

EVACUATION TREES WITH CONTRAFLOW AND DIVERGENCE CONSIDERATIONS

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## Title

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CONSIDERATIONS

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The supervisory committee certifies that this thesis complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

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## ABSTRACT

In this thesis, we investigate how to evacuate people using the available road transportation network efficiently. To successfully do that, we need to design evacuation model that is fast, safe, and seamless. We enable the first two criteria by developing a macroscopic, time-dynamic evacuation model that aims to maximize the number of people in relatively safer areas of the network at each time point; the third criterion is optimized by constructing an evacuation tree, where the vehicles are evacuated using a single path to safety. Divergence and contraflow policies have been incorporated to enhance the network capacity. Divergence enables specific nodes to diverge their flows into two or more streets, while contraflow allows certain streets to reverse their flow, effectively increasing their capacity. We investigate the performance of these policies in the evacuation networks obtained, and present results on two benchmark networks of Sioux Falls and Chicago.

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# 1. INTRODUCTION

An active or imminent disaster disrupts the regular functions of a community, and causes human, material, economic, and environmental damages. These damages often exceed the societal capabilities to cope using their own resources (IRFC, 2011); more specifically, imminent disasters also impose operational challenges on the evacuation process manager, as well as governments. The decisions that need to be made in very short notice include where to position evacuation personnel and equipment, how to mobilize and utilize the available resources, how to route people to safety, and when to schedule all operations. Hence, efficient evacuation planning through proper mathematical modeling is of utmost importance to protect people from potential dangers.

Over the last decade, we have seen evacuation processes due to man-made problems, such as nuclear reactor meltdowns (e.g., Fukushima-Daiichi in 2011, or Chernobyl in 1986), chemical accidents and spills, terrorist attacks, wildfires, dam failures. It is more common though to put to practice our evacuation plans for natural disasters, such as hurricanes (e.g., Hurricanes Andrew in 1992, Katrina in 2005, and Harvey and Irma in 2017), tsunamis (e.g., Japan in 2011), volcanic eruptions, and tornadoes. Specifically, people in the United States living along the gulf of Mexico and the Atlantic ocean coast are always carefully monitoring weather conditions during hurricane season, and are asked to always have a plan for evacuating. Hence, a properly and well executed evacuation plan will not only serve to save many human lives, but will also help these communities recover and bounce back fast.

The United States Federal Emergency Management Agency (FEMA) reports that the number of disasters that requires evacuation annually has grown to around 45 to 75 (TRB, 2008). Effective traffic management is listed as one of the important capabilities that need to be achieved for the mass evacuation of people in an endangered area (DHS, 2013). Evacuation traffic management is critical as human lives are at stake; if the evacuation is not planned properly, conditions will become (even) more chaotic. For the most obvious example, if the evacuation schedule is such that the demand for a certain street is suddenly higher than the capacity of the evacuation network, then it can cause congestion and render many evacuees helpless.

This thesis is organized as follows. In the next chapter, a brief literature review on posing the evacuation process as an optimization problem is presented. Then, this study proceeds with introducing a new optimization framework to solve the problem. The framework is based on the concept of an evacuation tree, and it is enhanced with contraflow and divergence considerations. Chapter 4 focuses on the numerical experiments performed on the benchmark networks of Sioux Falls and Chicago for different configurations of the problem. Finally, concluding remarks are offered in the last chapter.

## 2. LITERATURE REVIEW

### 2.1. Evacuation Literature

Evacuation is a much broader research topic than merely considering it as “moving people from a hazardous area to a safe area”. It is also a widely interdisciplinary research area that has been studied by a very diverse group of academicians and practitioners. Although evacuation models are similar to traffic flow models, seeing as both treat evacuees as vehicles and both use the underlying transportation network, the conditions and stochastic nature of an evacuation process, makes them more difficult and computationally challenging.

This chapter is outlined as follows. First, we provide details on the assumptions and goals of different evacuation models. Then, we proceed with a description of the different categories and types of evacuation models that are prominently used in theory and practice. In our work, we touch upon some network design concepts, seeing as devising an evacuation plan is similar to designing a transportation network: hence, a brief literature review on the network design problem is provided. Staging and shelter location problems are also discussed in this chapter, as well as previous contraflow research. Finally, we discuss simulation-based and human factors-based related literature.

#### 2.1.1. Assumptions and Goals of Evacuation Models

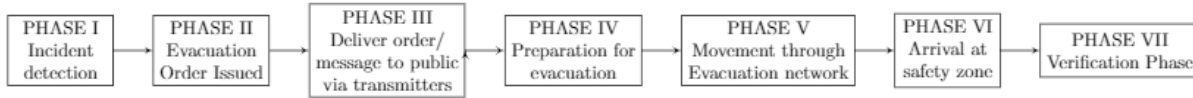
Hamacher and Tjandra (2002) defines the evacuation (as an emergency process) as the removal of residents from a given area that is considered as a danger zone to different areas, designated as safe zones, as quickly as possible and with utmost reliability. Evacuation can be precautionary (i.e., evacuation done prior to the disaster) or life-saving, which is performed during and after the disaster (e.g., rescuing of the injured evacuees in and around the damaged area, route clearance). Evacuations also arise in many, different systems of different scales (e.g., buildings, city, or airplanes). The structure of the system plays a very important role in optimal evacuation planning; it is clear that evacuating a building versus evacuating a city are two different processes. In essence, the following information is vital and should be estimated (if not readily available), prior to and while planning the evacuation for a particular system according to Hamacher and Tjandra (2002).

- system layout/ geographic information;
- evacuee behavior pattern estimation under alarmed conditions;
- occupant distribution (socioeconomic data, age, special accommodations);
- source and location of hazard, propagation rate, affecting factors;
- safe destinations;
- availability of emergency service facilities and personnel.

Once we have this information we can design the evacuation plan for a system. Stepanov and J. M. G. Smith (2009) briefly explain the phases which are involved in evacuation as flows:

1. In the first phase (Phase I) the imminent threat is detected.
2. In Phase II, the decision makers should assess the threat and make an informed decision whether to evacuate or not. The decision is made based on the severity of the threat and the availability of the infrastructure to sustain the evacuation process.
3. If a decision to evacuate is made, Phase III is disseminating the decision to the affected population.
4. Phase IV involves the affected population deciding whether to evacuate or not, based on their hazard perceptions and their related, previous experience.
5. Phase V includes the actual evacuation process taking place through predetermined evacuation routes. The time evacuees need to move towards safety area which is known as ‘egress time’.
6. Finally, in Phase VI evacuees arrive in a designated, safe location; at the same time, Phase VII is under way with the process planner verifying that all evacuees have safely reached the designated areas.

Our work then can be viewed as **precautionary**, and fits within **Phases IV-VI** in the above framework.



**Figure 2.1.** Evacuation Phases  
Source: (Stepanov and J. M. G. Smith, 2009)

### 2.1.2. Types of Evacuation Models

Evacuation models can be divided into two main categories: *macroscopic* and *microscopic*. Macroscopic models are used for large-scale transportation networks and are based on the concepts of dynamic network flows optimization and the Cell Transmission Model (CTM). In these, flows represent the traffic in the transportation network. Objectives that are typically used include (i) maximizing the number of people reaching safety in a given time interval (Hoppe and Tardos, 1994), and (ii) minimizing the clearance time (that is, the time it takes for everyone reach safety) (Han, Yuan, and Urbanik, 2007). Macroscopic models are “big picture” models, in that they do not consider individual differences—that is, all evacuees are treated as a homogeneous, single entity. On the other hand, microscopic models extensively use transportation engineering approaches to model each individual entity characteristics and their unique effects on the transportation network (S.A. Boxill, 2000). Due to this higher level of detail and the huge amount of data necessary, these models tend to use simulation approaches, or stochastic optimization models in which a probability is assigned to different evacuee movements. In our work, we focus on a **macroscopic** view of the problem.

### 2.1.3. Static Traffic Assignment Models

Static traffic assignment models have been used by evacuation planners to estimate current and future use of transportation network for evacuation purposes. Static models are based on the formulation provided by Beckmann, McGuire, and Winsten (1955). Static models for large evacuation networks can be solved to optimality using exact solution methods. Despite having these advantages, static models are not able to capture the traffic dynamics that can change over time. These dynamics can be captured by Dynamic Traffic Assignment Models (DTA).

#### 2.1.4. Cell Transmission Model Based Dynamic Traffic Assignment Models

The Cell Transmission Model was proposed by Daganzo (1994) and Daganzo (1995) to simulate traffic on a single highway link. This model is a transformation of the differential equations of the hydrodynamic model used by Lighthill and Whitham (1955) and Richards (1956) by assuming a piece-wise linear relationship between flow and density at a cell level. The resulting network then consists of cells. A cell has a length which can be traversed in a unit of the time interval at free-flow speed. The model depends on two sets of equations, which express the conservation of flow at the node and flow propagation on the links. There are two unknowns in this model. The first is occupancy  $d_{it}$ , which represents the number of entities in a cell  $i$  (also called holdover arcs) present at time  $t$ . The second is the flows in the cells from cell  $i$  to  $j$  at time interval  $t$ , represented as  $f_{ijt}$ . Even though CTM is formulated as a linear programming problem, it can grow extremely large.

Such models were used by A. K. Ziliaskopoulos (2000) to formulate the System Optimum Dynamic Traffic Assignment (SO DTA) problem as a linear programming problem for a single destination. Peeta and A. K. Ziliaskopoulos (2001) classify DTA models into four broad methodological groups: mathematical programming, optimal control, variational inequality, and simulation-based. Of these, the first three are analytical approaches. The drawback of these models is that they can get intractable for large-scale, real-life problems and may not represent time-dependent traffic characteristics as compared to simulation model.

#### 2.1.5. Dynamic Network Flows Based Models

To represent the time-varying characteristics of the evacuation traffic conditions, evacuee vehicular flow can be modeled using a linear dynamic network flow model. L. R. Ford and Fulkerson (1958) were the first to introduce the notion of a time-expanded graph to solve the dynamic network flow problem. Hamacher and Tjandra (2002) and Kimms and Maiwald (2017) also employ dynamic network flow models on the time-expanded networks from the original time-static network.

Dynamic maximum flow problems, earliest arrivals, quickest flows and dynamic minimum cost flows are but some of the types of the problems considered in dynamic network flow models. The sizes of time-expanded networks grow fast, therefore they become computationally intractable for real-life problems. Except for special cases, dynamic network flow problems are  $\mathcal{NP}$ -hard, or there is no known polynomial time algorithm to solve them.



## 2.2. Staging and Routing Literature

In evacuation problems, there are essential requirements which are not shared by other dynamic network flow problems. For example, it is preferable to evacuate cities in zones, rather than all of the area at once, especially in larger regions. This is reinforced by the fact that different areas of the transportation network might suffer different levels of danger over different time periods. To evacuate these areas in an optimized fashion, a *staged* evacuation strategy could be beneficial.

Bayram (2016) has provided a comparative overview of a recent evacuation using transportation modeling. The author discussed available research on macroscopic approaches in static/dynamic, deterministic/stochastic/robust evacuation modeling that consider different evacuee behavior assumptions, traffic assignment, shelter assignment methodologies and supply and demand management strategies. Author also presented a review of works related to solution methods to solve a large scale network and using simulation or optimization to assign shelters, decide routes to shelters, or even reallocate shelters.

To address staging evacuation, Sbayti and Mahmassani (2006) took a simulation-based approach to investigate the benefits of the staging evacuation over a simultaneous evacuation. The authors proposed a system optimal dynamic traffic assignment formulation for scheduling evacuation trips between a selected set of origins and safe destination with the objective of minimizing the total network clearance time. This study proved useful for a small portion of a large network while contending with the larger network flows. Liu, Lai, and Chang (2006) also studied staged evacuation strategies using an optimization model which decides the optimal sequence for starting the evacuation of a different affected zone and eventually minimizes the zonal risk parameter.

In contrast to staging, problems of routing evacuees to safer destinations have been extensively studied. Hanif D. Sherali, Carter, and Antoine G. Hobeika (1991) examined the combined shelter location and traffic routing problem using static nonlinear mixed integer programming problem such that the system-wide evacuation time is minimized. Cova and Johnson (2003) also developed static min-cost network flow model with dual objective. The primary objective is to route evacuee's vehicles to their closest evacuation zone exit, and the secondary is to minimize the intersection merging conflicts so that, each arc could be assigned travel time that is unaffected by the congestion. Murray-Tuite and Mahmassani (2003) have added consideration of household

behavior into the evacuation modeling. To capture the household behavior, they provide two static linear programs in which first determines the household meeting location and second assigns routes to the family's vehicles. Using the formulation of Hanif D. Sherali, Carter, and Antoine G. Hobeika (1991) for shelter location, M. Ng and S. Waller (2009) developed a two-level stochastic evacuation model to consider uncertainties in evacuation planning. Similarly, Bish, H. D. Sherali, and A. G. Hobeika (2014) proposed mixed integer linear programming formulation based on Cell Transmission Model to minimize the network clearance time using household staging and routing. These household level instructions used to structure the demand to avoid the congestions.

To minimize the design and flow related costs Üster, Wang, and Yates (2017) provide Strategic Evacuation Network Design (SEND) that prescribes shelter regions and capacities, intermediate locations which will provide a planning tool for emergency response planes while incorporating evacuation time consideration as well as cost constraints. Andreas and J. C. Smith (2009) came up with the novel approach of routing where the evacuation routes for the all origins combined to form an evacuation tree. They also provide solution strategy based on Benders decomposition. To consider Evacuees' route choice behavior in the development of optimal investment strategies, M. W. Ng, Park, and S. T. Waller (2010) provide 'Hybrid Bi-level Model', in which, upper level is a shelter assignment which occurs in system optimal manner whereas evacuees are free to choose how to reach their assigned shelter in the lower level. Since it is a static model, it lacks realism for evacuation planning. Na and Banerjee (2015) proposed a different approach to tackle evacuation problem, in which, they prioritize evacuees based on the severity level of injuries and assign evacuation vehicle to evacuees to take them from staging area to shelter location. Kimms and Maiwald (2017) proposed exact network flow formulation which is essentially the cell-based evacuation model in which they tried to incorporate the advantages of both 'Cell Transmission based' approach as well as 'Dynamic Traffic Assignment' approach to reduce the computational cost.

Bayram, Tansel, and Yaman (2015) developed a static nonlinear mixed integer evacuation model which locates the available shelter, which is no longer than the shortest path to the nearest shelter site by more than a degree of tolerance and assigns evacuees to the selected shelter so that the total evacuation time is minimized. In this study, authors have also provided solution method which uses second-order cone programming techniques. Based on this study, authors Bayram and Yaman (2017a) proposed a scenario based two-stage stochastic evacuation planning model which

considers uncertainty in the evacuation demand, the disruption in the road network and evacuation sites. The objective of this study is also to minimize the evacuation time by locating the shelter and assigning evacuees to the shelter. Similar to the previous study authors used the second-order cone programming approach to solve this model. Authors Bayram and Yaman (2017b) in this study proposed, exact algorithm based on Benders decomposition to solve the scenario based static two-stage stochastic evacuation planning model from (Bayram and Yaman, 2017a). Since these are all static models, they do not represent the actual evacuation scenario. The evacuation personnel is expected to make modification using the real-time information if disaster hits.

### **2.3. Contraflow Literature**

The concept of reversing the lane capacity has been in use for a long time. It is the reversal of the lane which is being underused or unused, to temporarily increase the capacity of the congested roads without constructing additional lanes. It has been demonstrated in many big cities during the rush hours; this method successfully reduces the traffic congestion. However, contraflow strategies deployed in different cities are developed for particular traffic pattern during specific time periods. For disaster management cases, implementation of this strategy will be handled by the local disaster management authorities. There are always some liability issues that concern the decision-makers about the negative impact of such options on the rest of the network and demand. Due to such concerns in 1996 during hurricane ‘Floyd’, the government of Florida rejected the plan to incorporate the contraflow strategy. Whereas On the other hand South Carolina and Georgia have successfully incorporated this strategy into their hurricane evacuation planning.

Kalafatas and Peeta (2009) incorporated contraflow strategy in evacuation planning to augment the capacity of the network under budget constraints. They developed the Mixed Integer Programming formulation then using the same model they performed sensitivity analysis on the network regarding budget constraint for contraflow operations and showed that beyond specific budget, they call it as ‘threshold budget’, the benefits regarding clearance time are negligible. For the solution of such kind of networks which uses contraflow network configuration, Tuydes and A. Ziliaskopoulos (2006) have developed ‘Tabu-search’ based heuristic approach to optimize the discrete network design evacuation problem focusing on contraflow strategies. Similarly, S. Kim, Shekhar, and Min (2008) have come up with a greedy heuristic which incorporates the street capacity constraint using macroscopic approach.

Some authors have tried to incorporate simulation approach in their studies. Theodoulou and Wolshon (2004) used CORSIM, a microscopic traffic simulation-based approach to evaluate the effectiveness of contraflow schemes in the evacuation planning of New Orleans. With the help of micro-scale traffic simulation, they were able to suggest alternative contraflow configuration at a detailed level. Also, they showed in case of New Orleans; contraflow strategy could increase the traffic flow over 53% over standard evacuation. Incorporating model of intersection crossing elimination first proposed by Cova and Johnson (2003), authors Xie, Lin, and Travis Waller (2010) proposed bi-level network optimization-simulation model which addresses the incorporation of contraflow and crossing elimination at the intersection. The upper level of the model aims at optimizing the system-wide network evacuation performance that is total evacuation time or network clearance time whereas lower level simulates dynamic evacuation flow. They used VISTA (Virtual Interactive System for Transport Algorithms) for simulating dynamic traffic assignment.

#### **2.4. Evacuation With the Help of Simulation Tools**

As the complexity of the evacuation model increased over the time modelers, and policymakers needed a more reliable tool that evaluates the model with many variables. There are three types of simulation approaches which are being used in evacuation planning. First is macro-simulation, second is micro-simulation, and third is meso-simulation (Abdelgawad and Abdulhai, 2009). The elaborate survey on features and characteristics of different evacuation models have been discussed in work done by Alsnih and Stopher (2004) and the authors also present how to design an evacuation plan using these simulation tools.

##### **2.4.1. Macro-Simulation Models**

Macro-simulation model makes no attempt track the detailed movement behavior such as car following, lane changing, etc.; they are based on the network flow equations (Pidd, De Silva, and Eglese, 1996). The majority of simulation models have developed to evacuate people in case of nuclear plant emergencies. Sheffi, Mahmassani, and Powell (1982) developed the ‘NETVAC1’ to simulate the traffic pattern during an emergency evacuation. It is a fixed time macro simulator which uses existing traffic flow model to simulate the evacuation process. It does not keep track of individual vehicle. The movements are determined at each simulation interval as the function of the changing traffic conditions. ‘MASSVAC’ is another macro-simulation model developed by Antoine G. Hobeika and C. Kim (1998) for the purpose of hurricane evacuation. This model is

based on three modules; a disaster characteristic module, population characteristics module and a network evacuation module. This model performs analysis at both the levels macroscopic as well as microscopic levels. Macroscopic The macro level considers the effect of the evacuation process on the network by focusing on major road arteries and provides different evacuation time under different disaster intensity levels, including severe traffic condition combinations. Whereas the micro level simulates the highway network in more detail in terms of allowing for different traffic conditions across certain intersections and varying traffic operational strategies to improve the evacuation process. Though this model is able to perform both macro and micro level, it is not explicit micro level mode. Another model is OREM (Oak Ridge Evacuation Modeling System) which is Windows-based simulation program developed by ‘Center for Transportation Analysis for Oak Ridge National Laboratory’. It simulates real-time evacuation plans from variety of disasters for large scale transportation networks.

#### **2.4.2. Micro-Simulation Models**

The purpose of micro-simulation model is to track detail movements of individual entities in the road network which is being simulated. The idea would be to take entities/people away from the affected area to safe destination either by their own or by guidance of police or traveller information guidance (Pidd, De Silva, and Eglese, 1996). The ability to incorporate fine details of individual movements makes it much easier to introduce real-life factors, such as traffic congestion, police intervention, and breakdown of vehicles, which might block the progress of an evacuation process. Micro-simulation models may be more informative but, on the other hand, require relatively much more time, extensive data and computer resources and very difficult to implement until recently.

#### **2.4.3. Meso-Simulation Models**

Meso-simulators are a compromise between the two approaches discussed above and they usually involve a discrete simulation which tracks the movements of groups of vehicles. One of the earliest approaches is the US federal emergency management agency’s I-Dynev system, which basically evolved to reduce the computational demands inherent in micro-simulation without losing the need for relatively detailed interaction (Pidd, De Silva, and Eglese, 1996). The Oak Ridge Evacuation Modeling System (OREMS) is a software package that can be used to model evacuation operations and planning and management scenarios for a variety of disasters (Chiu et al., 2007). The Evacuation Traffic Information System is a Web-based system tool for sharing information

among states and agencies. The ETIS tool is designed to help state and local managers anticipate state-to-state traffic. It is not a modeling simulation tool, but rather a tool to share information during an evacuation that may help decision makers make adjustments in their evacuation routing (Transportation, 2006).

## **2.5. Human Behavior Literature**

Travel behavior and how evacuees respond, plays an important role as soon as an evacuation order is issued. These behaviors should be modelled during planning for evacuation operations, since it is not accurate to assume that human behavior in the face of a life-threatening situation will be entirely predictable or controllable. Mathematical models have to dynamically change destination and route choices as they occur over the span of the emergency. Hence, for a model to be effective and applicable, it should have the following characteristics (Barrett, Ran, and Pillai, 2000).

- It must be capable of determining the number of evacuees based on demographic conditions and the type of the disaster.
- It should allow the planners to accurately determine origins and destinations for a particular moment in time.
- It should accurately reflect human behavior during an emergency.
- It should be sensitive to the changes in the demand characteristics of different phases of emergency, before, during and after the disaster.

Alsnihi and Stopher (2004) in their research stated several reasons why households may opt not to evacuate, such as wanting to stay back to protect their property, mimicking neighbour behavior when they have yet to evacuate, the inconvenience factors associated with an evacuation, or having no or limited knowledge of the severity of the imminent disaster. How to properly model and capture attitudes such as those is still a big concern that needs further exploration. There have been studies on whether the evacuees to be sent to the safest place or the nearest shelter. Shelters are generally built to protect evacuees from the effect of disaster as well as providing food and medical first aid treatments. Where to build these shelter for different types of disasters are generally guided by FEMA-310 (1998) and American Red Cross (2002). Even having built shelters at the potential locations, demands at these shelters are not fixed. Demand uncertainty refers to the uncertainty

associated with the number of people using the public shelters during evacuation. Kulshrestha et al. (2011) in their study provided the mathematical model that captures these uncertainties and provide the shelter locations for demand uncertainties. According to Southworth and Chin (1987), destination selection procedures, assuming the evacuee's behaviours can be modeled in four ways:

- Evacuees are assumed to exit the threatened area by moving towards the nearest exit;
- Evacuees will disperse and not choose similar exit points; this will depend on location of friends and relatives and the travel speed of the approaching hazard;
- Evacuees will move towards pre-specified destinations, depending on the evacuation plan in operation;
- Evacuees will depart the area given the underlying traffic conditions of the network at the time of evacuation (allows for myopic evacuee behaviour).

These destination selection criteria will significantly affect the evacuation operation.

### 3. METHODOLOGY

Since it is not desirable to build the transportation network just for the evacuation purposes, we have to use the existing transportation network efficiently and allow evacuees to reach the destination in stipulated time. In our approach, we are trying to evacuate the maximum number of evacuees in the specified amount of time, and we are achieving it by minimizing the cost. The cost of evacuating people in the network is determined by the sum of penalties incurred at the nodes, where penalties are assessed by the amount of time the evacuees spent in the area which is affected, and it is the non-decreasing function of time. Also, to evacuate more number of evacuee, we have incorporated two strategies to augment the capacity of the network. First is divergence and the other is contraflow strategy. With the prior knowledge of demands (population) at the affected nodes, arc capacities, transit time, and penalty function; we will generate the ‘*a priori*’ evacuation network.

#### 3.1. Fundamentals

Our work is based on the concept of ‘Evacuation tree’ generation, introduced by (Andreas and J. C. Smith, 2009). In the evacuation tree model, the aim is to achieve ‘seamless’ evacuation, meaning it is not allowed to evacuate people from the same location to multiple paths to the safety, because of the difficulties in communicating and coordinating such plans. The evacuation tree network structure is an effective way to evacuate but not efficient as it under-utilizes the available resources which might help to leave the network faster. Hence, in our model we are incorporating the tree concept and, to increase the capacity of the lanes, we are using the notion of lane-reversing aka contraflow, as well as divergence at certain nodes depending upon the budgetary conditions. Here divergence is the condition where, if a node has more than one incoming arcs then it will be eligible to spread out the flow on different outgoing paths, and the number of outgoing arcs will always be less than or equal to the number of incoming arcs.

The evacuation tree enforces a restriction on the paths used by the evacuees. It requires all the people from the same source to follow a common path. The idea of tree generation can be compared with heuristic developed by (Lu, George, and Shekhar, 2005) which assigns the evacuees to clusters which are then routed through the network by shortest path search. The difference



between these two approaches is that in the clustering technique evacuees at the intermediate node can be routed through the different paths, but evacuation tree model requires evacuees entering in the node will leave the node using only one exit.

It is very much intuitive to use multiple arcs leaving the node to reach the safety faster. Using more than one arcs to leave the node can be useful in case of emergency and especially at the node which is more central, that means most of the routes to the safety contain that node. In proposed evacuation method we are allowing some of the tree nodes of the network to diverge further depending upon the budget availability. Also, as discussed earlier we are using contraflow strategy to augment the capacity of the evacuation network. Each contraflow decision will be associated with the available budget. The budget will be consist of availability of the disaster management personnel, communication, and lane-reversal resources.

### 3.2. Notation

Let  $G(V, E)$  be the transportation network, where  $V$  represents the set of nodes (or, intersections) and  $E$  the set of arcs (or, streets) connecting any two nodes. In this work, we are considering the time expanded network, and hence we deal with an instance of the graph at any time  $t = 1, \dots, T$  where  $T$  is the total amount of time available for the evacuation process.

We also assume that all nodes in the graph are divided into 3 zones. Let those zones be  $S_1$ ,  $S_2$ , and  $S_3$  respectively where  $S_1 \cup S_2 \cup S_3 = V$  and  $S_1 \cap S_2 \cap S_3 = \emptyset$ . The intuition behind the zones is that  $S_1$  (zone 1) is the closest to the disaster and is in imminent danger.  $S_3$  (zone 3) contains all nodes that are considered safer. Finally,  $S_2$  (zone 2) is the intermediary zone which directly connects vehicles in the danger zone to the safe zone. Finally, each node has some initial number of vehicles which are in need of evacuation. A good evacuation policy then will try to make sure that all vehicles in the transportation network at time  $t = 0$  reach a safe area  $i \in S_3$  before the final time step  $T$ .

Another assumption we make is that nodes have no capacity consideration, whereas arcs have a limited capacity. This capacity  $u_{ij}$  is treated in a time-expanded way, implying that vehicles traversing the street count towards that upper limit, no matter when they started the traversal. Hence, we also assume that every street has a known (and deterministic) travel time  $\omega_{ij}$ . Last, for every node  $i \in S_1 \cup S_2$ , we assume that there is a danger factor  $r_{it}$ , which is dependent on the time step  $t$ , such that  $r_{it_1} \geq r_{it_2}, \forall t_1 \geq t_2$ . For nodes  $i \in S_3$ ,  $r_{it}$  instead represents a “safety” factor and

is used as a reward in our objective function for reaching a safe area. The danger (resp., safety) factors of all nodes in  $S_1 \cup S_2$  (resp.,  $S_3$ ) are assumed to be inputs to our problem, and are used as parameters in our model.

### 3.2.1. Sets

In this section, we introduce all sets necessary for our mathematical program, presented later in (3.1).

1.  $G(V, E)$ : Transportation Network where,  $V$  being the set of nodes (Intersections) and  $E$  being the Arcs (streets) connecting two nodes.
2.  $FS(i) = \{j \in V : (i, j) \in E\}$ : nodes adjacent through a street starting from intersection  $i \in V$ .
3.  $RS(i) = \{j \in V : (j, i) \in E\}$ : nodes adjacent through a street ending in intersection  $i \in V$ .
4.  $t = 1, \dots, T$ : Set of discrete time periods.

### 3.2.2. Parameters

In this section, we provide a list of all parameters used in this model.

$u_{i,j}$  = Capacity of the arc  $(i, j) \in E$ .

$\omega_{i,j}$  = Traverse time of the street between  $i$  to  $j$ .

$T$  = Total time available to evacuate the network.

$r_{it}$  = Penalty (Reward) for being at node  $i \in V$  during time  $t = 1, \dots, T$ .

$\bar{C}_{ij}$  = Cost of reversing the arcs  $(i, j) \in E$ .

$\hat{C}_i$  = Cost of allowing divergence at node  $i$ .

$\bar{B}$  = Budget of contraflow allowed.

$\hat{B}$  = Budget of divergences allowed.

### 3.2.3. Decision Variables

This section introduces all the decision variables we need to define for the optimization model. The decisions made in this work include the streets to be used, the streets to be reversed, the intersections to be diverged, as well as the flows and demands on streets and intersections, respectively.

$f_{ijt}$  = a continuous variable representing the vehicular flow on arc  $(i, j) \in E$  at time  $t = 1, \dots, T$ .

$d_{i,t}$  = a continuous variable representing the remaining demand at node  $i \in V$  at time  $t = 1, \dots, T$ .

$x_{i,j}$  = binary variable representing whether an arc  $(i, j)$  is used in the evacuation plan ( $x_{i,j} = 1$ ) or not ( $x_{i,j} = 0$ ).

$y_{i,j}$  = binary variable representing whether an arc  $(i, j)$  is reversed in the evacuation plan ( $y_{i,j} = 1$ ) or not ( $y_{i,j} = 0$ ).

$m_i$  = integer variable, representing the number of divergences a node  $i \in V$  is allowed (i.e.,  $m_i = 0$  implies no divergence,  $m_i > 0$  implies as many divergences from the plan).

### 3.2.4. Mathematical Model

After the definitions in the previous subsections, we are now ready to present the optimization model, shown in (3.1).

$$\min \sum_{i \in V} \sum_{t=1}^T r_{it} d_{it} \quad (3.1a)$$

$$s.t. \quad \sum_{\tau=\max\{0,t-\omega_{ij}+1\}}^{\min\{t,T-\omega_{ij}\}} f_{ij\tau} \leq u_{ij}x_{ij} + u_{ji}y_{ji}, \quad \forall (i,j) \in E, \forall t = 1, 2, \dots, T-1, \quad (3.1b)$$

$$\sum_{j \in FS(i)} f_{ijt} - \sum_{j \in RS(i)} f_{jit-\omega_{ji}} + d_{it} - d_{it-1} = 0, \quad \forall i \in V, \forall t = 1, 2, \dots, T-1, \quad (3.1c)$$

$$\sum_{j \in FS(i)} x_{ij} \leq \sum_{j \in RS(i)} x_{ji}, \quad \forall i \in V, \quad (3.1d)$$

$$\sum_{j \in FS(i)} x_{ij} = 1 + m_i, \quad \forall i \in V, \quad (3.1e)$$

$$\sum_{i \in V} \hat{C}_i m_i \leq \hat{B}, \quad (3.1f)$$

$$\sum_{(i,j) \in E} \bar{C}_{ij} y_{ij} \leq \bar{B}, \quad (3.1g)$$

$$y_{ji} \leq x_{ij}, \quad \forall (i,j) \in E, \quad (3.1h)$$

$$\sum_{i \in S, j \in S} x_{ij} \leq |S| - 1, \quad \forall S \subset V, \quad (3.1i)$$

$$f_{ijt} \geq 0, \quad \forall (i,j) \in E, \forall t = 1, \dots, T, \quad (3.1j)$$

$$d_{it} \geq 0, \quad \forall i \in V, \forall t = 1, \dots, T, \quad (3.1k)$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in E, \quad (3.1l)$$

$$y_{ij} \in \{0, 1\}, \quad \forall (i,j) \in E, \quad (3.1m)$$

$$m_i \in \mathbb{Z}^*, \quad \forall i \in V. \quad (3.1n)$$

### 3.2.5. Explanation of the Optimization Model

We are using a dynamic network flow model to represent our evacuation problem. The objective function is presented in (3.1a) and minimizes the total cost of evacuation. The cost is a penalty incurred by an evacuee by spending time in a danger zone. The penalty increases with time; if the evacuee stays in a danger zone for a longer time, he gets penalized more. This Objective function is subjected to the capacity constraint (3.1b) and the flow balance constraint (3.1c). The capacity constraint ensures the capacity of each street is respected means the flow on the arc stays

less than or equal to the capacity of the arc. The capacity constraint allows flow on the arc  $x_{ij}$  only if it has selected by the model that is  $x_{ij} = 1$ . Similarly, if the arc is selected for contraflow, its capacity gets augmented in the equation so that total capacity of the street be, capacity of the street  $x_{ij}$  and the capacity of the  $y_{ij}$ . Equation (3.1c) is the time expanded version of flow balance constraint for network problems. Our model assumes that the flow in the network is uninterrupted flow meaning it does not experience any delay while in transit. Also, however much flow enters through the one end of the arc at time  $t$ , appears on the other end after time;  $t$ -(the transit time of the street).

Constraints (3.1d) and (3.1e) are to restrict the number of outgoing arcs. Constraint (3.1d) limits the number of outgoing arcs to be less than or equal to the number of incoming arcs in the node. Constraint (3.1e) allows outgoing arcs from node  $i$  to at least equal to '1' and if the model allows that node to diverge, then the model selects an integer value for  $m$  which decides the number of arcs that might leave the node.

The number of divergences depends upon the budget availability, the constraint (3.1f) restricts the number divergence within the available budget. Similarly, constraint (3.1g) restricts the number of lanes to be selected for contraflow within the budget. Equation (3.1h) considers only those lanes for contraflows which have  $x_{ij}$  selected, no other than these arcs are allowed to be the contraflow lane. Constraint (3.1i) which is a cycle braking constraint which avoids cycles in the network. Finally, (3.1j)–(3.1n) are variable restrictions, in accordance to their definition in subsection 3.2.3.

## 4. RESULTS

### 4.1. Experimental Setup

To check how our optimization model with different cost functions performs, we used two benchmark transportation network available online at <https://github.com/bstabler/TransportationNetworks>. The two networks that contained all information necessary for our models (that is, coordinates for all nodes, demands, street capacities) were the ones named “Sioux Falls” and “Chicago Sketch”, and hence they are the two that we will be focusing on in this section.

For obtaining all results present in this work, we used a personal computer enabled with Intel Core i7-5500U at 2.39 GHz. All the codes were written in Python version 3.6. For the optimization models, the commercial optimization solver of Gurobi (version 7.5) was used, with its Gurobi-Python solver interface. Finally, for visualization purposes for all network outputs, the NetworkX Python package was used.

Before we start solving the model, we introduced two artificial nodes, *Sin* and *Sout*. *Sin* is connected to all the zone-1 and zone-2 nodes and *Sout* is connected to zone-3 nodes. Mathematical significance of *Sin* is, it will allow a node to initiate the evacuation process whereas *Sout* will collect all the evacuees that reach the zone-3. After that, we performed series of experiments on the selected networks to check the usefulness of our proposed evacuation method.

In our research we are considering three types of *Sink* costs  $(-1, -t, t - T)$ , i.e., in our case, it is *Sout* node, to check the behavior of the evacuation model. There is a philosophy behind selecting these costs. Cost  $-1$  affects all the evacuees arriving at the *Sout* nodes will get counted the same way, meaning, there is no reward for reaching early. The cost  $-t$  changes per time, making it more negative as time passes, and that makes the model force evacuees to evacuate early so that it minimizes the objective value by reaching the safe area early. Cost  $t - T$  behaves similar to the cost  $-t$ . But, here, evacuees get rewarded if they arrive early as this cost increases as time passes hence reaching early at the *Sout* minimizes the cost of objective function value. First, we tested our approach on the smaller network (Sioux Falls) to check our proposed model; then we tested it on, the bigger network (Chicago).

Notations  $a\_b\_c$  represents available budget for  $a$  number of divergences and  $b$  number of contraflows. This combination will be appended by the cost ‘ $c$ ’ for which this combination is being used, e.g., 5.5-1 represents a budget for 5 divergences and 5 contraflows under the influence of cost -1. Sometimes we have represented  $t - T$  cost as cost ‘3’ in our experiment just to save some space. In our experiment, we are not allowing nodes which belong to danger-zone to bifurcate just to avoid chaos. Whereas lanes which are connected to the danger zones are allowed to be used as contraflow lanes.

In our research, we are considering the cost as a ‘valid cost’ if it produces consistent results. That means as the budget of divergences and contraflows increases, the network clearance time, as well as danger-zone clearance time, should not be worse than the clearance time it had when we had a lower budget. As we will show in our experiments, while all costs perform similarly, the only consistent policy comes from using a cost of  $t - T$ .

## 4.2. The Sioux Falls Network

### 4.2.1. The Original Sioux Falls Network

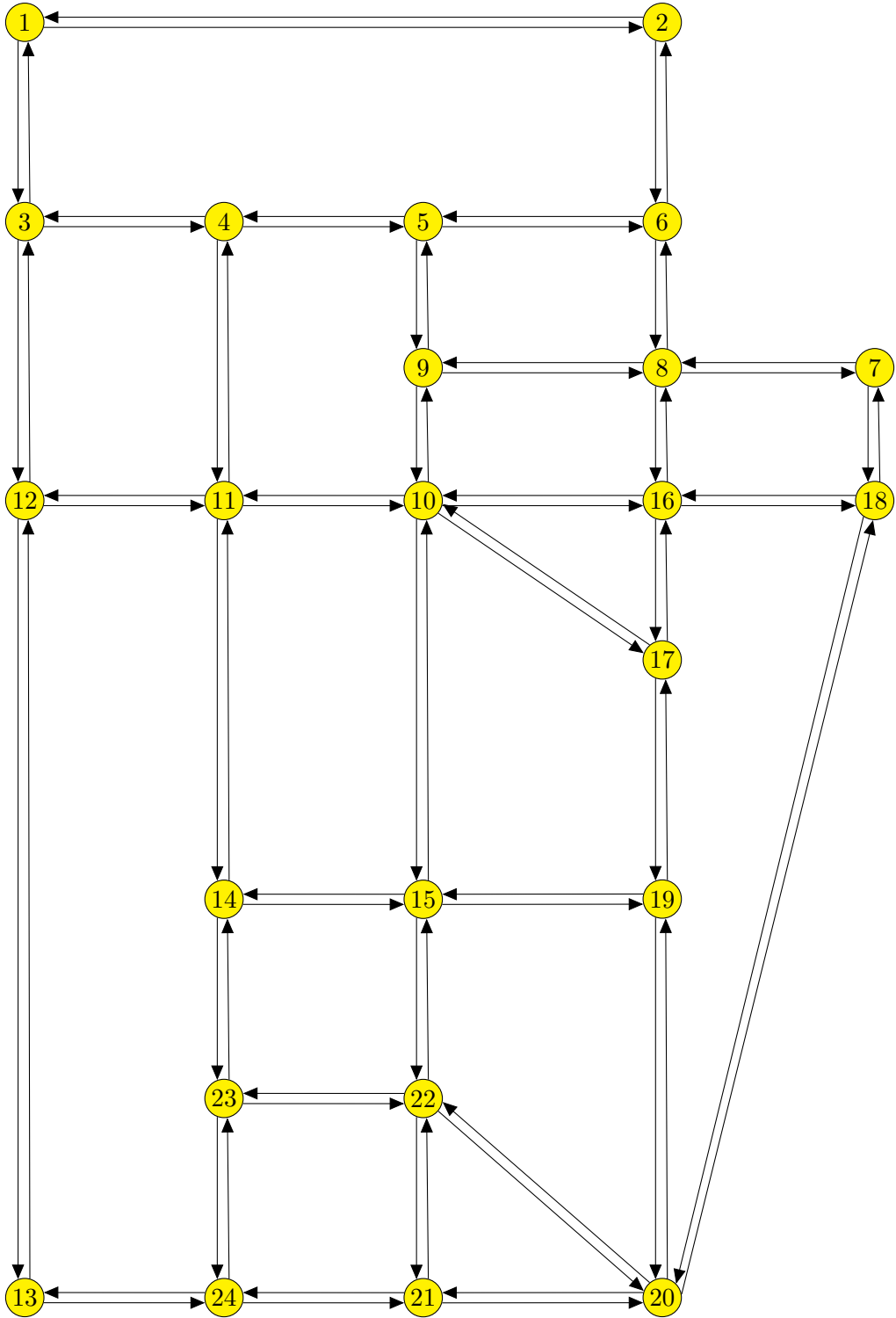


Figure 4.1. Sioux Falls transportation network



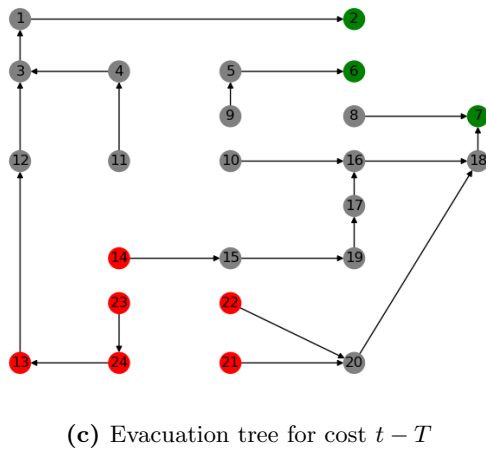
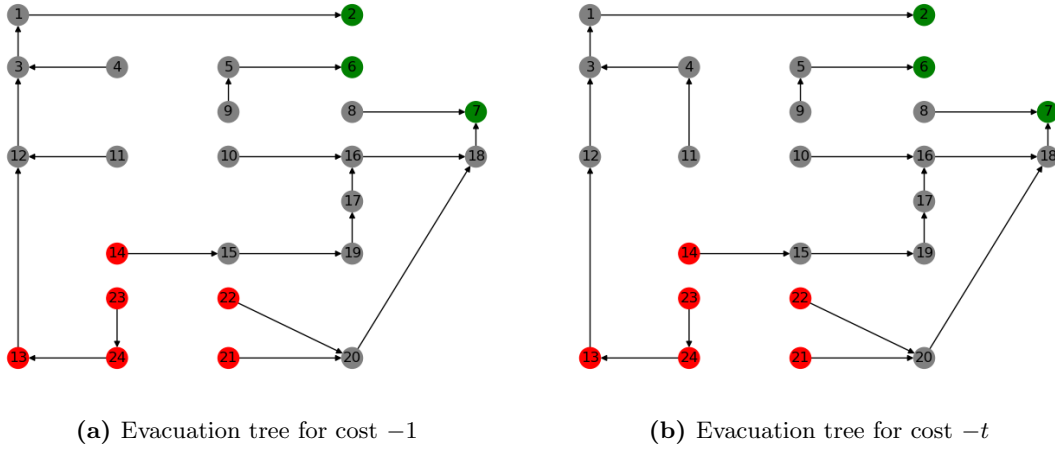
The Sioux Falls benchmark network has 24 nodes and 76 arcs with the total demand to be evacuated is 36060. We performed experiments on it with a dedicated budget for the allowed number of divergences as well as contraflows. Initially, we checked for a tree structure generated by all the types of *Sink* costs  $(-1, -t, t - T)$ , i.e., for the sink node *Sout*. The cost ‘-1’ is a static cost, means it does not change with the time, whereas cost ‘-t’ and ‘t - T’ are the dynamic costs. We ran the model for Sioux Falls network with total available time,  $T=100$  and checked the results for evacuation trees as shown in the figure 4.2. Different costs produce different evacuation tree network to achieve minimum total objective function value.

The model was run on the Sioux Falls network with different divergence and contraflow budgets. Initially we checked the model output just for divergence and contraflows with budget constraints. The resultant model output network is then generated as shown in the figures 4.3, 4.4, 4.5, 4.7, 4.8 and 4.9.

We must notice that there are three different types of colors have been used to represent three zones in the network. The red nodes indicate the ‘zone-1’ that is the ‘danger zone’, gray nodes indicate the ‘zone-2’ that is the intermediate zone, and finally, green nodes indicate the ‘zone-3’ the safe zone. When the node is selected for divergence by our model, then the node is represented in yellow. Similarly; when the arc is selected by the model to be a contraflow arc, then the arc is depicted in red.

We then checked the effect of both divergences and contraflows when used together; we are calling this method as ‘Coupled,’ on the evacuation of the same network. The outputs generated by the model for different coupled combinations are shown in figures 4.11, 4.12, and 4.13.

### 4.2.2. Evacuation Trees



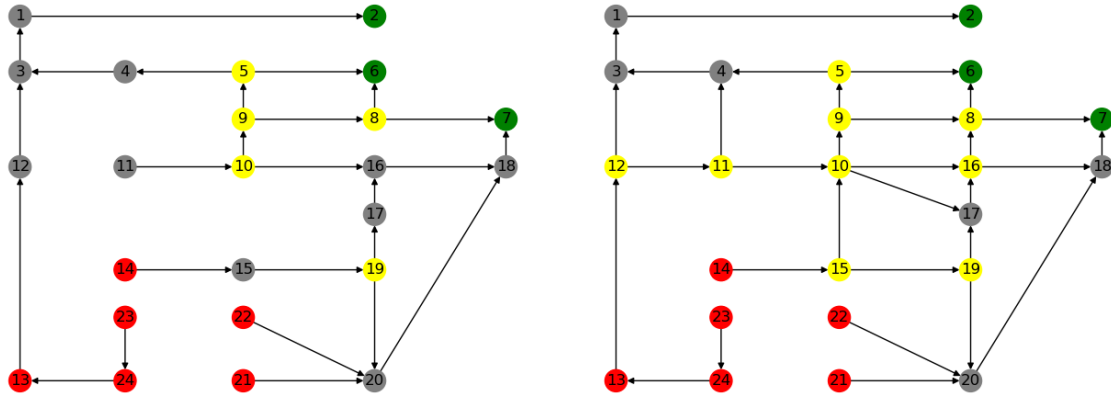
**Figure 4.2.** Evacuation tree generated by different costs

The figures 4.2a, 4.2b and 4.2c represent the evacuation network tree structure generated by the model for Sioux falls network using  $-1$ ,  $-t$  and  $t - T$  costs respectively. These tree networks have been generated by our model when given ‘zero’ budget for divergences as well as contraflows.

In the next section we will see how our model behaves when given budget for divergences and contraflows using different costs.

### 4.2.3. Network Evacuation Using Divergence Schemes

#### 1. Divergence schemes for cost: $-1$

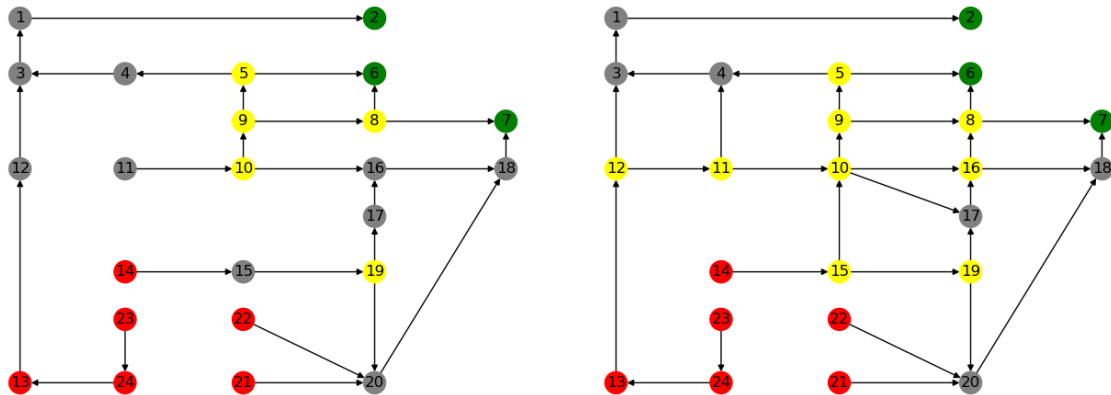


(a) 5 divergences allowed

(b) 10 divergences allowed

**Figure 4.3.** Divergence schemes used by the model for cost:  $-1$

#### 2. Divergence schemes for cost: $-t$

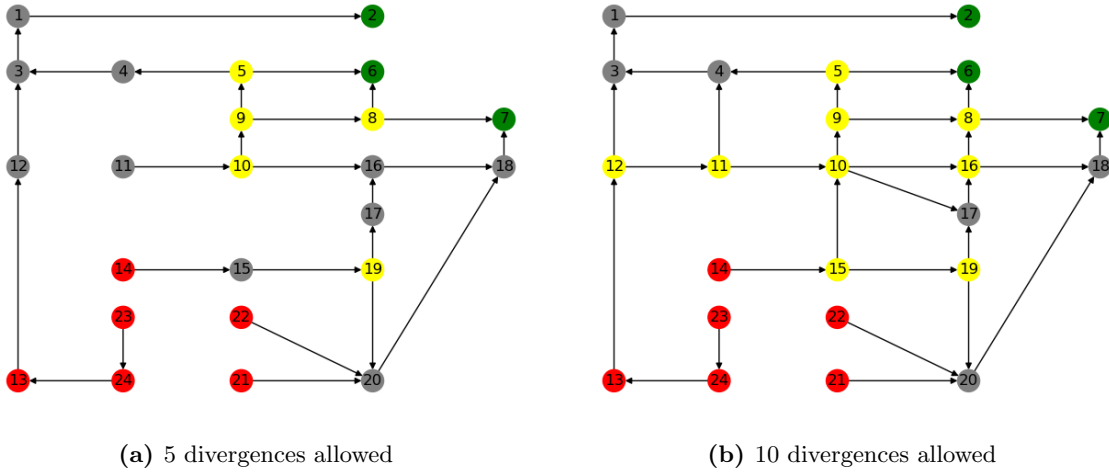


(a) 5 divergences allowed

(b) 10 divergences allowed

**Figure 4.4.** Divergence schemes used by the model for cost:  $-t$

3. Divergence schemes for cost:  $t - T$



**Figure 4.5.** Divergence schemes used by the model for cost:  $t - T$

**Table 4.1.** Nodes used in divergence scheme for cost:  $-1$

Nodes	Divergences	
	5	10
5	x	x
8	x	x
9	x	x
10	x	xx
11		x
12		x
15		x
16		x
19	x	x

**Table 4.2.** Nodes used in divergence scheme for cost:  $-t$

Nodes	Divergences	
	5	10
5	x	x
8	x	x
9	x	x
10	x	xx
11		x
12		x
15		x
16		x
19	x	x

**Table 4.3.** Nodes used in divergence scheme for cost:  $t - T$ 

Nodes	Divergences	
	5	10
5	x	x
8	x	x
9	x	x
10	x	xx
11		x
12		x
15		x
16		x
19	x	x

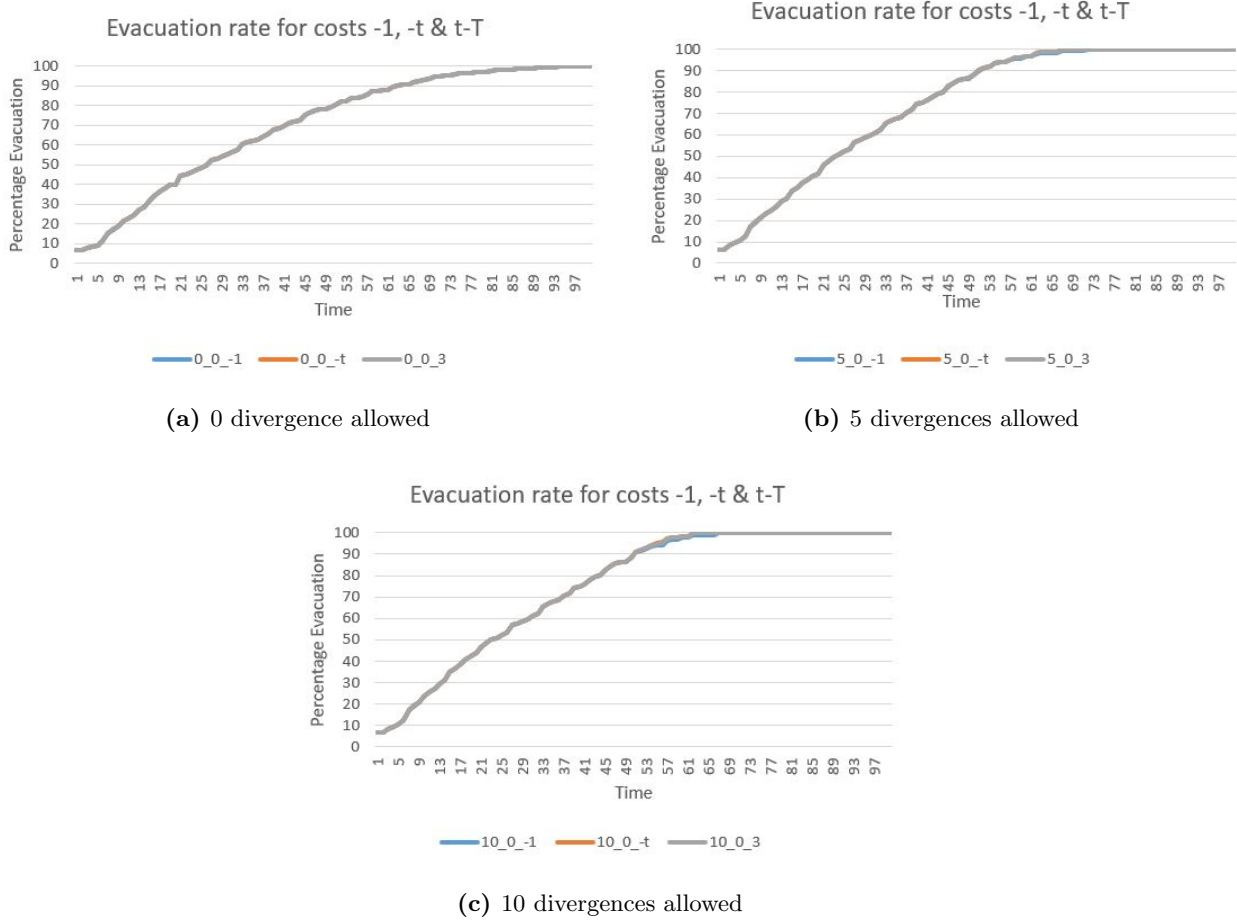
**Table 4.4.** Total network clearance by consideration of just divergences

Combination	Sink Costs		
	-1	-t	$t - T$
0_0	98	98	98
5_0	72	67	67
10_0	67	67	67

**Table 4.5.** Danger zone clearance by consideration of just divergences

Combination	Sink Costs		
	-1	-t	$t - T$
0_0	56	56	56
5_0	56	56	56
10_0	56	56	56

## Divergence schemes



**Figure 4.6.** Comparison of network evacuation rate using divergences

### 4.2.3.1. Analysis of Divergence Results

The results for the effect of presence of divergences with evacuation trees are displayed in the figures 4.3, 4.4 and 4.5. The evacuation networks generated by all three types of costs are similar. The nodes considered by all three costs for generating evacuation network under the budget considerations are exactly same as recorded in the tables 4.1, 4.2 and 4.3.

When compared with evacuation trees, the presence of divergence in the evacuation trees demonstrates the benefit of early network evacuation of the city as analyzed in the table 4.4. Figure 4.6 shows the comparison of the rates of evacuation for all three types of the costs. For particular divergence budget, there is no significant difference between the rates of evacuation for all three types of the costs. When divergence budget is 5 nodes, costs  $-t$  and  $t - T$  evacuate network at

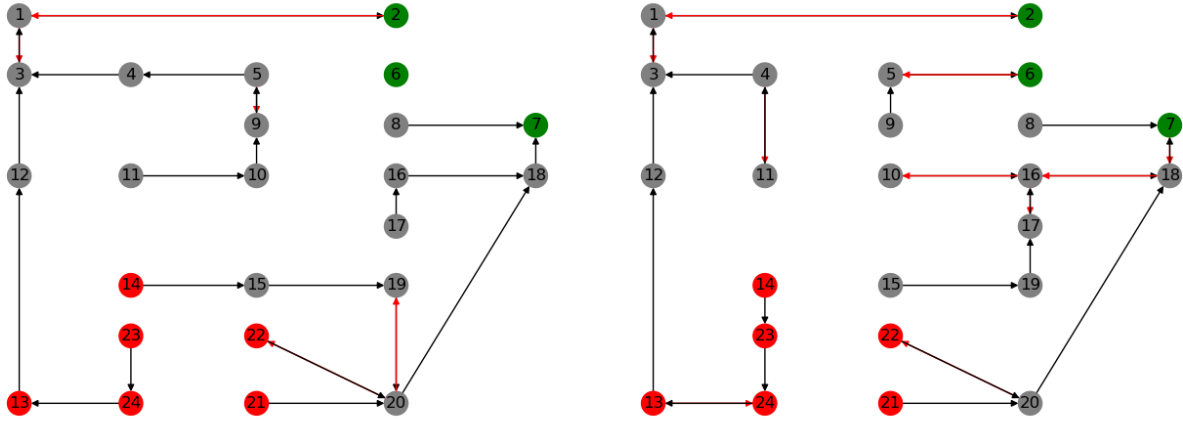
$t = 67$  and cost  $-1$  evacuates network at  $t = 72$ , and when the budget is increased to 10 nodes, all the costs evacuates the network at  $t = 67$  which is early, as compared to  $t = 98$  when the divergences are not allowed.

From the tables 4.1, 4.2 and 4.3, we can compare the nodes which have been used in different cost for the divergence schemes. It is apparent from the tables that, all the nodes which have been used when 5 divergences are allowed also been used when we allow budget for 10 divergences. Nodes 11, 12, 15, 16 which are not important at lower divergence budget but, become important as the divergence budget increases from 5 to 10. The node 10 which gains importance as we increase the budget; its out-degree increases from 1 to 2 which means this node is now able to spread out the incoming flow in three different directions as it can be seen in the figures 4.3, 4.4 and 4.5. Figure 4.6 shows how evacuees are being evacuated; we can see that all the costs evacuate at the same rate. At no budget, the graph is flatter, but as the budget increases, we can see the little hump at the end. It is because as the budget for divergence increases it allows the model to push evacuees who are close to the safe zone to safety faster.

Results in table 4.5 show the evacuation time for the danger zone (the zone-1). According to results which we got, it represents that there is no difference between evacuation time of danger zone. With or without the divergence budget, model evacuates the evacuee present at the danger zone at the same time that is at  $t = 56$ .

#### 4.2.4. Network Evacuation Using Contraflow Schemes

1. Contraflow schemes for cost: -1

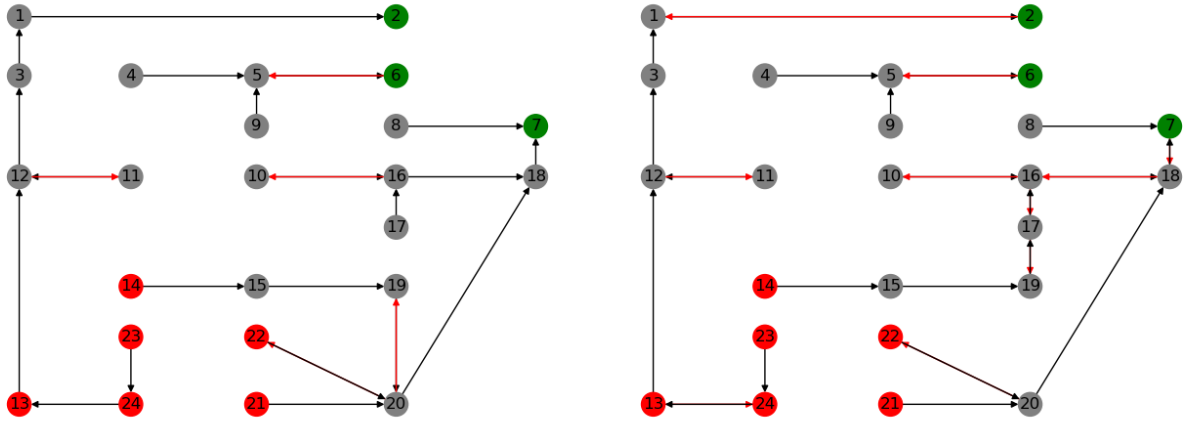


(a) 5 contraflows allowed

(b) 10 contraflows allowed

**Figure 4.7.** Contraflow schemes used by the model for cost: -1

2. Contraflow schemes for cost: -t



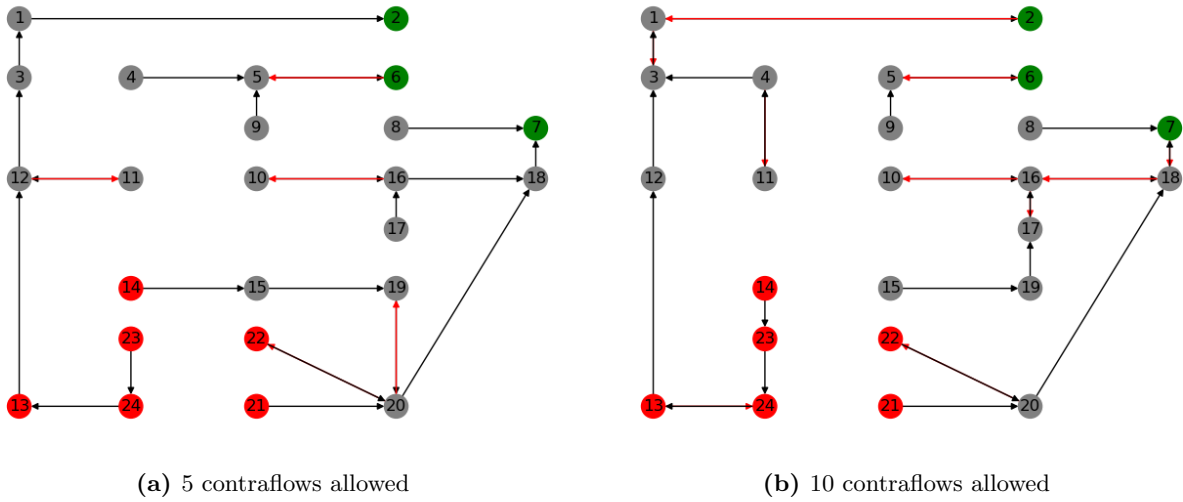
(a) 5 contraflows allowed

(b) 10 contraflows allowed

**Figure 4.8.** Contraflow schemes used by the model for cost: -t



3. Contraflow schemes for cost:  $t - T$



**Figure 4.9.** Contraflow schemes used by the model for cost:  $t - T$

**Table 4.6.** Arcs used in contraflow scheme for cost:  $-1$

Arcs	Contraflows	
	5	10
(1,3)	x	x
(2,1)	x	x
(4,11)		x
(5,9)	x	
(6,5)		x
(7,18)		x
(13,24)		x
(16,10)		x
(16,17)		x
(18,16)		x
(20,19)	x	
(20,22)	x	x

**Table 4.7.** Arcs used in contraflow scheme for cost:  $-t$

Arcs	Contraflows	
	5	10
(2,1)		x
(6,5)	x	x
(7,18)		x
(12,11)	x	x
(13,24)		x
(16,17)		x
(16,10)	x	x
(17,19)		x
(18,16)		x
(20,19)	x	
(20,22)	x	x

**Table 4.8.** Arcs used in contraflow scheme for cost:  $t - T$

Arcs	Contraflows	
	5	10
(1,3)		x
(2,1)		x
(4,11)		x
(6,5)	x	x
(7,18)		x
(12,11)	x	
(13,24)		x
(16,10)	x	x
(16,17)		x
(18,16)		x
(20,19)	x	
(20,22)	x	x

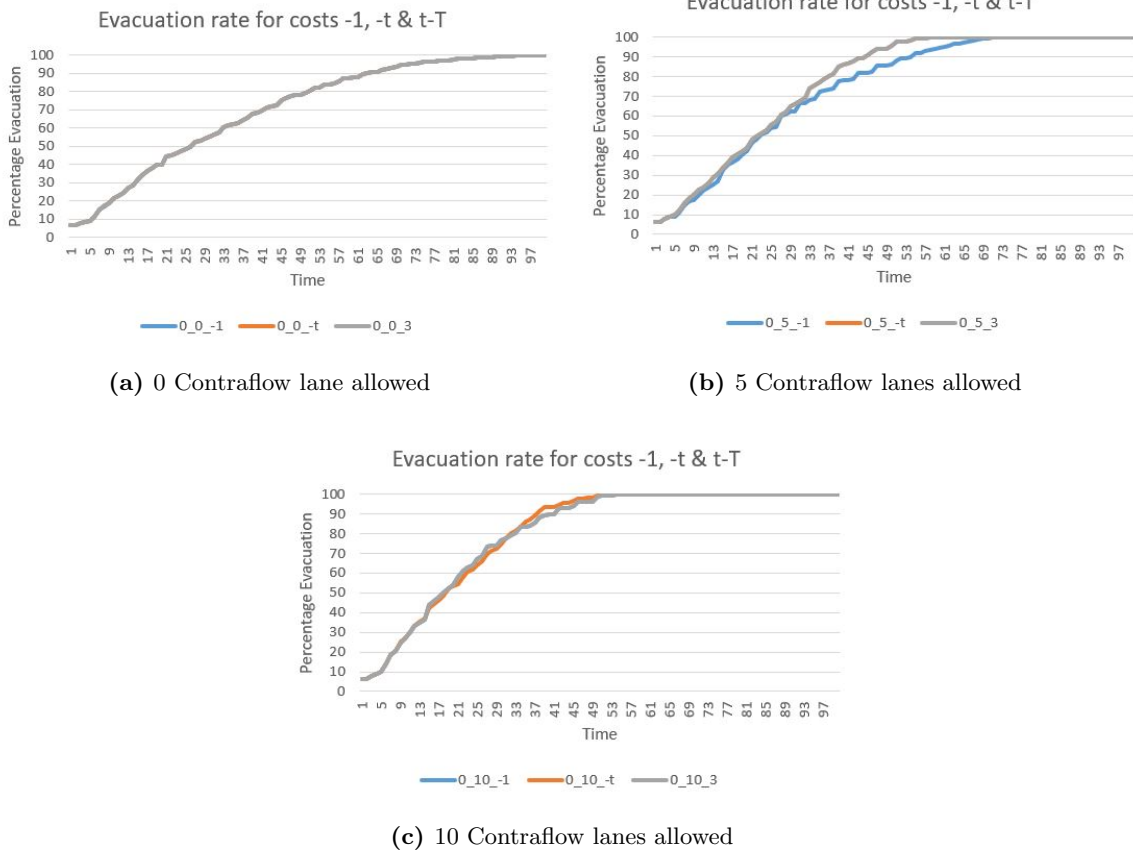
**Table 4.9.** Total network clearance by consideration of just contraflows

Combination	Sink Costs		
	-1	$-t$	$t - T$
0_0	98	98	98
0_5	72	62	62
0_10	54	54	54

**Table 4.10.** Danger zone clearance by consideration of just contraflows

Combination	Sink Costs		
	-1	$-t$	$t - T$
0_0	56	56	56
0_5	39	39	39
0_10	33	31	33

## Contraflow schemes



**Figure 4.10.** Comparison of network evacuation rate using contraflows

### 4.2.5. Analysis of Contraflow Results

From figures 4.7, 4.8 and 4.9, it is evident that, costs  $-1$ ,  $-t$  and cost  $t-T$  produce different evacuation network as well as they consider different arcs for reversal.

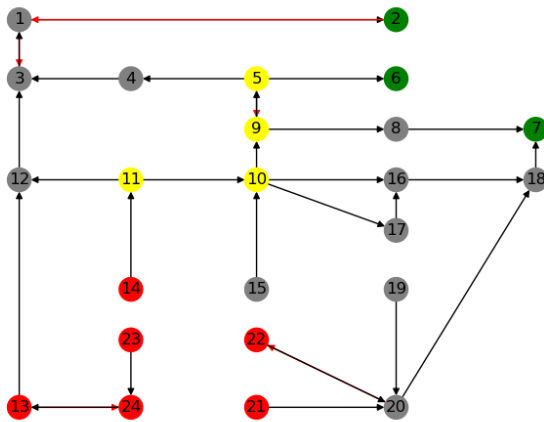
If compared with the evacuation tree, adding contraflows in the evacuation network demonstrates the benefit of early evacuation as noted in the table 4.9. Here, figure 4.10 shows the comparison between the rate of evacuation for all three costs with different budget constraints. As we keep adding the budget for divergence, we can see the considerable decrease in the danger zone evacuation time as shown in table 4.10. When we have contraflow budget of 5 lanes, costs  $-t$  and  $t-T$  evacuate network at  $t = 67$  and cost  $-1$  evacuates network at  $t = 72$ , and when the budget is increased to 10 lanes, all the costs evacuates the network at  $t = 54$  which is early, as compared to  $t = 98$  when the contraflows are not allowed.

From tables 4.6, 4.7 and 4.8 it is evident that arc (20,22) must be an important arc as it has been used by all three cost schemes for all the combinations. Also, arc (20,19) is used when we have a lower budget, but as budget increases, this arc becomes unimportant for all the costs. Similarly, for cost  $-1$  the arc (5,9) and for cost  $t - T$  arc (12,11) have been used when we allow lower budget for contraflows. It is interesting to note that from figure 4.10 how the rate of evacuation changes as the budget for contraflows is increased.

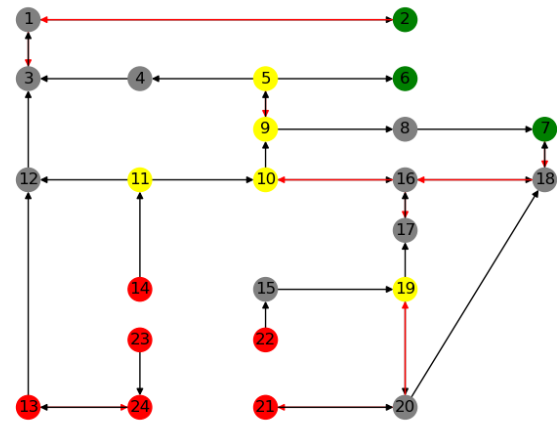
It is interesting to note that, the network generated by the cost  $t - T$  at the lower budget is similar to the network generated by the cost  $-t$  and as we increase the budget it becomes similar to the network generated by the cost  $-1$ . Results in the table 4.10 show that the time required to evacuate the danger zone is indeed dependent on the presence of contraflows. For lower budget i.e. when 5 contraflow lanes are allowed, the danger-zone evacuation time reduces to  $t = 39$  for all three costs from  $t = 59$ . If we further increase the contraflow budget, the danger-zone evacuation time further decreases  $t = 33$  for costs  $-1$  and  $t - T$ , and  $t = 31$  for cost  $-t$ .

#### 4.2.6. Evacuation Using Divergences and Contraflows Together, ‘The Coupled Scheme’

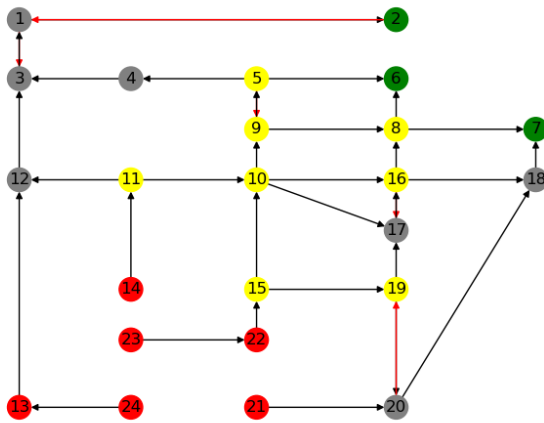
In this section, we have incorporated divergence and contraflow schemes together for the Sioux Falls network and checked if it evacuates the network faster rather than just using contraflow and divergence. We are calling it as coupled scheme, and from now on we refer it as the same.



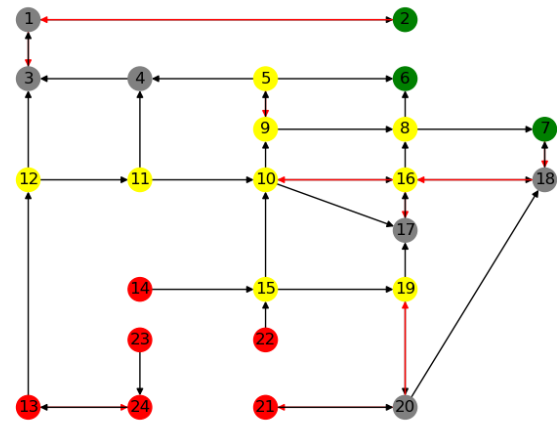
(a) 5 Divergences and 5 Contraflows allowed



(b) 5 Divergences and 10 Contraflows allowed

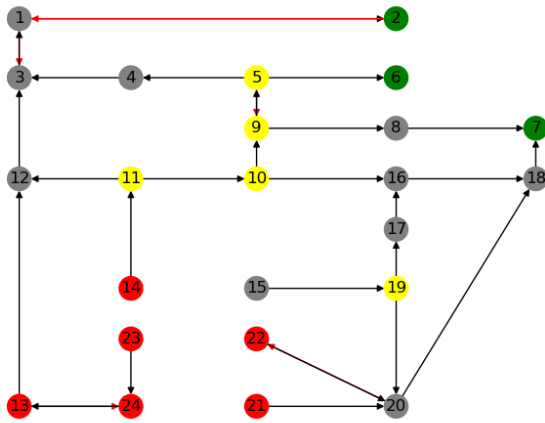


(c) 10 Divergences and 5 Contraflows allowed

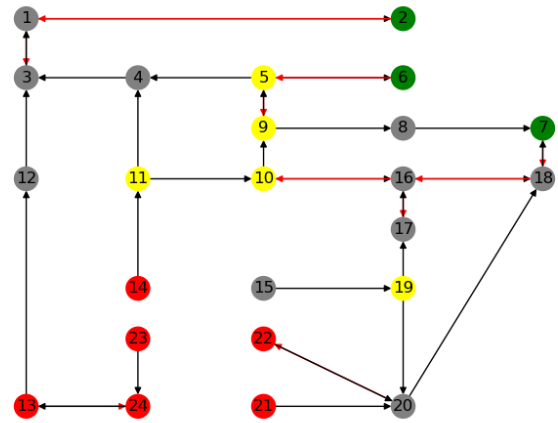


(d) 10 Divergences and 10 Contraflows allowed

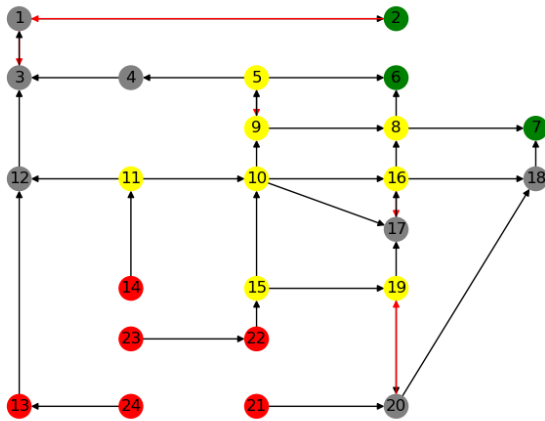
**Figure 4.11.** Contraflow schemes used by the model for cost:  $-1$



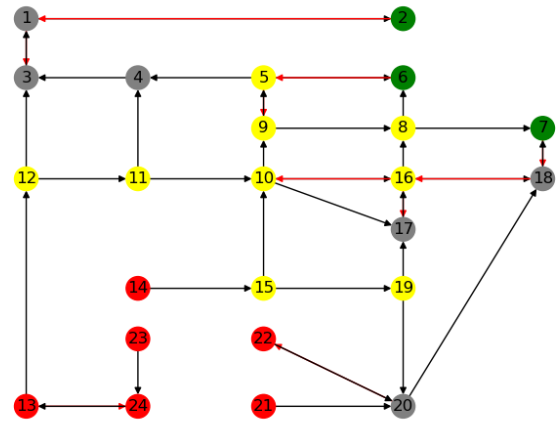
(a) 5 Divergences and 5 Contraflows allowed



(b) 5 Divergences and 10 Contraflows allowed

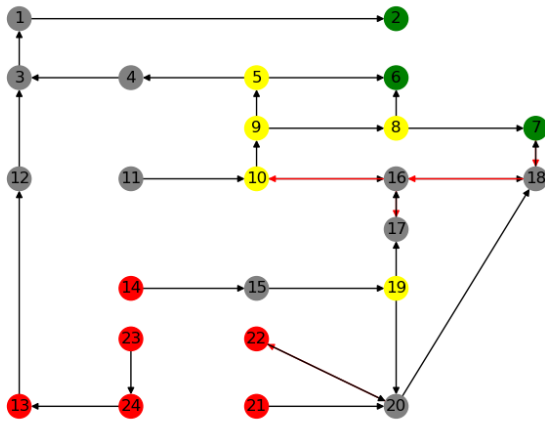


(c) 10 Divergences and 5 Contraflows allowed

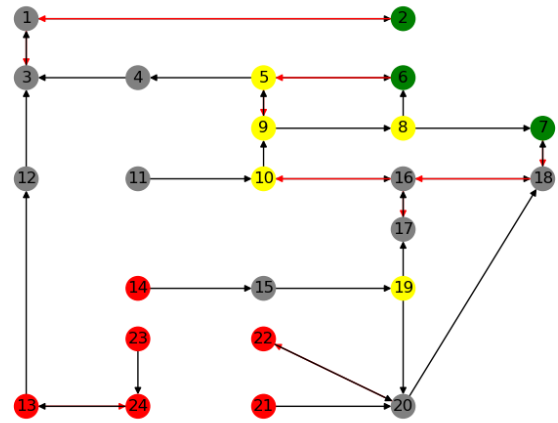


(d) 10 Divergences and 10 Contraflows allowed

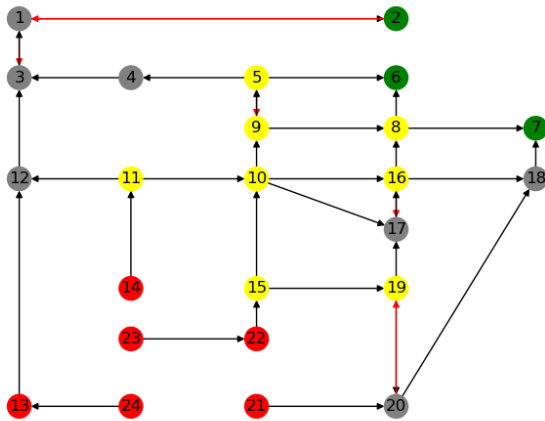
**Figure 4.12.** Contraflow schemes used by the model for cost:  $-t$



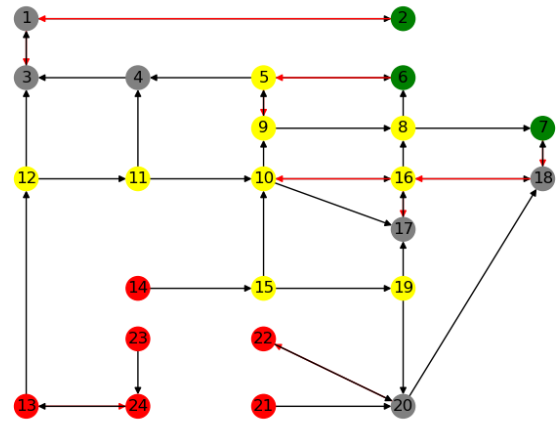
(a) 5 Divergences and 5 Contraflows allowed



(b) 5 Divergences and 10 Contraflows allowed



(c) 10 Divergences and 5 Contraflows allowed



(d) 10 Divergences and 10 Contraflows allowed

**Figure 4.13.** Contraflow schemes used by the model for cost:  $t - T$

**Table 4.11.** Arcs used in coupled scheme for cost:  $-1$

Arcs	Contraflows allowed					
	0.5	0.5	5.5	5.10	10.5	10.10
(1,3)	x	x	x	x	x	x
(2,1)	x	x	x	x	x	x
(4,11)		x				
(5,9)	x		x	x	x	x
(6,5)		x				
(7,18)		x		x		x
(13,24)		x	x	x		x
(16,10)		x		x		x
(16,17)		x		x	x	x
(18,16)		x		x		x
(20,19)	x			x	x	x
(20,21)				x		x
(20,22)	x	x	x			

**Table 4.12.** Nodes used in coupled scheme for cost:  $-1$

Nodes	Nodes Selected					
	5.0	10.0	5.5	5.10	10.5	10.10
5	x	x	x	x	x	x
8	x	x			x	x
9	x	x	x	x	x	x
10	x	xx	xx	x	xx	xx
11		x	x	x	x	x
12		x				x
15		x			x	x
16		x			x	x
19	x	x		x	x	x



**Table 4.13.** Arcs used in coupled scheme for cost:  $-t$ 

Arcs	Contraflows allowed					
	0.5	0.10	5.5	5.10	10.5	10.10
(1,3)			x	x	x	x
(2,1)		x	x	x	x	x
(5,9)			x	x	x	x
(6,5)	x	x		x		x
(7,18)		x		x		x
(12,11)	x	x				
(13,24)		x	x	x		x
(18,16)		x		x		x
(16,10)	x	x		x		x
(16,17)		x		x	x	x
(17,19)		x				
(20,19)	x				x	
(20,22)	x	x	x	x		x

**Table 4.14.** Nodes used in coupled scheme for cost:  $-t$ 

Nodes	Divergence allowed					
	5.0	10.0	5.5	5.10	10.5	10.10
5	x	x	x	x	x	x
8	x	x			x	x
9	x	x	x	x	x	x
10	x	xx	x	x	xx	xx
11		x	x	x	x	x
12		x				x
15		x			x	x
16		x			x	x
19	x	x	x	x	x	x

**Table 4.15.** Arcs used in coupled scheme for cost:  $t - T$ 

Arcs	Contraflows allowed					
	0.5	0.10	5.5	5.10	10.5	10.10
(1,3)		x		x	x	x
(2,1)		x		x	x	x
(4,11)		x				
(5,9)				x	x	x
(6,5)	x	x		x		x
(7,18)		x	x	x		x
(12,11)	x					
(13,24)		x		x		x
(16,10)	x	x	x	x		x
(16,17)		x	x	x	x	x
(18,16)		x	x	x		x
(20,19)	x				x	
(20,22)	x	x	x	x		x

**Table 4.16.** Nodes used in coupled scheme for cost:  $t - T$ 

Nodes	Divergence allowed					
	5.0	10.0	5.5	5.10	10.5	10.10
5	x	x	x	x	x	x
8	x	x	x		x	x
9	x	x	x	x	x	x
10	x	xx	x	x	xx	xx
11		x		x	x	x
12		x				x
15		x			x	x
16		x			x	x
19	x	x	x	x	x	x

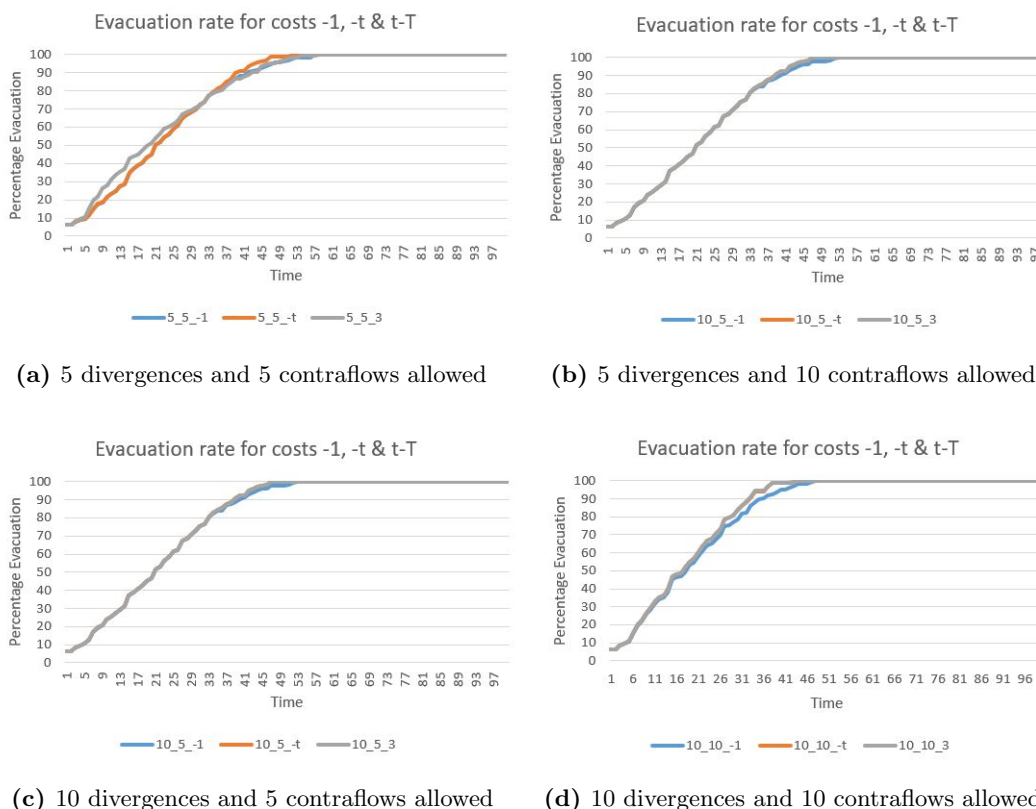
**Table 4.17.** Total network clearance time for Sioux Falls network with all costs.

Combination	Sink Costs		
	-1	-t	t - T
0_0	98	98	98
0_5	72	62	62
0_10	54	54	54
5_0	72	67	67
5_5	57	52	62
5_10	48	45	48
10_0	67	67	67
10_5	53	48	48
10_10	48	48	48

**Table 4.18.** Danger zone clearance time for Sioux Falls network with all costs.

Combination	Sink Costs		
	-1	-t	t - T
0_0	56	56	56
0_5	39	39	39
0_10	33	31	33
5_0	56	56	56
5_5	31	31	39
5_10	25	31	31
10_0	56	56	56
10_5	31	31	31
10_10	31	31	31

## Evacuation rates



**Figure 4.14.** Comparison of network evacuation rate using coupled scheme

### 4.2.7. Analysis of Coupled Scheme

Figures 4.11, 4.12, and 4.13 shows the evacuation networks generated by the model when couples schemes were used with cost  $-1$ ,  $-t$ , and  $t - T$  respectively. Tables 4.11, 4.13, and 4.15 shows the lanes used for the contraflows by the model for all three types of costs. According to table 4.11 lanes (2,1), (1,3) and lane (5,9) are very important for the cost  $-1$  as these lanes keep appearing frequently in all the combinations. Similarly, lanes (1,3), and (2,1) are important for the costs  $-t$  and  $t - T$ . There are some lanes which have been used only when the contraflows were used with no divergences for example, lane (4,11) when  $-1$  and  $t - T$  cost were considered and for cost  $-t$  lane (17,19) was used only once. As budget increases, costs  $-1$  and  $-t$  produces exactly the same evacuation networks for 5-10, 10.5 and 10-10 combinations. In case of divergence, node 10 plays a very significant role because the budget for divergences increased to 10, the node 10 is diverged further, allowing it to spread out incoming flow over to the different nodes faster.

If We check the graphs produced by the rate of evacuation as shown in the figure 4.14, we can see that, as the budgets increase, the rate of evacuation also increase for all the three costs.

Now lets consider the effect of different costs on the evacuation rate. We have seen the effect of presence of divergences as well as contraflows in the evacuation network similarly, from table 4.17 we can check that as budget for divergences and contraflow increases the evacuation network clearance decreases. But if we check the combination  $5_{-10_-} - t$  which is not even a highest budget we have for this combination, we see that network clearance time is lowest among all the combinations. Also, from table 4.18,  $5_{-10_-} - 1$  has the lowest danger zone clearance time among all the combinations. As costs  $-1$  and  $t$  produce inconsistent results we do not consider it to be valid for our experiment whereas cost  $t - T$  has always produced consistent results and hence we select cost  $t - T$  as the best cost to be considered for the experiment.

### 4.3. The Chicago Network

To check out model on Chicago network we used available benchmark ‘Chicago Sketch’ network which has 933 nodes and 2950 arcs with the total demand to be evacuated is 12805. We again tested this model against same three costs  $(-1, -t, t - T)$  and checked which cost performs better. We ran the model for Chicago network with total available time,  $T=150$  and checked the results.

The results tabulated in tables 4.19, 4.21, 4.23 show, the arcs used by the model for contraflow scheme when  $-1, -t$  and  $t - T$  were used respectively. Similarly the results shown in the tables 4.20, 4.22 and 4.24 are the nodes used for divergence scheme when costs  $-1, -t$  and  $t - T$  were used. The results for coupled scheme are tabulated in 4.25, 4.26, 4.27, 4.28, 4.29 and 4.30.

**Table 4.19.** Arcs used in contraflow scheme for Chicago network at cost:  $-1$

Arcs	Contraflows allowed			
	0_5_-1	0_10_-1	0_15_-1	20_0_-1
(424, 425)	x	x	x	x
(426, 441)	x	x	x	x
(431, 432)			x	
(434, 435)			x	
(437, 556)		x	x	x
(438, 535)	x	x	x	x
(439, 438)		x		x
(440, 439)		x	x	x
(441, 440)		x	x	x
(443, 897)				x
(489, 631)				x
(507, 646)				x
(513, 902)	x			
(526, 527)				x
(528, 526)				x
(552, 550)				x
(582, 660)		x	x	
(587, 400)			x	x
(617, 599)				x
(660, 902)		x		
(711, 713)			x	x
(769, 771)			x	x
(773, 775)			x	
(775, 589)			x	
(777, 778)	x	x	x	x

**Table 4.20.** Nodes used in divergence scheme for Chicago network at cost:  $-1$ 

Nodes	Divergence allowed			
	5_0_-1	10_0_-1	15_0_-1	20_0_-1
398	x	x	x	x
427			x	x
428	x	x	x	x
436	x	x	x	x
535	x	x	x	x
550				x
584		x	x	x
589			x	x
599			x	x
608		x	x	x
615			x	x
622		x	x	x
648		x	x	x
718				x
902		x	x	x
903		x		

**Table 4.21.** Arcs used in contraflow scheme for Chicago network at cost:  $-t$ 

Arcs	Contraflows allowed			
	0_5_-t	0_10_-t	0_15_-t	0_20_-t
(424, 425)	x	x	x	x
(426, 441)	x	x	x	x
(427, 428)		x	x	x
(431, 432)		x	x	x
(432, 433)				x
(433, 434)				x
(434, 435)		x	x	x
(437, 556)				x
(438, 535)		x	x	x
(443, 897)				x
(528, 526)		x	x	x
(541, 902)		x	x	x
(587, 400)			x	x
(641, 648)				x
(769, 771)				x
(777, 778)	x	x	x	x
(777, 779)		x	x	x

**Table 4.22.** Nodes used in divergence scheme for Chicago network at cost:  $-t$

Nodes	Divergence allowed			
	5_0_-t	10_0_-t	15_0_-t	20_0_-t
398	x	x	x	
401				x
428		x	x	x
436	x	x	x	x
437			x	
441			x	
535	x	x	x	x
564				x
584			x	x
589			x	
599			x	x
604				x
608		x	x	x
615			x	
622		x	x	x
631				x
643				x
648		x	x	
686				x
696				x
718				x
902	x	x	x	x

**Table 4.23.** Arcs used in contraflow scheme for Chicago network at cost:  $t - T$

Arcs	Contraflows allowed			
	0_5_3	0_10_3	0_15_3	0_20_3
(424, 425)	x	x	x	x
(425, 426)				x
(426, 441)	x	x	x	x
(437, 556)	x	x	x	x
(438, 535)	x	x	x	x
(439, 438)		x	x	x
(441, 440)		x	x	x
(440, 439)		x	x	x
(507, 508)				x
(528, 526)			x	x
(541, 902)			x	x
(553, 550)				x
(587, 400)		x	x	x
(594, 596)			x	x
(596, 612)			x	x
(711, 713)			x	x
(768, 584)				x
(769, 771)		x	x	x
(777, 778)	x	x	x	x
(777, 779)				x



**Table 4.24.** Nodes used in divergence scheme for Chicago network at cost:  $t - T$

Nodes	Divergence allowed			
	5_0_3	10_0_3	15_0_3	20_0_3
392				x
393		x		
398	x	x	x	x
413				x
427			x	x
428	x	x	x	x
436	x	x	x	x
437				x
440			x	x
441			x	
535	x	x	x	x
550				x
584		x	x	x
589		x	x	x
596				x
599			x	x
608			x	x
614				x
615		x	x	x
622		x	x	x
648				x
716			x	
779	x			
902		x	x	x

**Table 4.25.** Nodes used in coupled scheme for Chicago network at cost: -1

Nodes	Divergence allowed															
	5_5	5_10	5_15	5_20	10_5	10_10	10_15	10_20	15_5	15_10	15_15	15_20	20_5	20_10	20_15	20_20
394															x	
400	x				x			x	x	x	x		x	x	x	x
401												x				
409											x			x		
415													x			
427												x				x
428	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
436	x	x	x		x	x			x	x		x	x	x		x
477														x		
488														x		
491												x				
504															x	
512						x										
527											x		x		x	x
550						x	x		x	x		x				x
551											x					
552							x						x	x	x	
556		x	x		x	x	x	x	x	x	x		x	x	x	x
560														x	x	x
562								x								
564												x		x		
571												x			x	
572															x	x
575														x		
577											x		x		x	
584				x				x	x	x	x	x	x	x	x	x
591										x			x			
596																x
599								x	x							
604														x		
607																x
608									x				x			
609										x				x		
610							x									
613								x								x
615					x				x		x					x
616						x		x						x		x
620											x	x				x
622	x				x	x	x	x	x	x	x		x	x	x	x
627													x			
631					x											
646										x						
648		x	x		x		x		x	x					x	
686											x		x		x	
694											x	x				x
696															x	
702													x			
706																x
716															x	
771													x			
871							x					x		x		
902	x	x			x	x	x		x	x		x	x	x		
903						x	x		x	x				x		

**Table 4.26.** Arcs used in coupled scheme for Chicago network at cost:  $-1$

Arcs	Contraflows allowed															
	5.5	5.10	5.15	5.20	10.5	10.10	10.15	10.20	15.5	15.10	15.15	15.20	20.5	20.10	20.15	20.20
(424, 425)	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x
(426, 441)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(427, 428)				x			x	x			x	x			x	x
(428, 431)				x				x				x	x			x
(431, 432)		x	x	x	x	x	x	x		x	x	x	x	x	x	x
(432, 433)				x			x	x				x				x
(433, 434)				x			x	x				x				x
(434, 435)		x	x	x		x	x	x		x	x	x		x	x	x
(438, 535)		x	x	x	x	x	x	x		x	x	x	x	x	x	x
(440, 439)	x			x			x	x				x				x
(441, 440)	x			x			x	x				x				x
(443, 897)			x	x				x			x	x			x	x
(528, 526)			x	x			x	x			x	x			x	x
(541, 902)			x	x				x			x	x			x	x
(587, 400)			x	x			x	x			x	x			x	x
(711, 713)			x	x			x	x			x	x			x	x
(769, 771)		x	x	x		x	x	x	x		x	x			x	x
(777, 778)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(777, 779)		x	x	x		x		x		x	x	x		x	x	x

**Table 4.27.** Nodes used in coupled scheme for Chicago network at cost:  $-t$

Nodes	Divergence allowed															
	5_5	5_10	5_15	5_20	10_5	10_10	10_15	10_20	15_5	15_10	15_15	15_20	20_5	20_10	20_15	20_20
392			x								x			x		x
394										x	x				x	x
400					x	x	x	x	x		x	x	x		x	
401												x				x
409			x							x		x		x		x
410													x			
413										x						
414																x
415														x		
427												x				x
428	x	x		x	x	x	x	x	x		x	x	x	x	x	x
431								x				x				x
435										x						
436	x	x			x			x	x				x	x		x
504																x
527						x					x				x	
537										x						
550											x					
551			x													
552					x				x			x				
556		x				x	x	x		xx	x	x		xx	x	
560														x		x
562			x									x		x		x
564														x		x
571															x	
572													x			
574											x					x
575															x	
576															x	
577													x			
584		x		x	x	x	x	x	x	x	x		x		x	
596												x				x
599											x	x				
604						x			x		x			x	x	x
609														x		
610										x		x		x		x
613									x	x						x
615			x									x				x
616							x	x			x	x				
619													x			
620									x				x			
622	x				x	x	x	x	x	x	x		x		x	
631													x		x	
636													x			
648	x									x						
650													x			
686					x	x	x		x		x		x	x	x	
691														x	x	
694										x					x	x
696														x		
702							x							x		
706										x			x			
716												x			x	
718															x	
725									x					x		
731														x		
871															x	
902	x				x				x				x	x		
903								x			x		x			

**Table 4.28.** Arcs used in coupled scheme for Chicago network at cost:  $-t$

Arcs	Contraflows allowed															
	5_5	5_10	5_15	5_20	10_5	10_10	10_15	10_20	15_5	15_10	15_15	15_20	20_5	20_10	20_15	20_20
(424, 425)	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x
(425, 426)			x							x		x				
(426, 441)	x	x	x	x	x	x	x	x	x		x		x		x	
(427, 428)			x	x			x	x			x	x			x	
(428, 431)														x		x
(431, 432)	x	x		x	x	x	x	x	x	x	x		x	x	x	x
(432, 433)				x			x	x		x	x	x		x	x	x
(433, 434)			x	x			x	x			x				x	x
(434, 435)		x	x	x		x	x	x			x	x			x	x
(435, 436)			x									x				
(438, 535)	x	x	x	x	x	x	x	x	x	x	x		x		x	
(439, 438)				x				x		x		x		x		
(440, 439)				x				x		x				x		x
(441, 440)			x	x				x		x		x		x		x
(443, 897)				x				x								x
(493, 497)												x				x
(494, 493)																x
(498, 533)												x				x
(507, 646)																x
(526, 541)												x				
(527, 543)			x									x				
(528, 526)		x		x		x	x	x			x	x			x	
(531, 529)												x				x
(532, 531)																x
(533, 532)																x
(540, 622)																x
(541, 902)		x		x		x	x	x			x			x	x	
(587, 400)				x			x	x			x				x	
(641, 648)			x	x				x				x				
(711, 713)			x	x				x				x				
(722, 724)												x				x
(768, 584)												x				
(769, 771)			x	x			x	x		x	x	x			x	x
(777, 778)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
(777, 779)		x	x	x		x	x	x			x	x		x	x	x

**Table 4.29.** Nodes used in coupled scheme for Chicago network at cost:  $t - T$

Nodes	Divergence allowed															
	5_5	5_10	5_15	5_20	10_5	10_10	10_15	10_20	15_5	15_10	15_15	15_20	20_5	20_10	20_15	20_20
392					x					x			x	x		
393						x										
394															x	x
400	x		x		x	x	x		x	x	x	x	x	x	x	x
409											x		x	x	x	x
413											x					
427							x		x	x		x	x	x	x	x
428	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
431															x	
435									x				x			
436	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
437					x	x			x				x			
527				x	x		x	x	x	x	x	x	x	x	x	x
550								x		x	x	x	x	x	x	x
551									x				x	x		
556			x	x			x	x	x	x	x	x	x	x	x	x
562															x	
584					x		x	x	x	x	x	x	x	x	x	x
593														x		
599												x			x	x
608													x	x		
610				x			x	x			x	x		x	x	x
613						x				x	x		x	x	x	x
614									x				x			
615		x			x	x			x	x	x		x	x		x
622	x	x			x	x		x	x	x	x	x	x	x		x
631										x						
648						x	x	x		x	x	x	x	x	x	x
716									x			x			x	x
871														x	x	x
902	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x
903												x			x	x

**Table 4.30.** Arcs used in coupled scheme for Chicago network at cost:  $t - T$

Arcs	Contraflows allowed															
	5_5	5_10	5_15	5_20	10_5	10_10	10_15	10_20	15_5	15_10	15_15	15_20	20_5	20_10	20_15	20_20
(424, 425)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(425, 426)								x				x				x
(426, 441)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(427, 428)			x	x		x	x	x			x	x			x	x
(428, 431)								x				x				x
(431, 432)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(432, 433)			x	x		x	x	x			x	x			x	x
(433, 434)			x	x		x	x	x			x	x			x	x
(434, 435)	x	x	x	x	x	x	x	x		x	x	x		x	x	x
(438, 535)		x	x	x		x	x	x	x	x	x	x	x	x	x	x
(439, 438)		x	x	x			x	x		x	x	x		x	x	x
(440, 439)		x	x	x			x	x		x	x	x		x	x	x
(441, 440)		x	x	x			x	x		x	x	x		x	x	x
(528, 526)				x			x	x			x	x				x
(541, 902)				x				x				x				x
(587, 400)				x				x				x				x
(641, 648)				x												
(711, 713)			x	x			x	x			x	x			x	x
(768, 584)				x												
(769, 771)			x	x				x		x		x		x	x	x
(777, 778)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
(777, 779)		x	x	x		x	x	x			x	x			x	x

**Table 4.31.** Network clearance time for Chicago network

Combinations	Sink Costs		
	-1	-t	t - T
0_0	127	127	127
0_5	119	118	119
0_10	118	118	118
0_15	121	118	118
0_20	118	118	118
5_0	118	118	118
5_5	118	118	118
5_10	118	118	118
5_15	118	118	118
5_20	118	118	118
10_0	120	118	118
10_5	118	118	118
10_10	118	118	118
10_15	118	118	118
10_20	118	118	118
15_0	118	118	118
15_5	122	118	118
15_10	118	118	118
15_15	118	118	118
15_20	122	118	118
20_0	118	121	118
20_5	118	118	118
20_10	118	118	118
20_15	118	118	118
20_20	118	118	118

**Table 4.32.** Danger zone clearance time for Chicago network

Combinations	Sink Costs		
	$-1$	$-t$	$t - T$
0_0	6	8	6
0_5	4	19	5
0_10	3	19	5
0_15	5	16	5
0_20	4	16	5
5_0	5	12	5
5_5	5	19	5
5_10	19	7	5
5_15	19	19	5
5_20	19	5	3
10_0	3	5	5
10_5	19	7	5
10_10	3	7	5
10_15	4	19	3
10_20	19	19	3
15_0	5	5	5
15_5	3	19	3
15_10	19	19	3
15_15	19	19	3
15_20	7	15	3
20_0	5	19	5
20_5	7	7	3
20_10	7	19	3
20_15	4	7	3
20_20	7	14	3



Network evacuation rates when only divergences are allowed

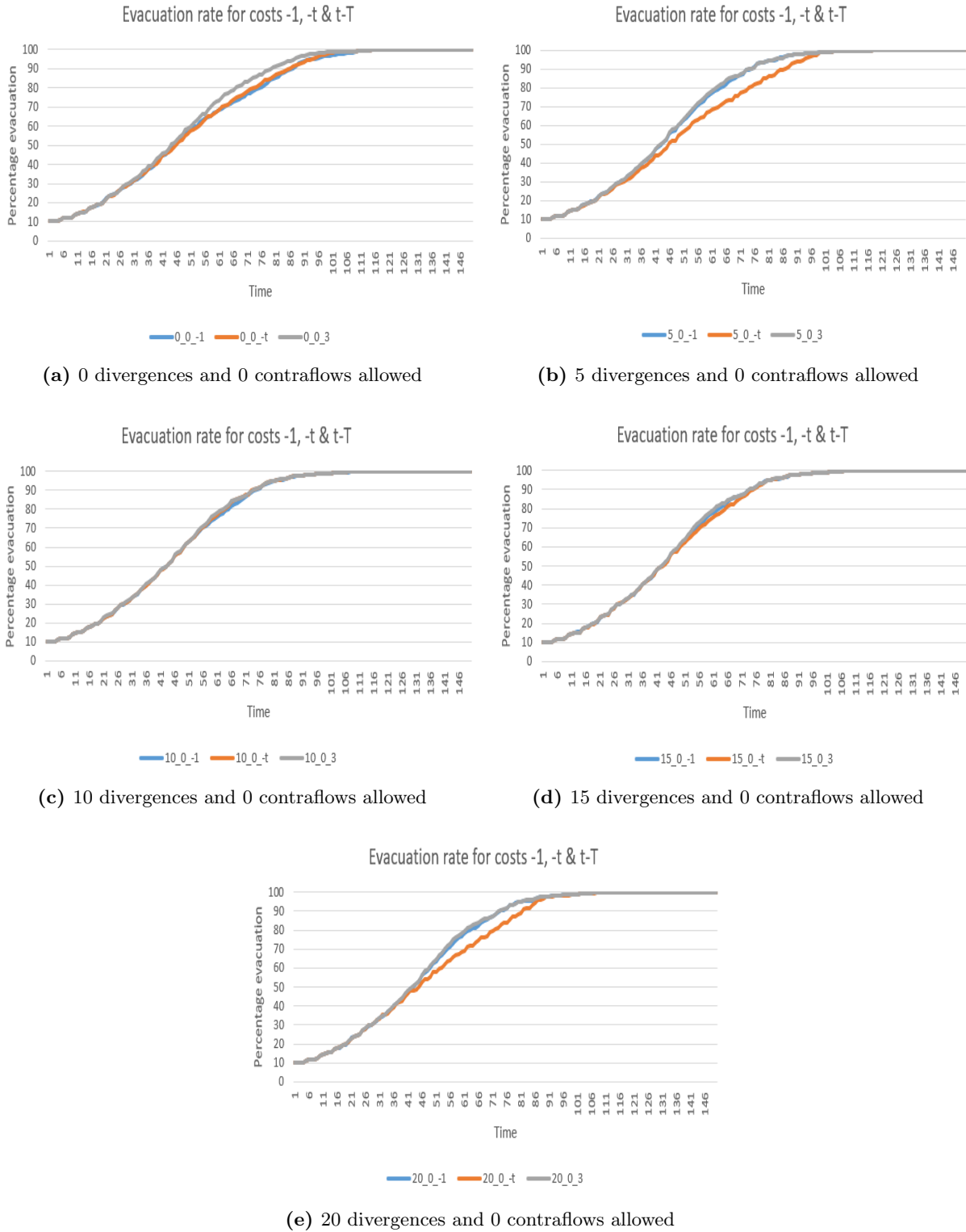


Figure 4.15. Comparison of network evacuation rate using divergence scheme

Network evacuation rates when only contraflows are allowed

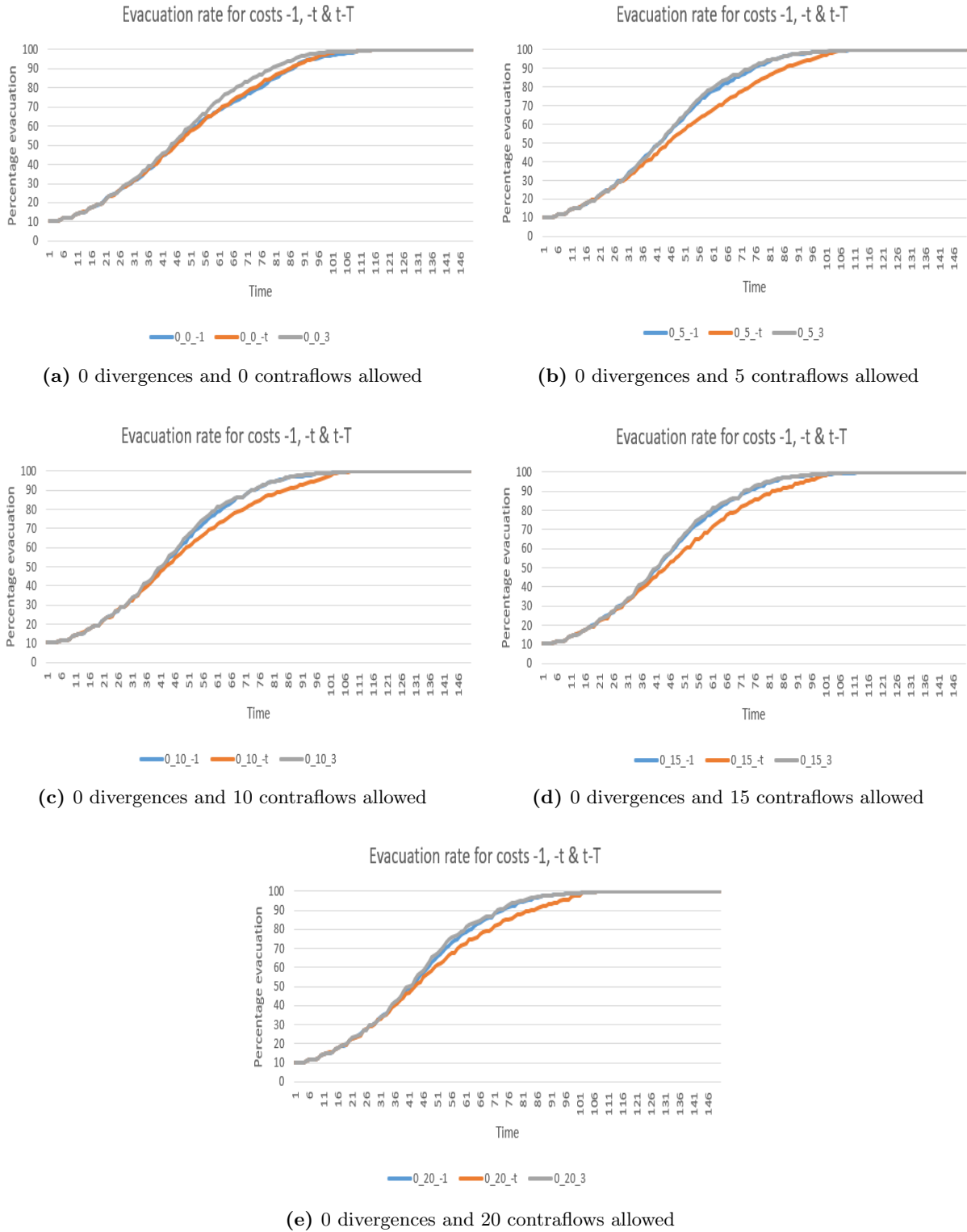
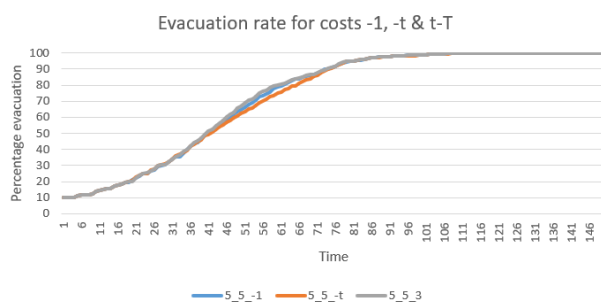
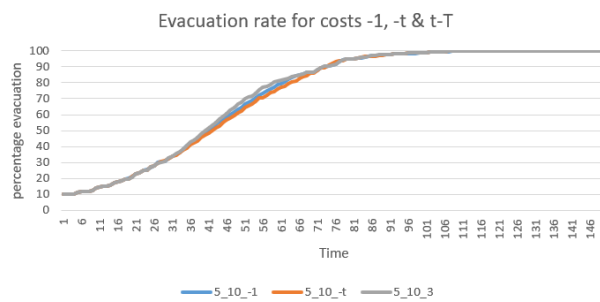


Figure 4.16. Comparison of network evacuation rate using contraflow scheme

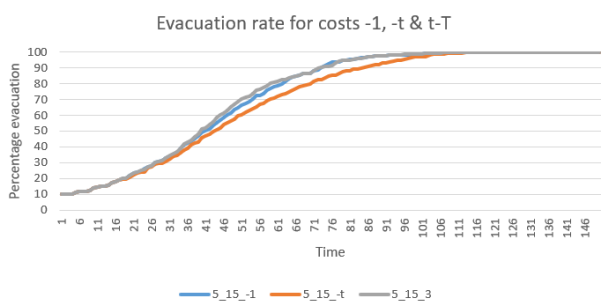
## Network evacuation rates when coupled schemes are allowed



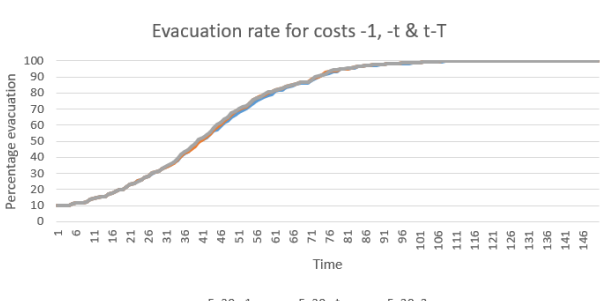
(a) 5 divergences and 5 contraflows allowed



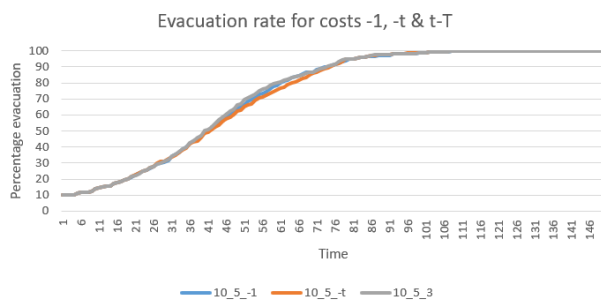
(b) 5 divergences and 10 contraflows allowed



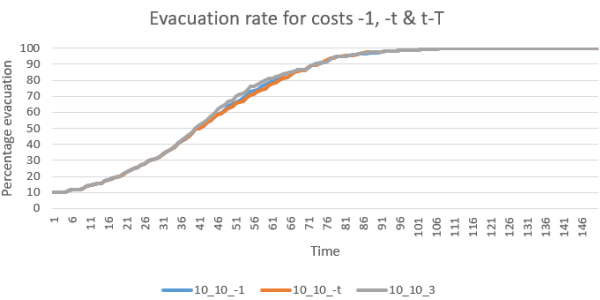
(c) 5 divergences and 15 contraflows allowed



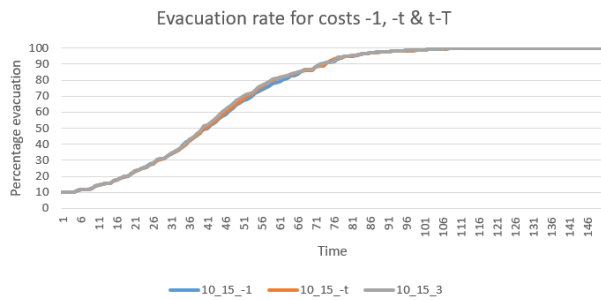
(d) 5 divergences and 20 contraflows allowed



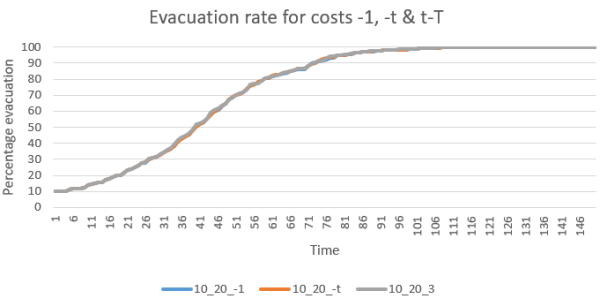
(e) 10 divergences and 5 contraflows allowed



(f) 10 divergences and 10 contraflows allowed

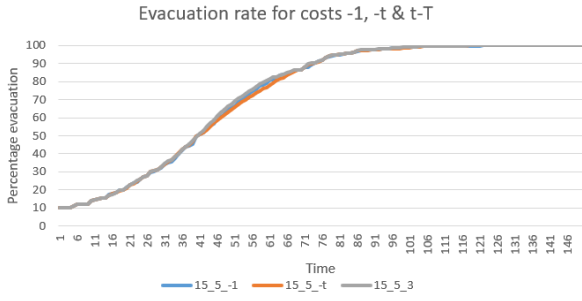


(g) 10 divergences and 15 contraflows allowed

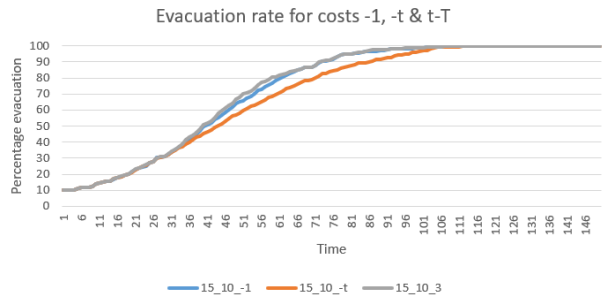


(h) 10 divergences and 20 contraflows allowed

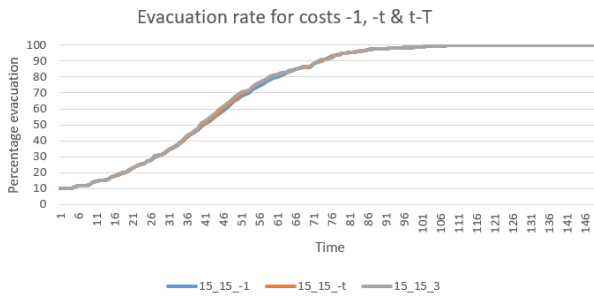
**Figure 4.17.** Comparison of network evacuation rate using coupled schemes (1)



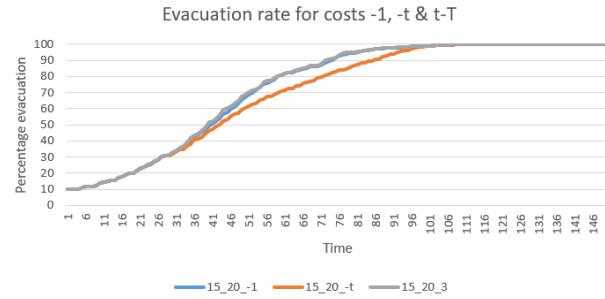
(a) 15 divergences and 5 contraflows allowed



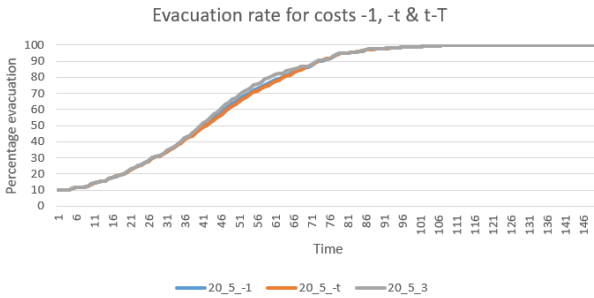
(b) 15 divergences and 10 contraflows allowed



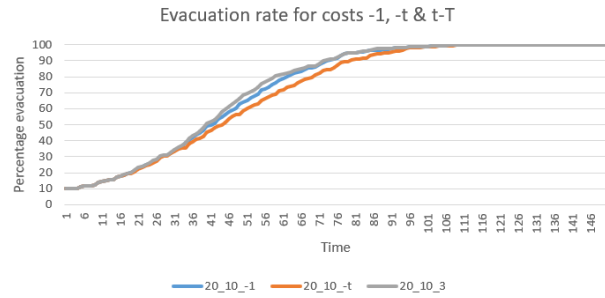
(c) 15 divergences and 15 contraflows allowed



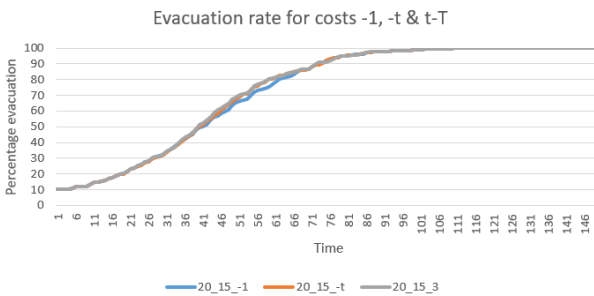
(d) 15 divergences and 20 contraflows allowed



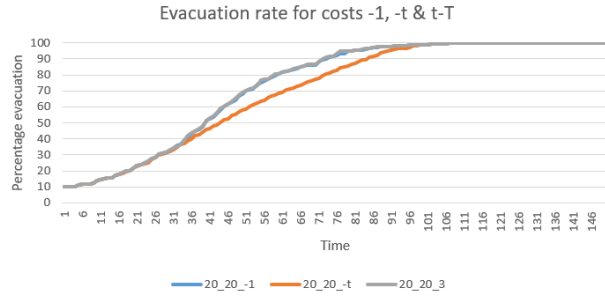
(e) 20 divergences and 5 contraflows allowed



(f) 20 divergences and 10 contraflows allowed



(g) 20 divergences and 15 contraflows allowed



(h) 20 divergences and 20 contraflows allowed

**Figure 4.18.** Comparison of network evacuation rate using coupled schemes (2)

### 4.3.1. Analysis of the Chicago Network Results

The Chicago network is a very complicated network, so we will keep our analysis to computer-generated output instead of providing any visual insights on the network. First we will consider the cases when we have budget only for contraflows see the table 4.19, 4.21, 4.23. From these tables, it is evident that there are some arcs which are being used by all three costs such as arc (424,425), (426,441), (438,535), and (777,778). We can state that these arcs must be significant when we have a budget only for contraflows as they appear in all the combinations. Similarly, there are some arcs which gets important for particular cost such as, arc (434,435) which is important for cost  $-1$ , cost  $t - T$  uses it once, whereas cost  $-t$  does not consider this arc at all. Similarly, arc (777,779) is considered by cost  $-t$  and as budget reaches to maximum  $t - T$  also uses this arc, but when the budget is less, it does not consider this arc for contraflow. Also, if we consider the pattern in which the lanes have been considered for contraflow, then we can say that cost  $-t$  and  $t - T$  produced very consistent patterns unlike cost  $-1$ . Adding contraflow lanes does reduce the network clearance time as we can see in the table 4.31. It is also interesting to note the danger zone clearance time from table 4.32. As the budget for contraflow increases, the danger-zone evacuation time for cost  $-1$  also decreases. However, if we check the danger-zone clearance time for cost  $-t$  we see that it the clearance time increases as budget increases and again decreases as we further increase the budget. It is happening because as increase the contraflow lanes, it evacuates people close to the safer zone first then it starts evacuating the danger zone. Whereas for cost  $t - T$  danger zone clearance time decreases as the budget increases.

Now we shall consider the case when we have a budget only for divergences. According to data in the tables 4.20, 4.22 and 4.24 we certainly can see some patterns followed by these three costs. Nodes 398, 428, 436, 535, 622, and 902 used by all three types of the costs for all the budgets. Again there are some nodes which get used under certain cost policy. For example, nodes 392, 393 and 413 are used only by cost  $t - T$  and nodes 631, 643, 686 and 696 used only by cost  $-t$  and finally node 903 is used only by cost  $-1$ . Addition of divergences in the evacuation trees gives us lower total network clearance time by the table 4.31. Evacuation tree gives the clearance time of  $t = 127$  for all the costs and addition of the divergences gives the lower clearance time of  $t = 118$  for all the cost and at all the budgets. However, if we check the danger-zone clearance

time, for cost  $-1$  and  $-t$  it is inconsistent as the budget is increased. But, in case of cost  $t - T$  it is rather consistent meaning, for evacuation tree the danger-zone clearance time is  $t = 6$ , as budget for divergence increases it drops to  $t = 5$  and remains constant for all divergence budget. When the budget for coupled scheme gets available for cost  $t - T$  the danger-zone clearance time further drops to  $t = 3$ .

So far we have seen the effects of adding budgets for contraflows and divergences in the evacuation tree. Now let's consider the case when we have a budget for contraflows as well as divergences together, the coupled scheme. Here we will go over only some important nodes and arcs used by all the costs. First we will consider the divergence that is nodes selected in coupled schemes for that we refer to tables 4.25, 4.26, 4.27. Nodes 400, 428, 436 seem to be important for all the costs since it keeps appearing in all the coupled combinations more than once. Take a look at node 584, it is not important at the lower budget for costs  $t - T$  and  $-1$ , but the opposite is true for cost  $-t$ . Node 648 is not that important for cost  $-t$  as it appears only twice in all the coupled combinations, for cost  $-1$  it is somewhat important and for cost  $t - T$  it is not used when the budget is lower, but as the budget goes up it starts using that node frequently. It is interesting to note that nodes which were important for all three costs when there was no budget for divergences and when the budget becomes available these nodes lose their importance. Consider the node 902 which was one of the important nodes when there was no budget, now it only gets used when the budget for contraflows is low by the costs  $-1$  and cost  $-t$ , but it is still an important node for cost  $t - T$  for almost all the combination. Contraflows also share some common arcs for all three costs and combinations. Arcs (424,425), (426,441), (431,432) and (777,778) are common important arcs for all the three costs and all the coupled combinations. Arc (769,771) and (777,779) gets available only after the budget for contraflows increases. Cost  $-t$  tends to produce inconsistent contraflow selections whereas cost  $-1$  and  $t - T$  produces consistent selections for contraflows. With the coupled scheme the network clearance time seem to be decreased, but it remains the same for all the coupled combinations, refer the table 4.31. Now check the danger-zone clearance time under the coupled scheme from table 4.32. danger-zone evacuation time for cost  $-1$  and  $-t$  frequently varies hence it is not consistent, but if we check the same by cost  $1 - T$ , we can see that it is pretty consistent. Higher the budget for divergences and contraflows, the danger-zone evacuation time is decreasing in the case of cost  $t - T$ .

## 5. CONCLUDING REMARKS

In this research, we model and analyze a city evacuation process that provides us with a strategic planning tool. The tool we used is mixed integer linear programming, which was solved using a commercial solver (Gurobi) and a network analysis software (NetworkX). Our model was based on the notion of an evacuation tree, which only allows people to use predefined streets to evacuate such that every vehicle has exactly one path to safety. We expanded on that notion by showing that allowing contraflow (arc reversals) and limited divergences (certain intersections that allow their flows to divert) also leads to good network clearance times. We tested our approach on two transportation networks, a smaller one based on the city of Sioux Falls, and a larger one based on the city of Chicago. Using this proposed evacuation method, we have investigated the effect of the presence of divergence as well as contraflows in an evacuation tree.

Using our approach we were able to show that, it is possible to achieve lower network clearance time using more resources and also get more evacuees to safety. It has also been demonstrated that after a certain budget limit (on the number of contraflows or divergences), it will not help to evacuate the network faster. We showed that using a dynamically updated cost of  $t - T$  helps to evacuate people in a consistent manner as compared to the other two costs we put to the test, equal to  $-1$  (static) and  $-t$  (dynamic).

To refine this method we further can incorporate diversion of a specific node using graph theoretic approach. For example, allowing a node to diverge if it has high betweenness centrality as a node with high betweenness centrality would have more control over the network. Finally for contraflows, we can consider lane dependent contraflow. With that, we do not allow the entire street to be reversed, but only a portion of the street leaving room for emergency vehicles to access the danger zones, if necessary.

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# APPENDIX

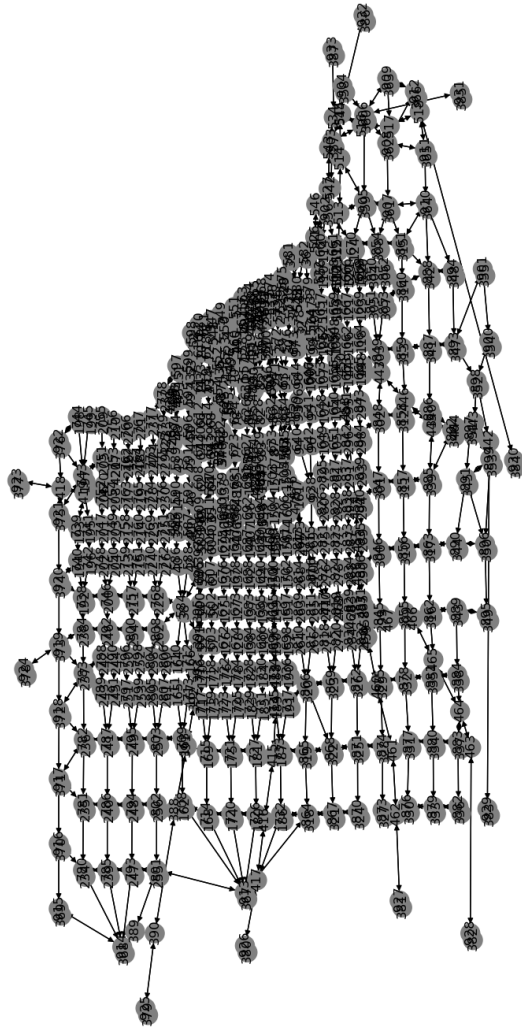


Figure A.1. Chicago original network