MULTIRESPONSE OPTIMIZATION METHODOLOGY CONSIDERING CORRELATED QUALITY CHARACTERISTICS

A Thesis
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agricultural and Applied Science

By

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In Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

Major Department:
Industrial and Manufacturing Engineering

September 2011

Fargo, North Dakota

North Dakota State University

Graduate School

Title

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The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

ABSTRACT

Thambidorai, Ganesh, M.S., Department of Industrial and Manufacturing Engineering, College of Engineering and Architecture, North Dakota State University, September 2011. Multiresponse Optimization Methodology Considering Correlated Quality Characteristics. Major Professor: Dr. Om Prakash Yadav.

Engineering problems often involve many conflicting quality characteristics that must be optimized simultaneously. Engineers are required to select suitable design parameter values which provide better trade-off among all quality characteristics. Multi-response optimization is one of the most essential tools for solving engineering problems involving multiple quality characteristics. Optimizing several quality characteristics when the quality characteristics are correlated makes the optimization process more complex.

The aim of this research is to evaluate the performance of several existing multi-response optimization methods and investigate their capabilities in dealing with correlated quality characteristics. This study also investigates the impact of uncertainty in terms of input parameter selection. A new multi-response optimization approach has been proposed for solving correlated quality characteristics. The proposed approach is compared with the existing methods and found more robust in terms dealing with uncertainty in target selection. The comparative study and application of the proposed approach is demonstrated by considering two examples from the literature having correlated quality characteristics.

ACKNOWLEDGEMENTS

I dedicate the thesis to my parents, family members and friends for the support and motivation they provided throughout my degree. I thank my advisor, Dr. Om Prakash Yadav because without his guidance it would have been impossible. I appreciate the support, suggestions and questions he had for me throughout the study. I thank Dr. John Cook for his suggestions and questions. I thank Dr. Canan Bilen Green for her questions. I also want to thank Dr. Suzen (Mechanical Department) for his suggestions.

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CHAPTER 1. INTRODUCTION

The increasing customer awareness and global competition have forced manufacturing companies to design highly reliable complex products faster and at a reasonable cost. Furthermore, the performance of these complex products is one of the indicators of competitiveness of the company. The overall performance of these complex products is determined based on their ability to satisfy several changing customer requirements and expectations. This requires incorporation of appropriate product (quality) characteristics in design and also the achievement of optimal tradeoff among several quality (sometimes conflicting) characteristics. Therefore, design engineers are constantly searching for different tools and techniques to optimize multiple quality characteristics. Any tool that helps to optimize multiple quality characteristics can help companies to survive in today's market. Multi-response optimization is one of the tools used in recent years for achieving better trade-off among multiple quality characteristics.

Generally multiresponse methods rely on response surface models (RSM) for developing the relation between design factors and responses. In RSM, higher order polynomial models are developed to establish a relationship between the design factors and responses by conducting designed experiments. The design factors are also known as controllable, design, or independent variables denoted by x. These design variables are within control of design engineers to make necessary changes in order to optimize quality characteristics. The responses are also known as dependent variables or quality characteristics denoted by Y, which should meet the customer functional requirement or expectations. These models are developed from the data collected by conducting physical or simulation experiments. It is important to note that these models are valid only to the experimental region. The experimental region is determined from the different levels selected for each design variable. A design engineer is interested in determining controllable factor settings within the experimental region such that it produces optimal values for all quality characteristics. The focus of this research is to understand and summarize different existing multiresponse optimization methods, perform a comparative study and finally propose a more effective multiresponse optimization methodology, which is capable of handling correlated responses.

1.1. Significance of the Problem

A single product has to meet several customer requirements and hence incorporate several quality characteristics in design. Design engineers are required to determine optimal design factor settings such that all objectives are satisfied by the product. In some cases, it is difficult to achieve the target for all quality characteristics, resulting in tradeoff between different objectives to achieve optimal product performance conditions. In order to achieve tradeoff between quality objectives, the design engineer has to prioritize these quality characteristics keeping in mind their importance in meeting functional requirements. This requires selection or assigning appropriate weights to each quality characteristic that introduces uncertainty in the decision making process to certain extent.

Several methods have been developed for optimizing single product quality characteristic such as, Design of experiments (Montgomery, 2005), Taguchi's robust design (Taguchi, 1986), and response surface methods (Myers et al., 2004). When these methods are used for solving multiple responses, engineering judgment is widely used for achieving tradeoff between different responses (Logothetis and Haigh, 1998; Jeypaul et al., 2005). Multiresponse optimization methods were later introduced to achieve a better tradeoff between the multiple responses. Myers and Carter (1973) introduced the constrained optimization technique in which two responses (primary and secondary) are optimized simultaneously. The primary response is treated as objective function and the secondary response is treated as constraints. In the optimization process, the primary response is either maximized or minimized depending on the nature of quality characteristic while satisfying the secondary response. Other multiresponse optimization methods developed later also combine multiple responses to form a composite objective function. In desirability function, a maximum overall desirability is modeled using the geometric mean of individual desirability to determine the optimum factor setting (Harrington, 1965; Derringer and Suich, 1980; Del Castillo et al., 1996). The quality loss function based multiresponse optimization models treat the deviation of expected response from target as a loss and use it as objective function in the model (Taguchi, 1986; Pignatiello, 1993; Riberto and Elsayed, 1995; Artiles-Leon, 1996-97; Bhamare et al., 2009). Using multiresponse optimization methods for solving multiple quality characteristics, some of the input parameters like weights, target, upper and lower response limits are required for optimization. The selection of input parameters depends entirely on designer engineers' prior knowledge and experience. This subjective input introduces uncertainty in the decision making process. Therefore, there is an urgent need to develop optimization approaches that is robust or insensitive to this uncertainty (subjective input parameter values). In real life, the products not only have multiple quality characteristics but some of these quality characteristics are correlated to each other making the tradeoff process more difficult and complex. There are two levels of correlation that need to be dealt with in optimization process. The first level includes correlation between controllable factors that must be eliminated while developing response models using response surface methods. The models developed including correlated design factors will not predict the exact relation between design factors and responses (Mendenhall and Sincich, 1996). Generally correlation between the design factors is solved by removing the variable which causes correlation from the analysis. This problem is also solved using principal component analysis (PCA) where correlated data set is transformed into uncorrelated data set. The second type of correlation represents correlation among the quality characteristics. The unique example of correlation between quality characteristics is weight and fuel efficiency in case of automotive design. There have been some efforts to deal with correlated quality characteristics using PCA (Antony, 2000; WU, 2004; Sibalija and Majstorovic, 2009). However, in a multiresponse optimization scenario, no major work has been done in the past to capture correlation in the optimization model.

1.2. Objectives of Research

The objectives of this research are as follows:

The first objective of this thesis is to perform a comparative study on existing multiresponse optimization methods. The comparison is performed to determine which method is more effective in dealing with uncertainty in parameter selection and to develop insight in terms of advantages and limitations of existing methods. Furthermore, the comparison is performed to understand their capability in dealing with correlated quality characteristics.

The second objective of this thesis is to propose multiresponse optimization methods which capture the correlation among quality characteristics. It is compared with existing method which captures correlation among the quality characteristics. The robustness in terms of parameter selection test is performed on the proposed method.

1.3. Overview of Research

This research is presented in the following order: Chapter 2 covers the background on conducting designed experiment and various methods involved in developing response surface models. The six multiresponse optimization methods are compared for sensitivity in parameter selection in Chapter 3. The proposed new method for solving multiple correlated quality characteristics is then presented, which is an improvement over the existing method in chapter 4. Conclusion with recommendation for future direction is presented in Chapter 5.

CHAPTER 2. BACKGROUND STUDY AND EXPERIMENTAL DESIGN

In multiresponse optimization, response surface models are used to build a relationship between independent and dependent variables. A parameter diagram is used to identify independent and dependent variables to build relationships. A parameter diagram classifies the functionality associated with the product functions such as different design parameters, output responses, and noise factors. Figure 1 shows the parameter diagram for a system.

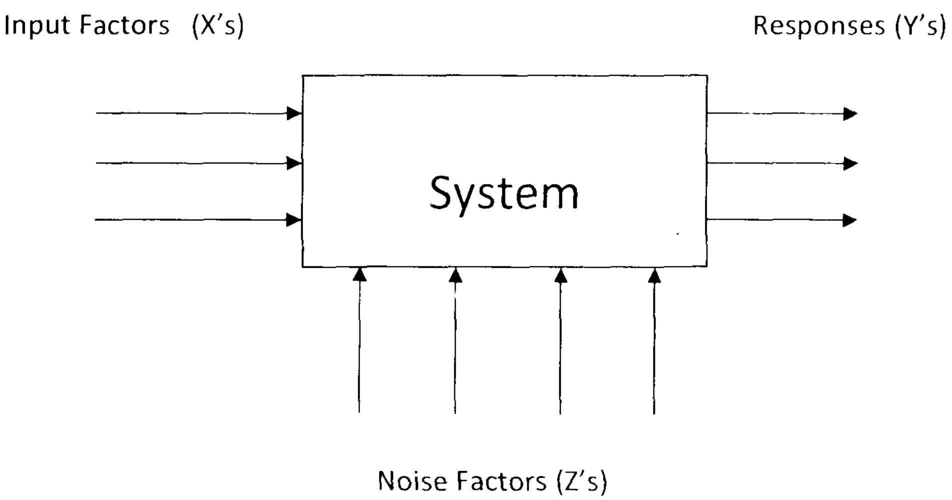

Figure 1. Parameter Diagram

The input factors are the design factors which are controlled by design engineers' to achieve the desired responses (quality characteristics). The noise factors are unwanted factors which affect the product performance but the design engineers' are not interested in them. There are two types of noise factors: known and controllable, known and uncontrollable. It is necessary for the design engineer to minimize or eliminate the effect of noise factors. Blocking is done to minimize the impact of known and controllable noise factors. Randomization is performed to minimize the effects of known and uncontrollable noise factors.

2.1. Design of Experiments

Statistically designed experiment was introduced by Sir Ronald Fisher in 1920 with the analysis technique called Analysis of variance (ANOVA) to improve the yield of agricultural crops (Phadke, 1989). Design of experiments was mainly used to study the effect of input design factors on a single quality characteristic. The optimal factor setting is determined by changing the design variable levels in a systematic way and measuring the effect on a quality characteristic. There are different types of experimental design available in the literature. The selection of a particular design depends on the objective of the experiment and also the availability of resources. The different types to experimental design and usages are given below:

Completely randomized design is used for studying one primary design factor without considering any noise factors in the analysis. The experiments are conducted randomly with different levels of the primary factor. Randomized block design is used for studying one primary factor by considering the noise factors in the analysis. Randomized block designs reduce residual error in an experiment by blocking the known and controllable noise factor. Latin square design is used to remove two nuisance source of variability. Graeco-Latin square design and Hyper Graeco-Latin square design is used to control three and four nuisance source of variability respectively.

A full factorial experiment considers all input factors which are always set at two levels (high and low). The experimental design is conducted for all possible combinations of high and low levels. If there are k factors and each factor is at two levels, a full fractional design consists of 2^k experiments for all possible combinations. The limitation of this experimental design is that the designer has to conduct comparatively more experiments. To reduce the number of experiments, fractional factorial experiment was developed where the higher order interactions are relaxed. In all the above experimental designs, the factor effect plots are used to determine the design factor settings to achieve optimal response.

2.2. Taguchi Robust Design

Taguchi worked with Ronald Fisher and C.R. Rao at Indian Statistical Institute where he was introduced to the concept of orthogonal arrays which laid the foundation for Taguchi's Robust Design. Taguchi's robust design involves orthogonal arrays with inner array and outer arrays. The inner array consists of design factors while the outer array has noise factors in the experimental design. The two arrays are crossed with each other so that each experiment includes both design variables and noise factors. The combination of the inner array and the outer array is called a crossed-array design. As the optimum values are determined from experiments which are conducted by including noise factors, the system will be robust to those included noise factors. Taguchi used signal to noise (SN) ratio and factor effects to determine the optimum value for responses. Robust design concepts focus on reducing variability around the mean to minimize the nonconformance. The limitations of using SN ratio is that the SN ratio might confound mean and variance (Myers et al., 2009) and it can be applied only to single response or single quality characteristic (Antony, 2000). Taguchi's robust design was often criticized by statisticians for limitation and weakness (Montgomery, 2005). In his philosophy of robust design, Taguchi states "as product quality characteristics departs from target it creates loss to the society". This loss could be early failure of the product or higher operating cost of the product. This philosophy has been widely accepted and plays a major role in imparting quality into the product design.

2.3. Response Surface Methods

Response surface methodology was developed by Box and Wilson in 1951 for improving the process of the chemical industry. This was done by sequential experiments using different factors such as temperature, pressure, time, and amount of reactants (Dean and Voss, 1999). Myers et al. (2004) define response surface methodology as "a collection of statistical design and numerical optimization techniques used to optimize process and product designs". Response surface methodology plays a major role in developing new products / processes and also for improving the performance of existing products / processes. Extensive work has been done in response surface methods with four review papers (Myers et al., 2004; Myers et al., 1989; Mead and Pike, 1975; Hill and Hunter, 1966). A simple response surface model is shown in the equation below:

$$Y = f\,(A,B) + € \qquad (2\text{-}1)$$

where, Y is the response, A and B are input controllable factors, and € is the random error. The expected value for Y is denoted by η, and η is the response surface as plotted against levels of factors A and B as shown in Figure 2. The design factor setting is selected from Figure 2 for which the value attains the desired response.

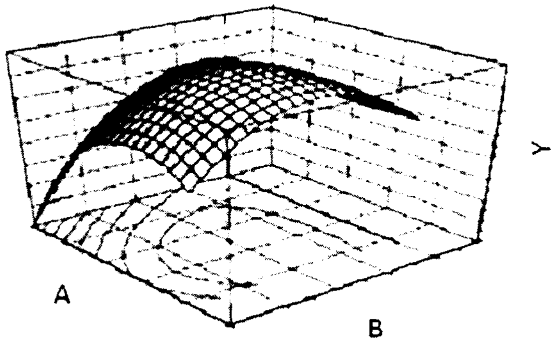

Figure 2. Response Surface Plot

In response surface methods, empirical models are developed between response and controllable factors using the data collected by conducting experiments. A model can be a simple linear model, second order model, model with higher polynomial with interaction between the control factors. A simple linear model is shown in the equation below:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon \qquad (2\text{-}2)$$

Higher degree polynomial models are used if there is a curvilinear relationship between the variables. A second order response surface model is shown in the equation below:

$$Y = \beta_0 + \sum_{i=1}^{K} \beta_i X_i + \sum_{i=1}^{K} \beta_{ii} X_{ii}^2 + \sum \sum_{i<j}^{K} \beta_{ij} X_i^2 X_j^2 + \epsilon \qquad (2\text{-}3)$$

where, Y is the response (quality characteristic), x_i is the i^{th} independent variable and β_i is the coefficients for corresponding i^{th} independent variable.

Box and Wilson (1951) introduced the concept of central composite design (CCD). It is the most common response surface design used for developing a second order response surface model. In general, CCD design consists of fractional factorial design of resolution V, 2K axial runs and n_c center runs. For design factors with two levels, the CCD design consists of: four runs at the center of the square, four runs at the corner of the squares and four axial runs. Three parts are used to estimate different components in the model: (1) factorial point estimates linear and two factor interaction terms (2) center point provide the information for the existence of curvature (3) the axial point consider the experimental design region.

Box and Behnken (1960) proposed a design for factors with three levels. It is a spherical design that is formed by combining two designs: incomplete block design and 2^k factorial design. For cuboidal region of interest, face centered composite design (FCC) was developed. The advantage of FCC is that it does not require as many center points as CCD. When designing an experiment, the designer should select the best design which satisfies experimental conditions.

2.4. Dual Response Surface

The dual response surface was introduced by Myers and Carter (1973) for optimizing two quality characteristics. In dual response optimization methods, one response called as the primary response is treated as an objective function and the other response which is known as the secondary response is treated as a constraint. The problem is solved by either maximizing or minimizing the primary response while satisfying the secondary response. The response surface models are shown below:

$$Y_P = a_0 + \sum_{i=1}^{K} a_i X_i + \sum_{i=1}^{K} a_{ii} X_i^2 + \sum\sum_{i<j}^{K} a_{ij} X_i^2 X_j^2$$

$$Y_s = b_0 + \sum_{i=1}^{K} b_i X_i + \sum_{i=1}^{K} b_{ii} X_i^2 + \sum\sum_{i<j}^{K} b_{ij} X_i^2 X_j^2 \qquad (2\text{-}6)$$

Taguchi emphasized the importance of considering both mean and deviation of a quality characteristic. Vining and Myers (1990) applied Taguchi's concept in dual response surface optimization scenario for robust optimization. They optimized a single quality characteristic but considered both mean and variance as primary and secondary response. The objective function and constraints are selected based on the type of quality characteristics: nominal the best (NTB), larger the better (LTB), smaller the better (STB). The general formulation for NTB quality characteristic is shown in equation (2-7) and the formulation of LTB and STB quality characteristics is shown in equation (2-8):

$$\text{Minimize} \quad \sigma \quad \text{S.t.} \quad \mu = \mu' \tag{2-7}$$

$$\text{Min/Max} \quad \mu \quad \text{S.t.} \quad \sigma = \sigma' \tag{2-8}$$

In the literature, two types of development occur on dual response surface methods. Researchers are interested to improve the solution strategy by trying different algorithms to obtain better solutions and also they are interested to improve the model formulation to achieve different objectives. Castillo and Montgomery (1993) used a nonlinear programming method called generalized reduced gradient algorithm for solving the dual response surface problems. Fuzzy modeling was used by Kim and Lin (1998) for solving dual response surface approaches. Their research measured the degree of satisfaction level for the deviation of mean from target and also magnitude of dispersion. A similar satisfaction level approach based on the desirability function was used by Koksoy (2005) for solving dual response problem. Dual response problems are widely used in industrial applications: Menon et al. (2002) used dual response surface methods for determining the optimal parameter setting which affects the performance of the spindle motor in hard disks. Kim and Rhee (2003) determined optimal operating condition for gas metal arc welding to achieve desired partial penetration.

CHAPTER 3. MULTIRESPONSE OPTIMIZATION METHODS

This chapter presents a literature review of six multiresponse optimization methods (constrained optimization, desirability approach, expected loss function, gradient loss function, standardized loss function and hybrid quality loss function). A comparative study of these six methods was performed by solving multiple quality characteristics problems from the literature to determine their sensitivity to variations in input parameters. Furthermore, these methods were used for solving correlated multiresponse problems in order to analyze their effectiveness in achieving better trade-off process.

3.1. Multiresponse Optimization Formulation Approaches

Solving multiple response problems using traditional methods require engineering judgment to achieve on optimal solution. However, engineering judgment creates uncertainty in the decision making process (Jeyapaul et al., 2005). Therefore, in order to deal with uncertainty and achieve a better tradeoff between multiple responses, the role of engineering judgment in the tradeoff process should be minimized. This requirement led to the development of simultaneous optimization of multiple quality characteristics, which known as multiresponse optimization methods.

Researchers have used different approaches for formulating multiresponse optimization models. These approaches are classified into three different categories: constrained optimization, desirability function, and loss function based optimization. Six different methods from the above categories are compared and analyzed in this chapter.

3.1.1. Constrained Optimization

Constrained optimization process maximizes or minimizes its objective function by assigning constraints to the optimization process. These constraints are imposed based on the availability of material or other resource. In multiresponse optimization, one response is treated as an objective function, which could be maximized or minimized depending upon the nature of quality characteristics and other responses are treated as constraints.

Contour plot optimization is a graphical method for solving multiresponse optimization problems using constraint optimization techniques. In contour plots, a single response is plotted using two design factors. The design factor which attains optimal response is identified from the contour plot. However, for multiresponse optimization, all individual contour plots drawn for each response are superimposed to form an overlaid contour plot. The region which satisfies all responses is identified and design factor setting is determined from the identified region. The limitation of this approach is that, as the number of responses increases, the overlaid contour plot is difficult to interpret. Additionally, only two design factors can be optimized using this graphical approach, also there is a great deal of subjectivity involved in selecting the design factor values from these plots (Montgomery, 2005).

The constrained optimization was developed based on the dual response optimization model (Myers and Carter, 1973) for solving multiple response problems. In this method, one response is treated as the primary response, which is considered as an objective function, and remaining responses are treated as secondary responses which are considered as constraints in the optimization model. The constrained optimization model can be expressed as:

$$\text{Max / Min } Y_i(X_k)$$

$$\text{S.t. } L_j < Y_j(X_k) < U_j$$

$$X_k \in R \quad (3\text{-}1)$$

where Y_i and Y_j represents primary and secondary responses, L_j and U_j are the lower and upper response limit for secondary responses. The lower and upper limits are marginally accepted values given by design engineers for each product quality characteristic. The performance of the product deteriorates when the response values are beyond the upper and lower limits. R is the experimental region through which the developed model is valid, j is the total number of responses given as constraints and k is the number of design factors.

3.1.2. Desirability Approach

The desirability-function-based optimization approach was originally proposed by Harrington (1965), where utility function is used to derive the desirability for each response. Later, Derringer and Suich (1980) modified this approach to incorporate different types of quality characteristics (NTB, LTB and STB). In desirability function, the desirability value for each quality characteristic is determined and the optimum factor setting is selected based on maximum desirability values. The desirability values provide a common metric for all responses, and it lies between 0 and 1.

$$0 \leq d_i \leq 1 \quad (3\text{-}2)$$

where, d_i is the desirability value for i^{th} response. A desirability value of 0 is assigned, if the response value is out of the acceptable range. A value of 1 is assigned, if the response value is on target. For response values, which lie between the target and acceptable range, Derringer and Suich (1980) proposed the following method to determine the desirability values. For STB quality characteristics with a target value (T), and upper limit value (U), the equation to determine the desirability value is given below:

$$d_i = \begin{cases} 0, & \hat{Y}_i(x) > U_i \\ \left(\frac{\hat{Y}_i(x) - U_i}{T_i - U_i}\right)^s, & T_i < \hat{Y}_i(x) \le U_i \\ 1, & \hat{Y}_i(x) < T_i \end{cases} \tag{3-3}$$

For LTB quality characteristics with a target value (T), and lower limit value (L), the equation to determine the desirability value is given below:

$$d_i = \begin{cases} 0, & \hat{Y}_i(x) < L_i \\ \left(\frac{\hat{Y}_i(x) - L_i}{T_i - L_i}\right)^t, & L_i < \hat{Y}_i(x) \le T_i \\ 1, & \hat{Y}_i(x) > T_i \end{cases} \tag{3-4}$$

For two side specification limit with target value T, lower limit value (L) and upper limit value (U), the equation to determine the desirability value is given below:

$$d_i = \begin{cases} 0, & \hat{Y}_i(x) < L_i \\ \left(\frac{\hat{Y}_i(x) - L_i}{T_i - L_i}\right)^s, & L_i < \hat{Y}_i(x) \le T_i \\ \left(\frac{\hat{Y}_i(x) - U_i}{T_i - U_i}\right)^t, & T_i \le \hat{Y}_i(x) < U_i \\ 0, & \hat{Y}_i(x) > U_i \end{cases} \tag{3-5}$$

where, $\hat{Y}_i(x)$ is the predicted response value for i^{th} response, s and t are the user specified weights. The design parameter is determined for the response which has maximum overall desirability value. The overall desirability value is calculated by the geometric mean of individual desirability value as given below:

$$D = (d_1 X d_2 \, X \ldots \ldots X d_n)^{1/n} \tag{3-6}$$

where D is the overall desirability value and n is the total number of quality characteristics. As the process contains non-differential points, Del Castillo et al. (1996) modified the solution approach by including differential points and solved using generalized reduced gradient procedure. In this approach, piecewise-continuous differential points are used to solve the non-differential points using local polynomial. The desirability function for non-differential points is given by:

$$d = \begin{cases} a_1 + b_1 Y & if \; Y_{min} < Y \; \leq T - \delta Y \\ f(Y) & T - \delta Y \; \leq Y \; \leq \; T + \delta Y \\ a_2 + b_2 Y & if \; T - \delta Y \; \leq Y \; \leq \; Y_{max} \\ 0 & otherwise \end{cases} \tag{3-7}$$

where, δY represents small range around the non-differential points. Refer the literature paper written by Del Castillo et al. (1996) for further reading on modified desirability function. The desirability approach has been used widely by researchers for optimizing problems based on multiple quality characteristics. Montgomery et al. (2000) used the desirability function approach for optimizing printed circuit board in an electronic industry. Tyan et al. (2004) integrated the tool and vehicle dispatching strategy, as a multiple performance measures in a fully automated fab environment, the dispatching strategy was developed using desirability function. Pasamontes and Callao (2006) used the desirability function to develop a single objective function which is known as a single global desirability function for solving multiresponse problems, and their research was applied to determine the optimum response value for amoxicillin in pharmaceuticals industry.

3.1.3. Loss Function Based Approach

The loss-functions are based on the economic importance of the quality characteristics. The deviation of response from the target is considered as loss. The loss-function based optimization approaches determine the factor settings, which minimizes the total loss for all quality characteristics. Taguchi (1986) proposed a quality loss-function approach, where he insisted that losses are caused when the product quality characteristics deviate from target. Taguchi's quality loss-function can be applied for problem requiring single response optimization, also for multiresponse optimization problems. Phadke (1989) used the quality loss function approach to study the surface defects and the thickness of poly-silicon wafer in a VLSI circuit board. The tradeoff between two responses was made using engineering knowledge and relevant experience. Tarng and Yang (1998) used Taguchi's signal to noise factors and loss-functions for solving multiresponse problem. Generally, when using Taguchi's loss function each quality characteristic is converted to a quality loss value and the weights are added to each loss value based on relevant experience. The total loss is calculated by the summation of weighted quality loss value for each quality characteristics.

Pignatiello (1993) proposed a loss function based multiresponse optimization approach to minimize the deviation and also to improve the robustness. The robustness is captured by including the variance-covariance matrix of responses. The general formulation of loss function proposed by Pignatiello (1993) is given below:

$$E(L(Y(X)) \;=\; E\left[(Y(X) - T)'C\big(Y(X) - T\big)\right] + trace[C\varepsilon_{Y(X)}] \qquad (3\text{-}8)$$

where, E(L(Y(X)) is the expected loss function, Y(x) is the response, T is the target matrix, C is the cost matrix, which is determined by the process parameter setting and scrap cost, and $\varepsilon_{Y(X)}$ is the variance - covariance matrix of responses. The first term in the expected loss function represents for bias and the second term accounts for variance. The advantage of this method is the addition of variance term but the cost matrix which is used as weights is difficult to obtain (Wurl and Albin,1999).

Riberto and Elsayed (1995) proposed a gradient loss function based approach that minimizes the total deviation of response from target, includes term for maximizing robustness, and to minimize fluctuation in design parameters. The gradient loss function proposed by Riberto and Elsayed (1995) is given below:

$$L = \sum_{i=1}^{q} w_i[(\hat{Y}_i - M_i)^2 + \hat{\sigma}_{Y_i}{}^2] \qquad (3\text{-}9)$$

The first term in equation (3-9) represent deviation of predicted response from target and the second term considers both the robustness and fluctuation in design parameters. The second term is expanded by Taylor expansion series.

$$\hat{\sigma}_{Y_i}{}^2 = \sum_{k=1}^{p} \hat{\sigma}_{x_k}{}^2 \left(\frac{\partial Y_i}{\partial x_k}\right)^2 + \sum_{k \neq 1} \hat{P}_{kl} \hat{\sigma}_{x_k}{}^2 \hat{\sigma}_{x_l}{}^2 \left(\frac{\partial Y_i}{\partial x_k}\right)\left(\frac{\partial Y_i}{\partial x_l}\right) \qquad (3\text{-}10)$$

where, w_i is the weights for each response, $\hat{\sigma}_{Y_i}{}^2$ is the predicted variance for i^{th} quality characteristic. $\hat{P}_{kl}$, $\hat{\sigma}_{x_k}{}^2$ and $\hat{\sigma}_{x_l}{}^2$ are the estimated correlation coefficient between x_k and x_l, estimated variances for process parameter x_k and x_l respectively. This approach is used for minimizing total cost in multiresponse optimization environment (Ribeiro et al., 2001).

The standardized loss function proposed by Artiles-Leon (1996-97) eliminates the difficulty in measuring the proportionality constant in Taguchi's loss-functions. Ideally, the value of K is measured from process scrap and it involves uncertainty in the measuring process. Standardized loss function considers the economic loss within the upper and lower limit region. When the response value is at target, a quality loss value of 0 is obtained. When the response is at upper or lower limits, a quality loss value is 1 is obtained. Taguchi's loss function for nominal-the-best quality characteristic is given below:

$$L(Y) = K\,(Y - T)^2 \tag{3-11}$$

where, L(Y) is the total loss value, K is the proportionality constant, Y is the quality characteristic value and T is the target value. The total loss value is unity when the predicted response value is at upper specification or at lower specification level. The value of proportionality constant is determined using the equation (3-11) by equating the total loss to unity and response to upper specification limit.

$$K = \frac{1}{(USL - T)^2} \tag{3-12}$$

Considering the NTB quality characteristic, the proportionality constant K can be equated as shown below:

$$K = \left(\frac{2}{USL - LSL}\right)^2 \tag{3-13}$$

As the total loss is equated to 1, the loss function is known as standardized loss function and for NTB quality characteristics it is:

$$SLoss(Y) = 4\left(\frac{Y - T}{USL - LSL}\right)^2 \tag{3-14}$$

This approach is extended to LTB and STB type by MA and Zhao (2004) and the standardized loss function for other types of quality characteristics are:

For LTB type

$$SLoss\ (Y) = \left(\frac{Y - Y_u}{Y_u - Y_l}\right)^2 \tag{3-15}$$

For STB type

$$SLoss\ (Y) = \left(\frac{Y - Y_l}{Y_u - Y_l}\right)^2 \tag{3-16}$$

Considering all three types of quality characteristics, the overall standardized loss function is formulated as:

Minimize

$$\left(\frac{Y - Y_u}{Y_u - Y_l}\right)^2 + 4\left(\frac{Y - T}{USL - LSL}\right)^2 + \left(\frac{Y - Y_l}{Y_u - Y_l}\right)^2$$

S.t.

$$X_k \in R \tag{3-17}$$

where, Y is the predicted response value, T is the target for NTB of quality characteristic, USL and LSL are the upper and lower specification limit, Y_u is the upper range above which the response are undesirable, and Y_l is the lower range below which the response are undesirable. K is the number of design factors involved in the optimization process and R is the experimental region through which the model is valid.

A loss function based approach known as Hybrid quality loss function (HQLF) proposed by Bhamare et al. (2009) is based on the concept of goal programming and quality loss function. The HQLF minimizes the undesirable deviation and maximizes desirable deviation using exponential transformation of deviational variable. In order to develop a composite quality loss function for all quality characteristics and achieve continuous function, an exponential transformation is used for the undesirable deviational variables and a negative exponential transformation is used for the desirable deviational variables, which are given as:

$$L = [(\exp(d))^2]$$

$$L = [(\exp(-d))^2] \tag{3-18}$$

For NTB quality characteristics both underachievement and overachievement are considered as undesirable deviation and the loss function is given below:

$$L = [(exp(d^+))^2 + (exp(d^-))^2] \tag{3-19}$$

For STB type of quality characteristics, underachievement is considered as desirable and over achievement is considered as undesirable and the loss function is given as:

$$L = [(\exp(d^+))^2 + (\exp(-d^-))^2] \tag{3-20}$$

For LTB type of quality characteristics, over achievement is considered as desirable and under achievement is considered as undesirable and the corresponding loss function is given as:

$$L = [(\exp(-d^+))^2 + (\exp(d^-))^2] \tag{3-21}$$

The overall hybrid quality loss function-based optimization model as given as:

Minimize

$$\sum_{i=1}^{n} w_{1i}[(exp(d_{1i}^+))^2 + (exp(d_{1i}^-))^2] +$$

$$\sum_{i=1}^{n} w_{2i}[(exp(d_{2i}^+))^2 + (exp(-d_{2i}^-))^2] +$$

$$\sum_{i=1}^{n} w_{3i}[(exp(-d_{3i}^+))^2 + (exp(d_{3i}^-))^2]$$

S.t.

$$f_{1i}(x_j) + d_{1i}^- - d_{1i}^+ = T_{1i}$$

$$f_{2i}(x_j) + d_{2i}^- - d_{2i}^+ = T_{2i}$$

$$f_{3i}(x_j) + d_{3i}^- - d_{3i}^+ = T_{3i}$$

$$X_k \in R \qquad (3\text{-}22)$$

where w_{1i}, w_{2i}, and w_{3i} are the weights attached for each response. d_{1i}^+, d_{2i}^+ and d_{3i}^- are undesired deviational variables. d_{1i}^-, d_{2i}^- and d_{3i}^+ are desired deviational variables. T_{1i}, T_{2i} and T_{3i} are the required target value for each response. $f_{1i}(x_j)$, $f_{2i}(x_j)$ and $f_{3i}(x_j)$ are the response surface models developed as a function of x.

3.2. Comparative Study

In this section, a comparative study is performed to analyze the sensitivity of parameter selection. Selection of input parameters for each quality characteristic is not an easy task, and it involves uncertainty due to design error or subjective judgment. The sensitivity analysis is performed by changing the input parameters like response range, targets and weights using reasonable alternatives. The criteria by which these methods evaluate multiresponse problems are explained in this section. This approach helps to determine which among these six methods are least sensitive to variation in input parameters and yet provide design factor setting that achieves better tradeoff.

Furthermore, the effect of correlation among multiple quality characteristics is also discussed in this comparative study. The correlation between the responses makes the optimization process more complex (Wu, 2004). The six methods are used to solve two correlated multiple response problems from literature, to study the impact of correlation on achieving optimal response values. The problems are selected based on responses having different magnitude, involving multiple correlation, and are of different quality characteristics. The quality loss value is used as performance measure to compare different multiresponse optimization methods. The quality loss values are calculated from the deviation between the response and the target. To avoid the influence of higher magnitude on quality loss values, the deviations are normalized. The normalized deviation assigns equal importance to all responses, and minimizes the domination of responses having higher magnitude. The normalized deviation and the quality loss values are calculated using the equation below:

For undesirable deviation:

$$N_i^- = \frac{d_i^-}{T_i}$$

$$\text{Loss} = (N_i^-)^2$$

For desirable deviation:

$$N_i^+ = \frac{d_i^+}{T_i}$$

$$\text{Loss} = \left(\frac{1}{N_i^+}\right)^2 \qquad (3\text{-}23)$$

where, $N_i^\pm$ is the normalized deviation value for i^{th} quality characteristic, T_i is the target value for i^{th} quality characteristic, $d^+$and d^- are the desirable and undesirable deviation of the quality characteristics.

3.3. Example 1

To evaluate the performance of the six multiresponse optimization methods and to investigate the sensitivity of these methods, we consider a dual response problem from

Vining (1998), which was originally presented by Box, Hunter and Hunter (1978). In this problem, engineers need to maximize the conversion of polymer (Y_1) and achieve a target value for thermal activity (Y_2). The controllable factors are: reaction time (X_1), reaction temperature (X_2) and the amount of catalyst (X_3). The acceptable value for Y_1 is greater than 80 and for Y_2, it is 55 to 60 with a target value of 57.5. The second order model is developed for two responses from the experimental data. The experiments are conducted within the experimental range: -1.682 and 1.682 for all design factors. The response surface models for these two quality characteristics are given below:

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83X_1{}^2 + 2.94X_2{}^2$$
$$- 5.19X_3{}^2 + 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3$$
$$Y_2 = 60.23 + 3.58X_1 + 2.23X_2 \qquad (3\text{-}24)$$

The problem explained above is solved using the six multiresponse optimization methods, in each method the sensitivity of the parameter selection is analyzed by changing the input response parameters such as response range, target, and weight for each quality characteristic. To calculate the quality loss for each predicted response, a target of 100 needs to be achieved for conversion of polymer and 57.5 for thermal activity.

3.3.1. Constrained Optimization

For solving the above problem using constrained optimization approach, the conversion of polymer (Y_1) is considered as the primary response and the thermal activity (Y_2) is considered as the secondary response in the model. The primary response (Y_1) is maximized while satisfying the secondary response (Y_2) between the acceptable range 55 and 60. The constrained optimization model for this problem is given below:

Maximize

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83X_1{}^2 + 2.94X_2{}^2 - 5.19X_3{}^2$$

$$+ \ 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3$$

S. t.

$$55 \leq 60.23 + 3.58X_1 + 2.23X_2 \leq 60$$

$$-1.682 \leq X_1 \leq 1.682$$

$$1.682 \leq X_2 \leq 1.682$$

$$1.682 \leq X_3 \leq 1.682$$

$$X_k \in R \qquad (3\text{-}25)$$

The above model is solved using Nonlinear programming software called as General Algebraic Modeling Software (GAMS) and the results are shown in Table 1. For alternative 1, this method attains a quality loss value of 0.0029 units.

Table 1. Constrained Optimization - Results

Alternatives	Response Range			Design Factor			Predicted Response	
	L_2	U_2	T_2	X_1	X_2	X_3	Y_1	Y_2
1	55	60	N/A	-0.906	1.682	-0.891	96.864	55
2	50	65	N/A	-1.682	1.682	-1.682	102.28	50.45

Furthermore, to analyze the sensitivity of this method, the lower and upper parameter value for Y_2 is widened to 50 and 65 as shown in alternative 2. The widening of response limits increased the predicted response value for Y_1 to 102.28 units and there is a decrease in Y_2 to 50.45 units which produces overall quality loss value of 0.0156 units.

Constraint optimization is highly sensitive to variations in lower and upper response limits, a small variation in this limit changes the entire optimal solution set. In this example, widening of response range for secondary response (thermal activity) has increased the solution space thereby it improves the predicted response value for the primary response (conversion of polymer). In terms of assigning priorities to the responses, this method doesn't allow the design engineer to assign priorities directly to each response. The only way to assign priorities for each response is to increase or decrease the response upper and lower limits. For example, to assign higher importance for Y_1, the response upper and lower limits for Y_2 has to be increased as shown in alternative 2. To increase the priority for Y_2, the upper and lower response limits for Y_2 has to be decreased. These priorities can be effectively assigned only if the process related information is known to the design engineer.

3.3.2. Desirability Approach

For using the desirability-function-based optimization model, it is required to calculate desirability value for each response. The desirability values for the conversion of polymer (d_1) and the thermal activity (d_2) are calculated using equations given below:

$$d_1 = \begin{cases} 0, & \hat{Y}_i(x) < L_i \\ \left(\dfrac{\hat{Y}_i(x) - L_i}{T_i - L_i}\right)^{w_1}, & L_i < \hat{Y}_i(x) \le T_i \\ 1, & \hat{Y}_i(x) > T_i \end{cases}$$

$$d_2 = \begin{cases} 0, & \hat{Y}_i(x) < L_i \\ (\frac{\hat{Y}_i(x) - L_i}{T_i - L_i})^{w_2}, & L_i < \hat{Y}_i(x) \le T_i \\ (\frac{\hat{Y}_i(x) - U_i}{T_i - U_i})^{w_2}, & T_i \le \hat{Y}_i(x) < U_i \\ 0, & \hat{Y}_i(x) > U_i \end{cases} \qquad (3\text{-}26)$$

The overall desirability function is obtained using the geometric mean and it is calculated from the equation below:

Maximize

$$(d_1 X d_2)^{1/2} \tag{3-27}$$

The problem is solved using the traditional desirability approach as proposed by Derringer and Suich (1980) and the optimal result is shown in Table 2. Alternative 1 show the result of the problem solved using the input parameter taken from the example and attains a quality loss value of 0.0087units. The response upper and lower limits with the target value are varied as shown in alternative 2.

Table 2. Desirability Function - Results

Alt	L_1	U_1	T_1	L_2	U_2	T_2	W_1	W_2	X_1	X_2	X_3	Y_1	Y_2
1	80	100	100	55	60	57.5	1	1	0	0	0	91.00	58.9
2	65	105	105	45	70	57.5	1	1	0	1.68	0	97.00	60.7
3	65	105	105	45	70	57.5	0.1	0.9	0	0	0	91.00	58.9
4	65	105	105	45	70	57.5	0.9	0.1	0	1.68	0	97.00	60.7

The variation in the input parameter, changes the optimal solution, and it attains quality loss value of 0.004 units. Furthermore, the user assigned priorities (or weights) are changed for two responses by assigning higher priorities to Y_2 in alternative 3 and assigning higher priorities to Y_1 in alternative 4. In desirability approach, the weights are assigned to the desirability value (d_i), which changes the overall desirability value (D), thereby changing the optimum predicted response values. Thus, the result from Table 2 shows that, this method is sensitivity for selecting input parameter values. Using desirability approach for solving LTB and STB quality characteristics, a desirability value of 1 is assigned for the response value at target, and also for the response value above and below target respectively. By assigning equal desirability value for response values at target and beyond target, the approach fails to determine the better solution with lower quality loss values

3.3.3. Expected Loss Function

For using the expected loss function based approach, the cost matrix and the covariance matrix has to be estimated from the experimental data. The cost matrix for this example is estimated as C = $\begin{pmatrix} 0.200 & 0.025 \\ 0.025 & 0.500 \end{pmatrix}$ and the covariance matrix is estimated as $\varepsilon_{Y(X)} = \begin{pmatrix} 11.16 & -0.71 \\ -0.71 & 2.20 \end{pmatrix}$. The loss function based model which has to be minimized is shown below:

Minimize

$$(Y_1 - 100 \quad Y_2 - 57.5)\begin{pmatrix} 0.100 & 0.025 \\ 0.025 & 0.500 \end{pmatrix}\begin{pmatrix} Y_1 - 100 \\ Y_2 - 57.5 \end{pmatrix} +$$

$$trace\left[\begin{pmatrix} 0.100 & 0.025 \\ 0.025 & 0.500 \end{pmatrix}\begin{pmatrix} 11.16 & -0.71 \\ -0.71 & 2.20 \end{pmatrix}\right]$$

S.t.

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83X_1^{\,2} +$$

$$2.94X_2^{\,2} - 5.19X_3^{\,2} + 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3$$

$$Y_2 = 60.23 + 3.58X_1 + 2.23X_2$$

$$-1.682 \le X_1 \le 1.682$$

$$1.682 \le X_2 \le 1.682$$

$$1.682 \le X_3 \le 1.682$$

$$X_k \in R \qquad (3\text{-}28)$$

The above model is solved using General Algebraic Modeling Software and the optimal solution is shown in Table 3. The alternative 1 shows the optimal response value solved using the input parameter from the model, attains quality loss value of 0.0014 units. In alternative 2, the target value for Y_1 is changed to 105 increases the predicted response value to 97.45, and attains a quality loss value of 0.0029 units. In alternative 3, the target value of Y_1 is changed to 90 attains the response value to 90. This shows the sensitivity of this method in terms of target selection.

Table 3. Expected Loss Function - Results

Alt	T_1	T_2	C		X_1	X_2	X_3	Y_1	Y_2
1	100	57.5	0.2 0.025	0.025 0.5	-0.379	1.682	-0.499	96.22	57. 8
2	105	57.5	0.2 0.025	0.025 0.5	0.087	1.682	-0.114	97.45	60.28
3	90	57.5	0.2 0.025	0.025 0.5	-0.531	1.248	-0.371	90	57.14
4	100	57.5	0.5 0.025	0.025 0.1	0.385	1.682	0.119	99.12	61.8

In alternative 4, the cost matrix is selected such that it assigns higher importance to response Y_1, which produces an overall quality loss value of 0.00567 units. The expected loss function is sensitivity to cost matrix, and selection of cost matrix plays a major role in deciding the optimum value. The different ways of selecting cost matrix and the effectiveness of cost matrix in optimization processes are discussed in (Vining, 1998).

3.3.4. Gradient Loss Function

In gradient loss function based approach the deviation between the predicted response and target is minimized and the robustness is achieved by minimizing the variation. In this method, the variation is calculated using Taylor expansion series. The model formulated is shown below:

Minimize

$$(\hat{Y}_1 - 100)^2$$

$$+ 0.1\begin{pmatrix} 1.03 - 3.66X_1 + 2.13\,X_2 + 11.37X_3 + 4.64 + \\ 5.88\,X_2 + 2.137X_1 - 3.87X_3 + 6.2 - 10.38\,X_3 + 11.37X_1 - 3.87X_2 \end{pmatrix}$$

$$+ (\hat{Y}_2 - 57.5)^2 + 0.1(\,3.58 + 2.23)$$

S.t.

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83X_1^{\,2} +$$

$$2.94X_2^{\,2} - 5.19X_3^{\,2} + 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3$$

$$Y_2 = 60.23 + 3.58X_1 + 2.23X_3$$

$$-1.682 \le X_1 \le 1.682$$

$$1.682 \le X_2 \le 1.682$$

$$1.682 \le X_3 \le 1.682$$

$$X_k \in R$$

where,

$$\frac{dY_1}{dX_1} = 1.03 - 3.66X_1 + 2.13\,X_2 + 11.37X_3,$$

$$\frac{dY_1}{dX_2} = 4.64 + 5.88\,X_2 + 2.137X_1 - 3.87X_3$$

$$\frac{dY_1}{dX_3} = 6.2 - 10.38\,X_3 + 11.37X_1 - 3.87X_2,$$

$$\frac{dY_2}{dX_1} = 3.58,$$

$$\frac{dY_2}{dX_2} = 0,$$

$$\frac{dY_2}{dX_3} = 2.23,$$

$$\hat{\sigma}_{x_1} = \hat{\sigma}_{x_2} = \hat{\sigma}_{x_3} = 0.1 \qquad (3\text{-}29)$$

The formulated model is solved using nonlinear programming software called General Algebraic Modeling Software and the results are shown in Table 4. This method produces quality loss of 0.0021 units, when it is solved using the input parameters from example, and the optimal response value is shown in alternative 1.

Table 4. Gradient Loss Function - Results

Alt	T_1	T_2	W_1	W_2	X_1	X_2	X_3	Y_1	Y_2
1	100	57.5	1	1	-0.077	1.682	-0.234	96.839	59.438
2	105	57.5	1	1	0.684	1.682	0.36	101.525	63.48
3	90	57.5	1	1	-0.862	1.682	0.173	89.97	57.5
4	100	57.5	0.2	0.8	-0.326	1.682	-0.434	96.28	58.096
5	100	57.5	0.8	0.2	0.387	1.682	0.12	99.104	61.85

The target value for Y_1 is changed to 105 as shown in alternative 2; the increase in target value, increases the deviation between responses and target, which changes the output optimal value to 101.53 units. In alternative 3, the target value is changed to 90, which attain the optimal solution at 89.97 units. The priorities assigned for each response are changed by assigning different weights. In alternative 4, higher priority is assigned to Y_2, which shifts the mean towards target attaining an overall quality loss value of 0.0015 units. In alternative 5, higher importance is assigned for Y_1, which increases the predicted response to 99.104 and attains an overall quality loss value of 0.0058 units. The change in response weights attains different optimal solution and weights should be applied only when the complete product related information's are known to design engineer. For example, if the cost loss due to thermal activity is lower than the cost loss due to the conversion of polymer, the weights applied as shown in alternative 4 can be used. This emphasizes the sensitivity of the method in selecting the target and weights.

The limitations of the two types of loss-function based approach (Expected loss function and gradient loss function) are: it does not classify different types of quality characteristics, especially LTB and STB. A target value is assigned for solving LTB and STB quality characteristics, assigning a target value shifts the focus of the optimization process to search for an optimal value near target, and it does not explore the solution beyond the assigned target.

3.3.5. Standardized Loss Function

For using the standardized loss function based approach, the conversion of polymer is maximized considering a target value of 100 and the target value of 57.5 is to be achieved for thermal activity. The Standardized loss function based model is formulated and it is given below:

Minimize

$$SLoss(y) = \left(\frac{Y_1(X) - 100}{100 - 80}\right)^2 + 4\left(\frac{Y_2(X) - 57.5}{60 - 55}\right)^2$$

S.t.

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83X_1{}^2 +$$

$$2.94X_2{}^2 - 5.19X_3{}^2 + 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3$$

$$Y_2 = 60.23 + 3.58X_1 + 2.23X_2$$

$$-1.682 \leq X_1 \leq 1.682$$

$$1.682 \leq X_2 \leq 1.682$$

$$1.682 \leq X_3 \leq 1.682$$

$$X_k \in R \tag{3-30}$$

The above model is solved using nonlinear optimization software called as General Algebraic Modeling Software and the result is shown in Table 5. The model is solved using the input parameters from example and the optimal result is shown in alternative 1 attains a quality loss value of 0.0015 units.

Table 5. Standardized Loss Function - Results

Alt	L_1	U_1	T_1	L_2	U_2	T_2	X_1	X_2	X_3	Y_1	Y_2
1	80	100	100	55	60	57.5	-0.436	1.682	-0.523	96.188	57.5
2	70	110	100	50	65	57.5	-0.435	1.682	-0.522	96.188	57.5

In this approach the upper and lower limit is increased to study the behavior in parameter selection. In alternative 3, the lower and upper limit value for Y_1 is changed to 70 and 110, and for Y_2 it is changed to 50 and 65. The change in response upper and lower limits as shown in alternative 2 attains a quality loss value of 0.0015 units. For this particular example, the result shows that the change in response range does not produce any significant impact in the solution. In this method, the denominator term in its objective function acts as weights which are assigned to each response. In alternative 1, the weight assigned by its denominator term equals $\left(\frac{1}{100-80}\right)^2 = 0.0025$ units for Y_1, and for Y_2 it is $4 * \left(\frac{1}{60-55}\right)^2 = 0.16$ units. The response range in alternative 1, assigns higher importance for Y_2 when compared to Y_1. The result of assigning higher importance for Y_2 attains the predicted response value exactly at target. In alternative 2, the response range is widened, which further increases the priority for Y_2. Thus, the standardized loss function method attains the same optimum predicted response value. This method is highly sensitivity to change in response upper and lower limits.

3.3.6. Hybrid Quality Loss Function

For using hybrid quality loss function two type of deviation has to be determined: the desirable deviation which is maximized and the undesirable deviation which is minimized. As the conversion of polymer is LTB type of quality characteristics, a target value of 100 is considered for optimization. The deviation calculated for the response values above 100 are considered as desirable deviation (d_1^+). The deviation calculated for response values below 100 are considered as undesirable deviation (d_1^-). As the thermal activity is NTB type, a target value of 57.5 has to be achieved. The response values greater than and less than 57.5 has to be minimized. The (d_2^-) and (d_2^+) represents the undesirable deviations which are greater than and less than 57.7 are minimized. The HQLF modeled for conversion polymer (Y_1) and thermal activity (Y_2) is given by:

Minimize

$$w_1\,\{(exp(d_1^-))^2 + (exp(-d_1^+))^2\} + w_2\{(exp(d_2^+))^2 + (exp(d_2^-))^2\}$$

S.t.

$$Y_1 = 81.09 + 1.03X_1 + 4.64X_2 + 6.2X_3 - 1.83{X_1}^2 +$$

$$2.94{X_2}^2 - 5.19{X_3}^2 + 2.13X_1X_2 + 11.37X_1X_3 - 3.87\,X_2X_3 + d_1^- - d_1^+ = 90$$

$$60.23 + 4.26X_1 + 2.23X_2 + d_2^- - d_2^+ = 57.5$$

$$-1.682 \le X_1 \le 1.682$$

$$1.682 \le X_2 \le 1.682$$

$$1.682 \le X_3 \le 1.682$$

$$d_1^-, d_{2i}^+ \geq 0$$

$$X_k \in R \tag{3-31}$$

The above model is solved using nonlinear optimization software called General Algebraic Modeling Software. In this method, the user assigned priorities are changed to analyze the effectiveness of tradeoff between the responses. The example problem is solved using HQLF and the result is shown in Table 6. In alternative 1, equal weights are assigned to responses and the optimum response values are obtained. This method produces quality loss value of 0.0027 units. In alternative 2 and 3, the weights are changed such that, alternative 2 assign higher importance to thermal activity and alternative 3 assigns higher importance to conversion of polymer.

Table 6. Hybrid Quality Loss Function - Results

Alt	T_1	T_2	W_1	W_2	X_1	X_2	X_3	Y_1	Y_2
1	100	57.5	1	1	0.049	1.682	-0.144	97.29	60.08
2	100	57.5	0.2	0.8	-0.036	1.682	-0.21	96.96	59.63
3	100	57.5	0.8	0.2	0.127	1.682	-0.083	97.63	60.4
4	100	57.5	1	1	-0.361	1.682	-0.464	96.23	57.65
5	100	57.5	0.8	0.2	-1.682	1.682	-1.682	102.28	50.45
6	105	57.5	1	1	-0.361	1.682	-0.463	96.24	57.65
7	90	57.5	1	1	-0.356	1.682	-0.461	96.23	57.66

The change in weight does not produce any significant impact in the optimal solution and produces quality loss value of 0.0023 and 0.0031 units for alternative 2 and 3 respectively. The major concern in this method is that, it gives higher preference to the deviation $(d_i^{\mp})$ which has higher magnitude. In this example, the conversion of polymer is higher in magnitude when compared to thermal activity. One of the possible remedy for this problem is to normalize the model. The response surface model is normalized before optimization and the solution is shown in alternative 4, 5, 6, and 7. Once it is normalized, the HQLF attains the target value for thermal activity when equal weights are assigned and produces a quality loss value of 0.0014 units. It also produces a better trade off when the weights are changed as shown in alternative 5. Furthermore, in alternative 6 and 7 the target value is changed to 105 and 90 for response variable Y_1 which attains the same solution as shown in alternative 4. Therefore, this method show is insensitiveness to target selection. The main advantage of using this method is that, it does not restrict the design engineer with a predefined target value for all types of quality characteristics. It also gives the flexibility to the design engineer for selecting the design parameter values. The HQLF method provides robustness in parameter selection, and it does not require any upper and lower response limits.

Six multiresponse optimization methods are used to determine the settings for the design factors: reaction time, reaction temperature, and amount of catalyst to achieve the optimum responses: conversion of polymer, and thermal activity. The comparative study is further extended to analyze the impact of correlation between the responses. Six multiresponse optimization methods are used for solving problems from literature which has correlated quality characteristics. Two problems from the literature are selected based on intensity of correlation, Example 2 has linear correlation between two responses, and Example 3 has multiple correlations between responses.

3.4. Example 2

To analyze the effects of correlation among the responses, we consider a correlated response problem from Kim and Lin (2006) to study the properties of colloidal gas aphirons (CGA - colloidal gas aphirons is the micro bubbles formed due the mixing of surfactant solutions). The colloidal gas aphirons is measured by three responses: stability (Y_1), volumetric ratio (Y_2) and temperature (Y_3). The design factors which affect the performance of colloidal gas aphirons properties are concentration of surfactant (X_1), concentration of salt (X_2) and time of stirring (X_3). The responses are: stability - LTB, volumetric ratio- STB and temperature - NTB types of quality characteristics. The optimum parameter value for stability is greater than 3, for volumetric ratio it is less than 0.6, and for temperature it is 15 and 45 with a target value of 30. A central composite design with eight factorial points, six axial points, and a center point is conducted and a second order model is developed for each response and the model are shown below:

$$Y_1 = 4.95 + 0.82X_1 - 0.45X_2 - 0.15{X_1}^2 + 0.28{X_2}^2 - 0.11X_1X_2 + 0.07X_1X_3$$

$$Y_2 = 0.46 + 0.13X_1 - 006X_2 + 0.05X_3 - 0.07{X_1}^2 - 0.04{X_3}^2$$

$$Y_3 = 28.36 - 1.48X_1 + 2.33X_3 - 0.15{X_1}^2 - 1.42{X_2}^2 - 0.71X_1X_3 \qquad (3\text{-}32)$$

In this example, a positive correlation is being reported between stability (Y_1) and volumetric ratio(Y_2). The correlation coefficient between the two responses is +0.865 but the two responses are of different quality characteristics: stability - LTB and volumetric ratio - STB type of quality characteristics. Table 7 shows the results of solving correlated response problem using six multiresponse optimization models. The results clearly show the impact of correlation between stability (Y_1) and volumetric ratio (Y_2) as their optimal values either increase or decrease simultaneously. Although these two design characteristics are of opposite nature (stability- LTB type and volumetric ratio-STB type), the change in their optimal values is simultaneous and in same direction because of the positive correlation between them.

Table 7. Example 2 – Results

Alter-native	Method	Design Factor			Response			Quality Loss
		X_1	X_2	X_3	Y_1	Y_2	Y_3	
1	Constrained Optimization	-1	1	-1	3.99	0.11	25.2	0.22
2	Desirability Approach	-1	-1	-1	4.5	0.22	26	1.59
3	Expected Loss Function	0.567	-1	1	6.19	0.58	27.98	22.11
4	Gradient Loss Function	0.3	-1	1	5.63	0.54	29.45	19.4
5	Standard Loss Function	-1	-1	-1	4.73	0.25	25.2	2.38
6	Hybrid Quality Loss Function	-1	1	-1	3.99	0.11	25.2	0.22

The optimal solution for both the responses are increased for the multiresponse optimization methods shown in alternative 3, and 4, and they are decreased for the multiresponse optimization methods shown in alternative 1, 2, 5 and 6. Furthermore, constraint optimization method and HQLF based model provided relatively better trade-off in terms of quality loss values. However, it is important to note that these two models also failed to address the problem of correlation between design characteristics in multi-response optimization process.

3.5. Example 3

To study the behavior of multiple correlations between responses we consider another problem from Schmidt et al. (1979) in which four responses are considered. In this problem, both positive and negative correlation occurs between different responses. The effects of calcium chloride (CaCl) and cysteine on gel texture parameters, and compressible water on the dialyzed whey protein concentrate were studied. The gel texture parameter consists of hardness, cohesiveness, and springiness. A central composite experiment was conducted within the experimental region -1.414 and 1.414. Multiple regression analysis was used to obtain the prediction equation to measure the effects of design factors such as: cysteine (X_1) and calcium chloride (X_2) on responses hardness (Y_1), cohesiveness (Y_2), springiness (Y_3) and compressible water (Y_4). All four responses are LTB type quality characteristics. The individual maximum value for responses hardness, cohesiveness, springiness and compressible water are 2.69, 0.68, 1.90 and 0.71 which are used as the target value for optimization. The lower and upper limits are 2.16 and 3.22 for hardness, 0.65 and 0.71 for cohesiveness, 1.82 and 1.98 for springiness and 0.61 and 0.83 for compressible water. Second order response surface model is developed for four quality characteristics as given below:

$$Y_1 = 1.526 - 0.575X_1 - 0.524X_2 - 0.717X_1^2 - 0.98X_2^2 + 0.318X_1X_2$$

$$Y_2 = 0.66 - 0.092X_1 - 0.01X_2 - 0.096X_1^2 - 0.058X_2^2 - 0.07X_1X_2$$

$$Y_3 = 1.776 - 0.25X_1 - 0.78X_2 - 0.156X_1^2 - 0.079X_2^2 + 0.01X_1X_2$$

$$Y_4 = 0.468 + 0.131X_1 + 0.073X_2 + 0.026X_1^2 + 0.024X_2^2 - 0.083X_1X_2 \qquad (3\text{-}33)$$

The correlation matrix for the four responses is calculated using Minitab 15 and the significant correlation coefficient values are shown in the Table 8. The above problem is solved using the six multiresponse optimization methods and the optimum results obtained from each method is shown in Table 9.

Table 8. Example 3 - Correlation Matrix

Responses	Y_1	Y_2	Y_3
Y_1	1	-	-
Y_2	-	1	-
Y_3	0.79	0.86	1
Y_4	-0.93		-0.84

Table 9. Example 3 – Results

Method	Responses				Quality loss
	Y_1	Y_2	Y_3	Y_4	
Constrained Optimization (-0.564, 0.415)	1.488	0.684	1.820	Inf sol	N/A
Desirability Approach (-1.414, 0)	2.28	0.59	1.75	0.33	0.21
Expected loss function (-0.741, -0.893)	2.46	0.59	1.89	0.28	0.16
Gradient Loss function (-0.164, -1.414)	2.235	0.554	1.778	0.373	0.24
Standard Loss function (-0.284, -0.166)	1.775	0.675	1.846	0.418	0.3
Hybrid Quality Loss Function (-0.245, -0.486)	2.68	0.53	1.81	0.24	0.15

The results clearly show the quality characteristics: cohesiveness and springiness are close to target, but the major impact of correlation is between hardness and compressible water. Although the two responses, hardness and compressible water are same type of quality characteristics (LTB type) but the optimal solution either increase for hardness or decrease for compressible water or vice versa due to negative correlation between them.

In standardized loss function approach the predicted response value for compressible water is greater than 0.4 units, which decreases the predicted response value for hardness to less than 2 units. Similarly for methods like expected loss function and hybrid quality loss function models the predicted response value for hardness is greater than 2.5 units, which decreases the predicted response value for compressible water to below 0.3 units. Further, the quality loss values are used to determine the effectiveness of the different model in terms of better tradeoff. The HQLF method provides relatively better tradeoff among different responses.

3.6. Discussion

Optimizing multiple quality characteristics using traditional methods involve uncertainty in the decision making process. The multiresponse optimization methods discussed in this chapter try to minimize the uncertainty in the decision making process. The sensitivity in selecting input parameter and the impact of correlation between the responses are analyzed. The six methods are sensitive for weights assigned to each response, and the way of assigning priority differs for each method. All methods except standardized loss function, and constraint optimization, allows the design engineer to assign priorities directly in its objective function. These methods are highly sensitive for assigning priority and the weight assignment should be considered as critical input. The standardized loss function and constraint optimization are highly sensitive to response upper and lower limits. These two methods use response limits as weights, and it should also be considered as critical input in the optimization process. All methods except HQLF are sensitive for variation in target parameter selection. In these methods, different optimal solution is obtained when the target value is varied. The change in the optimal solution indicates the sensitiveness in the target selection parameter.

Furthermore, assigning the target values for LTB and STB quality characteristics in expected loss function, gradient loss function, and standardized loss function based approaches hinder the optimization process for searching optimal values beyond the target value. These methods consider the two types of deviation (desirable and undesirable) as total deviation and it minimizes the total deviation. The total deviation, which is considered as objective function is not a continuous function, but it is in a form of step function. The two limitations (minimizes total deviation and continuous function) in these methods have been addressed in HQLF. The HQLF maximizes the desirable deviation, minimizes undesirable deviation, and it uses exponential data transformation on original deviation variables to address the objective function continuity problem.

3.7. Conclusion

This chapter presented a comparative study on multi-response optimization methods. The result shows that all methods were sensitive in assigning priorities for each quality characteristic. All methods, except HQLF method were sensitive for selecting target values. This chapter also explored the effectiveness of these existing methods in dealing with correlated multiresponse optimization problems. It was found that none of the existing methods were capable to deal with correlated problems effectively. Therefore, this chapter concludes that correlation between responses affects the optimization process. A multi-response optimization model capturing correlation between responses is proposed in Chapter 4, and the proposed method is evaluated by solving examples from literature.

CHAPTER 4. MULTIRESPONSE OPTIMIZATION WITH CORRELATED QUALITY CHARACTERISTICS

This chapter discusses different optimization technique for solving multiple correlated quality characteristics. A multiresponse optimization method have been proposed for capturing correlation based on Awad and Kovach (2011) model, and it is used for solving correlated multiple quality characteristics problems. Furthermore, the effectiveness of the proposed method is compared with existing methods proposed by Awad and Kovach (2010) model, and PCA based methods.

In multiresponse optimization methods, correlation occurs in two different ways: it occurs between the design factors, and between the responses. An article published by Coleman et al. (1966) on equal opportunity in public education has correlated design factors. Correlation between the design factors has to be identified and eliminated before developing the response surface model. The most common method for eliminating the correlation between design factors is to identify the design factor causing correlation, and eliminating them from analysis. The model when developed without removing correlation between design factors increases the design factor coefficient value. The increase in design factor coefficients will not predict the exact relationship between the response and design factors (Mendenhall and Sincich, 1996). In practical applications, the correlation also occur between the noise factors, Hejazi et al. (2011) used PCA to develop a response model for solving problems with correlated noise factors, and correlation occurs between different quality characteristics, Sibalija and Majstorovic (2009) used PCA based approach for solving correlated quality characteristics in a thermo-sonic copper wire bonding process, Darwish and Al-Dekhial (1999) developed a statistical model for spot welding process in which the design factors such as failure load and nugget area are correlated with process parameter. The following methods are used for solving multiple quality characteristics problem by considering correlation in their optimization process.

4.1. Principal Component Analysis

Su and Tong (1997) proposed a multiresponse method based on PCA for solving correlated multiple quality characteristics. PCA was used to transform the correlated response data into uncorrelated data set. The optimal factor level is determined from the uncorrelated data set. In their proposed approach, a quality loss value is calculated for each response using Taguchi's quality loss function. To minimize the impact of scale parameter, the quality loss value is normalized using the equation given below:

$$y_{ij} = \frac{L_i^+ - L_{ij}}{L_i^+ - L_i^-} \qquad (4\text{-}1)$$

where, L_{ij} is the quality loss value for i^{th} response at j^{th} trial respectively, y_{ij} is the normalized quality loss value calculated for i^{th} response at j^{th} trial respectively, L_i^+ is the maximum loss value obtained for i^{th} response and L_i^- is the minimum loss value obtained for i^{th} response. Using PCA on the normalized quality loss value, the parameters such as Eigen values, Eigenvectors, and percentage of variation are obtained from the data. These parameters are used to calculate a multiresponse performance index using the equation given below:

$$\Omega_{kj} = \sum_{i=1}^{p} a_{ki} y_{ij} \qquad (4\text{-}2)$$

where, Ω_{kj} is the multiresponse performance index corresponding to k^{th} Eigen value and j^{th} trial, a_{ki} is the element of the Eigen vector corresponding to k^{th} Eigen value and i^{th} response. The optimum factor level combination is selected from the multiresponse performance index. The larger the multiresponse performance index implies better the quality of the product. Kaiser (1960) study has been used to select the components whose Eigen values are greater than one. For every Eigen value greater than one, a multiresponse performance index is obtained. For more than one multiresponse performance index, Su and Tong (1997) suggested tradeoff for selecting optimal design factor level setting but the author did not address this issue in their analysis.

The drawback in this approach is that, it uses only one principal component for which Eigen value is greater than one. This is not applicable in the present day manufacturing condition because most of the problems occur with more than one principal component having Eigen values greater than one, and considering the first principal component does not produce optimal solution (Fung and Kang, 2005).

Fung and Kang (2005) proposed a multiresponse optimization method which considers all principal components with Eigen value greater than one, and also reduces the uncertainty in the decision making process. Their research used Taguchi method and PCA for optimizing frictional properties of PBT composites in the injection molding process. Frictional coefficients and surface roughness are two quality characteristics involved in the optimization process. Their research also used coefficient of determination to integrate all principal components to form a comprehensive index. The correlation coefficient is calculated for the normalized response values using the equation given below:

$$R_{jl} = \left(\frac{Cov(y_j, y_l)}{\sigma_j\, \sigma_l}\right) \text{ where, } j \neq l \tag{4-3}$$

where, R_{jl} is the correlation coefficient between the normalized j^{th} and l^{th} responses, σ_j and σ_l are the standard deviation of the response sequence. The coefficient of determination is used as weights to integrate all principal components. The coefficient of determination is multiplied with principal component, and it is added to form a comprehensive index. The higher the comprehensive index values, better the quality of the product.

Liao (2006) proposed a method to address the limitations in Su and Tong (1997) by considering all principal components obtained from the analysis. The multiresponse optimization method proposed by Liao (2006) uses Taguchi's quality loss function, and weighted principal component for solving multiple correlated quality characteristics. In this approach, the explained variance is used as weight to integrate all principal components into single overall multiresponse performance index and it is given by:

$$\Omega_{kj} = \sum_{i=1}^{p} a_{ki} y_{ij}$$

$$\text{Overall MPI} = \sum_{k=1}^{n} V_k\, \Omega_{kj} \tag{4-4}$$

where, a_{ki} is the elements of the eigenvector corresponding to k^{th} Eigen-value and i^{th} response respectively. y_{ij} is the normalized quality loss value calculated for i^{th} response at j^{th} trial respectively. Ω_{kj} is the principal component score corresponding to k^{th} Eigen value and j^{th} trial respectively. V_k is the explained variance for k^{th} principal components. The major difference between Liao (2006), and Fung and Kang (2005) method is that, Fung and Kang (2005) used coefficient of determination to integrate the principal components having Eigen values greater than one, whereas Liao (2006) used explained variation obtained from the analysis to integrate all principal components.

Wu (2004) proposed a method based on PCA and grey relational analysis for solving correlated multiple quality characteristics. The grey system was proposed by Deng (1982), and the grey relational analysis is a sub part of the grey system. It is a useful technique to deal with incomplete, poor, and uncertain data (Wu, 2004). Also, the research uses the proportional quality loss value instead of Taguchi's quality loss value used by Su and Tong (1997). The proportional quality loss uses signal to noise (SN) ratio as the performance measure. The SN ratio is changed from starting parameter condition to optimal parameter condition after initial process optimization. The change in the parameter condition results in a new quality loss value (L'). The proportional quality loss value is the ratio of the new quality loss value to the average quality loss value.

$$PQL = \frac{L'}{L} \tag{4-5}$$

Furthermore, the proportional quality loss is normalized to eliminate the domination due to scale parameter using following equation:

$$NPQL_{ik} = \frac{PQL_{ik} - \min PQL_{ik}}{\max PQL_{ik} - \min PQL_{ik}} \tag{4-6}$$

where, NPQL is the normalized proportional quality loss value, PQL_{ik} is the proportional quality loss value for i^{th} response and k^{th} trial. The PCA is performed on normalized proportional quality loss values. The principal component score is obtained using the

Eigen vectors and the normalized proportional quality loss values is shown in the equation below:

$$Y_{ik} = \sum_{i=1}^{n} EV_i \, (NPQL_{ik}) \qquad (4\text{-}7)$$

where, EV_i is the Eigen vector obtained from principal component analysis for i^{th} quality characteristic and Y_{ik} is the principal component score for i^{th} quality characteristics and k^{th} trial. In grey relational analysis, the principal component score is transformed to a set of comparable sequences using the equation shown below:

$$Z_{ik} = \frac{max|Y_{ik}| - |Y_{ik}|}{max|Y_{ik}| - min|Y_{ik}|} \qquad (4\text{-}8)$$

where, Z_{ik} is the standard multiresponse performance statistics and Y_{ik} is the principal component score for i^{th} quality characteristic and k^{th} trial. The grey relational coefficients and grey relational grade are determined from the equation below:

$$\xi_{ik} = \frac{minmin|Z_{ik} - Z_0(i)| + \varsigma \, maxmax|Z_i(k) - Z_0(i)|}{|Z_i(k) - Z_0(i)| + \varsigma \, maxmax|Z_i(k) - Z_0(i)|}$$

$$\gamma_k = \sum_{i=1}^{p} \omega_i \xi_i(k) \qquad (4\text{-}9)$$

where, ξ_i is the grey relational coefficient for i^{th} quality characteristic, $Z_0(i)$ is the ideal sequence with a value of 1, ς is the distinguished coefficient with a value of 0.5, γ_k is the grey relational grade for k^{th} treatment, and ω_i is the percentage variance of i^{th} component in principal component analysis. The higher the grey relation grade, the better quality of the product can be achieved. The method proposed by Wu (2004) is used by Sibalija and Majstorovic (2009) for solving correlated responses in thermo-sonic copper wire bonding process. The wire bonding process has three correlated quality characteristics namely: pull test average, pull test minimum, and process yield which are controlled by nine design factors.

The drawbacks of PCA based approach is that, the optimal factor setting is obtained only at the design factor levels. The levels are selected when conducting the experiments. These methods do not explore the optimum solution between the design factor levels. Implementing PCA based approach, requires significant amount of experimental data which is costly to obtain (Awad and Kovach, 2011), and these methods requires rigorous computation. Also these methods use Taguchi's quality loss value, which creates uncertainty when calculating proportionality constant values.

4.2. Awad and Kovach Method

A multiresponse optimization method for solving multiple quality characteristics has been proposed by Awad and Kovach (2011), which maximizes the overall multivariate process capability indices. Awad and Kovach (2011) used the mean, variance of each response, and covariance between multiple responses to determine the multivariate process capability index. We consider this method for analysis as it captures covariance of two quality characteristics. The general form of multivariate process capability index is described as:

$$\widehat{MC}_{pm} = \left(\frac{Vol.of\ specification\ region}{vol.of\ process\ spread\ region}\right)^{\frac{1}{v}} \tag{4-10}$$

where, v is the number of quality responses and $\widehat{MC}_{pm}$ is the multivariate process capability index. As the specification region is ellipsoidal, Chan et al. (1991) proposed a formula to measure the multivariate process capability index which is given below:

$$\widehat{MC}_{pm} = \sqrt{\frac{nv}{\sum_{i=1}^{n}[Y_i - T]^{-1}A^{-1}[Y_i - T]}} \tag{4-11}$$

where, Y_i is the i^{th} vector of dimension v, and n is the sample size of the data collected. In the above equation, as n and v are constants, the denominator term is minimized to increase the overall process capability index. Awad and Kovach (2011) derived a generalized model using the denominator term in the above equation and it is minimized to increase the multivariate process capability index. The generalized model proposed by Awad and Kovach (2011) is given below:

Minimize

$$z = \sum_{i=1}^{v} \hat{\sigma}_i^2 (\hat{y}_i - T_i)^2 + \sum_{i=1}^{v}\sum_{j=1}^{v} 2\sigma_{ij}(\hat{y}_i - T_i)(\hat{y}_j - T_j) \quad where\ i \neq j$$

S.t.

$$y_i = f(x_j)$$

$$X \in R \tag{4-12}$$

where, $\hat{\sigma}_i$ is the fitted response of standard deviation for i^{th} quality characteristic, $\hat{y}_i$ is the fitted mean response of i^{th} quality characteristic, T_i is the target values for i^{th} quality characteristic, σ_{ij} is the covariance between the i^{th} and j^{th} response respectively, and z is the denominator term in the equation (4-11). The covariance (σ_{ij}) between two responses is estimated from the response value, which are obtained from experimental data. This approach uses multivariate process capability indices as the performance measures, which is often used for monitoring processes and determining quality control issues. To use this model for product design, the model has to be customized to fit the product design issues as the model is not a perfect fit for design optimization. Moreover, this method does not classify different types of quality characteristics (LTB, STB and NTB). The optimization model (equation 4-12) requires a target value for each response to determine the optimal design factor setting. Assigning a target value during optimization, especially for LTB and STB type of quality characteristics restrict the optimization process near the target, and it will not explore the optimum values above and below target (Bhamare et al., 2009).

In summary, the PCA based approaches were able to determine the optimal factor setting only at the factor levels, which are considered in the designed experiment, and it requires more trial runs for determining optimal solution. The Awad and Kovach (2011) model does not classify different types of quality characteristics (LTB, STB and NTB), and it also uses the target value for all quality characteristics, which minimizes the total deviation (both desirable and undesirable).

4.3. Proposed Method

To address the limitations of PCA based approach and the model proposed by Awad and Kovach (2011), a multiresponse optimization method has been proposed in this section. The proposed method combines both HQLF model (Bhamare et al., 2009) and model proposed by Awad and Kovach (2011) to achieve better trade-off and to provide robust methodology. The integration of HQLF model treats the deviations from target value as desirable and undesirable deviation. This consideration facilitates the model to explore the optimal solution above or below the assigned target for LTB or STB type quality characteristics respectively. The proposed method further captures the correlation between the responses, and determines factor settings, which are least affected by correlation. The inclusion of the model suggested by Awad and Kovach (2011) further minimizes the overall variance of all quality characteristics, and hence making the model more robust to random variability. To develop an overall model, the variance of two independent variables is calculated, and it is shown below:

$$Var(Z_1 + Z_2) = Var(Z_1) + Var(Z_2) \qquad (4\text{-}13)$$

where, Z_1 and Z_2 are the two independent variable. If these two variables are dependent, the overall variation of these two variables can be calculated using the equation below:

$$Var(Y_1 + Y_2) = Var(Y_1) + Var(Y_2) + 2Cov(Y_1, Y_2)$$

$$Var(Y_1) = E((Y_1 - \mu_1)^2)$$

$$Cov\,(Y_1, Y_2) = (Y_1 - \mu_1)(Y_2 - \mu_2)$$

$$Var(Y_1 + Y_2) = E((Y_1 - \mu_1)^2) + E((Y_2 - \mu_2)^2) + 2(Y_1 - \mu_1)(Y_2 - \mu_2) \qquad (4\text{-}14)$$

where, Y_1 and Y_2 are the random dependent variables associated with first and second quality characteristic. μ_1 and μ_2 are the mean values associated with the first and second quality characteristic. If the mean associated with each quality characteristic is replaced by the target value, the equation (4-14) can be modified as:

$$Var(Y_1 + Y_2) = E((Y_1 - t_1)^2) + E((Y_2 - t_2)^2) + 2(Y_1 - t_1)(Y_2 - t_2) \qquad (4\text{-}15)$$

without loss of generality, the random variable Y_i is replaced by fitted mean response $\hat{y}_i$. Equation (4-15) can be generalized for solving multiple responses problem is shown below:

$$Z = Var(\textstyle\sum_{i=1}^{n} y_i) = \sum_{i=1}^{n}\{(\hat{y}_i - t_i)^2\} + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\hat{y}_i - t_i)(\hat{y}_j - t_j) \qquad (4\text{-}16)$$

The term $(\hat{y}_i - t_i)$ in the above equation represents the difference between optimal response and assigned target value. Furthermore, to avoid the optimization process from focusing near the assigned target for all types of quality characteristics, and to maximize the desirable deviation for LTB and STB quality characteristics, the above equation is merged with hybrid quality loss function (Bhamare et al., 2009). The Hybrid quality loss function minimizes the undesirable deviation and maximizes the desirable deviation using exponential transformation. The loss value for undesirable deviations is calculated using the equation below:

$$L(d) = [(exp(d))^2] \tag{4-17}$$

To calculate the total loss value for the desirable deviation, a negative exponential transformation is used as shown below:

$$L(d) = [(exp(-d))^2] \tag{4-18}$$

To achieve continuous nonlinear objective function, the exponential data transformation for the deviation variable is used in the objective function (Bhamare et al., 2009). The HQLF concept is applied on equation (4-16), and it is modeled for all types of quality characteristics as shown below:

For LTB quality characteristics

$$z = \sum_{i=1}^{v}[(exp(-d_{1i}^{+}))^2 + (exp(d_{1i}^{-}))^2] + \sum_{i=1}^{v}\sum_{j=1}^{v} 2\{exp(d_{1i}^{-})\}\{exp(d_{1j}^{-})\} \quad \text{where } i \neq j$$

For STB quality characteristics

$$z = \sum_{i=1}^{v}[(exp(d_{2i}^{+}))^2 + (exp(-d_{2i}^{-}))^2] + \sum_{i=1}^{v}\sum_{j=1}^{v} 2\{exp(d_{2i}^{+})\}\{exp(d_{2j}^{+})\} \text{ where } i \neq j$$

For NTB quality characteristics where both side deviations are undesirable:

$$z = \sum_{i=1}^{v}[(exp(d_{3i}^{+}))^2 + (exp(d_{3i}^{-}))^2] + \sum_{i=1}^{v}\sum_{j=1}^{v} 2\{exp(d_{3i}^{-}) + exp(d_{3i}^{+})\}$$

$$\{exp(d_{3j}^{-}) + exp(d_{3j}^{-})\} \qquad \text{where } i \neq j \qquad (4\text{-}19)$$

where v is the number of quality responses, d_{1i}^{+}, d_{2i}^{+} and d_{3i}^{+} are undesired deviational variables for mean response model, d_{1i}^{-}, and d_{2i}^{-} are desired deviational variables for mean response model. To include the random variability in the proposed approach, the equation (4-19) is merged with the Awad and Kovach (2011) model shown in equation (4-12). The integrated objective function of the proposed model is formulated below.

For LTB quality characteristics:

$$z = \sum_{i=1}^{n}[(exp(d_{\sigma i}^{+}))^2 + (exp(-d_{\sigma i}^{-}))^2][\{exp(-d_{\mu i}^{+})\}^2 + \{exp(d_{\mu i})\}^2]$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n} 2\sigma_{ij}\{exp(d_{\mu i}^{-}) + exp(-d_{\mu i}^{+})\}\{exp(d_{\mu j}^{-}) + exp(-d_{\mu j}^{+})\} \qquad \text{where } i \neq j$$

For STB quality characteristics

$$z = \sum_{i=1}^{n}[(exp(d_{\sigma i}^{+}))^2 + (exp(-d_{\sigma i}^{-}))^2][\{exp(d_{\mu i}^{+})\}^2 + \{exp(-d_{\mu i})\}^2]$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n} 2\sigma_{ij}\{exp(-d_{\mu i}^{-}) + exp(d_{\mu i}^{+})\}\{exp(-d_{\mu j}^{-}) + exp(d_{\mu j}^{+})\} \qquad \text{where } i \neq j$$

For NTB quality characteristics

$$z = \sum_{i=1}^{n}[(exp(d_{\sigma i}^{+}))^2 + (exp(-d_{\sigma i}^{-}))^2][\{exp(d_{\mu i}^{+})\}^2 + \{exp(d_{\mu i}^{-})\}^2]$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}2\sigma_{ij}\{exp(d_{\mu i}^{-})+exp(d_{\mu i}^{+})\}\{exp(d_{\mu j}^{-})+exp(d_{\mu j}^{+})\} \qquad \text{where } i \neq j$$

The overall model for all three types of quality characteristics is formulated and it is shown below:

Minimize

$$z=\sum_{i=1}^{v}\{(exp(-d_{\sigma i}^{+}))^{2}+(exp(d_{\sigma i}^{-}))^{2}\}\{\left(exp(-d_{\mu i}^{+})\right)^{2}+\left(exp(d_{\mu i})\right)^{2}\}$$

$$+\sum_{i=1}^{v}\{(exp(d_{\sigma i}^{+}))^{2}+(exp(-d_{\sigma i}^{-}))^{2}\}\{\left(exp(d_{\mu i}^{+})\right)^{2}+\left(exp(-d_{\mu i})\right)^{2}\}$$

$$+\sum_{i=1}^{v}\{(exp(d_{\sigma i}^{+}))^{2}+(exp(d_{\sigma i}^{-}))^{2}\}\left\{\left(exp(d_{\mu i}^{+})\right)^{2}+\left(exp(d_{\mu i})\right)^{2}\right\}$$

$$+\sum_{i=1}^{v}\sum_{j=1}^{v}2\sigma_{ij}\{exp(d_{\mu i}^{-})+exp(d_{\mu i}^{+})\}\{exp(d_{\mu j})+exp(d_{\mu j}^{+})\} \quad \text{where } i \neq j$$

S.t.

$$f_1(x_j)+d_{\mu i}^{-}-d_{\mu i}^{+}=T_{1i}$$

$$f_2(x_j)+d_{\mu i}^{-}-d_{\mu i}^{+}=T_{2i}$$

$$f_3(x_j)+d_{\mu i}^{-}-d_{\mu i}^{+}=T_{3i}$$

$$f_4(x_j)+d_{\sigma i}^{-}-d_{\sigma i}^{+}=T_{4i}$$

$$f_5(x_j)+d_{\sigma i}^{-}-d_{\sigma i}^{+}=T_{4i}$$

$$f_6(x_j)+d_{\sigma i}^{-}-d_{\sigma i}^{+}=T_{4i}$$

$$d_1^{\mp},d_2^{\mp},d_3^{\mp},d_4^{\mp},d_5^{\mp},d_6^{\mp} \geq 0$$

$$X_k \in R \tag{4-17}$$

where, v is the number of quality responses, $d_{\sigma i}^{\pm}$ is the desirable and undesirable deviation for standard deviation model for i[th] quality characteristics, $d_{\mu i}^{\pm}$ is the desirable and the undesirable deviation (mean response) for i^{th} quality characteristics, σ_{ij} is the covariance between i^{th} and j^{th} quality characteristic, which is determined from experimental data. T_{1i}, T_{2i} and T_{3i} are the required target value for each response, $f_{1i}(x_j), f_{2i}(x_j)$ and $f_{3i}(x_j)$ are the response surface model as a function of x.

The main advantage of proposed approach is that it classifies all quality characteristics into three different categories (NTB, LTB, and STB), and it explores the optimal response values beyond the assigned target by maximizing the desirable deviation for LTB and STB type of quality characteristics. The applicability of the proposed model is demonstrated by considering two examples, and comparing the result with existing approaches. Furthermore, to study the superiority, and robustness of the proposed approach, the sensitivity analysis is performed by considering different values of input parameter (target).

4.5. Example 4

To compare the effectiveness of the proposed model, we consider a problem discussed in Kim and Lin (2006), and Awad and Kovach (2011) to measure the properties of colloidal gas aphirons (colloidal gas aphirons is the micro bubbles formed due the mixing of surfactant solutions) and it is measured using three responses. These three responses are, stability (Y_1)--LTB, volumetric ratio (Y_2)—STB, and temperature (Y_3)—NTB type of quality characteristics. The design factors which affect the performance of CGA properties are concentration of surfactant (X_1), concentration of salt (X_2), and time of stirring (X_3). The optimum parameter values for Y_1 is greater than 3, Y_2 is less than 0.6, and Y_3 is between 15 and 45, with a target value of 30. Design engineers are interested in achieving stability close to 7 and volumetric ratio close to 0.1 as targets. The two quality characteristics: stability and volumetric ratio are positively correlated with correlation coefficient of 0.865. Using experimental data, a second order models are developed for each response as shown in the equation below:

$$Y_{\mu 1} = 4.95 + 0.82X_1 - 0.45X_2 - 0.15X_1^{\ 2} + 0.28X_2^{\ 2} - 0.11X_1X_2 + 0.07X_1X_3$$

$$Y_{\mu 2} = 0.46 + 0.13X_1 - 006X_2 + 0.05X_3 - 0.07X_1^{\ 2} - 0.04X_3^{\ 2}$$

$$Y_{\mu 3} = 28.36 - 1.48X_1 + 2.33X_3 - 0.15X_1^{\ 2} - 1.42X_2^{\ 2} - 0.71X_1X_3 \qquad (4\text{-}23)$$

The experiments are replicated to determine the variance within the experiments. Response surface model for standard deviation is developed for three quality characteristics. The desired standard deviation for stability and volumetric ratio is 0 with an upper bound value of 0.1. The acceptable standard deviation for temperature is 1 with an upper bound value of 2. The second order models developed for three standard deviations are given below:

$$Y_{\sigma 1} = 0.06 + 0.11X_2 + 0.06X_3 + 0.12X_1^{\ 2} + 0.11X_3^{\ 2} - 0.10X_1X_3 + 0.05X_2X_3$$

$$Y_{\sigma 2} = 0.02 - 0.01X_1 + 0.01X_3 - 0.01X_3 + 0.02X_3^{\ 2} - 0.01X_1X_3 + 0.02X_2X_3$$

$$Y_{\sigma 3} = 6.1 - 1.5X_1 + 0.5X_2 + 4.8X_3 + 2.3X_2^{\ 2} - 0.65X_1X_3 - 0.67X_1X_2X_3 \qquad (4\text{-}24)$$

4.5.1. PCA Based Approach

A weighted principal component method proposed by Liao (2006) is used to solve the above multiresponse problem. The quality loss value is determined for all quality characteristics, and the proportionality constant is set to unity for calculation purpose. The quality loss value is normalized for each quality characteristic using the equation shown in (4-1), and normalized quality losses (NQL) are shown in Table 10. The PCA is performed using the statistical software, Minitab 15 on normalized quality loss data to calculate Eigen value, explained variation, and Eigen vector for each principal component, and it is shown in Table 11.

Table 10. PCA Data Summary

X_1	X_2	X_3	NQL_1	NQL_2	NQL_3	z_1	z_2	z_3	MPI
-1	-1	-1	0.398	0.962	0.649	0.481	-0.386	-1.060	0.149
1	-1	-1	0.927	0.277	0.262	-0.426	-0.167	-0.892	-0.351
-1	1	-1	0.068	1.000	0.336	0.692	-0.081	-0.795	0.393
1	1	-1	0.746	0.567	0.000	-0.140	0.154	-0.914	-0.061
-1	-1	1	0.373	0.795	0.995	0.439	-0.763	-0.993	-0.003
1	-1	1	1.000	0.108	0.768	-0.513	-0.698	-0.923	-0.586
-1	1	1	0.000	0.988	0.951	0.829	-0.681	-0.854	0.276
1	1	1	0.740	0.400	0.731	-0.135	-0.596	-0.932	-0.311
-1	0	0	0.258	0.918	1.000	0.606	-0.740	-0.997	0.111
1	0	0	0.754	0.277	0.802	-0.218	-0.695	-0.871	-0.396
0	-1	0	0.869	0.000	0.884	-0.477	-0.839	-0.778	-0.607
0	1	0	0.455	0.775	0.951	0.360	-0.723	-1.029	-0.041
0	0	-1	0.557	0.815	0.556	0.253	-0.328	-1.054	0.023
0	0	1	0.602	0.277	0.951	-0.087	-0.842	-0.791	-0.359
0	0	0	0.587	0.429	0.926	0.025	-0.782	-0.881	-0.270

Table 11. Explained Variance and Eigen Vector

Principal Component	Eigen Value	Explained Variation	Cumulative Variation	Eigen Vector
PC1	1.9118	0.637	0.637	(0.709, 0.021, -0.705)
PC2	1.0103	0.337	0.974	(0.687, 0.244, -0.684)
PC3	0.0779	0.026	1.000	(0.158, -0.97, -0.187)

Using the Eigen vectors from Table 11 and the normalized quality loss value from Table 10, a relation between the principal component and the response is derived which is known as principal component score (z_i) and it is shown below:

$$z_1 = -0.709NQL_1 + 0.021NQL_2 - 0.705NQL_3$$

$$z_2 = 0.687NQL_1 + 0.244NQL_2 - 0.684NQL_3$$

$$z_3 = 0.158NQL_1 - 0.97NQL_2 - 0.187NQL_3$$

$$MPI = 0.637z_1 + 0.337z_2 + 0.026z_3 \quad (4\text{-}25)$$

Furthermore, the overall multiresponse performance index (MPI) is calculated using explained variation for integrating all principal component score. The overall multiresponse performance index is calculated using the equation shown in (4-25). The principal component scores, and the overall performance index are calculated for all experiments and the obtained values are shown in Table 10. The larger the multiresponse performances index, the better quality of the product. Considering the multiresponse performance index, the optimal factor setting is selected by computing main effects of each design factor level. The design factor setting which attains optimal response value is shown in Table 12. Furthermore, the design factor value is used in the standard deviation models to determine the optimal standard deviation values.

4.5.2. Awad and Kovach Method

The problem is solved using multi-response optimization model suggested by Awad and Kovach (2011), the covariances between the quality characteristic are determined using the response value obtained from experimental design. The optimization model for the Example 2 is shown below:

Minimize

$$y_{\sigma 1}^2(y_{\mu 1} - 7)^2 + y_{\sigma 2}^2(y_{\mu 2} - 0.1)^2 + y_{\sigma 3}^2(y_{\mu 3} - 30)^2$$

$$+2(0.095)(y_{\mu 1} - 7)(y_{\mu 2} - 0.1)$$

$$+ 2(-0.95)(y_{\mu 1} - 7)(y_{\mu 3} - 30)$$

$$+ 2(-0.05)(y_{\mu 2} - 0.1)(y_{\mu 3} - 30)$$

S.t.

$$4.95 + 0.82X_1 - 0.45X_2 - 0.15X_1^2 + 0.28X_2^2 - 0.11X_1X_2 + 0.07X_1X_3 = y_{\mu 1}$$

$$0.46 + 0.13X_1 - 006X_2 + 0.05X_3 - 0.07X_1^2 - 0.04X_3^2 = y_{\mu 2}$$

$$28.36 - 1.48X_1 + 2.33X_3 - 0.15X_1^2 - 1.42X_2^2 - 0.71X_1X_3 = y_{\mu 3}$$

$$0.06 + 0.11X_2 + 0.06X_3 + 0.12X_1^2 + 0.11X_3^2 - 0.10X_1X_3 + 0.05X_2X_3 = y_{\sigma 1}$$

$$0.02 - 0.01X_1 + 0.01X_3 - 0.01X_3 + 0.02X_3^2 - 0.01X_1X_3 + 0.02X_2X_3 = y_{\sigma 2}$$

$$6.08 - 1.53X_1 + 0.5X_2 + 4.85X_3 + 2.26X_2^2 - 0.65X_1X_3 - 0.67X_1X_2X_3 = y_{\sigma 3}$$

$$-1 \le X_1 \le 1$$

$$-1 \le X_2 \le 1$$

$$-1 \le X_3 \le 1 \qquad (4\text{-}26)$$

The model is solved using GAMS nonlinear programming software. The design factor setting, corresponding optimal value for each quality characteristic, and the quality loss value is shown in Table 12.

4.5.3. Proposed Method

The multiresponse optimization problem is solved using the proposed method and the model developed for the example problem is given below:

Minimize

$$Z = \{(exp(-d_4^-))^2 + (exp(d_4^+))^2\}\{(exp(-d_1^+))^2 + (exp(d_1^-))^2\} +$$

$$\{(exp(-d_5^-))^2 + (exp(d_5^+))^2\}\{(exp(-d_2^-))^2 + (exp(d_2^+))^2\}$$

$$+ \{ (exp(-d_6^-))^2 + (exp(d_6^+))^2\}\{(exp(d_3^-))^2 + (exp(d_3^+))^2\}$$

$$+ 2\,(0.095)\,\{exp(-d_1^+) + exp(d_1^-)\}\{exp(-d_2^-) + exp(d_2^+)\}$$

$$+ 2(-0.95)\,\{exp(-d_1^+) + exp(d_4^+)\}\{exp(d_3^-) + exp(d_3^+)\}$$

$$+ 2(-0.05)\,\{exp(-d_2^-) + exp(d_2^+)\}\{exp(d_3^-) + exp(d_3^+)\}$$

S.t.

$$4.95 + 0.82X_1 - 0.45X_2 - 0.15X_1{}^2 + 0.28X_2{}^2 - 0.11X_1X_2 + 0.07X_1X_3 + d_1^- - d_1^+ = 7$$

$$0.46 + 0.13X_1 - 006X_2 + 0.05X_3 - 0.07X_1{}^2 - 0.04X_3{}^2 + d_2^- - d_2^+ = 0.1$$

$$28.36 - 1.48X_1 + 2.33X_3 - 0.15X_1{}^2 - 1.42X_2{}^2 - 0.71X_1X_3 + d_3^- - d_3^+ = 30$$

$$0.06 + 0.11X_2 + 0.06X_3 + 0.12X_1{}^2 + 0.11X_3{}^2 - 0.1X_1X_3 + 0.05X_2X_3 + d_4^- - d_4^+ = 0$$

$$0.02 - 0.01X_1 + 0.01X_3 - 0.01X_3 + 0.02X_3{}^2 - 0.01X_1X_3 + 0.02X_2X_3 + d_5^- - d_5^+ = 0$$

$$6.08 - 1.53X_1 + 0.5X_2 + 4.85X_3 + 2.26X_2{}^2 - 0.65X_1X_3 - 0.67X_1X_2X_3 + d_6^- - d_6^+ = 1$$

$$-1 \le X_1 \le 1$$

$$-1 \le X_2 \le 1$$

$$-1 \le X_3 \le 1$$

$$d_1^{\mp}, d_2^{\mp}, d_3^{\mp}, d_4^{\mp}, d_5^{\mp}, d_6^{\mp} \ge 0 \qquad (4\text{-}27)$$

In the above model, (d_1^-, d_1^+) are the undesirable and the desirable deviations for stability, (d_2^+, d_2^-) are the undesirable and the desirable deviation for volumetric ratio, and (d_3^-, d_3^+) are the undesirable deviation for temperature, (d_4^+, d_5^+, d_6^+) and (d_4^-, d_5^-) are the undesirable deviation and the desirable deviation for standard deviation constraints for stability, volumetric ratio and temperature respectively. The model is solved using GAMS nonlinear optimization software. Table 12 show the results obtained by solving the multiresponse optimization problem considering three different approaches (Awad and Kovach (2011), proposed method, and PCA based approach).

Table 12. Example 4 - Results

Parameter /Method	Awad and Kovach (2011)	Proposed Method	PCA based approach
Design Setting			
X_1	1	0.212	-1
X_2	-0.16	-0.112	1
X_3	-1	-1	-1
Mean			
$Y_{\mu1}$	5.67	5.16	3.95
$Y_{\mu2}$	0.47	0.40	0.19
$Y_{\mu3}$	25.07	25.84	24.5
Standard Deviation			
$Y_{\sigma1}$	0.32	0.13	0.19
$Y_{\sigma3}$	0.04	0.05	0.04
$Y_{\sigma3}$	0.22	1	4.2
Quality Loss	14.47	9.18	11.31

Quality loss value is used as performance measure to compare the effectiveness of the models. Quality loss values are calculated using the equation shown in equation (3-23). The result shows that, for temperature ($Y_{\sigma3}$), the solution attained by PCA based approach and Awad and Kovach (2011) are not within the desirable range. Furthermore, none of the method attains solution within the desirable range for stability ($Y_{\sigma1}$) but the proposed method is close to the target value. The PCA-based method also has limitations in solving multiple quality characteristics problems. These limitations are: it requires more complex calculation to determine optimal solution, the optimal parameter values are selected from factor levels only, and do not explore the optimal points between the factor level combinations. PCA does not minimize the variance directly instead uses Taguchi's robustness concept when experiments are conducted. The other methods (Awad and Kovach, (2011), and proposed method) considered in this analysis explore the design factor setting between the factor-levels to achieve better solution. Among the methods compared, the proposed method achieves better tradeoff by producing lower quality loss value. The ratio between the stability and volumetric ratio is used to analyze the effect of correlation among the two quality characteristics. Higher the ratio between the optimal response values for stability and volumetric ratio indicates lesser correlation effect. The proposed method produces higher ratio of 12.74 units between stability and volumetric ratio, which is higher than Awad and Kovach (2011) method with value of 12.01 units.

Furthermore, assigning the target value for LTB and STB quality characteristics is relatively difficult and it involves subjective input (expert's opinion) for selecting a particular target. Therefore, to investigate the robustness in terms of parameter selection of the two models (Awad and Kovach, (2011) and proposed method), we perform a sensitivity analysis by changing the target value especially for LTB and STB type of quality characteristics. The purpose of the sensitivity analysis is to understand the behavior of the model in dealing with the subjective input (target) or uncertainty. The target value for LTB and STB type responses is varied to perform the sensitivity analysis. Table 13 and Table 14 show the results of sensitivity analysis for Awad and Kovach (2011) model and the proposed method. The result of the sensitivity analysis clearly shows the robustness of the proposed method for varying target value.

Table 13. Sensitivity Analysis - Awad and Kovach Method

Target			Design			Mean			Standard Deviation			Quality
Y_1	Y_2	Y_3	X_1	X_2	X_3	$Y_{\mu 1}$	$Y_{\mu 2}$	$Y_{\mu 3}$	$Y_{\sigma 1}$	$Y_{\sigma 2}$	$Y_{\sigma 3}$	Loss
7	0.1	30	1.00	-0.16	-1.0	5.64	0.44	25.07	0.32	0.05	0.22	14.46
5	0.3	30	-0.56	0.00	0.31	4.43	0.38	29.99	0.15	0.03	8.56	8.74
3	0.3	30	-1.00	0.04	0.11	3.96	0.26	30.01	0.20	0.03	8.23	3.78
5	0.5	30	-0.52	-1.0	0.86	5.12	0.45	30.00	0.12	0.01	12.81	13.15
3	0.5	30	-0.87	-0.15	0.17	4.17	0.31	30.01	0.16	0.03	8.31	5.52

Table 14. Sensitivity Analysis - Proposed Method

Target			Design			Mean			Standard Deviation			Quality
Y_1	Y_2	Y_3	X_1	X_2	X_3	$Y_{\mu 1}$	$Y_{\mu 2}$	$Y_{\mu 3}$	$Y_{\sigma 1}$	$Y_{\sigma 2}$	$Y_{\sigma 3}$	Loss
7	0.1	30	0.212	-0.11	-1.0	5.16	0.4	25.84	0.13	0.05	0.95	9.18
5	0.3	30	0.219	-0.08	-1.0	5.15	0.40	25.84	0.13	0.05	1.00	9.11
3	0.3	30	0.224	-0.06	-1.0	5.14	0.40	25.84	0.13	0.05	1.00	9.08
5	0.5	30	0.225	-0.06	-1.0	5.14	0.40	25.84	0.13	0.05	1.00	9.08
3	0.5	30	0.229	-0.05	-1.0	5.14	0.40	25.85	0.14	0.05	1.00	9.07

As shown in these two tables, the model proposed by Awad and Kovach (2011) produces high variation in the optimal results for all quality characteristics when the target value is changed. The quality loss value also shows a large variation when the target value is changed. On the other hand, the proposed approach provides almost similar results with minimum variation in both optimal response and quality loss values, which demonstrates the robustness for subjective uncertainty. This clearly proves the superiority of proposed method over existing methods, which includes the model proposed by Awad and Kovach (2011).

4.6. Example 5

To study the behavior and to demonstrate the applicability of the proposed methods, another correlated multiple quality characteristics problem is considered for analysis. This study is based on chemical filtration process taken from Kovach and Cho 2008. The effectiveness of the chemical filtration process is determined by measuring dosages. In order to optimize the chemical filtration process, engineers are interested in three responses: filtration time (STB), filtration volume (NTB), and filtration purity (LTB). The three responses are controlled by two input design factors: chemical temperature (X_1), and pressure (X_2). It is required to reduce the filtration time to less than 7 seconds; filtration volume has to be maintained at a target value of 10 within the allowable tolerance of ±0.5 ml. Filtration purity needs to be as high as 100% if possible. The response surface model is developed by conducting experiments, and the models are shown below:

$$Y_{\mu 1} = 2.1754 - 0.2219X_1 - 0.1493X_2 - 0.1656{X_1}^2 - 0.2911{X_2}^2 - 0.0862X_1X_2$$

$$Y_{\mu 2} = 10.0005 + 0.0495X_1 + 0.0492X_2 - 0.0139{X_1}^2 - 0.005{X_2}^2 - 0.0325X_1X_2$$

$$Y_{\mu 3} = 95.11 + 0.52X_1 + 0.65X_2 - 0.175{X_1}^2 - 0.135{X_2}^2 - 0.071X_1X_2 \qquad (4\text{-}28)$$

The experiments are replicated to study the variance parameter involved in the experiment. The standard deviation model is developed for three quality characteristics. The desired standard deviation values for filtration time, filtration volume and filtration purity is 0. A second order model is developed for three quality characteristics, which is given below:

$$Y_{\sigma 1} = 0.169 + 0.00189X_1 + 0.0045X_2 - 0.0538X_1^{\,2} - 0.0515X_2^{\,2} + 0.00253X_1X_2$$

$$Y_{\sigma 2} = 0.0478 - 0.0003X_1 - 0.013771X_2 + 0.0093X_1^{\,2} + 0.00103X_2^{\,2} + 0.02212X_1X_2$$

$$Y_{\sigma 3} = 0.41 + 0.011X_1 + 0.013X_2 - 0.14X_1^{\,2} - 0.19X_2^{\,2} + 0.034X_1X_2 \qquad (4\text{-}29)$$

For optimization purpose, a target value of 1, 10, and 100 is assigned to filtration time, filtration volume, and filtration purity. The three quality characteristics: filtration time, filtration volume, and filtration purity are correlated each other. Table 15, shows the significant correlation coefficient values for three quality characteristics calculated using Minitab 15. The filtration time and filtration volume are negatively correlated; filtration time and filtration purity are negatively correlated; filtration volume and filtration purity are positively correlated.

Table 15. Example 5 - Correlation Matrix

Responses	Y_1	Y_2
Y_2	-0.623	
Y_3	-0.585	0.86

The above problem is solved using the proposed method, PCA based method, and Awad and Kovach (2011) method. The design factor setting which attains optimal response value is shown in Table 16. The design setting for chemical temperature and pressure are determined to produce optimal filtration time, filtration volume, and filtration purity. In order to compare the effectiveness of each method in achieving optimal solution, overall quality loss values are calculated using the equation (3-23). The result shows that among the three methods, the proposed method provides lesser quality loss value.

Table 16. Example 5 – Results

Parameter /Method	Awad and Kovach (2011)	Proposed approach	PCA based approach
Design Setting			
X_1	1.259	0.857	0
X_2	1.159	1.386	-1.68
Mean			
$Y_{\mu 1}$	0.944	0.995	1.605
$Y_{\mu 2}$	10.04	10.053	9.904
$Y_{\mu 3}$	95.96	95.991	93.63
Standard Deviation			
$Y_{\sigma 1}$	0.02	.042	0.016
$Y_{\sigma 3}$	0.08	0.064	0.074
$Y_{\sigma 3}$	0	0	0
Quality Loss	0.012	0.008	0.376

Furthermore, the robustness in terms of variation in input parameters of proposed method and Awad and Kovach (2011) models is studied by analyzing the sensitivity of input parameter for LTB and STB type of quality characteristics. The analysis showed that, the Awad and Kovach (2011) model produces high variation in optimal results and the proposed method is insensitivity to parameter variation, and achieves better tradeoff between the quality characteristics. This demonstrates the robustness of the proposed model for subjective uncertainty.

The results of these two examples and the sensitivity analysis show the consistency and repeatability of the proposed approach In all the cases, the proposed approach achieves better result. Furthermore, the proposed method is more robust and reliable in attaining more consistent optimal solution for multiresponse optimization problems. For LTB and STB quality characteristics, the proposed methods explore the solution space beyond the target values to determine optimal solution.

4.7. Conclusion

This chapter presents alternative multiresponse optimization approach for solving multiple correlated quality characteristics. The proposed method achieves better tradeoff between multiple quality characteristics in design optimization, and the mechanics of the proposed approach is illustrated using two correlated multiple quality characteristic examples from literature. The proposed approach is also compared with two other optimization techniques, which consider correlation in the analysis. The comparison shows the superiority of the proposed method. The sensitivity analysis shows that the proposed method is highly robust to subjective input and hence provides better trade-off consistently under uncertain conditions as well.

CHAPTER 5. CONCLUSION AND FUTURE DIRECTION

5.1 Conclusion

In this research, a comparative study was performed to investigate the effectiveness and ability of existing multiresponse optimization methods for achieving better tradeoff among multiple quality characteristics. Furthermore, the comparative study was extended to analyze the sensitivity in selecting parameters for optimization. The constraint optimization, desirability function based approach, and standard quality loss function methods are sensitive in selecting response upper and lower limits. All loss function based methods except hybrid quality loss function method are sensitive to target values assigned in the model.

Furthermore, all the multiresponse optimization methods compared are sensitive to the priority assigned for each quality characteristic. The sensitivity of these models to input parameters shows that parameter selection is a more critical input and it is highly subjected to expert's engineering judgment. When considering all loss function based multiresponse optimization methods, the HQLF method showed certain advantages in exploring the optimal region beyond the target value. One of the reasons for this advantage is that HQLF based model includes desirable deviation in objective function whereas other loss function based models minimize total deviation including both undesirable deviation and desirable deviations.

The comparative study also investigated the effectiveness of these six multiresponse optimization methods in dealing with correlated multiple response problems. The six multiresponse optimization methods discussed in this study were found incapable to deal with correlated responses effectively. The results showed that the correlation between the quality characteristics (especially when correlated quality characteristics are opposite in nature) significantly affect the optimal solution.

A multiresponse optimization method has been proposed for addressing the correlation issue between quality characteristics. The proposed method simultaneously minimizes the deviation between mean and target, increases the robustness in terms of random variability and also minimizes the effect of correlation between quality characteristics. The validation and comparative study of the proposed model showed that the proposed method attains lower quality loss values resulting in better tradeoff among the correlated quality characteristics. The study further demonstrated that propose model is insensitive to uncertainty in input parameter values and showed significant superiority over existing methods including one proposed by Awad and Kovach (2011)

5.2 Recommendations for Future Research

In this research, the problems from literature are selected based on the response surface models which are developed already. These problems do not include noise factors in the optimization. The inclusion of noise factors will increase the input variable in the response surface model and will predict the practical operating scenario.

The proposed model can be integrated to reliability based robust design approaches in product design and development process. While integrating, the advantages of proposed approach should be combined with reliability models. In the product development process, the proper functioning of a product depends on the performance of the subsystem. The proposed approach can be used for optimizing different quality characteristics from different sub systems of a product.

REFERENCES

Antony, J., (2000), "Multi-response optimization in industrial experiments using taguchi's quality loss function and principal component analysis", Quality and Reliability Engineering International, 16, 3-8.

Artiles-Leon, Neol, (1996-97), "A pragmatic approach to multiple response problems using loss functions" Quality Engineering, 213-220.

Awad, M., and Kovach, J., (2011), "Multiresponse optimization using multivariate process capability index", Quality and Reliability Engineering International, 27, 465-477.

Bhamare, S. S., Yadav, O. P., and Rathore A.,(2009), "A hybrid quality loss function based multi-objective design optimization", Quality Engineering, 21, 277-289.

Box,G.E.P., and Behnken, D.W., (1960), "Some new three level designs for the study of quantitative variables", Technometrics, 2, 455-475.

Box,G.E.P., Hunter, W.G., and Hunter, J.S., (1978), "Statistics for experimenters: an introduction to design, data analysis, and model building", John Wiley, New York.

Box, G. E. P. and Wilson, K. B., (1951), "On the Experimental Attainment of Optimum Conditions", Journal of the Royal statistical Society, Series B, 13, 1-45.

Chan, L.K., Cheng, S.W., and Spring, F.A. ,(1991), " A multivariate measure of process capability. International Journal of Modeling and Simulation", 11, 1-6.

Chiao, C., and Hamda, M.,(2001), "Analyzing experiments with correlated multiple responses" Journal of Quality Technology, 33, 451-465.

Coleman, J. S., Campbell, E., Hobson, C., McPartland, J., Mood, A., Weinfeld, F., and York, R., (1966), "Equality of educational opportunity", Government Printing Office, Washington, D.C.

Darwish, S.M., and Al-Dekhial, S.D., (1999), "Statistical models for spot welding of commercial aluminum sheets", International Journal of Machine Tools Manufacture, 39, 1589–1610.

Dean, A., and Voss, D., (1999), "Design and Analysis of Experiments", Springer Verlag, New York, USA.

Del Castillo, E., and Montgomery, D., (1993), "A nonlinear programming solution to the dual response problem", Journal of Quality Technology, 25, 199-204.

Del Castillo, E., Montgomery, D., and Mccarville, D. R., (1996), "Modified desirability function for multiple response optimization", Journal of Quality Technology, 28, 337-345.

Deng, J. L., (1982), "Control problems of grey systems", Syst Control Letters, 5, 288–294

Derringer, G., and Suich, R., (1980), "Simultaneous optimization of several response variable", Journal of Quality Technology, 12, 214-219.

Fung, C.P., and Kang, P.C., (2005), "Multi-response optimization in friction properties of PBT composites using Taguchi method and principle component analysis", Journal of Materials Processing Technology,170, 602-610.

Harrington, E.C., (1965), "The Desirability Function", Industrial Quality Control, 21, 494-498.

Hejazi, Mahdi Bashiri, Kazem Noghondarian, and Atkinson, (2011), "Multiresposne optimization with consideration of probabilistics covariates", Quality and Reliability Engineering International, 27, 437-449.

Hill, W. J., and Hunter, W. G., (1966), "A Review of Response Surface Methodology: A Literature Survey", Technometrics , 8, 571-590.

Jeyapaul, R., Shahabudeen, P., and Krishnaiah, K., (2005), "Quality management research by considering multi-response problems in the Taguchi method – a review", The International Journal of Advanced Manufacturing Technology, 26, 1331 - 1337.

Kaiser, H.F., (1960), "The application of electronic computer to factor analysis", Education Psychological Measurement, 20, 141-151

Kim, D., and Rhee, S., (2003), "Optimization of GMA welding process using the dual response approach", International Journal of Production Research, 41, 4505-4515.

Kim, K. J., and K.J.Lin, D., (1998). Dual Response Surface Optimization: A Fuzzy Modeling approach. Journal of Quality Technology, 30, 1-10.

Kim, K. J., and Lin, D., (2006), "Optimization of multiple responses considering both location and dispersion effects", European Journal of Operational Research, 169, 133-145.

koksoy, O., (2005), "Dual response optimization: the desirability approach", International Journal of Industrial Engineering, 12(4), 335-342.

Kovach, J., and Byung R.C., (2008), "Solving multiresponse optimization problems using quality function based robust design", Quality Engineering, 20, 346-360.

Liao, H. C., (2006), "Multi-response optimization using weighted principal component", International Journal of Advance Manufacturing Technology, 27, 720-725.

Logothetis, and Haigh.A., (1988), "Characterizing and optimizing multi-response processes by the Taguchi method", Quality and Reliability Engineering International, 4(2), 159- 169.

MA, Yizhong, and Fengyu Zhao, (2004), "An improved multivariate loss function approach to optimization", Journal of System science and systems engineering, 13(3), 318-325.

Mead, R., and Pike, D. J., (1975). A Review of Response Surface Methodology from a Biometric Viewpoint, Biometrics , 31, 803-851.

Mendenhall, W., and Sincich, T., (1996), "A second course in statistics regression analysis -fifth edition, Prentice Hall, New Jersy.

Menon, R., Tong, L. H., Zhijie, L., and Ibrahim, Y., (2002), "Robust design of a spindle motor: a case study", Reliability Engineering and System Safety, 75(3), 313-319.

Montgomery, D.C., Keats, J.B., Perry, L.A., Thompson, J.R., and Messina, W.S., "Using statistically designed experiments for process development and improvement: an application in electronics manufacturing", Robotics and Computer Integrated Manufacturing, 16, 55–63.

Montgomery, D. C., (2005), "Design and analysis of experiments", John Wiley and Sons, NewYork.

Myers, R., and Carter, W., (1973), "Response Surface Techniques for Dual Response Systems", Technometrics , 15, 301-317.

Myers, R. H., Khuri, A. I., and Carter, W. H., (1989), "Response Surface Methodology: 1966-1988", Technometrics, 31, 137-157.

Myers, R. H., Montgomery, D. C., Vining, G. G., Borror, C. M., and Kowalski, S. M., (2004), "Response Surface Methodology: A Retrospective and Literature Survey", Journal of Quality Technology, 36, 53-77.

Myers, R., Montgomery, D., and Anderson, (2009), "Response Surface Methodology: Process and Product Optimization using Designed Experiments" John Wiley and Sons, Inc, USA

Pasamontes, A., and Callao, M.P., (2006), "Optimization by means of responses surface of an analytical sequence using a sequential injection system" Talanta, 68, 1617-1622.

Phadke, M. S.,(1989), "Quality Engineering Using Robust Design" Prentice-Hall, Englewood Cliffs, NewJersy .

Pignatiello, J.J., (1993), "Strategies for robust Multi-response quality engineering", IIE Transactions, 25, 5-15.

Ribeiro, J.L., Fogliatto,F.S., and Caten C.S.T., (2001), "Minimizing manufacturing and quality costs in multiresponse optimization." Quality Engineering, 559-569.

Riberto, J., and Elsayed, E., (1995), "A Case study on Process Optimization using Gradient Loss Function" International Journal of Production Research, 33(12), 3233-3248.

S.Phadke, M., (1989), "Quality engineering using robust design" Prentice Hall, New Jersy.

Schmidt, R.H., Illingworth, Cornell, and Deng J.C., (1979), "Multiple regression and response surface analysis of the effects of calcium chloride and cysteine on heat induced whey protein gelation", Journal of Agricultural food chemistry, 529-532.

Sibalija, T. V., and Majstorovic, V.D., (2009), "Multi-response optimisation of thermosonic copper wire-bonding process with correlated responses", International Journal of Advance Manufacturing Technology, 42, 363-371.

Su, C.T., and Tong, L.I., (1997), "Multi-response robust design by principal component analysis", Total Quality Management, 8(6), 409-416.

Taguchi, G., (1986), "Introduction to quality engineering: designing quality into the products and process", Kraus International publication, New York.

Tarng, Y. S., and Yang, W.H., "Application of the Taguchi method to the optimization of the submerged Arc Welding Process", Materials and Manufacturing Processes, 13(3), 455-467.

Tyan, J.C., Du, T.C., Chen, J.C., Chang, I.H., (2004), "Multiple response optimization in a fully automated FAB: An integrated tool and vehicle dispatching strategy", Computers and Industrial Engineering, 46, 121–139.

Vining, G.G., and Myers, R. H., (1990), "Combining Taguchi and Response Surface Philosophies: A Dual Response Approach", Journal of Quality Technology, 22, 38-45.

Vining, G.G., (1998), "A Compromise Approach to Multiresponse Optimization", Journal of Quality Technology, 30, 309-313.

Wu, F.C., (2004), "Optimizing robust design for correlated quality characteristics", International Journal of Advanced Manufacturing Technology, 24, 1-8.

Wurl, R.C., and Albin, S.L., (1999), "A comparison of multiresponse optimization: sensitivity to parameter selection", Quality Engineering, 11(3), 405-415.