HEDGING DEFAULT AND PRICE RISKS IN COMMODITY TRADING

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ABSTRACT

Many risk factors exist in the commodity markets, especially those related to price and quantity. Recently, the risk of counterparty default has been increasing. The purpose of this study is to develop a portfolio-hedging model to hedge both price and default risks using exchange traded commodity futures and option contracts. Two approaches are taken to determine the optimal hedge ratios (HR) using futures and options: an analytical approach that mathematically derives closed-form mean-variance (E-V) maximizing solutions, and an empirical approach that uses stochastic optimization and Monte Carlo simulation under mean-value-at-risk (E-VaR) framework. Based on the analytical approach, we proved that utility-maximizing solutions exists. The empirical approach suggests that naïve HR (HR of one) leads to a suboptimal result. The minimum-variance, E-V, and minimum VaR objective functions generated the same optimization results. Additionally, a copula is applied instead of a linear correlation, and resulted a higher put option HR.

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CHAPTER 1. INTRODUCTION

1.1. Overview of Study

Many risk factors in any business operation caused instability for the firm's profitability. The instability creates more difficulty for the firm to operate in today's highly competitive and risky business environment. Managing risk is one of the most important functions of the business operation. Risk management is particularly important for commodity trading because the industry is highly competitive and risky, and the firm or trader does not control the price for the commodity that they are trading. The uncontrollable price fluctuation of the traded commodity is called price risk. In order to mitigate the risk, traders hedge the price risk with commodity derivatives that are traded at public exchanges. Also, traders can approach risk management mathematically via the portfolio theory. The main objective of this study is to develop a hedging model by applying the portfolio theory and to determine the optimal hedging decision for traders who are experiencing price and default risks. There are two approaches to analyze the optimal hedging decision. Theoretical analysis and empirical analysis. The theoretical analysis is derived, mathematically, as the optimal hedging decision. Whereas the empirical analysis uses stochastic optimization.

1.2. Problem Statement

Risks for commodity trading are price risk, quantity risk, weather risk, and operational risk as well as counterparty or default risk, and many others. Two types of risks are considered in this study: price and default risk. The price risk refers to fluctuation in the asset's price. To reduce the price risk, a trader hedges the commodity price with commodity derivatives such as futures or options. The hedging is an act of taking the opposite position in similar and related markets to reduce the price risk. When the underlying position is hedged, the loss from one market is offset by another market. Default risk is also considered in this study. The default risk describes when the counterparty defaults on the contractual obligation. Default risk is a special type of quantity risk has varying sizes for the quantity that is traded. Recently, the occurrence of defaults is increasing, and traders are more cautious about the default risk in the commodity market.

Due to a significant increase in demand for commodities, particularly in China, many traders encountered defaults from Chinese buyers. For example, China has significant purchasing power in the world soybean market because it is the largest buyer of world soybeans. China imported approximately 70.40 million metric tons during the 2013 to 2014 marketing year (Production, Supply, and Distribution). This purchasing power allows Chinese buyers to default on soybean shipments. The soybean defaults occurred during the 2003 to 2004 marketing year and in 2014 (Solot, 2006; and Thukral & Shuping, 2014). Figure 1.1 shows the soybean-default and soybean-crush margins in China's Dalian region (Thomson Reuter Eikon 2016d). The figure illustrates that a lower soybean-crush margin increases the chance and number of cancellations.

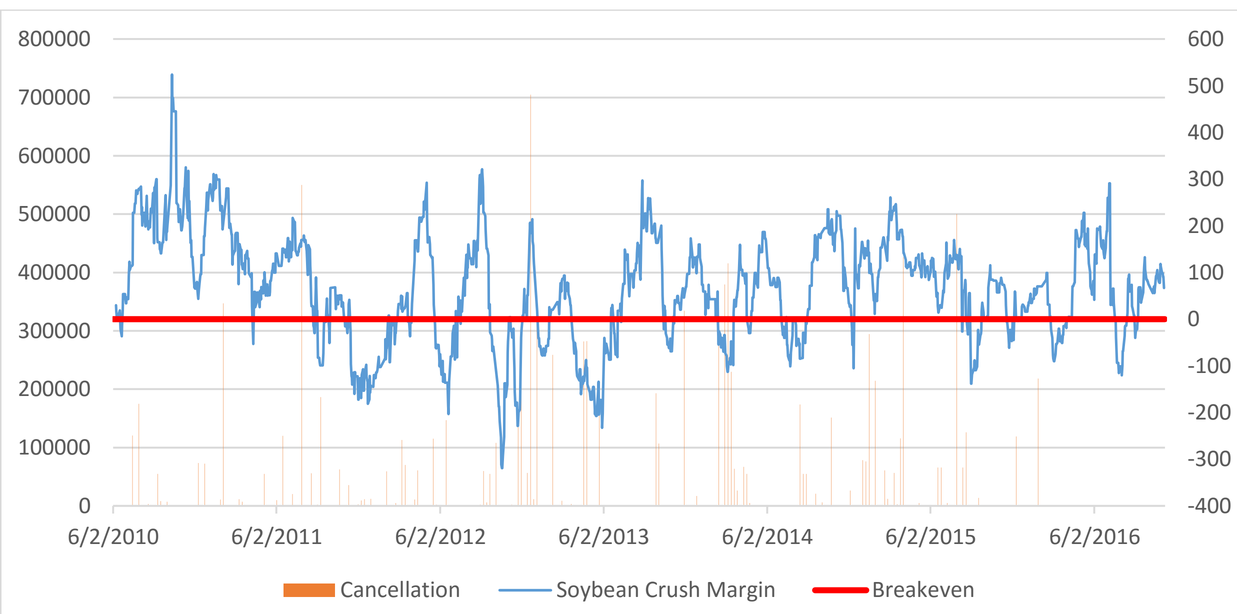

Figure 1.1. Default and Soybean-Crush Margins in Dalian, China.

Similarly, corn and distiller dried grain (DDG) are defaulted by many Chinese buyers due to the import ban of MIR162 varieties in late 2013 to 2014("China Rejects," 2013; Farm City Elevator, Inc., 2014). Corn and DDG are one of many cases of defaults that occurred with in agricultural products. Many other instances of defaults are seen not only in agricultural products, but also for other commodities. [1]

As an illustration to see the default's effect on the trade's profit and loss, suppose that a trader sells grain to an overseas buyer and hedges the entire sale with a futures contract. In this case, the trader has a short position in the cash market and a long position for the futures market. Suppose that the price declined, causing the buyer to default on the purchase contract. The default worsens situation because the loss from the futures position is magnified by a decreased return for the cash position. The futures contract has a limited ability to hedge both the price and default risks. Therefore, we introduce a put option to the hedging model in order to, primarily, hedge the default risk.

1.3. Portfolio Model of Hedging

Portfolio theory originates from Markowitz's (1952) seminal paper. One of the main objectives is to determine the optimal allocation for a risk-averse investor who is considering the portfolio's expected and variance return. When portfolio theory is applied to hedging for commodities, its main objective is to find optimal HR for the futures and option to either

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[1] Although this thesis addressed the buyer's default risk with international trading, similar problems exist for the seller's risk in some countries. As an example, the following message was received from a trader who worked with the former Soviet Union (FSU):

> *We started this year to actively originate and sell third party grain. Since June, we have purchased 650,000 tons for $100 million. In 2017, we want to trade 2 million tons for $300 million.*
>
> *While we measure and manage the price risk of our open trading position through a VaR calculation, I am concerned about our counterparty risk. For example, we buy grain forward from Ukrainian farmers and sell it forward to international traders. If grain prices spike in the meantime, farmers might default and we sit on losses, which our trading margin cannot cover. We therefore need to implement a system to measure and manage counterparty risk (Wilson, W., W., personal communication, October 31 2016)*

maximize or minimize the objective function. Researchers can specify several objective functions. This study includes but not limited to minimum variance, mean-variance (E-V), semivarinace, lower partial moments (LPM), mean-Gini coefficient, minimum value-at-risk (VaR), and mean-VaR (E-VaR). In this study, the E-V and E-VaR are used. Each objective function has advantages and disadvantages. For example, variance minimization is utilized to reduce the portfolio's variance. Similarly, the E-V framework maximizes the risk adjusted expected return function which approximates the trader's utility. The variance treats upside and downside risk equally. The equal treatment is the biggest disadvantage of using variance to measure risk because a risk averse trader prefers upside over downside risks. In contrast to variance, VaR is a downside risk measure and only considers the downside risk. This study focuses on E-V and E-VaR for the theoretical and empirical approaches respectively.

Traditionally, portfolio-hedging models assume a known cash position and only hedge with a futures contract. However, default risk is special type of quantity risk, and for this reason we have a put option in the study's objective function. The default risk is incorporated in the model by assuming that default risk has a probability distribution. Including put options offers more flexibility for hedging, and the asymmetric payoff allows the trader to hedge more efficiently hedge default risk. Integrating the default risk and put option with the hedging model's development is a major improvement compared to the traditional hedging model.

1.4. Theoretical Approach

A theoretical hedging model which incorporates the default risk and includes the futures and put options is specified. Constructing the theoretical model is based on Bullock and Hayes' (1992) paper. The theoretical model uses the E-V framework. The cash, futures, and default distributions are assumed to be normal. In order to derive the first and second moments of profit, Bullock and Hayes' (1992) theorem is applied. This theorem allows us to split the total payoff function at strike price of the put option. This theorem is useful to bypass the problem caused by the put option's asymmetric payoff. The major result from the theoretical analysis is the existence of a global optimum solution that maximizes the E-V function. Global optimum is proved by showing that the optimal futures and put option HRs exist and that the Hessian matrix is negative definite. This result is powerful because, at least theoretically, a solution exists.

1.5. Empirical Approach

The empirical analysis uses Monte Carlo simulation and stochastic optimization to estimate the optimal HRs under default risk. This analysis is conducted for soybeans and corn. The price distributions are lognormally distributed, and the default risk is assumed to be a Bernoulli distribution. The E-VaR is utilized as the objective function because using the VaR as a risk measure is appealing. The E-VaR is appealing because it has an expected profit component while, the VaR is a downside risk measure. The base case optimization result is derived to create a standard for analysis. Alternative hedging scenarios and different objective functions are assumed in order to compare the optimization results. Additionally, sensitivity analyses are conducted to see the effect of changing the variable's value and the correlation assumption to the optimization result.

From this analysis, traditional naïve hedging is a suboptimal hedging strategy, and surprisingly, the alternative objective functions lead to the equivalent optimization result. Another important result comes from using the copula function instead of the Pearson linear-correlation function. Copula leads to more realistic optimization results because the copula preserves correlation structure between random variables.

1.6. Thesis Organization

This thesis is organized as follows. Chapter 2 is the review of the related literature and the background of the problem. The chapter discusses defaults for both agricultural and non-agricultural markets as well as the academic literature for the hedging's portfolio model and its findings. Chapter 3 explains the theoretical model. The chapter focuses on mathematical constructions of the different hedging models and developing the theoretical model for analysis. Chapter 4 contains a detailed explanation about the empirical model and the data used for the analysis. In Chapter 5, both the theoretical and empirical Results are presented and discussed. Chapter 6 contains the study's Conclusions and implications.

CHAPTER 2. LITERATURE REVIEW

2.1. Introduction

Risk management is one of the paramount aspects of running successful agribusinesses. The managers can reduce risk by taking the opposite position for a commodity derivative such as futures or options in related markets to reduce the risk of physical-commodity price changes. The study's main focus is to derive the optimal HR by considering the existence of one of the counterparties' default risk. In commodity marketing, the default risk is a special case of production, or quantity, risk. If the buyer defaults on the agreed sale, the seller gains zero revenue. However, quantity risk can be hedged with commodity derivatives by taking the position based on portfolio-hedging models. The HR is the futures or options position that is taken with respect to the cash position's size. For example, if the HR is one, the size of the futures or option contract is equal to the cash position's size. In order to find the optimal HR, the hedger first needs to think about the definition of the risk for moving forward.

2.2. Risk in Agriculture and Commodity Marketing

In this study, the definitions for risk and uncertainty are treated differently to evaluate the default risk's effect on the HR. A risky event has probabilistic outcome whereas uncertainty is an event that cannot be associated with probability (Chavas, 2004). In other words, a risk has an associated probability distribution, but uncertainty does not have probability distribution. This distinction is important to analyze the default risk's impact on the optimal HR. Difficulty arises when the default event is uncertain because without knowing the likelihood of a default, building an analytical model is almost impossible. Because risk and uncertainty are defined, we first review the most common forms of risk in agriculture.

The most notable risks in agriculture are related to price and quantity. Price risk originates from fluctuating commodity prices. By definition, the price risk has associated probability distributions; therefore, the price risk can be measured by using the standard deviation or variance of a commodity's historical price or return during a specified time period. A higher price risk means greater standard deviations, and lower a price risk means lower standard deviations. Farmers and producers are subject to significant price risks if the cash market in which they participate has a high standard deviation for the price. For farmers, understanding of the behavior of the cash market where that they participate is a critical element of risk management (Tomek & Peterson, 2001). Farmers hedge the price risk of their cash position by using the commodity futures and options that are traded at centralized exchanges, such as the Chicago Board of Trade (CBOT), or take forward contract to set a delivery price and time. Without considering the basis risk, transaction cost, and storage cost, the farmer can hedge the entire cash position, which implies a HR of one, by selling an equal amount of futures in the futures market. The HR of one removes the return fluctuation that is caused by the physical commodity's price movement if the basis risk is zero. Any loss incurred from the cash position due to a decreased price is offset by profit from the short futures position and vice versa.

Similar to the price risk, yield risk arises from the quantity's fluctuation. This risk is felt by a commodity's farmers or producers and leads to revenue fluctuations. When a farmer hedges the price risk using futures and options to obtain more stable revenue, he/she has difficulty determining the hedging position's optimal size due to yield risk. They end up either under-hedging or over-hedging relative to the actual production at the end of harvest period. If the producer under-hedges or over-hedges compared to the actual harvest, the hedge may not be optimum.

2.3. Forward vs Futures Contracts

There is difference between a forward contract and a futures contract. Both forward and futures contracts are agreements between buyers and sellers. A forward contract is an agreement between two parties for the delivery of product at a specified future date and price which are written in the contract (Kolb & Overdahl, 2007). In contrast, a futures contract is a highly standardized type of forward contract with more specified contract terms (Kolb & Overdahl, 2007). Moreover, the futures contract is subject to the margin mechanism and clearinghouse procedures that do not exist with the forward contract. The futures contract has margin mechanism and counterparty risk is eliminated by clearinghouse.

The biggest difference between a forward and futures contract is that a forward contract is subject to the counterparty's default while the futures contract is not. The forward contract is subject to default risk because there are no or limited guarantees in the agreement. Default risk exists because the forward contract is done privately without a clearinghouse; at the same time, because the agreement is done privately, the contractual terms and specifications are negotiable. The reason why a futures contract is not subject to default is because the futures trading goes through organized exchanges such as the Chicago Mercantile Exchange(CME), and are required to go through a clearinghouse. The clearinghouse's job is to assure that every trader who participates in the futures trading honors the obligation. The clearinghouse matches the position of every buyer to every seller and the position of every seller to every buyer, and every trader in the market only has an obligation to the clearinghouse (Kolb & Overdahl, 2007). In addition to trading through the clearinghouse, traders must deposit money with a broker when they initiating trade in the futures market. This deposit, the margin, is a financial protection which forces the trader to follow the contractual agreement (Kolb & Overdahl, 2007).

2.4. Defaults in Commodity Marketing and Trading

Many risk types exist in the production and marketing of agriculture. They include yield and price risks. Default risk, or counterparty risk, has become more prevalent in agricultural marketing and commodity trading in general; hence, traders and risk managers find it necessary to hedge counterparty's default risk, i.e., contract non-performance risk. This section is an overview of defaults instances for agricultural and non-agricultural backgrounds.

2.4.1. Definition of Default

The first step in risk management is to clearly define the risk. The default risk is when one of the parties involved with a transaction reneges on the obligation. Jarrow and Turnbull (1995) identify two sources of default risk. The default occurs when payment is less than agreed payment or the writer of the derivative security defaults (Jarrow and Turnbull, 1995). Zhu and Pykhtin (2007) have similar definition that the counterparty credit risk is when the counterparty defaults before the contract matures and does not make all the promised payments.

2.4.2. Types of Counterparty Credit Risk

In this study, two types of counterparty credit risk exist in agriculture. The first type is a strategic default by one of the parties, and the second type is a non-strategic default. The difference between these two defaults types is simple. A strategic default occurs when defaulting on the purchasing contract is more beneficial than honoring the original contract. Historically, the strategic default occurs when market volatility increases tremendously. Non-strategic default takes place when exogenous, uncontrollable, factors, such as an import or export restriction, cause the counterparty to default on the purchasing and sales contracts. These default types are the dominant ones that occur in the commodity marketing and trading.

2.4.3. Impact of Defaults on Traders

The default's impact on the commodity traders is large because of a synergizing effect that comes from both the cash and futures positions. As an example, a U.S. based commodity trader sold soybeans to a crusher in China for $10 a bushel. The trader in the United States is short cash, and the trader hedges the short-cash position with a long-futures position. Suppose that the soybean price drops to $5 bushel, and that the buyer defaults on the contract. The trader has to find new buyer for the cash commodity at lower price than the originally agreed-price also losing the profit from the long-futures position ($5 bushel in this example). This simple example illustrates where the loss is synergized rather than offsetting because no return is coming from the cash position.

2.4.4. Chinese Buyers Defaulting on Soybean

The defaults for the grain-procurement agreement have become more prevalent in the agriculture sector, especially for international commodity trading. The most well-known default case for international commodity trading is by China's soybean buyers. Historically, China is the world's largest importer of soybeans. According to the Foreign Agricultural Service (FAS) at United States Department of Agriculture (USDA), China imported 70.364 million metric tons during the 2013 to 2014 marketing year, and this is approximately 62 percent of total world soybean export (U.S. Department of Agriculture, Foreign Agricultural Service, 2015c). The amount imported by China is almost six times greater than the second largest soybean importer, European Union, of 12.538 million metric tons in 2013/2014 marketing year (U.S. Department of Agriculture, Foreign Agricultural Service, 2015c). Due to the amount of soybeans that China imports each year, the country has huge buying power and advantages when buying soybeans from the world market. Figure 2.1 shows Chinese soybeans imports and its percentage compared to total world export.

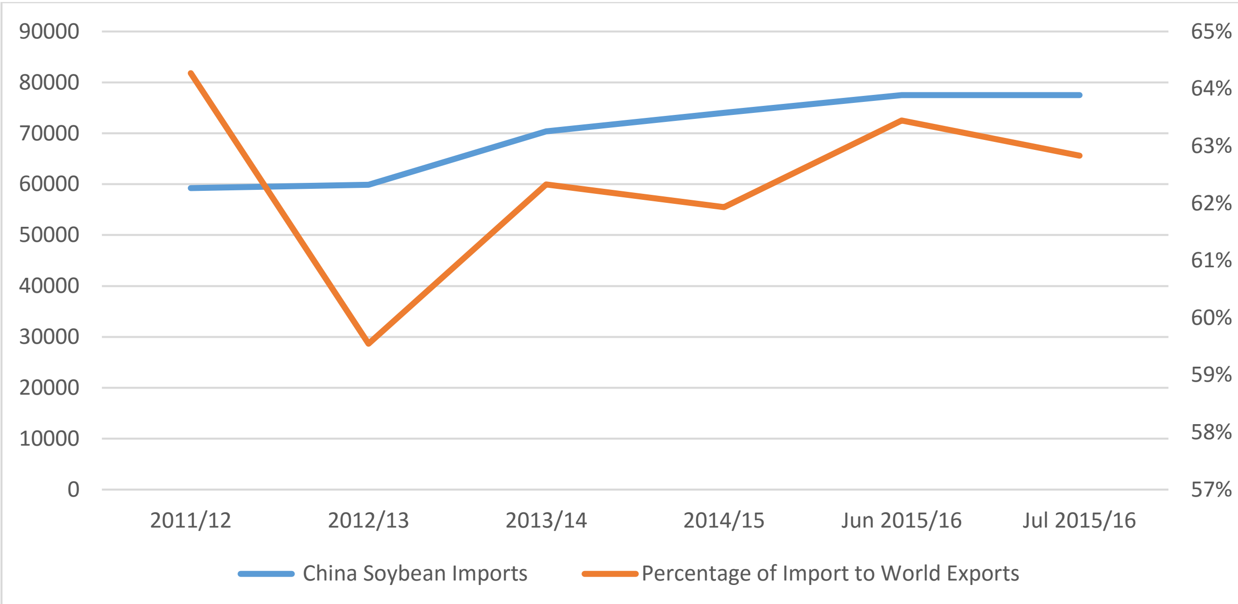

Figure 2.1. Chinese Soybean Imports and the Percentage of Imports to World Exports.

During the early 2000s, the soybean-crushing margin, the profit margin for crushing soybeans, in China was considerably high, but the margin decreased when the soybean price started rising. The margin turned and stayed negative during 2003 and 2004 (Solot, 2006). In April 2004, many Chinese soybean crushers contracted to purchase soybeans at $10 a bushel; however, during the delivery time in June to August, the price of soybean dropped to $6 a bushel, which leading to a default on the contract (Solot, 2006).

In 2014, some Chinese soybean buyers defaulted on soybean cargo because they failed to obtain a letter of credit, a document which is issued by a bank to guarantee that the seller receives full payment when the delivery condition is fulfilled. One goal for letter of credit is to reduce credit risks. When a buyer is unable to fulfill the purchase's payment, the bank steps in and pays the outstanding amount. Because Chinese importers could not obtain letter of credit from banks and had losses with soybean crushing, the Chinese importers defaulted on at least

500,000 tons, approximately $300 million, of the U.S. and Brazilian soybean cargo; this action was the biggest default that occurred since 2004 (Thukral & Shuping, 2014).

The Chinese soybean buyers' 2014 default case raises questions about why soybean exporters did not confirm the letters of credit before signing a purchasing agreement. After the 2004 default, soybean exporters stopped shipping soybeans to China without confirming the importer's letter of credit. This practice slowly started again and lead to the 2014 default case (Thukral & Shuping, 2014). Also, some trading firms relaxed the letter-of-credit requirement and accepted deposits from clients, particularly clients with well-established relationships (Topham & Shuping, 2014). With China's 2014 soybean default, some soybean cargo changed the destination of ocean shipping after the default incident. The two soybean shipments that were sold by the Japanese trading firm Marubeni Corp. contained Brazilian soybeans that were headed to China: those soybeans were rerouted to the United States (Plume, 2014).

The cancellation was derived from the USDA's export-sales reporting system, an export sales reporting program which monitors the 39 commodity sales made to foreign country from the United States on a daily and weekly basis. Commodity exporters are required to report the sale of more than 100,000 metric tons of a commodity to a single destination per day or more than 200,000 metric tons of total sales for single commodity to the one destination in a reporting week (U.S. Department of Agriculture, Foreign Agricultural Service, 2015a). These data are a proxy to illustrate the amount of cumulative sales cancellations for a single commodity that occurred during a particular week. Figure 2.2 represents the cancellation of U.S. soybean sales that were destined for China from 1990 to 2016.

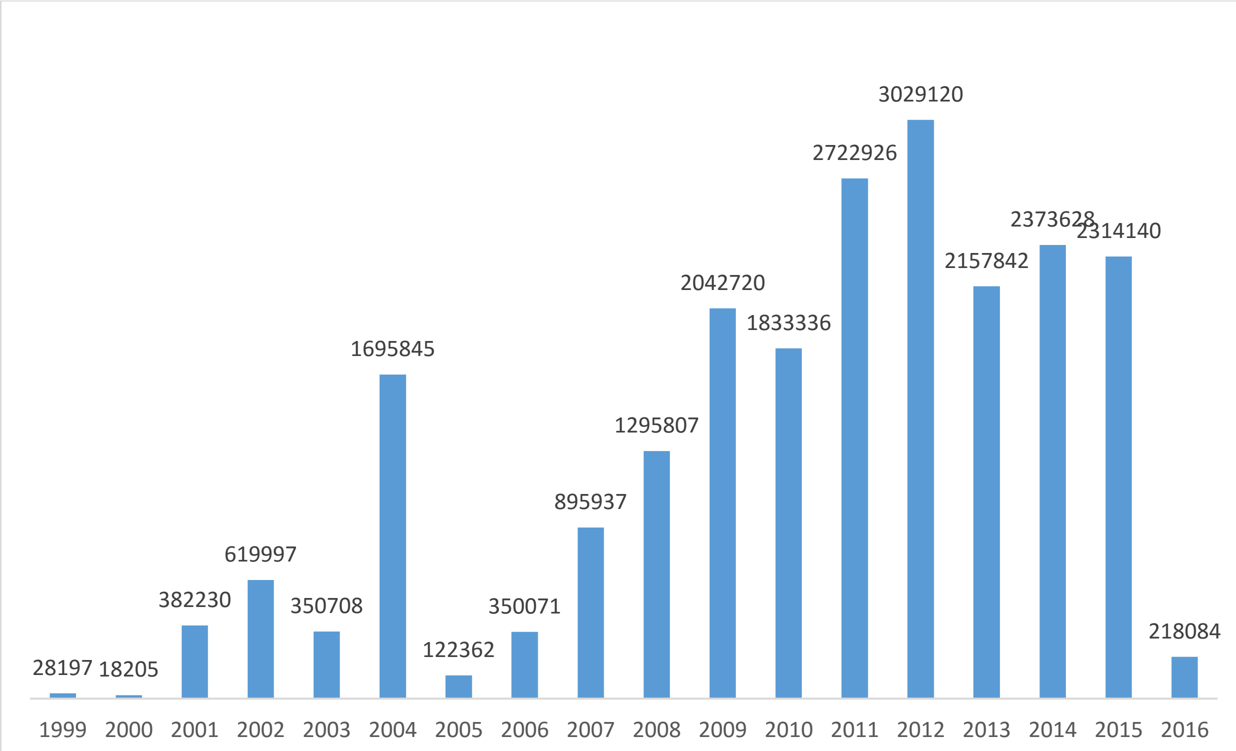

Figure 2.1. Cancellation of U.S. Soybean Sales to China in Metric Tons.

Since 2005, there was an upward trend for China's cancellation of U.S. soybean sales: the peak was in 2012. From 2013 to 2015, the cancellation level stayed above 2 million metric tons. Although soybean exporters may incur losses from Chinese soybean importers' defaults, the Chinese soybean market is the most lucrative one in the world due to the country's quantity of soybean imports. Because of the quantity of China's imports, Chinese soybean buyers have strong negotiating power over the commodity-trading firms that sell to China. Therefore, credit risk management is becoming more important in the world of commodity trading. According to personal communication with W. W. Wilson Chinese buyers purchased the U.S. soybeans exported from New Orleans, Louisiana region around $1.10 to $1.35 basis price. The soybeans basis price dropped to $0.28 due to cheap ocean freight rate and the competition from Brazil, the Chinese buyers may default on the original soybean contract (personal communication, November 23, 2016).

2.4.5. MIR162 Corns, DDG, and Sorghum Exports to China

The United States produced approximately 351 million metric tons of corn, 36% of the world's corn production, during 2013 to 2014 crop year (U.S. Department of Agriculture, Foreign Agricultural Service, 2015c). In terms of the corn trade, China is not as large of an importer as with soybeans. China imported 3.277 million metric tons of corn in the 2013/2014 crop year (U.S. Department of Agriculture, Foreign Agricultural Service, 2015b). Although China does not import as much corn as it does soybeans, China significantly influences the world's corn market. Figure 2.3 shows U.S. corn exports to China.

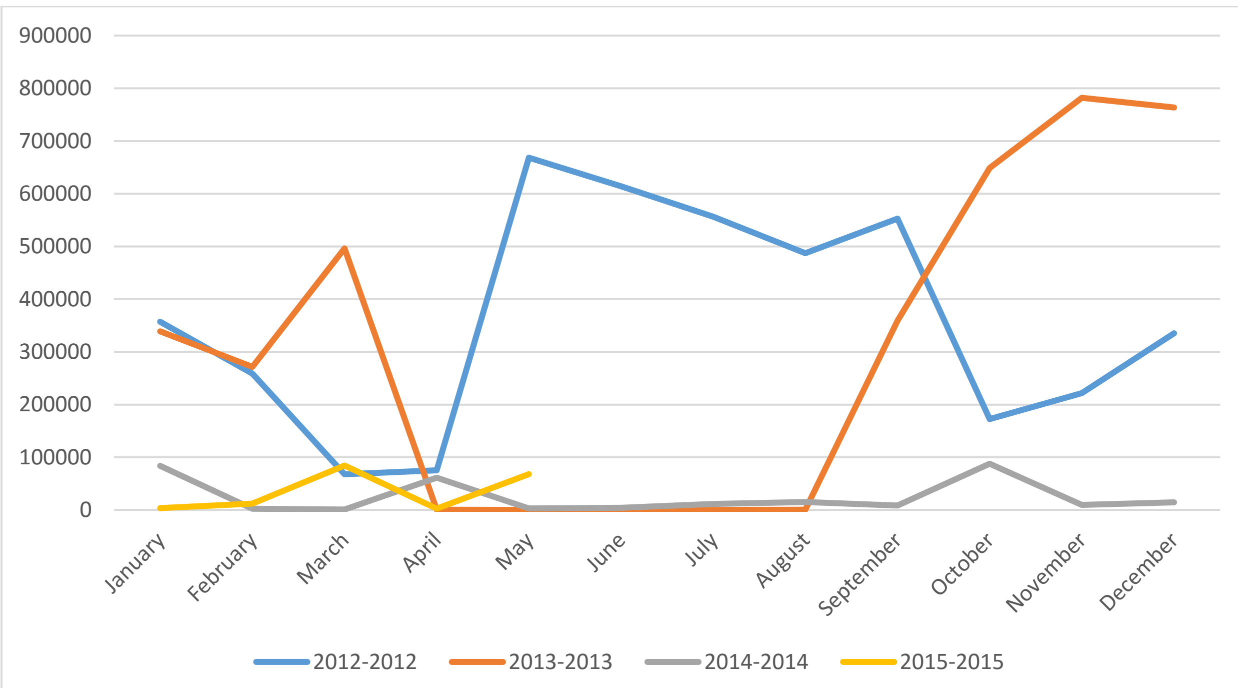

Figure 2.3. U.S. Corn Exports to China in Metric Tons.

A major default case for the corn market is an import rejection of U.S. corn by the Chinese government due to an unapproved genetically modified (GM) trait that was found in imported corn. This specific, unapproved GM corn found was SYN-IR162-4, the so-called MIR162 trait, developed by Syngenta Seeds, Inc. The MIR162 corn for approved in major corn markets, except China, and the first rejection of the U.S. corn occurred on November 18, 2013

(Farm City Elevator, Inc., 2014). According to the weekly report published by Informa Economics (2014), the purpose of restricting GM corn is to increase the consumption of China's domestic corn. The landed value of the U.S. corn in China is significantly cheaper than China's domestic-government-supported corn price which incentivizes consumers to purchase other overseas grains for feedstuffs. Figure 2.4 shows the cancelation of the U.S. corn sales to China from 1999 to 2016 in metric tons. In 2014, there was a huge spike in the cancellation of U.S. corn, primarily because of the MIR162 trait's rejection by the Chinese government.

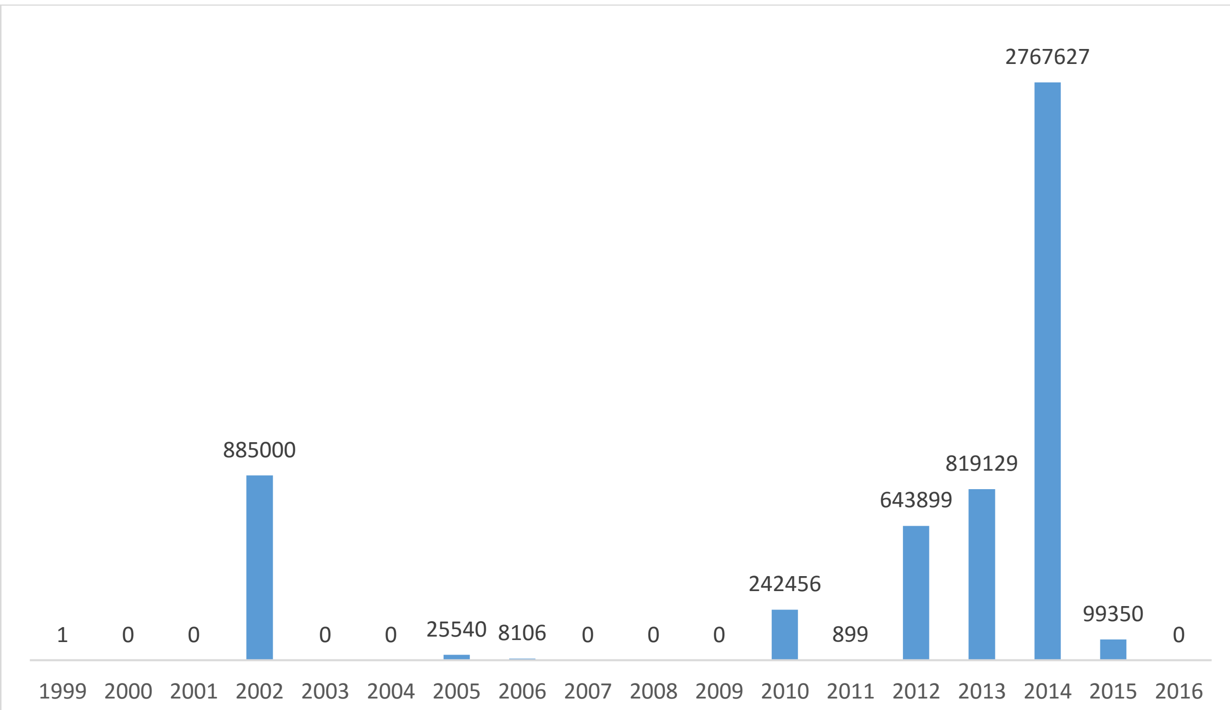

Figure 2.4. Cancellation of U.S. Corn Sales to China in Metric Tons.

In addition to the U.S. corn exports that were affected by the China's import rejection due to the unapproved MIR162 varieties, the byproduct of corn crushing, which is called distillers' dried grain (DDG), and sorghum exports to China also affected by rejections. DDG is a byproduct of ethanol which is produced with a process called crushing. The physical crushing is the process of converting corn into ethanol and DDG. Usually, DDG is used for a livestock feed because it is a rich, high-protein source. On December 27, 2013, Xinhua reported that two batches, approximately 758 tons, of the U.S. DDG with soluble were rejected because Chinese officials found them to contain the MIR162 strain ("China Rejects", 2013). Because MIR162 corn varieties are unapproved GM varieties, China imposed inspection of DDGs imports in July 2014, and any shipment with DDG purchases after August 18th is required to have MIR162 free certificate (Informa Econmics, 2014). Chinese authorities' inspection apparently caused a decrease in the U.S. DDG exports to China as which is shown in the Figure 2.5 (United States Department of Agriculture, Foreign Agricultural Service, 2015b).

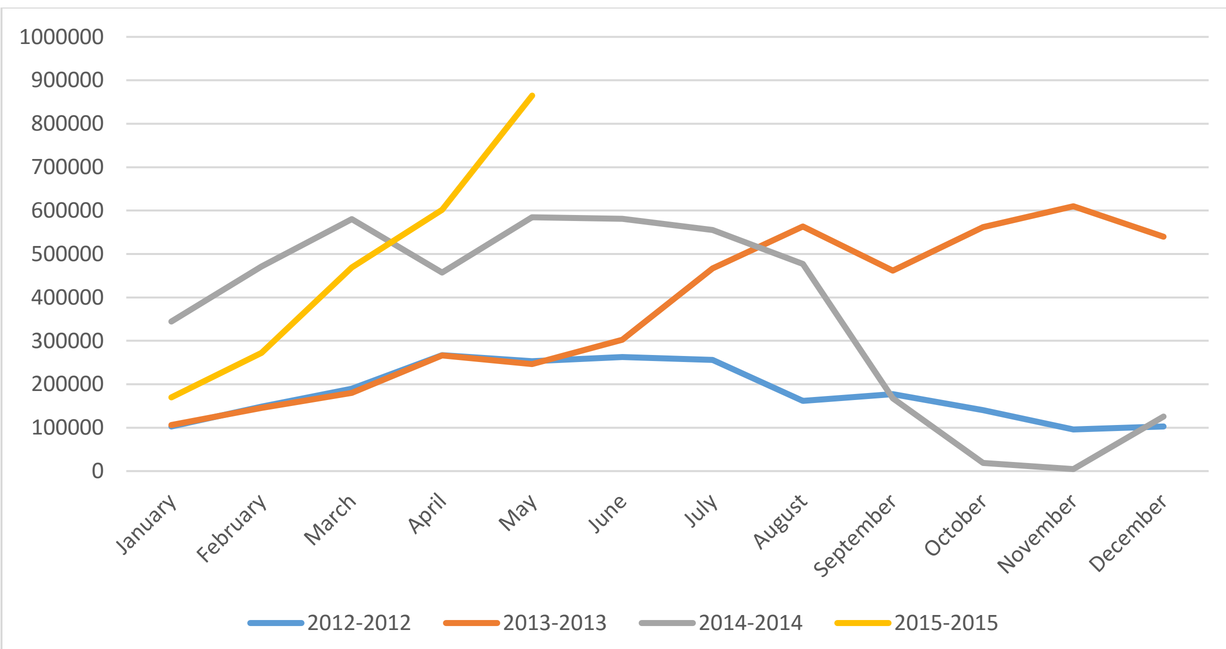

Figure 2.5. U.S. DDG Exports to China.

After the import ban for U.S. corn and the DDG inspection, U.S. sorghum exports to China increased more than 15-fold in 2014, and some Kansas grain elevators offer a 10% premium for sorghum above the corn price because sorghum does not have a futures market and because corn is used as reference for sorghum prices (Kesmodel, 2015). This phenomenon indicated that sorghum is a substitute for China's corn-import demand. Chinese importers moved their interest to purchase U.S. sorghum as an alternative, cheaper feedstuff, and the U.S.-corn import restrictions likely had a positive impact on U.S. sorghum exports to China (Informa Economics). The Figure 2.6 shows the increase in U.S. sorghum imported by China (United States Department of Agriculture, Foreign Agricultural Service, 2015c).

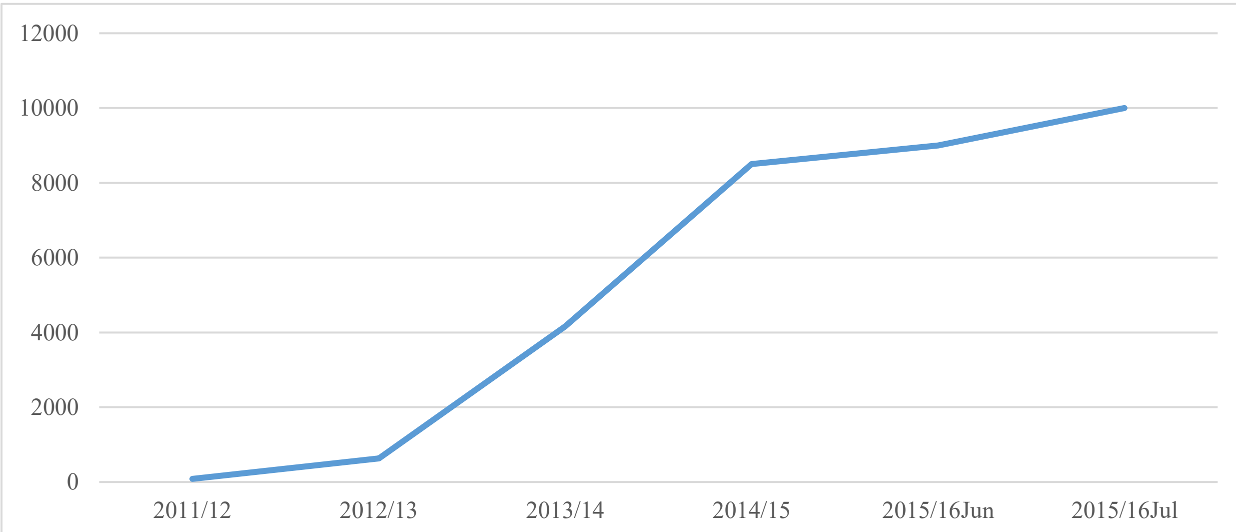

Figure 2.6. U.S. Sorghum Imports by China in Metric Ton.

Because the imported U.S. corn contained unapproved MIR162, there were many interesting consequences for corn, DDG, and sorghum. MIR162 corn was approved by Chinese officials during at the end of 2014. On December 22, 2014, Syngenta announced that MIR162 received formal import approval from China's regulatory authorities (Syngenta AG, 2014).

2.4.6. Defaults for the Wheat Market

Another major loss in the grain market is caused when one of the largest wheat-exporting countries, such as the Russian Federation, bans grain exports and establishes quotas. This decision has effects on wheat importers because they may have to find a new seller with a higher price. According to the U.S. Wheat Associates (2011), governments' export bans cause contractual prohibition and cancellation, creating a higher price than originally agreed upon for importers. Russia is known for banning grain exports, particularly in 2010, to deal with poor production due to drought and rising food prices. Additionally, Russia changed the tariff rates for wheat that was exported overseas (Blas, 2010; Global Agricultural Information Network, 2014; Kolesnikova, 2010; Kramer 2010; Vassilieva & Pyrtel, 2007). Because Russia is one of the largest wheat producers, a buyer is forced to find a different wheat seller. In 2010, Many Russian wheat buyers had to find an alternative seller with price that was $100 per metric ton above the original purchase price (U.S. Wheat Associates, 2011). According to the U.S. Wheat Associates (2015) Russia's state intervention creates extreme price volatility which is shown in Figure 2.7. The figure shows the effect of five Russian interventions from 2007 to 2014, on the Kansas wheat-futures price. The increases were as follows: 30% in two 2014 interventions, 45% with the 2012 intervention, and 100% for the 2007 and 2008 interventions (U.S. Wheat Associates, 2015).

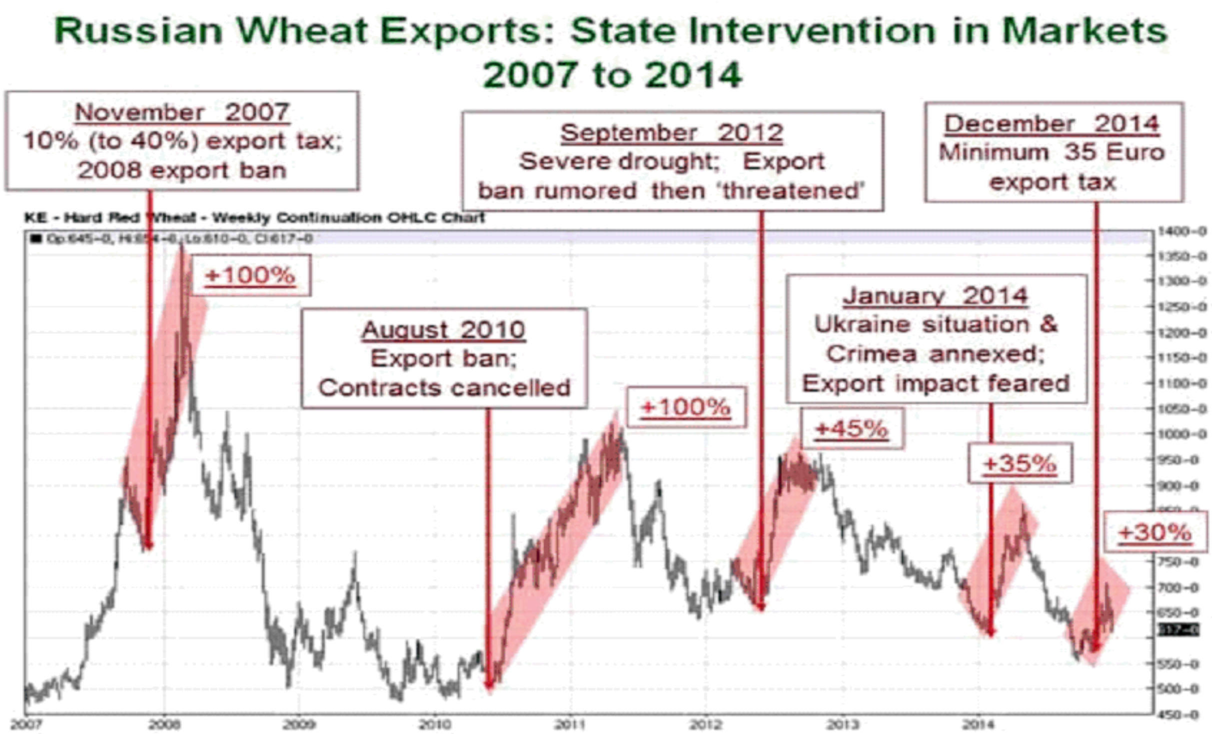

Figure 2.7. Russia's Wheat-Export Intervention and Kansas Wheat Futures Prices.

An extreme price move causes a supplier to default on the agreed selling contract. In 2012, the rise in the global commodity prices, triggered by a drought in the United States incentivized grain suppliers to default on the delivery contract because that contract created a loss from rising commodity prices (Hogan & Saul, 2012). With international grain trading, there is a function to force an agreement between counterparties. Most international grain contracts requires as 10% performance bond which the seller has to pay to buyer when a contract default occurs (Hogan & Saul, 2012). With a 10% performance bond, the seller defaults when better than keeping original agreement (Hogan & Saul, 2012). Defaulting on the sales contract may have a negative influence on the long-term relationship, but with extreme price movements defaulting on the sales agreement may be more beneficial for suppliers.

2.4.7. Defaults for the Cotton Market

An incidence of supplier default happened with cotton trading due to an unexpected price rise in the global cotton market. According to Pirrong (2014) and Kub (2012), the cotton market's contract performance risk emerged in during late 2010 to 2011. At that time, the world cotton price tripled, which led sellers to default on physical purchasing contracts; also when prices came back down, consumers defaulted on the contracts (Kub, 2012; Pirrong, 2014). With this unexpected rise in the cotton price, Glencore suffered when suppliers defaulted on purchasing contracts and had to purchasing cotton at a higher price (Kub, 2012).

2.4.8. Non-Agriculture Default Cases

A similar default case happened with the iron-ore and coal trade in China. According to Wong and Fabi (2012), there were at least six coal-cargo defaults as well as defaulting iron-ore contracts because of the price drop. Wong and Fabi (2012) said that the default was sparked by the drop in global, thermal coal benchmark prices to two-year lows and the increased anticipation of a steeper fall for the benchmark price. The comment made by a Singapore-based iron-ore trader in Wong and Fabi's (2012) article was a typical example of a buyer defaulting on a contract when the price is fell:

> We ourselves have had one of our buyers default on us after just a few hours. We sold the cargo to an end-user in China and a few hours later the buyer came back, saying "the market's falling too fast we want a lower price'." (p. 2)

We analyzed the default cases in both the agricultural and non-agricultural markets, and cancellation and default were not uncommon events. In the next section, an overview of the portfolio model of hedging is discussed.

2.5. Portfolio Hedging Models

The conceptual framework used to determine the optimal HR builds on portfolio optimization or the so-called modern portfolio theory, which was first published in Markowitz (1952). This model focused on the fact that investment's future return is dependent on the selection of the portfolio by considering the expected return and the variance of return as portfolio's desirable and undesirable aspects, respectively. With an analytical model that has finite number of securities, Markowitz (1952) formulated an investment rule called the expected return – variance of return (E-V) rule. This rule states that an investor chooses an efficient portfolio from a set of possible combinations for the portfolio's expected return and variance. This rule implies that an investor can select a portfolio by considering its E-Vs. Therefore, the investor can select a portfolio with the minimum variance of return or can accept more return variance to obtain a higher expected return. E-V is the portfolio's risk-return tradeoff. Hedging the cash position with futures and options by determining the optimal HR is the portfolio optimization, and there are different approaches to use when setting the objective function.

In order to calculate the optimal HR, both the theoretical framework and estimation procedures of the optimal HRs are important. Many of the HR's theoretical models are different and based on the objective function's maximization or minimization. The portfolio hedging models can be classified as minimum variance, utility maximization, and risk-adjusted return functions. With the minimum-variance hedging model, the hedger selects a HR that minimizes return's variance. Under the utility-maximization framework, the objective is to maximize the utility. The risk-adjusted return is similar to the expected utility maximization; however, risk-adjusted return is not associated with any specific utility functions.

2.5.1. Minimum-Variance Hedging Model

The minimum-variance hedging model does not reflect the return which the hedger seeks from the selected portfolio. With the portfolio theory of hedging, the position in the cash market is fixed, and the hedger has required to decide how much of the fixed cash position should be hedged (Ederington, 1979). Johnson (1960) developed a minimum-variance hedging model and theoretically analyzed how the hedger takes a position in the futures market to reduce the portfolio's variance of return which is caused by the commodity's price risk. Similarly, Ederington (1979) developed the minimum variance hedging model and conducted an empirical analysis assuming cash position in the Government National Mortgage Association's (GNMA) 8% Pass-Through Certificates, Treasury-Bill (T-Bill), wheat, and corn, and these were hedged with corresponding futures contracts. Both Johnson (1960) and Ederington (1979) derived the optimal HR from the variance of return equation, which is the ratio of the covariance of the spot and futures market, and the variance of the futures market. The derived, optimal HR equation was extended to formulate a measure of hedge effectiveness. The hedge's effectiveness is measured by the square of the correlation the spot and futures market. If the correlation for the commodity's price movement in the spot and futures market is one, the loss from one market is exactly offset by the profit made from the other market (Johnson, 1960). With this measure of hedge effectiveness, the lower the correlation for the price movement in the spot and futures market is an indication of a bad hedge because the loss from the spot market due to adverse price movement may not be offset by the profit made with the futures market. Ederington (1979) found that GNMA futures market was a more effective hedging instrument than the T-Bill market for risk-avoidance purposes.

2.5.2. Mean-Variance Hedging Model

Instead of only focusing on minimizing the portfolio's variance, the E-V model focuses on both the portfolio's expected return and variance to determine the optimal HR. One may consider that the minimum-variance HR is just a special case of the E-V HR. The minimum-variance HR is consistent with the E-V framework when the hedger is infinitely risk averse or when the expected futures' price change is zero. When the expected future price change is zero, it is pure martingale process (Chen, Lee, & Shrestha, 2003). Many studies incorporate the E-V approach to find the optimal HR (Blank, Carter, & Schmeising, 1991; Cecchetti, Cumby, & Figlewski, 1988; Howard & D'Antonio, 1984; Hsin, Kuo, and Lee, 1994). These studies are utility maximization under the E-V framework which aims to enhance the minimum-variance hedging model. With the E-V framework, the utility function, which the hedger is willing to maximize, is defined in terms of the expected return and variance for the hedged portfolio. Because only the first two moments of distributions are involved with the E-V utility function, E-V framework can be easily applied for risk analysis (Chavas, 2004).

The theoretical, optimal HR was derived in Blank et al. (1991): it maximized the expected utility of the hedger and included the expected return and variance for the hedged portfolio as well as the hedger's risk-aversion parameter. From this optimal HR formula, Blank et al. (1991) concluded that the formula had two sources of demand for the futures: hedging demand and speculative demand. The hedging demand for the utility-maximizing HR was the same as the minimum-variance HR, and the speculative demand showed hedger's expectation for the hedge's return because for speculative demand formula includes hedger's risk parameter (Blank et al., 1991). Blank et al.'s (1991) approach was the utility-maximization approach under the E-V framework where the hedger tried to maximize the utility based on the mean, variance, and risk aversion parameter.

Because many researchers illustrated the unrealistic nature of minimum variance hedging, Cecchetti et al. (1988) argued that the minimum-variance hedge was not optimal because it did not consider the expected return and the time-varying distribution of the spot and futures prices. Cecchetti et al. (1988) assumed that the hedger had log utility and tried to maximize the expected-utility function with the time-varying joint distribution of spot and futures returns which is estimated using the autoregressive conditional heteroskedasticity (ARCH) model. Cecchetti et al. (1988) also argued that the hedge effectiveness should be measured in terms of the expected utility or the return from the certainty-equivalent. Using the empirical result based on hedging the 20-year Treasury bond in the post-sample and in-sample periods, Cecchetti et al. (1988) found that log-utility maximization is better than the minimum-variance hedge when they compared the returns' certainty-equivalent.

Hsin et al's (1994) model hedges the currency exchange-rate risk with the currency's futures and options. Under the assumptions of the negative-exponential utility function, hedger's constant absolute risk aversion (CARA) and the normal return distribution, the expected utility maximization depends on the utility function with portfolio's mean and variance (Hsin et al, 1994). Additionally, Hsin et al. (1994) measured the hedge effectiveness as the difference between the certainty-equivalent of the hedged and spot position. Hsin et al. (1994) conducted an empirical study and concluded that the currency futures are a better hedging tool than the currency options regardless of whether the options are synthetic futures or delta/gamma hedges.

Under the E-V framework, Howard and D'Antonio's (1984) approach includes the risk-free return in the expected utility function. This approach is rather unique approach compared to other approaches. The hedger can hold a risk-free asset in the hedged portfolio to reduce the portfolio's risk. The purpose of holding futures position is not solely on the reduce portfolio risk, but it also aims to improve the risk-return characteristic (Howard & D'Antonio, 1984). Therefore, the hedger maximizes the utility function which depends on the differences for the expected return from the spot and futures positions, the return from the risk-free asset, and the portfolio's standard deviation for spot and futures positions (Howard & D'Antonio, 1984). The major finding from Howard and D'Antonio's (1984) is risk-return relative. The risk-return relative shows spot and future relative return attractiveness. The relationship between risk-return relative and the correlation for spot and futures prices show is important. This relationship defines the hedger's activity taking a futures position. If the risk-return relative is greater than the correlation, the hedger long futures contract. If risk-return relative is smaller than correlation coefficient, the hedger shorts futures contract. If risk return-relative is equal to the correlation coefficient, the hedger does not hold any futures position (Howard & D'Antonio, 1984).

The expected utility-maximization approach starts with an assumption for the utility function. Lence (1995, 1996) investigated the value of better approximation for the minimum-variance hedge by maximizing the expected utility of the risk-averse hedger's terminal wealth. One of Lence (1995, 1996) contributions was relaxing the assumptions that are ignored by most hedging literature: including the transaction cost and margin requirement. Additionally, Lence (1995, 1996) assumed that the hedger can borrow and lend capital and can also invest his/her own capital into an investment, yielding a certain return. Both Lence (1995, 1996) measured hedge effectiveness in terms of the hedger's opportunity cost to select a suboptimal return instead of the optimal return. Lence (1995) assumed that the distributions for the return from spot, futures, and alternative investment were joint normal distributions and that the utility function was CARA. Lence (1995) conducted a simulation of the hedger's behavior, considering the estimation risk, and found that value of better estimation for the minimum-variance hedge is insignificant and that the minimum variance hedge with a relaxed assumption resulted in the optimal hedge being significantly different than the usual assumptions. Lence (1996) relaxed the assumptions made in Lence (1995), one by one, to see the effect of changing the minimum-variance hedge's value. The biggest assumption changes made from Lence (1995) to Lence (1996) were stochastic production and not allowing all initial wealth to be invested into production. From the simulation results, Lence (1996) concluded that, with increased risk-averseness, the stochastic production reduced the optimal HR and the opportunity cost of not hedging at all to futures. The alternative investment opportunity induced the optimal HR to be proportional to the correlation between the alternative investment and futures prices.

2.5.3. Mean-Value-at-Risk Framework

E-VaR is a framework that adjusts the expected return with portfolio's VaR. The objective function is classified as risk-adjusted return function. This framework is gaining popularity for portfolio selection and hedging. This framework is used for the empirical analysis in Chapter 4. One of the early studies using E-VaR was conducted by Alexander and Baptista (2002). They compared the E-VaR portfolio selection to E-V, assuming a multivariate normal distribution of assets. Alexander and Baptista (2002) concluded that, as the VaR confidence interval increases, the minimum VaR converges to the minimum variance while the E-VaR converges to E-V. Alexander and Baptista (2002) also proved that E-VaR approximately maximizes the expected utility of a risk-averse agent.

When hedging, Awudu, Wilson, and Dahl (2016) used the E-VaR framework to hedge input and output price risks for ethanol. Using a stochastic optimization, they determined the E-VaR maximizing HR for three different hedging strategies: short corn, long corn, and hedging the crush margin. With the short-corn strategy, the producer sold all outputs and was left to buy corn. The long-corn strategy assumes that corn was purchased and that the output was sold as futures. The third strategy was to hedge the crush margin. With this strategy, the producer did not sell output and, instead, purchased inputs. The optimization result concluded that short corn was the best strategy because it had the highest E-VaR value.

2.5.4. Other Hedging Frameworks

For the E-V analysis to be consistent with the expected utility-maximization principle, restrictions need to be imposed on the utility's function and return distribution. Chen et al. (2003) stated that the utility function had to be a quadratic function and that the return's distribution to be a normal distribution. If these assumptions were not made, then the HR may not be optimal with respect to the expected utility-maximization framework (Chen et al., 2003).

All types of hedging paradigms, the minimum-variance hedge, E-V, expected utility maximization, and risk-adjusted return have strengths and weaknesses. Minimum variance only focuses on the hedged portfolio's variance. E-V approach has a restriction for the utility function and the return's distribution to be consistent with the expected utility maximization. The E-V framework is negative utility function. Therefore, one of the most important specifications is the utility function and for the portfolio's return distribution. Considering the restrictive assumptions about the hedging models mentioned so far, there is high demand to develop a hedging model which has less-restrictive assumptions about the hedger's utility function and return distribution.

Hedging models which lessens assumptions about specific utility functions and the return's distribution utilizes the semivariance, lower partial moment (LPM), Gini coefficient, minimum value-at-risk (VaR), and E-VaR. The purpose of applying semivariance in the hedging model is to hedge against the downside risk instead of portfolio's entire risk. With the E-V framework, the return's variance is used to quantify the portfolio's risk; however, variance treats both upside and downside risks equivalently. Therefore, under the E-V framework, the hedger has to sacrifice possible gain. Realistically, the hedger and investor favor upside risk and dislike portfolio's downside risk. A theoretical treatment of risk management's semivariance is done by Hogan and Warren (1974). Turvey and Nayak (2003) formulate a semivariance-minimizing hedging model for agricultural commodities to hedge the downside risk, and this model is free of prior assumptions about the distribution's shape. Utilizing the numerical approach instead of the econometric approach to calculate the HR, Turvey and Nayak (2003) conclude that a minimum semivariance hedge protects the hedger from downside risk more than the minimum variance hedging model. In general, the magnitude of the protection obtained with semivariance over the minimum variance is uncertain because the semivariance hedge is highly responsive to the distribution of the cash and forward positions as well as the hedger's target return.

The LPM hedging model's focus is same as the semivariance model. The LPM focuses on the portfolio's downside risk. Bawa and Lindenberg (1977) developed the theoretical framework of LPM, and it does not assume return's distribution. The LPM framework is a generalized framework, and the mean-variance and semivariance frameworks are special cases of the LPM framework (Bawa & Lindenberg, 1977; Eftekhari, 1998). Eftekhari (1998) formulated a hedging model which minimizes the LPM and numerically computed the LPM-minimizing HR and the minimum-variance HR to hedge the Financial Times Stock Exchange 100 (FTSE-100) index's return with FTSE-100 index futures. Eftekhari (1998) concluded that the hedger should use minimum variance if he/she is interested in hedging the return's volatility and use the LPM to hedge the downside risk.

Yitzhaki (1982, 1983), Lerman and Yitzhaki (1984), and Shalit and Yitzhaki (1984) proposed an approach to use Gini's mean difference, which measures income inequality, as a measure of variability in the field of finance. The mean-Gini framework makes no assumptions regarding the hedger's utility function and return distribution. This framework is consistent with first-degree and second-degree stochastic dominance (Cheung, Kwan, & Yip, 1990). Cheung et al. (1990) compared the efficient frontiers of E-V and the mean-Gini framework by using five foreign currencies' futures and options. Cheung et al. (1990) concluded that currency futures are a better hedging instrument than currency options with the minimum-variance and minimum mean-Gini hedging approaches. The E-V framework futures were a better hedging instrument; however, the mean-Gini approached had better options as a hedging instrument. Although the frameworks obtained different conclusions for the hedging instrument, it is difficult to ignore the loosened assumption for the utility function and return distribution.

In order to solve the problems with minimum variance, E-V, and the expected utility-maximization frameworks, researchers created semivariance, LPM, mean-Gini coefficient, and E-VaR hedging models. Each newly framed models had different assumptions and specifications, and these models had a big advantage: not relying on the return's distribution. The portfolio hedging model has the goal of either minimizing risk or maximizing the utility function depending on the model specifications. One of the most important assumptions to make when formulating a hedging model is the hedger's utility function and return distribution. The different assumptions and specifications for the utility function and return's distribution of the hedging models may lead to totally different optimal HRs. Comparing the hedging models can be done by using hedge effectiveness or the certainty equivalent of returns for each model. Based on the current research done about the portfolio model of hedging, there is no best model to hedge the risk due to the different assumptions made.

2.5.5. Hedging with Options

Instead of hedging a commodity's price risk with a futures contract, option contracts are a useful hedging instrument for the industry. Bullock, Wilson, and Dahl (2003) analyzed a bread baker's hedging demand for futures and options with the E-V framework. Bullock et al. (2003) derived three conclusions regarding the use of options as a hedging instrument. The demand for options as a hedging tool is always zero; options are a less-effective hedging instrument than futures because the delta is lower than one while the futures' hedging demand is not affected, including options that are available as the portfolio's hedging instrument (Bullock et al., 2003). Finally, the existence of bias in the futures or options markets as well as the difference between the firm's expected and actual price, allows non-zero speculative demand for options to be an optimal solution (Bullock et al., 2003).

Bullock and Hayes (1992) applied the E-V framework and used the futures and put options as the hedging instrument. Utilizing the statistical theorem, the investor only needs to focus on the price distributions' mean and variance. The study found that futures are a primary hedging instrument for the cash position and are used for speculation when the price distribution's mean changes. In particular, the put option is the speculative instrument when the price distribution's variance changes. The study also proved the existence of undiversifiable risk because the relationship between the period-two futures price and put option price is not one-to-one relationship because there are many futures prices for a worthless options price. Due to the undiversifiable risk, the delta neutral hedge needs adjustment. All these results are consistent with and without including the basis risk in the payoff function.

2.6. Previous Literature About Default Risk

In the field of the counterparty credit risk, most of the previous literature focuses on three main areas: the systematic risk of default risk in derivative contracts and regulating the over-the-counter (OTC) market, the evaluation of derivative contracts that are subject to counterparty risk, and the risk-management strategy with default risk (Korn, 2008). The focus of this study is to find the optimal HR by considering the price and default risk, so the problem is categorized as a risk-management strategy with default risk. Forward and futures contracts are similar; however, a forward contract is subject to counterparty risk, whereas a futures contract is not subject to counterparty risk.

The problem of hedging the counterparty's default risk is similar to hedging quantity risk. The problem is similar because the forward contract's seller loses a portion or the entire revenue that was supposed to come from the sale using the forward contract. This study's main goal is to find a solution for the portfolio model of hedging by considering price and default risk.

2.6.1. Risk Management with Counterparty Default Risk

Previous literature about the default risk's effect on a firm's risk-management strategy is most closely related to this study's problem. Mahul and Cummins (2008) used the expected utility-framework to analyze the firm's hedging and production decision when the company uses vulnerable, which is subject to counterparty default, forward and options contracts as hedging instruments. Mahul and Cummins (2008) assumed that the counterparty's default risk is endogenous: the counterparty's default depends on the hedging firm's decision. Mahul and Cummins (2008) showed, mathematically, that, when vulnerable and subject to counterparty credit risk, a forward contract is the only hedging instrument. The vulnerable forward contract encourages the hedging firm's production to be lower than the non-vulnerable forward contract. Mahul and Cummins (2008) also showed that, under certain conditions, the optimal forward contract is less than the firm's production. When the vulnerable-option contract, a long-put option, can be used as hedging instrument, the optimal HR contract is one (Mahul & Cummins, 2008).

Similar to Mahul and Cummins (2008), Korn (2008) used the expected utility-maximization framework to solve for the optimal hedging and production decisions of a risk-averse, competitive firm under the price uncertainty with an exogenous default risk. The firm's hedging decision does not affect the counterparty's default. Korn (2008) found that, if the expected profit for a forward contract is zero, the HR is independent of default risk: that is, a HR of one is the optimal. This result also implies that the HR is the same with and without default risk if the forward contract's expected profit from is zero. Korn (2008) extended the basic model to include a stochastic recovery rate. The stochastic recovery rate implies only some partition of gain from forward contract is lost due to default. With the stochastic recovery rate, Korn (2008) obtained a counterintuitive result, a HR above one with a stochastic recovery rate and an exogenous default risk. This conclusion was different from Mahul and Cummins' (2008) endogenous default risk. The possible explanation was that the endogenous default-risk assumption was affected by absolute size of a hedging company's forward contract which induces under-hedging: additionally, the hedge's speculative and default components have an effect on the forward position's absolute size (Korn, 2008). Furthermore, Korn (2008) suggested using default-risky credit derivative if forward contract is subject to default risk to diversity default risk between the forward contract and credit derivative.

Another approach taken by industry professionals to measure credit exposure is credit valuation adjustment (CVA) which is primarily used for OTC derivative contracts. The CVA allows banks to measure market value of credit exposure of derivative contract, and it is defined as difference between the risk-free portfolio value and true portfolio value subject to the default risk (Zhu and Pykhtin 2007). According to Zhu and Pykhtin (2007), there are three main components for calculating the distribution of a counterparty's credit exposures: scenario generation, instrument valuation, and portfolio aggregation. The first step to calculate the credit exposure is to generate a scenario for a future market with a by computer simulation by using evolution models of a risk factor for a fixed set of dates. The instrument is valued using the scenario created for each simulation date and risk factor in the first step. Portfolio aggregation is the sum of all contract-level counterparty credit exposure when considering the netting agreement for each simulation date and risk factor. Modeling the credit exposure and calculating the CVA allows a firm to quantify and hedge its counterparty credit exposure.

In this study, the put option is used as the hedging instrument. Mahul and Cummins (2008) introduce the vulnerable option, subject to counterparty risk, into the hedging. The option allows traders and risk managers to flexibly manage risk when the only incurred loss is the option premium. The Theoretical Hedging Model with a put option is described in the Chapter 3.

2.7. Summary

The purpose of this chapter is to review previous studies about risk management. Several risks exist in agribusiness, primarily a commodity's price and quantity. These risks induce variability into the profit that is measured by the profit's standard deviation. These risks are conventionally hedged with commodity futures and option contracts; however, hedging the default risk, a special case of quantity risk, is difficult for both industry professionals and academic researchers. In recent years, the size and frequency of cancellations in the agricultural market have increased. At the international level, the cancellation of soybean and corn sales grown. There are many factors and reasons for this default. Regardless of the reason for the defaults, risk managers and traders need to increasingly manage that default risk.

In order to hedge a risks, researchers developed portfolio hedging models based on Markowitz's (1952) seminal paper. Each one has different specification of the objective function that is subject of optimization. The minimum variance's objective is to minimize the portfolio's variance whereas the E-V maximizes the utility function that considers the portfolio's return and variance. The E-V framework requires the quadratic utility function and a normal return distribution to be consistent with the expected utility-maximization. Similarly, the E-VaR is discussed as an example of risk-adjusted return function. Managing the default risk is studied in few papers, such as Korn (2008) and Mahul and Cummins (2008). Both Korn (2008) and Mahul and Cummins (2008) analyzed the theoretical hedging decision of a risk-averse firm with default risk. Additionally, CVA is the latest methodology utilized to evaluate the counterparty's credit risk.

CHAPTER 3. THEORETICAL HEDGING MODEL

3.1. Introduction

Several portfolio hedging models were reviewed in the Chapter 2. Those models are classified into three approaches: minimum variance, expected utility-maximization and risk-adjusted return. The results from these models can be used by hedgers for decision making. This chapter extends portfolio hedging model to include the counterparty's default risk; the model uses the option contract as the hedging instrument. The default risk is a special case of quantity risk which the firm faces when counterparty defaults on the originally agreed sales for each transaction. This chapter is organized in five sections: first, the basic portfolio model for hedging the price risk is explained in detail; second, the models to hedge both price and quantity risk with a futures contract are developed; third, the basic models are extended to include options; fourth, two default risk models are discussed. Last, the mathematical analysis of the default risk model that hedges the price and default risks with futures and options is conducted.

3.2. Portfolio Model of Hedging the Price Risk

The portfolio model of hedging is a common agricultural problem: the model analyzes the hedging price risk with forward, futures, and/or options contracts. One of the main assumptions made in the models for hedging the price risk is that the hedger has a fixed and known amount of inventory and is only interested in hedging the price of that inventory. Traditionally, portfolio models are developed using the one of minimum variance, E-V, or utility maximization frameworks. Based on a seminal paper by Markowitz (1952), Johnson (1960) and Ederington (1979) applied the minimum variance to hedging commodity price risk and the GNMA 8% Pass-Through Certificates, respectively. Blank et al.'s (1991) developed a hedging model depends on cash and futures prices. The Blank et al. (1991) portfolio model of hedging with a single-inventory price risk starts with the following model:

$$\tilde{V} = (\tilde{p}_2 - p_1) + h(f_1 - \tilde{f}_2) \tag{1}$$

where the tilde represents random variables; $\tilde{p}_2$, p_1, $\tilde{f}_2$, and f_1 are the cash price for period two, the cash price for period one, the future price for period two, and the futures price for period one; h represents the size of the futures contract, and $\tilde{V}$ is the change in portfolio's value. Assuming that the hedger has a long cash position and based on equation (1), portfolio's expected value is as follows:

$$E(\tilde{V}) = E(\tilde{p}_2) - p_1 + h(f_1 - E(\tilde{f}_2)) \tag{2}$$

In addition to the portfolio's expected value, the portfolio's variance based on equation (1) is as follow:

$$var(\tilde{V}) = \sigma_{p_2}^2 + h^2\sigma_{f_2}^2 - 2h\sigma_{p_2f_2} \tag{3}$$

where $\sigma_{p_2}^2$ and $\sigma_{f_2}^2$ are the variance of the cash and futures prices for period two, and $\sigma_{p_2f_2}$ is the covariance between the cash and future prices for period two. The minimum-variance framework finds an h that minimizes the portfolio's variance, $var(\tilde{V})$. By taking the derivative of equation (3) and setting the results equal to zero, h, which minimizes the variance, is determined.

$$h^* = \frac{\sigma_{p_2f_2}}{\sigma_{f_2}^2} \tag{4}$$

Hence, the risk-minimizing HR depends on the covariance between the cash and futures prices for period two as well as variance of the futures price for period two, $\tilde{p}_2$.

For a numerical example, assume that the size of the cash position is 1. Suppose that the covariance between the cash and futures price, $\sigma_{p_2f_2}$, is 70 cents a bushel and that the variance of the futures price for period two is 100, hence the minimum-variance HR is 0.7. This HR indicates that the firm should hedge 70% of its inventory to minimize the portfolio's variance. Formulating portfolio's variance and deriving the optimal HR is essentially same for Blank et al. (1991), Johnson (1960), and Ederington (1979).

Another approach taken by Blank et al. (1991) was expected utility maximization using revenue. They used the E-V framework where the utility function is specified as follows:

$$\mathrm{U(R)} = E(R) - \phi \mathrm{var(R)} \tag{5}$$

where U() is the hedger's utility that depends on portfolio's expected return adjusted by the portfolio's variance. Alternatively, one can specify different types of utility or objective functions to be maximized or minimized. For example, Turvey and Nayak (2003) minimized semivariance; Eftekhari (1998) minimized the lower partial moment; and Cheung, Kwan, and Yip (1990) maximized the mean-Gini function. The portfolio hedging model, either maximizing or minimizing, a particular objective function comprised of first, second, or higher moments of portfolio's return. The ϕ variable is a risk-aversion parameter of the hedger and is assumed to be negative. Blank et al. (1991) substituted equation (2) and equation (3) into equation (5) and derived the following equation:

$$\mathrm{U}(\tilde{V}) = E(\tilde{p}_2) - p_1 + h\left(f_1 - E(\tilde{f}_2)\right) + \phi[\sigma_{p_2}^2 + h^2\sigma_{f_2}^2 - 2\mathrm{h}\sigma_{p_2f_2}] \tag{6}$$

By taking the derivative of equation (6), Blank et al. (1991) derived the utility maximizing h:

$$h^* = \frac{\sigma_{p_2f_2}}{\sigma_{f_2}^2} - \frac{f_1 - E(\tilde{f}_2)}{\phi 2\sigma_{f_2}^2} \tag{7}$$

The first component on the right-hand side of equation (7) is the slope of the price's regression in period two, $\tilde{p}_2$; the futures price in period two, $\tilde{f}_2$; and the hedger's demand for hedging, $\frac{\sigma_{p_2f_2}}{\sigma_{f_2}^2}$ (Blank et al., 1991). Moreover, right-hand side of equation (7) is same as the variance-minimizing HR in equation (4) and indicates hedging demand for the futures. The second component on the right-hand side of equation (7) is the futures' speculative demand which indicates the hedger's expectation for the return of the hedged position (Blank et al., 1991). Similar to the hedging demand of futures, the second part of equation (7) is referred to as the future's speculative demand. The minimum-variance and utility-maximizing optimal HRs are derived by dividing equations (5) and (7) by the cash position. For equation (6) to be in line with the expected utility-maximization principle, the utility function must be quadratic, and the portfolio's return is assumed to have a normal distribution. Because the E-V framework depends on the portfolio's variance, square of the portfolio's standard deviation, equation (5) is a quadratic function. Therefore, the E-V framework is one type of utility function for the expected utility-maximization framework that has a restrictive assumption when compared to other types of utility or objective functions.

This section illustrated the construction of the portfolio model for hedging the price risk of a fixed cash position with futures contracts. In order to find the optimal HR, the firm's revenue function needs to be defined first. Once the revenue function is defined, two moments of the revenue function, the expected value and variance, are derived. The optimal HR is derived by minimizing the revenue's variance or maximizing the utility function which is contingent on the model's framework.

3.3. Hedging Price and Output Risk with Portfolio Models

Most hedging models focus on the commodity price risk which can be hedged directly with a particular commodity's futures or options contracts; the risk can be cross-hedged with the derivative contract of a related, but not the same, commodity. One of the main assumptions made in a portfolio model about hedging is the inventory quantity or if the demand is fixed. From the grower's point of view, fixed quantity is unrealistic because the risk with the amount of production exists. Quantity risk makes producer difficult to determine the optimal size of the hedging position. In order to hedge both price and production risk using a futures contract, Blank et al. (1991), McKinnon (1967), and Robinson and Barry (1999) developed portfolio models which account for price and production risks. These output-hedging models are formulated from the producer's point of view which is similar to the seller trying to hedge the output risk that is caused by the buyer's cancellation or default.

3.3.1. Blank et al.'s (1991) Model with Uncertain Output

Blank et al. (1991) assumed that the expected change in the basis was not relevant. Blank et al.'s (1991) model for the output risk defined the payoff function as follows:

$$\tilde{R} = \tilde{p}_2\tilde{q}_2 - h(\tilde{f}_2 - f_1) \tag{8}$$

where $\tilde{R}_2$, $\tilde{p}_2$, $\tilde{q}_2$, and $\tilde{f}_2$ are random variables that represent the revenue, price, production, and futures price in period two. The h variable is the size of the futures position, and $f_1$is the futures price in period one. Period one is the planting time, and period two is the harvest. Based on equation (8), the hedger's variance is shown as equation (9):

$$var(\tilde{R}) = var(\tilde{p}_2\tilde{q}_2) - 2hcov(\tilde{p}_2\tilde{q}_2, \tilde{f}_2) + h^2 var(\tilde{f}_2) \tag{9}$$

where $var(\tilde{R})$, $var(\tilde{p}_2\tilde{q}_2)$, and $var(\tilde{f}_2)$ are the variance for the entire revenue, the variance for the hedger's revenue in period two, and the variance for the futures price in period two, respectively. The $cov(\tilde{p}_2\tilde{q}_2, \tilde{f}_2)$ is the covariance between the revenue and the futures price in period two. From equation (9), the portfolio's variance depends on the variance of revenue in period two, the covariance between price and production in period two and futures price in period two, and the futures price variance in period two. Blank et al. (1991) did not derive an optimal futures position or hedge to minimize the portfolio's variance. Mathematically, it is difficult to derive an h which minimizes variance from equation (9) due to the relationship specification between the product of price and production in period two. One needs to clearly define the relationship between production and the output price to simplify equation (9). If production and price are independent, one can simplify equation (9). The connection between the output price and production is one of the most important relationships to define.

3.3.2. Robinson and Barry (1999)

In contrast to Blank et al. (1991), Robinson and Barry (1999) defined the relationship between output price and production. Blank et al. (1991) emphasized the importance of the link between price and production, but Robinson and Barry (1999) defined the relationship to be independent. The constructed model is almost equivalent to Blank et al. (1991). It is assumed that a firm purchases a single input at a certain price and that the input is converted into a stochastic output. Therefore, the expected output is as follows:

$$E[f(x) + v] = f(x) \tag{10}$$

where x is input, f is an output function, and v is the randomness of the production function with a mean of zero and variance σ_v^2. Based on equation (10), the profit function for the firm is as follows:

$$\pi = (p + \varepsilon)[f(x) + v - h] + p_f h - p_x x - B \tag{11}$$

where p, p_f, p_x, and B are the cash price, futures price, input price, and fixed cost, respectively. Similar to the random element for production, v, the ε variable is the price's randomness for the output with a mean of zero and a variance of σ_ε^2. The h variable is the amount of forward sale. Because the firm's profit depends on two random variables, v and ε, the firm's profit, π, is also random. Robinson and Barry (1999) assumed that random components, v, and ε, for production and price, respectively, were statistically independent, implying that v does not affect the probability of ε, and ε does not affect the probability of v. This specification about the relationship between price and production is one of the Robinson and Barry's (1999) major differences from Blank et al. (1991) other than the fixed costs in the profit function. The assumption of independence between price and production makes it easier to derive the profit's variance. For example, suppose that K and L are statistically independent random variables:

$$var(KL) = E(KL)^2 - [E(KL)]^2 = E(K^2)E(L^2) - [E(K)E(L)]^2 \quad (12)$$

Based on equations (11) and (12), the profit's variance is derived and is substituted into the E-V framework or certainty equivalent model shown in equation (13):

$$y_{CE} = E(y) - \frac{\lambda}{2}\sigma^2(y) \quad (13)$$

where $E(y)$, λ, and $\sigma^2(y)$ are the expected profit, risk-aversion parameter, and variance of profit, respectively. y_{CE} is the certainty equivalent. Equation (13) is equivalent to equation (5). The primary difference is the equation (13)'s risk-aversion parameter; because risk-aversion parameter is arbitrary, equations (5) and (13) are identical. By plugging the expected profit and profit's variance into equation (13), the certainty equivalent model becomes:

$$y_{CE} = p[f(x) - h] + p_f h - p_x x - B - \frac{\lambda}{2}[p^2\sigma_v^2 + f(x)^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\sigma_v^2 + h^2\sigma_\varepsilon^2 - hf(x)\sigma_\varepsilon^2] \quad (14)$$

By defining the relationship between price and production, Robinson and Barry's (1999) model derived a utility function based on the E-V framework. Robinson and Barry optimized equation (14) to derive the optimal size of hedge position h.

$$\mathrm{h} = \mathrm{f(x)} - \frac{(p - p_f)}{\lambda\sigma_\varepsilon^2} \quad (15)$$

The correlation assumption in Robinson and Barry (1999) is the independence of price and production. Realistically, price and production may not be statistically independent. When the market is supplied with a large production, the commodity's price should decrease. When the market lacks a supply, the commodity's price should rise. Hence, there should be a correlation between price and production that affects the portfolio's variance. The firm which Robinson and Barry (1999) assumed in their model operates under perfect competition, so the assumption of statistical independence between price and the firm's output seems to be a reasonable assumption because the firm is a price taker.

According to Robinson and Barry (1999), the size of h depends on the difference for $p - p_f$, the hedger's market bias. If the expected spot price, p, is equal to the futures price, p_f, the entire production is hedged because the risk-averse firm prefers a certain price rather than a uncertain price. If the expected spot price, p, is different from the future price, p_f, the firm either over-hedges or under-hedges, depending on the sign of $p - p_f$, and the risk-averse firms sells output in the spot market if a risk premium exists in the spot-market price (Robinson and Barry, 1999). Additionally, the more risk-averse the firm is, the larger the amount of production that is being hedged with a constant $p - p_f$. If the difference is positive, h $<$ f(x), and if the difference is negative, h $>$ f(x). This model does not incorporate the basis risk. By dividing equation (15) by the size of production, the optimal HR can be derived.

$$\frac{h}{\mathrm{f(x)}} = 1 - \frac{(p - p_f)}{\lambda \mathrm{f(x)} \sigma_\varepsilon^2} \tag{16}$$

3.3.3. McKinnon (1967)

Another portfolio model which includes the quantity risk was developed by McKinnon (1967). His model started with similar assumptions as Blank et al. (1991) and Robinson and Barry (1999). McKinnon's model is specified as follows:

$$\mathrm{Y} = \mathrm{PX} + (P_f - P)X_f \tag{17}$$

where Y, P, and X are random variables for the farmer's income, output price, and output at harvest, respectively. X_f is the size of the futures position. McKinnon (1967) assumed the existence of normal backwardation, hence, the expected value of the output price, P, was the futures price, P_f:

$$\mathrm{E(P)} = P_f \tag{18}$$

Robinson and Barry (1999) made an assumption that price and output are statistically independent; however, under McKinnon's (1967) model, the relationship between price and output is a bivariate, normal distribution. The bivariate, normal assumption between price and output allows people to deriving the variance for the farmer's income from equation (16) by only using the first and second moments of price, P, and output, X as well as the correlation between price and output without making specific assumption about higher-order moments. Using equation (17) and (18), the farmer's variance for income farmer, Y, is as follows:

$$\sigma_y^2 = P_f^2\sigma_x^2 + \mu_x^2\sigma_p^2 + 2P_f\mu_x\rho\sigma_x\sigma_p + (1+\rho^2)\sigma_x^2\sigma_p^2 - 2X_fP_f\sigma_x\sigma_p - 2X_f\mu_x\sigma_p^2 + X_f^2\sigma_p^2 \tag{19}$$

where μ_x, σ_x, and σ_p are the mean and standard deviation of the output as well as the standard deviation for the output price at harvest. Variable ρ is the correlation between the output, X, and the price, P. In this model, the correlation between the output price and output is one of the most important variables. Intuitively, output should be negatively correlated with the market price because a large supply should cause the output price to decrease while a lack of supply should cause the price to increase. However, the output price may depend on a larger geographical area's production rather than the single farm which was considered in this model. Because the price may depend on a larger geographical area, empirically, the correlation between the farm's output and the price may not be negatively correlated. Using equation (18), McKinnon (1967) derived a variance-minimizing futures position, X_f^*, which leads to the optimal HR.

$$X_f^* = \rho P_f \frac{\sigma_x}{\sigma_p} + \mu_x \tag{20}$$

Equation (20) implies that the variance-minimizing future position depends on the correlation. The variable in equation (20), which can fall below zero, is only the correlation. If HR is below 1, the correlation must be below zero. If the correlation is positive, the variance-minimizing HR is greater than 1. When the correlation is equal to zero, the HR is equal to 1. Because the optimal HR is of futures position to the cash position, the optimal HR is derived by dividing equation (20) by the expected output, μ_x, as follows:

$$\frac{X_f^*}{\mu_x} = \rho \frac{\sigma_x / \mu_x}{\sigma_p / P_f} + 1 \tag{21}$$

The numerator, $\frac{\sigma_x}{\mu_x}$, is the relative variability of production, X, and similarly, $\frac{\sigma_p}{P_f}$, is the relative variability of price. McKinnon (1967) indicated that the correlation between the price and the output, output relative variability, or price relative variability determines the variance-minimizing HR. From equation (21), McKinnon (1967) concluded that the large numerator is compared to the denominator, the smaller the optimal HR is. Furthermore, a smaller negative correlation between price and production lowers the optimal HR. Hence, a smaller negative correlation acts as a natural hedge.

This section covered three portfolio models considering price and quantity risks. All models started by defining the payoff function and derived the expected profit in addition to the variance for the profit function. The expected profit function and the variance for the payoff functions were used to develop the portfolio model. The framework used in McKinnon (1967) and Robinson and Barry (1999) are minimum variance and E-V, respectively. The next section compares the model's assumptions and the derived HR equation.

3.3.4. Comparison of Assumptions and the Hedge-Ratio Equation

All the quantity-risk portfolio models described so far are two-period model, except Robison and Barry (1999), and all portfolio models do not assume the basis risk. In the two-period model, the firm initiates a hedge during the planting period and lifts the hedge at harvest. Additionally, only McKinnon (1967) assumed the backwardation which says that the expected cash price is the futures price during the harvesting period.

The statistical relationship between the output price and quantity is one of the most important factors that affect the optimal HR. Blank et al. (1991), Robinson and Barry (1999), and McKinnon (1967) emphasized the importance of the correlation between price and production. Blank et al. (1991) mentioned that the portfolio's variance is affected by the correlation between price and product; however, they did not specifically define using the optimal HR. Robinson and Barry (1999) assumed statistical independence between the output price and output, whereas McKinnon (1967) assumed a bivariate normal distribution.

Due to different setups for each model, the optimal HR that minimizes the portfolio's variance or maximizes the E-V framework's utility function is slightly different. The biggest difference between the models is a correlation between the cash price and output. The optimal HR was derived in Robinson and Barry (1999) and McKinnon (1967). The HR derived in McKinnon (1967) depended on the correlation and ratio of the relative variation for the output and price. The correlation connected price and output in the HR because they are not statistically independent. Robinson and Barry's (1999) HR depended on the bias for the price, price variance, and risk-aversion parameters. The biggest difference between the equations for the optimal HR that were derived in these two models was that Robinson and Barry (1999) did not have any output risk. The statistical relationship was assumed to be independent for Robinson and Barry's (1999) model, but there was a bivariate, normal distribution in the McKinnon (1967) model. The Table 3.1 shows the assumptions about correlation and the optimal HR for each model.

Table 3.1

Different Correlation Assumptions and Optimal Hedge Ratio in Quantity Risk Models

	Blank et al. (1991)	McKinnon (1967)	Robinson and Barry (1999)
Correlation between price and output	No particular assumption made on correlation, but correlation is important factor in determining the optimal HR	Price and output are normal bivariate distribution	Price and output are statistically independent
Optimal HR formula		$\frac{X_f^*}{\mu_x} = \rho \frac{\sigma_x / \mu_x}{\sigma_p / P_f} + 1$	$\frac{h}{\mathrm{f(x)}} = 1 - \frac{(p - p_f)}{\lambda \mathrm{f(x)} \sigma_\varepsilon^2}$

3.4. Portfolio Model for Hedging with Futures and Options

This section discusses the portfolio model of hedging that includes an option. The models discussed so far only had a futures contract as the hedging instrument for the commodity price. By including commodity options with an arsenal of hedging instruments, the hedgers have more flexibility when constructing their hedging strategies. Commodity options allows the hedger to create a nonlinear payoff function, the most advantageous aspect of using options when hedging the price risk. The nonlinear payoff function sets a price floor or ceiling that keeps the profit from falling below or rising above the floor and ceiling, respectively. Therefore, an option is very useful as an alternative tool for the commodity futures. More detailed models that hedge the price risk using options as well as models that hedge both the price and quantity risks using options are discussed.

3.4.1. Hedging the Price Risk with Futures and Options

The model in this section is based on Bullock and Hayes' (1992) study; they extended the study done by Wolf (1987) which included options as a hedging instrument. A goal of Bullock and Hayes (1992) was to develop hedging models that rely on first and second moments as the information source for the hedging decision. Bullock and Hayes (1992) developed a two-period hedging model that included futures and options for an investor who is storing cash grain. They developed two models: with and without basis risk. Two assumptions were made for the model without basis risk. Those assumptions were that the future and spot prices converged in period two and that there was an increasing marginal cost function:

$$\tilde{\pi} = (\tilde{p}_2 - p_1)I - \frac{1}{2}cI^2 + (\tilde{p}_2 - f_1)X + [q_1 - \max(0, k - \tilde{p}_2)]R \qquad (22)$$

Where the tilde stands for the random variable while p_t, f_t, and q_1 are the cash price, futures price, and put-option premiums, respectively at time t. The random spot price in period two follows a normal distribution: $\tilde{p}_2 \sim N(\mu_p, \sigma_p^2)$; c is the marginal cost; and I, X, and R are the size of the inventory, futures, and put-option positions, respectively. The investor's utility is assumed to be negative utility which can be estimated using E-V. After applying the theorem, which is explained in section 3.5, the expected profit is derived:

$$\mathrm{E}(\tilde{\pi}) = (\mu_p - p_1)I - \frac{1}{2}cI^2 + (\mu_p - f_1)X + (q_1 - \alpha h_2)\mathrm{R} \tag{23}$$

By applying same theorem, Bullock and Hayes (1992) derived the portfolio's variance:

$$var(\tilde{\pi}) = \alpha\sigma_{p-}^2(I + X + R)^2 + (1-\alpha)\sigma_{p+}^2(I + X)^2 + \alpha(1-\alpha)[h_1(I + X) + h_2 R]^2 \tag{24}$$

where α is the probability that the cash price for period two is below the strike price. The h_1 and h_2 variables are defined as $h_1 = \mu_{p+} - \mu_{p-} > 0$ and $h_2 = k - \mu_{p-} > 0$; μ and σ represent the expected and standard deviation for the subscript's variable. Additionally, the positive and negative signs in the subscript indicate the conditional expectation and conditional standard deviation above and below the strike prices, respectively. Therefore, σ_{p-}^2 is the standard deviation for the spot price in period two when the price is below the strike price.

As a standard approach for the maximization problem, Bullock and Hayes (1992) substituted equation (23) and (24) into the E-V framework, and solved for I, X, and R using a partial derivative and the Hessian Matrix to maximize utility. The optimal solutions of I, X, and R were derived as follows:

$$I^* = \frac{f_1 - p_1}{c} \tag{25}$$

$$X^* = \frac{\mu_p - f_1}{\lambda V_{11}} - \frac{V_{12}}{V_{11}}R^* - I^* \tag{26}$$

$$R^* = \frac{q_1 - \alpha h_2}{\lambda V_{22}} - \frac{V_{12}}{V_{22}}(I^* + X^*) \tag{27}$$

where $V_{11} = \sigma_p^2 = \alpha\sigma_{p-}^2(1-\alpha)\sigma_{p+}^2 + \alpha(1-\alpha)h_1^2$, $V_{12} = \alpha\sigma_{p-}^2 + \alpha(1-\alpha)h_1 h_2$, and $V_{22} = \alpha\sigma_{p-}^2 + \alpha(1-\alpha)h_1^2$.

These equations show important implications about the optimal size of the physical inventory, futures position, and option position. The future and put-option positions have hedging and speculative demands in the formula. The inventory's size is not dependent on the first and second moments of the cash and futures prices (Bullock & Hayes, 1992).The first terms in equation (26) and (27) in the right-hand side are the speculative demand, and the second terms in the equations, $\frac{V_{11}}{V_{11}}$ and $\frac{V_{12}}{V_{22}}$, are hedging demand. If the inventory position, I, is a fixed position, equations (26) and (27) change their form. With the assumptions of fixed inventory, the closed-formula solution for the optimal futures and options is as follows:

$$X^* = \frac{\gamma_1 - \beta_1\gamma_2}{1 - \beta_1\beta_2} - I_0 \tag{28}$$

$$R^* = \frac{\gamma_2 - \beta_2\gamma_2}{1 - \beta_1\beta_2} \tag{29}$$

where $\gamma_1 = \frac{\mu_p - f_1}{\lambda V_{11}}$, $\gamma_2 = \frac{q_1 - \alpha h_2}{\lambda V_{22}}$, $\beta_1 = \frac{V_{12}}{V_{11}}$, and $\beta_2 = \frac{V_{12}}{V_{22}}$.

Based on the solution, a few theoretically important results were derived with assumption of no basis risk. Bullock and Hayes (1992) obtained six important observations. The first result compared the usefulness of futures and options as hedging instruments. The analysis found that a futures contract is the preferred hedging instrument for a fixed inventory. This observation was done by taking a partial derivative of the optimal future and option position with respect to the inventory. While the partial derivative of the optimal future position with respect to inventory was -1, the partial derivative of the optimal put-option position was zero.

The second observation was about regarding risk-neutral hedging or so-called a delta hedging. The second proposition stated that the optimal put-option hedging demand is not equal to the inverse of the optimal hedging demand for futures. The unequal put-option hedging demand occurs because there is an undiversifiable market risk that cannot be hedged by taking opposite the put-option position, and the risk-neutral hedging becomes the inverse size of the futures adjusted by this undiversifiable risk.

The third observation compared an effect of change in expected speculative return to the optimal future and option positions. Interestingly, the change for the expected return from both futures and options leads to a larger change to the optimal options than the optimal futures position. The observation is made in position two and three in Bullock and Hayes (1992). The proposition two stated that the change for the in expected futures' speculative return is larger for the optimal option position than the optimal futures position. The proposition three was similar to the proposition two: changing the expected options' speculative return leads to bigger change for the optimal option position than for the optimal futures position.

The fourth observations, made with the no-basis risk assumption, related how changes with the expected spot price and standard deviation affected the futures and options' speculative return. The result was proven by taking the partial derivative of the future and option speculative returns by mean and standard deviation of spot price in period two. The futures contract is a primary speculative instrument when change in the price's expected value in period two occurs. When the expected spot price changes by one unit, the speculative return from futures is greater than the speculative return from the put option. In contrast to how one unit of change with the expected spot price alters the speculative return for the future and put options, the put option has a larger speculative return when the spot price's standard deviation in period two changes by one unit. These observations tell that which choice, futures or put option, is a better speculative tool when important information (i.e., mean and standard deviation) changes (Bullock & Hayes, 1992).

All the observations made so far were done with the assumption of no basis risk. Bullock and Hayes (1992) assumed that price discovery is done for the futures market and that basis price, which reflects local supply and demand, localizes the future price into the spot price. The model also assumed that basis and future prices are independent variables because the basis price consists of local price factors while the where future price reflects all market factors:

$$\tilde{\pi} = (\tilde{p}_2 - p_1)I - \frac{1}{2}cI^2 + \left(\tilde{f}_2 - f_1\right)X + [q_1 - \max(0, k - \tilde{f}_2)]R \tag{30}$$

where b_t is the basis price for period t and $\tilde{p}_2 = \tilde{f}_2 - \tilde{b}_2$. Both the future and spot prices follow a normal distribution: $\tilde{f}_2 \sim N(\mu_f, \sigma_f^2)$ and $\tilde{b}_2 \sim N(\mu_b, \sigma_b^2)$, respectively. By applying the same theorem that was applied with the no-basis risk assumption, the expected profit and profit's variance are derived as follows:

$$E(\tilde{\pi}) = \left(\mu_f - \mu_b - p_1\right)\mathrm{I} - \frac{1}{2}cI^2 + \left(\mu_f - f_1\right)X + (q_1 - \hat{\alpha}\hat{h}_2)R \tag{31}$$

$$\begin{aligned} var(\tilde{\pi}) = \sigma_b^2 I^2 + \hat{\alpha}\sigma_{f-}^2(I + X + R)^2 + (1 - \hat{\alpha})\sigma_{f+}^2(I + X)^2 \\ + \hat{\alpha}(1 - \hat{\alpha})[\hat{h}_1(I + X) + \hat{h}_2 R]^2 \end{aligned} \tag{32}$$

where $\hat{\alpha}$ is the probability that the future price in period two is less than the strike price. The $\hat{h}_1$ and $\hat{h}_2$ variables are defined as $\hat{h}_1 = \mu_{f+} - \mu_{f-}$ and $\hat{h}_2 = k - \mu_{f-}$, respectively. μ_f and σ_f^2 with the and negative signs are the conditional mean and standard derivations which are similar to the conditional mean and standard deviation that are defined for the spot price in period two.

The procedures to find the optimal physical inventory, futures position, and put-option position are determined by substituting into the E-V framework and taking a partial derivative:

$$I^* = \frac{f_1 - \mu_b - p_1}{c + \lambda\sigma_b^2} \tag{33}$$

$$X^* = \frac{\mu_f - f_1}{\lambda \hat{V}_{11}} - \frac{\hat{V}_{12}}{\hat{V}_{11}} R^* - I^* \tag{34}$$

$$R^* = \frac{q_1 - \hat{\alpha}\hat{h}_1}{\lambda \hat{V}_{22}} - \frac{\hat{V}_{12}}{\hat{V}_{22}} (I^* + X^*) \tag{35}$$

where $\hat{V}_{11} = \sigma_f^2 = \hat{\alpha}\sigma_{f-}^2 + (1-\hat{\alpha})\sigma_{f+}^2 + \hat{\alpha}(1-\hat{\alpha})\hat{h}_1^2$, $\hat{V}_{12} = \hat{\alpha}\sigma_{f-}^2 + \hat{\alpha}(1-\hat{\alpha})\hat{h}_1\hat{h}_2$, and $\hat{V}_{22} = \hat{\alpha}\sigma_{f-}^2 + \hat{\alpha}(1-\hat{\alpha})\hat{h}_2^2$. The next step is to solve for the optimal futures and put-option position:

$$X^* = \frac{\hat{\gamma}_1 - \hat{\beta}_1\hat{\gamma}_2}{1 - \hat{\beta}_1\hat{\beta}_2} - I^* \tag{36}$$

$$R^* = \frac{\hat{\gamma}_2 - \hat{\beta}_2\hat{\gamma}_1}{1 - \hat{\beta}_1\hat{\beta}_2} \tag{37}$$

Although basis risk is introduced, the closed-form solutions for the optimal futures and option position are the same except that they depend on the futures price instead of the spot price. The solution for the optimal physical inventory position is dependent on the futures price and the risk premium. All the results obtained with the no-basis risk still hold for the basis risk. The investor adjusts the inventory's size with respect to changes in the mean and standard deviation for the basis price. The optimal physical-inventory position is directly proportional to changes with the expected basis price, but the absolute inventory size is inversely proportional to changes with the standard deviation for the basis price (Bullock & Hayes, 1992).

This study focused on the behavior of an investor whose goal is to maximize the utility with profit that comes from the physical inventory. This fixed inventory is subject to price risk, and it can be hedged with futures and option contracts. One of the research's main findings is that futures are a better hedging instrument. The investor speculates with a futures contract when the price's expected value changes and speculates with options when the price's volatility changes. Similarly, Bullock, Wilson, and Dahl (2007) conducted a study for a bread-manufacturing firm with the E-V framework. They found that futures contract is a primary hedging instrument and that options are used as a speculative tool when the hedger has market bias with either the in futures or options premium. The existence of undiversifiable market risk is proven. This undiversifiable risk cannot be hedged through both futures and put option.

Our goal in this study is to hedge the risk occurring in the cash flow. For some readers, visualizing how a default affects the firm's revenue may be difficult. Suppose a company in U.S. sold corn to a buyer in China using a forward contract. The seller hedges price risk with the long corn futures position. If the cash price of corn goes down and there is rationale for the buyer to purchase cheaper corn somewhere else, the buyer may default on the forward contract. Then, the seller either renegotiates a new price or finds a different buyer. In either case, the buyer is very likely to receive a lower price because the market's corn price is lower than the price when forward contract was written. Because the new price is lower than the forward contract's price, the seller's cash flow is lower. Since the cash flow from the spot market is lower, the loss from long futures position due to price decline is not completely offset. Therefore, the original hedging plan becomes ineffective when the buyer defaults on the contract.

3.5. Default Risk Models

Hedging the default risk has a relatively small amount of literature to review when compared to the pricing for futures and options contracts that are subject to the default risk (Korn, 2008). The default risk is a special case of quantity risk because the counterparty's default forces the non-defaulting party to not be paid for revenue that was supposed to come from the defaulting party. In the quantity risk model, the firm's output is not fixed, or it is random. The random output leads to random revenue for the firm. A similarity between the default risk and output risk is that changes in the output amount affect the revenue for the hedging firm or non-defaulting party.

The quantity risk that is typically studied in agriculture is the producer's production or yield risk. A crop's annual production depends on many factors and is subject to great variability. One year, a producer may have a great harvest, and other years, the producer may have a poor harvest. Because the producer's revenue comes from selling the crop at a given price, the quantity risk significantly influences the producer's revenue. Hence, the default risk is encompassed by the quantity risk because both risks measure the output variability and affect the non-defaulting party's revenue. This study focuses on the non-defaulting party hedging both the price and quantity risks using exchange-traded futures and options. Previous papers investigated the decision making for a risk-averse firm with counterparty risk. Mahul and Cummins (2008) studied a risk-averse firm's optimal production and hedging decision that confronted price risk when the hedging instrument was subject to a counterparty credit risk. Korn's (2008) study was similar to Mahul and Cummins (2008).

3.5.1. Mahul and Cummins (2008)

Mahul and Cummins (2008) used a two-period model where a competitive firm produces a single, fixed output, q, and this firm can hedge the price risk with a forward contract which is subject to counterparty credit risk. The futures contracts were standardized contracts that were traded at a central exchange, but the forward contracts were over-the-counter contracts. Mahul and Cummins conducted a theoretical analysis of the hedging firm's behavior under the utility-maximization principle. The competitive firm's objective was to maximize the von Neumann-Morgenstern utility function, u, of the profit, $\tilde{\pi}$:

$$\tilde{\pi} = q\tilde{p} - c(q) + \min[n(p_0 - \tilde{p}), \widetilde{K}] \tag{38}$$

where variables with a tilde were random. The p_0 and $\tilde{p}$ variables were the forward price and market price at which the firm expects to sell output. $\widetilde{K}$ was the contract writer's asset value at the contract's maturity date. The $c(q)$ function was a cost function with assumptions of $c(0) \geq 0, c'(q) > 0,$ and $c''(q) > 0$. The n variable was the number of contracts that were purchased or sold by the firm. Mahul and Cummins (2008) assumed that the forward-contract writer defaults when the realized payoff function, I(p), was greater than the realized asset value, K. With this model, the hedging firm was assumed to have zero probability default, but the forward contract's holder (buyer) defaulted on the forward contract when the forward contract's payoff was positive for the hedging firm and negative for the buyer. Equation (38) represents the payoff function for the trading firm that sells output to a third party and uses a forward contract to hedge the price risk. The forward contract that is subject to counterparty credit risk is a vulnerable forward contract. Hence, the writer defaults when $\mathrm{I(p)} > K$, and the hedging firm only receives a payment of K. The writer is solvent, able to pay the full commitment, when $\mathrm{I(p)} \leq \mathrm{K}$.

Because Mahul and Cummins (2008) focused on a risk-averse firm's hedging and production decision with a counterparty risk of the over-the-counter (OTC) forward contract, the probability a contract writer's default must be defined. That probability is defined by the joint cumulative distribution function (CDF), $\Psi(\mathrm{p}, \mathrm{K})$, of settlement prices $\tilde{p}$ and $\widetilde{K}$. The cumulative distribution is defined on the two-dimensional support, $[0, p^{max}] \times [0, K^{max}]$, where $p^{max} > 0$ and $K^{max} > 0$. Mahul and Cummins (2008) also defined the conditional CDF, G, for variable $\widetilde{K}$ if the settlement price is equal to the realized settlement price; that is, $\tilde{p} = p$. Hence, $\mathrm{G}(\mathrm{I(p)}|\tilde{p} = p)$ is the probability of the contract writer's default for the payoff, I(p). Mahul and Cummins (2008) assumed that default is an endogenous event because the default is determined by the firm's hedging decision, and the counterparty only defaults when the payoff is positive. Mahul and Cummins (2008) rewrote equation (38) as follows:

$$\tilde{\pi} = \mathrm{q}\tilde{p} - \mathrm{c(q)} + \tilde{x}\mathrm{n}(p_0 - \mathrm{p}) \text{ with } \mathrm{x} = \begin{cases} 1, & p \geq p_0 - K/n \\ \mathrm{K}/(\mathrm{n}(p_0 - p)), & otherwise \end{cases} \quad (39)$$

Therefore, the firm's profit depends on variable $\tilde{x}$ which is comprised of variables p_0, K, and n. Equation (39) implies how the default risk is contingent on the firm's hedging strategy because x consists of n.

Because the hedging firm's objective is to maximize a utility function, the firm's expected utility function is as follows:

$$\mathrm{U} \equiv \mathrm{Eu}(\tilde{\pi}) = \int_0^{p^{max}} \int_0^{K^{max}} u(\pi) d\Psi(p, K) \quad (40)$$

where E is the expectation operator. Plugging equation (39) into the equation (40), the utility function, U, becomes

$$U \equiv \int_0^{p^{max}} \left\{ \int_0^{n(p_0 - p)} u(qp - c(q) + K) dG(K|\tilde{p} = p) + u(qp - c(q) + n(p_0 - p))\bar{G}(n(p_0 - p)|\tilde{p} = p) \right\} d\Phi(p) \quad (41)$$

where $\overline{G}(.|.) = 1 - G(.|.)$ is the probability that the counterparty does not default. The first term on the right-hand side of equation (41) is the hedging firm's utility level under counterparty's default while the second term signifies the hedging firm's utility level with a full commitment (Mahul & Cummins, 2008).

The necessary and sufficient conditions for a unique, maximum of U with respect to n and q are as follows:

$$\frac{\partial U}{\partial n} = E\left[(p_0 - \tilde{p})u'\left(q\tilde{p} - c(q) + n(p_0 - \tilde{p})\right)\bar{G}(n(p_0 - \tilde{p})|\tilde{p})\right] = 0 \quad (42)$$

$$\frac{\partial U}{\partial q} \equiv E\left\{\left(\tilde{p} - c'(q)\right)u'(\tilde{\pi})\right\} = 0 \quad (43)$$

Based on equations (42), equation (43), the contract writer's probability of default is zero when $\tilde{p} > p_0$. Mahul and Cummins (2008) derived the following equation:

$$c'(q) = p_0 + \frac{1}{Eu'(\tilde{\pi})}\int_0^{p_0}\left[(p - p_0)\int_0^{n(p_0-p)} u'(pq - c(q) + K)dG(K|\tilde{p} = p)\right]d\Phi(p) \quad (44)$$

If the default probability is zero, the second term on the right-hand side of equation (44) disappears, and the hedging firm selects the output where the marginal cost equals the forward price. If the default probability is nonzero, the second term on right-hand side of equation (44) is negative, hence optimal production under a forward contract that is subject to credit risk is less than a forward contract which is not subject to the counterparty credit risk (Mahul & Cummins, 2008). The optimal HR has not been derived yet. For the later part of this model, they designed an option contract based on a vulnerable forward and a derived optimal position for that option.

Substituting the output, q, into the variable, n, of equation (44), an equation associated with the optimal hedging decision is derived:

$$\frac{\partial U}{\partial n} = u'(qp_0 - c(q))\{(p_0 - E\tilde{p})E\bar{G}(q(p_0 - \tilde{p}|\tilde{p})) + cov(\tilde{p}, G(q(p_0 - \tilde{p}|\tilde{p}))\} \quad (45)$$

The optimal hedge which maximizes the utility of the hedging firm consists of hedging, speculative, and default components. The hedging component is equal to the output and the speculative component which depends on the difference between the expected settlement price and the forward price, $p_0 - E\tilde{p}$. The hedging firm decreases the hedge if $p_0 - E\tilde{p}$ is negative and increases the hedge if $p_0 - E\tilde{p}$ is positive. The default component is contingent on the correlation between the settlement price and counterparty's default probability given the realized settlement price; therefore, the hedging firm increases the forward hedge if the covariance is positive and decreases that hedge if the covariance is negative. When the default probability is differentiated with the settlement price, variations in the default probability with one unit of change in the settlement price can be measured:

$$\frac{dG(q(p_0 - p)|\tilde{p} = p)}{dp} = -qg(q(p_0 - p)|\tilde{p} = p) + G_p(q(p_0 - p)|\tilde{p} = p) \tag{46}$$

where $G_p(q(p_0 - p)|\tilde{p} = p)$ is the partial derivative with respect to the settlement price which is $\frac{\partial G(q(p_0 - p)|\tilde{p}=p)}{\partial p}$. From equation (46), Mahul and Cummins (2008) reached the conclusion that, assuming the forward price is less than or equal to the expected settlement price, the hedging firm requires partial coverage of $n^* < q$ if the increased settlement price, p, decreases the riskiness of the counterparty's asset value, K, by first-degree stochastic dominance (FSD); or if the settlement price, p, and the counterparty's asset value, K, are statistically independent.

In addition to analyzing the risk-averse firm's hedging decision with a vulnerable forward contract, Mahul and Cummins (2008) developed a vulnerable options contract with strike-price continuum, $[0, p^{max}]$, instead of a single strike price. This options contract has an underlying value which depends on the forward contract's payoff. (Mahul & Cummins, 2008).

If the hedging firm sells n forward contracts with a forward price, p_0, this vulnerable hedging instrument has an associated payoff and premium, $[I(.), P]$, where I is a non-negative payoff function of the forward contract for all realized settlement prices, p, and P is the hedging instrument's option premium. Assuming that the vulnerable hedging instrument is fairly priced, the premium for the options is as follows:

$$P = Emin[I(\tilde{p}) + n(p_0 - \tilde{p}), \widetilde{K}] \tag{47}$$

The vulnerable options contract needs to maximize the firm's utility with constraints, the non-negativity of the payoff function, and equation (48):

$$\underset{I(.)}{Max} \int_0^{p^{max}} \int_0^{K^{max}} u(qp - c(q) + \min[I(p) + n\,(p_0 - p), K] - P) d\Psi(\mathrm{p}, \mathrm{K}) \tag{48}$$

subject to the non-negativity of the payoff function and constrained by equation (47). Equation (48) is maximized with respect to the fixed premium, P. Mahul and Cummins (2008) derived two results when maximizing the problem in equation (48). The first one was that positive probability of the counterparty's full performance on the forward contract. The optimal hedging-instrument contract payoff, $I^*(p)$, gave full marginal coverage with the strike price:

$$\hat{p} \in [0, p^{max}]: I^*(p) = (q - n)\max(\hat{p} - p, 0) \tag{49}$$

The second conclusion obtained with equation (48) was that, with an unbiased, vulnerable hedging contract and n forward contracts, the optimal strike price for the vulnerable hedging contract satisfied:

$$\begin{gathered} \hat{p} < p^{max} \; if \; cov[u'(qp + I^*(\tilde{p}) + n(p_0 - \hat{p}) - P), G(I^*(\tilde{p}) + n(p_0 - \tilde{p}|\tilde{p})] > 0 \\ \hat{p} = p^{max} \; otherwize \end{gathered} \tag{50}$$

The covariance sign in equation (50) is important for the optimal strike price of vulnerable options. The covariance sign in equation (50) also depends on the counterparty's default probability as a decreasing or increasing function of the settlement price, p. The partial derivative of the default probability with respect to the settlement price is as follows:

$$\begin{aligned} &\frac{\partial G(I^*(p) + n(p_0 - p)|\tilde{p} = p)}{\partial p} \\ &\qquad = [I'^*(p) - n] g(I^*(p) + n(p_0 - p)|\tilde{p} = p) + G_p(I^*(p) + n(p_0 - p)|\tilde{p} \\ &\qquad = p) \end{aligned} \tag{51}$$

Assuming vulnerable options contract based on the vulnerable forward contract is available, fairly priced, and hedging firm takes forward position, the hedging firm's optimal hedging strategy is for long a put vulnerable options contract with HR equals to $1-\frac{n}{q}$ (Mahul and Cummins, 2008). This result relies on the one of two conditions. Increasing the settlement price, p, decreases the riskiness of the contract writer's asset value, $\widetilde{K}$, by FSD or the settlement price, $\tilde{p}$, and contract writer's asset value, $\widetilde{K}$, are statistically independent random variables.

3.5.2. Korn (2008)

This section gives a detailed explanation of Korn's (2008) research. This study was similar to Mahul and Cummins (2008) because it used the expected utility-maximization framework with a similar, basic payoff function. The differences between Korn (2008) and Mahul and Cummins (2008) were how they extended the basic payoff function. Both studies used the utility-maximization principle and assumed that a forward contract is subject to default risk. Instead of designing the options contract as Mahul and Cummins (2008) did where the underlying value was based on the vulnerable forward contract, Korn (2008) extended the basic payoff function to include the credit derivative and stochastic recovery rate. Also, model did not include the exchange-traded option as a hedging instrument.

The basic model assumes that a forward contract is subject to the default risk, and this model is extended to include a credit derivative for hedging counterparty's creditworthiness in the second section (Korn, 2008). Additionally, the assumption of a stochastic recovery rate is included with the model, so the hedger can recover a portion of the loss incurred from forward contract's default. Korn (2008) does not derive the closed-form solution for the optimal forward position and quantity, but theoretically he analyzes the hedging firm's production and hedging decision when the forward contract is a defaultable, subject to the default risk, contract.

In this model, the firm is a profit maximizer with the von Neumann-Morgenstern utility function, and the firm can hedge production with a defaultable forward contract at time zero and lift the hedge at time one:

$$\tilde{\pi} = \tilde{P}Q - c(Q) + h\left(F - \tilde{P}\right) - \tilde{I}\max[h\left(F - \tilde{P}\right), 0] \qquad (52)$$

Where the tilde represents random variable; $\tilde{\pi}$ is the firm's profit; and $\tilde{P}$ and Q are the competitive market price in period one and the output quantity, respectively. An increasing, strictly convex, twice differentiable cost function for production is represented by c. F is the forward price, and h is the size of the forward position sold in period zero. $\tilde{I}$ is a random variable with Bernoulli distribution, and it characterizes the counterparty's default for the forward contract. If $I = 0$, the counterparty did not default on the forward contract, and if $I = 1$, the counterparty defaulted on forward contract.

The hedging firm maximizes the expected value of the von Neumann-Morgenstern utility function for profit, U, with respect to the size of forward position, h, and output quantity, Q,:

$$\max_{Q, h} E[U(\tilde{\pi})] \qquad (53)$$

By solving for the optimal output quantity and forward position's size, which maximizes equation (53), Korn (2008) derived three propositions relating the HR, forward position, and output quantity.

The Korn (2008) first showed the relationship between the forward contract's expected profit and the optimal HR, $\frac{h^*}{Q^*}$. The expected profit from the forward contract, $\left(F - E(\tilde{P})\right)$, is proportional to the HR. If the forward contract's expected profit is zero, the optimal HR is one. If the expected profit form the forward contract is positive and negative, the optimal HR is above and below one respectively (Korn, 2008). This relationship between the expected profit from forward contract and the HR is independent of whether the counterparty defaults on the forward contract; however, the forward contract's default risk does affect the optimal HR if the forward contract's expected is non-zero (Korn, 2008).

The second proposition relates the optimal default-risky forward position and the optimal output quantity. The proposition states that, if the default-risky forward position is either strictly greater than zero or strictly lower than zero, then the marginal cost for the optimal output is greater than forward price or less than the forward price, respectively (Korn, 2008). This proposition implies that the marginal cost of production at the optimal output cannot be equal to the forward price if the forward position's size is non-zero.

If the forward contract is not subject to default risk, the firm seeks more profit to maximize the utility by simultaneously making the opposite move for production and the forward position. This move by the hedging firm is similar to how arbitragers profit from market inefficiencies. The hedging firm increases the optimal production to the point where the marginal cost equals the forward price because increased production is sold with a forward contract to take advantage of price difference. If production's marginal cost is higher than the forward price, the hedging firm decreases the optimal output quantity to the point where the marginal cost equals the forward price. The hedging firm can purchase a decreased amount of optimal output using a forward contract leading to a decreased total cost because the forward price is lower than the marginal cost of production (Korn, 2008).

When the forward contract is subject to default risk, the hedging firm loses the revenue that is supposed to come from the forward contract. The second proposition states that, if a forward position is taken, the marginal cost of production at the optimum cannot be equal to the forward price. When the counterparty defaults on the forward contract, the revenue from the forward position is zero because of the assumed zero recovery rate. Hence, the second proposition states reducing the size of the defaultable forward position and increases the optimal output to increase the total revenue in order to cope with a possible revenue loss when the counterparty defaults on the forward contract (Korn, 2008). The second proposition indicates the relationship between the optimal output and the forward position via the marginal production cost; however, the proposition does not show the optimal HR that maximize the utility (Korn, 2008).

The third proposition derived from equation (53) proves the connection among default-free, default-risky, and optimal production levels. Korn's (2008) third proposition states that the optimal production is lower for a default-risky forward contract than for a default-free forward contract when the expected profit for both default-risky and default-free forward contracts is zero. The goal of using forward contracts is to reduce the revenue fluctuations that arise from price risk; however, the forward contract's default risk increases the revenue fluctuation, and the forward contract is no longer a good hedging tool to reduce the price risk. The risk-averse hedging firm that uses a default-risky forward contract, which is not good hedging instrument, decreases the optimal production level due to the forward contract's unreliability to reduce the price risk (Korn, 2008).

The third proposition also has an effect on the optimal forward contract that is sold on the market. The first proposition states that, when the expected profit from a forward contract is zero the HR is one, and third proposition states that the optimal production level is lower for a forward contract with default risk. Hence, by combining the first and third propositions, default-risky forward position's size is smaller than the default-free forward position (Korn, 2008). The default-risky forward position size is smaller than default-free forward position because when the expected profit of a forward contract is zero, HR for both default-risky and default-free positions are one by the first proposition. Also, the optimal production level's size for the default-risky forward contract is lower than the optimal production level for the default-free forward contract. Therefore, the hedging firm has to reduce the amount of output that is sold with the default-risky forward position to the optimal production in order to keep the optimal HR of one, leading to a lower default-risky forward position than the default-free forward position. To analyze the firm's behavior, Korn (2008) extended equation (52) to include a credit derivative, the value of counterparty's derived creditworthiness, as another hedging tool.

Korn (2008) appended the credit derivative on equation (52) to hedge a revenue loss coming from the counterparty's default on the forward contract. The credit derivatives are contracts where the value is derived from a referenced entity's creditworthiness. The credit derivative is like an insurance for entity's creditworthiness, and the purpose is to transfer the credit risk from the derivative's buyer to its seller. The value of soybean, corn, and crude-oil futures contracts traded at the Chicago Mercantile Exchange (CME) and the New York Mercantile Exchange (NYMEX) are derivative contracts where the underlying value is derived from soybeans, corn, and crude oil. A great example of credit derivatives is a swap contract called a credit default swap (CDS). A swap contract allows cash flow between two parties, the buyer and seller, for a fixed time period which is specified in the contract. The CDS's seller insures that the buyer will be compensated for a possible default, creditworthiness decreasing to nil, of the referenced debt or bond, and CDS's buyer has to make a series of payments to the seller as a fee for protection, like a series of insurance premiums, against default.

Korn (2008) assumed that the hedging firm enters the credit -derivative market with the premium paid, K, at time zero. This credit derivative's function is similar to the CDS where the buyer is compensated against an entire loss which the buyer, the hedging firm, incurs with a default. However, Korn (2008) assumed that the credit derivative is also subject to possible default by the seller. After extending equation (52) to include the credit derivative, the firm's payoff function as follows:

$$\tilde{\pi} = \tilde{P}Q - c(Q) + h\big(F - \tilde{P}\big) - \tilde{I}\max\big[h\big(F - \tilde{P}\big), 0\big] + \mathrm{z}[\tilde{I}(1 - \tilde{J})\max\big[\big(\mathrm{F} - \tilde{P}\big), 0\big] - \mathrm{K}(1 + \mathrm{r})] \tag{54}$$

J is random variables with a Bernoulli distribution. J represents the probability of default for the credit derivative. The hedging firm buys z credit derivatives at time zero to hedge a possible loss from a default on a forward contract, and r is the risk-free rate for period zero to one (Korn, 2008). After extending equation (52) to include credit derivative, the decision variables for equation (54) in the utility-maximization problem are the output, the number of forward contracts and the credit derivative:

$$\max_{Q, h, z} E[U(\tilde{\pi})] \tag{55}$$

Based on equation (55), Korn (2008) derived two propositions relating the optimal HR and the expected profit from the forward contract when credit derivatives are available.

The fourth proposition states that, when the credit derivative is available as a hedging instrument for the forward contract's default, the optimal HR is one if the expected profit from selling the forward contract leads to zero expected profit and vice versa. This proposition implies the first proposition which, when derived without the credit derivative, still holds (Korn, 2008). Hence, this proposition leads to the same conclusion as first proposition: regardless of the counterparty's credit riskiness for the forward contract and the availability of a credit derivative, the optimal HR is one if and only if zero expected profit comes from the forward contract.

Furthermore, Korn (2008) derived the link between the ratio of the optimal credit derivative position and the forward position as well as the default risk on the credit derivative. The fifth proposition started with the assumption that the expected profit from the forward position is zero and compared the ratio of the optimal credit derivate and the forward position when the credit derivative was default-free or default-risky. With a default-free credit derivative, the fifth proposition stated that, if and only if zero expected profit comes from a credit derivative, the ratio of the optimal credit derivative and the forward position is one. If the credit derivative is not default-free, the ratio of the optimal credit derivative and the forward position is strictly greater than and smaller than zero and one, respectively (Korn, 2008). When the credit derivative is default-free, the hedging firm is guaranteed to get back its money which was lost with default-risky forward contract's default by paying a premium for the credit derivative. If a forward contract is not defaulted, the hedging firm only loses the price of buying the credit derivative. Because the hedging firm gets the entire loss back from the default-free credit derivative when the forward contract defaults, this situation is identical to selling a default-free forward at the default-risky forward price minus the cost of buying the credit derivative. When a credit derivative is subject to the default risk, the hedging firm still buys a credit derivative to protect profit; however, firm does not hedge entirely with the credit derivative. If a credit derivative is subject to the default risk, the firm should not only take a position for either a forward or credit derivative, but also spread among forward contract and credit derivatives (Korn, 2008).

Korn (2008) assumed a zero recovery rate for both equations (52) and (54), meaning that the firm does not recover any profit that is supposed to come from the forward contract. The stochastic recovery rate is the rate at which the firm can recover a portion of the loss from the forward contract's default. The hedging firm faces two types of risk: the probability of defaulting on the forward contract and the amount of loss that comes from the default (Korn, 2008). The payoff function with a stochastic recovery rate is an extension of equation (52):

$$\tilde{\pi} = \tilde{P}Q - c(Q) + h\left(F - \tilde{P}\right) - \tilde{I}(1 - \tilde{R})\max[h\left(F - \tilde{P}\right), 0] \tag{56}$$

where R represents the stochastic recovery rate of the forward contract, which lies between zero and one, and it may be independent of the price, P, and forward default, I. $R = 1$ means the entire default loss is recovered, that is the same as buying a free credit derivative, and $R = 0$ means equation (56) is the same as equation (52). From equation (56), Korn (2008) derived a proposition relating the optimal HR, the ratio of the optimal forward position and quantity, and the stochastic recovery rate with the assumption of expected profit from a forward contract being zero.

The proposition states that, under zero expected profit from a forward contract and a stochastic recovery rate, the optimal forward position is greater than the optimal production that is the optimal HR is greater than one. This result is rather counterintuitive from a risk-management perspective of reducing the risk and diversification. The first proposition stated that, with zero expected profit, the optimal HR is one regardless of the default risk, so when there is an opportunity, a non-zero recovery rate, for the firm to recover some loss which comes from defaulting on a forward contract, the firm should allocate more resources to the forward contract in order to gain more profit. Although this proposition is a surprising result, the effect of changing the stochastic recovery rate to the optimal HR is insignificant (Korn, 2008).

Korn (2008) derived several fundamental results about the default risk and HR. Similar to Mahul and Cummins (2008), Korn (2008) did not derive the closed-form solution of the optimal HR but, theoretically, analyzed the risk-averse firm's response with instrument that is subject to the default risk. The strongest result obtained for the basic model was the first proposition which says that, under zero expected profit, the optimal HR is one irrespective of whether the forward contract defaults. The basic model was extended to include a credit derivative to hedge the credit risk and a stochastic recovery rate if there was a default for the forward contract in order to make the basic model more realistic. The basic results that were obtained by extending the basic model were that firms diversify credit risk when a credit derivative is available and over-hedge a forward contract with the non-zero recovery-rate assumption in order to maximize the profit.

3.5.3. Summary of the Default Risk Models

This section explained the basic portfolio model of hedging with the forward contract's default risk which the hedging firm uses to hedge the price risk. Both studies, Mahul and Cummins (2008) as well as Korn (2008), conducted a theoretical analysis of the hedging firm's behavior. Both studies lacked a closed-formula solution for the optimal HR which maximizes the utility function. The closed-formula solution of the optimal HR is important when the model is applied to hedge the default risk. As an alternative approach for further research, one could model a firm's activity of hedging price risk by utilizing futures contracts that consider basis risk and default-risky forward contracts (Mahul & Cummins, 2008).

The default-risk problem for commodity trading is the existence of a counterparty's possible default on the forward contract which is troublesome for the commodity-trading firm that wants to secure the revenue source. If a buyer defaults on the forward contract, the seller has to find a new buyer or to renegotiate a new price. In either case, the commodity-trading firm is likely to have lower-than-expected revenue, hence, hedging for revenue loss with futures or options is paramount aspects of risk management when trading a commodity.

3.6. Theoretical Model for the Hedging Default Risk

In this section, an analytical model for the default risk that is based on the E-V framework is explored. The hedger has three markets to consider, the spot market, futures market, and options market, for a particular commodity. The model solves for the optimal futures and option positions to maximize the E-V utility function. The maximization problem is, essentially, a global-maximization problem. This problem is solved with the following steps: first, the hedger's payoff function is defined; second, the expected value and variance for the payoff function are derived; third the expected value and variance are inserted into the E-V framework; and fourth, the solution for the optimal futures and option positions are found, proving that the Hessian matrix is a negative-definite matrix.

The quantity-risk model developed for the analytical solution is built on Bullock and Hayes (1992). Suppose a hedging company sold grain on the cash market and took positions for futures and/or options in period one. The hedge placed in period one is closed in period two. The hedging company's payoff function is defined as follows:

$$\tilde{\pi} = \tilde{p}_2\tilde{d} + p_1(Q - \tilde{d}) + (\tilde{f}_2 - f_1)X + [Prem + \max(0, K - \tilde{f}_2)]R \quad (57)$$

The tilde represents random variables. $\tilde{\pi}$ is the hedger's profit. The p_t and f_t variables are the cash price and futures price for period t. Because this model is a one period model period one and two are used. Q and $\tilde{d}$ are the initial quantity sold and the random amount of default or cancellation that occurred during the transaction, respectively. X and R are the futures position and put-option positions, respectively. Cash, futures, and default distribution is assumed to be normally distributed. $Prem$ and K are the premium for the put option and the strike price for the put option, respectively.

In equation (57), only the put option is considered. The put option is only considered because of the redundancy with both calls and the put option in the model. This redundancy places both call and put options in the model at the same time which is meaningless because a combination of futures and option positions can create another payoff function using the synthetic option strategy. When futures, put options, and call options are in the model, one instrument becomes redundant because a combination of payouts for any two can imitate the third instrument's payout (Bullock & Hayes, 1992).

For example, the payout of the combination of futures and long-put option positions is identical to long call position. This identical payout is the synthetic call option. The Figure 3.1 shows the payout for the futures, put option, and synthetic-call option. The orange and green lines are payout functions for long futures and long-put option, respectively. The grey line is the net payout, which is the sum of the futures and long-put payout. The shape of the net-payout position is identical to the long call option.

Figure 3.1. Synthetic Option Payoff.

The next step is to derive the expected value and the variance of equation (57) to insert them into the E-V framework. The theorem derived in Bullock and Hayes (1992) is applied to equation (57) in order to accomplish this task. This theorem's advantage is that the hedger only needs to rely on the random variables' first and second moments (Bullock & Hayes, 1992). Because the option's payoff function is nonlinear, adding options as a hedging instrument makes the hedger's payoff becomes a nonlinear function. Because equation (57) is a nonlinear function, the theorem is useful to apply to equation (57). The theorem allows the payoff function to be split into two parts, the futures price below the option's strike price and the futures price above the strike price. This theorem is as follows:

If $\mathrm{g(p)} = g_1(p)\ \forall p \leq k, g(p) = g_2(p)\ \forall p > k, and\ p$ is a random variable with cumulative distribution function, F(p), then

$$\mathrm{E[g(p)]} = \alpha \mathrm{E}[g_1(p)|\ p \leq k] + (1-\alpha)\mathrm{E}[g_2(p)|p > k]$$
$$\mathrm{var[g(p)]} = \alpha\ \mathrm{var}[g_1(p)|\ p \leq k] + (1-\alpha)\ \mathrm{var}[g_2(p)|p > k]$$
$$+\alpha(1-\alpha)\{\ \mathrm{E}[g_2(p)|p > k] - \ \mathrm{E}[g_1(p)|p \leq k]\}^2,$$

where, $\alpha = \mathrm{F(k)} = prob(\ p \leq k)$.

The theorem splits a function into two parts. In this study, equation (57) is split into two parts based on $\tilde{f}_2$ and K. This theorem shows the relationship between the hedger's payoff when $\tilde{f}_2$ is below K and when $\tilde{f}_2$ is above K. This theorem is useful because the payoff function with an option is a non-smooth function without a derivative, at a point where the futures price is equal to the strike price (Bullock & Hayes, 1992).

Another implication of this theorem is the relationship between the option's delta and α, and how delta is incorporated with the derivation for the expected value and variance. An option's delta is the partial derivative of the theoretical option-price formula by the underlying price, and it measures the change in the option's premium with respect to the underlying price's change. For example, if an option's delta at a particular strike is one, then a one-dollar change in the underlying price will change the option premium by a dollar for the call option. If the delta is 0.5, then a one-dollar change in the underlying price will alter the premium by 50 cents. The delta also indicates the option's likelihood to be in-the-money, that is, the option's probability of being profitable. According to Bittman (2008), the delta for in-the-money options is large, but the at-the-money and out-of-the-money options have a small delta. In the theorem, the delta for the option with strike price, K, is captured by variable, α, which is the probability of $\tilde{f}_2$ falling below the strike price, K. Hence, α measures how likely the option is to be in-the-money. Because the model uses a put option for hedging, α, is equal to the negative of delta, which ranges from -1 to 0 (Bullock & Hayes, 1992).

Simplifying the expected value of equation (57) is done by applying the total probability theorem. Suppose that there are n mutually exclusive events, $B_1, \dots B_n$; then, the expected value of Z is equal to the sum of the conditional expectations' product for random variable Z with probability of B_i where $1 \leq i \leq n$.

$$E[Z] = \sum_{i=1}^{n} E[Z|\, B_i] * P(B_i) \tag{58}$$

By applying equation (58) to equation (57), the expected value is derived as follows:

$$E[\pi] = \mu_{p_2d} + p_1(Q - \mu_d) + (\mu_{f_2} - f_1)X + RPrem + \alpha RH \tag{59}$$

where μ represents the expected value for the subscript's random variable. For example, μ_{p_2d} is the expected value for the product of the cash price in period two and the default quantity which is the expected loss of possible revenue. $H = K - \mu_{f_2-}$. The positive and negative signs in the subscripts mean that there is conditional expectations when the strike price is below and above the K, respectively. Similarly, by substituting equation (57) into the theorem's variance portion, the portfolio's variance is derived as follows:

$$\begin{aligned} Var(\pi) = {} & \alpha(\mathrm{Var}(p_2 d)_- - \mathrm{p}_1^2\mathrm{Var}(d)_- + (\mathrm{X}-\mathrm{R})^2\mathrm{Var}(f_2)_- - 2p_1\mathrm{Cov}(p_2 d, d)_- \\ & + 2(\mathrm{X}-\mathrm{R})\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - 2p_1(\mathrm{X}-\mathrm{R})\mathrm{Cov}(\mathrm{d}, f_2)_-) \\ & + (1-\alpha)(\mathrm{Var}(p_2 d)_+ + \mathrm{p}_1^2\mathrm{Var}(d)_+ + \mathrm{X}^2\mathrm{Var}(f_2)_+ \\ & - 2p_1\mathrm{Cov}(p_2 d, d)_+ + 2\mathrm{X}\mathrm{Cov}(p_2 d, f_2)_+ - 2p_1\mathrm{X}\mathrm{Cov}(d, f_2)_+) \\ & + \alpha(1-\alpha)(\mathrm{T} - p_1\mathrm{D} + \mathrm{XF} - \mathrm{RH})^2, \end{aligned} \tag{60}$$

where, $Var()$, is the variable's variance and $Cov()$ represents covariance. The positive and negative signs are conditions for the variance and covariance the where futures price is above and below the strike price, respectively. $T = \mu_{p_2 d+} - \mu_{p_2 d-}$, $\mathrm{D} = \mu_{d+} - \mu_{d-}$, $F = \mu_{f_2+} - \mu_{f_2-}$. Plugging equations (59) and (60) into the E-V framework, the utility function for hedging company is established:

$$\begin{aligned} \mathrm{U} = {} & \mu_{p_2 d} + p_1(Q - \mu_d) + (\mu_{f_2} - f_1)\mathrm{X} + \mathrm{RPrem} + \alpha\mathrm{RH} - \frac{\lambda}{2}[\alpha(\mathrm{Var}(p_2 d)_- \\ & - \mathrm{p}_1^2\mathrm{Var}(d)_- + (\mathrm{X}-\mathrm{R})^2\mathrm{Var}(f_2)_- - 2p_1\mathrm{Cov}(p_2 d, d)_- \\ & + 2(\mathrm{X}-\mathrm{R})\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - 2p_1(\mathrm{X}-\mathrm{R})\mathrm{Cov}(\mathrm{d}, f_2)_-) \\ & + (1-\alpha)(\mathrm{Var}(p_2 d)_+ + \mathrm{p}_1^2\mathrm{Var}(d)_+ + \mathrm{X}^2\mathrm{Var}(f_2)_+ \\ & - 2p_1\mathrm{Cov}(p_2 d, d)_+ + 2\mathrm{X}\mathrm{Cov}(p_2 d, f_2)_+ - 2p_1\mathrm{X}\mathrm{Cov}(d, f_2)_+) + \alpha(1 \\ & - \alpha)(\mathrm{T} - p_1\mathrm{D} + \mathrm{XF} - \mathrm{RH})^2], \end{aligned} \tag{61}$$

where λ is the hedging firm's risk-averse coefficient.

Obtaining the optimal solution is a difficult task with all the variables involved in equation (61); therefore, Mathematica 10 is used to symbolically solve the equation. The partial derivative of U with respect to the futures and put-option positions is as follows:

$$\begin{aligned} \frac{\partial \mathrm{U}}{\partial X} = {} & (\mu_{f_2} - f_1) - \frac{\lambda}{2}[\alpha(2\mathrm{Cov}(p_2\mathrm{d}, f_2)_- + 2(\mathrm{X}-\mathrm{R})\mathrm{Var}(f_2)_- - 2p_1\mathrm{Cov}(\mathrm{d}, f_2)_-) \\ & + (1-\alpha)(2\mathrm{Cov}(p_2\mathrm{d}, f_2)_+ - 2p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ + 2\mathrm{X}\mathrm{Var}(f_2)_+) + \alpha(1 \\ & - \alpha)(2\mathrm{FT} + 2\mathrm{F}^2\mathrm{X} - 2\mathrm{FRH} - 2p_1\mathrm{DF})] \end{aligned} \tag{62}$$

$$\frac{\partial \mathrm{U}}{\partial R} = \mathrm{Prem} + \alpha\mathrm{H} - \frac{\lambda}{2}[\alpha(2p_1\mathrm{Cov}(\mathrm{d}, f_2)_- - 2\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - (\mathrm{X} - \mathrm{R})\mathrm{Var}(f_2)_-) + \alpha(1-\alpha)(2\mathrm{H}^2\mathrm{R} - 2\mathrm{HT} - 2\mathrm{FHX} + 2p_1\mathrm{DH})] \tag{63}$$

By solving these equations, the simultaneous optimal closed-formula solution for the futures and put-option positions that maximizes utility is derived with the condition that the default amount is less than or equal to the quantity:

$$\begin{aligned} X^* = -((-(&\mathrm{Prem} + \alpha\mathrm{H} + \alpha(1-\alpha)\lambda\mathrm{HT} + \alpha\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - \alpha(1-\alpha)\lambda p_1\mathrm{DH} \\ &- \alpha\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_-)(\mathrm{FH}\alpha(1-\alpha)\lambda + \alpha\lambda\mathrm{Var}(f_2)_-) \\ &+ (-\mathrm{H}^2\alpha(1-\alpha)\lambda - \alpha\lambda\mathrm{Var}(f_2)_-)\big(-\mathrm{FT}\alpha(1-\alpha)\lambda \\ &- \alpha\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - (1-\alpha)\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+ - f_1 + \mathrm{DF}p_1(1-\alpha)\alpha \\ &+ \alpha\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ + (1-\alpha)\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ + \mu_{f_2}\big)) \\ &/(-(\mathrm{FH}(1-\alpha)\alpha\lambda + \alpha\lambda\mathrm{Var}(f_2)_-)^2 \\ &+ (-\mathrm{H}^2\alpha(1-\alpha)\lambda - \alpha\lambda\mathrm{Var}(f_2)_-)(-\mathrm{F}^2(1-\alpha)\alpha\lambda - \alpha\lambda\mathrm{Var}(f_2)_- \\ &- (1-\alpha)\lambda\mathrm{Var}(f_2)_+))) \end{aligned} \tag{64}$$

$$\begin{aligned} R^* = -((&\alpha\mathrm{F}^2\mathrm{Prem} + \mathrm{F}^2\mathrm{H}\alpha^2 - \mathrm{F}^2\mathrm{H}\alpha^3 + \alpha^2\mathrm{F}^2\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- \\ &- \alpha^2\mathrm{FH}\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - \mathrm{F}^2\alpha^3\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- \\ &+ \mathrm{FH}\alpha^3\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_- - \mathrm{FH}\alpha\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+ \\ &+ 2\mathrm{FH}\alpha^2\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+ - \mathrm{FH}\alpha^3\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+ - \mathrm{FH}\alpha f_1 + \mathrm{FH}\alpha^2 f_1 \\ &- \mathrm{F}^2\alpha^2\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_- + \mathrm{FH}\alpha^2\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_- \\ &+ \mathrm{F}^2\alpha^3\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_- - \mathrm{FH}\alpha^3\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_- + \mathrm{FH}\alpha\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ \\ &- 2\mathrm{FH}\alpha^2\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ + \mathrm{FH}\alpha^3\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+ + \alpha\mathrm{PremVar}(f_2)_- \\ &+ \mathrm{H}\alpha^2\mathrm{Var}(f_2)_- - \mathrm{FT}\alpha^2\lambda\mathrm{Var}(f_2)_- + \mathrm{HT}\alpha^2\lambda\mathrm{Var}(f_2)_- \\ &+ \mathrm{HT}\alpha^2\lambda\mathrm{Var}(f_2)_- + \lambda\mathrm{FT}\alpha^3\mathrm{Var}(f_2)_- - \lambda\mathrm{HT}\alpha^3\mathrm{Var}(f_2)_- \\ &- \alpha\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+\mathrm{Var}(f_2)_- + \alpha^2\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+\mathrm{Var}(f_2)_- \\ &- \alpha f_1\mathrm{Var}(f_2)_- + \mathrm{DF}\alpha^2\lambda p_1\mathrm{Var}(f_2)_- - \mathrm{DH}\alpha^2\lambda p_1\mathrm{Var}(f_2)_- \\ &- \mathrm{DF}\alpha^3\lambda p_1\mathrm{Var}(f_2)_- + \mathrm{DH}\alpha^3\lambda p_1\mathrm{Var}(f_2)_- \\ &+ \alpha\lambda p_1\mathrm{Cov}(p_2\mathrm{d}, f_2)_+\mathrm{Var}(f_2)_- - \alpha^2\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_+\mathrm{Var}(f_2)_- \\ &+ \mathrm{PremVar}(f_2)_+ + \mathrm{H}\alpha\mathrm{Var}(f_2)_+ - \alpha\mathrm{PremVar}(f_2)_+ - \mathrm{H}\alpha^2\mathrm{Var}(f_2)_+ \\ &+ \alpha\mathrm{HT}\lambda\mathrm{Var}(f_2)_+ - 2\mathrm{HT}\alpha^2\lambda\mathrm{Var}(f_2)_+ + \alpha^3\mathrm{HT}\lambda\mathrm{Var}(f_2)_+ \\ &+ \alpha\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_+\mathrm{Var}(f_2)_+ - \alpha^2\lambda\mathrm{Cov}(p_2\mathrm{d}, f_2)_-\mathrm{Var}(f_2)_- \\ &- \mathrm{DH}\alpha\lambda p_1\mathrm{Var}(f_2)_+ + 2\mathrm{DH}\alpha^2\lambda p_1\mathrm{Var}(f_2)_+ - \mathrm{DH}\alpha^3\lambda p_1\mathrm{Var}(f_2)_+ \\ &- \alpha\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_-\mathrm{Var}(f_2)_+ + \alpha\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_-\mathrm{Var}(f_2)_+ \\ &+ \alpha^2\lambda p_1\mathrm{Cov}(\mathrm{d}, f_2)_-\mathrm{Var}(f_2)_+ + \mathrm{FH}\alpha\mu_{f_2} - \mathrm{FH}\alpha^2\mu_{f_2} \\ &+ \alpha\mu_{f_2}\mathrm{Var}(f_2)_-) \\ &/(-\alpha(1-\alpha)\lambda(\alpha\mathrm{F}^2\mathrm{Var}(f_2)_- - 2\mathrm{FH}\alpha\mathrm{Var}(f_2)_- + \alpha\mathrm{H}^2\mathrm{Var}(f_2)_- \\ &+ \mathrm{H}^2\mathrm{Var}(f_2)_+ - \alpha\mathrm{H}^2\mathrm{Var}(f_2)_+ + \mathrm{Var}(f_2)_-\mathrm{Var}(f_2)_+))) \end{aligned} \tag{65}$$

The next step is to derive the Hessian matrix for the utility function and to prove that equation (63) and (65) are global optimum solutions. First the Hessian matrix is constructed, and the Hessian matrix needs to be negative definite matrix. The following equation is the Hessian matrix for equation (61):

$$\begin{pmatrix} \frac{-1}{2}\lambda(2\mathrm{F}^2\alpha(1-\alpha)+2\alpha\mathrm{Var}(f_2)_- + 2(1-\alpha)\mathrm{Var}(f_2)_+) & \frac{-1}{2}\lambda(-2\mathrm{FH}\alpha(1-\alpha)-2(1-\alpha)\mathrm{Var}(f_2)_-) \\ \frac{-1}{2}\lambda(-2\mathrm{FH}\alpha(1-\alpha)-2(1-\alpha)\mathrm{Var}(f_2)_-) & \frac{-1}{2}\lambda(2\mathrm{H}^2\alpha(1-\alpha)+2\alpha\mathrm{Var}(f_2)_-) \end{pmatrix} \tag{66}$$

For a symmetric matrix, A, is a negative definite matrix if and only if every leading principle minor satisfies the following condition:

$$(-1)^i A_i > 0 \tag{67}$$

The leading principle minors, A_1 and A_2, must satisfy condition (67). Equation (68) and (69) show that matrix, equation (66), satisfies the condition which proves that the optimal futures and put options position is global optimum.

$$(-1)A_1 = \frac{1}{2}\lambda(2\alpha\mathrm{F}^2\alpha(1-\alpha)+2\alpha\mathrm{Var}(f_2)_- + 2(1-\alpha)\mathrm{Var}(f_2)_+) > 0 \tag{68}$$

$$(-1)^2 A_2 = \alpha(1-\alpha)\lambda^2(\mathrm{H}^2(1-\alpha)\mathrm{Var}(f_2)_+ + \mathrm{Var}(f_2)_-(\alpha(\mathrm{F}-\mathrm{H})^2 + \mathrm{Var}(f_2)_+) > 0 \tag{69}$$

Hence, the Hessian matrix is negative definite, and the solutions are global optimum solutions if $\alpha \in (0,1)$. This is a powerful result because the optimal futures and put option position is guaranteed to maximize the utility function. Additionally, optimal futures and put-option positions heavily depend on the strike price, conditional variance of price, and the probability of put option to expire with profit.

3.7. Summary

This chapter covered hedging models that were based on the portfolio theory developed in Markowitz's (1952) seminal study. These approaches are only hedging the price risk with the minimum-variance and E-V frameworks. Similarly, Blank et al. (1991), McKinnon (1967), and Robinson and Barry (1999) extended the model to include the output risk which significantly affects the revenue. One of the most important implications that was derived was the relationship between the commodity's output and price. The derived optimal HR formula depended on the correlation between price and output. Mahul and Cummins (2008) further extended the model to analyze the optimal production and hedging decision of a risk-averse firm with vulnerable forward and options contracts that were based on a vulnerable forward contract. Korn (2008) similarly analyzed the risk-averse firm's behavior when the forward contract is subject to default risk and extended model to include the credit derivative and stochastic recovery rate. Based on the Bullock and Hayes (1992), the analytical section derived the closed-formula solutions which maximizes E-V utility function if $\alpha \in (0,1)$. The solution is derived by taking partial derivative of the utility function and showing the Hessian matrix is negative definite. This is powerful result because the optimal futures and put option positions are global optimum solutions.

CHAPTER 4. EMPIRICAL MODEL FOR HEDGING THE PRICE AND DEFAULT RISKS

4.1. Introduction

In the previous chapter, the optimal solutions to maximizes the E-V utility function were presented. The chapter's objective is to conduct an empirical analysis of the optimal HR by considering a buyer's (counterparty's) possible default. The empirical analysis uses a Monte Carlo simulation for random sampling and Evolver to solve the optimal HR. For the empirical analysis, a stylized and simplified model is constructed due to the availability of public data. Sensitivity analyses are conducted to observe the impact of a changing value for a single variable as well as a changing assumption about the dependency between price variables in the payoff function. The empirical analysis is conducted for corn and soybeans by using cash and futures price data, and data about Chinese buyer's cancellation of U.S. crop purchases.

4.2. The Payoff Function's Specifications

The empirical analysis starts by specifying the payoff function. This function is different than the equation (57) in Chapter 3. This model assumes that a U.S.-based international grain trader agreed to sell grain to a buyer in China. The Chinese buyer's default risk is inherent in this trade, and the default can occur anywhere and anytime in the supply chain. To make this model simpler, the default occurs either before or after the seller purchases the grain that is shipped to China. An illustration for this relationship is shown in Figure 4.1. Suppose that the U.S. international grain trader agreed to sell one dry-bulk cargo of grain to the Chinese buyer, and that the trader has a short-cash position. This short-cash position is exposed to the price risk; hence, the trader immediately takes a long-futures position to hedge the short-cash position's price risk. The Chinese buyer's default is assumed to occur before the trader buys physical grain or after the trader purchases the physical grain which is loaded in dry-bulk cargo and shipped to China. If the default occurs before the grain is purchased, the trader's cash position becomes zero immediately. If default occurs after the grain is bought, the trader has to find an alternative buyer at a resale price, a lower price than the original one from Chinese buyer.

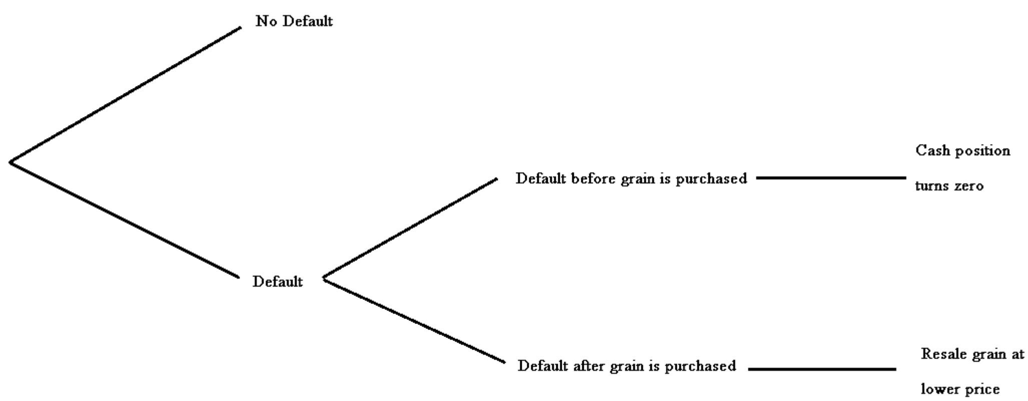

Figure 4.1. Default Tree for the Cash Market.

The empirical analysis's payoff function is defined as follows:

$$\pi = Q\left[\left(\tilde{P}_2 - P_1\right)\left(1 - \tilde{D}_p\right) + \tilde{D}_p\left(\tilde{l}_q + \tilde{l}_q\left(\tilde{P}_2 - \tilde{\Delta} - P_1\right)\right)\right] + \mathrm{X}\left(\tilde{f}_2 - f_1\right) + \mathrm{R}[\mathrm{Prem} + \max\left(0, \mathrm{K} - \tilde{f}_2\right)], \tag{70}$$

where the tilde indicates that the variable is random. P_2, P_1, f_2, and f_1 are the cash price in period two, the cash price in period one, the futures price in period two, and the futures price in period one, respectively. Q is the fixed-quantity amount sold in period one. K and Prem are the strike price and the option premium for the put option, respectively. For simplicity, this model implicitly assumes that the trader allows put option to expire, and the put option's value is derived only from the intrinsic portion. This implicit assumption imposes the trader not to resell the put option to lift the hedge. Specifically, variable D represents the counterparty's default probability. Default probability follows a Bernoulli distribution with a default probability, p. If

$\widetilde{D}_p = 1$, the buyer defaults, and if $\widetilde{D}_p = 0$, the buyer does not default. The default is assumed to be independent variable. The default is not correlated with other variables in the model. In this model, the buyer Similarly, l_q represents whether the default occurs before or after the trader purchase grain. The probability of defaulting after the purchase is q, and the probability of defaulting before the grain is purchased is the complement of default occurring before $1 - q$. If $l_q = 1$, the default occurs after the purchase, and if $l_q = 0$, the default occurs before the purchase. The default before probability is conditional probability that is before or after default is determined given that the buyer defaults on the grain. A factor that affects the grain's resale price is Δ. In this model, if the default occurs after the grain is purchased, the trader has to find a new buyer to sell the already purchased grain at a lower price. X and R are the futures and put-option positions, in bushels.

4.3. Specifications for the European Put-Option Premium

For this empirical model, the put-option premium is derived from the option-pricing formula for commodity contracts that were developed by Fischer Black in the paper Black (1976). This European option-pricing model extends the Black-Scholes pricing formula and applies the formula to a futures contract. Due to its simplicity to derive the option premium, Black's (1976) European option pricing is utilized instead of using an American-option pricing model. According to Hull (2005), Black's model assumes a lognormal distribution for the futures price, and the put-option premium is defined as follows:

$$\mathrm{p} = e^{-rT}[KN(-d_2) - F_0N(-d_1)] \tag{71}$$

where d_1 and d_2 are defined as follows:

$$d_1 = \frac{\ln(F_0/K) + \sigma^2\, T/2}{\sigma\sqrt{T}} \tag{72}$$

$$d_2 = \frac{\ln(F_0/K) - \sigma^2\, T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{73}$$

where F_0 and K are the futures price at time zero and the strike price. σ and T are the annualized return's volatility for futures and time to maturity, respectively. The theoretical put-option premiums derived from equation (71) for the different strike prices are used for empirical model.

4.4. Definition of Value-at-Risk

The problem outlined in the empirical section is similar to the analytical section. The biggest difference between the sections is the approach to finding the optimal futures and options positions which maximize the objective function. In the empirical section, the data used for the analysis are fit with an empirical distribution which represents the data points during a given time interval. The theoretical framework used in the analytical section is the E-V framework; however, the E-VaR is used for the empirical analysis. Mathematically, VaR with confidence levels, β, is defined as follows:

$$VaR_\beta(X) = \min\{x|F(x) \geq \beta\}, \tag{74}$$

where α is the confidence level between $(0,1)$ and F is the portfolio's return distribution. Equation (74) implies that the VaR at confidence level α is the portfolio's minimum loss that has a probability of occurrence that is greater than or equal to β. Similarly, the 5% confidence-level VaR can be interpreted as there being a 5% of probability that the portfolio incurs a loss that is greater than or equal to $\min\{x|F(x) \geq \beta\}$. The VaR is one of the downside risk metrics; other such risk metrics include semivariance and lower partial moments

For illustration suppose that there is a portfolio with a normal distribution of returns; the mean is 10% and the standard deviation of 3%. The 5% confidence-level VaR is shown in Figure 4.2. The figure implies that there is 5% chance that the portfolio incurs a loss that is greater than or equal to 5.06%.

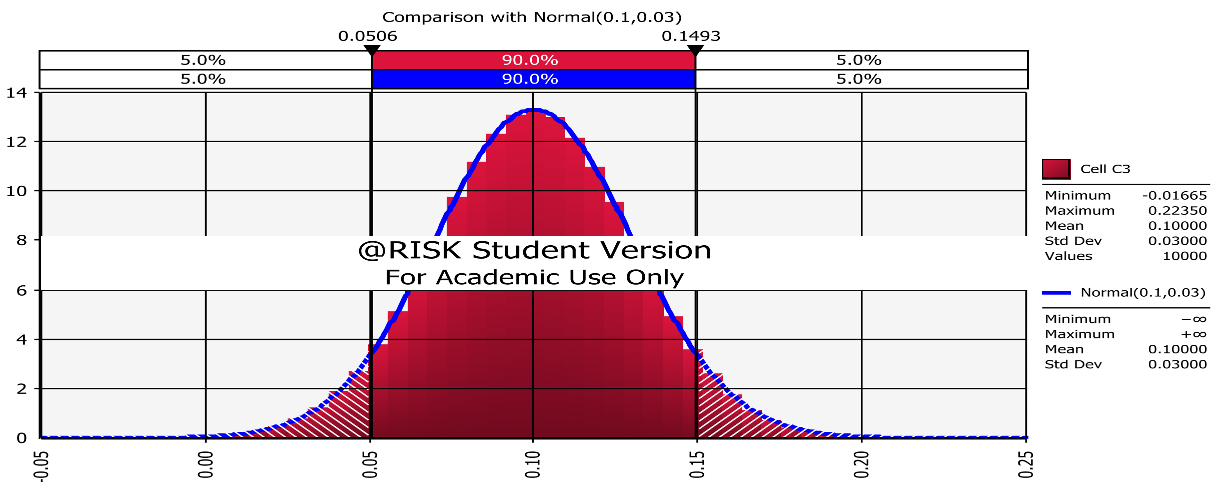

Figure 4.2. Example of the VaR.

4.5. Mean-VaR Framework

The E-V framework is used to derive the analytical solution; however, the empirical analysis utilizes the E-VaR framework. Conceptually, both the E-V and E-VaR frameworks are similar because they adjust the portfolio's expected return by risk. Because the variance measures the portfolio's volatility, both the upside and downside volatilities are treated equally as the same risk. Intuitively, a risk-averse investor favors an upside risk and dislikes the portfolio's downside risk; hence the variance may not be accurately adjusting the expected return for the risk that investors are taking to seek that return.

One advantage of E-VaR over E-V is a more accurate specification for the risk measure because VaR is a downside risk metrics. The E-V framework uses the variance is risk measure. Therefore, the VaR may be more appropriate as a risk measure compared to the variance when adjusting the expected return. Additionally, the E-V framework assumes that the return distribution is normally distributed. Awudu et al. (2016) use the E-VaR framework for an ethanol producer's hedging because of the VaR's increasing acceptance. Also, with the E-V framework, the risk averse-investor can make a decision to allocate the capital inefficiently (Tsao, 2010). The theoretical framework for E-VaR is based on Awudu et al (2016):

$$U = E(\pi) - \frac{\lambda}{2} VaR_{\beta}(\pi) \tag{75}$$

where $E(\pi)$ is the portfolio's expected profit, λ is the risk-averse coefficient, and $VaR_{\beta}(\pi)$ is the portfolio's, β, confidence-level VaR. For simplicity, the normal distribution is assumed when estimating the VaR. The empirical analysis's objective is find the optimal position size for the futures and put options to maximize equation (75).

4.6. Sensitivity Analysis

A sensitivity analysis is conducted to measure how much the result changes with respect to varying values or the assumption of a single independent variable when all other independent variables are fixed. For the sensitivity analysis, the following analyses are conducted after obtaining a base case optimization result for the empirical model: the put option's strike price, the probability of default and the before-default probability, the price volatility for the cash and futures prices, and the copula.

4.7. Correlation Between Price Distributions

This section describes the copula function's mathematical foundation. The copula function describes joint distributions' dependence structure, and this function is more flexible compared to the Pearson linear-correlation coefficient and the Spearman correlation coefficient. Each method's characteristics are described in the following paragraphs.

The Pearson linear correlation is a coefficient between two random variables; however, it has restrictive assumptions compared to the other correlation measures. Assumptions made when calculating the Pearson linear-correlation coefficient are a normal distribution and a linear relationship between random variables. The existence of an outlier in the dataset has a large effect when estimating the correlation between random variables.

The Spearman correlation coefficient makes less-restrictive assumptions than the Pearson correlation coefficient. The spearman correlation is the distribution-free correlation measure between random variables, assuming a monotone, nonlinear relationship between two variables. The Spearman correlation is measured by ranking the dataset instead of an actual value, that is, using each sample point's rank in the dataset instead of using the sample point's actual value for the correlation estimate.

The copula also measures the dependence between random variables. It is flexible and more accurately captures the random variables' correlation by joining the marginal distribution of the random variables with the copula function (McNeil, Frey, & Embrechts 2005; Vose 2008). The Sklar theorem proved the existence of a copula function for a joint distribution, and when the copula function and marginal distributions are merged, the combination is equivalent to the joint distribution (McNeil et al., 2005). They defined that, if there are n random variables and each random variable has joint-distribution function F with a continuous marginal distribution, then there exists copula function C of F such that

$$\mathrm{F}(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n) \tag{76}$$

Different copula functions are used for the risk analysis. There are two copula classes: the Archimedean and Elliptical copulas. With the Archimedean copula, there are Clayton copula, Gumbel Copula, and Frank copula. With the elliptical copula, there are Normal (Gaussian) copula and Student t copula. The copula functions have advantages and disadvantages (Vose, 2008). According to Vose (2008), the Archimedean copula function is defined as follows:

$$\mathrm{C}(\mathrm{u},\mathrm{v}) = \varphi^{-1}(\varphi(u) + \varphi(v)), \tag{77}$$

where φ is the copula generator. Each copula function in the Archimedean copula class has a different specification for the copula generator. The elliptical copula function is defined as follows:

$$\rho(\mathrm{X},\mathrm{Y}) = \sin\left(\frac{\pi}{2}\tau\right), \tag{78}$$

where τ is the Kendall's tau between X and Y.

Another type of copula that is used for the empirical analysis is the empirical copula. The empirical copula does not assume any functional form for the dependency's structure; therefore, it is a nonparametric copula. The empirical copula uses observations to construct marginal distributions which are substituted in the copula instead of fitting a copula function to the dataset (Wilson, Awudu, Skadberg, Dahl, & Chen, 2016). Because empirical-copula estimation is nonparametric, it is the most general form of a dependency measure between random variables.

4.8. Data

Several datasets are collected for the empirical analysis: cash price, freight on board (FOB), cost insurance and freight (CIF), and the corn and soybeans' ocean-freight rates to Dalian from the Pacific Northwest (PNW) and U.S. Gulf are collected using Thomson Reuter Eikon. These price are daily data. The corn and soybeans' price data range from January 5, 2009, to September 1, 2016 (Thomson Reuter Eikon 2016a & 2016b). Both the corn and soybeans' CME futures daily price data that are collected from (Data Transmission Network) DTN Prophetx for the same time period as the cash price (Data Transmission Network Prophetx, 2016). Corn and soybean sales' cancellation data are collected from the USDA's FAS export-reporting system. These sales cancellation data are used to estimate the default probability.

4.8.1. Corn and Soybean Prices

The cash price is defined as the minimum cost of shipping grain at given date from the PNW or U.S. Gulf to Dalian, China. Prices are defined as the price at the origin plus the ocean-freight rate to Dalian at a given date. The corn FOB at the PNW is not available; hence, track CIF is used. For soybeans, the FOB prices at PNW and U.S. Gulf are to calculate cash price. For example, the corn cash price is represented as $\min[PNW_{FOB,t} + O_{PNW,t}, GULF_{FOB,t} + O_{GULF,t}]$, where $O_{L,t}$ is the ocean-freight rate from location, L, to Dalian at time t. $PNW_{CIF,t}$ and $GULF_{FOB,t}$ are the PNW CIF and the U.S. Gulf FOB prices at time t. The Figures 4.3 and 4.4 show the cash and futures price for corn and soybeans, respectively.

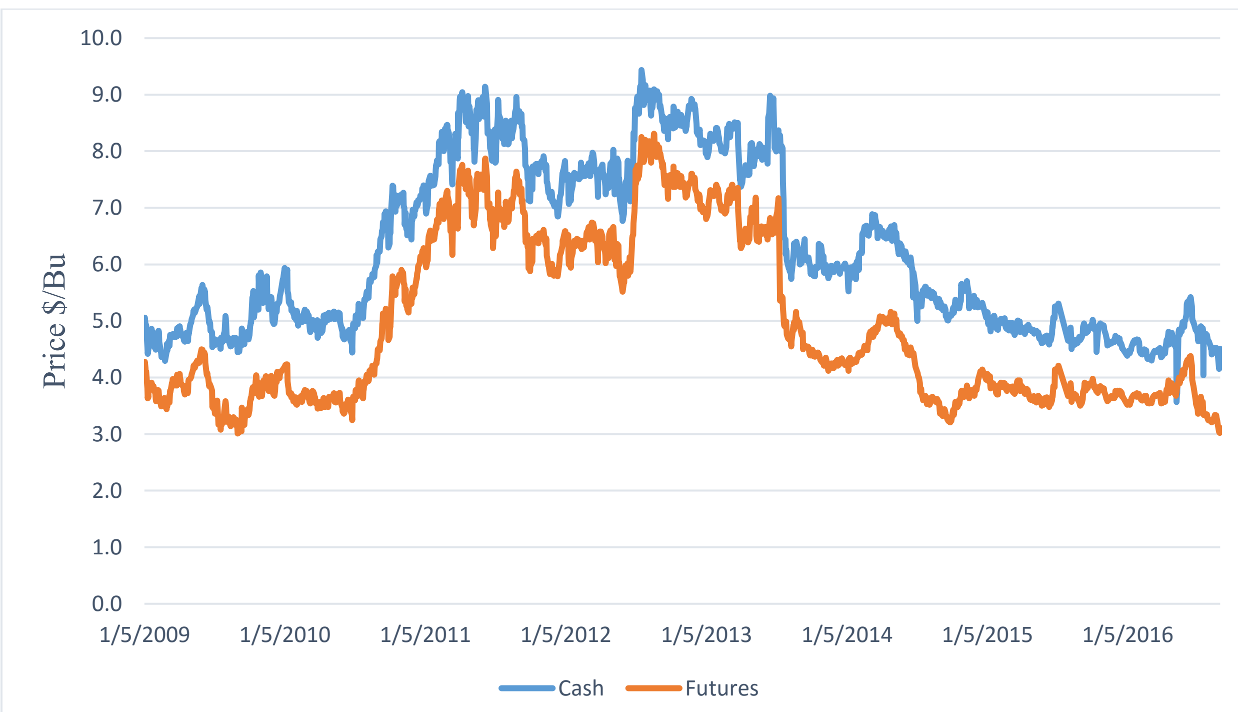

Figure 4.3. Corn's Cash and Futures Prices.

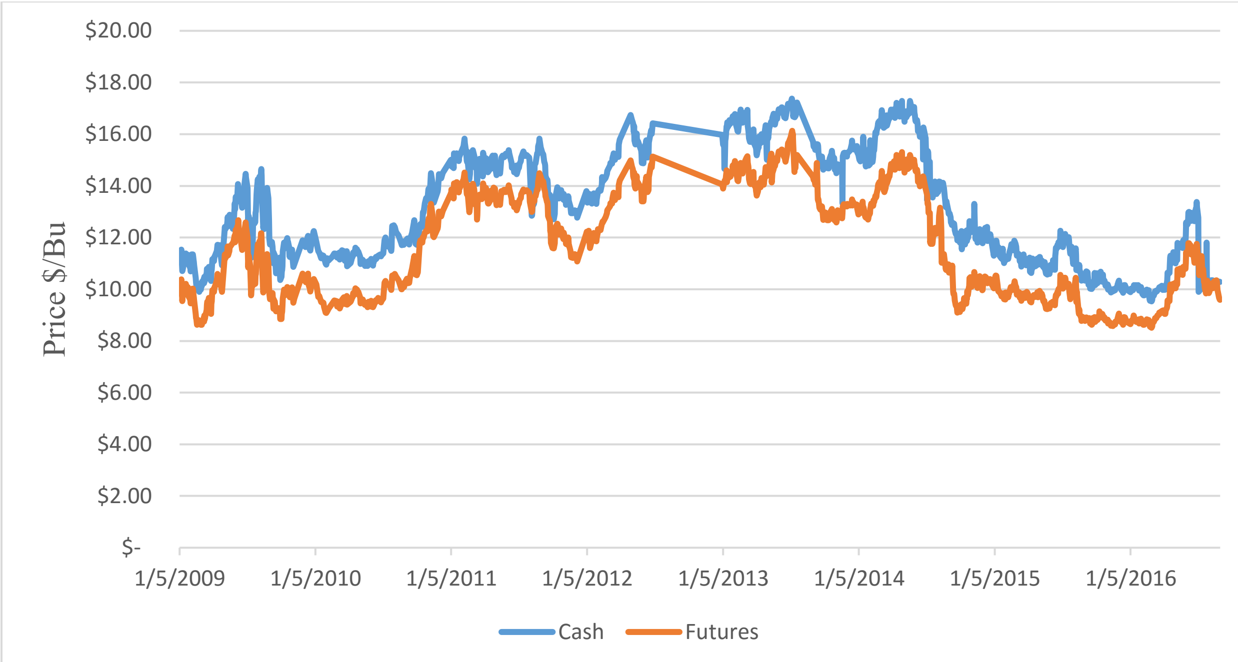

Figure 4.4. Soybeans' Cash and Futures Prices.

4.8.2. Price Distributions

The futures and cash prices are assumed to be lognormally distributed. The mean of the lognormal distribution is the latest price of data and standard deviation is estimated from the dataset. This price distribution is more forward looking than the fitted distribution, and the forward-looking distribution is better than the best-fit distribution for two reasons. The price distributions are not confined within the data, and the optimization result is not affected by where the latest price lies on the fitted distribution. For the corn's cash-price distribution, the mean is $4.50, and the standard deviation is $1.49. The corn's futures-price distribution has a mean of $3.11 and a standard deviation of $1.50. The linear correlation coefficient between the cash and futures price is 0.98. The soybean's cash-price distribution has a mean of $9.92 and a standard deviation of $2.21. The soybean's futures-price distribution has a mean of $8.88 and a standard deviation of $2.12. The linear correlation between the soybean's cash and futures price is 0.98.

4.8.3. Best-Fit Copula

The copula function that fits the futures and cash prices is approximated by using maximum likelihood estimation (MLE). Based on the Akaike Information Criterion (AIC) test, the best-fit copula function for corn is the ClaytonR copula with parameter θ = 6.55. ClaytonR is a Clayton copula that is reflected at both the x and y axes (Palisade Corporation, 2016). The Clayton copula is one of the Archimedean copula functions. The characteristics of the Clayton copula are a high correlation level at the lower end (Vose, 2008). The best-fit copula for the soybean's cash and futures price is the Frank copula with parameter θ = 28.23. The Frank copula is an Archimedean copula and has an even correlation structure (Vose, 2008). Figures 4.5 and 4.6 show the correlation structure for corn and soybeans, respectively.

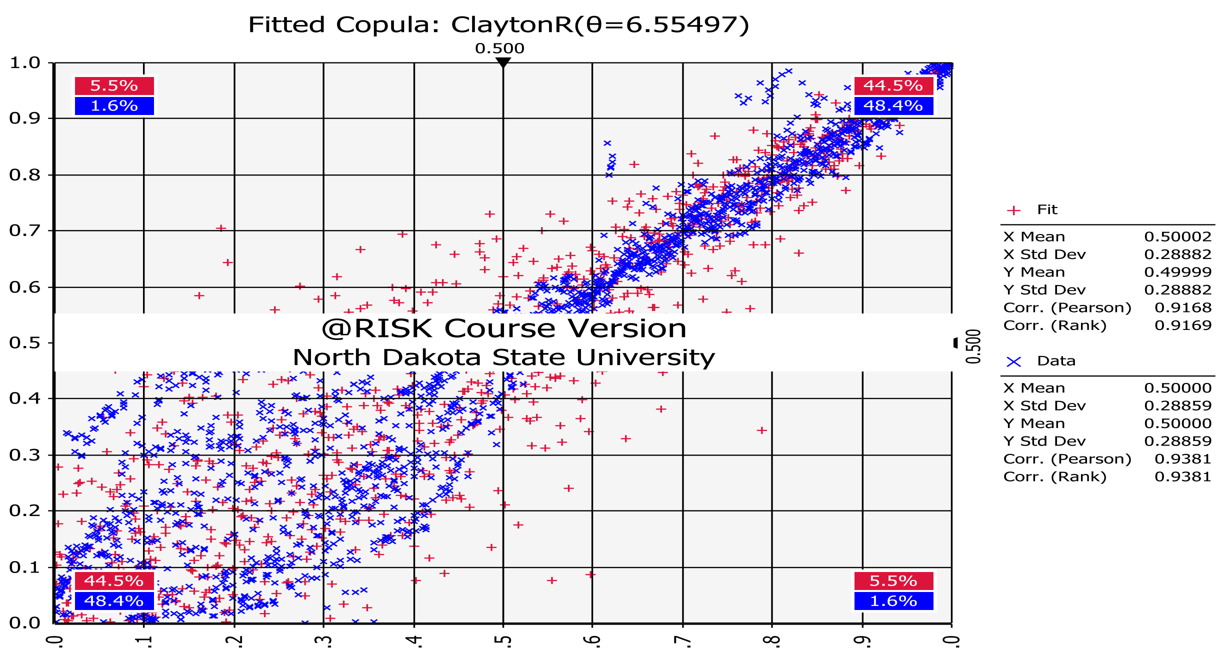

Figure 4.5. Corn's Best-Fit Copula.

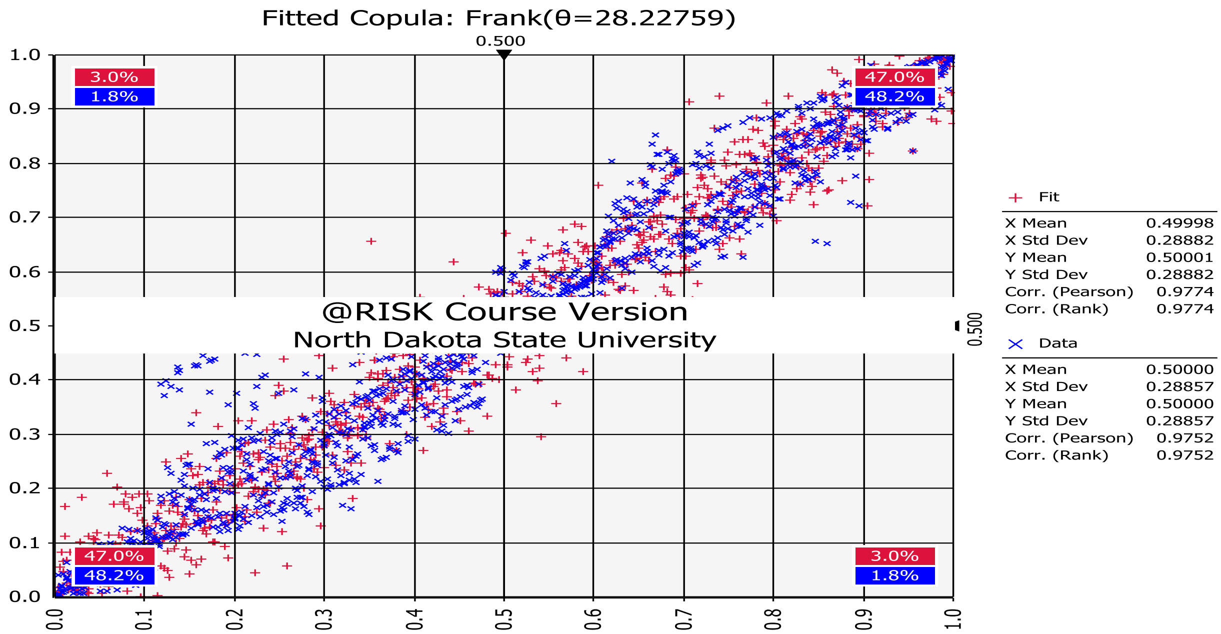

Figure 4.6. Soybean's Best-Fit Copula.

4.8.4. Probability of Default

The cancellation data for U.S. soybean sales to China are collected from the USDA's FAS. The weekly data are from the January 7, 1999, to February 25, 2016. These data are aggregate sales cancellations, from Chinese buyers, for U.S. corn and soybeans that occurred during a particular week. The data are recorded in metric tons. These statistics are the best publicly available data to estimate the default probability. The probability of default is estimated by dividing the total number of default weeks that occurred by the data's total number of weeks. The Figures 4.7 and 4.8 show the Chinese buyers' cancellation of U.S. corn and soybean exports.

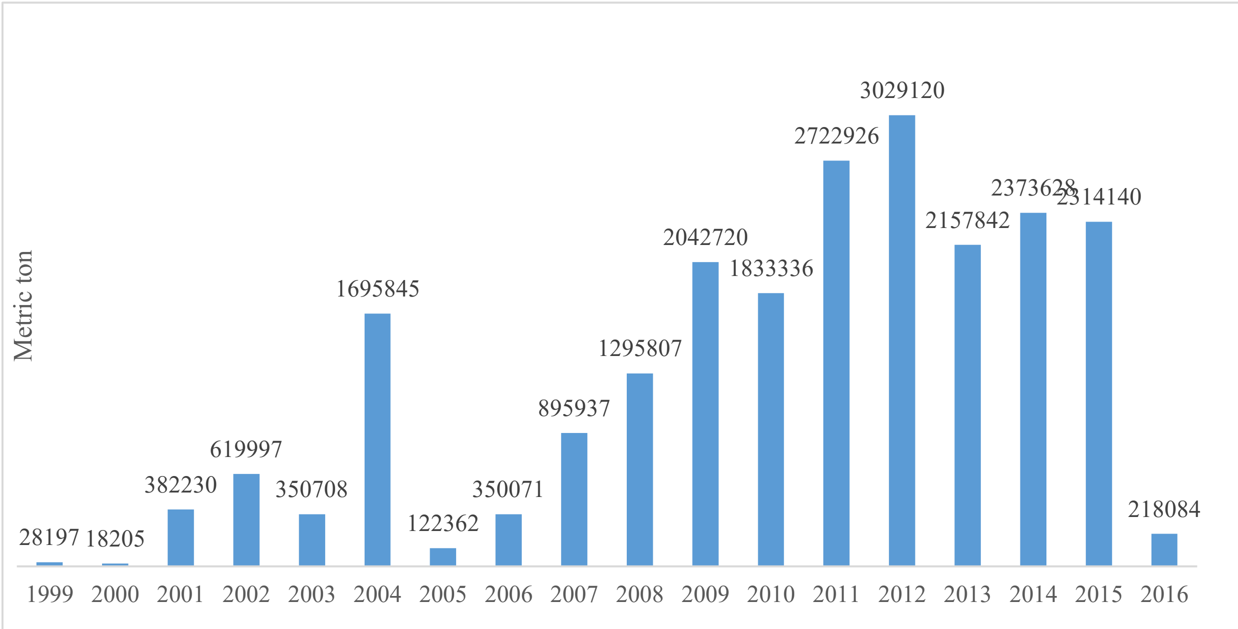

Figure 4.7. U.S.-Corn Export Cancellations by China.

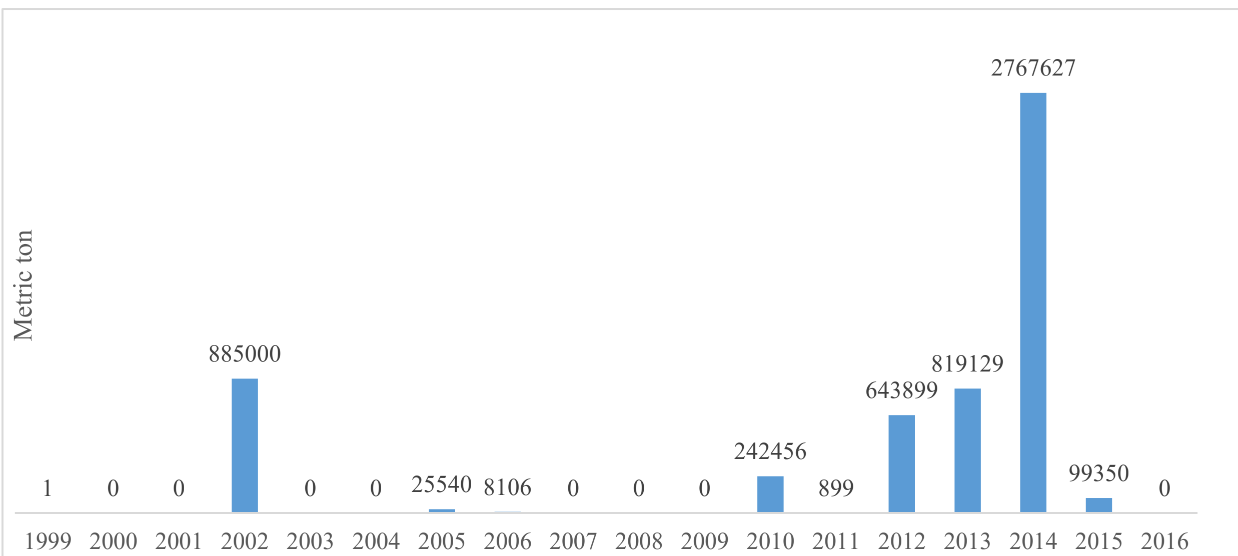

Figure 4.8. U.S.-Soybean Export Cancellations by China.

4.9. Summary

This chapter described the model, data, and approaches used for the empirical analysis which was applied to both corn and soybeans. The empirical model employed the VaR as risk metrics and the E-VaR as the objective function instead of the E-V framework that was used for the theoretical model. Conceptually, the E-VaR was better than the E-V because VaR is a downside risk measurement. Cash-price data were estimated with the minimum origination cost at the PNW or U.S. Gulf. Based on the price and sales-cancellation data, the price distributions, copula, and default probabilities were estimated. The price distributions were assumed to be lognormal distributions with the mean equal to the latest price and standard deviation that were estimated with the data. The best-fit copula functions, ClaytonR and Frank, were estimated with the cash and futures prices for corn and soybeans, respectively.

CHAPTER 5. RESULTS

5.1. Introduction

Counterparty default risk has become more prevalent with commodity marketing and has implications for traders. Intuitively, once a trader sells grain to a buyer, the trader is short on cash commodities and, normally, long in the futures market to hedge the price risk. In addition, this hedge may be supplemented with a long put option to hedge the buyer's default risk of the buyer. This intuitive understanding is important to the comprehend assumptions as well as, the analytical and stochastic optimization results. This chapter's purpose is to show the findings from the analytical and empirical models.

In Chapter 3, the analytical hedging model was developed to derive the optimal HR for the futures and put option under the E-V framework in order to hedge the price and the default risk with the assumption of normality for prices and defaults. In Chapter 4, the empirical hedging model was developed using the E-VaR framework. The E-VaR framework had the 5% confidence-level VaR as a measure of risk instead of standard deviation. An advantage of the E-VaR technique over the E-V method is that VaR is a downside risk measure, whereas the standard deviation treats the upside and downside equally. Conceptually, the E-V framework may overstate risk. Proving that the E-VaR metod is superior for portfolio selection compared to the E-V framework is outside this study's scope. For the empirical analysis, prices are assumed to follow a lognormal distribution, and default risk is represented by the Bernoulli distribution. Due to the lack of publicly available data about defaults, the empirical models are highly stylized and simplified to capture the default risk's effect on the optimal HRs. The analytical model's solution is derived in a closed formula, and the solution for the empirical model is derived by stochastic optimization.

Results from theoretical model are discussed first. Theoretical model to hedging derived E-V maximizing global optimum solution. In the empirical section, the result from the base case model is presented, followed by the sensitivity analysis for the model variables. The optimization results are presented first for corn followed by soybeans.

5.2. Analytical Model Result

Based on the paper Bullock and Hayes (1992), theoretical model under E-V framework is developed to find optimal futures and put option HRs. Major result from theoretical model is existence of the global optimum solution if probability of the futures price below strike price is not 0 and 1. In theoretical model, cash price, futures price, and default distributions are assumed to be normal. By applying the theorem from Bullock and Hayes (1992) and taking partial derivative on equation (61), optimal futures position and put option positions are derived. Hessian matrix is proved to be a negative definite matrix. Additionally, the optimum futures and put-option positions heavily depends on the strike price and option delta.

5.3. Empirical Model Using the Stochastic Simulation's Result

This section illustrates the results from the empirical analysis of the default risk models for soybean and corn. The model used for the empirical analysis differs from the analytical model. The biggest difference is the use of stochastic optimization rather than mathematically deriving the optimal solutions. In addition, instead of the E-V framework, the E-VaR framework is used, focusing on the portfolio's downside risk. The cash and futures price distributions are assumed to be lognormally distributed instead of having a normal distribution which is used for the analytical model.

5.3.1. Assumptions About the Corn Empirical Base Case Model for Corn

The base case model for corn assumes that a trader sells one cargo, which is approximately 1,968,400 bushels, of corn to China. The trade is assumed to take 90 days if the counterparty's risk is nil. The probability of a buyer's default is estimated by using cancellation data from the USDA FAS' export-reporting system. The default probability is approximately 12%. The probability of the buyer defaulting before the trader buys the cash grain is assumed to be 30%. Cash-price is defined as the minimum cost to ship to Dalian, China from PNW or U.S. Gulf. Costs are defined as the FOB plus the ocean-freight rate to Dalian. The FOB rate at the PNW is not available; therefore, the track CIF is used. The futures price is the close price at the CME corn futures' price. For period one, the cash price is $4.51 per bushel, and the futures price is $3.11 per bushel; they are used as mean of lognormal price distributions. The distribution's standard deviation is estimated from the data and is $1.49 and $1.50 for the cash and futures price, respectively. These lognormal price distributions are considered as ex-ante distributions than the best-fit distribution because the best fit distribution is confined and restricted by the price data.

The salvage rate, or resale factor, is the rate at which the trader decreases the price for a new buyer if the first buyer defaults on the grain after the trader purchased it for the shipment. The salvage-rate distribution is assumed to be triangular with mean of -$30 per metric ton, a minimum of -$50 per metric ton, and a maximum of zero. The salvage rates are suggested by Cenex Harvest States (CHS) traders. The salvage rate is converted from dollars per metric ton to dollars per bushel. The risk-aversion coefficient is assumed to be two, so the dollar value for the expected profit and risk is symmetrical.

The put-option premium is derived from the Black option-pricing model[2] which is $0.2 per bushel for at-the-money (ATM) strike price of $3.10. The option premium is based on the price of the nearby corn-futures contracts that are traded at the CME. Volatility is estimated by calculating the return when taking the natural logarithm of the ratio for the current price and the previous price: $(\frac{P_t}{P_{t-1}})$. The annualized volatility is the 21 periods standard deviation of return multiplied by $\sqrt{252}$, the number of trading days for the futures in a single year. The estimated volatility is 24.1%, and the days to expiration is 120 days. The risk-free interest rate for a 3 months' treasury bill is 0.33%, which is obtained from the U.S. Department of the Treasury (2016).

5.3.2. Base case Empirical Model Result for Corn

Based on the stochastic simulation, the optimal HR are 1.05 and 0.36 for the futures and put options, respectively. The value of E-VaR for base case optimization is a negative $1.62 million. For comparison, the alternative assumptions regarding the objective function and the HRs' restrictions are evaluated. Table 5.1 summarizes the base case optimization result.

For comparison, the alternative hedging approach is specified, including no hedge, naïve hedge, and a naïve hedge with optimal put options. When neither the futures nor the put option is available, the objective function's value is a negative $4.9 million. When the futures HR is set at 1 and the put-option HR is 0, the objective function's value is a negative $1.71 million. This type of hedge is the naïve hedging that is utilized by traders to hedge the price risk. These results indicate that the futures position improved the trader's risk-adjusted return. Instead of setting the put-option HR to 0, it is optimized to find the maximum E-VaR value when the futures HR is

[2] The Black-76 option pricing model is used to calculate the put option premium

one. This approach's objective is to see the optimal-hedging decision trader once the price risk is hedged by the futures position. When the put-option HR is optimized, the put-option HR is 0.27, and the E-VaR is a negative 1.63 million. The base case result is the most superior results because it does not have restrictions for the possible HRs to maximize the objective function.

The maximum E-V, minimum VaR, and minimum variance led to the equivalent-optimization results, although their objective functions are different. This result may be due to the risk calculation's assumption. The variance and VaR are assumed to be normally distributed around the average profit and the profit's variance for all iterations. Because the variance is extremely large compared to expected profit, the equivalent-optimization result for the E-V and the minimum variance is acceptable. Also, the minimum VaR result is similar to the E-V and minimum variance because the normal distribution is symmetric around the mean.

Table 5.1

Base Case Stochastic Optimization Result (Corn)

	Futures HR	Put-Option HR	Objective Function	5% VaR
Base Case	1.05	0.36	-$1,622,635	$1,760,446
Future HR=1, Put-Option HR=0	1	0	-$1,714,164	$1,600,305
Future HR=1, Put-Option HR = Optimized	1	0.27	-$1,627,700	$1,702,112
No Hedge	0	0	-$4,892,339	$4,778,435
E-V	0.83	-0.32	-836,067,195,956	$1,504,000
Min VaR	0.83	-0.32	$1,503,999	$1,504,000
Min Variance	0.83	-0.32	836,066,858,233	$1,504,000

5.4. Sensitivity Analysis with Empirical Model for Corn

This section shows the results for the sensitivity analyses of the stochastic optimization on the corn's base case results. The purpose is to observe the effect of changing the variable's value to the optimal HRs. Sensitivity analyses were conducted on the put option's strike price, the futures and cash-price volatilities, the risk-averse coefficient, the default probability, and the before-default probability. Additionally, the correlation assumptions for the random variables were changed from the Pearson linear correlation to the best-fit copula and the empirical copula.

5.4.1. Strike-Price Sensitivity Analysis for Corn

Table 5.2 and Figure 5.1 summarizes the results. The sensitivity analysis for the strike price shows an increase, from 0.95 to 1.05, for the futures HR when the strike price increases from $2.10 to $3.20. However, the futures HR decreases with a strike price of $3.30 or higher. The put-option HR exhibits a similar pattern. The HR for the put option peaked at $2.90, and the HR deceased after $3.0. For the put-option HR, the higher strike price implies a greater chance that the option is in-the-money, but the optimization decided to the decrease long exposure for put option. The futures and put-option HRs peaked at different strike prices. The futures price peaks near ATM, whereas the put option peaks before the ATM price.

Table 5.2

Sensitivity Analysis: Put-Option Strike Price (Corn)

Strike Price	Futures HR	Put-Option HR	E-VaR
$2.10	0.95	0.23	-$1,669,965
$2.20	0.96	0.27	-$1,664,976
$2.30	0.97	0.30	-$1,658,689
$2.40	0.98	0.33	-$1,651,328
$2.50	0.99	0.36	-$1,643,394
$2.60	1.01	0.38	-$1,635,597
$2.70	1.02	0.39	-$1,628,755
$2.80	1.03	0.39	-$1,623,633
$2.90	1.04	0.39	-$1,620,766
$3.00	1.04	0.38	-$1,620,455
$3.10 (Base Case)	1.05	0.36	-$1,622,635
$3.20	1.05	0.33	-$1,626,929
$3.30	1.04	0.31	-$1,632,694
$3.40	1.04	0.28	-$1,639,329
$3.50	1.03	0.24	-$1,646,252
$3.60	1.02	0.21	-$1,652,914
$3.70	1.02	0.18	-$1,658,971
$3.80	1.01	0.15	-$1,664,229
$3.90	0.99	0.13	-$1,668,566
$4.00	0.98	0.10	-$1,671,986
$4.10	0.97	0.08	-$1,674,545

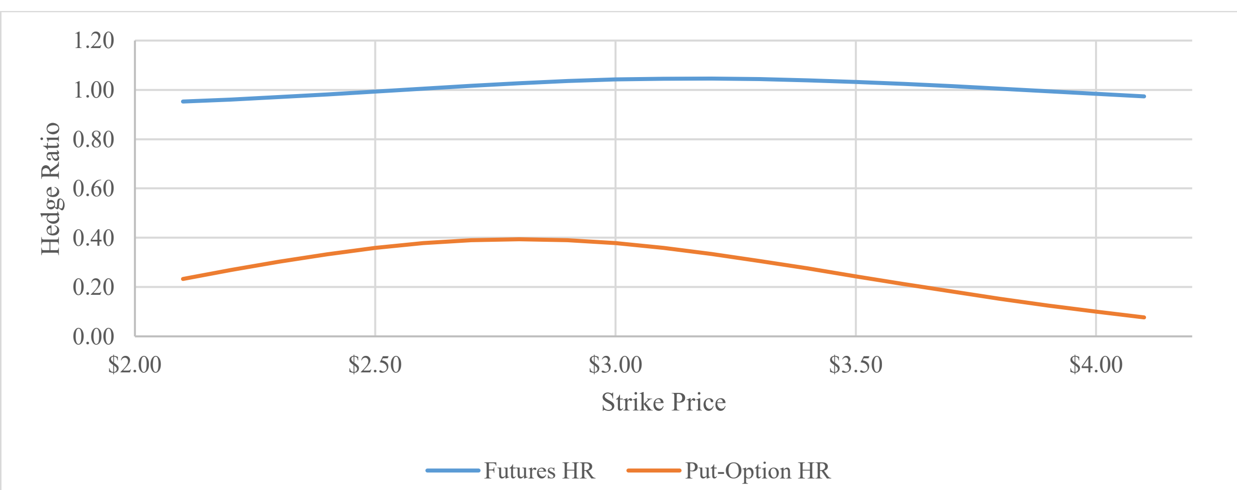

Figure 5.1. Sensitivity Analysis: Put-Option Strike Price (Corn).

5.4.2. Default-Probability Sensitivity Analysis for Corn

The base case default probability is approximately 12% which is based on the dataset. The sensitivity analysis for the default probability ranges from 0% to 40%. If there is zero chance of default, the only risk that the trader needs to hedge is the cash-price risk. The HR for the futures is 1.01, hence it is equivalent to a naïve hedge, and the put-option HR of 0.14 allows a further decrease for the portfolio's price risk. As the default probability increases, the put-option HR increases to hedge the loss of the buyer's default on the cash position. Although the put-option HR increases with a higher default probability, the E-VaR increases. The futures HR increases slightly as the default probability rises. These results are shown in Table 5.3 and Figure 5.2.

Table 5.3

Sensitivity Analysis: Default Probability (Corn)

Default Probability	Futures HR	Put-Option HR	E-VaR
0%	1.01	0.14	-$1,124,291
2%	1.02	0.18	-$1,216,828
4%	1.03	0.22	-$1,303,816
6%	1.03	0.25	-$1,373,385
8%	1.04	0.29	-$1,478,238
10%	1.04	0.33	-$1,548,511
12%	1.04	0.35	-$1,626,489
14%	1.05	0.38	-$1,664,378
16%	1.05	0.42	-$1,748,541
18%	1.05	0.44	-$1,824,599
20%	1.06	0.47	-$1,863,584
22%	1.06	0.49	-$1,935,079
24%	1.06	0.51	-$1,999,729
26%	1.06	0.53	-$2,050,405
28%	1.06	0.55	-$2,095,438
30%	1.06	0.57	-$2,147,349
32%	1.06	0.59	-$2,198,327
34%	1.06	0.61	-$2,249,442
36%	1.07	0.63	-$2,272,008
38%	1.06	0.64	-$2,336,229
40%	1.07	0.66	-$2,381,557

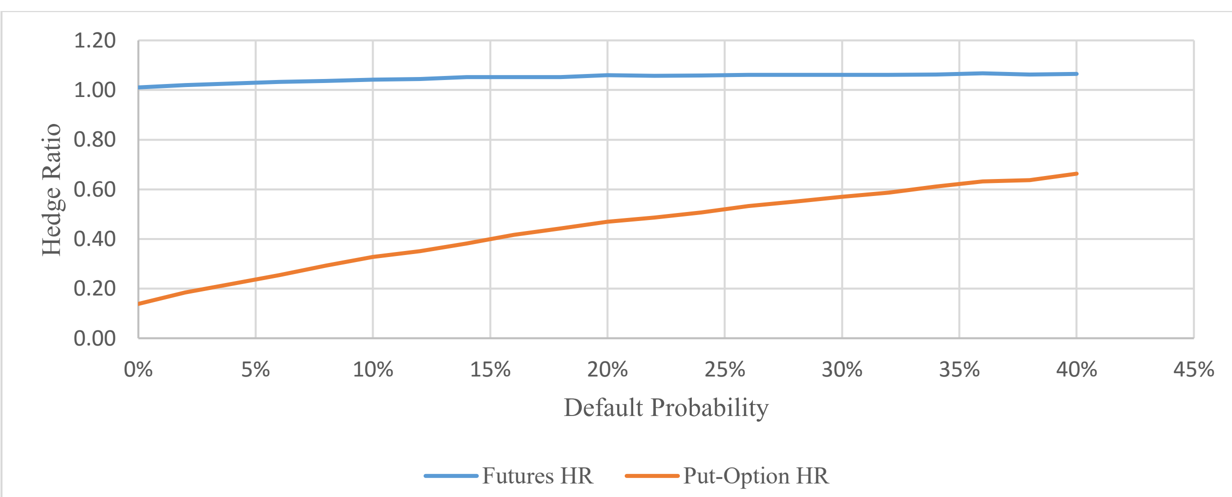

Figure 5.2. Sensitivity Analysis: Default Probability (Corn).

5.4.3. Before-Default Probability Sensitivity Analysis for Corn

The before-default probability is defined as the likelihood of the buyer defaulting on the purchase contract before the trader purchases the grain for shipment. This assumption is important because default can occur any time before the cargo arrives at its destination, and the problem worsens if the default occurs after the grain is purchased for shipment. If the buyer defaults after the trader purchases grain, the trader has to find new buyer and has to take a lower price. Otherwise, the trader's cash position becomes zero immediately. The base case before-default probability is assumed to be 30%. The increased before probability has similar effects as the increased default probability where the futures HR is almost constant while the put-option HR and the E-VaR increase. Both the default probability and the before-default probability's sensitivity results illustrate that the put option is used to hedge a default loss. This result is also consistent with the initial hypothesis that the futures is used to hedge the price risk and that the put option is used to hedge the default risk. The before-default probability's sensitivity analysis is shown in Table 5.4 and Figure 5.3.

Table 5.4

Sensitivity Analysis: Probability of Default Before Cash Purchases (Corn)

Before Probability	Futures HR	Put-Option HR	E-VaR
0%	1.05	0.25	-$1,503,700
2%	1.05	0.26	-$1,511,292
4%	1.05	0.27	-$1,515,927
6%	1.05	0.28	-$1,539,118
8%	1.05	0.29	-$1,540,067
10%	1.05	0.29	-$1,543,161
12%	1.05	0.30	-$1,551,934
14%	1.05	0.31	-$1,578,098
16%	1.05	0.31	-$1,573,784
18%	1.05	0.32	-$1,574,760
20%	1.05	0.33	-$1,581,605
22%	1.05	0.33	-$1,591,237
24%	1.05	0.34	-$1,600,954
26%	1.05	0.35	-$1,611,127
28%	1.05	0.36	-$1,607,418
30% (Base Case)	1.05	0.36	-$1,622,635
32%	1.04	0.37	-$1,650,003
34%	1.04	0.37	-$1,642,083
36%	1.04	0.38	-$1,636,940
38%	1.05	0.38	-$1,637,236
40%	1.04	0.39	-$1,658,193

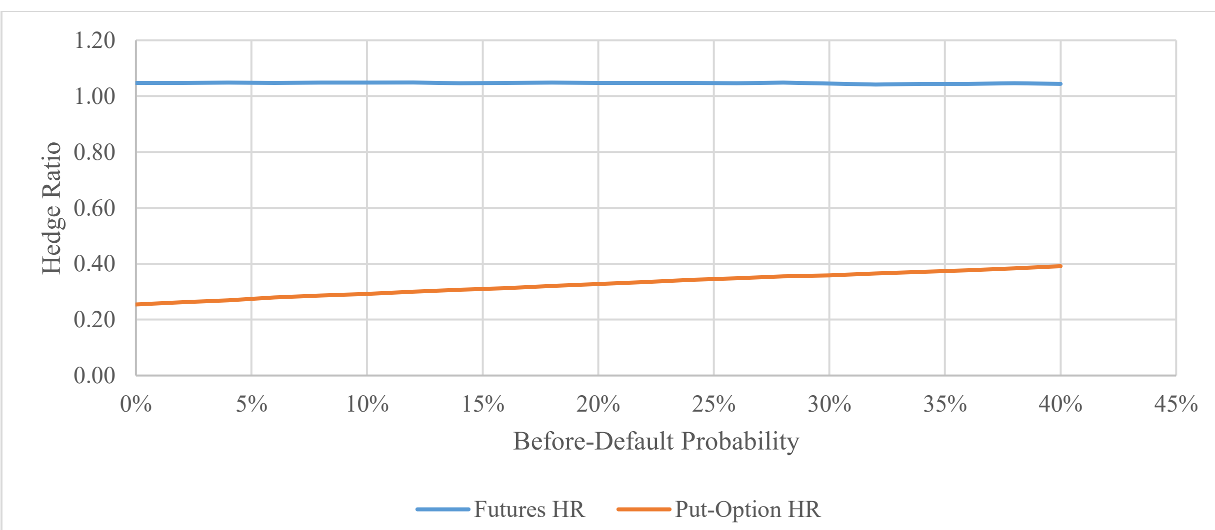

Figure 5.3. Sensitivity Analysis: Probability of Default Before Cash Purchases (Corn).

5.4.4. Risk-Averse Coefficient Sensitivity Analysis (Corn)

The risk-averse coefficient measures the trader's risk-averse. This coefficient has large influence on the risk and return tradeoff for the E-VaR model. The higher the risk-averse coefficient, the weight of risk component gets larger compared to the expected profit component. When the risk-averse coefficient is two, the dollar values for the risk and return are symmetric. The tradeoff is asymmetric when the risk-averse coefficient is not equal to two. The base case risk coefficient is set at two. As the risk-averse coefficient increases, the futures HR and the put-option HR decrease nonlinearly. The E-VaR value also decreases. The put-option HR turns from the long position to the short position. Table 5.4 and Figure 5.3 show the detailed results for this sensitivity analysis.

Table 5.5

Sensitivity Analysis: Risk –Averse Coefficient (Corn)

Coefficient	Futures HR	Put-Option HR	E-VaR
2 (Base Case)	1.05	0.36	-$1,622,635
3	0.96	0.09	-$2,453,975
4	0.92	-0.02	-$3,242,383
5	0.90	-0.08	-$4,015,574
6	0.89	-0.12	-$4,781,498
7	0.88	-0.15	-$5,543,363
8	0.88	-0.17	-$6,302,725
9	0.87	-0.19	-$7,060,432
10	0.87	-0.20	-$7,816,987

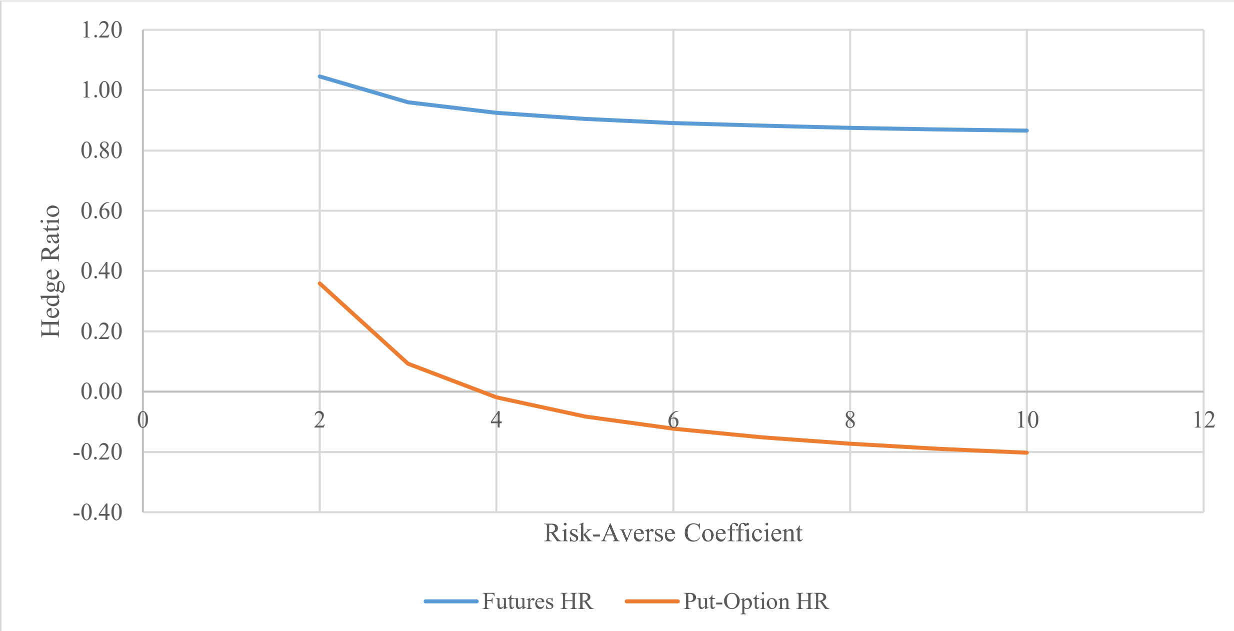

Figure 5.4. Sensitivity Analysis: Risk-Averse Coefficient (Corn).

5.4.5. Corn-Price Volatility's Sensitivity Analysis

The volatility is measured as the standard deviation of the price which is different than the traditional measure of volatility. The traditional volatility measure is the asset's annualized return. Although this approach is different than the traditional methods, it is convenient and has a similar influence to the optimization results. The base case's standard deviation is approximately $1.49 per bushel. The standard deviation's value is multiplied by the scalar multiplier to change the its level. The base case's standard deviation is assumed to be 1, and the scalar multiple ranges from 0.9 to 1.1 of the base case standard deviation with 0.02 increments. For example, the base case standard deviation is multiplied by a scalar factor of 0.9 to decrease the price standard deviation. From Table 5.6 and Figure 5.5, the higher cash-price standard deviation leads to increases futures and put-option HRs. This result is easy to explain because high-cash price standard deviation means a higher price risk as well as greater losses when the buyer defaults. Hence, larger futures and put-option positions are optimal for the trader.

Table 5.6

Sensitivity Analysis: Cash-Price Standard Deviation (Corn)

Cash Price Standard Deviation	Futures HR	Put-Option HR	E-VaR
$1.34	0.93	0.28	-$1,523,455
$1.37	0.95	0.29	-$1,539,686
$1.40	0.97	0.31	-$1,555,660
$1.43	1.00	0.32	-$1,571,378
$1.46	1.02	0.34	-$1,586,840
$1.49 (Base Case)	1.05	0.36	-$1,602,047
$1.52	1.07	0.38	-$1,616,999
$1.55	1.10	0.39	-$1,631,699
$1.58	1.12	0.41	-$1,646,148
$1.61	1.15	0.43	-$1,660,348
$1.64	1.17	0.45	-$1,674,302

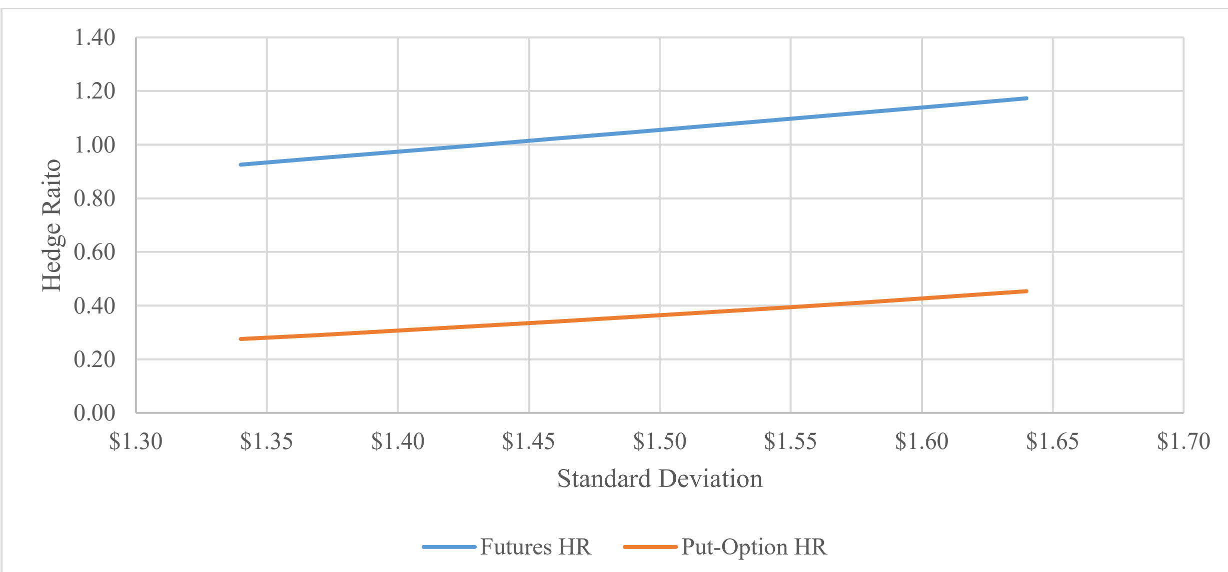

Figure 5.5. Sensitivity Analysis: Cash-Price Standard Deviation (Corn).

The approach for the corn futures' price-volatility sensitivity analysis is identical to the cash-price volatility. The futures-price volatility is important because a high futures price volatility increases the price risk to take futures positions for hedging. Moreover, the futures contract is the underlying asset for the put option. The base case for the futures' standard deviation is set at $1.50 per bushel. The futures and the put-option HRs linearly decreases when standard deviation increases. This result is intuitive because, as the standard deviation increase for the futures, the entire portfolio's volatility increases, leading to a reduced value for the E-VaR. Table 5.7 and Figure 5.6 illustrates the results.

Table 5.7

Sensitivity Analysis: Future-Price Standard Deviation (Corn)

Futures Price Standard Deviation	Futures HR	Put-Option HR	E-VaR
1.35	1.18	0.43	-$1,548,712
1.38	1.15	0.42	-$1,558,667
1.41	1.13	0.40	-$1,569,002
1.44	1.10	0.39	-$1,579,692
1.47	1.07	0.37	-$1,590,714
1.50 (Base Case)	1.05	0.36	-$1,602,047
1.53	1.02	0.34	-$1,613,669
1.56	1.00	0.33	-$1,625,561
1.59	0.98	0.32	-$1,637,706
1.62	0.95	0.30	-$1,650,086
1.65	0.93	0.29	-$1,662,686

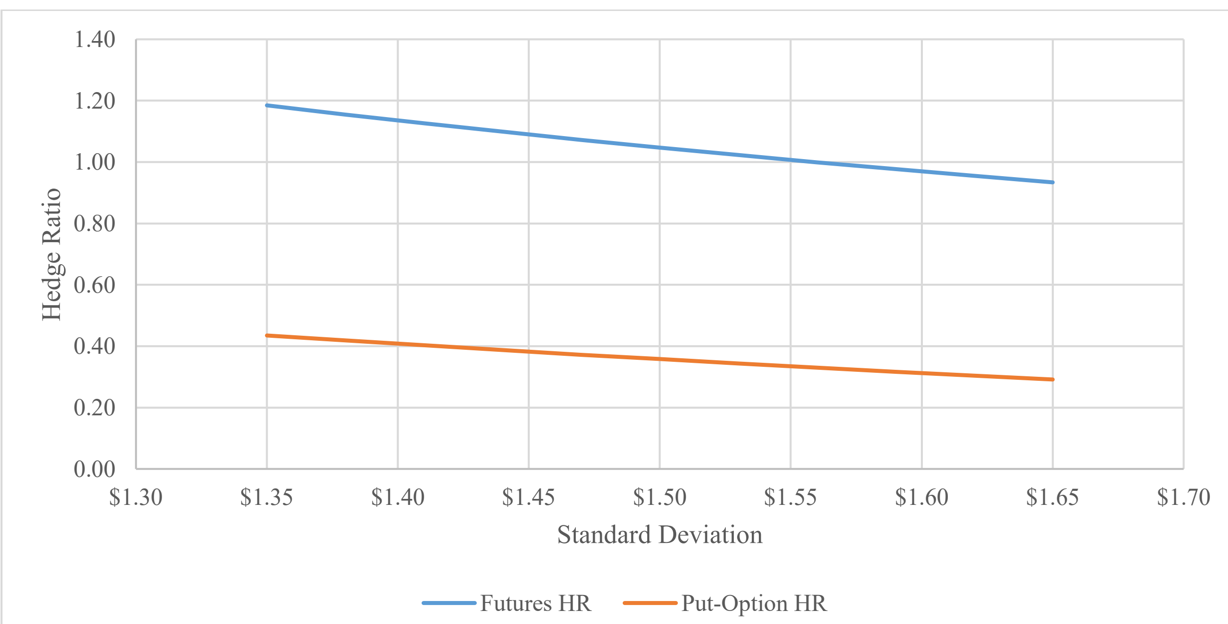

Figure 5.6. Sensitivity Analysis: Futures-Price Standard Deviation (Corn).

5.4.6. Corn-Copula Sensitivity Analysis

Copula is an alternative measure of dependency between random variables, and it is more flexible than the linear correlation, especially at the tail of the distributions. Hence, the simulation results may be more appropriate when using the copula than when using the Pearson linear correlation. As an alternative analysis, the best-fit and empirical copula is used. The linear correlation between the cash and futures prices for corn is 0.98. Using the Palisade @Risk fit copula function, the best-fit copula between the cash and futures prices is the ClaytonR copula. The empirical copula is non-parametric copula that it does not assume any functional form. Because the empirical copula is a nonparametric copula, the optimization result is more realistic and reliable. Table 5.8 summarizes the results. The base case HR is 1.05 for the future and 0.36 for the put option. The optimization result from the best-fit copula is 1.16 for the futures and 0.79 for the put option. Similarly, empirical copula resulted 1.14 for the futures and 0.74 for the put option. Considering the optimization results of the best-fit and empirical copula, the copula simulations reflect a larger default loss than the linear correlation does. This result may suggest that linear correlation is biased because it assumes a normal distribution and is affected by data outliers. One strength of the copula is that it captures the data’s tail dependency. The copula results suggest that, when default risk is present, the trader’s optimal decision is to take larger long positions for both the futures and put options when compared to the linear-correlation optimization result.

Table 5.8

Sensitivity Analysis: Copula (Corn)

	Future HR	Put-Option HR	E-VaR
Base case	1.05	0.36	-$1,622,635
Best-Fit Copula	1.16	0.79	-$1,726,883
Empirical Copula	1.14	0.74	-$1,912,802

5.5. Assumptions About the Empirical Base Case Model for Soybeans

The base case optimization model for soybeans is similar the corn model. The biggest differences are the values for the inputs and price distributions. The soybean's salvage rate is assumed to be the same as corn. The hedging firm is short 1,837,185 bushels of soybeans for the cash market. The futures and cash price for period one are taken from the dataset's latest day. The prices are \$10.29 and \$9.59 per bushel for cash and futures, respectively. The probability of default for soybeans is approximately 29%, and the probability of default before the soybeans are purchased is 30%. The risk-averse level for the soybean base case model is set at 2. The strike price for the put option is set at \$9.60 with an option premium of \$0.53 a bushel. The interest rate and days to expiration are the same as corn. The historical annualized volatility for soybeans is 20.1%.

5.5.1. Base Case Empirical-Model Results for Soybeans

The optimal HRs to maximize the E-VaR are 0.86 and -0.02 for the futures and put option, respectively. The negative HR indicates a short position. The objective is to maximize E-VaR, hence, the base case optimization result suggests selling the put option and collecting a premium is optimal. This strategy increases the overall profitability, although there is a risk of incurring unlimited loss from shorting the put option. With this optimization, E-VaR value is a negative 4.97 million. For a naïve hedge, the E-VaR value is 7.76 million losses. If the put-option position is optimized and the futures HR set at 1, the put-option HR is 0.2. These optimization results show that when a naïve hedge is placed to completely hedge the market risk, the small, short put-option position hedges the counterparty's default risk. Because the base case does not have any restrictions for the available HRs, the E-VaR value is superior compared to restricted optimizations.

The optimal futures and put-option HRs are equivalent across the E-V, minimum VaR, and minimum variance objective functions. These optimization results are same as they are for corn, and the same arguments can be used to explain why. The large profit variance caused the E-V and minimum variance to have same optimization result. Also, because a normal distribution is assumed for the VaR calculation, the symmetry may have caused the optimization result to be same as it is for the E-V and minimum variance.

Table 5.9

Base Case Stochastic-Optimization Result (Soybeans)

	Futures HR	Put-Option HR	Objective function	5%VaR
Base Case	0.86	-0.02	-$4,972,533	$2,547,237
Future HR=1, Put-Option HR = 0	1.00	0.00	-$5,105,703	$2,510,885
Future HR=1, Put-Option HR =Optimized	1.00	0.20	-$5,023,369	$2,535,023
No Hedge	0.00	0.00	-$7,764,498	$6,455,709
E-V	0.95	-0.01	-2,296,754,082,400	$2,492,782
Min VaR	0.95	-0.01	$2,492,782	$2,492,782
Min Variance	0.95	-0.01	2,296,751,549,358	$2,492,782

5.5.2. Soybean Strike-Price Sensitivity Analysis

The base case soybean strike price is $9.60, and the sensitivity ranges from positive to negative $1 per bushel. The base case optimization result is 0.86 for futures and -0.02 for the put option. The increased strike price lowers the futures HR and the put-option HR nonlinearly. The put-option HR is long when the strike price is $8.60, but put- option HR gradually decreases to turn the short position. Interestingly, the E-VaR decreases as the strike price increases, and creating a trough at $9.50. Table 5.10 and Figure 5.7 illustrates the optimization results.

Table 5.10

Sensitivity Analysis: Put-Option Strike Price (Soybeans)

Strike Price	Futures HR	Put-Option HR	E-VaR
$8.60	0.89	0.10	-$4,968,465
$8.70	0.89	0.09	-$4,969,019
$8.80	0.89	0.08	-$4,969,634
$8.90	0.89	0.07	-$4,970,280
$9.00	0.89	0.06	-$4,970,935
$9.10	0.88	0.05	-$4,971,563
$9.20	0.88	0.03	-$4,972,113
$9.30	0.88	0.02	-$4,972,539
$9.40	0.87	0.01	-$4,972,789
$9.50	0.87	-0.01	-$4,972,808
$9.60 (Base Case)	0.86	-0.02	-$4,972,533
$9.70	0.85	-0.04	-$4,971,895
$9.80	0.84	-0.05	-$4,970,828
$9.90	0.83	-0.07	-$4,969,257
$10.00	0.82	-0.09	-$4,967,095
$10.10	0.81	-0.11	-$4,964,255
$10.20	0.80	-0.13	-$4,960,639
$10.30	0.78	-0.15	-$4,956,157
$10.40	0.77	-0.18	-$4,950,719
$10.50	0.75	-0.20	-$4,944,243
$10.60	0.73	-0.23	-$4,936,624

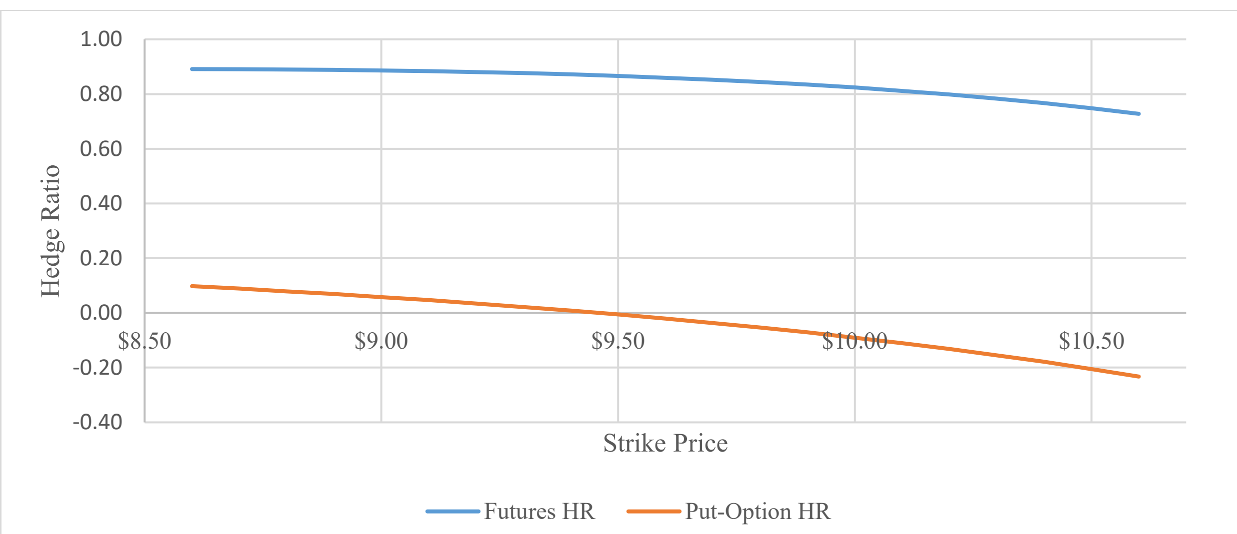

Figure 5.7. Sensitivity Analysis: Put-Option Strike Price (Soybeans).

5.5.2. Default-Probability Sensitivity Analysis for Soybeans

The base case default probability is approximately 29%. For soybeans, the futures HR declines as the default probability increases, and the put-option HR remains constant across different default-probability levels. For the soybeans' empirical optimization, the put option may not be used to hedge the loss from the buyer default. Although the put option does not change, the futures HR decreases as the default probability decreases. Table 5.11 and Figure 5.8 show the results from the sensitivity analysis.

Table 5.11

Sensitivity Analysis: Default Probability (Soybeans)

Default Probability	Futures HR	Put-Option HR	E-VaR
0%	0.98	-0.02	-$4,122,874
2%	0.97	-0.02	-$4,235,506
4%	0.97	-0.01	-$4,328,172
6%	0.96	-0.02	-$4,421,541
8%	0.95	-0.01	-$4,510,834
10%	0.94	-0.02	-$4,605,744
12%	0.93	-0.02	-$4,688,283
14%	0.92	-0.02	-$4,752,523
16%	0.90	-0.03	-$4,842,803
18%	0.90	-0.02	-$4,907,616
20%	0.89	-0.02	-$4,981,750
22%	0.89	-0.01	-$5,022,535
24%	0.88	-0.02	-$5,096,077
26%	0.87	-0.02	-$5,155,023
28%	0.86	-0.03	-$5,225,959
30%	0.85	-0.02	-$5,277,511
32%	0.84	-0.02	-$5,348,441
34%	0.83	-0.02	-$5,392,441
36%	0.83	-0.01	-$5,425,395
38%	0.82	-0.02	-$5,475,298
40%	0.81	-0.03	-$5,533,890

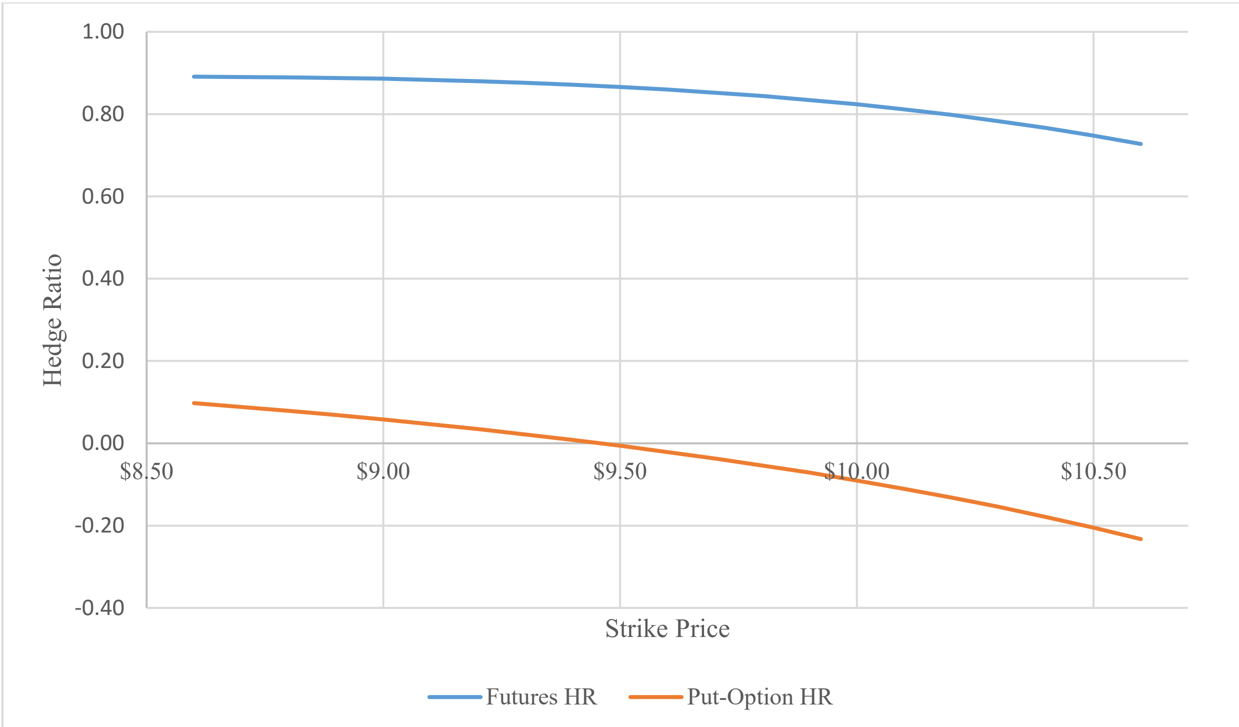

Figure 5.8. Sensitivity Analysis: Default Probability (Soybeans).

5.5.3. Soybean Before Default Probability Sensitivity Analysis

The influence of the before-default probability on the HRs is the same as the default probability. The futures HR decreases, and the put-option HR stayed almost constant. Table 5.12 and Figure 5.9 show the sensitivity analysis result.

Table 5.12

Sensitivity Analysis: Probability of Default Before Cash Purchase (Soybeans)

Before Probability	Futures HR	Put-Option HR	E-VaR
0%	0.98	-0.02	-$4,419,272
2%	0.97	-0.02	-$4,475,387
4%	0.97	-0.02	-$4,520,163
6%	0.95	-0.02	-$4,582,849
8%	0.95	-0.02	-$4,609,263
10%	0.94	-0.02	-$4,669,707
12%	0.93	-0.02	-$4,709,852
14%	0.92	-0.02	-$4,751,447
16%	0.91	-0.03	-$4,785,778
18%	0.91	-0.01	-$4,805,701
20%	0.90	-0.02	-$4,849,199
22%	0.89	-0.02	-$4,876,716
24%	0.89	-0.01	-$4,894,691
26%	0.88	-0.02	-$4,925,826
28%	0.87	-0.02	-$4,944,970
30% (Base Case)	0.86	-0.02	-$4,974,106
32%	0.85	-0.03	-$4,999,299
34%	0.85	-0.02	-$5,011,076
36%	0.83	-0.02	-$5,042,614
38%	0.83	-0.02	-$5,049,169
40%	0.82	-0.02	-$5,065,189

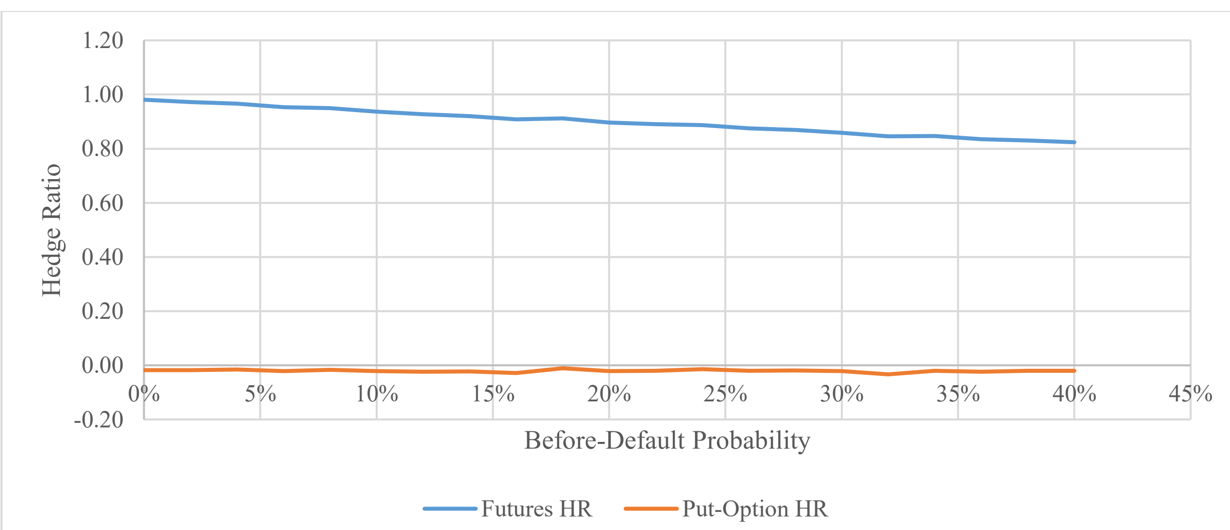

Figure 5.9. Sensitivity Analysis: Probability of Default Before Cash Purchase (Soybeans).

5.5.4. Risk-Averse Coefficient Sensitivity Analysis (Soybeans)

The base case risk-averse coefficient is set at 2. As the risk-averse coefficient increases, the futures HR increases, and the put-option HR increases slightly. The futures HR rises because taking larger futures offsets the cash-price risk. This result is shown by the large decrease for the E-VaR due to asymmetries with the risk and return tradeoff. Table 5.13 and Figure 5.10 show the optimization results.

Table 5.13

Sensitivity Analysis: Risk-Averse Coefficient (Soybeans)

Coefficient	Futures HR	Put-Option HR	E-VaR
1	0.76	-0.03	-$3,669,107
2 (Base Case)	0.86	-0.02	-$4,972,533
3	0.89	-0.02	-$6,236,901
4	0.90	-0.02	-$7,492,175
5	0.91	-0.02	-$8,743,872
6	0.92	-0.02	-$9,993,793
7	0.92	-0.02	-$11,242,702
8	0.92	-0.02	-$12,490,980
9	0.93	-0.02	-$13,738,838
10	0.93	-0.02	-$14,986,403

Figure 5.10. Sensitivity Analysis: Risk-Averse Coefficient (Soybeans).

5.5.5. Soybean-Price Volatility's Sensitivity Analysis

The base case value for the soybeans' cash-price standard deviation is approximately $2.21 per bushel. The increased standard deviation results in higher futures and put-option HRs. This finding is the same as the result for corn because an increased cash-price standard deviation causes the price risk and the loss from the buyer's default to increase. Table 5.14 and Figure 5.11 show the results for the sensitivity analysis for cash price.

Table 5.14

Sensitivity Analysis: Cash-Price Standard Deviation (Soybeans)

Cash Price Standard Deviation	Futures HR	Put-Option HR	E-VaR
$1.99	0.76	-0.06	-$4,632,606
$2.03	0.78	-0.05	-$4,700,162
$2.08	0.80	-0.04	-$4,767,860
$2.12	0.82	-0.04	-$4,835,696
$2.16	0.84	-0.03	-$4,903,663
$2.21 (Base Case)	0.86	-0.02	-$4,971,758
$2.25	0.88	-0.01	-$5,039,976
$2.30	0.90	0.00	-$5,108,313
$2.34	0.92	0.00	-$5,176,766
$2.38	0.94	0.01	-$5,245,331
$2.43	0.97	0.02	-$5,314,005

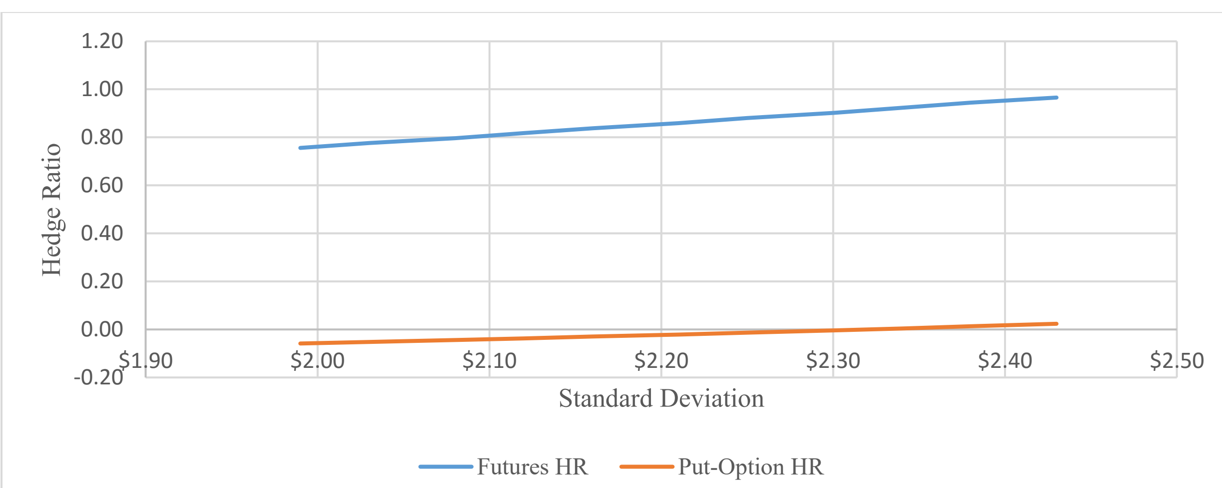

Figure 5.11. Sensitivity Analysis: Cash-Price Standard Deviation (Soybeans).

The base case soybean futures' standard deviation is $2.12 per bushel. The sensitivity analysis for futures price shows the decreased futures HR and the increased put-option HR. The put-option HR turns from short to long as the futures price standard deviation increases. In contrast to soybeans, the future HR decreases, whereas the futures HR increases for corn. Table 5.15 and Figure 5.12 show the results of the sensitivity analysis for cash price.

Table 5.15

Sensitivity Analysis: Futures-Price Standard Deviation (Soybeans)

Futures Price Standard Deviation	Futures HR	Put-Option HR	E-VaR
$1.90	0.90	-0.12	-$5,088,299
$1.95	0.89	-0.10	-$5,063,833
$1.99	0.89	-0.07	-$5,039,909
$2.03	0.88	-0.05	-$5,016,570
$2.07	0.87	-0.05	-$4,993,846
$2.12 (Base Case)	0.86	-0.02	-$4,971,758
$2.16	0.85	-0.01	-$4,950,320
$2.20	0.84	0.01	-$4,929,540
$2.24	0.83	0.02	-$4,909,422
$2.29	0.82	0.03	-$4,889,964
$2.33	0.81	0.04	-$4,871,164

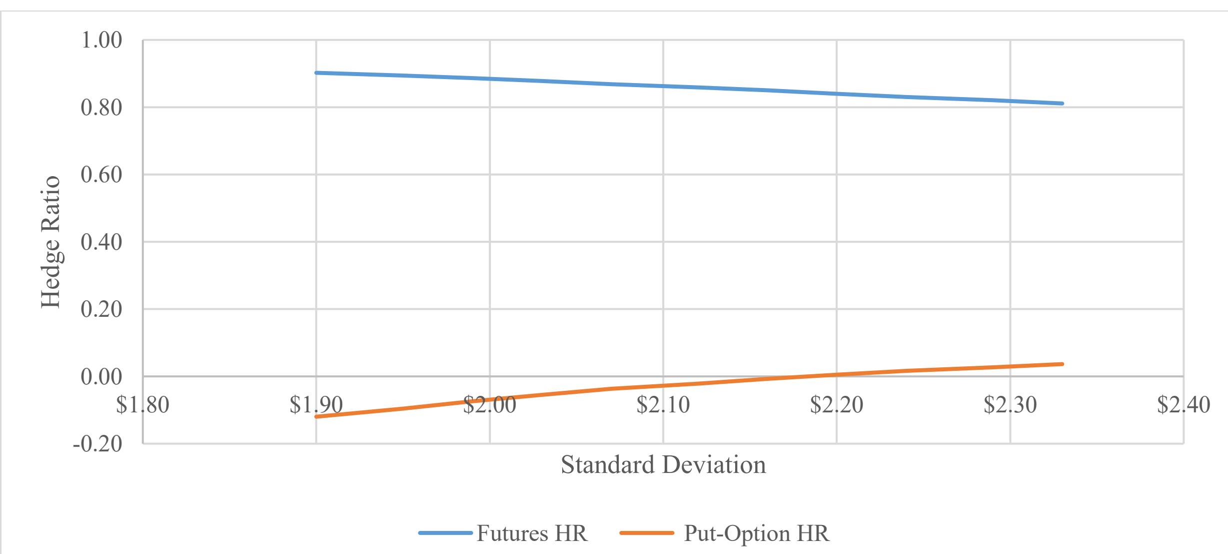

Figure 5.12. Sensitivity Analysis: Futures-Price Standard Deviation (Soybeans).

5.5.6. Copula Sensitivity Analysis for Soybeans

The correlation between the cash and futures price is 0.98, and the best-fit copula is the Frank copula with a dependency parameter that equals 28.228. There are no significant changes for the optimization result among the base case, best-fit copula, and empirical copula, except that the put-option HR for the empirical copula is positive. For the best-fit copula, 0.84 and -0.01 for futures HR and put option HR, respectively. The empirical copula results are for 0.86 for the futures HR and 0.05 for the put-option HR. Table 5.16 summarizes the optimization results.

Table 5.16

Sensitivity Analysis: Copula (Soybeans)

	Futures HR	Put-Option HR	E-VaR
Base Case	0.86	-0.02	-$4,972,533
Best-Fit Copula	0.84	-0.01	-$5,134,146
Empirical Copula	0.86	0.05	-$5,226,591

5.6. Summary

This chapter summarized the analytical and empirical optimization results using models from Chapters 3 and 4. Chapter 3 focused on the analytical model which used the derived optimal-hedging position under the E-V framework. The empirical-model approach utilized the Monte Carlo simulation for data generation and stochastic optimization.

The theoretical solutions to maximize the E-V objective function is derived by taking partial derivative of the utility function and proving the Hessian matrix is negative definite matrix. Although the solutions are not tested numerically, this result was powerful because the existence of the global optimum solution was proved. Additionally, the optimum solutions heavily depended on strike price and option delta.

With stochastic optimization, the base case optimization for corn resulted in 1.05 for the future and 0.36 for the put option. Soybeans had 0.86 and -0.02 for the futures and put options, respectively. The futures contract was primarily used to hedge the price risk. Both corn and soybeans optimization results reflected that the futures contract is primarily used to hedge price risk because their futures HRs are close to 1. For corn, the optimization resulted in the long-put option, which is used to the hedge loss if default occurs. The soybean optimization resulted in a small, short-put option. This result suggested that the trader should collect option premiums to maximize the E-VaR value although the trader risks a large downside potential. The empirical model developed to derive the optimal HR was highly stylized and complex. Therefore, identifying the connection between the variables and the optimal HR was a challenging task. When the correlation assumption changed from a Pearson linear correlation to a copula, the long-put option doubled for corn, but there was no significant change for soybeans.

The sensitivity analysis was conducted to determine the influence of a single variable's value on the optimal HR. The strike price for the put option, the default probability, the before-default probability, the risk-averse coefficient, and the volatility for cash and futures prices were considered. Several interesting consequences were obtained from the sensitivity analysis. The most interesting result was the effect of changing the default and before-default probabilities to the put-option HR. The put option HR remained constant for varying probability levels. Also, the same sensitivity led to different optimization results for between corn and soybeans. This results were shown with the default probability, before-default probability, risk-averse coefficient, and futures-price standard deviation (volatility). For example, as the futures-price standard deviation increased, the futures HR decreased. The soybeans' put-option HR increased, but it decreased for the corn. Although these results are interesting, pinpointing the cause for the different optimization results is difficult due to the empirical model's complexity.

Another sensitivity was conducted around the correlation. Typically, the correlation matrix is constructed using a Pearson linear correlation. The copula is a more flexible dependency measure and captures the tail dependency between random variables. The best-fit copula and the empirical copula were used. The best-fit copula for corn and soybeans are ClaytonR and Frank copula, respectively. The empirical copula is a nonparametric copula which does not assume any functional form. Both corn and soybeans increased the put-option HR when the correlation assumption was changed to a copula. This increase was significant for corn. The futures HR increased slightly for corn and stayed constant for soybeans. This optimization result indicated that the linear correlation is biased and does not capture the dependency at the distribution's tail.

CHAPTER 6. CONCLUSION

6.1. Introduction

Risk management is one of the most vital to run successful commodity-trading business. Risk and uncertainty are treated differently. In risk management, the risk is represented by using a probability density distribution; whereas uncertainty cannot be represented by utilizing probability density distribution. Usually, risk is measured in terms of the asset price's variability. The most common risks in the commodity market are price and quantity risk. The price risk refers to changes in the market price, and affects the position or asset's value. The quantity risk is caused by production-level variability. A farmer's typical production risk pertains to many variables, including weather, crop diseases, and many other factors. These risks are hedged with the exchange-traded futures or options contracts. Usually, when the traders deliver grain, they take a fixed short position in the cash market and an equal and opposite long position in the futures market in order to hedge the price risk.

Incorporating default risk to the hedging model is newer approach when compared to traditional hedging models used in the commodity trading. The counterparty may default before the contract maturity or back out from payments obligations that were stated on the contract (Zhu & Pykhtin, 2007). Traditionally, the cash position is assumed to be fixed; however, once the default risk is introduced, the cash position cannot be fixed. Therefore, the default risk is considered to be a special type of quantity risk. For the trader, the default risk affects the total revenue for the grain's sale.

To efficiently hedge the price and yield risks, researchers and practitioners base the theoretical foundation on Markowitz's (1952) seminal study; several portfolio-optimization models for hedging, such as minimum variance, E-V, and E-VaR framework are developed.

These models are used to derive the optimal HR for the futures and options that optimize the objective function. The quantitative risk analysis and financial modeling are used to determine the optimal hedging decisions.

6.2. Problem Statement

There is significant attention on loan defaults as well as sovereign and corporate bonds. The attention to default risk is shown by relying on the large rating agencies' credit ratings. In commodity marketing, the default risk's impacts on traders have become significant. Commodity-market defaults are caused by many different factors, particularly market-price swing and the government's trade restrictions.

Historically, China has been known for defaulting in both the agricultural and non-agricultural markets. China imports approximately 62% of the world soybeans (U.S. Department of Agriculture, 2015c). This statistic implies that the China has significant buying power for the world's soybean market. Historically, China defaulted on soybean imports in April 2004 due to a negative crash margin (Solot, 2006). During 2014, Chinese soybean buyers defaulted because they were unable to obtain a letter of credit from the banks (Thukral & Shuping, 2014). In November 2014, the Chinese government imposed a ban on U.S. corn due to unapproved GM corn varieties called MIR162. This action led to defaults for both corn and its byproduct, DDG, as well as a significant increase for the price of a substitute commodity, sorghum ("China Rejects," 2013; Kesmodel, 2015; Farm City Elevator Inc., 2014).

Defaults were also common with other agricultural and non-agricultural markets, including wheat, cotton, and iron ore. In the wheat market the Russian Federation and Ukraine banned wheat exports and created quotas during 2007, 2010, and 2012 (Blas, 2010; Global Agricultural Information Network, 2014; Kolesnikova, 2010, Kramer 2010; Vassilieva & Pyrtel, 2007). In the cotton market, Pirrong (2014) and Kub (2012) pointed out that there was default risk in 2010 and 2011 due to increased price volatility. In the non-agricultural market, defaults were common for China for iron ore market due to the price drop (Wong & Fabi, 2012).

6.3. Conclusions from the Theoretical Result

In Chapter 3, Bullock and Hayes' (1992) was extended and used to derive the closed-form expression for the optimal-futures and put-option positions. This portfolio hedging model included price and default risk, assuming that they were normally distributed. This traditional approach was utilized to find the optimal HRs without using a simulation and stochastic optimization. This solution's approach was fairly simple because the objective function only depends on first and second moments. Once the first and second moments were substituted into the mean-variance (E-V) framework, the optimal futures and option positions were obtained by taking a partial derivative.

The portfolio theory was originally developed by Markowitz (1952). The portfolio's assets are optimized by maximizing or minimizing the objective function. In general, there are three approaches for portfolio optimization; minimum-variance, utility-maximization, and risk-adjusted return. The objective of the minimum variance approach is to minimize the portfolio's variance. Johnson (1960) and Ederington (1979) the applied minimum-variance hedging model to the commodities, Government National Mortgage Association (GNMA) 8% Pass-Through Certificate, and treasury bill (T-Bill). The utility-maximization approach associates the objective function with the hedger's utility function. For example, the E-V framework is developed by applying Taylor-series expansion to the risk-averse hedger's negative utility function of the. Here, the minimum-variance is a special E-V case. The risk-adjusted return is similar to the utility-maximization except that its objective function may not be associated with the hedger's utility. The mean-value-at-risk (E-VaR) is similar to E-V except that the risk is measured by value-at-risk (VaR).

In Chapter 3, the theoretical model is derived by using the E-V framework. Several conclusions are derived. One of the most important conclusions is the existence of the global optimum under the price and default risk assumptions. The existence is proven because the Hessian matrix that is constructed with the model is a negative definite matrix if probability of the put option to be ITM is not 0 and 1. This result is powerful because the optimal solution that maximizes the objective function may not exist. Moreover, the variables such as strike price, conditional price variance, and probability of the option expiring with profit, equivalently option delta, influence the optimal-futures and put-option results. This conclusion can be verified with equations (68) and (69).

6.4. Conclusions from the Empirical Results

Chapter 5 shows the empirical model's results which consist of data generated with the Monte Carlo simulation and stochastic optimization under the E-VaR framework. For analyses, the base case optimization result is derived followed by the sensitivity analyses, which changes values of variables and correlation assumption. These empirical analyses are conducted on both soybeans and corn using publicly available historical data.

For clarification, the empirical model is created as follows. The trader is assumed to agree on a sale to the customer who may default on that sale. Most importantly, the trader has an ability to hedge the price and default risks by using exchange traded futures and put options. When the sales agreement is made, the trader does not have any grain inventory to ship; hence, the grain must be purchased. Here, the trader has a short position in the corn cash market; the buyer may or may not default before or after the grain is purchased. If the buyer defaults before the trader buys the grain, the cash position immediately goes to zero, but the trader has to find a new buyer. The new buyer is assumed to give at most the first price If the buyer does not default, the grain arrives at the destination, and the transaction is complete.

Specifically, the soybean and corn models are different; the probability of default by buyer. The fixed short-cash position, one cargo worth of cash, is taken by the trader who assumes that the price and default risks that cargo in 90-days trading window. Using stochastic optimization, the model determines the optimal futures and put-option HRs to maximize the E-VaR function. The E-VaR is an appealing objective function because it considers the portfolio's profit and downside risk. For comparison, the minimum variance, maximum E-V, and minimum VaR are used to derive the optimal HRs.

6.4.1. Empirical Results: Corn

With no hedge, the E-VaR is a \$4.89 million loss. When the strategy is a naïve hedge, the E-VaR is a \$1.71 million loss. When the futures HR is naïve and the put-option HR is optimized, the E-VaR is a \$1.63 million loss. The base case HRs for corn are 1.05 futures HR and 0.36 put-option HR with E-VaR value of \$1.62 million loss. This indicates futures HR is almost the same as a naïve hedge (HR=1.0) to completely remove the price risk, and the long put option is used to hedge the loss from a default. Based on the E-VaR value, the base case result is superior to when a restriction is imposed on the HR. This observation indicates that the flexibility is important to improve the trader's profitability; however, improvements from the long futures position are larger than the long put option, which may imply that the price risk is larger than the default risk. The optimal HRs are at 0.83 for the futures and -0.32 for the put option when minimum variance, minimum VaR, and E-V are used. This optimization result comes from the significantly large revenue variance and normality assumption for the VaR calculation.

Sensitivity analysis ranges from the strike price to the price volatility one-at-a-time. This approach consists of changing a single variable's value while keeping the other variables the same. Sensitivity analyses are conducted with the following variables: strike price, default probability, default before probability, risk-averse coefficient, cash and futures price volatilities, and copula. The base case strike price is the ATM price. The price volatility and default probability are estimated from the dataset. The before default probability and risk-averse coefficient are given initial value for the estimation. For corn particularly interesting results are seen for the strike price and the copula as an alternative correlation structure.

As the corn put option strike price increased, the HR peaked near an ATM price of \$3.10. The futures HR increased to and peaked at \$3.20, but the put-option HR peaked faster at \$2.90. The put-option HR decreased as the option became more in-the-money (ITM). This result may imply that the option premium and moneyness of the option are important factors. As the option became more ITM, the option premium increased to the point where it was expensive to buy. The optimization algorithm reduced the long put-option HR because it is not profiting enough to justify the cost of paying a premium. Although it was difficult to pinpoint why the futures HR moved similarly to the put-option HR as strike price increased, the result was highly likely because the ITM option has a lower delta meaning that put option is likely to make a profit when position is closed. Hence, it is optimal to reduce the long futures exposure.

The correlation between prices is one of the most important assumptions to test. The copula is used as an alternative correlation measure. The best-fit and empirical copula are utilized for the analysis. The empirical copula is a nonparametric copula, and based on the AIC test, the best-fit copula for the cash and futures price is the Frank copula. A linear correlation has limiting assumptions including the linearity, normality assumption and no outliners. The copula preserves correlation structure, and it allows the copula to be at least as good as a linear correlation.

Optimization using the best-fit copula resulted in 1.16 and 0.79 for futures the HR and put-option HR respectively. The empirical copula resulted in 1.14 for the futures HR and 0.74 for the put-option HR. Because the base case futures HR is 1.05, the copula assumption significantly contributed to a change in the future HR, and the put-option HR doubled. The base case put option HR was 0.36. The put-option HR was 0.79 and 0.74 for the best-fit copula and empirical copulas respectively. The correlation structure is preserved in a copula, so copula may capture the correlation that is not by the linear correlation because the linear correlation has limiting assumptions. Furthermore, this results may suggest asymmetric option payoff is suitable to capture the unlimited upside potential while incurring limited loss.

6.4.2. Empirical Results: Soybean

If the trader does not hedge, the E-VaR value is a $7.76 million loss. For the naïve hedge, the E-VaR is a $5.11 million loss. The naïve hedge futures HR with optimized put-option HR scenario resulted for 0.2 put-option HR with the E-VaR is a $5.02 million loss. The optimal HRs for soybeans are 0.86 and -0.02 for the futures and put options respectively, and the E-VaR is a $4.97 million loss. The short position for the put option suggests that collecting premiums and taking a possible unlimited loss is more profitable than the long put option to hedge the default risk. This result is similar to corn because the base case result is superior compared to the other scenarios. This is high likely because of the flexibility to determine the optimal positions. Although the E-V, minimum VaR, and minimum variance are different objective functions, the optimization results are equivalent. The futures HR is 0.95, and the put-option HR is -0.01. This result is the same as corn because of the significantly large variance and the normal distribution assumption for the VaR calculation.

Particularly interesting sensitivity analyses were the strike price, default and before default probabilities, risk-averse coefficient, and futures price volatility. They were interesting because they had different results than the corn-sensitivity analyses. Due to the empirical model's complexity, pinpointing the cause for the differences was difficult. The futures and put-option HR did not create a peak which appeared for corn. The default and before-default probability sensitivities had almost the exactly opposite result as corn. For the corn, the futures HR stayed constant while the put-option HR increased linearly for the default and before-default probabilities; however, the futures HR decreased, and the put-option HR stayed constant for soybeans. With soybeans, as the risk-averse coefficient increased, the futures HR increased slightly, and the put-option HR stayed constant; both the futures and put-option HR decreased for corn. For the futures-price volatility, the put-option HR increased as the volatility increased instead of decreasing as with corn. In contrast, the futures HR decreased for both soybeans and corn as the price volatility increased.

For soybeans, the optimization with a copula was similar to that of the corn. The futures HR stayed constant. The futures HR was 0.86 for the base case, 0.84 for the best-fit Frank copula, and 0.86 for the empirical copula. The put-option HR was -0.01 and 0.05 for the best-fit copula and empirical copula respectively while the base case put option HR was -0.02. The derived conclusion from this result was the same as it was for corn where the optimization result using the copula may be superior because it preserves the correlation structure. The increased put-option HR when a copula is used suggest that the copula captures the tail dependence more than the linear correlation. Taking long or decreasing the short put-option position is better to exploit the price movement at the tail of the distribution while limiting the losses.

6.5. Implications from the Empirical Analysis

Three important conclusions and implications can be derived from the empirical model. The alternative objective functions, E-V, minimum variance, and minimum VaR, showed almost equivalent optimization results. At least conceptually, the E-VaR seems to be superior to the alternative objective functions used for this study because it utilizes downside risk and incorporates the expected profit. Determining which objective function is best or superior to another one is outside this research's scope. Interestingly, the optimal HRs are equivalent for corn and soybeans when one can easily hypothesize that the optimization result should be different between the objective functions.

Second, the base case optimization implies that the traditional naïve hedging approach is not optimal based on the E-VaR's value. While the naïve approach for hedging is to take equal and opposite positions in futures or option market, this technique is not optimal based on the E-VaR for both corn and soybeans. This is because E-VaR's value is the highest for base cases. Hence, the trader who is simultaneously hedging the price and default risk should not thoughtlessly take a naïve hedging position.

Third, the copula may be a preferable correlation measure over the Pearson linear correlation, although it is difficult to confirm this finding with the optimization results. The linear correlation has restrictive assumptions, but the copula keeps the correlation structure between random variables. When using the copula, both the corn and soybean optimization results increased in the put-option HR. This optimization result may suggest that the cash and futures prices are more correlated at the distribution's tail. The optimization result to decrease short exposure or take long put option positions is due to higher correlation at the distribution's tail. Nonetheless, the copula's popularity is increasing for financial risk management and is becoming the industry's best practice.

6.6. Contribution to the Literature

This study's main contribution to the literature is developing portfolio optimization models by considering the buyer's default risk. The first contribution is an analytical solution under the E-V framework. Optimal HRs are mathematically derived based on Bullock and Hayes' (1992) paper. A major contribution from the theoretical model is that a global-optimum solution exits under E-V framework with the price and default risk. This result is powerful because, at least theoretically, the utility maximizing solution exists when the option creates a nonlinear payoff.

The second contribution comes from the empirical model's setup and copula. The empirical model utilizes the E-VaR under a default risk with ex-ante price distributions. The E-VaR model is an extension of the E-V model, and using the E-VaR is, at least conceptually, better than the E-V model because risk is overstated with the E-V model. Additionally, default risk, a special case of quantity risk, is incorporated with the model. In the empirical model, the default risk is assumed to be Bernoulli distribution with the probability derived from the data. The cash and futures prices are lognormal distributions with means equal to the dataset's most recent price and estimated standard deviation from the data; therefore, the price distributions are ex-ante distributions. Additionally, several important implications arise concluded from the base case optimization, alternative objective function, and sensitivity analyses.

6.7. Summary and Further Research

In commodity trading, a trader's need to hedge the default risk using exchange-traded futures and options is increasing. This study analyzed the price and default risks' effect on the trader who sold a physical commodity overseas and hedged the price risk. Theoretical and empirical portfolio models were developed to determine the optimal futures and put-option HRs. The theoretical approach derived the optimal HRs via the E-V framework. With this approach, the global optimum results were proven to exist. The empirical approach used a stochastic optimization to derive the optimal HRs under the E-VaR framework assumption. From this analysis, the naïve hedging was not optimal, and alternative objective functions led to an equivalent optimization result. Also, the copula assumption may be better ways to measure the correlation than the Pearson linear correlation. Due to a lack of a publicly available default data, the models used for this study were simple and stylized; however, high quality default data may allow the trader to create more realistic risk models to evaluate the optimal HRs.

Both analytical and empirical models are stylized and simplified. The theoretical model is developed from seller's point of view; hence it does not incorporate the buyer's utility function. By incorporating both the buyers and seller's utility functions to solve for the optimal HR that simultaneously maximizes their utility functions. Several improvements can be made on the empirical model, especially, on the accuracy of default data. The data accuracy is the biggest hurdle for creating the empirical model because firm level default data is usually a proprietary information. From the accurate default data, one may be able to estimate a correlation and the best-fit copula between commodity prices and default probability. Also, one may an apply limited dependent model to estimate default probability. Additionally, if the trader is able to resell the put option before it expires, the overall put option purchasing cost decreases significantly. The empirical model assumed that the 90 days trading window and the put option has 120 days to expiration. Therefore, the put option has 30 days of time value left at the end of trading window and can be sold in the option market. There could be many unspecified improvements that may or may not improve the models to estimate the optimal HRs.

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