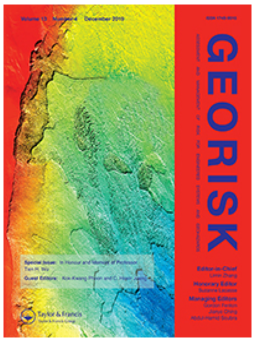

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ISSN: 1749-9518 (Print) 1749-9526 (Online) Journal homepage: https://www.tandfonline.com/loi/ngrk20

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**To cite this article:** Michael A. Hicks, Divya Varkey, Abraham P. van den Eijnden, Tom de Gast & Philip J. Vardon (2019) On characteristic values and the reliability-based assessment of dykes, Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 13:4, 313-319, DOI: 10.1080/17499518.2019.1652918

**To link to this article:** https://doi.org/10.1080/17499518.2019.1652918

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GEORISK
2019, VOL. 13, NO. 4, 313–319
https://doi.org/10.1080/17499518.2019.1652918

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# On characteristic values and the reliability-based assessment of dykes

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**ABSTRACT**
A case study involving the assessment and re-design of an existing dyke, founded on a layered soil, has compared deterministic analysis based on 5-percentile property values and a reliability-based random finite element analysis consistent with the requirements of Eurocode 7. The results show that a consideration of the spatial nature of soil variability generally leads to higher computed factors of safety and, for those dyke sections requiring remedial action, to more economic designs. Back-figured characteristic values are shown to be considerably higher than the 5-percentile soil properties; hence, a reduction in over-conservatism is achieved.

**ARTICLE HISTORY**
Received 30 November 2018
Accepted 4 August 2019

**KEYWORDS**
Characteristic values; dykes; Eurocode 7; slope reliability; spatial variability

## 1. Introduction

Around 1 billion euros per year are required to maintain and upgrade the Dutch dyke network, which protects around 40% of the Netherlands from inundation. This includes 14,000 km of rural dykes, which are currently maintained and upgraded using rules mainly derived from research on primary dykes (a very different type of structure). The current strategy for determining when maintenance and/or upgrading are needed is based on assessment using partial factors and reliability-based characteristic values derived only from the point statistics of the material properties. This paper reports a recent reliability-based assessment of a dyke ring in the west of the Netherlands, based on statistics derived from laboratory and site investigation data. In particular, for a selected dyke cross-section, deterministic solutions for the factor of safety are compared with probability distributions of factor of safety based on reliability analyses using (a) only the point statistics, and (b) random fields.

## 2. Background

Dutch stability assessments of rural dykes are based on the Eurocode 7 (EC7) philosophy of partial factors and characteristic values of soil properties, in which the partial factors are defined by the code and the characteristic values are chosen by the engineer (CEN 2004). In particular, they adopt a statistical approach to deriving characteristic values.

Extracts from Section 2.4.5.2 of EC7, "Characteristic values of geotechnical parameters", were reviewed by Hicks (2012) and Hicks and Nuttall (2012). In particular, they highlighted Clause (11), which gives guidelines for when statistical methods are used (see Table 1). It infers that characteristic values should be selected so as to give a structure reliability (relative to the limit state) of at least 95%. Although this appears to be contradicted by the two parts of the footnote, the first part of which refers to mean values and the second part which refers to the soil property distribution, it was demonstrated that the clause and both parts of the footnote are entirely consistent, and explained by a consideration of the scale of fluctuation $\theta$ (the distance over which soil properties are significantly correlated) and the size of the problem domain $D$ (e.g. the extent of the failure surface), relative to a modified "effective" property distribution governing the limit state. In Figure 1, which, for simplicity, shows a single soil property $X$ represented by a normal distribution, 3 scenarios are possible for the characteristic value $X_k$:

(1) For very small values of $\theta/D$, there is considerable averaging of soil property values over the potential failure surface. This leads to a narrow "effective" property distribution centred about the mean ($X_m$) of the underlying distribution. In this case, the 5 percentile of the modified distribution represents a cautious estimate of the mean (cf. part 1 of footnote);
(2) For very large values of $\theta/D$, failures tend to be local and there is a large range of possible solutions. This leads to the "effective" distribution tending towards the underlying distribution, from which the characteristic value is the 5 percentile (cf. part 2 of footnote).

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**Table 1.** Clause (11): Extract from Section 2.4.5.2 of Eurocode 7 (CEN 2004).

(11) If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%.
NOTE: In this respect, a cautious estimate of the mean value is a selection of the mean value of the limited set of geotechnical parameter values, with a confidence level of 95%; where local failure is concerned, a cautious estimate of the low value is a 5% fractile.

(3) For intermediate values of $\theta/D$ (i.e. the usual scenario), $X_k$ is problem-dependent and there are 2 competing factors: (a) the averaging of soil properties over the potential failure surface leads to a narrower "effective" property distribution; (b) the tendency for failure to be attracted to semi-continuous weaker zones leads to a reduced mean ($X_m$*) relative to the underlying distribution.

Note that Scenario 3, as illustrated in Figure 1, is the general case, whereas Scenarios 1 and 2 (not shown) are limiting cases. Moreover, although the mean is reduced in Scenario 3, because the modified distribution is narrower than the underlying distribution, the 5-percentile characteristic value is generally greater than in Scenario 2; that is, relative to the underlying distribution, $X_k$ corresponds to a percentile greater than 5%.

Various approaches have been proposed for selecting the characteristic values of soil properties; for example, as reported by Orr (2017) and Shen et al. (2018). However, for reasons of simplicity, engineering practice often uses the 5 percentile of the underlying distribution as the characteristic value, regardless of the value of $\theta/D$ or the geotechnical application. The implications of this simplification are demonstrated below, through use of a reliability-based random finite element approach consistent with the requirements of Eurocode 7.

## 3. Case history

The Starnmeer polder is situated in the province of North Holland and is managed by the water board Hoogheemraadschap Hollands Noorderkwartier (HHNK). It was originally drained in 1643, covers an area of 580 hectares, and is contained within a ring dyke of around 13 km in length. Recently, HHNK initiated a stability assessment of the dyke. This was performed by dividing the dyke into 10 sections and, for each section, the factor of safety ($F$) against slope failure was determined for a representative cross-section using the limit equilibrium software D-Geo Stability (Deltares 2018). This revealed that 5 of the 10 sections do not comply with current safety requirements. Indeed, not only did they return factors of safety below the required $F$; in some cases, factors of safety as low as 0.5 were reported even though the dyke has remained stable for hundreds of years.

In this paper, the authors investigate the assumptions made in analysing a dyke cross-section which returned a factor of safety of 0.59 based on design property values. Figure 2 shows that the 3.8 m high dyke is composed

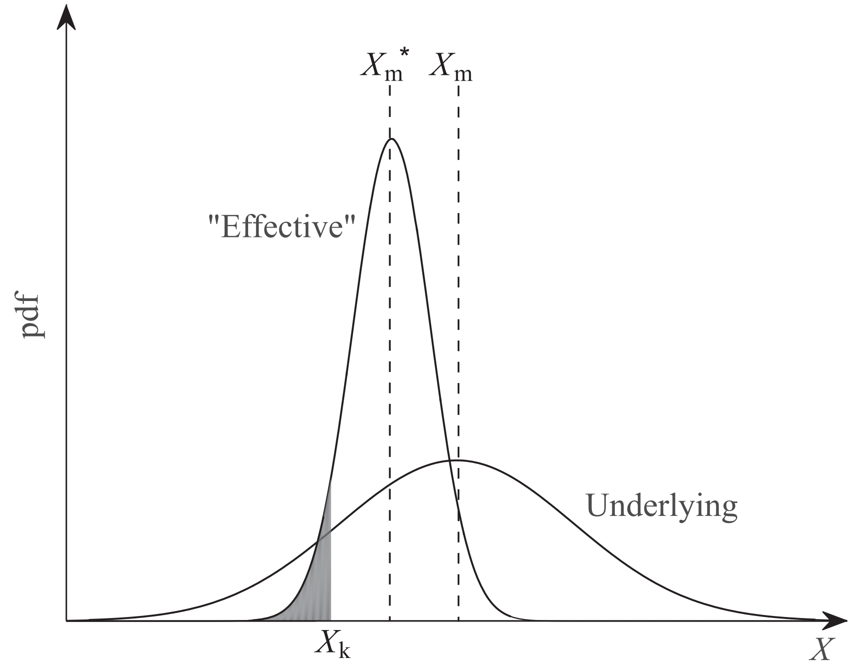

**Figure 1.** Derivation of characteristic property value satisfying EC7: underlying distribution of $X$, and "effective" distribution accounting for influence of spatial correlation and problem being analysed.

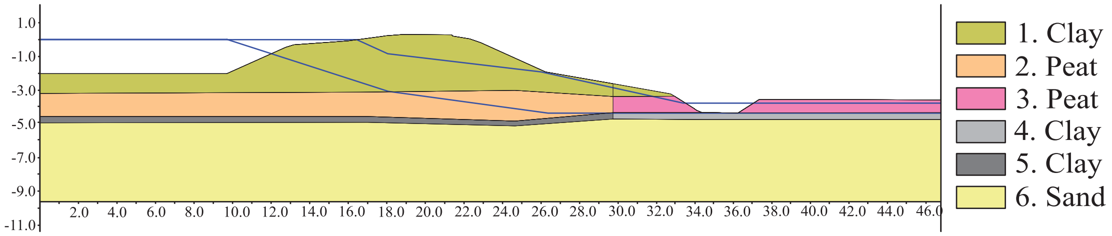

**Figure 2.** Dyke cross-section (scale in metres).

**Table 2.** Unit weights and shear strength parameter values used in analysis of dyke cross-section. (Layers 1–6 refer to Figure 2; layers 7–8 refer to Figure 5.)

| | | $c'$ | | | | | $\tan \phi'$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Layer | $\gamma$ (kN/m$^3$) | Mean (kPa) | 5-percentile value (kPa) | COV | Partial factor | Design value (kPa) | Mean | 5-percentile value | COV | Partial factor | Design value |
| *(a) Layers 1–6* | | | | | | | | | | | |
| 1 | 13.9* | 4.4 | 1.1 | 0.773 | 1.20 | 0.917 | 0.580 | 0.506 | 0.081 | 1.15 | 0.429 |
| 2 | 9.8 | 3.2 | 1.0 | 0.656 | 1.20 | 0.833 | 0.398 | 0.361 | 0.058 | 1.15 | 0.310 |
| 3 | 9.9 | 2.0 | 0.5 | 0.775 | 1.20 | 0.417 | 0.358 | 0.279 | 0.145 | 1.15 | 0.241 |
| 4 | 15.0 | 4.5 | 1.7 | 0.544 | 1.20 | 1.417 | 0.559 | 0.547 | 0.012 | 1.15 | 0.465 |
| 5 | 15.0 | 5.4 | 2.9 | 0.352 | 1.20 | 2.417 | 0.601 | 0.594 | 0.007 | 1.15 | 0.503 |
| 6 | 20.0 | 0.0 | 0.0 | 0.000 | – | 0.000 | 0.637 | 0.637 | 0.000 | 1.20 | 0.531 |
| *(b) Layers 7–8* | | | | | | | | | | | |
| 7 | 17.0 | 6.2 | 1.6 | 0.773 | 1.20 | 1.333 | 0.531 | 0.463 | 0.081 | 1.15 | 0.403 |
| 8 | 20.0 | 0.0 | 0.0 | 0.000 | – | 0.000 | 0.637 | 0.637 | 0.000 | 1.20 | 0.531 |

*$\gamma = 6.9$ kN/m$^3$ above phreatic surface.

of clay, and is founded on a peat layer underlain by a thin clay layer and a thick sand layer. Table 2(a) summarises the unit weights and shear strength properties used in the original assessment, based on the results of extensive laboratory (triaxial and direct simple shear) tests on soils from Starnmeer (Kames 2015). In this table, the mean and 5-percentile values for the cohesion ($c'$) and tangent of the friction angle ($\tan \phi'$), for each material zone indicated in Figure 2, are reported, as well as the respective partial factors and design property values used in the stability analysis (in which the design value is equal to the characteristic value divided by the partial factor). Also shown in the table are the coefficients of variation (COV = standard deviation/mean) of $c'$ and $\tan \phi'$, which have been back-figured from the respective mean and 5-percentile values assuming a lognormal distribution, and are on the conservative (high) side due to soil samples coming from the Starnmeer area as a whole rather than from the specific cross-section being analysed. Note that no test results were reported for the bottom (sand) layer, and that the 5-percentile value of $\tan \phi'$ adopted for this layer is the value suggested by NEN 9997-1 (2011) for a moderately packed sand.

**Table 3.** Factors of safety $F$ for dyke cross-section based on deterministic and stochastic analyses.

| Deterministic analysis | | Stochastic analysis | | |
|---|---|---|---|---|
| | | | $F$ corresponding to CDF of 0.05 | |
| Property values | $F$ | $\theta_h$ (m) | without partial factors | with partial factors |
| Mean | 1.31 | 0.5 | 1.10 | – |
| 5-percentile | 0.66 | 6.0 | 0.98 | 0.85 |
| Design | 0.54 | 12.0 | 0.98 | – |

### 3.1. Re-analysis of dyke stability

Table 3 summarises the results of a re-evaluation of the stability of the dyke section by the authors. These results have been obtained using an in-house finite element software, developed at TU Delft, that computes the factor of safety using the strength reduction method, and they are based on the same cross-sectional geometry and material properties used previously. Moreover, the authors have assumed the same external water level and phreatic surfaces (represented by the blue lines in Figure 2) as in the original assessment, in which the higher phreatic surface relates to all soil layers except for layer 6, for which the lower phreatic surface is used. Figure 3 illustrates the significance of the underlying peat layer, by showing the computed failure mechanism based on homogenous soils.

Firstly, Table 3 lists the deterministic factors of safety obtained using the mean, 5-percentile and design property values for the different material zones (from Table 2(a)). Based on the design properties, $F = 0.54$, which compares favourably with the D-Geo Stability solution of 0.59, as well as with an $F$ of 0.56 obtained by the

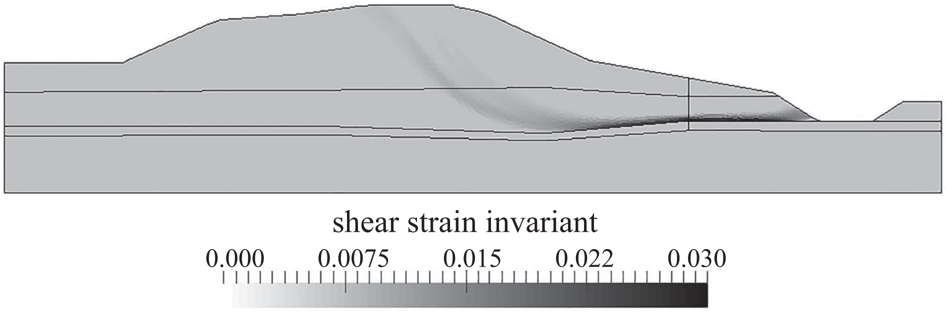

**Figure 3.** Plastic shear strain contours at failure based on homogeneous soil layers.

authors using the commercial finite element code PLAXIS. Each of these solutions takes account of the uncertainty in the design property values by basing them on characteristic values representing the 5 percentile of the property distribution; that is, by adopting the approach called Scenario 2 in Section 2. However, as discussed, this is not consistent with the intention of EC7, as illustrated in Figure 1, in that the characteristic values take no account of the spatial nature of the soil variability nor of the problem being analysed.

Hence, Table 3 also shows stochastic results accounting for the spatial variability of soil property values within the material zones. These have been computed with the same in-house finite element software, but now implemented within a Monte Carlo simulation in which each realisation uses different random fields of soil property values for each material zone, a procedure often referred to as the random finite element method (RFEM) (Fenton and Griffiths 2008). The random fields have here been generated by covariance matrix decomposition using a Markov autocorrelation function; see van den Eijnden and Hicks (2017) for details. The RFEM process uses the same point statistics as listed in Table 2, but additionally, for each soil property and each material zone, vertical and horizontal scales of fluctuation are specified to quantify the distance over which property values are significantly correlated. As insufficient data are available for the cross-section, the vertical scale of fluctuation ($\theta_v$) has been taken as 0.5 m for all properties and all material zones. This is a conservative (high) estimate based on a range of 0.2–0.5 m reported by de Gast, Vardon, and Hicks (2017) for similar soils found at the Leendert de Boerspolder site in South Holland. Three values for the horizontal scale of fluctuation ($\theta_h$) have initially been considered; 0.5, 6.0 and 12.0 m, to investigate the sensitivity of the solution to this statistical measure. For each value of $\theta_h$, an RFEM analysis involving 500 realisations has been conducted, in which, for each realisation, the point and spatial statistics have been used to generate uncorrelated random fields of $c'$ and $\tan \phi'$ for each material zone, and the factor of safety of the dyke then computed using the strength reduction method. This gives 500 factors of safety, from which a cumulative distribution function (cdf) of $F$ can be plotted.

Figure 4 shows the cdf of $F$ computed using RFEM for each value of $\theta_h$ (as a solid curve), based on the soil property statistics given in Table 2(a). Also indicated in the figure are the factors of safety obtained from deterministic analyses based only on the mean, median and 5-percentile values, as well as that obtained based on the design property values. The cdf of $F$ from a stochastic analysis based only on the point statistics (i.e. with no spatial averaging) is included, to highlight the significance of spatial averaging in the RFEM analyses.

Figure 4 shows that for a reliability ($R$) of 95%, a conservative estimate of $F = 0.98$ is obtained when $\theta_h = 6.0$ m. In order to determine the value of $F$ corresponding to the design property values, for each material zone the property distribution for $c'$ has been scaled down by a partial factor of 1.20 and the property distribution for tan $\phi'$ has been scaled down by a partial factor of 1.15 (or 1.20, in the case of the sand layer). These "design" property distributions have then been used in a further RFEM analysis (with $\theta_h = 6.0$ m), to give a new cdf (shown as a broken curve) and a value of $F = 0.85$ corresponding to $R = 95\%$ (Table 3). This value represents a significant (57%) increase in $F$ when accounting for the spatial nature of the soil variability, although, as it is still less than the $F = 0.95$ safety requirement (based on the IPO-class, i.e. design class, this dyke section belongs to (Kames 2015)), some upgrading of the dyke section is needed.

### 3.2. Re-design of the dyke section

Figure 5 shows an initial proposal for the re-design of the dyke section, following on from the original stability assessment of $F = 0.59$ (using D-Geo Stability). This involves moving the ditch further away, infilling the original ditch with sand, and constructing a clay berm over the sloping face to increase the resistance against failure.

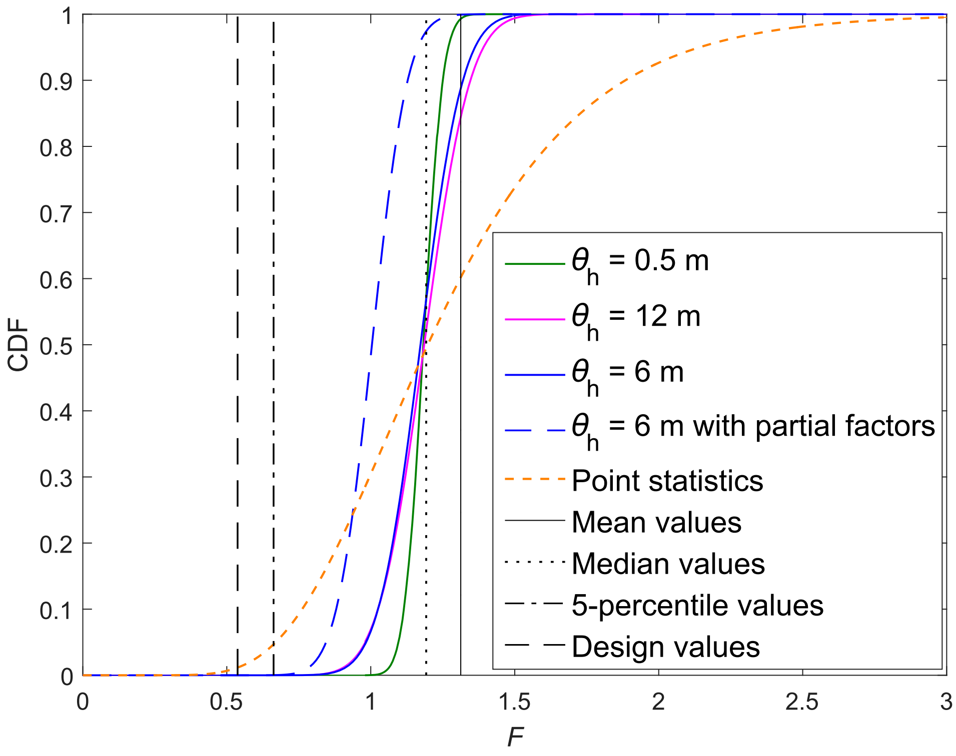

**Figure 4.** Comparison of deterministic and stochastic solutions for factor of safety.

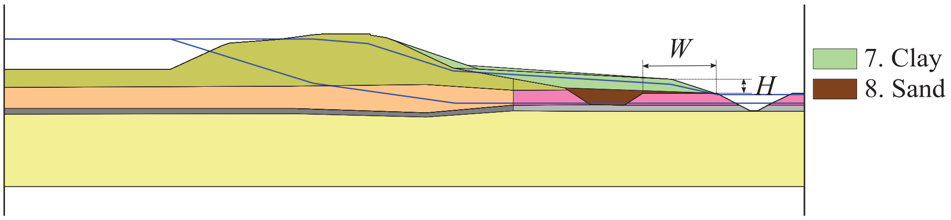

**Figure 5.** Initial re-design for dyke cross-section.

The unit weights and shear strength properties for the sand infill and clay fill are summarised in Table 2(b). This led to increased deterministic factors of safety, based on the design property values, of $F = 1.33$ using D-Geo Stability and $F = 1.21$ using the in-house software. However, an RFEM analysis based on the design property distributions, $\theta_v = 0.5$ m, and $\theta_h = 6.0$ m, for the cross-section in Figure 5, gave $F = 1.531$ for $R = 95\%$, an increase of 27% relative to the deterministic in-house solution.

Table 4 shows the results of further RFEM analyses, corresponding to a range of berm heights and berm widths (as quantified by the distance between old and new ditches), see Figure 5. These results show how $F$ corresponding to $R = 95\%$, with and without partial factors, varies as a function of the berm geometry. In particular, it highlights how a berm with a height of $H/2$ and inter-ditch spacing of $W/3$ gives a factor of safety (with partial factors) satisfying the safety requirement (i.e. $F = 1.015 > 0.95$). This represents a significant saving relative to the

**Table 4.** Factors of safety corresponding to $R = 95\%$ for various berm designs.

| | *H* | 2*H*/3 | 3*H*/5 | | *H*/2 | | 0*H* | |
|---|---|---|---|---|---|---|---|---|
| | without partial factors | without partial factors | without partial factors | with partial factors | without partial factors | with partial factors | without partial factors | with partial factors |
| *W* | 1.789 | 1.461 | 1.268 | 1.083 | 1.197 | 1.027 | 0.968 | 0.826 |
| 2*W*/3 | 1.736 | 1.377 | 1.265 | 1.080 | 1.193 | 1.021 | – | – |
| *W*/2 | 1.724 | 1.375 | 1.259 | 1.079 | 1.186 | 1.016 | – | – |
| *W*/3 | 1.647 | 1.360 | 1.249 | 1.071 | 1.181 | 1.015 | – | – |

original re-design (Figure 5), both in terms of volume of fill required and impact on neighbouring property.

### 3.3. Characteristic values

The above analysis and re-design of the dyke section using RFEM is fully consistent with EC7, in that it is based on characteristic soil property values giving a 95% reliability of the structure, factored down by the required partial factors. Note that, even though the characteristic soil properties have not been calculated explicitly during the analyses (i.e. the 5 percentile of the "effective" distribution), it is the reliability-based factor of safety that is needed in the safety assessment. Moreover, calculating characteristic values for a problem in which there are two soil properties and multiple soil layers is not straightforward. In contrast to the simple illustration given in Figure 1, in which the characteristic property is a single value, for this dyke section the characteristic values for each material zone are represented by a surface in $c'$–tan $\phi'$ space; in other words, there are many combinations of $c'$ and tan $\phi'$ that give the same reliability for the structure. Nonetheless, it is informative to back-calculate percentiles (of the underlying property distributions) representing the characteristic values and, for illustrative purposes, a simple approach has here been adopted.

Specifically, a single characteristic percentile has been back-figured, which, when applied to the distributions of $c'$ and tan $\phi'$ for each material zone, gives characteristic values that return the correct factor of safety for $R = 95\%$. This percentile has been determined by conducting multiple deterministic analyses, in which, for any given realisation, the shear strength parameters for all material zones are sampled from the same percentile location in the respective property distributions (i.e. each material zone is treated as homogeneous). Thus, in realisation $r$, the input $(X_i)_r$ for a parameter $X_i$ (i.e. either $c'$ or tan $\phi'$) is calculated using

$$(X_i)_r = \exp\left(\mu_{\ln X_i} + \sigma_{\ln X_i} \times k_r\right) \qquad (1)$$

where $\mu_{\ln X_i}$ and $\sigma_{\ln X_i}$ are the mean and standard deviation of the natural logarithm of $X_i$, respectively, and $k_r$ is the standard score, computed using

$$k_r = \Phi^{-1}(r/N) \qquad (2)$$

where $\Phi^{-1}$ is the inverse of the standard normal cumulative distribution function and $N$ is the total number of realisations. Figure 6 compares the cdf of $F$ obtained using this approach with the cdf of $F$ obtained using RFEM with $\theta_v = 0.5$ m and $\theta_h = 6.0$ m, for the original dyke cross-section in Figure 2. For $F = 0.98$, corresponding to $R = 95\%$ in the RFEM analysis, the value of $r/N$ is 0.34. Hence, for this particular dyke section and loading

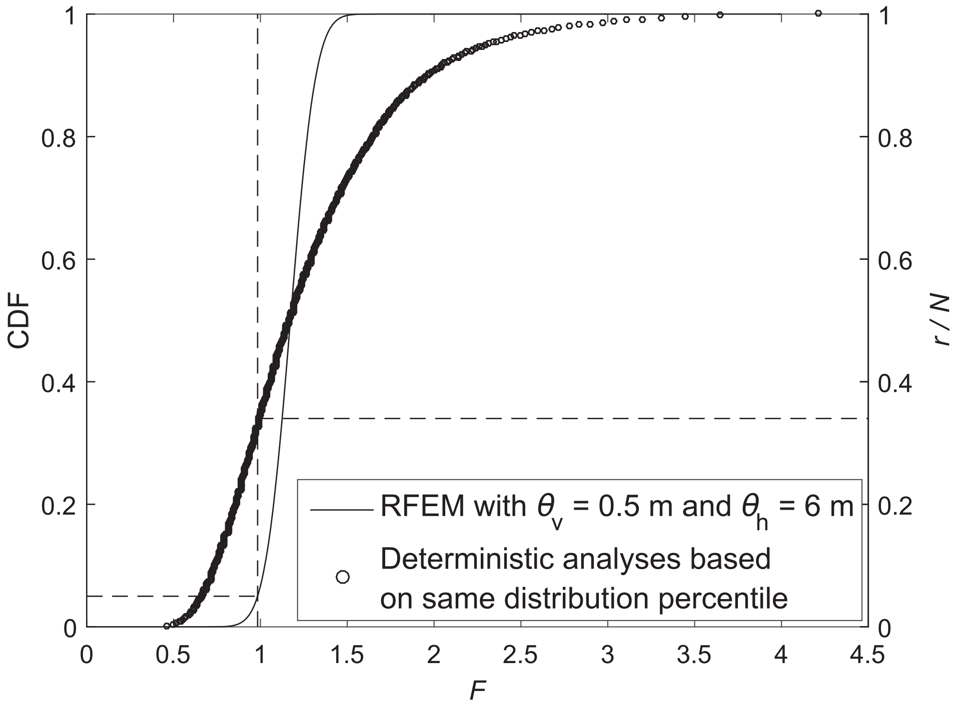

**Figure 6.** Comparison of factor of safety distribution obtained using RFEM with deterministic analyses based on same distribution percentile.

conditions, the characteristic percentile that may be used for both the $c'$ and tan $\phi'$ distributions (for all material zones) is 34%.

Note that no correlation has been assumed between $c'$ and tan $\phi'$ in this research, although previous studies have mainly suggested a negative correlation between these two parameters, which would result in a narrower cdf of $F$ (Vardon, Liu, and Hicks 2016) and thereby to a higher characteristic percentile. Thus, the characteristic percentile of 34% for this particular dyke section is likely to be a conservative estimate.

## 4. Conclusions

A comparison has been made between using a deterministic assessment method and the random finite element method to assess the stability and re-design of an historic dyke in the Netherlands, based on a reliability-based framework consistent with Eurocode 7. It has been shown that a proper consideration of spatial variability, such as with the random finite element method, can lead to higher factors of safety and, for those structures requiring attention, to less costly and less intrusive mitigation measures. The advantage of the proposed approach is that it satisfies the requirements of Eurocode 7 without the need to explicitly select or calculate the characteristic property values. Nevertheless, for the particular dyke section analysed in this paper, and for illustrative purposes only, characteristic soil property values consistent with Eurocode 7 were back-calculated and found to be the 34 percentiles of the respective property distributions. This represents a significant increase in strength capacity over simpler interpretations of Eurocode 7 based only on the point statistics. However, given the problem-dependent nature of characteristic values, as well as the desire for simpler validated approaches amenable to practice, further studies are recommended for a more general insight.

## Acknowledgements

The authors are grateful for the assistance of Henk van Hemert (STOWA) and Niels Tenhage (Hoogheemraadschap Hollands Noorderkwartier).

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This work is part of the research programme Reliable Dykes with project number 13864 which is financed by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO), and was carried out on the Dutch National e-infrastructure with the support of SURF Foundation.

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