Novel formulation and decomposition-based optimization for strategic supply chain management under uncertainty

by

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Abstract

This thesis proposes a novel synergy of the classical scenario and robust approaches used for strategic supply chain optimization under uncertainty. Two novel formulations, namely the naïve robust scenario formulation and the affinely adjustable robust scenario formulation, are developed, which can be reformulated into tractable deterministic optimization problems if the uncertainty is bounded by the infinity-norm. The two formulations are applied to a classical farm planning problem and an energy and bioproduct supply chain problem. The case study results demonstrate that, compared to the scenario formulation, the proposed formulations can achieve the optimal expected economic performance with smaller number of scenarios, and they can correctly indicate the feasibility of a problem. The results also show that the affinely adjustable robust scenario formulation can better address uncertainties than the naïve robust scenario formulation.

Next, a strategic optimization problem for an industrial chemical supply chain from DuPont was studied. The supply chain involves one materials warehouse, five manufacturing plants, five regional product warehouses and five market locations. Each manufacturing plant produces up to 23 grades of final products from 55 grades of primary raw materials. The goal of the strategic optimization is to determine the capacities of the five plants to maximize the total profits of the supply chain system while satisfying uncertain customer demands at the different market locations. A mathematical model is developed to relate the material and product flows in the supply chain, based on which the classical scenario approach and the affinely adjustable robust scenario formulation were developed to address the uncertainty in the demands. The case study results show the advantages of the affinely robust scenario formulation over the scenario formulation.

Using the affinely adjustable robust scenario formulation often results in problems with very large sizes, which cannot be solved by regular optimization solvers efficiently. In order to exploit the decomposable structure of the formulation, Dantzig-Wolfe decomposition is studied in the thesis. Two approaches to implement Dantzig-Wolfe decomposition are developed, and both approaches involve the solution of a sequence of linear programming (LP) and mixed-integer linear programming (MILP) subproblems. The computational study of the industrial chemical supply chain shows that a combination of the two Dantzig-Wolfe approaches can achieve an optimal or a near-optimal solution much more quickly than a state-of-the-art commercial LP/MILP solver, and the computational advantage increases with the increase of number of scenarios involved in the problem.

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Chapter 1

Introduction

1.1 Background

With modern enterprises commonly incorporating multiproduct and multi-site facilities, the need for supply chain analysis is vital to remain competitive. Supply chain management is defined as a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements (Simchi-Levi et al., 1999). A simple example of a supply chain network is given in Figure 1.1 which shows potential layers and channels that can be used.

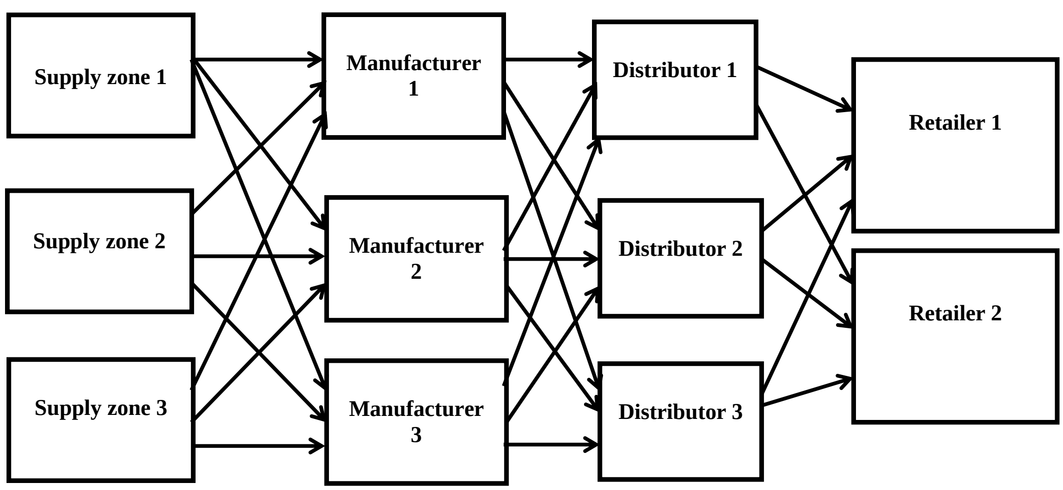

Figure 1.1. Example of a network for a supply chain problem

The design and planning of supply chains typically involve many decisions which can be classified under the appropriate levels of operational, tactical and strategic decision-making. McNair and Vangermeersch (1998) provide descriptions of the three levels. Operational decisions are focused on short-term problems and strive to ensure that product is available for when and where the customer demands it. The tactical perspective can be bounded by a six month to three year time frame and decisions are focused on improving performance by reducing the required resources and by eliminating bottle-necks and nonvalue-added activities. The strategic level defines the processes and activities that will form a competitive advantage for industries. The decisions are related to the total quantity and type of capacity needed, the location of infrastructure, and the attainment and segmentation of infrastructure. Each level of decision-making is vital to the overall success of the supply chain.

Within the process systems engineering community (PSE), supply chain optimization (SCO) has emerged as a major research direction. Oil and gas, chemical, biorefinery, and carbon capture and storage networks are a few examples of the fields of research within the PSE community. There are not only measures based off financial flow or customer responsiveness, but also increased competition and shorter product life-cycles, the need for improved sustainability and environmental impacts, and future regulations and compliance requirements that dictate the need of SCO research (Papageorgiou, 2004). SCO models can be either mathematical programming or simulation-based. Mathematical programming models are for optimizing important decisions involving unknown configurations, such as supply chain network design or distribution planning. Simulation models cover the dynamic operation of a fixed configuration featuring operational uncertainty, in order to gauge performance measures.

1.2 Supply Chain Optimization and Consideration of Uncertainty

The necessity to account for uncertainty in supply chains has been accepted as a significant issue (Sahinidis, 2004). Sources of uncertainty can be related to raw material supplies, transportation and logistics, production and operation uncertainties, product prices, emissions, etc. Models that account for these uncertainties can be large in size and difficult to solve. Approaches to optimization under uncertainty will have objectives to minimize deviations of goals, minimize expectations, or most commonly minimize overall costs. These main approaches can be based off of fuzzy programming (which includes flexible and possabilistic programming) or stochastic programming (which includes recourse models, robust stochastic programming, and probabilistic models). Within recourse models, by considering a finite number of uncertain parameters sampled, the scenario formulation can be used. The general recourse model for a two-stage stochastic problem, the scenario formulation and robust formulation will be described below.

1.2.1 General Recourse Model

A standard formulation for a two-stage stochastic formulation with recourse is given below (Birge and Louveaux, 2011)

Problem (P)

$$\min_{x \in X} \ c^T x + E_{\xi \in \Xi}\left[Q(x,\xi)\right] \tag{1.1}$$

$$s.t. \quad Ax \le b \ , \tag{1.2}$$

where $x \in X \subset \mathbb{R}^{n_x}$ represents the first-stage decision variables, which can be either continuous or integer depending on the problem, and are made before the realizations of uncertainty. Related to strategic supply chain optimization under uncertainty, the first-stage variables are typically design decisions, such as whether or not to develop a unit in the network, the capacity of a plant or warehouse or the mode of transportation. These design variables are decided before the actual realization of the uncertain parameters. $c \in \mathbb{R}^{n_x}$ symbolizes the costs related to the first-stage decisions, such as the cost related to capacities of the manufacturing plants. $A \in \mathbb{R}^{m_1 \times n_x}$, $b \in \mathbb{R}^{m_1}$, and constraint (1.2) includes the limitations of the first-stage decisions, such as the topology relationship for the units in the supply chain network, maximum capacity limits of plants and warehouses, etc. The costs related to the second-stage decisions are $Q(x,\xi) = \min\{q(\xi)^T y : T(\xi)x + Wy \le h(\xi)\}$, in which $\xi \in \Xi$ denotes the uncertain parameters that are realized when making the second-stage decisions, such as raw material and product prices, customer demands, product conversions, etc. $y \in \mathbb{R}^{n_y}$ denotes the second-stage continuous variables and in relation to strategic supply chain optimization, they represent the operational decisions such as material and product transportation plans for the various layers of the supply chain. $q(\xi) \in \mathbb{R}^{n_y}$, $T(\xi) \in \mathbb{R}^{m_2 \times n_x}$, $W \in \mathbb{R}^{m_2 \times n_y}$, $h(\xi) \in \mathbb{R}^{m_2}$ are parameters in relation to the second stage, ξ in the parentheses after the parameters indicates that the they are dependent on the realization of uncertainty. W is a known constant assumed to be independent of the uncertain parameters here which means Problem P has fixed-recourse (Birge and Louveaux, 2011). $E_{\xi \in \Xi}\left[Q(x,\xi)\right]$ in the objective function of Problem P, denotes the expected second-stage cost over different realizations of uncertainty.

1.2.2 Scenario Formulation

Since Problem P is intractable, as it assumes full knowledge of the uncertain parameters, it can be transformed approximately into a tractable optimization problem for practical solution. A common approach to approximate the general problem, is to address a finite number of realizations of uncertainty within the optimization. These realizations are typically called scenarios, and it leads to the formulation outlined below.

Problem (S)

$$\min_{\substack{x\in X,\\ y_1,\dots,y_s}} \; c^T x + \sum_{\omega=1}^{s} p_\omega q_\omega^T y_\omega \tag{1.3}$$

$$s.t. \quad Ax \le b\,, \tag{1.4}$$

$$T_\omega x + W y_\omega \le h_\omega, \quad \omega = 1,\dots,s. \tag{1.5}$$

The parameters dependent on uncertainty are characterized by *s* scenarios (indexed by subscript ω) and the relevant probabilities are denoted by p_ω. The second-stage variables are now explicitly optimized for *s* groups for Problem S. If the number of scenarios is 1, and the uncertain parameters realize their expected value, the resulting formulation becomes the so-called expected value (or deterministic) formulation.

1.2.3 Robust Formulation

Another approach to approximate the general Problem P, is to address the "worst-case scenario" (i.e., the scenario in which the constraint is most likely to be violated), as opposed to addressing a finite number of predetermined scenarios. The resulting problem is called a robust optimization problem (Ben-Tal et al., 2004), and is typically motivated by applications in which feasibility is more important than optimality of the solution. The robust optimization problem is written in the form

Problem (R)

$$\min_{x\in X,\, y} \; c^T x + \overline{q}^T y \tag{1.6}$$

$$s.t. \quad Ax \le b\,, \tag{1.7}$$

$$\max_{\xi\in\Xi}\{t_i^T(\xi)x + w_i^T y - h_i(\xi)\} \le 0\,, \quad i = 1,\dots,m_2\,. \tag{1.8}$$

For convenience, the group of second-stage constraints (1.8) are expressed as m_2 individual inequality constraints. Vectors $t_i, w_i \in \mathbb{R}^{n_y}$ $(i = 1, ..., m_2)$ are obtained from disassembling the matrices *T* and *W*, i.e., $[t_1 \cdots t_{m_2}]^T = T$, $[w_1 \cdots w_{m_2}]^T = W$, $h_i \in \mathbb{R}$ $(i = 1, ..., m_2)$, $[h_1 \cdots h_{m_2}]^T = h$, a bar '-' over parameters or variables indicates the nominal (usually is set to the expected value) of that parameter or variable.

1.2.4 Affinely Adjustable Formulations

In order to reduce the conservativeness of robust optimization, an affinely adjustable robust formulation is developed by Ben-Tal et al. (2004). Here the second-stage decision variables are adjusted for different realizations of uncertainty according to an affine function of the uncertain parameters, $y = U\xi + v$. The second-stage decisions can then be optimized through matrix $U \in \mathbb{R}^{n_y \times n_\xi}$ and vector $v \in \mathbb{R}^{n_y}$. This approximation to the general problem P is given by

Problem (AAR)

$$\min_{x \in X,\, U,\, v} \; c^T x + \bar{q}^T \left(U\bar{\xi} + v \right) \tag{1.9}$$

$$s.t. \quad Ax \le b\,, \tag{1.10}$$

$$\max_{\xi \in \Xi} \{ t_i^T(\xi) x + w_i^T (U\xi + v) - h_i(\xi) \} \le 0, \quad i = 1, ..., m_2. \tag{1.11}$$

Problem (AAR) contains more decision variables than problem R, but by having more flexibility in deciding the second-stage decision variables, increased optimality of solution can be achieved. As shown later in Chapter 3, this formulation can be reformulated into a tractable, single-level, convex optimization problem with the assumption on the uncertainty set Ξ (Ben-Tal and Nemirovski, 1999).

1.2.5 Advantages/Disadvantages of the Scenario Formulation and Robust Formulation

The scenario formulation is widely used in SCO research and manages to retain flexibility in of determining the second-stage decisions according to different realizations of uncertainty. Often it will achieve a good estimation of the expected performance and return reasonable solutions. The drawback to using the scenario formulation is that it cannot guarantee feasibility of the solution, since not all uncertainty realizations for each parameter can be included. By increasing the number of uncertainty realizations, the chances of obtaining a feasible solution would rise. However, this could cause the size of the formulation to drastically increase, making it computationally intractable. It is difficult to identify how many scenarios are required for a reliable solution.

The robust formulation was developed to guarantee feasibility against a given set of uncertainty realizations (if a feasible solution exists). The shortcoming of the formulation is that it usually cannot accurately predict expected performances, since only the worst-case uncertainty realization is addressed in the problem. As a result, the robust formulation should only be used in applications where the feasibility of the problem is of much greater importance than its optimality. Since the objective function of the affinely adjustable robust formulation still involves the nominal costs instead of expected second-stage costs, this formulation may still lead to poor solutions as well.

1.3 Decomposition Algorithms

With the large problem sizes and complexities involved in optimization under uncertainty, there is a focus on developing efficient solution procedures. Incorporation of decomposition algorithms is a promising solution to this problem. Two-stage stochastic linear models have a problem structure that can allow it to be solved by a variety of different methods (Birge, 1985). Benders decomposition (Benders, 1962), is a popular approach for exploiting the structure of a model that contains complicating variables. Geoffrion (1972) proposes generalized Benders decomposition which can be applied to a wider array of problems. In stochastic programming literature, benders decomposition is referred to as the L-shaped method. Multi-cut strategies can be applied when discrete distributions of the uncertain parameters are considered. For continuous uncertain parameter distributions, sample-based decomposition and approximation schemes, as well as gradient-based algorithms can be used. Dantzig and Wolfe (1960) developed a decomposition algorithm for linear programs that involves an iterative process of solving subproblems and generating columns. For integer programming, algorithms contain ideas from lift-and-project and reformulation-linearization techniques for Benders-like decomposition approaches (Sahinidis, 2004).

1.4 Research Objectives

The objective of the thesis is to first develop a novel formulation in order to solve two-stage strategic supply chain optimization problems under uncertainty. The formulation is to be a synergy of the classical scenario formulation and robust formulation used to solve two-stage stochastic supply chain problems. The solutions obtained by the new formulation should provide better estimations of performance (which is a weakness in the robust formulation and sometimes the scenario formulation) and feasibility (which the scenario formulation cannot guarantee). This hybrid formulation will then be compared to the classical approaches for case studies of varying complexities. The cases solved will be linear programming (LP) and mixed-integer linear programming (MILP) problems. It should be noted that independent uniform distributions (further explained in Chapter 3) are assumed for uncertain parameters in all case studies in the thesis, as the focus of the thesis is not related to figuring out the structure of uncertainty.

The mathematical model for an industrial chemical supply chain problem will be developed from data provided by DuPont. The supply chain network consists of one materials warehouse, five manufacturing plants, five regional product warehouses and five market locations. Each manufacturing plant produces up to 23 grades of final products from 55 grades of primary raw materials. The goal is to determine the capacities of the five plants to maximize the total profits of the supply chain system while satisfying uncertain customer demands at the different market locations. The developed hybrid formulation will be used to address the uncertainties and solve this strategic optimization problem.

In addition to developing the novel formulation, the Dantzig-Wolfe decomposition algorithm will be studied and applied to the new hybrid formulation for an industrial case study. The objective is to show the benefits of the decomposition algorithm for MILP problems compared to using the state-of-the-art CPLEX solver (IBM CPLEX, 2014).

To summarize, the major contributions of the thesis are:

- Develop a hybrid formulation based on the synergy of the scenario and robust formulations, in order to improve the solutions achieved for strategic supply chain optimization problems under uncertainty.
- Formulate the mathematical model for a large-scale industrial chemical supply chain problem, and apply the developed hybrid formulation to address uncertainties.
- Develop Dantzig-Wolfe decomposition procedures to improve computing times when using the novel hybrid formulation.

1.5 Thesis Structure

The thesis is arranged as follows: Chapter 2 presents a review of the literature. In chapter 3, the developed novel formulations will be discussed, and their benefits will be displayed in two case studies. In Chapter 4, the mathematical equations for an industrial sized case study are developed, and the benefits of the hybrid formulation are again shown. In chapter 5, the Dantzig-Wolfe decomposition algorithm will be discussed along with its results compared to those of the CPLEX solver for the industrial case study. Lastly, in Chapter 6 the conclusions and future works are presented.

Chapter 2

Literature Review

2.1 Supply Chain Design and Planning

In this section, literature related to supply chain design and planning is reviewed. It has been divided such that deterministic models are examined in section 2.1.1 and studies that consider uncertainty are examined in section 2.1.2.

2.1.1 Deterministic Models

Through coordinated planning in supply chain design, costs can be minimized, market-size can increase and other objectives may be achieved such as a focus on sustainable supply chains and waste management. Numerous studies related to the operational, tactical and strategic levels of decision-making have been made at static and multi-period levels.

Within early research, an optimization-based decision support system was developed for the company Nabisco to manage problems involving facility selection, equipment location and utilization, manufacturing and distributing the products (Brown et al., 1987). The resulting MILP model allows for savings on transportation and production costs. Camm et al. (1997) performed studies to aid Procter and Gamble's North American supply chain. Annual savings of $200 million were resulted by combining integer programming, network optimization and geographical information systems. Sabri and Beamon (2000) develop an integrated multi-objective supply chain model at the strategic and operational levels. A performance measurement system is used that incorporates cost, customer service levels, and flexibility. A MILP model is developed by

Tsiakis and Papageorgiou (2008) to determine the optimal configuration of a production and distribution network that is subject to capacity and financial constraints. The operational constraints consider quality, production, and supply restrictions and the financial constraints include production costs, transportation costs, and duties for the material flow. Out-sourcing production is considered when demand cannot be satisfied. Sustainable supply chains under the emission trading scheme are studied by Chaabane et al. (2012). A MILP model is used that considers life cycle assessment in addition to the material balance constraints of the supply chain. Marvin et al. (2013) proposes a MILP model for determining the locations and capacities of economical biomass facilities. The feasibility of meeting the governmental biofuel mandates in 2015 is examined. Akgul et al. (2012) examine the economical and environmental performance of a static biofuel supply chain system. Potential greenhouse gas savings and the impact of carbon tax are analyzed. Within pharmaceutical research, Susarla and Karimi (2012) examine an industrial scale planning problem for a multinational pharmaceutical enterprise. Their model incorporates procurement, production, distribution, the effects of international tax differentials, inventory holding costs, material shelf-lives, and waste treatment and disposal. The oil and gas industry features work performed by Stebel et al. (2012) in which they aim to reduce the gap between scheduling activities in pipeline networks for supply of petroleum products. Their MILP model is applied to a case study involving a Brazilian Oil company. An optimization-based supply chain network is proposed by Elia et al. (2013) for nationwide, statewide, and regional analyses of natural gas to liquids systems across the United States. The large-scale MILP model is solved by minimizing the costs of fuel production.

For multi-period models at the operational and tactical levels of SCO problems, the time horizon is usually a few days to several months and faces decisions such as equipment changeovers and inventory management. Wilkinson et al. (1996) propose an aggregate model as an approximate formulation to allocate production over a week-long horizon. Equipment changeovers and intermediate storage is considered in the model. McDonald and Karimi (1997) consider a deterministic MILP formulation for scheduling of a semi-continuous process. Safety stock and short fall penalties are included in the inventory costs and multiple facilities are considered for geographically distributed customers. Another MILP formulation is proposed by Timpe and Kallrath (2000) for a multi-site production network. Equipment items can operate in different modes and the model is flexible in the fact that timescales for production and distribution can be altered. Jin-Kwang et al. (2000) focus on operational decisions, specifically a supply chain for sales, intermittent deliveries, production shortfalls, delivery delays, inventory profiles and job changeovers, for their multi-period optimization model. More recently, Lim et al. (2012) design a rice mill complex based on fluctuating thermal and electrical demands, diverse energy supply options, varying product demands, resource availability and product degradation. Inventory has the potential to degrade at each time period and 12 months are considered. Multi-period models at the strategic decision making, covers matters such as capacity, production and distribution, regularly involves a time horizon of a few years. A multi-period model was proposed by Liu and Sahinidis (1996) which focuses on a two-stage stochastic model for process planning under uncertainty. Up to four time periods are considered as well as 5^{24} scenarios. Bok et al. (1998) extend the previous model to consider the investment for long-term capacity expansions. They use a mixed-integer nonlinear programming (MINLP) model to trade-off between the expected net present value, its expected square of deviation and the anticipated square of excess capacity. Iyer and Grossmann (1998) determine the optimal selection and expansion of processes using a MILP model. Up to eight scenarios are considered and 10 time periods are used. Another MILP model is described by Papageorgiou et al. (2001) for a pharmaceutical industry problem. Product development, capacity expansion and investment strategy are included in the optimization with a time period of several years. Jackson and Grossmann (2003) outline a multi-site, multiproduct plant in which nonlinear process models are used. A success story of a multi-period model for a chemical industry is revealed by Kallrath (2002). A horizon of up to 15 years was used and the paper features a sensitivity analysis outlining varying product demands. Within the oil industry, MINLP multi-period formulations are developed for large-scale operations by Neiro and Pinto (2004) and Schulz et al. (2005). Sundaramoorthy et al. (2012) examine a multi-period MILP model in which they determine the capacities for integrated continuous facilities for potential products facing clinical trial uncertainty.

For further studies, Papageorgiou (2009) outlines the advances and opportunities of supply chain optimization for the process industries. Works related to both static and multi-period models are examined. Nikolopoulou and Ierapetritou (2012) present a review on the optimal design of sustainable chemical processes and supply chains. They focus on sustainable supply chains related to energy efficiency and waste management, environmentally sustainable supply chains and sustainable water management. Awudu and Zhang (2012) address studies related to biofuel supply chain management in relation to uncertainties and sustainability concepts.

2.1.2 Consideration of Uncertainty

Through consideration of uncertainty (e.g., raw material supplies, product demands and prices, or conversion factors) the accuracy of the solution can increase with the trade-off that the model is usually large in size and difficult to solve. Two-stage stochastic formulations with recourse and robust optimization are common approaches to solve these types of programming problems. Gupta and Maranas (2000) develop a two-stage stochastic model for a multi-site midterm supply chain planning problem. Demand is included as the uncertain parameter and production decisions are considered as the first-stage variables. Guillen et al. (2006) also consider demand uncertainty for the design of a chemical supply chain. Their model maximizes profits through a multi-stage stochastic MILP model. Salema et al. (2007) use a scenario-based approach for the design of a reverse logistics network that incorporates capacity limits, multi-product management, and uncertainties on product demands and returns. Within the oil and gas industry, Ribas et al. (2010) use a two-stage stochastic model with a finite number of scenarios to address the development of a strategic planning model for a Brazil oil supply chain. The sources of uncertainty include crude oil production, demand for refined products and market prices. Terrazas-Moreno et al. (2012) utilize a MILP model for a two-stage stochastic problem with endogenous uncertainty for the design of a large-scale chemical process with integrated sites and random process failures. Gebreslassie et al. (2012) propose a bicriterion, multi-period stochastic MILP model for the optimal design of a hydrocarbon biorefinery supply chain. The objective is minimization of expected annualized costs and of the financial risks with supply and demand uncertainties. Han and Lee (2012) study a multi-period stochastic programming model for planning carbon capture and storage network which includes CO_2 utilization and disposal. A two-stage approach is used to account for uncertainties in product prices, operating costs, and CO_2 emissions. Chu and You (2013) examine a two-stage stochastic programming problem for integration of scheduling and dynamic optimization of a production process under uncertainty.

Robust optimization, which seeks to attain a feasible solution to an uncertain system, has not been widely studied within the PSE community, especially at the strategic level (Grossmann and Guillén-Gosálbez, 2010). At the operational level, Janak et al. (2007) consider robust optimization for a short-term scheduling problem for multipurpose batch processes. Robust optimization has been examined more so in control applications (Goulart et al., 2008) and operations research literature (Kuhn et al., 2011).

2.2 Decomposition Strategies

This section is divided such that decomposition strategies from literature are reviewed in section 2.2.1. Next, in section 2.2.2, the Dantzig-Wolfe decomposition algorithm is outlined for a typical linear programming problem.

2.2.1 Decomposition Strategies Review

Due to the nature of the large problem sizes when accounting for uncertainty, it becomes necessary to consider the implementation of decomposition strategies, so that the problems can be solved within reasonable computing times. In literature, there are many works in the PSE community related to Lagrangian decomposition and Benders decomposition techniques.

Jackson and Grossman (2003) consider a Lagrangian based method on a multi-site, multiproduct network in which nonlinear process models are applied. They compare temporal and spatial decomposition methods, in which the effectiveness of the temporal method is detailed. Neiro and Pinto (2004) use a similar Lagrangian strategy for their large-scale petroleum supply chain model and soon after, the work is extended by Chen and Pinto (2008). You and Grossmann (2008, 2010) extend the strategy to nonlinear models with stochastic inventories. Iyer and Grossmann (1998) suggest a bilevel decomposition technique to reduce the computational times of their mixed-integer linear programming (MILP) formulation involving the selection and expansion of processes. Jin-Kwang and Grossmann (2000) use the bilevel approach on a problem involving operational decisions such as sales, deliveries, production shortfalls, etc. You et al. (2011) later performed a comparison of Lagrangian and bilevel decomposition schemes involving a multi-period MILP model for a multisite system. Their results show that the bilevel decomposition gives faster computational times and a smaller optimality gap for their specific problem. Liu and

Sahinidis (1996) test both a Branch-and-Bound and a Benders decomposition algorithm on their two-stage stochastic multi-period model. Terrazas-Moreno et al. (2012) and Gebreslassie et al. (2012), whose papers were discussed in the previous section, use Benders decomposition and a multi-cut L-shaped method, respectively, to overcome the complexities of their MILP models. A slight deviation from the Benders algorithm is used by Egging (2013), in which a large-scale, multi-period stochastic model is applied to a global natural gas problem. Vaskan et al. (2013) propose a decomposition method for MILP problems based off a bilevel decomposition strategy. A lower bounding master problem is solved followed by an upper bounding slave problem where the pixels of the master problem are disaggregated. This technique is applied to a case study on sewage sludge amendment in Catalonia.

An alternative approach, not as commonly implemented within supply chain optimization problems or for MILP models, is the Dantzig-Wolfe decomposition algorithm which consists of a restricted master problem that contains the active columns from the solutions of the subproblems. Pimental et al. (2010) utilize the Dantzig-Wolfe algorithm, along with a branch-and-price algorithm, where a MILP model is applied to multi-item capacitated lot sizing problem with setup times. The subproblems of the Dantzig-Wolfe algorithm are defined by items and then by periods. The algorithm has also been featured in such research areas as security-constrained unit commitment problems (Fu et al., 2005) and moral-hazard programs (Prescott, 2004).

2.2.2 Dantzig-Wolfe Decomposition Algorithm

This section introduces the Dantzig-Wolfe algorithm based on the discussion in Bertsimas and Tsitsiklis (1997).

2.2.2.1 Overview of the Restricted Master Problem and Subproblems

A typical linear programming is considered for the procedure to implement the Dantzig-Wolfe decomposition algorithm. The problem is shown by

$$\min c_1^T x_1 + c_2^T x_2 \tag{2.1}$$

$$s.t. \quad D_1 x_1 + D_2 x_2 = b_0, \tag{2.2}$$

$$x_1 \in P_1, \tag{2.3}$$

$$x_2 \in P_2, \tag{2.4}$$

where,

$$P_i = \{x_i \geq 0 : F_i x_i = b_i\}, \qquad i = 1, 2. \tag{2.5}$$

Here, x_1 and x_2 are decision variables that are vectors of dimensions n_1 and n_2, respectively. The vectors $b_0, b_1,$ and b_2 have dimensions $m_0, m_1,$ and m_2, respectively. x_1 satisfies m_1 constraints, x_2 satisfies m_2 constraints, and together x_1 and x_2 satisfy m_0 coupling constraints. Next, it is assumed without loss of generality that sets P_1 and P_2 are bounded, as two-stage strategic supply chain optimization problems can always be formulated with bounded feasible sets. For $i = 1, 2,$ x_i^j, where $j \in J_i$ is defined as the extreme points of set P_i. The resolution theorem implies that any element of a bounded polyhedron can be expressed as a combination of its extreme points (Lasdon, 1970). Therefore any element x_i of P_i can be expressed as follows

$$x_i = \sum_{j \in J_i} \lambda_i^j x_i^j, \tag{2.6}$$

where, $\sum_{j \in J_i} \lambda_i^j = 1, i = 1, 2$ and $\lambda_i^j \geq 0, \forall i, j$.

The original problem can now be written as

$$\min \sum_{j\in J_1} \lambda_1^j c_1^T x_1^j + \sum_{j\in J_2} \lambda_2^j c_2^T x_2^j \tag{2.7}$$

$$s.t. \ \sum_{j\in J_1} \lambda_1^j D_1 x_1^j + \sum_{j\in J_2} \lambda_2^j D_2 x_2^j = b_0, \tag{2.8}$$

$$\sum_{j\in J_1} \lambda_1^j = 1, \tag{2.9}$$

$$\sum_{j\in J_2} \lambda_2^j = 1, \tag{2.10}$$

$$\lambda_i^j \geq 0, \quad \forall i, j. \tag{2.11}$$

This formulation is known as the master problem. It is equivalent to the original formulation and is a linear programming problem with decision variables λ_i^j. Another way to represent the constraints (2.8), (2.9), and (2.10) is shown below which shows more clearly the structure of each column

$$\sum_{j\in J_1} \lambda_1^j \begin{bmatrix} D_1 x_1^j \\ 1 \\ 0 \end{bmatrix} + \sum_{j\in J_2} \lambda_2^j \begin{bmatrix} D_2 x_2^j \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b_0 \\ 1 \\ 1 \end{bmatrix}. \tag{2.12}$$

The number of extreme points used in the master problem can be extremely large. Instead, a restricted master problem will be used, in which the number of variables are set to equal zero at the start. The solution to the restricted master problem is defined as obj_{RMP}. Promising variables, based off of the solution to subproblems, will enter the restricted master problem when the algorithm is executed.

Suppose there is a basic feasible solution to the master problem which has a basis matrix *B*. It is assumed that B^{-1} is available, along with the dual vector denoted by $p^T = c_B^T B^{-1}$. Since there are $m_0 + 2$ equality constraints, the vector p has a dimension of $m_0 + 2$. The first m_0

components are the dual variables associated with constraint (2.8) denoted by π. The last two components, denoted by μ_1 and μ_2, are the dual variables associated with the "convexity" constraints (2.9) and (2.10). Altogether, $p = \begin{pmatrix} \pi & \mu_1 & \mu_2 \end{pmatrix}$. To decide if the current basic feasible solution is optimal, the reduced costs of the different variables are checked whether or not they are negative. The reduced cost of variable λ_1^j is given by

$$c_1^T x_1^j - \begin{bmatrix} \pi^T & \mu_1 & \mu_2 \end{bmatrix} \begin{bmatrix} D_1 x_1^j \\ 1 \\ 0 \end{bmatrix} = \left(c_1^T - \pi^T D_1 \right) x_1^j - \mu_1. \tag{2.13}$$

Instead of checking the reduced costs of each variable λ_1^j, the following linear programming problem is formed

$$\min \left(c_1^T - \pi^T D_1 \right) x_1 \tag{2.14}$$

$$s.t. \;\; x_1 \in P_1. \tag{2.15}$$

This optimization problem is known as the pricing problem. From its solution there are two scenarios to consider:

(1) If the optimal cost of the pricing problem is finite and smaller than μ_1, then there is an extreme point x_1^j that satisfies $\left(c_1^T - \pi^T D_1 \right) x_1^j < \mu_1$. The reduced cost of the variable λ_1^j is negative and $\begin{bmatrix} D_1 x_1^j \\ 1 \\ 0 \end{bmatrix}$ is the column generated to enter the basis of the restricted master problem.

(2) If the optimal cost of the subproblem is finite and no smaller than μ_1, then $\left(c_1^T - \pi^T D_1 \right) x_1^j \geq \mu_1$ for all extreme points x_1^j. The reduced cost of each variable λ_1^j is nonnegative.

The same methodology is applied to the second subproblem

$$\min\left(c_2^T - \pi^T D_2\right) x_2 \tag{2.16}$$

$$s.t. \;\; x_2 \in P_2, \tag{2.17}$$

and once solved, either the optimal cost is greater than or equal to μ_2 or there is a variable λ_2^j with a negative reduced cost that enters the basis for the restricted master problem. The restricted master problem and pricing problems described above refer to the optimization problems solved for the phase 2 component of the Dantzig-Wolfe algorithm. In order to begin this phase, a basic feasible solution is required.

2.2.2.2 Phase 1 Problem

In order to begin the algorithm, a basic feasible solution to the master problem is required. A phase 1 feasibility problem is performed for each of the polyhedra P_1 and P_2, to find their respective extreme points, x_1^1 and x_2^1. By letting y be a vector of auxiliary variables with a dimension of m_0, the auxiliary master problem is formed

$$\min \sum_{t=1}^{m_0} y_t \tag{2.18}$$

$$s.t. \;\; \sum_{i=1,2}\left(\sum_{j\in J_i} \lambda_i^j D_i x_i^j\right) + y = b_0, \tag{2.19}$$

$$\sum_{j\in J_1} \lambda_1^j = 1, \tag{2.20}$$

$$\sum_{j\in J_2} \lambda_2^j = 1, \tag{2.21}$$

$$\lambda_i^j \geq 0, y_t \geq 0 \quad \forall i, j, t. \tag{2.22}$$

Now, a basic feasible solution is obtained by letting $\lambda_1^1 = \lambda_1^2 = 1, \ \ \lambda_i^j = 0$ for $j \neq 1,$ and $y = b_0 - D_1 x_1^1 - D_2 x_2^1.$ The decomposition algorithm can be used to solve the auxiliary master problem. If the optimal cost reaches zero, then the optimal solution to the auxiliary master problem is a basic feasible solution to the master problem.

2.2.2.3 Termination Criteria

The phase 1 problem involves solving the auxiliary master problem and the subproblems until the following condition is met

$$\sum_{t=1}^{m_0} y_t \leq \varepsilon, \tag{2.23}$$

where ε is the required tolerance. If the solution to the auxiliary master problem is positive, then the master problem is infeasible and the algorithm will terminate. If the tolerance criterion is satisfied, then the basic feasible solution to the master problem is used for the phase 2 problem.

Here for phase 2, the restricted master problem and subproblems are solved until the following is achieved

$$UBD - LBD \leq \varepsilon. \tag{2.24}$$

UBD stands for the upper bound and is updated during each iteration as the solution to the objective function of the restricted master problem

$$UBD = obj_{RMP}. \tag{2.25}$$

LBD stands for the lower bound and can be updated for every iteration by the expression

$$LBD = obj_{RMP} + \sum_{n=1,2} Obj_{PP_n} - \mu_1 - \mu_2, \tag{2.26}$$

where Obj_{PP_n} is the solution to the respective objective function of the pricing problems, μ_1 and μ_2 are the multipliers obtained from the "convexity" constraints of the restricted master problem. If the termination criterion is satisfied, then the following optimal solution to the problem would be returned $\left(\sum_{j \in J_1} \lambda_1^j x_1^j , \sum_{j \in J_2} \lambda_2^j x_2^j \right)$.

In this chapter, literature related to supply chain design and planning was reviewed for deterministic studies and for those in which uncertainty is considered. Scenario formulations have been utilized for strategic supply chain optimization problems under uncertainty, but there is little work performed in this area related to robust formulation. This reinforces the need to developing a hybrid robust scenario formulation, as seen in Chapter 3, to improve on the quality of solutions obtained for strategic supply chain optimization problems. Next, the focus was on decomposition techniques and the Dantzig-Wolfe decomposition algorithm was specifically outlined. This methodology has the potential to obtain solutions faster than the state-of-the-art CPLEX solver for two-stage optimization problems and is studied further in Chapter 5.

Chapter 3

Novel Robust Scenario Formulations

In section 3.1, the hybrid robust scenario formulations are derived based on the need to improve on optimality and feasibility of the generated solution. Next in section 3.2, the classical and new formulations are applied to two case study problems in order to show the benefit of the robust scenario formulations.

3.1 Robust Scenario Formulations

The scenario formulation and robust formulation that are used as approximations to the general Problem (P), have inherent disadvantages. The scenario formulation cannot guarantee optimality or feasibility of the generated solution, whereas the robust formulation can guarantee feasibility but at the expense that it will generally produce an overly conservative solution. Next, one of the main contributions of the thesis is presented, in which hybrid formulations are developed in the next sections by combining the classical formulations to achieve better solutions.

3.1.1 Naïve Robust Scenario Formulation

The naïve robust scenario formulation is developed from the synergy of formulation (S) and formulation (R). The infinite number of uncertainty realizations are approximated by considering a finite number of scenarios, and each scenario now contains a set of uncertainty realizations as opposed to a single realization. The formulation is given by

Problem (NRS)

$$\min_{\substack{x\in X,\\ y_1,\dots,y_s}} \; c^T x + \sum_{\omega=1}^{s} p_\omega \bar{q}_\omega^T y_\omega \tag{3.1}$$

$$s.t. \quad Ax \le b\,, \tag{3.2}$$

$$\max_{\xi_\omega \in \Xi_\omega} \{ t_{i,\omega}^T(\xi_\omega)x + w_i^T y_\omega - h_{i,\omega}(\xi_\omega)\} \le 0\,,\;\; i=1,\dots,m_2\,,\;\; \omega=1,\dots,s\,, \tag{3.3}$$

$$\left(t_{j,\omega}^{(eq)}(\xi_\omega)\right)^T x + \left(w_{j,\omega}^{(eq)}\right)^T y_\omega - h_{j,\omega}^{(eq)}(\xi_\omega) = 0\,,\; \xi_\omega \in \Xi_\omega\,,\; j=1,\dots,m_2^{(eq)}\,, \omega=1,\dots,s\,. \tag{3.4}$$

For formulation (NRS), the uncertainty region is expressed by Ξ and is covered by s uncertainty subregions Ξ_s for the s scenarios, that is, $\bigcup_{\omega=1}^{s} \Xi_\omega \supset \Xi$. This shows that a scenario is now associated with a set of uncertainty realizations as opposed to a single uncertainty realization as what is used for Formulation S. The deterministic inequalities shown by eq. (1.5) for formulation S become the "robust inequalities" in eq. (3.3) for Problem (NRS), and the second-stage cost coefficient q_ω now becomes the nominal value of the coefficient $\bar{q}_\omega$. The equality constraints are now explicitly addressed in eq. (3.4) since they have to be satisfied for all realizations of uncertainty, as opposed to a single worst-case realization.

Problem (NRS) can be transformed into a tractable problem if the assumption that uncertainty is bounded by the infinity-norm is made. The inequality constraints of eq. (3.3) are rewritten to separate the deterministic and uncertain elements. This is shown by

$$\max_{\xi_{i,\omega} \in \Xi_{i,\omega}} \left\{ t_{d,i,\omega}^T x_{d,i} + \xi_{i,\omega}^T x_{u,i} + w_i^T y_\omega \right\} \le 0\,, \qquad i=1,\dots,m_2\,, \quad \omega=1,\dots,s\,. \tag{3.5}$$

Here the uncertainty subregion is expressed by $\Xi_{i,\omega} = \left\{ \xi_{i,\omega} : \left\| M_{i,\omega}\left(\xi_{i,\omega} - \bar{\xi}_{i,\omega}\right)\right\|_\infty \le \delta_{i,\omega} \right\} \subset \mathbb{R}^{l_{\xi,i}}$, where $l_{\xi,i} \le n_\xi$ (as a constraint may not involve all the uncertain parameters), $M_{i,\omega}$ is an invertible weighting matrix, and $\delta_{i,\omega} > 0$. A visualization of the infinity-norm assumption is shown in Figure 3.1 for an example with two uncertain parameters and nine scenarios.

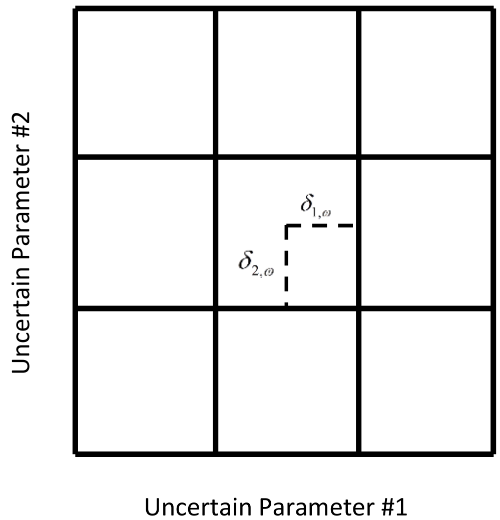

Figure 3.1. Example of uncertainty bounded by the infinity-norm

From this example based on the box distribution, invertible matrix $M_{i,\omega}$ would be a diagonal matrix. Typically this assumption is used for cases in which the uncertain parameters are independent of each other. If the parameters are dependent on each other, then the uncertainty region and uncertainty subregions would be rotated and $M_{i,\omega}$ would be a non-diagonal matrix. The uncertain parameters in the uncertainty subregion Ξ_{ω} are now described in different subregions $\Xi_{1,\omega}$, $\Xi_{2,\omega}$, …, $\Xi_{m_2,\omega}$. $\left[t_{d,i,\omega}^T \;\; \xi_{i,\omega}^T\right] = \left[t_{i,\omega}^T \;\; h_{i,\omega}\right]$, in which $t_{d,i,\omega} \in \mathbb{R}^{n_x+1-l_{\xi,i}}$ contains the deterministic elements in $\left[t_{i,\omega}^T \;\; h_{i,\omega}\right]$ and $\xi_{i,\omega}$ contains the uncertain elements. $\left[x_{d,i}^T \;\; x_{u,i}^T\right] = \left[x^T \;\; -1\right]$, in which $x_{d,i} \in \mathbb{R}^{n_x+1-l_{\xi,i}}$ contains the elements in $\left[x^T \;\; -1\right]$ that are associated with $t_{d,i,\omega}$ and $x_{u,i} \in \mathbb{R}^{l_{\xi,i}}$ contains the elements in $\left[x^T \;\; -1\right]$ that are associated with $\xi_{i,\omega}$. By separating the deterministic parts from the optimization operation, the inequality is now shown by

$$t_{d,i,\omega}^T x_{d,i} + w_i^T y_\omega + \max_{\xi_{i,\omega} \in \Xi_{i,\omega}} \left\{ \xi_{i,\omega}^T x_{u,i} \right\} \le 0, \quad i = 1,...,m_2, \quad \omega = 1,...,s. \tag{3.6}$$

Since constraint (3.6) involves an optimization problem, Problem (NRS) is a bilevel optimization problem that is difficult to solve. The following proposition can be used to reduce the overall formulation to a single-level problem.

Proposition 1: *From the optimization problem*

$$\max_{\phi} \quad \left(\phi - \bar{\phi}\right)^T x \tag{3.7}$$

$$s.t. \quad \left\| M\left(\phi - \bar{\phi}\right)\right\|_\infty \le \delta, \tag{3.8}$$

where *M* is invertible and $\delta \ge 0$, the optimal objective value is $\delta \left\| (M^{-1})^T x \right\|_1$.

Proof of Proposition 1 can be found in Appendix A. Based off of this solution, inequality (3.6) can be written as

$$t_{d,i,\omega}^T x_{d,i} + w_i^T y_\omega + \delta_{i,\omega} \left\| \left(M_{i,\omega}^{-1}\right)^T x_{u,i} \right\|_1 \le 0, \quad i = 1,...,m_2, \quad \omega = 1,...,s. \tag{3.9}$$

This inequality can be reformulated into no more than $2^{l_{\xi,i}}$ linear inequalities. This reformulation is shown in section 3.1.3. Usually, $l_{\xi,i} \ll n_\xi$, which will not result in a much larger problem.

From the equality constraints in eq. (3.4), they can also be expressed by separating the deterministic and uncertain elements

$$\left(t_{d,j,\omega}^{(eq)}\right)^T x_{d,j}^{(eq)} + \left(\xi_{j,\omega}^{(eq)}\right)^T x_{u,j}^{(eq)} + \left(w_j^{(eq)}\right)^T y_\omega = 0, \quad \xi_{j,\omega}^{(eq)} \in \Xi_{j,\omega}^{(eq)}, \quad j = 1,...,m_2^{(eq)}, \quad \omega = 1,...,s. \tag{3.10}$$

$\Xi_{j,\omega}^{(eq)}$ is the uncertainty subregion for the uncertain parameters involved in the equation indexed by j and ω. No assumptions are necessary on this subregion for its reformulation. $\left[\left(t_{d,j,\omega}^{(eq)}\right)^T \ \left(\xi_{j,\omega}^{(eq)}\right)^T\right] = \left[\left(t_{j,\omega}^{(eq)}\right)^T \ h_{j,\omega}^{(eq)}\right]$, where $t_{d,j,\omega}^{(eq)}$ contains the deterministic elements in $\left[\left(t_{j,\omega}^{(eq)}\right)^T \ h_{j,\omega}^{(eq)}\right]$ and $\xi_{j,\omega}^{(eq)}$ contains the uncertain elements. $\left[\left(x_{d,j}^{(eq)}\right)^T \ \left(x_{u,j}^{(eq)}\right)^T\right] = \left[x^T \ -1\right]$, where $x_{d,j}^{(eq)}$ contains the elements in $\left[x^T \ -1\right]$ associated with $t_{d,j,\omega}^{(eq)}$ and $x_{u,j}^{(eq)}$ contains the elements in $\left[x^T \ -1\right]$ associated with $\xi_{j,\omega}^{(eq)}$.

Assuming that the uncertain components in $\xi_{j,\omega}^{(eq)}$ are not linearly dependent on each other, then

$$x_{u,j}^{(eq)} = 0, \qquad j = 1,\ldots,m_2^{(eq)}, \tag{3.11}$$

has to hold for eq. (3.10), otherwise it would mean that the "uncertain part" $\left(\xi_{j,\omega}^{(eq)}\right)^T x_{u,j}^{(eq)}$ has different values for different realizations of $\xi_{j,\omega}^{(eq)}$. Similarly, the "deterministic part" must hold as well

$$\left(t_{d,j,\omega}^{(eq)}\right)^T x_{d,j}^{(eq)} + \left(w_j^{(eq)}\right)^T y_\omega = 0, \qquad j = 1,\ldots,m_2^{(eq)},\ \omega = 1,\ldots,s. \tag{3.12}$$

Therefore eq. (3.10) can be transformed into eq. (3.11) and (3.12). Problem (NRS) can now be expressed as the following problem, if uncertainty is bounded by the infinity-norm

Problem (NRS_IN)

$$\min_{\substack{x\in X,\\ y_1,\ldots,y_s}} \ c^T x + \sum_{\omega=1}^{s} p_\omega \bar{q}_\omega^T y_\omega \tag{3.13}$$

$$s.t. \quad Ax \le b, \tag{3.14}$$

$$t_{d,i,\omega}^T x_{d,i} + w_i^T y_\omega + \delta_{i,\omega} \left\| \left(M_{i,\omega}^{-1} \right)^T x_{u,i} \right\|_1 \le 0\,, \quad i = 1,...,m_2\,,\ \ \omega = 1,...,s, \tag{3.15}$$

$$x_{u,j}^{(eq)} = 0\,, \quad j = 1,...,m_2^{(eq)}, \tag{3.16}$$

$$\left(t_{d,j,\omega}^{(eq)} \right)^T x_{d,j}^{(eq)} + \left(w_j^{(eq)} \right)^T y_\omega = 0\,, \quad j = 1,...,m_2^{(eq)}\,,\ \omega = 1,...,s\,. \tag{3.17}$$

3.1.2 Affinely Adjustable Robust Scenario Formulation

To improve on formulation (NRS), which can still be overly conservative, an affinely adjustable robust scenario formulation is proposed. This formulation is a hybrid of formulation (S) and formulation (AAR). Here, the second-stage decision variables are adjusted affinely with respect to the uncertainty realizations in a scenario. These adjustments are expressed by $y_\omega = \left(U_\omega \xi_\omega + v_\omega \right)$ for scenario ω, where U_ω and v_ω are the constant coefficients for the affine relationship for scenario ω. This will allow for less restriction when determining the second-stage decisions, and the formulation is written in the form

Problem (AARS)

$$\min_{x \in X, U_1, v_1, \cdots, U_s, v_s} \quad c^T x + \sum_{\omega=1}^{s} p_\omega E_{\xi_\omega} \left[\bar{q}_\omega^T \left(U_\omega \xi_\omega + v_\omega \right) \right] \tag{3.18}$$

$$s.t. \quad Ax \le b\,, \tag{3.19}$$

$$\max_{\xi_\omega \in \Xi_\omega} \{ t_{i,\omega}^T(\xi_\omega) x + w_i^T \left(U_\omega \xi_\omega + v_\omega \right) - h_{i,\omega}(\xi_\omega) \} \le 0\,, \quad i = 1,...,m_2\,,\ \omega = 1,...,s\,, \tag{3.20}$$

$$\left(t_{j,\omega}^{(eq)}(\xi_\omega) \right)^T x + \left(w_{j,\omega}^{(eq)} \right)^T \left(U_\omega \xi_\omega + v_\omega \right) - h_{j,\omega}^{(eq)}(\xi_\omega) = 0\,, \quad \xi_\omega \in \Xi_\omega\,,\ j = 1,...,m_2^{(eq)}\,, \omega = 1,...,s\,. \tag{3.21}$$

The inequality constraint (3.23) is now over all the uncertain parameters, as dependence of the second-stage decisions are on all uncertain parameters. Due to these affine functions for different uncertainty regions, problem (AARS) essentially uses piece-wise affine function over the entire uncertainty region for the adjustments of second-stage decisions. This will lead to a less conservative approach than problem (AAR), which contains a single affine function over an uncertainty region. If only one scenario is used for problem (AARS), then it will degrade into problem (AAR).

Similar to problem (NRS), this hybrid formulation can also be transformed into a tractable problem if the uncertainty is bounded by the infinity-norm. Expected second-stage costs in the objective function can be rewritten as

$$E_{\xi_\omega}\left[\bar{q}_\omega^T\left(U_\omega\xi_\omega+v_\omega\right)\right]=\bar{q}_\omega^T\left(U_\omega E_{\xi_\omega}\left(\xi_\omega\right)+v_\omega\right)=\bar{q}_\omega^T\left(U_\omega\bar{\xi}_\omega+v_\omega\right), \tag{3.22}$$

where $\bar{\xi}_\omega$ is the expected value of ξ_ω for scenario ω. Next, the inequality constraints (3.20), can be written as

$$\max_{\xi_\omega\in\Xi_\omega}\left\{t_{d,i,\omega}^T x_{d,i}+\xi_{i,\omega}^T x_{u,i}+w_i^T\left(U_\omega\xi_\omega+v_\omega\right)\right\}\le 0, \quad i=1,\dots,m_2, \quad \omega=1,\dots,s. \tag{3.23}$$

From the assumption that uncertainty is bounded by the infinity-norm, that is, $\Xi_\omega=\{\xi:\left\|M_\omega(\xi_\omega-\bar{\xi}_\omega)\right\|_\infty\le\delta_\omega\}$, where M_ω is invertible and $\delta_\omega>0$. By allowing $\xi_{i,\omega}=P_{i,\omega}\xi_\omega$ for convenience, the inequality becomes

$$t_{d,i,\omega}^T x_{d,i}+w_i^T v_\omega+\max_{\xi_\omega\in\Xi_\omega}\left\{\xi_\omega^T\left(P_{i,\omega}^T x_{u,i}+U_\omega^T w_i\right)\right\}\le 0, \quad i=1,\dots,m_2, \quad \omega=1,\dots,s. \tag{3.24}$$

Again, by following Proposition 1, it can now be written by

$$t_{d,i,\omega}^T x_{d,i}+w_i^T v_\omega+\delta_\omega\left\|(M_\omega^{-1})^T\left(P_{i,\omega}^T x_{u,i}+U_\omega^T w_i\right)\right\|_1\le 0, \quad i=1,\dots,m_2, \quad \omega=1,\dots,s. \tag{3.25}$$

Similarly, the transformation for the equality constraints (3.21) can be made by first rewriting the constraints as

$$\left(t_{d,j,\omega}^{(eq)}\right)^T x_{d,j}^{(eq)}+\left(\xi_{j,\omega}^{(eq)}\right)^T x_{u,j}^{(eq)}+\left(w_j^{(eq)}\right)^T\left(U_\omega\xi_\omega+v_\omega\right)=0,$$

$$\xi_{j,\omega}^{(eq)}\in\Xi_{j,\omega}^{(eq)}, \ j=1,\dots,m_2^{(eq)}, \ \omega=1,\dots,s. \tag{3.26}$$

By letting $\xi_{j,\omega} = P^{(eq)}_{j,\omega}\xi_\omega$, it now becomes

$$\left(t^{(eq)}_{d,j,\omega}\right)^T x^{(eq)}_{d,j} + \left[\left(x^{(eq)}_{u,j}\right)^T P^{(eq)}_{j,\omega} + \left(w^{(eq)}_j\right)^T U_\omega\right]\xi_\omega + \left(w^{(eq)}_j\right)^T v_\omega = 0\,,$$

$$\xi^{(eq)}_{j,\omega} \in \Xi^{(eq)}_{j,\omega}\,,\; j = 1,\ldots,m^{(eq)}_2\,,\; \omega = 1,\ldots,s\,. \quad (3.27)$$

From here, similar to problem (NRS), the deterministic and uncertain parts must hold to 0 and can be written by the following two equations

$$\left(x^{(eq)}_{u,j}\right)^T P^{(eq)}_{j,\omega} + \left(w^{(eq)}_j\right)^T U_\omega = 0\,,\quad j = 1,\ldots,m^{(eq)}_2\,,\; \omega = 1,\ldots,s\,, \quad (3.28)$$

$$\left(t^{(eq)}_{d,j,\omega}\right)^T x^{(eq)}_{d,j} + \left(w^{(eq)}_j\right)^T v_\omega = 0\,,\quad j = 1,\ldots,m^{(eq)}_2\,,\; \omega = 1,\ldots,s\,. \quad (3.29)$$

The result of the transformation can be seen below as Problem (AARS) can now be described by

Problem (AARS_IN)

$$\min_{x\in X, U_1, v_1, \cdots, U_s, v_s} \; c^T x + \sum_{\omega=1}^{s} p_\omega \bar{q}^T_\omega \left(U_\omega \bar{\xi}_\omega + v_\omega\right) \quad (3.30)$$

$$s.t. \quad Ax \le b\,, \quad (3.31)$$

$$t^T_{d,i,\omega} x_{d,i} + w^T_i v_\omega + \delta_\omega \left\| (M^{-1}_\omega)^T \left(P^T_{i,\omega} x_{u,i} + U^T_\omega w_i\right)\right\|_1 \le 0\,,\; i = 1,\ldots,m_2\,,\quad \omega = 1,\ldots,s\,, \quad (3.32)$$

$$\left(x^{(eq)}_{u,j}\right)^T P^{(eq)}_{j,\omega} + \left(w^{(eq)}_j\right)^T U_\omega = 0\,,\quad j = 1,\ldots,m^{(eq)}_2\,,\; \omega = 1,\ldots,s\,, \quad (3.33)$$

$$\left(t^{(eq)}_{d,j,\omega}\right)^T x^{(eq)}_{d,j} + \left(w^{(eq)}_j\right)^T v_\omega = 0\,,\quad j = 1,\ldots,m^{(eq)}_2\,,\; \omega = 1,\ldots,s\,. \quad (3.34)$$

Compared to formulation (RS_IN), problem (AARS_IN) will involve more decision variables and constraints, so it will be more difficult to solve, but it should improve on the optimality of the generated solutions.

3.1.3 Reformulation of the 1-Norm function in Inequality Constraints

Constraint (3.15) for formulation (NRS_IN) and constraint (3.32) for formulation (AARS_IN) contain the 1-norm function which is nonlinear. These constraints can be reformulated into linear constraints, so that the overall problem is a LP or MILP problem as opposed to a nonlinear optimization problem. To show the reformulation used for the inequality constraints with a 1-norm, the following example is used

$$\|z\|_1 \le b, \tag{3.35}$$

where $z = (z_1, ..., z_n) \in R^n$ and b is any non-zero value. From the definition of 1-norm

$$\|z\|_1 = \sum_{i=1}^{n} |z_i| = \max_{j=1,...,2^n-1} \left\{ \sum_{i=1}^{n} (-1)^{\mathrm{mod}\left(\lfloor j/2^{i-1} \rfloor, 2\right)} z_i \right\}, \tag{3.36}$$

where $\mathrm{mod}(x, y)$ finds the remainder from the division of *x* by *y*. $\lfloor \cdot \rfloor$ is a function returns the largest integer that is smaller than its argument. Inequality (3.38) now becomes

$$\max_{j=0,...,2^n-1} \left\{ \sum_{i=1}^{n} (-1)^{\mathrm{mod}\left(\lfloor j/2^{i-1} \rfloor, 2\right)} z_i \right\} \le b, \tag{3.37}$$

which can then be transformed into the following 2^n linear inequalities,

$$\sum_{i=1}^{n} (-1)^{\mathrm{mod}\left(\lfloor j/2^{i-1} \rfloor, 2\right)} z_i \le b, \quad j = 0, ..., 2^n - 1. \tag{3.38}$$

As an example, if $n = 3$, constraint (3.38) would be transformed into 8 linear inequalities in the following form

$$(-1)^{\mathrm{mod}(j,2)} z_1 + (-1)^{\mathrm{mod}\left(\lfloor j/2 \rfloor, 2\right)} z_2 + (-1)^{\mathrm{mod}\left(\lfloor j/4 \rfloor, 2\right)} z_3 \le b, \quad j = 0, ..., 7, \tag{3.39}$$

which would be

$$z_1 + z_2 + z_3 \le b, \quad j = 0, \tag{3.40}$$

$$-z_1 + z_2 + z_3 \le b, \quad j = 1, \tag{3.41}$$

$$z_1 - z_2 + z_3 \le b, \quad j = 2, \tag{3.42}$$

$$-z_1 - z_2 + z_3 \le b, \quad j = 3, \tag{3.43}$$

$$z_1 + z_2 - z_3 \le b, \quad j = 4, \tag{3.44}$$

$$-z_1 + z_2 - z_3 \le b, \quad j = 5, \tag{3.45}$$

$$z_1 - z_2 - z_3 \le b, \quad j = 6, \tag{3.46}$$

$$-z_1 - z_2 - z_3 \le b, \quad j = 7. \tag{3.47}$$

It should be noted that 2^n linear equalities are not always required for the 1-norm transformation. For example if all the elements in z are known to be non-negative, then it can be reformulated into simply one linear inequality.

3.2 Case Studies

The purpose of the case studies section is to evaluate the solutions of the novel robust scenario formulations in comparison with the solutions of the scenario formulation and expected value formulation (which is a special scenario formulation with one scenario that involves the expected values of the uncertain parameters).

All uncertain parameters are assumed to be independently and uniformly distributed, so only the ranges of the uncertain parameters are given when explaining the subsequent case study problems. As this article is not focused on scenario generation, a simple approach is used to construct the scenarios for the different formulations. The range of each uncertain parameter is divided into n_s subintervals, and the uncertainty region is divided into $n_s^{n_\xi}$ subregions (where

n_ξ is the total number of uncertain parameters), which lead to $n_s^{n_\xi}$ scenarios. For the affinely adjustable robust scenario formulation, each scenario addresses the relevant uncertainty subregion; for the S formulation, each scenario addresses the mean values of the uncertain parameters over the relevant uncertainty subregion. These uncertainty subregions can be readily represented using the infinity norm, so the affinely adjustable robust scenario formulation can be transformed to the computationally tractable problem (AARS_IN) as described. Figure 3.2 displays how the scenario formulation uses a finite number of realizations each made by a recourse action, to address uncertainty. Figure 3.3 shows how the novel robust scenario formulations are represented by groups of uncertainty realizations. The formulation (NRS_IN) incorporates actions for each uncertainty group, and the formulation (AARS_IN) incorporates an affine policy for each group.

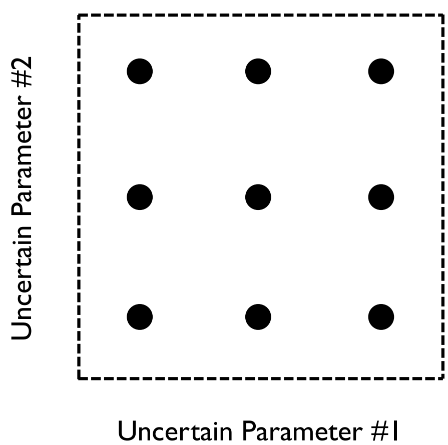

Figure 3.2. Scenario generation for the scenario formulation

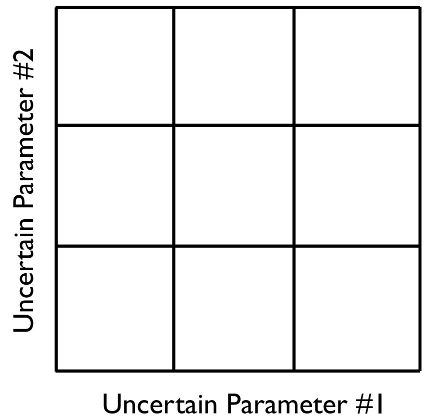

Figure 3.3. Scenario generation for the robust scenario formulations

The two case study problems were modeled using GAMS 22.9.3 (Rosenthal, 2008), and they were solved on a machine with 3.40 GHz CPU and Linux operating system using CPLEX 12.4.

3.2.1 Case Study 1 – Farm Planning Problem

Problem 1 is a classical farm planning problem from the stochastic programming literature (Birge and Louveaux, 2011). In this problem, a farmer needs to plan the allocation of his land area for raising three crops: wheat, corn and sugar beets. The goal of the planning is to achieve the best overall profit while reserving a certain amount of wheat and corn for feeding cattle. If the harvested wheat or corn is not enough for feeding cattle, it can be purchased on the market at a relatively high price. Sugar beets are the most profitable crop among the three, but the selling price drops after a certain amount (i.e., the quota on the sugar beet production) of sugar beets has been sold. Although it is not a typical SCO problem, the farm planning problem has similar features to the strategic supply chain planning problem of interest.

The deterministic formulation for the farm planning problem is described below. The list of indices and sets, parameters and variables that are used in the formulation can be found below in Table 3.1.

Table 3.1 Indices and sets, parameters, and variables for the farm planning problem

Indices and Sets
$m \in \Omega$ - wheat, corn, sugar beets
$m \in \Omega_1$ - wheat, corn
$m \in \Omega_2$ - sugar beets
Parameters
c_m^{pl} - Planting cost for crop *m*, $/acre
c_m^{pur} - Purchasing cost of crop *m*, $/t
c_m^{sell} - Selling cost of crop *m*, $/t
$c_m^{sell,h}$ - Selling cost of sugar beets below the quota, $/t
$c_m^{sell,l}$ - Selling cost of sugar beets above the quota, $/t
F_m - Amount of crop *m* reserved for feeding cattle, t
L - Total land area, acre
Q_m - Quota on sugar beet production, t
Y_m - Yield of crop *m*, t/acre
Variables
w_m - Amount of crop *m* sold on the market, t
w_m^h - Amount of crop *m* sold below the quota, t
w_m^l - Amount of crop *m* sold above the quota, t
x_m - Area allocated for crop *m*, acre
y_m - Amount of crop *m* purchased on the market, t

Objective function in which costs are minimized

$$\min \quad \sum_{m\in\Omega} c_m^{pl}\cdot x_m + \sum_{m\in\Omega_1}\left(c_m^{pur}\cdot y_m - c_m^{sell}\cdot w_m\right) - \sum_{m\in\Omega_2}\left(c_m^{sell,h}\cdot w_m^h + c_m^{sell,l}\cdot w_m^l\right) \tag{3.48}$$

The constraints to the model are described by equations (3.49)-(3.55). The total amount of crops planted is bounded by the total available area

$$\sum_{m\in\Omega} x_m \le L. \tag{3.49}$$

Material balance ensuring that the amount of corn and wheat is sufficient to feed the cattle requirements

$$Y_m\cdot x_m + y_m - w_m \ge F_m\,, \quad m\in\Omega_1. \tag{3.50}$$

Material balance for the amount of sugar beets planted and sold

$$w_m^h + w_m^l \le Y_m\cdot x_m\,, \quad m\in\Omega_2. \tag{3.51}$$

The amount of sugar beets sold at the favourable price is bounded by a quota

$$w_m^h \le Q_m\,, \quad m\in\Omega_2. \tag{3.52}$$

The first and second-stage variables are to be non-negative

$$x_m \ge 0\,, \quad m\in\Omega, \tag{3.53}$$

$$y_m, w_m \ge 0\,, \quad m\in\Omega_1, \tag{3.54}$$

$$w_m^h, w_m^l \ge 0\,, \quad m\in\Omega_2. \tag{3.55}$$

Formulations (S), (NRS_IN), and (AARS_IN) can be found in Appendix B. The first-stage decision variables for this problem are the areas planned for each of the crops. The second-stage operational variables are the amounts of crops sold or purchased on the market. The objective function aims to minimize the costs associated with planting and purchasing the materials.

3.2.1.1 Results and Discussion – Uncertain Case A

In this case, the amount of wheat and corn that is required for feeding cattle is uncertain; specifically, $F_{\text{wheat}} = 300 \pm 300$ t, $F_{\text{corn}} = 340 \pm 320$ t. In addition, the selling price of the sugar beets below the quota is changed to 27 $/t. Figure 3.4 depicts the uncertainty regions and subregions used for formulations (NRS_IN) and (AARS_IN) with 9 scenarios.

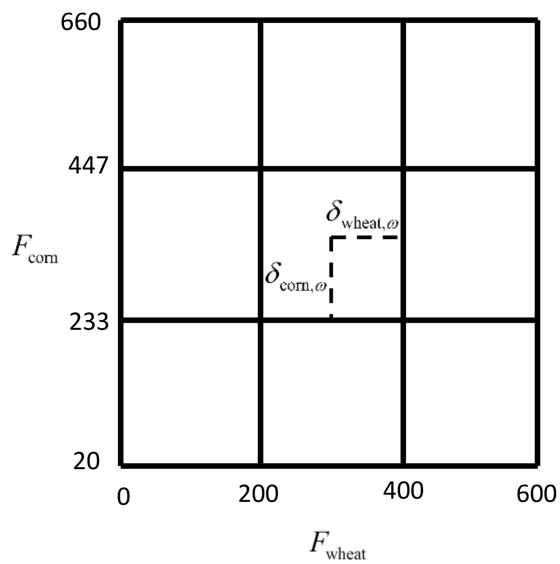

Figure 3.4 Uncertainty region for uncertain case A

Here, the uncertainty region can be defined by $\Xi = \left\{ \xi : \left\| M \left(\xi - \bar{\xi} \right) \right\|_{\infty} \le \delta \right\}$, where the uncertain parameters are denoted by $\xi = \begin{pmatrix} F_{\text{wheat}} \\ F_{\text{corn}} \end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix} 300 \\ 340 \end{pmatrix}$, $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\delta = \begin{pmatrix} 300 \\ 320 \end{pmatrix}$.

The uncertainty subregions are defined as $\Xi_{i,\omega} = \left\{ \xi_{i,\omega} : \left\| M_{i,\omega} \left(\xi_{i,\omega} - \bar{\xi}_{i,\omega} \right) \right\|_{\infty} \le \delta_{i,\omega} \right\}$, where $\xi_{i,\omega} = \begin{pmatrix} F_{\text{wheat},\omega} \\ F_{\text{corn},\ \omega} \end{pmatrix}$, $\bar{\xi}_{i,\omega} = \begin{pmatrix} \bar{F}_{\text{wheat},\omega} \\ \bar{F}_{\text{corn},\ \omega} \end{pmatrix}$, $M_{i,\omega} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \forall \omega$, and $\delta_{i,\omega} = \begin{pmatrix} \delta_{wheat} \\ \delta_{corn} \end{pmatrix}, \forall \omega$. As an example based on Figure 3.4, if 9 scenarios are used then $\xi_{i,\omega} = \begin{pmatrix} F_{\text{wheat},\omega} \\ F_{\text{corn},\ \omega} \end{pmatrix}$, the nominal values can be calculated by the midpoints of each subregion (e.g. $\bar{\xi}_{i,'1'} = \begin{pmatrix} 100 \\ 126.5 \end{pmatrix}$ if $\omega = 1$ refers to the bottom left subregion), $M_{i,\omega} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \omega = 1,...,9$, and $\delta_{i,\omega} = \begin{pmatrix} 100 \\ 106.67 \end{pmatrix}, \omega = 1,..,9$.

Table 3.2 summarizes the results of expected value formulation (EV), scenario formulation (S), naïve robust scenario formulation (NRS_IN) (with uncertainty bounded by the infinity-norm), affinely adjustable robust scenario formulation (AARS_IN) (with uncertainty bounded by the infinity-norm). The results include formulation sizes, solution times, optimal decisions obtained (i.e., the crop area allocation results), predicted expected profit, and the achieved expected profit. The predicted expected profits are predicted by the formulation if the first-stage decisions are implemented. The achieved expected profits are the profits that can actually be achieved by implementation of the first-stage variables. To estimate the achieved expected profits, the expected second-stage cost is approximated over a large number of uncertainty realizations. In this article, 99 realizations of each uncertain parameter were sampled for the estimation of the achieved expected profits for each case study.

Table 3.2 Solution results of the farm planning problem for uncertain case A

	Formulation [a]			
	EV	S	NRS_IN	AARS_IN
Number of Scenarios	1	9	9	9
Number of Variables	12	60	60	366
Number of Constraints	6	38	38	506
Solution Time (s)	0.02	0.02	0.02	0.06
Crop Area Allocation Result (acre)				
Wheat	120	200	240	240
Corn	113	113	149	149
Sugar Beets	267	187	111	111
Predicted Expected Profit [b] ($)	30,600	25,933	- 9,400	25,733
Achieved Expected Profit [c] ($)	20,700	24,833	25,733	25,733

Note: [a] EV: Expected value formulation; S: Scenario formulation; NRS_IN: Naïve robust scenario formulation; AARS_IN: Affinely adjustable robust scenario formulation. [b] The predicted expected profit is the expected profit predicted by the formulation at its solution. [c] The achieved expected profit refers to the expected profit that can be achieved with the obtained area allocation, which is estimated by using 99^2=9801 sampled scenarios.

It can be found that formulation (EV) obtains the lowest achieved expected profit, even though it predicts a much higher value. This formulation allocates most of the available area for sugar beets (which are most profitable) leaving wheat and corn areas that are just enough for the expected cattle feeding needs. Therefore, in the uncertainty realizations with higher cattle feeding needs, the farmer might have to purchase wheat and corn from the market at high prices, which can significantly reduce the overall expected profit. Formulation S considering 9 scenarios achieves a better expected profit, because it allocates more wheat area to hedge against higher cattle feeding needs. But it still overestimates the expected profit it can achieve (because of its incapability of considering all uncertainty realizations). The two robust scenario formulations (NRS_IN) and (AARS_IN) achieve the highest expected profit by allocating sufficient wheat and corn areas. In addition, formulation (AARS_IN) gives a perfect prediction of the expected profit, while formulation (NRS_IN) gives a poor prediction due to its inherent conservativeness.

Although formulation (AARS_IN) provides the best performance, it leads to the largest problem size and longest required solution time with the same number of scenarios considered for each of the formulations. Therefore, it is beneficial to show whether formulation (S) or (NRS_IN) might achieve equivalently good performance with increased number of scenarios. The expected profits predicted and achieved are shown in Figures 3.5-3.7 for formulations (S), (NRS_IN), and (AARS_IN), respectively, using increased number of scenarios.

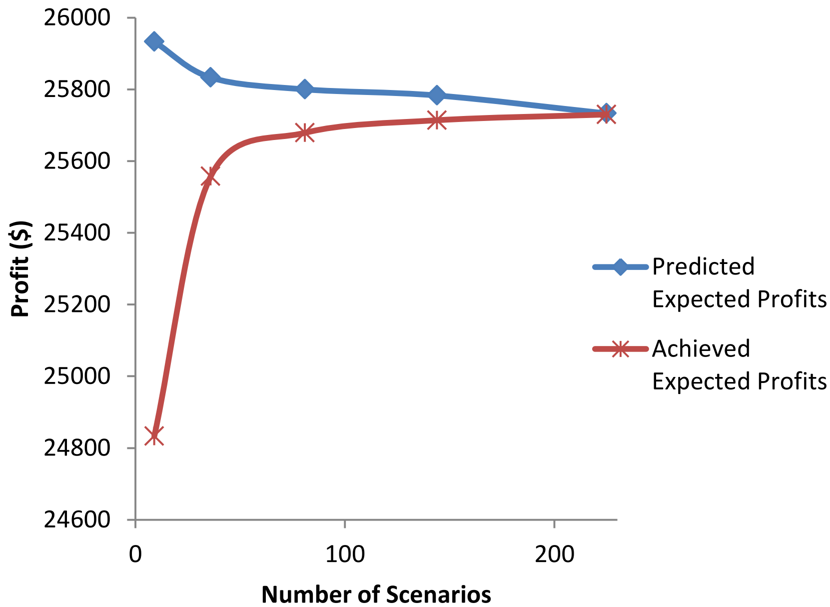

Figure 3.5. Predicted and achieved expected profits with formulation (S)

In Figure 3.5, it can be seen that, as the number of scenarios increase, the expected profit predicted by formulation (S) approaches the achieved expected profit. These profits converge at $25,733 with 225 scenarios, and this profit is exactly the same as the predicted and achieved profits of the (AARS_IN) formulation with 9 scenarios. It should be noted that formulation (S) with 225 scenarios requires 1356 variables and 902 constraints, whereas formulation (AARS_IN) with 9 scenarios only involves 366 variables and 506 constraints. Thus, formulation (S) can achieve the same performance as formulation (AARS_IN) at the expense of solving a larger problem.

Figure 3.6 displays that formulation (NRS_IN) achieves a consistently good expected performance, and its prediction of the expected profits improves as the number of scenarios increases. But due to its conservative nature, the profits still do not converge with 1000 scenarios.

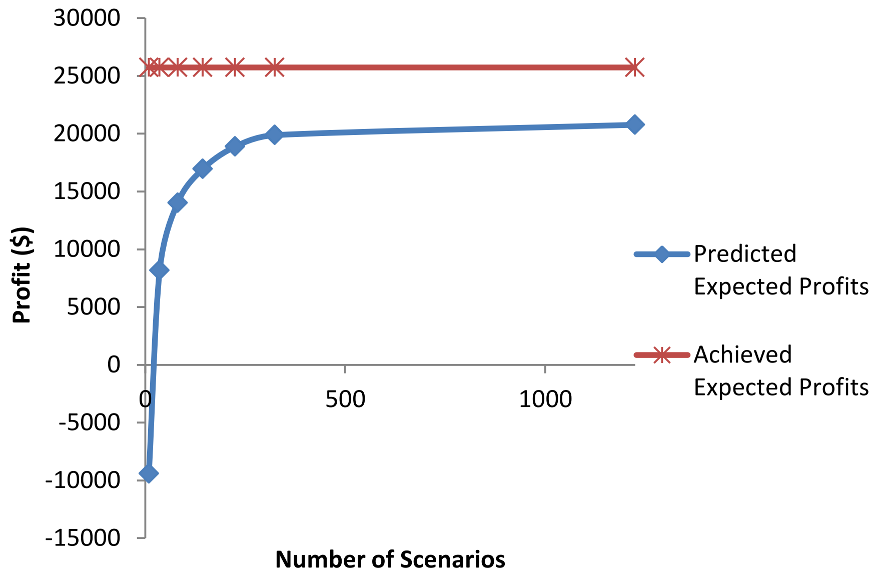

Figure 3.6. Predicted and achieved expected profits with formulation (NRS_IN)

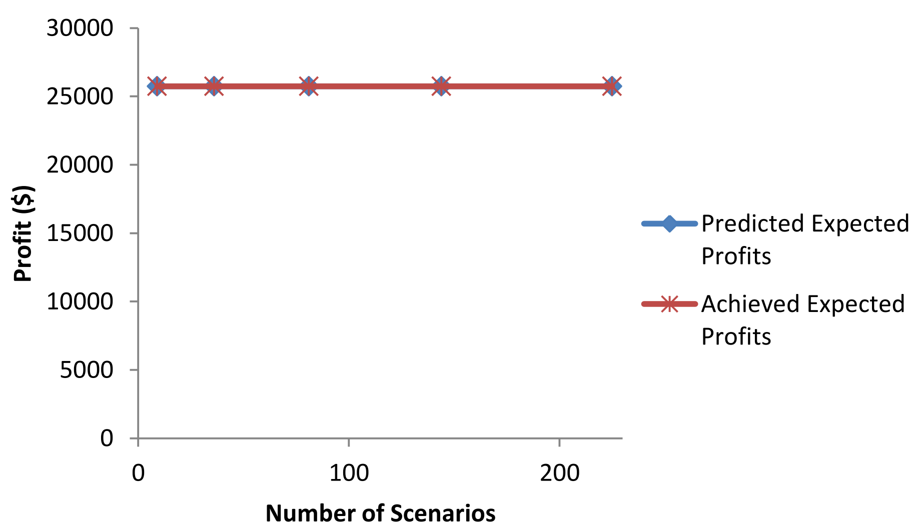

Figure 3.7. Predicted and achieved expected profits with formulation (AARS_IN)

Figure 3.7 shows that formulation (AARS_IN) provides consistently good predicted and achieved expected profits as the number of scenarios increase.

3.2.1.2 Results and Discussion – Uncertain Case B

In this case, the yields of the crops are uncertain and range between $\pm 20\%$ of their nominal values; specifically, $Y_{\text{wheat}} = 2.5 \pm 0.5$ t/acre, $Y_{\text{corn}} = 3.0 \pm 0.6$ t/acre, $Y_{\text{sugar beets}} = 20 \pm 4$ t/acre. Wheat and corn can no longer be purchased on the market, so there may be insufficient wheat or corn to feed cattle if the yield of wheat or corn is lower than anticipated. In addition, the allocated planting areas of crops need to be multiples of 5 acres, so an allocation decision is modelled in the optimization as 5 acres times an integer variable. The uncertainty region can be defined by $\Xi = \left\{ \xi : \left\| M \left(\xi - \bar{\xi} \right) \right\|_{\infty} \le \delta \right\}$, where the uncertain parameters are denoted by $\xi = \begin{pmatrix} Y_{wheat} \\ Y_{corn} \\ Y_{sugar\ beets} \end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix} 2.5 \\ 3.0 \\ 20 \end{pmatrix}$, $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $\delta = \begin{pmatrix} 0.5 \\ 0.6 \\ 4 \end{pmatrix}$. The uncertainty subregions can be defined by the approach shown in section 3.2.1.1.

Table 3.3 summarizes the results of the different formulations for uncertain case B. Formulation (EV) leads to an infeasible area allocation. Formulation (S) leads to infeasible result as well when 27 scenarios are considered, and it achieves a feasible result (which is also optimal) when the number of scenarios considered increases to 12,167. Formulations (NRS_IN) and (AARS_IN) achieve the optimal result with 27 scenarios, while formulation (NRS_IN) gives a poor prediction. As in Uncertain Case A, formulation (S) here needs to solve a larger problem to achieve the same performance to formulation (AARS_IN); and in this case, solution time for S several orders of magnitude larger than that of formulation (AARS_IN).

Table 3.3 Solution results of the farm planning problem for uncertain case B

	Formulation [a]				
	EV	S		NRS_IN	AARS_IN
Number of Scenarios	1	27	12,167 [e]	27	27
Number of Variables [b]	3/10	3/114	3/48,674	3/168	3/816
Number of Constraints	6	110	48,670	110	1,946
Solution Time (s)	0.03	0.03	109.86	0.05	0.06
Crop Area Allocation Result (acres)					
Wheat	120	140	150	150	150
Corn	115	135	145	145	145
Sugar Beets	265	225	205	205	205
Predicted Expected Profit ($) [c]	78,200	69,700	65,450	47,010	65,450
Achieved Expected Profit ($) [d]	Infeasible	Infeasible	65,450	65,450	65,450

Note: [a] EV: Expected value formulation; S: Scenario formulation; NRS_IN: Naïve robust scenario formulation; AARS_IN: Affinely adjustable robust scenario formulation. [b] Number of integer variables/Number of continuous variables. [c] The predicted expected profit is the expected profit predicted by the formulation at its solution. [d] The achieved expected profit refers to the expected profit that can be achieved with the obtained area allocation, which is estimated by using 99^2=9801 sampled scenarios. [e] The scenario formulation keeps generating infeasible allocation results until the number of scenarios is increased to 23^3=12,167.

3.2.2 Case Study 2 – Energy and Bioproduct Supply Chain

The energy and bioproduct supply chain optimization problem is adapted from Čuček et al. (2010). The supply chain network involves four layers. At layer 1, different biomass materials are harvested from 10 supply zones and then sent to up to six preprocessing centres. At layer 2, the materials go through different preprocessing procedures (e.g., drying, compaction and collection) in the preprocessing centres and are then sent to up to three main plants. At layer 3, materials are converted into different final products at the main processing plants. A number of technologies are available for the main processing. At layer 4, the final products are shipped to three demand locations, including two local cities, denoted by j_1 and j_2, and one export location, denoted by j_3. The superstructure of the supply chain network is shown in Figure 3.8. The dashed line denotes a railway that joins the preprocessing centres with the processing plants. The export location j_3 is located in the north of the region shown in the figure.

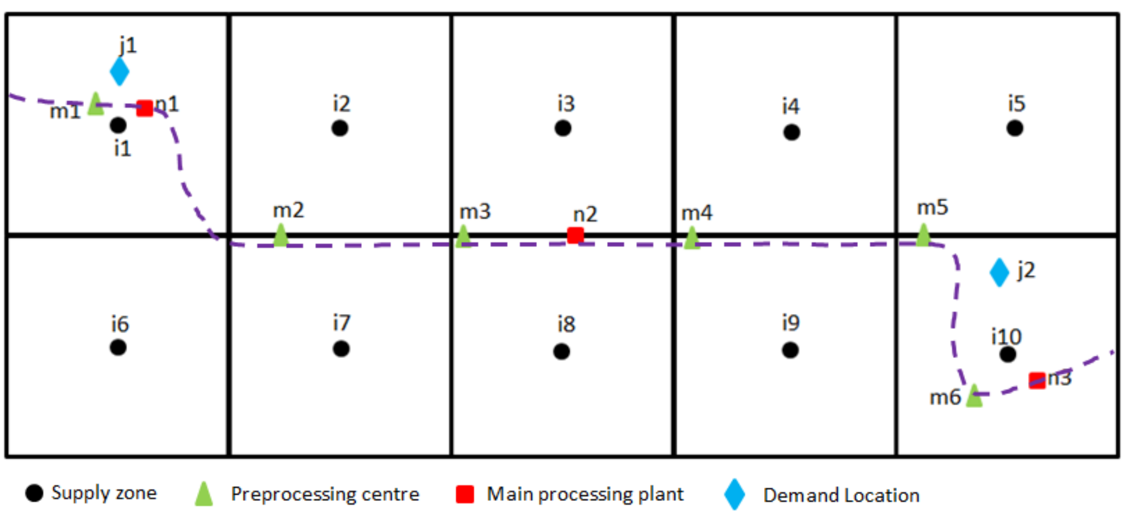

Figure 3.8. Superstructure of the energy and bioproduct supply chain network

The goal of the strategic SCO problem is to determine the optimal configuration of the supply chain network and the technologies used in the processing plants, such that the total profit is maximized and the customer demands at the three demand locations are satisfied. The first-stage decisions are whether or not specific units or technologies are to be included in the supply chain, and are represented by binary variables in the optimization. The second-stage decisions are material or product flows that determine the operation of the supply chain, and are represented by continuous variables in the optimization.

The deterministic formulation for the energy and bioproduct supply chain problem is described below. The list of nomenclature for the problem are provided in Table 3.4.

Table 3.4 Nomenclature for the energy and bioproduct supply chain problem

Sets and Subsets

I - supply zones, $I = \{i_1,\dots,i_{10}\}$

J - Demand locations, $J = \{j_1, j_2, j_3\}$

J^o - Subset for local demand locations, $J^o = \{j_1, j_2\}$

J^e - Subset for export location, $J^e = \{j_3\}$

M - Preprocessing centres, $M = \{m_1,\dots,m_6\}$

N - Process plants, $N = \{n_1, n_2, n_3\}$

PI - Set of raw materials, *PI* = {corn, corn-stover, wood chips, MSW, manure, timber}

PP - Set of produced products, *PP* = {electricity, heat, bioethanol, digestate, DDGS, boards}

PT - Pairs of intermediated products and technologies, *PT* = {(corn, dry grind), (corn stover, digestion), (corn stover, incineration), (wood chips, incineration), (MSW, incineration), (manure, incineration), (manure, digestion), (timber, sawing)}

PIPT - Groups of intermediate products, their produced products and related technology, *PIPT* = {(Corn, bioethanol, dry grind), (Corn, DDGS, dry grind), (Corn stover, digestate, digestion), (Corn stover, electricity, incineration), (Corn stover, heat, incineration), (wood chips, electricity, incineration), (wood chips, heat, incineration), (MSW, electricity, incineration), (MSW, heat, incineration), (manure, digestate, digestion), (manure, electricity, digestion), (manure, heat, digestion), (timber, boards, sawing)}

T - Technology options, *T* = {dry grind, digestion, sawing, incineration}

Superscripts

conv – conversion
fix – fixed part of investment costs
inv – investment costs
L1 – harvesting layer
L2 – preprocessing layer
L3 – main processing layer
L4 – demand layer
LO – lower bound
op – operating costs
price – price of products
road – road conditions
tr – transportation
UP – upper bound
var – variable part of investment costs

Indices

i – supply zones
j – demand locations
j^o – local level
j^e – export level
m – preprocessing centres
n – main processing plants
pi – raw materials
pp – produced products
t – technology options

Parameters

$A_{i,pi}$ – available area of material, km^2

$c_{pi}^{op,L2}$ – operating costs of material in preprocessing centre, €/t

$c_{pi,t}^{op,L3}$ – operating costs of material in main plant, €/t

$c^{fix,inv,L2}$ – fixed investment costs of preprocessing centre, €/y

$c_t^{fix,inv,L3}$ – fixed investment costs of main plant technology, €/y

$c_t^{var,inv,L3}$ – variable investment costs of main plant technology, €/t

c_{pp}^{price} – price of produced product, €/t or €/MWh or €/MJ

c_{pi} – cost of raw material, €/t

$c_p^{tr,La,Lb}$ – cost coefficient for transportation from layer a to layer b, €/(t·km)

$D_{x,y}^{La,Lb}$ – distance between object x in layer a and object y in layer b, km

$Dem_{j,pp}^{LO}$ – lower bound of product demand, t/y or MWh/y or MJ/y

$Dem_{j,pp}^{UP}$ – upper bound of product demand, t/y or MWh/y or MJ/y

$f_{pi}^{conv,L2}$ – conversion factor through preprocessing centre

$f_{pi,pp,t}^{conv,L3}$ – conversion factor through main plant

$f_{x,y}^{road,La,Lb}$ – road condition factor of object x in layer a and object y in layer b

HY_{pi} – yield of raw material, t/(t·km)

$q^{L2,UP}$ – overall capacity of preprocessing centres, t/y

$q_{pi}^{L1,L2,UP}$ – capacity of preprocessing centres for each raw material, t/y

$q_t^{L3,UP}$ – capacity of technology t, t/y

Continuous Variables

c^{tr} – total transportation costs, €/y

c^{op} – total operating costs, €/y

c^{inv} – total investment costs, €/y

$q_{i,pi}^{L1}$ – rate of raw material harvesting, t/y

$q_{i,m,pi}^{L1,L2}$ – rate of material entering preprocessing centre, t/y

$q_{m,n,pi}^{L2,L3}$ – rate of material exiting the preprocessing centre, t/y

$q_{n,pi,t}^{L2,L3}$ – rate of material sent to technology option, t/y

$q_{m,pi,pp,t}^{L2,L3}$ – rate of product production, t/y

$q_{n,j,pp}^{L3,L4}$ – rate of product sent to demand location, t/y

Binary Variables

y_m^{L2} – selection or rejection of preprocessing centres

$y_{n,t}^{L3}$ – selection or rejection of main plants

Equation (3.59) describes the objective function in which total profits are to be maximized

$$\begin{aligned}
&\text{Objective} = \text{Revenue} - \text{Total Costs} \\
&= \sum_{n\in N}\sum_{j^o\in J}\sum_{pp\in PP} q_{n,j^o,pp}^{\text{L3,L4}} \cdot c_{pp}^{\text{price}} + \sum_{n\in N}\sum_{j^e\in J}\sum_{pp\in PP} 0.9 \cdot \left(q_{n,j^e,pp}^{\text{L3,L4}} \cdot c_{pp}^{\text{price}} \right) \\
&- \sum_{i\in I}\sum_{pi\in PI} q_{i,pi}^{\text{L1}} \cdot c_{pi} - c^{\text{tr}} - c^{\text{op}} - c^{\text{inv}}
\end{aligned} \tag{3.59}$$

where 0.9 represents the discount factor for selling products at the export location j_3.

The total transportation cost considering distance and road conditions is

$$\begin{aligned}
c^{\text{tr}} &= \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI} D_{i,m}^{\text{L1,L2}} \cdot f_{i,m}^{\text{road,L1,L2}} \cdot c_{pi}^{\text{tr,L1,L2}} \cdot q_{i,m,pi}^{\text{L1,L2}} \\
&+ \sum_{m\in M}\sum_{n\in N}\sum_{pi\in PI} D_{m,n}^{\text{L2,L3}} \cdot f_{m,n}^{\text{road,L2,L3}} \cdot c_{pi}^{\text{tr,L2,L3}} \cdot q_{m,n,pi}^{\text{L2,L3}} \\
&+ \sum_{n\in N}\sum_{j\in J}\sum_{pp\in PP} D_{n,j}^{\text{L3,L4}} \cdot f_{n,j}^{\text{road,L3,L4}} \cdot c_{pp}^{\text{tr,L3,L4}} \cdot q_{n,j,pp}^{\text{L3,L4}}.
\end{aligned} \tag{3.60}$$

The total operating cost for preprocessing centres and processing plants is

$$c^{\text{op}} = \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI} c_{pi}^{\text{op,L2}} \cdot q_{i,m,pi}^{\text{L1,L2}} + \sum_{n\in N}\sum_{(pi,t)\in PT} c_{pi,t}^{\text{op,L3}} \cdot q_{n,pi,t}^{\text{L2,L3}}, \tag{3.61}$$

and the total investment cost for the preprocessing centres and processing plants is

$$c^{\text{inv}} = \sum_{m\in M} c^{\text{fix,inv,L2}} \cdot y_m^{\text{L2}} + \sum_{n\in N}\sum_{t\in T}\left(c_t^{\text{fix,inv,L3}} \cdot y_{n,t}^{\text{L3}} + \sum_{(pi,t)\in PT} c_t^{\text{var,inv,L3}} \cdot q_{n,pi,t}^{\text{L2,L3}} \right). \tag{3.62}$$

It should be noted that $\sum_{(pi,t)\in PT} c_t^{\text{var,inv,L3}} \cdot q_{n,pi,t}^{\text{L2,L3}}$ denotes total variable investment cost that is dependent on material flow rate.

The amount of each biomass material that can be harvested at each supply zone is subject to the capacity of that supply zone,

$$q_{i,pi}^{\text{L1}} \le HY_{pi} \cdot A_{i,pi}, \quad \forall pi \in PI, \forall i \in I, \tag{3.63}$$

and the mass balance for the materials harvested and sent out for preprocessing is

$$q_{i,pi}^{\mathrm{L1}} = \sum_{m\in M} q_{i,m,pi}^{\mathrm{L1,L2}}, \quad \forall pi \in PI, \forall i \in I. \tag{3.64}$$

The material flows going through the preprocessing centres are subject to preprocessing capacities as

$$\sum_{i\in I} q_{i,m,pi}^{\mathrm{L1,L2}} \le q_{pi}^{\mathrm{L1,L2,UP}} \cdot y_m^{\mathrm{L2}}, \quad \forall m \in M, \forall pi \in PI, \tag{3.65}$$

$$\sum_{i\in I} \sum_{pi\in PI} q_{i,m,pi}^{\mathrm{L1,L2}} \le q^{\mathrm{L2,UP}} \cdot y_m^{\mathrm{L2}}, \quad \forall m \in M, \tag{3.66}$$

and the inlet and outlet material flows of the preprocessing centres are subject to the following mass balance that taking into account the loss of mass in the preprocessing,

$$\sum_{i\in I} q_{i,m,pi}^{\mathrm{L1,L2}} \cdot f_{pi}^{\mathrm{conv,L2}} = \sum_{n\in N} q_{m,n,pi}^{\mathrm{L2,L3}}, \quad \forall m \in M, \forall pi \in PI. \tag{3.67}$$

The materials sent into the main processing plants are processed using different technologies,

$$\sum_{m\in M} q_{m,n,pi}^{\mathrm{L2,L3}} = \sum_{(pi,t)\in PT} q_{n,pi,t}^{\mathrm{L2,L3}}, \quad \forall n \in N, \forall pi \in PI, \tag{3.68}$$

and the processing is subject to the capacities of the technologies,

$$\sum_{(pi,t)\in PT} q_{n,pi,t}^{\mathrm{L2,L3}} \le q_t^{\mathrm{L3,UP}} \cdot y_{n,t}^{\mathrm{L3}}, \quad \forall n \in N, \forall t \in T. \tag{3.69}$$

The materials are converted into the products in the processing plants at specific conversion rates,

$$q_{n,pi,t}^{\mathrm{L2,L3}} \cdot f_{pi,pp,t}^{\mathrm{conv,L3}} = q_{n,pi,pp,t}^{\mathrm{L2,L3}}, \quad \forall n \in N, \forall (pi,pp,t) \in PIPT. \tag{3.70}$$

The final products sent to demand locations are limited by the products generated in the processing center,

$$\sum_{(pi,pp,t)\in PIPT} q_{n,pi,pp,t}^{\mathrm{L2,L3}} \ge \sum_{j\in J} q_{n,j,pp}^{\mathrm{L3,L4}}, \quad \forall n \in N, \forall pp \in PP, \tag{3.71}$$

and it is also bounded by a certain amount of customer demands it has to satisfy (i.e., contracted demands) as well as the capacity of the market at the demand locations,

$$Dem_{j,pp}^{\mathrm{LO}} \le \sum_{n \in N} q_{n,j,pp}^{\mathrm{L3,L4}} \le Dem_{j,pp}^{\mathrm{UP}}, \quad \forall j \in J, \forall pp \in PP. \tag{3.72}$$

The overall optimization model can be expressed as

maximize Objective

subject to Constraints (3.63)-(3.72),

All continuous variables are non-negative.

Formulations (S), (NRS_IN), and (AARS_IN) can be found in the Appendix C. The parameter values used for the following cases are shown in Appendix D.

3.2.2.1 Results and Discussion – Uncertain Case A

In this case, the upper demand limits for electricity and the yield of corn stover are assumed to be uncertain; specifically, $Dem_{j,\mathrm{electricity}}^{\mathrm{UP}} = 200,000 \pm 150,000$ MWh and $HY_{\mathrm{corn\ stover}} = 840 \pm 300$ $\mathrm{t/(km^2 \cdot y)}$. In addition, the capacity of incineration in the processing plants is $q_{\mathrm{incineration}}^{\mathrm{L3,UP}} = 390,000$ t/year. Here, the uncertainty region can be defined by $\Xi = \left\{ \xi : \left\| M \left(\xi - \bar{\xi} \right) \right\|_{\infty} \le \delta \right\}$, where the uncertain parameters are denoted by

$\xi = \begin{pmatrix} Dem^{\mathrm{UP}}_{j,\mathrm{electricity}} \\ HY_{\mathrm{corn\ stover}} \end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix} 200,000 \\ 840 \end{pmatrix}$, $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\delta = \begin{pmatrix} 150,000 \\ 300 \end{pmatrix}$.

The uncertainty subregions are defined as $\Xi_{i,\omega} = \left\{ \xi_{i,\omega} : \left\| M_{i,\omega} \left(\xi_{i,\omega} - \bar{\xi}_{i,\omega} \right) \right\|_{\infty} \le \delta_{i,\omega} \right\}$, where $\xi_{i,\omega} = \begin{pmatrix} Dem^{\mathrm{UP}}_{j,\mathrm{electricity},\omega} \\ HY_{\mathrm{corn\ stover},\omega} \end{pmatrix}$, $\bar{\xi}_{i,\omega} = \begin{pmatrix} \overline{Dem}^{\mathrm{UP}}_{j,\mathrm{electricity},\omega} \\ \overline{HY}_{\mathrm{corn\ stover},\omega} \end{pmatrix}$, $M_{i,\omega} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \forall \omega$, and $\delta_{i,\omega} = \begin{pmatrix} \delta_{electricity} \\ \delta_{corn\ stover} \end{pmatrix}, \forall \omega$.

Table 3.5 summarizes the results of each formulation. Whereas all of the formulations lead to feasible configurations of the supply chain network, the expected profits achieved by formulations (S), (NRS_IN) and (AARS_IN) are more than 15% better than the one achieved by formulation (EV). In addition, formulation (S) overestimates the expected profit because of its incomplete consideration of uncertainty, while formulations (NRS_IN) and (AARS_IN) underestimate the expected profit because of their inherent conservativeness.

Table 3.5 Solution results of the energy and bioproduct problem for uncertain case A

	Formulation [a]			
	EV	S	NRS_IN	AARS_IN
Number of Scenarios	1	25	25	25
Number of Variables [b]	18/670	18/16,078	18/16,078	18/59,428
Number of Constraints	376	9,088	9,088	84,013
Solution Time (s)	0.16	4.02	4.85	2,420.25
Preprocessing Centres to be Developed	1, 2, 4, 5, 6	1, 2, 4, 5, 6	1, 2, 3, 4, 5, 6	1, 2, 4, 5, 6
Processing Technologies to be Applied [c]				
Drygrind	1, 3	1, 3	1, 3	1, 3
Digestion	1		1	
Incineration	1	1, 3	1, 3	1, 3
Sawing	3	3	3	3
Predicted Expected Profits [d] (million $)	78.66	76.23	65.29	75.08
Achieved Expected Profits [e] (million $)	65.77	76.00	75.98	76.00

Note: [a] EV: Expected value formulation; S: Scenario formulation; NRS_IN: Naïve robust scenario formulation; AARS_IN: Affinely adjustable robust scenario formulation. [b] Number of binary variables/Number of continuous variables. [c] Processing technology – Processing plant. [d] The predicted expected profit is the expected profit predicted by the formulation at its solution. [e] The achieved expected profit refers to the expected profit that can be achieved with the obtained area allocation, which is estimated by using 99^2=9801 sampled scenarios.

3.2.2.2 Results and Discussion – Uncertain Case B

In this case, both the lower and upper demand limits for electricity are assumed to be uncertain; specifically, $Dem_{j,\text{electricity}}^{\text{LO}} = 90,000 \pm 56,000$ $Dem_{j,\text{electricity}}^{\text{UP}} = 500,000 \pm 10,000$ MWh, and MWh. In addition, the capacity of incineration in the processing plants is $q_{\text{incineration}}^{\text{L3,UP}} = 290,000$ t/year. Here, the uncertainty region can be defined by $\Xi = \left\{ \xi : \left\| M\left(\xi - \bar{\xi} \right) \right\|_{\infty} \le \delta \right\}$, where the uncertain parameters are denoted by $\xi = \begin{pmatrix} Dem_{j,\text{electricity}}^{\text{LO}} \\ Dem_{j,\text{electricity}}^{\text{UP}} \end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix} 90,000 \\ 500,000 \end{pmatrix}$, $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\delta = \begin{pmatrix} 56,000 \\ 10,000 \end{pmatrix}$. The uncertainty subregions can be defined by the approach shown in section 3.2.2.1.

Table 3.6 summarizes the results of each formulation. It can be seen that formulation (EV) and formulation (S) with 25 scenarios lead to infeasible supply chain configurations, whereas formulations (NRS_IN) and (AARS_IN) lead to a feasible and optimal supply chain configuration. Although it is featured with high conservativeness, formulation (NRS_IN) even gives a very good prediction of the expected profit in this case. The key difference between the two supply chain configurations is the number of plants that are equipped with incineration technology (for electric power generation). The infeasible configuration only has incineration in processing plant 2, and the optimal configuration has incineration in both processing plant 1 and 3. When the number of scenarios addressed increased to 2809, formulation (S) results in the optimal configuration; but this solution is obtained through a very large scale MILP problem, which requires more than 46 hours. Thus formulation S is outperformed by formulation (AARS_IN), which requires less than one hour to obtain the optimal configuration.

Table 3.6 Solution results of the energy and bioproduct problem for uncertain case B

	Formulation [a]				
	EV	S		NRS_IN	AARS_IN
Number of Scenarios	1	25	2809[f]	25	25
Number of Variables [b]	18/670	18/16,078	18/1,803,406	18/16,078	18/59,428
Number of Constraints	376	9,088	1,019,680	9,088	84,013
Solution Time (s)	0.17	21.80	166,890.05	13.75	2,245.73
Preprocessing Centres to be Developed	1, 2, 3, 4, 6	1, 2, 3, 4, 6	1, 2, 3, 5, 6	1, 2, 3, 5, 6	1, 2, 3, 5, 6
Processing Technologies to be Applied [c]					
Drygrind	2	2	2	2	2
Digestion	1	1	1	2	1
Incineration	2	2	1, 3	1, 3	1, 3
Sawing	3	3	3	3	3
Predicted Expected Profits [d] (million $)	57.01	57.01	57.01	56.80	57.01
Achieved Expected Profits [e] (million $)	Infeasible	Infeasible	57.01	57.01	57.01

Note: [a] EV: Expected value formulation; S: Scenario formulation; NRS_IN: Naïve robust scenario formulation; AARS_IN: Affinely adjustable robust scenario formulation. [b] Number of binary variables/Number of continuous variables. [c] Technology – Processing plant. [d] The predicted expected profit is the expected profit predicted by the formulation at its solution. [e] The achieved expected profit refers to the expected profit that can be achieved with the obtained area allocation, which is estimated by using 99^2=9801 sampled scenarios. [f] The scenario formulation (S) keeps generating infeasible planning results until the number of scenarios is increased to 53^2=2,809.

3.2.2.3 Results and Discussion – Uncertain Case C

In this case, both the lower demand limit for electricity and the yield of corn stover are assumed to be uncertain; specifically, $Dem_{j,\text{electricity}}^{\text{LO}} = 80,000 \pm 18,000$ MWh and $HY_{\text{corn stover}} = 840 \pm 300\ \text{t/(km}^2 \cdot \text{y)}$. In addition, the capacity of incineration in the processing plants is $q_{\text{incineration}}^{\text{L3,UP}} = 500,000$ t/year. These parameters were selected such that no feasible configuration exists for the given superstructure of the supply chain network, as the purpose is to investigate whether the optimization formulations can identify the infeasibility. Here, the uncertainty region can be defined by $\Xi = \left\{ \xi : \left\| M\left(\xi - \bar{\xi}\right) \right\|_{\infty} \le \delta \right\}$, where the uncertain parameters are denoted by $\xi = \begin{pmatrix} Dem_{j,\text{electricity}}^{\text{LO}} \\ HY_{\text{corn stover}} \end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix} 80,000 \\ 840 \end{pmatrix}$, $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\delta = \begin{pmatrix} 18,000 \\ 300 \end{pmatrix}$. The uncertainty subregions can be defined by the approach shown in section 3.2.2.1.

Table 3.7 summarizes the results of each formulation. It can be seen that formulations (NRS_IN) and (AARS_IN) with 9 scenarios indicate the infeasibility of the problem within low computing times. Formulation (EV) and formulation (S) do not identify the infeasibility and report infeasible configurations as a solution. Formulation (S) does not report infeasibility even when it addresses 2500 uncertainty realizations (which requires almost 17 hours to solve). This result demonstrates that when a feasible solution does not exist, formulations (NRS_IN) and (AARS_IN) can report infeasibility efficiently and effectively.

Table 3.7 Solution results of the energy and bioproduct problem for uncertain case C

	Formulation [a]				
	EV	S		NRS_IN	AARS_IN
Number of Scenarios	1	9	2500	9	9
Number of Variables [b]	18/670	18/5,806	18/1,605,028	18/5,806	18/21,412
Number of Constraints	376	3,280	907,513	3,280	30,301
Solution Time (s)	0.16	2.99	61,044.77	0.13	113.01
Preprocessing Centres to be Developed	1, 2, 3, 4, 6	1, 2, 3, 4, 6	1, 2, 3, 4, 6	-	-
Processing Technologies to be Applied [c]					
Drygrind	2	2	2	-	-
Digestion	1	1	1		
Incineration	2	2	2		
Sawing	3	3	3		
Predicted Expected Profits [d] (million $)	57.98	57.98	57.98	Infeasibility indicated	Infeasibility indicated
Achieved Expected Profits [e] (million $)	Infeasible	Infeasible	Infeasible	-	-

Note: [a] EV: Expected value formulation; S: Scenario formulation; NRS_IN: Naïve robust scenario formulation; AARS_IN: Affinely adjustable robust scenario formulation. [b] Number of binary variables/Number of continuous variables. [c] Technology – Processing plant. [d] The predicted expected profit is the expected profit predicted by the formulation at its solution. [e] The achieved expected profit refers to the expected profit that can be achieved with the obtained area allocation, which is estimated by using 99^2=9801 sampled scenarios

In this chapter, the classical scenario approach, which commonly provides good optimality and the robust approach, which can guarantee feasibility of a problem (if a feasible solution exists) are combined to develop hybrid formulations. Two robust scenario formulations were generated, namely the naïve robust scenario formulation and the affinely adjustable robust formulation. The formulations were then applied to a farm planning problem and an energy and bioproduct supply chain optimization problem. The results demonstrate that the proposed formulations can effectively avoid infeasibility of the problem or report infeasibility for a situation when no feasible solution exists. They also outperform the classical scenario approach by generating the optimal solutions in a smaller number of scenarios and shorter solution times.

Chapter 4

Optimization of an Industrial Chemical Supply Chain

In this chapter, a case study at the industrial level is considered. With collaboration and data provided from DuPont, the mathematical optimization model is derived for an industrial chemical supply chain problem. The derivation of the model is shown in section 4.1. Next in section 4.2, the goal is to again show the benefits of formulation (AARS_IN), but for the more complex and realistic problem. Formulation (NRS_IN) is not considered for this case study problem, since its results from the previous chapter were too conservative. Since formulation (AARS_IN) can better address uncertainties, formulation (NRS_IN) can be neglected at this stage.

4.1 The Industrial Chemical Supply Chain

The supply chain case study problem is formulated from data provided by DuPont. The supply chain involves 55 grades of Primary Raw Material (PRM) which when purchased, can either be transported to the PRM warehouse or to any of the 5 on-site PRM warehouses. From the on-site warehouses, the raw material is then processed in one of the 5 production plants and then sent to the on-site final product warehouses. The final products (FP), which are classified under one of the appropriate 23 possible FP grades, can be transported to regional warehouses for additional storage or to the 5 regional markets to be sold to the end users. The superstructure of the supply chain problem is shown in Figure 4.1.

The goal of this industrial strategic SCO problem is to determine the capacities for each of the plants z_i, such that the total profits are maximized and the customer demands at the 5 regional markets are satisfied. Final product demands at each of the regional markets are the uncertain parameters considered for this case study. Capacities at the plants are the first-stage decision variables and are represented by continuous variables in the optimization. The second-stage variables are the raw material or product flows that determine the operation of the supply chain, and are also represented by continuous variables in the optimization.

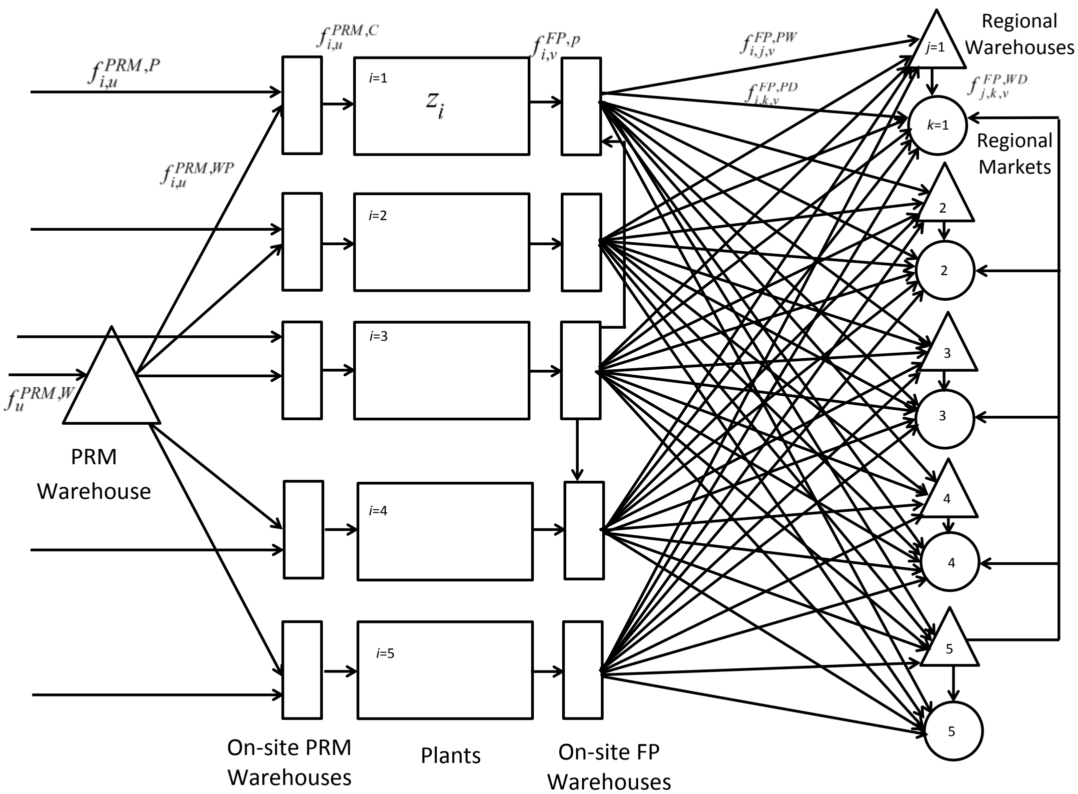

Figure 4.1. Diagram of the industrial chemical supply chain network.

The variables and parameters for this model are listed in Table 4.1. Following, the deterministic optimization model is given.

Table 4.1 Nomenclature for the industrial chemical supply chain problem

Indices and Sets

$i \in I = \{1,\ldots,5\}$ - Plants
$j \in J = \{1,\ldots,5\}$ - Regional warehouses
$k \in K = \{1,\ldots,5\}$ - Regional markets
$n \in N$ – Time periods
$nu \in NU$ – Number of uncertain parameters
$s \in S$ – Scenarios
$u \in U = \{1,\ldots,55\}$ - PRM grades
$v \in V = \{1,\ldots,23\}$ - FP grades
$w \in W = \{1,\ldots,41\}$ – Impurities
$(i, j) \in \Omega$ - FP shipment routes for plants to regional warehouses
$(i,k) \in \Theta$ - FP shipment routes for plants to regional markets
$(i,v) \in \Psi$ - FP grades available for production at plant *i*
$(j,k) \in \Pi$ - FP shipment routes for regional warehouses to regional markets

Parameters

$a_i^{\text{avg,FP}}$ - Average additive factor for plant *i*
b_i^{td} - Intercept related to minimum turndown for plant *i*
b_i^{wl} - Intercept related to waste limit for plant *i*
C^{pen} - Penalty cost for not meeting demand requirements, \$/t
C^{cap} - Capacity cost, \$/t
C_i^{fix} - Fixed cost for plant *i*, \$MM
C_i^{var} - Other variable costs for plant *i*, \$/t
$C_{i,k}^{\text{fr,FP,PD}}$ - Freight cost of FP from plant *i* to market *k*, \$/t
$C_{i,j}^{\text{fr,FP,PW}}$ - Freight cost of FP from plant *i* to regional warehouse *j*, \$/t
$C_{j,k}^{\text{fr,FP,WD}}$ - Freight cost of FP from regional warehouse *j* to market *k*, \$/t
C_i^{PI} - Plant *i* on-site inventory cost, \$/t
C_j^{WI} - Regional warehouse *j* inventory cost, \$/t
$C_{i,u}^{\text{PRM}}$ - Cost of PRM grade *u* for plant *i*, \$/t
C_i^{RM2} - Cost of RM2 for plant *i*, \$/t
C_i^{RM3} - Cost of RM3 for plant *i*, \$/t
C_i^{waste} - Cost of waste for plant *i*, \$/t
$D_{i,v}^{\text{FP}}$ - Demand of FP grade *v* at plant *i*, t
$D_{j,v}^{\text{FP}}$ - Demand of FP grade *v* at regional warehouse *j*, t
$D_{v,k}^{\min}$ - Minimum demand of FP grade *v* from at

$P_{k,v}^{\text{FP}}$ - Price of FP grade *v* for market *k*, \$/t
$\hat{P}_{j,v}^{\text{FP}}$ - Estimated price of FP grade *v* for regional warehouse *j*, \$/t
$q_{i,u}^{\text{RM2}}$ - RM2 to PRM grade *u* ratio for plant *i*, QPU
$q_{i,u}^{\text{RM3}}$ - RM3 to PRM grade *u* ratio for plant *i*, QPU
q_u^{waste} - Waste to PRM grade *u* ratio, %
$q_{u,w}^{\text{imp}}$ - Impurity *w* content in PRM grade *u*, %
$q_{i,j}^{\text{fr,PW}}$ - Proportion of FP shipped from plant *i* to the regional warehouse *j*, %
$Q_{i,w}^{\text{imp}}$ - Maximum impurity *w* limit for PRM at plant *i*, % or PPM
Q_i^{imp} - Maximum limit for total blend at plant *i*, %
r_i^{inc} - Income tax rate for plant *i*, %
$r_{i,k}^{\text{du}}$ - Duty rate for shipments from plant *i* to market *k*, %
$\hat{r}_{i,j}^{\text{du}}$ - Estimated duty rate for shipments from plant *i* to regional warehouse *j*, %
r^{tp} - Transfer price rate, %
$R_{i,v}^{\text{FP}}$ - Target inventory day supply of FP grade *v* for plant *i*, day
$R_{j,v}^{\text{FP}}$ - Target inventory day supply of FP grade *v* for regional warehouse *j*, day
$R_{i,u}^{\text{PRM,P}}$ - Target inventory of PRM grade *u* for plant *i*, t
$R_u^{\text{PRM,W}}$ - Target inventory of PRM grade *u* for the

market k, t E_u^{PRM} - Effective percentage in PRM grade u, for generating FP, % m_i^{td} - Slope related to minimum turndown for plant i m_i^{wl} - Slope related to waste limit for plant i M_u^{PRM} - PRM grade u availability, t O_i - Scheduled outage at plant i, d/y	PRM warehouse, t U_i - Uptime for plant i, % $X_{i,v}^{\text{FP}}$ - Beginning inventory of FP grade v at plant i, t $X_{j,v}^{\text{FP}}$ - Beginning inventory of FP grade v at regional warehouse j, t $X_{i,u}^{\text{PRM,P}}$ - Beginning inventory of PRM grade u at plant i, t $X_u^{\text{PRM,W}}$ - Beginning inventory of PRM grade u at the PRM warehouse, t Y_i^{FP} - Yield of FP at plant i, % $Z_i^{\max}$ - Maximum allowable capacity at plant i, t

Variables

c_i^{fr} - Freight cost for plant i, $

c_i^{du} - Duty cost for plant i, $

c_i^{I} - Inventory cost for plant i, $

c_i^{PRM} - PRM cost for plant i, $

c_i^{RM2} - RM2 cost for plant i, $

c_i^{RM3} - RM3 cost for plant i, $

c_i^{waste} - Waste cost for plant i, $

c_i^{OPVC} - Other plant i variable costs, $

c_i^{cap} - Capacity cost for plant i, $

$c_{v,k}^{\text{pen}}$ - Penalty cost of FP grade v at market k, $

$f_{i,k,v}^{\text{FP,PD}}$ - Shipment of FP grade v from plant i to market k, t

$f_{i,j,v}^{\text{FP,PW}}$ - Shipment of FP grade v from plant i to regional warehouse j, t

$f_{j,k,v}^{\text{FP,WD}}$ - Shipment of FP grade v from regional warehouse j to market k, t

$f_{i,u}^{\text{PRM,c}}$ - PRM grade u consumed at plant i, t

$f_{i,u}^{\text{PRM,P}}$ - PRM grade u purchased and sent to plant i, t

$f_u^{\text{PRM,W}}$ - PRM grade u purchased and sent to the PRM warehouse, t

$f_{i,u}^{\text{PRM,WP}}$ - Shipments of PRM grade u from the PRM warehouse to plant i, t

$f_{i,v}^{\text{FP,p}}$ - Amount of FP grade v produced from plant i, t

$y_{v,k}$ - Slack variable to penalize demand not satisfied of grade v at location k, t

z_i - Capacity of plant i, t

The total profit objective function to be maximized is

$$\max \sum_{i \in I} \left[\left(\left(\text{Revenue}_i - \text{TC}_i \right) \left(1 - r_i^{\text{inc}} \right) \right) \right] \tag{4.1}$$

Revenue is calculated at each plant *i*, based on the FP flow rates from the plants to the regional markets and the FP flow rates from the plants to the regional warehouses

$$\text{Revenue}_i = \sum_{(i,k)\in\Theta} \sum_{v \in V} \left(f_{i,k,v}^{\text{FP,PD}} \cdot P_{k,v}^{\text{FP}} \right) + \sum_{(i,j)\in\Omega} \sum_{v \in V} \left(f_{i,j,v}^{\text{FP,PW}} \cdot \hat{P}_{j,v}^{\text{FP}} \right), \quad i \in I. \tag{4.2}$$

The individual cost terms featured in the objective function are summed

$$\text{TC}_i = c_i^{\text{fix}} + c_i^{\text{cap}} + c_i^{\text{fr}} + c_i^{\text{du}} + c_i^{\text{I}} + c_i^{\text{PRM}} + c_i^{\text{RM2}} + c_i^{\text{RM3}} + c_i^{\text{waste}} + c_i^{\text{OPVC}}, \quad i \in I. \tag{4.3}$$

Capacity costs are considered at each of the plants

$$c_i^{\text{cap}} = z_i \cdot C^{\text{cap}}, \quad i \in I. \tag{4.4}$$

Freight costs are calculated based on the FP flow rates from the plants to regional warehouses, plants to regional markets, and from the regional warehouses to the regional markets

$$\begin{aligned} c_i^{\text{fr}} &= \sum_{(i,k)\in\Theta} \sum_{v \in V} \left(f_{i,k,v}^{\text{FP,PD}} \cdot C_{i,k}^{\text{fr,FP,PD}} \right) + \sum_{(i,j)\in\Omega} \sum_{v \in V} \left(f_{i,j,v}^{\text{FP,PW}} \cdot C_{i,j}^{\text{fr,FP,PW}} \right) \\ &+ \sum_{(i,j)\in\Omega} \sum_{(j,k)\in\Pi} \sum_{v \in V} \left(f_{j,k,v}^{\text{FP,WD}} \cdot C_{j,k}^{\text{fr,FP,WD}} \cdot q_{i,j}^{\text{fr,PW}} \right), \; i \in I. \end{aligned} \tag{4.5}$$

Duty costs are calculated by subjecting the amount of final products produced by duty rates and a transfer price rate

$$c_i^{\text{du}} = r^{\text{tp}} \cdot \left[\sum_{(i,k)\in\Theta} \sum_{v \in V} \left(f_{i,k,v}^{\text{FP,PD}} \cdot P_{k,v}^{\text{FP}} \cdot r_{i,k}^{\text{du}} \right) + \sum_{(i,j)\in\Omega} \sum_{v \in V} \left(f_{i,j,v}^{\text{FP,PW}} \cdot \hat{P}_{j,v}^{\text{FP}} \cdot \hat{r}_{i,j}^{\text{du}} \right) \right], \quad i \in I. \tag{4.6}$$

Inventory costs are equated at the on-site plant warehouses and the regional warehouses for the storage of final products

$$c_i^{\text{I}} = \sum_{(i,v)\in\Psi} \left(f_{i,v}^{\text{FP,p}} \cdot C_i^{\text{PI}} \right) + \sum_{(i,j)\in\Omega} \sum_{v \in V} \left(f_{i,j,v}^{\text{FP,PW}} \cdot C_j^{\text{WI}} \right), \quad i \in I. \tag{4.7}$$

Equations (4.9)-(4.13) calculate the costs of PRM, Raw Material 2, Raw Material 3, waste and other pant variable costs, respectively at each of the plants

$$c_i^{\text{PRM}} = \sum_{u \in U} \left(f_{i,u}^{\text{PRM,c}} \cdot C_{i,u}^{\text{PRM}} \right), \quad i \in I, \tag{4.8}$$

$$c_i^{\text{RM2}} = \sum_{u \in U} \left(f_{i,u}^{\text{PRM,c}} \cdot q_{i,u}^{\text{RM2}} \right) \cdot C_i^{\text{RM2}}, \quad i \in I, \tag{4.9}$$

$$c_i^{\text{RM3}} = \sum_{u \in U} \left(f_{i,u}^{\text{PRM,c}} \cdot q_{i,u}^{\text{RM3}} \right) \cdot C_i^{\text{RM3}}, \quad i \in I, \tag{4.10}$$

$$c_i^{\text{waste}} = \sum_{u \in U} \left(f_{i,u}^{\text{PRM,c}} \cdot q_u^{\text{waste}} \right) \cdot C_i^{\text{waste}}, \quad i \in I, \tag{4.11}$$

$$c_i^{\text{OPVC}} = C_i^{\text{var}} \cdot \left[\sum_{(i,v) \in \Psi} \left(f_{i,v}^{\text{FP,p}} \right) \right], \quad i \in I. \tag{4.12}$$

Equations (4.14)-(4.26) describe the constraints for the supply chain problem.

The amount of each grade of PRM consumed is subject to a turndown limit at each plant

$$\sum_{u \in U} f_{i,u}^{\text{PRM,c}} \le \left(\sum_{(i,v) \in \Psi} \left[\left(\frac{f_{i,v}^{\text{FP,p}}}{a_i^{avg,FP} / 24 / 365 / \left(\left(365 - O_i \right) \cdot U_i \right)} \right) \right] \cdot m_i^{\text{td}} + b_i^{\text{td}} \right) \cdot 24 \cdot 365 \cdot \left(\left(365 - O_i \right) \cdot U_i / 365 \right), \quad i \in I. \tag{4.13}$$

The amount of each grade of PRM consumed is subject to a waste limit at each plant

$$\sum_{u \in U} f_{i,u}^{\text{PRM,c}} \le \left(\sum_{(i,v) \in \Psi} \left[\left(\frac{f_{i,v}^{\text{FP,p}}}{a_i^{avg,FP} / 24 / 365 / \left(\left(365 - O_i \right) \cdot U_i \right)} \right) \right] \cdot m_i^{\text{wl}} + b_i^{\text{wl}} \right) \cdot 24 \cdot 365 \cdot \left(\left(365 - O_i \right) \cdot U_i / 365 \cdot y_i \right) / y_i, \quad i \in I. \tag{4.14}$$

Material balance relationship between the FP capability from PRM consumption and FP production is

$$\sum_{u \in U} \left(f_{i,u}^{\text{PRM,c}} \cdot E_u^{\text{PRM}} \right) \cdot a_i^{\text{avg,FP}} \cdot Y_i^{\text{FP}} = \sum_{(i,v) \in \Psi} \left(f_{i,v}^{\text{FP,p}} \right), \quad i \in I. \tag{4.15}$$

The ending FP inventory at the plants is subject to a target inventory limit

$$\sum_{(i,j)\in\Omega}\left(f_{i,j,v}^{\text{FP,PW}}\right)+\sum_{(i,k)\in\Theta}\left(f_{i,k,v}^{\text{FP,PD}}\right)\le X_{i,v}^{\text{FP}}+f_{i,v}^{\text{FP,p}}-D_{i,v}^{\text{FP}}\cdot R_{i,v}^{\text{FP}}/365,\quad (i,v)\in\Psi. \tag{4.16}$$

The ending FP inventory at the regional warehouses is subject to a target inventory limit

$$\sum_{(j,k)\in\Pi}\left(f_{j,k,v}^{\text{FP,WD}}\right)\le X_{j,v}^{\text{FP}}+\sum_{i\in I}\left(f_{i,j,v}^{\text{FP,PW}}\right)-D_{j,v}^{\text{FP}}\cdot R_{j,v}^{\text{FP}}/365,\quad j\in J,\ v\in V. \tag{4.17}$$

Constraints (4.18) and (4.19) show that the amount of impurity *w* within the PRM consumed at plant *i* must be less than the impurity limit of the PRM consumption for each individual impurity (4.18) and for the total product mix (4.19)

$$\sum_{u\in U}\left(f_{i,u}^{\text{PRM,c}}\cdot q_{u,w}^{\text{imp}}\right)\le\sum_{u\in U}\left(f_{i,u}^{\text{PRM,c}}\cdot Q_{i,w}^{\text{imp}}\right),\quad i\in I,\ w\in W, \tag{4.18}$$

$$\sum_{u\in U}\left(f_{i,u}^{\text{PRM,c}}\cdot E_{u}^{\text{PRM}}\right)\le\sum_{u\in U}\left(f_{i,u}^{\text{PRM,c}}\cdot Q_{i}^{\text{imp}}\right),\quad i\in I. \tag{4.19}$$

The PRM purchased and sent to the plants and PRM warehouse must be less than the PRM availability

$$\sum_{i\in I}\left(f_{i,u}^{\text{PRM,P}}\right)+f_{u}^{\text{PRM,W}}\le M_{u}^{\text{PRM}},\quad u\in U. \tag{4.20}$$

PRM ending inventory at the PRM warehouse must be greater than the target inventory

$$X_{u}^{\text{PRM,W}}-R_{u}^{\text{PRM,W}}+f_{u}^{\text{PRM,W}}-\sum_{i\in I}\left(f_{i,u}^{\text{PRM,WP}}\right)\ge 0,\quad u\in U. \tag{4.21}$$

PRM ending inventory at each of the plants must be greater than the target inventory

$$f_{i,u}^{\text{PRM,c}}\le X_{i,u}^{\text{PRM,P}}+f_{i,u}^{\text{PRM,P}}+f_{i,u}^{\text{PRM,WP}}-R_{i,u}^{\text{PRM,P}},\quad i\in I,\ u\in U. \tag{4.22}$$

FP shipped to customers at the regional markets must meet the demand requirements in place at each location and for each FP grade

$$\sum_{(i,k)\in\Theta}f_{i,k,v}^{\text{FP,PD}}+\sum_{(j,k)\in\Pi}f_{j,k,v}^{\text{FP,WD}}\ge D_{v,k}^{\min},\quad k\in K,\ v\in V. \tag{4.23}$$

The determined capacity, which is the first-stage decision variable must be lower than the maximum allowable capacity limit

$$z_i \le Z_i^{\max}, \quad i \in I. \tag{4.24}$$

Production of FP at each of the production plants must be lower than the determined capacity

$$\sum_{(i,v)\in\Psi} f_{i,v}^{\mathrm{FP,p}} \le z_i, \quad i \in I. \tag{4.25}$$

Formulations (S) and (AARS_IN) are provided within the Appendix E. The parameter values used for the case study are listed in Appendix F.

4.2 Case Studies

As was the case in Chapter 3, the uncertain parameters are assumed to be independently and uniformly distributed. The same method is again used in constructing the scenarios for the formulations. The case study problem was modeled using GAMS 23.9.2, and solved on a machine with 3.40 GHz CPU and Linux operating system using CPLEX 12.4. A relative termination criterion of 10^{-4} was used for all the problems.

4.2.1 Results and Discussion – Uncertain Case A

In this case, the lower demand limits at the regional markets are used as the uncertain parameters, and have nominal values represented by $D_{v,k}^{\min(a)}$ which are shown in Appendix D. The ranges of ±40% for locations 1, 2 and 3, and ±30% for locations 4 and 5 are used and there are no penalty costs used in this case. Production of the final product cannot be lower than the demand limits, or the result is infeasible. The uncertainty regions are defined in Appendix E.

Table 4.2 summarizes the results of the expected value formulation (EV), formulation (S) and affinely adjustable robust scenario formulation (AARS_IN) (with uncertainty bounded by the infinity norm). The results include formulation sizes, solution times, optimal decisions obtained (i.e., capacity results), predicted expected profits and achieved expected profits. The predicted expected profits are predicted by the formulation if the first-stage decisions are implemented. The achieved expected profits are the profits that can actually be achieved by implementation of the first-stage variables. To estimate the achieved expected profits, the expected second-stage cost is approximated over a large number of uncertainty realizations. In this chapter, 99 realizations of each uncertain parameter were sampled for the estimation of the achieved expected profits for each case study.

In Table 4.2, formulation (EV) and formulation (S) with 9 scenarios both lead to an infeasible solution, whereas formulation (AARS_IN) leads to a result that is feasible and optimal. The difference in results is that formulation (AARS_IN) predicts a higher capacity for plants 2, 3 and 4 which results in the demands satisfied for each of the final product grades. When the number of scenarios is increased to 1225 for formulation (S), the result is still an infeasible solution and requires 37 minutes to solve. Formulation (AARS_IN) considering 9 scenarios requires only 6 minutes reaching the optimal solution.

Table 4.2 Solution results of the industrial chemical case study for uncertain case A

	Formulation[a]			
	EV	S		AARS_IN
Number of Scenarios	1	9	1,225	9
Number of Variables	2,335	20,927	2,846,911	73,901
Number of Constraints	1,021	9,101	1,237,261	132,941
Solution Time (s)	0.09	0.51	2,245.53	346.72
Capacity[b] at Location(kt)				
1	8870	7890	7940	7400
2	880	1060	1160	1170
3	440	530	570	580
4	2120	2240	2130	2270
5	440	1010	1430	1310
Predicted Expected Profits[c] (billion $)	22.03	21.86	21.73	21.70
Achieved Expected Profits[d] (billion $)	Infeasible	Infeasible	Infeasible	21.71

Note: [a] EV, expected value formulation; S, scenario formulation; AARS_IN, affinely adjustable robust scenario formulation. [b] Allowable capacities at each processing plants. [c] Expected profit predicted by the formulation at its solution. [d] Expected profit that can be achieved with the obtained capacities, as estimated using 99^2 = 9801 sampled uncertainty realizations.

4.2.2 Results and Discussion – Uncertain Case B

In this case, the lower demand limits for the final product are again assumed to be uncertain; specifically the nominal demand values for regional markets 1, 2, and 3 have a range of ±40%, and regional markets 4 and 5 have a range of ±30%. The nominal demand values are again represented by $D_{v,k}^{\min(a)}$. Penalty costs, $C^{pen} = 200{,}000\$/\mathrm{t}$, are introduced for this case in which the lower demand limits are not satisfied. This is included so that feasible solutions can be obtained, even if the demand limits are not satisfied. A high penalty cost is used to reflect the market share that is lost by not meeting the customer demands. To calculate the total penalty costs, the following equation is used

$$c_{v,k}^{\mathrm{pen}} = y_{v,k} \cdot C^{\mathrm{pen}}, \quad v \in V, k \in K, \tag{4.26}$$

where $y_{v,k}$ is introduced as a second-stage variable to represent the amount of final products that cannot meet the demand requirements. Constraint (4.23) is then updated to consider this new variable

$$y_{v,k} + \sum_{(i,k)\in\Theta} f_{i,k,v}^{\mathrm{FP,PD}} + \sum_{(j,k)\in\Pi} f_{j,k,v}^{\mathrm{FP,WD}} \geq D_{v,k}^{\min}, \quad k \in K,\ v \in V. \tag{4.27}$$

The objective function is also updated to include the penalty costs

$$\max \sum_{i\in I} \left[\left(\left(\mathrm{Revenue}_i - \mathrm{TC}_i\right)\left(1 - r_i^{\mathrm{inc}}\right) - \sum_{v\in V} \sum_{k\in K} c_{v,k}^{\mathrm{pen}} \right) \right] \tag{4.28}$$

In Table 4.3, it can be seen that formulation (EV) obtains the worst achieved expected profits, although it predicts the highest profits. This is due to its allocation of a high capacity in plant 1, which will allow producing final product grades which are highly profitable. However, from the lower capacity predictions for plants 2, 3, 4, and 5, the penalty costs will be very high for not meeting the demand requirements of numerous final product grades. Formulation (S) considering 9 scenarios, allocates more capacity to plants 2, 3, 4 and 5, which causes a reduction in the penalty costs leading to better achieved expected profits. However, it still overestimates the predicted expected profits, due to its inability to consider the entire range for uncertainty realizations. Formulation (AARS_IN) provides a nearly perfect prediction of expected profits from considering 9 scenarios. The capacity is assigned to each plant in a manner that reduces the total amounts of capacity and penalty costs.

Table 4.3 Solution results of the industrial chemical case study for uncertain case B

	Formulation[a]		
	EV	S	AARS_IN
Number of Scenarios	1	9	9
Number of Variables	2,452	21,972	77,016
Number of Constraints	1,023	9,111	137,091
Solution Time (s)	0.09	0.53	340.24
Capacity[b] at Location (kt)			
1	8870	7890	7400
2	880	1060	1170
3	440	530	580
4	2120	2240	2270
5	440	1010	1310
Predicted Expected Profits[c] (billion $)	22.03	21.86	21.69
Achieved Expected Profits[d] (billion $)	-31.55	19.57	21.71

Note: [a] EV, expected value formulation; S, scenario formulation; AARS_IN, affinely adjustable robust scenario formulation. [b] Allowable capacities at each processing plants. [c] Expected profit predicted by the formulation at its solution. [d] Expected profit that can be achieved with the obtained capacities, as estimated using 99^2 = 9801 sampled uncertainty realizations.

4.2.3 Results and Discussion – Uncertain Case C

In this case, the lower demand limits are again used as the uncertain parameters. The nominal values are increased and are shown by $D_{v,k}^{\min(b)}$, and again have a range of ±40% for locations 1, 2 and 3, and ±30% for locations 4 and 5. A penalty cost is no longer used in this case, as it is assumed that production below the demand limits is not allowed. The parameters were selected such that no feasible operation exists, as the goal is to investigate whether the optimization formulations can identify this infeasibility.

Table 4.4 summarizes the results of each formulation. Formulation (EV) and formulation (S) considering 9 scenarios do not identify the infeasibility and report infeasible first-stage decisions. When the number of scenarios addressed is increased to 1225, the S formulation still does not indicate infeasibility, and requires 41 minutes to solve. Formulation (AARS_IN) is able to indicate the infeasibility of the problem using 9 scenarios and within a short time. This demonstrates that the robust scenario formulation can report infeasibility effectively and efficiently when a feasible solution does not exist.

Table 4.4 Solution results of the industrial chemical case study for uncertain case C

	Formulation[a]			
	EV	S		AARS_IN
Number of Scenarios	1	9	1,225	9
Number of Variables	2,335	20,927	2,846,911	73,901
Number of Constraints	1,021	9,101	1,237,261	132,941
Solution Time (s)	0.12	0.49	2,466.30	433.51
Capacity[b] at Location (kt)				
1	6810	5350	4670	-
2	1330	1600	1750	
3	660	800	870	
4	2284	2600	2810	
5	1630	2350	2600	
Predicted Expected Profits[c] (billion $)	20.97	20.47	20.19	Infeasibility indicated
Achieved Expected Profits[d] (billion $)	Infeasible	Infeasible	Infeasible	-

Note: [a] EV, expected value formulation; S, scenario formulation; AARS_IN, affinely adjustable robust scenario formulation. [b] Allowable capacities at each processing plants. [c] Expected profit predicted by the formulation at its solution. [d] Expected profit that can be achieved with the obtained capacities, as estimated using 99^2 = 9801 sampled uncertainty realizations.

In this chapter, the mathematical model for an industrial chemical supply chain problem was formulated from data provided from DuPont. The problem modeled involves 55 grades of primary raw materials that are converted to 23 grades of final products at five plant facilities. From the plants, the final products can either be transported to regional warehouses for additional storage or to customers at five regional markets. With the determined capacities at each of the plants being the first-stage variables, the goal was to maximize profits for the supply chain system considering demand uncertainties. After completion of the mathematical model, the classical scenario approach and the affinely adjustable robust scenario formulation were used to solve three uncertain cases. The results demonstrate that the affinely adjustable robust scenario formulation can effectively avoid infeasibility, can report infeasibility (for cases in which no feasible solution can be obtained), and can obtain optimal solution with a small number of scenarios.

Chapter 5

Application of Dantzig-Wolfe Decomposition Algorithm

Since the AARS_IN formulation is of a greater size than the scenario formulation (using the same number of scenarios), it becomes even more important to consider the use of a decomposition algorithm to improve the computing times of generating the optimal solution. The Dantzig-Wolfe algorithm developed by Dantzig and Wolfe (1960) was chosen to be studied for this thesis, as it has not received much attention for solving two-stage stochastic problems within the PSE community. In section 5.1, the Dantzig-Wolfe decomposition algorithm will be described for solving a class of decomposable optimization problems. Section 5.2 will introduce an alternative formulation of this class of optimization problems, resulting in a different set of Dantzig-Wolfe subproblems. Lastly, section 5.3 will show and discuss computational results.

5.1 Dantzig-Wolfe Decomposition Algorithm for Stochastic Problems

This section describes the Dantzig-Wolfe subproblems and solution procedure for decomposable optimization problems in the following form

Problem (SP1)

$$\min c_0^T x_0 + \sum_{\omega=1}^{s} c_\omega^T x_\omega \tag{5.1}$$

$$s.t.\ \ A_0 x_0 + A_\omega x_\omega \le b,\ \ \omega = 1,\dots s, \tag{5.2}$$

$$x_0 \in X_0 = \left\{ x_0 \in \mathbb{R}^{n_{x0}} : B_0 x_0 \le d_0 \right\}, \tag{5.3}$$

$$x_\omega \in X_\omega = \left\{ x_\omega \in \mathbb{R}^{n_x} : B_\omega x_\omega \le d_\omega \right\},\ \ \omega = 1,\dots,s. \tag{5.4}$$

Note the formulations (S), (NRS_IN), and (AARS_IN) discussed in Chapter 3 all exhibit the structure of Problem (SP1). Here, x_0 would represent the first-stage design decisions and x_ω would represent the second-stage operational decisions. The first-stage variables could involve integer variables. However, when integer variables are involved, the solution obtained by Dantzig-Wolfe decomposition may be suboptimal due to a lack of strong duality (Dantzig and Wolfe, 1960). The typical strategy for deriving the master problem, involves adding multiple columns corresponding to a scenario at each iteration of the algorithm (Birge, 1985). For this Dantzig-Wolfe decomposition procedure, the set X_ω can be defined as follows

$$X_\omega = \left\{ x_\omega \in \mathbb{R}^{n_x} : x_\omega = \sum_{j \in J} \lambda_\omega^j x_\omega^j, \lambda_\omega^j \geq 0, \sum_{j \in J} \lambda_\omega^j = 1, \forall j \in J \right\}, \ \forall \omega \in \{1, ..., s\}. \tag{5.5}$$

The master problem for Problem (SP1) can now be written as

Problem (MP1)

$$\min c_0^T x_0 + \sum_{\omega=1}^{s} c_\omega^T \left(\sum_{j \in J} \lambda_\omega^j x_\omega^j \right) \tag{5.6}$$

$$s.t. \ A_0 x_0 + A_\omega \left(\sum_{j \in J} \lambda_\omega^j x_\omega^j \right) \leq b, \ \omega = 1, ..., s, \tag{5.7}$$

$$\sum_{j \in J} \lambda_\omega^j = 1, \ \omega = 1, ..., s, \tag{5.8}$$

$$\lambda_\omega \geq 0, \ \omega = 1, ..., s, \tag{5.9}$$

$$x_0 \in X_0. \tag{5.10}$$

5.1.1 Restricted Master Problem and Pricing Problems

The master problem considers the set of all extreme points, which will make it a very large problem. To reduce the size of the master problem, the following set is introduced $J^k = \{1,...,N_k\} \subset J$, so that only a subset of extreme points are considered. This restricted master problem is shown by

Problem (RMP1)

$$\min c_0^T x_0 + \sum_{\omega=1}^{s} c_\omega^T \left(\sum_{j \in J^k} \lambda_\omega^j x_\omega^j \right) \tag{5.11}$$

$$s.t. \;\; A_0 x_0 + A_\omega \left(\sum_{j \in J^k} \lambda_\omega^j x_\omega^j \right) \le b, \;\; \omega = 1,...,s, \tag{5.12}$$

$$\sum_{j \in J^k} \lambda_\omega^j = 1, \;\; \omega = 1,...,s, \tag{5.13}$$

$$\lambda_\omega \ge 0, \;\; \omega = 1,...,s, \tag{5.14}$$

$$x_0 \in X_0. \tag{5.15}$$

The optimal objective value is denoted as obj_{RMP}. The Lagrange multipliers from constraint (5.12), which are denoted by π_ω^k, are used to construct the pricing problems. The subset of extreme points is generated through solving pricing problems, which are decomposed into a set of pricing subproblems and are solved for $\omega = 1,...,s$

Problem (PP1$_\omega$)

$$\min \left(c_\omega^T + \left(\pi_\omega^k \right)^T A_\omega \right) x_\omega \tag{5.16}$$

$$s.t. \;\; x_\omega \in X_\omega. \tag{5.17}$$

Here, the optimal objective value is denoted as $obj_{PP_{\omega}^{k}}$ for $\omega = 1,...,s$. The subproblems are solved in order to add promising extreme points to the restricted master problem. Solving the restricted master problem and the corresponding subproblems is known as phase 2 of the Dantzig-Wolfe algorithm.

5.1.2 Phase 1 Feasibility Problem

The first step of the algorithm is to obtain an initial set of extreme points to be used. To do so, an initial phase 1 feasibility problem is solved

Problem (FP1)

$$\min \sum_{\omega=1}^{s} \sum_{t=1}^{m_0} y_{t,\omega} \tag{5.18}$$

$$s.t.\ \ A_0 x_0 + A_{\omega} x_{\omega} \leq b + y_{\omega},\ \ \omega = 1,...,s, \tag{5.19}$$

$$y_{\omega} \geq 0,\ \ \omega = 1,...,s, \tag{5.20}$$

$$x_0 \in X_0, \tag{5.21}$$

$$x_{\omega} \in X_{\omega},\ \ \omega = 1,...,s. \tag{5.22}$$

Here the slack variables are denoted by $y_{\omega} = \left(y_{1,\omega},..., y_{m_0,\omega} \right) \in \mathbb{R}^{m_0}$, $\forall \omega = 1,...s$. The corresponding pricing problems would have the following form and would be required to be solved for $\omega = 1,...,s$

Problem (SFP1$_{\omega}$)

$$\min \left(\pi_{\omega}^{k} \right)^{T} x_{\omega} \tag{5.23}$$

$$s.t.\ \ x_{\omega} \in X_{\omega}. \tag{5.24}$$

5.1.3 Termination Criteria

For this phase 1 problem, the convergence criterion is simply $\sum_{\omega=1}^{s}\sum_{t=1}^{m_0} y_{t,\omega} \le \varepsilon_1$. If this is satisfied, then a basic feasible solution is obtained for the phase 2 problem. Every iteration for the phase 2 problem, the upper and lower bounds are updated based on the following

$$UBD = obj_{RMP}, \tag{5.25}$$

$$LBD = obj_{RMP} + \sum_{\omega=1}^{s}\left(obj_{PP_{\omega}^{k}} - \mu_{\omega}^{k}\right). \tag{5.26}$$

Here, μ_{ω}^{k} is the multiplier obtained from constraint (5.13) of the restricted master problem. The tolerance criterion for the phase 2 problem is

$$UBD - LBD \le \varepsilon_2 \cdot \left|UBD\right|. \tag{5.27}$$

If the condition is met, then the optimal solution $\left(x_0^k, \sum_{j\in J^k} \lambda_1^k x_1^j, \ldots, \sum_{j\in J^k} \lambda_s^k x_s^j\right)$ is returned.

To summarize the Dantzig-Wolfe algorithm, the following steps are taken as outlined in Table 5.1.

Table 5.1 Dantzig-Wolfe decomposition algorithm for problem SP1

(1) **Initialization (Phase 1):** Select a large positive number M, such that $UBD = M$ and $LBD = -M$. Set the tolerance ε and initial iteration counter as $k = 1$. Solve the phase 1 feasibility problem (problem FP1 and SFP1$_{\omega}$) to obtain the initial set of extreme points for when the convergence criterion, $\sum_{\omega=1}^{s}\sum_{t=1}^{m_0} y_{t,\omega} \le \varepsilon_1$, is satisfied. If the convergence criterion cannot be met, then the problem is infeasible.

(2) Restricted Master Problem: Solve the phase 2 restricted master problem and set $UBD = obj_{RMP}$. Set the Lagrange multipliers $\pi_1^k,...,\pi_\omega^k$, $\mu_1^k,...,\mu_s^k$ from the constraints of the problem.

(3) Pricing Problem: For all $\omega = \{1,...,s\}$, solve the pricing problems to obtain the optimal solutions. Set $LBD = obj_{RMP} + \sum_{\omega=1}^{s}\left(obj_{PP_\omega^k} - \mu_\omega^k\right)$.

(4) Termination Check: If $UBD - LBD \le \varepsilon_2 \cdot |UBD|$ return $\left(x_0^k, \sum_{j \in J^k} \lambda_1^k x_1^j, ..., \sum_{j \in J^k} \lambda_s^k x_s^j\right)$ as the optimal solution. Otherwise set $k = k + 1$ and repeat steps 2-4.

5.2 Alternative Formulation for Dantzig-Wolfe Decomposition

Using the Dantzig-Wolfe algorithm for formulation (SP1) leads to suboptimal solutions, which are much worse than the true solution. A second approach to apply the Dantzig-Wolfe decomposition was studied to see if more favourable results could be achieved. The Dantzig-Wolfe procedure is similar, but the subproblems to be solved are different than those described in section 5.1. In this this Problem (SP1) is reformulated as

Problem (SP2)

$$\min c_0^T x_0 + \sum_{\omega=1}^{s} c_\omega^T x_\omega \tag{5.28}$$

$$s.t.\ \ A_0 x_{0,\omega} + A_\omega x_\omega \le b,\ \ \omega = 1,...,s, \tag{5.29}$$

$$x_{0,\omega} \in X_{0,\omega} = \left\{ x_{0,\omega} \in \mathbb{R}^{n_{x_0}} : B_0 x_{0,\omega} \le d_0 \right\}, \tag{5.30}$$

$$x_\omega \in X_\omega = \left\{ x_\omega \in \mathbb{R}^{n_x} : B_\omega x_\omega \le d_\omega \right\},\ \ \omega = 1,...,s, \tag{5.31}$$

$$x_0 = x_{0,\omega},\ \ \omega = 1,...,s. \tag{5.32}$$

The key difference of the alternative formulation is to introduce $x_{0,\omega}$ which represents the first stage-variables and constraint (5.32) to ensure that the capacities will be the same for each scenario. The second-stage variables are again represented by x_ω. For this Dantzig-Wolfe decomposition procedure, the set $X_{0,\omega}$ can be defined as follows

$$X_{0,\omega} = \left\{ x_{0,\omega} \in \mathbb{R}^{n_{x_0}} : x_{0,\omega} = \sum_{j \in J} \lambda_\omega^j x_{0,\omega}^j, \lambda_\omega^j \ge 0, \sum_{j \in J} \lambda_\omega^j = 1, \forall j \in J \right\},\ \ \forall \omega \in \{1,...,s\}. \tag{5.33}$$

The master problem is now formulated

Problem (MP2)

$$\min \sum_{\omega=1}^{s} c_{0,\omega}^{T}\left(\sum_{j\in J}\lambda_{\omega}^{j}x_{0,\omega}^{j}\right)+\sum_{\omega=1}^{s}c_{\omega}^{T}\left(\sum_{j\in J}\lambda_{\omega}^{j}x_{\omega}^{j}\right) \tag{5.34}$$

$$s.t.\;\; x_0=\sum_{j\in J}\lambda_{\omega}^{j}x_{0,\omega}^{j},\;\; \omega=1,\ldots,s, \tag{5.35}$$

$$\sum_{j\in J}\lambda_{\omega}^{j}=1,\;\; \omega=1,\ldots,s, \tag{5.36}$$

$$\lambda_{\omega}\geq 0,\;\; \omega=1,\ldots,s. \tag{5.37}$$

5.2.1 Restricted Master Problem and Pricing Problems

To reduce the size of the master problem, the following set is again introduced $J^k=\{1,\ldots,N_k\}\subset J$. This restricted master problem is shown by

Problem (RMP2)

$$\min \sum_{\omega=1}^{s} c_{0,\omega}^{T}\left(\sum_{j\in J^k}\lambda_{\omega}^{j}x_{0,\omega}^{j}\right)+\sum_{\omega=1}^{s}c_{\omega}^{T}\left(\sum_{j\in J^k}\lambda_{\omega}^{j}x_{\omega}^{j}\right) \tag{5.38}$$

$$s.t.\;\; x_0=\left(\sum_{j\in J^k}\lambda_{\omega}^{j}x_{0,\omega}^{j}\right),\;\; \omega=1,\ldots,s, \tag{5.39}$$

$$\sum_{j\in J^k}\lambda_{\omega}^{j}=1,\;\; \omega=1,\ldots,s, \tag{5.40}$$

$$\lambda_{\omega}\geq 0,\;\; \omega=1,\ldots,s. \tag{5.41}$$

Here the optimal objective value is denoted by obj_{RMP}. The Lagrange multipliers from constraint (5.39), which are denoted by π_{ω}^{k}, are used to construct the pricing problems. The pricing problems are decomposed into a set of pricing subproblems and are solved for $\omega = 1,...,s$

Problem (PP2$_{\omega}$)

$$\min\left(c_{0,\omega}^{T} + \left(\pi_{\omega}^{k}\right)^{T}\right)x_{0,\omega} + c_{\omega}^{T}x_{\omega} \tag{5.42}$$

$$s.t.\ \ A_0 x_{0,\omega} + A_{\omega}x_{\omega} \le b,\ \ \omega = 1,...,s, \tag{5.43}$$

$$x_{\omega} \in X_{\omega}, \tag{5.44}$$

$$x_{0,\omega} \in X_{0,\omega}. \tag{5.45}$$

The optimal objective values for this formulation are denoted by $obj_{PP_{\omega}^{k}}$ for $\omega = 1,...,s$.

5.2.2 Phase 1 Feasibility Problem

The initial phase 1 feasibility problem is formulated as

Problem (FP2)

$$\min \sum_{\omega=1}^{s}\sum_{t=1}^{n_{x0}} y_{t,\omega} \tag{5.46}$$

$$s.t.\ \ x_0 = \left(\sum_{j\in J^k} \lambda_{\omega}^{j} x_{0,\omega}^{j}\right) + y_{\omega},\ \ \omega = 1,...,s, \tag{5.47}$$

$$y_{\omega} \ge 0,\ \ \omega = 1,...,s. \tag{5.48}$$

Here the slack variables are denoted by $y_{\omega} = \left(y_{1,\omega},...,y_{n_{x_0},\omega}\right) \in \mathbb{R}^{n_{x0}}$, $\forall \omega = 1,...s$. The corresponding subproblems would have the following form and would be required to be solved for $\omega = 1,...,s$

Problem (SFP2$_\omega$)

$$\min\left(\pi_\omega^k\right)^T x_{0,\omega} \tag{5.49}$$

$$s.t.\ A_0 x_{0,\omega} + A_\omega x_\omega \le b, \tag{5.50}$$

$$x_\omega \in X_\omega, \tag{5.51}$$

$$x_{0,\omega} \in X_{0,\omega}. \tag{5.52}$$

5.2.3 Termination Criteria

For the phase 1 problem, the convergence criterion is $\sum_{\omega=1}^{s}\sum_{t=1}^{n_{x_0}} y_{t,\omega} \le \varepsilon_1$. If this is satisfied, then a basic feasible solution is obtained for the phase 2 problem. For every iteration, the upper and lower bounds are updated based on the following

$$UBD = obj_{RMP}, \tag{5.53}$$

$$LBD = obj_{RMP} + \sum_{\omega=1}^{s}\left(obj_{PP_\omega^k} - \mu_\omega^k\right). \tag{5.54}$$

Here, μ_ω^k is the multiplier obtained from constraint (5.40) of the restricted master problem. The tolerance criterion for the phase 2 problem is

$$UBD - LBD \le \varepsilon_2 \cdot \left|UBD\right|. \tag{5.55}$$

If the condition is satisfied, then the optimal solution $\left(\sum_{j\in J^k}\lambda_1^k x_{0,1}^j, \ldots, \sum_{j\in J^k}\lambda_s^k x_{0,s}^j, \sum_{j\in J^k}\lambda_1^k x_1^j, \ldots, \sum_{j\in J^k}\lambda_s^k x_s^j\right)$ is returned.

5.3 Computational Results

The Dantzig-Wolfe decomposition procedure described in section 5.1 is referred to as DWD1 and the procedure described in section 5.2 is referred to as DWD2. These algorithmic procedures were applied to formulation (AARS_IN) for uncertain case A of the industrial chemical supply chain problem described in Chapter 4. Though the capacity will now be represented by integer variables, such that the solution of the first-stage variables will be a multiple of 500. This is done so that the capacity cannot simply be set to an arbitrary number. This means that the first-stage variables denoted by x_0 within the DWD1 and DWD2 procedures will be represented by $500 \cdot z_i^{\text{int}}$ for the industrial case study as opposed to z_i. Formulation (AARS_IN) is used, with the goal of improving its computing times when compared to the CPLEX solver. The pricing problems solved were LP formulations and the restricted master problems were MILP formulations. Note that neither the DWD1 nor DWD2 procedure will guarantee achieving the optimal solution because the loss of strong duality due to the presence of the integer variables. The major purpose of this case study is to see whether the DWD procedure can effectively generate satisfactory solutions. The full list of equations for the DWD1 and DWD2 subproblems are provided in Appendix G.

The case study problems were modeled using GAMS 24.1.1, and solved on a machine with 3.20 GHz CPU and Linux operating system using CPLEX 12.5.0.1 with a tolerance of 10^{-3}. A relative termination criterion of $\varepsilon_1 = 10^{-3}$ was used for the phase 1 termination criteria and $\varepsilon_2 = 10^{-3}$ was used for the phase 2 termination criteria of the DWD procedures.

In the case studies, DWD1 tends to obtain a feasible solution quickly but generates poor sub-optimal solutions. The DWD2 procedure would fail in generating an initial feasible solution, but if a feasible solution is provided, it tends to generate high quality sub-optimal/optimal solutions. This motivated the idea of combining both the DWD1 and DWD2 procedures. The sub-optimal solutions quickly obtained by DWD1 are used for the phase 2 procedure of DWD2.

Shown in Table 5.2 are the optimal objective values for the CPLEX solver, the DWD1 and DWD2 algorithms for 9, 25, 49, 81, 121, and 169 scenarios. Also displayed are the required computing times for CPLEX, DWD1, DWD2, and the total time summed for the two decomposition algorithms. It is important to note the optimal objective values obtained from the DWD1 algorithm differ from the CPLEX results. Whereas, the optimal solution can be obtained from the DWD2 algorithm, except when using 49 scenarios where a high quality sub-optimal solution is obtained. This shows the significance of combining the DWD1 and DWD2 procedures to obtain higher quality solutions.

Table 5.2 Dantzig-Wolfe decomposition results

Scenarios	9	25	49	81	121	169
CPLEX objective (billion $)	21.22	21.23	21.23	21.23	21.23	21.23
CPLEX time (s)	4,087	17,173	47,533	73,630	111,593	318,567
DWD1[a] objective (billion $)	17.80	17.81	18.54	19.21	19.21	19.21
DWD2[b] objective (billion $)	21.21	21.22	20.92	21.22	21.22	21.22
DWD1 time (s)	90	343	1,319	5,472	9,138	10,424
DWD2 time (s)	3,940	11,486	22,226	40,998	53,296	75,778
Total DWD time (s)	4,030	11,829	23,545	46,470	62,434	86,202

Note: [a] Algorithm DWD1 solves problems FP1 and SFP1$_\omega$ until phase 1 convergence is satisfied, and problems RMP1 and PP1$_\omega$ until phase 2 convergence is satisfied. [b] Algorithm DWD2 uses the solution from DWD1 as the initial feasible solution and solves problems RMP2 and PP2$_\omega$ until phase 2 convergence is satisfied.

Also from Table 5.2, it can be seen that with 25 scenarios, it is beneficial to use the decomposition formulations as 3.3 hours is required compared to 4.8 hours for CPLEX. As the number of scenarios increase, the CPLEX time increases dramatically. The decomposition time is more reasonable as only 24 hours are required for the 169 scenarios. This shows the importance of using the DWD1 procedure together with DWD2, in order to achieve a high quality solution more quickly than the state-of-the-art CPLEX solver.

In this chapter, two approaches to the Dantzig-Wolfe decomposition algorithm were developed to exploit the decomposable structure of the formulation. The sub-optimal solutions generated from the first procedure were then used for the initial extreme points required for phase 2 of the second procedure. This led to obtaining high quality sub-optimal/optimal solutions. From comparing the results of the Dantzig-Wolfe procedure to the CPLEX, it was found that at 25 scenarios the computing times for the Dantzig-Wolfe procedures were faster than that of CPLEX. As the number of scenarios increase, the benefit of using the Dantzig-Wolfe decomposition algorithm over the CPLEX solver grows dramatically.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, a novel framework is proposed to solve two-stage stochastic programs with recourse that come from strategic supply chain optimization under uncertainty. The framework integrates the classical scenario approach, which commonly provides good optimality and the robust approach, which can guarantee feasibility of a problem (if a feasible solution exists). Two robust formulations were generated, namely the naïve robust scenario formulation and the affinely adjustable robust formulation. In the two hybrid formulations, a scenario represents a group of uncertainty realizations instead of a single realization and it was shown that both formulations can be transformed into tractable optimization problems if uncertainty is assumed to be bounded by the infinity-norm. The formulations were applied to a farm planning problem and an energy and bioproduct supply chain optimization problem. The results demonstrate that the proposed formulations can effectively avoid infeasibility of the problem or report infeasibility for a situation when no feasible solution exists. They also outperform the classical scenario approach by generating the optimal solutions in a smaller number of scenarios and shorter solution times. The affinely adjustable robust scenario formulation outperforms the naïve robust scenario formulation, as the latter is often overly conservative and gives poor performance predictions.

Next, with collaboration from DuPont, a mathematical model for an industrial chemical supply chain problem was formulated. The problem modeled involves 55 grades of primary raw materials that are converted to 23 grades of final products at five plant facilities which all have raw material and product storage warehouses. From the plants, the final products can either be transported to regional warehouses for additional storage or to customers at five regional markets.

The raw materials consumed are subject to turndown limits, waste limits, impurity restrictions, etc. whereas the final products must meet demand requirements at the five regional markets. With the determined capacities at each of the plants as the first-stage variables, the goal was to maximize profits for the supply chain system considering demand uncertainties. After completion of the mathematical model, the classical scenario approach and the affinely adjustable robust scenario formulation were used to solve three uncertain cases. The results demonstrate again that the affinely adjustable robust scenario formulation can effectively avoid infeasibility, can report infeasibility (case in which no feasible solution can be obtained), and can obtain optimal solution with a small number of scenarios.

Due to the large-scale nature of the industrial case study problem, there is a need to consider decomposition techniques for the affinely adjustable robust scenario formulation. Two approaches to the Dantzig-Wolfe decomposition algorithm were developed to exploit the decomposable structure of the formulation. The sub-optimal solutions generated from the first procedure were then used for the phase 2 stage of the second procedure, which led to obtaining high quality sub-optimal/optimal solutions. From comparing the results of the Dantzig-Wolfe procedure to the CPLEX solver at 9, 25, 49, 81, 121, and 169 scenarios, it was found that at 25 scenarios the computing times for the Dantzig-Wolfe procedures were faster than that of CPLEX. As the number of scenarios increase, the benefit of using the Dantzig-Wolfe decomposition algorithm over the state-of-the-art CPLEX solver grows dramatically.

6.2 Future Work

A weakness to the Dantzig-Wolfe decomposition algorithm is that for MILP problems, it cannot guarantee to converge to the optimal solution. In terms of possible future work, one area is to use Benders decomposition for the industrial chemical supply chain problem. The benefit of using Benders decomposition is not only that may it provide better solution times than the Dantzig-Wolfe formulations, but it can also be guaranteed to converge to the optimal solution for problems involving first-stage integer variables.

Further work may involve addressing ellipsoidal uncertainty regions. Uncertainty should be addressed with an ellipsoidal region if joint confidence regions that are normally distributed are used to characterize uncertain parameters. This will change the reformulation of the robust scenario formulations developed in the thesis. It is known that, if an ellipsoidal uncertainty region is used then uncertainty can be represented by the 2-norm and the resulting robust scenario formulations can be transformed into a deterministic formulation involving the 2-norm (Bertsimas et al., 2004). A challenging issue with incorporating ellipsoidal uncertainty would be how to generate reasonable uncertainty subregions for each scenario.

With collaboration from DuPont, there may be a desire to consider multi-period operation of the supply chain. This multi-period problem would then be an operational problem, which only contains continuous variables and is decomposable over different time periods. It is well known that Dantzig-Wolfe decomposition is ideal for solving this type of optimization problem, while Benders decomposition is not.

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Appendix A
Proof of Proposition 1

Proposition 1: *From the optimization problem*

$$\max_{\phi} \quad \left(\phi - \bar{\phi}\right)^T x \tag{A.1}$$

$$s.t. \quad \left\| M\left(\phi - \bar{\phi}\right) \right\|_\infty \leq \delta, \tag{A.2}$$

where M is invertible and $\delta \geq 0$, the optimal objective value is $\delta \left\| (M^{-1})^T x \right\|_1$.

Let $M\left(\phi - \bar{\phi}\right) = z = \left(z_1, \cdots, z_n\right)$, $(M^{-1})^T x = r = \left(r_1, \cdots, r_n\right)$, then the problem to solve becomes

$$\max_{z} \quad z^T r \tag{A.3}$$

$$s.t. \quad \|z\|_\infty \leq \delta. \tag{A.4}$$

A feasible solution of this optimization problem is

$$z_i = \begin{cases} \delta, & \text{if} \quad r_i \geq 0, \\ -\delta, & \text{if} \quad r_i < 0. \end{cases} \tag{A.5}$$

With this solution

$$z^T r = \sum_{i=1}^{n} \delta |r_i| = \delta \|r\|_1. \tag{A.6}$$

So $\delta \|r\|_1$ is a lower bound on the optimal objective value. When $\|z\|_\infty \leq \delta$,

$$z^T r = \sum_{i=1}^{n} z_i r_i \leq \sum_{i=1}^{n} |z_i||r_i| \leq \sum_{i=1}^{n} \delta |r_i| = \delta \|r\|_1, \tag{A.7}$$

so $\delta \|r\|_1$ is an upper bound on the optimal objective value. Therefore, $\delta \|r\|_1$ is the optimal objective value of problem (A.3) with constraint (A.4). Thus, $\delta \|(M^{-1})^T x\|_1$ is the optimal objective value of problem (A.1) with constraint (A.2).

Appendix B

Farm Planning Case Study – Equations and Parameter Values

Formulation (S), formulation (NRS_IN) and formulation (AARS_IN) are provided for the farm planning problem – uncertain case A.

B.1 Scenario Formulation (S)

$$\min \quad \sum_{m\in\Omega} c_m^{pl}\cdot x_m + \sum_{s\in S}\left[\Pr_s\cdot\left(\begin{array}{l}\sum_{m\in\Omega_1}\left(c_m^{pur}\cdot y_{m,s} - c_m^{sell}\cdot w_{m,s}\right)\\ -\sum_{m\in\Omega_2}\left(c_m^{sell,h}\cdot w_{m,s}^{h} + c_m^{sell,l}\cdot w_{m,s}^{l}\right)\end{array}\right)\right] \tag{B.1}$$

subject to

$$\sum_{m\in\Omega} x_m \le L, \tag{B.2}$$

$$Y_m\cdot x_m + y_{m,s} - w_{m,s} \ge F_{m,s}, \quad m\in\Omega_1, s\in S, \tag{B.3}$$

$$w_{m,s}^{h} + w_{m,s}^{l} \le Y_m\cdot x_m, \quad m\in\Omega_2, s\in S, \tag{B.4}$$

$$w_{m,s}^{h} \le Q_m, \quad m\in\Omega_2, s\in S, \tag{B.5}$$

$$x_m \ge 0, \quad m\in\Omega, \tag{B.6}$$

$$y_{m,s}, w_{m,s} \ge 0, \quad m\in\Omega_1, s\in S, \tag{B.7}$$

$$w_{m,s}^{h}, w_{m,s}^{l} \ge 0, \quad m\in\Omega_2, s\in S. \tag{B.8}$$

Here, $s\in S$ is defined as the set of scenarios, $\Pr_s$ denotes the probability of a given scenario s occurring. The first stage variables are the amounts of land allocated to each crop denoted by x_m. The second stage variables are the amounts of corn and wheat purchased ($y_{m,s}$) and sold ($w_{m,s}$), and the amounts of sugar beets sold below ($w_{m,s}^{h}$) and above ($w_{m,s}^{l}$) the quota. The amounts of corn and wheat required to feed cattle are the uncertain parameters and is denoted by $F_{m,s}$.

B.2 Naïve Robust Scenario Formulation (NRS_IN)

The naïve robust scenario formulation is a hybrid of formulation (S) and formulation (R). The intermediate representation is shown below by formulation (NRS)

$$\min \quad \sum_{m\in\Omega} c_m^{pl}\cdot x_m + \sum_{s\in S}\left[\Pr_s \cdot \left(\begin{array}{l} \sum_{m\in\Omega_1}\left(c_m^{pur}\cdot y_{m,s} - c_m^{sell}\cdot w_{m,s}\right) \\ -\sum_{m\in\Omega_2}\left(c_m^{sell,h}\cdot w_{m,s}^{h} + c_m^{sell,l}\cdot w_{m,s}^{l}\right)\end{array}\right)\right] \tag{B.9}$$

subject to

$$\sum_{m\in\Omega} x_m \le L, \tag{B.10}$$

$$Y_m \cdot x_m + y_{m,s} - w_{m,s} \ge \max_{F_{m,s}}\left(F_{m,s} - \bar{F}_{m,s}\right) + \bar{F}_{m,s}, \qquad m\in\Omega_1,\, s\in S, \tag{B.11}$$

$$w_{m,s}^{h} + w_{m,s}^{l} \le Y_m \cdot x_m, \qquad m\in\Omega_2,\, s\in S, \tag{B.12}$$

$$w_{m,s}^{h} \le Q_m, \qquad m\in\Omega_2,\, s\in S, \tag{B.13}$$

$$x_m \ge 0, \qquad m\in\Omega, \tag{B.14}$$

$$y_{m,s},\, w_{m,s} \ge 0, \qquad m\in\Omega_1,\, s\in S, \tag{B.15}$$

$$w_{m,s}^{h},\, w_{m,s}^{l} \ge 0, \qquad m\in\Omega_2,\, s\in S. \tag{B.16}$$

Here, $s \in S$ is defined as the set of scenarios, $\Pr_s$ denotes the probability of a given scenario s occurring. With the assumption of uniform distribution, $\Pr_s = \frac{1}{|S|}, s \in S$ where $|S|$ refers to the cardinality of set S. The first stage variables are the amounts of land allocated to each crop denoted by x_m. The second stage variables are the amounts of corn and wheat purchased ($y_{m,s}$) and sold ($w_{m,s}$), and the amounts of sugar beets sold below ($w^h_{m,s}$) and above ($w^l_{m,s}$) the quota. The amounts of corn and wheat required to feed cattle are the uncertain parameters and is denoted by $F_{m,s}$.

By applying Proposition 1 to constraint (B.11), the bilevel optimization problem is reduced to a single-level problem. This problem is shown below as formulation (NRS_IN)

$$\min \quad \sum_{m \in \Omega} c^{pl}_m \cdot x_m + \sum_{s \in S} \left[\Pr_s \cdot \left(\begin{array}{l} \sum_{m \in \Omega_1} \left(c^{pur}_m \cdot y_{m,s} - c^{sell}_m \cdot w_{m,s} \right) \\ - \sum_{m \in \Omega_2} \left(c^{sell,h}_m \cdot w^h_{m,s} + c^{sell,l}_m \cdot w^l_{m,s} \right) \end{array} \right) \right] \tag{B.17}$$

subject to

$$\sum_{m \in \Omega} x_m \leq L, \tag{B.18}$$

$$Y_m \cdot x_m + y_{m,s} - w_{m,s} \geq \delta_{m,s} \cdot \left\| \left(M^{-1}_{m,s} \right)^T \right\|_1 + \left(\bar{F}_{m,s} \right), \quad m \in \Omega_1, s \in S, \tag{B.19}$$

$$w^h_{m,s} + w^l_{m,s} \leq Y_m \cdot x_m, \quad m \in \Omega_2, s \in S, \tag{B.20}$$

$$w^h_{m,s} \leq Q_m, \quad m \in \Omega_2, s \in S, \tag{B.21}$$

$$x_m \geq 0, \quad m \in \Omega, \tag{B.22}$$

$$y_{m,s}, w_{m,s} \geq 0, \qquad m \in \Omega_1, s \in S, \qquad \text{(B.23)}$$

$$w^h_{m,s}, w^l_{m,s} \geq 0, \qquad m \in \Omega_2, s \in S. \qquad \text{(B.24)}$$

The constraints containing the 1-Norm functions can be reformulated into linear equations based on the example provided in section 3.1.3. The $\left(M^{-1}_{m,s}\right)^T$ values are provided in Table B.1.

Table B.1 $\left(M^{-1}_{m,s}\right)^T$ values for the farm planning problem – uncertain case A

$\left(M^{-1}_{\text{wheat},s}\right)^T$	$1, \forall s$
$\left(M^{-1}_{corn,s}\right)^T$	$1, \forall s$

B.3 Affinely Adjustable Robust Scenario Formulation (AARS_IN)

The uncertain parameters, $F_{m,s}$, are now described by $\sum_{nu \in NU} \left(\beta_{m,s,nu} \cdot \xi_{s,nu} + \gamma_{m,s}\right)$ to separate the deterministic and uncertain elements. $nu \in NU$ is the set introduced as the number of uncertain elements. Here, $nu \in NU = \{1, 2\}$, since there are two uncertain parameters (corn required to feed cattle and wheat required to feed cattle). The values for $\beta_{m,s,nu}$ and $\gamma_{m,s}$ are shown below in Table B.2.

Table B.2 $\beta_{m,s,nu}$ and $\gamma_{m,s}$ values for the farm planning problem – uncertain case A

$\beta_{'wheat',s,'1'}$	$1, \forall s$
$\beta_{'wheat',s,'2'}$	$0, \forall s$
$\beta_{'corn',s,'1'}$	$0, \forall s$
$\beta_{'corn',s,'2'}$	$1, \forall s$
$\gamma_{m,s}$	$0, \forall s, m \in \Omega_1$

The second stage variables are now represented by affine functions of uncertain parameters, where $y_{m,s}$ in Problem (S) is replaced by $\Phi^{pur}_{m,s,nu} \cdot \xi_{s,nu} + \phi^{pur}_{m,s}$, $w_{m,s}$ is replaced by $\Phi^{sell}_{m,s,nu} \cdot \xi_{s,nu} + \phi^{sell}_{m,s}$, $w^{h}_{m,s}$ is replaced by $\Phi^{sell,h}_{m,s,nu} \cdot \xi_{s,nu} + \phi^{sell,h}_{m,s}$ and $w^{l}_{m,s}$ is replaced by $\Phi^{sell,l}_{m,s,nu} \cdot \xi_{s,nu} + \phi^{sell,l}_{m,s}$.

The intermediate formulation (AARS) formulation is given below and is a hybrid of formulation (S) and formulation (AAR)

$$\min \quad \sum_{m\in\Omega} c^{pl}_m \cdot x_m + \sum_{s\in S}\left[\Pr_s \cdot \left(\begin{array}{l}\displaystyle\sum_{m\in\Omega_1}\left(c^{pur}_m \cdot \left(\Phi^{pur}_{m,s,nu}\cdot \overline{\xi}_{s,nu} + \phi^{pur}_{m,s}\right) - c^{sell}_m \cdot \left(\Phi^{sell}_{m,s,nu}\cdot \overline{\xi}_{s,nu} + \phi^{sell}_{m,s}\right)\right) \\ \displaystyle -\sum_{m\in\Omega_2}\left(c^{sell,h}_m \cdot \left(\Phi^{sell,h}_{m,s,nu}\cdot \overline{\xi}_{s,nu} + \phi^{sell,h}_{m,s}\right) + c^{sell,l}_m \cdot \left(\Phi^{sell,l}_{m,s,nu}\cdot \overline{\xi}_{s,nu} + \phi^{sell,l}_{m,s}\right)\right)\end{array}\right)\right] \tag{B.25}$$

subject to

$$\sum_{m\in\Omega} x_m \le L, \tag{B.26}$$

$$Y_m x_m + \sum_{nu\in NU}\left(\begin{array}{l}\displaystyle\max_{\xi_{s,nu}}\left[\left(\xi_{s,nu} - \overline{\xi}_{s,nu}\right)\cdot\left(-\beta_{m,s,nu} + \Phi^{pur}_{m,s,nu} - \Phi^{sell}_{m,s,nu}\right)\right] \\ +\left(-\beta_{m,s,nu} + \Phi^{pur}_{m,s,nu} - \Phi^{sell}_{m,s,nu}\right)\cdot \overline{\xi}_{s,nu} + \phi^{pur}_{m,s} - \phi^{sell}_{m,s}\end{array}\right) \ge 0, \quad m\in\Omega_1,\ s\in S, \tag{B.27}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,h}_{m,s,nu} + \Phi^{sell,l}_{m,s,nu}\right)\cdot \overline{\xi}_{s,nu} + \phi^{sell,h}_{m,s} + \phi^{sell,l}_{m,s}\right) \le Y_m x_m, \qquad m\in\Omega_2,\ s\in S, \tag{B.28}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,h}_{m,s,nu}\right)\cdot \overline{\xi}_{s,nu} + \phi^{sell,h}_{m,s}\right) \le Q_m, \qquad m\in\Omega_2,\ s\in S, \tag{B.29}$$

$$x_m \ge 0, \qquad m\in\Omega, \tag{B.30}$$

$$\sum_{nu\in NU}\left(\max_{\xi_{s,nu}}\left[\left(\xi_{s,nu} - \overline{\xi}_{s,nu}\right)\cdot \Phi^{pur}_{m,s,nu}\right] + \Phi^{pur}_{m,s,nu}\cdot \overline{\xi}_{s,nu} + \phi^{pur}_{m,s}\right) \ge 0, \quad m\in\Omega_1,\ s\in S, \tag{B.31}$$

$$\sum_{nu\in NU}\left(\max_{\xi_{s,nu}}\left[\left(\xi_{s,nu}-\overline{\xi}_{s,nu}\right)\cdot\Phi^{sell}_{m,s,nu}\right]+\Phi^{sell}_{m,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{sell}_{m,s}\right)\geq 0,\quad m\in\Omega_1,\ s\in S, \tag{B.32}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,h}_{m,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\phi^{sell,h}_{m,s}\right)\geq 0,\quad m\in\Omega_2,\ s\in S, \tag{B.33}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,l}_{m,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\phi^{sell,l}_{m,s}\right)\geq 0,\quad m\in\Omega_2,\ s\in S. \tag{B.34}$$

By applying Proposition 1 to constraints (B.27), (B.31), and (B.32) the bilevel optimization problem is reduced to a single-level problem. This problem is shown below as formulation (AARS_IN)

$$\min\ \sum_{m\in\Omega}c^{pl}_m\cdot x_m+$$
$$\sum_{s\in S}\left[\Pr_s\cdot\left(\begin{array}{l}\sum_{m\in\Omega_1}\left(c^{pur}_m\cdot\left(\Phi^{pur}_{m,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{pur}_{m,s}\right)-c^{sell}_m\cdot\left(\Phi^{sell}_{m,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{sell}_{m,s}\right)\right)\\ -\sum_{m\in\Omega_2}\left(c^{sell,h}_m\cdot\left(\Phi^{sell,h}_{m,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{sell,h}_{m,s}\right)+c^{sell,l}_m\cdot\left(\Phi^{sell,l}_{m,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{sell,l}_{m,s}\right)\right)\end{array}\right)\right] \tag{B.35}$$

subject to

$$\sum_{m\in\Omega}x_m\leq L, \tag{B.36}$$

$$Y_m x_m+\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M^{-1}_{s,nu}\right)^T\cdot\left(-\beta_{m,s,nu}+\Phi^{pur}_{m,s,nu}-\Phi^{sell}_{m,s,nu}\right)\right\|_1\\ +\left(-\beta_{m,s,nu}+\Phi^{pur}_{m,s,nu}-\Phi^{sell}_{m,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\phi^{pur}_{m,s}-\phi^{sell}_{m,s}\end{array}\right)\geq 0,\quad m\in\Omega_1,\ s\in S, \tag{B.37}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,h}_{m,s,nu}+\Phi^{sell,l}_{m,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\phi^{sell,h}_{m,s}+\phi^{sell,l}_{m,s}\right)\leq Y_m x_m,\quad m\in\Omega_2,\ s\in S, \tag{B.38}$$

$$\sum_{nu\in NU}\left(\left(\Phi^{sell,h}_{m,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\phi^{sell,h}_{m,s}\right)\leq Q_m,\quad m\in\Omega_2,\ s\in S, \tag{B.39}$$

$$x_m\geq 0,\quad m\in\Omega, \tag{B.40}$$

$$\sum_{nu\in NU}\left(-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{m,s,nu}^{pur}\right)\right\|_{1}+\Phi_{m,s,nu}^{pur}\cdot\overline{\xi}_{s,nu}+\phi_{m,s}^{pur}\right)\geq 0,\quad m\in\Omega_{1},\, s\in S, \tag{B.41}$$

$$\sum_{nu\in NU}\left(-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{m,s,nu}^{sell}\right)\right\|_{1}+\Phi_{m,s,nu}^{sell}\cdot\overline{\xi}_{s,nu}+\phi_{m,s}^{sell}\right)\geq 0,\quad m\in\Omega_{1},\, s\in S, \tag{B.42}$$

$$\sum_{nu\in NU}\left(\left(\Phi_{m,s,nu}^{sell,h}\right)\cdot\overline{\xi}_{s,nu}+\phi_{m,s}^{sell,h}\right)\geq 0,\quad m\in\Omega_{2},\, s\in S, \tag{B.43}$$

$$\sum_{nu\in NU}\left(\left(\Phi_{m,s,nu}^{sell,l}\right)\cdot\overline{\xi}_{s,nu}+\phi_{m,s}^{sell,l}\right)\geq 0,\quad m\in\Omega_{2},\, s\in S. \tag{B.44}$$

Constraints (B.41)-(B.44) are to ensure that the affine functions for the second stage variables are non-negative. For formulation (AARS_IN), $\delta_{\omega}=\begin{pmatrix}\delta_{1,\omega}\\ \delta_{2,\omega}\end{pmatrix}$ and $\left(M_{\omega}^{-1}\right)^{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. To obtain the appropriate values from the vector and matrix, $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1}\right)^{T}$ were used as seen in the above equations. Their values are shown in Table B.3.

Table B.3 $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1}\right)^{T}$ values for the farm planning problem – uncertain case A

$\delta_{s,'1'}$	$\delta_{wheat},\forall s$
$\delta_{s,'2'}$	$\delta_{corn},\forall s$
$\left(M_{s,'1'}^{-1}\right)^{T}$	$1,\forall s$
$\left(M_{s,'2'}^{-1}\right)^{T}$	$1,\forall s$

B.4 Parameter Values

Table B.4 below displays the parameter values used for the deterministic formulation in the farm planning case study.

Table B.4 Parameters and their values for the farm planning problem

Crop, *m*,	Wheat	Corn	Sugar Beets
Total land area, L (acre)	500		
Planting Cost, c_m^{pl} ($/acre)	150	230	260
Purchase price, c_m^{pur} ($/T)	238	210	-
Selling price, c_m^{sell} ($/T)	170	150	-
Selling price (under quota), $c_m^{sell,h}$ ($/T)	-	-	36
Selling price (over quota), $c_m^{sell,l}$ ($/T)	-	-	10
Yield, Y_m (T/acre)	2.5	3	20
Reserved for feeding cattle, F_m (T)	300	340	-
Quota on production, Q_m (T)	-	-	6,000

Appendix C

Energy and Bioproduct Supply Chain Case Study – Equations

Formulation (S), formulation (NRS_IN) and formulation (AARS_IN) are provided for the energy and bioproduct supply chain – uncertain case A.

C.1 Scenario Formulation (S)

$$\max \sum_{s\in S}\left[\Pr_s \cdot \left(\begin{array}{l} \sum_{n\in N}\sum_{j^o\in J}\sum_{pp\in PP} q^{\mathrm{L3,L4}}_{n,j^o,pp,s}\cdot c^{\mathrm{price}}_{pp} + \sum_{n\in N}\sum_{j^e\in J}\sum_{pp\in PP} 0.9\cdot\left(q^{\mathrm{L3,L4}}_{n,j^e,pp,s}\cdot c^{\mathrm{price}}_{pp}\right) \\ -\sum_{i\in I}\sum_{pi\in PI} q^{\mathrm{L1}}_{i,pi,s}\cdot c_{pi} - c^{\mathrm{tr}}_s - c^{\mathrm{op}}_s - c^{\mathrm{inv}}_s \end{array}\right)\right] \tag{C.1}$$

subject to

$$\begin{aligned} c^{\mathrm{tr}}_s &= \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI} D^{\mathrm{L1,L2}}_{i,m}\cdot f^{\mathrm{road,L1,L2}}_{i,m}\cdot c^{\mathrm{tr,L1,L2}}_{pi}\cdot q^{\mathrm{L1,L2}}_{i,m,pi,s} \\ &+ \sum_{m\in M}\sum_{n\in N}\sum_{pi\in PI} D^{\mathrm{L2,L3}}_{m,n}\cdot f^{\mathrm{road,L2,L3}}_{m,n}\cdot c^{\mathrm{tr,L2,L3}}_{pi}\cdot q^{\mathrm{L2,L3}}_{m,n,pi,s} \\ &+ \sum_{n\in N}\sum_{j\in J}\sum_{pp\in PP} D^{\mathrm{L3,L4}}_{n,j}\cdot f^{\mathrm{road,L3,L4}}_{n,j}\cdot c^{\mathrm{tr,L3,L4}}_{pp}\cdot q^{\mathrm{L3,L4}}_{n,j,pp,s}, \end{aligned} \tag{C.2}$$

$$c^{\mathrm{op}}_s = \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI} c^{\mathrm{op,L2}}_{pi}\cdot q^{\mathrm{L1,L2}}_{i,m,pi,s} + \sum_{n\in N}\sum_{(pi,t)\in PT} c^{\mathrm{op,L3}}_{pi,t}\cdot q^{\mathrm{L2,L3}}_{n,pi,t,s}, \tag{C.3}$$

$$c^{\mathrm{inv}}_s = \sum_{m\in M} c^{\mathrm{fix,inv,L2}}\cdot y^{\mathrm{L2}}_m + \sum_{n\in N}\sum_{t\in T}\left(c^{\mathrm{fix,inv,L3}}_t\cdot y^{\mathrm{L3}}_{n,t} + \sum_{(pi,t)\in PT} c^{\mathrm{var,inv,L3}}_t\cdot q^{\mathrm{L2,L3}}_{n,pi,t,s}\right), \tag{C.4}$$

$$q^{\mathrm{L1}}_{i,pi,s} \le HY_{pi,s}\cdot A_{i,pi}, \qquad \forall pi\in PI, \forall i\in I, \forall s\in S, \tag{C.5}$$

$$q^{\mathrm{L1}}_{i,pi,s} = \sum_{m\in M} q^{\mathrm{L1,L2}}_{i,m,pi,s}, \qquad \forall pi\in PI, \forall i\in I, \forall s\in S, \tag{C.6}$$

$$\sum_{i\in I} q^{\mathrm{L1,L2}}_{i,m,pi,s} \le q^{\mathrm{L1,L2,UP}}_{pi}\cdot y^{\mathrm{L2}}_m, \qquad \forall m\in M, \forall pi\in PI, \forall s\in S, \tag{C.7}$$

$$\sum_{i\in I}\sum_{pi\in PI} q^{\mathrm{L1,L2}}_{i,m,pi,s} \le q^{\mathrm{L2,UP}} \cdot y^{\mathrm{L2}}_{m}, \qquad \forall m \in M, \forall s \in S, \tag{C.8}$$

$$\sum_{i\in I} q^{\mathrm{L1,L2}}_{i,m,pi,s} \cdot f^{\mathrm{conv,L2}}_{pi} = \sum_{n\in N} q^{\mathrm{L2,L3}}_{m,n,pi,s}, \qquad \forall m \in M, \forall pi \in PI, \forall s \in S, \tag{C.9}$$

$$\sum_{m\in M} q^{\mathrm{L2,L3}}_{m,n,pi,s} = \sum_{(pi,t)\in PT} q^{\mathrm{L2,L3}}_{n,pi,t,s}, \qquad \forall n \in N, \forall pi \in PI, \forall s \in S, \tag{C.10}$$

$$\sum_{(pi,t)\in PT} q^{\mathrm{L2,L3}}_{n,pi,t,s} \le q^{\mathrm{L3,UP}}_{t} \cdot y^{\mathrm{L3}}_{n,t}, \qquad \forall n \in N, \forall t \in T, \forall s \in S, \tag{C.11}$$

$$q^{\mathrm{L2,L3}}_{n,pi,t,s} \cdot f^{\mathrm{conv,L3}}_{pi,pp,t} = q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}, \quad \forall n \in N, \forall (pi,pp,t) \in PIPT, \forall s \in S, \tag{C.12}$$

$$\sum_{(pi,pp,t)\in PIPT} q^{\mathrm{L2,L3}}_{n,pi,pp,t,s} \ge \sum_{j\in J} q^{\mathrm{L3,L4}}_{n,j,pp,s}, \qquad \forall n \in N, \forall pp \in PP, \forall s \in S, \tag{C.13}$$

$$Dem^{\mathrm{LO}}_{j,pp} \le \sum_{n\in N} q^{\mathrm{L3,L4}}_{n,j,pp,s} \le Dem^{\mathrm{UP}}_{j,pp,s}, \qquad \forall j \in J, \forall pp \in PP, \forall s \in S, \tag{C.14}$$

$$q^{\mathrm{L1}}_{i,pi,s}, q^{\mathrm{L1,L2}}_{i,m,pi,s}, q^{\mathrm{L2,L3}}_{m,n,pi,s}, q^{\mathrm{L2,L3}}_{n,pi,t,s}, q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}, q^{\mathrm{L3,L4}}_{n,j,pp,s} \ge 0, \ \forall i,m,n,j,pi,pp,t,s. \tag{C.15}$$

Here, $s \in S$ is defined as the set of scenarios, Pr_s denotes the probability of a given scenario s occurring and is again calculate by $\mathrm{Pr}_s = \frac{1}{|S|}, s \in S$. The first stage variables are the binary variables denoted by y^{L2}_{m} and $y^{\mathrm{L3}}_{n,t}$. The second stage variables are flow rates of the materials and products denoted by $q^{\mathrm{L1}}_{i,pi,s}$, $q^{\mathrm{L1,L2}}_{i,m,pi,s}$, $q^{\mathrm{L2,L3}}_{m,n,pi,s}$, $q^{\mathrm{L2,L3}}_{n,pi,t,s}$, $q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}$, and $q^{\mathrm{L3,L4}}_{n,j,pp,s}$. The uncertain parameters are denoted by $HY_{pi,s}$ and $Dem^{\mathrm{UP}}_{j,pp,s}$.

C.2 Naïve Robust Scenario Formulation (NRS_IN)

Formulation (NRS) is a hybrid of formulation (S) and formulation (R). This intermediate formulation can be formed in the similar method shown in Appendix B. Once Proposition 1 has been applied to the constraints containing uncertain parameters, formulation (NRS_IN) is shown below

$$\max \sum_{s \in S} \left[\Pr_s \cdot \left(\begin{array}{l} \sum_{n \in N} \sum_{j^o \in J} \sum_{pp \in PP} q^{\mathrm{L3,L4}}_{n,j^o,pp,s} \cdot c^{\mathrm{price}}_{pp} + \sum_{n \in N} \sum_{j^e \in J} \sum_{pp \in PP} 0.9 \cdot \left(q^{\mathrm{L3,L4}}_{n,j^e,pp,s} \cdot c^{\mathrm{price}}_{pp} \right) \\ - \sum_{i \in I} \sum_{pi \in PI} q^{\mathrm{L1}}_{i,pi,s} \cdot c_{pi} - c^{\mathrm{tr}}_s - c^{\mathrm{op}}_s - c^{\mathrm{inv}}_s \end{array} \right) \right] \tag{C.16}$$

subject to

$$\begin{aligned} c^{\mathrm{tr}}_s &= \sum_{i \in I} \sum_{m \in M} \sum_{pi \in PI} D^{\mathrm{L1,L2}}_{i,m} \cdot f^{\mathrm{road,L1,L2}}_{i,m} \cdot c^{\mathrm{tr,L1,L2}}_{pi} \cdot q^{\mathrm{L1,L2}}_{i,m,pi,s} \\ &+ \sum_{m \in M} \sum_{n \in N} \sum_{pi \in PI} D^{\mathrm{L2,L3}}_{m,n} \cdot f^{\mathrm{road,L2,L3}}_{m,n} \cdot c^{\mathrm{tr,L2,L3}}_{pi} \cdot q^{\mathrm{L2,L3}}_{m,n,pi,s} \\ &+ \sum_{n \in N} \sum_{j \in J} \sum_{pp \in PP} D^{\mathrm{L3,L4}}_{n,j} \cdot f^{\mathrm{road,L3,L4}}_{n,j} \cdot c^{\mathrm{tr,L3,L4}}_{pp} \cdot q^{\mathrm{L3,L4}}_{n,j,pp,s}, \end{aligned} \tag{C.17}$$

$$c^{\mathrm{op}}_s = \sum_{i \in I} \sum_{m \in M} \sum_{pi \in PI} c^{\mathrm{op,L2}}_{pi} \cdot q^{\mathrm{L1,L2}}_{i,m,pi,s} + \sum_{n \in N} \sum_{(pi,t) \in PT} c^{\mathrm{op,L3}}_{pi,t} \cdot q^{\mathrm{L2,L3}}_{n,pi,t,s}, \tag{C.18}$$

$$c^{\mathrm{inv}}_s = \sum_{m \in M} c^{\mathrm{fix,inv,L2}} \cdot y^{\mathrm{L2}}_m + \sum_{n \in N} \sum_{t \in T} \left(c^{\mathrm{fix,inv,L3}}_t \cdot y^{\mathrm{L3}}_{n,t} + \sum_{(pi,t) \in PT} c^{\mathrm{var,inv,L3}}_t \cdot q^{\mathrm{L2,L3}}_{n,pi,t,s} \right), \tag{C.19}$$

$$q^{\mathrm{L1}}_{i,pi,s} \le -\delta_{i,pi,s} \cdot \left\| \left(M^{-1}_{i,pi,s} \right)^T \right\|_1 + \left(\overline{HY}_{pi,s} \cdot A_{i,pi} \right), \quad \forall pi \in PI, \forall i \in I, \forall s \in S, \tag{C.20}$$

$$q^{\mathrm{L1}}_{i,pi,s} = \sum_{m \in M} q^{\mathrm{L1,L2}}_{i,m,pi,s}, \qquad \forall pi \in PI, \forall i \in I, \forall s \in S, \tag{C.21}$$

$$\sum_{i \in I} q^{\mathrm{L1,L2}}_{i,m,pi,s} \le q^{\mathrm{L1,L2,UP}}_{pi} \cdot y^{\mathrm{L2}}_m, \qquad \forall m \in M, \forall pi \in PI, \forall s \in S, \tag{C.22}$$

$$\sum_{i \in I} \sum_{pi \in PI} q^{\mathrm{L1,L2}}_{i,m,pi,s} \le q^{\mathrm{L2,UP}} \cdot y^{\mathrm{L2}}_m, \qquad \forall m \in M, \forall s \in S, \tag{C.23}$$

$$\sum_{i \in I} q^{\mathrm{L1,L2}}_{i,m,pi,s} \cdot f^{\mathrm{conv,L2}}_{pi} = \sum_{n \in N} q^{\mathrm{L2,L3}}_{m,n,pi,s}, \qquad \forall m \in M, \forall pi \in PI, \forall s \in S, \tag{C.24}$$

$$\sum_{m \in M} q^{\mathrm{L2,L3}}_{m,n,pi,s} = \sum_{(pi,t) \in PT} q^{\mathrm{L2,L3}}_{n,pi,t,s}, \qquad \forall n \in N, \forall pi \in PI, \forall s \in S, \tag{C.25}$$

$$\sum_{(pi,t) \in PT} q^{\mathrm{L2,L3}}_{n,pi,t,s} \le q^{\mathrm{L3,UP}}_t \cdot y^{\mathrm{L3}}_{n,t}, \qquad \forall n \in N, \forall t \in T, \forall s \in S, \tag{C.26}$$

$$q^{\mathrm{L2,L3}}_{n,pi,t,s} \cdot f^{\mathrm{conv,L3}}_{pi,pp,t} = q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}, \quad \forall n \in N, \forall (pi,pp,t) \in PIPT, \forall s \in S, \tag{C.27}$$

$$\sum_{(pi,pp,t)\in PIPT} q^{\mathrm{L2,L3}}_{n,pi,pp,t,s} \geq \sum_{j\in J} q^{\mathrm{L3,L4}}_{n,j,pp,s}, \qquad \forall n \in N, \forall pp \in PP, \forall s \in S, \tag{C.28}$$

$$Dem^{\mathrm{LO}}_{j,pp} \leq \sum_{n\in N} q^{\mathrm{L3,L4}}_{n,j,pp,s} \leq -\delta_{j,pp,s} \cdot \left\| \left(M^{-1}_{j,pp,s} \right)^T \right\|_1 + \left(\overline{Dem}^{\mathrm{UP}}_{j,pp,s} \right), \quad \forall j \in J, \forall pp \in PP, \forall s \in S, \tag{C.29}$$

$$q^{\mathrm{L1}}_{i,pi,s}, q^{\mathrm{L1,L2}}_{i,m,pi,s}, q^{\mathrm{L2,L3}}_{m,n,pi,s}, q^{\mathrm{L2,L3}}_{n,pi,t,s}, q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}, q^{\mathrm{L3,L4}}_{n,j,pp,s} \geq 0, \ \forall i,m,n,j,pi,pp,t,s. \tag{C.30}$$

The first stage variables are the binary variables denoted by y^{L2}_m and $y^{\mathrm{L3}}_{n,t}$. The second stage variables are flow rates of the materials and products denoted by $q^{\mathrm{L1}}_{i,pi,s}$, $q^{\mathrm{L1,L2}}_{i,m,pi,s}$, $q^{\mathrm{L2,L3}}_{m,n,pi,s}$, $q^{\mathrm{L2,L3}}_{n,pi,t,s}$, $q^{\mathrm{L2,L3}}_{n,pi,pp,t,s}$, and $q^{\mathrm{L3,L4}}_{n,j,pp,s}$. The uncertain parameters are denoted by $HY_{pi,s}$ and $Dem^{\mathrm{UP}}_{j,pp,s}$.

C.3 Affinely Adjustable Robust Scenario Formulation (AARS_IN)

Formulation (AARS) is composed of formulation (S) and formulation (AAR). For simplicity, constraints (C.5) and (C.6) are combined and constraints (C.12) and (C.13) are combined to form the following

$$HY_{pi,s} \cdot A_{i,pi} \leq \sum_{m\in M} q^{\mathrm{L1,L2}}_{i,m,pi,s}, \qquad \forall pi \in PI, \forall i \in I, \forall s \in S, \tag{C.31}$$

$$\sum_{(pi,pp,t)\in PIPT} q^{\mathrm{L2,L3}}_{n,pi,t,s} \cdot f^{\mathrm{conv,L3}}_{pi,pp,t} = \sum_{j\in J} q^{\mathrm{L3,L4}}_{n,j,pp,s}, \quad \forall n \in N, \forall pp \in PP, \forall s \in S. \tag{C.32}$$

This reduces the number of second stage variables to four and now they are repalced by affine functions, where $q^{\mathrm{L1,L2}}_{i,m,pi,s}$ is replaced by $\Phi^{\mathrm{L1,L2}}_{i,m,pi,s,nu} \cdot \xi_{s,nu} + \phi^{\mathrm{L1,L2}}_{i,m,pi,s}$, $q^{\mathrm{L2,L3}}_{m,n,pi,s}$ is replaced by $\Phi^{\mathrm{L2,L3}}_{m,n,pi,s,nu} \cdot \xi_{s,nu} + \phi^{\mathrm{L2,L3}}_{m,n,pi,s}$, $q^{\mathrm{L2,L3}}_{n,pi,t,s}$ is replaced by $\Phi^{\mathrm{L2,L3}}_{n,pi,t,s,nu} \cdot \xi_{s,nu} + \phi^{\mathrm{L2,L3}}_{n,pi,t,s}$ and $q^{\mathrm{L3,L4}}_{n,j,pp,s}$ is replaced by $\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu} \cdot \xi_{s,nu} + \phi^{\mathrm{L3,L4}}_{n,j,pp,s}$.

The uncertain parameters, $HY_{pi,s}$ is now replaced by $\sum_{nu\in NU}\left(\beta_{pi,s,nu}\cdot\xi_{s,nu}+\gamma_{pi,s}\right)$, and $Dem^{\mathrm{UP}}_{j,pp,s}$ is replaced by $\sum_{nu\in NU}\left(\beta_{j,pp,s,nu}\cdot\xi_{s,nu}+\gamma_{j,pp,s}\right)$ to distinguish the deterministic and uncertain elements. The set $nu\in NU=\{1,2\}$ is introduced. The values for β and γ are shown below in Table C.1.

Table C.1 β and γ values for the energy and bioproduct supply chain – uncertain case A

$\beta_{'corn\ stover',s,'1'}$	$1,\forall s$
$\beta_{'corn\ stover',s,'2'}$	$0,\forall s$
$\beta_{pi,s,nu}$	$0,\forall s,nu,pi\neq corn\ stover$
$\beta_{j,'electricity',s,'2'}$	$1,\forall j,s$
$\beta_{j,'electricity',s,'1'}$	$0,\forall j,s$
$\beta_{j,pp,s,nu}$	$0,\forall j,s,nu,pp\neq electricity$
$\gamma_{'corn\ stover',s}$	$0,\forall s$
$\gamma_{pi,s}$	$\overline{HY}_{pi},\forall s,pi\neq corn\ stover$
$\gamma_{j,'electricity',s}$	$0,\forall j,s$
$\gamma_{j,pp,s}$	$\overline{Dem}^{\mathrm{UP}}_{j,pp},\forall j,s,pp\neq electricity$

Once Proposition 1 has been applied to the constraints containing uncertain parameters as seen in Appendix B, formulation (AARS_IN) is given as

$$\max\sum_{s\in S}\left[\Pr_s\cdot\left(\begin{array}{l}\sum_{n\in N}\sum_{j^o\in J}\sum_{pp\in PP}\sum_{nu\in NU}\left(\Phi^{\mathrm{L3,L4}}_{n,j^o,pp,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{\mathrm{L3,L4}}_{n,j^o,pp,s}\right)\cdot c^{\mathrm{price}}_{pp}\\+\sum_{n\in N}\sum_{j^e\in J}\sum_{pp\in PP}\sum_{nu\in NU}0.9\cdot\left(\left(\Phi^{\mathrm{L3,L4}}_{n,j^e,pp,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{\mathrm{L3,L4}}_{n,j^e,pp,s}\right)\cdot c^{\mathrm{price}}_{pp}\right)\\-\sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI}\sum_{nu\in NU}\left(\left(\Phi^{\mathrm{L1,L2}}_{i,m,pi,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{\mathrm{L1,L2}}_{i,m,pi,s}\right)\cdot c_{pi}\right)-c^{\mathrm{tr}}_s-c^{\mathrm{op}}_s-c^{\mathrm{inv}}_s\end{array}\right)\right] \quad \text{(C.33)}$$

subject to

$$
\begin{aligned}
c_s^{\text{tr}} &= \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI}\sum_{nu\in NU} D_{i,m}^{\text{L1,L2}} \cdot f_{i,m}^{\text{road,L1,L2}} \cdot c_{pi}^{\text{tr,L1,L2}} \cdot \left(\Phi_{i,m,pi,s,nu}^{\text{L1,L2}} \cdot \overline{\xi}_{s,nu} + \phi_{i,m,pi,s}^{\text{L1,L2}}\right) \\
&+ \sum_{m\in M}\sum_{n\in N}\sum_{pi\in PI}\sum_{nu\in NU} D_{m,n}^{\text{L2,L3}} \cdot f_{m,n}^{\text{road,L2,L3}} \cdot c_{pi}^{\text{tr,L2,L3}} \cdot \left(\Phi_{m,n,pi,s,nu}^{\text{L2,L3}} \cdot \overline{\xi}_{s,nu} + \phi_{m,n,pi,s}^{\text{L2,L3}}\right) \\
&+ \sum_{n\in N}\sum_{j\in J}\sum_{pp\in PP}\sum_{nu\in NU} D_{n,j}^{\text{L3,L4}} \cdot f_{n,j}^{\text{road,L3,L4}} \cdot c_{pp}^{\text{tr,L3,L4}} \cdot \left(\Phi_{n,j,pp,s,nu}^{\text{L3,L4}} \cdot \overline{\xi}_{s,nu} + \phi_{n,j,pp,s}^{\text{L3,L4}}\right),
\end{aligned}
\tag{C.34}
$$

$$
\begin{aligned}
c_s^{\text{op}} &= \sum_{i\in I}\sum_{m\in M}\sum_{pi\in PI}\sum_{nu\in NU} c_{pi}^{\text{op,L2}} \cdot \left(\Phi_{i,m,pi,s,nu}^{\text{L1,L2}} \cdot \overline{\xi}_{s,nu} + \phi_{i,m,pi,s}^{\text{L1,L2}}\right) \\
&+ \sum_{n\in N}\sum_{(pi,t)\in PT}\sum_{nu\in NU} c_{pi,t}^{\text{op,L3}} \cdot \left(\Phi_{n,pi,t,s,nu}^{\text{L2,L3}} \cdot \overline{\xi}_{s,nu} + \phi_{n,pi,t,s}^{\text{L2,L3}}\right),
\end{aligned}
\tag{C.35}
$$

$$
c_s^{\text{inv}} = \sum_{m\in M} c^{\text{fix,inv,L2}} \cdot y_m^{\text{L2}} + \sum_{n\in N}\sum_{t\in T}\left(\begin{aligned} & c_t^{\text{fix,inv,L3}} \cdot y_{n,t}^{\text{L3}} \\ & + \sum_{(pi,t)\in PT}\sum_{nu\in NU} c_t^{\text{var,inv,L3}} \cdot \left(\Phi_{n,pi,t,s,nu}^{\text{L2,L3}} \cdot \overline{\xi}_{s,nu} + \phi_{n,pi,t,s}^{\text{L2,L3}}\right)\end{aligned}\right),
\tag{C.36}
$$

$$
\sum_{nu\in NU}\left(\begin{aligned} & -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1}\right)^T \cdot \left(\sum_{m\in M} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}} - A_{i,pi} \cdot \beta_{pi,s,nu}\right)\right\|_1 \\ & + \left(\sum_{m\in M} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}} - A_{i,pi} \cdot \beta_{pi,s,nu}\right) \cdot \overline{\xi}_{s,nu} + \sum_{m\in M} \phi_{i,m,pi,s}^{\text{L1,L2}} - A_{i,pi} \cdot \gamma_{pi,s}\end{aligned}\right) \le 0,
\tag{C.37}
$$

$$\forall pi \in PI, \forall i \in I, \forall s \in S,$$

$$
\sum_{nu\in NU}\left(\begin{aligned} & \delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1}\right)^T \cdot \left(\sum_{i\in I} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}}\right)\right\|_1 \\ & + \left(\sum_{i\in I} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}}\right) \cdot \overline{\xi}_{s,nu} + \sum_{i\in I} \phi_{i,m,pi,s}^{\text{L1,L2}}\end{aligned}\right) \le q_{pi}^{\text{L1,L2,UP}} \cdot y_m^{\text{L2}},
\tag{C.38}
$$

$$\forall m \in M, \forall pi \in PI, \forall s \in S,$$

$$
\sum_{nu\in NU}\left(\begin{aligned} & \delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1}\right)^T \cdot \left(\sum_{i\in I}\sum_{pi\in PI} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}}\right)\right\|_1 \\ & + \left(\sum_{i\in I}\sum_{pi\in PI} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}}\right) \cdot \overline{\xi}_{s,nu} + \sum_{i\in I}\sum_{pi\in PI} \phi_{i,m,pi,s}^{\text{L1,L2}}\end{aligned}\right) \le q^{\text{L2,UP}} \cdot y_m^{\text{L2}},
\tag{C.39}
$$

$$\forall m \in M, \forall s \in S,$$

$$
\sum_{i\in I} \Phi_{i,m,pi,s,nu}^{\text{L1,L2}} \cdot f_{pi}^{\text{conv,L2}} - \sum_{n\in N} \Phi_{m,n,pi,s,nu}^{\text{L2,L3}} = 0, \quad \forall m \in M, \forall pi \in PI, \forall s \in S, \forall nu \in NU,
\tag{C.40}
$$

$$\sum_{i\in I}\phi^{\mathrm{L1,L2}}_{i,m,pi,s}\cdot f^{\mathrm{conv,L2}}_{pi}-\sum_{n\in N}\phi^{\mathrm{L2,L3}}_{m,n,pi,s}=0,\qquad \forall m\in M,\forall pi\in PI,\forall s\in S, \tag{C.41}$$

$$\sum_{m\in M}\Phi^{\mathrm{L2,L3}}_{m,n,pi,s,nu}-\sum_{(pi,t)\in PT}\Phi^{\mathrm{L2,L3}}_{n,pi,t,s,nu}=0,\quad \forall n\in N,\forall pi\in PI,\forall s\in S,\forall nu\in NU, \tag{C.42}$$

$$\sum_{m\in M}\phi^{\mathrm{L2,L3}}_{m,n,pi,s}-\sum_{(pi,t)\in PT}\phi^{\mathrm{L2,L3}}_{n,pi,t,s}=0,\quad \forall n\in N,\forall pi\in PI,\forall s\in S, \tag{C.43}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M^{-1}_{s,nu}\right)^{T}\cdot\left(\sum_{(pi,t)\in PT}\Phi^{\mathrm{L2,L3}}_{n,pi,t,s,nu}\right)\right\|_{1}\\+\left(\sum_{(pi,t)\in PT}\Phi^{\mathrm{L2,L3}}_{n,pi,t,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\sum_{(pi,t)\in PT}\phi^{\mathrm{L2,L3}}_{n,pi,t,s}\end{array}\right)\le q^{\mathrm{L3,UP}}_{t}\cdot y^{\mathrm{L3}}_{n,t},\quad \forall n\in N,\forall t\in T,\forall s\in S, \tag{C.44}$$

$$\sum_{(pi,pp,t)\in PIPT}\Phi^{\mathrm{L2,L3}}_{n,pi,t,s,nu}\cdot f^{\mathrm{conv,L3}}_{pi,pp,t}-\sum_{j\in J}\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu}=0,\quad \forall n\in N,\forall pp\in PP,\forall s\in S,\forall nu\in NU, \tag{C.45}$$

$$\sum_{(pi,pp,t)\in PIPT}\phi^{\mathrm{L2,L3}}_{n,pi,t,s}\cdot f^{\mathrm{conv,L3}}_{pi,pp,t}-\sum_{j\in J}\phi^{\mathrm{L3,L4}}_{n,j,pp,s}=0,\qquad \forall n\in N,\forall pp\in PP,\forall s\in S, \tag{C.46}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M^{-1}_{s,nu}\right)^{T}\cdot\left(\beta_{j,pp,s,nu}-\sum_{n\in N}\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu}\right)\right\|_{1}\\+\left(\beta_{j,pp,s,nu}-\sum_{n\in N}\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu}\right)\cdot\overline{\xi}_{s,nu}-\sum_{n\in N}\phi^{\mathrm{L3,L4}}_{n,j,pp,s}+\gamma_{j,pp,s}\end{array}\right)\ge 0,\quad \forall j\in J,\forall pp\in PP,\forall s\in S, \tag{C.47}$$

$$Dem^{\mathrm{LO}}_{j,pp}\le\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M^{-1}_{s,nu}\right)^{T}\cdot\left(\sum_{n\in N}\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu}\right)\right\|_{1}\\+\left(\sum_{n\in N}\Phi^{\mathrm{L3,L4}}_{n,j,pp,s,nu}\right)\cdot\overline{\xi}_{s,nu}+\sum_{n\in N}\phi^{\mathrm{L3,L4}}_{n,j,pp,s}\end{array}\right),\ \forall j\in J,\forall pp\in PP,\forall s\in S, \tag{C.48}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M^{-1}_{s,nu}\right)^{T}\cdot\left(\Phi^{\mathrm{L1,L2}}_{i,m,pi,s,nu}\right)\right\|_{1}\\+\Phi^{\mathrm{L1,L2}}_{i,m,pi,s,nu}\cdot\overline{\xi}_{s,nu}+\phi^{\mathrm{L1,L2}}_{i,m,pi,s}\end{array}\right)\ge 0,\quad \forall pi\in PI,\forall i\in I,\forall m\in M,\forall s\in S, \tag{C.49}$$

$$\sum_{nu \in NU} \begin{pmatrix} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{m,n,pi,s,nu}^{\text{L2,L3}} \right) \right\|_1 \\ +\Phi_{m,n,pi,s,nu}^{\text{L2,L3}} \cdot \overline{\xi}_{s,nu} + \phi_{m,n,pi,s}^{\text{L2,L3}} \end{pmatrix} \geq 0, \quad \forall pi \in PI, \forall n \in N, \forall m \in M, \forall s \in S, \qquad \text{(C.50)}$$

$$\sum_{nu \in NU} \begin{pmatrix} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{n,pi,t,s,nu}^{\text{L2,L3}} \right) \right\|_1 \\ +\Phi_{n,pi,t,s,nu}^{\text{L2,L3}} \cdot \overline{\xi}_{s,nu} + \phi_{n,pi,t,s}^{\text{L2,L3}} \end{pmatrix} \geq 0, \quad \forall pi \in PI, \forall n \in N, \forall t \in T, \forall s \in S, \qquad \text{(C.51)}$$

$$\sum_{nu \in NU} \begin{pmatrix} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{n,j,pp,s,nu}^{\text{L3,L4}} \right) \right\|_1 \\ +\Phi_{n,j,pp,s,nu}^{\text{L3,L4}} \cdot \overline{\xi}_{s,nu} + \phi_{n,j,pp,s}^{\text{L3,L4}} \end{pmatrix} \geq 0, \quad \forall n \in N, \forall j \in J, \forall pp \in PP, \forall s \in S. \qquad \text{(C.52)}$$

Here, constraints (C.49)-(C.52) are to ensure that the second-stage affine functions are non-negative. The values for $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1}\right)^T$ are shown in Table C.2.

Table C.2 $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1}\right)^T$ values for the energy and bioproduct supply chain – uncertain case A

$\delta_{s,'1'}$	$\delta_{electricity}, \forall s$
$\delta_{s,'2'}$	$\delta_{corn\ stover}, \forall s$
$\left(M_{s,'1'}^{-1}\right)^T$	$1, \forall s$
$\left(M_{s,'2'}^{-1}\right)^T$	$1, \forall s$

Appendix D

Energy and Bioproduct Supply Chain Case Study – Parameters

Tables D.1 to D.11provide the required parameter values for the energy and bioproduct supply chain case study.

Table D.1 Availability of raw materials in each supply zone ($A_{i,pi}$)

Zone	Corn (km^2)	Corn stover (km^2)	Wood chips (km^2)	Timber (km^2)	Manure (t/day)	MSW $\text{kg/capita} \cdot \text{day}$
1	20 (80)	20 (80)	-	-	4	-
2	20 (80)	20 (80)	-	-	4	-
3	65 (75)	65 (75)	15	15	2.75	-
4	30 (40)	30 (40)	60	60	1.25	-
5	40 (90)	40 (90)	10	10	2.75	1
6	25 (100)	25 (100)	-	-	2.75	1
7	65 (100)	65 (100)	-	-	3	-
8	45 (85)	45 (85)	15	15	2	-
9	-	-	100	100	-	-
10	10 (70)	10 (70)	30	30	2.5	-

Note: The values in parenthesis are used for Uncertain Case A of Problem 2. Zones 5 and 6 have populations of 30,000 and 28,000, respectively.

Table D.2 Yields (HY_{pi}) and costs (c_{pi}) of raw materials

	Corn	Corn stover	Wood chips	Timber	Manure	MSW
Yield ($\text{t}/(\text{km}^2 \times \text{y})$)	730	840	9.6	96	-	-
Cost (€/t)	85	30	40	90	10	-48

Note: Yield of manure and MSW is reflected in Table 1.

Table D.3 Upper limit of product demands ($Dem_{j,pp}$) and product prices (c_{pp}^{price})

	Heat	Electricity	Bioethanol	DDGS	Digestate	Boards
Maximum Demand	6.26×10^8 MJ/y	87,000 MWh/y	3,480 t/y	No upper bound	No upper bound	No upper bound
Price	0.017 €/MJ	100 €/MWh	550 €/t	120 €/t	24 €/t	300 €/t

Note: Minimum demands are zero unless indicated in the case studies.

Table D.4 Preprocessing and main processing costs ($c^{fix,inv,L2}$, $c^{fix,inv,L3}$, $c^{\text{var},inv,L3}$, $c_{pi}^{op,L2}$, $c_{pi,t}^{op,L3}$)

	Fixed Investment Cost (€/y)	Variable Investment Cost (€/t)	Operating Cost (€/t)
Preprocessing Centres	50,000	-	-
Corn stover – compressing	-	-	3.0
Corn grains - drying	-	-	25.0
Timber - drying	-	-	25.0
Main Plant			
Dry grind process	370,000	0.0047	40.0
Anaerobic digestion	30,658	7.0	15.3
Incineration	1,300,000	20.32	23.0
Sawing	20,000	3.07	7.43

Table D.5 Conversion factors of materials to products ($f_{pi}^{conv,L2}$, $f_{pi,pp,t}^{conv,L3}$)

Biomass	Weight Loss Ratio for Drying	Heat (MJ/t)	Electricity (MWh/t)	Bioethanol (t/t)	Boards (t/t)	Digestate (t/t)	DDGS (t/t)
Corn	0.2	-	-	0.323 (dry grind)	-	-	0.25 (dry grind)
Wood chips	-	7,200 (incineration)	1.4 (incineration)	-	-	-	-
MSW	-	4,600 (incineration)	0.9 (incineration)	-	-	-	-
Corn stover	-	8,000 (incineration)	1.55 (incineration)	-	-	0.4 (digestion)	-
Manure	-	1,350 (digestion)	0.26 (digestion)	-	-	0.4 (digestion)	-
Timber	0.41	-	-	-	0.8 (sawing)	-	-

Note: Related technology to produce the product is shown underneath the conversion factors.

Table D.6 Transportation costs for materials and products ($c_p^{tr,La,Lb}$)

Material	Tractor with trailer (€/ t · km)	Rail (€/ t · km)	Tank trailer (€/ t · km)	MSW Truck (€/ t · km)
Corn	0.142	0.064	-	-
Wood chips	0.24	0.064	-	-
MSW	-	-	-	1.15
Corn stover	0.75	0.064	-	-
Manure	-	-	0.132	-
Timber	0.15	0.064	-	-
Bioethanol	-	-	0.12	-
Digestate	-	-	0.132	-
DDGS	-	-	0.132	-
Boards	-	0.064	-	-

Note: Rail option is used for transportation from preprocessing centres to processing plants and for transporting boards.

Table D.7 Distances between supply zones and preprocessing centres ($D_{x,y}^{L1,L2}$)

	m_1	m_2	m_3	m_4	m_5	m_6
i_1	12	9	16	22	36	38
i_2	16	6	7	17	26	28
i_3	24	14	7	8	17	18
i_4	33	24	16	6	8	8
i_5	42	33	25	15	6	3
i_6	2	8	16	21	36	40
i_7	11	6	7	17	26	30
i_8	21	14	7	8	17	22
i_9	31	24	16	6	8	14
i_{10}	41	33	25	15	6	12

Note: m_1, …, m6 denote the six preprocessing centres, respectively. i_1, …, i_{10} denote the ten supply zones, respectively. Road condition factors are set to be 1 for all routes.

Table D.8 Distances between preprocessing centres and main plants ($D_{x,y}^{L2,L3}$)

	n_1	n_2	n_3
m_1	2	22	41
m_2	8	14	34
m_3	16	5	25
m_4	26	6	15
m_5	36	16	6
m_6	40	19	3

Note: n_1, …, n_3 denote the three main plants, respectively. m_1, …, m_6 denote the six preprocessing centres. Road condition factors are set to be 1 for all routes.

Table D.9 Distances between main plants and customer locations ($D_{x,y}^{L3,L4}$)

	j_1	j_2	j_3
n_1	3	40	34
n_2	21	20	27
n_3	42	3	32

Note: j_1, …, j_3 denote the three demand locations, respectively. n_1, …, n_3 denote the three main plants. Road condition factors are set to be 1 for all routes.

Table D.10 Preprocessing capacities for different materials ($q_{pi}^{L1,L2,UP}$, $q^{L2,UP}$)

Material	Capacity (t/y)
Corn	500,000
Corn stover	160,000
Wood chips	3,000
Timber	30,000
Manure	10,000
MSW	30,000
Total raw materials	500,000

Table D.11 Capacities of different processing technologies ($q_t^{L3,UP}$)

Technology	Capacity (t/y)
Dry grind	250,000
Anaerobic digestion	160,000
Incineration	390,000, 290,000, 500,000
Sawing	20,000

Note: Different incineration capacities are used for the three respective case studies.

Appendix E

Industrial Chemical Supply Chain Case Study - Equations

Formulation (S), and formulation (AARS_IN) are provided for the industrial and chemical supply chain case study – uncertain case A

E.1 Scenario Formulation (S)

$$\max \sum_{i\in I}\sum_{s\in S}\left[\mathrm{Pr}_s\cdot\left(\left(\mathrm{Revenue}_{i,s}-\mathrm{TC}_{i,s}\right)\left(1-r_i^{\mathrm{inc}}\right)\right)\right] \tag{E.1}$$

subject to

$$\mathrm{Revenue}_{i,s} = \sum_{(i,k)\in\Theta}\sum_{v\in V}\left(f_{i,k,v,s}^{\mathrm{FP,PD}}\cdot P_{k,v}^{\mathrm{FP}}\right)+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(f_{i,j,v,s}^{\mathrm{FP,PW}}\cdot \hat{P}_{j,v}^{\mathrm{FP}}\right),\quad i\in I,\quad s\in S, \tag{E.2}$$

$$\mathrm{TC}_{i,s} = c_i^{\mathrm{fix}}+c_i^{\mathrm{cap}}+c_{i,s}^{\mathrm{fr}}+c_{i,s}^{\mathrm{du}}+c_{i,s}^{\mathrm{I}}+c_{i,s}^{\mathrm{PRM}}+c_{i,s}^{\mathrm{RM2}}+c_{i,s}^{\mathrm{RM3}}+c_{i,s}^{\mathrm{waste}}+c_{i,s}^{\mathrm{OPVC}},\quad i\in I,\ s\in S, \tag{E.3}$$

$$c_i^{\mathrm{cap}} = z_i\cdot C^{\mathrm{cap}},\quad i\in I, \tag{E.4}$$

$$c_{i,s}^{\mathrm{fr}} = \sum_{(i,k)\in\Theta}\sum_{v\in V}\left(f_{i,k,v,s}^{\mathrm{FP,PD}}\cdot C_{i,k}^{\mathrm{fr,FP,PD}}\right)+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(f_{i,j,v,s}^{\mathrm{FP,PW}}\cdot C_{i,j}^{\mathrm{fr,FP,PW}}\right) + \sum_{(i,j)\in\Omega}\sum_{(j,k)\in\Pi}\sum_{v\in V}\left(f_{j,k,v,s}^{\mathrm{FP,WD}}\cdot C_{j,k}^{\mathrm{fr,FP,WD}}\cdot q_{i,j}^{\mathrm{fr,PW}}\right),\quad i\in I, s\in S, \tag{E.5}$$

$$c_{i,s}^{\mathrm{du}} = r^{\mathrm{tp}}\cdot\left[\sum_{(i,k)\in\Theta}\sum_{v\in V}\left(f_{i,k,v,s}^{\mathrm{FP,PD}}\cdot P_{k,v}^{\mathrm{FP}}\cdot r_{i,k}^{\mathrm{du}}\right)+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(f_{i,j,v,s}^{\mathrm{FP,PW}}\cdot \hat{P}_{j,v}^{\mathrm{FP}}\cdot \hat{r}_{i,j}^{\mathrm{du}}\right)\right],\quad i\in I,\ s\in S, \tag{E.6}$$

$$c_{i,s}^{\mathrm{I}} = \sum_{(i,v)\in\Psi}\left(f_{i,v,s}^{\mathrm{FP,p}}\cdot C_i^{\mathrm{PI}}\right)+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(f_{i,j,v,s}^{\mathrm{FP,PW}}\cdot C_j^{\mathrm{WI}}\right),\quad i\in I,\ s\in S, \tag{E.7}$$

$$c_{i,s}^{\mathrm{PRM}} = \sum_{u\in U}\left(f_{i,u,s}^{\mathrm{PRM,c}}\cdot C_{i,u}^{\mathrm{PRM}}\right),\quad i\in I,\ s\in S, \tag{E.8}$$

$$c_{i,s}^{\mathrm{RM2}} = \sum_{u\in U}\left(f_{i,u,s}^{\mathrm{PRM,c}}\cdot q_{i,u}^{\mathrm{RM2}}\right)\cdot C_i^{\mathrm{RM2}},\quad i\in I,\ s\in S, \tag{E.9}$$

$$c_{i,s}^{\mathrm{RM3}} = \sum_{u\in U}\left(f_{i,u,s}^{\mathrm{PRM,c}}\cdot q_{i,u}^{\mathrm{RM3}}\right)\cdot C_i^{\mathrm{RM3}},\quad i\in I,\ s\in S, \tag{E.10}$$

$$c_{i,s}^{\text{waste}} = \sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}} \cdot q_u^{\text{waste}}\right)\cdot C_i^{\text{waste}}, \quad i\in I,\ s\in S, \tag{E.11}$$

$$c_{i,s}^{\text{OPVC}} = C_i^{\text{var}} \cdot \left[\sum_{(i,v)\in\Psi}\left(f_{i,v,s}^{\text{FP,p}}\right)\right], \quad i\in I,\ s\in S, \tag{E.12}$$

$$\sum_{u\in U} f_{i,u,s}^{\text{PRM,C}} \le \left(\sum_{(i,v)\in\Psi}\left[\left(\frac{f_{i,v,s}^{\text{FP,p}}}{a_i^{avg,FP}/24/365/\left(\left(365-O_i\right)\cdot U_i\right)}\right)\right]\cdot m_i^{\text{td}} + b_i^{\text{td}}\right)$$
$$\cdot 24\cdot 365\cdot\left(\left(365-O_i\right)\cdot U_i/365\right), \quad i\in I,\ s\in S, \tag{E.13}$$

$$\sum_{u\in U} f_{i,u,s}^{\text{PRM,C}} \le \left(\sum_{(i,v)\in\Psi}\left[\left(\frac{f_{i,v,s}^{\text{FP,p}}}{a_i^{avg,FP}/24/365/\left(\left(365-O_i\right)\cdot U_i\right)}\right)\right]\cdot m_i^{\text{wl}} + b_i^{\text{wl}}\right)$$
$$\cdot 24\cdot 365\cdot\left(\left(365-O_i\right)\cdot U_i/365\right), \quad i\in I,\ s\in S, \tag{E.14}$$

$$\sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}}\cdot E_u^{\text{PRM}}\right)\cdot a_i^{\text{avg,FP}}\cdot Y_i^{\text{FP}} = \sum_{(i,v)\in\Psi}\left(f_{i,v,s}^{\text{FP,p}}\right), \quad i\in I,\ s\in S, \tag{E.15}$$

$$\sum_{(i,j)\in\Omega}\left(f_{i,j,v,s}^{\text{FP,PW}}\right) + \sum_{(i,k)\in\Theta}\left(f_{i,k,v,s}^{\text{FP,PD}}\right) \le X_{i,v}^{\text{FP}} + f_{i,v,s}^{\text{FP,p}} - D_{i,v}^{\text{FP}}\cdot R_{i,v}^{\text{FP}}/365,\ (i,v)\in\Psi,\ s\in S, \tag{E.16}$$

$$\sum_{(j,k)\in\Pi}\left(f_{j,k,v,s}^{\text{FP,WD}}\right) \le X_{j,v}^{\text{FP}} + \sum_{i\in I}\left(f_{i,j,v,s}^{\text{FP,PW}}\right) - D_{j,v}^{\text{FP}}\cdot R_{j,v}^{\text{FP}}/365, \quad j\in J,\ v\in V,\ s\in S, \tag{E.17}$$

$$\sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}}\cdot q_{u,w}^{\text{imp}}\right) \le \sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}}\cdot Q_{i,w}^{\text{imp}}\right), \quad i\in I,\ w\in W,\ s\in S, \tag{E.18}$$

$$\sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}}\cdot E_u^{\text{PRM}}\right) \le \sum_{u\in U}\left(f_{i,u,s}^{\text{PRM,c}}\cdot Q_i^{\text{imp}}\right), \quad i\in I,\ s\in S, \tag{E.19}$$

$$\sum_{i\in I}\left(f_{i,u,s}^{\text{PRM,P}}\right) + f_{u,s}^{\text{PRM,W}} \le M_u^{\text{PRM}}, \quad u\in U,\ s\in S, \tag{E.20}$$

$$X_u^{\text{PRM,W}} - R_u^{\text{PRM,W}} + f_u^{\text{PRM,W}} - \sum_{i\in I}\left(f_{i,u,s}^{\text{PRM,WP}}\right) \ge 0, \quad u\in U,\ s\in S, \tag{E.21}$$

$$f_{i,u,s}^{\text{PRM,c}} \le X_{i,u}^{\text{PRM,P}} + f_{i,u,s}^{\text{PRM,P}} + f_{i,u,s}^{\text{PRM,WP}} - R_{i,u}^{\text{PRM,P}}, \quad i\in I,\ u\in U,\ s\in S, \tag{E.22}$$

$$\sum_{(i,k)\in\Theta} f_{i,k,v,s}^{\text{FP,PD}} + \sum_{(j,k)\in\Pi} f_{j,k,v,s}^{\text{FP,WD}} \ge \sum_{nu\in NU}\left(\eta_{k,nu}\cdot \overline{D}_{v,k}^{\text{min}}\cdot \xi_{s,nu}\right), \quad k\in K,\ v\in V,\ s\in S, \tag{E.23}$$

$$z_i \le Z_i^{\text{max}}, \quad i\in I, \tag{E.24}$$

$$\sum_{(i,v)\in\Psi} f_{i,v,s}^{\text{FP,p}} \le z_i, \quad i\in I,\ s\in S, \tag{E.25}$$

$$z_i\,,\, f^{\mathrm{FP,PD}}_{i,k,v,s}\,,\, f^{\mathrm{FP,PW}}_{i,j,v,s}\,,\, f^{\mathrm{FP,WD}}_{j,k,v,s}\,,\, f^{\mathrm{PRM,C}}_{i,u,s}\,,\, f^{\mathrm{PRM,P}}_{i,u,s}\,,\, f^{\mathrm{PRM,W}}_{u,s}\,,\, f^{\mathrm{PRM,WP}}_{i,u,s}\,,\, f^{\mathrm{FP,p}}_{i,v,s} \ge 0,\ \ \forall i, j, k, v, u, s. \quad \text{(E.26)}$$

Here, $s \in S$ is defined as the set of scenarios, $\Pr_s$ denotes the probability of a given scenario s occurring and is calculated by $\Pr_s = \dfrac{1}{|S|}, s \in S$. The first stage variables are the determined capacities of the plants z_i. The second stage variables are flow rates of the primary raw materials and the final products denoted by $f^{\mathrm{FP,PD}}_{i,k,v,s}$, $f^{\mathrm{FP,PW}}_{i,j,v,s}$, $f^{\mathrm{FP,WD}}_{j,k,v,s}$, $f^{\mathrm{PRM,C}}_{i,u,s}$, $f^{\mathrm{PRM,P}}_{i,u,s}$, $f^{\mathrm{PRM,W}}_{u,s}$, $f^{\mathrm{PRM,WP}}_{i,u,s}$, and $f^{\mathrm{FP,p}}_{i,v,s}$. The uncertainty in demand is now represented through the expression $\sum_{nu \in NU} \left(\eta_{k,nu} \cdot \overline{D}^{\min}_{v,k} \cdot \xi_{s,nu} \right)$, where $\overline{D}^{\min}_{v,k}$ represents the nominal values used in the deterministic formulation. $\xi_{s,nu}$ are the uncertain parameters that is multiplied by the nominal demand value and $\eta_{k,nu}$ is introduced so that the uncertain parameters are multiplied for the correct regional market locations. These ξ and η values are shown below in Table E.1.

Table E.1 ξ and η values for the industrial chemical supply chain – uncertain case A

$\xi_{s,'1'}$	$\{1 \pm 0.4\}$
$\xi_{s,'2'}$	$\{1 \pm 0.3\}$
$\eta_{k,'1'}$	$1, k = 1, 2, 3$
$\eta_{k,'1'}$	$0, k = 4, 5$
$\eta_{k,'2'}$	$0, k = 1, 2, 3$
$\eta_{k,'2'}$	$1, k = 4, 5$

Here, the uncertainty region can be defined by $\Xi = \left\{\xi : \left\| M\left(\xi - \bar{\xi}\right)\right\|_\infty \le \delta\right\}$, where the uncertain parameters are denoted by $\xi = \begin{pmatrix}\xi_1 \\ \xi_2\end{pmatrix}$, the nominal values are denoted by $\bar{\xi} = \begin{pmatrix}1 \\ 1\end{pmatrix}$, $M = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$, and $\delta = \begin{pmatrix}0.4 \\ 0.3\end{pmatrix}$.

The uncertainty subregions are defined as $\Xi_{i,s} = \left\{\xi_{i,s} : \left\| M_{i,s}\left(\xi_{i,s} - \bar{\xi}_{i,s}\right)\right\|_\infty \le \delta_{i,s}\right\}$, where $\xi_{i,s} = \begin{pmatrix}\xi_{1,s} \\ \xi_{2,s}\end{pmatrix}$, $\bar{\xi}_{i,s} = \begin{pmatrix}\bar{\xi}_{1,s} \\ \bar{\xi}_{2,s}\end{pmatrix}$, $M_{i,s} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}, \forall s$, and $\delta_{i,s} = \begin{pmatrix}\delta_1 \\ \delta_2\end{pmatrix}, \forall s$.

E.2 Adjustable Robust Scenario Formulation (AARS_IN)

Formulation (AARS) is a hybrid of formulation (S) and (AAR). The second stage variables here are now replaced by affine functions, in which $f^{\text{FP,PD}}_{i,k,v,s}$ is replaced by $\Phi^{\text{FP,PD}}_{i,k,v,s,nu} \cdot \xi_{s,nu} + \phi^{\text{FP,PD}}_{i,k,v,s}$, $f^{\text{FP,PW}}_{i,j,v,s}$ is replaced by $\Phi^{\text{FP,PW}}_{i,j,v,s,nu} \cdot \xi_{s,nu} + \phi^{\text{FP,PW}}_{i,j,v,s}$, $f^{\text{FP,WD}}_{j,k,v,s}$ is replaced by $\Phi^{\text{FP,WD}}_{j,k,v,s,nu} \cdot \xi_{s,nu} + \phi^{\text{FP,WD}}_{j,k,v,s}$, $f^{\text{PRM,C}}_{i,u,s}$ is replaced by $\Phi^{\text{PRM,c}}_{i,u,s,nu} \cdot \xi_{s,nu} + \phi^{\text{PRM,c}}_{i,u,s}$, $f^{\text{PRM,P}}_{i,u,s}$ is replaced by $\Phi^{\text{PRM,P}}_{i,u,s,nu} \cdot \xi_{s,nu} + \phi^{\text{PRM,P}}_{i,u,s}$, $f^{\text{PRM,W}}_{u,s}$ is replaced by $\Phi^{\text{PRM,W}}_{u,s,nu} \cdot \xi_{s,nu} + \phi^{\text{PRM,W}}_{u,s}$, $f^{\text{PRM,WP}}_{i,u,s}$ is replaced by $\Phi^{\text{PRM,WP}}_{i,u,s,nu} \cdot \xi_{s,nu} + \phi^{\text{PRM,WP}}_{i,u,s}$ and $f^{\text{FP,p}}_{i,v,s}$ is replaced by $\Phi^{\text{FP,p}}_{i,v,s,nu} \cdot \xi_{s,nu} + \phi^{\text{FP,p}}_{i,v,s}$.

The intermediate formulation (AARS) can be expressed through a method shown in Appendix B. Once Proposition 1 has been applied to the constraints containing an optimization expression, formulation (AARS_IN) is given as

$$\max \sum_{i\in I}\sum_{s\in S}\left[\Pr_s \cdot \left(\left(\text{Revenue}_{i,s} - \text{TC}_{i,s}\right)\left(1 - r_i^{\text{inc}}\right)\right)\right] \tag{E.27}$$

subject to

$$\begin{aligned}\text{Revenue}_{i,s} = &\sum_{(i,k)\in\Theta}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,k,v,s,nu}^{\text{FP,PD}} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{\text{FP,PD}}\right)\cdot P_{k,v}^{\text{FP}}\right)\\ &+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,j,v,s,nu}^{\text{FP,PW}} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{\text{FP,PW}}\right)\cdot \hat{P}_{j,v}^{\text{FP}}\right), \quad i\in I, \quad s\in S,\end{aligned} \tag{E.28}$$

$$\begin{aligned}\text{TC}_{i,s} = c_i^{\text{fix}} + c_i^{\text{cap}} + \overline{c}_{i,s}^{\text{fr}} + \overline{c}_{i,s}^{\text{du}} + \overline{c}_{i,s}^{\text{I}} + \overline{c}_{i,s}^{\text{PRM}} + \overline{c}_{i,s}^{\text{RM2}} + \overline{c}_{i,s}^{\text{RM3}} + \overline{c}_{i,s}^{\text{waste}} + \overline{c}_{i,s}^{\text{OPVC}},\\ i\in I,\ s\in S,\end{aligned} \tag{E.29}$$

$$c_i^{\text{cap}} = z_i \cdot C^{\text{cap}}, \quad i\in I, \tag{E.30}$$

$$\begin{aligned}\overline{c}_{i,s}^{\text{fr}} = &\sum_{(i,k)\in\Theta}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,k,v,s,nu}^{\text{FP,PD}} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{\text{FP,PD}}\right)\cdot C_{i,k}^{\text{fr,FP,PD}}\right)\\ &+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,j,v,s,nu}^{\text{FP,PW}} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{\text{FP,PW}}\right)\cdot C_{i,j}^{\text{fr,FP,PW}}\right)\\ &+\sum_{(i,j)\in\Omega}\sum_{(j,k)\in\Pi}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{j,k,v,s,nu}^{\text{FP,WD}} \cdot \overline{\xi}_{s,nu} + \phi_{j,k,v,s}^{\text{FP,WD}}\right)\cdot C_{j,k}^{\text{fr,FP,WD}} \cdot q_{i,j}^{\text{fr,PW}}\right), \quad i\in I, \quad s\in S,\end{aligned} \tag{E.31}$$

$$\overline{c}_{i,s}^{\text{du}} = r^{\text{tp}} \cdot \left[\begin{aligned}&\sum_{(i,k)\in\Theta}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,k,v,s,nu}^{\text{FP,PD}} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{\text{FP,PD}}\right)\cdot P_{k,v}^{\text{FP}} \cdot r_{i,k}^{\text{du}}\right)\\ &+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,j,v,s,nu}^{\text{FP,PW}} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{\text{FP,PW}}\right)\cdot \hat{P}_{j,v}^{\text{FP}} \cdot \hat{r}_{i,j}^{\text{du}}\right)\end{aligned}\right], \quad i\in I,\ s\in S, \tag{E.32}$$

$$\begin{aligned}\overline{c}_{i,s}^{\text{I}} = &\sum_{(i,v)\in\Psi}\left(\left(\sum_{nu\in NU}\Phi_{i,v,s,nu}^{\text{FP,p}} \cdot \overline{\xi}_{s,nu} + \phi_{i,v,s}^{\text{FP,p}}\right)\cdot C_i^{\text{PI}}\right)\\ &+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,j,v,s,nu}^{\text{FP,PW}} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{\text{FP,PW}}\right)\cdot C_j^{\text{WI}}\right), \quad i\in I,\ s\in S,\end{aligned} \tag{E.33}$$

$$\overline{c}_{i,s}^{\text{PRM}} = \sum_{u\in U}\left(\left(\sum_{nu\in NU}\Phi_{i,u,s,nu}^{\text{PRM,c}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,c}}\right)\cdot C_{i,u}^{\text{PRM}}\right), \quad i\in I,\ s\in S, \tag{E.34}$$

$$\overline{c}_{i,s}^{\text{RM2}} = \sum_{u\in U}\left(\left(\sum_{nu\in NU}\Phi_{i,u,s,nu}^{\text{PRM,c}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,c}}\right)\cdot q_{i,u}^{\text{RM2}}\right)\cdot C_i^{\text{RM2}}, \quad i\in I,\ s\in S, \tag{E.35}$$

$$\overline{c}_{i,s}^{\text{RM3}} = \sum_{u\in U}\left(\left(\sum_{nu\in NU}\Phi_{i,u,s,nu}^{\text{PRM,c}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,c}}\right)\cdot q_{i,u}^{\text{RM3}}\right)\cdot C_i^{\text{RM3}}, \quad i\in I,\ s\in S, \tag{E.36}$$

$$\overline{c}_{i,s}^{\text{waste}} = \sum_{u\in U}\left(\left(\sum_{nu\in NU}\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot\overline{\xi}_{s,nu}+\phi_{i,u,s}^{\text{PRM,c}}\right)\cdot q_u^{\text{waste}}\right)\cdot C_i^{\text{waste}},\quad i\in I,\ s\in S, \tag{E.37}$$

$$\overline{c}_{i,s}^{\text{OPVC}} = C_i^{\text{var}}\cdot\left[\sum_{(i,v)\in\Psi}\left(\sum_{nu\in NU}\Phi_{i,v,s,nu}^{\text{FP,p}}\cdot\overline{\xi}_{s,nu}+\phi_{i,v,s}^{\text{FP,p}}\right)\right],\quad i\in I,\ s\in S, \tag{E.38}$$

$$\begin{aligned}&\sum_{nu\in NU}\left(\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{u\in U}\Phi_{i,u,s,nu}^{\text{PRM,c}}\right)\right\|_1+\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\right)\cdot\overline{\xi}_{s,nu}+\sum_{u\in U}\phi_{i,u,s}^{\text{PRM,c}}\right)\\&\le\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\cdot A_i\right)\right\|_1+\\\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\cdot A_i\right)\cdot\overline{\xi}_{s,nu}+\left(\sum_{(i,v)\in\Psi}\phi_{i,v,s}^{\text{FP,p}}\cdot A_i\right)\end{array}\right),\quad i\in I,\ s\in S,\end{aligned} \tag{E.39}$$

where,

$$A_i=\left(\left(\frac{1}{a_i^{\text{avg,FP}}/24/365/\left(\left(365-O_i\right)\cdot U_i\right)}\right)\cdot m_i^{\text{td}}+b_i^{\text{td}}\right)\cdot 24\cdot 365\cdot\left(\left(365-O_i\right)\cdot U_i/365\right),\quad i\in I.$$

$$\begin{aligned}&\sum_{nu\in NU}\left(\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{u\in U}\Phi_{i,u,s,nu}^{\text{PRM,c}}\right)\right\|_1+\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\right)\cdot\overline{\xi}_{s,nu}+\sum_{u\in U}\phi_{i,u,s}^{\text{PRM,c}}\right)\\&\le\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\cdot B_i\right)\right\|_1+\\\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\cdot B_i\right)\cdot\overline{\xi}_{s,nu}+\left(\sum_{(i,v)\in\Psi}\phi_{i,v,s}^{\text{FP,p}}\cdot B_i\right)\end{array}\right),\quad i\in I,\ s\in S,\end{aligned} \tag{E.40}$$

where,

$$B_i=\left(\left(\frac{1}{a_i^{\text{avg,FP}}/24/365/\left(\left(365-O_i\right)\cdot U_i\right)}\right)\cdot m_i^{\text{wl}}+b_i^{\text{wl}}\right)\cdot 24\cdot 365\cdot\left(\left(365-O_i\right)\cdot U_i/365\right),\quad i\in I.$$

$$\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot E_u^{\text{PRM}}\right)\cdot a_i^{\text{avg,FP}}\cdot Y_i^{\text{FP}}-\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}=0,\quad i\in I,\ s\in S,\ nu\in NU, \tag{E.41}$$

$$\sum_{u\in U}\left(\phi_{i,u,s}^{\mathrm{PRM,c}}\cdot E_u^{\mathrm{PRM}}\right)\cdot a_i^{\mathrm{avg,FP}}\cdot Y_i^{\mathrm{FP}}-\sum_{(i,v)\in\Psi}\phi_{i,v,s}^{\mathrm{FP,p}}=0,\quad i\in I,\ s\in S, \tag{E.42}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\right)+\sum_{(i,k)\in\Theta}\left(\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}\right)-\Phi_{i,v,s,nu}^{\mathrm{FP,p}}\right)\right\|_1\\ +\left(\left(\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\right)+\sum_{(i,k)\in\Theta}\left(\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}\right)-\Phi_{i,v,s,nu}^{\mathrm{FP,p}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{(i,j)\in\Omega}\left(\phi_{i,j,v,s}^{\mathrm{FP,PW}}\right)+\sum_{(i,k)\in\Theta}\left(\phi_{i,k,v,s}^{\mathrm{FP,PD}}\right)-\phi_{i,v,s}^{\mathrm{FP,p}}\end{array}\right) \tag{E.43}$$

$$\leq X_{i,v}^{\mathrm{FP}}-D_{i,v}^{\mathrm{FP}}\cdot R_{i,v}^{\mathrm{FP}}/365,\quad (i,v)\in\Psi,\ s\in S,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\begin{array}{l}\sum_{(j,k)\in\Pi}\left(\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}\right)\\ -\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\right)\end{array}\right)\right\|_1\\ +\left(\sum_{(j,k)\in\Pi}\left(\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}\right)-\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{(j,k)\in\Pi}\left(\phi_{j,k,v,s}^{\mathrm{FP,WD}}\right)-\sum_{(i,j)\in\Omega}\left(\phi_{i,j,v,s}^{\mathrm{FP,PW}}\right)\end{array}\right)\leq X_{j,v}^{\mathrm{FP}}-D_{j,v}^{\mathrm{FP}}\cdot R_{j,v}^{\mathrm{FP}}/365, \tag{E.44}$$

$$j\in J,\ v\in V,\ s\in S,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot q_{u,w}^{\text{imp}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot Q_{i,w}^{\text{imp}}\right)\right)\right\|_{1}+\\ \left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot q_{u,w}^{\text{imp}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot Q_{i,w}^{\text{imp}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{u\in U}\left(\phi_{i,u,s}^{\text{PRM,c}}\cdot q_{u,w}^{\text{imp}}\right)-\sum_{u\in U}\left(\phi_{i,u,s}^{\text{PRM,c}}\cdot Q_{i,w}^{\text{imp}}\right)\end{array}\right)\leq 0, \quad i\in I,\ w\in W,\ s\in S, \tag{E.45}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\begin{array}{l}\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot E_{u}^{\text{PRM}}\right)\\ -\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot Q_{i}^{\text{imp}}\right)\end{array}\right)\right\|_{1}+\\ \left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot E_{u}^{\text{PRM}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\text{PRM,c}}\cdot Q_{i}^{\text{imp}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{u\in U}\left(\phi_{i,u,s}^{\text{PRM,c}}\cdot E_{u}^{\text{PRM}}\right)-\sum_{u\in U}\left(\phi_{i,u,s}^{\text{PRM,c}}\cdot Q_{i}^{\text{imp}}\right)\end{array}\right)\leq 0, \quad i\in I,\ s\in S, \tag{E.46}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{\text{PRM,P}}\right)+\Phi_{u,s,nu}^{\text{PRM,W}}\right)\right\|_{1}\\ +\left(\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{\text{PRM,P}}\right)+\Phi_{u,s,nu}^{\text{PRM,W}}\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{i\in I}\left(\phi_{i,u,s}^{\text{PRM,P}}\right)+\phi_{u,s}^{\text{PRM,W}}\end{array}\right)\leq M_{u}^{\text{PRM}}, \quad u\in U,\ s\in S, \tag{E.47}$$

$$X_{u}^{\text{PRM,W}}-R_{u}^{\text{PRM,W}}+\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{u,s,nu}^{\text{PRM,W}}-\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{\text{PRM,WP}}\right)\right)\right\|_{1}\\ +\left(\Phi_{u,s,nu}^{\text{PRM,W}}-\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{\text{PRM,WP}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\phi_{u,s}^{\text{PRM,W}}-\sum_{i\in I}\left(\phi_{i,u,s}^{\text{PRM,WP}}\right)\end{array}\right)\geq 0, \quad u\in U,\ s\in S, \tag{E.48}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}-\Phi_{i,u,s,nu}^{\mathrm{PRM,P}}-\Phi_{i,u,s,nu}^{\mathrm{PRM,WP}}\right)\right\|_{1}+\\ \left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}-\Phi_{i,u,s,nu}^{\mathrm{PRM,P}}-\Phi_{i,u,s,nu}^{\mathrm{PRM,WP}}\right)\cdot\overline{\xi}_{s,nu}\\ +\phi_{i,u,s,nu}^{\mathrm{PRM,c}}-\phi_{i,u,s}^{\mathrm{PRM,P}}-\phi_{i,u,s}^{\mathrm{PRM,WP}}\end{array}\right)\leq X_{i,u}^{\mathrm{PRM,P}}-R_{i,u}^{\mathrm{PRM,P}}, \tag{E.49}$$

$$i\in I,\ u\in U,\ s\in S,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{(i,k)\in\Theta}\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}+\sum_{(j,k)\in\Pi}\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}-D_{k,v}^{\min}\cdot\eta_{k,nu}\right)\right\|_{1}\\ +\left(\sum_{(i,k)\in\Theta}\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}+\sum_{(j,k)\in\Pi}\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}-\overline{D}_{k,v}^{\min}\cdot\eta_{k,nu}\right)\cdot\left(\overline{\xi}_{s,nu}\right)\\ +\phi1_{i,k,v,s}^{\mathrm{FP,PD}}+\phi3_{j,k,v,s}^{\mathrm{FP,WD}}\end{array}\right)\geq 0, \tag{E.50}$$

$$k\in K,\ v\in V,\ s\in S,$$

$$z_i\leq Z_i^{\max},\quad i\in I, \tag{E.51}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\mathrm{FP,p}}\right)\right\|_{1}\\ +\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\mathrm{FP,p}}\right)\cdot\overline{\xi}_{s,nu}+\left(\sum_{(i,v)\in\Psi}\phi_{i,v,s,nu}^{\mathrm{FP,p}}\right)\end{array}\right)\leq z_i,\quad i\in I,\ s\in S, \tag{E.52}$$

$$\sum_{nu\in NU}\left(-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}\right)\right\|_{1}+\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}}\cdot\overline{\xi}_{s,nu}+\phi_{i,k,v,s}^{\mathrm{FP,PD}}\right)\geq 0, \tag{E.53}$$

$$i\in I,\ k\in K,\ v\in V,\ s\in S,$$

$$\sum_{nu\in NU}\left(-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\right)\right\|_{1}+\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}}\cdot\overline{\xi}_{s,nu}+\phi_{i,j,v,s}^{\mathrm{FP,PW}}\right)\geq 0, \tag{E.54}$$

$$i\in I,\ j\in J,\ v\in V,\ s\in S,$$

$$\sum_{nu\in NU}\left(-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}\right)\right\|_{1}+\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}}\cdot\overline{\xi}_{s,nu}+\phi_{j,k,v,s}^{\mathrm{FP,WD}}\right)\geq 0, \tag{E.55}$$

$$j\in J,\ k\in K,\ v\in V,\ s\in S,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\text{PRM,c}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\text{PRM,c}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,c}} \right) \geq 0, \quad i \in I,\ u \in U,\ s \in S, \tag{E.56}$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\text{PRM,P}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\text{PRM,P}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,P}} \right) \geq 0, \quad i \in I,\ u \in U,\ s \in S, \tag{E.57}$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{u,s,nu}^{\text{PRM,W}} \right) \right\|_1 + \Phi_{u,s,nu}^{\text{PRM,W}} \cdot \overline{\xi}_{s,nu} + \phi_{u,s}^{\text{PRM,W}} \right) \geq 0, \quad u \in U,\ s \in S, \tag{E.58}$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\text{PRM,WP}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\text{PRM,WP}} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{\text{PRM,WP}} \right) \geq 0, \quad i \in I,\ u \in U,\ s \in S, \tag{E.59}$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,v,s,nu}^{\text{FP,p}} \right) \right\|_1 + \Phi_{i,v,s,nu}^{\text{FP,p}} \cdot \overline{\xi}_{s,nu} + \phi_{i,v,s}^{\text{FP,p}} \right) \geq 0, \quad i \in I,\ k \in K,\ v \in V,\ s \in S. \tag{E.60}$$

Here constraints (E.52)-(E.59) are to ensure that the affine functions for the second stage variables are non-negative. The values for $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1} \right)^T$ are shown in Table E.2.

Table E.2 $\delta_{s,nu}$ and $\left(M_{s,nu}^{-1} \right)^T$ values for the industrial supply chain – uncertain case A

$\delta_{s,'1'}$	$\delta_1, \forall s$
$\delta_{s,'2'}$	$\delta_2, \forall s$
$\left(M_{s,'1'}^{-1} \right)^T$	$1, \forall s$
$\left(M_{s,'2'}^{-1} \right)^T$	$1, \forall s$

Appendix F

Industrial Chemical Supply Chain Case Study - Parameters

Tables F.1 to F.35 provide the parameter values used for the industrial chemical supply chain case study problem. The capacity cost (C^{cap}) is a scalar set to \$1500 \$/t and the transfer price rate (r^{tp}) is 96.00%.

Table F.1 Income tax rates (r_i^{inc})

Plant	1	2	3	4	5
Income Tax Rate	10.0%	30.0%	30.0%	30.0%	2.5%

Table F.2 Duty rates ($r_{i,k}^{\text{du}}$) from plants to regional markets

Ship From / Ship To	Region1	Region2	Region3	Region4	Region5
PLANT1	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT2	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT3	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT4	2.5%	6.5%	2.5%	1.2%	0.0%
PLANT5	1.8%	0.0%	0.6%	1.2%	6.0%

Table F.3 Estimated duty rates ($\hat{r}_{i,j}^{\text{du}}$) from plants to regional warehouses

Ship From / Ship To	Region1	Region2	Region3	Region4	Region5
PLANT1	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT2	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT3	2.5%	6.5%	0.0%	1.2%	0.0%
PLANT4	2.5%	6.5%	2.5%	1.2%	0.0%
PLANT5	1.8%	0.0%	0.6%	1.2%	6.0%

Table F.4 Maximum allowable capacities ($Z_i^{\max}$)

Plant	1	2	3	4	5
Capacity (t)	12,000,000	7,600,000	5,600,000	4,000,000	7,200,000

Table F.5 RM2 to PRM grade ratio ($q_{i,u}^{\mathrm{RM2}}$)

PRM NAME	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
PRM-1	0.25	0.23	0.21	0.22	0.25
PRM-2	0.25	0.23	0.21	0.22	0.25
PRM-3	0.25	0.23	0.21	0.22	0.25
PRM-4	0.25	0.23	0.21	0.22	0.25
PRM-5	0.25	0.23	0.21	0.22	0.25
PRM-6	0.25	0.23	0.21	0.22	0.25
PRM-7	0.24	0.23	0.20	0.21	0.25
PRM-8	0.25	0.23	0.21	0.22	0.25
PRM-9	0.24	0.22	0.20	0.21	0.25
PRM-10	0.25	0.23	0.20	0.22	0.25
PRM-11	0.24	0.22	0.20	0.21	0.24
PRM-12	0.24	0.23	0.20	0.21	0.25
PRM-13	0.25	0.23	0.21	0.22	0.25
PRM-14	0.10	0.10	0.10	0.10	0.10
PRM-15	0.24	0.22	0.20	0.21	0.24
PRM-16	0.22	0.21	0.19	0.20	0.23
PRM-17	0.25	0.23	0.21	0.22	0.25
PRM-18	0.25	0.23	0.20	0.22	0.25
PRM-19	0.24	0.23	0.20	0.21	0.25
PRM-20	0.24	0.22	0.20	0.21	0.24
PRM-21	0.25	0.23	0.21	0.22	0.25
PRM-22	0.25	0.23	0.21	0.22	0.25
PRM-23	0.25	0.23	0.21	0.22	0.25
PRM-24	0.24	0.22	0.20	0.21	0.24
PRM-25	0.25	0.23	0.21	0.22	0.25
PRM-26	0.23	0.21	0.19	0.20	0.23
PRM-27	0.23	0.22	0.19	0.20	0.24
PRM-28	0.22	0.21	0.19	0.20	0.23
PRM-29	0.23	0.21	0.19	0.20	0.23
PRM-30	0.23	0.22	0.19	0.20	0.24
PRM-31	0.22	0.21	0.19	0.19	0.23
PRM-32	0.23	0.21	0.19	0.20	0.23
PRM-33	0.23	0.21	0.19	0.20	0.23
PRM-34	0.22	0.21	0.19	0.19	0.23
PRM-35	0.22	0.21	0.19	0.20	0.23
PRM-36	0.22	0.21	0.19	0.20	0.23
PRM-37	0.22	0.21	0.19	0.20	0.23
PRM-38	0.22	0.21	0.18	0.19	0.23
PRM-39	0.22	0.21	0.19	0.19	0.23
PRM-40	0.00	0.00	0.00	0.00	0.00
PRM-41	0.23	0.21	0.19	0.20	0.23
PRM-42	0.22	0.21	0.19	0.20	0.23
PRM-43	0.22	0.21	0.19	0.20	0.23
PRM-44	0.23	0.21	0.19	0.20	0.23
PRM-45	0.22	0.21	0.18	0.19	0.23
PRM-46	0.22	0.21	0.18	0.19	0.23
PRM-47	0.22	0.21	0.18	0.19	0.23
PRM-48	0.23	0.21	0.19	0.20	0.23
PRM-49	0.21	0.20	0.18	0.18	0.21
PRM-50	0.23	0.21	0.19	0.20	0.23

PRM-51	0.23	0.21	0.19	0.20	0.23
PRM-52	0.17	0.16	0.14	0.15	0.17
PRM-53	0.22	0.21	0.18	0.19	0.22
PRM-54	0.23	0.21	0.19	0.20	0.23
PRM-55	0.21	0.20	0.18	0.18	0.21

Table F.6 RM3 to PRM grade ratio ($q_{i,u}^{\text{RM3}}$)

PRM NAME	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
PRM-1	0.11	0.11	0.15	0.13	0.11
PRM-2	0.12	0.12	0.15	0.14	0.12
PRM-3	0.13	0.13	0.17	0.16	0.14
PRM-4	0.13	0.13	0.17	0.16	0.14
PRM-5	0.14	0.14	0.18	0.17	0.14
PRM-6	0.13	0.13	0.17	0.16	0.13
PRM-7	0.12	0.12	0.16	0.15	0.12
PRM-8	0.15	0.15	0.18	0.18	0.15
PRM-9	0.10	0.10	0.14	0.12	0.09
PRM-10	0.19	0.18	0.24	0.23	0.19
PRM-11	0.17	0.16	0.21	0.20	0.17
PRM-12	0.17	0.17	0.21	0.21	0.18
PRM-13	0.19	0.19	0.25	0.23	0.19
PRM-14	0.70	0.60	0.66	0.80	0.70
PRM-15	0.22	0.21	0.27	0.27	0.22
PRM-16	0.44	0.39	0.54	0.51	0.44
PRM-17	0.30	0.28	0.34	0.35	0.30
PRM-18	0.22	0.21	0.26	0.27	0.22
PRM-19	0.21	0.20	0.27	0.25	0.21
PRM-20	0.25	0.23	0.29	0.29	0.25
PRM-21	0.24	0.23	0.30	0.29	0.25
PRM-22	0.24	0.22	0.29	0.28	0.24
PRM-23	0.24	0.22	0.29	0.28	0.24
PRM-24	0.22	0.21	0.27	0.26	0.29
PRM-25	0.30	0.28	0.34	0.35	0.30
PRM-26	0.37	0.33	0.43	0.42	0.39
PRM-27	0.31	0.29	0.38	0.37	0.38
PRM-28	0.41	0.36	0.49	0.47	0.42
PRM-29	0.36	0.33	0.43	0.42	0.43
PRM-30	0.37	0.34	0.45	0.43	0.37
PRM-31	0.47	0.42	0.57	0.54	0.52
PRM-32	0.41	0.36	0.49	0.47	0.48
PRM-33	0.40	0.36	0.48	0.46	0.47
PRM-34	0.46	0.41	0.55	0.53	0.52
PRM-35	0.42	0.38	0.51	0.49	0.49
PRM-36	0.44	0.39	0.54	0.51	0.44
PRM-37	0.42	0.37	0.49	0.48	0.44
PRM-38	0.44	0.39	0.53	0.51	0.49
PRM-39	0.45	0.40	0.55	0.52	0.45
PRM-40	0.00	0.00	0.00	0.00	0.00
PRM-41	0.38	0.34	0.47	0.44	0.38
PRM-42	0.46	0.41	0.56	0.53	0.52
PRM-43	0.44	0.39	0.54	0.51	0.44

PRM-44	0.39	0.35	0.48	0.45	0.39
PRM-45	0.48	0.42	0.57	0.55	0.53
PRM-46	0.48	0.42	0.57	0.55	0.53
PRM-47	0.48	0.42	0.57	0.55	0.49
PRM-48	0.44	0.39	0.53	0.51	0.44
PRM-49	0.50	0.45	0.59	0.58	0.58
PRM-50	0.39	0.35	0.48	0.45	0.39
PRM-51	0.41	0.36	0.49	0.47	0.48
PRM-52	0.38	0.33	0.38	0.44	0.37
PRM-53	0.49	0.43	0.60	0.56	0.49
PRM-54	0.39	0.35	0.47	0.45	0.46
PRM-55	0.52	0.46	0.61	0.60	0.57

Table F.7 Waste to PRM grade ratio (q_u^{waste})

PRM NAME	WASTE FACTOR
PRM-1	4.00%
PRM-2	4.40%
PRM-3	5.00%
PRM-4	5.00%
PRM-5	5.00%
PRM-6	5.50%
PRM-7	6.50%
PRM-8	6.50%
PRM-9	5.80%
PRM-10	11.50%
PRM-11	9.20%
PRM-12	9.30%
PRM-13	10.30%
PRM-14	4.31%
PRM-15	16.00%
PRM-16	40.72%
PRM-17	16.60%
PRM-18	11.86%
PRM-19	14.70%
PRM-20	15.00%
PRM-21	14.90%
PRM-22	14.70%
PRM-23	14.70%
PRM-24	14.23%
PRM-25	16.60%
PRM-26	31.70%
PRM-27	25.00%
PRM-28	36.30%
PRM-29	29.59%
PRM-30	30.22%
PRM-31	42.10%
PRM-32	34.70%
PRM-33	33.70%
PRM-34	40.00%
PRM-35	36.20%

PRM-36	40.24%
PRM-37	36.50%
PRM-38	40.00%
PRM-39	41.90%
PRM-40	34.00%
PRM-41	34.63%
PRM-42	40.20%
PRM-43	40.72%
PRM-44	35.00%
PRM-45	43.00%
PRM-46	43.00%
PRM-47	43.00%
PRM-48	38.20%
PRM-49	48.00%
PRM-50	35.00%
PRM-51	34.70%
PRM-52	52.00%
PRM-53	46.36%
PRM-54	32.95%
PRM-55	50.00%

Table F.8 Impurity content ($q_{u,w}^{\text{imp}}$) in PRM

PRM grade / Impurity	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.00%	0.76%	0.00%	0.51%	0.01%	0.14%	0.03%	0.05%	0.34%	0.02%	0.78%	0.26%	0.33%
2	0.00%	0.42%	0.00%	0.30%	0.02%	0.13%	0.01%	0.03%	0.37%	0.02%	1.15%	0.25%	1.00%
3	0.00%	2.88%	0.00%	0.40%	0.03%	0.13%	0.04%	0.03%	0.36%	0.04%	1.30%	0.25%	0.80%
4	0.00%	2.19%	0.00%	0.33%	0.04%	0.05%	0.08%	0.03%	0.32%	0.10%	1.06%	0.11%	0.02%
5	0.00%	0.64%	0.00%	0.90%	0.00%	0.10%	0.05%	0.05%	0.00%	0.01%	2.00%	0.38%	1.00%
6	0.00%	2.50%	0.00%	0.40%	0.04%	0.12%	0.04%	0.12%	0.38%	0.06%	1.20%	0.18%	0.65%
7	0.00%	0.66%	0.00%	0.11%	0.00%	0.21%	0.05%	0.06%	0.21%	0.04%	0.72%	0.58%	1.08%
8	0.00%	0.66%	0.00%	0.45%	0.09%	0.11%	0.04%	0.04%	0.30%	0.02%	1.70%	0.20%	1.80%
9	0.00%	2.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.00%	5.50%	0.00%	1.10%	0.07%	0.18%	0.40%	1.30%	0.27%	0.03%	1.40%	0.21%	0.20%
11	11.64%	4.40%	0.00%	1.00%	0.04%	0.13%	0.39%	1.10%	0.23%	0.03%	1.15%	0.16%	0.15%
12	0.00%	1.80%	0.00%	0.38%	0.02%	0.11%	0.03%	0.05%	0.36%	0.04%	1.95%	0.17%	2.30%
13	0.00%	8.30%	0.00%	1.40%	0.10%	0.08%	1.15%	0.51%	0.33%	0.00%	1.00%	0.13%	0.11%
14	0.00%	0.84%	0.00%	0.22%	0.02%	0.15%	0.04%	0.03%	0.38%	0.05%	0.61%	0.62%	0.93%
15	0.00%	8.74%	0.00%	0.67%	0.15%	0.18%	0.34%	1.37%	0.12%	0.13%	1.30%	0.46%	0.09%
16	0.00%	36.02%	0.00%	0.84%	0.03%	0.23%	0.36%	1.05%	0.13%	0.07%	0.64%	0.25%	0.10%
17	0.00%	8.20%	0.00%	1.90%	0.32%	0.17%	5.10%	0.27%	0.04%	0.00%	1.70%	0.29%	0.02%
18	0.00%	4.24%	0.00%	1.79%	0.06%	0.23%	0.12%	0.04%	0.27%	0.07%	3.08%	0.24%	0.24%
19	0.00%	9.76%	0.00%	0.76%	0.02%	0.21%	0.78%	1.65%	0.17%	0.06%	0.76%	0.29%	0.13%
20	10.00%	6.00%	0.00%	1.60%	0.08%	0.65%	1.30%	1.20%	0.15%	0.05%	2.00%	0.35%	0.25%
21	0.00%	11.20%	0.00%	1.10%	0.15%	0.16%	0.85%	1.77%	0.14%	0.00%	1.70%	0.23%	0.18%
22	0.00%	11.70%	0.00%	1.00%	0.15%	0.16%	0.87%	1.72%	0.15%	0.01%	1.35%	0.23%	0.22%
23	0.00%	11.70%	0.00%	1.00%	0.15%	0.16%	0.87%	1.72%	0.15%	0.01%	1.35%	0.23%	0.22%
24	0.00%	7.87%	0.00%	1.15%	0.04%	0.18%	0.06%	0.11%	0.33%	0.21%	0.83%	0.22%	0.23%
25	0.00%	8.20%	0.00%	1.90%	0.32%	0.17%	5.10%	0.27%	0.04%	0.00%	1.70%	0.29%	0.02%
26	0.00%	20.00%	0.00%	1.40%	0.09%	0.29%	0.40%	0.70%	0.18%	0.20%	1.36%	0.32%	0.24%

27	0.00%	20.00%	0.00%	0.85%	0.04%	0.07%	0.14%	0.75%	0.24%	0.19%	0.60%	0.10%	0.40%
28	0.00%	27.30%	0.00%	0.72%	0.04%	0.37%	0.22%	0.87%	0.16%	0.09%	1.50%	0.36%	0.55%
29	0.00%	20.94%	0.00%	1.67%	0.02%	0.10%	0.13%	0.61%	0.20%	0.06%	1.04%	0.21%	0.52%
30	0.00%	23.37%	0.00%	2.63%	0.06%	0.32%	0.19%	0.62%	0.19%	0.05%	1.37%	0.16%	0.06%
31	0.00%	37.69%	0.00%	0.87%	0.02%	0.07%	0.10%	1.39%	0.15%	0.06%	0.67%	0.11%	0.46%
32	0.00%	28.60%	0.00%	1.30%	0.03%	0.06%	0.19%	1.03%	0.13%	0.16%	0.75%	0.08%	0.27%
33	0.00%	26.00%	0.00%	1.00%	0.12%	0.77%	0.33%	1.00%	0.13%	0.14%	0.90%	0.70%	0.09%
34	0.00%	31.50%	0.00%	1.30%	0.02%	0.17%	0.21%	1.00%	0.19%	0.08%	1.60%	0.20%	0.70%
35	0.00%	30.40%	0.00%	1.00%	0.05%	0.21%	0.72%	0.30%	0.08%	0.14%	0.95%	0.26%	0.11%
36	0.00%	35.45%	0.00%	1.06%	0.02%	0.04%	0.54%	0.39%	0.28%	0.08%	1.05%	0.06%	0.12%
37	0.00%	24.90%	0.00%	1.33%	0.08%	0.70%	0.60%	0.86%	0.13%	0.18%	1.35%	0.66%	0.40%
38	0.00%	33.40%	0.00%	0.84%	0.01%	0.06%	0.12%	1.15%	0.16%	0.04%	0.57%	0.15%	0.10%
39	0.00%	37.60%	0.00%	0.46%	0.11%	0.12%	0.23%	0.95%	0.09%	0.09%	0.90%	0.32%	0.06%
40	0.00%	21.74%	2.90%	2.20%	0.03%	0.12%	0.15%	0.48%	0.19%	0.14%	4.05%	0.18%	1.38%
41	0.00%	27.95%	0.00%	0.80%	0.11%	0.68%	0.37%	0.79%	0.13%	0.08%	0.59%	0.60%	0.03%
42	0.00%	36.50%	0.00%	0.90%	0.02%	0.06%	0.12%	1.45%	0.15%	0.06%	0.85%	0.08%	0.46%
43	0.00%	36.02%	0.00%	0.84%	0.03%	0.23%	0.36%	1.05%	0.13%	0.07%	0.64%	0.25%	0.10%
44	0.00%	29.98%	0.00%	0.96%	0.02%	0.28%	0.60%	0.86%	0.12%	0.07%	0.72%	0.23%	0.06%
45	0.00%	36.30%	0.00%	0.82%	0.02%	0.42%	0.33%	1.55%	0.19%	0.06%	0.80%	0.40%	0.30%
46	0.00%	36.30%	0.00%	0.82%	0.02%	0.42%	0.33%	1.55%	0.19%	0.06%	0.80%	0.40%	0.30%
47	0.00%	34.50%	0.00%	1.10%	0.07%	1.25%	1.40%	0.90%	0.10%	0.13%	0.76%	0.30%	0.14%
48	0.00%	32.30%	0.00%	0.88%	0.08%	0.09%	0.24%	0.88%	0.18%	0.03%	2.37%	0.15%	0.04%
49	0.00%	36.10%	0.00%	0.85%	0.08%	0.15%	0.54%	0.33%	0.13%	0.14%	0.62%	0.19%	0.15%
50	0.00%	29.98%	0.00%	0.96%	0.02%	0.28%	0.60%	0.86%	0.12%	0.07%	0.72%	0.23%	0.06%
51	0.00%	28.60%	0.00%	1.30%	0.03%	0.06%	0.19%	1.03%	0.13%	0.16%	0.75%	0.08%	0.27%
52	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
53	0.00%	43.65%	0.00%	0.48%	0.01%	0.13%	0.49%	1.04%	0.11%	0.04%	0.48%	0.18%	0.08%
54	0.00%	27.05%	0.00%	1.22%	0.03%	0.06%	0.18%	0.98%	0.15%	0.16%	0.72%	0.08%	0.29%
55	0.00%	36.30%	0.00%	0.82%	0.02%	0.42%	0.33%	1.55%	0.19%	0.06%	0.80%	0.40%	0.30%

Table F.8 Continued

PRM grade / Impurity	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	0.00%	0.02%	0.0013%	0.0021%	0.0000%	0.0900%	0.0018%	0.0005%	0.0023%	0.0096%	0.0246%	0.0000%	0.0018%	0.0044%	0.0000%
2	0.00%	0.03%	0.0018%	0.0044%	0.0200%	0.1400%	0.0025%	0.0008%	0.0070%	0.0250%	0.0218%	0.0000%	0.0017%	0.0000%	0.0000%
3	0.00%	0.03%	0.0012%	0.0038%	0.0200%	0.1400%	0.0025%	0.0003%	0.0070%	0.0180%	0.0251%	0.0000%	0.0017%	0.0000%	0.0000%
4	0.00%	0.02%	0.0039%	0.0018%	0.0000%	0.3800%	0.0658%	0.0004%	0.0003%	0.0027%	0.0224%	0.0000%	0.0017%	0.0068%	0.0000%
5	0.00%	0.00%	0.0072%	0.0045%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
6	0.00%	0.03%	0.0025%	0.0030%	0.0200%	0.0200%	0.0125%	0.0025%	0.0200%	0.0093%	0.0311%	0.0000%	0.0018%	0.0007%	0.0000%
7	0.00%	0.03%	0.0031%	0.0030%	0.0000%	0.0000%	0.1665%	0.0010%	0.0018%	0.0023%	0.0053%	0.0000%	0.0008%	0.0010%	0.0000%
8	0.00%	0.03%	0.0055%	0.0055%	0.0200%	0.1800%	0.0030%	0.0008%	0.0128%	0.0135%	0.0214%	0.0078%	0.0017%	0.0003%	0.0000%
9	0.00%	0.00%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
10	0.00%	0.03%	0.0300%	0.0010%	0.0000%	0.0000%	0.2000%	0.0010%	0.0000%	0.0040%	0.0210%	0.0000%	0.0019%	0.0004%	0.0000%
11	0.00%	0.02%	0.0130%	0.0013%	0.0000%	0.2500%	0.4500%	0.0001%	0.0017%	0.0025%	0.0000%	0.0000%	0.0028%	0.0020%	0.0000%
12	0.00%	0.02%	0.0080%	0.0060%	0.0000%	0.3000%	0.0085%	0.0063%	0.0285%	0.0550%	0.0247%	0.0000%	0.0018%	0.0010%	0.0000%
13	0.00%	0.03%	0.0120%	0.0005%	0.0000%	0.0000%	0.0660%	0.0000%	0.0000%	0.0000%	0.0206%	0.0000%	0.0017%	0.0000%	0.0000%
14	0.00%	0.02%	0.0047%	0.0027%	0.0200%	0.1400%	0.0100%	0.0041%	0.0257%	0.0206%	0.0258%	0.0000%	0.0018%	0.0003%	0.0000%
15	0.00%	0.00%	0.0053%	0.0025%	0.0000%	0.0000%	0.0114%	0.0049%	0.0143%	0.0006%	0.0318%	0.0000%	0.0022%	0.0330%	0.0000%
16	0.00%	0.02%	0.0096%	0.0006%	0.0000%	0.0000%	0.0000%	0.0039%	0.0339%	0.0059%	0.0156%	0.0000%	0.0020%	0.0155%	0.0000%
17	0.00%	0.02%	0.0030%	0.0005%	0.0100%	0.0700%	0.0370%	0.0013%	0.0048%	0.0005%	0.0206%	0.0000%	0.0017%	0.0005%	0.0000%
18	0.00%	0.00%	0.0223%	0.0016%	0.0000%	0.0000%	0.0243%	0.0110%	0.0787%	0.0124%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
19	0.00%	0.04%	0.0076%	0.0012%	0.0000%	0.0000%	0.0000%	0.0029%	0.0271%	0.0083%	0.0223%	0.0000%	0.0033%	0.0241%	0.0000%

20	0.00%	0.03%	0.0160%	0.0025%	0.0000%	0.2000%	0.3000%	0.0010%	0.0000%	0.0040%	0.0210%	0.0000%	0.0019%	0.0004%	0.0000%
21	0.00%	0.02%	0.0010%	0.0010%	0.0100%	0.1000%	0.0600%	0.0005%	0.0000%	0.0004%	0.0210%	0.0210%	0.0018%	0.0001%	0.0010%
22	0.00%	0.02%	0.0012%	0.0005%	0.0100%	0.1200%	0.0680%	0.0009%	0.0027%	0.0011%	0.0197%	0.0210%	0.0017%	0.0003%	0.0010%
23	0.00%	0.02%	0.0012%	0.0005%	0.0100%	0.1200%	0.0680%	0.0009%	0.0027%	0.0011%	0.0197%	0.0210%	0.0017%	0.0003%	0.0010%
24	0.00%	0.03%	0.0240%	0.0020%	0.2000%	0.7300%	0.0126%	0.0200%	0.0881%	0.0132%	0.0226%	0.0150%	0.0019%	0.0037%	0.0000%
25	0.00%	0.02%	0.0030%	0.0005%	0.0100%	0.0700%	0.0370%	0.0013%	0.0048%	0.0005%	0.0206%	0.0000%	0.0017%	0.0005%	0.0000%
26	0.02%	0.02%	0.0140%	0.0020%	0.0000%	0.8000%	0.0600%	0.0155%	0.0630%	0.0200%	0.0220%	0.0000%	0.0000%	0.0190%	0.0163%
27	0.00%	0.02%	0.0030%	0.0020%	0.2000%	0.7000%	0.0090%	0.0060%	0.0520%	0.0135%	0.0200%	0.0610%	0.0020%	0.0085%	0.0010%
28	0.00%	0.02%	0.0115%	0.0018%	0.0000%	0.6300%	0.0210%	0.0195%	0.0732%	0.0130%	0.0154%	0.0000%	0.0022%	0.0187%	0.0000%
29	0.00%	0.03%	0.0369%	0.0017%	0.2000%	0.6000%	0.0342%	0.0101%	0.0784%	0.0093%	0.0195%	0.0200%	0.0020%	0.0111%	0.0000%
30	0.00%	0.00%	0.0175%	0.0010%	0.0000%	0.0000%	0.0228%	0.0110%	0.1000%	0.0078%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
31	0.00%	0.00%	0.0043%	0.0009%	0.2000%	0.0200%	0.0161%	0.0025%	0.0291%	0.0071%	0.0146%	0.0000%	0.0022%	0.0268%	0.0030%
32	0.01%	0.02%	0.0030%	0.0015%	0.2000%	0.6500%	0.0070%	0.0050%	0.0500%	0.0085%	0.0160%	0.0000%	0.0022%	0.0150%	0.0010%
33	0.01%	0.02%	0.0045%	0.0019%	0.2000%	0.6000%	0.0100%	0.0117%	0.0900%	0.0075%	0.0163%	0.0150%	0.0024%	0.0180%	0.0010%
34	0.00%	0.00%	0.0420%	0.0014%	0.2000%	0.3746%	0.0101%	0.0150%	0.1008%	0.0093%	0.0166%	0.0240%	0.0024%	0.0285%	0.0028%
35	0.00%	0.02%	0.0210%	0.0018%	0.2000%	0.6000%	0.2000%	0.0026%	0.0300%	0.0060%	0.0200%	0.0000%	0.0030%	0.0250%	0.0000%
36	0.00%	0.02%	0.0122%	0.0021%	0.0000%	0.0000%	0.0461%	0.0034%	0.0340%	0.0074%	0.0162%	0.0000%	0.0019%	0.0283%	0.0000%
37	0.01%	0.01%	0.0165%	0.0025%	0.0000%	0.8000%	0.0210%	0.0115%	0.0500%	0.0125%	0.0180%	0.0000%	0.0025%	0.0229%	0.0000%
38	0.00%	0.02%	0.0170%	0.0015%	0.2000%	0.1000%	0.0216%	0.0065%	0.0560%	0.0076%	0.0159%	0.0000%	0.0023%	0.0114%	0.0000%
39	0.00%	0.00%	0.0037%	0.0017%	0.0000%	0.0000%	0.0079%	0.0034%	0.0099%	0.0004%	0.0220%	0.0000%	0.0015%	0.0228%	0.0000%
40	0.00%	0.14%	0.0450%	0.0030%	0.0000%	0.0000%	0.0190%	0.0028%	0.0238%	0.0061%	0.0000%	0.0000%	0.0000%	0.0205%	0.0000%
41	0.00%	0.00%	0.0055%	0.0017%	0.0000%	0.0000%	0.0000%	0.0099%	0.0000%	0.0078%	0.0000%	0.0000%	0.0000%	0.0154%	0.0000%
42	0.00%	0.00%	0.0045%	0.0015%	0.2000%	0.4000%	0.0120%	0.0020%	0.0300%	0.0070%	0.0146%	0.0000%	0.0022%	0.0280%	0.0030%
43	0.00%	0.02%	0.0096%	0.0006%	0.0000%	0.0000%	0.0000%	0.0039%	0.0339%	0.0059%	0.0156%	0.0000%	0.0020%	0.0155%	0.0000%
44	0.01%	0.01%	0.0032%	0.0008%	0.0000%	0.0000%	0.0145%	0.0035%	0.0220%	0.0095%	0.0315%	0.0000%	0.0035%	0.0185%	0.0000%
45	0.00%	0.00%	0.0185%	0.0015%	0.2000%	0.0200%	0.0010%	0.0055%	0.0550%	0.0105%	0.0140%	0.0000%	0.0026%	0.0230%	0.0010%
46	0.00%	0.00%	0.0185%	0.0015%	0.2000%	0.0200%	0.0010%	0.0055%	0.0550%	0.0105%	0.0140%	0.0000%	0.0026%	0.0230%	0.0010%
47	0.01%	0.02%	0.0100%	0.0016%	0.0000%	0.5700%	0.0121%	0.0089%	0.0324%	0.0100%	0.0143%	0.0000%	0.0000%	0.0237%	0.0000%
48	0.00%	0.02%	0.0113%	0.0000%	0.0000%	0.0000%	0.0059%	0.0042%	0.0284%	0.0360%	0.0100%	0.0000%	0.0010%	0.0200%	0.0000%
49	0.01%	0.05%	0.0159%	0.0010%	0.2000%	1.0000%	0.0136%	0.0027%	0.0173%	0.0014%	0.0058%	0.0188%	0.0084%	0.0402%	0.0030%
50	0.01%	0.01%	0.0032%	0.0008%	0.0000%	0.0000%	0.0145%	0.0035%	0.0220%	0.0095%	0.0315%	0.0000%	0.0035%	0.0185%	0.0000%
51	0.01%	0.02%	0.0030%	0.0015%	0.2000%	0.6500%	0.0070%	0.0050%	0.0500%	0.0085%	0.0160%	0.0000%	0.0022%	0.0150%	0.0010%
52	0.00%	0.00%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
53	0.00%	0.02%	0.0048%	0.0008%	0.0000%	0.0000%	0.0000%	0.0019%	0.0171%	0.0052%	0.0140%	0.0000%	0.0021%	0.0152%	0.0000%
54	0.00%	0.02%	0.0030%	0.0016%	0.2000%	0.6590%	0.0074%	0.0052%	0.0504%	0.0094%	0.0167%	0.0110%	0.0022%	0.0138%	0.0010%
55	0.00%	0.00%	0.0185%	0.0015%	0.2000%	0.0200%	0.0010%	0.0055%	0.0550%	0.0105%	0.0140%	0.0000%	0.0026%	0.0230%	0.0010%

Table F.8 Continued

PRM grade / Impurity	29	30	31	32	33	34	35	36	37	38	39	40	41
1	6.58%	0.0034%	0.09%	0.07%	0.67%	43.42%	44.74%	0.00%	0.58%	0.04%	0.12%	0.01%	10.53%
2	6.55%	0.0062%	0.05%	0.05%	1.37%	238.10%	88.10%	0.00%	0.00%	0.03%	0.08%	0.02%	8.93%
3	0.95%	0.0050%	0.10%	0.10%	1.16%	27.78%	12.50%	0.00%	0.00%	0.07%	0.13%	0.03%	2.34%
4	1.37%	0.0057%	0.15%	0.14%	0.34%	0.91%	14.61%	0.00%	0.31%	0.12%	0.17%	0.04%	5.02%
5	7.81%	0.0117%	0.10%	0.05%	1.00%	156.25%	0.00%	0.00%	0.00%	0.05%	0.10%	0.00%	15.63%
6	4.80%	0.0055%	0.20%	0.12%	1.03%	26.00%	15.20%	0.00%	0.03%	0.08%	0.24%	0.04%	6.40%
7	8.33%	0.0061%	0.11%	0.06%	1.29%	163.64%	31.06%	0.00%	0.15%	0.06%	0.11%	0.00%	15.91%
8	5.30%	0.0110%	0.16%	0.15%	2.10%	272.73%	45.45%	0.00%	0.04%	0.13%	0.18%	0.09%	11.36%
9	0.00%	0.0000%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	22.35%	0.0310%	1.77%	0.49%	0.47%	3.64%	4.91%	0.00%	0.01%	0.53%	1.79%	0.08%	30.91%
11	25.00%	0.0143%	1.53%	0.43%	0.38%	3.41%	5.23%	0.00%	0.05%	0.47%	1.53%	0.04%	33.86%
12	2.78%	0.0140%	0.10%	0.08%	2.66%	127.78%	20.00%	0.01%	0.06%	0.06%	0.13%	0.02%	4.44%

13	6.14%	0.0125%	1.76%	1.27%	0.44%	1.33%	3.98%	0.00%	0.00%	1.39%	1.78%	0.11%	20.00%
14	3.57%	0.0074%	0.09%	0.08%	1.30%	110.12%	44.64%	0.00%	0.04%	0.06%	0.11%	0.02%	7.74%
15	15.67%	0.0078%	1.86%	0.53%	0.21%	0.99%	1.42%	0.01%	0.38%	0.58%	1.89%	0.18%	19.50%
16	2.92%	0.0101%	1.44%	0.40%	0.23%	0.28%	0.36%	0.01%	0.04%	0.65%	0.04%	0.05%	3.90%
17	3.29%	0.0035%	5.69%	5.44%	0.06%	0.24%	0.49%	0.00%	0.01%	6.50%	5.71%	0.38%	65.49%
18	0.94%	0.0239%	0.22%	0.18%	0.51%	5.67%	6.37%	0.01%	0.00%	0.20%	0.22%	0.07%	3.73%
19	16.86%	0.0087%	2.44%	0.82%	0.30%	1.30%	1.79%	0.00%	0.25%	0.93%	2.46%	0.02%	24.85%
20	6.00%	0.0185%	2.58%	1.40%	0.40%	4.17%	2.50%	0.00%	0.01%	1.62%	2.60%	0.09%	41.67%
21	15.80%	0.0020%	2.77%	1.02%	0.32%	1.61%	1.25%	0.00%	0.00%	1.18%	2.79%	0.18%	23.39%
22	14.70%	0.0017%	2.74%	1.04%	0.37%	1.88%	1.28%	0.00%	0.00%	1.20%	2.76%	0.18%	22.14%
23	14.70%	0.0017%	2.74%	1.04%	0.37%	1.88%	1.28%	0.00%	0.00%	1.20%	2.76%	0.18%	22.14%
24	1.40%	0.0260%	0.21%	0.13%	0.56%	2.92%	4.19%	0.02%	0.05%	0.12%	0.24%	0.05%	2.16%
25	3.29%	0.0035%	5.69%	5.44%	0.06%	0.24%	0.49%	0.00%	0.01%	6.50%	5.71%	0.38%	65.49%
26	3.50%	0.0160%	1.19%	0.53%	0.42%	1.20%	0.90%	0.02%	0.10%	0.72%	1.23%	0.13%	5.50%
27	3.75%	0.0050%	0.93%	0.20%	0.64%	2.00%	1.20%	0.01%	0.04%	0.23%	0.95%	0.05%	4.45%
28	3.19%	0.0133%	1.13%	0.28%	0.71%	2.01%	0.59%	0.03%	0.07%	0.41%	1.15%	0.06%	3.99%
29	2.90%	0.0386%	0.76%	0.17%	0.72%	2.47%	0.96%	0.01%	0.05%	0.21%	0.78%	0.03%	3.51%
30	2.66%	0.0185%	0.87%	0.25%	0.26%	0.27%	0.82%	0.02%	0.00%	0.35%	0.87%	0.09%	3.45%
31	3.68%	0.0052%	1.51%	0.14%	0.61%	1.21%	0.40%	0.00%	0.07%	0.22%	1.53%	0.04%	3.95%
32	3.60%	0.0045%	1.25%	0.24%	0.40%	0.94%	0.45%	0.01%	0.05%	0.34%	1.27%	0.05%	4.27%
33	3.85%	0.0064%	1.45%	0.48%	0.22%	0.35%	0.50%	0.02%	0.07%	0.68%	1.48%	0.18%	5.12%
34	3.17%	0.0434%	1.23%	0.25%	0.89%	2.22%	0.62%	0.03%	0.09%	0.38%	1.25%	0.03%	3.84%
35	0.99%	0.0228%	1.07%	0.79%	0.19%	0.36%	0.26%	0.00%	0.08%	1.20%	1.09%	0.07%	3.36%
36	1.10%	0.0143%	0.94%	0.57%	0.40%	0.34%	0.79%	0.01%	0.08%	0.92%	0.96%	0.03%	2.61%
37	3.45%	0.0190%	1.54%	0.71%	0.53%	1.61%	0.52%	0.02%	0.09%	1.07%	1.57%	0.13%	5.86%
38	3.43%	0.0184%	1.27%	0.14%	0.26%	0.29%	0.48%	0.01%	0.03%	0.21%	1.29%	0.02%	3.77%
39	2.52%	0.0054%	1.29%	0.36%	0.15%	0.16%	0.23%	0.01%	0.06%	0.58%	1.31%	0.18%	0.00%
40	2.21%	0.0116%	2.37%	1.49%	0.24%	0.41%	0.29%	0.02%	0.07%	2.58%	2.39%	0.12%	6.67%
41	2.83%	0.0072%	1.27%	0.48%	0.16%	0.11%	0.47%	0.02%	0.06%	0.73%	1.27%	0.17%	4.15%
42	3.97%	0.0060%	1.59%	0.16%	0.61%	1.26%	0.41%	0.00%	0.08%	0.23%	1.61%	0.03%	4.30%
43	2.92%	0.0101%	1.44%	0.40%	0.23%	0.28%	0.36%	0.01%	0.04%	0.65%	0.04%	0.05%	3.90%
44	2.87%	0.0040%	1.48%	0.66%	0.18%	0.20%	0.40%	0.01%	0.06%	0.95%	1.52%	0.03%	4.87%
45	4.29%	0.0200%	1.90%	0.36%	0.49%	0.83%	0.52%	0.01%	0.06%	0.61%	1.91%	0.03%	5.18%
46	4.27%	0.0200%	1.90%	0.36%	0.49%	0.83%	0.52%	0.01%	0.06%	0.61%	1.91%	0.03%	5.18%
47	2.61%	0.0116%	2.37%	1.49%	0.24%	0.41%	0.29%	0.02%	0.07%	2.58%	2.39%	0.12%	6.67%
48	2.74%	0.0113%	1.20%	0.33%	0.21%	0.12%	0.54%	0.01%	0.06%	0.51%	1.21%	0.12%	3.48%
49	0.90%	0.0169%	0.94%	0.63%	0.29%	0.42%	0.37%	0.01%	0.11%	1.18%	0.95%	0.15%	2.39%
50	2.87%	0.0040%	1.48%	0.66%	0.18%	0.20%	0.40%	0.01%	0.06%	0.95%	1.52%	0.03%	4.87%
51	3.60%	0.0045%	1.25%	0.24%	0.40%	0.94%	0.45%	0.01%	0.05%	0.34%	1.27%	0.05%	4.27%
52	0.00%	0.0000%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
53	2.37%	0.0055%	1.54%	0.51%	0.19%	0.18%	0.25%	0.00%	0.03%	0.93%	0.03%	0.02%	3.49%
54	3.63%	0.0046%	1.19%	0.23%	0.44%	1.13%	0.59%	0.01%	0.05%	0.32%	1.21%	0.05%	4.30%
55	4.29%	0.0200%	1.90%	0.36%	0.49%	0.83%	0.52%	0.01%	0.06%	0.61%	1.91%	0.03%	5.18%

Table F.9 Proportion of FP shipped from plant to the regional warehouse ($q_{i,j}^{\text{fr,PW}}$)

Shipped from	RegWrhs1	RegWrhs2	RegWrhs3	RegWrhs4	RegWrhs5
PLANT1	0.0%	0.0%	23.9%	43.5%	0.0%
PLANT2	35.3%	7.7%	72.8%	56.4%	29.6%
PLANT3	0.0%	0.0%	2.9%	0.2%	25.9%
PLANT4	24.9%	10.1%	0.4%	0.0%	44.6%
PLANT5	39.9%	82.1%	0.0%	0.0%	0.0%

Table F.10 Maximum impurity limit ($Q_{i,w}^{\text{imp}}$)

Impurity / Plant	Plant1	Plant2	Plant3	Plant4	Plant5
Impurity-1	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-2	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-3	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-4	1.50%	1.20%	1.50%	1.50%	1.00%
Impurity-5	0.21%	0.12%	0.10%	0.16%	0.08%
Impurity-6	100.0%	0.45%	0.20%	0.25%	100.0%
Impurity-7	1.00%	1.00%	1.10%	100.0%	0.35%
Impurity-8	1.00%	100.00%	100.00%	100.00%	100.00%
Impurity-9	100.0%	1.08%	100.0%	100.0%	100.0%
Impurity-10	0.25%	0.10%	0.20%	0.20%	100.0%
Impurity-11	1.50%	1.50%	2.00%	2.50%	100.0%
Impurity-12	1.00%	100.0%	100.0%	100.0%	100.0%
Impurity-13	1.00%	100.0%	100.0%	100.0%	100.0%
Impurity-14	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-15	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-16	100.00%	100.00%	100.00%	100.00%	100.00%
Impurity-17	100.00%	100.00%	100.00%	100.00%	100.00%
Impurity-18	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-19	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-20	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-21	50	90	60	42.5	1,000,000
Impurity-22	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-23	100	120	75	130	600
Impurity-24	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-25	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-26	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-27	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-28	100.0%	100.0%	100.0%	100.0%	100.0%
Impurity-29	100.0%	100.0%	4.0%	100.0%	100.0%
Impurity-30	183	500	100	150	500
Impurity-31	100.0%	100.0%	100.0%	2.55%	100.0%
Impurity-32	100.0%	1.85%	100.0%	100.0%	100.0%
Impurity-33	100.0%	100.0%	100.0%	100.0%	1.60%
Impurity-34	100.0%	100.0%	1.00%	100.0%	100.0%
Impurity-35	100.0%	100.0%	0.66%	100.0%	100.0%
Impurity-36	1,000,000.00	1,000,000.00	1,000,000.00	52	1,000,000.00
Impurity-37	100.0%	100.0%	0.07%	100.0%	100.0%
Impurity-38	1.20%	100.0%	100.0%	100.0%	100.0%
Impurity-39	100.00%	1.85%	100.00%	100.00%	100.00%
Impurity-40	100.0%	0.12%	100.0%	100.0%	100.0%
Impurity-41	100.0%	100.0%	100.0%	100.0%	100.0%

Table F.11 Price of final product ($P^{FP}_{k,v}$) in $/t

FP GRADE	Region1	Region2	Region3	Region4	Region5
Grade-1	5,000	5,000	5,100	5,300	5,300
Grade-2	5,000	5,100	5,100	5,300	5,300
Grade-3	-	-	-	-	-
Grade-4	-	-	-	-	-
Grade-5	-	-	-	-	-
Grade-6	-	-	-	-	-
Grade-7	5,000	5,000	5,100	5,300	5,200
Grade-8	5,000	5,000	5,100	5,300	5,200
Grade-9	5,000	5,000	5,100	5,300	5,200
Grade-10	5,000	5,100	5,100	5,300	5,200
Grade-11	5,000	5,100	5,100	5,300	5,200
Grade-12	5,000	5,000	5,200	5,300	5,200
Grade-13	5,000	5,100	5,200	5,200	5,200
Grade-14	5,000	5,000	5,100	5,300	5,200
Grade-15	5,000	5,000	5,100	5,300	5,200
Grade-16	5,000	5,100	5,100	5,300	5,200
Grade-17	5,000	5,100	5,100	5,300	5,200
Grade-18	5,000	5,000	5,100	5,200	5,200
Grade-19	5,000	5,000	5,100	5,200	5,200
Grade-20	5,000	5,100	5,100	5,200	5,200
Grade-21	5,000	5,000	5,100	5,300	5,200
Grade-22	5,000	5,100	5,100	5,200	5,200
Grade-23	5,000	5,000	5,100	5,300	5,200

Table F.12 Estimated price of FP at regional warehouse ($\hat{P}^{FP}_{j,v}$) in $/t

FP GRADE	RW1	RW2	RW3	RW4	RW5
Grade-1	5,000	5,000	5,100	5,300	5,300
Grade-2	5,000	5,100	5,100	5,300	5,300
Grade-3	-	-	-	-	-
Grade-4	-	-	-	-	-
Grade-5	-	-	-	-	-
Grade-6	-	-	-	-	-
Grade-7	5,000	5,000	5,100	5,300	5,200
Grade-8	5,000	5,000	5,100	5,300	5,200
Grade-9	5,000	5,000	5,100	5,300	5,200
Grade-10	5,000	5,100	5,100	5,300	5,200
Grade-11	5,000	5,100	5,100	5,300	5,200
Grade-12	5,000	5,000	5,200	5,300	5,200
Grade-13	5,000	5,100	5,200	5,200	5,200
Grade-14	5,000	5,000	5,100	5,300	5,200
Grade-15	5,000	5,000	5,100	5,300	5,200
Grade-16	5,000	5,100	5,100	5,300	5,200
Grade-17	5,000	5,100	5,100	5,300	5,200
Grade-18	5,000	5,000	5,100	5,200	5,200
Grade-19	5,000	5,000	5,100	5,200	5,200
Grade-20	5,000	5,100	5,100	5,200	5,200
Grade-21	5,000	5,000	5,100	5,300	5,200
Grade-22	5,000	5,100	5,100	5,200	5,200
Grade-23	5,000	5,000	5,100	5,300	5,200

Table F.13 Fixed costs (C_i^{fix}) and other plant variable costs (C_i^{var})

	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
Other Plant Variable Cost of FG - $/T	200	250	275	300	400
FIXED COST ($MM)	30.00	120.00	40.00	100.00	20.00

Table F.14 Freight cost from plants to regional markets ($C_{i,k}^{\text{fr,FP,PD}}$) in $/t

Ship From / Ship To	Region1	Region2	Region3	Region4	Region5
PLANT1	181	212	194	83	251
PLANT2	201	258	215	166	138
PLANT3	455	455	455	383	176
PLANT4	205	205	275	196	135
PLANT5	53	45	226	226	226

Table F.15 Freight cost from plants to regional warehouses ($C_{i,j}^{\text{fr,FP,PW}}$) in $/t

Ship From / Ship To	RW1	RW2	RW3	RW4	RW5
PLANT1	181	212	194	83	251
PLANT2	201	258	215	166	138
PLANT3	455	455	455	383	176
PLANT4	205	205	275	196	135
PLANT5	53	45	226	226	226

Table F.16 Freight cost from regional warehouses to regional markets ($C_{j,k}^{\text{fr,FP,WD}}$) in $/t

Ship From / Ship To	Region1	Region2	Region3	Region4	Region5
RW1	105	-	-	-	-
RW2	-	-	-	-	-
RW3	-	-	105	-	-
RW4	-	-	-	105	-
RW5	105	105	105	105	105

Table F.17 Plant on-site inventory cost (C_i^{PI}) in $/t

PLANT	**Cost**
PLANT1	9.5
PLANT2	23.0
PLANT3	0.0
PLANT4	20.0
PLANT5	3.1

Table F.18 Regional warehouse inventory cost (C_j^{WI}) in $/t

Warehouse	**Cost**
RW1	9.5
RW2	23.0
RW3	0.0
RW4	20.0
RW5	3.1

Table F.19 Cost of primary raw material ($C_{i,u}^{\text{PRM}}$) in $/t

PRM NAME	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
PRM-1	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-2	2,062.00	2,060.00	2,065.00	2,070.00	2,073.00
PRM-3	2,062.00	2,060.00	2,065.00	2,070.00	2,073.00
PRM-4	10,076.00	10,074.00	10,079.00	10,084.00	10,087.00
PRM-5	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-6	2,072.00	2,070.00	2,075.00	2,080.00	2,083.00
PRM-7	2,062.00	2,060.00	2,065.00	2,070.00	2,073.00
PRM-8	2,552.00	2,553.00	2,558.00	2,563.00	2,566.00
PRM-9	1,962.00	1,960.00	1,965.00	1,970.00	1,973.00
PRM-10	2,562.00	2,560.00	2,565.00	2,570.00	2,573.00
PRM-11	2,562.00	2,560.00	2,565.00	2,570.00	2,573.00
PRM-12	1,962.00	1,960.00	1,965.00	1,970.00	1,973.00
PRM-13	1,957.00	1,955.00	1,960.00	1,965.00	1,968.00
PRM-14	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-15	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-16	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-17	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-18	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-19	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-20	1,562.00	1,560.00	1,565.00	1,570.00	1,573.00
PRM-21	1,562.00	1,560.00	1,565.00	1,570.00	1,573.00
PRM-22	752.00	753.00	758.00	763.00	766.00
PRM-23	10,051.00	10,052.00	10,057.00	10,062.00	10,065.00
PRM-24	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-25	557.00	555.00	560.00	565.00	568.00
PRM-26	562.00	560.00	565.00	570.00	573.00
PRM-27	612.00	570.32	556.40	584.61	673.00
PRM-28	662.00	660.00	665.00	670.00	673.00
PRM-29	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-30	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-31	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-32	612.00	570.32	556.40	584.61	673.00
PRM-33	572.00	570.00	575.00	580.00	583.00
PRM-34	10,071.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-35	372.00	370.00	375.00	380.00	383.00
PRM-36	10,066.00	10,064.00	10,069.00	10,074.00	10,077.00
PRM-37	362.00	360.00	365.00	370.00	373.00
PRM-38	462.00	460.00	465.00	470.00	473.00
PRM-39	372.00	370.00	375.00	380.00	383.00
PRM-40	362.00	360.00	365.00	370.00	373.00
PRM-41	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-42	1,511.00	594.99	548.58	600.00	1,512.00
PRM-43	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-44	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-45	362.00	360.00	365.00	370.00	373.00
PRM-46	362.00	360.00	365.00	370.00	373.00
PRM-47	262.00	260.00	265.00	270.00	273.00
PRM-48	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-49	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-50	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00

PRM-51	10,071.00	10,069.00	10,074.00	10,079.00	10,082.00
PRM-52	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-53	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00
PRM-54	10,061.00	10,014.00	10,019.00	10,024.00	10,027.00
PRM-55	10,061.00	10,059.00	10,064.00	10,069.00	10,072.00

Table F.20 Cost of RM2 (C_i^{RM2}), RM3 (C_i^{RM3}), and waste (C_i^{waste}) in $/t

	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
RM-2 Price	200	250	300	275	400
RM-3 Price	300	250	350	275	375
Waste Price	40	35	100	110	55

Table F.21 Average additive factors ($a_i^{\text{avg,FP}}$), minimum turndown slope (m_i^{td}) and intercept (b_i^{td}), waste limit slope (m_i^{wl}) and intercept (b_i^{wl}), scheduled outage (O_i), plant uptime (U_i), Yield of FP (Y_i^{FP}) and maximum limit for total blend (Q_i^{imp})

	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
Average additive factor	1.078	1.046	1.022	1.077	1.070
Minimum turndown slope	3.50	4.20	4.00	3.00	2.40
Minimum turndown intercept	0	0	0	0	0
Waste limit slope	2.00	2.50	2.50	2.00	2.50
Waste limit intercept	15	31	23	27	6
Scheduled outage (d/y)	10.0	15.0	15.0	10.67	8.0
Plant uptime	95.00 %	90.00 %	85.00 %	93.33 %	95.00 %
Yield	98.0 %	98.0 %	98.0 %	98.0 %	98.0 %
Product blend limit	100.0%	80.0 %	80.0 %	90.0 %	100.0%

Table F.22 Effective percentage in PRM for generating FP (E_u^{PRM})

PRM NAME	**PRODUCT Mix**
PRM-1	96.00%
PRM-2	0.00%
PRM-3	95.00%
PRM-4	95.00%
PRM-5	95.00%
PRM-6	94.50%
PRM-7	93.50%
PRM-8	93.50%
PRM-9	94.20%
PRM-10	88.50%

PRM-11	77.86%
PRM-12	90.70%
PRM-13	89.70%
PRM-14	95.69%
PRM-15	84.00%
PRM-16	59.28%
PRM-17	83.40%
PRM-18	88.14%
PRM-19	85.30%
PRM-20	73.89%
PRM-21	85.10%
PRM-22	85.30%
PRM-23	85.30%
PRM-24	85.77%
PRM-25	83.40%
PRM-26	68.30%
PRM-27	75.00%
PRM-28	63.70%
PRM-29	70.41%
PRM-30	69.78%
PRM-31	57.90%
PRM-32	65.30%
PRM-33	66.30%
PRM-34	60.00%
PRM-35	63.80%
PRM-36	59.77%
PRM-37	63.50%
PRM-38	60.00%
PRM-39	58.10%
PRM-40	66.00%
PRM-41	65.37%
PRM-42	59.80%
PRM-43	59.28%
PRM-44	65.00%
PRM-45	57.00%
PRM-46	57.00%
PRM-47	57.00%
PRM-48	61.80%
PRM-49	52.00%
PRM-50	65.00%
PRM-51	65.30%
PRM-52	48.00%
PRM-53	53.64%
PRM-54	67.05%
PRM-55	50.00%

Table F.23 Beginning inventory of FP at plants ($X_{i,v}^{FP}$) in t

FP Grade	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
Grade-1	35,000	0	0	10,000	0
Grade-2	30,000	30,000	0	1,800	10,000
Grade-3	0	0	0	0	0
Grade-4	0	0	0	0	0
Grade-5	0	0	0	0	0
Grade-6	0	0	0	0	0
Grade-7	0	18,000	0	1,600	0
Grade-8	0	0	0	25	0
Grade-9	0	200	0	20,000	0
Grade-10	0	45,000	0	0	0
Grade-11	0	8,000	0	0	2,000
Grade-12	0	0	0	500	0
Grade-13	0	6,000	0	0	0
Grade-14	0	25,000	0	43,000	3,000
Grade-15	0	0	0	13,000	0
Grade-16	0	0	0	45,000	1,500
Grade-17	0	0	8,000	0	0
Grade-18	350	15,000	0	750	7,500
Grade-19	0	60	0	19,000	5,000
Grade-20	0	0	10,000	0	0
Grade-21	0	500	0	25,000	0
Grade-22	0	0	40,000	0	0
Grade-23	0	150	0	1,000	0

Table F.24 Beginning inventory of FP at regional warehouses ($X_{j,v}^{FP}$) in t

FP Grade	RegWrhs1	RegWrhs2	RegWrhs3	RegWrhs4	RegWrhs5
Grade-1	1,000	13,000	60,000	0	0
Grade-2	50,000	30,000	35,000	55,000	100
Grade-3	0	0	0	0	0
Grade-4	0	0	0	0	0
Grade-5	0	0	0	0	0
Grade-6	0	0	0	0	0
Grade-7	4,500	325	3,500	1,100	19,000
Grade-8	0	25	0	0	0
Grade-9	30,000	6,000	16,000	5,000	0
Grade-10	26,000	4,500	35,000	9,000	10,000
Grade-11	3,000	15,000	20,000	350	1,900
Grade-12	2,500	200	1,000	0	0
Grade-13	10,000	3,000	1,300	650	1,600
Grade-14	48,000	30,000	43,000	19,000	3,000
Grade-15	0	0	0	0	0
Grade-16	1,000	0	0	0	2,000
Grade-17	0	0	0	0	0
Grade-18	8,000	6,000	12,000	4,000	1,000
Grade-19	6,500	700	10,000	3,000	0
Grade-20	0	0	0	0	800
Grade-21	13,000	3,500	8,500	2,000	500
Grade-22	0	0	8,000	150	13,000
Grade-23	7,000	350	1,900	75	0

Table F.25 Beginning inventory of PRM at plants ($X_{i,u}^{\mathrm{PRM,P}}$) in t

PRM NAME	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
PRM-1	0	0	0	0	0
PRM-2	80,088	0	0	2,280	0
PRM-3	0	0	0	0	0
PRM-4	0	0	0	0	0
PRM-5	0	0	0	0	0
PRM-6	0	0	0	0	20,216
PRM-7	0	0	0	0	0
PRM-8	0	0	0	0	181,952
PRM-9	0	0	0	0	149,928
PRM-10	0	0	0	83,632	0
PRM-11	2,144	0	0	0	12,312
PRM-12	0	0	0	77,688	0
PRM-13	259,592	0	2,592	53,456	80,800
PRM-14	0	0	0	0	0
PRM-15	0	0	0	0	0
PRM-16	0	0	0	0	0
PRM-17	0	0	0	0	0
PRM-18	0	0	0	0	0
PRM-19	0	0	0	0	0
PRM-20	0	0	0	317,704	0
PRM-21	0	0	0	0	0
PRM-22	168,456	0	0	336,232	53,720
PRM-23	0	0	0	0	0
PRM-24	0	0	0	0	39,440
PRM-25	0	128,256	11,136	209,488	0
PRM-26	0	0	6,576	145,056	0
PRM-27	0	0	28,312	0	0
PRM-28	37,584	604,992	0	6,296	27,928
PRM-29	0	0	0	0	0
PRM-30	0	0	0	0	0
PRM-31	0	0	0	0	0
PRM-32	0	19,968	45,752	0	0
PRM-33	57,440	0	488	0	0
PRM-34	0	0	0	0	0
PRM-35	4,816	0	80,000	0	0
PRM-36	0	0	0	0	0
PRM-37	119,912	263,808	0	40,000	0
PRM-38	0	0	0	0	0
PRM-39	0	0	0	0	0
PRM-40	0	0	0	33,800	0
PRM-41	0	0	0	0	0
PRM-42	0	57,112	285,776	4,872	0
PRM-43	0	0	0	0	0
PRM-44	0	0	0	0	0
PRM-45	0	275,528	0	0	0
PRM-46	0	0	0	0	0
PRM-47	0	244,136	12,000	0	0
PRM-48	0	0	0	0	0
PRM-49	0	0	0	0	0
PRM-50	0	0	0	0	0

PRM-51	0	0	0	0	0
PRM-52	0	0	0	0	0
PRM-53	0	0	0	0	0
PRM-54	0	0	0	0	0
PRM-55	0	0	0	0	0

Table F.26 Beginning inventory of PRM at PRM warehouse ($X_u^{\text{PRM,W}}$) in t

PRM NAME	PRM Warehouse
PRM-1	0
PRM-2	0
PRM-3	0
PRM-4	0
PRM-5	0
PRM-6	0
PRM-7	0
PRM-8	0
PRM-9	0
PRM-10	0
PRM-11	0
PRM-12	0
PRM-13	0
PRM-14	0
PRM-15	0
PRM-16	0
PRM-17	0
PRM-18	0
PRM-19	0
PRM-20	0
PRM-21	0
PRM-22	0
PRM-23	0
PRM-24	0
PRM-25	0
PRM-26	0
PRM-27	0
PRM-28	0
PRM-29	0
PRM-30	0
PRM-31	0
PRM-32	0
PRM-33	0
PRM-34	0
PRM-35	0
PRM-36	0
PRM-37	0
PRM-38	0
PRM-39	0
PRM-40	0
PRM-41	0
PRM-42	0
PRM-43	0

PRM-44	0
PRM-45	0
PRM-46	0
PRM-47	0
PRM-48	0
PRM-49	0
PRM-50	0
PRM-51	0
PRM-52	0
PRM-53	0
PRM-54	0
PRM-55	0

Table F.27 Demand parameters for uncertain case A and B ($D_{v,k}^{\min(a)}$) in kt

	Regional Market				
FP Grade	1	2	3	4	5
1	147	1	343	60	56
2	181	361	213	638	15
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	6	25	21	7	26
8	1	0	0	0	0
9	53	99	53	39	57
10	43	172	249	172	119
11	84	28	126	23	35
12	4	14	6	0	0
13	28	67	7	7	18
14	186	329	263	158	158
15	0	0	0	0	77
16	0	13	0	0	270
17	0	0	0	0	49
18	38	84	85	57	49
19	4	116	56	53	11
20	0	0	0	0	60
21	23	83	50	9	145
22	0	0	2	2	301
23	3	43	11	1	4

Table F.28 Demand parameters for uncertain case C ($D_{v,k}^{\min(b)}$) in kt

	Regional Market				
FP Grade	1	2	3	4	5
1	220	1	513	89	84
2	270	539	318	954	23
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	8	37	31	10	39
8	1	0	0	0	0
9	78	149	78	58	86
10	64	256	371	256	178
11	126	42	188	35	52
12	5	21	9	0	0
13	42	99	10	10	26
14	278	492	392	235	235
15	0	0	0	0	115
16	0	19	0	0	404
17	0	0	0	0	73
18	56	126	127	86	73
19	6	173	84	78	17
20	0	0	0	0	90
21	35	123	75	14	217
22	0	0	3	3	450
23	4	64	16	1	5

Table F.29 Reference demand of FP at plants ($D_{i,v}^{\text{FP}}$) in t

FP Grade	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
1	285,000	0	0	80,000	0
2	266,400	245,400	0	25,000	83,000
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	0	15,000	0	13,000	0
8	0	0	0	200	0
9	0	2,000	0	167,000	0
10	0	351,000	0	0	0
11	0	65,000	0	0	15,000
12	0	0	0	3,114	0
13	0	52,000	0	0	0
14	0	190,000	0	153,000	25,000
15	0	0	0	110,000	0
16	0	0	0	370,000	10,000
17	0	0	70,000	0	0

18	2,500	125,000	0	7,000	59,000
19	0	500	0	150,000	40,000
20	0	0	80,000	0	0
21	0	4,000	0	216,000	0
22	0	0	331,600	0	0
23	0	150	0	7500	0

Table F.30 Reference demand of FP at regional warehouse ($D_{j,v}^{FP}$) in t

FP Grade	RW1	RW2	RW3	RW4	RW5
1	10,000	1,000	490,000	0	0
2	250,000	420,000	270,000	450,000	800
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	3,000	35,000	30,000	9,000	15,000
8	300	0	0	0	0
9	25,000	130,000	65,000	40,000	0
10	38,000	225,000	300,000	75,000	87,000
11	120,000	25,000	180,000	3,000	15,000
12	2,000	20,000	9,000	0	0
13	25,000	75,000	10,000	5,000	13,000
14	250,000	400,000	350,000	170,000	23,000
15	0	0	0	0	0
16	0	8,000	0	0	16,000
17	0	0	0	0	0
18	50,000	65,000	100,000	30,000	8,000
19	6,000	55,000	70,000	20,000	0
20	0	0	0	0	6,000
21	30,000	110,000	70,000	10,000	3,000
22	0	0	60,000	1,000	100,000
23	2,000	60,000	15,000	600	0

Table F.31 Target inventory day supply of FP at the plants ($R_{i,v}^{FP}$)

FP GRADE	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
Grade-1	35.00	35.00	35.00	35.00	35.00
Grade-2	35.00	35.00	35.00	35.00	35.00
Grade-3	35.00	35.00	35.00	35.00	35.00
Grade-4	35.00	35.00	35.00	35.00	35.00
Grade-5	35.00	35.00	35.00	35.00	35.00
Grade-6	35.00	35.00	35.00	35.00	35.00
Grade-7	35.00	35.00	35.00	35.00	35.00
Grade-8	35.00	35.00	35.00	35.00	35.00
Grade-9	35.00	35.00	35.00	35.00	35.00
Grade-10	35.00	35.00	35.00	35.00	35.00
Grade-11	35.00	35.00	35.00	35.00	35.00
Grade-12	35.00	35.00	35.00	35.00	35.00

Grade-13	35.00	35.00	35.00	35.00	35.00
Grade-14	35.00	35.00	35.00	35.00	35.00
Grade-15	35.00	35.00	35.00	35.00	35.00
Grade-16	35.00	35.00	35.00	35.00	35.00
Grade-17	35.00	35.00	35.00	35.00	35.00
Grade-18	35.00	35.00	35.00	35.00	35.00
Grade-19	35.00	35.00	35.00	35.00	35.00
Grade-20	35.00	35.00	35.00	35.00	35.00
Grade-21	35.00	35.00	35.00	35.00	35.00
Grade-22	35.00	35.00	35.00	35.00	35.00
Grade-23	35.00	35.00	35.00	35.00	35.00

Table F.32 Target inventory day supply of FP at the regional warehouses ($R^{\mathrm{FP}}_{j,v}$)

FP GRADE	RW1	RW2	RW3	RW4	RW5
Grade-1	35.00	35.00	35.00	35.00	35.00
Grade-2	35.00	35.00	35.00	35.00	35.00
Grade-3	35.00	35.00	35.00	35.00	35.00
Grade-4	35.00	35.00	35.00	35.00	35.00
Grade-5	35.00	35.00	35.00	35.00	35.00
Grade-6	35.00	35.00	35.00	35.00	35.00
Grade-7	35.00	35.00	35.00	35.00	35.00
Grade-8	35.00	35.00	35.00	35.00	35.00
Grade-9	35.00	35.00	35.00	35.00	35.00
Grade-10	35.00	35.00	35.00	35.00	35.00
Grade-11	35.00	35.00	35.00	35.00	35.00
Grade-12	35.00	35.00	35.00	35.00	35.00
Grade-13	35.00	35.00	35.00	35.00	35.00
Grade-14	35.00	35.00	35.00	35.00	35.00
Grade-15	35.00	35.00	35.00	35.00	35.00
Grade-16	35.00	35.00	35.00	35.00	35.00
Grade-17	35.00	35.00	35.00	35.00	35.00
Grade-18	35.00	35.00	35.00	35.00	35.00
Grade-19	35.00	35.00	35.00	35.00	35.00
Grade-20	35.00	35.00	35.00	35.00	35.00
Grade-21	35.00	35.00	35.00	35.00	35.00
Grade-22	35.00	35.00	35.00	35.00	35.00
Grade-23	35.00	35.00	35.00	35.00	35.00

Table F.33 Target inventory of PRM at the plants ($R^{\mathrm{PRM,P}}_{i,u}$) in t

PRM NAME	PLANT1	PLANT2	PLANT3	PLANT4	PLANT5
PRM-1	0	0	0	0	0
PRM-2	0	0	0	0	0
PRM-3	0	0	0	0	0
PRM-4	0	0	0	0	0
PRM-5	0	0	0	0	0
PRM-6	0	0	0	0	0
PRM-7	0	0	0	0	0
PRM-8	0	0	0	0	80,000

PRM-9	0	0	0	0	120,000
PRM-10	0	0	0	0	0
PRM-11	0	0	0	0	0
PRM-12	0	0	0	0	0
PRM-13	120,000	0	0	80,000	40,000
PRM-14	0	0	0	0	0
PRM-15	0	0	0	0	0
PRM-16	0	0	0	0	0
PRM-17	0	0	0	0	0
PRM-18	0	0	0	0	0
PRM-19	0	0	0	0	0
PRM-20	0	0	0	0	0
PRM-21	0	0	0	0	0
PRM-22	80,000	16,000	16,000	240,000	80,000
PRM-23	0	0	0	0	0
PRM-24	0	0	0	0	0
PRM-25	0	40,000	8,000	112,000	0
PRM-26	0	0	0	0	0
PRM-27	0	0	16,000	0	0
PRM-28	0	0	0	0	0
PRM-29	0	0	0	0	0
PRM-30	0	0	0	0	0
PRM-31	0	0	0	0	0
PRM-32	0	16,000	32,000	0	0
PRM-33	0	0	0	0	0
PRM-34	0	0	0	0	0
PRM-35	0	0	40,000	0	0
PRM-36	0	0	0	0	0
PRM-37	64,000	160,000	0	40,000	0
PRM-38	0	0	0	0	0
PRM-39	0	0	0	0	0
PRM-40	0	0	0	0	0
PRM-41	0	0	0	0	0
PRM-42	0	0	0	0	0
PRM-43	0	0	0	0	0
PRM-44	0	0	0	0	0
PRM-45	0	80,000	0	0	0
PRM-46	0	0	0	0	0
PRM-47	0	80,000	0	0	0
PRM-48	0	0	0	0	0
PRM-49	0	0	0	0	0
PRM-50	0	0	0	0	0
PRM-51	0	0	0	0	0
PRM-52	0	0	0	0	0
PRM-53	0	0	0	0	0
PRM-54	0	0	0	0	0
PRM-55	0	0	0	0	0

Table F.34 Target inventory of PRM at the PRM warehouse ($R_u^{\mathrm{PRM,W}}$) in t

PRM NAME	PRM Warehouse
PRM-1	0
PRM-2	0
PRM-3	0
PRM-4	0
PRM-5	0
PRM-6	0
PRM-7	0
PRM-8	0
PRM-9	0
PRM-10	0
PRM-11	0
PRM-12	0
PRM-13	0
PRM-14	0
PRM-15	0
PRM-16	0
PRM-17	0
PRM-18	0
PRM-19	0
PRM-20	0
PRM-21	0
PRM-22	0
PRM-23	0
PRM-24	0
PRM-25	0
PRM-26	0
PRM-27	0
PRM-28	0
PRM-29	0
PRM-30	0
PRM-31	0
PRM-32	0
PRM-33	0
PRM-34	0
PRM-35	0
PRM-36	0
PRM-37	0
PRM-38	0
PRM-39	0
PRM-40	0
PRM-41	0
PRM-42	0
PRM-43	0
PRM-44	0
PRM-45	0
PRM-46	0
PRM-47	0
PRM-48	0
PRM-49	0
PRM-50	0

PRM-51	0
PRM-52	0
PRM-53	0
PRM-54	0
PRM-55	0

Table F.35 PRM availability (M_u^{PRM}) in t

PRM NAME	PRM Availability
PRM-1	0
PRM-2	50,000
PRM-3	0
PRM-4	0
PRM-5	0
PRM-6	30,000
PRM-7	0
PRM-8	600,000
PRM-9	400,000
PRM-10	60,000
PRM-11	0
PRM-12	0
PRM-13	1,000,000
PRM-14	0
PRM-15	0
PRM-16	0
PRM-17	0
PRM-18	0
PRM-19	0
PRM-20	150,000
PRM-21	350,000
PRM-22	2,500,000
PRM-23	0
PRM-24	0
PRM-25	1,000,000
PRM-26	650,000
PRM-27	520,000
PRM-28	0
PRM-29	0
PRM-30	0
PRM-31	0
PRM-32	1,000,000
PRM-33	0
PRM-34	0
PRM-35	250,000
PRM-36	0
PRM-37	1,400,000
PRM-38	0
PRM-39	0
PRM-40	0
PRM-41	0
PRM-42	390,000
PRM-43	0

PRM-44	0
PRM-45	2,500,000
PRM-46	0
PRM-47	625,000
PRM-48	0
PRM-49	0
PRM-50	0
PRM-51	0
PRM-52	0
PRM-53	0
PRM-54	0
PRM-55	0

Appendix G

Dantzig-Wolfe Subproblems for Formulation (AARS_IN)

Here, the Dantzig-Wolfe decomposition formulations for the results obtained in Chapter 5 are shown for the industrial case study problem. The formulations for the DWD1 algorithm are first described. Capacity as an integer variable is represented by z_i^{int} and the set $G^h = \{1, \ldots, N_h\} \subset G$ is introduced to denote the columns generated for the restricted master problem (this is consistent to $J^k = \{1, \ldots, N_k\} \subset J$ used in Chapter 5).

G.1 Subproblems for DWD1

The phase 1 feasibility problem is shown below

Problem (FP1)

$$\min \sum_{i \in I} \sum_{s \in S} AV_{i,s} \tag{G.1}$$

subject to

$$\sum_{g \in G^h} \lambda_{s,g}{}^{(h)} \le 1, \quad s \in S, \tag{G.2}$$

$$-\left(\sum_{g \in G^h} \lambda_{s,g}{}^{(h)} \right) \le -1, \quad s \in S, \tag{G.3}$$

$$\sum_{nu \in NU} \left(\begin{array}{l} \delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\sum_{(i,v) \in \Psi} \sum_{g \in G^h} \left(\Phi_{i,v,s,nu,g}^{FP,p}{}^{(h)} \cdot \lambda_{s,g}{}^{(h)} \right) \right) \right\|_1 \\ + \left(\sum_{(i,v) \in \Psi} \sum_{g \in G^h} \left(\Phi_{i,v,s,nu,g}^{FP,p}{}^{(h)} \cdot \lambda_{s,g}{}^{(h)} \right) \right) \cdot \overline{\xi}_{nu} + \left(\sum_{(i,v) \in \Psi} \sum_{g \in G^h} \left(\phi_{i,v,s,g}^{FP,p}{}^{(h)} \cdot \lambda_{s,g}{}^{(h)} \right) \right) \end{array} \right) \tag{G.4}$$

$$\le 500 \cdot z_i^{\text{int}} + AV_{i,s}, \quad i \in I, s \in S,$$

$$500 \cdot z_i^{\text{int}} \le Z_i^{\max}, \quad i \in I, \tag{G.5}$$

$$z_i^{\text{int}} \in \left\{ z_i \in Z^{n_{x0}} : z_i \ge 0 \right\}, \ i \in I, \tag{G.6}$$

$$\lambda_{s,g}{}^{(h)} \geq 0, \;\; s \in S, g \in G^h, \tag{G.7}$$

where $AV_{i,s}$ is the artificial slack variable required for the phase 1 problem. Through solving the restricted master problem, Lagrange multipliers can be obtained. $\rho_{i,s}^{(h)}$(consistent to π_{ω}^{k} used in Chapter 5) is obtained from constraint (G.4) and using these multipliers, the related phase 1 subproblems can be formulated and solved for $s \in S$

Problem (SFP1$_s$)

$$\min \sum_{nu \in NU} \left(\begin{array}{l} \delta_{s,nu} \cdot \sum_{(i,v)\in\Psi} \left(-\rho_{i,s}^{(h)} \cdot \left\| \left(M_{s,nu}^{-1} \right)^{T} \left(\Phi_{i,v,s,nu}^{FP,p} \right) \right\|_{1} \right) + \\ \sum_{(i,v)\in\Psi} \left(-\rho_{i,s}^{(h)} \cdot \Phi_{i,v,s,nu}^{FP,p} \cdot \overline{\xi}_{s,nu} \right) + \sum_{(i,v)\in\Psi} \left(-\rho_{i,s}^{(h)} \cdot \phi_{i,v,s}^{FP,p} \right) \end{array} \right) \tag{G.8}$$

subject to

$$\begin{aligned} \text{Revenue}_{i,s} = & \sum_{(i,k)\in\Theta} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,k,v,s,nu}^{FP,PD} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{FP,PD} \right) \cdot P_{k,v}^{FP} \right) \\ & + \sum_{(i,j)\in\Omega} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,j,v,s,nu}^{FP,PW} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{FP,PW} \right) \cdot \hat{P}_{j,v}^{FP} \right), \quad i \in I, \end{aligned} \tag{G.9}$$

$$\text{RC}_{i,s} = \overline{c}_{i,s}^{fr} + \overline{c}_{i,s}^{du} + \overline{c}_{i,s}^{I} + \overline{c}_{i,s}^{PRM} + \overline{c}_{i,s}^{RM2} + \overline{c}_{i,s}^{RM3} + \overline{c}_{i,s}^{waste}, \quad i \in I, \tag{G.10}$$

$$\begin{aligned} \overline{c}_{i,s}^{fr} = & \sum_{(i,k)\in\Theta} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,k,v,s,nu}^{FP,PD} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{FP,PD} \right) \cdot C_{i,k}^{fr,FP,PD} \right) \\ & + \sum_{(i,j)\in\Omega} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,j,v,s,nu}^{FP,PW} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{FP,PW} \right) \cdot C_{i,j}^{fr,FP,PW} \right) \\ & + \sum_{(i,j)\in\Omega} \sum_{(j,k)\in\Pi} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{j,k,v,s,nu}^{FP,WD} \cdot \overline{\xi}_{s,nu} + \phi_{j,k,v,s}^{FP,WD} \right) \cdot C_{j,k}^{fr,FP,WD} \cdot q_{i,j}^{fr,PW} \right), \quad i \in I, \end{aligned} \tag{G.11}$$

$$\overline{c}_{i,s}^{du} = r^{tp} \cdot \left[\begin{array}{l} \sum_{(i,k)\in\Theta} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,k,v,s,nu}^{FP,PD} \cdot \overline{\xi}_{s,nu} + \phi_{i,k,v,s}^{FP,PD} \right) \cdot P_{k,v}^{FP} \cdot r_{i,k}^{du} \right) \\ + \sum_{(i,j)\in\Omega} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,j,v,s,nu}^{FP,PW} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{FP,PW} \right) \cdot \hat{P}_{j,v}^{FP} \cdot \hat{r}_{i,j}^{du} \right) \end{array} \right], \quad i \in I, \tag{G.12}$$

$$\overline{c}_{i,s}^{I} = \sum_{(i,j)\in\Omega} \sum_{v\in V} \left(\left(\sum_{nu\in NU} \Phi_{i,j,v,s,nu}^{FP,PW} \cdot \overline{\xi}_{s,nu} + \phi_{i,j,v,s}^{FP,PW} \right) \cdot C_{j}^{WI} \right), \quad i \in I, \tag{G.13}$$

$$\overline{c}_{i,s}^{PRM} = \sum_{u\in U} \left(\left(\sum_{nu\in NU} \Phi_{i,u,s,nu}^{PRM,c} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{PRM,c} \right) \cdot C_{i,u}^{PRM} \right), \quad i \in I, \tag{G.14}$$

$$\overline{c}_{i,s}^{RM2} = \sum_{u\in U} \left(\left(\sum_{nu\in NU} \Phi_{i,u,s,nu}^{PRM,c} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{PRM,c} \right) \cdot q_{i,u}^{RM2} \right) \cdot C_{i}^{RM2}, \quad i \in I, \tag{G.15}$$

$$\overline{c}_{i,s}^{RM3} = \sum_{u\in U} \left(\left(\sum_{nu\in NU} \Phi_{i,u,s,nu}^{PRM,c} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{PRM,c} \right) \cdot q_{i,u}^{RM3} \right) \cdot C_{i}^{RM3}, \quad i \in I, \tag{G.16}$$

$$\overline{c}_{i,s}^{waste} = \sum_{u\in U} \left(\left(\sum_{nu\in NU} \Phi_{i,u,s,nu}^{PRM,c} \cdot \overline{\xi}_{s,nu} + \phi_{i,u,s}^{PRM,c} \right) \cdot q_{u}^{waste} \right) \cdot C_{i}^{waste}, \quad i \in I, \tag{G.17}$$

$$\begin{array}{l} \sum_{nu\in NU} \left(\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^{T} \cdot \left(\sum_{u\in U} \Phi_{i,u,s,nu}^{PRM,c} \right) \right\|_{1} + \sum_{u\in U} \left(\Phi_{i,u,s,nu}^{PRM,c} \right) \cdot \overline{\xi}_{s,nu} + \sum_{u\in U} \phi_{i,u,s}^{PRM,c} \right) \\ \leq \sum_{nu\in NU} \left(\begin{array}{l} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^{T} \cdot \left(\sum_{(i,v)\in\Psi} \Phi_{i,v,s,nu}^{FP,p} \cdot A_{i} \right) \right\|_{1} + \\ \left(\sum_{(i,v)\in\Psi} \Phi_{i,v,s,nu}^{FP,p} \cdot A_{i} \right) \cdot \overline{\xi}_{s,nu} + \left(\sum_{(i,v)\in\Psi} \phi_{i,v,s}^{FP,p} \cdot A_{i} \right) \end{array} \right), \quad i \in I, \end{array} \tag{G.18}$$

$$\begin{array}{l} \sum_{nu\in NU} \left(\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^{T} \cdot \left(\sum_{u\in U} \Phi_{i,u,s,nu}^{PRM,c} \right) \right\|_{1} + \sum_{u\in U} \left(\Phi_{i,u,s,nu}^{PRM,c} \right) \cdot \overline{\xi}_{s,nu} + \sum_{u\in U} \phi_{i,u,s}^{PRM,c} \right) \\ \leq \sum_{nu\in NU} \left(\begin{array}{l} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^{T} \cdot \left(\sum_{(i,v)\in\Psi} \Phi_{i,v,s,nu}^{FP,p} \cdot B_{i} \right) \right\|_{1} + \\ \left(\sum_{(i,v)\in\Psi} \Phi_{i,v,s,nu}^{FP,p} \cdot B_{i} \right) \cdot \overline{\xi}_{s,nu} + \left(\sum_{(i,v)\in\Psi} \phi_{i,v,s}^{FP,p} \cdot B_{i} \right) \end{array} \right), \quad i \in I, \end{array} \tag{G.19}$$

$$\sum_{u\in U} \left(\Phi_{i,u,s,nu}^{PRM,c} \cdot E_{u}^{PRM} \right) \cdot a_{i}^{avg,FP} \cdot Y_{i}^{FP} - \sum_{(i,v)\in\Psi} \Phi_{i,v,s,nu}^{FP,p} = 0, \quad i \in I, \ nu \in NU, \tag{G.20}$$

$$\sum_{u\in U}\left(\phi_{i,u,s}^{PRM,c}\cdot E_{u}^{PRM}\right)\cdot a_{i}^{avg,FP}\cdot Y_{i}^{FP}-\sum_{(i,v)\in\Psi}\phi_{i,v,s}^{FP,p}=0,\quad i\in I, \tag{G.21}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{FP,PW}\right)+\sum_{(i,k)\in\Theta}\left(\Phi_{i,k,v,s,nu}^{FP,PD}\right)-\Phi_{i,v,s,nu}^{FP,p}\right)\right\|_{1}\\ +\left(\left(\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{FP,PW}\right)+\sum_{(i,k)\in\Theta}\left(\Phi_{i,k,v,s,nu}^{FP,PD}\right)-\Phi_{i,v,s,nu}^{FP,p}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{(i,j)\in\Omega}\left(\phi_{i,j,v,s}^{FP,PW}\right)+\sum_{(i,k)\in\Theta}\left(\phi_{i,k,v,s}^{FP,PD}\right)-\phi_{i,v,s}^{FP,p}\end{array}\right) \tag{G.22}$$

$$\leq X_{i,v}^{FP}-D_{i,v}^{FP}\cdot R_{i,v}^{FP}/365,\quad (i,v)\in\Psi,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{(j,k)\in\Pi}\left(\Phi_{j,k,v,s,nu}^{FP,WD}\right)-\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{FP,PW}\right)\right)\right\|_{1}\\ +\left(\sum_{(j,k)\in\Pi}\left(\Phi_{j,k,v,s,nu}^{FP,WD}\right)-\sum_{(i,j)\in\Omega}\left(\Phi_{i,j,v,s,nu}^{FP,PW}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{(j,k)\in\Pi}\left(\phi_{j,k,v,s}^{FP,WD}\right)-\sum_{(i,j)\in\Omega}\left(\phi_{i,j,v,s}^{FP,PW}\right)\end{array}\right) \tag{G.23}$$

$$\leq X_{j,v}^{FP}-D_{j,v}^{FP}\cdot R_{j,v}^{FP}/365,\quad j\in J,\ v\in V,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot q_{u,w}^{\mathrm{imp}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot Q_{i,w}^{\mathrm{imp}}\right)\right)\right\|_{1}+\\ \left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot q_{u,w}^{\mathrm{imp}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot Q_{i,w}^{\mathrm{imp}}\right)\right)\cdot\overline{\xi}_{s,nu}\\ +\sum_{u\in U}\left(\phi_{i,u,s}^{\mathrm{PRM,c}}\cdot q_{u,w}^{\mathrm{imp}}\right)-\sum_{u\in U}\left(\phi_{i,u,s}^{\mathrm{PRM,c}}\cdot Q_{i,w}^{\mathrm{imp}}\right)\end{array}\right)\leq 0, \tag{G.24}$$

$$i\in I,\ w\in W,$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\begin{array}{l}\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot E_{u}^{\mathrm{PRM}}\right)\\-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot Q_{i}^{\mathrm{imp}}\right)\end{array}\right)\right\|_{1}\\+\left(\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot E_{u}^{\mathrm{PRM}}\right)-\sum_{u\in U}\left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}}\cdot Q_{i}^{\mathrm{imp}}\right)\right)\cdot\overline{\xi}_{s,nu}\\+\sum_{u\in U}\left(\phi_{i,u,s}^{\mathrm{PRM,c}}\cdot E_{u}^{\mathrm{PRM}}\right)-\sum_{u\in U}\left(\phi_{i,u,s}^{\mathrm{PRM,c}}\cdot Q_{i}^{\mathrm{imp}}\right)\end{array}\right)\leq 0,\quad i\in I, \tag{G.25}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{PRM,P}\right)+\Phi_{u,s,nu}^{PRM,W}\right)\right\|_{1}+\\\left(\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{PRM,P}\right)+\Phi_{u,s,nu}^{PRM,W}\right)\cdot\overline{\xi}_{s,nu}+\sum_{i\in I}\left(\phi_{i,u,s}^{PRM,P}\right)+\phi_{u,s}^{PRM,W}\end{array}\right)\leq M_{u}^{PRM},\quad u\in U, \tag{G.26}$$

$$X_{u}^{PRM,W}-R_{u}^{PRM,W}+\sum_{nu\in NU}\left(\begin{array}{l}-\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{u,s,nu}^{PRM,W}-\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{PRM,WP}\right)\right)\right\|_{1}+\\\left(\Phi_{u,s,nu}^{PRM,W}-\sum_{i\in I}\left(\Phi_{i,u,s,nu}^{PRM,WP}\right)\right)\cdot\overline{\xi}_{s,nu}\\+\phi_{u,s}^{PRM,W}-\sum_{i\in I}\left(\phi_{i,u,s}^{PRM,WP}\right)\end{array}\right)\geq 0,\,u\in U, \tag{G.27}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^{T}\cdot\left(\Phi_{i,u,s,nu}^{PRM,c}-\Phi_{i,u,s,nu}^{PRM,P}-\Phi_{i,u,s,nu}^{PRM,WP}\right)\right\|_{1}+\\\left(\Phi_{i,u,s,nu}^{PRM,c}-\Phi_{i,u,s,nu}^{PRM,P}-\Phi_{i,u,s,nu}^{PRM,WP}\right)\cdot\overline{\xi}_{s,nu}+\phi_{i,u,s,nu}^{PRM,c}-\phi_{i,u,s}^{PRM,P}-\phi_{i,u,s}^{PRM,WP}\end{array}\right) \tag{G.28}$$

$$\leq X_{i,u}^{PRM,P}-R_{i,u}^{PRM,P},\quad i\in I,\ u\in U,$$

$$\sum_{nu \in NU} \left(\begin{array}{l} -\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\sum_{(i,k)\in\Theta} \Phi_{i,k,v,s,nu}^{FP,PD} + \sum_{(j,k)\in\Pi} \Phi_{j,k,v,s,nu}^{FP,WD} - D_{k,v}^{\min} \cdot \eta_{k,nu} \right) \right\|_1 \\ + \left(\sum_{(i,k)\in\Theta} \Phi_{i,k,v,s,nu}^{FP,PD} + \sum_{(j,k)\in\Pi} \Phi_{j,k,v,s,nu}^{FP,WD} - \bar{D}_{k,v}^{\min} \cdot \eta_{k,nu} \right) \cdot \left(\bar{\xi}_{s,nu} \right) \\ + \phi_{i,k,v,s}^{FP,PD} + \phi_{j,k,v,s}^{FP,WD} \end{array} \right) \geq 0, \tag{G.29}$$

$$k \in K,\ v \in V,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}} \right) \right\|_1 + \Phi_{i,k,v,s,nu}^{\mathrm{FP,PD}} \cdot \bar{\xi}_{s,nu} + \phi_{i,k,v,s}^{\mathrm{FP,PD}} \right) \geq 0, \tag{G.30}$$

$$i \in I,\ k \in K,\ v \in V,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}} \right) \right\|_1 + \Phi_{i,j,v,s,nu}^{\mathrm{FP,PW}} \cdot \bar{\xi}_{s,nu} + \phi_{i,j,v,s}^{\mathrm{FP,PW}} \right) \geq 0, \tag{G.31}$$

$$i \in I,\ j \in J,\ v \in V,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}} \right) \right\|_1 + \Phi_{j,k,v,s,nu}^{\mathrm{FP,WD}} \cdot \bar{\xi}_{s,nu} + \phi_{j,k,v,s}^{\mathrm{FP,WD}} \right) \geq 0, \tag{G.32}$$

$$j \in J,\ k \in K,\ v \in V,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\mathrm{PRM,c}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\mathrm{PRM,c}} \cdot \bar{\xi}_{s,nu} + \phi_{i,u,s}^{\mathrm{PRM,c}} \right) \geq 0, \tag{G.33}$$

$$i \in I,\ u \in U,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\mathrm{PRM,P}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\mathrm{PRM,P}} \cdot \bar{\xi}_{s,nu} + \phi_{i,u,s}^{\mathrm{PRM,P}} \right) \geq 0, \tag{G.34}$$

$$i \in I,\ u \in U,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{u,s,nu}^{\mathrm{PRM,W}} \right) \right\|_1 + \Phi_{u,s,nu}^{\mathrm{PRM,W}} \cdot \bar{\xi}_{s,nu} + \phi_{u,s}^{\mathrm{PRM,W}} \right) \geq 0, \quad u \in U, \tag{G.35}$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,u,s,nu}^{\mathrm{PRM,WP}} \right) \right\|_1 + \Phi_{i,u,s,nu}^{\mathrm{PRM,WP}} \cdot \bar{\xi}_{s,nu} + \phi_{i,u,s}^{\mathrm{PRM,WP}} \right) \geq 0, \tag{G.36}$$

$$i \in I,\ u \in U,$$

$$\sum_{nu \in NU} \left(-\delta_{s,nu} \cdot \left\| \left(M_{s,nu}^{-1} \right)^T \cdot \left(\Phi_{i,v,s,nu}^{\mathrm{FP,p}} \right) \right\|_1 + \Phi_{i,v,s,nu}^{\mathrm{FP,p}} \cdot \bar{\xi}_{s,nu} + \phi_{i,v,s}^{\mathrm{FP,p}} \right) \geq 0, \tag{G.37}$$

$$i \in I,\ k \in K,\ v \in V.$$

Here, $\mathrm{RC}_{i,s}$ refers to the remaining cost terms that are not explicitly calculated in the restricted master problem. Proceeding to the phase 2 problem, the restricted master problem formulation is

Problem (RMP1)

$$\begin{aligned}\mathrm{Obj}_{RMP} &= \sum_{i\in I}\sum_{s\in S}\left[\Pr_s\cdot\left(\left(\begin{array}{l}\sum_{g\in G^h}\left(\mathrm{RC}_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)+c_i^{fix}\\ -\sum_{g\in G^h}\left(\mathrm{Revenue}_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\end{array}\right)\cdot\left(1-r_i^{inc}\right)\right)\right]\\ &+\sum_{(i,v)\in\Psi}\sum_{nu\in NU}\sum_{s\in S}\left[\left(\Pr_s\cdot\left(\overline{\xi}_{s,nu}\cdot\left(C_i^{PI}+C_i^{\mathrm{var}}\right)\right)\cdot\left(1-r_i^{inc}\right)\right)\cdot\left(\sum_{g\in G^h}\left(\Phi_{i,v,s,nu,g}^{FP,p}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\right]\\ &+\sum_{(i,v)\in\Psi}\sum_{s\in S}\left[\left(\Pr_s\cdot\left(C_i^{PI}+C_i^{\mathrm{var}}\right)\cdot\left(1-r_i^{inc}\right)\right)\cdot\left(\sum_{g\in G^h}\left(\phi_{i,v,s,g}^{FP,p}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\right]\end{aligned} \qquad \text{(G.38)}$$

subject to

$$\sum_{g\in G^h}\lambda_{s,g}{}^{(h)}\le 1,\quad s\in S, \qquad \text{(G.39)}$$

$$-\left(\sum_{g\in G^h}\lambda_{s,g}{}^{(h)}\right)\le -1,\quad s\in S, \qquad \text{(G.40)}$$

$$\sum_{nu\in NU}\left(\begin{array}{l}\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{(i,v)\in\Psi}\sum_{g\in G^h}\left(\Phi_{i,v,s,nu,g}^{FP,p}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\right\|_1\\ +\left(\sum_{(i,v)\in\Psi}\sum_{g\in G^h}\left(\Phi_{i,v,s,nu,g}^{FP,p}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\cdot\overline{\xi}_{nu}\\ +\left(\sum_{(i,v)\in\Psi}\sum_{g\in G^h}\left(\phi_{i,v,s,g}^{FP,p\ (h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\end{array}\right)\le 500\cdot z_i^{\mathrm{int}},\ i\in I, s\in S, \qquad \text{(G.41)}$$

$$500\cdot z_i^{\mathrm{int}}\le Z_i^{\max},\quad i\in I, \qquad \text{(G.42)}$$

$$z_i^{\mathrm{int}}\in\left\{z_i\in Z^{n_{x_0}}: z_i\ge 0\right\},\ i\in I, \qquad \text{(G.43)}$$

$$\lambda_{s,g}{}^{(h)}\ge 0,\ s\in S, g\in G^h. \qquad \text{(G.44)}$$

The Lagrange multipliers are again obtained through solving the restricted master problem. $\rho_{i,s}^{(h)}$ is obtained from constraint (G.41) (consistent to π_ω^k used in Chapter 5), $\mu_s{}^{(h)+}$ from (G.39), and

$\mu_s^{(h)-}$ from (G.40). The multipliers are required for the pricing problems and for the convergence criteria. The objective function for the phase 2 pricing problems is shown below and the constraints are the same as those in the phase 1 subproblems. The problem is solved for $s \in S$

Problem (PP1$_s$)

$$\begin{aligned}\text{Obj}_{PP_s} = \sum_{(i,v)\in\Psi}\sum_{nu\in NU}&\left[\begin{array}{l}\left(\text{Pr}_s\cdot\left(\sum_{u\in U}\left(\overline{\xi}_{s,nu}\cdot\left(C_i^{PI}+C_i^{\text{var}}\right)\right)\right)\cdot\left(1-r_i^{inc}\right)\right)\\ -\left(\delta_{s,nu}\cdot\left(\rho_{i,s}^{(h)}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\left(\Phi_{i,v,s,nu}^{FP,p}\right)\right\|_1\right)+\left(\rho_{i,s}^{(h)}\cdot\left(\Phi_{i,v,s,nu}^{FP,p}\right)\cdot\overline{\xi}_{s,nu}\right)\right)\end{array}\right]\\ +\sum_{(i,v)\in\Psi}\sum_{nu\in NU}&\left[\left(\text{Pr}_s\cdot\left(C_i^{PI}+C_i^{\text{var}}\right)\cdot\left(1-r_i^{inc}\right)\right)-\rho_{i,s}^{(h)}\cdot\phi_{i,v,s}^{FP,p}\right]\\ +\sum_{i\in I}&\left[\text{Pr}_s\cdot\left(\left(\text{RC}_{i,s}-\text{Revenue}_{i,s}\right)\left(1-r_i^{inc}\right)\right)\right]\end{aligned} \tag{G.45}$$

subject to,

constraints (G.9)-(G.37).

G.2 Subproblems for DWD2

The DWD1 procedure provides a feasible sub-optimal solution that can be used as the initial extreme points for the DWD2 procedure. This means that the phase 1 problem for DWD2 is not required. The phase 2 restricted master problem is

Problem (RMP2)

$$
\begin{aligned}
\text{Obj}_{RMP} &= \sum_{i\in I} c_i^{fix}\cdot\left(1-r_i^{inc}\right) \\
&+\sum_{i\in I}\sum_{s\in S}\left[\Pr_s\cdot\left(\left(\sum_{g\in G^h}\left(\text{RC}_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)-\sum_{g\in G^h}\left(\text{Revenue}_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\left(1-r_i^{inc}\right)\right)\right] \\
&+\sum_{i\in I}\sum_{s\in S}\left[\left(\left(\Pr_s\cdot C^{cap}\right)\cdot\left(1-r_i^{inc}\right)\right)\cdot\left(\sum_{g\in G^h}\left(z_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\right)\right]
\end{aligned} \tag{G.46}
$$

subject to

$$500\cdot z_i^{\text{int}}-\sum_{g\in G^h}\left(z_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\le 0,\quad i\in I,\ s\in S, \tag{G.47}$$

$$-500\cdot z_i^{\text{int}}+\sum_{g\in G^h}\left(z_{i,s,g}{}^{(h)}\cdot\lambda_{s,g}{}^{(h)}\right)\le 0,\quad i\in I,\ s\in S, \tag{G.48}$$

$$\sum_{g\in G^h}\lambda_{s,g}{}^{(h)}\le 1,\quad s\in S, \tag{G.49}$$

$$-\left(\sum_{g\in G^h}\lambda_{s,g}{}^{(h)}\right)\le -1,\quad s\in S, \tag{G.50}$$

$$z_i^{\text{int}}\in\left\{z_i\in Z^{n_{x_0}}:z_i\ge 0\right\},\ i\in I, \tag{G.51}$$

$$\lambda_{s,g}{}^{(h)}\ge 0,\ s\in S, g\in G^h. \tag{G.52}$$

The Lagrange multipliers are obtained through solving the restricted master problem. $\rho_{i,s}^{(h)+}$ is obtained from constraint (G.47), $\rho_{i,s}^{(h)-}$ is obtained from (G.48) (these are consistent to π_ω^k used in Chapter 5), $\mu_s{}^{(h)+}$ from (G.49), and $\mu_s{}^{(h)-}$ from (G.50). The phase 2 pricing problems are shown below and solved for $s\in S$

Problem (PP2$_s$)

$$\begin{aligned}\text{Obj}_{PP_s} &= \sum_{i\in I}\left[\left(\left(\Pr_s \cdot C^{cap}\right)\cdot\left(1-r_i^{inc}\right)+\left(\rho_{i,s}^{(h)+}-\rho_{i,s}^{(h)-}\right)\right)\cdot\left(z_{i,s}\right)\right] \\ &+\sum_{i\in I}\left[\Pr_s\cdot\left(\left(\text{RC}_{i,s}-\text{Revenue}_{i,s}\right)\left(1-r_i^{inc}\right)\right)\right]-\mu_s^{(h)+}+\mu_s^{(h)-}\end{aligned} \tag{G.53}$$

subject to

(G.9, G.11, G.12, G.14-G.37),

$$\text{RC}_{i,s} = \overline{c}_{i,s}^{fr}+\overline{c}_{i,s}^{du}+\overline{c}_{i,s}^{I}+\overline{c}_{i,s}^{PRM}+\overline{c}_{i,s}^{RM2}+\overline{c}_{i,s}^{RM3}+\overline{c}_{i,s}^{waste}+\overline{c}_{i,s}^{OPVC}, \quad i\in I, \tag{G.54}$$

$$\begin{aligned}\overline{c}_{i,s}^{I} &= \sum_{(i,v)\in\Psi}\left(\left(\sum_{nu\in NU}\Phi_{i,v,s,nu}^{FP,p}\cdot\overline{\xi}_{s,nu}+\phi_{i,v,s}^{FP,p}\right)\cdot C_i^{PI}\right) \\ &+\sum_{(i,j)\in\Omega}\sum_{v\in V}\left(\left(\sum_{nu\in NU}\Phi_{i,j,v,s,nu}^{FP,PW}\cdot\overline{\xi}_{s,nu}+\phi_{i,j,v,s}^{FP,PW}\right)\cdot C_j^{WI}\right), \quad i\in I,\end{aligned} \tag{G.55}$$

$$\overline{c}_{i,s}^{OPVC} = C_i^{\text{var}}\cdot\left[\sum_{(i,v)\in\Psi}\left(\sum_{nu\in NU}\Phi_{i,v,s,nu}^{FP,p}\cdot\overline{\xi}_{s,nu}+\phi_{i,v,s}^{FP,p}\right)\right], \quad i\in I, \tag{G.56}$$

$$z_{i,s}\le Z_i^{\max}, \quad i\in I, \tag{G.57}$$

$$\begin{aligned}\sum_{nu\in NU}\left(\delta_{s,nu}\cdot\left\|\left(M_{s,nu}^{-1}\right)^T\cdot\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\right)\right\|_1+\left(\sum_{(i,v)\in\Psi}\Phi_{i,v,s,nu}^{\text{FP,p}}\right)\cdot\overline{\xi}_{s,nu}+\left(\sum_{(i,v)\in\Psi}\phi_{i,v,s,nu}^{\text{FP,p}}\right)\right) \\ \le z_{i,s}, \quad i\in I.\end{aligned} \tag{G.58}$$