333 students participated in this study. The researcher selected a small sample size, and as a result, this study cannot be generalized to all students taking AP calculus AB.

The researcher did not have the opportunities to interview students and teachers in this study due to time limitations of the participating teachers. AP calculus AB is an intense, time-consuming course for teachers and the teachers could only spare a minimal amount of time for the research study. In the future, interviews should be conducted, and more information could be collected. Lastly, due to time constraints, the researcher was not able to begin the study at the start of AP calculus AB. It is important for future studies to observe and assess students throughout the course in order to obtain a stronger understanding of the uses of the technology, including its influence on other important areas of calculus to see the significance of using the technology throughout the course.

Recommendations

This study was designed to identify whether or not CAS and GC students differed in performance in high school AP calculus AB based on the type of instruction they received. The use of CAS must be integrated into the calculus curriculum in order to have maximum positive impact (Cooley, 1991; Heid & Edwards, 2001; King, 1997; Palmiter, 1991). This study looked at the integral half of the curriculum and the effect was positive. In another study, the researcher could look at the whole curriculum of calculus to improve significance in their findings. In addition, the researcher could prepare specific lessons using CAS and then distribute a set of tasks based on those lessons and see if the conceptual and procedural knowledge of the students varied.

The other recommendations are: (a) curriculum developers should embed CAS technology when designing the calculus curriculum, (b) calculus textbook writers should include a larger section dedicated to CAS and problems addressed to CAS for appropriate units, (c) education policy makers should provide recourses which are needed by both the teachers and students, in implementing CAS technologies into the calculus course, and (d) mathematics educators should use CAS materials when teaching a calculus course.

The population sample size of this study was 128 students and lasted three months. A much larger randomized sample size from a larger population, in a longer period of time may yield more generalizable results. Lastly, the researcher did not have the opportunity to interview the students or teachers in this study due to time limitations. In the future, if participants and teachers are interviewed, more inferences can be made and better justified. The interviews would most likely provide more support for the conclusions made in this study.

APPENDIX A: OBSERVATION RUBRIC

Name of School: ___________________________ District: ________________

Date of Observation: ____________ Time Start: _________ Time End: ____________

Teacher Name: _________________ Gender: _____________

Teacher Ethnicity:
____ American Indian or Alaskan Native
____ Asian
____ Hispanic or Latino
____ Black or African-American
____ Native Hawaiian or Other Pacific Islander
____ White, Non-Hispanic

Course Title: _________________________

Course Period: __________________

Students: _____ Number of Males _______ Number of Females

Classroom Organization:

___ Traditional Rows
___ Small cluster of 3-5 student desks
___ Desks arranged so that students face each other
___ Desks in circles or semi-circles

Pattern of Access to Technology:

___ Teacher access only ___ 2 students per device
___ One presentation station ___ 3-5 students per device
___ 1 student per device ___ More than 5 students per device

Average Length of Time Using Technology:

___ Less than 5 minutes ___ 10 – 15 minutes
___ 5 – 10 minutes ___ More than 15 minutes

Proportion of Students Using Technology:

___ Fewer than 1 in 10 ___ ½ or more; fewer than ¾
___ 1/10 or more; fewer than ¼ ___ ¾ or more; fewer than 90%
___ ¼ or more; fewer than ½ ___ 90% or more

Teacher time using technology:

___ No technology used ___ 1 – 25%
___ 26 – 50% ___ 51 – 75%
___ 76 – 100%

Student time using technology:

___ No technology used ___ 1 – 25%
___ 26 – 50% ___ 51 – 75%
___ 76 – 100%

Instructional technology level:

___ Instruction demonstrates advanced graphing calculator use (GC)

___ Instruction demonstrates CAS use

Additional Comments:

APPENDIX B: TECHNOLOGY SURVEY AND RANKING SCALE

To what extent is the calculator used during each of the following:

Demonstrations during teaching of concepts	1	2	3	4	5	6	N/A
When analyzing functions	1	2	3	4	5	6	N/A
When analyzing graphs of functions	1	2	3	4	5	6	N/A
When modeling questions dealing with graphs	1	2	3	4	5	6	N/A
While solving real world word problems	1	2	3	4	5	6	N/A
While solving equations	1	2	3	4	5	6	N/A
When performing integration	1	2	3	4	5	6	N/A
When analyzing what the integral means	1	2	3	4	5	6	N/A
When estimating with finite sums	1	2	3	4	5	6	N/A
When analyzing or finding maximum area	1	2	3	4	5	6	N/A
When working on anti-derivatives	1	2	3	4	5	6	N/A
When finding the area under curves	1	2	3	4	5	6	N/A
When working with definite integrals	1	2	3	4	5	6	N/A
When evaluating sums and limits of integrals	1	2	3	4	5	6	N/A
When solving integrals with trigonometric functions	1	2	3	4	5	6	N/A

The following is a key to the ranking scale system:

1. Using calculator for computational purposes only
2. Using built-in functions of calculator or "Black Box" version.
3. Using calculator for graphical illustrations only.
4. Using built-in functions of calculator with reference to graphical illustrations.
5. Using graphical representations and symbolic manipulation functions.
6. Using graphical representations, symbolic manipulation, and animation functions.

APPENDIX C: PRODCEDURAL CALCULUS TASKS

1. *Compute the following definite integrals:*

   a. $$\int_1^2 4x^3\,dx$$

   b. $$\int_0^{\frac{1}{2}} e^{2x}\,dx$$

2. *Consider:* $f(x)=\begin{cases} x^2, & x\le 1 \\ x+3, & x>1 \end{cases}$

   a. *Find* $\lim_{x\to 1^+} f(x)$

   b. *What is the derivative of f(x) from the left at x = 1?*

3. *Using the graph of* $\frac{dy}{dx}=f'(x)=(x-1)(x-2)^2(x-3)^3$, *sketch a graph of* $y=f(x)$.

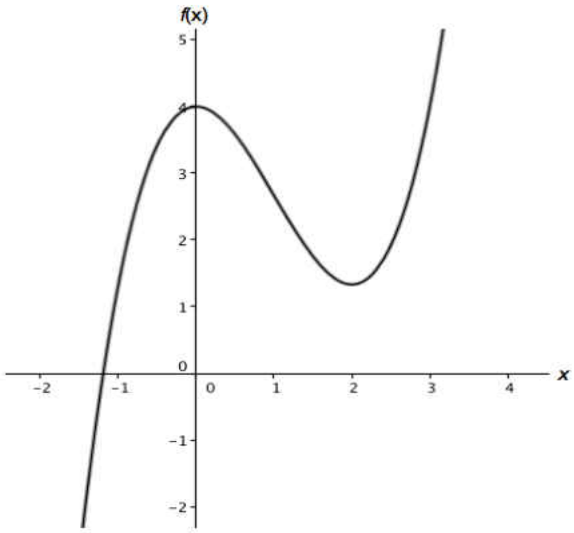
4. *Find the maximum slope of the graph of* $y=-x^3+3x^2+9x-27$.

APPENDIX D: CONCEPTUAL CALCULUS TASKS

1. Given below is the graph of a function.

a. On what intervals is f(x) increasing? Decreasing?

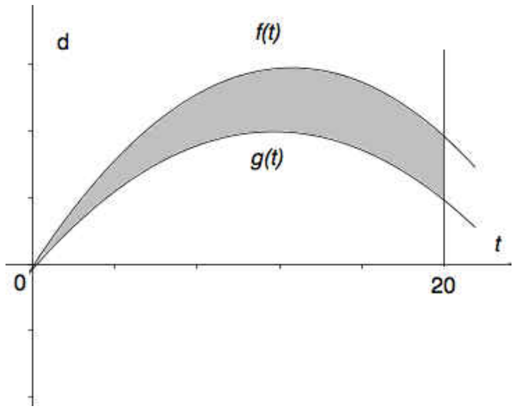
b. Estimate any points at which a local maximum or a local minimum occurs.



2. Why is the following computation obviously wrong?

$$\int_{-1}^{1} \frac{dx}{x^2} = \int_{-1}^{1} x^{-2}dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{1} = \left.\frac{-1}{x}\right|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$$

3. Suppose that on a typical day the rate at which the electric company accumulates revenue (in dollars per minute) is given for any time in the day by the function f(t). The rate at which the company's operating costs accumulate (in dollars per minute) is given for any time in the day by the function g(t). These functions are graphed below.



a. How could you use calculus to find the company's revenue and cost functions?

b. How could you use calculus to compute the shaded area?

c. What does the shaded area represent economically?

4. A compact car in motion on the freeway uses f(x) gallons of gas in driving x miles.

a. For what values of x, if any, is f'(x) positive? For what values of x, if any, is f'(x)negative?

b. A motor home uses g(x) gallons of gas in driving x miles. How would you expect the graphs of f(x) and g(x) to compare?

5. If f is an odd function on [-a, a], evaluate $\int_{-a}^{a} b + (f(x))\,dx$.

APPENDIX E: IRB APPROVAL

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Zyad Bawatneh**

Date: **December 14, 2011**

Dear Researcher:

On December 14, 2011, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	UCF Initial Review Submission Form / Exempt #1
Project Title:	The Impact on Students' Performance of Using CAS in Secondary School Calculus
Investigator:	Zyad Bawatneh
IRB Number:	SBE-11-08048
Funding Agency:	None

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 12/14/2011 12:10:45 PM EST

Janice M Turchin

IRB Coordinator

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