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A HETEROSKEDASTICITY-ROBUST *F*-TEST STATISTIC FOR INDIVIDUAL EFFECTS

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□ *We derive the asymptotic distribution of the standard* F*-test statistic for fixed effects, in static linear panel data models, under both non-normality and heteroskedasticity of the error terms, when the cross-section dimension is large but the time series dimension is fixed. It is shown that a simple linear transformation of the* F*-test statistic yields asymptotically valid inferences and under local fixed (or correlated) individual effects, this heteroskedasticity-robust* F*-test enjoys higher asymptotic power than a suitably robustified Random Effects test. Wild bootstrap versions of these tests are considered which, in a Monte Carlo study, provide more reliable inference in finite samples.*

Keywords Bootstrap; *F*-test; Heteroskedasticity; Non-normality; Random effects.

JEL Classification C12; C15; C23.

1. INTRODUCTION

In an earlier article, Orme and Yamagata (2006) added to the already large literature on the analysis of variance testing, by establishing that, in a static linear panel data model, the standard *F-test* for individual effects remains asymptotically valid (large N, fixed T) under non-normality of the error term. Moreover, their (local) asymptotic analysis, supported by Monte Carlo evidence, showed that under (pure) local random effects

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both the *F-test* and *Random Effects test* (*RE-test*) will have similar power whilst under local fixed effects, or random effects which are correlated with the regressors, the *RE-test* procedure will have lower asymptotic power than the *F-test* procedure.

The key result in the above article (Proposition 1, p. 409) is, essentially, the asymptotic equivalence of the appropriately centred *F-test* statistic and the numerator (test indicator) in the *RE-test* statistic, under homoskedastic, but not necessarily normally distributed, errors. However, it is straightforward to verify (Proposition 1 in Section 3.2 below) that this asymptotic equivalence continues to hold under general heteroskedasticity of the errors.[1] The analysis which produces this result also predicts that, under certain forms of neglected heteroskedasticity, the standard (homoskedastic-based) *F* and *RE* tests will be, either, asymptotically under or over-sized. For example: (i) under cross-sectional heteroskedasticity only, both tests will be asymptotically oversized; (ii) under time series heteroskedasticity *and* serial independence of the errors, both tests will be asymptotically undersized, but under symmetric time series conditional heteroskedasticity such as GARCH, where the squared error terms exhibit positive correlation, both tests will be asymptotically oversized; and (iii) furthermore, in the singular case of independently and identically distributed (i.i.d.) data, over *both* the cross-section and time dimensions, then even if the errors are conditionally heteroskedastic, the standard *F* and *RE* tests remain asymptotically valid. The assumptions in this article explicitly allow for independently but not identically distributed data and, therefore, unconditional heteroskedasticity in the errors.

Given the result of Proposition 1, below, Wooldridge's (2010, p. 299) heteroskedastic-robust *RE-test* suggests a number of possible transformations of the standard *F-test* statistic which will recover its asymptotic validity under general heteroskedasticity of unknown form. Moreover, this transformation, or correction, involves simple functions of the pooled model's residuals (i.e., the restricted residuals). Following the literature on heteroskedasticity robust inference, restricted residuals are employed as advocated, for example, by Davidson and MacKinnon (1985) and Godfrey and Orme (2004), who report reliable sampling performance of tests of linear restrictions in the linear model when employing restricted residuals in the construction of heteroskedasticity robust standard errors.[2]

Importantly, though, the *F* and *RE* heteroskedastic-robust tests, so constructed, retain the qualitative properties that were reported by Orme and Yamagata (2006). Specifically: (i) under (pure) local random effects,

[1]Orme and Yamagata (2006) did not cover the case of heteroskedastic errors in the linear model, although their analysis did allow for heteroskedastic individual effects.

[2]As Wooldridge (2010, p. 300) points out, standard tests for individual effects essentially test for non zero correlation in the errors; thus, constructing autocorrelation robust procedures would appear to be counter productive.

both tests have the same asymptotic power; and, (ii) under local fixed effects, or random effects which are correlated with the regressors, the *RE-test* procedure will have lower asymptotic power than the *F-test* procedure.

The plan of this article is as follows. In order to make the current article self-contained, Section 2 reproduces Orme and Yamagata (2006, Section 2) and introduces the notation and standard test statistics as discussed widely in econometric texts; for example Baltagi (2008). Section 3 details the assumptions and asymptotic analysis. The latter provides a description of the asymptotic behaviour of the *F-test* statistic, its heteroskedasticity robust transformation, its relationship with the *RE-test* statistic (under both the null and local alternatives), and predictions concerning the asymptotic significance levels of the unadjusted *F-test* test under certain forms of neglected heterokedasticity. All proofs of the main results are relegated to the Appendix. Section 4 illustrates the main findings by reporting the results of a small Monte Carlo study. This also includes an evaluation of a wild bootstrap procedure scheme, based on Mammen (1993) and Davidson and Flachaire (2008), which might be employed in order to provide closer agreement between the desired nominal and the empirical significance level of the proposed test procedures. Section 5 concludes.

2. THE NOTATION, MODEL, AND TEST STATISTICS

We consider the static linear panel data model

$$\mathbf{y}_i = \alpha_i \boldsymbol{\iota}_T + \mathbf{X}_i \boldsymbol{\beta}_1 + \mathbf{u}_i, \qquad i = 1, \ldots, N, \tag{1}$$

where $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT})'$, $\mathbf{u}_i = (u_{i1}, \ldots, u_{iT})'$, $\boldsymbol{\iota}_T$ is a $(T \times 1)$ vector of ones, and $\mathbf{X}_i = (\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT})'$ a $(T \times K)$ matrix. The innovations, u_{it}, have zero mean and uniformly bounded variances and the α_i are the individual effects. By stacking the N equations of (1), the model for all individuals becomes

$$\mathbf{y} = \mathbf{D}\boldsymbol{\alpha} + \mathbf{X}\boldsymbol{\beta}_1 + \mathbf{u}, \tag{2}$$

where $\mathbf{y} = (\mathbf{y}_1', \ldots, \mathbf{y}_N')'$ and $\mathbf{u} = (\mathbf{u}_1', \ldots, \mathbf{u}_N')'$ are both $(NT \times 1)$ vectors, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)'$ is a $(N \times 1)$ vector, $\mathbf{D} = [\mathbf{I}_N \otimes \boldsymbol{\iota}_T]$ is a $(NT \times N)$ matrix, $\mathbf{X} = (\mathbf{X}_1', \ldots, \mathbf{X}_N')'$ is a $(NT \times K)$ matrix, and $[\mathbf{D}, \mathbf{X}]$ has full column rank. Thus, for the purposes of the current exposition, $\mathbf{x}_{it} = (x_{it1}, \ldots, x_{itK})'$, $(K \times 1)$, contains no time invariant regressors, in particular a constant term corresponding to an overall intercept. In the context of fixed effects this allows estimation of $\boldsymbol{\beta}_1$, as follows.

In general, define the projection matrices, $\mathbf{P_B} = \mathbf{B}(\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'$ and $\mathbf{M_B} = \mathbf{I}_{NT} - \mathbf{P_B}$, for any $(NT \times S)$ matrix $\mathbf{B}$ of full column rank, with $\tilde{\mathbf{B}} = \mathbf{M_D}\mathbf{B}$ being the residual matrix from a multivariate least squares regression of $\mathbf{B}$

on $\mathbf{D}$ which is, of course, the within transformation. Then the fixed effects (least squares dummy variable) estimator of $\boldsymbol{\beta}_1$ in (2) is given by

$$\tilde{\boldsymbol{\beta}}_1 = (\mathbf{X}'\mathbf{M_D X})^{-1}\mathbf{X}'\mathbf{M_D y} = (\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}})^{-1}\widetilde{\mathbf{X}}'\tilde{\mathbf{y}}. \tag{3}$$

The null model of no individual effects is the pooled regression model of

$$\begin{aligned}\mathbf{y} &= \beta_0 \boldsymbol{\iota}_{NT} + \mathbf{X}\boldsymbol{\beta}_1 + \mathbf{u},\\ &= \mathbf{Z}\boldsymbol{\beta} + \mathbf{u},\end{aligned} \tag{4}$$

where $\mathbf{Z} = [\boldsymbol{\iota}_{NT}, \mathbf{X}] = (\mathbf{Z}_1', \ldots, \mathbf{Z}_N')'$, and $\mathbf{Z}_i$ has rows $\mathbf{z}_{it}' = (1, x_{it1}, \ldots, x_{itK}) = \{z_{itj}\}$, $j = 1, \ldots, K+1$. The (pooled) regression of $\mathbf{y}$ on $\mathbf{Z}$ delivers the Ordinary Least Squares (OLS) estimator $\hat{\boldsymbol{\beta}} = \left(\hat{\beta}_0, \hat{\boldsymbol{\beta}}_1'\right)' = \left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\mathbf{Z}'\mathbf{y}$.

The standard *F-test* for fixed effects requires estimation of both (2), treating the α_i as unknown parameters, and (4) whilst the standard *RE-test* only requires estimation of (4). In order to provide a framework in which to investigate the limiting behaviour of the *F-test* and *RE-test* statistics, under both fixed and random effects, the individual effects are assumed to have the form $\boldsymbol{\alpha} = \beta_0 \boldsymbol{\iota}_N + \boldsymbol{\delta}$, $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_N)'$. Fixed effects correspond to the α_i, $i = 1, \ldots, N$, being fixed unknown parameters (or, equivalently, $\delta_1 \equiv 0$ with β_0 and δ_i, $i = 2, \ldots, N$, being the fixed unknown parameters). The case of random effects is accommodated when the δ_i, $i = 1, \ldots, N$ are random variables. Equations (1) and (2) will be employed to characterise the data generation process, with the restrictions of $H_0 : \boldsymbol{\delta} = \delta_1 \boldsymbol{\iota}_N$ providing the null model of no individual effects (notice that $\boldsymbol{\delta} = \mathbf{0}$ belongs to this set of restrictions). Specifically, when considering the alternative of fixed effects, the $(N-1)$ restrictions placed on (2) are $H_0 : \mathbf{H}\boldsymbol{\alpha} = \mathbf{0}$, where $\mathbf{H} = [\boldsymbol{\iota}_{N-1}, -\mathbf{I}_{N-1}]$, whilst for random effects the null is $H_0 : var(\delta_i) = 0$.

The standard *F* and *RE* test statistics are defined as follows.

F-Test Statistic

This is constructed as

$$F_N = \frac{(RSS_R - RSS_U)/(N-1)}{RSS_U/(N(T-1)-K)}, \tag{5}$$

where $RSS_R = \hat{\mathbf{u}}'\hat{\mathbf{u}}$ is the restricted sum of squares (from the pooled regression (4)) with $\hat{\mathbf{u}} = \mathbf{M_Z y}$, and $RSS_U = \tilde{\mathbf{u}}'\tilde{\mathbf{u}}$ is the unrestricted sum of squares (from the fixed effects regression (2)) with $\tilde{\mathbf{u}} = \mathbf{M}_{\tilde{\mathbf{X}}}\tilde{\mathbf{y}}$, the residual vector from regressing $\tilde{\mathbf{y}}$ on $\tilde{\mathbf{X}}$. If normality, homoskedasticity, and strong exogeneity were imposed such that, conditional on $\mathbf{X}$,

$\mathbf{u}_i \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_T)$, $i = 1, \ldots, N$, then a standard *F-test* would be exact. In the case of non-normal, but homoskedastic, errors Orme and Yamagata (2006) demonstrated that a standard *F-test* would be asymptotically valid.

RE-Test Statistic

The usual *RE-test* statistic is[3]

$$R_N = \sqrt{\frac{NT}{2(T-1)}} \left[\frac{\hat{\mathbf{u}}' (\mathbf{I}_N \otimes \mathbf{A}) \hat{\mathbf{u}}}{\hat{\mathbf{u}}' \hat{\mathbf{u}}} \right] = \sqrt{\frac{1}{2NT(T-1)}} \left[\frac{\hat{\mathbf{u}}' (\mathbf{I}_N \otimes \mathbf{A}) \hat{\mathbf{u}}}{\hat{\mathbf{u}}' \hat{\mathbf{u}} / NT} \right], \quad (6)$$

where $\mathbf{A} = \mathbf{A}' = \boldsymbol{\iota}_T \boldsymbol{\iota}_T' - \mathbf{I}_T$, so that

$$\mathbf{u}' (\mathbf{I}_N \otimes \mathbf{A}) \mathbf{u} = \sum_{i=1}^{N} \mathbf{u}_i' \mathbf{A} \mathbf{u}_i = \sum_i \sum_t \sum_{s \neq t} u_{it} u_{is}.$$

R_N has a limit standard normal distribution, as $N \to \infty$, under H_0 and homoskedasticity but not necessarily normality of the errors.

3. ASYMPTOTIC PROPERTIES OF F_N

In this section we describe the properties of F_N, under both local fixed and random effects, by (i) deriving its asymptotic distribution, and (ii) establishing its asymptotic relationship with R_N. In the subsequent analysis asymptotic theory is employed in which $N \to \infty$ and T is fixed. To facilitate this, the following sections detail the assumptions that are made, which are of the sort found in, for example, (White, 2001, p. 120).

3.1. Assumptions

A1: (i) $\{\mathbf{X}_i, \mathbf{u}_i\}_{i=1}^N$ is an independent sequence;
(ii) $E\left(u_{it} \mid \mathbf{X}_i, u_{i,t-1}, u_{i,t-2}, \ldots\right) = 0$, almost surely, for all i and t.

A2: (i) $E\left(\left|z_{isj} u_{it}\right|^{2+\eta}\right) \leq \Delta < \infty$ for some $\eta > 0$, all $s, t = 1, \ldots, T$, $j = 1, \ldots, K+1$, and all $i = 1, \ldots, N$;
(ii) $E\left(\left|z_{itj}\right|^{4+\eta}\right) \leq \Delta < \infty$ for some $\eta > 0$, all $t = 1, \ldots, T$, $j = 1, \ldots, K+1$, and all $i = 1, \ldots, N$;
(iii) $E\left(\mathbf{Z}'\mathbf{Z}/N\right)$ is uniformly positive definite;
(iv) $E(\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}}/N)$ is uniformly positive definite;
(v) $\mathbf{V}_N = N^{-1} \sum_{i=1}^N \sum_{t=1}^T E\left(u_{it}^2 \mathbf{z}_{it} \mathbf{z}_{it}'\right)$ is uniformly positive definite;
(vi) $\widetilde{\mathbf{V}}_N = N^{-1} \sum_{i=1}^N \sum_{t=1}^T E\left(u_{it}^2 \tilde{\mathbf{x}}_{it} \tilde{\mathbf{x}}_{it}'\right)$ is uniformly positive definite.

[3]See, for example, Breusch and Pagan (1980) or Honda (1985).

Assumption A1 imposes independent sampling of cross-section units and also, A1(ii), a strong exogeneity assumption on $\mathbf{X}_i$, so that $E(\widetilde{\mathbf{X}}_i'\mathbf{u}_i) = \mathbf{0}$; thus ruling out (for example) lagged dependent variables. Assumption A1(ii) also constrains the u_{it} to be conditionally serially uncorrelated, and thus serially uncorrelated, but not necessarily serially independent. In particular, this resembles a martingale difference assumption, but is more direct (see, for example, White (2001, p. 54)) and accommodates most models of heteroskedasticity (including time series conditional heteroskedasticity such as GARCH and its relatives). If it were strengthened to that of u_{it} being serially independent, conditionally on $\mathbf{X}_i$, GARCH processes, for example, would be ruled out. Together with Assumption A2, which explicitly allows for rather general heteroskedasticity in the disturbances, we obtain consistency and asymptotic normality of both the pooled and fixed effects least squares regression estimators ($\hat{\boldsymbol{\beta}}$ and $\tilde{\boldsymbol{\beta}}_1$, respectively), and also consistency of the corresponding heteroskedasticity-robust covariance matrix estimators.[4] These results follow for the fixed effects estimator because Assumption A2(i) and (ii) also imply that $E\left[\left|\tilde{x}_{isj}u_{it}\right|^{2+\eta}\right]$ and $E\left[\left|\tilde{x}_{itj}\tilde{x}_{isl}\right|^{2+\eta}\right]$ are both uniformly bounded. Thus, in particular, $\frac{1}{\sqrt{N}}\mathbf{Z}'\mathbf{u}$, $\frac{1}{\sqrt{N}}\widetilde{\mathbf{X}}'\mathbf{u}$, $\frac{1}{N}\mathbf{Z}'\mathbf{Z}$ and $\frac{1}{N}\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}}$ are all $O_p(1)$, with $\mathbf{V}_N^{-1/2}\frac{1}{\sqrt{N}}\mathbf{Z}'\mathbf{u} \xrightarrow{d} N(\mathbf{0}, \mathbf{I}_{K+1})$ and $\widetilde{\mathbf{V}}_N^{-1/2}\frac{1}{\sqrt{N}}\widetilde{\mathbf{X}}'\mathbf{u} \xrightarrow{d} N(\mathbf{0}, \mathbf{I}_K)$, as $N \to \infty$, T fixed. If Assumption A1(ii) is weakened to $E\left(\mathbf{X}_i'\mathbf{u}_i\right) = \mathbf{0}$, or even $E\left(\mathbf{x}_{it}u_{it}\right) = \mathbf{0}$ (zero contemporaneous correlation), $\tilde{\boldsymbol{\beta}}_1$ is not guaranteed to be consistent and, when it is inconsistent, the *F-test* is asymptotically invalid anyway, even under normality; for example, in the presence of lagged dependent variables—see the discussion in (Wooldridge, 2010, Sections 10.5 and 11.6). Furthermore, note that Assumptions A1(ii) and A2(v) imply that $\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}E(u_{it}^2) = \frac{1}{N}\sum_{i=1}^{N}E(\sum_{t=1}^{T}u_{it})^2$ is uniformly positive.

For the purposes of this article, in addition, we assume as follows:

A3: (i) $E\left|u_{it}\right|^{4+\eta} \leq \Delta < \infty$ for some $\eta > 0$, all $t = 1, \ldots, T$, and all $i = 1, \ldots, N$;
(ii) $var\left(N^{-1/2}\mathbf{u}'\left(\mathbf{I}_N \otimes \mathbf{A}\right)\mathbf{u}\right) = N^{-1}\sum_{i=1}^{N}E\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2$ is uniformly positive.

A4: (i) $\alpha_i = \beta_0 + \frac{\delta_i}{N^{1/4}}$, $i = 1, \ldots, N$;
(ii) the δ_i are independent, satisfying $E\left[u_{it}\delta_i\right] = 0$ and $E\left|\delta_i\right|^{4+\eta} \leq \Delta < \infty$, for all $i = 1, \ldots, N$;
(iii) $N^{-1}\sum_{i=1}^{N}E\left[\delta_i^2\right]$ is uniformly positive, where $\boldsymbol{\delta}' = (\delta_1, \ldots, \delta_N)$.

[4] See, for example, (White, 2001, Exercises 3.14, 5.12 and Chapter 6). Assumption A2(ii) is also required to obtain a heteroskedasticity robust *F-test*.

Assumption A3 justifies the limit distribution obtained in Proposition 1 below, and as a consequence also that of R_N. (In fact, Assumption A3(i) and Assumption A2(ii) actually imply Assumption A2(i), using the Cauchy–Schwartz inequality.) Assumption A4 characterizes the alternative data generation process and permits the investigation of asymptotic power, under local individual effects, by restricting the test criteria under consideration to be $O_p(1)$ with well defined limit distributions. Together with Assumptions A3(i) and A2(ii), Assumption A4(ii) implies $E\left|u_{it}\delta_i\right|^{2+\eta} \leq \Delta < \infty$ and $E\left|z_{itj}\delta_i\right|^{2+\eta} \leq \Delta < \infty$, for some $\eta > 0$, and all $i = 1, \ldots, N,\ t = 1, \ldots, T,\ j = 1, \ldots, K+1$. As well as fixed effects (with the δ_i being nonstochastic) it also accommodates local heteroskedastic random effects, but which are uncorrelated with $\mathbf{u}_i$. If the δ_i are also distributed independently of $\mathbf{X}_i$, then we have "pure" random effects whilst if the δ_i are correlated with $\mathbf{X}_i$ then we have "correlated" random effects. (As pointed out by Wooldridge (2010, p. 287), in microeconometric applications of panel data models with individual effects, the term fixed effect is generally used to mean correlated random effects, rather than α_i being strictly nonstochastic.)

3.2. The Asymptotic Distribution of F_N

The results concerning the limiting behavior of both the *F-test* and *RE-test* are driven by the following lemma, which also substantiates the asymptotic validity of Wooldridge's (2010, p. 299) heteroskedasticity-robust test for unobserved effects; see Section 3.4.

Lemma 1. *Define*

$$H_N = \frac{\mathbf{u}'\left(\mathbf{I}_N \otimes \mathbf{A}\right)\mathbf{u}}{\sqrt{NT\left(T-1\right)}} = \frac{1}{\sqrt{NT(T-1)}}\sum_{i=1}^{N}\mathbf{u}_i'\mathbf{A}\mathbf{u}_i$$

and

$$\kappa_N = var\left(H_N\right) = \frac{1}{NT\left(T-1\right)}\sum_{i=1}^{N}E\left\{\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right\}^2.$$

Then under Assumptions A1 and A3,

$$\kappa_N^{-1/2}H_N \xrightarrow{d} N(0,1),$$

for fixed T, as $N \to \infty$.

The expression for κ_N, whilst correct, is quite general as it simply exploits the fact that the u_{it} are serially uncorrelated. Assumption A1(ii), however, implies something a little stronger and this affords a more refined expression for κ_N which is discussed in Section 3.3. Before that discussion, however, the asymptotic distribution of F_N, under non-normality and heteroskedasticity, is given by the following proposition.

Proposition 1. *Define* $\bar{\sigma}_N^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} E(u_{it}^2)$.

(i) *Under model (2) and Assumptions A1 to A4,* $\sqrt{N}(F_N - 1) = O_p(1)$, *with*

$$\bar{\sigma}_N^2\sqrt{N}(F_N - 1) = \sqrt{\frac{T}{T-1}}H_N + \lambda_N + o_p(1),$$

where H_N *is given in Lemma 1 and* $\lambda_N = O(1)$ *is defined by*

$$\lambda_N = E[\zeta_1'\zeta_1/N] = \mu_N - \boldsymbol{\rho}_N'\boldsymbol{\Sigma}_N^{-1}\boldsymbol{\rho}_N \geq 0,$$
$$\zeta_1 = \mathbf{D}\boldsymbol{\delta} - \mathbf{Z}\boldsymbol{\Sigma}_N^{-1}\boldsymbol{\rho}_N,$$

$\boldsymbol{\Sigma}_N = E[\mathbf{Z}'\mathbf{Z}/N]$, $\boldsymbol{\rho}_N = E[\mathbf{Z}'\mathbf{D}\boldsymbol{\delta}/N]$, $\mu_N = E[\boldsymbol{\delta}'\mathbf{D}'\mathbf{D}\boldsymbol{\delta}/N]$.

(ii) *Furthermore, if* $\omega_N = \frac{\bar{\sigma}_N^2}{\sqrt{\kappa_N/2}}$, *where* κ_N *is defined in Lemma 1, then*

$$\omega_N\sqrt{N}(F_N - 1) - \frac{\lambda_N}{\sqrt{\kappa_N/2}} \xrightarrow{d} N\left(0, \frac{2T}{T-1}\right).$$

Given our assumptions, note that both ω_N and λ_N are $O(1)$ satisfying

$$\frac{\frac{1}{NT}\sum_{i=1}^{N}\mathbf{u}_i'\mathbf{u}_i}{\sqrt{\frac{1}{2NT(T-1)}\sum_{i=1}^{N}\{\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\}^2}} - \omega_N \xrightarrow{p} 0$$

and

$$\frac{\boldsymbol{\delta}'\mathbf{D}'\mathbf{M_Z}\mathbf{D}\boldsymbol{\delta}}{N} - \lambda_N \xrightarrow{p} 0,$$

respectively, with ω_N uniformly positive by Assumption, although neither ω_N or λ_N need necessarily converge. The special case of no individual effects, with $\boldsymbol{\delta} = \delta_1\boldsymbol{\iota}_N$, yields $\lambda_N \equiv 0$, as it should (this includes the case of $\boldsymbol{\delta} = \mathbf{0}$).

As exploited by Orme and Yamagata (2006), it is easy to show that if ξ_N has an F distribution with $n_1 = N - 1$ and $n_2 = N(T-1) - K$ degrees of freedom, then $\xi_N^* = \sqrt{\frac{N(T-1)}{2T}}(\xi_N - 1) \sim N(0,1)$, or approximately for large N, $\xi_N \overset{A}{\sim} N\left(1, \frac{2T}{N(T-1)}\right)$. Therefore, by Proposition 1, we can employ the following approximation, under the null,

$$F_\omega \equiv \hat{\omega}_N \{F_N - 1\} + 1 \overset{A}{\sim} F(n_1, n_2), \tag{7}$$

for any choice of $\hat{\omega}_N$ satisfying $\hat{\omega}_N - \omega_N \overset{p}{\to} 0$, implying that F_ω can be used in an asymptotically valid "standard" *F-test* procedure.

Before proceeding to derive a suitable $\hat{\omega}_N$, note that under pure local random effects, with $E[\delta_i|\mathbf{X}_i] = 0$ and $E\left[\delta_i^2|\mathbf{X}_i\right] = \tau^2$, $\boldsymbol{\rho}_N = \frac{T}{N}\sum_{i=1}^{N} E[\delta_i \bar{\mathbf{z}}_i] = \mathbf{0}$ with $\bar{\mathbf{z}}_i = T^{-1}\sum_{t=1}^{T}\mathbf{z}_{it}$ so that $\lambda_N = TE\left[\frac{\delta'\delta}{N}\right] = T\tau^2$. In this case, we immediately obtain the following Corollary to Proposition 1 (the proof is omitted).

Corollary 1. *Under the alternative of (pure) local random effects, and under the assumptions of Proposition 1,*

$$\hat{\omega}_N\sqrt{N}(F_N - 1) - \frac{T\tau^2}{\sqrt{\kappa_N/2}} \overset{d}{\to} N\left(0, \frac{2T}{T-1}\right)$$

for any choice of $\hat{\omega}_N$ *satisfying* $\hat{\omega}_N - \omega_N \overset{p}{\to} 0$.

Therefore, a robust *F-test*, based on F_ω, will have nontrivial asymptotic local power against pure random effects. In fact, and analogous to Orme and Yamagata (2006), a stronger result will be established in Section 3.4. There it is shown that, under (pure) local random effects, a robust *F-test* procedure based on F_ω will possess the same asymptotic power as a suitably "robustified" *RE-test*, of the sort proposed by Wooldridge (2010, p. 299) or Häggström and Laitila (2002). However, under "correlated" local random effects a robust *F-test* will possess higher asymptotic power than a robust *RE-test*.

3.3. Asymptotically Valid *F*-Test Statistics

As noted above, an asymptotically valid *F-test* can be constructed if there is a $\hat{\omega}_N$ available satisfying $\hat{\omega}_N - \omega_N \overset{p}{\to} 0$. Using restricted OLS (i.e., pooled) residuals a natural choice for $\hat{\omega}_N$ might be

$$\hat{\omega}_N = \frac{\hat{\sigma}_N^2}{\sqrt{\hat{\kappa}_N/2}},$$

where $\hat{\sigma}_N^2 = \hat{\mathbf{u}}'\hat{\mathbf{u}}/(NT - K - 1)$ and

$$\hat{\kappa}_N = \frac{1}{NT(T-1)}\sum_{i=1}^{N}\left\{\hat{\mathbf{u}}_i'\mathbf{A}\hat{\mathbf{u}}_i\right\}^2 = \frac{1}{NT(T-1)}\sum_{i=1}^{N}\left\{\sum_t\sum_{s\neq t}\hat{u}_{it}\hat{u}_{is}\right\}^2.$$

Indeed, this choice is justified in Proposition 2 below; c.f., Wooldridge (2010, p. 299).

However, another (perhaps more efficient) choice for $\hat{\kappa}_N$, and thus $\hat{\omega}_N$, emerges if we exploit Assumption A1(ii).[5] To see this, first note that $\sum_t\sum_{s\neq t} u_{it}u_{is} = 2\sum_{t=2}^{T} w_{it}$, where $w_{it} = u_{it}\sum_{s=1}^{t-1} u_{is}$, so that κ_N can equivalently be expressed as

$$\kappa_N = \frac{4}{NT(T-1)}\sum_{i=1}^{N} E\left[\left(\sum_{t=2}^{T} w_{it}\right)^2\right]. \tag{8}$$

Now, from Assumption A1(ii), $E[w_{it}w_{it-m}] = 0$, for all $t \geq 3$ and $m = 1, \ldots, t-1$, so that (8) becomes

$$\kappa_N = \frac{4}{NT(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T} E\left(w_{it}^2\right),$$

where

$$\sum_{t=2}^{T} w_{it}^2 = \sum_{t=2}^{T}\sum_{s=1}^{t-1} u_{it}^2 u_{is}^2 + 2\sum_{t=3}^{T}\sum_{s=2}^{t-1}\sum_{r=1}^{s-1} u_{it}^2 u_{is}u_{ir}. \tag{9}$$

A further simplification arises if, in addition to A1(ii), we can assume as follows:

A1(iii): $E\left(u_{it}^2 u_{is}u_{ir}\right) = 0$, for $t > s > r$.

In this case (8) is

$$\begin{aligned}\kappa_N &= \frac{4}{NT(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{s=1}^{t-1} E\left(u_{it}^2u_{is}^2\right)\\ &= \frac{2}{NT(T-1)}\sum_{i=1}^{N}\sum_t\sum_{s\neq t} E\left(u_{it}^2u_{is}^2\right).\end{aligned}$$

[5]We shall not, here, consider alternative estimators of $\bar{\sigma}_N^2$, although this is possible.

This assumption, however, restricts the admissibility of certain forms of time series conditional heteroskedasticity, as it rules out an asymmetric GARCH process for u_{it}^2.[6]

The same expression for κ_N emerges if A1 (iii) is strengthened to the following assumption:

A1 (iii)′: $E\left(u_{it}^2|\mathbf{X}_i, u_{i,t-1}, u_{i,t-2}, \ldots\right) = E\left(u_{it}^2|\mathbf{X}_i\right)$, almost surely, for all i and t.

This implies A1 (iii) because by iterative expectations, and for $t > s > r$,

$$\begin{aligned} E\left(u_{it}^2 u_{is} u_{ir}|\mathbf{X}_i\right) &= E\left[E\left(u_{it}^2|\mathbf{X}_i, u_{i,t-1}, u_{i,t-2}, \ldots\right) u_{is} u_{ir}|\mathbf{X}_i\right] \\ &= E\left(u_{it}^2|\mathbf{X}_i\right) E(u_{is} u_{ir}|\mathbf{X}_i) = 0, \end{aligned}$$

and, for the subsequent analysis in Section 3.5, it will be useful to note that in this case the u_{it}^2 are conditionally serially uncorrelated and κ_N can also be expressed as

$$\kappa_N = \frac{2}{NT\,(T-1)} \sum_{i=1}^{N} \sum_{t} \sum_{s \neq t} E\left(E\left(u_{it}^2 \mid \mathbf{X}_i\right) E\left(u_{is}^2 \mid \mathbf{X}_i\right)\right).$$

Whilst still allowing general forms of heteroskedasticity, A1 (iii)′ does rule out time series conditional heteroskedasticity processes.

Finally, consider a strengthening of A1 (iii) to the following assumption:

A1 (iii)″ $\{u_{it}\}_{t=1}^{T}$ is a sequence of serially independent random variables, for all $i = 1, \ldots, N$.

Then (8) becomes

$$\kappa_N = \frac{2}{NT\,(T-1)} \sum_{i=1}^{N} \sum_{t} \sum_{s \neq t} E\left(u_{it}^2\right) E\left(u_{is}^2\right).$$

The preceding discussion suggests differing possible consistent estimators for κ_N, and thus for ω_N, according to: (i) whether, or not, Assumption A1 (ii) is fully exploited; or, (ii) whether one of the additional A1 (iii), A1 (iii)′, or A1 (iii)″ is adopted. These are described in the following proposition.

[6]See, for example, Goncalves and Kilian (2004).

Proposition 2. *Define* $\hat{\sigma}_N^2 = \hat{\mathbf{u}}'\hat{\mathbf{u}}/(NT-K-1)$, $\hat{w}_{it} = \hat{u}_{it}\sum_{s=1}^{t-1}\hat{u}_{is}$, *and*

$$\hat{\kappa}_N^{(1)} = \frac{1}{NT(T-1)}\sum_{i=1}^{N}\left(\sum_t\sum_{s\neq t}\hat{u}_{it}\hat{u}_{is}\right)^2 = \frac{4}{NT(T-1)}\sum_{i=1}^{N}\left(\sum_{t=2}^{T}\hat{w}_{it}\right)^2$$

$$\hat{\kappa}_N^{(2)} = \frac{4}{NT(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{w}_{it}^2$$

$$\hat{\kappa}_N^{(3)} = \frac{2}{NT(T-1)}\sum_{i=1}^{N}\sum_t\sum_{t\neq s}\hat{u}_{it}^2\hat{u}_{is}^2 = \frac{4}{NT\,(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{s=1}^{t-1}\hat{u}_{it}^2\hat{u}_{is}^2.$$

Under model (2) and Assumptions A1 to A4, we have the following situation:

1. $\hat{\sigma}_N^2 - \bar{\sigma}_N^2 \xrightarrow{p} 0$ *and* $\hat{\kappa}_N^{(j)} - \kappa_N \xrightarrow{p} 0$, $j = 1, 2$.

Under model (2), Assumptions A1–A4 and either A1(iii), A1(iii)′, or A1(iii)″, we have the following situation:

2. $\hat{\sigma}_N^2 - \bar{\sigma}_N^2 \xrightarrow{p} 0$ *and* $\hat{\kappa}_N^{(j)} - \kappa_N \xrightarrow{p} 0$, $j = 1, 2, 3$.

From this analysis, it follows that asymptotically valid choices for $\hat{\omega}_N$ include $\hat{\omega}_N^{(j)} = \hat{\sigma}_N^2/\sqrt{\hat{\kappa}_N^{(j)}/2}$, $j = 1, 2, 3$, where, specifically,

$$\hat{\omega}_N^{(1)} = \frac{\hat{\sigma}_N^2}{\sqrt{\frac{2}{NT(T-1)}\sum_{i=1}^{N}\left(\sum_{t=2}^{T}\hat{w}_{it}\right)^2}}, \tag{10}$$

$$\hat{\omega}_N^{(2)} = \frac{\hat{\sigma}_N^2}{\sqrt{\frac{2}{NT(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{w}_{it}^2}}, \tag{11}$$

$$\hat{\omega}_N^{(3)} = \frac{\hat{\sigma}_N^2}{\sqrt{\frac{2}{NT(T-1)}\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{s=1}^{t-1}\hat{u}_{it}^2\hat{u}_{is}^2}}, \tag{12}$$

depending on assumptions made about the u_{it}, $t = 1, \ldots, T$. Robust *F-test* statistics can then be constructed as $F_\omega^{(m)} = \hat{\omega}_N^{(m)}\{F_N - 1\} + 1$, $m = 1, 2, 3$, and approximate inferences obtained based on (7). Note that $\hat{\omega}_N^{(1)}$ is very general, whereas $\hat{\omega}_N^{(2)}$ is tailored to the main assumptions of the article. Thus we might expect better sampling behavior from using the latter, rather than the former, under the maintained assumptions A1–A4. Finally, $\hat{\omega}_N^{(3)}$ is *only* valid under rather more restrictive assumptions.

3.4. The Relationship between F_N and R_N

Under the null of no individual effects, it is straightforward to show that

$$\frac{1}{\sqrt{N}}\left[\frac{\hat{\mathbf{u}}'\left(\mathbf{I}_N\otimes\mathbf{A}\right)\hat{\mathbf{u}}}{\hat{\mathbf{u}}'\hat{\mathbf{u}}/NT}\right]=\frac{1}{\sqrt{N}}\frac{\mathbf{u}'\left(\mathbf{I}_N\otimes\mathbf{A}\right)\mathbf{u}}{\bar{\sigma}_N^2}+o_p\left(1\right).$$

From (6), Lemma 1, and Proposition 1, therefore, we can write

$$\begin{aligned}R_N&=\frac{1}{\sqrt{2}}\frac{H_N}{\bar{\sigma}_N^2}+o_p(1)\\&=\sqrt{\frac{T-1}{2T}}\sqrt{N}\left(F_N-1\right)+o_p(1),\end{aligned}$$

under the null, so that

$$R_\omega\equiv\hat{\omega}_N R_N\xrightarrow{d}N(0,1)\tag{13}$$

for any choice of $\hat{\omega}_N$ satisfying $\hat{\omega}_N-\omega_N\xrightarrow{p}0$; for example, $\hat{\omega}_N^{(2)}$ under assumptions A1–A4 of this article. Moreover, this also substantiates Wooldridge's (2010, p. 299) suggestion for a heteroskedasticity-robust *RE* test statistic constructed as $\hat{\omega}_N^{(1)}R_N$; or, under under the more restrictive assumptions A1(iii), A1(iii)′, or A1(iii)″, $\hat{\omega}_N^{(3)}R_N$ as proposed by Häggström and Laitila (2002).

The following proposition extends this result to the case of local individual effects (fixed or random).

Proposition 3. *Under model (2) and Assumptions A1 to A4,*

$$\hat{\omega}_N R_N=\left\{\sqrt{\frac{(T-1)}{2T}}\right\}\hat{\omega}_N\sqrt{N}\left[F_N-1\right]-\sqrt{\frac{T}{2\left(T-1\right)}}\frac{\gamma_N}{\sqrt{\kappa_N/2}}+o_p\left(1\right),$$

for any choice of $\hat{\omega}_N$ *satisfying* $\hat{\omega}_N-\omega_N\xrightarrow{p}0$, *where* $\gamma_N=O(1)$ *defined by*

$$\begin{aligned}\gamma_N&=E(\boldsymbol{\zeta}_2'\boldsymbol{\zeta}_2/N)=\boldsymbol{\rho}_N'\boldsymbol{\Sigma}_N^{-1}\tilde{\boldsymbol{\Sigma}}_N\boldsymbol{\Sigma}_N^{-1}\boldsymbol{\rho}_N\geq 0,\\\boldsymbol{\zeta}_2&=\tilde{\mathbf{Z}}\boldsymbol{\Sigma}_N^{-1}\boldsymbol{\rho}_N,\\\tilde{\boldsymbol{\Sigma}}_N&=E(\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}/N),\end{aligned}$$

and the limit distribution of $\omega_N\sqrt{N}\left[F_N-1\right]$ *is given by Proposition 1.*

Again, γ_N need not converge, but it is $O(1)$ and $\gamma_N - \frac{\delta'\mathbf{D}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}(\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}})(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{D}\delta}{N} \xrightarrow{p} 0$. As with Proposition 1, $\gamma_N \equiv 0$ obtains under $H_0 : \boldsymbol{\delta} = \delta_1 \boldsymbol{\iota}_N$, as it should, since $\left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\mathbf{Z}'\mathbf{D}\boldsymbol{\delta} = (\delta_1, \mathbf{0}')'$ and the top-left, $(1, 1)$, element of $\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}$ is 0. As discussed above, under the alternative of (pure) local random effects $\boldsymbol{\rho}_N = \mathbf{0}$, and we obtain the following Corollary, which is immediate from Corollary 1 given Proposition 3.

Corollary 2. *Under the alternative of (pure) local random effects, and under the assumptions of Proposition 1,*

$$\hat{\omega}_N R_N - \left\{\sqrt{\frac{T(T-1)}{2}}\right\} \frac{\tau^2}{\sqrt{\kappa_N/2}} \xrightarrow{d} N(0, 1),$$

for any choice of $\hat{\omega}_N$ *satisfying* $\hat{\omega}_N - \omega_N \xrightarrow{p} 0$.

Thus, since under (pure) local random effects, $\hat{\omega}_N R_N - \sqrt{\frac{N(T-1)}{2T}}\hat{\omega}_N (F_N - 1) = o_p(1)$, both the robust *RE* and robust *F-test* procedures, based on (13) and (7), respectively, will have identical asymptotic power functions. However, under local fixed effects or random effects which are correlated with $\mathbf{X}_i$, the robust *F-test* can have greater asymptotic power. In particular, when individual effects are correlated with the mean values of the regressors, $\boldsymbol{\rho}_N \neq \mathbf{0}$ and is $O(1)$, implying $\gamma_N > 0$ so that a test based on R_N (but suitably robust to heteroskedasticity) should have lower asymptotic local power than one based on F_N. This makes intuitive sense, since F_N is designed to test for individual effects which are correlated with $\bar{\mathbf{z}}_i$, whereas R_N is constructed on the assumption that the individual effects are uncorrelated with all regressor values. The importance of distinguishing between individual effects which are correlated or uncorrelated with regressors, rather than simply labelling them fixed or random, is discussed by Wooldridge (2010, Section 10.2).

3.5. Analysis of the Standard *F*-Test and RE-test

Given the analysis above certain predictions can be made concerning the asymptotic behaviour, under the null hypothesis, of both the standard *F-test*, based on F_N, and *RE-test*, based on R_N, under specific assumptions about the data and/or forms of heteroskedasticity.

Serial Independence

Suppose $\{u_{it}, \mathbf{x}'_{it}\}_{t=1}^{T}$ are serially independent, or assumption A1(iii)$'$ holds, with $E(u_{it}^2|\mathbf{X}_i) = h_{it} > 0$. Consider, first, the case of $E[h_{it}] = \sigma^2 < \infty$, so that the errors are unconditionally homoskedastic. Then, $\kappa_N = 2\sigma^4$

and $\omega_N = 1$. In this very restricted case, then, both the *F-test* and *RE-test*, based on F_N and R_N, respectively, remain asymptotically valid without any adjustment. In particular, this result is true if the $\left(u_{it}, \mathbf{x}'_{it}\right)$ are i.i.d., but the u_{it} are conditionally heteroskedastic.

Cross-Section Heteroskedasticity

In this case, we rule out time series heteroskedasticity and adopt assumption A1 (iii)′ with

$$h_i \equiv E\left[u_{it}^2 | \mathbf{X}_i\right], \qquad \text{for all } i \text{ and } t, \tag{14}$$

so that $\sigma_i^2 = E(h_i) > 0$ is the unconditional variance and $E\left(u_{it}^2 u_{is}^2\right) = E(h_i^2) < \infty$. Here, both the *F-test* based on F_N and *RE-test* based on R_N, without adjustment, will be asymptotically oversized (in that, asymptotically, both will reject a correct null of no individual effects too often for any given nominal significance level)[7] To demonstrate the result, one need only show that $\omega_N < 1$ which is evidently true because

$$\frac{1}{N}\sum_{i=1}^{N} E(h_i^2) - \left\{\frac{1}{N}\sum_{i=1}^{N} E(h_i)\right\}^2 \geq \frac{1}{N}\sum_{i=1}^{N} \sigma_i^4 - \left\{\frac{1}{N}\sum_{i=1}^{N} \sigma_i^2\right\}^2 > 0. \tag{15}$$

The same prediction is true in the case of unconditional heteroskedasticity, by which we mean $h_i \equiv \sigma_i^2$, since the first (weak) inequality in (15) can be replaced with equality.

Time Series Heteroskedasticity

Here we consider two scenarios which afford tractable results. Under the first scenario, assumption A1 (iii)′ is, again, adopted. The second scenario allows for a GARCH process, but under the symmetry assumption of A1 (iii).

(i) Consider unconditional time series heteroskedasticity, so that A1 (iii)′ holds and

$$\sigma_t^2 \equiv E\left[u_{it}^2 \,|\, \mathbf{X}_i\right], \qquad \text{for all } i \text{ and } t, \tag{16}$$

are constants. Here

$$\omega_N^2 = \frac{\left(\frac{1}{T}\sum_{t=1}^{T} \sigma_t^2\right)^2}{\frac{1}{T(T-1)}\sum_t \sum_{s \neq t} \sigma_t^2 \sigma_s^2} > 1,$$

[7]Indeed, this particular conclusion explains some of the finite sample Monte Carlo results obtained by Häggström and Laitila (2002).

because

$$\left(\frac{1}{T}\sum_{t=1}^{T}\sigma_t^2\right)^2 - \frac{1}{T(T-1)}\sum_{t}\sum_{s\neq t}\sigma_t^2\sigma_s^2$$
$$= \frac{1}{T-1}\left(\frac{1}{T}\sum_{t=1}^{T}\sigma_t^4 - \left(\frac{1}{T}\sum_{t=1}^{T}\sigma_t^2\right)^2\right) > 0.$$

This implies that both the F and *RE-test* procedures, without adjustment, will be asymptotically undersized.

To obtain a similar result for conditional time series heteroskedasticity, with $h_t \equiv E\left[u_{it}^2\,|\,\mathbf{X}_i\right]$, for all i and t, and $\sigma_t^2 = E(h_t) > 0$, A(iii)$'$ needs to be strengthened to A(iii)$''$ (serial independence) so that $E\left(u_{it}^2 u_{is}^2\right) = \sigma_t^2\sigma_s^2 < \infty$.

(ii) In order to provide a succinct analysis for the conditional time series heteroskedastic case, we restrict u_{it} to be a stationary time series, for all i, such that A1(iii) holds either by implication of A1(iii)$'$ or by direct supposition. Thus, (symmetric) ARCH/GARCH specifications are allowed for but certain asymmetric ARCH/GARCH models with leverage are not. Exploiting stationarity, and heteroskedasticity in the time series dimension only, we express the unconditional variance and covariances as $E[u_{it}^2] = \sigma^2 > 0$, $E[u_{it}^2 u_{i,t-j}^2] = \gamma_j > 0$, say, so that

$$\omega_N = \frac{\sigma^2}{\sqrt{\frac{2}{T(T-1)}\sum_{t=2}^{T}\sum_{j=1}^{t-1}\gamma_j}},$$

where

$$\frac{2}{T\,(T-1)}\sum_{t=2}^{T}\sum_{j=1}^{t-1}\gamma_j = \frac{2}{T\,(T-1)}\sum_{t=2}^{T}\sum_{j=1}^{t-1}\left(\gamma_j - \sigma^4\right) + \sigma^4.$$

Thus, if the u_{it}^2 are (serially) positively correlated, $\gamma_j - \sigma^4 > 0$ and $\omega_N < 1$ so that both the F and *RE-test* procedures, without adjustment, will be asymptotically oversized. The converse is true if the u_{it}^2 are (serially) negatively correlated. In the particular case of symmetric ARCH/GARCH processes, and with the usual positivity constraints on the parameters, the u_{it}^2 will be (serially) positively correlated,[8] so that the unadjusted F and *RE-test* procedures will be asymptotically oversized.

[8]He and Terasvirta (1999) establish that the autocorrelation function of the squared process is positive.

In order to shed light on the relevance of the preceding asymptotic analysis, the next section reports the results of a small Monte Carlo experiment which illustrates the asymptotic robustness of the *F-test* to non-normality/heteroskedasticity and its power properties relative to the *RE-test.*

4. MONTE CARLO STUDY

The Monte Carlo study investigates the sampling behavior of the test statistics considered above, (7) and (13), for differing choices of $\hat{\omega}_N$, including $\hat{\omega}_N \equiv 1$. As our analytical results suggest, the tests are justified when $N \to \infty$ with T fixed, we consider $(N, T) = (20, 5)$, $(50, 5)$, $(100, 5)$, $(50, 10)$, $(50, 20)$.

4.1. Monte Carlo Design

The model employed is

$$y_{it} = \alpha_i + \sum_{j=1}^{3} z_{it,j}\beta_j + u_{it},\ u_{it} = \sigma_{it}\varepsilon_{it}, \tag{17}$$

where $z_{it,1} = 1$, $z_{it,2}$ is drawn from a uniform distribution on $(1, 31)$ independently for i and t, and $z_{it,3}$ is generated following Nerlove (1971), such that

$$z_{it,3} = 0.1t + 0.5z_{it-1,3} + \upsilon_{it},$$

where the value $z_{i0,3}$ is chosen as $5 + 10\upsilon_{i0}$, and υ_{it} (and υ_{i0}) is drawn from the uniform distribution on $(-0.5, 0.5)$ independently for i and t, in order to avoid any normality in regressors. These regressor values are held fixed over replications. Also, observe that the regression design is not quadratically balanced.[9] Without loss of generality, the coefficients are set as $\beta_j = 1$ for $j = 1, 2, 3$. The i.i.d. standardised errors for ε_{it} are drawn from: the standard normal distribution (SN); the t distribution with five degrees of freedom (t_5); and, the chi-square distribution with six degrees of freedom (χ^2_6).

We consider the following five specifications for σ_{it}:[10]

[9]See the discussion in Orme and Yamagata (2006).

[10]We also considered an ARCH(1) specification. However, the associated results are not reported since they are qualitatively similar to the results for the GARCH(1,1) specification, which are presented below.

1. Homoskedasticity (HET0)

$$\sigma_{it} = \sigma = 1;$$

2. Cross-sectional one-break-in-volatility heteroskedasticity (HET1)

$$\begin{aligned}\sigma_{it} &= \sigma_1, \qquad i = 1, \ldots, N_1, \quad t = 1, \ldots, T \\ &= \sigma_2, \qquad i = N_1 + 1, \ldots, N, \quad t = 1, \ldots, T\end{aligned}$$

with $N_1 = \lceil N/2 \rceil$, where $\lceil A \rceil$ is the largest integer not less than A, $\sigma_1 = 0.5$, and $\sigma_2 = 1.5$.
3. Time series one-break-in-volatility heteroskedasticity (HET2)

$$\begin{aligned}\sigma_{it} &= \sigma_1, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T_1 \\ &= \sigma_2, \qquad i = 1, \ldots, N, \quad t = T_1 + 1, \ldots, T\end{aligned}$$

with $T_1 = \lceil T/2 \rceil$, $\sigma_1 = 0.5$, and $\sigma_2 = 1.5$.
4. Conditional heteroskedasticity depending on a regressor (HET3)

$$\sigma_{it} = \eta_c[(z_{it,2} - 1)/30]/c, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T$$

$\eta_c[\cdot]$ is the inverse of the cumulative distribution function of chi-squared distribution with degrees of freedom c. Since $z_{it,2}$ is drawn from a uniform distribution on $(1, 31)$, σ_{it} has mean 1 and variance $2/c$, so it is easy to control the degree of heteroskedasticity through the choice of c. We employ $c = 1$.
5. Time Series conditional heteroskedasticity, GARCH(1,1) (HET4)

$$u_{it} = \sigma_{it}\varepsilon_{it}, \qquad t = -49, \ldots, T, \quad i = 1, \ldots, N,$$

where

$$\sigma_{it}^2 = \phi_0 + \phi_1 u_{i,t-1}^2 + \phi_2 \sigma_{i,t-1}^2.$$

The value of parameters are chosen to be $\phi_0 = 0.5$, $\phi_1 = 0.25$, and $\phi_2 = 0.25$, and $u_{i,-50} = 0$ with the first 50 observations being discarded, so that the unconditional variance is $E\left(u_{it}^2\right) = \phi_0/\left(1 - \phi_1 - \phi_2\right)$.
6. Time series conditional heteroskedasticity, GJR-GARCH(1,1) (HET5)

$$u_{it} = \sigma_{it}\varepsilon_{it}, \qquad t = -49, \ldots, T, \quad i = 1, \ldots, N,$$

where

$$\sigma_{it}^2 = \phi_0 + \phi_1 \sigma_{i,t-1}^2 + \phi_2 \left(\left|u_{i,t-1}\right| - \phi_3 u_{it-1}\right)^2.$$

The value of parameters are chosen to be $\phi_0 = 0.3$, $\phi_1 = 0.5$, $\phi_2 = 0.2$, and $\phi_3 = 0.23$, and $u_{i,-50} = 0$ with the first 50 observations being discarded.[11]

For power comparisons, the individual effects are generated according to

$$\alpha_i = \tau_i \left[\sqrt{R^2} g_i(\bar{\mathbf{z}}_i) + \sqrt{1 - R^2} \varphi_i \right], \tag{18}$$

where the φ_i are i.i.d. $N(0,1)$, $g_i(\bar{\mathbf{z}}_i) = \boldsymbol{\iota}_3'(\bar{\mathbf{z}}_i - \bar{\bar{\mathbf{z}}})/s$ with $\boldsymbol{\iota}_3 = (1,1,1)'$, $\bar{\bar{\mathbf{z}}}$ being overall average of $\mathbf{z}_{it}$, s being the standard deviation of $\boldsymbol{\iota}_3'\bar{\mathbf{z}}_i$, and the R^2 is from the regression of (18). With this set up, the variance of inside of the square brackets is always unity across designs. We consider two combinations of (τ_i, R^2): (i) $(\tau_i, R^2) = (0, 0)$, which is a simple null model specification, with $\alpha_i \equiv 0$, and (ii) $(\tau_i, R^2) = (v_\alpha, 1)$, which is simple fixed effects specification (given that the z_{it} are fixed over replications).[12] To control the power, we consider $v_\alpha^2 = 0.1$.

4.2. Asymptotic Tests

Four versions of the *FE* and *RE* test statistics are considered, constructed using $\hat{\omega}_N^{(0)} \equiv 1$ and $\hat{\omega}_N^{(m)}$, $m = 1, 2, 3$, as defined at (10)–(12), and all are based on the restricted estimator, $\hat{\boldsymbol{\beta}}$.[13]

1. *F-test* statistics (denoted F_ω in the Tables)

$$F_\omega^{(m)} = \hat{\omega}_N^{(m)}(F_N - 1) + 1, \qquad m = 0, 1, 2, 3, \tag{19}$$

where

$$F_N = \frac{(RSS_R - RSS_U)/(N-1)}{RSS_U/(N(T-1) - K)} \equiv F_\omega^{(0)},$$

is the standard *F-test* statistic. The corresponding test procedure, for each separate statistic (19), employs critical vales from an F distribution with n_1 and n_2 degrees of freedom, respectively, where $n_1 = N - 1$ and $n_2 = N(T-1) - K$. That is, for each $m = 0, 1, 2, 3$, reject H_0 if $F_\omega^{(m)} > c_{N,\alpha}$, where $\Pr\left(\xi > c_{N,\alpha}\right) = \alpha$, for chosen α, and $\xi \sim F(n_1, n_2)$

[11] Note that the parameters chosen for specifications 5 and 6 ensure that $E\left|u_{it}\right|^{4+\eta}$ exists for all error distributions; see Ling and McAleer (2002).

[12] We also considered a pure random effects specification, $\tau_i = v_\alpha$, $R^2 = 0$, and the results show that the power properties of the modified fixed effects test and the modified random effects test are very similar.

[13] The estimator $\tilde{\omega}_N$, based on the unrestricted estimator (i.e., fixed effects estimator), is also considered, but the finite sample performance of the tests considered is monotonically inferior to that based on the estimator of $\hat{\omega}_N$.

2. One sided (positive) *RE-test* statistics (denoted R_ω in the tables)

$$R_\omega^{(m)} = \hat{\omega}_N^{(m)} R_N, \qquad m = 0, 1, 2, 3, \tag{20}$$

where

$$R_N = \sqrt{\frac{NT}{2(T-1)}} \left[\frac{\hat{\mathbf{u}}' (\mathbf{I}_N \otimes \mathbf{A}) \hat{\mathbf{u}}}{\hat{\mathbf{u}}' \hat{\mathbf{u}}} \right] \equiv R_\omega^{(0)}$$

is the one-sided (positive) standard RE-test statistic. The corresponding test procedure, for each separate statistic (20), employs critical values from a $N(0, 1)$ distribution. That is, for each $m = 0, 1, 2, 3$, reject H_0 if $R_\omega^{(m)} > z_\alpha$, where $\Pr(Z > z_\alpha) = \alpha$, for chosen α, and $Z \sim N(0, 1)$.

4.3. Bootstrap Tests

As is well known, asymptotic theory can provide a poor approximation to actual finite sample behaviour and that bootstrap procedures often lead to more reliable inferences.[14] Therefore, we also consider a simple wild bootstrap procedure scheme, based on Mammen (1993) and Davidson and Flachaire (2008), which might be employed in order to provide closer agreement between the desired nominal and the empirical significance level of the proposed test procedures and which has proved effective in previous studies; see, for example, Godfrey and Orme (2004). The wild bootstrap is implemented using the following steps:

1. Estimate the models (2) and (4) to get $\hat{u}_{it}$, $i = 1, \ldots, N$, and construct test statistics $F_\omega^{(m)}$ and $R_\omega^{(m)}$, $m = 0, 1, 2, 3$;
2. Repeat the following B times:

   (a) Generate $u_{it}^* = v_{it} \hat{u}_{it}$, where the v_{it} are i.i.d., over i and t, taking the discrete values ± 1 with an equal probability of 0.5;

   (b) Construct

$$y_{it}^* = \mathbf{z}_{it}' \hat{\boldsymbol{\beta}} + v_{it} \hat{u}_{it} = \mathbf{z}_{it}' \hat{\boldsymbol{\beta}} + u_{it}^*, \tag{21}$$

   obtain restricted and unrestricted OLS residuals $\hat{u}_{it}^* = y_{it}^* - \mathbf{z}_{it}' \hat{\boldsymbol{\beta}}^*$ and $\tilde{u}_{it}^* = \tilde{y}_{it}^* - \tilde{\mathbf{x}}_{it}' \tilde{\boldsymbol{\beta}}_1^*$, respectively, and the restricted and unrestricted residual sums of squares (RSS_R^* and RSS_U^*, respectively);

[14]See Godfrey (2009) for an excellent guide to bootstrap test procedures for regression models.

(c) Construct the bootstrap test statistics

$$F_{\omega}^{*(m)} = \hat{\omega}_N^{*(m)}(F_N^* - 1) + 1, \qquad F_N^* = \frac{(RSS_R^* - RSS_U^*)/(N-1)}{RSS_U^*/(N(T-1)-K)} \equiv F_{\omega}^{*(0)}$$

and

$$R_{\omega}^{*(m)} = \hat{\omega}_N^{*(m)} R_N^*, \qquad R_N^* = \sqrt{\frac{NT}{2(T-1)}}\left[\frac{\hat{\mathbf{u}}^{*\prime}(\mathbf{I}_N \otimes \mathbf{A})\hat{\mathbf{u}}^*}{\hat{\mathbf{u}}^{*\prime}\hat{\mathbf{u}}^*}\right] \equiv R_{\omega}^{*(0)},$$

where $\hat{\omega}_N^{*(m)}$, $m = 1, 2, 3$ is constructed as in (10)–(12) but using $\hat{u}_{it}^*$, and $\hat{\omega}_N^{*(0)} \equiv 1$;

3. Calculate the proportion of bootstrap test statistics, $F_{\omega}^{*(m)}$ (respectively, $R_{\omega}^{*(m)}$), from the B repetitions of Step 2c that are at least as large as the actual value of $F_{\omega}^{(m)}$ (respectively, $R_{\omega}^{(m)}$). Let this proportion be denoted by $\hat{p}^{(m)}$ and the desired significance level be denoted by α. The asymptotically valid rejection rule, for each m, is that H_0 is rejected if $\hat{p}^{(m)} \leq \alpha$.

The sampling behavior of all the above tests are investigated using 5000 replications of sample data and $B = 200$ bootstrap samples, employing a nominal 5% significance level.[15]

Observe that the wild bootstrap scheme imposes symmetry on the u_{it}^*. Because of this, it is readily shown that $\hat{\omega}_N^{*(m)} - \hat{\omega}_N^{(3)} = o_{p^*}(1)$, in probability, $m = 1, 2, 3$, signifying that, for any $\delta > 0$, $P^*\left(\left|\hat{\omega}_N^{*(m)} - \hat{\omega}_N^{(3)}\right| > \delta\right) = o_p(1)$, as $N \to \infty$, T fixed, where P^* is the probability measure induced by the wild bootstrap conditional on the sample data. It can also be established that, for example, $\hat{\omega}_N^{(3)}\sqrt{N}\left(F_N^* - 1\right) \xrightarrow{d^*} N\left(0, \frac{2T}{T-1}\right)$, in probability, implying that $\sup_x \left|P^*(\hat{\omega}_N^{(3)}\sqrt{N}\left(F_N^* - 1\right) \leq x) - \mathscr{D}_T(x)\right| = o_p(1)$, where $\mathscr{D}_T(x)$ denotes the distribution function of a $N\left(0, \frac{2T}{T-1}\right)$ random variable. Combining these results, we obtain

$$\sup_x \left|P^*(\hat{\omega}_N^{*(m)}\sqrt{N}\left(F_N^* - 1\right) \leq x) - P(\hat{\omega}_N^{(m)}\sqrt{N}\left(F_N - 1\right) \leq x)\right| = o_p(1),$$
$$m = 1, 2, 3,$$

which justifies the asymptotic validity of the wild bootstrap scheme for $F_{\omega}^{*(m)}$, $m = 1, 2, 3$, notwithstanding the fact the u_{it} may not be asymmetrically distributed. This will not be the case, however, for the

[15] It is often advocated that $(B+1)/100$ should be an integer. However, running the experiments with $B = 199$ does not change the results.

unadjusted *F-test* statistic $\sqrt{N}\left(F_N^* - 1\right)$. Thus, it will useful to investigate how the wild bootstrap performs in finite samples when the true errors are asymmetric.[16]

4.4. Results

Before looking at the results from the Monte Carlo study, and drawing on the discussion in Godfrey et al. (2006), it is important to define criteria to evaluate the performance of the different tests considered. Given the large number of replications performed, the standard asymptotic test for proportions can be used to test the null hypotheses that the true significance level is equal to its nominal value. In practice, however, what is important is not that the significance level of the test is identical to the chosen nominal level, but rather that the true and nominal rejection frequencies stay reasonably close, even when the test is only approximately valid. Following Cochran's (1952) suggestion, we shall regard a test as being robust, relative to a nominal value of 5%, if its actual significance level is between 4.5% and 5.5%. Considering the number of replications used in these experiments, estimated rejection frequencies within the range 3.9% to 6.1% are viewed as providing evidence consistent with the robustness of the test, according to our definition.[17]

Under the null, with homoskedastic standard normal errors (reported in Table 1, $H_0 : \alpha_i = 0$), the rejection frequencies of both the asymptotic $F_\omega^{(0)} \equiv F_N$ and $F_\omega^{(3)}$ tests are close to the nominal significance level of 5%. The asymptotic *F-test* based on $F_\omega^{(2)}$, however, tends to under reject the null when T is relatively large, whilst $F_\omega^{(1)}$ suffers from large size distortion with empirical significance levels being considerably smaller than the nominal 5%. The size properties of the R_ω tests, for different $\hat{\omega}_N$, are qualitatively similar to those of the F_ω tests, but tend to have empirical significance levels that are smaller than those of the corresponding F_ω tests. Turning our attention to the bootstrap tests, all the modified fixed and random effects tests control the empirical significance levels very well. The results are qualitatively similar for t_5 and χ_6^2 errors and, confirming the analysis of Orme and Yamagata (2006), $F_\omega^{(0)} \equiv F_N$ appears quite robust to non-normality, whilst in these cases as well the bootstrap tests provide very close agreement between nominal and empirical significance levels, even for $F_\omega^{*(0)} \equiv F_N^*$ when the errors are asymmetric. Given these results, we now just compare the power of the bootstrap tests. All bootstrap F_ω tests have

[16]Similarly to Goncalves and Kilian (2004), this derives from the asymptotic invalidity of the wild bootstrap scheme when employed to estimate asymptotic standard errors associated with nonpivotal statistics.

[17]Employing a standard asymptotic test these bounds are calculated as $4.5 - 1.96\sqrt{\frac{4.5\times 95.5}{5000}} = 3.9$ and $5.5 + 1.96\sqrt{\frac{5.5\times 94.5}{5000}} = 6.1$.

TABLE 1 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under homoskedastic errors (HET0)

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	5.8	2.8	4.7	5.9	6.2	5.9	5.9	6.1	29.0	18.0	25.4	29.4	30.4	29.9	29.8	30.4
50, 5	5.2	2.8	4.5	5.2	5.7	5.8	5.8	5.7	46.8	38.6	44.4	47.1	47.9	48.8	48.3	47.8
100, 5	4.7	3.1	4.1	4.8	5.3	5.2	5.3	5.3	72.1	66.6	70.8	72.3	73.0	74.3	73.6	73.0
50, 10	4.3	1.9	3.1	4.3	4.5	4.7	4.4	4.5	94.2	87.9	92.6	94.2	94.4	94.0	94.4	94.4
50, 20	4.7	1.6	3.4	4.8	5.1	5.2	5.0	5.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	5.1	1.5	3.7	5.2	6.1	5.9	6.1	6.1	23.9	10.8	18.8	24.4	27.0	25.4	26.1	26.9
50, 5	4.6	2.3	3.9	4.5	5.6	5.6	5.6	5.5	32.5	20.4	29.3	32.7	35.9	35.1	35.3	35.8
100, 5	4.4	2.8	3.8	4.6	5.3	5.2	5.1	5.3	55.8	44.7	52.6	55.9	57.6	57.1	57.5	57.7
50, 10	4.1	1.7	2.9	4.0	4.5	4.8	4.5	4.5	89.7	77.7	87.2	89.8	90.5	89.6	90.3	90.5
50, 20	4.6	1.6	3.4	4.6	5.2	5.3	5.0	5.2	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	4.8	2.4	3.9	5.3	5.5	5.4	5.5	5.4	30.1	20.7	27.6	31.5	31.9	31.7	31.4	31.7
50, 5	4.6	2.7	4.0	4.9	5.3	5.0	5.3	5.2	47.9	40.3	46.4	49.0	49.2	50.5	50.0	49.1
100, 5	5.3	3.5	4.5	5.3	5.9	5.9	5.9	5.8	72.6	68.2	71.4	72.7	73.3	74.7	74.0	73.3
50, 10	5.2	2.1	4.0	5.3	5.7	5.4	5.6	5.7	93.6	87.1	92.0	93.6	94.0	93.3	93.5	93.9
50, 20	4.8	1.5	3.4	4.8	5.1	5.2	5.1	5.1	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	4.4	1.5	3.2	4.6	5.7	5.6	5.4	5.4	24.2	12.7	21.2	25.8	28.6	26.7	27.7	28.6
50, 5	4.1	2.3	3.6	4.5	5.5	5.1	5.3	5.4	32.9	21.8	30.2	33.9	36.4	35.1	36.1	36.5
100, 5	5.0	3.3	4.3	5.0	5.8	6.1	5.8	5.9	56.6	47.1	55.1	57.7	58.8	58.7	59.2	58.8
50, 10	5.0	2.0	3.8	5.0	5.9	5.4	5.6	5.8	89.3	77.6	87.1	89.7	90.1	88.3	89.9	90.1
50, 20	4.6	1.5	3.4	4.5	5.2	5.2	5.1	5.2	100.0	99.8	100.0	100.0	100.0	100.0	100.0	100.0
	χ^2_6								χ^2_6							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	4.5	2.3	3.6	4.4	4.7	5.1	4.7	4.5	30.1	19.7	27.4	31.0	31.5	31.6	32.2	31.4
50, 5	5.1	2.3	3.6	4.8	4.9	5.0	5.0	4.9	46.3	38.4	44.2	46.6	47.2	49.1	47.6	47.3
100, 5	4.9	3.0	4.0	4.8	5.0	5.3	5.2	5.0	72.8	67.8	72.3	73.6	74.4	75.5	74.9	74.3
50, 10	4.5	1.9	3.3	4.4	5.1	5.4	5.0	5.1	93.0	86.9	91.5	93.3	93.3	92.7	93.4	93.4
50, 20	5.2	1.8	3.6	4.9	5.1	5.5	5.4	5.1	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0

Continued

TABLE 1 Continued

	$H_0: \alpha_i = 0$								$H_1: var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	χ_6^2								χ_6^2							
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	4.3	1.4	2.9	3.9	4.7	5.4	4.6	4.4	24.3	11.4	20.3	25.2	27.8	26.8	27.5	27.6
50, 5	4.6	1.8	3.1	4.1	4.9	4.9	4.9	4.8	31.5	20.5	27.8	31.6	34.2	33.3	34.1	34.2
100, 5	4.8	2.8	3.9	4.6	5.1	5.4	5.4	4.9	57.0	45.9	53.9	57.5	59.2	58.2	58.6	59.3
50, 10	4.4	1.7	3.2	4.3	5.1	5.3	5.0	5.1	89.1	76.5	86.7	89.3	89.8	88.1	89.5	89.9
50, 20	5.1	1.7	3.6	4.8	5.2	5.5	5.4	5.2	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0

Notes: The model employed is $y_{it} = \alpha_i + \sum_{j=1}^{3} z_{it,j}\beta_j + u_{it}$, $u_{it} = \sigma_{it}\varepsilon_{it}$, where $z_{it,1} = 1$, $z_{it,2}$ is drawn from a uniform distribution on $(1, 31)$ independently for i and t, and $z_{it,3}$ is generated following Nerlove (1971), such that $z_{it,3} = 0.1t + 0.5z_{it-1,3} + v_{it}$, where the value $z_{i0,3}$ is chosen as $5 + 10v_{i0}$, and v_{it} (and v_{i0}) is drawn from the uniform distribution on $(-0.5, 0.5)$ independently for i and t, in order to avoid any normality in regressors. These regressor values are held fixed over replications. $\beta_j = 1$ for $j = 1, 2, 3$. The i.i.d. standardized errors for ε_{it} are drawn from: the standard normal distribution (SN); the t distribution with five degrees of freedom (t_5); and, the chi-square distribution with six degrees of freedom (χ_6^2). For estimating size of the tests, $\alpha_i = 0$ and power is investigated using $\alpha_i = \sqrt{0.1}g(z_i)$ where $g_i(z_i)$ is the standardised value of $\sum_{j=1}^{3}\sum_{t=1}^{T} z_{it,j}$, so that the regressors and α_i are correlated. F_ω is the modified F-test and R_ω is the modified random effects test, and F_ω^* and R_ω^* are their wild bootstrap tests, with different choice of $\hat{\omega}_N^{(m)}$, $m = 0, 1, 2, 3$ with $\hat{\omega}_N^{(0)} \equiv 1$; see section 4.2 and 4.3 Here $\sigma_{it} = 1$. The sampling behaviour of the tests are investigated using 5000 replications of sample data and 200 bootstrap samples, employing a nominal 5% significance level.

very similar power, as do the bootstrap R_ω tests. However, the power of the bootstrap F_ω tests are uniformly higher than power of the corresponding bootstrap R_ω tests which is as expected given the analysis in Section 3.4 because of the correlation between regressors and individual effects.

The above results indicate that, even when the errors are homoskedastic, a wild bootstrap procedure still offers reliable finite sample inference for all variants of the *FE* and *RE* tests considered. Now let us look at the results under various heteroskedastic schemes. Table 2 reports the results under cross-sectional one-break-in-volatility scheme (HET1). First, and as predicted by the analysis in Section 3.5, both the $F_\omega^{(0)} \equiv F_N$ and $R_\omega^{(0)} \equiv R_N$ tests reject the correct null too often. On the other hand, the empirical significance levels of the other F_ω and R_ω tests are very similar to those presented in homoskedastic case. As before, however, the bootstrap F_ω^* and R_ω^* tests provide close agreement between nominal and empirical significance levels, across all error distributions, so again it is sensible to focus only on the power properties of these tests. In contrast to the power properties under homoskedastic errors, under the HET1 scheme the power of bootstrap F_ω^* tests appear different across different

TABLE 2 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under cross-sectional one-break-in-volatility heteroskedastic scheme (HET1)

	$H_0: \alpha_i = 0$								$H_1: var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	9.4	2.7	4.9	6.6	6.1	5.8	5.9	5.9	26.9	14.2	18.4	21.0	20.5	22.8	20.5	20.0
50, 5	9.2	2.8	4.7	5.6	5.8	5.5	5.8	5.7	37.6	21.2	26.3	28.8	28.0	31.6	29.1	27.6
100, 5	9.1	3.2	4.7	5.4	5.5	5.4	5.5	5.5	55.2	40.4	43.3	44.7	44.2	49.5	46.2	44.1
50, 10	9.1	1.8	3.8	5.2	5.2	4.9	5.1	5.2	82.7	68.1	73.3	74.5	74.1	81.5	78.0	74.1
50, 20	8.7	1.2	2.9	4.5	4.8	4.8	4.9	4.8	99.8	98.5	99.5	99.4	99.5	99.7	99.7	99.5
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	8.6	1.6	3.7	5.6	6.1	5.6	5.8	5.9	22.5	8.3	13.8	16.6	18.0	19.8	18.2	17.7
50, 5	8.8	2.0	4.1	5.2	5.8	5.6	5.7	5.7	27.9	10.3	16.3	19.0	20.9	21.4	21.0	20.7
100, 5	8.7	2.8	4.3	5.1	5.5	5.3	5.4	5.4	42.7	24.9	29.7	31.9	32.9	36.3	34.0	32.6
50, 10	8.7	1.6	3.5	5.0	5.2	4.9	5.2	5.2	75.5	53.9	63.3	65.7	66.1	73.4	69.3	66.0
50, 20	8.6	1.2	3.0	4.6	4.8	4.7	4.9	4.8	99.7	97.6	99.0	99.1	99.0	99.5	99.5	99.0
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	8.5	2.7	4.6	6.0	5.8	5.6	5.4	5.6	26.6	16.0	20.0	21.9	21.7	23.5	22.4	21.1
50, 5	8.6	2.9	4.3	5.5	5.2	5.3	5.4	5.1	39.3	24.4	28.6	30.9	30.7	33.1	31.5	30.3
100, 5	10.4	3.3	4.8	6.2	5.8	6.0	6.1	5.8	56.9	43.2	46.4	47.4	47.3	52.3	49.5	47.1
50, 10	9.2	1.8	3.9	5.2	5.6	5.2	5.5	5.5	82.1	68.0	73.7	74.6	73.6	80.4	77.1	73.5
50, 20	8.8	1.2	3.0	4.7	5.1	5.3	4.7	5.1	99.7	98.2	99.0	99.3	99.2	99.4	99.4	99.2
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	7.6	1.8	3.6	5.1	5.9	5.5	5.6	5.6	22.4	9.3	14.6	17.6	19.2	21.0	19.4	18.4
50, 5	8.1	2.1	3.8	4.9	5.3	5.2	5.2	5.2	28.6	11.7	17.3	20.5	21.9	22.7	22.0	21.6
100, 5	9.9	2.8	4.4	5.7	6.0	6.0	6.0	5.9	44.5	27.2	32.4	35.1	35.6	38.8	37.1	35.5
50, 10	8.8	1.7	3.7	5.1	5.5	5.1	5.6	5.6	75.3	56.3	64.6	66.5	67.1	72.7	70.2	67.0
50, 20	8.7	1.3	3.0	4.7	5.1	5.3	4.8	5.0	99.5	96.9	98.5	99.0	98.9	99.0	99.1	98.9
	χ_6^2								χ_6^2							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	8.4	2.6	4.5	5.5	5.1	5.4	5.0	4.9	26.4	15.1	18.8	20.8	19.9	23.0	21.1	19.3
50, 5	8.4	2.2	3.7	4.5	4.5	4.9	4.7	4.4	36.6	21.2	25.7	27.6	27.5	30.2	28.4	27.1
100, 5	9.5	3.0	4.4	5.2	5.5	5.5	5.3	5.4	57.3	41.0	44.5	45.9	45.7	50.6	47.6	45.6
50, 10	9.1	1.7	3.4	4.7	4.8	5.1	5.0	4.8	81.4	67.6	72.7	74.0	73.9	80.0	76.7	73.9
50, 20	8.5	1.6	3.4	5.1	4.7	5.1	4.7	4.7	99.7	98.4	99.4	99.5	99.4	99.5	99.6	99.4

Continued

TABLE 2 Continued

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	χ_6^2								χ_6^2							
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	7.6	1.4	3.2	4.7	5.3	5.2	5.2	5.0	22.2	8.5	14.1	16.2	17.8	19.7	18.5	17.3
50, 5	7.9	1.7	3.1	4.0	4.5	4.9	4.7	4.4	26.7	10.6	15.5	18.3	19.8	20.2	20.1	19.5
100, 5	9.1	2.5	4.1	4.9	5.4	5.4	5.4	5.4	43.9	25.9	30.2	32.9	33.2	37.1	34.8	33.0
50, 10	8.9	1.6	3.1	4.4	4.9	5.0	5.0	4.8	74.6	53.7	63.2	65.7	66.1	71.5	69.5	66.0
50, 20	8.4	1.6	3.4	5.1	4.8	5.2	4.7	4.8	99.6	97.3	99.0	99.1	99.1	99.3	99.4	99.0

Notes: See notes to Table 1. The DGP is identical to that for Table 1 except $\sigma_{it} = \sigma_1$, $i = 1, \ldots, N_1$, $t = 1, \ldots, T$, and $\sigma_{it} = \sigma_2$, $i = N_1 + 1, \ldots, N$, $t = 1, \ldots, T$ with $N_1 = \lceil N/2 \rceil$, where $\lceil A \rceil$ is the largest integer not less than A, $\sigma_1 = 0.5$ and $\sigma_2 = 1.5$.

variants. For example, $F_\omega^{*(0)} \equiv F_N^*$ and $F_\omega^{*(3)}$ have similar powers but are slightly lower than that of $F_\omega^{*(2)}$, which is again slightly exceeded by that of $F_\omega^{*(1)}$. This feature is qualitatively similar for the R_ω^* tests, but is less striking. Finally, the results confirm again that F_ω^* has higher power than that of R_ω^*.

Table 3 reports the test results under time-series one-break-in-volatility scheme (HET2). In contrast to the results with HET1 scheme, but still consistent with prediction of Section 3.5, both the $F_\omega^{(0)} \equiv F_N$ and $R_\omega^{(0)} \equiv R_N$ tests reject the null too infrequently, especially for $N = 20, 50, 100$ and $T = 5$. As before the bootstrap versions control the size very well, and, interestingly, the power ranking of the bootstrap tests is different than that obtained under HET1. In fact, the $F_\omega^{*(0)} \equiv F_N^*$ and $F_\omega^{*(3)}$ tests (respectively, $R_\omega^{*(0)} = R_N$ and $R_\omega^{*(3)}$ tests) still have similar powers but they are now slightly higher than those of the $F_\omega^{*(2)}$ and $F_\omega^{*(1)}$ tests (respectively, $R_\omega^{*(2)}$ and $R_\omega^{*(1)}$ tests), which are in this case comparable.

Based on the analysis in Section 3.5 it is possible to derive approximate null rejection frequencies of the $F_\omega^{(0)} \equiv F_N$ test analytically, under the simple heteroskedastic schemes of HET1 and HET2. Given the "population" value of ω_N, and a nominal significance level of $\alpha \times 100\%$, the rejection frequency of the F_N test is, approximately, $\Pr[F_N > c_{\alpha,n1,n2}]$, where $\Pr[F_{n1,n2} > c_{\alpha,n1,n2}] = \alpha$ and $F_{n1,n2} \sim F(n_1, n_2)$. But this is identical to $\Pr[F_{n1,n2} > q]$, where $q = \omega_N(c_{\alpha,n1,n2} - 1) + 1$. More precisely, consider first the case of HET1 where a little calculation shows that, since N is always even in our experiments, $\omega_N = 0.781$. Using $\alpha = 0.05$, it is then straightforward to obtain q and $\Pr[F_{n1,n2} > q]$. Similar calculations can be undertaken for the case HET2 but, here, ω_N varies according to whether T

TABLE 3 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under time-series one-break-in-volatility heteroskedastic scheme (HET2)

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	3.1	3.1	4.3	5.2	5.5	5.6	5.7	5.6	27.2	20.4	25.8	32.7	33.3	29.5	29.7	33.5
50, 5	3.2	3.6	4.5	5.1	5.5	5.9	5.6	5.5	44.9	41.5	46.0	51.5	52.2	49.8	49.2	52.2
100, 5	2.9	3.7	4.5	4.8	5.5	5.5	5.5	5.5	70.7	68.6	71.0	75.7	76.2	74.3	73.2	76.3
50, 10	3.8	1.8	3.1	4.5	4.7	4.7	4.8	4.7	85.8	75.2	80.5	87.0	86.8	85.7	83.9	86.9
50, 20	4.2	1.6	3.3	4.5	5.1	5.2	5.4	5.1	99.9	99.5	99.7	99.9	99.9	99.8	99.9	99.9
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	2.6	2.0	3.6	4.5	5.7	5.4	5.7	5.7	21.3	12.1	19.1	26.5	30.5	25.2	26.1	30.6
50, 5	2.7	2.7	3.8	4.5	5.6	5.7	5.7	5.7	29.8	23.3	29.7	36.0	39.6	35.2	35.6	39.5
100, 5	2.7	3.3	4.1	4.4	5.6	5.5	5.4	5.6	53.3	48.6	53.4	59.8	62.3	57.5	57.7	62.3
50, 10	3.5	1.6	3.0	4.1	4.7	4.7	4.8	4.7	78.9	62.1	71.7	80.2	81.5	78.0	76.9	81.5
50, 20	4.1	1.6	3.2	4.4	5.1	5.2	5.3	5.1	99.8	99.0	99.5	99.8	99.9	99.7	99.7	99.9
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	3.0	3.4	4.0	4.9	5.4	5.4	5.4	5.5	29.1	23.3	29.6	35.8	36.2	32.2	33.2	36.3
50, 5	2.9	3.4	4.0	4.6	5.2	5.2	5.3	5.1	46.5	44.4	48.4	53.8	54.6	51.5	51.1	54.5
100, 5	3.3	4.4	5.0	5.5	6.0	6.0	6.1	6.0	71.9	70.7	72.9	76.9	77.7	75.1	75.0	77.6
50, 10	4.4	2.2	4.2	5.0	5.7	5.3	5.7	5.7	85.3	75.9	80.7	86.8	86.9	84.8	83.9	86.9
50, 20	4.0	1.7	3.2	4.5	5.1	5.2	4.9	5.1	99.9	99.1	99.5	99.9	99.9	99.7	99.6	99.9
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	2.3	1.9	2.8	3.9	5.4	5.4	5.3	5.3	22.9	13.9	21.8	29.3	32.9	27.8	29.4	33.1
50, 5	2.3	2.7	3.4	4.0	5.3	5.3	5.4	5.3	31.0	25.7	32.4	38.4	41.8	37.1	38.2	41.7
100, 5	3.0	3.9	4.5	5.0	5.8	6.1	6.1	5.8	55.5	52.6	56.7	63.2	64.8	60.5	61.0	64.9
50, 10	4.1	2.0	3.9	4.7	5.7	5.4	5.8	5.8	79.2	63.6	72.9	80.7	81.9	78.0	77.5	81.8
50, 20	3.9	1.6	3.2	4.3	5.0	5.2	5.0	5.0	99.9	98.7	99.3	99.9	99.9	99.5	99.5	99.8
	χ_6^2								χ_6^2							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	3.2	2.8	3.8	4.3	4.6	5.2	4.7	4.6	28.8	22.7	27.9	35.1	35.9	31.1	32.0	35.9
50, 5	3.4	3.0	3.8	4.2	4.8	4.6	4.7	4.7	44.2	41.4	45.2	51.0	52.2	49.4	49.0	52.1
100, 5	3.3	3.7	4.5	4.6	5.1	5.0	5.2	5.0	71.8	70.7	72.9	77.4	77.6	75.6	74.6	77.6
50, 10	4.4	2.3	3.7	4.9	5.3	5.5	5.4	5.3	84.1	74.7	79.8	85.5	85.8	83.8	82.8	85.8
50, 20	4.8	2.0	3.5	4.8	4.8	5.2	4.8	4.8	99.9	99.2	99.7	99.9	99.9	99.9	99.9	99.9

Continued

TABLE 3 Continued

	$H_0: \alpha_i = 0$								$H_1: var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	χ_6^2								χ_6^2							
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	2.5	1.9	2.7	3.4	4.5	5.1	4.6	4.4	22.1	14.3	21.1	28.1	32.1	26.8	27.8	32.2
50, 5	2.9	2.5	3.2	3.7	4.7	4.9	4.6	4.6	27.9	22.2	28.1	35.0	38.3	33.9	34.4	38.5
100, 5	3.1	3.4	4.1	4.4	5.2	5.2	5.2	5.1	54.7	50.2	54.8	62.0	64.1	59.0	59.1	64.2
50, 10	4.2	1.9	3.4	4.6	5.1	5.5	5.4	5.1	77.8	61.4	71.5	79.7	81.0	76.9	77.0	81.0
50, 20	4.7	2.0	3.5	4.7	4.7	5.0	4.8	4.7	99.9	98.7	99.4	99.9	99.9	99.7	99.8	99.9

Notes: See notes to Table 1. The DGP is identical to that for Table 1 except $\sigma_{it} = \sigma_1$, $i = 1, \ldots, N$, $t = 1, \ldots, T_1$, $\sigma_{it} = \sigma_2$, $i = 1, \ldots, N$, $t = T_1 + 1, \ldots, T$ with $T_1 = \lceil T/2 \rceil$, $\sigma_1 = 0.5$, and $\sigma_2 = 1.5$.

is even ($\omega_N = 1.02$) or odd ($\omega_N = 1.13$). From these calculations we obtain the following (approximate) significance levels for our choices of (N, T):

	Approximate significance levels of F_N				
	$T = 5$			$N = 50$	
	$N = 20$	$N = 50$	$N = 100$	$T = 10$	$T = 20$
HET1:	8.8%	9.2%	9.4%	9.2%	9.2%
HET2:	3.5%	3.4%	3.3%	4.8%	4.8%

As can be seen, the obtained empirical significance levels, for F_N, are qualitatively very similar to these predicted values.

Table 4 summarises the results under conditional heteroskedasticity depending on a regressor $z_{it,2}$ (HET3), where $\sigma_{it} = \eta_1[(z_{it,2} - 1)/30]$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, and $\eta_1[\cdot]$ is the inverse of the cumulative distribution function of the χ_1^2 distribution. Since the $z_{it,2}$ are initially i.i.d. draws from a uniform distribution on $(1, 31)$, the values of $\sigma_{it}(z_{it,2})$ are realisations from a χ_1^2 distribution. This means that even though for a given N (and T) σ_{it} will be held fixed for each replication of data, possibly yielding a realisation of $\omega_N \neq 1$, as N increases a Law of Large Numbers implies that the given realisation of ω_N will converge to unity. For example, when $N = 20$ and $T = 5$, $\omega_N = 1.36$, yielding a predicted (approximate) significance level for F_N of 1.9%, which explains the under-rejection of this test in our experiments. For larger sample sizes, the value of ω_N does, indeed, tend to unity, and the empirical significance level of F_N converges to the nominal level, as expected. Due to the larger average error variance encountered here, than that under other heteroskedastic schemes, the power of the tests

TABLE 4 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under conditional heteroskedasticity depending on a regressor (HET3)

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	2.1	3.9	4.6	5.3	5.7	5.5	5.8	5.4	9.8	13.1	18.5	20.1	20.8	17.5	20.6	20.5
50, 5	5.7	3.1	4.1	4.8	5.4	5.1	5.1	5.1	18.5	13.9	16.9	18.9	19.8	19.0	19.6	19.4
100, 5	5.6	3.9	4.9	5.3	6.0	6.3	5.9	5.8	30.0	25.9	29.7	31.1	32.7	32.7	32.5	32.4
50, 10	5.5	2.6	4.0	5.1	5.5	5.5	5.6	5.5	47.5	31.4	40.5	45.2	46.3	43.5	45.3	46.1
50, 20	5.4	2.2	3.9	5.1	5.5	5.4	5.5	5.5	80.6	64.2	75.6	79.6	80.0	77.7	79.7	80.0
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	1.7	2.5	3.4	4.4	5.9	6.0	5.9	6.0	7.2	7.8	13.4	16.0	18.9	15.8	18.6	18.6
50, 5	5.1	2.5	3.3	4.1	5.6	5.3	5.2	5.3	13.1	7.6	10.8	12.6	15.4	13.8	15.1	14.8
100, 5	5.3	3.5	4.4	4.9	6.0	6.3	6.0	5.8	21.7	16.1	20.7	22.7	25.2	23.9	24.9	24.9
50, 10	5.3	2.5	3.7	4.8	5.5	5.4	5.5	5.4	41.0	22.6	33.0	38.5	40.5	36.5	38.8	40.2
50, 20	5.3	2.2	3.7	5.0	5.4	5.4	5.4	5.4	76.9	57.5	71.6	75.9	77.0	73.8	76.8	76.9
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	1.6	4.1	4.7	5.2	5.4	5.4	5.3	5.4	10.4	15.5	20.8	22.5	23.5	19.8	22.8	23.1
50, 5	5.6	3.7	4.6	5.3	5.7	5.7	5.4	5.6	18.7	16.6	20.0	21.5	23.1	22.2	23.3	22.9
100, 5	4.4	3.4	4.2	4.5	5.4	5.4	5.4	5.3	30.9	28.8	33.0	35.1	36.7	35.5	36.4	36.3
50, 10	6.1	2.9	4.2	5.5	5.9	5.8	5.9	5.8	49.7	35.6	44.8	48.5	49.4	46.8	48.7	49.4
50, 20	4.6	1.8	3.2	4.6	5.1	5.5	5.0	5.1	79.4	65.1	76.2	79.6	79.8	77.5	79.5	79.7
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	1.3	2.4	3.1	3.8	5.6	5.7	5.3	5.3	7.7	9.2	15.2	17.9	21.2	17.4	20.4	20.8
50, 5	5.1	2.7	3.7	4.3	5.7	5.6	5.4	5.5	13.3	9.1	12.4	14.9	17.6	15.4	17.0	17.3
100, 5	4.2	3.1	3.8	4.3	5.3	5.2	5.3	5.2	21.9	18.0	23.3	25.6	28.0	26.4	27.8	27.6
50, 10	5.7	2.4	3.8	5.2	5.9	5.8	5.9	5.8	43.3	26.7	38.0	42.8	44.4	40.0	43.0	44.2
50, 20	4.6	1.7	3.2	4.4	5.1	5.5	5.0	5.1	76.3	59.8	72.8	76.5	77.4	73.7	76.8	77.3
	χ_6^2								χ_6^2							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	1.8	4.4	4.9	5.3	5.7	6.0	5.5	5.5	10.6	13.2	17.9	19.3	20.7	17.6	20.3	20.2
50, 5	5.4	3.4	4.3	4.8	5.4	5.0	5.0	5.0	16.9	13.3	15.8	17.2	18.5	18.1	18.6	18.0
100, 5	5.4	3.7	4.2	4.7	5.6	5.5	5.5	5.4	27.8	25.1	28.1	29.9	31.4	31.1	31.0	31.0
50, 10	6.0	2.5	3.7	4.6	5.0	5.5	5.5	4.9	47.9	33.7	41.5	46.1	47.3	46.6	46.7	47.2
50, 20	4.5	1.5	3.1	3.9	4.2	4.5	4.7	4.2	80.0	66.7	77.0	80.5	80.9	79.4	80.9	80.8

Continued

TABLE 4 Continued

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	χ_6^2								χ_6^2							
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	1.4	2.7	3.2	4.0	5.7	5.9	5.6	5.6	8.0	8.2	13.1	14.8	18.4	15.3	17.9	18.0
50, 5	4.8	2.8	3.6	4.2	5.3	5.1	5.1	4.9	12.3	6.9	9.7	11.4	14.4	12.6	13.8	14.1
100, 5	5.2	3.4	3.9	4.4	5.6	5.4	5.4	5.3	20.5	14.7	19.0	21.2	23.5	21.8	22.8	23.1
50, 10	5.7	2.3	3.5	4.4	5.1	5.5	5.4	5.0	40.9	23.7	33.2	38.6	40.6	38.6	39.7	40.3
50, 20	4.4	1.4	3.0	3.8	4.2	4.4	4.7	4.2	76.5	60.5	72.9	77.0	78.0	75.8	77.9	77.9

Notes: See notes to Table 1. The DGP is identical to that for Table 1 except $\sigma_{it} = \eta_c[(z_{it,2} - 1)/30]/c$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, where $\eta_c[\cdot]$ is the inverse of the cumulative distribution function of chi-squared distribution with degrees of freedom c. Since $z_{it,2}$ is drawn from a uniform distribution on $(1, 31)$, σ_{it} has mean 1 and variance $2/c$, so it is easy to control the degree of heteroskedasticity through the choice of c. We employ $c = 1$.

are lower although, qualitatively, the results are very similar to those under HET0 but with $F_\omega^{*(0)} = F_N^*$ and $F_\omega^{*(3)}$ (respectively, $R_\omega^{*(0)} = R_N^*$ and $R_\omega^{*(3)}$) enjoying a slight power advantage and the F_ω^* tests being more powerful than their R_ω^* counterparts.

The results under symmetric conditional heteroskedasticity, GARCH(1,1) (HET4), are reported in Table 5. Similar to the results obtained under HET1, and as predicted by the analysis of Section 3.5, the $F_\omega^{(0)} = F_N$ test rejects a correct null too frequently but the empirical significance levels of other variants of the F_ω tests are very similar to those presented in homoskedastic case. Again, all the bootstrap F_ω^* tests control the empirical significance levels very well, and the power rankings are, from the lowest, $F_\omega^{*(0)} = F_N^*$ and $F_\omega^{*(3)}$, followed by $F_\omega^{*(2)}$, then $F_\omega^{*(1)}$. The same comments apply to the bootstrap R_ω^* tests, which again exhibit lower power than their F_ω^* counterparts. The results under asymmetric conditional heteroskedasticity, GJR-GARCH(1,1) (HET5), are summarised in Table 6. In contrast to GARCH model, GJR-GARCH is an asymmetric model of heteroskedasticity with leverage, and $E\left(u_{it}^2 u_{is} u_{ir}\right) \neq 0$ in general, rendering $\hat{\omega}_N^{(3)}$ inconsistent, meanwhile $\hat{\omega}_N^{(1)}$ and $\hat{\omega}_N^{(2)}$ remain consistent. Despite this, the experimental results are qualitatively very similar to those under GARCH model. All the bootstrap F_ω^* tests, including $F_\omega^{*(3)}$, control the empirical significance levels very well, and the power rankings of the F_ω^* and R_ω^* tests are very similar to those obtained under the symmetric GARCH models.

TABLE 5 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under conditional heteroskedasticity, GARCH(1,1) (HET4)

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	7.2	2.3	4.5	6.0	6.0	5.8	5.8	5.8	30.5	17.6	24.3	27.5	27.5	28.8	28.1	27.1
50, 5	7.9	2.3	4.4	5.5	5.8	5.6	5.4	5.7	47.2	34.9	39.1	41.2	40.9	46.2	42.8	40.7
100, 5	8.8	2.9	5.1	6.3	6.0	5.8	6.0	6.0	71.1	60.8	62.9	64.4	63.4	70.4	66.2	63.3
50, 10	6.8	1.8	3.9	5.2	5.7	5.9	5.5	5.7	92.7	85.3	89.9	90.9	90.8	92.6	91.9	90.8
50, 20	5.6	1.6	3.6	4.9	5.3	5.6	5.3	5.3	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	6.5	1.3	3.5	5.3	6.0	5.7	5.8	5.8	25.7	10.1	17.9	22.5	24.2	24.6	24.5	23.8
50, 5	7.4	1.9	3.7	5.1	5.8	5.4	5.5	5.7	33.7	18.4	24.9	28.6	29.2	32.3	30.4	29.0
100, 5	8.6	2.6	4.5	5.9	6.0	5.8	5.9	6.0	55.6	40.3	45.9	48.1	49.0	53.2	50.5	49.0
50, 10	6.5	1.6	3.7	5.1	5.6	5.8	5.5	5.4	88.6	74.0	83.2	86.0	85.7	87.4	87.0	85.7
50, 20	5.5	1.6	3.6	4.8	5.3	5.6	5.3	5.3	100.0	99.8	100.0	99.9	100.0	100.0	100.0	100.0
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	7.9	1.9	4.2	5.7	5.3	5.1	5.1	5.1	32.7	20.8	27.0	29.7	29.0	31.4	29.8	28.5
50, 5	9.2	2.6	4.4	5.8	5.4	5.3	5.1	5.3	49.7	36.5	41.1	42.7	41.4	47.1	43.6	41.1
100, 5	11.5	3.5	5.6	6.5	6.3	6.2	6.4	6.3	70.8	59.0	59.9	60.3	59.3	67.3	62.2	59.2
50, 10	8.2	1.9	4.0	5.5	5.6	5.5	5.3	5.5	91.9	82.8	86.9	87.8	86.8	90.6	88.8	86.8
50, 20	6.9	1.5	3.8	5.3	5.4	5.5	5.5	5.3	99.9	99.3	99.6	99.4	99.3	99.7	99.7	99.3
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	7.4	1.3	3.2	5.1	5.4	5.1	5.2	5.2	27.7	12.0	19.8	24.0	25.7	26.3	26.2	25.2
50, 5	8.7	2.0	3.7	5.3	5.4	5.1	5.2	5.3	36.4	20.0	26.3	29.2	30.5	33.3	31.4	30.2
100, 5	11.1	3.0	5.0	6.2	6.3	6.3	6.4	6.3	56.9	39.5	44.6	46.3	46.6	52.4	48.8	46.4
50, 10	8.0	1.8	3.6	5.3	5.6	5.4	5.3	5.6	87.7	72.1	80.2	82.3	81.9	85.4	84.4	81.9
50, 20	6.8	1.5	3.7	5.2	5.4	5.5	5.5	5.3	99.9	99.0	99.4	99.4	99.2	99.7	99.6	99.1
	χ_6^2								χ_6^2							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	6.9	1.9	3.3	4.4	3.7	4.2	3.7	3.4	29.8	17.2	23.2	26.2	25.1	28.0	25.9	24.6
50, 5	8.1	2.0	3.3	4.7	4.3	4.2	4.1	4.2	46.2	33.1	36.9	38.9	37.6	44.6	40.3	37.3
100, 5	9.3	1.8	3.0	4.6	3.8	4.4	3.5	3.7	68.3	56.2	57.3	58.7	57.3	66.5	60.3	57.0
50, 10	7.4	1.2	2.9	4.5	4.3	4.7	4.1	4.3	92.4	84.4	87.6	89.2	88.4	92.3	89.9	88.3
50, 20	6.4	1.1	2.7	4.5	4.8	4.8	4.4	4.8	100.0	99.8	99.9	99.9	99.8	100.0	99.9	99.8

Continued

TABLE 5 Continued

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	χ^2_6								χ^2_6							
	R_ω				R^*_ω				R_ω				R^*_ω			
20, 5	6.1	1.2	2.2	3.9	3.7	4.1	3.9	3.5	24.5	9.7	16.5	20.5	22.3	23.6	22.7	21.7
50, 5	7.6	1.5	2.5	4.0	4.2	4.4	4.0	4.1	33.6	17.1	22.8	26.1	27.0	30.2	27.7	26.5
100, 5	9.0	1.5	2.7	4.2	3.8	4.4	3.6	3.8	53.1	35.4	40.1	42.6	43.0	48.8	44.6	42.9
50, 10	7.2	1.0	2.6	4.4	4.3	4.8	4.2	4.3	87.8	72.4	81.1	84.0	83.3	86.3	85.1	83.2
50, 20	6.3	1.1	2.7	4.4	4.8	4.8	4.4	4.8	100.0	99.6	99.8	99.9	99.8	100.0	99.9	99.8

Notes: See notes to Table 1. The DGP is identical to that for Table 1 except $u_{it} = \sigma_{it}\varepsilon_{it}$, $t = -49, \ldots, T, i = 1, \ldots, N$, where $\sigma^2_{it} = \phi_0 + \phi_1 u^2_{i,t-1} + \phi_2\sigma^2_{i,t-1}$. The value of parameters are chosen to be $\phi_0 = 0.5$, $\phi_1 = 0.25$ and $\phi_2 = 0.25$.

5. CONCLUSIONS

This article has provided an asymptotic analysis of the sampling behaviour of the standard F-test statistic for fixed effects, in a static linear

TABLE 6 Rejection frequencies of the asymptotic and wild-bootstrap modified F-tests and modified random effects tests under conditional heteroskedasticity, GJR-GARCH(1,1) (HET5)

	$H_0 : \alpha_i = 0$								$H_1 : var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	SN								*SN*							
N, T	F_ω				F^*_ω				F_ω				F^*_ω			
20, 5	7.7	2.4	4.8	6.4	6.2	5.6	5.9	6.1	29.3	16.7	23.0	26.2	26.1	27.9	26.6	25.7
50, 5	8.4	2.5	4.3	5.6	5.6	5.5	5.4	5.5	45.1	32.1	36.8	38.5	37.6	43.7	39.6	37.2
100, 5	9.9	3.0	4.9	6.4	6.2	6.1	5.8	6.0	68.3	56.8	59.2	60.4	59.1	66.4	61.4	59.1
50, 10	7.6	1.4	3.8	5.8	5.7	5.9	5.8	5.7	90.9	80.9	86.0	87.7	86.8	90.3	88.6	86.8
50, 20	6.5	1.3	3.5	5.2	5.4	5.2	5.2	5.4	100.0	99.8	99.9	100.0	99.9	100.0	99.9	99.9
	R_ω				R^*_ω				R_ω				R^*_ω			
20, 5	7.0	1.4	3.7	5.5	6.3	5.6	6.1	6.1	24.7	9.5	17.0	21.2	23.0	23.8	23.1	22.5
50, 5	7.8	1.9	3.7	5.2	5.6	5.3	5.3	5.4	32.5	16.3	22.3	26.2	26.8	30.0	27.6	26.6
100, 5	9.6	2.4	4.5	6.1	6.1	6.0	5.7	6.1	53.3	35.9	41.7	44.8	45.5	49.8	46.7	45.3
50, 10	7.3	1.3	3.6	5.6	5.7	5.8	5.9	5.7	85.5	69.6	78.5	81.6	81.3	84.5	82.8	81.1
50, 20	6.4	1.3	3.5	5.2	5.5	5.2	5.2	5.5	100.0	99.6	99.9	99.9	99.9	99.9	99.9	99.9

Continued

TABLE 6 Continued

	$H_0: \alpha_i = 0$								$H_1: var(\alpha_i) = 0.1$, α_i correlated with regressors							
	Asymptotic tests				Bootstrap tests				Asymptotic tests				Bootstrap tests			
ω	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$	1	$\hat{\omega}_N^{(1)}$	$\hat{\omega}_N^{(2)}$	$\hat{\omega}_N^{(3)}$	1	$\hat{\omega}_N^{*(1)}$	$\hat{\omega}_N^{*(2)}$	$\hat{\omega}_N^{*(3)}$
	t_5								t_5							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	8.2	1.9	4.3	6.2	5.6	5.0	5.4	5.4	31.5	20.1	26.1	28.0	27.4	30.2	28.7	26.9
50, 5	10.0	2.6	4.6	6.3	5.7	5.5	5.3	5.5	48.0	35.1	38.2	40.0	38.9	44.9	40.8	38.5
100, 5	12.6	3.6	5.4	6.8	6.5	6.1	6.3	6.4	67.8	54.9	56.2	56.8	55.3	63.5	58.3	55.0
50, 10	9.3	1.6	3.6	5.6	5.5	5.5	5.1	5.5	90.2	79.2	82.4	83.8	82.2	88.4	84.9	82.1
50, 20	8.0	1.5	3.3	5.5	5.4	5.1	5.3	5.4	99.8	98.9	99.0	99.1	98.7	99.5	99.2	98.7
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	7.6	1.2	3.3	5.4	5.7	5.1	5.3	5.4	26.5	11.6	19.3	23.1	24.5	25.8	25.2	24.0
50, 5	9.6	1.9	3.7	5.8	5.7	5.2	5.2	5.5	35.8	18.0	25.1	27.7	28.9	32.3	29.9	28.6
100, 5	12.0	2.9	4.8	6.4	6.5	6.1	6.2	6.4	54.9	36.0	41.1	43.8	42.8	49.1	45.3	42.6
50, 10	9.0	1.5	3.3	5.4	5.5	5.5	5.0	5.5	85.0	67.9	75.4	77.3	76.5	82.3	79.4	76.4
50, 20	7.9	1.4	3.3	5.5	5.4	5.1	5.2	5.4	99.7	98.3	98.9	98.9	98.5	99.4	99.0	98.4
	χ_6^2								χ_6^2							
N, T	F_ω				F_ω^*				F_ω				F_ω^*			
20, 5	6.5	2.2	3.4	4.7	4.2	4.8	4.4	4.1	32.1	19.6	26.3	29.0	28.5	31.5	29.7	28.2
50, 5	7.2	2.6	3.8	4.8	4.7	5.0	4.7	4.6	49.5	38.0	42.9	44.3	43.9	49.6	46.2	43.5
100, 5	7.6	2.3	3.4	4.4	4.1	5.1	4.2	4.1	74.0	64.3	67.0	68.0	67.4	73.1	69.9	67.3
50, 10	6.6	1.6	3.3	4.5	4.7	5.4	4.7	4.7	94.6	88.0	91.7	92.8	92.4	94.1	93.4	92.3
50, 20	5.7	1.3	3.1	4.5	4.7	5.0	4.6	4.7	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0
	R_ω				R_ω^*				R_ω				R_ω^*			
20, 5	5.9	1.3	2.5	4.1	4.3	5.0	4.6	4.1	26.3	10.9	19.6	23.4	25.2	26.1	25.7	24.8
50, 5	6.7	2.0	3.2	4.3	4.7	4.9	4.5	4.6	35.1	20.1	26.6	30.2	31.1	33.9	31.9	30.8
100, 5	7.5	2.0	3.1	4.1	4.1	5.1	4.2	4.0	57.6	42.8	48.6	50.3	51.5	56.0	53.0	51.3
50, 10	6.3	1.4	3.1	4.3	4.8	5.5	4.7	4.7	90.7	77.6	86.1	88.4	88.0	89.6	89.4	87.9
50, 20	5.6	1.2	3.1	4.4	4.7	5.0	4.7	4.7	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0

Notes: See notes to Table 1. The DGP is identical to that for Table 1 except $u_{it} = \sigma_{it}\varepsilon_{it}$, $t = -49, \ldots, T, i = 1, \ldots, N$, where $\sigma_{it}^2 = \phi_0 + \phi_1\sigma_{i,t-1}^2 + \phi_2(|u_{i,t-1}| - \phi_3 u_{it-1})^2$. The value of parameters are chosen to be $\phi_0 = 0.3$, $\phi_1 = 0.5$, $\phi_2 = 0.2$ and $\phi_3 = 0.23$.

panel data model, under both non-normality and heteroskedasticity of the error terms, when the number of cross-sections, N, is large and T, the number of time periods, is fixed. First, it has been shown that a linear transformation of the commonly cited F and *RE* tests (using a simple function of restricted residuals) provides asymptotically valid test procedures, when employed in conjunction with the usual F and *standard normal* critical values (respectively). Second, it has been shown that the asymptotic relationship between the heteroskedastic robust *F-test* and the *RE-test* statistics, carries over from the homoskedastic case. That is, under (pure) local random effects, they share the same asymptotic power, whilst under local fixed (or correlated) individual effects the heteroskedastic robust *F-test* enjoys higher asymptotic power. Third, we have provided qualitative predictions about the approximate true significance levels of the standard *F* and *RE Tests* in the presence of certain forms of heteroskedasticity. These theoretical findings are supported by Monte Carlo evidence. Finally, although asymptotic theory does not always provide a good approximation to finite sample behaviour, our experiments show that all the wild bootstrap versions of these tests, employing the resampling scheme advocated by Davidson and Flachaire (2008), yield reliable inferences in the sense of close agreement between nominal and actual significance levels. There are slight differences in the power properties of these tests, although none dominates across the different models of heteroskedasticity considered. Thus, for example, the wild boostrap version of the unadjusted *F-test* appears to behave quite favourably under homoskedasticity and general heteroskedasticity both in terms of finite sample significance levels and power, and even under asymmetric errors for which it is not asymptotically justified.

APPENDIX

In what follows $\|\mathbf{C}\| = \sqrt{tr\left(\mathbf{C}'\mathbf{C}\right)} = \sqrt{\sum_i \sum_j c_{ij}^2}$ denotes the Euclidean norm of a matrix $\mathbf{C} = \left\{c_{ij}\right\}$.

Proof of Lemma 1. Write $W_i = \frac{\mathbf{u}_i'\mathbf{A}\mathbf{u}_i}{\sqrt{T(T-1)}}$, which are independent, so that $H_N = \frac{1}{\sqrt{N}}\sum_{i=1}^N W_i$ and $E\left[W_i\right] = 0$, by Assumption A1(ii). Since $\|\mathbf{A}\| = \sqrt{T(T-1)}$, $\left|W_i\right| = \frac{\left|\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right|}{\sqrt{T(T-1)}} \leq \frac{\|\mathbf{u}_i\|^2\|\mathbf{A}\|}{\sqrt{T(T-1)}} = \|\mathbf{u}_i\|^2$. Thus, by Minkowski's inequality and Assumption A3(i), for some $\eta > 0$,

$$E\left|W_i\right|^{2+\eta} \leq E\left|\sum_{t=1}^T u_{it}^2\right|^{2+\eta} \leq \left[\sum_{t=1}^T \left\{E\left|u_{it}^2\right|^{2+\eta}\right\}^{\frac{1}{2+\eta}}\right]^{2+\eta} = O(1),$$

so that $\kappa_N = \frac{1}{2N}\sum_{i=1}^N E\left(W_i^2\right) = O(1)$. With Assumption A3(ii), a standard (Liapounov) Central Limit Theorem yields $\kappa_N^{-1/2}H_N \xrightarrow{d} N(0,1)$. □

Proof of Proposition 1. The method of proof is nearly identical to that of (Orme and Yamagata, 2006, Proposition 1) but where, now, our assumptions allow for heteroskedasticity.

(i) Let $S_N = (RSS_R - RSS_U)/(N-1)$ and $\tilde{\sigma}^2 = RSS_U/(N(T-1) - K)$, so that

$$\bar{\sigma}_N^2 \sqrt{N}(F_N - 1) = \frac{\bar{\sigma}_N^2}{\tilde{\sigma}^2} \sqrt{N} \left(S_N - \tilde{\sigma}^2 \right). \tag{22}$$

We first show that $\tilde{\sigma}^2 - \bar{\sigma}_N^2 = o_p(1)$, so that (since $\bar{\sigma}_N^2$ is uniformly positive by Assumption A2(v)) $\tilde{\sigma}^2/\bar{\sigma}_N^2 \xrightarrow{p} 1$. Following (Orme and Yamagata, 2006, Proof of Proposition 1), we can write

$$\begin{aligned}
\tilde{\sigma}^2 &= \frac{N}{N(T-1)-K} \frac{\mathbf{u}'(\mathbf{M}_{\tilde{\mathbf{X}}} - \mathbf{P}_\mathbf{D})\mathbf{u}}{N} \\
&= \frac{N}{N(T-1)-K} \left\{ \frac{\mathbf{u}'\mathbf{u}}{N} - \frac{\mathbf{u}'\mathbf{P}_{\tilde{\mathbf{X}}}\mathbf{u}}{N} - \frac{\mathbf{u}'\mathbf{P}_\mathbf{D}\mathbf{u}}{N} \right\} \\
&= \frac{\mathbf{u}'\mathbf{M}_\mathbf{D}\mathbf{u}}{N(T-1)} + O_p(N^{-1})
\end{aligned}$$

because $\frac{\mathbf{u}'\mathbf{u}}{N}$, $\frac{\mathbf{u}'\mathbf{P}_\mathbf{D}\mathbf{u}}{N}$, and $\mathbf{u}'\mathbf{P}_{\tilde{\mathbf{X}}}\mathbf{u}$ are all $O_p(1)$ and $\frac{N}{N(T-1)-K} = \frac{1}{T-1} + O(N^{-1})$. Therefore,

$$\begin{aligned}
\tilde{\sigma}^2 - \bar{\sigma}_N^2 &= \frac{\mathbf{u}'\mathbf{M}_\mathbf{D}\mathbf{u}}{N(T-1)} - \frac{T\bar{\sigma}_N^2}{T-1} + \frac{\bar{\sigma}_N^2}{T-1} + O_p(N^{-1}) \\
&= \frac{1}{T-1} \left\{ T \left(\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} u_{it}^2 - \bar{\sigma}_N^2 \right) - \left(\frac{1}{NT} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} u_{it} \right)^2 - \bar{\sigma}_N^2 \right) \right\} \\
&\quad + O_p(N^{-1}) \\
&= o_p(1),
\end{aligned}$$

because, by Assumption A2(i) and A1(ii), both terms inside the $\{\cdot\}$ above are $o_p(1)$. Thus, provided $\sqrt{N}\left(S_N - \tilde{\sigma}^2\right) = O_p(1)$, (22) yields

$$\bar{\sigma}_N^2 \sqrt{N}(F_N - 1) = \sqrt{N}\left(S_N - \tilde{\sigma}^2\right) + o_p(1),$$

but from exactly the same argument employed by Orme and Yamagata (2006, pp. 418–419) $\sqrt{N}\left(S_N - \tilde{\sigma}^2\right) = O_p(1)$ with

$$\sqrt{N}\left(S_N - \tilde{\sigma}^2\right) = \frac{1}{(T-1)} \frac{1}{\sqrt{N}} \left[\mathbf{u}'(\mathbf{I}_N \otimes \mathbf{A})\mathbf{u} \right] + \lambda_N + o_p(1).$$

Thus, (22) can be expressed as

$$\bar{\sigma}_N^2\sqrt{N}(F_N-1)=\frac{1}{\sqrt{N}}\frac{\mathbf{u}'(\mathbf{I}_N\otimes\mathbf{A})\mathbf{u}}{T-1}+\lambda_N+o_p(1),$$
$$=\sqrt{\frac{T}{T-1}}H_N+\lambda_N+o_p(1).$$

(ii) By Lemma 1,

$$\omega_N\sqrt{N}(F_N-1)-\frac{\lambda_N}{\sqrt{\kappa_N/2}}\xrightarrow{d}N\left(0,\frac{2T}{T-1}\right),$$

and the result follows. This completes the proof. □

Proof of Proposition 2. 1. First, for $\hat{\sigma}_N^2$, by the Triangle Inequality, $\left|\hat{\sigma}_N^2-\bar{\sigma}_N^2\right|\le\left|\hat{\sigma}^2-\frac{\mathbf{u}'\mathbf{u}}{NT}\right|+\left|\frac{\mathbf{u}'\mathbf{u}}{NT}-\bar{\sigma}_N^2\right|=o_p(1)$, since, as previously noted, $\frac{\mathbf{u}'\mathbf{u}}{NT}=\bar{\sigma}_N^2+o_p(1)$ and $\hat{\sigma}_N^2-\frac{\mathbf{u}'\mathbf{u}}{NT}=o_p(1)$ by the arguments of Orme and Yamagata (2006, p. 422).

Second, for $\hat{\kappa}_N^{(1)}$, from the proof of Lemma 1, we have that

$$\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2-\frac{1}{N}\sum_{i=1}^{N}E\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2\xrightarrow{p}0.$$

Therefore, by the Triangle Inequality, it remains to show that $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\mathbf{u}}_i'\mathbf{A}\hat{\mathbf{u}}_i\right)^2-\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2\xrightarrow{p}0$. Since, $\hat{\mathbf{u}}_i=\mathbf{u}_i+\hat{\mathbf{v}}_i$, where $\hat{\mathbf{v}}_i=\boldsymbol{\iota}_T\delta_i/N^{1/4}-\mathbf{Z}_i(\hat{\boldsymbol{\beta}}-\boldsymbol{\beta})$, we can write

$$\hat{\mathbf{u}}_i'\mathbf{A}\hat{\mathbf{u}}_i=\mathbf{u}_i'\mathbf{A}\mathbf{u}_i+2\mathbf{u}_i'\mathbf{A}\hat{\mathbf{v}}_i+\hat{\mathbf{v}}_i'\mathbf{A}\hat{\mathbf{v}}_i$$
$$=\mathbf{u}_i'\mathbf{A}\mathbf{u}_i+S_i,\quad \text{say},$$

so that

$$\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\mathbf{u}}_i'\mathbf{A}\hat{\mathbf{u}}_i\right)^2=\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2+\frac{1}{N}\sum_{i=1}^{N}S_i^2+\frac{2}{N}\sum_{i=1}^{N}\mathbf{u}_i'\mathbf{A}\mathbf{u}_iS_i.$$

Now, $\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{u}_i'\mathbf{A}\mathbf{u}_i\right)^2=O_p(1)$, and we shall show that $\frac{1}{N}\sum_{i=1}^{N}S_i^2=o_p(1)$ so that, by Cauchy–Schwartz, $\frac{1}{N}\sum_{i=1}^{N}\mathbf{u}_i'\mathbf{A}\mathbf{u}_iS_i=o_p(1)$; then we are done.

Again by Cauchy–Schwartz, $\frac{1}{N}\sum_{i=1}^{N}S_i^2=o_p(1)$ if it can be shown that (i) $\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{u}_i'\mathbf{A}\hat{\mathbf{v}}_i\right)^2=o_p(1)$; and (ii) $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\mathbf{v}}_i'\mathbf{A}\hat{\mathbf{v}}_i\right)^2=o_p(1)$, and we take each of these in turn:

(i) First, by repeated application of Cauchy–Schwartz, noting that $\|\mathbf{A}\|^2 = T(T-1)$,

$$\frac{1}{N}\sum_{i=1}^{N}\left|\mathbf{u}_i'\mathbf{A}\hat{\mathbf{v}}_i\right|^2 \leq \frac{T(T-1)}{N}\sum_{i=1}^{N}\|\mathbf{u}_i\|^2\|\hat{\mathbf{v}}_i\|^2$$
$$\leq T(T-1)\sqrt{\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{u}_i\|^4\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4}.$$

Now, $E\|\mathbf{u}_i\|^4$ is uniformly bounded, by Assumption A3(i), so by Markov's Inequality, $\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{u}_i\|^4 = O_p(1)$, and it suffices to show that $\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4 = o_p(1)$.

Now,

$$\|\hat{\mathbf{v}}_i\|^2 = \frac{T\delta_i^2}{\sqrt{N}} - 2\frac{\delta_i}{N^{1/4}}\boldsymbol{\iota}_T'\mathbf{Z}_i(\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}) + (\hat{\boldsymbol{\beta}}-\boldsymbol{\beta})'\mathbf{Z}_i'\mathbf{Z}_i(\hat{\boldsymbol{\beta}}-\boldsymbol{\beta})$$
$$= S_{i1} + S_{i2} + S_{i3}, \quad \text{say,}$$

so that, by Cauchy–Schwartz, $\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4 = o_p(1)$ if $\frac{1}{N}\sum_{i=1}^{N}S_{im}^2 = o_p(1)$, for $m = 1, 2, 3$. Clearly, $\frac{1}{N}\sum_{i=1}^{N}S_{i1}^2 = \frac{T}{N}\frac{1}{N}\sum_{i=1}^{N}\delta_i^2 = o_p(1)$, by Assumption A4(ii) and, by repeated use of Cauchy–Schwartz,

$$\frac{1}{N}\sum_{i=1}^{N}S_{i2}^2 \leq 4\frac{T}{\sqrt{N}}\left\|\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}\right\|^2\frac{1}{N}\sum_{i=1}^{N}\|\delta_i\mathbf{Z}_i\|^2$$
$$= o_p(1)$$

because $\|\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}\| = o_p(1)$, $\frac{1}{N}\sum_{i=1}^{N}\|\delta_i\mathbf{Z}_i\|^2 = \frac{1}{N}\sum_{i=1}^{N}\sum_t\sum_j\left|\delta_i z_{itj}\right|^2 = O_p(1)$, by an application of Markov's Inequality, Cauchy–Schwartz, and Assumptions A2(ii) and A4(ii). Finally,

$$\frac{1}{N}\sum_{i=1}^{N}S_{i3}^2 \leq \left\|\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}\right\|^4\frac{1}{N}\sum_{i=1}^{N}\left\|\mathbf{Z}_i'\mathbf{Z}_i\right\|^2$$

where $\|\mathbf{Z}_i'\mathbf{Z}_i\|^2 = \sum_j\sum_k\left\{\sum_t z_{itj}z_{itk}\right\}^2$ and an application of Markov's Inequality, Minkowski's Inequality, Cauchy–Schwartz, and Assumption A2(ii) yields $\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{Z}_i'\mathbf{Z}_i\|^2 = O_p(1)$ and $\frac{1}{N}\sum_{i=1}^{N}S_{i3}^2 = o_p(1)$. Thus, $\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4 = o_p(1)$.

(ii) It immediately follows that $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\mathbf{v}}_i'\mathbf{A}\hat{\mathbf{v}}_i\right)^2 \leq T(T-1)\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4 = o_p(1)$, and we are done.

Third, for $\hat{\kappa}_N^{(2)}$, by Assumption A3(i), and Minkowski's Inequality $E|\sum_{t=2}^{T} w_{it}^2|^{1+\eta}$ is uniformly bounded so that $\frac{1}{N}\sum_{i=1}^{N}\sum_{t=2}^{T} w_{it}^2 - \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} E\left(w_{it}^2\right) \xrightarrow{p} 0$. Thus, by the Triangle Inequality, it remains to show that $\frac{1}{N}\sum_{i=1}^{N}\sum_{t=2}^{T} \hat{w}_{it}^2 - \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} w_{it}^2 \xrightarrow{p} 0$. Since $\hat{u}_{it} = u_{it} + \hat{v}_{it}$, $\hat{v}_{it} = \delta_i/N^{1/4} - \mathbf{z}_{it}'(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$, we can write

$$\begin{aligned}\hat{w}_{it} &= w_{it} + \hat{v}_{it}\sum_{s=1}^{t-1} u_{is} + \hat{v}_{it}\sum_{s=1}^{t-1}\hat{v}_{is} + u_{it}\sum_{s=1}^{t-1}\hat{v}_{is} \\ &= w_{it} + \hat{g}_{it}, \quad \text{say.}\end{aligned}$$

Thus, by Cauchy–Schwartz, it suffices to show that $\frac{1}{N}\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{g}_{it}^2 = o_p(1)$. It will be useful to note that

$$\begin{aligned}\sum_{t=2}^{T}\hat{g}_{it}^2 &\le \sum_{t=1}^{T}\left(\left|\hat{v}_{it}\right|\sum_{t=1}^{T}\left|u_{it}\right| + \left|\hat{v}_{it}\right|\sum_{t=1}^{T}\left|\hat{v}_{it}\right| + \left|u_{it}\right|\sum_{t=1}^{T}\left|\hat{v}_{it}\right|\right)^2 \\ &= \sum_{t=1}^{T}\left(S_{it1} + S_{it2} + S_{it3}\right)^2, \quad \text{say,}\end{aligned}$$

so that, now, it is sufficient to demonstrate that , $\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} S_{itm}^2 = o_p(1)$, $m = 1, 2, 3$.

By Cauchy–Schwartz, we have

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} S_{it1}^2 &= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{v}_{it}^2\left(\sum_{t=1}^{T}\left|u_{it}\right|\right)^2 \\ &\le \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{v}_{it}^2\right)^2 \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|u_{it}\right|\right)^4}, \\ \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} S_{it2}^2 &\le \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{v}_{it}^2\right)^2 \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|\hat{v}_{it}\right|\right)^4},\end{aligned}$$

and

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} S_{it3}^2 \le \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T} u_{it}^2\right)^2 \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|\hat{v}_{it}\right|\right)^4}.$$

Both $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|u_{it}\right|\right)^4$ and $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T} u_{it}^2\right)^2$ are $O_p(1)$, by Markov's Inequality, Minkowski's Inequality, and Assumption A3(i). Thus, it suffices to show that $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{v}_{it}^{2}\right)^{2}$ and $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|\hat{v}_{it}\right|\right)^{4}$ are both $o_p(1)$. The former is identical to $\frac{1}{N}\sum_{i=1}^{N}\|\hat{\mathbf{v}}_i\|^4 = o_p(1)$, by the proof of 2(i), above, and the latter is $o_p(1)$ by Assumptions A2(ii) and A4(ii) and the consistency of $\hat{\boldsymbol{\beta}}$. This completes the proof of part 3.

2. As in previous proofs, by Assumption A3(i) and the Triangle Inequality it suffices to show that

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}\hat{u}_{it}^{2}\hat{u}_{is}^{2} - \frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}u_{it}^{2}u_{is}^{2} \xrightarrow{p} 0.$$

Again, since $\hat{u}_{it} = u_{it} + \hat{v}_{it}$, $\hat{v}_{it} = \delta_i/N^{1/4} - \mathbf{z}_{it}'(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$, we can write

$$\begin{aligned}
&\frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}\hat{u}_{it}^{2}\hat{u}_{is}^{2} - \frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}u_{it}^{2}u_{is}^{2}\\
&\quad = 2\frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}u_{it}^{2}V_{is} + \frac{1}{N}\sum_{i=1}^{N}\sum_{t}\sum_{s\neq t}V_{it}V_{is}\\
&\quad = S_{N1} + S_{N2}, \quad \text{say,}
\end{aligned}$$

where $V_{it} = 2u_{it}\hat{v}_{it} + \hat{v}_{it}^{2}$, and it suffices to show that $S_{Nm} = o_p(1)$, $m = 1, 2$. Now,

$$\begin{aligned}
\left|S_{N1}\right| &\leq 2\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}u_{it}^{2}\sum_{t=1}^{T}\left|V_{it}\right|\\
&\leq 2\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}u_{it}^{2}\right)^{2}\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|V_{it}\right|\right)^{2}}.
\end{aligned}$$

Thus, since $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t}u_{it}^{2}\right)^{2} = O_p(1)$, it suffices to show that $\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t}\left|V_{it}\right|\right)^{2} = o_p(1)$, or that $\frac{1}{N}\sum_{i=1}^{N}\sum_{t}V_{it}^{2} = o_p(1)$ since $\left(\sum_{t}\left|V_{it}\right|\right)^{2} \leq T\sum_{t}V_{it}^{2}$. But this is true because

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}V_{it}^{2} \leq \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{v}_{it}^{4} + 4\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}u_{it}\hat{v}_{it}^{3} + 4\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}u_{it}^{2}\hat{v}_{it}^{2}.$$

The first term on the right-hand side is $o_p(1)$ as are the latter two terms by an application of Cauchy–Schwartz.

Second,

$$\left|S_{N2}\right| \leq \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\left|V_{it}\right|\right)^{2} = o_p(1)$$

by the preceding result, and this completes the proof. □

Proof of Proposition 3. We can write $R_N = \frac{1}{\sqrt{2}}\frac{\widehat{H}_N}{\hat{\sigma}^2}$, where $\hat{\sigma}^2 = \hat{\mathbf{u}}'\hat{\mathbf{u}}/NT$ and

$$\widehat{H}_N = \frac{1}{\sqrt{NT(T-1)}}\left[\hat{\mathbf{u}}'\left(\mathbf{I}_N \otimes \mathbf{A}\right)\hat{\mathbf{u}}\right]$$
$$= \frac{1}{\sqrt{NT(T-1)}}\left[\mathbf{y}'\mathbf{M}_{\mathbf{Z}}\left(\mathbf{I}_N \otimes \mathbf{A}\right)\mathbf{M}_{\mathbf{Z}}\mathbf{y}\right].$$

By Proposition 1, it is sufficient to show that

$$\widehat{H}_N = H_N + \sqrt{\frac{T-1}{T}}\lambda_N - \sqrt{\frac{T}{T-1}}\gamma_N + o_p(1)$$

and

$$\hat{\sigma}^2 - \bar{\sigma}_N^2 = o_p(1)$$

and the result follows.

Establishing the former follows exactly the argument as in Orme and Yamagata (2006, Proof of Proposition 2), and $\hat{\sigma}^2 - \bar{\sigma}_N^2 = o_p(1)$, was established above. This completes the proof. □

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