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Locating-Domination in Complementary Prisms.

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Locating-Domination in Complementary Prisms

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by

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ABSTRACT

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Let $G = (V(G),\ E(G))$ be a graph and $\overline{G}$ be the complement of G. The complementary prism of G, denoted $G\overline{G}$, is the graph formed from the disjoint union of G and $\overline{G}$ by adding the edges of a perfect matching between the corresponding vertices of G and $\overline{G}$. A set $D \subseteq V(G)$ is a locating-dominating set of G if for every $u \in V(G)\backslash D$, its neighborhood $N(u) \cap D$ is nonempty and distinct from $N(v) \cap D$ for all $v \in V(G)\backslash D$ where $v \neq u$. The locating-domination number of G is the minimum cardinality of a locating-dominating set of G. In this thesis, we study the locating-domination number of complementary prisms. We determine the locating-domination number of $G\overline{G}$ for specific graphs G and characterize the complementary prisms with small locating-domination numbers. We also present bounds on the locating-domination numbers of complementary prisms.

DEDICATION

First, I would like to dedicate this thesis to my loving husband, Jonathan Taylor Holmes. He has supported and inspired me to follow my dreams through the entire time we've known each other. Also, to my parents; Gregory Vaughn Stone, Carol Ann Dixon and Roger Dean Dixon. Their love and support has always pushed me to follow my own path. Finally, to all of my dear friends, who have stuck with me through the years.

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CONTENTS

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1 INTRODUCTION

The purpose of this thesis is to study selected domination parameters of a family of graphs known as complementary prisms. In Section 1.1, we introduce the basic terminology of graph theory utilized in this paper. In Section 1.2, we introduce the definitions of each of the domination parameters discussed in this paper. In Section 1.3, we define perfection in graphs. In Section 1.4, we define the complementary prism graph.

1.1 Basic Graph Theory Terminology

As defined in [2], a graph $G = (V(G), E(G))$ is a nonempty, finite set of elements called *vertices* together with a (possibly empty) set of unordered pairs of distinct vertices of G called *edges*. The *vertex set* of G is denoted by $V(G)$ and the edge set of G is denoted by $E(G)$. In Figure 1, we have an example of a graph.

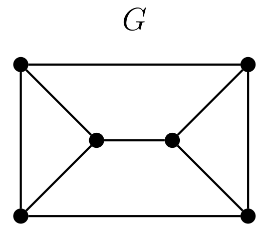

Figure 1: A Graph G

In this paper, we will be studying *simple graphs*, which are graphs for which there exists at most one edge between any two vertices. Given any graph G, the *order* of G, denoted $n(G) = |V(G)|$, is the number of vertices in G. The *size* of G, denoted

$m(G) = |E(G)|$, is the number of edges in G. For example, for the graph G in Figure 1, the order $n(G) = 6$ and the size $m(G) = 9$. The *complement* of G, denoted $\overline{G}$, is a graph with $V(\overline{G}) = V(G)$ and $E(\overline{G}) = \{ab|ab \notin E(G)\}$. For example, consider the graphs G and $\overline{G}$ shown in Figure 2.

Figure 2: A Graph G and $\overline{G}$

For any vertices $v, u \in V(G)$, u and v are *adjacent* if $uv \in E(G)$. A *u-v path* is a finite alternating sequence $\{u = v_0, e_1, v_1, e_2 \ldots e_k, v_k = v\}$ of vertices and edges such that $e_i = v_{i-1}v_i$ for $i = 1...k$ and $e_i = e_j$ if and only if $i = j$. Among all u-v paths, the number of edges in a shortest length u-v path is known as the *distance* from u to v, denoted by $dist(u, v)$. For any vertex $v \in V(G)$, the *open neighborhood* of v is $N(v) = \{u \in V(G) \mid uv \in E(G)\}$, and the *closed neighborhood* $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V(G)$, its *open neighborhood* is $N(S) = \cup_{v \in S} N(v)$, and its *closed neighborhood* is $N[S] = N(S) \cup S$. The *degree* of a vertex v is $deg_G(v) = |N(v)|$. The *minimum degree* of G is $\delta(G) = \min\{deg_G(v)|v \in V(G)\}$. The *maximum degree* of G is $\Delta(G)$ =max$\{deg_G(v)|v \in V(G)\}$. A vertex of degree zero is an *isolated vertex*, these are also known as *isolates*. A vertex of degree one is called a *leaf* or a *pendant*, and its neighbor is called a *support vertex*. For any leaf vertex v and support vertex

w, the edge vw is called a *pendant edge.*

Given $S \subseteq V(G)$, and $v \in S$, a vertex $w \in V(G)$ is an *S-private neighbor of* v if $N_G(w) \cap S = \{v\}$. The *S-external private neighborhood of* v, denoted $epn(v, S)$, is the set of all S-private neighbors of v in $V(G) \setminus S$. For any $S \subseteq V(G)$, the subgraph of G *induced* by S is denoted $\langle S \rangle$. If $S \subseteq V(G)$ and $uv \in E(G)$ for every $u, v \in S$, then S forms a *clique* of order $|S|$, and $\langle S \rangle$ is called a *complete graph* of order $|S|$. If $uv \notin E(G)$ for every $u, v \in S$, then S is an *independent set* of order $|S|$ and $\langle S \rangle$ is called an *empty graph* of order $|S|$. For any graph G, the *corona* of G, denoted $G \circ K_1$, is formed by adding for each $v \in V(G)$, a new vertex v', and a pendant edge vv'. A set $P \subseteq V(G)$ is a *packing* if $N[u] \cap N[v] = \emptyset$ for every $u, v \in P$. The *join* of simple graphs G and H, denoted $G + H$, is the graph obtained by the disjoint union of G and H by adding the edges $\{xy : x \in V(G), y \in V(H)\}$. A *matching* M in a graph G is a set of pairwise non-adjacent edges. A *perfect matching* is a matching which matches all the vertices in the graph.

Given a graph G with vertex set $V(G)$, a *proper coloring* of G is a partitioning of $V(G)$ into independent sets. These sets are called *color classes.* A proper coloring of G that has a minimum number of color classes is called a $\chi(G)$-*coloring* and the number of color classes in such a coloring is $\chi(G)$. For other definitions and terminology related to graph theory, the interested reader is referred to [2, 7, 5, 13].

1.2 Domination Parameters

A set $S \subseteq V(G)$ is a *dominating set* (abbreviated DS) if $N[S] = V(G)$ and is a *total dominating set* (abbreviated TDS) if $N(S) = V(G)$. The minimum cardinality of any DS (respectively, TDS) of G is the *domination number* $\gamma(G)$ (respectively, *total domination number* $\gamma_t(G)$). A DS of G with cardinality $\gamma(G)$ is called a $\gamma(G)$*-set*, and a $\gamma_t(G)$*-set* is defined similarly. A set $S \subseteq V(G)$ is a *locating-dominating set* (abbreviated LDS) of G, if for every $u \in V(G) \backslash D$, its neighborhood $N(u) \cap D$ is nonempty and distinct from $N(v) \cap D$ for all $v \in V(G) \backslash D$ where $v \neq u$. The *locating-domination number* of G, denoted $\gamma_L(G)$, is the minimum cardinality of a locating-dominating set of G. An LDS of G with cardinality $\gamma_L(G)$ is called a $\gamma_L(G)$*-set*. See Figure 3 for an example of an LDS for the path P_6, where the darkened vertices represent the $\gamma_L(G)$-set, L. Notice that $N(v_2) \cap L = \{v_1\}, N(v_3) \cap L = \{v_4\}$ and $N(v_5) \cap L = \{v_4,\ v_6\}$, so each of the vertices, v_2, v_3, and v_5 have unique neighborhoods $V(G) \cap L$. If a set L locating-dominates a set X, then we denote this as $L \succ_L X$.

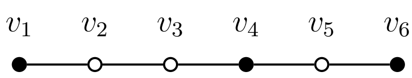

Figure 3: Locating-Dominating Set in P_6

Since an LDS is a dominating set, we have the following observation.

Observation 1 *For any graph G, $\gamma(G) \leq \gamma_L(G)$.*

A set $S \subseteq V(G)$ is a *connected dominating set* (abbreviated CDS) of G, if S is a dominating set and the induced subgraph $\langle S \rangle$ is connected. The *connected domination* *number* $\gamma_c(G)$ is the minimum cardinality of a CDS of G. A CDS of G with cardinality $\gamma_c(G)$ is called a $\gamma_c(G)$-*set*. It is obvious that $\gamma(G) \leq \gamma_c(G)$ and if $\gamma(G) = 1$, then $\gamma(G) = \gamma_c(G) = 1$. Also, since any nontrivial connected dominating set is also a total dominating set, $\gamma(G) \leq \gamma_t(G) \leq \gamma_c(G)$ for any graph G with $\Delta(G) < n - 1$. For examples of connected dominating sets in graphs see Figure 4 where the darkened vertices represent the CDS.

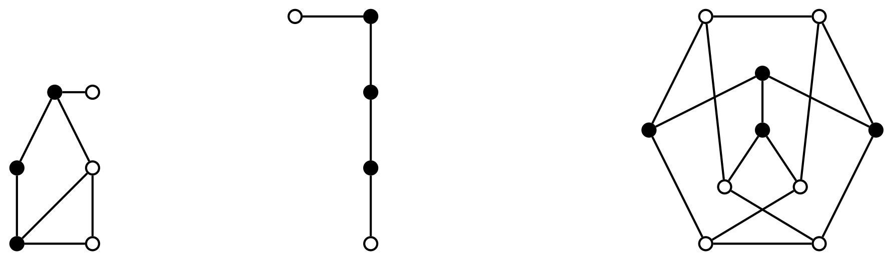

Figure 4: Connected Domination in Graphs

For more information related to domination in graphs, the interested reader is referred to [7, 8].

1.3 Perfect Graphs

A *clique* is a set of pairwise adjacent vertices in G. The *clique number* $\omega(G)$ is the maximum order of a clique in G. A graph is properly colored if no two adjacent vertices are assigned the same color. A graph G is *perfect* if $\chi(G) = \omega(G)$ for every induced subgraph H of G.

Proposition 2 (***The Perfect Graph Theorem*** [13]) *A graph G is perfect if and only if $\overline{G}$ is perfect.*

Observation 3 [13] *If $k \geq 2$, then $\chi(C_{2k+1}) > \omega(C_{2k+1})$ and $\chi(\overline{C}_{2k+1}) > \omega(\overline{C}_{2k+1})$. Therefore, odd cycles of order ≥ 5 are not perfect.*

Observation 3 prompted the following:

Proposition 4 (***Strong Perfect Graph Theorem*** [13]) *A graph G is perfect if and only if both G and $\overline{G}$ have no induced subgraph that is a cycle of length 5 or greater.*

1.4 Complementary Prisms

Complementary prisms were first introduced by Haynes, Henning, Slater, and van der Merwe in [9]. For a graph G, its *complementary prism*, denoted $G\overline{G}$, is formed from a copy of G and a copy of $\overline{G}$ by adding a perfect matching between corresponding vertices. For each $v \in V(G)$, let $\overline{v}$ denote the vertex v in the copy of $\overline{G}$. Formally, $G\overline{G}$ is formed from $G \cup \overline{G}$ by adding the edge $v\overline{v}$ for every $v \in V(G)$. For any graph G, we denote its complementary prism by $G\overline{G}$. Complementary prisms generalize several well-known graphs. For instance, the corona $K_n \circ K_1$ is the complementary prism $K_n\overline{K}_n$. Another example, is the Petersen graph, which is the complementary prism $C_5\overline{C}_5$. These are illustrated in Figure 5.

To aid in the discussion of complementary prisms, we will use the following terminology: For a set $P \subseteq V(G)$, let $\overline{P}$ be the corresponding set of vertices in $V(\overline{G})$. For a vertex $v \in V(G)$, let $\overline{v}$ represent the corresponding vertex in $V(\overline{G})$.

In this thesis, we will explore locating-domination in complementary prisms. We will also characterize the graphs G for which the complementary prism $G\overline{G}$ is perfect.

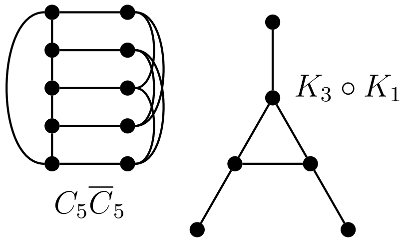

Figure 5: Examples of Complementary Prisms

2 LITERATURE REVIEW

In this chapter, we review the literature on complementary prisms. In Section 2.1, we will examine the complementary product first introduced in [9] and will see how complementary prisms are a subset of this. In Section 2.2, we will review the work on domination and total domination in complementary prisms seen in [9, 10]. This work will include, but is not limited to, characterizations of complementary prisms with small domination and total domination numbers as well as bounds.

2.1 The Complementary Product of Two Graphs

In [9], Haynes, Henning, Slater, and van der Merwe introduced a generalization of the Cartesian product of two graphs. Let G_1 and G_2 be graphs with $V(G_1) = \{u_1,\ u_2, \ldots, u_n\}$ and $V(G_2) = \{v_1,\ v_2, \ldots, v_p\}$. The *Cartesian product* of the graphs G_1 and G_2, symbolized by $G_1 \Box G_2$, is the graph formed from G_1 and G_2 in the following manner:

The graph $G_1 \Box G_2$ has np vertices. Each of these vertices has a label taken from $V(G_1) \times V(G_2)$. In $G_1 \Box G_2$, two vertices $(u_i,\ v_j)$ and $(u_r,\ v_s)$ are adjacent if and only if one of the following conditions hold:

(1) $i = r$, and $v_j v_s \in E(G_2)$.

(2) $j = s$, and $u_i u_r \in E(G_1)$.

For each i, the induced subgraph on the vertices (u_i, v_j) for $1 \leq j \leq p$ is a copy of G_2, and for each j, the induced subgraph on the vertices (u_i, v_j) for $1 \leq i \leq n$ is a copy of G_1. In less formal terms, $G_1 \Box G_2$ can either be viewed as the graph formed by taking each vertex of G_1, replacing it with a copy of G_2 and matching the corresponding vertices and taking each vertex of G_2, replacing it with a copy of G_1 and matching the corresponding vertices.

In [9], the *complementary product* of two graphs is defined. Let R be a subset of $V(G)$ and S be a subset of $V(H)$. The complementary product (symbolized by $G(R)\square H(S)$) has the vertex set $V(G(R)\square H(S)) = \{(u_i,\ v_j) \mid 1 \le i \le n, 1 \le j \le p\}$. The edge $(u_i, v_j)(u_h, v_k)$ is in $E(G(R)\square H(S))$ if one of the following conditions hold.

(1) If $i = h$, $u_i \in R$, and $v_jv_k \in E(H)$, or if $i = h$, $u_i \notin R$ and $v_jv_k \notin E(H)$.

(2) If $j = k$, $v_j \in S$, and $u_iu_h \in E(G)$, or if $j = k$, $v_j \notin S$, and $u_iu_h \notin E(G)$.

In other words, for each $u_i \in V(G)$, we replace u_i with a copy of H if u_i is in R and with a copy of its complement $\overline{H}$ if u_i is not in R, and for each $v_j \in V(H)$, we replace each v_j with a copy of G if $v_j \in S$ and a copy of $\overline{G}$ if $v_j \notin S$.

In the case where $R = V(G)$ (respectively, $S = V(H)$), the complementary product $G(R)\square H(S)$ is written $G\square H(S)$ (respectively, $G(R)\square H$). To put it more informally, $G\square H(S)$ is the graph obtained by replacing each vertex $v \in V(H)$ with a copy of G if $v \in S$ and by a copy of $\overline{G}$ if $v \notin S$, and replacing each u_i with a copy of H. In the extreme case where $R = V(G)$, and $S = V(H)$, the complementary product $G(V(G))\square H(V(H)) = G\square H$ is simply the same as the Cartesian product $G\square H$. See Figure 6 for an illustration of $C_4(\{u_1,\ u_4\}) \square C_3(\{v_3\})$.

A complementary prism $G\overline{G}$ is the complementary product $G\square K_2(S)$ with $|S| = 1$.

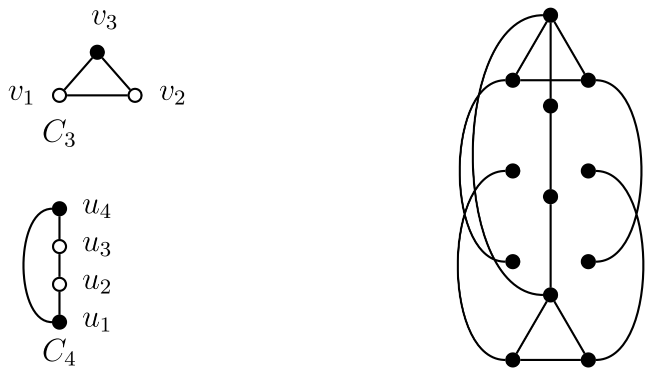

Figure 6: $C_4(\{u_1,\ u_4\}) \ \Box\ C_3(\{v_3\})$

2.2 Domination and Total Domination in Complementary Prisms

In [10], Haynes, Henning, and van der Merwe studied domination and total domination in complementary prisms, they obtained the following results. When G is a complete graph K_n, the graph tK_2, the corona $K_t \circ K_1$, a cycle C_n, or a path P_n, they obtained the exact values of $\gamma(G\overline{G})$ and $\gamma_t(G\overline{G})$, where tK_2 is the graph of t disjoint copies of K_2.

Proposition 5 [10]

(1) *If* $G = K_n$, *then* $\gamma(G\overline{G}) = n$.

(2) *If* $G = tK_2$, *then* $\gamma(G\overline{G}) = t + 1$.

(3) *If* $G = K_t \circ K_1$ *and* $t \geq 3$, *then* $\gamma(G\overline{G}) = \gamma(G) = t$.

(4) *If* $G = C_n$ *and* $n \geq 3$, *then* $\gamma(G\overline{G}) = \lceil (n + 4)/3 \rceil$.

(5) *If* $G = P_n$ *and* $n \geq 2$, *then* $\gamma(G\overline{G}) = \lceil (n + 3)/3 \rceil$.

Proposition 6 [10]

(1) *If* $G = K_n$*, then* $\gamma_t(G\overline{G}) = n$.

(2) *If* $G = tK_2$*, then* $\gamma_t(G\overline{G}) = n = 2t$.

(3) *If* $G = K_t \circ K_1$ *and* $t \geq 3$*, then* $\gamma_t(G\overline{G}) = \gamma_t(G) = t$.

(4) *If* $G \in \{C_n,\ P_n\}$ *with order* $n \geq 5$*, then*

$$\gamma_t(G\overline{G}) = \begin{cases} \gamma_t(G), & \text{if } n \equiv 2 \pmod{4} \\ \gamma_t(G) + 2, & \text{if } G = C_n \text{ and } n \equiv 0 \pmod{4} \\ \gamma_t(G) + 1, & \text{otherwise.} \end{cases}$$

They characterized graphs G for which the domination number $\gamma(G\overline{G})$ and the total domination number $\gamma_t(G\overline{G})$ of a complementary prism are small.

Proposition 7 [10] *Let* G *be a graph of order* n*. Then,*

(1) $\gamma(G\overline{G}) = 1$ *if and only if* $G = K_1$.

(2) $\gamma(G\overline{G}) = 2$ *if and only if* $n \geq 2$ *and* G *has a support vertex that dominates* $V(G)$ *or* $\overline{G}$ *has a support vertex that dominates* $V(\overline{G})$.

Proposition 8 [10] *Let* G *be a graph of order* $n \geq 2$*, with* $|E(G)| = |E(\overline{G})|$*. Then*

(1) $\gamma_t(G\overline{G}) = 2$ *if and only if* $G = K_2$.

(2) $\gamma_t(G\overline{G}) = 3$ *if and only if* $n \geq 3$ *and* $G = K_3$ *or* G *has a support vertex that dominates* $V(G)$ *or* $\overline{G}$ *has a support vertex that dominates* $V(\overline{G})$.

They found the following upper and lower bounds on the parameters $\gamma(G\overline{G})$ and $\gamma_t(G\overline{G})$.

Proposition 9 [10] *For any graph G,* $\max\{\gamma(G), \gamma(\overline{G})\} \leq \gamma(G\overline{G}) \leq \gamma(G) + \gamma(\overline{G})$*.*

Proposition 10 [10] *If G and $\overline{G}$ are without isolates, then* $\max\{\gamma_t(G), \gamma_t(\overline{G})\} \leq \gamma_t(G\overline{G}) \leq \gamma_t(G) + \gamma_t(\overline{G})$*.*

Finally, they characterized graphs G for which $\gamma(G\overline{G}) = max\{\gamma(G), \gamma(\overline{G})\}$ and $\gamma_t(G\overline{G}) = max\{\gamma_t(G), \gamma_t(\overline{G})\}$.

Proposition 11 [10] *A graph G satisfies $\gamma(G\overline{G}) = \gamma(G) \geq \gamma(\overline{G})$ if and only if G has an isolated vertex or there exists a packing P of G such that $|P| \geq 2$ and $\gamma(G \setminus P) = \gamma(G) - |P|$.*

Proposition 12 [10] *Let G be a graph such that neither G nor $\overline{G}$ has an isolated vertex. Then $\gamma_t(G\overline{G}) = \gamma_t(G) \geq \gamma_t(\overline{G})$ if and only if $G = \frac{n}{2}K_2$ or there exists an open packing $P = P_1 \cup P_2$ in G satisfying the following conditions:*

(1) $|P| \geq 2$*;*

(2) $P_1 \cap P_2 = \emptyset$*;*

(3) *if $P_1 \neq \emptyset$, then P_1 is a packing in G;*

(4) *if $P_1 = \emptyset$, then $|P| \geq 3$ or $G[P] = \overline{K}_2$;*

(5) $\gamma_t(G \setminus N[P_1] \setminus P_2) = \gamma_t(G) - 2|P_1| - |P_2|$*.*

3 LOCATING-DOMINATION IN COMPLEMENTARY PRISMS

In this chapter, we present the major results of this thesis. We will parallel the work done in [10] for domination and total domination and will obtain analogous results for the locating-domination number $\gamma_L(G\overline{G})$ of a complementary prism.

3.1 Locating-Domination Number of $G\overline{G}$ for a Specific Graph G

In this section, we determine the locating-domination number of the complementary prism $G\overline{G}$ for selected graphs G. Since every LDS must also be a DS leads us the the following observation:

Observation 13 *Every LDS of a graph G must include all of the isolated vertices of G.*

First, we find the locating-domination number of $G\overline{G}$, when G is a complete graph.

Proposition 14 *If G is the non-trivial complete graph K_n, then $\gamma_L(G\overline{G}) = n$.*

Proof. For $G = K_n$, the complementary prism $G\overline{G}$ is the corona $K_n \circ K_1$. Any $\gamma_L(G\overline{G})$-set must contain each leaf or its support vertex. Therefore $\gamma_L(G\overline{G}) \geq n$. The set of leaves forms an LDS, so $\gamma_L(G\overline{G}) \leq n$. Hence $\gamma_L(G\overline{G}) = n$. □

Next, we obtain the locating-domination number of $G\overline{G}$, when G is a complete bipartite graph.

Proposition 15 *Let G be the complete bipartite graph $K_{r,s}$, where $r + s = n$ and $1 \leq r \leq s$.*

$$\gamma_L(G\overline{G}) = \begin{cases} n, & if\ r = 1 \\ n-1, & if\ r = 2 \\ n-2, & otherwise. \end{cases}$$

Proof. Let $G = K_{r,s}$, $1 \leq r \leq s$, where R and S are the bipartite sets of G with cardinality r and s, respectively. Let $R = \{x_1,\ x_2,\ ...,\ x_r\}$ and $S = \{y_1,\ y_2,\ ...,\ y_s\}$. Let L be a $\gamma_L(G\overline{G})$-set.

First let $r = 1$, that is, $G = K_{1,s}$, $1 \leq s$. Clearly $V(G)$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq n$.

To see that $\gamma_L(G\overline{G}) \geq n$, note that $\overline{x}_1$ is a leaf in $G\overline{G}$. This implies that at least one of x_1 and $\overline{x}_1$ is in L.

If $x_1 \in L$, then x_1 can locating-dominate at most one of its neighbors. Thus, there are $n-1$ vertices in $N_{G\overline{G}}(x_1)$ that must either be in L or have another neighbor in L. Hence, $\gamma_L(G\overline{G}) \geq 1 + n - 1 = n$.

If $x_1 \notin L$, then $\overline{x}_1 \in L$. This implies that at least one of y_i and $\overline{y}_i$ is in L to dominate y_i, $1 \leq i \leq n-1$. And again $\gamma_L(G\overline{G}) \geq n$. Thus, if $G = K_{1,s}$, $\gamma_L(G\overline{G}) = n$.

Now assume that $2 \leq r \leq s$. We first show that $|L \cap (S \cup \overline{S})| \geq s - 1$. Assume that there are two vertices in S, say y_i and y_j, where none of $y_i,\ y_j,\ \overline{y}_i$, and $\overline{y}_j$ are in L. Then $N_{G\overline{G}}(y_i) \cap L = N_G(y_i) \cap L = R \cap L = N_{G\overline{G}}(y_j) \cap L$. Thus, there exists at most one vertex, $y_i \in S$ such that y_i and $\overline{y}_i$ are in $V(G)\backslash L$. This implies that $|L \cap (S \cup \overline{S})| \geq s - 1$ as desired.

Case I: $2 = r \leq s$. To show that $\gamma_L(G\overline{G}) \leq n-1$, we note that $R \cup (\overline{S} \backslash \{\overline{y}_1,\ \overline{y}_2\}) \cup \{y_1\}$ is an LDS for $G\overline{G}$. To see this, notice that $N_{G\overline{G}}(\overline{x}_i) \cap L = \{x_i\}$, for $i \in \{1,\ 2\}$. Also $N_{G\overline{G}}(y_2) \cap L = \{x_1,\ x_2\}$, $N_{G\overline{G}}(\overline{y}_1) \cap L = \{y_1,\ \overline{y}_i | i \geq 3\}$, $N_{G\overline{G}}(\overline{y}_2) \cap L = \{\overline{y}_i | i \geq 3\}$. For $i \geq 3$, $N_{G\overline{G}}(y_i) \cap L = \{x_1,\ x_2, \overline{y}_i\}$. Thus, every vertex in $V(G\overline{G})\backslash L$ is locating-dominated by L. Hence $\gamma_L(G\overline{G}) \leq |R| + |S| - 2 + 1 = r + s - 1 = n - 1$.

Next we want to show $\gamma_L(G\overline{G}) \geq n-1 = s+1$. We have shown that $|L \cap (S \cup \overline{S})| \geq s-1$. Assume to the contrary that $\gamma_L(G\overline{G}) \leq s$. Hence $|L \cap (R \cup \overline{R})| = 1$. Without loss of generality, either $L \cap (R \cup \overline{R}) = \{x_1\}$ or $L \cap (R \cup \overline{R}) = \{\overline{x}_1\}$. In the former, $\overline{x}_2$ is not dominated by L, a contradiction. In the later, at least one vertex from $S \cup \overline{S}$ is not dominated by L, a contradiction. And so, $\gamma_L(G\overline{G}) \geq s+1 = s+r-1 = n-1$.

Case II: $3 \leq r \leq s$. We show that $(\overline{R} \backslash \{\overline{x}_1,\ \overline{x}_2\}) \cup (\overline{S} \backslash \{\overline{y}_1,\ \overline{y}_2\}) \cup \{x_1,\ y_1\}$ is an LDS of $G\overline{G}$. To see this, notice that $N_{G\overline{G}}(x_2) \cap L = \{y_1\}$, $N_{G\overline{G}}(y_2) \cap L = \{x_1\}$, $N_{G\overline{G}}(\overline{x}_1) \cap L = \{x_1,\ \overline{x}_i | i \geq 3\}$, $N_{G\overline{G}}(\overline{x}_2) \cap L = \{\overline{x}_i | i \geq 3\}$, $N_{G\overline{G}}(\overline{y}_1) \cap L = \{y_1,\ \overline{y}_i | i \geq 3\}$, $N_{G\overline{G}}(\overline{y}_2) \cap L = \{\overline{y}_i | i \geq 3\}$. And for $i \geq 3$, $N_{G\overline{G}}(x_i) \cap L = \{y_1,\ \overline{x}_i\}$, and $N_{G\overline{G}}(y_i) \cap L = \{x_1, \overline{y}_i\}$. Thus, every vertex in $V(G\overline{G}) \backslash L$ is locating-dominated by L. Hence, $\gamma(G\overline{G}) \leq |R| - 2 + 2 + |S| - 2 = r + s - 2 = n - 2$.

Next we show that $\gamma_L(G\overline{G}) \geq n-2$. We have shown $|L \cap (S \cup \overline{S})| \geq s-1$. A similar argument for $R \cup \overline{R}$ will lead to $|L \cap (R \cup \overline{R})| \geq r-1$. Thus, $\gamma_L(G\overline{G}) \geq s-1+r-1 = r+s-2 = n-2$. $\square$

Now we will explore the locating-domination numbers of paths and cycles.

Proposition 16 *If* $G \in \{P_n,\ C_n\}$ *for* $5 \leq n \leq 7$*, then*

$$\gamma_L(G\overline{G}) = \begin{cases} 4, & if\ n \in \{5,\ 6\} \\ 5, & if\ n = 7 \end{cases}$$

Proof. *Case I:* $n = 5$. First assume that $G \in \{P_5,\ C_5\}$ with the vertices of G labeled sequentially $v_1,\ v_2,\ v_3,\ v_4,\ v_5$. Then the set $\{v_2,\ v_4,\ \overline{v}_3,\ \overline{v}_5\}$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq 4$.

To show that at least four vertices are necessary to locating-dominate $G\overline{G}$, we assume to the contrary that $\gamma_L(G\overline{G}) \leq 3$. Let L be a $\gamma_L(G\overline{G})$-set. If $L \subseteq V(G)$ (respectively, $L \subseteq V(\overline{G})$), then at most three vertices are dominated in $\overline{G}$ (respectively, G), a contradiction. Hence $L \cap V(G) \neq \emptyset$ and $L \cap V(\overline{G}) \neq \emptyset$.

Case Ia: $G = C_5$. Without loss of generality, assume that $|L \cap V(G)| = 1$ and $|L \cap V(\overline{G})| = 2$. Then there are at least two vertices in G, say v_i and v_j, such that $N_{G\overline{G}}(v_i) \cap L = N_{G\overline{G}}(v_j) \cap L = L \cap V(G)$, contradicting that L is an LDS of $G\overline{G}$.

Case Ib: $G = P_5$. First assume that $|L \cap V(G)| = 1$ and $|L \cap V(\overline{G})| = 2$. Let $L \cap V(G) = \{v_i\}$. Since L dominates $G\overline{G}$, v_i is not an endvertex of the path. If $v_i = v_3$, then to dominate $G\overline{G}$, $\overline{v}_1$ and $\overline{v}_5$ are in L. But then $N_{G\overline{G}}(v_2) \cap L = N_{G\overline{G}}(v_3) \cap L$, a contradiction. Without loss of generality, the other possibility is that $v_i = v_2$. Then to dominate $G\overline{G}$, $L = \{v_2,\ \overline{v}_4,\ \overline{v}_5\}$. Again, $N_{G\overline{G}}(v_1) \cap L = N_{G\overline{G}}(v_3) \cap L$, a contradiction.

Now assume that $|L \cap V(G)| = 2$ and $|L \cap V(\overline{G})| = 1$. First assume that $L \cap V(\overline{G})$ is an endvertex of G. Without loss of generality, assume that $\overline{v}_1 \in L$. Since L must dominate, $L = \{\overline{v}_1,\ v_2,\ v_4\}$ or $\{\overline{v}_1,\ v_2,\ v_5\}$. And so $N_{G\overline{G}}(\overline{v}_3) \cap L = N_{G\overline{G}}(\overline{v}_5) \cap L$ or $N_{G\overline{G}}(\overline{v}_3) \cap L = N_{G\overline{G}}(\overline{v}_4) \cap L$. In both cases, we have a contradiction.

Now assume that the vertex in $V(\overline{G}) \cap L$ is not an endvertex. If $V(\overline{G}) \cap L \in \{\overline{v}_2,\ \overline{v}_4\}$, then $G\overline{G}$ cannot be dominated in three. So assume $V(\overline{G}) \cap L = \{\overline{v}_3\}$. To dominate $G\overline{G}$, $L = \{\overline{v}_3,\ v_2,\ v_4\}$. And so $N_{G\overline{G}}(\overline{v}_1) \cap L = N_{G\overline{G}}(\overline{v}_5) \cap L$, a contradiction.

Hence, $\gamma_L(G\overline{G}) \geq 4$. Therefore, $\gamma_L(G\overline{G}) = 4$.

Case II: $n = 6$. First assume that $G \in \{P_6,\ C_6\}$ with the vertices of G labeled sequentially $v_1,\ v_2,\ v_3,\ v_4,\ v_5,\ v_6$. Then the set $\{v_2,\ v_5,\ \overline{v}_1,\ \overline{v}_4\}$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq 4$. If $G = C_6$, then since $4 = \lceil \frac{n+4}{3} \rceil = \gamma(G\overline{G}) \leq \gamma_L(G\overline{G})$, we have $\gamma_L(G\overline{G}) = 4$. Thus, assume $G = P_6$.

To show that at least four vertices are necessary to locating-dominate $G\overline{G}$, we assume to the contrary that $\gamma_L(G\overline{G}) \leq 3$. Let L be a $\gamma_L(G\overline{G})$-set. If $L \subseteq V(G)$ (respectively, $L \subseteq V(\overline{G})$), then at most three vertices are dominated in $\overline{G}$ (respectively, G), a contradiction. Hence $L \cap V(G) \neq \emptyset$ and $L \cap V(\overline{G}) \neq \emptyset$.

First let $|V(G) \cap L| = 1$ and $|V(\overline{G}) \cap L| = 2$. There does not exist a dominating set which meets this condition.

Next let $|V(G) \cap L| = 2$ and $|V(\overline{G}) \cap L| = 1$. In order for L to dominate $G\overline{G}$, the vertex in $V(G) \cap L$ must either be $\overline{v}_1$ or $\overline{v}_6$. Without loss of generality, let $V(\overline{G}) \cap L = \{\overline{v}_1\}$. Since L is a DS, it follows that $L = \{\overline{v}_1,\ v_2,\ v_5\}$. Then $N_{G\overline{G}}(v_4) \cap L = N_{G\overline{G}}(v_6) \cap L$, a contradiction.

Hence, $\gamma_L(G\overline{G}) \geq 4$. Therefore, $\gamma_L(G\overline{G}) = 4$.

Case III: $n = 7$. First assume that $G \in \{P_7,\ C_7\}$ with the vertices of G labeled sequentially $v_1,\ v_2,\ v_3,\ v_4,\ v_5,\ v_6,\ v_7$. Then the set $\{v_1,\ v_4,\ v_7,\ \overline{v}_2,\ \overline{v}_5\}$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq 5$.

To show that at least five vertices are necessary to locating-dominate $G\overline{G}$, we assume to the contrary that $\gamma_L(G\overline{G}) \leq 4$. Let L be a $\gamma_L(G\overline{G})$-set. If $L \subseteq V(G)$ (respectively, $L \subseteq V(\overline{G})$), then at most four vertices are dominated in $\overline{G}$ (respectively, G), a contradiction. Hence, $L \cap V(G) \neq \emptyset$ and $L \cap V(\overline{G}) \neq \emptyset$.

First let $|V(G) \cap L| = 1$ and $|V(\overline{G}) \cap L| = 3$ or $|V(G) \cap L| = 2$ and $|V(\overline{G}) \cap L| = 2$. There does not exist a dominating set which meets this condition.

Assume that $|V(G) \cap L| = 3$ and $|V(\overline{G}) \cap L| = 1$. Then there exist at least two vertices, $\overline{v}_i$ and $\overline{v}_j$ in $V(\overline{G})$, such that $N_{G\overline{G}}(\overline{v}_i) \cap L = V(\overline{G}) \cap L = N_{G\overline{G}}(\overline{v}_j) \cap L$, contradicting the fact that L is an LDS of $G\overline{G}$.

Hence, $\gamma_L(G\overline{G}) \geq 5$. Therefore, $\gamma_L(G\overline{G}) = 5$.□

For paths and cycles of order $n \geq 8$ consider Figure 7 where the darkened vertices represent a $\gamma_L(G\overline{G})$-set, L. For $n = k \equiv 0\ (mod\ 3)$, the set $L = \{\overline{v}_i,\ v_j,\ v_{k-3},\ \overline{v}_{k-1},\ v_k | i \equiv 1\ (mod\ 3), j \equiv 0\ (mod\ 3)\}$ is the LDS of $G\overline{G}$. For $n \equiv 1,\ 2\ (mod\ 3)$ let $n = k + l$ where $n \equiv l\ (mod\ 3)$. Then $L \cup \{\overline{v}_n\}$ is the LDS of $G\overline{G}$. The pattern is shown for paths and the same pattern applies to cycles. These observations lead us to the following conjecture.

Conjecture 17 *If $G \in \{P_n,\ C_n\}$ for $n \geq 8$, then*

$$\gamma_L(G\overline{G}) = \left\lfloor \frac{2n}{3} \right\rfloor.$$

3.2 Complementary Prisms with Small Locating-Domination Number

In this section, we consider complementary prisms with small locating-domination numbers.

Proposition 18 *For a graph G of order n and its complementary prism $G\overline{G}$,*

(1) $\gamma_L(G\overline{G}) = 1$ *if and only if* $n = 1$.

(2) $\gamma_L(G\overline{G}) = 2$ *if and only if* $n = 2$.

(3) $\gamma_L(G\overline{G}) = 3$ *if and only if* $n \in \{3,\ 4\}$ *such that* $G \notin \{K_4,\ \overline{K}_4,\ K_{1,3},\ \overline{K}_{1,3}\}$.

Proof. (1) If $|V(G)| = 1$, then $G\overline{G} = K_2$. Thus, $\gamma_L(G\overline{G}) = 1$. Now assume that $\gamma_L(G\overline{G}) = 1$, and without loss of generality, S is a $\gamma_L(G\overline{G})$-set and $S \subseteq V(G)$. Since S must locating-dominate $\overline{G}$ in $G\overline{G}$, it follows that $|V(\overline{G})| = 1$ and $G = K_1$.

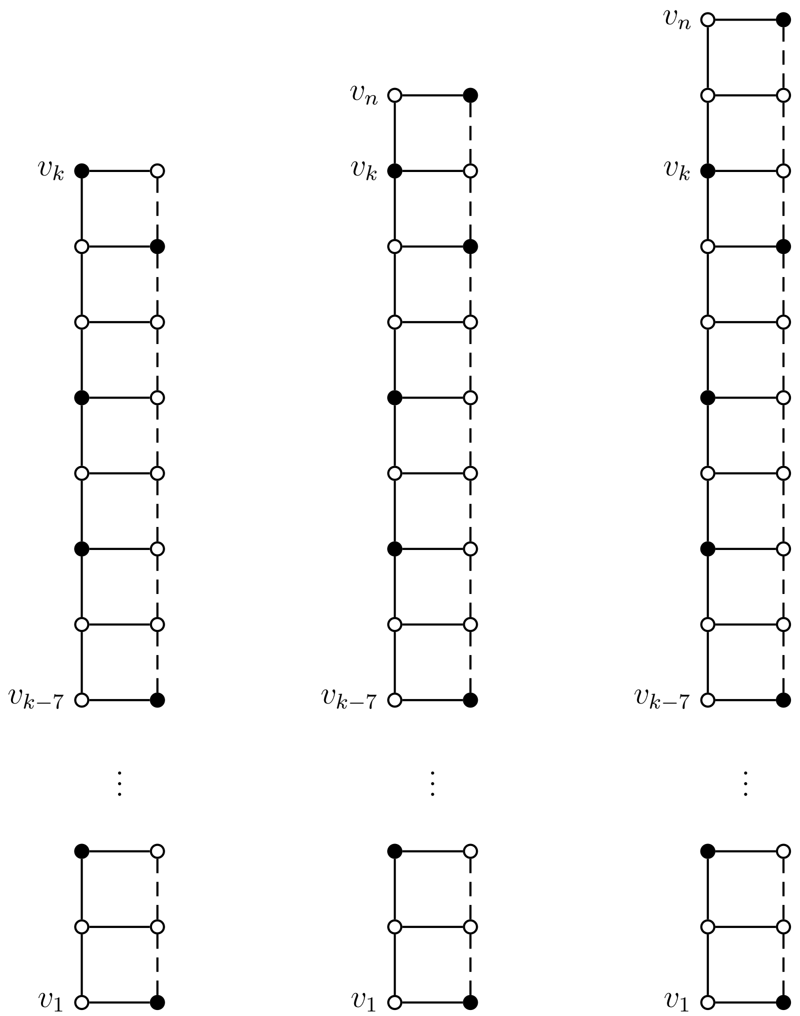

Figure 7: Locating-Domination where $G \in \{P_n, C_n\}$ for $n \geq 8$

(2) If $|V(G)| = 2$, then $G \in \{K_2, \ \overline{K}_2\}$ so $G\overline{G} = P_4$ and $\gamma_L(G\overline{G}) = 2$.

Assume that $\gamma_L(G\overline{G}) = 2$, and let S be a $\gamma_L(G\overline{G})$-set. If $S \subseteq V(G)$, then since S must dominate $\overline{G}$, it follows that $|V(G)| = 2$ and so $G\overline{G} = P_4$. Now assume $S \cap V(G) = 1$ and $S \cap V(\overline{G}) = 1$. Without loss of generality, let $S = \{x, \ \overline{y}\}$. We consider two cases:

Case I: $\overline{y} = \overline{x}$. Then $\{x\} \succ_L V(G)\backslash\{x\}$ and $\{\overline{x}\} \succ_L V(\overline{G})\backslash\{\overline{x}\}$. Let $w \in V(G)\backslash\{x\}$. Then w is adjacent to x and $\overline{w}$ is adjacent to $\overline{x}$, a contradiction. Thus, $V(G)\backslash\{x\} = \emptyset$, that is, $|V(G)| = 1$. Then $\gamma_L(G\overline{G}) = 1$, a contradiction.

Case II: $\overline{x} \neq \overline{y}$. Then $x \succ_L V(G)\backslash\{x,\ y\}$ and $\overline{y} \succ_L V(\overline{G})\backslash\{\overline{x},\ \overline{y}\}$. Without loss of generality, we may assume that $xy \in E(G\overline{G})$ and $\overline{xy} \notin E(G\overline{G})$. Let $w \in V(G)\backslash\{x,\ y\}$. Then $N_{G\overline{G}}(w) \cap S = \{x\} = N_{G\overline{G}}(\overline{x}) \cap S$, contradicting that S is an LDS of $G\overline{G}$. Hence $V(G)\backslash\{x,\ y\} = \emptyset$, that is, $|V(G)| = 2$.

(3) Let $n \in \{3,\ 4\}$. By (2), $\gamma_L(G\overline{G}) \geq 3$. If $n = 3$, then $V(G)$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq 3$ and hence $\gamma_L(G\overline{G}) = 3$. If $n = 4$, then again $V(G)$ is an LDS of $G\overline{G}$, so $\gamma_L(G\overline{G}) \leq 4$. If $G \in \{K_4,\ \overline{K}_4,\ K_{1,3},\ \overline{K}_{1,3}\}$, then by Propositions 14 and 15, $\gamma_L(G\overline{G}) = 4$. So assume $G \notin \{K_4,\ \overline{K}_4,\ K_{1,3},\ \overline{K}_{1,3}\}$.

Figure 8 illustrates an LDS of $G\overline{G}$ for all remaining graphs G on four vertices. The darkened vertices represent the LDS. Since each has an LDS of cardinality three, $\gamma_L(G\overline{G}) \leq 3$. Hence for those graphs, $\gamma_L(G\overline{G}) = 3$.

Again by Propositions 14 and 15 for $G \in \{K_4,\ \overline{K}_4,\ K_{1,3},\ \overline{K}_{1,3}\}$, $\gamma_L(G\overline{G}) = 4$. Assume that G is a graph of order n such that $\gamma_L(G\overline{G}) = 3$. We only need to show that $n \in \{3,\ 4\}$. Clearly $n \geq 3$ by part (2) of this proof. Let L be a $\gamma_L(G\overline{G})$-set. If $L \subseteq V(G)$ or $L \subseteq V(\overline{G})$, then it follows that $n = 3$. Hence assume that $L \cap V(G) \neq \emptyset$ and $L \cap V(\overline{G}) \neq \emptyset$. Without loss of generality, let $L = \{x,\ y,\ \overline{z}\}$ and consider two cases:

Case I: $\overline{z} \in \{\overline{x},\ \overline{y}\}$. Assume without loss of generality, $\overline{z} = \overline{x}$. Then $\{\overline{x}\} \succ_L V(\overline{G})\backslash\{\overline{x},\ \overline{y}\}$ in $G\overline{G}$, implying that there is at most one vertex in $V(\overline{G})\backslash\{\overline{x},\ \overline{y}\}$ in $G\overline{G}$. Hence $n = 3$.

Case II: $\overline{z} \notin \{\overline{x},\ \overline{y}\}$. Thus, $\{\overline{z}\} \succ_L V(\overline{G})\backslash\{\overline{x},\ \overline{y},\ \overline{z}\}$. This implies that there is at most one vertex in $V(\overline{G})\backslash\{\overline{x},\ \overline{y},\ \overline{z}\}$. Hence, $n \leq 4$. □

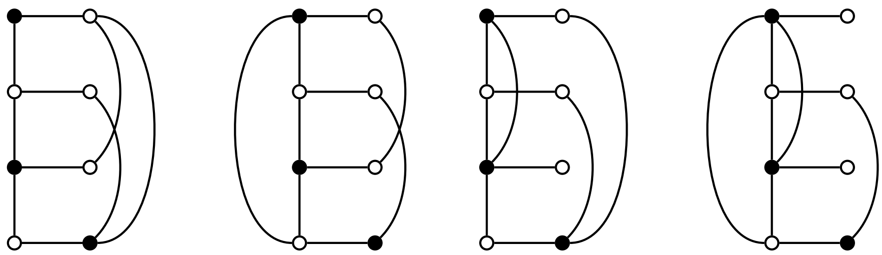

Figure 8: LDS of $G\overline{G}$ when $n = 4$ and $G \notin \{K_4,\ \overline{K}_4,\ K_{1,3},\ \overline{K}_{1,3}\}$

3.3 Bounds on the Locating-Domination Number for $G\overline{G}$

Similar to the bounds seen in Proposition 9 and Proposition 10 for domination and total domination respectively, $\gamma_L(G\overline{G})$ is bounded below by $\max\{\gamma_L(G),\ \gamma_L(\overline{G})\}$ and above by $\gamma_L(G) + \gamma_L(\overline{G})$.

Proposition 19 *For any graph G,* $\max\{\gamma_L(G),\ \gamma_L(\overline{G})\} \leq \gamma_L(G\overline{G}) \leq \gamma_L(G)+\gamma_L(\overline{G})$.

Proof. By Proposition 14, if $G = K_n$, then $\max\{\gamma_L(G),\ \gamma_L(\overline{G})\} = n = \gamma_L(G\overline{G}) \leq 2n - 1 = \gamma_L(G) + \gamma_L(\overline{G})$. Thus, we may assume G is not complete. Let D be a $\gamma_L(G\overline{G})$-set, and let $D_1 = D \cap V(G)$ and $D_2 = D \cap V(\overline{G})$. Assume, without loss of generality, that $\gamma_L(G) \geq \gamma_L(\overline{G})$. If D_1 locating-dominates G, then we are finished. So assume there exists a set $T \subseteq V(G)$ such that T is not locating-dominated by D_1. Thus, T is located and/or dominated by D_2. Also, each vertex in D_2 is adjacent to at most one vertex in T. Thus, $|T| \leq |D_2|$. But $D_1 \cup T$ is a locating-dominating set of G. So $\gamma_L(G) \leq |D_1 \cup T| = |D_1| + |T| \leq |D_1| + |D_2| = |D| = \gamma_L(G\overline{G})$.

For the upper bound, let S_1 be a $\gamma_L(G)$-set and S_2 be a $\gamma_L(\overline{G})$-set, and $S = S_1 \cup S_2$. Also, let $x \in V(G) \backslash S_1$ and $\overline{y} \in V(\overline{G}) \backslash S_2$. Then,

$$N_{G\overline{G}}(x) = \begin{cases} N_G(x) \cap S_1 \cup \{\overline{x}\}, & if\ \overline{x} \in S_2 \\ N_G(x) \cap S_1, & otherwise \end{cases}, and$$

$$N_{G\overline{G}}(\overline{y}) = \begin{cases} N_{\overline{G}}(\overline{y}) \cap S_2 \cup \{y\}, & if\ y \in S_1 \\ N_{\overline{G}}(\overline{y}) \cap S_2, & otherwise. \end{cases}$$

Since S_1 and S_2 locating-dominate G and $\overline{G}$, respectively, and $N_{G\overline{G}}(x) \cap S_1 \neq \emptyset \neq N_{G\overline{G}}(\overline{y}) \cap S_2$, S is an LDS of $G\overline{G}$. $\square$

The lower bound is sharp, for $G \in \{\overline{K}_n,\ K_{r,s}\}$ when $3 \leq r \leq s$. The upper bound is sharp when $G = P_5$.

4 MISCELLANIOUS RESULTS

4.1 Perfection in Complementary Prisms

In this section, we explore perfection in complementary prisms. We know that if G is perfect, then $\overline{G}$ is perfect by Proposition 2. We characterize the perfect complementary prisms.

Proposition 20 *A graph $G\overline{G}$ is perfect if and only if $G \in \{K_n,\ \overline{K}_n\}$.*

Proof. Let $G \in \{K_n,\ \overline{K}_n\}$. Then $G\overline{G} = K_n \circ K_1$ and $\overline{G\overline{G}}$ is the graph obtained from the join of $K_n + \overline{K}_n$ by removing a perfect matching between the vertices of K_n and the vertices of $\overline{K}_n$. Clearly $G\overline{G}$ has no induced C_5. Let $H = G\overline{G}$. To see that $\overline{H}$ has no induced C_5, we note that any induced cycle of length five or more in $\overline{H}$ must include at least three vertices from the copy of K_n. These three vertices form a triangle, so there is no induced cycle in $\overline{H}$ with length five or more. Hence neither H nor $\overline{H}$ has an induced cycle of length five or more, so by Proposition 4, $H = G\overline{G}$ is perfect.

Let $G\overline{G}$ be perfect. For any graph G either G or $\overline{G}$ is connected. Assume to the contrary G is connected and is not complete. Therefore, there exists a pair of vertices, x and z of distance two apart. Thus, G contains an induced P_3. Let $\langle x, y, z\rangle$ be the induced P_3 in G. Then $\overline{xz} \in E(\overline{G})$ and $\overline{x}, x, y, z, \overline{z}$ is an induced C_5 in $G\overline{G}$. Hence by Proposition 4, $G\overline{G}$ is not perfect which yields a contradiction. Thus, G is complete. $\square$

4.2 Connected Domination in Complementary Prisms

This section provides some results in connected domination of complementary prisms.

Observation 21 *A graph G must be connected to have a connected dominating set.*

Proposition 22 *If G complete bipartite graph, $K_{r,s}$ when $2 \leq r \leq s$, then $\gamma_c(G\overline{G}) = 4$.*

Proof. To show $\gamma_c(G\overline{G}) \leq 4$, we note that the set $C = \{\overline{x}_i,\ x_i,\ y_i,\ \overline{y}_i\}$ is a CDS of G. Hence, $\gamma_c(G\overline{G}) \leq 4$.

Next we show that $\gamma_c(G\overline{G}) \geq 4$. Since $\overline{G}$ is disconnected, any CDS, say C, of $G\overline{G}$ must include at least one vertex from $V(G)$ and two vertices from $V(\overline{G})$ (one from each component of $\overline{G}$). However, no matter which bipartite set of G contains the vertex of C, $\langle C \rangle$ is disconnected. Hence, we need at least one more vertex in C to connected dominate G. Thus, $\gamma_c(G\overline{G}) \geq 4$. $\square$

Proposition 23 *For any graph G, $max\{\gamma(G),\ \gamma(\overline{G})\} \leq \gamma_c(G\overline{G}) \leq \gamma_c(G)+\gamma_c(\overline{G})+1$.*

Proof. The lower bound is easy to see given $\gamma(G\overline{G}) \leq \gamma_c(G\overline{G})$ and by Theorem 9, $max\{\gamma(G),\ \gamma(\overline{G})\} \leq \gamma(G\overline{G})$. For the upper bound, let S be $\gamma_c(G)$-set and T be a $\gamma_c(\overline{G})$-set. If there exists a $v \in S$ where $\overline{v} \in T$, then $S \cup T$ is a CDS and $\gamma_c(G\overline{G}) \leq |S| + |T|$. If no such pair exists, let $\overline{u} \in T$ such that $\overline{u} \neq \overline{v}$. Then either $\overline{uv} \in E(\overline{G})$ or $uv \in E(G)$. Thus, $S \cup T \cup \{u\}$ or $S \cup T \cup \{\overline{v}\}$ is a CDS of $G\overline{G}$ implying that $\gamma_c(G\overline{G}) \leq |S| + |T| + 1$. $\square$

5 CONCLUDING REMARKS

This thesis presented results on locating-dominating parameters and connected domination parameters. Also we explored perfection in complementary prisms. Some unsolved problems from the future would include:

- Finding the locating-domination of $G\overline{G}$ when G is a tree.
- Characterizing graphs where the bounds of Theorem 19 are sharp.
- Investigating which complementary prisms have a small chromatic number.
- Finding bounds on the chromatic number of $G\overline{G}$.
- Investigating which complementary prisms are Hamiltonian.

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