OPTIMIZATION OF SOYBEAN BUYING STRATEGIES USING DERIVATIVES

A Thesis
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agriculture and Applied Science

By

Nathaniel David Moody

In Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

Major Department:
Agribusiness and Applied Economics

December 2017

Fargo, North Dakota

North Dakota State University
Graduate School

Title

Optimization of Soybean Buying Strategies Using Derivatives

By

Nathaniel David Moody

The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

SUPERVISORY COMMITTEE:

William Wilson
Chair

William Nganje

Ruilin Tian

Approved:

12/11/2017
Date

William Nganje
Department Chair

ABSTRACT

The portfolio model of hedging framework, based off Markowitz (1952), is used to determine the best portfolio of futures, basis, and option contracts to hedge a soybean purchase from PNW 28 weeks into the future. Eighteen options are incorporated including in-the-money, at-the-money, and out-of-the-money call and puts with different expiration dates. Futures and option pricing data is extracted from ProphetX from November of 2013 to December of 2016. Expected utility objectives including mean-variance, CVaR, Mean-CVaR, and Mean-CVaR with copula are maximized using linear programming optimization methods. A two-stage model is built to simulate hedging scenarios while measuring various statistics. Under high risk aversion, a standard futures hedge performs the best. Buyers with lower risk aversion should explore option strategies. In-the-money calls, collars, strangles, and short butterfly strategies all perform well.

ACKNOWLEDGEMENTS

I'd like to thank my graduate advisor, Dr. Wilson, for all his support over the past 16 months. That includes support related to this thesis as well as in the "real world." I've met many interesting, successful people through him and can't thank him enough for his time invested in me. Thank you to Bruce Dahl for helping me with any technical issues and providing some much need football conversation when I needed a break. I'd also like to thank Dr. Jeremy Jackson. Without Jeremy, I wouldn't have applied or even known about this program. I also enjoyed his classes even though he wrongfully received many poor reviews. Thank you to Dr. Nganje and Dr. Tian for taking time to be on my committee. Your input and time are appreciated.

Thank you to all the students and friends I have met in this program. I'd like to think most of us have challenged each other intellectually and have grown accordingly. Also, shout out to Dr. Nganje for thinking he could whoop me in racquetball the first time we played. I enjoyed the friendly competition and happy to come out on top.

Lastly, thank you to my parents for always supporting me through whatever endeavors I had in mind next. Fargo has become a place to call home and my roots will always be in North Dakota wherever I end up.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDIX TABLES

Table Page

CHAPTER 1. INTRODUCTION

1.1. Overview

Large price swings in the commodity markets can bring vigorous joy or bitter despair to commodity trading firms, buyers, and importers. Just before the dawn of financial crisis of 2008, commodity prices went wild in February that year leading to large profits and losses depending on which side of the trade firms were on. Buyers and sellers confront numerous strategies for combating these risks. Examples include basis contracts, HTA (hedged-to-arrive) contracts where futures are locked in, regular futures contracts, and a plethora of derivative strategies. Altogether, the problem can be approached as a portfolio of alternatives. The question is what amount of each of alternative should be procured at different levels of risk aversion. Risk aversion determines how much one is willing to lose relative to a profit opportunity. The more a hedger is willing to lose to have a probability of reducing cost by a certain figure, the more risk tolerant that hedger is. A hedging model is developed which utilizes stochastic optimization to determine the best alternatives to include in a hedging portfolio. Important features include the use of copulas and Mean-CVaR (condition value-at-risk) objective functions. These can both be found is recent related literature. Once developed, the model can be used to determine the optimal allocation to each of the hedging alternatives. It also can evaluate VaR (value-at-risk), CVaR, and the impacts of higher or lower aversion and volatility.

The theoretical model can be applied to any type of commodity hedging problem while the empirical model is used for a generic soybean purchase from PNW (Pacific North West). This is an application of portfolio analysis common in the financial industry. Important features include the allocations to cash, basis, futures, and options. The greeks are also of great importance, particularly delta and gamma.

It is now broadly recognized there is a need for enterprise risk management within firms involved in commodity trading. Some firms adopted VaR to theoretically measure the 1st or 5th percentile of their loss distribution. Options can provide a similar risk profile to futures but leave the upside potential for a case like 2008. However, VaR has its own shortcomings as it was the primary risk measure used in the banking system before 2008. This chapter introduces the problems commodity trading firms face in dealing with price risk. It also introduces some possible solutions and alternative ways of thinking about hedging.

1.2. Portfolio Model of Hedging

In one of the most famous papers published in the financial industry to this day, Markowitz (1952) outlined the benefits of diversification in an investment portfolio. The biggest contribution from the early paper was the derivation of the portfolio variance. Only three statistics were needed to compute the variance in a two stock portfolio. These are the respective weight of the security in the portfolio, its standard deviation of past prices or returns, and the covariance with the other security in the portfolio. If two stocks aren't perfectly correlated and have the same amount of risk, adding the second one would theoretically reduce the variance of the portfolio.

The methodology was intended to be applied to a portfolio of stocks. Researches and firms could now optimize for a certain level of return or risk given a choice of stocks and assumptions about risk and return. His later work (Markowitz & Stuart, 1959) outlined the derivation of expected utility assuming a quadratic utility function. This brought risk and return together in one utility function to optimize for utility versus just returns or risk. Utility is a function of risk, return, and risk aversion.

The same methodology was then applied to hedging problems. A short cash and long futures position can be looked at as a two security portfolio with positive correlation but not perfect. Ederington (1979) explored the hedging effectiveness of various securities with their spot market equivalents. He found that the spot and futures do not move perfectly together thus should not be hedged one to one. Later studies focused on different markets, different objective functions, and different ways to measure risk. There is much more literature that is reviewed in Chapter 2 relative to this subject. After the financial crisis coupled with extreme volatility in the commodities markets, commodity trading companies like Gavilon, Cargill, and numerous others built out risk monitoring procedures within their companies. Prior, many firms didn't have a grasp on the inherent risk in these markets. In theory, the portfolio model of hedging methodology can be applied to any portfolio of securities.

1.3. Commodity Trading Today

The world of commodity traders are split into two categories: hedgers and speculators. Hedgers are the firms we look at in this study. They take physical positions in a commodity and "hedge" their risk using the futures or other derivatives. Speculators do not take a physical position in the commodity. They buy and sell futures and other derivatives with the sole intent to make a profit when they close the position. It's very similar to betting on a stock.

The two are broken apart because one category can be ultra-sophisticated and the other not so much. The not-so-sophisticated traders are group of traders that are targeted in this study. The typical hedging strategy is to always take an equal and opposite position in the underlying physical commodity. If a Chinese soybean buyer agrees to purchase a couple hundred thousand tonnes of beans in a few months, they take a long position in the futures market while also hedging their currency risk. The way traders like this make money is basically arbitrage. They buy low (U.S), and sell high (China). This kind of arbitrage determines the way commodities move throughout the world. After taking out shipping costs, they're left with essentially a price-riskless profit.

Speculators are often the ones taking opposite positions of the hedgers but also involved in options trading and complex derivatives. This isn't to say that all hedgers use the basic "equal and opposite" position but most do. Speculators use standard futures but are also trading options, swaps, swaptions, and other related securities. This gives them much more flexibility to bet on characteristic of the underlying besides price. For example, volatility is easy to go long or short by using a butterfly spread which is explained in chapters 2 and 4. The same principles that are used by speculators can be used by hedgers. In today's world, it's my opinion that most elevator mangers, flour mills, etc. don't understand how these derivatives behave and see them as too risky. There may be an opportunity for hedgers to make more money by applying more sophisticated hedging strategies.

1.4. Problem Statement

There are many risk factors in most businesses today. A shift in consumer preferences, a sudden hike in input costs, and dismal consumer confidence can all derail future revenue and profit expectations. No matter the size and scale, unforeseen events can bring even the largest, most stable businesses to their knees. Just recall what happened to the some of the big banks during the financial crisis only a decade ago.

For commodity trading firms, risks can be amplified by the price swings in commodities. Commodity trading firms buy and source crops, metals, oil, and a plethora of other commodities in order to sell or transform them. They make a margin between the prices they bought and sold minus any transportation, storage, or other costs incurred before selling the commodity or refined product.

Price fluctuations in these commodities are one of the main sources of risk for commodity trading companies. Input levels for end users are usually known well in advance. Even if that's not the case, the time to transport commodities internationally can take weeks. Price swings can happen quickly and violently over that short of a time period. A hedger has various alternatives to deal with or combat this risk. They can operate unhedged, hedge with various forward or basis contracts, hedge with futures contracts, or deploy some sort of derivative hedging strategy. The futures market was created for firms involved in businesses like this to hedge their price risk. It works great for a firm which intends to lock in prices at certain levels and have little regard for potential margin calls. However, generally hedgers would like to be able to take advantage of favorable price moves over this period of uncertainty while still controlling their risk. They also are weary of margin calls during unfavorable price moves.

Hedging with derivative contracts can solve some of these problems. Swaps currently don't require any margin. Options provide a price floor (ceiling) with unlimited risk on the upside (downside) for a cost. Depending on current volatility, the time period needed to hedge, and the degree of "moneyness" desired can make them expensive to buy and sell. One solution to offset those costs is to simultaneously sell similar options. However, this opens up more price risk to one side or cuts short what would have been an unlimited upside. In other words, there is no free lunch.

Today, hedgers primarily use the futures market to hedge their price risk. They eliminate all price risk inherent in the futures market but give away any would be profits as well. Can you imagine firms that went short on oil in 2008 when it was around $140 looking to go higher? That exact situation happened to Delta Airlines before oil prices tumbled down to less than $40 a barrel. They took a $1.9 billion hedging loss during that year. Even though they bought oil cheaper in the cash market on its way down, management and shareholders were uneasy about a hedging loss of that magnitude. Delta's earnings came in negative that year and the hedging loss was the primary cause. (Kelly, 2015) There are plenty of other instances where large trading firms have gotten caught on the wrong side of a hedge.

On the other hand, options have asymmetric payoffs. Firms can lock in a price minimum or maximum net price for a small fee called the option premium. They are basic instruments one at a time but loading a multitude of them into one portfolio can make for a headache. They also require an ample amount of margin compared to the cost of them.

1.5. Objective, Procedures, and Hypothesis

The goal of this study is to find out which alternatives would be an optimal hedge under risk preferences using different measurement techniques. Securities included to hedge with are a futures contract, basis contract, and eighteen options. The options include calls and puts, cover various degrees of moneyness, and also include different maturities. Moneyness refers to how far in or out-of-the-money the options are. The pricing data used is from November of 2013 and ends in December of 2016. All data points are accounted for and no extrapolation is necessary.

The previous literature has sparsely covered the opportunities for employing a portfolio model of hedging using options. The literature that has covered the topic is either theoretical in nature or only uses one to two options for the hedges. All pricing is based on models as well. In this study actual option prices were gathered from DTN. In general, the model is a large-scale optimization hedging problem. It uses two stages so to measure the return and risk of the hedge over the respective period. Volatility is the main input into the price of an option besides its degree of moneyness. Implied volatility is sourced from the underlying prices.

If the objective is to minimize risk, futures should be the best hedge. However, using alternative objective functions and risk preferences should make options strategies more attractive. Because the data starts right after the large drop is crop prices, particular soybeans after the harvest of 2013, the problem is based on a low volatility environment. Strategies that involve some upside while limiting downside are expected to perform the best. Collar and butterfly strategies are the first to come to mind that fit that profile.

1.6. Organization

The organization of this thesis is outlined as follows: next, chapter two provides a detailed overview of the previous research that went into this problem. It starts from the dawn of portfolio theory and ends with sophisticated semi-variance measures. Chapter three introduces the theoretical model. It dives deeper into the model and provides a blueprint for designing hedging problems focused on derivatives. Some comments about risk aversion and the effects of high or low risk tolerance are also made. Chapter four provides the empirical model. It goes into detail about the different option strategies that are included in this study. The method for gathering data is also outlined. Lastly, correlations, margins, and optimization procedures are covered in the chapter. Chapter five shows the main findings of the study. The results are compared to a base case scenario while strategies are also compared across different objective functions. Chapter six provides the summary and conclusions. Any shortcomings, limitations and comments for further research are provided here.

CHAPTER 2. LITERATURE REVIEW

2.1. Introduction

It is important for agribusiness firms to control price risk, which is often mitigated by taking an opposite position in the specific commodity via a futures, forward, or options contract. This is done to lock in a price to counteract future price movements of the relevant commodity. Buyers today participate in the futures and options market for those reasons. Generally, it is safe to assume that futures and cash prices move together but not perfectly. The difference lies within the basis.

Buyers tends to be short in the cash market. This means they must buy grain in the future. The objective of a hedge is to reduce price risk. A hedge offsets losses in the cash market through gains in the futures market. Consequently, gains in the cash market can also be offset by losses in the futures market. It's easy to see that a buyer hedged completely has zero price risk assuming futures and cash prices are perfectly correlated. Gains and losses refer to profit unless otherwise stated.

This study focuses on a portfolio model of hedging that is designed to reduce risk while leaving upside potential for profits. In the next section a history of Modern Portfolio Theory (MPT) is provided. Section three expands on the idea and explains how it can be used in a portfolio model of hedging framework. Section four introduces Post-Modern Portfolio Theory (PMPT) and illustrates its use and possible superiority to MPT. Section five describes more recent approaches to portfolio optimization. Finally, section six describes and outlines the characteristics of the hedging "vehicles" that are be used in this study.

2.2. Modern Portfolio Theory (MPT)

One of the main questions for individuals and professional asset managers is where to put their money. In today's globalized world, even a retail investor can get access to investments all over the world. Money managers are not only expected to achieve high returns, but also to keep the risk of a portfolio in check. They also must decide where to invest and how much to allocate to each investment. Over the last 60 plus years, investment professionals have answered most of these questions. There are hundreds of contributors to today's knowledge on this topic. Only the early contributions that are critical to MPT are highlighted in this chapter. A review of some recent innovations in the area are explored while parallels are drawn to a Portfolio Model of Hedging framework.

Markowitz (1952) is widely considered the father of MPT. He published the first paper that related risk and return in a portfolio model developing the E-V framework (expected return vs variance of returns). He identified the positive relationship that existed between risk and return. To achieve better returns, one must take on more risk. Identification and use of the correlation coefficients between assets to further interpret the risk of a portfolio is still used today. In a two-asset portfolio, the coefficient of correlation between the assets has a positive relationship with overall risk. A diversified portfolio is one with assets that usually exhibit negative correlation or low positive correlation with other assets held. Using his model, the portfolio could be optimized for a certain level of risk, which is measured by the variance or standard deviation in returns, or a specific level of return. An individual could further control their risk by introducing a risk-free asset into the portfolio. All of these factors play a key role in the analysis of a portfolio. These principles led to the formulation of the efficient frontier. Although Markowitz (1952) outlined the underlying theory behind the efficient frontier, it wasn't until Merton (1972) that the frontier was derived and shown as a hyperbola. A graph is shown later in this chapter.

Around the same time, Roy (1952) was working on a similar theory, independent from Markowitz. He developed the safety-first criterion which measured the chance of a portfolio returning less than a certain extreme loss. The purpose of the criterion is to minimize the chance of a portfolio falling below that level. Today, there is a similar measure used is practice called the “margin of safety.” The term was coined by Benjamin Graham in his well-known book *Security Analysis*.

Sharpe (1964) was one of a handful of scientists to be credited with the formulation of the Capital Asset Pricing Model (CAPM). The CAPM is a model that combines the efficient frontier with a capital market line. The capital market line intersects the y-axis at the risk-free rate and is drawn tangent to the efficient frontier. The tangent point lies above the minimum variance point. Tobin (1958) is credited with deriving the capital market line. Sharpe et al. (1964) allowed for the use of investing or borrowing at the risk-free rate. Today, this is known as leveraging which is to borrow money (at the risk-free rate or a different, higher rate depending on the risk of the borrower to the creditor) and invest in risky assets. When an investor is short the risk-free asset and long the risky asset at a level higher than their equity position, more risk is now injected into the portfolio. This creates an opportunity for higher returns but also heavier losses. Sharpe also derived systemic and unsystematic risk for an individual security. He proposed a relationship exists between economic activity and returns on an asset. This eventually led to an asset’s “Beta” being used in the CAPM model, which used in finance classes today. Beta is derived by regressing returns or prices of an individual stock against economic activity (usually the S&P 500).

Sharpe (1966) continued developing the CAPM. Thirty-four open ended mutual funds were studied using ex post data. Returns and variability of returns were plotted, and the reward-to-variability ratio was coined. Today, this is known as the Sharpe ratio. The ratio is simply a risk-adjusted rate of return. It's calculated by the ratio of returns, in excess of the risk-free rate, over the standard deviation of returns. Although the term "Sharpe ratio" wasn't officially coined until Sharpe (1994), the concept was illustrated 30 years prior.

Merton (1972) derived and graphed the efficient frontier for a three-asset portfolio. Previously, only two asset portfolios were studied, and none derived the efficient frontier as a hyperbola shown below. The efficient frontier is derived, on a graph with risk on the x-axis and return and the y-axis, by changing the allocation to each security. Any point above the most leftward point (minimum variance portfolio) is considered "efficient." The points below the minimum variance portfolio represent asset being sold short (borrowing a share of a stock to sell it and buying it back at a later date). Any point (combination of risk and return) on or within the efficient frontier is obtainable while points that lie outside the frontier are not obtainable.

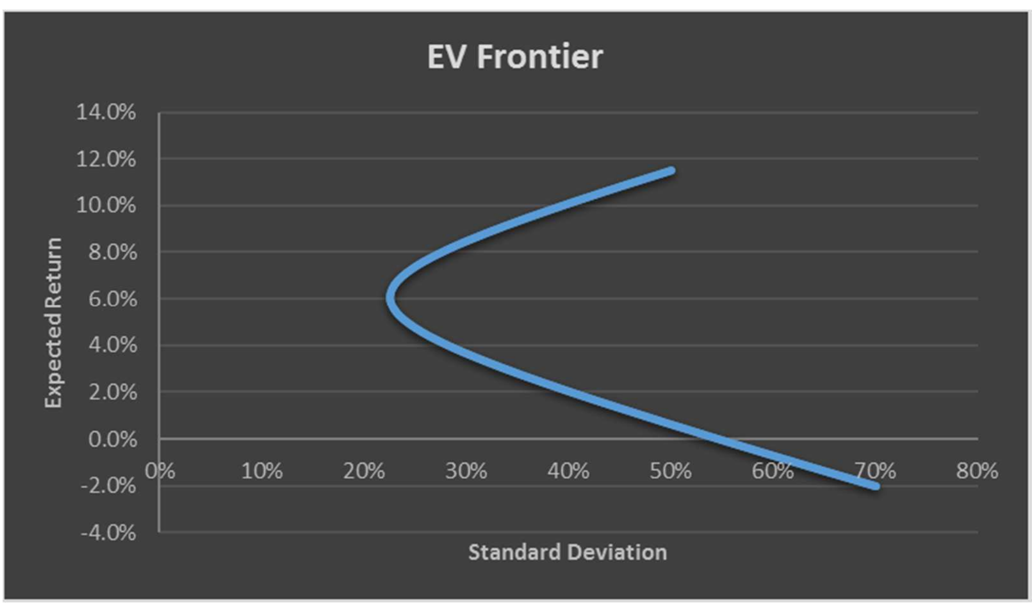

Figure 2.1: EV Frontier

2.3. Early Advancements in the Portfolio Model of Hedging

The underlying theory for a portfolio model was introduced by Markowitz (1952). He proposed a model where risk and return where connected. Risk was measured by the variance or standard deviation of returns over a specified time period. Returns were measured as the percentage change over that same time period. Essentially, high standard deviations and correlation coefficients (or covariance) characterizes a higher risk portfolio. The opposite is true for lower levels of risk. A security with low risk and high returns is considered optimal.

Traditional theory, at this time, assumed hedgers would either be completely hedged or unhedged. Ederington (1979) identified that spot and futures prices in T-bills, GNMAs (Government National Mortgage Association), wheat and corn do not move perfectly together. Therefore, they should not be hedged perfectly or have a hedge ratio of one (HR = 1). This hedge is referred to as a naïve hedge following previous literature. This is true even for a risk-minimizing individual. He argues that hedging should be treated the same as any other investment decision. The objective is to maximize return while taking an appropriate amount of risk. Ederington (1979) proved that minimizing risk is done by having a hedge ratio of less than one in all securities and time periods studied. He also found that the risk-minimizing hedge ratio was lower as the time period of the hedge grew longer in the T-Bill and GNMA market. The optimal hedge ratio is further reviewed in the next section.

2.3.1. Hedging

"Organized commodity futures trading facilitates two kinds of activity – speculation and hedging." "Most commodity trading theorists have visualized the hedger as a dealer in the "actual" commodity who desires "insurance" against the price risks he faces." (Johnson, 1960) In the literature, hedging and speculation are divided by whether there exists an underlying, physical commodity in relation to the transaction. Speculators do not have an underlying cash position. They are in the market to speculate on future price movements and profit from those positions.

2.3.2. Minimum Variance Hedge

Johnson (1960) derived the minimum variance hedge ratio (MV HR) from price levels. His formulation for the risk minimizing hedge ratio is derived by taking a first derivative of the portfolio variance formula. This results in the ratio of the covariance in spot and futures prices over the variance in futures. This is called this the "minimum variance hedge ratio" (MV HR). Empirically, this HR is the same as the slope coefficient of an OLS (ordinary least squares) regression on futures and cash prices.

Wilson (1982) used price changes in wheat to determine optimal hedge ratios for eight different spot markets using three different futures' markets. He measured the ratio of variance of the optimal hedged portfolio over the naïve hedged portfolio variance to illustrate how effective the hedge was over a short and long-term hedge. Using the portfolio of hedging framework, he analyzed the effectiveness of a single futures market, two market, and three market hedge. He concluded that a two-market hedge can significantly increase the effectiveness of a hedge in some cases while hedging in three markets provides almost zero marginal benefit. He also found that hedging using short term contracts versus longer term displays a higher hedging effectiveness coefficient.

Brown (1985) also used price changes (returns) to derive the MV HR. He identified an assumption of OLS that may be violated when price levels are used to estimate the HR. He argued that the "residuals of price level regressions often exhibit significant degrees of autocorrelation." If autocorrelation is present in the residuals, the estimated MV hedge ratio derived may be inefficient.

Myers and Thompson (1989) further argued that even returns may not be appropriate to use in all situations. Instead of standard OLS they used a more flexible GLS (generalized least squares) model. Using a GLS model allows more of the residuals to follow an alternative distribution other than normal. They tested a storage hedging model using corn, soybeans, and wheat prices. Their results indicate the simple regression model using price levels was not appropriate. The OLS model using price changes derived a MV HR that was very similar to the GLS model. They note that although they were similar, other data could indicate problems using standard OLS.

There is speculative element that is introduced when the hedger has future price expectations. Blank, Carter, & Schmeising (1991) used a hedge ratio that combines the risk-minimizing hedge ratio plus a speculative element called the "optimal hedge ratio." In reality, hedgers often do have future price expectations and therefore are not driven solely to minimize their price risk. However, most buyers still completely hedge themselves (HR =1) even though theory may suggest a different hedge ratio. To adjust for this, expected return can be added into the objective function while keeping the risk aspect present. This method is further discussed in the next section.

2.3.3. Expected Utility E(U) and Mean-Variance Hedge (E-V)

Minimizing variance can be one objective of a hedger. However, the risk-minimizing hedge ratio does not account for the expected return of the hedge. If a buyer expects grain prices in the future to drop, they may want to hedge less than what the risk-minimizing hedge ratio would indicate. If prices do drop, the firm would "save" some profits by not hedging earlier.

They would also benefit from the drop in cash prices. Less price risk is generally more attractive to a hedger than more price risk but, higher returns are also more attractive. Both metrics can be incorporated to form an optimal hedge ratio using a mean-variance (E-V) framework. Blanc (1991) derived the optimal hedge ratio which uses the risk-minimizing HR and a speculative element mentioned in the previous section. This HR incorporates bias into the objective function. "Although, it can be shown that if the futures price follows a pure martingale process, then the optimal mean-variance HR is the same as the MV hedge ratio" (Chen, Lee and Shrestha, 2003). In other words, if returns are normally distributed, the optimal HRs are the same.

The simplistic mean-variance objective function is typically measured by return minus one-half times variance. E-V is essentially a subset of expected utility. "For a mean-variance framework to be consistent with the expected utility maximization principle, either the utility function needs to be quadratic or the returns should be jointly normal" (Chen et al. 2003). If neither of these conditions hold true, the optimal HR may not be correct. In that case, the optimal hedge ratio can be derived from the expected utility function (Chen et al. 2003). One characteristic of an E-V framework is assuming constant risk aversion. In an expected utility framework, the quadratic risk aversion parameter can change. Instead of having one-half multiplied by variance, the objective function changes to phi (quadratic risk aversion parameter) over two multiplied by variance. This is shown in chapter three.

Most literature regarding a mean-variance hedge incorporates a risk-return metric (Howard and D'Antonio, 1984; Cecchetti, Cumby, & Figlewski, 1998; Hsin, Kuo, & Lee, 1994). These studies maximize expected utility using the E-V framework. The Sharpe ratio is one such metric used in previous literature. Theoretically, there is no connection between the Sharpe ratio and optimal HR derived by Blanc (1991). However, empirically the optimal hedge ratio also provides the highest Sharpe ratio available for that "portfolio."

Kahl (1983) studied how the assumption of random cash prices affects optimal hedging strategies. He argues risk aversion shouldn't matter when determining the optimal hedge if cash prices are unbiased. He points out that if cash prices are stochastic instead of determined simultaneously with futures prices, then risk aversion is independent of the optimal hedge. Intuitively, this conclusion is parallel to Blanc's (1991) derivation of the optimal hedge ratio. It is also in line with Chen et al. (2003) above.

Howard and D'Antonio, 1984 used an E-V framework with a risk-free asset incorporated into the problem following early modern portfolio theorists to further control the amount of risk taken. When a risk-free asset is added to a portfolio, it further reduces the risk but at a cost to expected return. The most significant contribution was the derivation of hedging effectiveness. Up to this point, the effectiveness of a hedge was measured by r2. Instead of measuring effectiveness with r2, they used a new metric referred to it as the risk-return relative in conjunction with ρ (rho) (or r). Both symbols represent correlation. They determined the risk-to-excess-return relative (λ) of futures versus the spot price was a better measure. The relationship between ρ and λ determine the optimal strategy for the hedger. If λ is less than ρ, the hedger should have a short futures position. A long futures position should be held if λ is greater than ρ. Lastly, if they are equal, no futures position should be held.

A similar approach was taken by Cecchetti et al. (1988). They studied treasury bonds and theoretically hedged in treasury futures. Where they differed from Howard and D'Antionio (1984) is they maximized expected utility instead of using the less flexible E-V framework. Past studies also implicitly assumed that the joint distribution of cash and futures price do not change in the future. They identified that “when there is time variation present in the joint distribution, regressing past data does not correctly estimate current risk.” They used a log-normal utility function and aimed to maximize utility as a hedger. This also maximized the certainty equivalent. They found that by assuming a log normal utility function, they outperformed the no hedge and MV hedge significantly.

Hsln et al. (1994) developed a model to measure the effectiveness of hedging currency in the futures market versus using options. Using assumptions consistent with the E-V framework, they measured effectiveness by the difference in the certainty equivalent just as Cecchetti et al. (1988) did above. They concluded, at that time, the futures market is a better platform to hedge currency than using options.

Another version of the mean-variance hedging strategies includes ways to by-pass assumptions about the utility function and return distributions. One method is to minimize the mean-extended Gini coefficient (MEG) which is consistent with first and second degree stochastic dominance, while the other maximizes the optimum MEG (M-MEG). The difference between the two is parallel to the differences between the MV hedge and the mean-variance hedge. One accounts for expected returns (M-MEG) and the other does not (MEG). Examples of this can be seen in (Cheung, Kwan, & Yip; 1990), (Shalit, 1995), and many others.

The last method to go over in this section is the generalized semivariance method (GSV) or lower partial moments. Semivariance is the variance of a certain part of the return distribution (usually the left side). Empirically, this is useful when the hedger uses only negative returns or negative returns below a threshold (i.e. <2%) in his model to determine risk exposure. This methodology is also consistent with stochastic dominance. Hedgers are trying to hedge their downside risk and generally do not care about their upside risk. This method lets the hedger focus on returns that are below that threshold instead of including all returns. Just like the MEG hedge ratio methodology, one can minimize the GSV or optimize the M-GSV. This was shown first by Fishburn (1977) and then by (Lien & Tse, 1998, 2000) (Chen, Lee, Shrestha; 2001).

2.4. Post-Modern Portfolio Theory

There are a couple differences between MPT and PMPT (Post Modern Portfolio Theory). Although the difference can be somewhat subjective, one key is the way variance is used in the models. MPT assumes a symmetrical measure of risk which implies investors have symmetrical risk tolerance. This means they view upside-risk the same on down-side risk. Surprise jumps on the upside are treated the same as large moves on the downside. This creates some problems because most investors don't care about their upside risk. That's why returns, or expected returns are measured. Investors should pay more attention to their down-side risk. This makes the problem more complex because now risk must be measured asymmetrically. There are other differences that you may see throughout this section but the main one is how risk is measured.

Rom and Ferguson (1994) published one of the first articles on PMPT. They explain some limitations of MPT, some of which Markowitz, Sharpe and others had actually identified. Markowitz suggests that a mean-variance approached may not be appropriate in all circumstances. He suggested a semivariance method would be preferred. However, modern portfolio theorists were constrained by the lack of computing power in their time. (Rom et al. 1994) identifies a minimum acceptable return parameter (MAR). Each MAR (minimum accepted return) has its own unique efficient frontier. This is different from MPT where there is only one efficient frontier for an entire portfolio. The way that downside risk is measured in PMPT is split into two components. The first component is the downside probability which is the probability that the return does not meet or exceed the MAR (10% in this study). The second is the average downside magnitude which measures the average shortfall below the MAR. In simpler terms, it measures the difference between the MAR and the average return when the return does not meet or exceed the MAR. Another limitation outlined by (Rom et al. 1994) was the assumption that returns follow a normal distribution. If the underlying asset does not follow the distribution assumed, it can create significant problems in estimation. In this study, instead of a two-parameter normal or lognormal distribution, they use a four-parameter lognormal distribution adding skewness and kurtosis. They argue this distribution follows actual returns much closer than two parameter distributions. The study consists of five assets with 10 plus years of data to estimate efficient portfolios using MPT versus PMPT. They show the difference between the MV portfolio using a mean-variance framework and the MV portfolio using a 10% MAR approach. The MV portfolio using standard mean-variance is shown to be inefficient compared the MV portfolio using a downside risk measure. This difference between the two methods in largely due to skewness in the estimated returns distribution.

Harlow (1991) was one of the first to optimize a portfolio using a semivariance measure of risk. There are numerus methods to measure downside risk in finance. These risk measures are referred to as lower partial moments (LPM). LPM, for an empirically discrete distribution, is measured by taking the probability that the return does not meet the target rate of return multiplied by the difference between target rate and the realized return raised to the nth power. LPM uses the expected values of the squared negative deviations relative to the target return. Hence n would be equal to two. Assuming a symmetric distribution of returns and a target rate of 0%, this formulation is equal to the regular variance calculation. Another advantage of LPM is the relaxed assumptions including investor preferences and return distribution. However, skewness and risk aversion (via target return) are still assumed.

2.4.1. VaR

The increased popularity of semivariance methods spawned a metric known as value-at-risk (VaR) in the 90's. VaR is defined as the worst loss over a target horizon that won't be exceeded with a given level of confidence (Jorion, 1997) under normal market conditions. VaR became popular in the investment industry because firms could now put a number on their risk exposure. This number is relative and only should be compared to portfolios of similar value and asset allocation. For example, a weekly VaR of $100 million at 95% confidence indicates the firm should not lose more than $100 million in any one-week period 95% of the time. However, there are times when those losses do exceed the VaR limit. In these cases, another metric called conditional value-at-risk (CVaR) can be used. CVaR is defined as the average loss when losses do exceed the VaR limit. Notice the similarity between VaR measures and the GSV methods above. Another reason VaR became the risk metric of choice was its flexibility. Using a monte-carlo simulation, one can define distribution parameters and correlation coefficients, they expect to see in the future, to compute their portfolio's VaR.

One of the first large scale applications of VaR was the development of RiskMetrics by JPMorgan. This software could use elements of portfolio theory to combine the risks of long, short, futures, options, and other derivative positions. This software was designed to output a firm wide risk measurement for the company and is still one of the primary risk measurement tools used by JPMorgan Chase today.

Rockafellar and Uryasev (2000) provided a brief synopsis of VaR in the late during the turn of the millennium. They showed the sizable differences between optimizing VaR and CVaR. Following a study done by Mausser and Rosen (1999), they had bull butterfly spread positions in Mitsubishi and Komatsu. This spread can be accomplished by purchasing a call option, selling two calls at a slightly higher strike price, and buying another call at an even higher strike price. The trade is generally profitable when the positions exhibit low volatility over the relative holding periods. Their entire portfolio value was about 10 million JPY. Minimizing for VaR and CVaR using historical data to run a Monte Carlo simulation, values of 205 K and -1.2 MM JPY respectively were computed. According to VaR, the portfolio is going to gain 205 thousand JPY or more 95% of the time. This is somewhat odd because the VaR is positive. The estimated CVaR for the same portfolio exhibits a much worse loss relative to regular VaR. This indicates the presence of a fat tail that must be present in the return distribution. At this time, VaR was a very popular measure of risk, but "it has undesirable mathematical characteristics like a lack of subadditivity and convexity. VaR is coherent only when it is based on the standard deviation of a normal distribution." (Rockafellar and Uryasev 2000). Mckay and Keefer (1996) and Mausser and Rosen (1999) showed the problem explicitly. They tried to minimize VaR and their results showed multiple local minimums. For more on the axioms of a coherent measure of risk see (Artzner, Delbaen, Eber, and Heath; 1998)

With a lone measure for volatility always comes an expected return aspect to incorporate into the optimization as previous studies have shown. Mean-VaR (EVaR) is just that. Campbell, Huisman, and Koedijk (2001) use a Sharpe ratio to maximize expected utility. By substituting VaR for variance, they optimize the Sharpe ratio of a portfolio consisting of a stock and bond index. Although, there is no mathematical connection between a mean-variance utility function and mean-VaR, they lead to the same optimal portfolio assuming a normal distribution of returns and a risk-free rate of zero.

Alexander and Baptista (2002) analyzed the difference between using E-V and EVaR for asset allocation assuming a multivariate normal distribution of returns. They were the first to optimize a portfolio based on VaR. They show the mean-VaR set is not mean-variance efficient as the EVaR set lies at a higher point on the efficient frontier than the mean-variance set. Only as the confidence level converges to 100% does the EVaR set equal the E-V set. If a portfolio manager elects to use VaR instead of variance, under the same assumptions, they should observe a larger standard deviation of returns.

Zabolotskyy and Vitlinskyy (2013) analyzed implications of risk aversion on the EVaR model. They find the risk aversion parameter (ρ) has a deeper effect on the model than just how VaR is weighted (recall the standard E-V objective function above). A measure of less than one is found to be inappropriate. While a measure of four significantly decreases the expected return and almost converge to the MV portfolio. Another finding is the affect ρ has on the portfolio's density function. As ρ decreases from four to one, the probability density function becomes very positively skewed with a fat right tail. Hence, a low ρ is not appropriate because of the steep increase in portfolio risk.

2.4.2. Copulas

When two univariate normal distributions (marginal distributions) are correlated, they can form one multivariate normal distribution (joint distribution). A copula is a type of multivariate distribution. Skylar's theorem states "any possible joint distribution can be written as a combination of the known marginal distributions as an object called a copula." Copulas have been heavily used in in the financial and risk management industry for over a decade. There are three main types of copulas. Archimedean copulas define can define different dependency structure from the left side of the distribution to the right. They are simple in that they only require one parameter. However, they can only model positively correlated assets. The distribution must be reflected across the x or y axes to model negatively correlated assets.

Elliptical copulas are similar to a normal distribution. Actually, a Gaussian copula is approximately the same as Pearson correlation. A student t's copula is similar but has fatter tails. The last type of copula is an empirical copula. This copula is derived from the exact underlying data. To use an empirical copula effectively, one must have an ample amount of data points and have a compelling case the data will follow a similar pattern in the future. In general, copulas offer more flexibility when defining the relationships between two univariate distributions.

One of the first papers to use copulas in a portfolio based approach was by David X. Li (2000) while working for the *RiskMetrics* division at JPMorgan. The infamous paper is known, among other names, as the paper that destroyed Wall Street. He uses a statistical survival technique that is usually used for survival analysis on human beings. He took that framework and applied it to credit defaults. For example, instead of measuring the probability that a husband and wife would survive for n years, he obtained "time-until-default" probabilities and used a copula function to derive a joint distribution of default probability between two securities. This paper also showed valuation examples of CDS (credit default swaps) using copula functions. CDS are widely blamed for their role in the 2008 recession.

Ozun and Cifter (2007) used copulas to derive the VaR of a two asset, emerging markets portfolio and compared it to an EWMA model (exponential weighted moving average). They pointed out that with increased variability in returns, the lesser appropriate normality becomes as an assumption. In practice, the assumption of normality is almost never appropriate. The appropriate model should decipher between marginality and dependencies among the assets held, while also capturing non-linear returns and extreme values. They found that the copula model was a better estimator of VaR with the least number of violations between 2001 and 2007.

Boubaker and Sghaier (2013) studied the difference between estimation models using standard mean-variance, mean-variance-copula, mean-CVaR, and mean-CVaR-copula. They use two portfolios. One consisting of US and French stock market indices, and the other with the US Dollar / Euro and Yen / Euro (USD/EUR and JPY/EUR) to conduct the study. They found that the optimal mean-CVaR portfolio has a lower risk than the optimal mean-CVaR-copula. In other words, a copula based approach may be able to capture more risk inherent in the underlying data. They used a Gumbel copula based on the AIC (Akaike information criterion) and BIC (Bayesian information criterion) scores of the raw data. Gumbel assumes more up-side dependence while Clayton assumes more down-side dependence. Both are in the Archimedean family of copulas. This is interesting because risk managers should be more wary of the inherent down-side dependence than up-side. Put another way, the stock market tends to become highly correlated in bear markets.

2.4.3. Recent Financial Portfolio Approaches

Since the early years of MPT, risk models have branched off in many different directions. A couple branches include risk parity and maximum diversification. Risk parity portfolios are constructed so each asset contributes the same amount of total risk to the portfolio. Maximum diversification hopefully defines itself as portfolio managers using this strategy attempt to diversify away all their systematic risk. Clarke, Silva, and Thorley (2011) provide one of the most robust asset allocation studies done to this date. They measured risk and return of a minimum-variance, risk parity, and maximum diversification stock portfolio from 1968 – 2012. They used 1,000 assets in their model and found that the risk parity portfolio outperformed both the other portfolios as well as an equally weighted and market value weighted portfolio. The risk parity portfolio averaged a 6.2% annualized return which was the same as the S&P 500 over that time frame. It also displayed less risk than the maximum diversification portfolio as measured by standard deviation of returns.

2.5. Post-Modern Portfolio Model of Hedging in Agribusiness

Manfredo and Leuthold (2001) used VaR to analyze the cattle feeding margin using a portfolio of assets. Those assets being: live cattle, feeder cattle, and corn. The two inputs are feeder cattle and corn, while the output is live cattle. The margin is the output minus the inputs. One shortcoming of their VaR model is the assumption of normality. Unlike previous studies discussed in this chapter, they use a parametric process using the standard deviation of the entire distribution to determine VaR of the portfolio. Using various VaR estimation methods within the criteria above and comparing actual violations against forecasted values, they determine the best model was RiskMetrics which used an exponentially weighted average to forecast volatilities and correlations.

Following the same idea, the methodology has since been applied to numerous agribusiness portfolios. The soybean crush margin, crack spread, corn milling, and flour milling have all been used as "portfolios" to measure price risk in agribusiness. Chen, Wilson, Larsen, and Dahl (2015) applied it to flour milling. They analyzed different long/short scenarios in the cash and futures wheat market as well as mill feeds to determine optimal hedge ratios. The main findings were longer term hedges should be accompanied by higher hedge ratios. Also, conventional techniques for determining VaR may overstate risk relative to copula models.

Chen, Wilson, Larsen, and Dahl (2016) optimized a portfolio of agriculture assets that included farmland, futures contracts, and agriculture related equities. Out of 37 assets, there was only four where the returns were best represented by a normal distribution according to a best fit AIC score using @Risk. Most other asset returns were best represented by a Logistic or Laplace distribution which are both characterized by excessive kurtosis and symmetry. Even those assumptions are stretching the limits because inherent negative skewness in stock prices is still not accounted for. The model used a Mean-VaR-copula framework with and without allocation restrictions. Their main finding was that farmland offers and exceptional rate of return while showing minimal amounts of risk.

Kimura (2016) used the portfolio of hedging model to solve a different problem; default risk. He identified a problem where producers would sell forward to buyers but if prices fell in the interim, sometimes those buyers would "default," or not fulfill the obligations of the contract. Prices would be lower now than they were when the buyers entered in to the forward contract. Defaulting would allow them to buy grain at lower prices. This is a huge problem because now the producers are forced to sell their grain at lower prices than before or store it. He incorporated a put option in the strategy for producers to hedge this risk. The producer would buy put options while simultaneously selling forward to hedge some of this default risk. That way, if a default did occur, the producer could still gain some lost profits from the put options.

2.6. Hedging Vehicles

Hedging in an important financial management tool used by agribusiness firms to mitigate risk. Hedging is defined as the use of securities or derivatives that are negatively correlated with current long or short positions to offset the returns of that position. For an agribusiness firm, they would want to hedge their position in the cash market. There are many different vehicles in which a grain buyer can use to hedge. Futures, forwards, and basis contracts are the most common. Other vehicles include options, spreads on options, swaps, ETFs (exchange trades funds), etc. The focus for this study is on futures, basis, and different combinations of options.

2.6.1. Futures and Forwards

Futures markets can be traced all the way back to the 1700's in Japan where the first futures market for rice existed. Earlier markets can be traced back to ancient times. Modern day futures on more prevalent agriculture commodities were not introduced until the 1970's by the CME. Although the CBOT opened in 1848, it was more than a century later when futures markets became widely used.

The CME and CBOT (now one exchange after a merger in 2007) are exchanges where futures' contracts are traded in a similar manner to stocks traded on the NYSE and the NASDAQ. They are standardized contracts where either the underlying asset is deliverable or can be settled in cash after the final trading day of the contract. Although most contracts are cash settled today. There are hundreds of different futures contracts traded in almost every market imaginable today. Crops, precious metals, currencies and stock indices all have a futures market. Similar to futures are forward contracts.

Forwards are similar but have a few main differences. First, forwards are not standardized. Regarding commodities; quantity, quality, time of delivery, and a handful of other items can all be agreed upon between the buyer and the seller. For example, if you make an agreement to sell stock to a friend at a certain date and price and they agree to buy it, you've just entered into a forward contract. Another difference is the futures market is run through a clearinghouse and requires a margin on both sides of the contract. Margin is a form of financial security for the counterparty as well as the exchange itself. When a contract is bought or sold, the holder puts down an initial margin as security for the exchange if an unfavorable price movement occurs. Typically, the initial margin requirement for soybean futures is around 3%. However, it changes based on current volatility and the contract end date. Farther out contracts require less margin than contracts expiring within six months. If the price movement is substantial enough, the margin account may fall below the maintenance margin threshold in which the holder has to add to their margin account or risk the position being closed. This, plus the guarantee from the clearinghouse, ensures that there is no risk of default from the other party involved. If for some reason the counterparty does default, the clearinghouse guarantees any payments owed.

On the other hand, forward contracts can be at risk of default. They also do not require any margin. Hedgers in the forward market have an incentive to default if the price of the underlying moves in an unfavorable direction. This is the reason more hedgers use the futures market instead of forward contracts. Previously, forward contracts were more prevalent but recently all buyers who have access to the futures market use the exchange instead. The hedgers refer to are buyers of grain. Be careful not to confuse these hedgers with producers.

2.6.2. Basis Contracts

Basis contracts are totally independent from futures or forward contracts. Basis is defined as the difference between the futures price of the current contract and the local cash price. It is different in each location and based on transportation, storage, and other local cost variables. There’s two reasons why buyers use basis contracts.

First, is to offset a sale from a producer. Buyers can lock in a basis price with the end user (i.e. PNW, GULF, or a food processing plant). In this situation, that buyer has zero basis risk. Second is to lock in an attractive basis. Basis tends to be seasonal in nature. It tends to be high right before the North American harvest season (late summer and early fall). Once harvest is in full swing, the basis tends to retreat to lower levels. However, other factors can overpower this pattern in any given year. Lastly, they can be used to secure future inventory from producers.

PNW (Pacific Northwest) and GULF are the two “cash export markets” available in the US. PNW is the cash price for export in Portland, OR while GULF is the cash price for export in New Orleans. Whether the basis moves up or down, the buyer can eliminate price risk related to the basis.

2.6.3. Hedged-to-Arrive Contracts

HTA contracts (hedged-to-arrive) are another tool buyers can use to alleviate some price risk. An HTA contract is essentially a forward contract without the basis included. Like regular forward contracts, they are not used today as much as they used to be. Just as in forward, futures, and basis contracts, a buyer can be short or long an HTA contract. If the futures and export basis price is high, the buyer may want to lock in their selling price with the end user. When they buy the grain from producers, low priced futures and low, local basis would be optimal.

HTA contracts are not used much, if at all, between a buyer and an end user, but are still used between buyers and producers. This is because producers are handling less grain than buyers would, and it may be cost effective to use the futures market.

2.6.4. Spreads

Spread contracts can also be used by agribusiness firms. However, their use as a risk mitigation tool is limited. Spread contracts are very liquid instruments in the highly-traded commodity futures. However, most if not all this trading is from speculators. General spreads mean much more to a buyer than spread contracts. A grain buyer should look at spreads and the cost of carry. If a buyer can store grain for cheaper than the calendar spread in futures, they can buy in the cash market and store it. As long as the carry is more than their storage cost, the buyer can make a risk-free profit (arbitrage).

Inter-calendar and inter-market spreads are other highly liquid contracts. The first is just the spread between contract months for the same commodity. The latter is the spread between different exchanges for the same commodity and contract month.

Inter-commodity spreads are the difference in prices between two commodities in the same (or close to the same) contract month. For example, the Nov – Dec spread for soybeans and corn is often one of the main indicators of how much of each commodity is expected to be planted in the next crop year. There are many other spreads a buyer can analyze. However, in my view, the use of these contracts is better classified as speculation versus hedging.

2.6.5. Options

Options are a more complicated tool used to mitigate risk. Options are popular because they offer more flexibility when hedging. An option is a contract that gives the buyer the right to buy or sell the underlying asset at a pre-determined price known as the strike price. The seller is then obligated to the sell the underlying at the strike price if exercised by the buyer. Buying an option is equivalent to holding a "long" position while the seller holds a "short" position. A call option gives the buyer the right to buy the underlying asset while a put option gives the buyer the right to sell it at the strike price. A grain buyer can buy a call option at a specified strike price that accomplishes relatively the same goal as a long futures or forward position would. The difference is the buyer sets a limit, at the strike price, on their downside risk for a premium (option price) while leaving unlimited upside potential.

Just as in the futures market, one must contribute capital to a margin account when buying and selling options. If the position moves against the holder, that margin can erode much quicker holding options instead of futures contracts. Margins can be much higher or lower for options accounts versus futures depending on the type of trading. If naked options are bought and sold, the margin is higher than it is for futures. However, if a collar strategy is used, where there is protection on the upside and downside from a substantial price movement in the underlying, the initial margin is much less. The required margin varies drastically based on each individual's account holdings. For example, the margins for the CME is determined by a VaR simulation model called SPAM (Standard Portfolio Analysis of Risk). The model uses 16 different "risk" scenarios to access the risk of an individual portfolio. It outputs an appropriate initial margin for the individual based on their option positions.

In the late 1950's and 60's, academics scrambled to find a sound formula to price options. They had come up with all kinds of variables that were frankly immeasurable. It wasn't until the early 70's that a sound formula to derive the value of an options was published (Black and Scholes, 1973). The price of options on commodities are usually determined using the Black-76 model (Black, 1976). The Black model is very similar to the more well-known Black Scholes option pricing model (Black and Scholes, 1973). There are only two differences between the two models. The Black model does not include a dividend like the Black-Scholes model does. Second, the Black model uses forward prices instead of spot prices. This is because spot prices have been known to follow seasonal patterns and therefore are non-random making the use forward prices more appropriate.

Recall from above that basis tends to be seasonal which feeds directly into spot prices. This is a problem because both models assume a normal distribution of returns (price changes). One other obvious area of seasonality in spot prices exists in the natural gas market. Typically, natural gas spot prices are higher in the winter than in the summer. However, forward prices in the same market may not show signs of seasonality because the expectations of spot price seasonality should be priced into the market. In general, one is assuming forward prices are random. Thus, the Black model is appropriate.

Besides the price of the underlying, the other factors that affect the price of an option are volatility and time to expiration. Volatility can be measured in two ways. First, by the variance or standard deviation of past returns. Second, by using implied volatility (IV). IV can be calculated using the Black model by inserting the current option premium into the model and solving for volatility (sigma σ). Volatility and option prices have a positive relationship regardless if the option is a put or a call. This relationship is measured by "Vega" in option pricing. It measures the change in the option price given 1% change in volatility.

Time to expiration is measured by the days until the options expires. Time is important because options exponentially decline in value as time to expiration decreases. This is known time decay as is measured by Theta. Theta is the amount of premium an option loses each day moving forward. Theta has a negative relationship with the price of options. This is considered a disadvantage for the holder of an option but an advantage for the seller.

Where options offer flexibility is when you pair them with other types of contracts or other options. Today there are plenty of well-known option trading strategies. Straddle, strangle, collar, protective put, iron condor and a butterfly spread are just a few. For example, if a grain buyer wants to protect himself from downside risk but still wants upside potential, he can employ a simple protective call strategy. They are already short in the cash market because they expect to buy grain from a producer or another grain distributor. To employ a protective call strategy, the buyer would only have to buy OTM call options (out of the money). They are now short cash and own an option to buy futures. However, this is not a perfect hedging strategy because, as stated above, futures and spot prices are not perfectly correlated. The hedge is less effective than a short futures and long call option position but in general accomplishes the same goal. This is from a buyer's perspective, so the strategy would be opposite for a producer. This strategy protects them from losing money after the price moves above the strike price but allows them to benefit from a downside move in prices as well.

2.6.6. Option Spreads

Options spreads are more complex. The more options that kept in a portfolio, the more complicated it becomes to track one's exposure. The greeks (delta, gamma, vega, and theta) are key to identifying where risk exposure lies in a portfolio of options. A brief outline of popular strategies common for speculators is presented below. This section is meant give the reader a brief outline of how options and option spreads are priced and how they behave. Lastly, a case for the potential use by hedgers is described.

One of the more popular strategies is the bull call spread. This is known as a vertical spread. A vertical spread is a put or call spread where the option contracts are traded with the same expiration date but different strike prices. Specifically, a bull call spread entails buying an ITM or ATM call option and selling an OTM one. The benefits of a bull spread include giving the user the upside potential with limited loss. Because you are buying and selling call options, the premium from the sold option helps pay for the premium of the call. However, the price you pay for these benefits comes with a cap on profit as well. This strategy is good when the buyer has future price expectations and doesn't want to deal with Vega or Theta risk. A bull call spread payoff function is illustrated below on the left. As you can see, the strike prices are set at $920 and $1120 in this example. The orange line considers just the payoff for the options. This can be looked at as a speculative payoff. The blue line considers the same position along with a short cash position. It can be viewed as the hedger's payoff.

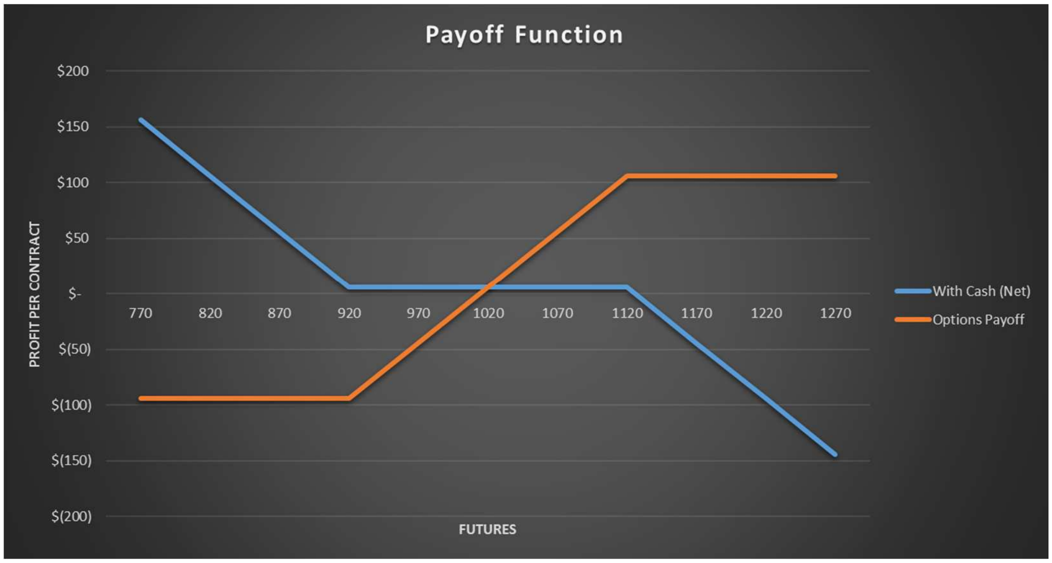

Figure 2.2: Bull Call Spread Payoff Function

One could also double the position on the spread to obtain a payoff function that doesn't exhibit as much downside risk as prices move against the buyer.

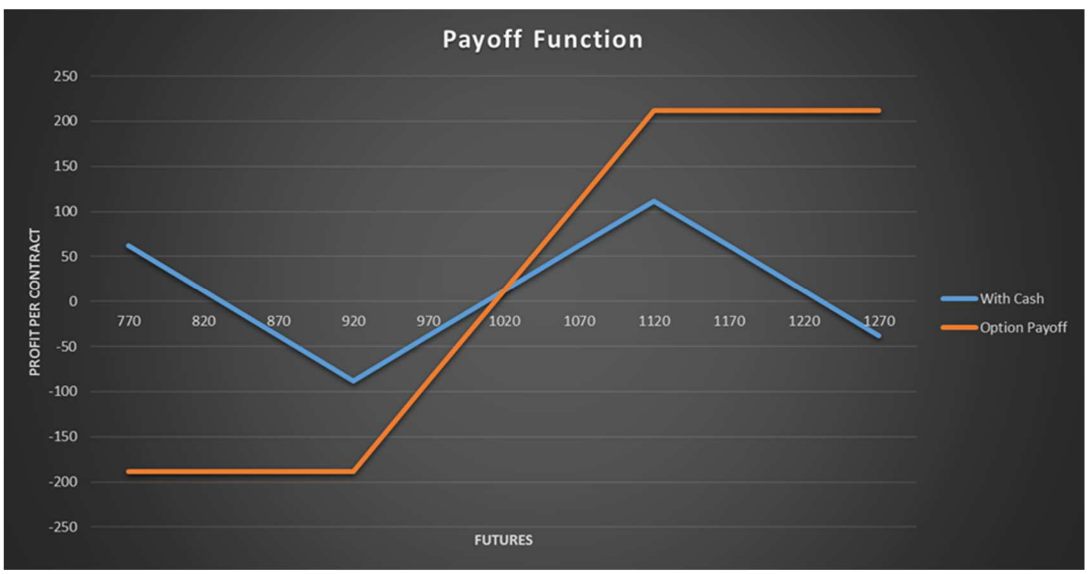

Figure 2.3: 2x Bull Call Spread Payoff Function

Assuming the same volatility for both options, the graphs are an accurate representation of total profit. Other similar spreads include a bear call spread as well as bull and bear put spreads.

Another strategy, similar to a vertical spread, is a horizontal spread (or calendar spread). A calendar spread entails buying and selling an option with the same strike price at different dates of expiration. One could buy the option further from expiration and sell the closer to expiration one. This is a "positive theta trade" as the holder makes money as time goes because shorter maturity options decay faster than longer maturity options. Decay refers to the value that is lost each day from holding an option or gained from selling one. However, a price and volatility swing in the underlying asset could also go against the holder. Even though this seems similar to the bull spread, the calendar spread exhibits a much different payoff function which is shown below. It actually is the same payoff function as a no hedge scenario with a little premium gained through the options strategy. If this same graph was shown two months after the hedge was placed, the end of the orange line would start to rise which would feed into the blue line as well. Assuming volatility stays constant, a "theta premium" would be collected.

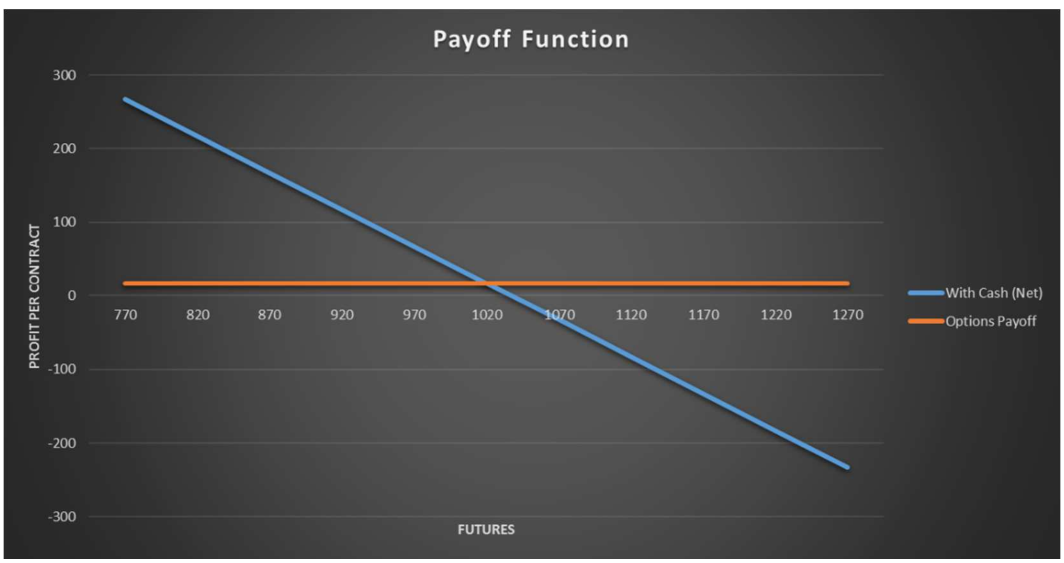

Figure 2.4: Long Calendar Spread

All three graphs above are static. Slight changes in strike prices, duration, and volatility can change these payoff functions quite drastically. To reiterate, the more options that are included in a portfolio, the more complex the payoff function becomes. Managing the "greeks" becomes an essential part of managing a portfolio of options.

Today, option spreads are mostly traded by speculators. For some of the large buyers, liquidity may be an issue. "Diversifying" their option holdings should solve any liquidity issues for large volume buyers. There use for a buyer without any future price expectations could be limited. There may be seasonal cases around the new crop (August – November) where option prices are less correlated and a diversified hedging strategy becomes optimal. In general, if options prices become less correlated to each other, there may be instances where it makes sense do some spreading from a risk mitigation stand point.

2.7. Conclusion

Markowitz (1952, 1959) is arguably still considered the most important advancement in financial research. The underlying theory has further been refined and advanced with the help of computing power not available in the 1950's, 60's, or 70's. Using LPM, a portfolio manager could now focus on purely the downside risk of a portfolio. This was another breakthrough because from a risk perspective as investors typically do not care about the "upside risk." In the mid 90's, JPMorgan developed the most comprehensive enterprise risk management (ERM) software available at that time. It was able to compute the risk of a portfolio that included everything from stocks and bonds to complex derivatives with non-linear payoffs. The software used VaR and CVaR as the "risk metrics."

Copulas are able to define a joint distribution between any two marginal distributions according to Skylar's theorem. They are more flexible as they can control for different magnitudes of correlation between the high and low side of the joint distribution. More recent optimization approaches have incorporated EVaR, Mean-CVaR, and copulas into one objective function. Other approaches include risk parity which spreads an equal amount of risk to each position.

Many these concepts have been used in agriculture scenarios thus far. The futures and options markets' in agriculture provide an opportunity for agribusinesses to treat their positions as a portfolio. Thus, the portfolio model of hedging is an appropriate method to measure risk exposure in this industry.

Futures markets have been around for a few centuries but were not open and utilized in the US until the latter part of the 20th century. The options market and other derivative markets have followed suit and their use has expanded rapidly; most notably leading up to the financial crisis of 2008. The options market has created the opportunity for money managers to create an unlimited amount of payoff functions that can be used to make excess returns in the right market scenarios. These options are also available for agribusinesses but instead of finding complex ways to make excess returns, these markets can provide complex portfolio alternatives to increase returns and reduce risk relative to just using the futures market.

CHAPTER 3. THEORETICAL MODEL

3.1. Introduction

A plethora of hedging frameworks were reviewed in Chapter 2. VaR, Copulas, and an introduction to hedging vehicles also were presented. Buyers can use these concepts to derive an optimal hedge ratio for their operation. This chapter uses the concepts reviewed in chapter 2 and applies them to a portfolio that includes futures, basis and option contracts. Futures generally don't follow a normal or lognormal distribution while options are extremely hard to model because of their inherent leverage and volatility. A framework is laid out to overcome these challenges while capturing performance metrics of the portfolio. The section is organized as follows: first a progression of objective functions are shown; next some assumptions and problems are laid out; after that the Black-76 pricing model is shown analytically; and finally the greeks are outlined and explained in more detail.

3.2. Risk and Return

Grain companies offer an excellent example for a portfolio model of hedging framework to be applied. Some have short or long positions in the futures market along with other derivative positions. The more positions that are held, especially in options or other derivatives, the more attention must be paid to control different characteristics of the portfolio. Markowitiz (1952, 1959), Johnson (1960), Ederington (1979), Brown (1985), and Myers and Thompson (1989) used the variance of a portfolio framework to derive MV hedges for a multitude of different assets. Blanc (1991) furthered their work by incorporating a speculative element using the E-V framework. Finally, Alexander and Baptista (2002) substituted VaR for variance to create the E-VaR framework.

The return of a hedge is fairly straightforward while risk can be subjective. There are many ways to measure risk as outlined above and in chapter 2. The use of expected return and variance is one of many ways a portfolio can be optimized:

$$E_p(\mathrm{R}) = \sum_{i}^{n} w_i * r_i \tag{3.1}$$

$$\sigma_p^2 = \sum_{i}^{n} w_i^2 * \sigma_i^2 \ + 2 * \sum_{i}^{n}\sum_{j}^{n} \sigma_i * \sigma_j * \rho_{(i,j)} \tag{3.2}$$

Where *Er(R)* is the expected return of the portfolio, *w* is the weight of the position relative to the portfolio, and *r* is the return of asset i over time period *n*. σ^2 is the variance of asset i, σ is the standard deviation, and ρ is the correlation coefficient between asset i and j. the double sum part of equation (3.2) represents the covariance multiplied by the weights of each position. Hence, assuming positively weighted assets and negative correlation, the variance is reduced. If it's assumed that two assets have a zero correlation, the right side of that equation should be equal to zero as well. Assuming the two assets are futures and basis prices where there is zero correlation between them and the portfolio is perfectly hedged, the variance of the portfolio is equal to the basis multiplied by the cash position:

$$\sigma_p^2 = \sigma_b^2 * w \tag{3.3}$$

3.3. Specification of Objective Functions

The ratio of futures to cash positions is usually one to one in practice. However, it's been shown that a hedge ratio of one may not be optimal is most cases. Johnson (1960) derived the risk minimizing hedge ratio:

$$HR^* = -\frac{w_s cov_{s,f}}{\sigma_f^2} \tag{3.4}$$

Where *HR** is the optimal hedge ratio, *cov* is the covariance of price changes between spot and futures, and σ is the standard deviation of the price changes. The MV hedge ratio depends largely on the covariance between spot and futures. The higher the covariance is, the more effective the hedge when short (long) cash and long (short) futures positions are held. However, minimizing risk is not the main objective of all grain buyers. When designing an optimal hedge, the return on that hedge should be taken into consideration. An approach taken by Blank (1991) does this by maximizing the quadratic utility function from the E-V framework:

$$U(R) = E(R) - \frac{\phi}{2} * \sigma_p^2 \quad (3.5)$$

Where *U(R)* is the utility of the hedger, *E(R)* is the expected return of the hedge, ϕ (phi) is the quadratic risk aversion parameter, and σ^2_p is the variance of the portfolio's return. The expected return and variance are taken from equations (3.1) and (3.2) respectively. This function adds risk preferences to the equation. Phi is also known as the risk aversion coefficient. A positive number indicates the hedger is risk averse and a negative indicates they are risk loving. If phi is positive it makes the right side of the quadratic utility function negative overall, subtracting from overall utility. The opposite is true if phi is negative.

The Allias paradox, designed by Maurice Allais (1953) is worth defining when risk preferences are brought into an objective function. The paradox shows the difference is preferences between expected value and a sure bet. For example, if a college student has 100% chance to win $500 or an 89% chance to win $500, 10% chance of winning $2500, and a 1% of winning nothing, what option do they pick? Most would take the “sure bet” of $500 even though the expected value of the other option is higher at $695. They wouldn’t want to take the small chance of winning nothing. Relating this to the problem, hedgers typically fear downside risk more than the “joy” of upside risk. Risk usually isn’t expressed when prices move in your favor.

The variance measures the deviations from the mean across the entire return distribution. This is why semi-variance methods are used. In a basic sense, the risk aversion parameter helps separate the individuals who would pick choice one versus those who would pick two.

Going further, another problem arises in the E-V framework. The E-V framework accounts for risk across the entire distribution of price changes. Hedgers generally don't care about the "up-side risk" of a portfolio. The want to know how much they can lose. The downside variance or semivariance frameworks, outlined in Chapter 2, were used because of this.

Wolf (1987) explored the use of options along with futures in a hedging problem that aimed to maximize utility using the E-V framework. He also included basis risk and quantity risk in his formulation. Quantity risk is the risk that a producer is over or under hedged because the yield on his crop at harvest time is still uncertain. This problem isn't as pertinent to a buyer and is not explored further.

A simulation was conducted using futures, a call and put option, and a risk-free asset. The probability distributions were assumed to be multivariate normal while a static volatility was used to price the options using Black's '76 model. The beta for the options was also computed.

$$B_c = \frac{cov_{(f,d_c)}}{\sigma_c} \tag{3.6}$$

Where *Bc* is the beta of the call option, and *dc* is the call option at date two. The same form can be used to find the beta of the put option. If beta is used to measure an option, it's always greater than one for a long call position and less than -1 for a long put position. The optimal call position without basis risk was then:

$$N_i^c = \frac{E_c d_i^c - rc_i}{\lambda \sigma_{(c_i)}} - q\beta_c \tag{3.7}$$

Where N_i^c is the number of call options purchased (sold), E is the expected value, r is the risk-free interest rate, λ is the risk aversion coefficient, q is the quantity of the cash position, and β is the beta of the option relative to the underlying futures. The same equation can be used to find the optimal value of put options.

Bullock and Hayes (1992) followed up Wolf (1987) with a similar approach. They modified the problem by endogenizing the variance-covariance matrix of portfolio returns. In the previous works, it was arbitrarily specified. They also used a constant absolute risk aversion (CARA) instead of a hyperbolic (HARA) utility function. The difference being that the individual has constant risk aversion across all levels of wealth with CARA. In a HARA utility function, the individual is less risk averse and the wealth function increases. They also only used a single put option in their problem citing a synthetic call option could be achieved holding short futures position and being short a put option. Generally, they found that investors (producers) should use futures to hedge their underlying position. They can then use options to speculate on future movements of the price and volatility.

3.3.1. VaR and CVaR

VaR was introduced to the world of risk as an easily interpretable number on their risky positions. This made it widely popular in the financial industry and other related fields. However, minimizing VaR brings about on a couple problems mentioned by Rockafellar and Uryasev (2000). They point out that VaR lacks subadditivity and convexity, two of the four characteristics of a coherent measure of risk described by Artzner et al. (1998). For a risk measure to display subadditivity, the risk of a two-asset portfolio can’t be greater than the risk of each individual position added together. This scenario can happen when VaR is used as the risk measure. Using variance as measure of risk, two assets can never be more risky than each held by themselves. In order for that scenario to be possible, the correlation coefficient between the assets would have to be greater than 1 or less than -1. This scenario if obviously impossible. This can be seen in equation (3.2). VaR is also a poor approximation of risk when assets exhibit any kind of "jump" process. This is the case because VaR isn't a coherent measure of risk when a function is discrete and also the jump could fall outside of the relevant confidence interval. VaR is only a coherent measure of risk when the underlying distributions exhibit normality.

Following Alexander, Coleman, Li (2006), CVaR can be used instead. The standard method for solving CVaR optimization problems is a linear programing (LP) formulation. By using Monte Carlo simulation, a piecewise linear function is used to formulate a continuously differentiable CVaR function. The problem can then be solved using LP software. One problem that CVaR can't solve is the errors inherent for large-scale problems. CVaR is a coherent measure of risk as shown by Pflug, (2000). "In addition, minimizing CVaR typically leads to a portfolio with a small VaR" (Alexander et al, 2006) Using linear programming (LP) methods, CVaR can be minimized with the use of a standard LP software. The approach is outlined in Rockafellar and Uryasec (2000) and again in Alexander et al. (2006). CVaR is defined as follows:

$$CVaR = (\frac{1}{1-C}) \int_{-1}^{VaR} p(x)dx \tag{3.8}$$

Where C is the critical value (i.e. 90%, 95%, 99%), VaR is the limit set by the hedger, and p(x)dx is the probability density function of the cost distribution.

Minimizing CVaR exhibits attractive characteristics but does not account for the mean return on the hedge. Alexander and Baptista (2002) substituted VaR for variance in equation (3.2) and maximized utility. If the underlying distributions exhibit normality than the minimum

CVaR portfolio is the same as the MV portfolio. Theoretically there is no connection between variance and VaR but the two have been used interchangeable in the literature starting with Huisman, and Koedijk (2001). The issue with VaR is the return distribution can exhibit fat left tails that may not be captured in the 95% or even the 99% confidence intervals. For a portfolio of options, that extreme left tail can be very important. Therefore, CVaR is substituted for VaR in the E-VaR framework. Maximizing the following utility function displays a utility measure based on downside risk and the extreme tail risk in a portfolio of options.

$$(R) = E(R) - \frac{\phi}{2} * CVaR \quad (3.9)$$

The E-CVaR framework is better designed to measure utility of a portfolio when derivatives are present versus E-VaR.

3.4. Understanding a Hedging Optimization Model

Assume that a stochastic model for changes in the underlying futures contract is given. Also assume that option prices can be retrieved, or another pricing mechanism is used such as Black-Scholes or Delta-Gamma approximation. It's also important that the futures contracts are indeed the underlying for the second, fourth, or nth deferred option contract. The futures curve can be flat, contango or in backwardation which can significantly skew the data if the active contract is the only contract used. The same goes if only a further deferred contract is used. It's important to model each underlying for which an option is present.

The CVaR optimization problem is a convex, nonlinear function. If the function is continuous, it is also continuously differentiable. This standard optimization framework was applied by Alexander et al. (2006) to a portfolio of derivatives. They used delta-gamma approximations to derive option prices. A pricing model was used because using price changes for options is impractical. The option value can increase by more than 1000% or decrease 100%

by the time it expires. Some kind of price approximation is necessary to achieve a practical problem that can be solved. It depends what kind of data is available to best price the options. Black-Scholes is the best pricing mechanism if all the uncertain variables are known. Delta-gamma approximations can be very close to Black-Scholes for short time periods if volatility data is not available. Delta approximation is the least accurate because it becomes a linear function. Options are not linear in nature.

One type of problem that CVaR has a tough time solving are large-scale problems. The optimization software is able to optimize for a given statistic of an objective function listed above. For example, the mean, variance, or skewness can be maximized, minimized, or set to a certain value. The mean of the E-V or E-CVaR framework can also be maximized to find an optimal allocation. When more and more alternatives are included in the optimization, there is a greater chance the optimization software finds a local maximum (minimum) instead of a global maximum (minimum). It's recommended to keep the problem at an appropriate size so not to run into multiple solutions for same problem.

The return of the portfolio can be approximated using equation (3.12). By approximating a quantity that must be hedged, the positions can be weighted using the number of contracts needed to hedge. Linear constraints that limit the hedged amount to the actual quantity can be enforced but is not essential. Most agriculture assets use 5,000 bushel contracts for both futures and options. The weights can then be computed by dividing the number of hedged bushels for that futures or option position by the entire quantity of bushels both hedged and unhedged. For example, a strangle options strategy can be used for high volatility environments where the price of the underlying is likely to move substantially over the life of the hedge. Buying calls equal to

100% of the short cash position while also buying puts in the same quantity results in a portfolio with 33.3% cash, 33.3% percent call options, and 33.3% puts.

3.5. Black-76 Option Pricing Model

After the Black-Scholes option pricing model was developed in 1973, other variations were developed in the years to come. The use of the option pricing model for commodities, developed by Fischer Black, is essential to this model (Black, 1976). In this section the model is outlined in detail. The price of a call and put option is theoretically shown below.

$$C = e^{-r\hat{t}}\left[\hat{F}\;\;(d_1) - X\;\;(d_2)\right] \tag{3.10}$$

$$P = e^{-r\hat{t}}\left[X\;\;(-d_2) - \hat{F}\;\;(-d_1)\right] \tag{3.11}$$

Where *C* and *P* are the price of the call and put respectively, *r* is the risk-free interest rate, *t* is a stochastic term that represents the time to expiration is years, *F* is a stochastic term that represents the current futures price, *X* is the current strike price, and Φ is the standard normal cumulative distribution function. Nielsen (1992) explains and interprets the d1 and d2 values. The price of call is a function of the discounted current forward price multiplied by a probability factor (d1) – the discounted exercise price multiplied by another probability factor (d2).

$$d_1 = \frac{ln\left(\frac{\hat{F}}{X}\right) + \left(\frac{\hat{\sigma}^2}{2}\right)\hat{t}}{\hat{\sigma}^2} \tag{3.12}$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{\hat{t}} \tag{3.13}$$

Where *ln* is the natural logarithm, and σ is a stochastic term that represents implied volatility. "Briefly stated, N(d2) is the risk-adjusted probability that the option will be exercised" (Nielsen, 1992). N(d1) is much more complicated. The risk-adjusted, expected value of taking delivery of grain, if the option expires in-the-money, is N(d1) multiplied by the forward price discounted at the risk-free rate. Essentially it represents the portion of the present value of taking delivery that exceeds the discounted forward price.

3.5.1. Put-Call Parity

Just before the Black-Scholes model was published, Stoll (1969) was the first to show the relationship between put and call options known as put-call parity. The discounted price of the call plus the strike should theoretically be equal to the underlying plus the price of the put option:

$$\frac{C+X}{(1+r)^t} = F + P \tag{3.14}$$

By rearranging the above equation and solving for X, the strike prices for each option can be extracted:

$$X = (F + P - C) * (1+r)^t \tag{3.15}$$

3.6. Variable Relationships

One of the main questions that has to be answered in a portfolio model is how to treat the relationships between variables. In a problem where it is not clear how variables relate to each other, it's acceptable to treat them as independent. When researching commodities or any publicly traded asset, an argument can usually be made on both sides to control relationships between variables or keep them independent. For example, in the stock market some researches define stocks as having two sources of risk; systematic and non-systematic. Systematic risk is the general risk in the market. Non-systematic risk is the idiosyncratic risk only relative to that individual company. These two categories of risk are well established terms because there is a positive correlation inherent in the stock market. Those correlations can vary based on size of the company and industry but generally stocks are positively correlated to each other over the long term.

Commodity prices are also correlated to each other. There are periods of time when supply and demand can be out of balance and prices in one commodity may jump or dive while the others don't move. However, over the long term the larger volume commodity prices are highly correlated. Below is a monthly price chart of corn, soybean, and wheat. All correlations are above .85 during this time period. In a model where prices are stochastic, these relationships have a high degree of influence on the results.

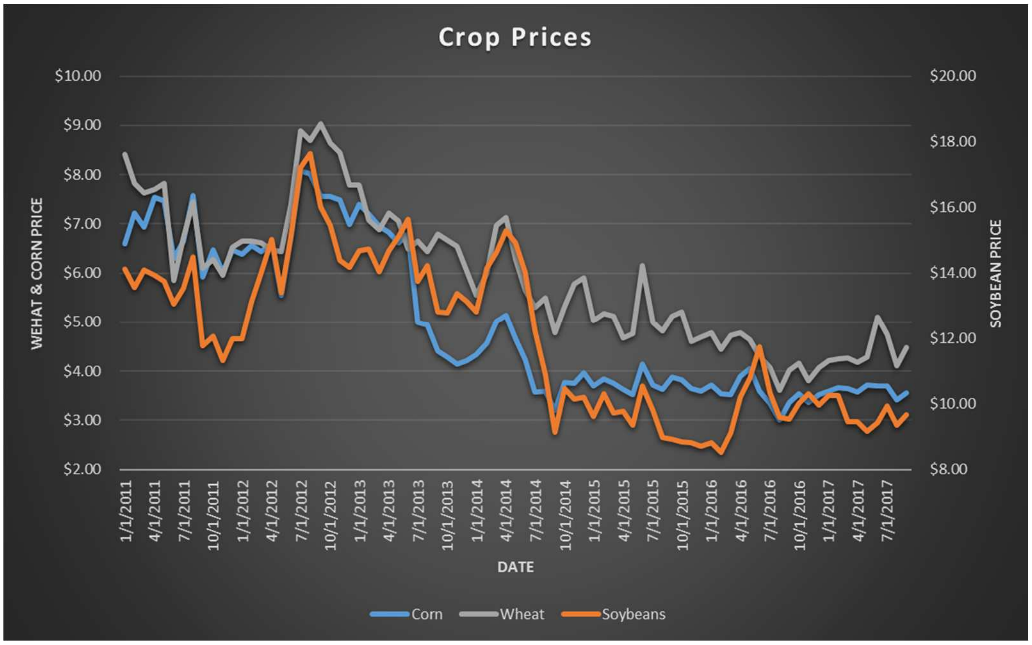

Figure 3.1: Crop Prices
Note: Corn, Soybean, and Wheat prices 2011 - 2017

Futures and a multitude of options also have positive relationships that should hold over the long term. Calls are positively correlated with a long futures position while puts are negatively correlated. Comparing the individual options to each other, call options should be positively correlated with other calls with the same underlying. The same goes for put options. The difference between the time to expiration between options would also affect their correlation. Generally, contracts close together exhibit higher correlation than further apart contracts. Similar parallels can be drawn between ITM, ATM, and OTM options. The options for the same expiration should still be positively correlated but less so than the same moneyless options where the expiration dates are close. The reason being that the value of ITM options is more reliant on the intrinsic value versus extrinsic. Intrinsic value of an options is the portion of the option's value that is made up from the degree it's ITM. Extrinsic value is portion of the option's value that's derived from the time and volatility. On the other hand, OTM options derive all of their value from the extrinsic value. Generally, a price move in the underlying causes the value of the ITM option to move more severely than an OTM option. A volatility spike causes the value of the OTM option to move more severely than the ITM one. In the previous section, the pricing model was defined analytically. Note that σ was defined as a stochastic term. Volatility also has a relationship with the option price that must hold as well. A fundamental aspect of any financial option pricing model is that volatility has a positive relationship with the value of the option.

3.6.1. Parametric vs Non-parametric

There are two general frameworks for correlating variables. Parametric is used when assumptions are made about the underlying distributions of the variables. Pearson correlation is common for the parametric category.

$$\boldsymbol{\rho} = \frac{\mathbf{n}\sum \mathbf{xy} - \sum(\mathbf{x})(\mathbf{y})}{\sqrt{[\mathbf{n}\sum \mathbf{x}^2 - \sum(\mathbf{x}^2)][\mathbf{n}\sum \mathbf{y}^2 - \sum(\mathbf{y}^2)]}} \tag{3.16}$$

Where ρ is the Pearson correlation coefficient, n is the number of values in each data set, and x and y are the correlated variables. The most prominent assumption is both variables follow a normal distribution. Linearity and homoscedasticity are two more assumptions a Pearson correlation make. In the financial world, it's rare to find two assets where a normal distribution would best characterize the return distribution of the asset. Even rarer is to find two assets where that relationship is stationary over an extended period of time.

Non-parametric correlation is more relaxed and does not make assumptions about the distribution of the variables. Spearman rank correlation is common for the non-parametric category. Spearman rank actually ranks the values in the data set from 1 to n. Then the difference in the ranks are inserted into the formula below:

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)} \tag{3.17}$$

One more parametric technique that measures correlation between two variables is through the use of a copula. Recall from chapter 2 that a copula is a multivariate joint distribution. In some instances, copulas can better define correlations because of their flexible nature. For example, it's well known that stocks became highly correlated in the financial crisis. The Clayton copula can better define this relationship than a Pearson or Spearman correlation. The dispersion becomes larger as values increase. This type of relationship could not be defined without a copula. There are other forms of copulas that can better define other relationships as well. The Clayton copula is shown below:

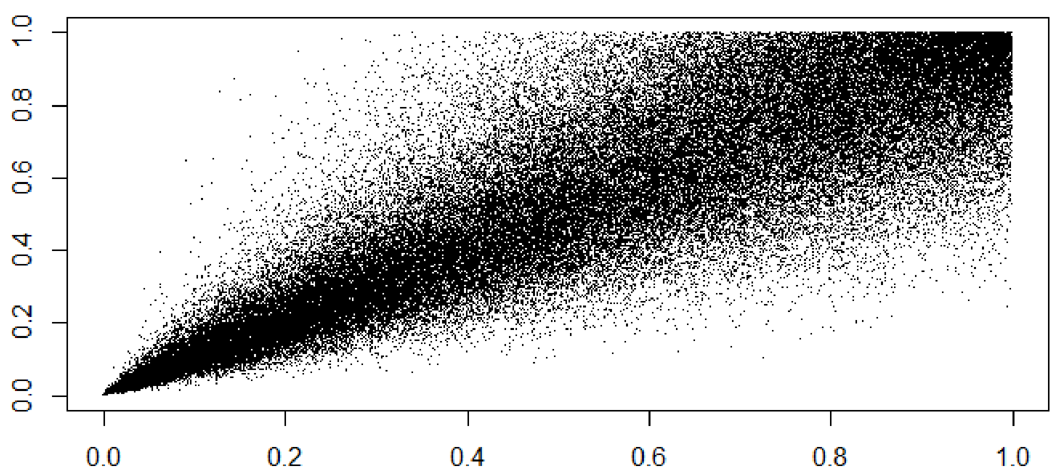

Figure 3.2: Clayton Copula Joint Distribution

3.7. The Greeks

Managing a portfolio of options can become complex as more positions are added. Instead of solely managing a strategy, most portfolio managers who deal with options also manage the "greeks" of the portfolio. Delta, gamma, vega, and theta are four of the more important greeks. They can describe how the portfolio is most likely to behave based on a variety of changes in the underlying variables. These variables are 1st, 2nd, or in some cases, 3rd order derivatives of variables that make up the Black pricing model.

Delta is the first order derivative of the value of the options relative to the price of underlying asset. It measures the amount of value change in the portfolio for a dollar change in the underlying. For example, if the portfolio Delta is .25 and the underlying goes up by one dollar, the value of the portfolio will increase .25%. Delta of an individual option can also be viewed as the probability it will expire in the money assuming a normal distribution for the underlying. Delta is positive for calls and negative for puts. Hence adding up the total delta value for all assets in a portfolio shows the delta of the portfolio:

$$\Delta_c = e^{-rt} \quad (d_1) \tag{3.18}$$

$$\Delta_{put} = e^{-rt} \quad (d_1) \tag{3.19}$$

$$\Delta_{port} = w_i * \Delta_i \tag{3.20}$$

Where *c*, *put*, and *port* represent the delta of a call, put, and a portfolio respectably. The delta of an underlying long (short) position is one (negative one) discounted back to present value. (Usually .97-.99) A fully hedged short cash position theoretically has a delta of zero. For a portfolio with basis included, the basis does not have a delta.

Gamma is the second order derivative of the value of the option relative to the price of the underlying asset. It represents the rate of change in delta with respect to the underlying. Gamma has its highest value for ATM options and converges to zero as the option becomes more OTM or ITM. A positive gamma indicates the asset or portfolio will increase in value if the underlying prices move substantially in either direction. A negative gamma indicates the asset or portfolio will benefit most from no movement in prices. Also, it generally decreases as time goes by. The gamma of a portfolio is also weighted proportionally to the assets held, just like delta:

$$\Gamma = e^{-rt}\frac{\phi(d_1)}{F\sigma\sqrt{t}} \tag{3.21}$$

Where Γ is gamma. Gamma is the same for a call and put. The only difference is d1, which is different for a call and put.

Vega is the first order derivative of the value of the option relative to the volatility. Vega is the amount of the option value that changes with a 1% change in volatility. It is also positive for a long position and negative for a short position. This is because any volatility is positive for the holder of the option as shown. Like gamma, vega is always greatest for ATM options. It also can be calculated the same way for calls and puts:

$$\nu = Fe^{-rt}\phi(d_1)\sqrt{t} \tag{3.22}$$

Where ν, the Greek letter nu, represents vega. It's also highly dependent on the time value. For a scenario where volatility spikes and there is an ample amount of time value left, the value of that option will also spike. However, for an option with very little time left before expiration, the same volatility spike may not affect the value of the option much depending on how close the price of the underlying is to the strike price.

The last greek important for a portfolio of options is theta. Theta is concerned with the time value of the option as an option with more time left until expiration is worth more, all else held equal. It's the first derivative of the value of the option relative to time, or days to expiration. It can be described as the amount of value an option loses in one day. Theta is almost always negative for a long position and positive for short position:

$$\Theta_c = -\frac{Fe^{-rt}\phi(d_1)\sigma}{2\sqrt{t}} + rFe^{-rt}\quad(d_1) - rXe^{-rt}\quad(d_2) \tag{3.23}$$

$$\Theta_p = -\frac{Fe^{-rt}\phi(d_1)\sigma}{2\sqrt{t}} - rFe^{-rt}\quad(-d_1) + rXe^{-rt}\quad(-d_2) \tag{3.24}$$

The formula is slightly different for a put and call. Theta for ATM money options have the highest value followed by ITM options while OTM options have the least theta value.

That rounds out the four main greeks. There are many more that are important but these four are adequate to capture the characteristics of a portfolio that includes options. The greeks for a portfolio can be computed the same way as equation (3.14) by substituting gamma, vega, or theta for delta.

3.8. Summary

The theoretical model is fairly straightforward. The returns from each position is calculated individually and then summed together to obtain the profit (cost) of the hedge. The risk is jointly determined with equation (3.2). VaR and CVaR are calculated after assumptions are made about the underlying distributions of each security and the inherent relationships are defined. The black model can be used as an alternative pricing mechanism. It can also be manipulated to solve for other variables if price is already known. The assumed relationships between prices have a significant effect on the model's results. There are a couple different options given for correlation. Lastly, a portfolio of derivatives must be managed properly. One of the ways to keep risk in check is to manage the greeks. Delta and gamma of a portfolio of derivatives play a crucial role in how the portfolio is managed.

CHAPTER 4. EMPIRICAL MODEL

4.1. Introduction

The futures market for commodities was created so that producers, elevators, and end users could reduce their price risk. Grain is constantly being bought and sold all over the world at all times of day. Prices can be very volatile leading up to the next USDA report or around harvest season. Buyers can be exposed to a great amount of price risk if they don't manage it well. Holding a long or short futures contract will cancel out any change in price from the date the contract is obtained through its expiration. The only other risk is basis risk. Basis risk can be mitigated as well but it has to be done with the physical buyer or seller outside of any futures market. Options add another alternative to mitigate price risk. Options provide the ability to create asymmetric payoff functions curtailed to the holder's desire. Last chapter I explained the nuances of those instruments and outlined their use for hedging. In this chapter, an empirical model will be presented using futures, basis, and options contracts for an optimal portfolio according to Mean-Variance, Mean-CVaR, and the maximization of CVaR. Then an overview of the Black-76 model will be shown and applied to the empirical model. A process for setting up the model will follow. Lastly, data sources and distributions for stochastic variables are illustrated.

4.2. The Problem

This problem is meant to replicate the situation of a large buyer of American produced soybeans. However, the problem only covers the hedging part and does not account for transportation from PNW to the final destination. It also doesn't cover any logistical challenges inherent in the transportation process. For example, Chinese companies buying beans typically use them to feed their livestock. Soybeans are also crushed for their oil and sometimes used in biodiesel. This model measures the efficiency of different hedging strategies these buyers can use to reduce risk and lower their buying cost. The model assumes 100,000 m/t (metric tonnes) of soybeans are to be bought.

A typical portfolio would consist of a short cash position and long futures and basis positions. Futures and cash prices are generally highly correlated. This means a long futures position can lock in a stable, future buying price. The theoretical transaction in the cash market takes place at the Port in Portland, Oregon (PNW). The basis at PNW is always positive, meaning soybeans are slightly more expensive at Portland than the futures market would indicate. The hedge is placed 28 weeks in advance of the purchase. The goal is to reduce risk while generating an appropriate return over the time period.

4.2.1. Alternative Solutions

The base case in this problem is assuming the buyer doesn't hedge their purchase at all. Meaning they are 100% short in the cash market and their buying cost is exposed if prices move higher. On the other end of the spectrum, they would greatly benefit from a down side move because their purchase would become cheaper from the port. A payoff function is show below:

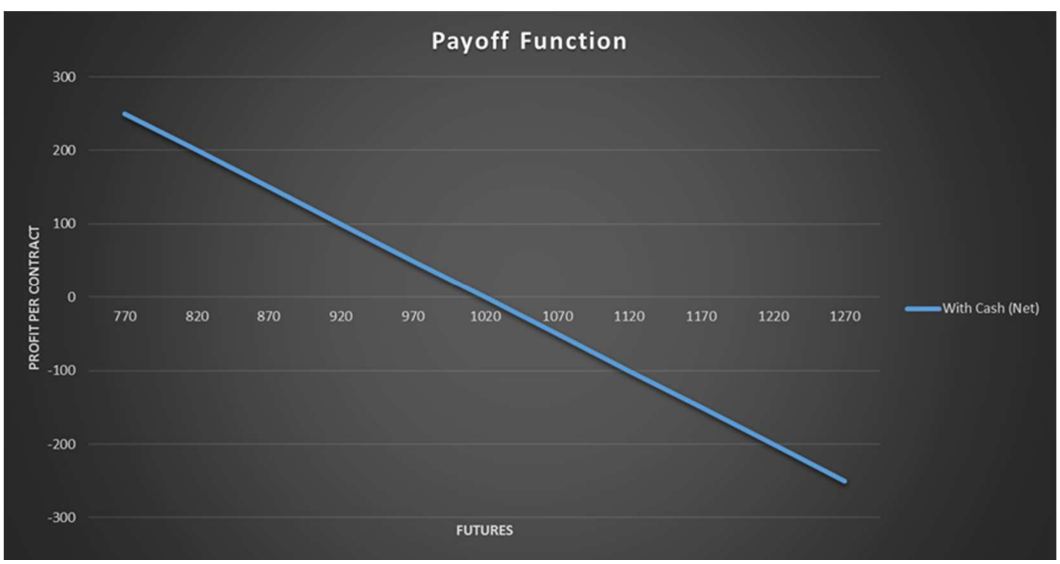

Figure 4.1: No Hedge

Probably the most popular strategy deployed when buying commodities from a port is to hedge the purchase one for one using a futures exchange. This means the buyer will hedge the approximate amount of beans they are buying with a long position in the futures market of equal volume. It's the most popular hedging strategy because it's easy to understand and removes all price risk associated with the futures price. Below is a visual representation of what the hedge would accomplish:

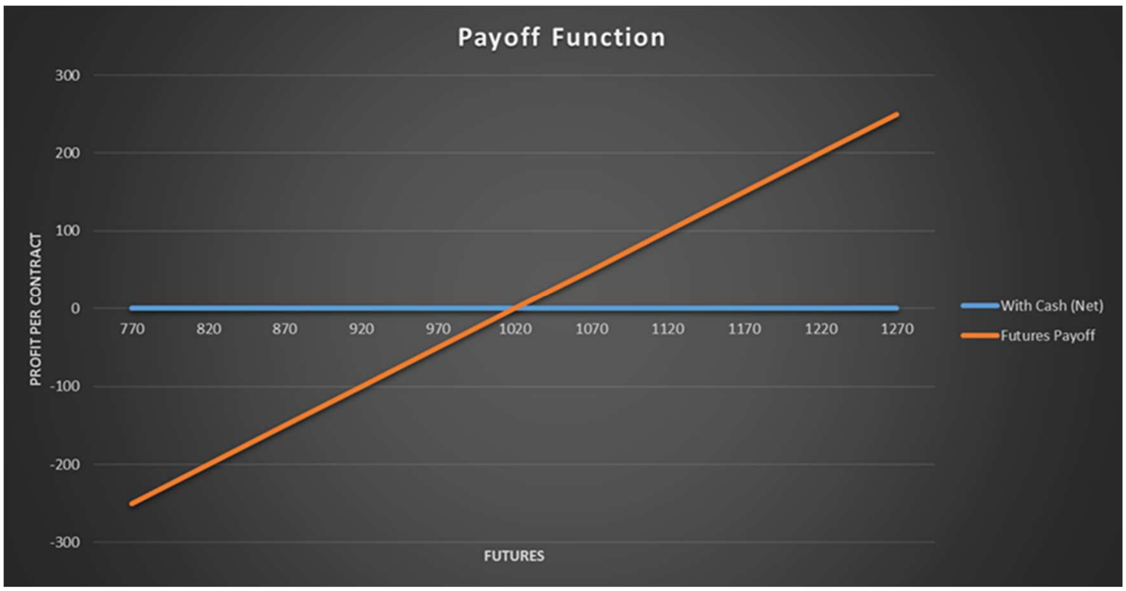

Figure 4.2: Futures Naïve Hedge

The hedger would have zero price risk associated with the futures price. However, a hedger doesn't have to hedge all of their position. They could under hedge, looking for a future price decline. They could also over hedge if they have strong beliefs prices will go up. Either way, a buyer can mitigate all price risk associated with the futures price. Basis risk would still be present.

Basis contracts are obtained directly from the seller. Typically, producers sell their grain to their local elevators. Elevators, who are largely owned by large grain companies, then ship that grain to PNW or GULF port for export. The grain arrives at the company's port location where the buyer would have a basis contract in place with the seller. Unlike a futures hedge, a basis contract is a deal between only two parties with no middle man. It would be unwise for a buyer to “over hedge” their position because they would be responsible for buying the entire volume stated in the contract. They could of course under hedge, looking for a drop in the PNW basis. The consequence of under hedging basis is there may not be extra beans available to buy if there’s no contract for purchase in place (basis contract). This would leave the buyer with less soybeans then they need (CME group, 2015)

Hedging with options adds another level of complexity to the problem. Options are attractive because they give the hedger a floor or ceiling where their average buying cost can’t surpass. The strategy also leaves room for the buyer to take advantage of a potential downward price move. Some of the more popular option hedging strategies for buyers would include a married call, bull call spread, and a long collar. Many more are explored including: straddles, strangles, butterfly spreads, and calendar spreads. I’ll go over each strategy as all are deployed in the study.

4.2.2. More Option Strategies

Building off of the brief overview of option spreads in chapter 2, a broader range of strategies are defined in this section. First a married call is a basic call option that behaves like a futures contract if prices move lower but set a cap on the average price the buyer will pay if prices move higher. The payoff function displayed below:

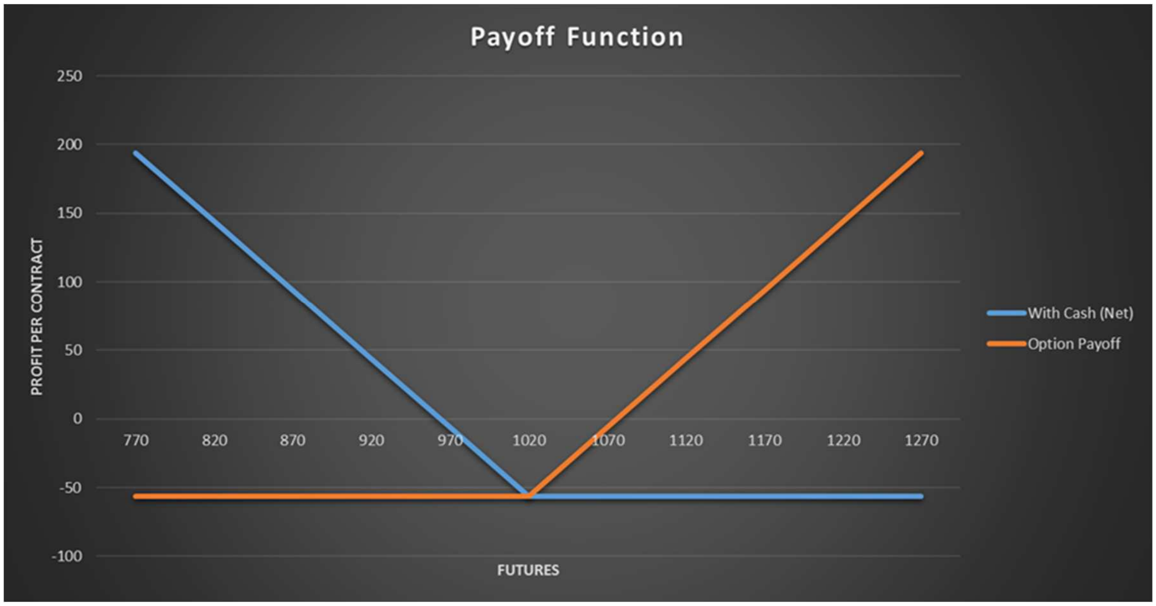

Figure 4.3: ATMC4 Hedge

Notice the similarities between the futures graph and this one. With an options contract, the hedger will pay a little more to hedge. The call option shown is an ATM option but OTM options are also popular. They put in cost floor for themselves when prices rise but still benefit if the market goes down. It costs a little more for this “floor” as shown by the graph.

Collar hedging strategies are another efficient strategy to employ. In a collar strategy, the hedger still buys an OTM call. This time, they also sell an OTM put simultaneously. This gives the hedger some upside and downside potential but sets floors and ceiling on cost as shown below:

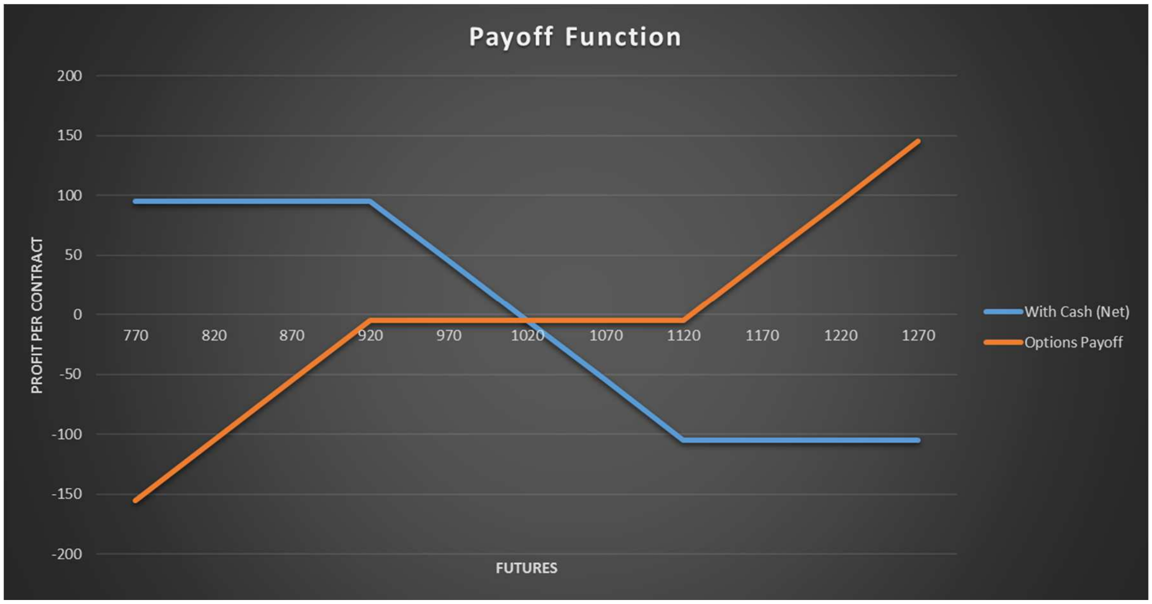

Figure 4.4: Collar Hedge

The strike price for the call is set at 1120 and the put at 920. This would be ideal for a buyer who has a downward bias for prices but doesn't want to take too much risk.

Straddles and Strangles are basically the same thing. A straddle consists of buying two ATM options. Call and puts are bought at the same strike price in the same quantity. This is a "long volatility" strategy where a rise in volatility benefits the hedger. The payoff function looks as follows:

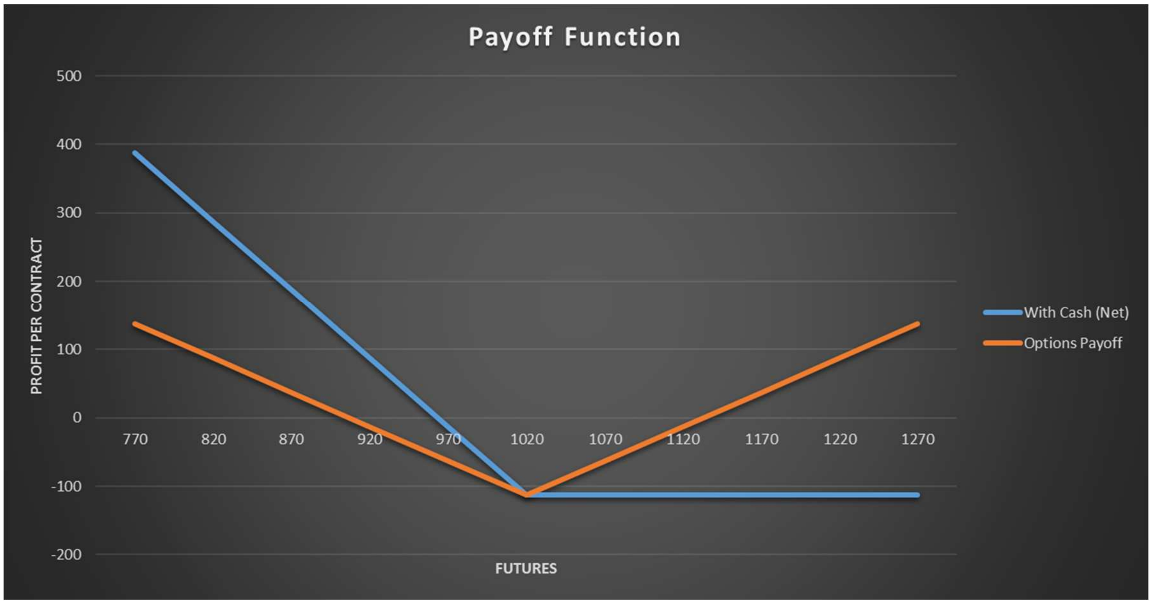

Figure 4.5: Long Straddle

It's almost identical to that of a married call. The only difference is the strategy costs more and the slope of the net payoff function to the left of the inflection point is double that of a regular call with a short cash position. The reason being buying a put on top of the call doubles the payoff if prices move in that direction. The only difference between a straddle and a strangle is the options purchased for the latter are OTM instead of ATM for a straddle.

The graph below portrays a "net" straddle. This is accomplished by buying 2 ATM calls instead of both a call and put like the graph above. These are all expensive strategies but can be advantageous if the environment turns to a highly volatile one in the future.

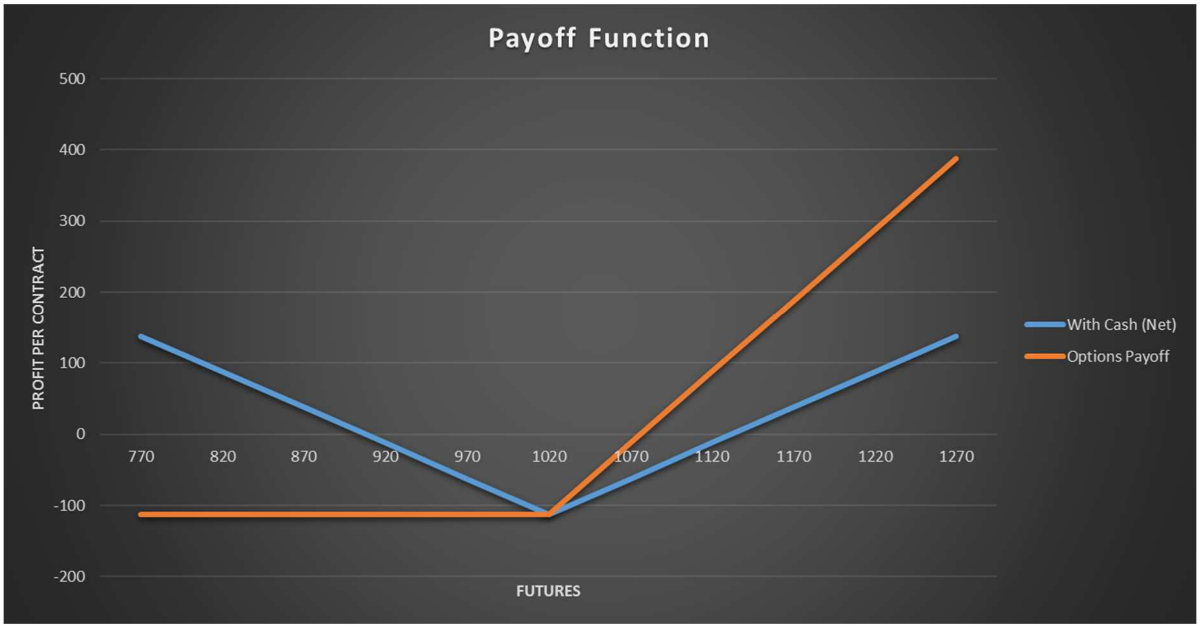

Figure 4.6: Net Long Straddle

The last strategy, or family of strategies, is the butterfly spread. Technically the strategy employed is called an iron butterfly because puts and calls are both used. In a standard long butterfly spread using calls, an ITM option is bought, two ATM options are sold, and an OTM option is also bought. The butterfly spread is similar to that of a straddle in that it's a volatility play. The only difference between a butterfly spread and an iron butterfly is the iron butterfly uses puts and calls where as a standard butterfly spread uses puts or calls. They both can represent the same payoff function. The graph on the top represents a typical short iron butterfly. It's not so much of a hedge as it is a speculative play. However, the graph on the bottom is a long strangle. The positions are very similar, yet the payoff functions look completely different.

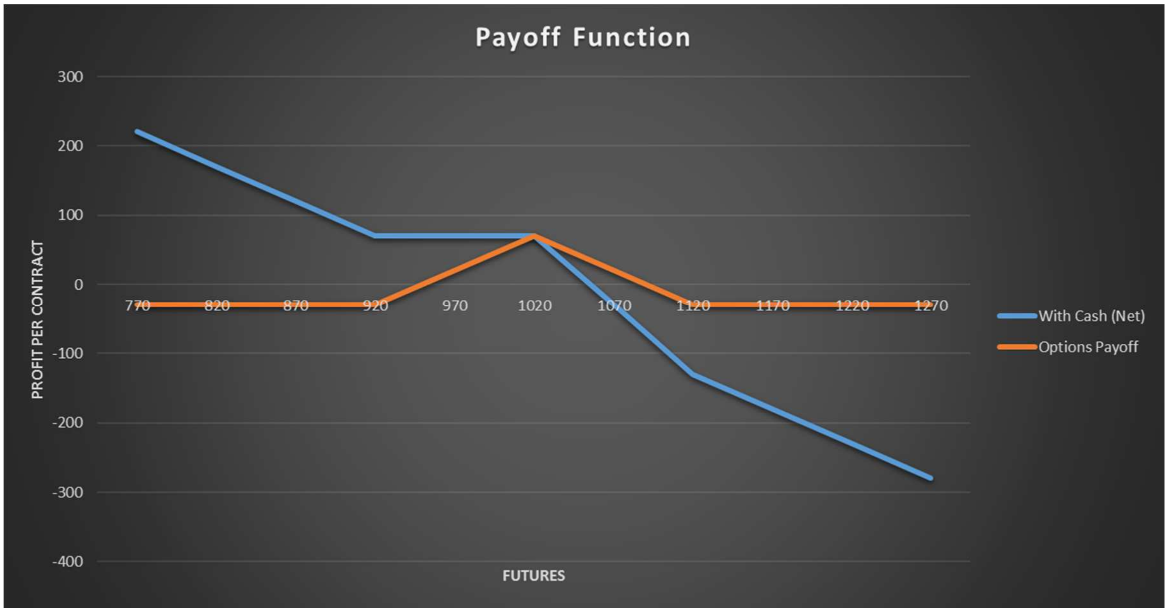

Figure 4.7: Long Iron Butterfly

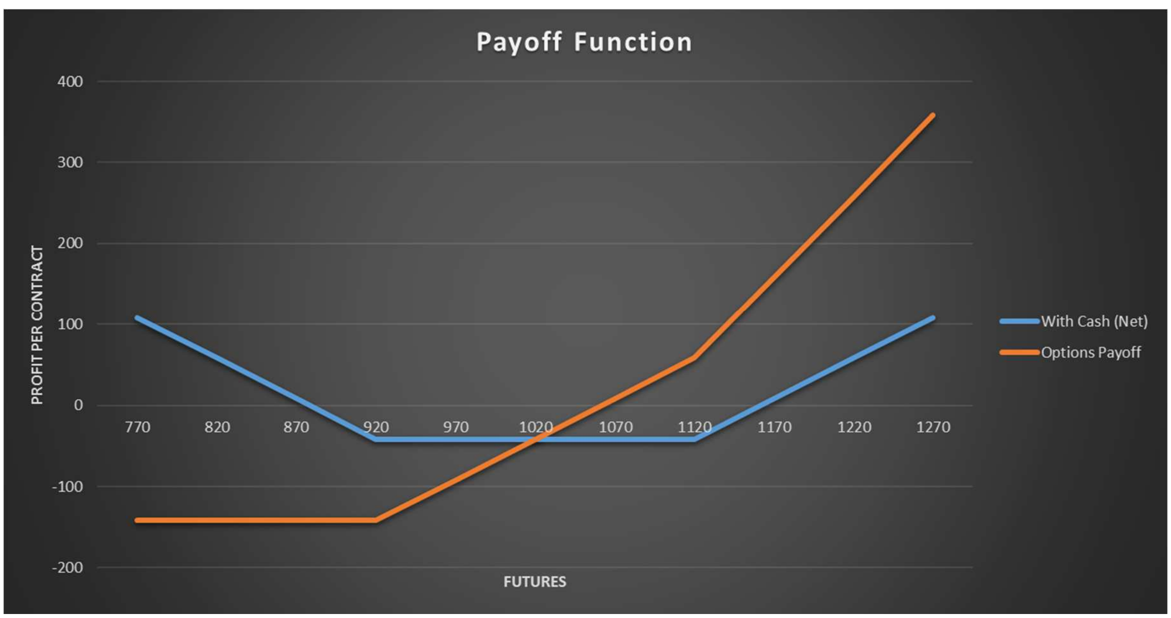

Figure 4.8: Net Long Strangle

The first function is accomplished by selling an ATM put and call while buying and OTM put and call. The OTM options act as protection for the ATM options that are sold. The second graph is similar except that instead of selling the ATM call, it's bought instead. The net position is actually identical to that of a long strangle.

To obtain a net payoff that looks closer to a long iron butterfly, we make a slight adjustment to the ATM positions. By selling 1 more ATM put and not taking a position in ATM call, the desired payoff function is achieved. Below is the payoff function achieved by selling two ATM puts and buying both an OTM call and put. Contrary to the goal of the "strangle like" position above, this strategy decreases overall cost when volatility is kept to a minimum.

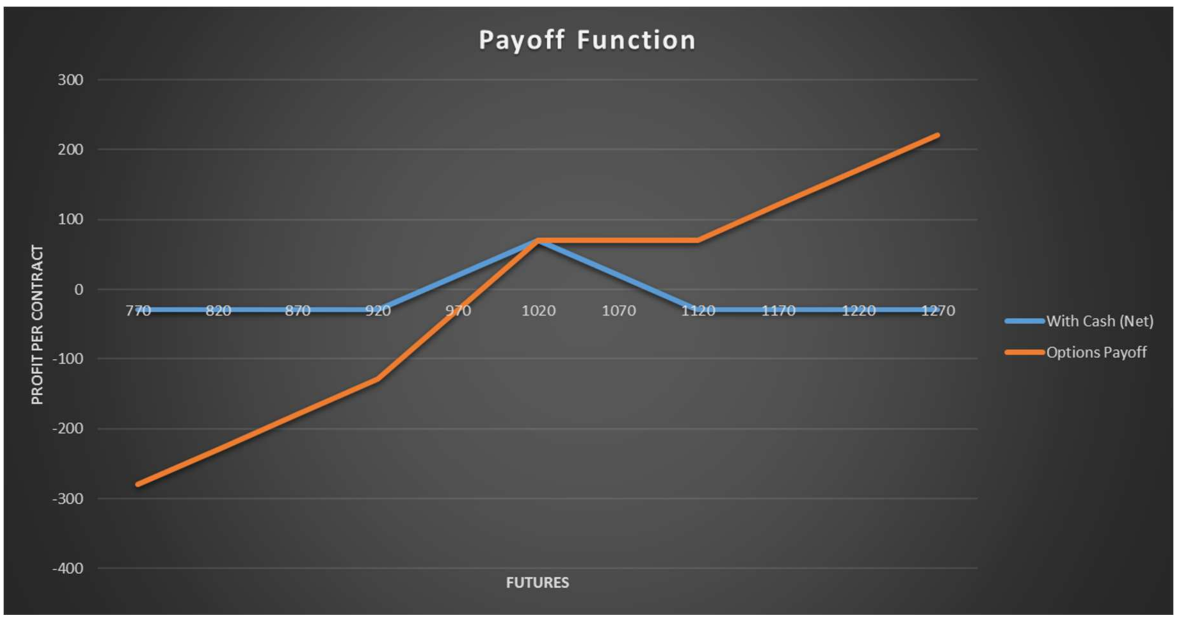

Figure 4.9: Net Long Iron Butterfly

Hedging strategies using options can be used in the same way they are used for speculators, which don't hold an underlying position. Nearly any option payoff function can be slightly retooled to achieve the desired qualities for a hedge. The strategies described above are used in the model to rank them against each other using the different objectives functions described in Chapter 3. This is ultimately how the decision is made on what the optimal hedge should entail.

To review, a table below shows the strategies above. For each hedge, the table is designed to show the net strategy from the hedger's perspective. For example, a straddle usually contains long positions in an ATM put and call. Since the hedger already has a short cash position equal to 100%, doubling the call position and not taking a put position nets out to a straddle.

Table 4.1: Option Hedging Table

Hedge	PNW	Futures	ATMC4	OTMC4	OTMP4	ATMP4	Delta	Gamma	Vega	Theta
Unhedged	(100%)						-0.98	0.00%	-	-
Futures	(100%)	100%					0.00	0.00%	-	-
Married Call	(100%)		100%				-0.25	0.13%	112,604	(18,176)
Collar ATM	(100%)		100%			(100%)	0.00	-0.02%	197	(2,932)
Collar OTM	(100%)			100%	(100%)		-0.17	-0.01%	3,690	(2,142)
Straddle	(100%)		200%			0%	-0.01	0.34%	450,417	(72,705)
Strangle	(100%)		100%	100%	100%	(100%)	0.01	0.08%	178,320	(30,639)
Butterfly Spread	(100%)		0%	100%	100%	(200%)	0.01	-0.16%	(271,505)	33,269

4.3. Model Overview

The goal of the model is to identify the most efficient hedge for a soybean buyer. Allocating capital to futures, options and basis contracts can be beneficial versus just using the futures market and locking in basis. This is done using eighteen generic options. Calls and puts are included for the second, fourth, and sixth deferred months. When deferred is used to describe a contract, it means it's not the "active" contract. For example, the next contract to expire after the active contract is the second deferred. If it's the beginning of September, the September contract would still be the active contract until it expires on the 3rd Friday of the month. The model uses the 15th day of the month instead for simplicity reasons. The November contract is called the second deferred contract until the September contract expires. The same process can be used to identify the months associated with the fourth and sixth deferred contracts and any time during the calendar year.

There are also in-the-money (ITM), at-the-money (ATM), and out-of-the-money (OTM) contracts associated with each calendar month. The ITM and OTM options are always a dollar away from the at-the-money strike price. The degree for which the options are ITM or OTM is arbitrary. One could choose a dollar, two dollars, 40 cents etc. This model uses a dollar which can be anywhere from 16% to 7% OTM or ITM depending on the underlying futures price. For example, if the futures price for the fourth deferred ATM call option is $10, then the ITM, fourth deferred call option (ITMC4) strike price is $9. The OTM, fourth deferred call (OTMC4) strike price is $11. The same process is used for the other calendar options as well as puts. To illustrate the same with put options, the prices will flip. If the ATM, second deferred put option (ATMP2) strike price is $10, the strike prices for the ITMP2 and OTMP2 options are $11 and $9 respectively.

The only futures contract that is included in the optimization part of the model is the fourth deferred contract. The fourth deferred contract is used because it expires between 24 and 32 weeks from the generic hedge date. Since the model uses a 28-week hedge, it was the most appropriate contract. Contracts for August are not included in the model for simplicity. Without August, there are two months between every other soybean contract. The contracts can then be modeled the same as the time to expiration is uniform across all contracts. Without that assumption, the August contract would pose a big problem for this model. For example, in the winter and planting months, the 2nd deferred contract is always between 43 and 84 days from expiring. This is because there is 21 trading days in each month. However, with a July, August, and September contract, the days to expiration for a 2nd deferred contract could range from 22 to 84. Taking out August reduces some time variability in the model.

Storage and transportation costs are not included in this model. If those two variables were included, it may make sense to include other futures contracts and explore hedging with spread contracts mixed in. The price of soybeans relies on supply and demand in the market. There are a plethora of exogenous variables that can affect supply and demand. Those factors are out of the scope of this study.

The model is made of a short cash soybean position and a combination of a fourth deferred futures contract, a basis contract, 9 calls, and 9 puts. The optimal, hedged portfolio consists of a mix of positive and negative exposure between these securities.

4.3.1. Modeling Futures and Basis Prices

One of the main inputs into the model is futures prices. Futures prices rarely follow a normal distribution for any given time period. There can be years when soybeans hover around $15 per bushel like they did from the summer of 2012 to the summer of 2014. They can also stay closer to $5 like the 3-year period from 1999 – 2002. The extreme volatility makes it very difficult to model prices. However, a distribution was fit on the active contract using the AIC score to determine the best fitting distribution. The nominal price in the first stage can be looked at as arbitrary. What matters to the model is the return of each asset. Therefore, it’s the price change that controls the return.

In the second stage, the model captures the price change over 28 weeks from the data. A distribution is then fit on that data, and uses it as a stochastic variable. 2nd, 4th, and 6th deferred futures prices are then modeled as spreads from the adjacent contract. For example, the 2nd deferred contract is modeled from the active and the spread between the active and 2nd deferred. The 4th deferred is modeled from the 2nd, which was just modeled, and the spread between the 2nd and 4th deferred contracts.

Using price distributions or price changes was also explored. The price distributions for each contract weren't able to mimic a practical market even with extremely positive correlation coefficients. Modeling price changes turned out just as poor as the price distributions or arguably worse. The prices the model output were sometimes two dollars away for consecutive contract months which is not practical. Spreads actually behave very well and mimic what the actual forward curve would look like. However, the spreads did tend to characterize a bimodal distribution. This will be talked about further in Chapter 6.

Basis was modeled exactly the same as the active futures contract. A best bit distribution was used for basis in the first stage of the model. Then a price change distribution was used in the 2nd stage. The PNW cash price is modeled as the sum of the active futures contract and basis in both stages.

4.3.2. Modeling Options

The vast majority of portfolio optimization literature has taken data for price changes from periods n to n+1. One can measure the risk and return of the portfolio over that time frame. The addition of options into a portfolio makes that approach inappropriate to apply. Price changes for soybeans can only change so drastically over 7-month period. A change from $8.55 to $11.80 can occur over a short period of time as it did in 2016 from February to July. That change entails a 38% increase in price over a period of 5 months. With options, an OTM call option purchased in February may cost $.10. In July the same option may be worth around $2.50. The holder of that option would see a return of 2500%. Using data with price changes that include 2500% returns all the way down to -100% creates problems for this framework.

In the first stage of the model, the option prices were modeled from the empirical data for all 18 options. There are inherent relationships that must hold for the majority of these option prices. The ITM options at the same strike price always have to be worth more than the ATM and OTM options. The ATMC4 premium should be less than the ATMC6 premium most of the time. There are times around harvest when the ATMC6 could be cheaper with new crop coming into the market, but it shouldn't happen often. To hold these relationships within reasonable bounds, the price changes were abandoned and, an option pricing model was used in the 2nd stage instead.

The Black-76 model was used to price the options during the second lag of the transaction. This is common in the literature so far and very easy to set up. Where this model differentiates from the literature is actual option price data was gathered and fit to a distribution in the first stage instead of using a pricing model.

4.3.3. Objective Functions

Four objective functions were used in the hedging model. The first is a standard quadratic utility function based on mean-variance shown in chapter 3.

$$U(R) = \sum_{i}^{n} w_i * r_i - \frac{\phi}{2} * \{\sum_{i}^{n} w_i^2 * \sigma_i^2 + 2 * \sum_{i}^{n} \sum_{j}^{n} \sigma_i * \sigma_j * \rho_{(i,j)}\} \quad (4.1)$$

Depending on the risk aversion parameter, a portion of the variance is subtracted from the mean to arrive at an expected utility of the hedger. Bullock and Hayes (1992) define a low risk aversion parameter as .0001 and high-risk aversion at .01. Since options are heavily used with this study, the risk aversion parameters used must be different. Option payoffs have much more variance than futures. If the same parameters in Bullock and Hayes (1992) are used, the variance will still dominate the utility function. After doing some testing, .00001 and .0000001 are used for low and high-risk aversion respectively. In the base case, phi is set at 1 but sensitivity analysis will be done with .00001 and .0000001 as well.

The second objective is to minimize conditional value-at-risk. This is the average loss of the losses that lie in the left five percent of the cost distribution. Most of the literature in Chapter 2 starts out my minimizing variance and then moves to include the mean and semivariance measures. CVaR does a better job of expressing the downside risk of the hedge versus using variance. Therefore, a minimum variance objective is not included in this study.

$$Min[(\frac{1}{1-C})\int_{-1}^{VaR} p(x)dx] \tag{4.2}$$

The next objective function is Mean-CVaR. Similar to mean-variance, Mean-CVaR subtracts a portion of the conditional value-at-risk from the mean cost. The risk aversion parameter can also be changed in this function. Although, because CVaR is a much higher number (less negative), the parameters that entail high and low risk aversion will be higher than that of Bullock and Hayes (1992). This will be discussed more in chapter 5.

$$ECVaR = \sum_{i}^{n} w_i * r_i - \frac{\phi}{2} * (\frac{1}{1-C})\int_{-1}^{VaR} p(x)dx \tag{4.3}$$

The last objective function used is a Mean-CVaR Copula. It's the same as the previous function but the variables are related by a t copula instead of a spearman correlation. This function should exhibit more risk than Mean-CVaR because of a potential fat left tail on the cost distribution.

4.4. Data

Data was gathered from two sources over a period of 38 months from November of 2013 to December of 2016. CME (Chicago Mercantile Exchange) soybean futures and option pricing data was extracted from ProphetX, a trading platform owned by DTN (Data Transfer Network).

The PNW cash prices for soybeans were gathered from Milling and Baking News. PNW basis is obviously the difference between the cash and futures prices.

That extraction of the futures and cash prices are straight forward. Milling and Baking News publishes PNW bids weekly and has been doing so for over a decade. The other futures prices were extracted into excel using the ProphetX add-in.

4.4.1. Options Data

Extracting option data was not as straight forward. Obviously, it's not practical to extract the premium over a long period of time for thousands of options traded over a 38-month period. Instead a systematic approach was taken where ITM, ATM, and OTM premiums were extracted. At every data point the strike price adjusted because the underlying price most likely moved as well. What was left was 18 series of option prices that had small variability in strike prices and time, but a substantial amount of variability in volatility.

The model constructed for gathering the option pricing data started with a table that displayed each month and its respective code (letter) alongside it. The non-soybean contract months were then eliminated along with August. In the non-contract months, the letter from the next month with a soybean contract was back filled. This series was used to identify the active contract in the first 15 days of each month. Another series was generated that displayed the active month code on a weekly basis that included the 15th day cutoff using the table already created.

The next step was to display the first part of the "option ticker" for the active contract on a weekly basis. The first part of the ticker included the symbol of the underlying which was "S", the contract month code (i.e. F for January), and finally the last two digits of the year the contract expired. The first part of an option ticker that expired in January of 2017 would then be "SF17."

After the first part of the option ticker is displayed for every listed date where a data point should be present, strike prices must be added. It's important to match up each calendar option with its respective futures contract. It should be noted that options that expire in different months than its underlying contract are available. However, there's not as much liquidity as options that expire in the same month as its underlying.

If the active contract is exclusively used as the underlying price series, the option prices will be more variable because the further deferred contracts are likely to be priced higher, or in some cases, lower than the active. It depends if the market is flat, in contango, or backwardation. In an extreme market with the latter cases, using the active contract for pricing options expiring in the sixth deferred month will extremely skew the data.

For Soybean options, there is typically a liquid option every 20 cents around the liquid ATM option. The liquid ATM option would be the closest, even, 20 cent option between the two whole dollar prices. For example, if the futures price is $9.73 per bushel, the liquid ATM price would be $9.80. Rounding the respective futures price to the nearest 20 cents displays the strike prices used for the option series.

The last step is to bring it all together and define the option ticker as a put or call. A put or call can be defined using "P" or "C" respectively. It can be concatenated using the first part of the option ticker followed by the put or call symbol, and finally the strike price. For example, for an ATM call option that expires in September of 2017, where the September futures price is at $9.42 and its currently June of 2017, the entire option ticker would read,"SU17C9400".

Once option tickers are made, they should change as the date and futures prices change following the steps outlined above. From there, extracting option prices is as easy as extracting futures prices for the respective commodity.

4.5. Treatment of Volatility

As was already mentioned above, it's impractical to use standard price change methodology to simulate returns for options. As such, the Black-76 model was used to price the options in the second stage of the problem. One concern was how to treat volatility. The best situation would be to extract volatility but unfortunately, that data was not available. It is possible to go back and measure realized volatility over a certain period of time. However, realized volatility tends to lag behind implied volatility and the two don't necessarily have to move together. Realized volatility is measured in high-sight while IV is forward looking.

Since there was no historical data for IV, @Risk and excel's solver tool were used to capture the IV distribution. First a cell that displayed the difference between the option prices at the first and second stages was included. Using solver, the implied volatility cell was set to the changing cell. The cell where the difference in prices was stored was set to 0 as the objective. Solver was not run. Instead @Risk was set to run solver after every iteration of the simulation. By making the implied volatility as an output cell, which was the changing cell, the distribution of implied volatilities was captured. The stochastic volatility replaced the static for all 18 options. This was appropriate since actual option prices were compared to derived prices. Implied volatility is measured with the Black-76 model for commodities. Therefore, the volatility extracted should exactly match the implied volatility distribution if actual data was available. The only problem was that the correlation between the volatilities and prices were unknown.

4.6. Correlations and Copulas

How relationships between variables are treated in any portfolio optimization model can drastically affect the optimization results. Whether two data series are positively correlated, negatively or totally independent are assumptions that must be made while modeling risk. Many times, the correlations between financial securities are not stable over time. Meaning they will change based on the time period and periodicity of the data. Fortunately, option prices are derived from the underlying futures contract. Futures prices are also highly correlated with each other. That should keep most of the relationships in this model fairly stable over time. The relationships that option prices have with other options in this model are not so straightforward. Generally, the call options should be positively correlated with futures and other calls. The opposite is true for puts. However, it's difficult to determine the degree of positive or negative correlation that should hold between an ITMC4 and OTMC6, for example. Those relationships may not be stable over time, especially in times of market "turmoil."

There are two types of correlation used in this model. The first one being a spearman rank correlation and second is a t copula. The spearman rank was defined in chapter 3. A t copula is in the elliptical copula family and is characterized by a bell shaped, symmetric distribution with slightly fatter tails than a standard normal distribution.

@Risk was used to derive the spearman rank correlation matrix for the four futures contracts, basis, and 18 options. The matrix was then attached to the price distributions. Along with prices, volatility is also a stochastic term in the model.

One problem involved adding those volatilities into the spearman correlation matrix. Since there was no real data for historical implied volatility, the correlation could not be derived in the same way as prices. @Risk was used to gather a "string" of spearman correlation coefficients. The only coefficients that were captured were between the option price and its respective implied volatility. A scatter plot function was used after a simulation had already been run to identify the correlation coefficients. That was done for all 18 options and then the volatilities were added to the correlation matrix. The correlation coefficients were then added to the matrix. There were only 18 added which left 468 correlations coefficients yet to be determined. The matrix, as it was, was not a self-consistent or a positive semi-definite matrix. @Risk does have a way to make the matrix positive semi-definite. First it finds the smallest eigenvalue in the matrix, E0. It then shifts the matrix so that E0 is now equal to 0. "It does this by adding the product of –E0 and the identity matrix (I) to the correlation matrix (C). C' = C – E0I. It then divides the new matrix by 1 – E0 so that the diagonal terms are equal to C'' = (1 / 1 – E0) *C' (@Risk User's Guide, Version 7).

@Risk also lets the user construct a weighted matrix to control which correlation coefficients are allowed to change. A weighted matrix was constructed using 100 in every cell that had a non-zero value in it. A 0 was placed in every other cell. A value of 100 placed in the weighted matrix meant that value could not change or could only change very slightly. A 0 meant the value was free to change to any value from (-1,1) that would make the matrix positive semi-definite. In general, cells with a value were held constant and cells without one were free to change as the program shifts the matrix. With only a few slight changes to the coefficients already in place, a positive semi-definite matrix was generated.

The stochastic terms in the second stage of the model had their own spearman correlation matrix which was derived from the empirical data.

4.6.1. Adding IV to the Copula

A similar process was done to derive a copula correlation for the problem. A copula was fit on the empirical option pricing data that was collected. The best fit copula was a t copula. This t copula is characterized by fatter tails and a wider distribution from the mean than a normal bivariate distribution or a Gaussian copula. The same steps were taken to derive the coefficients for IV up until a weighted matrix is needed to correct the "inconsistent" matrix. @Risk does not let the user use a weighted matrix in the same way as it does for a Pearson or Spearman correlation matrix. Instead, the "RiskCorrectCorrmat" function was used. This function does the same thing as the program would do with an inconsistent Pearson or Spearman matrix and a weighted one. The weighted matrix was designed so all non-zero values were set to change freely while known values where held stable. @Risk was able to generate a positive semi-definite matrix from using the methods described. The t copula that was derived looks very similar to the Spearman matrix used with the previous methods.

4.7. Other Notable Variables

There's two other notable variables in the model that have not been discussed. They are the interest rate and days to expiration (DTE) for the options. The interest rate was set at a static rate of 2.75%. The 10-year US Treasury bond yield is typically used as the risk-free rate in practice. The rate fluctuated between 1.35% and 3% over the past five years. One could argue that the rate has been kept artificially low by the central bank since the financial crisis. 2.75% was a fair rate for that time period.

The other variable is the days to expiration (DTE). DTE is another stochastic variable in the model. The distribution is one of uniformity for the number of days between each contract month. For example, assuming there are 21 trading days in each month and thus 252 trading days in a year, the 2nd deferred contract has between 43 and 84 trading days left. The 4th deferred had between 127 and 168 DTE. One of the goals of this model is to solve a generic situation versus a more specific problem. Therefore, stochastic DTE was preferred to static. The DTE variables at each expiration are perfectly correlated with each other.

4.8. Profit, Loss, and Margins

The overall goal of this problem to mitigate the price risk of soybean purchases by hedging with futures, basis, and option contracts. Therefore, there must be an adequate way to measure profit and loss on the hedge. There are two stages incorporated in the model. Aggregating the profit or loss from each futures, basis, and options contracts will display the profit or loss from the entire hedge. One crucial factor that hasn't been mentioned yet is margins. Margins are the reason speculators like to use the futures market and also the reason hedgers fear it.

4.8.1. Margins

Margins in the futures and options markets are self-defining. They represent the amount of money the hedger (speculator) must put on "margin" for their desired position. Anyone who takes positions in the futures and options market has to have an account with the clearing house involved in the transaction. Of course, this is assuming these are exchange traded contracts. For OTC (over-the-counter) trades, no margin is required unless there is an agreement between the two parties. Basis contracts are only traded OTC so there are no margin obligations built in to them. The account holds the capital as collateral in case the position moves in a disadvantageous way. The probability of these scenarios can be very difficult to calculate. Hence futures contract margin requirements change from time to time.

On the other hand, option margins can change frequently and drastically depending on the volatility in the market. Each option has its own margin requirement. The margin for a long call option position is equal to that of the buying price. The margin for a short call position is:

$$\delta = O + [.2 * F * C - \max(X - F, 0) * C], O + (.1 * F * C) \tag{4.4}$$

Where δ is the margin requirement for a short call position, O is the price of the call option, and C is the number of contracts needed to hedge. This is the formula the CBOE (Chicago Board of Options Exchange) uses. Basically, the margin requirement is the price of the option plus twenty percent of the underlying futures position minus the amount the option is OTM or the price of the option plus ten percent of the underlying futures position. Whichever is greater is the initial margin. The margin requirement for put positions are identical to that of calls. The only thing that changes in the formula is the strike price is subtracted from the futures price because the OTM portion is opposite for puts and calls. Generally, for short positions, the margins for the options included turns out to be about three times the price for ITM and ATM options and about five times for OTM ones.

For a portfolio of options, the margin requirements can become complex. There is no formula that can derive the margin for an options portfolio. Instead a methodology called SPAN (Standard Portfolio Analysis of Risk) is used by the CME. It's a simulation based VaR model that accesses the risk of the portfolio in 16 different profit and loss scenarios. This type of program is necessary to treat all customers' positions fairly. For example, if a speculator sells 100 OTM puts that are five percent OTM, they'll have to deposit around five times the selling price into their trading account. However, if the same speculator sells those same options while simultaneously buying 100 OTM puts that are ten percent OTM, the risk of loss is now much smaller. After the underlying breaks below the ten percent OTM strike price, there's no risk of losing any more. A graph is shown below for this position. This is a true collar without an underlying cash position.

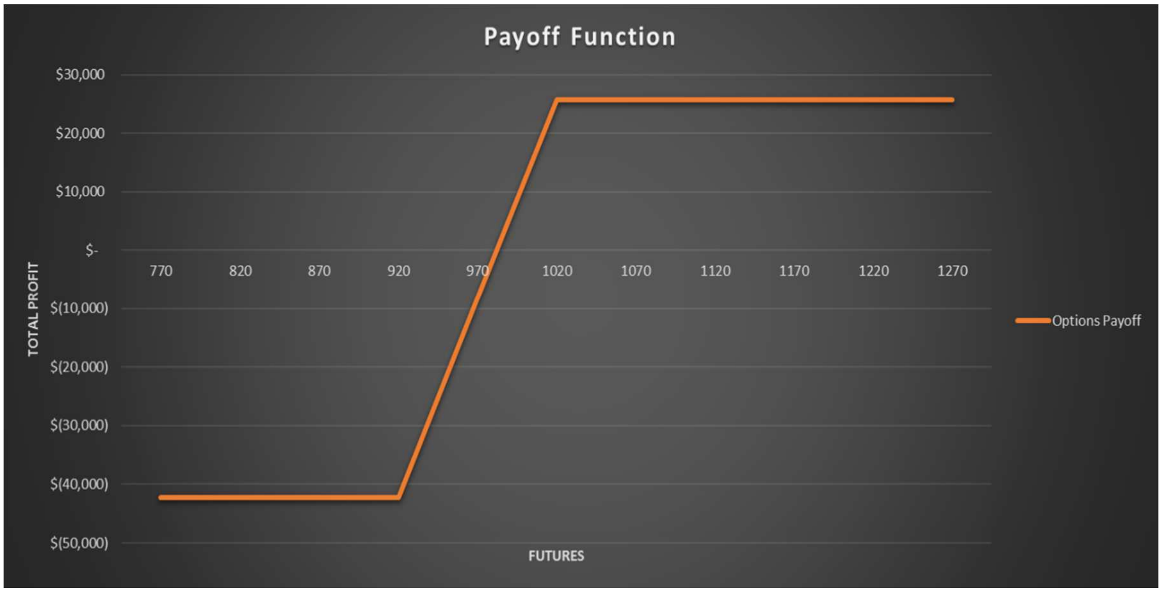

Figure 4.10: Bull Put Spread
Note: Example of Maximum Loss

If the margins were completely independent from each other, the total initial margin requirement would be $1.4M. The notional contract value of the underlying would be $5.1M. The most this customer can lose on this trade is $42K. That's 3% of what the independent margin position would have been. With the extra downside protection, the margin will mostly likely be the maximum amount they can lose or slightly less. This is a relatively simple example. The positions can get much more complex and therefore a simulation model is needed to compute that risk. For simple strategies, the initial margin in calculated with the formula above. For more complex strategies, the VaR is used as a proxy. The margin is not a factor in any of the objective functions but because of the importance in practice, margins for each position will be listed with the results in the next chapter.

4.9. Simulation and Optimization Procedures

Once the model is all set up, simulations have to be run in order to rank the hedging strategies against each other. The first step is to create "allocation" cells or portfolio weights for each of the securities used. This can be done a few separate ways. This model used a percentage term as the weight based on the amount needed to hedge relative to the underlying short cash position. The weight is based on the number of underlying contracts needed for an effective hedge. For example, if a scenario is created where the hedger wishes to create a naïve hedge (HR = 1) with futures and basis contracts, the weights would be set to a positive 100% for each position. If the hedger wanted to "Texas" hedge or hedging extra while creating a net long position, 150% or 200% percent can be entered. Texas hedging doesn't have to end up as a long position. It just refers to the speculative behavior outside of a typical hedge.

Weights for options are derived in the same way. For example, if 100% is the input for the weight of ATMC4, the number of underlying contracts needed to hedge 100,000 metric tonnes (mt) one to one is 734.8 contracts. To come to that number, 100,000 mt must be converted to bushels. Since 1 mt is equal to 36.74 bushels, 100,000 mt is equal to 3,674,000 bushels. One soybean contract represents 5,000 underlying bushels for futures and options. To get to the 734.8 contracts, we take the number of bushels to hedge divided by the number of bushels in one soybean contract. 100% allocation to any of the futures, basis, and options contracts will result in 734.8 contracts hedged.

4.9.1. Optimization

The objective functions defined in Chapter 3 were used as "goals" for the adjustment cells. The adjustment cells are the weights that were just defined above. The optimization tool in @Risk uses the methods outlined in sections 3.3 and 3.4 to determine the optimal allocation for the hedge. The only constraints used in the model are that the weights can't be below -300% or above 300%. The constraints are arbitrary. Some risk tolerant strategies allocate a maximum allocation to options and a limit was necessary for simplicity. The number of iterations per simulation is set at 5,000 in this model. Then the optimizer runs as many simulations as it takes to adequately maximize or minimize the respective objective function by changing the allocations.

4.10. Summary

There's a core group of options strategies that are explored through section 4.2. The most impactful assumption in the empirical model is the way the contracts are modeled. In this study, past data is taken and analyzed to create an optimal solution based on that data. One could also use different assumptions regarding the shape of the active futures distribution and the correlations between each contract. Three objective functions are studied while E-CVaR also has a separate copula model. Actual option pricing data was extracted for this study. Another generalized way to make a similar model would be to use the Black model to price at both time periods one and two while making assumptions about future volatility. Margins and margin calls carry extensive risk when dealing with options. However, they're very difficult to model because they're based off stress testing. The optimization procedure is a standard linear programing optimization searching for a global maximum. The less securities within an optimization problem, the higher probability a global maximum will be reached instead of a local maximum.

CHAPTER 5. RESULTS

5.1. Introduction

Soybean buyers can typically make money on the difference in location basis. The arbitrage principle of grain trading involves buying low at one location and selling higher at another. One should be left with a "risk-free" profit after such a trade. However, there may exist an opportunity to increase profits while limiting risk with the use of derivative contracts. International buyers are typically involved in buying and selling hundreds of thousands of tonnes of grain around the world. They should have a large "think tank" built that could tip them off to the direction prices are going. Still, large grain trading firms routinely hedge their positions on a one-to-one basis. The results of this study show some opportunities for buyers to reduce cost while limiting risk by using options.

In chapter two, the portfolio model of hedging blueprint was given. Ways measure risk and the differences between them were explored. The biggest difference being variance versus semivariance is variance measures the deviations from the mean on both sides of the distribution. Semivariance only measures the amount of left-tail risk within the same distribution. Some measures identify the mean of the left side of the distribution and take a variance for the downside moves relative to that mean. Other measures capture the tenth, fifth, or first percentile of the distribution to identify a "worst loss" statistic. VaR (value-at-risk) and CVaR (conditional value-at-risk) are used for those measurements. The contrast between how risk is measured should differentiate E-V, E-CVaR, and E-CVaR copula substantially. Scientists and practitioners have been and still do measure variance over the entire distribution of a variable. This treats upside-risk the same as downside. Correlations between variables were also outlined in detail in chapters two, three, and four. Using a Pearson (linear), Spearman, or some form of Copula correlation can significantly affect the end results.

Various option strategies were also illustrated in chapters two and four. There are many different approaches to setting up a trade or hedge. These approaches can be applied to a hedging problem. When the objective is to make more money or cut costs, it's worthwhile to explore securities with asymmetric payoffs to conform to one's view of future market prices. This is possible without taking undesired amounts of risk.

This chapter is split between the core results and sensitivity analysis. The next section lays out the base case scenario. The base case in comprised of a no hedge and futures hedge scenario. After that, alternative option hedging strategies are analyzed using the E-V framework. Next, the results from minimizing CVaR are presented. To round out the objective functions, the results from E-CVaR and E-CVaR with copula are shown. All objectives have an ARAP (absolute risk aversion parameter) comparison following the initial results. Synthetic long positions are also explored. After that, further sensitivity analysis involving risk aversion and volatility are shown. Finally, a summary at the end highlights the main results in the chapter.

5.2. Results

Many of the following results are heavily dependent on the time period used to collect data. The data was collected over a time with low volatility in soybean prices. Prices stuck right around $10 a bushel. Also, the market was predominantly in backwardation over that time period. The main input throughout the entire model is the assumption that the active futures contract follows a price distribution that characterizes a gamma distribution. The futures prices in this model are based off the active contract. The other main assumption is that the Black-76 model is an accurate price approximation tool for the second stage of the hedging model. Overall these assumptions all fit the data from the period studied.

The base case results are reported first, followed by E-V alternatives. CVaR, E-CVaR and E-CVaR copula results are reported last. A futures hedge is compared against option hedges to follow. Optimized basis HRs are included in every strategy.

5.2.1. Base Case

The base case represents an unhedged scenario as well as a naïve hedge with futures, basis, and futures and basis together. Table 5.1 compares the results from these cases as well as an ATMC4. The red one-hundred percent indicates a short position. The red in this table indicates the inherent short cash position at PNW since the hedger buys soybeans in the future. It also indicates other short positions throughout this chapter. The regular 100% figures for futures, basis, and the call option indicates a one-to-one (naïve) hedge with the short cash position. One-hundred percent indicates the position is fully hedged. The percentage of each position can be interpreted as the position relative to the short cash position. A position of 150% means the futures or options underlying position includes 50% more than the cash position. The E-V column shows the results of the mean-variance function with an ARAP parameter equal to one. Delta and gamma are shown on the right-hand side. It's helpful to compare those statistics across strategies. Lastly, under the table are symbols used to scale the objective functions and initial margin. For example, the E-V function is in billions. Those symbols are used throughout this chapter. The E-V and initial margin numbers are in dollars.

The unhedged position is the least attractive according to E-V, followed by a basis only hedge and the option hedge. The change from an unhedged to basis only hedge is not significant.

By adding a futures hedge, the E-V objective increases by 90%. The goal is to maximize the E-V objective.

Table 5.1: Base Case Results

Hedge	PNW	Futures	PNW Basis	ATMC4	E-V	Initial Margin	Delta	Gamma
Unhedged	(100%)				(28,195)	-	-0.98	0.00%
Basis	(100%)		100%		(26,899)	-	-0.98	0.00%
Futures	(100%)	100%			(3,333)	1,125	0.00	0.00%
F & B	(100%)	100%	100%		(2,263)	1,125	0.00	0.00%
ATMC4	(100%)		100%	100%	(12,567)	2,064	-0.25	0.26%
*thousands, **millions, ***billions					***	*		

Margins are also an important aspect of hedging even though it is not included in any objectives within this study. The table shows no margin requirement for the unhedged and basis hedge. This should be straightforward after the discussion in chapter two. The next two hedges are hedged with futures. In this model, futures require 3% of the underlying contract value as initial margin. The ATMC4 option is slightly more expensive. The margin reflects the cost to buy the option.

Recall from chapter 3 the delta of a short cash position is equal to -.98. Both futures hedges take an equal and opposite position, hence the delta is zero. The option strategy has a delta of -.25 and gamma of .08%. The delta of the option alone is .48 and is combined with a short cash position with a delta of -.98. Assuming equal positions, the delta of the portfolio should be -.25. All greeks listed in this chapter are the greeks of each strategy or portfolio. Delta shows price bias in the hedge. A negative delta expresses a negative price bias. In other words, the hedger benefits more from a downside move then an upside move. Lastly, the gamma of the options hedge is .26%. All of the gamma is coming from the options since futures positions do not have a gamma value. Gamma indicates the change in delta relative to the change in price in the underlying. Gamma is always greatest for an ATM position relative to an ITM or OTM position. It can also be a good indicator of overall risk no matter if it is positive or negative. This overall risk includes upside and downside risk. A large positive gamma indicates the hedge performs better when a large price move happens. A negative gamma indicates the hedge makes money if the underlying price is inactive in the future.

Table 5.2 shows a married call option table that compares the optimized strategies to naïve hedges. Recall from chapter four a married call strategy pairs a long call position with a short position in the underlying. In this case, the underlying is a short cash position. A naïve hedge for futures and basis (F & B) along with an optimized trial that maximized the E-V function is used to illustrate the optimal positions. The strategies with 100% in the options positions are not optimized but they are compared to the optimized version directly below. The E-V heading is highlighted to show which function is being optimized. For the remainder of the chapter, the black highlighted column heading shows which function is being optimized. The ARAP is listed at the bottom of each table from now on. That parameter can make a substantial difference in how the model achieves the optimal portfolio in the E-V and Mean-CVaR objective functions.

The optimal position sizes increase as the options go from ITM to OTM. Also, the deltas of those optimized positions are hovering around zero and are all negative. This should be expected since the market model is in backwardation and a delta neutral portfolio should contain less risk in comparison to a delta positive or negative portfolio. The gamma of each option portfolio is positive. As the allocation increases, gamma is expected to increase as well. This is also known as delta-hedging which entails making the delta of the hedged portfolio equal to zero. A position of 200% in the ATM option would be perfectly delta neutral.

CVaR and the Mean-CVaR values are also listed in this table for reference. In chapter three, these objective functions were illustrated. The E-V and Mean-CVaR measure of utility is slightly different because of the risk measure used. The E-V incorporates all risk while Mean-CVaR only incorporates downside risk.

Table 5.2: Married Call Results

Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	E-V	CVaR	Mean-CVaR	Initial Margin	Delta	Gamma
F & B	(100%)	100%	100%				(2,263)	(4,398)	(2,072)	$ 1,125	0.00	0.00%
F & B	(100%)	99%	91%				(2,213)	(4,379)	(2,060)	$ 1,116	0.00	0.00%
ITMC4	(100%)		100%	100%			(7,507)	(5,362)	(2,141)	$ 4,322	-0.14	0.20%
ITMC4	(100%)		94%	130%			(6,320)	(5,165)	(1,888)	$ 5,607	-0.02	0.26%
ATMC4	(100%)		100%		100%		(12,567)	(6,453)	(2,618)	$ 2,064	-0.25	0.26%
ATMC4	(100%)		97%		155%		(10,324)	(6,201)	(2,180)	$ 3,207	-0.09	0.40%
OTMC4	(100%)		100%			100%	(18,858)	(8,354)	(3,854)	$ 865	-0.37	0.21%
OTMC4	(100%)		96%			197%	(15,719)	(8,185)	(3,490)	$ 1,702	-0.16	0.41%
ARAP = 1							***	*	*	*		

ARAP = Absolute Risk Aversion Parameter

In all four of the cases below, the optimized E-V values are higher than their respective naïve hedges. This indicates a better performing hedge. The futures hedge is the best performing followed by the ITM, ATM, then OTM strategies. CVaR and Mean-CVaR values are also displayed. Those values are also higher for the optimized E-V strategies than their naïve hedges. The basis HR is always less than one in the optimized results. This is most likely due to the slightly positive correlation between the call options and the basis. Some of the basis risk can be hedged efficiently with options. This theme that shows up throughout the chapter.

5.2.2. E-V Alternative Strategies

Next are the results for some alternative strategies using mean-variance. Table 5.3 shows a married call, a collar, a multi-calendar collar, an all married call, all 4th deferred, and finally a short butterfly spread. This table includes the hedge ratio defined as the sum of all positions. For example, a naïve collar hedge with 100% and -100% positions in the ATM call and put respectively has a hedge ratio of zero. Also included are the other greeks; vega and theta. They paint a better picture of where the risk is coming from in each strategy. Vega displays the effect a change in volatility will have on the portfolio. A positive vega indicates a long vega position while a negative vega is the opposite. In both married call strategies, vega is high and positive while theta is also high but negative. A 1% increase in volatility will increase (decrease) the value of that portfolio by vega. Theta displays whether time is in the hedgers favor or not. A negative theta indicates eroding time value while a positive theta indicates the position is valued higher as time goes. Of course, that assumes prices stay the same. Theta specifies the amount that is gained (lost) every day all else held equal. The last new item on this graph are the stars (*) on the right side. When present, it denotes that VaR is used as the initial margin. Recall from chapter four the reasoning for using VaR.

The futures and basis positions indicate a perfect hedge is the optimal strategy. Skipping strategy two (listed on the left side of the table) because it was in the last set of results, the collar nearly shows a naïve collar. The put position is slightly larger than the call, making gamma slightly negative. Strategy 3 benefits from a stable futures price. Strategy 4 has six options within it. The multi-calendar collar is made up of three different collar hedges at each date of expiration. The results from table 5.3 indicate a long collar in the 2nd deferred, a short collar in the 4th deferred with a larger position in the put, and a long collar in the sixth deferred with a larger position in the put relative to the call. This nets out to an 8% hedge ratio with a portfolio that is delta and gamma neutral. Strategy five incorporates all the 4th deferred call position into one portfolio. A large long position is taken in the ITM call while a lesser negative position is shown for ATM and a small long position in the OTM call. The hedge ratio is close to 100% but the gamma of the portfolio is only .1%. In table, 5.2 a hedge ratio of 100% in the ATM strategy has

Table 5.3: E-V Alternative Strategies

	Hedge	PNW	Futures	PNW Basis	ATMC2	ITMC4	ATMC4	OTMC4	ATMC6	ATMP2	OTMP4	ATMP4	ITMP4	ATMP6	Hedge Ratio	E-V	Initial Margin	Delta	Gamma	Vega	Theta	
1	F & B	(100%)	100%	100%											100%	(2,263)	1,125	0.00	0.00%	-	-	
2	ATMC4	(100%)		97%			155%								155%	(10,324)	3,207	-0.09	0.40%	270,531	(43,668)	
3	Collar ATM4	(100%)		93%			94%					(103%)			(9%)	(2,148)	3,905	0.00	-0.05%	(19,289)	28	*
4	Multi Calendar Collar ATM	(100%)		99%	106%		(29%)		16%	(103%)		58%		(57%)	(8%)	(655)	2,082	0.00	0.00%	(5,899)	(4,214)	*
5	All C4	(100%)		93%		253%	(177%)	9%							85%	(5,026)	4,437	0.06	0.10%	244,074	(41,286)	*
6	All 4th	(100%)		98%		55%	32%	(5%)			(73%)	(5%)	(29%)		(25%)	(2,112)	3,845	-0.01	-0.01%	(13,491)	1,084	*
7	Short Butterfly Spread 4	(100%)		59%			46%	60%			48%	(141%)			13%	(2,475)	4,213	0.00	-0.15%	(147,149)	18,193	*
	ARAP = 1															***	*					

a gamma of .26%. The negative ATM position combined with positive ITM and OTM positions reduces the gamma closer to neutral. The reasoning behind the negative ATM call positions is an attempt to indirectly lower the gamma of the portfolio. This is possible since the gamma is greatest for an ATM options. Gamma decreases in bell-curve like fashion as the options goes more ITM or OTM. Strategy six is another six-option strategy. It includes every 4th deferred position. The largest positions are taken in the ITM options while lesser positions are allocated to the ATM and OTM. The hedge ratio is -25% which seems like a "Texas hedge." In this case, taking a net short position in futures and options while already having a short cash position. However, the delta and gamma are both approximately zero. The last strategy is the short butterfly spread. This would be somewhere between an iron butterfly and a butterfly spread. The ATM position nets out to a -95% in the ATM put while the OTM position are about half that and positive. This strategy is also delta neutral but has a negative gamma. This makes perfect sense since no price movement is the best-case scenario for a short butterfly strategy. The most favorable strategy, according to the table, is strategy four. This creates a portfolio that is nearly delta and gamma neutral. It's close to vega and theta neutral as well. Strategy six is the next best performing followed by the collar strategy and the futures hedge. The results would be similar to a variance minimization objective since the ARAP is equal to one. It makes sense that the least risky portfolio would be approximately delta, gamma, vega, nd theta neutral. It's difficult to tell whether having six different option positions within one portfolio is worth the time to manage it. The collar performed very well and only has two option positions within it. If there's one take away from this table, a portfolio that is delta and gamma neutral are the best for highly risk averse hedgers. Vega and theta neutral would be bonuses. The more options that are included in a portfolio, the easier it is to achieve neutrality in the greeks. Tables 5.4 and 5.5 show the difference risk aversion has on the utility function. In the table 5.4, the optimal positions are nearly identical to the results in table 5.2. The objective functions also display the same results. Thus, table 5.2 exhibits extremely high-risk aversion. An ARAP of 1 in an E-V framework approximately show the same results as a minimization of the variance.

Table 5.4: E-V High Risk Aversion

Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	E-V	CVaR	Initial Margin	Delta	Gamma
Unhedged	(100%)						(280,392)	(13,388)	0	-0.98	0.00%
Futures	(100%)	100%					(32,414)	(5,216)	1,125	0.00	0.00%
F & B	(100%)	100%	100%				(22,844)	(4,460)	1,125	0.00	0.00%
F & B	(100%)	99%	93%				(21,996)	(4,381)	1,112	0.00	0.00%
ITMC4	(100%)		100%	100%			(74,737)	(5,362)	4,322	-0.13	0.20%
ITMC4	(100%)		99%	129%			(62,543)	(5,159)	5,562	-0.02	0.26%
ATMC4	(100%)		100%		100%		(126,700)	(6,453)	2,064	-0.25	0.26%
ATMC4	(100%)		100%		157%		(102,342)	(6,217)	3,242	-0.09	0.40%
OTMC4	(100%)		100%			100%	(187,770)	(8,354)	865	-0.37	0.21%
OTMC4	(100%)		96%			198%	(156,591)	(8,192)	1,716	-0.27	0.41%
ARAP = .00001							*	*	*		

An ARAP of 1 and .00001 display the same results and reinforces this fact. The next table shows the exact same strategies but with low risk aversion. The ARAP is set to .0000001 for table 5.5. First, notice how the basis allocation drops form the nineties to the seventies. Part of the reason is because 55% of the underlying basis price change distribution is below zero percent. Meaning that the data analyzed show a negative move is basis values over a 28-week time frame 55% of the time. A risk tolerant buyer is likely to not lock in as many basis contracts if the probability of the basis moving lower is greater than 50%. This makes prices cheaper when the time comes to buy the physical beans. This is still an E-V optimization which means the right tail of the cost function is still counted as “risk.”

Table 5.5: E-V Low Risk Aversion

Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	E-V	CVaR	Initial Margin	Delta	Gamma
Unhedged	(100%)						(2,749)	(13,388)	-	-0.98	0.00%
Futures	(100%)	100%					(159)	(5,216)	1,125	0.00	0.00%
F & B	(100%)	100%	100%				(100)	(4,460)	1,125	0.00	0.00%
F & B	(100%)	103%	72%				(98)	(4,430)	1,158	0.01	0.00%
ITMC4	(100%)		100%	100%			(205)	(5,362)	4,322	-0.13	0.20%
ITMC4	(100%)		71%	152%			127	(5,598)	6,560	0.04	0.30%
ATMC4	(100%)		100%		100%		(637)	(6,453)	2,064	-0.25	0.26%
ATMC4	(100%)		78%		200%		25	(6,604)	4,127	-0.01	0.51%
OTMC4	(100%)		100%			100%	(1,533)	(8,354)	865	-0.37	0.21%
OTMC4	(100%)		74%			253%	(879)	(8,438)	2,192	-0.10	0.52%
ARAP = .0000001							*	*	*		

The next piece to look at are the positions. The futures position increased to over 100%. This was probably due to a slight upward bias within the 4th deferred futures contract. The call position also increased across the board. The optimal ITM, ATM, and OTM call positions increased by 33%, 43%, and 55% respectively. The main change is the preferred portfolio changes from a futures hedge to an option hedge. Specifically, the ITM option was the best performing according to E-V. The optimized ITM strategy has the highest value for E-V. It was followed by the ATM strategy and the futures hedge. However, the ITM strategy also requires the most initial margin since ITM options are expensive to buy. Keep in mind the deltas of all the portfolios. The hedge ratios all increased which means the delta increased as well. Each of the deltas were pushed to slightly positive territory except for the OTM call. This follows the pattern of moving toward a delta neutral portfolio. The optimized ATM strategy is perfectly “delta hedged” and nets out to a perfect straddle position for the hedger. The gamma of the portfolios also increases because the position sizes increased. The cash and futures positions are gamma neutral since the deltas of each do not change individually under any circumstances. They are equal to one or negative one minus a slight time value component. This is why the delta is -.98 and .98 instead. A large move in prices either way benefit, or at least won’t hurt, the hedger in a gamma positive portfolio. It is easiest to interpret gamma relative to another portfolio's gamma value.

Tables 5.6 and 5.7 continue the comparison of risk aversion for the E-V function in alternative strategies. Strategy one in table 5.6 behaves in an equivalent manner to the same strategy in 5.3. When risk aversion decreases, the allocations change drastically but delta and gamma stay approximately the same. The large ATM position could be because the mean profit (cost) of that options is slightly higher than the ITM.

Table 5.6: E-V Alternatives with High Risk Aversion

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	E-V	CVaR	Initial Margin	Delta	Gamma	
1	All Call	(100%)		52%	200%	(148%)	76%			(58,077)	(5,350)	9,297	-0.01	0.08%	
2	Collar ATM4	(100%)		54%		97%			(100%)	(25,428)	(5,065)	4,025	0.00	-0.03%	*
3	Collar OTM4	(100%)		65%			147%	(151%)		(50,195)	(7,198)	6,041	-0.07	-0.02%	*
4	Strangle 4th	(100%)		57%		48%	67%	31%		(25,715)	(5,314)	4,248	0.01	-0.12%	*
	ARAP = .00001									*	*	*			

Table 5.7: E-V Alternatives with Low Risk Aversion

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	E-V	CVaR	Initial Margin	Delta	Gamma	
1	All Call	(100%)		100%	92%	206%	(209%)			293	(6,141)	8,247	0.02	0.07%	
2	Collar ATM4	(100%)		61%		202%			(4%)	26	(6,630)	4,176	0.00	0.51%	*
3	Collar OTM4	(100%)		60%			242%	(38%)		(807)	(8,127)	6,799	-0.08	0.42%	*
4	Strangle 4th	(100%)		61%		117%	51%	167%	(153%)	(1)	(5,685)	4,481	0.02	0.06%	*
	ARAP = .00001									*	*	*			

Nearly a perfect collar can be seen in table 5.6 for ATM. The OTM collar was also close to equal although the positions were 50% larger. In table 5.7, both collars switch to straddles. In table 5.7, the ATM straddle is nearly a perfect straddle. A perfect straddle would have a call position of 200% and no put position for a portfolio that already contains a short position in the underlying. It's a delta neutral strategy with highly positive gamma. The strategy losses if prices stay the same and wins if prices move substantially in either direction. The OTM version for the risk tolerant hedger did something similar. The hedge ratio is still net long about 200% but larger positions were taken in both options. Each OTM collar / straddle has a slightly negative delta. Besides the graph of the payoff function, one can tell if the strategy is a collar or straddle by gamma. A collar has a gamma near zero while a straddle will have a moderate to highly positive gamma value.

The short butterfly strategy in table 5.6 created a net short ATM position equal to 83% while the OTM positions were substantially lower than 100%. Table 5.7 shows a strangle for the same strategy with a higher allocation to the ATM put than the call. The OTM put has an extremely large position compared to the OTM put. This could be a product of the backwardation bias in the model. The same method to differentiate between a collar and straddle can be used for a short butterfly and strangle.

The best performing strategy in table 5.6 was the short butterfly by a thin margin over the ATM collar. However, the futures strategy in table 5.4 still performed slightly better than each of these. The first strategy in table 5.7 was the best performing followed by the ATM straddle and OTM strangle. The All call strategy was the best performing strategy overall considering table 5.5 as well.

For a highly risk averse hedger, futures is the best hedge. A collar and short butterfly work well too. A hedger who is more tolerant to risk prefers the "all call" strategy. ITM married call, ATM straddle, and ATM married call would all be considered good hedges. A risk tolerant hedger prefers a non-neutral gamma portfolio while a risk averse one prefers gamma neutral.

5.2.3. CVaR Maximization

The next set of results are from a maximization of the CVaR objective function. This is used instead of variance because a hedger only cares about downside risk. Therefore, it's more appropriate to do a semivariance maximization. Table 5.8 shows these results. First, notice again how the amount of basis hedged is predominantly around 70 to 80% for most strategies. Also, the same pattern for the married call strategy emerges in this table that match the E-V tables above. The married call strategies are displayed as strategies three, four, and five. The optimal ITM allocation is less than the ATM and OTM and also performs better according to CVaR. The same pattern has been observed above.

The next set of strategies are the four collar simulations. ATM and OTM for the 2nd and 4th deferred options were used. The best performing was the ATM 2nd deferred collar followed by the ATM 4th deferred. The 2nd and 4th deferred OTM collar strategies followed in the same order. It's not surprising that the ATM collars perform better than the OTM ones but why do the 2nd deferred collars perform better than the 4th deferred? Part of the reason has to do with one of the potential shortcomings of the study. The left tails of the spreads between contracts couldn't be modeled in the same fashion the data was dressed in. This is discussed further in chapter six. The second answer comes from the table in appendix 1A. There, some statistics are listed describing the underlying characteristics of each futures and options contract. The mean in the table assumes a position equal to 100%. Notice that the means are greater for the 2nd deferred call options relative to the 4th deferred. The probability that the profit from each hedged position is listed below the mean. Those probabilities are also higher for the 2nd deferred options. The same pattern can be seen moving from the 4th deferred call to the 6th deferred. However, the mean in the 6th deferred put options are slightly higher than they are for the 4th deferred.

Appendix table 1A also displays mean prices for every security in the model on the first line, their standard deviations, the mean implied volatility for each option and its standard deviation. To be clear the standard deviation in the fourth line down is the standard deviation of the implied volatility distribution. This gives slightly better insight into the deviations from the mean in those distributions. Lastly the profit (cost) of the hedge is This is the case for ITM, ATM, and OTM. The means profit (cost) of the 100% position displays a more negative value in all the ATM calls versus the ITM and OTM. One other interesting piece is the volatility is highest in the 4^{th} deferred options. Intuitively, the closest options to expiration should have the highest volatility. Vega is amplified by the amount of time value left in an option. This would be an abnormal market scenario. The volatility distributions become wider for 2^{nd} and 6^{th} deferred options relative to 4^{th} as is seen by the standard deviation for volatility. Lastly, the price of the options should increase as they move farther from maturity given extra time value. Although this is barely the case for the OTM calls, the ITM and ATM mean call prices are smaller in the 6^{th} deferred month than the 4^{th} deferred. This would be another abnormal scenario.

The puts have some of the same characteristics and some different. First, there's not much of a discrepancy between the probability the option expires ITM at different dates of expiration. Second, the volatility eases as maturity is farther out. This is what's supposed to happen. The standard deviation of those volatilities shows the same pattern as the call. The ATM has small deviations from the mean relative to ITM or OTM. The mean prices are larger as DTE (days to expiration) increases. This is also how option chains are supposed to work in the real world.

Table 5.8: CVaR

	Hedge	PNW	Futures	PNW Basis	ITMC2	ATMC2	OTMC2	ITMC4	ATMC4	OTMC4	ITMC6	ATMC6	OTMC6	OTMP2	ATMP2	ITMP2	OTMP4	ATMP4	Hedge Ratio	CVaR	Initial Margin	
1	Unhedged	(100%)																	0%	(13,091)	-	
2	F & B	(100%)	99%	78%															99%	(4,383)	-	
3	ITMC4	(100%)		77%				116%											116%	(5,094)	5,014	
4	ATMC4	(100%)		75%					135%										135%	(6,134)	-	
5	OTMC4	(100%)		68%						144%									144%	(8,082)	1,246	
6	Collar OTM2	(100%)		77%			231%							(108%)					123%	(5,157)	4,298	*
7	Collar ATM2	(100%)		76%		126%									(81%)				45%	(2,653)	1,922	*
8	Collar OTM4	(100%)		77%						131%							(144%)		(13%)	(7,169)	6,041	*
9	Collar ATM4	(100%)		98%					109%									(92%)	17%	(4,934)	3,788	*
10	Straddle (Turns to Collar)																					
11	Strangle 2nd	(100%)		73%		99%	34%								98%	(111%)			120%	(2,841)	2,120	*
12	Strangle 4th	(100%)		71%					101%	94%							63%	(98%)	160%	(5,559)	4,418	*
13	All ITM Call	(100%)		87%	146%			(58%)			26%								114%	(2,486)	1,831	*
14	All ATM Call	(100%)		87%		173%			(76%)			37%							134%	(3,036)	2,331	*
15	All OTM Call	(100%)		99%			197%			(115%)			117%						199%	(5,701)	4,892	*
																				*	*	

Table 5.8.1: Supplemental Information to table 5.8

	Hedge	Cash	Basis	Futures	Options	Delta	Gamma	Vega	Theta
1	Unhedged	100%				-0.98	0.00%	-	-
2	F & B	36%	28%	36%		0.00	0.00%	-	-
3	ITMC4	34%	26%		40%	-0.07	0.23%	125,395	(20,637)
4	ATMC4	32%	24%		44%	-0.14	0.35%	205,221	(33,126)
5	OTMC4	32%	22%		46%	-0.25	0.30%	188,504	(30,946)
6	Collar OTM2	19%	15%		66%	-0.11	0.31%	191,602	(70,843)
7	Collar ATM2	26%	20%		54%	0.01	0.19%	70,577	(25,484)
8	Collar OTM4	22%	17%		61%	-0.08	-0.04%	(26,197)	1,106
9	Collar ATM4	25%	25%		50%	0.01	0.02%	38,256	(8,628)
10	Straddle (Turns to Collar)								
11	Strangle 2nd	19%	14%		66%	0.00	0.04%	26,758	(11,267)
12	Strangle 4th	19%	13%		68%	-0.20	0.07%	123,390	(22,380)
13	All ITM Call	24%	21%		55%	0.00	0.19%	73,388	(30,554)
14	All ATM Call	21%	18%		60%	-0.14	0.40%	180,109	(70,421)
15	All OTM Call	16%	16%		68%	-0.13	0.22%	207,934	(58,830)

In table 5.8, the CVaR results show the call position in the collar decrease moving from 2nd deferred to 4th while the puts increase the negative position (selling puts) resulting in a smaller hedge ratio. The reasoning for this is along the same lines as the previous page. The 2nd deferred calls have a higher probability of profitability than the 4th deferred. Also observe that the gammas of the 4th deferred collars are approximately zero from table 5.8.1 which provides supplemental information to table 5.8. This reinforces that the idea that a gamma neutral portfolio is also less “risky” than not.

The next two strategies listed in the table are the two strangles. First, it’s important to point out the options involved for a strangle are the same as a short butterfly. When optimizing for CVaR, a strangle performs better than a short butterfly. This isn’t shown because the same four options are used for a strangle and short butterfly. If the optimizer sells ATM calls and puts, the strategy is a short butterfly. If instead it forms something closer to an ATM collar between those two positions, a strangle is formed. In both cases the OTM positions should be positive. The 2nd deferred strategy performs better than the 4th deferred. This time when moving from the 2nd to 4th the net HR increases rather than decreases as was observed above. In actuality, the positions are very similar besides the OTM call and ATM put. There's no identifiable reason for why the optimizer made those changes.

Lastly in table 5.8, the ITM, ATM, and OTM are grouped together. The closer to expiration option portfolio performed better than the further deferred as has been observed so far. The net HR also went up following the previous results. The last similarity is the large position in the ITM options relative to the ATM and OTM. The interesting point about these three strategies is a negative 4th deferred position in all three. Recall form table 5.3 that similar type allocations were observed. There's no identifiable rational for this behavior. If the 4th deferred position displayed a larger negative, it would characterize a butterfly spread. However, since it's significantly smaller than the ITM position, all three of these strategies net to a straddle like payoff function with higher profitability potential if futures prices go down rather than up. Although, it performs better than a standard married call strategy, the initial margin is more expensive and maybe less attractive in practice. Here's an example of the ATM strategy:

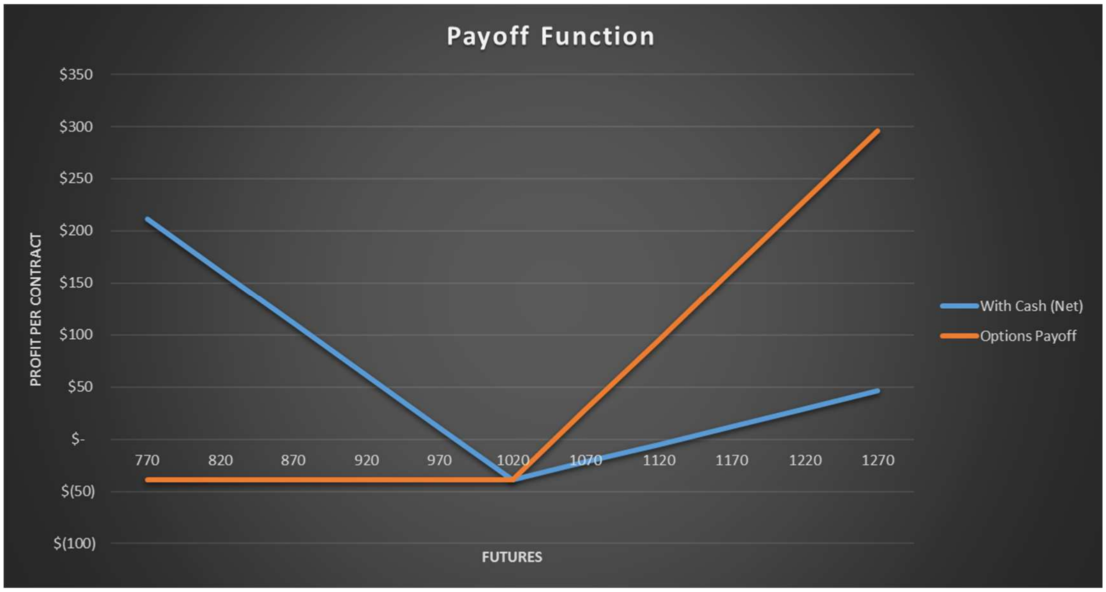

Figure 5.1: Max CVaR ATM All Call Payoff Function

One attractive aspect of optimizing for CVaR is risk preferences do not matter. The downfall is profit (cost) is not accounted for. So far, the optimal positions and strategies have changed when risk preferences changed and that will continue to be the case. Overall, both strategies that included 2nd deferred options instead of 4th deferred performed the best. The next best performing strategies were 13, 14, and 15. All three are highly gamma positive. The ITM version is delta neutral while the ATM an OTM have a negative delta.

Out of the more practical strategies, the collars performed slightly better than the strangles with ATM collar displaying more attractiveness than OTM. According to the results in table 5.8, excluding the 2nd deferred strategies, the all call strategies are the best. If capital for margins are an issue, the collar is the best hedge which is delta and gamma neutral.

5.2.4. Mean-CVaR

The last objective function with linear assumptions built into the correlations is E-CVaR. Two tables are shown. Table 5.9 reflects high risk aversion with the ARAP equal to ten. The parameter in table 5.10 is equal to two and represents low risk aversion or risk tolerance. The parameters for high and low risk aversion are vastly different between E-CVaR and E-V. The reason being that the absolute value of CVaR is much smaller than the absolute value of the variance. Therefore, after some testing, two is used for high risk tolerance and ten is used for high risk aversion.

Table 5.9: Mean-CVaR Risk Averse

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	Initial Margin	Delta	Gamma	Vega	Theta	
1	Unhedged	(100%)								0%	(65,408)	-	-0.98	0.00%	-	-	
2	F & B	(100%)	99%	87%						99%	(21,752)	1,119	0.00	0.00%	-	-	
3	ITMC4	(100%)		88%	117%					117%	(25,589)	5,057	-0.07	0.09%	127,567	(20,995)	
4	ATMC4	(100%)		86%		139%				139%	(29,805)	2,867	-0.13	0.15%	217,423	(35,096)	
5	OTMC4	(100%)		76%			158%			158%	(39,871)	1,365	-0.23	0.15%	226,342	(37,158)	
6	All Call 4	(100%)		71%	113%	16%	(31%)			98%	(24,605)	5,224	-0.07	0.07%	113,198	(18,624)	
7	Collar ATM4	(100%)		48%		132%			(60%)	72%	(26,369)	5,307	-0.01	0.10%	156,168	(26,264)	*
8	Collar OTM4	(100%)		72%			157%	(139%)		18%	(36,606)	7,155	-0.07	0.02%	53,892	(11,825)	*
9	Strangle OTM4	(100%)		86%		98%	89%	94%	(100%)	181%	(27,499)	5,639	0.00	0.05%	147,427	(25,735)	*
	ARAP = 10										*	*					

Table 5.9 shows equivalent results to what has been seen so far for the married call positions. Strategy six follows table 5.3 with similar allocations. Allocations for ITM are greater than the ATM and OTM with the OTM position being negative. This results in a hedge ratio at nearly 100% while still staying more gamma neutral than would be a single married call position.

The collar shows a large bias towards the long call versus short position. The HR is at 72%. This is a drastic change from the CVaR or E-V optimizations. The delta of the portfolio is neutral with a moderately positive gamma. The OTM collar had a hedge ratio similar to the CVaR objective but with a larger position in the long call than short put. The strategy is slightly delta negative and is gamma neutral. Lastly, the strangle is nearly a perfect strangle with 100% allocated to the OTM options as well as the ATM call with a -100% to the ATM put. This give the hedger delta neutrality with a slightly negative gamma.

Table 5.10: Mean-CVaR Risk Tolerant

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	Initial Margin	Delta	Gamma	Vega	Theta	
1	Unhedged	(100%)									(13,045)	-	-0.98	0.00%	-	-	
2	F & B	(100%)	100%	82%						100%	(4,244)	1,125	0.00	0.00%	-	-	
3	ITMC4	(100%)		87%	121%					121%	(4,436)	5,250	-0.05	0.24%	137,517	(22,632)	
4	ATMC4	(100%)		82%		158%				158%	(5,259)	3,264	-0.09	0.41%	281,771	(45,483)	
5	OTMC4	(100%)		76%			186%			186%	(7,552)	1,609	-0.18	0.39%	314,356	(51,607)	
6	All Call 4	(100%)		86%	86%	65%	(19%)			132%	(4,508)	5,087	-0.04	0.15%	114,397	(18,676)	
7	Collar ATM4	(100%)		86%		142%			(60%)	82%	(4,996)	(4,084)	0.00	0.20%	185,869	(31,038)	*
8	Collar OTM4	(100%)		75%			212%	300%		512%	(7,046)	(9,160)	-0.19	0.58%	1,191,964	(181,859)	*
9	Strangle OTM4	(100%)		73%		110%	12%	300%	(80%)	342%	(4,301)	(5,804)	-0.12	0.43%	849,410	(127,322)	*
	ARAP = 2										*	*					

Moving to table 5.10, the married call HRs are similar to what's been demonstrated. Lower risk aversion makes for higher hedge ratios. Skipping to the collars, the ATM is very similar to the risk averse strategy. A 10% jump in the hedge ratio is experienced. The OTM collar changes quite drastically. It actually "blows up" into an ultra-long volatility strangle like strategy. The reason being the right tail is so long in this case that even though the chances of this strategy being profitable are only about 39%, the mean is significantly positive and outweighs the risk in that objective function. A graph of the cost function is shown below.

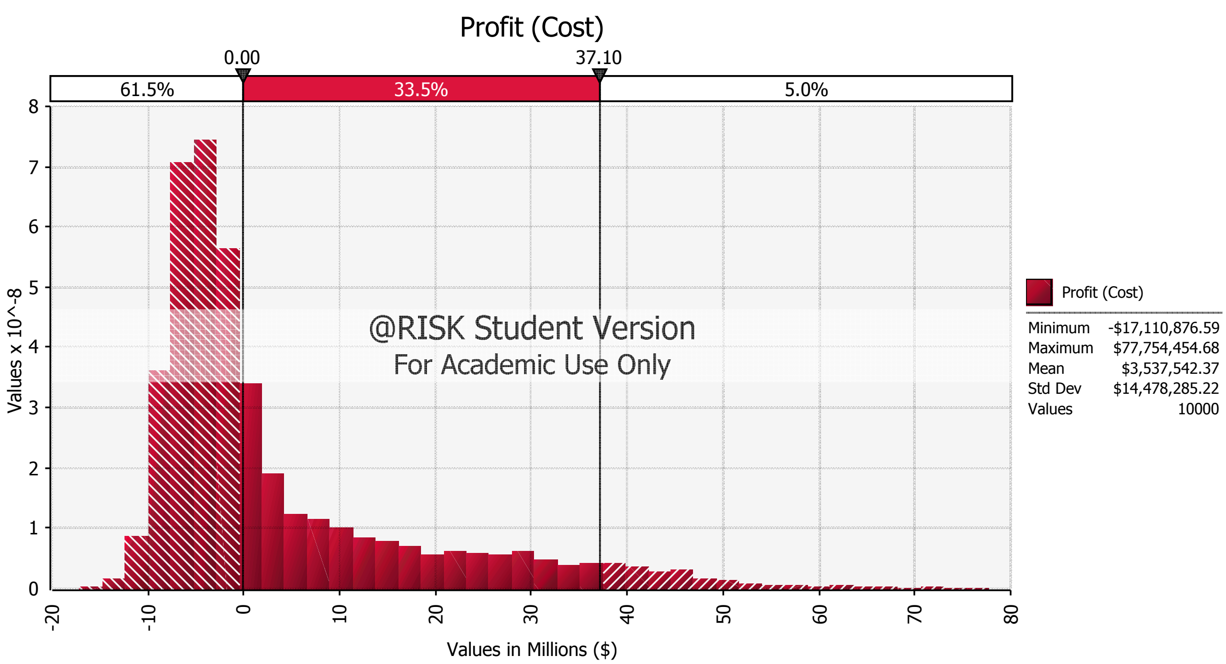

Figure 5.2: Ultra-Long Volatility Cost Distribution

This strategy will cost the hedger a little under $4 M in margin initially. It still has a negative delta because a downward price move would be much more beneficial than an upward one. The gamma is highly positive which isn't surprising considering the allocations observed. Also notice the slightly higher OTM put position versus the call. This may be because the forward curve in this study is in backwardation according to the means of the distributions. The last strategy is the strangle. The OTMP4 position also blows up to the maximum position allowed. A negligible position is taken in the OTMC4 while the ATM options form a slightly delta positive collar. The payoff function for this strategy is very similar to a long put position. The cash position is nearly hedged with the ATM options and the OTMP4 provides a large benefit with a downward price move.

The futures hedge is the best strategy according to table 5.10. The same result can be observed for 5.9. Where this simulation differs slightly is the 2nd through 6th best performing strategies. This includes the ITM married call, ATM collar, the all call, and strangle. However, the differences are minimal and not worth going over in detail. A rank table is provided to the right in descending order to illustrate this.

Table 5.11: CVaR Rank

Hedge	**5.8**	**5.7**
Unhedged	9	9
F & B	1	1
ITMC4	3	3
ATMC4	6	6
OTMC4	8	8
All Call 4	4	2
Collar ATM4	5	4
Collar OTM4	7	7
Strangle OTM4	2	5

The collar, strangle, short butterfly and all call strategies all performed similar or better than a futures hedge at some point in this study. What's interesting is they are all used in different environments. The ATM collar is used when the hedger is not willing to take any price risk. A strangle is deployed when a large price move is expected in either direction. The short butterfly is used in the exact opposite scenario. That strategy performs best in an environment where prices don't change, and volatility stays to a minimum.

Comparing the two tables, the basis position moves slightly lower as risk aversion pulls back. This is similar to what happened with the E-V results. The results for the married call strategies are also similar. The position sizes in each table are lower for E-CVaR relative to E-V. It's difficult to compare the high-risk aversion because of the different scales but it is appropriate to compare the low risk aversion tables. The difference between the married call strategies in E-V and E-CVaR for ITM, ATM, and OTM are 12%, 18%, and 42% respectively. The reason being the right tail of the return distribution is not factored into E-CVaR like it is in E-V. The options in the portfolio amplify that right tail to extreme levels because options can be very lucrative during a big price swing if one is on the right side of the trade. In general, the E-V optimizer sees this as more risk and hedges more accordingly. The E-CVaR only considers the downside risk which does not have a long left tail. The figure illustrates this result:

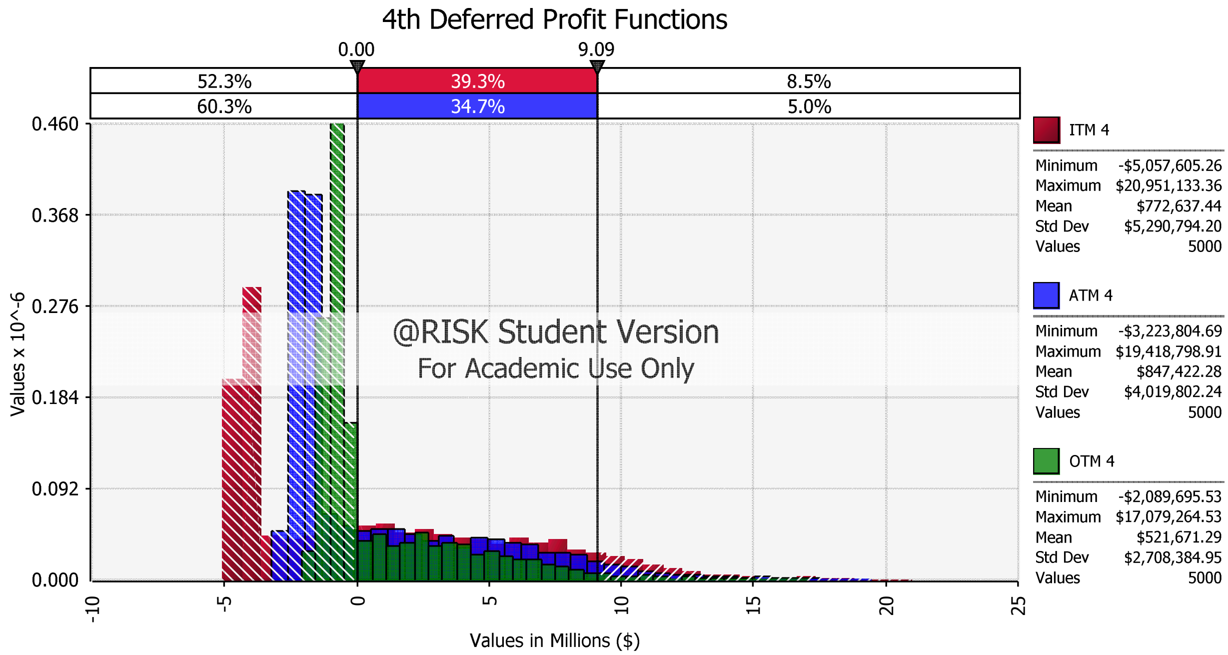

Figure 5.3: 4th Deferred Profit (Cost) Distributions

The figure displays the profit distributions assuming 100% position in each of the options. Specifically, notice the max for all of these distributions varies between $17 M and $21 M while the minimum is between -$5.1 M for an ITM option and -$2 M for OTM one. Also, pay attention to the high point on the “left tail.” The ITM has the highest minimum because it’s already in the money and most likely expires profitable. However, the right to buy an ITM option is expensive and the reason why it displays the highest loss compared to ATM and ITM. Buying an OTM option has the least risk of loss but the probability it expires in the money is substantially less than an ITM or ATM. The left tail of the green column (OTM) goes up to eight on the y axis. To get a better look at the rest of each distribution, it’s cut off at .46.

Comparison between E-V and Mean-CVaR shows higher allocations to the married call strategies for E-V in both risk scenarios. Basis HRs were near 100% for E-V but much lower for E-CVaR. The ATM collar resulted in a near perfect collar in E-V versus higher allocation to the call and lower to the put for E-CVaR. The strategy for E-CVaR actually nets out to a slight straddle as can be seen by a moderately positive gamma. The ATM collars for E-CVaR showed similar allocations to each other. They were right “between” the risk averse and risk tolerant strategies for E-V. The E-CVaR strategies could be considered a hybrid between a collar and strangle. The OTM collars for high risk aversion have similar allocations. The risk tolerant hedger would prefer a straddle in E-V and an “ultra-strangle” for E-CVaR. The OTM put position was -38% for E-V versus 300% for E-CVaR. The E-V strangle allocations were about half of what they were for E-CVaR in the risk averse scenario. The E-CVaR case was nearly a perfect strangle. The ATM allocations flipped moving from E-V to E-CVaR. A net short position by 36% was taken in E-V versus a net long position of 30% in E-CVaR. The behavior for the risk tolerant hedger was to decrease the OTM call position and increase the OTM put in both E-V and E-CVaR. The E-CVaR objective shows a much more drastic shift compared to E-V. The OTM put position for E-CVaR is double what it is for E-V while the OTM call position is almost non-existent relative to E-V.

This last set of result comparisons attempts to compare low risk aversion against one another for two different objective functions. It is difficult to tell whether both respective ARAPs involve the same amount of risk aversion (.0000001 and 2 for E-V and E-CVaR respectively). Generally, the E-V objectives are more apt to buy calls than E-CVaR. E-CVaR is also more apt to buy puts. Notice the lower delta values in table 5.8 versus 5.7. While both functions prefer delta neutrality, the E-V results show more neutrality relative to E-CVaR. The reason being is the E-V objective won’t take one-sides bets. It sees a long right tail in the profit (cost) distribution as risk. Therefore, it balances the payoff functions to hedge where a price swing in one direction is not any more beneficial than it would be the other direction.

5.2.5. Mean-CVaR Copula

The last set of results uses a t copula as the joint distribution instead of a joint bivariate normal distribution to relate the variables included. Tables 5.12 and 5.13 illustrate the results. It shows naïve hedges for E-CVaR versus the same hedges assuming a student t copula correlation. Table 5.11 shows the results for E-CVaR and 5.13 shows it with correlations defined by a t copula.

Table 5.12: Mean-CVaR Naïve Hedging Strategies

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	Mean Return	CVaR	Initial Margin	
1	Unhedged	(100%)								0%	$(61,081)	$ 51	$(12,226)	$ -	
2	F & B	(100%)	100%	100%						100%	$(24,640)	$ 127	$ (4,953)	$ 1,125	
3	ITMC4	(100%)		100%	100%					100%	$(24,497)	$ 767	$ (5,053)	$ 4,322	
4	ATMC4	(100%)		100%		100%				100%	$(29,029)	$ 843	$ (5,974)	$ 2,064	
5	OTMC4	(100%)		100%			100%			100%	$(38,098)	$ 516	$ (7,723)	$ 865	
6	Collar ATM4	(100%)		100%		100%			(100%)	0%	$(28,128)	$ (471)	$ (5,531)	$ 4,232	*
7	Collar OTM4	(100%)		100%			100%	(100%)		0%	$(35,936)	$ (665)	$ (7,054)	$ 5,904	*
8	Strangle OTM4	(100%)		100%		100%	100%	100%	(100%)	200%	$(27,392)	$ 1,220	$ (5,722)	$ 4,441	*
9	Short Butterfly 4th	(100%)		100%		0%	100%	100%	(200%)	0%	$(32,948)	$ (931)	$ (6,403)	$ 5,048	*
	ARAP = 10										*		*	*	

Table 5.13: Mean-CVaR Copula Naïve Hedging Strategies

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	Mean Return	CVaR	Initial Margin	
1	Unhedged	(100%)								0%	$(65,408)	$ 45	$(13,091)	$ -	
2	F & B	(100%)	100%	100%						100%	$(21,865)	$ 127	$ (4,398)	$ 1,125	
3	ITMC4	(100%)		100%	100%					100%	$(25,735)	$ 533	$ (5,254)	$ 4,322	
4	ATMC4	(100%)		100%		100%				100%	$(31,675)	$ 590	$ (6,453)	$ 2,064	
5	OTMC4	(100%)		100%			100%			100%	$(41,520)	$ 303	$ (8,365)	$ 865	
6	Collar ATM4	(100%)		100%		100%			(100%)	0%	$(25,344)	$ (472)	$ (4,974)	$ 3,871	*
7	Collar OTM4	(100%)		100%			100%	(100%)		0%	$(38,997)	$ (644)	$ (7,671)	$ 6,518	*
8	Strangle OTM4	(100%)		100%		100%	100%	100%	(100%)	200%	$(28,065)	$ 782	$ (5,770)	$ 4,551	*
9	Short Butterfly 4th	(100%)		100%		0%	100%	100%	(200%)	0%	$(29,338)	$ (873)	$ (5,693)	$ 4,612	*
	ARAP = 10										*		*	*	

The first nuance of the two tables is the copula makes the mean returns (cost) of the hedges more positive or negative relative to table 5.12. The t copula has fatter tails than a normal distribution which explains why the means are amplified in table 5.12. Measuring by CVaR, the unhedged, married calls, and OTM collar are all less (more negative) than their parallel copula strategy. The Mean-CVaR column has lower values in the unhedged, F&B, ATM collar strategies. Six out of ten strategies have a lower E-CVaR objective in table 5.12. Therefore, no determination can be made whether copulas over or under estimate risk. The portfolio allocations by security type and greeks of the portfolio is located in appendix table 2A for reference.

The previous set of results displayed the naïve hedges for E-CVaR versus E-CVaR copula. It demonstrated the slight difference a copula makes. The next set of results displays the optimized tables for Mean-CVaR copula with different levels of risk tolerance. Tables 5.14 and 5.15 have the same strategies displayed with the addition of an all call strategy. The 4th deferred short butterfly isn't displayed since it's the same as the strangle with different allocations.

First the futures position is very different than what has been seen previously. The optimal futures position moves lower in each table. This is more in line with related literature in chapter 2 that generally concludes a minimum and/or optimal futures HR is less than one to one.

Table 5.14: Mean-CVaR Copula Optimized Strategies (10)

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	CVaR	Initial Margin	Delta	Gamma	
1	Unhedged	(100%)								0%	(61,081)	(12,226)	-	-0.98	0.00%	
2	F & B	(100%)	87%	99%						87%	(22,460)	(4,514)	976	-0.07	0.00%	
3	ITMC4	(100%)		100%	105%					105%	(24,070)	(4,979)	4,552	-0.11	0.21%	
4	ATMC4	(100%)		100%		131%				132%	(27,880)	(5,801)	2,717	-0.15	0.34%	
5	OTMC4	(100%)		100%			160%			160%	(36,216)	(7,412)	1,387	-0.22	0.33%	
6	All Call 4	(100%)		100%	110%	(15%)	9%			104%	(24,112)	(4,979)	4,841	-0.10	0.18%	
7	Collar ATM4	(100%)		86%		107%			(72%)	35%	(24,836)	(4,959)	3,806	-0.04	0.08%	*
8	Collar OTM4	(100%)		99%			161%	(97%)		64%	(34,242)	(6,788)	5,542	-0.10	0.13%	*
9	Strangle OTM4	(100%)		100%		66%	101%	81%	(104%)	144%	(25,967)	(5,330)	4,207	-0.01	0.04%	*
	ARAP = 10										*	*	*			

Table 5.15: Mean-CVaR Copula Optimized Strategies (2)

	Hedge	PNW	Futures	PNW Basis	ITMC4	ATMC4	OTMC4	OTMP4	ATMP4	Hedge Ratio	Mean-CVaR	CVaR	Initial Margin	Delta	Gamma	
1	Unhedged	(100%)								0%	(12,176)	(12,226)	0	-0.98	0.00%	
2	F & B	(100%)	86%	100%						86%	(4,402)	(4,513)	972	-0.07	0.00%	
3	ITMC4	(100%)		86%	112%					112%	(4,188)	(5,066)	4,830	-0.08	0.22%	
4	ATMC4	(100%)		100%		149%				149%	(4,700)	(5,953)	3,075	-0.11	0.38%	
5	OTMC4	(100%)		100%			242%			242%	(6,433)	(7,704)	2,094	-0.11	0.50%	
6	All Call 4	(100%)		100%	78%	63%	(29%)			112%	(4,126)	(5,118)	4,680	-0.07	0.12%	
7	Collar ATM4	(100%)		93%		145%			(23%)	122%	(4,621)	(5,562)	4,302	-0.06	0.31%	*
8	Collar OTM4	(100%)		100%			240%	300%		540%	(5,294)	(10,067)	8,639	-0.08	0.60%	*
9	Strangle OTM4	(100%)		96%		111%	100%	121%	(63%)	269%	(4,208)	(6,290)	5,049	-0.03	0.19%	*
	ARAP = 2										*	*	*			

The basis HR in the risk tolerant table (5.15) is higher than what has been seen so far. It's been in the 70 to 80% HR range and is now 90 – 100%. Comparing the Mean-CVaR to the Mean-CVaR with copula for the risk averse hedger between tables 5.9 and 5.14, the married call strategies generally follow the same pattern as above. The optimal ITM allocation are higher than ATM and OTM and also is a better hedge according to E-CVaR copula. The allocations are similar to E-CVaR (table 5.9). The ITM and ATM allocation dropped 12% and 8% respectively while the OTM allocation was approximately the same.

The collar strategies behaved differently than anticipated. The HR dropped significantly by decreasing the call allocation and increasing the short put slightly. The opposite happened for the OTM collar. The call position increased slightly while the short put position decreased by 42%. The allocations for the strangle and short butterfly are similar to E-CVaR with spearman correlation. The deltas of the E-CVaR copula portfolios are slightly more negative for copula correlation. Gamma and vega are positive across the board and theta is negative. That information is available in appendix table 3A.

For the risk tolerant hedger, tables 5.10 and 5.15 are compared. The married call positions are again less by about 10% for ITM and ATM but the OTM position shot up 56%. The ATM collar HR increased substantially after the optimized short call position dropped significantly. This increases delta to zero and dropped gamma by .11%. The OTM allocations were similar to table 5.10. Lastly, the strangle doesn't blow up with the copula. The positions are similar to a perfect strangle with a larger position in the ATM call then put.

One other result worth pointing out is the allocation to options increases as risk aversion increases. Tables 5.16 show the ratio of options as a percentage of the overall portfolio between high risk aversion and low risk aversion with an E-CVaR objective. This shouldn't be a surprise, but the allocation increases 13% on average with an E-CVaR copula function versus a 7% average increase in the same trials with spearman correlation. Table 5.16 provides the ratio of option allocation for each strategy. Supplemental information is provided in appendix tables A3 and A4.

Table 5.16: Options Allocation

Hedge	Spearman	Copula
ITMC4	1.03	1.09
ATMC4	1.09	1.08
OTMC4	1.08	1.23
All Call 4	0.99	1.15
Collar ATM4	0.92	0.95
Collar OTM4	1.18	1.29
Strangle OTM4	1.11	1.05
Average	**1.06**	**1.12**

5.3. Futures versus Synthetic Positions

In the results so far, futures are not paired up with options in any strategy. Recall discussion about global maximum versus local maximum. When a futures contract is paired with an ATM collar position, both strategies have the same payoff function. There is little to no discrepancy between a 25% futures and 75% collar position versus a 75% futures and 25% collar position when measured by E-V or Mean-CVaR. The one difference would be volatility. In a futures position, the only variable that matters is price. With a long collar position, volatility has a significant impact on the value of that position. For example, imagine taking an ATM collar position when volatility is abnormally low in the market. Seven months later or prior, whenever the hedge is unwound, volatility jumps in the market. The hedger is able to unwind their option position at a profit, mainly because volatility is high in the market. In the same scenario a futures hedge would not provide that volatility premium. However, they wouldn't sacrifice any option premium if volatility stayed the same or dropped.

Table 5.17 shows the difference between the objectives for three different strategies where a long futures and long synthetic futures position (ATM collar) can be interchanged. The table displays values for E-V, CVaR, and Mean-CVaR at the two levels of risk aversion. The strategy that includes a real futures position performs better than the parallel synthetic futures position according to E-V, CVaR, and Mean-CVaR. Actually, all strategies that include a futures contract versus a synthetic futures position perform better. In this case, a synthetic futures position is made up of a long ATM call and short ATM put. One can conclude that anytime a synthetic futures position is created for an option strategy, it is optimal to substitute a futures contract is its place. However, many options strategies observed so far do not display perfect synthetic positions. Some are far from, especially as risk aversion decreases. The deltas are all neutral and displayed on the right side as well.

The nuance of options compared to futures are the gamma, vega and theta values are not seen in a futures position. From this table the differences are not shown. Look back at a strategy that compared a futures hedge with an ATM collar hedge. The vega and theta values are zero for a futures hedge but not for an options hedge. Also reference table 5.8 where the ITM and ATM hedges outperformed the futures hedge. Sophisticated option traders prefer options because they have more control over these three variables.

Table 5.17: Futures Versus Synthetic Option Positions

Hedge	PNW	Futures	PNW Basis	ATMC4	OTMC4	OTMP4	ATMP4	E-V (.0000001)	EV (.00001)	CVaR	Mean-CVaR (10)	Mean-CVaR (2)	Delta
F & B	(100%)	100%	100%					(95,211)	(22,107)	(4,460)	(4,333)	(22,173)	0.00
ATM4	(100%)		100%	100%			(100%)	(690,739)	(22,357)	(5,038)	(5,510)	(25,660)	0.00
Strangle w/futures	(100%)	100%	100%		100%	100%		476,059	(89,185)	(5,348)	(3,953)	(25,345)	0.00
Strangle	(100%)		100%	100%	100%	100%	(100%)	(116,180)	(91,702)	(5,937)	(5,141)	(28,890)	0.01
Short butterfly w/futures	(100%)	100%	100%	(100%)	100%	100%	(100%)	(554,105)	(28,246)	(5,127)	(5,413)	(25,922)	0.01
Short butterfly	(100%)		100%		100%	100%	(200%)	(1,149,669)	(28,500)	(5,703)	(6,588)	(29,400)	0.01
ARAP = 10									*	*	*	*	

5.4. Sensitivity Analysis

Indirect sensitivity analysis is the main subject of this study. The comparison between different objective functions that resulted in different allocations to similar strategies were shown above. In this section the risk aversion parameter and volatility are discussed. The decisions for the appropriate values assigned to risk aversion parameters for E-V and E-CVaR are discussed.

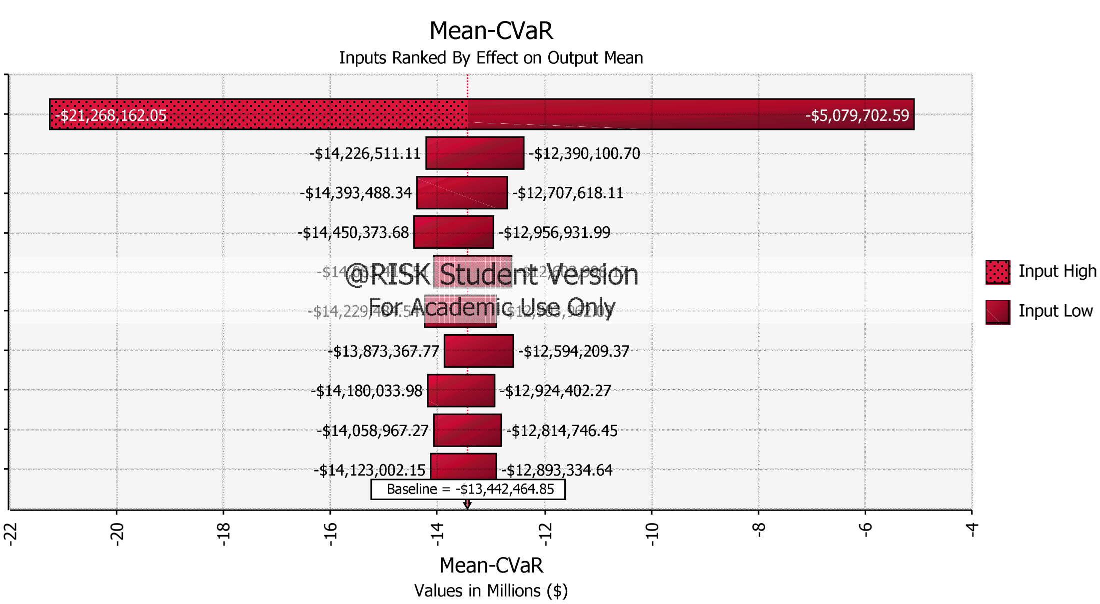

Figure 5.4: ARAP Sensitivity Analysis

Tables that display the effect of a changing ARAP are displayed. Sensitivity analysis to changes in volatility are also shown. The risk aversion parameter is the most sensitive parameter is the entire model. By creating a uniform distribution from two to ten and inserting it into the model, sensitivity analysis by @Risk confirms this. The top output shows the effect the risk aversion parameter has on the mean of the E-CVaR objective.

5.4.1. ARAP Sensitivity

The rest of the outputs are a mixture of options and futures contracts. The graph above shows the change in the mean of the objective given a high and low-risk aversion parameter. The right side represents a low input (two) while the right side represents a high input (ten). Therefore, it's important to have strong assumptions about what that parameter is set to for different risk preferences. There is a loose guide to what numbers would be appropriate. Recall Bullock and Hayes (1992) used .01 and .0001 for low and high-risk aversion for E-V optimization respectively. However, they are using futures which have substantially shorter tails within their payoff distributions than options would have. An ARAP of 1, .01, and .0001 all allocated approximately the same positions to a married call and collar hedging strategy in testing. .00001 deviated slightly from those allocations observed and .0000001 deviated quite substantially without showing an abundance of "irrationality." Therefore, those parameters were chosen to represent low and high-risk aversion respectively.

The standard risk aversion parameters for E-CVaR were developed the same way. Two and ten were identified as high and low risk aversion parameters. The absolute value of CVaR is much smaller than the variance. Therefore, the ARAP has to be much larger to compare similar levels of risk aversion across different objective functions. Generally, CVaR has been between three and ten million in this study. The mean of the cost distribution generally hovers around zero to one million. With an ARAP equal to ten, CVaR is multiplied by five (first its divided by 2; see equation 3.9) and subtracted from the mean. In this case CVaR is responsible for over 97% of the of the objective's value. However, when the ARAP is set to two, the optimizer allocates the max long positions for certain strategies. That results in a mean of closer to four million. CVaR is multiplied by one and subtracted from the mean. This results in a nearly a 75% contribution from CVaR and the rest is from the mean.

To supplement the discussion above, a table 5.18 is provided below to show the changes in allocation to a married call and collar strategy while the ARAP changes. Notice the absolute changes in the ARAP are smaller as the table goes on. The reason is the Mean-CVaR objective function in this model have CVaR and mean profit (cost) values that are relatively close from an absolute value point of view. The objective specifies phi (ARAP) over two multiplied by the variance. The closer phi gets to one, the more sensitive the equation is. Recall equation 3.9 from above. In the E-V function, this problem doesn't exist since the variance is so much greater than the mean profit (cost) value.

Table 5.18: ARAP Sensitivity

	Married Call	**Collar**	
ARAP	***ATMC4***	***ATMC4***	***ATMP4***
12	130%	105%	-70%
10	131%	107%	-70%
8	132%	109%	-66%
6	134%	113%	-63%
4	136%	119%	-58%
3	141%	125%	-48%
2	149%	140%	-34%
1.50	165%	179%	48%
1.25	184%	229%	226%
1	238%	300%	300%

First, the married call column shows an increase in allocation as the ARAP becomes smaller. This is observed in the tables above as well. Looking at the collar strategy as the ARAP changes is quite fascinating. At high levels of risk aversion, the optimizer allocates less to the put than the call, but a collar strategy is still deployed. As the ARAP decreases, the call allocation increases while it decreases for the put. Jumping from 2 to 1.5 the strategy changes to a straddle, illustrating a long position in the put. More allocation to each of those positions is observed as the ARAP goes to one.

5.4.2. Sensitivity to High Volatility

Sensitivity testing is also done on volatility. This model uses data from a low volatility environment. Since volatility is an important part of options pricing, the effects of different volatility assumptions should be known. The rest of the model assumed low volatility in the first and second stage. The first stage still assumes the same prices and spreads while the volatility increases in the second stage. Table 5.19 shows how allocations change moving from low to high volatility. The tests show how hedge ratios change for three strategies based on 25% and 60% increases in volatility. The active futures price change, the spreads in the 2^{nd} stage, as well as the volatility distributions were all multiplied by 25 and 60% for the tests.

Table 5.19: Sensitivity to Increasing Volatility

Volatility Increase	**Married Call**	**Collar**		**Short Butterfly / Strangle**			
	ATMC4	ATMC4	ATMP4	ATMC4	OTMC4	OTMP4	ATMP4
0%	131%	107%	-72%	66%	101%	81%	-104%
25%	143%	130%	-54%	96%	55%	106%	-104%
60%	162%	152%	-26%	116%	50%	113%	-85%

First, the married call HR increases as volatility increases. This should be obvious since the options are still the same price in the first stage but offer a larger profit in the second stage with the same or more downside protection. The allocations in the collar increase for the call and decreases for the put. This is the same observation that was made for the ARAP sensitivity. It can be concluded that the hedge ratio should increase with higher volatility or lower risk aversion. The last strategy tested was the strangle / short butterfly. In a high volatility scenario, the ATMC4 position increases to approximately a perfect collar while the OTM put allocation increases and the OTM call decreases. A perfect collar is 100% allocation to the call and -100% allocation to the put. With more volatility entering the market, it would make sense to build up larger positions in the OTM money options. However, this may be another effect that a backward futures market has on the model. For all three strategies, it's safe to assume the optimal hedge ratio increases and volatility increases.

5.5. Summary

The base case results show a futures hedge is the best hedge for a risk averse hedger. However, as the ARAP shrinks the optimal hedge consists of options instead based on the large upside potential. According to all three objective functions, ITM options are better hedges than ATM or OTM for all pertinent strategies. Those positions increase as the ARAP becomes smaller. Delta moves closer to zero or to slightly negative values while gamma generally moves higher or away from zero as risk aversion decreases. Closer to expiration options offer a better hedge according to the model. In the E-V results, strategies that consisted of six options performed the best. The ATM collar and strangle / short butterfly strategies performed approximately as well as the future hedge. The E-V alternatives in tables 5.6 and 5.7 show gamma increases and risk aversion decreases.

For the maximization of CVaR, the futures hedge was the best performing besides the 2nd deferred options and a couple "all call" strategies. The "all call" strategies aren't as practical since the same payoff function can be achieved using less options. Although the hedger has more control over delta and gamma in that strategy.

The selected strategies with an E-CVaR objective behave in a similar manner to E-V and CVaR maximization. The ITM allocations are less than the ATM and OTM optimal allocations. The optimal hedge ratios are also larger for a risk tolerant hedger. The ATM collar and strangle strategies both perform well in all tests. Generally, delta is neutral or slightly negative while gamma is moderately positive. Gamma increases as risk tolerance increases. The parallel copula objective had specific effects on the allocations. The call positions decreased around the board. The HR of the collar increased while the strangle didn't blow up in the copula version as it did assuming spearman correlation. Except for the collar, gamma decreased for all strategies while delta increased. This may indicate the spearman correlation overestimates risk relative to copula correlation.

Comparing the E-V and E-CVaR allocations, married call allocations were higher for E-V. This indicates E-V overestimates the risk. Lower delta values (negative) were observed for E-V while E-CVaR delta values were closer to zero. It seems the E-V objective function creates symmetrical payoff strategies for both levels of risk aversion while E-CVaR is more apt to create "one-side bets" in risk tolerant scenarios.

The ARAP has a significant effect on the model. The parameter sets how much utility is derived from the mean and how much is from the variance or CVaR. As the ARAP decreases, the optimal allocations increase. At a high level of risk tolerance, the hedger switches to more of a speculator as seen by the switch from a collar to a straddle at lower and higher levels of risk aversion. Volatility has a positive relationship with the hedge ratio. In the married call and collar strategy, the net hedge ratio increased as volatility increased.

In general, a futures hedge, collar, ITM married call, or short butterfly / strangle strategy are the best performing hedges. Futures are the best hedge for a risk averse buyer while various option strategies are more attractive for lower risk averse buyers. Each of these is optimal for different volatility biases in the market. A futures and collar hedge are optimal when no bias is present. A short butterfly is best when low volatility is expected. A strangle or straddle is best when volatility is expected to increase which may accompany a large price move in either direction.

CHAPTER 6. CONCLUSION

6.1. Introduction

The risks from operating a business largely reliant on commodities is a risky venture. Thankfully, there are futures and derivative markets in many different commodity markets. Oil, corn, soybeans, and gold to name a few. Firms involved in trading and using these commodities often hedge their price risk one-to-one in the futures market. Therefore, they eliminate the futures price risk and lock in their net buying price. However, firms also want to make money. They also don't want to go long near the top of a market only to see it drop off a cliff and take a large hedging loss on their books. Even though they'll be able to buy the physical commodity at much lower prices, a large hedging loss is not something any company likes to see. This is where options and other derivatives can aide firms because options provide asymmetric payoffs. One can take advantage of any favorable move and still set a ceiling on the net price they'll pay in the future. Options can also be bought and sold together to create more attractive positions.

In a seminal paper around the world of finance and investment, Markowitz (1952) published what's now known as modern portfolio theory. He derived the variance of an n asset portfolio for the first time. The variance of an entire stock portfolio could be measured with a few simple statistics. One problem was traders only cared about their downside risk. Thus, many different downside risk measures were adopted. In another famous work, Markowitz (1959) illustrated the mean-variance objective which is a quadratic utility function. One could now measure utility from return and risk in one objective value. This methodology has been adopted in the commodity trading (hedging) world and the framework has been applied to many hedging problems involving futures. However, there has not been much research in the realm of options within a hedging portfolio. Through the derivation of the greeks, traders are able to manage a portfolio of options with very specific qualities.

The goal of this study is to find out which alternatives would be an optimal hedge under risk preferences using different measurement techniques. Allocations are optimized and statistics are displayed from the portfolio. The previous chapter identified the problem and potential solutions for it. The theoretical and empirical models were presented including a background on options strategies. Results were displayed, analyzed and interpreted for many scenarios. This chapter reviews the problem in the study and explicitly identifies the problem. It briefly touches on the models from chapters three and four. Prominent results are discussed and implications are identified. An outline of the contribution this study makes to the existing literature follows with limitations and suggestions for further research as well.

6.1.1. Problem Statement

There are many risk factors in commodity oriented business. One of the main risks for a buyer or end user is if prices move higher. This increases the cost of inputs and thus lowers margins. Buyers are faced with many alternatives to hedge with. Various forward contracts, exchange-traded futures contracts, and a plethora of derivative strategies can be used. One solution is to hedge the risk with the futures market. A hedger can take an equal and opposite position in a futures contract and effectively lock in the price they pay for the commodity at a certain time in the future today. There are still risks left to manage. Basis can’t be hedged on an exchange. It must be hedged with a forward contract with the opposite party directly. The other risk are margin calls. In a futures contract, a hedger only has to initially put up around 3% of the notional value. If that position moves against the hedger, a margin call may happen after a certain price level is reached. This can be a strain on the working capital of the firm.

One other problem with strictly futures oriented strategies is firms don't like to take a large hedging loss. Even though the futures hedge is meant to lock in prices, it doesn't look good to absorb a large hedging loss on the books. Recall what happened to Delta Airlines in 2008 when prices shot up, looking to go higher and Delta hedged their short cash position when oil was well above $120. Shortly after, Delta revamped its approach to hedging which involved more reliance on derivatives including swaps and options.

6.1.2. Commodity Trading Today

Commodity traders are divided into two groups; hedgers and speculators. Hedgers are the subject of this study. They're the ones taking physical positions and shipping commodities around the globe. Speculators do not involve themselves in the physical assets. They only "speculate" on price in the futures and derivatives markets. Speculators are often blamed for moving the market outside of "equilibrium." However, the fact is that the opportunity to hedge would not be there without speculators in the market. Speculators provide liquidity, the main driver for the existence of a market.

Hedgers and speculators also differ in their trading sophistication. Hedgers often use the futures market to hedge their underlying position one-to-one. This reduces futures price risk to near zero. The door is still open to going the wrong way on a hedge and taking a large hedging loss. Speculators and sophisticated hedgers manage their risk with other alternatives. This includes a combination of futures, options, swaps, and other related derivatives. Hedgers can employ these strategies to put themselves in a more favorable position based on biases they may have. This may or may not satisfy their preferences for risk.

6.2. Methodology

Along with Markowitiz (1952, 1959), Johnson (1960), Ederington (1979), Brown (1985), and Myers and Thompson (1989) used the variance of a portfolio to derive MV hedge ratios for a variety of financial assets in a portfolio. That framework can be used to optimize a portfolio according to other statistics as well. Hedgers are typically worried about the return of the hedge as well as risk. Therefore, the objective function should incorporate this. The E-V utility function is one way to compare portfolios against each other. The E-V framework can be unpopular because it penalizes the objective for long right tail the same way it does for a long left tail. Hedgers typically welcome upside risk while trying to avoid downside risk.

Rom and Ferguson (1994), Harlow (1991), both provided semivariance measures to account for downside risk aversion only. Mausser and Rosen (1999), Rockafellar and Uryasev (2000) both provided some insight to value-at-risk and its behavior during stochastic optimization. VaR became the preferred risk measure of the financial industry in the early 2000s. Alexander and Baptista (2002) analyzed a Mean-VaR objective function, similar to E-V but had VaR replace the variance. (Artzner, Delbaen, Eber, and Heath; 1998) identified four axioms that qualified a risk measure as coherent. VaR lacked two of these characteristics including subadditivity. The lack of subadditivity means the risk of two assets together can be more than the risk of each asset independently. This axiom can be violated using VaR. Conditional value-at-risk (CVaR) is a coherent measure of risk and is used as the preferred risk measure instead.

For a model consisting of options, one must use some sort of price approximation tool if option pricing data cannot be extracted. Futures pricing data must be extracted from a data source. Delta-gamma approximation and the Black-76 model are two models that work well. Various assumptions have to be made for each model.

Once the model has pricing mechanisms, the objective functions are formed. E-V, CVaR, E-CVaR, and E-CVaR with copula are maximized in this study. An optimization tool is also needed. The tool is used to find optimal allocations using different objective functions under different risk preferences. Greeks of the portfolio can be measured to give the hedger a better idea of where their risk lies. These objective functions are compared and contrasted to identify the best hedging strategy available.

6.3. Empirical Model

Common option strategies are used to evaluate selected hedging strategies using different risk preferences and objective functions. Married call, collar, short butterfly spread, and a strangle are the core strategies studied. Payoff functions are shown at the beginning of chapter 4.

The model is specified in two stages. In the first stage, random draws are taken from price distributions for the 4^{th} deferred futures contract, the basis, and all 18 options. The 4^{th} deferred futures contract refers to the 4^{th} soybean futures contract from expiration. The active, 2^{nd} deferred and 3^{rd} deferred will all expire prior to the 4^{th} deferred contract. Those distributions were derived from pricing data extracted from ProphetX. The data is correlated and specified using a spearman correlation matrix and a t copula matrix. Details on the procedures to correlate the prices are explained in chapter 4.

In the second stage, a pricing model is used to determine prices of the options at 28 weeks. Price change distributions are used for the active futures and basis distributions. Since futures are priced at both stages, the Black-76 model is used to price the options. Volatility is a stochastic variable derived from the implied volatility needed to make the price in stage one equal to the modeled price is stage two. That distribution is then used as an input into the model in the second stage. To obtain the profit or cost of the hedge, allocations are made according to the size of the underlying cash position. 100% indicates a HR of one. The model is set up to capture the mean, variance, greeks, etc. Objectives are set up to capture the E-V, VaR, CVaR and Mean-CVaR values. A separate model with the copula version is also used. The greeks are determined through the equations at the end of chapter 3.

Once the model is set up and the objective functions are all defined, the optimization process can begin. Using the Risk Optimizer tool within @Risk, any combination of options can be chosen to allocate capital to with the goal of maximizing the chosen objective function.

6.4. Results

The base case results in table 5.1 for E-V indicate futures and basis offer the best hedge. Individual hedges with basis only and futures only are also shown. Hedging basis reduces risk by about 4.4%. The rest of the risk is associated with the futures price. The ATM married call hedge is much more attractive than the unhedged position. A futures hedge is preferred to the married call hedge by a wide margin and it also costs less as measured by the initial margin.

Other base case results compared naïve hedges to optimized hedges according to E-V in table 5.2. The results show a preference for a larger HR than one. The optimized results show a larger allocation to OTM followed by ATM and ITM. The delta values are all fairly close to zero meaning a price move in the underlying will not affect the net value of the hedged portfolio everything else held equal. The more OTM an option is, the more allocation has to be given to that option to become delta neutral. Gamma is highly positive in all scenarios. A price swing either way will benefit the hedger.

The alternative E-V strategies in table 5.3 point to a multi-calendar collar as the best strategy, followed by strategy 6 (all 4th deferred options included), the ATM collar, and finally the short butterfly spread. The first two strategies aren't very practical. They're reported to demonstrate that the optimization model with improve as more options are added to the problem. The collar was nearly a perfect collar. The short butterfly position was about a half of a typical butterfly spread. A net short position in the ATM call and puts were approximately 100% with about 50% allocated to each OTM option.

Tables 5.4 through 5.7 show the effect risk aversion has the optimal hedge ratios for E-V. Table 5.2 and 5.4 and nearly identical. Tables 5.4 and 5.6 represents high risk aversion while tables 5.5 and 5.7 represent low risk aversion. For a risk tolerant hedger optimal HRs and gamma increase. The best performing strategies also change to options based strategies rather than futures.

The CVaR objective results (table 5.8) shows smaller allocations to the three married calls than E-V. The ATM collars are also much more call heavy while the OTM collars show more equal positions between to the two. The strangles look similar to the strangles for E-V. In the ITM, ATM, and OTM all call strategies, a large position is taken in the ITM option, a smaller but substantially short position is taken in the ATM option, while smaller long positions are taken in the OTM. This is an effort to stay gamma neutral which is positively correlated with a less negative CVaR value. Futures was the best hedge again. The ATM collar and strangle also performed well.

Mean-CVaR results are similar to E-V and CVaR. The married call allocations behave the same between ITM, ATM, and OTM and across risk preference scenarios. The ATM collar is weighted heavily toward the call. The payoff actually looks like a straddle for both risk scenarios. The OTM straddle with high risk aversion is close to having a HR of zero with long and short allocations around 150%. The same strategy for risk tolerance blows up into an ultra-long volatility straddle. The risk averse strangle is nearly perfect while the risk tolerant strangle allocates the maximum allowed to the OTM put while carrying a negligible position in OTM call. It's challenging to spot a difference in delta, but gamma is greater is the risk tolerant scenarios.

E-CVaR with a t copula as opposed to spearman correlation showed mixed results as to which one over or underestimated risk. The optimized married call positions showed smaller HR's with a copula except for the OTM call is the risk tolerant table (table 5.15). Both collars behaved similarly to the E-CVaR without a copula. The strangle changed allocations slightly and all positions are close to 100%, even in the risk tolerant scenario. The best performing hedge for the risk averse strategy was the "all call" strategy followed by the ITM married call, strangle, and ATM collar. Optimal option HR's increased by 6% moving from a spearman correlation to a t copula.

6.4.1. Results Summary

Comparing the allocations of all four objective functions, E-CVaR with copula has the smallest HRs for the married call positions with the largest optimal positions showing up in the E-V objective. The ATM collar allocations are different across the board. In the E-V tables, approximate collar and straddles are observed for risk averse and risk tolerant hedgers respectively. CVaR approximately shows a perfect collar as well. Allocations increased for the call and decreased for the put in the E-CVaR objectives, placing the HR closer to 80% under both risk preferences. The HR shrank to 30% for the risk averse hedger and grew to 120% for the risk tolerant scenario using E-CVaR copula. Lastly the short butterfly with an E-V objective in a risk averse scenario changes to a straddle under low risk aversion. The gamma goes from -.12% to .06%. CVaR and both E-CVaR objectives under high risk aversion have similar allocations to a perfect strangle. The same strategy with a copula for E-CVaR behaves the same with larger positions in both OTM options and 50% higher allocation to the ATM call than put. It also has a significantly higher HR and gamma value. The E-CVaR with spearman correlation is still a strangle but with a max OTM put position and no OTM call position. The ATM options become slightly call biased.

Futures is the best hedge for a risk averse hedger. Options strategies provide better hedges for buyers with low risk aversion. Objective functions with variance as a risk measurement (E-V) prefer symmetric payoff functions (straddle, strangle, butterfly) while semivariance objectives (CVaR, E-CVaR) will prefer asymmetric payoffs in some scenarios. ITM calls, collars, strangles, and short butterflies all provide well performing hedges using the four objective functions. The personal biases of the risk tolerant hedger should further decide which option hedging strategy is best for themselves.

6.4.2. Implications

This study separates risk averse and risk tolerant hedgers into two categories to compare potential preferred strategies. Futures are the best hedge under high risk aversion for all strategies. For low risk averse hedgers, options are more attractive. The options strategy ultimately chosen by the hedger would be based on personal biases. ITM married calls, ATM collars, strangles, and short butterfly spreads were the best performing core strategies in this study. In practice, these can be coupled with swaps and even more derivatives.

This result may have large implications on the grain trading industry. Some firms apt to take more risk or who under pressure to increase their bottom line may be inclined to deploy derivative hedging strategies instead of futures. There's also an argument for risk averse hedgers to pursue derivative strategies. Delta was mentioned in chapter one because they naively hedged oil when it was close to its peak price and had an enormous hedging loss on the books. This scenario can be avoided with derivatives. They later hired a new director or hedging and employed complex strategies like "cap-swap double-down extendable" which basically entails a swap with a less attractive price in exchange for an option to buy additional quantities of the underlying at the swap fixed price that is also extendable for a certain amount of time under the agreement. They started making money in their hedging department. (Kelly, 2015)

6.5. Limitations

The model became quite complex. Deriving and deciding on the appropriate distribution along with developing correlations was the most difficult part. One also has to assume the black-76 pricing model is an accurate pricing model to use in the second stage. A brief but detailed list of limitations follows.

First, the active contract is defined by a gamma distribution with 50% of the prices under $10.11. It has a long right tail with a maximum price at $16.50 per bushel. This shape of this distribution affects the deferred contracts which affects the option prices in the second stage. The deferred contracts were modeled as spreads from the active. The empirical distributions showed bimodal-like distributions in the left tail of the spreads that could not be model appropriately. This may be the reason the 2nd deferred hedges performed better than the 4th and 6th. The 2nd deferred contracts had a higher probability of expiring ITM and there was little probability a large negative price move (modeled as the spread) could occur between the 2nd and 4th deferred contracts.

This model correlates parameters for implied volatility to the rest of the stochastic variables. A distribution was extracted but questionable methods were used to add on to the existing correlation and copula matrix. In a perfect world, implied volatility data would be available for extraction and correlations could be derived through standard procedures.

The last limitation of the model involves margins. For strategies that included calls and puts, especially more than one of each, there's no formula to determine margins. They are determined through simulation models in practice. Accurate margin data would be helpful to a hedger when deciding which strategy to deploy. The model also doesn't use margins in the utility functions. In practice, a large initial margin is unattractive and should be considered when determining optimal hedging strategies.

6.6. Summary

Buyers today typically place their hedges one to one against the underlying commodity. Speculators are known for more advanced derivative strategies. Some large corporations may employ these types of strategies, but the bulk of hedging today is done through the futures market. A portfolio model of hedging framework is used to construct an optimal hedge. The framework is based on Markowitz (1952) which is the most prominent publication in finance and more specifically, portfolio theory to date. CVaR is a downside risk measurement that is used instead of VaR. The model has two stages where prices are determined from a random pull of a stochastic option price in the first stage and priced by the Black-76 model in the second stage. Futures, basis, and options are all used to determine the best hedge according to an optimization procedure using @Risk. Implied volatility is derived based on the different prices observed in each stage. Those differences are fit to a distribution and put back into the model. Spearman correlations and a t copula are both used to relate prices and volatility. E-V, CVaR, E-CVaR, and E-CVaR with copula are used as objective functions to maximize. The greeks are captured after every simulation.

The results show a futures hedge is best for a risk averse hedger. Various option strategies perform better than futures for a risk tolerant buyer. The best option strategy should be decided based on the buyer's biases. Married calls, collars, and strangles / short butterflies all performed well in low risk aversion scenarios. For practical reasons, risk averse hedgers should also explore options strategies as they can "hedge out" any large hedging loss.

6.7. Suggestions for Further Research

The intent of this study was to determine the optimal hedging strategies using futures, basis, and options. Historical option data is very scarce and having real options prices was a priority in this study. However, as of this study no robust portfolio of hedging models involving the wide array of options included in this study has been published to my knowledge. An easy application for further research would be to set the same model up with no bias. Meaning to use a uniformly distributed active contract and normal distributions for the spreads and price change variables. Use the Black-76 model to price the options at both stages while using a symmetrical distribution for implied volatility. Some kind of simulation model for calculated margins similar to (SPAN) should be imbedded within the model as well. Further research can also be done to include other types of derivatives including swaps, swaptions, and other exotic derivatives if possible.

The most intriguing related research would be to find the optimal moneyness of options to hedge with. Meaning to find the perfect option in the chain that is ITM or OTM. This model explicitly assumes each are a dollar from the ATM prices. I'm not sure how a model would be constructed or are aware of any research related to the subject either.

BILIOGRAPHY

@Risk 7.5 User Guide. (2016). 152. Retrieved October 15, 2017.

CBOE Margin Calculator (2017). Retrieved September 30, 2017, from http://www.cboe.com/trading-tools/calculators/margin-calculator

Alexander, G. J., & Baptista, A. M. (2002). Economic implications of using a mean-VaR model for portfolio selection: A comparison with mean-variance analysis. *Journal of Economic Dynamics and Control*, *26*(7), 1159-1193.

Alexander, S., Coleman, T. F., & Li, Y. (2006). Minimizing CVaR and VaR for a portfolio of derivatives. *Journal of Banking & Finance*, *30*(2), 583-605.

Allais, M. (1953). L'extension des théories de l'équilibre économique général et du rendement social au cas du risque. *Econometrica, Journal of the Econometric Society*, 269-290.

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, *9*(3), 203-228.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), 637-654.

Black, F. (1976). The pricing of commodity contracts. *Journal of financial economics*, *3*(1-2), 167-179.

Blank, S. C., C.A. Carter, and B.H. Schmiesing. Futures and Options Markets – Trading in Commodities and Financials, Englewood Cliffs, NJ: Prentice Hall, 1991.

Boubaker, H., & Sghaier, N. (2013). Portfolio optimization in the presence of dependent financial returns with long memory: A copula based approach. *Journal of Banking & Finance*, *37*(2), 361-377.

Brown, S. L. (1985). A Reformulation of the Portfolio Model of Hedging. *American Journal of Agricultural Economics, 67*(3), 508.

Bullock, D. W., & Hayes, D. J. (1992). Speculation and hedging in commodity options: A modification of Wolf's portfolio model. *Journal of Economics and Business*, *44*(3), 201-221.

C. (n.d.). Basis Trading for Cash Merchandisers. *CBOT Marketing and Education Publications*.

Campbell, R., Huisman, R., & Koedijk, K. (2001). Optimal portfolio selection in a Value-at-Risk framework. *Journal of Banking & Finance*, *25*(9), 1789-1804.

Cecchetti, S. G., Cumby, R. E., & Figlewski, S. (1988). Estimation of the optimal futures hedge. *The Review of Economics and Statistics*, *70*(4), 623-630.

Chen, S., Lee, C., & Shrestha, K. (2001). On a Mean? Generalized Semivariance Approach to Determining the Hedge Ratio. *Journal of Futures Markets, 21*(6), 581-598.

Chen, S. S., Lee, C. F., & Shrestha, K. (2003). Futures hedge ratios: a review. *The Quarterly Review of Economics and Finance*, *43*(3), 433-465.

Chen, S., Wilson, W. W., Larsen, R., & Dahl, B. (2015). Investing in agriculture as an asset class. *Agribusiness*, *31*(3), 353-371.

Chen, S., Wilson, W., Larsen, R., & Dahl, B. (2016). Risk Management for Grain Processors and "Copulas". *Canadian Journal of Agricultural Economics/Revue canadienne d'agroeconomie*, *64*(2), 365-382.

Cheung, C. S., Kwan, C. C., & Yip, P. C. (1990). The hedging effectiveness of options and futures: A mean-Gini approach. *Journal of Futures Markets*, *10*(1), 61-73.

Clarke, R., De Silva, H., & Thorley, S. (2011). Minimum-variance portfolio composition. *The Journal of Portfolio Management*, *37*(2), 31-45.

Correlation (Pearson, Kendall, Spearman). (n.d.). Retrieved October 30, 2017, from http://www.statisticssolutions.com/correlation-pearson-kendall-spearman/

Data Transmission Network (DTN) ProphetX. 2017. Chicago Mercantile Exchange Soybean Futures and Option prices. Retrieved from DTN ProphetX

Ederington, L. H. (1979). The Hedging Performance of the New Futures Markets. The Journal of Finance, 34(1), 157-170.

Fishburn, P. C. (1997). Mean-Risk Analysis with Risk Associated with Below Target Returns. *The American Economic Review, 67*, 116-126

Geman, H. (2005). Commodities and commodity derivatives. *Modeling and Pricing for Agriculturals, Metals and Energy, Chichester (Grande-Bretagne): Wiley Finance*.

Harlow, W. V. (1991). Asset Allocation in a Down-side Risk Framework. *Financial Analysis Journal, 47*(5), 28-40

Holton, G. (2016, October 19). Black (1976) Option Pricing Formula. Retrieved September 20, 2017, from https://www.glynholton.com/notes/black_1976/

Howard, C. T., & D'Antonio, L. J. (1984). A risk-return measure of hedging effectiveness. *Journal of Financial and Quantitative Analysis*, *19*(01), 101-112.

Hsin, C. W., Kuo, J., & Lee, C. F. (1994). A new measure to compare the hedging effectiveness of foreign currency futures versus options. *Journal of Futures Markets*, *14*(6), 685-707.

Johnson, L. L. (1960). The Theory of Hedging and Speculation in Commodity Futures. *The Review of Economic Studies, 27*(3), 139.

Jorian, P. (1997). *Value at Risk: The New Benchmark for Controlling Derivatives Risk*. Chicago, ND: Irwin Professional Publishing.

Kahl, K. H. (1983). Determination of the Recommended Hedging Ratio. *American Journal of Agricultural Economics, 65*(3), 603.

Kelly, K. (2015). *The Secret Club that Runs the World: Inside the Fraternity of Commodities Traders*. Penguin.

Kimura, N. (2016). *Hedging Default and Price Risk in Commodity Trading.* Unpublished master's thesis, North Dakota State University, Fargo, North Dakota.

Li, D. X. (2000). On default correlation: A copula function approach. *The Journal of Fixed Income*, *9*(4), 43-54.

Lien, D., & Tse, Y. K. (1998). Hedging time-varying downside risk. *Journal of Futures Markets,18*(6), 705-722.

Lien, D., & Tse, Y. K. (2000). Hedging downside risk with futures contracts. *Applied Financial Economics, 10*(2), 163-170.

Manfredo, M. R., & Leuthold, R. M. (2001). Market risk and the cattle feeding margin: An application of Value-at-Risk. *Agribusiness*, *17*(3), 333-353.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance,7*(1), 77-91.

Mausser, H. and Rosen, D. (1999). Beyond VaR: From Measuring Risk to Managing Risk. *ALGO Research Quarterly*. Vol.1, 2, 5-20.

McKay, R., & Keefer, T. E. (1996). VaR is a dangerous technique. *Corporate Finance Searching for Systems Integration Supplement. Sep*, 30.

Merton, R. C. (1972). An Analytic Derivation of the Efficient Portfolio Frontier. The Journal of Financial and Quantitative Analysis, 7(4), 1851.

Myers, R. J., & Thompson, S. R. (1989). Generalized Optimal Hedge Ratio Estimation. *American Journal of Agricultural Economics, 71*(4), 858.

Nielsen, L. T. (1992). *Understanding N (d1) and N (d2): Risk-adjusted probabilities in the Black-Scholes model*. INSEAD.

Ozun, A., & Cifter, A. (2007). Portfolio value-at-risk with time-varying copula: evidence from the Americas.

Pflug, G. C. (2000). Some remarks on the value-at-risk and the conditional value-at-risk. In *Probabilistic constrained optimization* (pp. 272-281). Springer US.

PNW Soybean Prices. (2016, December 30). *Milling and Baking News*, 40

Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *The Journal of Risk, 2*(3), 21-41

Roy, A. D. (1952). Safety First and the Holding of Assets. Econometrica, 20(3), 431.

Rom, B. M., & Ferguson, K. W. (1994). Post-Modern Portfolio Theory Comes of Age. *The Journal of Investing, 3*(3), 11-17.

Shalit, H., & Yitzhaki, S. (1984). Mean-Gini, portfolio theory, and the pricing of risky assets. *The Journal of Finance*, *39*(5), 1449-1468.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*. The Journal of Finance, 19(3), 425-442.

Sharpe, W. F. (1994). The Sharpe Ratio. *The Journal of Portfolio Management, 21*(1), 49-58.

Standard Portfolio Analysis of Risk (SPAN) - CME Group. (n.d.). Retrieved November 30, 2017, from http://www.cmegroup.com/clearing/risk-management/span-overview.html

Stoll, H. R. (1969). The relationship between put and call option prices. *The Journal of Finance*, *24*(5), 801-824.

Stuart, A., & Markowitz, H. M. (1959). Portfolio Selection: Efficient Diversification of Investments. *Or, 10*(4), 253.

Wilson, W. W. (1982). *Hedging effectiveness of US wheat futures markets*. Department of Agricultural Economics, North Dakota Experiment Station, North Dakota State University.

Wolf, A. (1987). Optimal hedging with futures options. *Journal of Economics and Business*, *39*(2), 141-158.

Zabolotskyy, T., & Vitlinskyy, V. (2013). The distribution of the characteristics of the maximum expected utility portfolio based on VaR: the impact of investor's risk aversion coefficient. *Economic Cybernetics*, (4-6 (82-84)), 4-1

APPENDIX

Table A1: Data Statistics

Contract	Active Futures	Price Change	2nd def	4th def	6th def	PNW Basis	Basis Change	ITMC2	ATMC2	OTMC2	ITMC4	ATMC4	OTMC4
Price	$ 1,057.4	0.0%	$ (16.0)	$ (20.4)	$ (12.6)	$ 116.1	-1.2%	$ 106.7	$ 35.9	$ 8.8	$ 117.6	$ 56.2	$ 23.5
St. Dev	$ 169.2	18.7%	$ 27.3	$ 28.9	$ 21.9	$ 36.4	35.1%	$ 7.4	$ 10.8	$ 6.6	$ 8.0	$ 9.7	$ 8.2
Volatility								19.4%	17.8%	18.5%	21.0%	19.6%	19.6%
St. Dev								2.5%	3.7%	3.8%	1.8%	2.0%	2.2%
Mean		$ (3,517)	$(130,359)	$ 130,711	$ (49,329)	$ (49,329)	$ (12,310)	$ (343,090)	$(1,204,564)	$(321,992)	$(1,098,864)	$(2,027,116)	$(865,109)
Prob > 0		47%	47%	54%	45%	45%	44%	51%	46%	32%	47%	39%	26%

Contract	ITMC6	ATMC6	OTMC6	OTMP2	ATMP2	ITMP2	OTMP4	ATMP4	ITMP4	OTMP6	ATMP6	ITMP6
Price	$ 104.1	$ 52.8	$ 25.6	$ 6.5	$ 35.9	$ 108.9	$ 18.7	$ 56.6	$ 123.8	$ 25.8	$ 66.6	$ 132.3
St. Dev	$ 26.9	$ 14.8	$ 8.5	$ 4.4	$ 9.9	$ 8.0	$ 6.9	$ 9.7	$ 9.3	$ 6.4	$ 8.5	$ 8.4
Volatility	19.9%	17.6%	17.8%	18.1%	17.0%	18.9%	17.3%	16.3%	14.7%	13.8%	12.8%	12.6%
St. Dev	3.7%	3.3%	3.2%	3.6%	4.5%	4.9%	2.2%	2.2%	3.0%	3.7%	3.4%	2.8%
Mean	$ (1,701,027)	$ (1,939,543)	$(940,677)	$(238,090)	$(1,082,317)	$(167,708)	$ (687,161)	$(1,573,749)	$ (336,682)	$(949,631)	$(1,134,755)	$ (81,767)
Prob > 0	43%	35%	23%	30%	42%	47%	29%	40%	45%	32%	41%	47%

Note: Description

The mean prices are listed in the first row. The standard deviation of those prices is listed directly below. Next, mean of the implied volatility distributions are listed for the options. The standard deviation of those distributions is listed below. Lastly, the mean profit assuming 100% position in each contract is listed. The probability that number is above zero is listed below.

Table A2: Supplemental Information to tables 5.12 and 5.13

	Hedge	Cash	Basis	Futures	Options	Delta	Gamma	Vega	Theta
1	Unhedged	100%				-0.98	0.00%	$ -	$ -
2	F & B	33%	33%	33%		0.00	0.00%	$ -	$ -
3	ITMC4	33%	33%		33%	-0.13	0.06%	$ 95,760	$(17,707)
4	ATMC4	33%	33%		33%	-0.25	0.08%	$ 112,638	$(19,294)
5	OTMC4	33%	33%		33%	-0.37	0.07%	$ 92,662	$(15,792)
6	Collar ATM4	25%	25%		50%	0.00	-0.02%	$ 306	$ (4,760)
7	Collar OTM4	25%	25%		50%	-0.17	-0.01%	$ 6,539	$ (3,530)
8	Strangle OTM4	17%	17%		67%	0.01	0.06%	$ 179,091	$(32,814)
9	Short Butterfly 4th	17%	17%		67%	0.01	-0.14%	$(270,543)	$ 30,082

Table A3: Supplemental Information to table 5.14

	Hedge	Cash	Basis	Futures	Options	Delta	Gamma	Vega	Theta
1	Unhedged	100%				-0.98	0.00%	$ -	$ -
2	F & B	35%	35%	30%		-0.07	0.00%	$ -	$ -
3	ITMC4	33%	33%		34%	-0.11	0.21%	$106,217	$(19,640)
4	ATMC4	30%	30%		40%	-0.15	0.34%	$195,225	$(33,440)
5	OTMC4	28%	28%		44%	-0.22	0.33%	$238,038	$(40,567)
6	All Call 4	30%	30%		40%	-0.10	0.18%	$114,752	$(21,241)
7	Collar ATM4	27%	24%		49%	-0.04	0.08%	$ 69,918	$(14,430)
8	Collar OTM4	22%	22%		56%	-0.10	0.13%	$161,027	$(29,696)
9	Strangle OTM4	18%	18%		64%	-0.01	0.04%	$ 78,297	$(16,817)

Table A4: Supplemental Information to table 5.15

Hedge	Cash	Basis	Futures	Options	Delta	Gamma	Vega	Theta
Unhedged	100%				-0.98	0.00%	$ -	$ -
F & B	35%	35%	30%		-0.07	0.00%	$ -	$ -
ITMC4	34%	29%		38%	-0.08	0.22%	$ 119,588	$ (22,113)
ATMC4	29%	29%		43%	-0.11	0.38%	$ 250,068	$ (42,834)
OTMC4	23%	23%		55%	-0.11	0.50%	$ 542,866	$ (92,517)
All Call 4	27%	27%		46%	-0.07	0.12%	$ 95,067	$ (17,091)
Collar ATM4	28%	26%		47%	-0.06	0.31%	$ 231,398	$ (39,877)
Collar OTM4	14%	14%		73%	-0.08	0.60%	$1,308,840	$(201,320)
Strangle OTM4	17%	16%		67%	-0.03	0.19%	$ 312,761	$ (51,705)