

EXAMINATION OF NORTH DAKOTA'S  
PRODUCTION, COST, AND PROFIT FUNCTIONS:  
A QUANTILE REGRESSION APPROACH

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Title

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## ABSTRACT

Marroquin, Jacklin Beatriz; M.S.; Department of Agribusiness and Applied Economics; College of Agriculture, Food Systems, and Natural Resources; North Dakota State University; December 2008. Examination of North Dakota's Production, Cost, and Profit Functions: A Quantile Regression Approach. Major Professor: Dr. Saleem Shaik.

This thesis estimates the production, cost, and profit functions for North Dakota agriculture using state-level input-output quantity and price data for the period 1960-2004. A Cobb-Douglas functional form with Hicks-neutral technology change is used to measure the contribution of capital, land, labor, materials, energy, and chemical inputs quantities and output quantity using the primal production function; contribution of capital quantity, land quantity, output quantity, labor price, materials price, energy price, and chemical price to cost using the dual restricted cost function; and the contribution of capital quantity, land quantity, labor price, materials price, energy price, chemical price, output price to profit using the dual restricted profit function. In contrast to previous studies, quantile regression is used to explore the linear or nonlinear relationship between the independent and dependent variable by estimating parameter coefficients at each quantile using time-series data.

Empirical findings suggest the cost function is the best model to examine the relationship between input prices, output quantity and cost using quantile regression for North Dakota agriculture. Further, the quantile regression suggests a linear and non-linear relationship between cost and certain independent variables.

## ACKNOWLEDGMENTS

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## CHAPTER 1. INTRODUCTION

### 1.0 Rationale and significance

In the last century, the United States, Northern Great Plains, and North Dakota's agriculture have undergone an impressive transformation with much debate about changes in their farm economic structure.<sup>1</sup> This thesis examines the changes in input resource use in the production of crops and livestock and the relationship between the uses of inputs to produce outputs using a primal or dual framework. Apart from this functional relationship, there is a growing interest in how these relations (linear or non-linear) have evolved over time and across quantiles due to changes in technology, consumer demand, and increased globalization.

Specifically, the changes in input use include increased capital investments in the earlier 1950-60s substituting for farm labor, investment in farm real estate in the 1970-80s, and use of fertilizer, chemicals, and energy in the 1990s followed by investment in breeding technology resulting in enhanced and disease resistant seeds. Concurrent to these structural changes, there was a decline in the number of small family farms, an increase in the average farm size, augmented capital investment leading to financial re-structuring for new and young farmers and ranchers, and increased risk faced by farmers. Similarly, changes with respect to output production involved shifting away from the traditional commodity crops to oil and vegetable production and livestock production including cattle and hog production.

The input and output changes in farm economic structure were examined for the U.S. agriculture sector using the primal production function [Marschak, and Andrews

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<sup>1</sup> Farm economic structure is defined as the relation between input and output using production, cost and profit function.

(1944); Mundlak (1963); Hoch (1958, 1962); Zellner, Kmenta, and Dreze (1966); Schmidt (1988)], and the dual cost function [Nerlove (1963); Fuss, and McFadden (1978); Diewert (1974); McElroy (1987)] or the profit function [Weaver (1983); Lopez (1985); Dixon, Garcia, and Anderson (1987); Antle (1984)]. Because the assumptions of cost minimization and profit maximization do not always hold, they have been rejected by Lin, Dean, and Moore (1974), Ray and Bhadra (1993), Pope and Chavas (1994), and Tauer and Stefanides (1998) due to biased estimates. Mundlak (1996) however, points out that, even when the underlying behavioral assumptions hold, the dual approach may still deem questionable as, “*estimates based on duality, unlike direct estimators of the production, do not utilize all the available information and therefore are statistically inefficient and the loss in efficiency may be sizeable*” (p. 431). An additional problem associated with the dual approach is the requirement for information on prices, information which when available shows little variation complicating estimation. This is primarily true when cross-sectional data are used.

Though there is a considerable body of literature pointing to the importance of evaluating U.S. agriculture, there is hardly any research which attempts to examine the relation between input and output using a production, cost, or profit function for the Northern Great Plains or North Dakota agriculture. This thesis closes the gap by estimating the relationship between aggregate inputs and outputs from a primal and dual approach using the underlying assumptions of production, profit maximization, and cost minimization for North Dakota agriculture sector from 1960-2004.

## **1.1 Theoretical aspects of quantile regression**

There is a widespread use of ordinary least square (OLS) in examining the changes in farm economic structure accounting for autocorrelation and heteroscedasticity, alternative functional forms, and estimation techniques. Most research involved estimation of the relationship between endogenous and exogenous variables at the mean. With the introduction of quantile regression (QR) methods by Koenker and Bassett (1978), the relationship between endogenous and exogenous variables can be estimated and examined at each quantile of the endogenous variable. In general, quantile regression proves to be extremely useful whenever one is interested in focusing on particular segments of the analyzed conditional distribution. This facilitates examining whether the relationship between the endogenous and exogenous differ across quantiles.

Growing recognition of the need for a more flexible, more complete analysis is a driving force in the use of quantile regression in the literature. Though quantile regression has been used in numerous studies such as wage inequality, urban-rural inequality, unemployment insurance, cash bonuses, unemployment and public-private sector wage gap, it has yet to be used in the agricultural sector. Quantile regression has been developed and applied to cross-section data; here quantile regression is applied to time-series data to examine the shape and the linear or non-linear relationship between the endogenous and exogenous variables in the estimation of the production, cost, and profit functions.

The rest of this paper is organized as follows: Chapter II summarizes the relevant literature on the production theory using production, cost, and profit functions. The chapter concludes with a brief revision of the expanding literature on quantile regression and its increasing application to a wide range of studies. Chapter III presents the conceptual

framework and data used in the empirical application. Chapter IV focuses on the specific features of the empirical model and the results of the production, cost, and profit functions. Finally in Chapter V, the conclusions are presented and scope for future research is proposed.

## **CHAPTER 2. LITERATURE REVIEW**

### **2.0 Background**

This chapter outlines a review of studies that feature the use of the OLS procedure to estimate the production, cost, and profit functions. A brief review of the use of quantile regression is presented. This chapter indicates there is a dearth of studies that utilize quantile regression in estimating production, cost, and profit functions in economics and agricultural economics literature in the United States. This shortage of literature seems to exist not only at the state level but also at the national level.

### **2.1 Production function**

Production functions have been used as a fundamental tool of economic analysis in the neoclassical tradition. Between 1950 and 1970, estimation of the production, cost, and profit function gained substantial interest as a means of gauging the overall performance of the agricultural sector in the U.S., and has since been widely experimented on since the Von Thünen application to production and agricultural economics (Humphrey, 1997).

Research efforts have used production functions to investigate differences in agricultural productivity among countries. Since the introduction of the meta-production function by Hayami and Ruttan in 1970, numerous studies have used this concept in related work [Kawagoe and Hayami (1985); Binswanger, et al., (1987); Lau and Yotopoulos (1989); Frisvold and Lomax (1991); Boskin and Lawrence (1992)]. Trueblood (1991) examines numerous studies that test the meta-production function hypothesis mainly differentiating between developed and less developed countries. Table 1 reprints the range and average of the estimated production elasticities at the inter-country level using

aggregate observations [Bhattacharjee (1955); Hayami and Ruttan (1970); Evenson and Kislev (1975); Kawagoe, Haymai, and Ruttan (1985); Antle (1984); Nguyen (1979); Yamada and Ruttan (1980)].

Table 1. Summary of the input elasticities estimated from production function.

	Variable	Elasticity range	Average
Conventional Inputs	Labor	0.30 - 0.54	0.42
	Livestock	0.25 - 0.35	0.26
	Fertilizer	0.12 - 0.20	0.13
	Land	0.02 - 0.20	0.08
Nonconventional Inputs	Machinery	0.05 - 0.15	0.25
	General Education	0.25 - 0.35	0.24
	Infrastructure	0.20	0.16
	Technical Education	0.14 - 0.17	0.13
	Research	0.09 - 0.13	0.11

Source: Trueblood, 1991.

Labor illustrates the widest range of estimates among inputs. The estimated range measures the marginal effect, representing the range by which output changes in response to one unit change in farm labor. Trueblood (1991) points out that, aside from the land coefficient which has been rather small and insignificant, the other conventional inputs -- livestock, fertilizer, and machinery -- were consistently statistically significant. Though the inclusion of livestock as an input is not clear, Bhattacharjee (as quoted in Trueblood), differentiates between 'productive' animals (cattle, pigs, sheep, and goats) and 'draft' animals (horses, mules, and buffaloes) and assigned weights and aggregated these animals as a representation of 'internal capital accumulation.'

Trueblood further points out that, whereas fertilizer indicates a strong correlation with research, the increase in usage of fertilizer is due largely in part to the Green

Revolution's new seed varieties. Additionally, a unit change in the use of mechanical capital as measured by the horsepower equivalent developed by Hayami-Ruttan further changes agricultural output between the ranges of 0.05-0.15. Similar statistical significance to conventional inputs held true for the nonconventional inputs/activities such as technical education, research, and infrastructure. These are not generally considered inputs that are controlled by farmers. Griliches (1964) found that in agriculture in the United States a given percentage increase in education, which improves the quality of labor, has the same output effect as an equal percentage increase in labor itself.

In examining the production structure between developed countries (DC's) and less developed countries (LDC's), Kawagoe, Hayami, and Ruttan (1985) found that, despite the dramatic technological developments in both DC's and LDC's during the past two decades, the productivity structure of world agriculture as measured by production elasticities of conventional and nonconventional inputs remained largely the same. Results of the Kawagoe, et al., (1985) indicate that there are significant differences in the production functions between the DC's and LDC's. The conventional input coefficients for the DC's are significantly larger than one, whereas the sum is not significantly different from one for the LDC's. The results indicate that LDC's displayed decreasing to constant (0.80-1.04) returns to scale with conventional inputs and increasing returns to scale (1.04-1.61) when including nonconventional inputs. Unlike the LDC's, the DC's displayed increasing returns to scale with both conventional and nonconventional input (Trueblood, 1991).



## 2.2 Cost function<sup>2</sup>

While empirical analysis of the agricultural production structure highlights the relationship between a single input and output, aggregate production, cost and profit function functions have also been estimated using an aggregation of the total products involved. Multiple-output production functions are often estimated in the dual form represented by their cost or profit functions. However, one of the major difficulties in estimating nonexperimental agricultural production function is that input data are not typically available by a crop basis according to Just, Zilberman, and Hochman (1983). These authors also point out that the problem of growing more than one crop without the specification of the allocated inputs has been addressed through single-equation joint production functions specifying the relationship between output quantities and aggregate input quantities or through corresponding relationships between quantities and prices resulting from duality under (expected) profit maximization.

Binswanger (1974a); Ray (1982); Lopez (1980) and Kako (1978) have all used a cost function approach in modeling agricultural production (Table 2). Results from Ray (1982) reveal farm capital; fertilizer; and feed, seed, and livestock all substitute for hired labor in varying measures. However, the degree of substitution between labor and capital is much smaller than between labor and fertilizer. The high degree of substitutability between labor and fertilizers is consistent with the steady decline in labor use and steep increase in fertilizer use. Ray also points out that farm capital is a substitute for all other inputs and the substitutability between labor and capital has declined, while between labor and fertilizer or labor and feed, seed and livestock has increased. Whereas farm labor has

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<sup>2</sup> Though the economic literature is rich with elasticity estimates of input prices, similar estimates are scant in the agricultural literature.

the highest price elasticity of demand, each input experienced increase in its price elasticity of demand over time, implying a relative greater use of purchased rather than farm-supplied inputs (Ray, 1982).

Table 2. Summary of input and output elasticities estimated from cost functions.

	Variable	Elasticity range
Inputs	Labor	-0.05 - 0.91
	Fertilizer	-0.13 - 0.95
	Land	-0.42 - 0.48
	Capital	-0.35 - 0.53
	Machinery	-0.54 - 1.09
	Feed, seed & livestock	-0.34
	Intermediate inputs	-0.41
	Miscellaneous	-0.16 - 1.90

Note: Author's calculations from Binswanger (1974a), Ray (1982), Lopez (1980) and Kako (1978).

In comparison, Binswanger (1974a), who also used a translog approximation for the cost function for U.S. agriculture, found significant substitutability between labor and land as well as between labor and machinery. His average elasticities of substitution were 0.204 between land and labor and 0.851 between labor and machinery. However, he found complementarity between labor and fertilizer as opposed to Ray's strong substitutability relation between labor and fertilizer. In considering the price elasticities of factor demands, Ray's measures for labor (-0.8389 at the mean) is similar to Binswanger's average value of -0.9109 (Ray, 1982).

In examining the structure of production and the derived demand for inputs in Canadian agriculture, Lopez (1980) aimed to expand from the previous agricultural and input demand functions. He used a more general functional form than those used by Kako (1978) and Binswanger (1974a), allowing for a fixed-proportion production function

(Leontief production function) as well as for constant returns to scale and homotheticity. Binswanger (1974b) was able to separate the effect of biased technical change on the observed factor shares from the effect of ordinary factor substitution due to factor prices, although, Lopez points that factor shares also can be affected by changes in the scale of production if the production function is nonhomothetic. Kako also assumes constant returns to scale, and, although he measured technical change biases, he did not thoroughly test for neutral or biased technical change (Lopez, 1980).

Lopez points out that the Leontief production function is an extreme situation where the input-output coefficients are independent of input prices or, equivalently, that all elasticities of substitution are zero. Lopez concludes that agricultural production entails a considerable degree of factor substitution in response to price changes. By rejecting constant returns to scale, Lopez points out that as the scale of production increases, efficiency in the use of factor of production increases (Lopez, 1980). His estimated own-price elasticities for labor, capital, land and structures, and intermediate inputs are inelastic, with values ranging between -0.280 and -0.897. These results are similar to those obtained by Kako for Japan using a translog cost function. The similarity between the own-price elasticities of labor and land is quite remarkable. Kako obtained values ranging from -0.401 to -0.465 for labor and -0.464 to -0.491 for land. The own-price elasticities for labor obtained by Binswanger (1974b) using the U.S. data was -0.911, which is higher than estimations by Lopez. Binswanger's estimate for land was -0.336, which was somewhat lower than those estimated by Lopez.

Lopez points out that all input pairs appear to be substitutes, and the highest degree of substitution occurs between labor and farm capital and between capital and intermediate

inputs. On the other hand, the substitution between labor and land and structures is very low and, indeed, it is not significantly different from zero. Thus labor and land and structures would not be substitutes for each other. Lopez's estimates are quite different from those obtained by Kako for Japan's rice production, even though the predominantly positive signs of his coefficients are consistent with those estimated by Lopez. Kako obtained a higher elasticity of substitution between land and labor ranging from 0.760 to 0.816, and a lower elasticity of substitution between labor and capital (0.934). These can be compared with Binswanger's elasticity estimates for U.S. agriculture of 0.204 for land and labor, 1.215 for land and machinery, and 0.851 for labor and machinery (Binswanger, 1974b).

### **2.3 Profit function<sup>3</sup>**

Several studies have reported output supply and input demand elasticity estimates for U.S. agriculture [Antle (1984); Vasavada and Chambers (1986); Shumway, Saez and Gottret (1988); Ball (1988); Huffman and Evenson (1989)]. A reprint of the ranges and averages of own-price output supply and input demand elasticities from different studies is reported in Table 3.

Elasticities are similar for a few categories with multiple estimates, such as output and feed grain supplies. However, in general, the elasticities vary widely among estimates. In modeling the supply response in a multiproduct framework, Ball (1988) uses disaggregated output data to approximate the agricultural technology by a restricted profit function.

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<sup>3</sup> Analogous to the cost function, the economic literature is rich with elasticity estimates of input and output prices, similar estimates are scant in the agricultural literature.

Table 3. Summary of input and output elasticities estimated from profit functions.

	Variable	Elasticity range	Average	
Output	Livestock	0.11 - 1.09	0.542	
Supply	Fluid milk	0.64	0.64	
	Grains	0.84	0.84	
	Food grains	0.31	0.31	
	Wheat	0.97	0.97	
	Feed grains	0.02 - 0.11	0.07	
	Oil seeds	0.10 - 0.43	0.27	
	Soy beans	1.31	1.31	
	Other crops	0.08 - 0.11	0.60	
	Input Demand	Machinery	-1.27 - 0.12	-0.42
		Real estate	-0.58 - 0.03	-0.26
Farm produced products		-1.16	-1.16	
Labor		-0.51 - 0.01	-0.34	
Hired labor		-1.50 - 0.10	-0.80	
Energy		-0.94 - 0.25	-0.48	
Fertilizer		-0.12	-0.12	
Fuel		-0.72	-0.72	
Materials		-0.34 - 0.08	-0.21	
Other purchased inputs		-2.9	-2.9	

Note: Author's calculations from Antle (1984), Vasavada and Chambers (1986), Shumway, Saez and Gottret (1988), Ball (1988) and Huffman and Evenson (1989).

Ball's findings indicate that the own-elasticities of supply are generally less than unity; only the supply function for livestock and 'other crops' are price elastic. The gross complementarity of outputs depicts interesting results suggesting that an increase in the price of a particular output would result in increased production of all outputs (Ball, 1988).

Similarly to Ball, Antle (1984) found that all own-price elasticities are negative as theory predicts, and most elasticities are absolutely less than 1. Antle compares elasticity estimates for different studies. Binswanger's aggregate models also produced a low demand elasticity for land (-0.34) but elasticities for labor, machinery, and fertilizer were near -0.9. Ray's aggregate model also produced a labor demand elasticity near -0.9 to 0.53 for capital and -0.13 for fertilizer. Weaver's multiproduct model for the Dakotas produced

higher demand elasticities, generally greater than 1 in absolute value, whereas Shumway's multiproduct model for Texas crops estimated -0.80 for fertilizer, -0.43 for labor, and -0.37 for machinery. Thus except for Weaver's study, these results present a picture of input demand inelasticity in U.S. agriculture. Antle also points out that, both Weaver's and Shumway's models produced inelastic supply functions, ranging from 0.4 to 0.73 in the former study and from 0.25 to 0.72 in the latter (Antle, 1984).

## **2.4 Quantile regression**

A commonality among the differing methods for analyzing longitudinal data is the use of the mean as the measure of centrality. However, estimating at the mean level has its disadvantages (Karlsson, 2006). Nonetheless, unlike linear regression, which estimates the mean value of the response variable for the given level of the predictor variables, quantile regression is an evolving body of statistical methods for estimating and drawing inferences about conditional quantile functions. Hence, more recent research has shifted from the using to mean to the median as a measure of centrality first introduced by Koenker and Bassett (1978).

In calculating regression curves for different quantiles, it is possible to get a distribution of quantile regression curves that show the distribution of the data for each time point, conditional on the specific time points. The quantile procedure makes it possible to study any changes over time in the shape of the entire conditional distribution of the data, and not only the change over time in the conditional mean or median. It gives a picture of how the individual subject performs in comparison with the overall performance

of the sample, thus providing a much more complete picture of the dataset (Karlsson, 2006).

Quantile regression offers a richer, more focused view of the applications than could be achieved by looking exclusively at conditional mean models. There have been recent advances in the use of quantile regression to complement classical linear regression analysis which abandons the idea of estimating separate means for grouped data. The OLS procedure is a standard approach to specify a linear regression model and estimate its unknown parameters by minimizing the sum of square errors, leading to an approximation of the mean function of the conditional distribution of the dependent variable. Ordinary least square is similar to quantile regression in that both specify a moment of the conditional distribution as a linear function of the conditioning variables. The least square estimator specifies and estimates the conditional mean function,  $E [Y|X = x] = x\beta$ , where  $Y$  is a univariate random variable and  $x$  is a vector of covariates with the associated parameter vector  $\beta$ . Quantile regression, first introduced by Koenker and Bassett (1978), specifies and estimates a family of conditional quantile functions,  $F_{y|x}^{-1}(\tau |x) = x\beta(\tau)$ , where  $F$  is the conditional distribution function of  $Y$  given  $X$ , and  $\tau$  is a quantile in the interval  $[0,1]$ . Thus quantile regression provides several summary statistics of the conditional distribution function, rather than just one characteristic, namely the mean (Centeno and Novo, 2006).

Krüger (2006) points out that quantile regression has the potential to uncover differences in the response of the dependent variable to changes in explanatory variables, thereby providing a large amount of information about the heterogeneity of the sample items. In addition, coefficient estimates obtained by quantile regression are more robust with respect to outliers of the dependent variable and, in the case of non-normal errors.

Fitzenberger, et al., (2001) suggested the quantile regression estimates may be even more efficient than least square estimates.

There is rapidly expanding empirical quantile regression literature in economics which can be taken as a persuasive case for the value of “*going beyond models for the conditional mean,*” in empirical economics (Buhai, 2005, p. 2). The context of quantile regression has been applied in labor economics: on union wage effects and returns to education, and labor market discrimination [Falaris (2003); Martins and Pereira (2004)].

Deaton (1997) has been credited with introducing quantile regression for demand analysis. His analysis explains the procedure as involving minimization that is similar to minimizing the sum of absolute errors in a median-regression context. Though this minimization problem can be solved through the use of linear programming, Deaton and other scholars have pointed out the estimation of the variance-covariance matrix of the estimators, and therefore hypothesis tests can become problematic in the presence of certain departures.

Though the application of quantile regression continues to expand, literature of its application to the agricultural industry is scant. Krüger (2006) examines the productivity dynamics beyond-the-mean in U.S. manufacturing industries. In his paper, Krüger uses quantile regression to explore the relation of current and lagged productivity levels of U.S. manufacturing industries. The results of the quantile regression specification give the overall impression that the degree of persistence tends to be larger in high-productivity industries compared to low-productivity industries. The result of persistence is established by the application of different empirical methods like unit root and stationarity tests, indices, and non-parametric estimation. Krüger concludes that productivity transitions are



characterized by a substantial degree of persistence which tends to be larger for high-productive industries than for industries with lower levels of productivity. The difference across different quantiles supports the notion of differential growth of industries in the U.S. manufacturing sector.

## **2.5 Conclusion**

This review of the literature initially focused on studies that used the OLS procedure to estimate production, cost, and profit functions. Emerging literature, however, estimates these functions via the use of quantile regression. Thus, by its initial focus on these relationships using quantile regression at the state level, this thesis makes a first contribution to knowledge and, by its subsequent estimation of the three functions (production via a primal framework and cost and profit functions via a dual framework), this thesis can be regarded as a pioneering endeavor.

### CHAPTER 3. METHODOLOGY AND DATA

Past and current econometric estimates have focused on the estimation of the production, cost, and profit function using the traditional OLS procedure. This thesis differs in that its main objective is to estimate the production, cost, and profit function at a state level using quantile regression. This is done by examining the changes in input resource use to produce output, cost as a function of input prices and output quantities, and profit as a function of input prices and output prices using a primal production function, a dual cost function, and a dual profit function, respectively.

Whereas duality has long received previous attention [Shephard (1953); McFadden (1962); Uzawa (1964)], its potential in econometric analysis was not recognized until Nerlove (1963) employed the Cobb-Douglas (CD) production function in the estimation of a cost function. After the seminal contributions of Fuss and McFadden (1978) and Diewert (1974), the duality approach became the preferred method of estimation. Though the debate between the duality approach and the primal methodology has not subsided, as a supporter of the primal approach, Mundlak (1996) points out that:

Much of the discussion on the estimation of the production functions is related to the fact that inputs may be endogenous and therefore direct estimators of the production functions may be inconsistent. One way to overcome this problem has been to apply the concept of duality. The purpose of this note is to point out that estimates based on duality, unlike direct estimators of the production function do not utilize all the available information and therefore are statistically inefficient and the loss in efficiency may be sizeable. (p. 4 31).

Paris and Caputo (2004) argue that a more efficient system is composed by both primal and dual relations that must be jointly estimated, and that only under special cases is it convenient to estimate either a primal (Mundlak's) or a dual (McElroy's) function.

### 3.0 Production function

Production theory assumes that the relationship between multiple outputs and inputs is reflected by the concept of a *transformation function*. With some additional assumptions and aggregation of all outputs, the input-output relationship is often reduced to a *production function* (Chambers, 1988). The production function represents the relation between nonallocable input vectors,  $x = (x_1, x_1, \dots, x_n) \in \mathfrak{R}_+^N$  used in the production of an output vector,  $y = (y_1, y_1, \dots, y_m) \in \mathfrak{R}_+^M$ .

Different functional forms can be applied in the context of agricultural production functions. The Cobb-Douglas functional form of production functions (multiplicative), initially proposed by Wicksell (1851-1926) and empirically estimated by Charles Cobb and Paul Douglas in 1928, is widely used to represent the relationship of an output to inputs. This research uses the Cobb-Douglas function to represent the production function characterized as:

$$y_t = f(x_{k,t} | \alpha) \text{ or } y_t = A \sum_{k=1}^K (x_{k,t}^{\alpha_k}) \quad (3.1.1)$$

where  $k = 1 \dots K$  (number of inputs and time  $1 \dots T$ ). Converting the inputs and output into logarithms and adding a stochastic error term, the production function can be represented as:

$$\begin{aligned} \ln y_t &= \alpha_0 + \sum_{k=1}^K \alpha_k \ln x_{k,t} \\ &= \alpha_0 + \alpha_1 \ln x_{1,t} + \dots + \alpha_k \ln x_{k,t} + \varepsilon_t \end{aligned} \quad (3.1.2)$$

where,  $\alpha_1, \dots, \alpha_k$ , are the input elasticities, and  $\varepsilon$  denotes the error term.

The Cobb-Douglas production function was the function of choice from the 1920s until the early 1950s when economists learned of its limitations and began exploring

alternatives. Included among the proposed alternatives was a less restrictive quadratic production function, the transcendental logarithmic production function and the constant elasticity of substitution (CES) function, both generalizations of the Cobb-Douglas production function. Additional types of functional forms included the multifactor CES, generalized production functions, variable elasticity of substitution functions, constant ratio elasticity of substitution-homothetic, and the generalized Leontief. Such developments in functional forms reflect the growing understanding that the functional forms used in production analysis may impose restrictions on the economic relationships (Capalbo and Antle, 1984). Though the CD production function imposes restrictions of unitary elasticities of substitution, constant production elasticities, and constant factor demand elasticities, a CD production functional form is used here to avoid a problem of degrees of freedom with non-Hicks neutral technology change and other functional forms.

The production function has been widely applied in the measurement of farm performance via the OLS procedure; this thesis estimates the CD production function by the means of a quantile regression approach. An OLS regression is based on the mean of the distribution of the regression's variable. This approach is used because one implicitly assumes that the possible difference in terms of the impact of the exogenous variables along the conditional distribution is unimportant. However, this may prove inadequate in some research. If exogenous variables influence parameters of the conditional distribution of the dependent variable other than at the mean, an analysis that disregards this possibility will be severely weakened (Koenker and Bassett, 1978).

For a single equation production function econometric model, the parameter coefficients are generally estimated as:

$$\hat{\alpha} = \min \sum_{t=1}^T (y_t - x'_{k,t} \alpha_k)^2 \quad (3.1.3)$$

where  $y_t$  is the endogenous variable and  $x_{k,t}$  is a vector of exogenous variables. Following Koenker and Bassett (1978), a single equation econometric model can be extended to quantile regression to examine the changes in coefficients across the distribution of endogenous model. Following Koenker and Hallock (2001, p. 146), the quantile regression provides parameter coefficients estimation for any quantile in the range of zero and one (0, 1) conditional on the exogenous variables, represented as:

$$\hat{\alpha}(\tau) = \min_{\alpha \in R^p} \sum_{t=1}^T \tau (y_t - x'_{k,t} \alpha_k)^2 \quad \text{for any quantile, } \tau \in (0, 1)$$

or

$$\hat{\alpha}(\tau) = \min_{\alpha \in R^p} \left[ \sum_{t \in \{y_t, x'_{k,t} \alpha_k\}} \tau |y_t - x'_{k,t} \alpha_k| + \sum_{t \in \{y_t, x'_{k,t} \alpha_k\}} (1-\tau) |y_t - x'_{k,t} \alpha_k| \right]$$

The quantile regression as defined in equation 3.1.4 is used as the basis for the empirical model presented here following a reduced-methodology. The quantile model specification follows equation 3.1.3 and can be represented as:

$$Q_\tau [y | x_k] = \alpha_{0,\tau} + \alpha_{k,\tau} x_k \quad (3.1.5)$$

where  $y$  is aggregate output,  $Q_\tau [y | x_k]$  is the  $\tau^{th}$  quantile of  $y$  conditional on covariate matrix,  $X_k$  that includes the quantities of capital, land, labor (hired, self-employed, and unpaid family labor), materials, energy, and chemicals. The coefficient  $\alpha_{k,\tau}$  represents the returns to covariates or inputs at the  $\tau^{th}$  quantile.

### 3.1 Cost function

The econometric model used to analyze cost is a model in which an explanatory variable represents total cost and exogenous variables represent factors that influence their level. Production quantity is the most important factor which determines the level of total cost. The cost function is the minimum cost of producing a given output level during a given period expressed as a function of input prices ( $w$ ) and output ( $y$ ). The cost function can be defined as:  $C(w, y) = \min \{w \cdot x: \text{s.t. } x \in V(y)\}$ .

Numerous algebraic equation forms can be applied in the context of agricultural cost functions. An advantage of such a function is its capability of handling multiple outputs. The CD functional form of the cost function is defined as a function of the input prices ( $w$ ) and output ( $y$ ) and can be represented as:

$$c_t = f(w_{k,t}, y | \beta, \gamma) \quad \text{or} \quad c_t = A \left( \sum_{k=1}^K (w_{k,t}^{\beta_k}), y^\gamma \right) \quad (3.2.1)$$

$k=1 \dots K$  (number of inputs and time) and  $t = 1 \dots T$  years. Log-linearizing and adding a stochastic error term, the cost function can be represented as:

$$\begin{aligned} \ln c_t &= \beta_0 + \sum_{k=1}^K \beta_k \ln w_{k,t} + \gamma \ln y_t + \varepsilon_t \\ &= \beta_0 + \beta_1 \ln w_{1,t} + \dots + \beta_K \ln w_{K,t} + \gamma \ln y_t + \varepsilon_t \end{aligned} \quad (3.2.2)$$

For a single equation cost function econometric model, the parameter coefficients are generally estimated as:

$$\hat{\beta}, \hat{\gamma} = \min \sum_{t=1}^T \left( c_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), y | \gamma \right) \right)^2 \quad (3.2.3)$$

where  $C_t$  is the endogenous variable and  $w_{k,t}$  and  $y$  are a vector of exogenous variables.

Following Koenker and Bassett, a single equation econometric model can be extended to quantile regression. Unlike the OLS, quantile regression is not limited to explaining the mean of the dependent variable. It can be employed to explain the determinants of the dependent variable at any point of the distribution of the dependent variable. Following Koenker and Hallock (2001, p. 146), the quantile regression provides parameter coefficients estimation for any quantile in the range of zero and one (0, 1) conditional on the exogenous variables, represented as:

$$\hat{\beta}(\tau), \hat{\gamma}(\tau) = \min_{\beta, \gamma \in R^p} \sum_{t=1}^T (\tau) \left( c_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), y | \gamma \right) \right)^2$$

for any quantile,  $\tau \in (0, 1)$

or

$$\hat{\beta}(\tau), \hat{\gamma}(\tau) = \min_{\beta, \gamma \in R^p} \left[ \begin{array}{l} \sum_{t \in \{t: c_t \geq (w_{k,t}^k | \beta_k, y | \gamma)\}} \tau \left| c_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), y | \gamma \right) \right| + \\ \sum_{t \in \{t: c_t \leq (w_{k,t}^k | \beta_k, y | \gamma)\}} (1-\tau) \left| c_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), y | \gamma \right) \right| \end{array} \right] \quad (3.2.4)$$

The quantile regression used is defined in equation 3.2.4 as the base for the empirical model presented here following a reduced-methodology. The quantile model specification follows equation 3.2.3 and can be represented as:

$$Q_\tau [c | w_{k,t}, y] = \beta_{0,\tau} + \beta_{k,\tau} w_k + \gamma_\tau y \quad (3.2.5)$$

where  $y$  is aggregate output,  $Q_\tau [c | w_{k,t}, y]$  is the  $\tau^{th}$  quantile of  $c$  conditional on covariate matrix,  $w_{k,t}$  that includes the quantities of capital and land, the price of labor (hired, self-

employed and, unpaid family labor), materials, energy, chemicals, and  $y$  includes the output over time and the coefficient  $\beta_{k,\tau}$  represents the returns to covariates at the  $\tau^{\text{th}}$  quantile.

### 3.2. Profit function

The genealogy of the profit function is slightly more distinct than that of either the cost or the production function. Hotelling (1932) clearly had conceptualized such a function in the 1930s. However, it was not until McFadden's work that the dual relationship between profit and production functions was exhaustively investigated. McFadden (1978) and Gorman (1968) were among the first to establish the existence of a duality between the profit and the direct technology (Chambers, 1988).

A profit function is the mathematical representation of the solution to an economic agent's optimization problem. Profit function maximizes profit during a given period expressed as a function of input prices and output prices and represented as:  $\pi(p, w) = \max \{pf(x) - w \cdot x\} = \max \{py - c(w, y)\}$  where  $p$  is the output price producers take as given in either maximizing profit for one output (short run) or maximizing profits by minimizing costs (long run).

Analogous to the production function and the cost function, different functional forms can be applied in the context of agricultural production functions. The CD functional form of the profit function can be represented as a relationship between input prices and output prices:

$$\pi_t = f(w_{k,t}, p_t | \beta, \delta) \quad \text{or} \quad \pi_t = A \left( \sum_{k=1}^K (w_{k,t}^{\beta_k}), p_t^\delta \right) \quad (3.3.1)$$



Log-linearizing and adding a stochastic error term, the profit function can be represented

as:

$$\begin{aligned}\ln \pi_t &= \beta_0 + \sum_{k=1}^K \beta_k \ln w_{k,t} + \delta \ln p_t + \varepsilon_t \\ &= \beta_0 + \beta_1 \ln w_{1,t} + \dots + \beta_K \ln w_{K,t} + \delta \ln p_t + \varepsilon_t\end{aligned}\quad (3.3.2)$$

For a single equation profit function econometric model, the parameter coefficients are generally estimated as:

$$\hat{\beta}, \hat{\delta} = \min \sum_{t=1}^T \left( \pi_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), p | \delta \right) \right)^2 \quad (3.3.3)$$

where  $\pi_t$  is the endogenous variable and  $w_{k,t}$  and  $p$  are a vector of exogenous variables and  $p$  represents the number of parameters to be estimated. Following Koenker and Hallock, (2001, p. 146), the quantile regression provides parameter coefficients estimation for any quantile in the range of zero and one (0, 1) conditional on the exogenous variables,

represented as:

$$\hat{\beta}(\tau), \hat{\delta}(\tau) = \min_{\beta, \delta \in R^p} \sum_{t=1}^T (\tau) \left( \pi_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), p | \delta \right) \right)^2$$

*for any quantile,  $\tau \in (0, 1)$*

or

$$\hat{\beta}(\tau), \hat{\gamma}(\tau) = \min_{\beta, \delta \in R^p} \left[ \begin{array}{c} \sum_{t \in \{t: \pi_t \geq (w_t^k, p | \beta, \delta)\}} \tau \left| \pi_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), p | \delta \right) \right| \\ + \\ \sum_{t \in \{t: \pi_t \leq (w_t^k, p | \beta, \delta)\}} (1-\tau) \left| \pi_t - \left( \sum_{k=1}^K (w_{k,t} | \beta_k), p | \delta \right) \right| \end{array} \right] \quad (3.3.4)$$

where  $k = 1 \dots K$  (number of inputs and time) and  $l = 1 \dots L$  (number of outputs). The quantile regression as defined in equation 3.3.5 is used as the basis for the empirical model

presented here following a reduced-methodology. The quantile model specification follows equation 3.3.3 and can be represented as:

$$Q_{\tau}[\pi | w_{k,t}, p] = \beta_{0,\tau} + \beta_{k,\tau} x_k + \delta_{\tau} p \quad (3.3.5)$$

where  $\pi$  is profit,  $Q_{\tau}[\pi | w_{k,t}, p]$  is the  $\tau^{\text{th}}$  quantile of  $\pi$  conditional on covariate matrix,  $w_{k,t}$  that includes the quantities of capital, and land, and the price of labor (hired, self-employed, and unpaid family labor), materials, energy, chemicals, and the price of output ( $p$ ), and the coefficient  $\beta_{k,\tau}$  which represents the returns to covariates at the  $\tau^{\text{th}}$  quantile.

### 3.3 Data

The input and output data for North Dakota's agriculture span a 45-year period from 1960-2004. Six categories of inputs and three categories of outputs were used in the estimation of the production, cost, and profit models. The six inputs include capital excluding land, land, two types of farm labor (hired and self-employed, and unpaid family labor), aggregated materials, energy, and agricultural chemicals (pesticides and fertilizers) to produce three outputs, specifically livestock, crops and other farm related outputs. The United States Department of Agriculture (USDA) uses Eltetö and Köves (1964) and Szulc (1964) (EKS) indices of relative levels of output and inputs among all states for a single base year. Indices of output and input quantities in each state are obtained relative to those in base state for each year by linking time-series quantity indices with estimates of relative output and input levels for base period equal to unity in Alabama in 1996 (Ball, 2008).

Annual time-series data for North Dakota were used to estimate the models. The output series was defined as aggregated quantity and price index (AO\_QI, AO\_PI)

comprised of livestock, crops, and other farm output. The independent variables include six conventional input price and quantity indices including capital (cap\_PI, cap\_QI), land (land\_PI, land\_QI), labor (labor\_PI, labor\_QI), materials (mat\_PI, mat\_QI), energy (eng\_PI, eng\_QI), and chemicals (chem\_PI, chem\_QI). The indices in each category are based on prices relative to levels in Alabama in 1996. All the input and output quantities are implicit quantities in value of \$1,000 in 1996, and input and output prices are indexed in 1996 dollars.

In the primal production function, physical input and output quantities are used in the estimation. Alternatively, when the dual cost function is estimated, input prices are used rather than quantities. Total cost was estimated by aggregating the individual input price multiplied by input quantities. In addition, in specifying cost and the profit functions, two inputs were treated as fixed (capital and land). Hicks-neutrality CD production, restricted cost and restricted profit function is used in the estimation of traditional OLS and quantile regression.

In estimating the profit maximizing function, total revenue was calculated by aggregating output price multiplied by output quantities of crops, livestock and other outputs. Profit was estimated by subtracting total cost from total revenues and specifying the profit maximizing model as a function of a subset of restricted input factors, four input prices, and output prices.

Tables 4, 5, and 6 presents the summary statistics for independent and dependent variables used in the estimation of production function, restricted cost function and restricted profit function respectively. The summary statistics includes the mean, standard deviation, minimum and maximum.

Table 4. Descriptive statistics of input and output quantity variables used in the estimation of the production function for North Dakota’s agriculture sector, 1960-2004

Variable	N	Mean	Std Dev	Minimum	Maximum
Output_QI	45	3,490,990	922,745	1,631,446	5,246,704
Capital_QI	45	872,684	144,655	676,890	1,176,491
Land_QI	45	1,274,286	38,350	1,219,174	1,334,053
Labor_QI	45	1,591,884	309,844	1,044,932	2,408,406
Materials_QI	45	1,669,789	214,355	1,262,242	2,073,637
Energy_QI	45	209,728	21,022	182,300	253,331
Chemicals_QI	45	432,095	239,555	82,429	989,574

Because this thesis makes use of the primal production function, aggregate output is modeled as a function of input quantities (Table 4), where N represents the sample size (number of years from 1960-2004). The mean scores represent the numerical average for the set of variables. The estimated average value for aggregate outputs stood at 3,490,990 with a minimum value of 1631,446 in 1961 and a maximum of 5,246,704 in 2003. The distribution around the mean of the aggregate outputs is 922,745, so  $\pm$  one standard deviation from the mean gives the range of 4,413,735 and 2,568,245 which represents approximately two thirds of output values.

In contrast from the input vector, aggregate materials ranked the highest average at 1,669,789 followed by labor at 1,591,884, land at 1,274,286, and capital at 872,684 followed by the use of chemicals at 432,095 and energy with the lowest amount at 209,728. The higher averages indicate the level of concentration relative to input use in North Dakota in the time period 1960-2004.

Unlike the averages for the input quantities where materials portrayed the highest average, the price of energy ranks highest with a total of 0.724 with a minimum price of 0.250 in 1966 and maximum of 1.303 in 2004 (Table 5).

Table 5. Descriptive statistics of the input quantity, input price, and output price variables used in the estimation of cost function for North Dakota's agriculture, 1960-2004

Variable	N	Mean	Std Dev	Minimum	Maximum
Cost	45	3,178,028	1,763,374	670,535	5,852,090
Capital_QI	45	872,684	144,655	676,890	1,176,491
Land_QI	45	1,274,286	38,350	1,219,174	1,334,053
Labor_PI	45	0.495	0.401	0.068	1.548
Materials_PI	45	0.684	0.310	0.224	1.109
Energy_PI	45	0.724	0.370	0.250	1.303
Chemicals_PI	45	0.529	0.236	0.165	0.860
Output_QI	45	3,490,990	922,745	1,631,446	5,246,704

The distribution around the mean of the energy prices is 0.370. Following the price of energy is the price of materials with an average of 0.684, followed by the price of chemicals and labor with average price 0.529 and 0.495, respectively. The aggregate output quantities over the 44 years averaged 3,490,990, with an average dispersion of the mean of 922,745. The total cost over the period, 1960-2004 averaged 3,178,028 with a standard deviation of 1,763,374 and an overall minimum cost of 670,535 in 1960 and a maximum cost of 5,852,090 in 2001.

Table 6. Descriptive statistics of the input quantity, input price, and output price variables used in the estimation of restricted profit function for North Dakota's agriculture, 1960-2004

Variable	N	Mean	Std Dev	Minimum	Maximum
Rprofit	45	1046680	460633	244975	1989283
Capital_QI	45	872684	144655	676890	1176491
Land_QI	45	1274286	38350	1219174	1334053
Labor_PI	45	0.495	0.401	0.068	1.548
Materials_PI	45	0.684	0.310	0.224	1.109
Energy_PI	45	0.724	0.370	0.250	1.303
Chemicals_PI	45	0.529	0.236	0.165	0.860
Output_PI	45	0.728	0.218	0.361	1.018

As noted from statistics of the cost function, the average price of energy ranks highest followed by the price of materials, capital, chemicals, labor and land with the lowest average price. However, the price of aggregate outputs is slightly larger than the price of energy (Table 6) at 0.728 with a distribution of 0.218 and a minimum average price of 0.361 in 1960 and a maximum of 1.018 in 1988. The restricted profit function was defined by distinguishing between input prices of variable and fixed inputs (land and capital). The average profit over the 45-year time period was 1,046,680 with an average dispersion of 640,633 and a lowest profit of 244,975 in 1961 and the highest of 1,989,283 in 1992.

## CHAPTER 4. EMPIRICAL MODEL AND RESULTS

The theoretical methodology described in the previous chapter was applied to measure the farm input and output change characterizing North Dakota agriculture from the time period 1960-2004. The models to be estimated include the production, cost, and profit function with primary emphasis on the bias between the traditional OLS procedure and the quantile regression. The different models are briefly presented with a complete depiction of the related variables.

The following input and output categories were used in the estimation of the production, cost, and profit function with prices relative to the level in Alabama in 1996, and implicit quantities in value \$1,000 in 1996 prices of Alabama. Included among the input variables are aggregate price and quantity of inputs, as well as disaggregated prices and quantities of capital excluding land, land, two types of labor (hired and self-employed, and unpaid family labor), aggregate materials, energy and chemicals including pesticides and fertilizers.

### 4.0 Production function for North Dakota's agriculture sector

The production function is estimated by traditional OLS and quantile regression using North Dakota state level data. The empirical representation of the Hick-neutral technical change of the production function as defined in equation 3.1.2 in chapter three can be represented as:

$$\begin{aligned} \ln AO\_QI_t = & \alpha_0 + \alpha_1 \ln cap\_QI_t + \alpha_2 \ln land\_QI_t \\ & + \alpha_3 \ln lab\_QI_t + \alpha_4 \ln mat\_QI_t \\ & + \alpha_5 \ln eng\_QI_t + \alpha_6 \ln chem\_QI_t + \alpha_7 T + \varepsilon_t \end{aligned} \quad (4.1.1)$$

where  $AO_{QI}$ ,  $Cap_{QI}$ ,  $Land_{QI}$ ,  $Lab_{QI}$ ,  $Mat_{QI}$ ,  $Eng_{QI}$ , and  $Chem_{QI}$  and  $T$  characterize aggregate output, capital, land, labor, aggregate materials, energy, chemicals, and technology, respectively, and  $\varepsilon$  represents the error term. Appendix 1 presents a detailed representation of the parameter coefficient, standard errors, t-value and probability for each quantile ranging from 10 to 90 percent. Because quantile regression presents snapshots at different points of a conditional distribution, they represent a parsimonious way of describing the whole distribution (Martins and Pereira, 2004).

The parameters obtained from the traditional OLS estimation expose no statistical significance between the agricultural inputs and aggregate output for the period 1960-2004 using aggregate state-level data as reported in Table 7. Unlike traditional OLS, quantile regression results provide parameter coefficients at each quantile. However, similar to the OLS, quantile regression also reveals no statistical significance between the six input variables and aggregate output production.

Table 7. Quantile regression estimates of the production function for North Dakota's agriculture sector, 1960-2004

Variable	<u>Selected quantiles</u>				OLS
	10	40	60	90	
Intercept	-608.006	-279.828	-311.407	-285.158	-277.8111
Capital_QI	-0.331	0.3394	0.2002	0.4158	0.24792
Land_QI	7.6021	-0.4794	-0.0955	-2.8731	-0.71105
Labor_QI	0.4357	-0.1174	0.2102	-0.1422	-0.04646
Materials_QI	0.6694	0.4151	0.1598	-0.1436	0.53489
Energy_QI	0.2306	0.002	0.0495	0.3328	0.09425
Chemicals_QI	-0.2306	-0.1154	0.0144	-0.168	-0.11482
Year	66.154	38.7423	42.0204	44.4127	38.563

Figure 1 presents a graphical summary of the quantile regression results for each input. Each panel in Figure 1 plots one coordinate of the parameter vector  $\beta(\tau)$  as a



function of  $\tau$ , which takes the value in  $[0,1]$ . The shaded area in each plot represents a 95 percent confidence band.

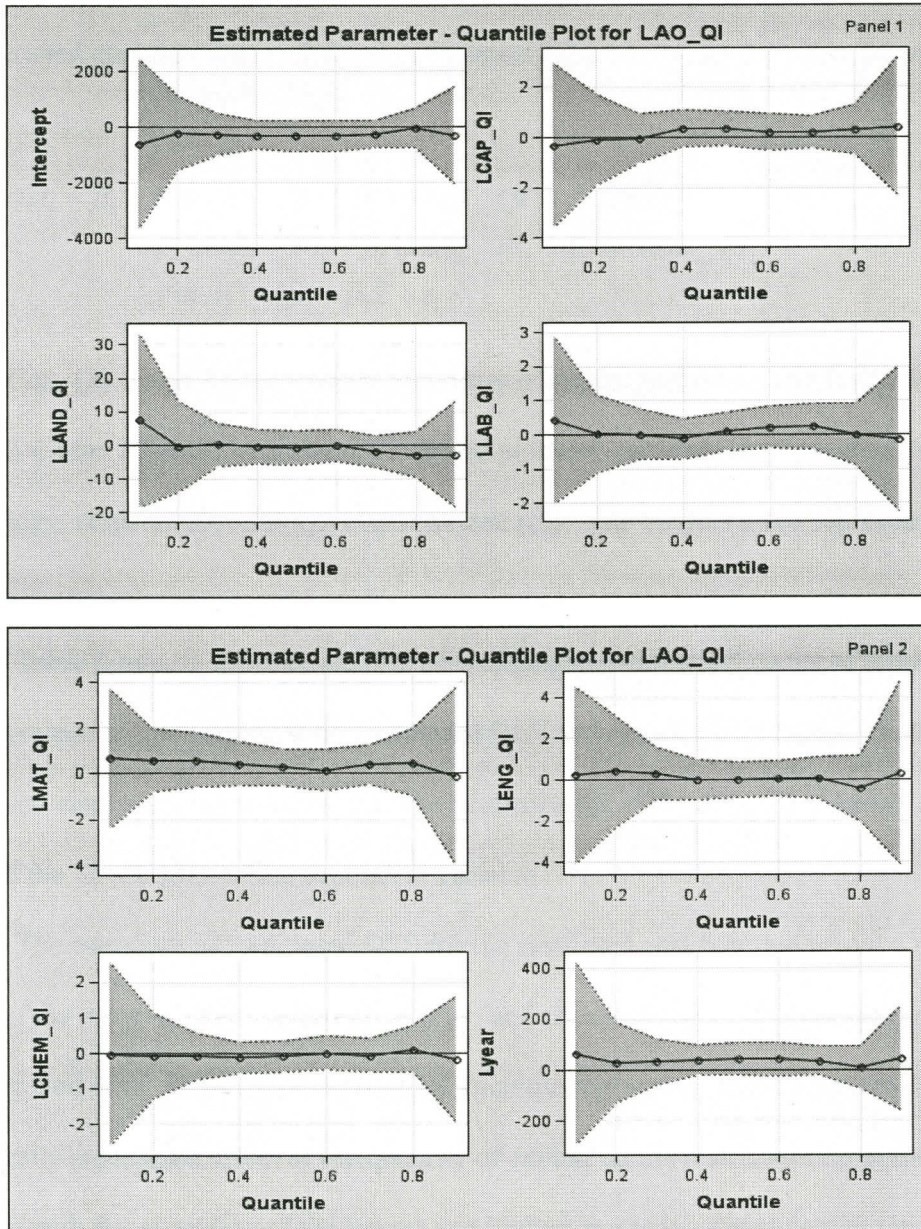


Figure 1. Graphical representation of the quantile regression estimates for the production function in North Dakota's agriculture sector, 1960-2004

#### 4.1 Restricted cost function for North Dakota's agriculture sector

Since the cost was greater than the revenue, the restricted cost function is estimated using state-level data. The empirical representation of the Hick-neutral technical change of the restricted cost function as defined in equation 3.2.2 in chapter three can be represented as:

$$\begin{aligned} \ln RCost_t = & \beta_0 + \beta_1 \ln cap\_QI_t + \beta_2 \ln land\_QI_t + \beta_3 \ln lab\_PI_t \\ & + \beta_4 \ln mat\_PI_t + \beta_5 \ln eng\_PI_t + \beta_6 \ln chem\_PI_t \\ & + \gamma \ln AO\_QI_t + \beta_T T + \varepsilon_t \end{aligned} \quad (4.2.1)$$

where  $Cap\_QI_t$ ,  $Land\_QI_t$  represents quantities of capital and labor, and  $Lab\_PI_t$ ,  $Mat\_PI_t$ ,  $Eng\_PI_t$ ,  $Chem\_PI_t$  and  $T$  represents the prices of labor, materials, energy, chemicals and technology, respectively;  $AO\_QI_t$  characterizes aggregate outputs and  $\varepsilon$  represents the error term.

Table 8 reports the restricted cost function parameter coefficients of input prices and aggregate output estimated by traditional OLS and quantile regression. Quantile regression can be employed to explain the determinants of the dependent variable at any point of the distribution of the dependent variable.

The estimates of traditional OLS for the restricted cost function reveal a significant effect of the price of labor, materials, energy, and chemicals as well as the technology on the restricted cost. Unlike traditional OLS, quantile regression illustrated a positive and statistically significant effect of the quantity of capital on the restricted cost at the 20<sup>th</sup>-80<sup>th</sup> quantile with the exception of the lowest and highest quantile. These results reveal considerable differences between those presented by OLS estimates and the estimates for specific quantiles. For comparison purposes, Table 9 presents the differences between the OLS and quantile regression.

Table 8. Quantile regression estimates of the restricted cost function for North Dakota's agriculture sector, 1960-2004

Variable	Estimated quantiles										OLS
	10	20	30	40	50	60	70	80	90	90	
Intercept	120.873	-153.999**	-107.936	-106.581	-125.806	-144.836*	-131.554	-89.740	-23.389	-178.159	
Capital_QI	0.836	0.654***	0.615**	0.637***	0.621***	0.665***	0.697***	0.574***	0.551	0.217	
Land_QI	-2.312	-0.623	-0.959	-0.707	-0.813	-1.103	-1.143	0.057	-0.655	2.517	
Labor_PI	0.304*	0.266***	0.268***	0.264***	0.269***	0.308***	0.319***	0.318***	0.314***	0.109	
Materials_PI	-0.005	0.372	0.530***	0.509***	0.501***	0.468***	0.463***	0.438***	0.638**	0.940	
Energy_PI	0.431	0.337**	0.224	0.281**	0.287***	0.329***	0.350***	0.378***	0.276	0.304	
Chemical_PI	0.091	-0.084	-0.080	-0.088	-0.104	-0.177	-0.187	-0.073	-0.124	-0.251	
Output_QI	0.067	0.057	0.071	0.078	0.052	-0.021	-0.019	0.016	0.064	0.0561	
Year	20.579	22.162***	16.765	16.068	18.879*	21.988**	20.253	12.688	5.216	20.232	

Note 1. Single, double and triple asterisks indicate significance at 10, 5, and 1 percent level, respectively.

Note 2. Figures rounded off to three decimal places.

Table 9. Differences between the ordinary least squares and quantile regression estimates for the restricted cost function

Variable	Estimated quantiles									
	OLS	10	20	30	40	50	60	70	80	90
Intercept	57.286	57.286	24.160	70.223	71.578	52.353	33.323	46.605	88.419	154.70
Capital_QI	-0.619	-0.619	-0.438	-0.398	-0.420	-0.404	-0.448	-0.481	-0.358	-0.335
Land_QI	0.205	0.205	1.894	3.47596	1.811	1.704	1.414	1.374	2.461	1.862
Labor_PI	-0.194	-0.194	-0.157	-0.159	-0.155	-0.160	-0.199	-0.209	-0.209	-0.205
Materials_PI	0.935	0.935	0.569	0.411	0.431	0.440	0.472	0.478	0.501	0.302
Energy_PI	-0.127	-0.127	-0.033	0.080	0.023	0.017	-0.025	-0.046	-0.074	0.028
Chemical_PI	0.160	0.160	0.167	0.171	0.164	0.147	0.074	0.064	0.178	0.127
Output_QI	-0.011	-0.011	-0.001	-0.0146	-0.022	0.004	0.035	0.037	0.040	-0.008
Year	-0.347	-0.347	-1.930	3.467	4.164	1.353	-1.756	-0.021	7.543	15.016

Note 1. Figures rounded off to three decimal places.

While the traditional OLS allows the effect of restricted cost to be estimated at the mean of the conditional distribution, it is important to examine the relationship at different points of the conditional distribution function. Quantile regression facilitates such an analysis, depicting the effect of each variable at precise quantiles, as illustrated by each panel in Fig 2, 3 and 4. Figure 2 illustrates the graphical representation of table 8; the first panel in Figure 2 reports the intercepts estimated through quantile regression and their 95% confidence interval. Estimates from the traditional OLS and quantile regression reveal a negative intercept at the mean as well as through quantiles 10<sup>th</sup> – 90<sup>th</sup>. Figure 2 also illustrates a marginal increasing trend in the price of labor, with a lower effect at the lowest quantiles and increasing towards the 90<sup>th</sup> quantile. However, less confidence can be associated to both extremes.

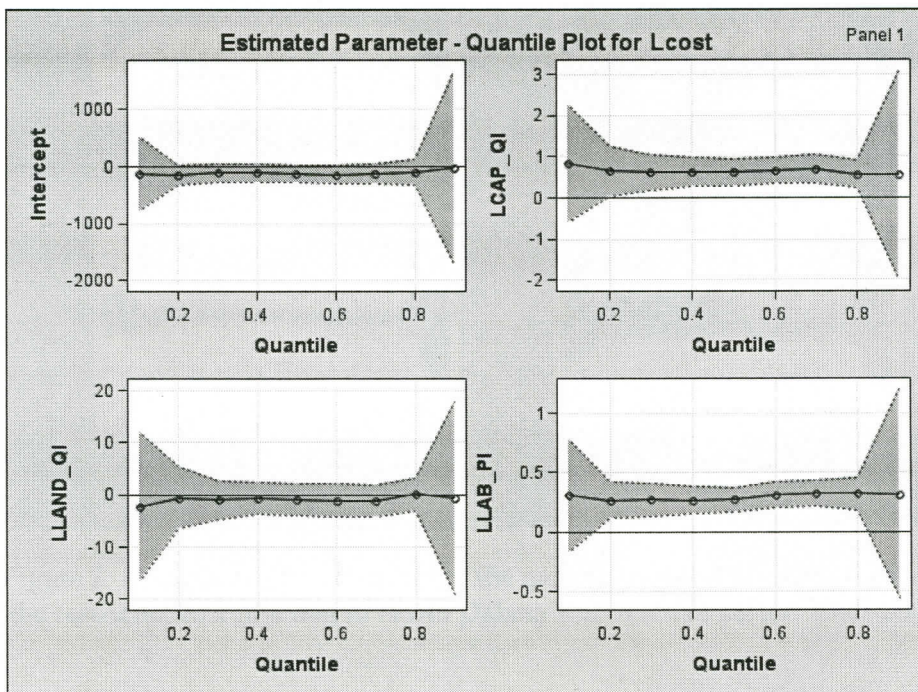


Figure 2. Graphical representation of the quantile regression estimates for the restricted cost function in North Dakota’s agriculture sector, 1960-2004

Similar to the price of labor, confidence also can be associated with the highest price of materials; Figure 3 reveals a statistical significant effect across each quantile for LMAT\_PI except at the 10<sup>th</sup>, 20<sup>th</sup> and 30<sup>th</sup> quantile. The parameter coefficient of materials indicates a positive and significant effect on the restricted cost. Quantile regression, however, reveals no significance between the price of materials and the total restricted cost at the lower quantiles but becomes significant at the higher quantiles. This suggests that the restricted cost at the bottom of the conditional distribution of the price of materials appears not to be benefiting from changes in price.

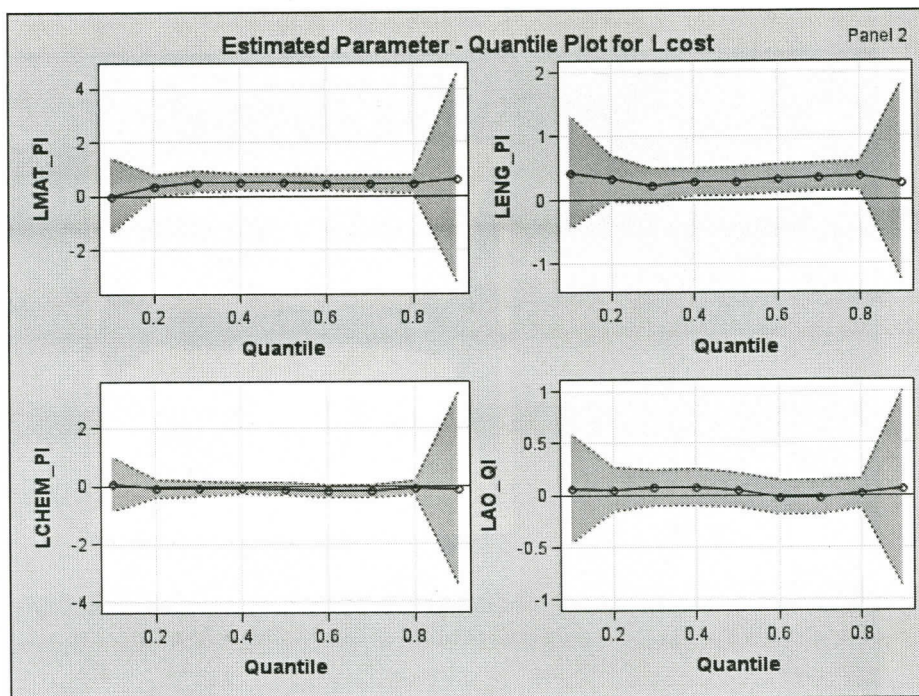


Figure 3. Graphical representation of the quantile regression estimates for the restricted cost function in North Dakota's agriculture sector, 1960-2004

The second panel in figure 3 reveals a statistical significance in the price of energy across most quantiles with the exception of 10<sup>th</sup>, 30<sup>th</sup> and 90<sup>th</sup> quantiles. The energy crisis -

- characterized by the rapid and drastic price increases as well as threatened shortages -- is significantly increasing the overall total cost of agricultural production in North Dakota due to higher input costs. While the traditional OLS confirm that an increase in the price of energy would result in a 0.30385 increase in total price, quantile regression, however, depicts a flatter positive marginal decrease at the lower quantiles implying that the price of energy is less sensitive at the lower quantiles as opposed to the higher quantiles. The OLS estimator, by focusing only on the central tendency of the distribution (i.e., the mean) does not allow for the possibility that the impact of explanatory variables can be differ across quantiles.

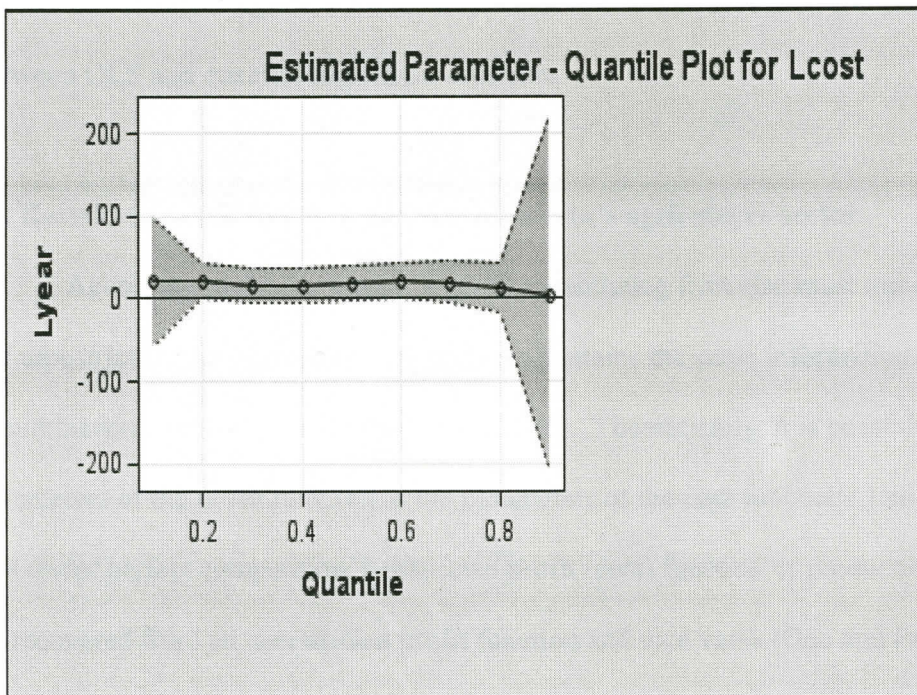


Figure 4. Graphical representation of the quantile regression estimates for the restricted cost function in North Dakota's agriculture sector, 1960-2004

The parameter coefficient on time variable used as a proxy for technology indicated a positive but not significant effect on the restricted cost. Quantile regression reveals no statistical significance at the 10<sup>th</sup>, 30<sup>th</sup>, 40<sup>th</sup>, 60<sup>th</sup>, 70<sup>th</sup>, 80<sup>th</sup> and 90<sup>th</sup> quantiles as indicated in table 9 and graphically presented in Figure 4. These results present major differences between OLS estimates and quantile regression estimates where only three out of the nine quantiles appear to significantly affect total cost.

Though most of the significant variables present a positive relationship with total cost, results from the OLS procedure depicts an inverse relationship between the price of chemicals, and the restricted cost of production. On the other hand, quantile regression depicts no statistical significance of the price of chemicals, revealing the major differences between OLS and quantile regression estimates.

#### **4.2 Restricted profit function for North Dakota's agriculture sector**

According to duality theory, a profit maximizing firm also must minimize cost, and the unrestricted profit maximization problem contains the same information as the cost minimization problem (Mas-Collel, et al., 1995). Theoretically, it is possible to link the parameters of the profit function to the parameters of the cost function. Lau (1976) proves that under perfect competition, a restricted profit (cost) function or production function can be recovered from an unrestricted profit function and vice versa (Gao and Featherstone, 2006). This thesis does not treat all inputs as variable and assumes both the land and capital input as fixed; accordingly a restricted profit function is used to model the relation between input price, fixed inputs quantity, and output price.

The empirical representation of the Hick-neutral technical change of the restricted profit function as defined in equation 3.3.2 in chapter three can be represented as:

$$\begin{aligned} \ln Rprofit_t = & \beta_0 + \beta_1 \ln cap\_QI_t + \beta_2 \ln land\_QI_t + \beta_3 \ln lab\_PI_t \\ & + \beta_4 \ln mat\_PI_t + \beta_5 \ln eng\_PI_t + \beta_6 \ln chem\_PI_t \\ & + \delta_1 \ln AO\_PI_t + \beta_T T + \varepsilon_t \end{aligned} \quad (4.3.1)$$

where  $Cap\_QI_t$  and  $Land\_QI_t$  represent quantities of land and capital,  $Lab\_PI_t$ ,  $Mat\_PI_t$ ,  $Eng\_PI_t$ ,  $Chem\_PI_t$ , and  $T$  represent price of labor, materials, energy, chemicals and technology, respectively;  $AO\_PI_t$  represents the price of aggregate outputs and  $\varepsilon$  represents the error term.

Table 10 reports the parameter coefficients of restricted profit function estimated by traditional OLS and quantile regression. While both fixed input quantities (capital, and land) reveal no statistical significance on the maximization of profits, the variable inputs indicate little statistical significance with the exception of the price of labor.

Table 10. Quantile regression estimates of the restricted profit function for North Dakota's agriculture sector, 1960-2004

Variable	Selected quantiles						OLS
	10	30	50	60	70	80	
Intercept	-622.436	-336.043	-258.978	-110.568	-271.076	-308.804	-205.103
Capital_QI	-0.136	0.4193	0.3943	0.289	0.7284	0.833	0.678
Land_QI	8.846	-1.950	-3.557	-4.559	-5.861	-6.307	-5.343
Labor_PI	-0.343	-0.268**	-0.317***	-0.254	-0.436***	-0.460**	-0.255**
Materials_PI	-0.5368	-1.023	-0.805	-1.015	-0.191	-0.152	-0.228
Energy_PI	0.177	-0.162	-0.212	-0.101	-0.468	-0.527	-0.471
Chemical_PI	0.117	0.156	0.171	0.240	-0.031	-0.018	-0.093
Output_PI	0.193	0.983**	0.844**	0.942***	0.637*	0.576	0.592**
Year	65.806	47.092	39.968	22.482	45.213	50.815	35.655*

Note 1 Single, double and triple asterisks indicate significance at 10, 5, and 1 percent levels, respectively.

Note 2. Figures rounded off to three decimal points.



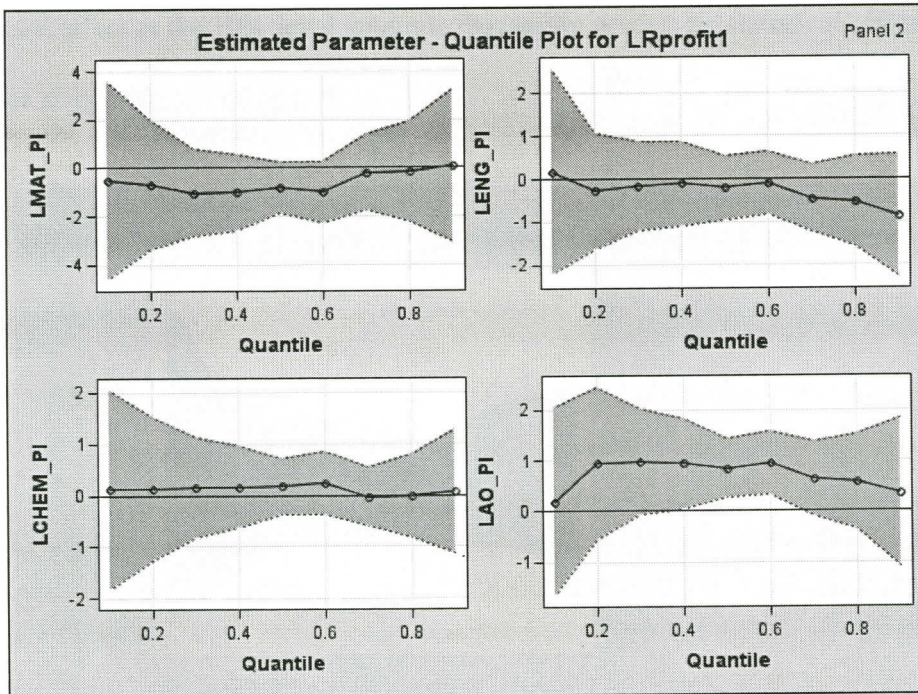
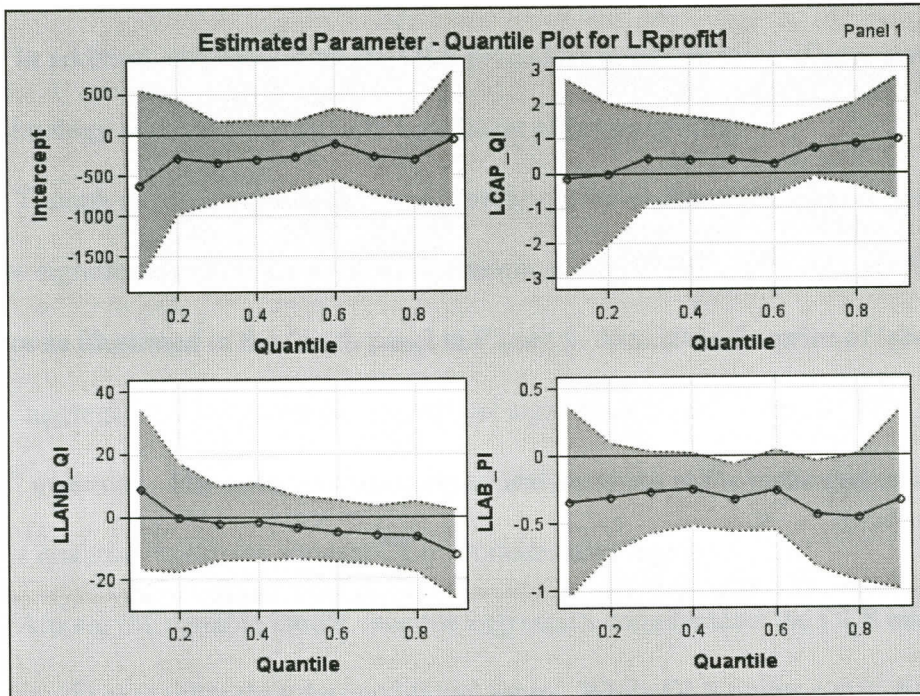


Figure 5. Graphical representation of the quantile regression estimates for the restricted profit function in North Dakota's agriculture sector

In addition, estimates from the OLS procedure posit that price of aggregate output and technology also has a statistically significant effect on profits.

Though quantile regression reveal similar estimates, the price of labor presents a negative significant effect on profit maximization at the 30<sup>th</sup>, 40<sup>th</sup>, 50<sup>th</sup>, 70<sup>th</sup>, and 80<sup>th</sup> quantiles as illustrated in the fourth panel in Figure 5. Similar to the price of labor, the price of aggregate outputs presents a statistical significance only at the 30<sup>th</sup>, 40<sup>th</sup>, 50<sup>th</sup>, 60<sup>th</sup> and 70<sup>th</sup> quantiles. The statistical significance and variation in the parameter estimates from the quantile regression suggest OLS estimates may be misleading.

Among the variable inputs, quantile regression complements the OLS inability to depict significance other than the conditional mean. While OLS points a statistical significant effect at the 10% level, quantile regression reveals no statistical effect across quantiles as illustrated in Figure 6.

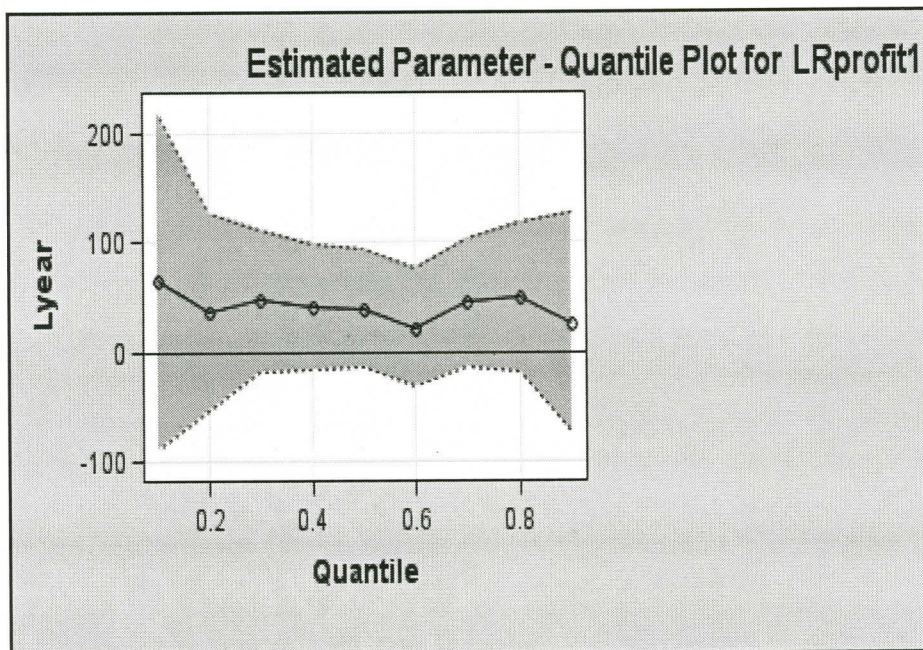


Figure 6. Graphical representation of the quantile regression estimates for the restricted profit function in North Dakota's agriculture sector

Table 11. Differences between the ordinary least square and quantile regression estimates for the restricted profit function

Variable	OLS	Selected quantiles					
		10	30	50	60	70	80
Intercept	-205.103	-417.333	-130.94	-53.875	94.535	-65.973	-103.701
Capital_QI	0.678	0.543	0.259	0.284	0.390	-0.050	-0.155
Land_QI	-5.343	-3.503	3.392	1.785	0.783	-0.518	-0.965
Labor_PI	-0.255**	-0.088	-0.013	-0.062	0.001	-0.181	-0.205
Materials_PI	-0.228	-0.308	-0.794	-0.576	-0.787	0.037	0.076
Energy_PI	-0.471	0.295	0.309	0.259	0.370	0.0040	-0.055
Chemical_PI	-0.093	-0.024	-0.063	-0.078	-0.147	0.062	0.075
Output_PI	0.592**	0.400	-0.391	-0.252	-0.349	-0.045	0.016

Each panel in Figures 5 and 6 plots each variable across quantiles 10<sup>th</sup> -90<sup>th</sup> depicting beneficial variables aiding in maximizing profits. Table 11 presents the unaccounted difference between the OLS and quantile regression. Appendix 2 reprints the quantile regression estimates of the input and output prices used in agricultural production in North Dakota since 1960-2004 at each particular quantile.

## CHAPTER 5. CONCLUSIONS AND DISCUSSIONS

Previously the contribution of capital, land, labor, materials, energy, and chemical inputs quantities and output quantity using the primal production function; contribution of capital quantity, land quantity, output quantity, labor price, materials price, energy price, and chemical price to cost using the dual restricted cost function; and the contribution of capital quantity, land quantity, labor price, materials price, energy price, chemical price, output price to profit using the dual restricted profit function had been examined for different sectors including agriculture in the United States. This thesis estimates the production, restricted cost, and restricted profit functions using North Dakota agriculture sector data from 1960-2004.

Second, this research utilizes quantile regression methods (Koenker and Bassett, 1978) to examine the relationship at different quantiles. Even though quantile regression has been developed and applied to cross-section data, here the quantile regression is applied to time-series data to examine the shape and the linear or non-linear relationship between the endogenous and exogenous variables in the estimation of the production, cost, and profit functions.

To summarize, the production function parameters obtained from the traditional OLS and quantile regression estimation reveal no statistical significance between the agricultural inputs and aggregate output for the period, 1960-2004 using aggregate state-level data for North Dakota agriculture. Similar results were indicated by restricted profit function wherein, the traditional OLS estimates reveal a statistical significance between the price of labor as well as the price of aggregate output. In addition, whereas OLS exposes a positive statistical significant relationship between profits and technology, quantile

regression depicts the price of labor and aggregate outputs as the variables having a statistical effect on profits at quantiles 30<sup>th</sup>-80<sup>th</sup> with no significance at either extreme.

Results from the traditional OLS and the quantile regression reveal highly significant variables between input prices and output. Whereas OLS depicts no statistical significance between the quantities of capital and labor, it does, however, reveal high significance between the price of labor, materials, energy, chemicals as well as technology, with no significance between profits and aggregate output. Slight differences can be noted upon comparing the results from the traditional OLS and the estimates from the quantile regression. With the exception of the price of labor, which is significant across each quantile, quantile regression depicts explicit quantiles (20<sup>th</sup> – 80<sup>th</sup>) wherein the quantity of capital used significantly affects total profits. An additional major distinction between the traditional OLS and quantile regression is the disparity presented by the OLS depicting a significant effect between the price of chemicals and total profits. Quantile regression, however, does not reveal any statistical significance between the price of chemicals and total profits across any quantile. In addition, quantile regression reveals a clearer representation depicting defined quantiles wherein each variable maintain a significant effect on profits.

Overall results seem to be reasonable and indicate that the cost function, in general, leads to valuable methods for of estimation of production parameters. In conclusion, results identify the restricted cost function as the most appropriate model for the data used in the analysis. Nonetheless, additional research is needed to understand the structure of production in the Midwest region, thus expanding the research to the broader Midwest states or the entire United States. Additional variables can be included to an extended time

period which would allow for the estimation of flexible functional forms, and thus exploring the importance of the choice of functional forms and quantifying the magnitude of the errors which may arise from the use of incorrect functional forms as noted by Bockstael and McConnell (1986); Ziemer, Musser, and Hill (1980); Sutherlan (1982). The short data set was a major limitation affecting the choice of functional form applied in the estimation of the farm economic structure for North Dakota from 1960-2004.

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## APPENDICES

Appendix I. Quantile regression estimates for the production function for North Dakota's agriculture sector, 1960-2004

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.1	Intercept	-608.006	3163.688	-7018.25	5802.235	-0.19	0.8486
0.1	LCAP_QI	-0.331	4.1734	-8.7872	8.1251	-0.08	0.9372
0.1	LLAND_QI	7.6021	33.3887	-60.0499	75.2541	0.23	0.8211
0.1	LLAB_QI	0.4357	2.9439	-5.5292	6.4006	0.15	0.8831
0.1	LMAT_QI	0.6694	4.8136	-9.0838	10.4227	0.14	0.8901
0.1	LENG_QI	0.2306	6.3027	-12.5398	13.0011	0.04	0.971
0.1	LCHEM_QI	-0.0251	2.9017	-5.9045	5.8543	-0.01	0.9932
0.1	Lyear	66.154	372.4871	-688.577	820.8845	0.18	0.86
0.2	Intercept	-208.152	428.114	-1075.59	659.289	-0.49	0.6297
0.2	LCAP_QI	-0.0832	0.5324	-1.162	0.9955	-0.16	0.8766
0.2	LLAND_QI	-0.2697	4.5162	-9.4203	8.881	-0.06	0.9527
0.2	LLAB_QI	0.0168	0.4384	-0.8715	0.9051	0.04	0.9696
0.2	LMAT_QI	0.5733	0.5112	-0.4625	1.6091	1.12	0.2693
0.2	LENG_QI	0.4187	0.7298	-1.0601	1.8975	0.57	0.5697
0.2	LCHEM_QI	-0.0618	0.3419	-0.7546	0.631	-0.18	0.8575
0.2	Lyear	28.3552	49.1209	-71.1732	127.8836	0.58	0.5673
0.3	Intercept	-256.863	356.3547	-978.906	465.18	-0.72	0.4756
0.3	LCAP_QI	-0.0349	0.4567	-0.9603	0.8904	-0.08	0.9395
0.3	LLAND_QI	0.1721	2.9514	-5.8079	6.1522	0.06	0.9538
0.3	LLAB_QI	0.0038	0.3008	-0.6056	0.6133	0.01	0.9899
0.3	LMAT_QI	0.5881	0.4159	-0.2545	1.4307	1.41	0.1657
0.3	LENG_QI	0.3184	0.5433	-0.7824	1.4193	0.59	0.5613
0.3	LCHEM_QI	-0.086	0.3012	-0.6963	0.5244	-0.29	0.777
0.3	Lyear	34.0666	42.0694	-51.1742	119.3073	0.81	0.4232
0.4	Intercept	-279.828	275.9322	-838.919	279.2643	-1.01	0.3171
0.4	LCAP_QI	0.3394	0.3963	-0.4636	1.1425	0.86	0.3973
0.4	LLAND_QI	-0.4794	2.4521	-5.4478	4.4891	-0.2	0.8461
0.4	LLAB_QI	-0.1174	0.306	-0.7374	0.5026	-0.38	0.7034
0.4	LMAT_QI	0.4151	0.3906	-0.3762	1.2065	1.06	0.2947
0.4	LENG_QI	0.002	0.4375	-0.8845	0.8884	0	0.9965
0.4	LCHEM_QI	-0.1154	0.2532	-0.6284	0.3976	-0.46	0.6511
0.4	Lyear	38.7423	32.5892	-27.2896	104.7743	1.19	0.2421
0.5	Intercept	-321.109	319.0208	-967.507	325.2884	-1.01	0.3207
0.5	LCAP_QI	0.3558	0.3741	-0.4022	1.1138	0.95	0.3477
0.5	LLAND_QI	-0.7757	2.7147	-6.2762	4.7249	-0.29	0.7767
0.5	LLAB_QI	0.0971	0.3271	-0.5656	0.7598	0.3	0.7682
0.5	LMAT_QI	0.2971	0.448	-0.6107	1.2049	0.66	0.5113
0.5	LENG_QI	0.0187	0.4896	-0.9734	1.0108	0.04	0.9697
0.5	LCHEM_QI	-0.0917	0.2688	-0.6364	0.453	-0.34	0.735



Appendix 1. (Continued)

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.5	Lyear	44.4559	37.2806	-31.0818	119.9936	1.19	0.2407
0.6	Intercept	-311.407	258.4783	-835.133	212.3203	-1.2	0.2359
0.6	LCAP_QI	0.2002	0.3605	-0.5304	0.9307	0.56	0.5821
0.6	LLAND_QI	-0.0955	2.1844	-4.5216	4.3306	-0.04	0.9654
0.6	LLAB_QI	0.2102	0.2893	-0.376	0.7964	0.73	0.4721
0.6	LMAT_QI	0.1598	0.4246	-0.7006	1.0201	0.38	0.7088
0.6	LENG_QI	0.0495	0.4401	-0.8422	0.9411	0.11	0.9111
0.6	LCHEM_QI	0.0144	0.2388	-0.4694	0.4982	0.06	0.9522
0.6	Lyear	42.0204	30.4053	-19.5867	103.6275	1.38	0.1753
0.7	Intercept	-252.596	252.9452	-765.111	259.92	-1	0.3245
0.7	LCAP_QI	0.1891	0.3961	-0.6135	0.9917	0.48	0.6359
0.7	LLAND_QI	-1.7545	2.1388	-6.088	2.5791	-0.82	0.4173
0.7	LLAB_QI	0.2378	0.3637	-0.4991	0.9747	0.65	0.5172
0.7	LMAT_QI	0.3869	0.4906	-0.6071	1.3809	0.79	0.4354
0.7	LENG_QI	0.0865	0.58	-1.0886	1.2617	0.15	0.8822
0.7	LCHEM_QI	-0.0655	0.2327	-0.5369	0.4059	-0.28	0.7799
0.7	Lyear	36.9628	29.879	-23.5779	97.5035	1.24	0.2239
0.8	Intercept	-27.8971	299.2698	-634.275	578.4811	-0.09	0.9262
0.8	LCAP_QI	0.2811	0.3615	-0.4512	1.0135	0.78	0.4416
0.8	LLAND_QI	-2.8094	2.6515	-8.1819	2.563	-1.06	0.2962
0.8	LLAB_QI	-0.0016	0.345	-0.7007	0.6974	0	0.9963
0.8	LMAT_QI	0.4788	0.6604	-0.8593	1.8168	0.72	0.473
0.8	LENG_QI	-0.4026	0.6879	-1.7965	0.9913	-0.59	0.5619
0.8	LCHEM_QI	0.1191	0.3026	-0.494	0.7321	0.39	0.6962
0.8	Lyear	9.9118	35.3108	-61.6346	81.4582	0.28	0.7805
0.9	Intercept	-285.158	410.8763	-1117.67	547.3566	-0.69	0.492
0.9	LCAP_QI	0.4158	0.6149	-0.8302	1.6617	0.68	0.5032
0.9	LLAND_QI	-2.8731	4.0876	-11.1553	5.4091	-0.7	0.4865
0.9	LLAB_QI	-0.1422	0.4167	-0.9864	0.7021	-0.34	0.7349
0.9	LMAT_QI	-0.1436	1.1798	-2.5342	2.247	-0.12	0.9038
0.9	LENG_QI	0.3328	1.1102	-1.9167	2.5822	0.3	0.766
0.9	LCHEM_QI	-0.168	0.3778	-0.9334	0.5974	-0.44	0.6591
0.9	Lyear	44.4127	48.1816	-53.2125	142.0379	0.92	0.3626

Appendix 2. Quantile regression estimates for ct Restricted Cost Function for North Dakota's agriculture sector, 1960-2004

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.1	Intercept	-120.87	264.177	-656.65	414.902	-0.46	0.65
0.1	LCAP_QI	0.8358	0.6658	-0.5146	2.1861	1.26	0.2175
0.1	LLAND_QI	-2.3119	6.336	-15.162	10.5381	-0.36	0.7173
0.1	LLAB_PI	0.3035	0.1757	-0.0527	0.6598	1.73	0.0926
0.1	LMAT_PI	-0.0052	0.5836	-1.1889	1.1784	-0.01	0.9929
0.1	LENG_PI	0.4313	0.3807	-0.3407	1.2033	1.13	0.2647
0.1	LCHEM_PI	0.0912	0.3969	-0.7137	0.8961	0.23	0.8195
0.1	LAO_QI	0.0668	0.2281	-0.3959	0.5294	0.29	0.7714
0.1	Lyear	20.5793	32.2154	-44.757	85.9151	0.64	0.527
0.2	Intercept	-154	77.4139	-311	3.0041	-1.99	0.0543
0.2	LCAP_QI	0.6541	0.2671	0.1124	1.1959	2.45	0.0193
0.2	LLAND_QI	-0.6231	2.4876	-5.6682	4.4221	-0.25	0.8037
0.2	LLAB_PI	0.2664	0.0657	0.1331	0.3996	4.05	0.0003
0.2	LMAT_PI	0.3715	0.2384	-0.1119	0.8549	1.56	0.1278
0.2	LENG_PI	0.3369	0.1654	0.0014	0.6723	2.04	0.0491
0.2	LCHEM_PI	-0.0842	0.1732	-0.4355	0.2672	-0.49	0.6299
0.2	LAO_QI	0.0567	0.0895	-0.1248	0.2382	0.63	0.5304
0.2	Lyear	22.162	9.6137	2.6647	41.6594	2.31	0.027
0.3	Intercept	-107.94	84.2072	-278.72	62.8439	-1.28	0.2081
0.3	LCAP_QI	0.615	0.2572	0.0934	1.1367	2.39	0.0221
0.3	LLAND_QI	-0.9586	2.4491	-5.9256	4.0084	-0.39	0.6978
0.3	LLAB_PI	0.2682	0.0702	0.1258	0.4106	3.82	0.0005
0.3	LMAT_PI	0.5298	0.2058	0.1123	0.9472	2.57	0.0143
0.3	LENG_PI	0.2241	0.16	-0.1004	0.5486	1.4	0.1699
0.3	LCHEM_PI	-0.0802	0.1416	-0.3674	0.2069	-0.57	0.5744
0.3	LAO_QI	0.0707	0.094	-0.1201	0.2614	0.75	0.4573
0.3	Lyear	16.7648	10.4997	-4.5296	38.0592	1.6	0.1191
0.4	Intercept	-106.58	94.0652	-297.35	84.1924	-1.13	0.2647
0.4	LCAP_QI	0.637	0.1861	0.2597	1.0144	3.42	0.0016
0.4	LLAND_QI	-0.7068	2.012	-4.7873	3.3737	-0.35	0.7274
0.4	LLAB_PI	0.2641	0.0621	0.1381	0.3901	4.25	0.0001
0.4	LMAT_PI	0.5093	0.1613	0.1821	0.8365	3.16	0.0032
0.4	LENG_PI	0.2809	0.1278	0.0217	0.5401	2.2	0.0345
0.4	LCHEM_PI	-0.0876	0.1139	-0.3186	0.1434	-0.77	0.4467
0.4	LAO_QI	0.078	0.0911	-0.1068	0.2628	0.86	0.3976
0.4	Lyear	16.0683	10.9066	-6.0514	38.188	1.47	0.1494
0.5	Intercept	-125.81	84.2912	-296.76	45.1443	-1.49	0.1443
0.5	LCAP_QI	0.6208	0.1705	0.2751	0.9665	3.64	0.0008
0.5	LLAND_QI	-0.8133	1.5325	-3.9214	2.2948	-0.53	0.5989
0.5	LLAB_PI	0.2691	0.0583	0.1509	0.3872	4.62	<.0001

Appendix 2. (Continued)

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.5	LMAT_PI	0.5009	0.1391	0.2187	0.7831	3.6	0.001
0.5	LENG_PI	0.2868	0.1067	0.0704	0.5032	2.69	0.0108
0.5	LCHEM_PI	-0.1037	0.1032	-0.313	0.1057	-1	0.322
0.5	LAO_QI	0.052	0.0778	-0.1059	0.2099	0.67	0.5084
0.5	Lyear	18.8788	10.969	-3.3673	41.1249	1.72	0.0938
0.6	Intercept	-144.84	80.4203	-307.94	18.2643	-1.8	0.0801
0.6	LCAP_QI	0.6649	0.1725	0.315	1.0148	3.85	0.0005
0.6	LLAND_QI	-1.1034	1.628	-4.4052	2.1983	-0.68	0.5022
0.6	LLAB_PI	0.3081	0.0577	0.191	0.4252	5.34	<.0001
0.6	LMAT_PI	0.4681	0.1702	0.1229	0.8134	2.75	0.0093
0.6	LENG_PI	0.3291	0.1136	0.0987	0.5595	2.9	0.0064
0.6	LCHEM_PI	-0.1774	0.1106	-0.4017	0.0469	-1.6	0.1175
0.6	LAO_QI	-0.0211	0.0989	-0.2217	0.1795	-0.21	0.8324
0.6	Lyear	21.9881	10.952	-0.2235	44.1997	2.01	0.0522
0.7	Intercept	-131.55	95.995	-326.24	63.1329	-1.37	0.179
0.7	LCAP_QI	0.6973	0.1765	0.3394	1.0551	3.95	0.0003
0.7	LLAND_QI	-1.1434	1.4633	-4.111	1.8242	-0.78	0.4397
0.7	LLAB_PI	0.3186	0.055	0.2072	0.4301	5.8	<.0001
0.7	LMAT_PI	0.4627	0.1647	0.1287	0.7967	2.81	0.008
0.7	LENG_PI	0.3501	0.1176	0.1117	0.5885	2.98	0.0052
0.7	LCHEM_PI	-0.187	0.1233	-0.4372	0.0631	-1.52	0.1382
0.7	LAO_QI	-0.0192	0.0853	-0.1921	0.1537	-0.22	0.8233
0.7	Lyear	20.2528	13.0804	-6.2755	46.7811	1.55	0.1303
0.8	Intercept	-89.74	117.65	-328.35	148.866	-0.76	0.4506
0.8	LCAP_QI	0.5742	0.185	0.199	0.9495	3.1	0.0037
0.8	LLAND_QI	0.0565	1.5791	-3.146	3.2591	0.04	0.9717
0.8	LLAB_PI	0.3181	0.066	0.1841	0.452	4.82	<.0001
0.8	LMAT_PI	0.4379	0.1662	0.1009	0.775	2.63	0.0123
0.8	LENG_PI	0.3776	0.1109	0.1526	0.6025	3.4	0.0016
0.8	LCHEM_PI	-0.073	0.1292	-0.3351	0.1892	-0.56	0.5759
0.8	LAO_QI	0.0158	0.0822	-0.151	0.1826	0.19	0.849
0.8	Lyear	12.6884	15.8592	-19.476	44.8523	0.8	0.4289
0.9	Intercept	-23.389	168.176	-364.47	317.688	-0.14	0.8902
0.9	LCAP_QI	0.5511	0.3294	-0.1169	1.2192	1.67	0.103
0.9	LLAND_QI	-0.655	2.5021	-5.7296	4.4195	-0.26	0.795
0.9	LLAB_PI	0.3139	0.0957	0.1198	0.5079	3.28	0.0023
0.9	LMAT_PI	0.6381	0.2764	0.0775	1.1988	2.31	0.0268
0.9	LENG_PI	0.2763	0.1685	-0.0654	0.618	1.64	0.1098
0.9	LCHEM_PI	-0.1242	0.2213	-0.573	0.3245	-0.56	0.578
0.9	LAO_QI	0.0637	0.1276	-0.1951	0.3225	0.5	0.6208
0.9	Lyear	5.216	24.0925	-43.646	54.0778	0.22	0.8298

Appendix 3. Quantile regression estimates for the Restricted Profit Function for North Dakota's agriculture sector, 1960-2004

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.1	Intercept	-622.44	700.59	-2043.3	798.427	-0.89	0.3802
0.1	LCAP_QI	-0.1358	1.7175	-3.6192	3.3475	-0.08	0.9374
0.1	LLAND_QI	8.8456	14.7582	-21.085	38.7765	0.6	0.5527
0.1	LLAB_PI	-0.343	0.4372	-1.2297	0.5438	-0.78	0.4379
0.1	LMAT_PI	-0.5358	2.3455	-5.2928	4.2212	-0.23	0.8206
0.1	LENG_PI	0.1767	1.1069	-2.0682	2.4217	0.16	0.874
0.1	LCHEM_PI	0.1174	1.5089	-2.9427	3.1775	0.08	0.9384
0.1	LAO_PI	0.1926	1.1607	-2.1614	2.5466	0.17	0.8691
0.1	Lyear	65.8058	91.1704	-119.1	250.708	0.72	0.4751
0.2	Intercept	-277.11	436.237	-1161.8	607.618	-0.64	0.5293
0.2	LCAP_QI	-0.0079	1.092	-2.2227	2.2068	-0.01	0.9942
0.2	LLAND_QI	-0.2963	9.4591	-19.48	18.8877	-0.03	0.9752
0.2	LLAB_PI	-0.31	0.2339	-0.7843	0.1644	-1.33	0.1934
0.2	LMAT_PI	-0.6749	1.5724	-3.8639	2.5141	-0.43	0.6703
0.2	LENG_PI	-0.273	0.8323	-1.9611	1.415	-0.33	0.7448
0.2	LCHEM_PI	0.1235	0.8503	-1.6009	1.8479	0.15	0.8853
0.2	LAO_PI	0.9447	0.6553	-0.3843	2.2737	1.44	0.158
0.2	Lyear	37.0346	55.8997	-76.335	150.405	0.66	0.5119
0.3	Intercept	-336.04	222.982	-788.27	116.185	-1.51	0.1405
0.3	LCAP_QI	0.4193	0.5862	-0.7696	1.6082	0.72	0.4791
0.3	LLAND_QI	-1.9502	5.4	-12.902	9.0014	-0.36	0.7201
0.3	LLAB_PI	-0.2682	0.1228	-0.5171	-0.0192	-2.18	0.0355
0.3	LMAT_PI	-1.0226	0.7652	-2.5745	0.5292	-1.34	0.1898
0.3	LENG_PI	-0.1622	0.4659	-1.1071	0.7827	-0.35	0.7298
0.3	LCHEM_PI	0.1559	0.3911	-0.6373	0.9492	0.4	0.6925
0.3	LAO_PI	0.9831	0.4384	0.094	1.8722	2.24	0.0312
0.3	Lyear	47.0919	29.516	-12.769	106.953	1.6	0.1193
0.4	Intercept	-295.7	200.234	-701.79	110.392	-1.48	0.1484
0.4	LCAP_QI	0.4168	0.5409	-0.6801	1.5138	0.77	0.4459
0.4	LLAND_QI	-1.6399	4.9892	-11.758	8.4786	-0.33	0.7443
0.4	LLAB_PI	-0.2469	0.115	-0.4802	-0.0136	-2.15	0.0386
0.4	LMAT_PI	-1.0037	0.6835	-2.3899	0.3825	-1.47	0.1506
0.4	LENG_PI	-0.1141	0.4272	-0.9804	0.7523	-0.27	0.791
0.4	LCHEM_PI	0.1663	0.315	-0.4726	0.8051	0.53	0.6008
0.4	LAO_PI	0.9384	0.3369	0.2551	1.6218	2.79	0.0085
0.4	Lyear	41.215	25.9137	-11.34	93.7704	1.59	0.1205
0.5	Intercept	-258.98	213.395	-691.76	173.808	-1.21	0.2328
0.5	LCAP_QI	0.3943	0.5162	-0.6526	1.4412	0.76	0.4499
0.5	LLAND_QI	-3.5571	4.9662	-13.629	6.5147	-0.72	0.4784
0.5	LLAB_PI	-0.3174	0.1264	-0.5738	-0.061	-2.51	0.0167

Appendix 3. (Continued)

Quantile	Parameter	Estimate	Stderr	Lowercl	Uppercl	Tvalue	Probt
0.5	LMAT_PI	-0.8045	0.6331	-2.0885	0.4796	-1.27	0.212
0.5	LENG_PI	-0.2121	0.3932	-1.0095	0.5852	-0.54	0.5928
0.5	LCHEM_PI	0.1712	0.2892	-0.4154	0.7578	0.59	0.5576
0.5	LAO_PI	0.8438	0.3475	0.1391	1.5486	2.43	0.0203
0.5	Lyear	39.9677	26.2528	-13.275	93.2108	1.52	0.1366
0.6	Intercept	-110.57	213.48	-543.53	322.39	-0.52	0.6077
0.6	LCAP_QI	0.2885	0.491	-0.7073	1.2843	0.59	0.5605
0.6	LLAND_QI	-4.5594	4.5668	-13.821	4.7024	-1	0.3248
0.6	LLAB_PI	-0.2535	0.172	-0.6024	0.0955	-1.47	0.1494
0.6	LMAT_PI	-1.015	0.6865	-2.4072	0.3773	-1.48	0.148
0.6	LENG_PI	-0.1014	0.3729	-0.8576	0.6548	-0.27	0.7873
0.6	LCHEM_PI	0.24	0.2857	-0.3395	0.8195	0.84	0.4065
0.6	LAO_PI	0.9416	0.3337	0.2648	1.6184	2.82	0.0077
0.6	Lyear	22.4816	26.7353	-31.74	76.7032	0.84	0.406
0.7	Intercept	-271.08	229.096	-735.7	193.552	-1.18	0.2445
0.7	LCAP_QI	0.7284	0.4499	-0.1841	1.6409	1.62	0.1142
0.7	LLAND_QI	-5.8607	4.3361	-14.655	2.9332	-1.35	0.1849
0.7	LLAB_PI	-0.4363	0.1787	-0.7987	-0.074	-2.44	0.0196
0.7	LMAT_PI	-0.1911	0.7209	-1.6531	1.2709	-0.27	0.7924
0.7	LENG_PI	-0.4675	0.3782	-1.2345	0.2995	-1.24	0.2244
0.7	LCHEM_PI	-0.0305	0.282	-0.6024	0.5414	-0.11	0.9145
0.7	LAO_PI	0.6372	0.3448	-0.0621	1.3366	1.85	0.0729
0.7	Lyear	45.2128	28.516	-12.62	103.046	1.59	0.1216
0.8	Intercept	-308.8	286.752	-890.36	272.756	-1.08	0.2887
0.8	LCAP_QI	0.8334	0.5746	-0.3319	1.9987	1.45	0.1556
0.8	LLAND_QI	-6.3074	5.2722	-17	4.3851	-1.2	0.2394
0.8	LLAB_PI	-0.4599	0.209	-0.8838	-0.0361	-2.2	0.0343
0.8	LMAT_PI	-0.1517	0.9765	-2.1321	1.8288	-0.16	0.8775
0.8	LENG_PI	-0.5269	0.5429	-1.6279	0.5741	-0.97	0.3383
0.8	LCHEM_PI	-0.0183	0.3863	-0.8017	0.7651	-0.05	0.9625
0.8	LAO_PI	0.576	0.4169	-0.2695	1.4215	1.38	0.1756
0.8	Lyear	50.815	34.3294	-18.808	120.438	1.48	0.1475
0.9	Intercept	-49.732	3272.86	-6687.4	6587.94	-0.02	0.988
0.9	LCAP_QI	0.995	9.2932	-17.852	19.8424	0.11	0.9153
0.9	LLAND_QI	-12.038	87.7714	-190.05	165.971	-0.14	0.8917
0.9	LLAB_PI	-0.3288	2.2458	-4.8835	4.2259	-0.15	0.8844
0.9	LMAT_PI	0.0911	15.8896	-32.135	32.3167	0.01	0.9955
0.9	LENG_PI	-0.8669	8.7747	-18.663	16.9289	-0.1	0.9218
0.9	LCHEM_PI	0.0635	6.7541	-13.634	13.7614	0.01	0.9926
0.9	LAO_PI	0.3626	5.8817	-11.566	12.2914	0.06	0.9512
0.9	Lyear	27.0237	426.329	-837.61	891.659	0.06	0.9498

Appendix 4a. Output dataset for North Dakota's agriculture sector, 1960-2004

YEAR	AO PI	AO QI	LS PI	LS QI	CR PI	CR QI	OFR PI	OFR QI
1960	0.3608	2111138	0.3315	617212	0.3910	1312806	0.1719	255225
1961	0.3800	1631446	0.3309	679801	0.4251	827156	0.1728	250715
1962	0.4182	2531060	0.3434	691924	0.4838	1609333	0.1751	241561
1963	0.3810	2352220	0.3307	750120	0.4296	1412320	0.1764	235337
1964	0.3732	2390996	0.3029	768569	0.4311	1445563	0.1784	204168
1965	0.3798	2601387	0.3308	732634	0.4278	1662335	0.1806	190406
1966	0.4168	2456978	0.3820	735136	0.4624	1530984	0.1849	190996
1967	0.4088	2432202	0.3781	694439	0.4509	1544009	0.1894	187211
1968	0.3849	2680657	0.3899	667547	0.4086	1801572	0.1976	178866
1969	0.4201	2674613	0.4369	646817	0.4430	1828486	0.2037	152316
1970	0.4587	2319645	0.4581	679458	0.4918	1478753	0.2170	117702
1971	0.4059	3206502	0.4689	717770	0.4089	2290973	0.2260	125632
1972	0.5078	2822442	0.5584	729583	0.5244	1901530	0.2295	124804
1973	0.8351	2915753	0.7380	751599	0.9403	1963014	0.2511	136522
1974	1.0004	2603690	0.5953	772889	1.2473	1686027	0.3076	136085
1975	0.8795	3036383	0.5613	749695	1.0764	2044443	0.3291	149238
1976	0.7366	3122074	0.6190	745250	0.8428	2119026	0.3440	152396
1977	0.6709	3038398	0.6331	700962	0.7384	2080968	0.3896	149727
1978	0.7423	3592179	0.8462	603102	0.7781	2688984	0.4118	155405
1979	0.8066	3455155	1.1220	655811	0.7947	2506305	0.4678	126489
1980	0.9172	2837269	1.0358	686056	0.9763	1882149	0.5620	96594
1981	0.8572	4165631	0.9640	703844	0.9086	3119675	0.6557	88159
1982	0.8026	4060383	0.9376	673468	0.8414	3044886	0.6649	98740
1983	0.8738	3551220	0.9389	725081	0.9388	2501290	0.6692	110635
1984	0.8796	3916759	0.9363	726044	0.9461	2827718	0.7034	127976
1985	0.8378	4240642	0.9358	724626	0.8894	3128187	0.6961	133399
1986	0.7899	4278699	0.9151	696148	0.8318	3190952	0.6382	138391
1987	0.8242	4011072	1.0716	677817	0.8424	2948837	0.6525	146390
1988	1.0178	2204960	1.1449	535034	1.1060	1333272	0.6926	226748
1989	0.9339	3129617	1.1708	537660	0.9656	2175411	0.7123	270567
1990	0.8752	4039209	1.2428	555000	0.8658	3030274	0.7405	299435
1991	0.8583	3991997	1.2069	560541	0.8513	2976086	0.7255	297821
1992	0.8426	4786900	1.1605	566975	0.8387	3752728	0.7256	314362
1993	0.9429	3910797	1.1976	599972	0.9642	2796006	0.7417	368092
1994	0.9215	4279405	1.1141	569958	0.9499	3212001	0.7537	341306
1995	0.9387	3967363	0.9740	597468	0.9992	2837551	0.7646	401657
1996	0.9432	4608169	0.9433	564137	1.0095	3502218	0.7764	359046
1997	0.8907	3992816	1.0799	509193	0.9109	2975483	0.8315	356572
1998	0.8180	4649566	1.0884	606884	0.8199	3431903	0.7615	432140
1999	0.7934	4367343	1.1262	573516	0.7809	3208119	0.7527	417209
2000	0.7960	4972253	1.2179	612124	0.7703	3758042	0.7400	429339
2001	0.7914	4707504	1.2852	609944	0.7484	3550809	0.7817	363597
2002	0.8278	4410288	1.2225	590566	0.8091	3300023	0.7691	336374
2003	0.8804	5246704	1.3289	574671	0.8609	4180368	0.7645	335697
2004	0.9467	4793069	1.5396	589324	0.9159	3648162	0.7495	385598

# Appendix 4b. Input dataset for North Dakota's agriculture sector, 1960-2004

YEAR	AI PI	AI QI	CAP PI	CAP QI	LAND PI	LAND QI	LAB PI	LAB QI	MAT PI	MAT QI	ENG PI	ENG QI	CHEM PI	CHEM QI
1960	0.1255	4839753	0.1523	820159	0.0138	1290008	0.0668	3310916	0.2349	1262242	182300	182300	0.194495512	82428.63722
1961	0.1292	4789455	0.1487	811303	0.0102	1295826	0.077328859	2408405.873	0.2375	1280214	184777	184777	0.196797957	90742.81197
1962	0.1408	4598373	0.1539	786762	0.0126	1303524	0.086619785	2216775.62	0.2627	1372252	187626	187626	0.173061092	107776.9692
1963	0.1340	5018913	0.1547	806090	0.0128	1310672	0.096018002	2057552.843	0.2243	1487004	186597	186597	0.1718533	105933.3745
1964	0.1476	4984757	0.1634	812367	0.0174	1317903	0.105477574	1941558.163	0.2483	1511542	186444	186444	0.171848464	120117.4543
1965	0.1540	4925643	0.1662	828071	0.0183	1324464	0.1114566904	1966889.646	0.2554	1454360	188687	188687	0.189489498	125347.3163
1966	0.1620	4999267	0.1782	851765	0.0243	1329610	0.126486482	1894957.261	0.2546	1517609	193285	193285	0.186985484	123156.722
1967	0.1711	4914996	0.1832	871326	0.0282	1332962	0.13965248	1766513.973	0.2621	1515215	198837	198837	0.164516295	230354.0809
1968	0.1851	4757068	0.1975	900106	0.0390	1334053	0.160299466	1686318.776	0.2655	1433710	197312	197312	0.249850844	170033.4464
1969	0.1940	4792281	0.2109	919164	0.0490	1332429	0.168905812	1658000.182	0.2780	1462021	204567	204567	0.233863583	169521.9044
1970	0.2132	4684608	0.2325	935693	0.0687	1328087	0.186961678	1530515.618	0.2672	1452713	189392	189392	0.223301865	188119.4732
1971	0.2193	4775700	0.2384	935019	0.0767	1322116	0.188556653	1559971.12	0.2851	1505148	192582	192582	0.26836368	189024.4361
1972	0.2254	4780352	0.2501	954457	0.0869	1315794	0.215215521	1555635.12	0.3082	1460998	185175	185175	0.246264456	196269.5928
1973	0.2892	4911378	0.2916	943988	0.1159	1310496	0.299630102	158557.808	0.4127	1595956	189854	189854	0.291099206	289942.3916
1974	0.3843	4897932	0.3415	973385	0.1390	1307595	0.300753862	1676211.566	0.5741	1505358	194653	194653	0.444362091	293839.1973
1975	0.3720	4907988	0.3159	1005917	0.0916	1308030	0.295159324	1532108.958	0.5978	1565682	237070	237070	0.556303578	297440.9657
1976	0.3541	5168961	0.3119	1056404	0.0641	1309732	0.290630102	1875427.142	0.6011	1584528	247919	247919	0.523056369	350038.7548
1977	0.4206	5290637	0.3823	1096657	0.1759	1309799	0.33221285	1841864.927	0.6075	1764133	243388	243388	0.516330726	416097.6469
1978	0.4289	5333984	0.4178	1120089	0.2069	1305082	0.307807899	1841864.927	0.6755	1990089	239488	239488	0.543765052	475151.9048
1979	0.4751	5959724	0.4734	1120089	0.2450	1293636	0.331476092	1507513.862	0.8321	1982657	1.0271	239658	0.611273861	514116.5357
1980	0.5836	5676800	0.5717	1176491	0.3399	1278248	0.364797343	1555044.331	0.9212	1805634	240695	240695	0.637198553	497066.4145
1981	0.6856	5385147	0.7012	1114220	0.5132	1262888	0.387833957	1719178.752	0.8625	1755743	238009	238009	0.655791324	432341.7977
1982	0.6917	5356521	0.7573	1069548	0.5707	1251502	0.387833957	1528425.26	0.8270	1828501	242135	242135	0.632490853	394883.1809
1983	0.7067	5328398	0.8323	1062396	0.6401	1246945	0.373745205	1528425.26	0.8700	1726132	242198	242198	0.606938622	472453.045
1984	0.7593	5304922	0.9043	1023249	0.6943	1247796	0.430094747	1783219.674	0.8360	1704729	218266	218266	0.53485596	578258.4906
1985	0.7212	5171042	0.8842	1001619	0.5965	1251544	0.412052332	1630963.578	0.7450	1692252	230882	230882	0.578877158	531290.6128
1986	0.6490	5002791	0.8049	928429	0.4421	1255694	0.448307661	1525043.076	0.7492	1747027	227323	227323	0.578877158	473285.0099
1987	0.6546	4979337	0.8104	867676	0.4005	1257662	0.493064994	1508068.535	0.7492	1580461	211289	211289	0.674933694	473256.4241
1988	0.6939	4829876	0.8848	850344	0.3872	1255620	0.449546933	1613812.46	0.8779	1580461	197362	197362	0.718667476	473256.4241
1989	0.6773	4537296	0.8966	817083	0.3692	1250487	0.328008332	1549174.713	0.9632	1422974	204692	204692	0.723214928	455091.2965
1990	0.7898	4600538	0.8991	809882	0.3710	1243943	0.667817972	1468484.527	0.9553	1527557	197362	197362	0.718667476	485424.811
1991	0.7099	4687639	0.9011	797575	0.3596	1237644	0.425948329	1500143.969	0.9592	1589178	203409	203409	0.773168784	472990.9623
1992	0.7424	4621661	0.9026	769753	0.3489	1233235	0.590837734	1373581.571	0.9139	1635188	199134	199134	0.760961686	511027.0878
1993	0.7858	4580321	0.9292	760048	0.3570	1231906	0.695000526	1328403.725	0.9316	1642546	184934	184934	0.742520326	657247.0796
1994	0.8529	4619375	0.9628	740101	0.3727	1232974	0.859799424	1262374.594	0.9614	1749843	197705	197705	0.800610455	536706.7061
1995	0.8772	4618131	1.0088	729222	0.3967	1233336	0.861976752	1235643.62	0.9874	1782427	199321	199321	0.840242944	586074.5435
1996	0.8410	4814474	1.0244	706214	0.4110	1237850	0.64863929	1522274.423	1.0396	1759615	194384	194384	0.859850954	627247.0796
1997	1.0662	4571056	1.0365	710528	0.4273	1239423	1.376412572	1225577.933	1.0616	1809300	205762	205762	0.846809994	690799.5939
1998	0.9797	4640720	1.0218	689821	0.4084	1239221	1.222490469	1188867.237	0.9520	1947723	214159	214159	0.735171027	751590.7275
1999	1.0460	4649015	1.0396	692834	0.4494	1237423	1.435244156	1239440.8	0.9520	1987203	200465	200465	0.783026788	675832.5616
2000	0.9612	4815764	1.1171	681019	0.4756	1234481	1.054038846	1275192.566	1.0007	2026499	214705	214705	0.717053963	813050.3841
2001	1.0896	4577433	1.1057	686186	0.4353	1230801	1.548301984	1044932.077	1.0079	2073637	207181	207181	0.72508179	864451.4434
2002	1.0279	4601955	1.0779	689546	0.3834	1226825	1.306366219	1097711.852	1.0293	2023192	211481	211481	0.713618471	87190.1483
2003	0.9672	4574553	1.0838	678714	0.3656	1229498	0.993974957	1143658.625	1.0742	1959443	225242	225242	0.733624532	871374.8956
2004	0.8983	4814634	1.0598	676890	0.3079	1219174	0.739201038	1270906.772	1.1094	2066731	209852	209852	0.751873055	989573.9647

## Appendix 5. SAS codes for the estimation of the production, cost, and profit function using quantile regression

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```

PROC IMPORT OUT = WORK.THESIS
    DATAFILE = "H:\Everything\North Dakota.xls"
    DBMS = EXCEL REPLACE;
    SHEET = "Sheet1$";
    GETNAMES = YES;
    MIXED = NO;
    SCANTEXT = YES;
    USEDATE = YES;
    SCANTIME = YES;

Run;
TITLE1 'Examine North Dakotas Production, Cost, and Profit Functions: A Quantile Regression Approach.';
DM 'clear log; clear output; '; Run;
TITLE2 'Read the SAS data set';
/*****/
Data THESIS; set TFP.PCPrF;
Run; quit;
TITLE2 'Contents of THESIS data file';
/*****/
PROC CONTENTS data= Thesis; run;
PROC CONTENTS short data= thesis; run;
TITLE2 'Names of Variables'
/*****/
/* ods html;
ods graphics on;
ods graphics off;
ods html close;
NAMES OF VARIABLES
roll = ROLLING REGRESSION PERIOD
state = NORTH DAKOTA STATE
year = YEAR 1960 TO 2004
st =STATE
AO_PI = P_AGGREGATE OUTPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
AGGREGATE OUTPUT: OUT_US.WK1 F/W/C
AO_QI = Q_AGGREGATE OUTPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)
LS_PI = P_LIVESTOCK (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996) LIVESTOCK &
PRODUCTS
LS_QI = Q_LIVESTOCK (VALUE $1000 IN 1996 PRICES OF ALABAMA)
CR_PI = P_CROPS (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
CR_QI = Q_CROPS (VALUE $1000 IN 1996 PRICES OF ALABAMA)
OFR_PI = P_FARM RELATED OUTPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
OFR_QI = Q_FARM RELATED OUTPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)
AI_PI =P_AGGREGATE INPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
AI_QI = Q_AGGREGATE INPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)
CAP_PI = P_CAPITAL EXCLUDING LAND (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
CAP_QI = Q_CAPITAL EXCLUDING LAND (VALUE $1000 IN 1996 PRICES OF ALABAMA)
LAND_PI = P_LAND INPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
LAND_QI = Q_LAND INPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)
LAB_PI = P_LABOR INPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
LAB_QI = Q_LABOR INPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)
MAT_PI = P_AGGREGATE MATERIALS (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
MAT_QI = P_ENERGY INPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
ENG_PI = P_ENERGY INPUT (PRICE RELATIVE TO LEVEL IN ALABAMA IN 1996)
ENG_QI = Q_ENERGY INPUT (VALUE $1000 IN 1996 PRICES OF ALABAMA)

```



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### Appendix 5. (Continued)

```
CHEM_PI = P_AGRICULTURAL CHEMICALS (PRICE RELATIVE TO LEVEL IN ALABAMA IN
1996)
CHEM_QI = Q_AGRICULTURAL CHEMICALS (VALUE_$1000_IN 1996 PRICES OF ALABAMA)/
TITLE2 'NEW DATASET computing the cost and profit';
DATA thesis1;
  SET thesis;
/*COMPUTING INDIVIDUAL COST AND TOTAL COST*/
C_CAP = CAP_PI*CAP_QI;
C_LAND = LAND_PI*LAND_QI;
C_LAB = LAB_PI*LAB_QI;
C_MAT = MAT_PI*MAT_QI;
C_ENG = ENG_PI*ENG_QI;
C_CHEM = CHEM_PI*CHEM_QI;
COST = (C_CAP+C_LAND+C_LAB+C_MAT+C_ENG+C_CHEM);
RCOST = (C_MAT+C_ENG+C_CHEM);
RCOST1 = (C_LAB+C_MAT+C_ENG+C_CHEM);
/*COMPUTING INDIVIDUAL Revenue AND TOTAL Revenue*/
R_CR = CR_PI*CR_QI;
R_LS = LS_PI*LS_QI;
R_OFR = OFR_PI*OFR_QI;
Revenue = (R_CR+R_LS+R_OFR);
Profit = Revenue - Cost;
RProfit = Revenue - RCOST;
RProfit1 = Revenue - RCOST1;
Run;
TITLE3 'Generate Logs of the variables';
DATA thesis2; SET thesis1;
/*COMPUTING Logs of Price data*/
Lyear = log(year);
LAI_PI = LOG(AI_PI);
LCAP_PI = LOG(CAP_PI);
LLAND_PI = LOG(LAND_PI);
LLAB_PI = LOG(LAB_PI);
LMAT_PI = log(MAT_PI);
LENG_PI = log(ENG_PI);
LCHEM_PI = log(CHEM_PI);
LAO_PI = LOG(AO_PI);
LCR_PI = log(CR_QI);
LLS_PI = log(LS_QI);
LOFR_PI = log(OFR_QI);
/*COMPUTING Logs of Quantity data*/
LAI_QI = LOG(AI_QI);
LCAP_QI = LOG(CAP_QI);
LLAND_QI = LOG(LAND_QI);
LLAB_QI = LOG(LAB_QI);
LMAT_QI = log(MAT_QI);
LENG_QI = log(ENG_QI);
LCHEM_QI = log(CHEM_QI);
LAO_QI = LOG(AO_QI);
LCR_QI = log(CR_QI);
LLS_QI = log(LS_QI);
/*COMPUTING Logs of Cost and Profit data*/
Lcost = log(cost);
```

---

## Appendix 5. (Continued)

```
LRcost = log(Rcost);
LRcost1 = log(Rcost1);
Lrevenue = log(revenue);
Lprofit = Lrevenue - Lcost;
LRprofit = Lrevenue - LRcost;
LRprofit1 = Lrevenue - LRcost1;
run;
DATA thesis3 negtiveprofit ; SET thesis2;
  if Profit < 0 then output negtiveprofit;
  else output thesis3;
run ;
DATA thesis4 negtiveprofit ; SET thesis2;
  if RProfit < 0 then output negtiveprofit;
  else output thesis4 ;
run ;
Title2 'Measures of Correlation between the variables';
/*****/
proc corr data = thesis1;
  var AO_QI cap_QI land_QI lab_QI mat_QI eng_QI chem_QI year;
run;
proc corr data = thesis1 ;
  var cost cap_PI land_PI lab_PI mat_PI eng_PI chem_PI AO_QI year;
run;
proc corr data = thesis1 ;
  var Rprofit cap_QI land_QI lab_PI mat_PI eng_PI chem_PI AO_PI year;
run;
proc corr data = thesis1 ;
  var Rprofit1 cap_QI land_QI lab_PI mat_PI eng_PI chem_PI AO_PI year;
run;
/*****/
ods html ;
ods graphics on;
*COBB_DOUGLAS;
ODS output ParameterEstimates = QuantCD_Prod;
proc quantreg data = thesis2 ci = resampling;
  model LAO_QI = Lcap_QI Lland_QI Llab_QI Lmat_QI Leng_QI Lchem_QI Lyear
    / quantile = .10 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 plot = quantplot;
  id year ;
Run; quit;
ODS output ParameterEstimates = QuantCD_Cost;
proc quantreg data = thesis2 ci = resampling;
  model Lcost = Lcap_PI Lland_PI Llab_PI Lmat_PI Leng_PI Lchem_PI LAO_QI Lyear
    / quantile = .10 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 plot = quantplot;
  id year;
Run; quit;
ODS output ParameterEstimates = QuantCD_RProfit1;
proc quantreg data = thesis4 ci = resampling;
  model LRprofit1 = Lcap_QI Lland_QI Llab_PI Lmat_PI Leng_PI Lchem_PI LAO_PI Lyear
    / quantile = .10 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 plot = quantplot;
  id year;
Run; quit;
ods graphics off;
ods html close;
```

---

## Appendix 6. SAS codes for the estimation of the production, cost, and profit function using ordinary least squares

---

```
TITLE2 'Estimation of the Production, Cost, and Profit Function using ordinary least squares.'  
/*****  
/* ods html;  
ods graphics on;  
ods graphics off ;  
ods html close;  
  
*COBB_DOUGLAS;  
  
ODS output ParameterEstimates = OLSCD_Prod;  
proc reg data = thesis2;  
  model LAO_QI = Lcap_QI Lland_QI Llab_QI Lmat_QI Leng_QI Lchem_QI Lyear ;  
  id year ;  
run; quit;  
  
ODS output ParameterEstimates = OLSCD_Cost;  
proc reg data = thesis2;  
  model Lcost = Lcap_PI Lland_PI Llab_PI Lmat_PI Leng_PI Lchem_PI LAO_QI Lyear ;  
  id year ;  
run;quit;  
  
ODS output ParameterEstimates = OLSCD_RProfit1;  
proc reg data = thesis4;  
  model LRprofit1 = Lcap_QI Lland_QI Llab_PI Lmat_PI Leng_PI Lchem_PI LAO_PI Lyear ;  
  id year ;  
run; quit ;  
ods graphics off;  
ods html close;
```

---