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The propagation of Hermite-Gaussian beam pairs in strong nonlocal planar waveguide

ShiMin Wu[a], Qing Wang[b,*], XingHui Gao[c,*], Yong Wang[d]

[a] School of Physics and Electronic Information, Gannan Normal University, Ganzhou 341000, China
[b] School of Physics and Electronic Information, Shangrao Normal University, Jiangxi 334000, China
[c] Key Lab of Information Processing and Transmission of Guangzhou, Guangzhou University, Guangzhou 510006, China
[d] Nanjing nriet Industrial Company Limited, Nanjing 211106, China

A B S T R A C T

Incoherent orthogonal polarized Hermite-Gaussian (HG) beam pairs are investigated in nonlocal planar waveguide. Using the variational approach, we discuss the existence and dynamics of Vector HG solitons analytically and confirm it by split-step Fourier method. The results show that a series of vector solitons, which were consisted of different-order HG beam pairs, can form when the total initial power is equal to the critical power. Whereas the beam widths will vibrate periodically during propagation.

Introduction

In recent years, many scholars have paid attention to spatial optical solitons in nonlocal nonlinear media and obtained a series of novel results. For example, Z.J. Yang investigated the interaction between the anomalous vortex beams [1], and the motion of interactional solitons in nonlocal media [2]. T.P. Horikisi found that the ring dark and antidark solitons can exist in a weak or strong nonlocality regime [3]. B.K. Esbensen exploited the formal analogy between nonlocal and quadratic solitons [4]. Q. Wang obtained the bistable elliptic optical soliton in anisotropic nonlocal competing nonlinear media [5].

The high-order optical beam, which features two or more peaks of optical intensity, has attracted much interest recently. For instance, Husebaut et al. experimentally discovered the stationary high-order soliton in nematic liquid crystal [6]. L.H. Zhong obtained the exact solution of HG solitons in nonlinear nonlocal media with exponential response function [7], Q. Wang got the Spiraling elliptic Laguerre–-Gaussian soliton in isotropic nonlocal nonlinear media [8]. Future investigations include vector-necklace-ring soliton cluster [9], elegant Ince–Gaussian beam [10,11], Bessel–Gaussian beam [12], HG soliton [13], variable sinh-Gaussian soliton [14], coupled super-Gaussian beam pairs, etc, in strong nonlocal media [15]. This paper studies the propagation of HG beam pairs in strongly planar waveguide with variational method, and obtains some valuable results.

Theoretical model and variational approximation

The propagation of incoherent orthogonal polarized beam pairs in nonlocal planar waveguide, can be modeled by the coupled normalized nonlocal nonlinear Schrodinger equation [14–16]:

$$i\frac{\partial\psi_j}{\partial z}+\frac{1}{2}\frac{\partial^2\psi_j}{\partial x^2}+\psi_j\int_{-\infty}^{+\infty}R(x-x')[|\psi_j(x',z)|^2+|\psi_{3-j}(x',z)|^2]dx'=0 \tag{1}$$

where ψ_j ($j=1,2$) are the two paraxial optical beams. x and z are the transverse and longitudinal coordinates, respectively. $R(x)$ represents the nonlocal response function.

The Lagrange density equation of Eq. (1) can be given as follow

$$L=\sum_{j=1,2}\frac{i}{2}\left(\psi_j^*\frac{\partial\psi_j}{\partial z}-\psi_j\frac{\partial\psi_j^*}{\partial z}\right)-\frac{1}{2}\left|\frac{\partial\psi_j}{\partial x}\right|^2+\frac{1}{2}|\psi_j|^2\int_{-\infty}^{+\infty}R(x-x')[|\psi_j(x',z)|^2+|\psi_{3-j}(x',z)|^2]dx' \tag{2}$$

Assuming that the trial beam shaped is HG function

$$\psi_j(x,z)=A_j(z)H_{n_j}\left[\frac{x}{a_j(z)}\right]\exp\left[i\theta_j(z)+ic_j(z)x^2-\frac{x^2}{2a_j^2(z)}\right], \tag{3}$$

where $A_j(z)$ ($j=1,2$), $\theta_j(z)$, $a_j(z)$, $c_j(z)$ represent the amplitude, phase, width, coefficient of wavefront curvature of the two beams, respectively.

The response function can be expanded as follow [3–5] under the strong nonlocal case

* Corresponding authors.
E-mail addresses: wangqingszu@sohu.com (Q. Wang), anderson02@163.com (X. Gao).

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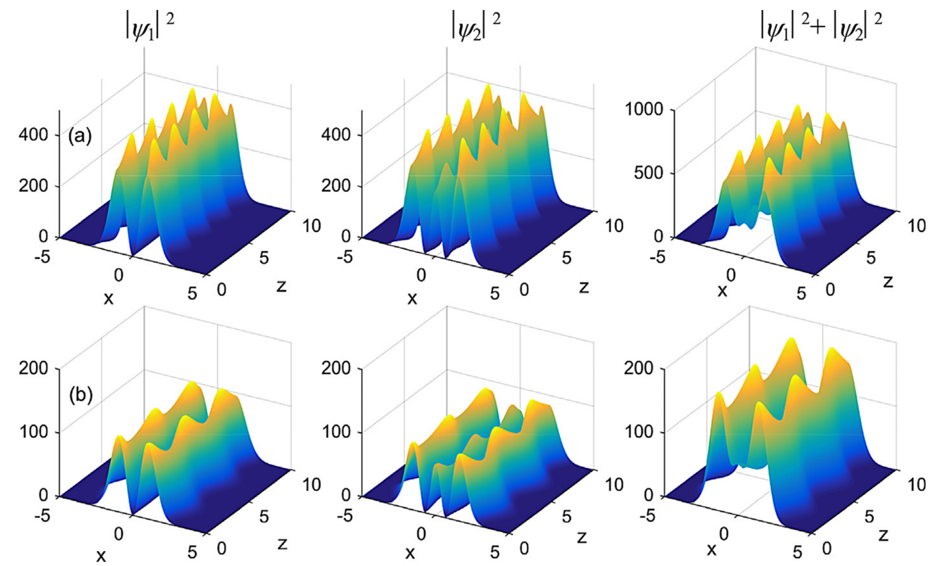

Fig. 1. Propagation of Vector HG breathers in strong nonlocal planar waveguide. The parameters are chosen as $n_1 = 1$, $n_2 = 2$. (a) $P_0 > P_c$ and (b) $P_0 < P_c$.

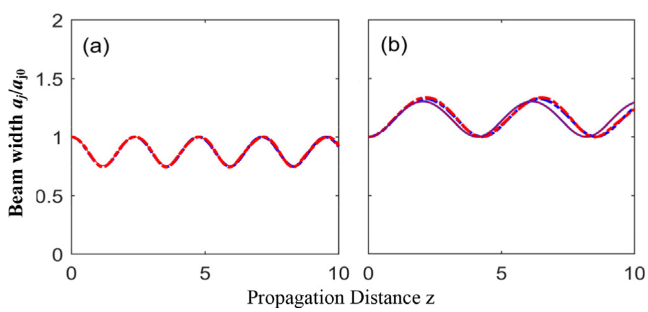

Fig. 2. Comparison of analytical solution (solid lines) with numerical solution (dashed lines). The blue and red lines represents the evolution trajectory of beam widths for ψ_1 and ψ_2. The parameters are chosen the same as in Fig. 1. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$R(x-x') \approx R_0-\frac{1}{2}\gamma(x-x')^2 \tag{4}$$

where $R_0 = R(0)$, $\gamma = -R^{(2)}(0)$.

Substituting Eqs. (3) and (4) into Eq. (2), and integrating it over x, yields

$$\begin{aligned} L = &-\sqrt{\pi}2^{n_1}n_1!A_1^2\left[\tfrac{(2n_1+1)}{2}a_1^3\tfrac{dc_1}{dz} + a_1\tfrac{d\theta_1}{dz} + \tfrac{1}{2}(2n_1+1)\left(\tfrac{1}{2a_1} + 2c_1^2a_1^3\right)\right] \\ &+ \tfrac{\pi}{2}2^{2n_1}(n_1!)^2A_1^4\left[R_0a_1^2-\tfrac{(2n_1+1)}{2}\gamma a_1^4\right] \\ &-\sqrt{\pi}2^{n_2}n_2!A_2^2\left[\tfrac{(2n_2+1)}{2}a_2^3\tfrac{dc_2}{dz} + a_2\tfrac{d\theta_2}{dz} + \tfrac{1}{2}(2n_2+1)\left(\tfrac{1}{2a_2} + 2c_2^2a_2^3\right)\right] \\ &+ \tfrac{\pi}{2}2^{2n_2}(n_2!)^2A_2^4\left[R_0a_2^2-\tfrac{(2n_2+1)}{2}\gamma a_2^4\right] \\ &+ \pi2^{n_1}n_1!2^{n_2}n_2!A_1^2A_2^2a_1a_2\left[R_0-\tfrac{(2n_1+1)}{4}\gamma a_1^2-\tfrac{(2n_2+1)}{4}\gamma a_2^2\right] \end{aligned} \tag{5}$$

The evolution equations of parameters for optical beams can be obtained by variational method

$$A_j^2a_j = \frac{P_{j0}}{2^{n_j}n_j!\sqrt{\pi}} \tag{6a}$$

$$\frac{da_j}{dz} = 2c_ja_j \tag{6b}$$

$$\frac{dc_j}{dz} = \frac{1}{2a_j^4}-2c_j^2-\frac{1}{2}\gamma P_{j0}-\frac{1}{2}\gamma P_{(3-j)0} \tag{6c}$$

$$\frac{d\theta_j}{dz} = -\frac{(2n_j+1)}{2a_j^2}-\frac{(2n_j+1)}{4}\gamma P_{j0}a_j^2 + R_0P_{j0}-\frac{(2n_{(3-j)}+1)}{4}\gamma P_{(3-j)0}a_{(3-j)}^2 + R_0P_{(3-j)0} \tag{6d}$$

where P_{j0} ($j = 1, 2$) represent the initial powers, a_{j0} and A_{j0} denote the initial fundamental mode beam widths and amplitudes, respectively. Combining Eqs. (6b) and (6c), the evolution rules of beam widths can be obtained

$$\frac{d^2a_j}{dz^2} = \frac{1}{a_j^3}-\gamma a_jP_0 \tag{7}$$

where $P_0 = P_{10} + P_{20}$ is the total incident power. Therefore the evolutions of two HG beams only depend on the total initial power. By setting $d^2a_1/dz_1{}^2|_{z=0} = 0$, which means that ψ_1 can keep its initial beam width unchanged during propagation, then we can get the critical power as follow

$$P_{c1} = \frac{1}{\gamma a_{10}^4} \tag{8}$$

Similarly, by setting $d^2a_2/dz_2{}^2|_{z=0} = 0$, the critical power of ψ_2 can be obtained

$$P_{c2} = \frac{1}{\gamma a_{20}^4} \tag{9}$$

When the total initial power is equal to the two critical powers, i.e., $P_0 = P_{c1} = P_{c2}$, the HG beam pairs will both keep their widths unchanged, which means that a stationary Vector HG soliton is form.

The evolution equations of beam widths can be obtained by solving Eq. (7)

$$a_j^2 = a_{j0}^2\left[\cos^2(\beta_0z) + \frac{P_{cj}}{P_0}\sin^2(\beta_0z)\right] \tag{10}$$

where $\beta_0 = (\gamma P_0)^{1/2} = [\gamma(P_{10} + P_{20})]^{1/2}$ is the propagation constant.

Numerical results

The split-step Fourier transform method is often used to simulate the propagation of optical beam in nonlinear media [17,18]. In this section, we also adopt this numerical method for confirming the analytically results. The step size in the propagation direction and the

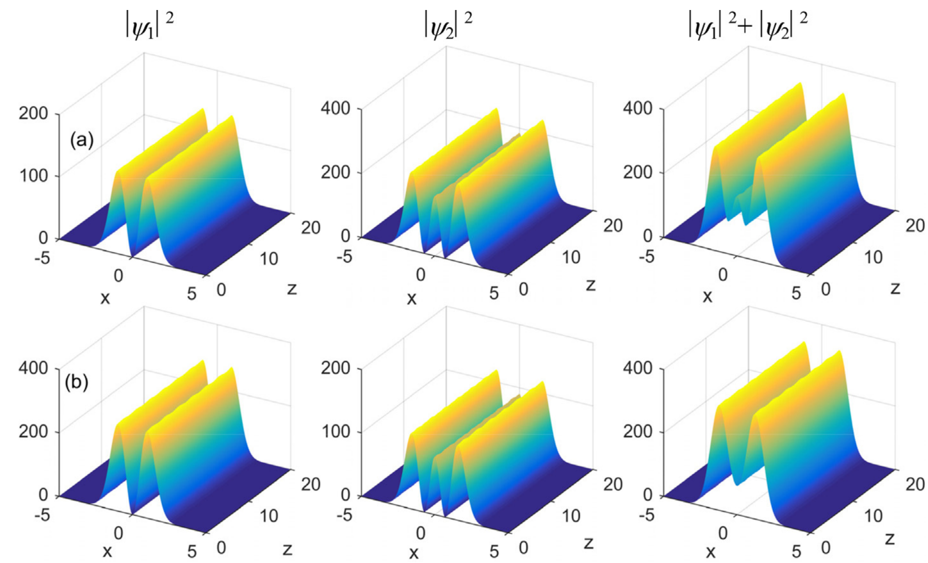

Fig. 3. Propagation of Vector HG soliton in strong nonlocal planar waveguide. The parameters are chosen as $n_1 = 1$, $n_2 = 2$, $P_0 = P_c$. (a) $P_1 = 1/2P_2$ and (b) $P_1 = 2P_2$.

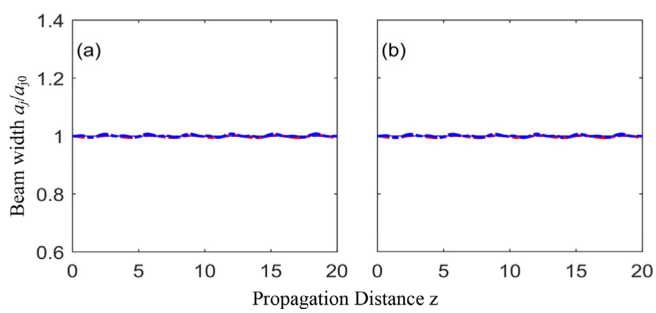

Fig. 4. Comparison of analytical solution (solid lines) with numerical solution (dashed lines). The blue and red lines represent the evolution trajectories of beam widths for ψ_1 and ψ_2. The parameters are chosen the same as in Fig. 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

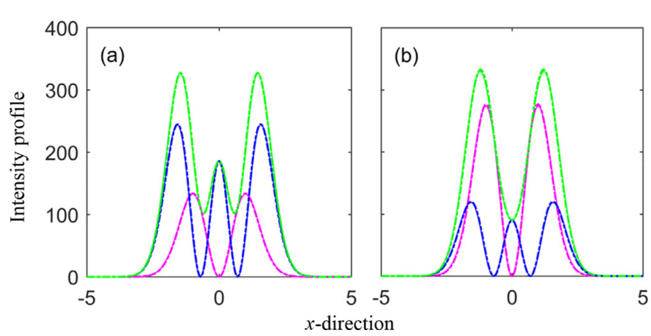

Fig. 5. The red, blue and green lines represent the intensity profiles of ψ_1, ψ_2, and $\psi_1 + \psi_2$, respectively. Solid and dashed lines are the input and output beam profiles, respectively. The parameters are the same as in Fig. 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

number of point in x-direction are 0.01 and 2048, respectively. The nonlocal response function is assumed as Gaussian-shaped. For convenience, we let $a_{10} = a_{20} = 1$ in our simulations, thus $P_{c1} = P_{c2} = P_c$.

The Vector HG breathers

Fig. 1. (a) and (b) display the propagation of the Vector HG

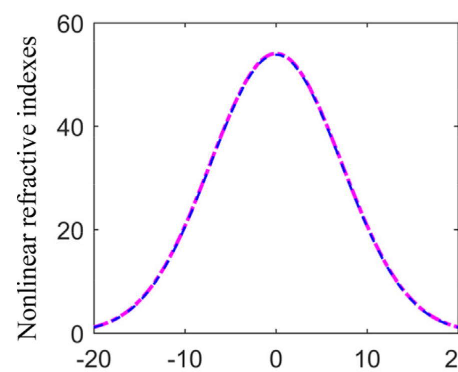

Fig. 6. Red and blue lines represent the induced nonlinear refractive indexes which are induced by the soliton in Fig. 3 (a) and (b), respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

breathers in strong nonlocal planar waveguide. Fig. 2 depicts the evolution trajectories of the beam widths. From Fig. 2(a), we can find that, when the total initial power is larger than the critical power, i.e. $P_0 > P_c$, the nonlinear effect overcomes the diffraction effect and the beam width begins to decrease. When $P_0 < P_c$ as seen in Fig. 2(b), the beam width begins to increase because that the diffraction effect is stronger than nonlinear effect. According to the definition of second-order moment theorem, the initial HG beam widths are $a_{j0}(2n_j + 1)^{1/2}$.

The Vector HG solitons

From Fig. 3, we can conclude that the stable Vector HG soliton can form in strong nonlocal planar waveguide when the total initial power is equal to the critical power. Fig. 4 displays that, the two beams both keep their widths unchanged during propagation. Obviously, the variational approximate solutions agree well with the numerical results.

By comparing Fig. 3(a) and (b), one can find that the Vector HG soliton can form in strong nonlocal planar waveguide with arbitrary P_1/P_2, such as $P_1 = 1/2P_2$ in Fig. 3(a) and $P_1 = 2P_2$ in Fig. 3(b).

Fig. 5 shows that the intensity profiles of input beams coincide with that of the output beams very well. In addition, we can found that the intensity profiles of $\psi_1 + \psi_2$ in (a) and (b) are different. However, the induced nonlinear refractive indexes are identical as seen from Fig. 6. This is because that, for strong nonlocality, the nonlinear refractive

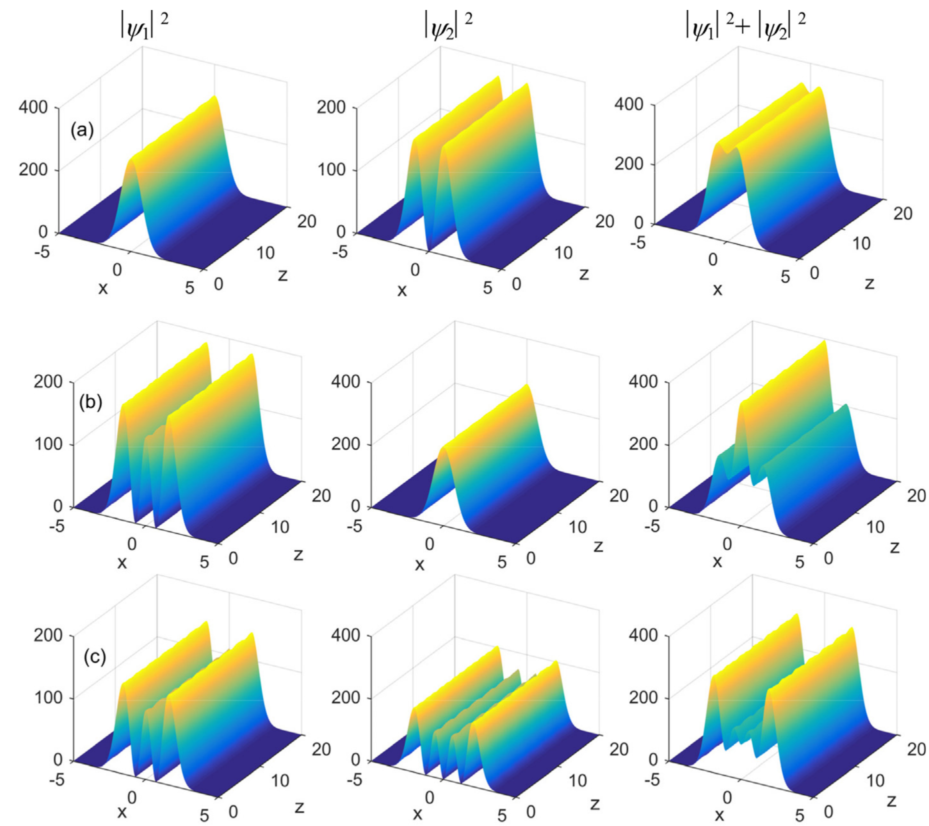

Fig. 7. Propagation of Vector HG soliton in strong nonlocal planar waveguide. The parameters are chosen as $P_0 = P_c$. (a) $n = 0$, $n_2 = 1$, (b) $n_1 = 2$, $n_2 = 0$, and (c) $n_1 = 2$, $n_2 = 3$.

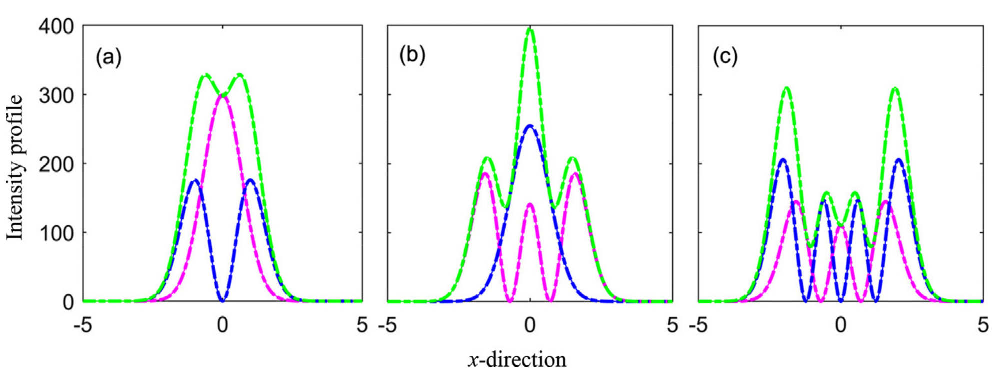

Fig. 8. The red, blue and green lines are the intensity profile of ψ_1, ψ_2 and$\psi_1 + \psi_2$, respectively. Solid and dashed lines are the input and output beam profiles, respectively. The parameters are chosen the same as in Fig. 7. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

index is only depended on the total initially power.

A series of Vector HG solitons which are consisted of different-order HG beam pairs

The numerical results in Fig. 7(a) (b) and (c) show that, as long as $P_0 = P_c$, the diffraction effect is exactly balanced by nonlinear effect and the different-order HG beam pairs can keep their initial widths. From Fig. 8(a) (b) and (c), we also find that the intensity profiles of input beams coincide well with that of the output beams.

Summary

We study analytically the self-trapping of incoherent orthogonally polarized HG beam pairs in (1 + 1)-dimensional strong nonlocal media and obtain the existence condition of Vector HG soliton. The evolution of the Vector HG solitons is also investigated numerically based on the split step beam propagation method. We find that, when $P_0 = P_c$, the Vector HG soliton can form in strong nonlocal planar waveguide with arbitrary P_1/P_2. However, when $P_0 \neq P_c$, the beam widths will vibrate during propagation. At last, we demonstrate the stable propagation of a series of vector solitons, which are consisted of different-order HG beam pairs, in strong nonlocal planar waveguide.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.rinp.2018.06.014.

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