COMPARISON OF PROPOSED K SAMPLE TESTS WITH DIETZ'S TEST FOR NONDECREASING ORDERED ALTERNATIVES FOR BIVARIATE NORMAL DATA

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The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

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ABSTRACT

Yanchun, Zhao, M.S., Department of Statistics, College of Science and Mathematics, North Dakota State University, April 2011. Comparison of Proposed k Sample Tests with Dietz's Test for Nondecreasing Ordered Alternatives for Bivariate Normal Data. Major Professor: Dr. Rhonda Magel. Co-advisor: Dr. Fu-Chih Cheng.

There are many situations in which researchers want to consider a set of response variables simultaneously rather than just one response variable. For instance, a possible example is when a researcher wishes to determine the effects of an exercise and diet program on both the cholesterol levels and the weights of obese subjects. Dietz (1989) proposed two multivariate generalizations of the Jonckheere test for ordered alternatives. In this study, we propose k-sample tests for nondecreasing ordered alternatives for bivariate normal data and compare their powers with Dietz's sum statistic.

The proposed k-sample tests are based on transformations of bivariate data to univariate data. The transformations considered are the sum, maximum and minimum functions. The ideas for these transformations come from the Leconte, Moreau, and Lellouch (1994). After the underlying bivariate normal data are reduced to univariate data, the Jonckheere-Terpstra (JT) test (Terpstra, 1952 and Jonckheere, 1954) and the Modified Jonckheere-Terpstra (MJT) test (Tryon and Hettmansperger, 1973) are applied to the univariate data. A simulation study is conducted to compare the proposed tests with Dietz's test for k bivariate normal populations (k=3, 4, 5). A variety of sample sizes and various location shifts are considered in this study. Two different correlations are used for the bivariate normal distributions. The simulation results show that generally the Dietz test performs the best for the situations considered with the underlying bivariate normal distribution. The estimated powers of MJT_{sum} and JT_{sum} are often close with the MJT_{sum}

generally having a little higher power. The sum transformation was the best of the three transformations to use for bivariate normal data.

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1. INTRODUCTION

Comparison of two-sample problems with multivariate censored data was investigated by Leconte, Moreau, and Lellouch (1994). They proposed a new rank test family which developed from weighted logrank tests to test the equality of two multivariate failure time distributions with censored observations; a transformation of the multivariate rank vectors to a univariate rank score is applied. Their test statistics are called multivariate weighted logrank tests.

Krogen and Magel (2000) used the above-mentioned ideas of Leconte, Moreau, and Lellouch, and developed a set of k-sample tests for bivariate censored data when testing the equality of k survival functions against the nondecreasing ordered alternatives (with at least one strict inequality).

The problem of comparing two groups on uncensored univariate data has been extensively investigated. The Mann-Whitney-Wilcoxon test (Mann and Whitney, 1947; Wilcoxon, 1945) is the most widely used noparametric technique. The problem of comparing several groups based on univariate data has also been developed in detail. The Kruskal-Wallis test (Kruskal and Wallis, 1952) is the most commonly applied nonparametric technique to test the equality of k population distribution functions. The Jonckheere-Terpstra (JT) test (Terpstra, 1952; Jonckheere, 1954) has been heavily used in practice for nondecreasing ordered alternatives with several samples. Tryon and Hettmansperger (Tryon and Hettmansperger, 1973) proposed a modified version of the JT test which gives a weight for each Mann-Whitney statistic based on the order of populations.

There are times when we may wish to test for nondecreasing treatment effects when we have k samples of bivariate uncensored data from k populations. Let us begin with three populations of interest such as a control population and two different treatment populations. We have two observations for each subject in three populations, and therefore, we have bivariate population distributions. A possible example is given to illustrate the issue. Suppose that researchers wish to determine the effect of an exercise and diet program on the cholesterol level and weight of obese subjects. A set of obese subjects who have volunteered for the study are randomly divided into three groups. The first group is the control group who do not receive any instructions at all. Subjects in the second group are not given any instructions on eating, but are given exercise instructions by professionals. Subjects in the third group receive both professional exercise instructions and diet instructions. This program is designed to last 4 months. The cholesterol level and weight of each subject are measured both at the beginning of the study and end of the study. The change in the cholesterol levels and the change in the weights are recorded for each subject with these changes equal to the values recorded in the beginning of the study subtracted by the values recorded end of the study. Researchers would like to see if there is a nondecreasing effect on changes in cholesterol and weight for these three population groups.

In order to test for nondecreasing treatment effects on several variables based on k samples, one could analyze each variable separately, but one may wish to analyze them collectively. Two approaches could be considered to do this. One approach is to use a test designed for multivariate data. Dietz (1989) has developed such a test for k populations. The second approach for testing nondecreasing treatment effects is to reduce the bivariate data to univarate data and then apply a test based on univariate data. Leconte, Moreau, and Lellouch (1994) have illustrated how to reduce censored bivariate data to censored univariate data as mentioned in the beginning. In this work, we use their ideas and apply these to uncensored data. The transformations used will be described in detail in Chapter 3.

Consequently, a question arises: which method is better to test for nondecreasing treatment effects among k samples with multivariate uncensored data? In this study, we compare tests developed from both methods and compare their powers under a variety of scenarios. The underlying distribution considered in this study is the bivariate normal. The related issues are investigated by simulation.

The rest of this thesis is structured as follows. Chapter 2 gives a review of the related literature. In Chapter 3, the proposed tests are described in detail. Chapter 4 gives the details of the simulation study. The simulation results are presented and analyzed in Chapter 5. Finally, conclusions are drawn in Chapter 6.

2. REVIEW OF LITERATURE

This chapter contains related literature about the proposed problem. A discussion of Dietz's test is included in this chapter since Dietz's test is a test designed for multivariate data. Many nonparametric univariate data tests are included because this research proposes procedures which reduce bivariate data to univariate data before testing. We also include a discussion as to what has been done with bivariate censored data.

2.1. Mann-Whitney-Wilcoxon Test

The Mann-Whitney-Wilcoxon test is a test for equal population location parameters using univariate data based on two independent samples. Wilcoxon(1945) proposed a rank sum test statistic for the case of equal sample sizes. Mann and Whitney (1947) developed the test statistic for the case of unequal sample sizes. For the Mann-Whitney-Wilcoxon test, the null hypothesis is H_0: $M_x=M_y$, and

the alterative hypothesis could be H_1: $M_x \neq M_y$ or H_1: $M_x < M_y$ or H_1: $M_x > M_y$.

The test statistic is

$$T = S - n_1(n_1 + 1)/2\,, \tag{1}$$

where S is the sum of the ranks assigned to the sample observations from population 1 (i.e., the X's), and n_1 is the sample size of population 1. In order to compute the test statistic, we need first combine the two samples and rank all sample observations from smallest to largest. If ties happen, we assign tied observations the mean of the ranks they would be assigned if there were no ties.

When both n_1 and n_2 are large, the large-sample approximation could be applied. The test statistic becomes

$$Z = (T - \frac{n_1 n_2}{2}) / \sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}. \quad (2)$$

The test statistic Z has approximately a standard normal distribution when H_0 is true. The mean of T is $n_1 n_2/2$, and its variance is $n_1 n_2 (n_1 + n_2 + 1)/12$.

2.2. Kruskal-Wallis One Way Analysis of Variance by Ranks

The Kruskal-Wallis test (Kruskal and Wallis, 1952) is a nonparametric test commonly used to test that three or more independent samples have been drawn from the same population.

The null hypothesis is H_0: The k population distribution functions are identical.

The alterative hypothesis is H_1: The k populations do not all have the same median.

The test statistic is

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} R_i^2 / n_i - 3(N+1), \quad (3)$$

where R_i is the sum of the ranks assigned to observations in the i^{th} sample, n_i is the i^{th} sample size, N is the total number of observations in the k samples. To calculate R_i, we need first rank all sample observations from the smallest to the largest and assign the tied observations the average of the rank positions they would have received as if there were no ties.

When sample sizes n_i i=1, 2,..., k and k are large, Kruskal (1952) has shown that H is distributed approximately as chi-square with k-1 degrees of freedom when the null hypotheses is true. Gabriel and Lachenbruch (1969) have researched how accurate the chi-square approximation is for small samples.

2.3. Jonckheere-Terpstra Test for Ordered Alternatives

The Jonckheere-Terpstra (JT) test, which was proposed by Terpstra(1952) and Jonckheere(1954), is a nonparametric test to test for differences in treatment effects among several independent samples if it can be assumed treatment effects are nondecreasing, the null hypothesis is H_0: $\tau_1 = \tau_2 = \ldots = \tau_k$

and the ordered alternative hypothesis is H_1: $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_k$ (At least one inequality is strict), where τ_k is k^{th} treatment effect.

The test statistic is

$$J = \sum_{i<j} U_{ij} , \tag{4}$$

where U_{ij} is the number of pairs of observations (a, b) in which X_{ia} is less than X_{jb}. In this case, X_{ia} is the a^{th} observation in the i^{th} treatment sample, $a=1,2,\ldots,n_i$ and X_{jb} is the b^{th} observation in the j^{th} treatment sample, $b=1,2,\ldots,n_j$.

When the sample sizes are large, the distribution of the JT test statistic is normal with mean $(N^2 - \sum_{i=1}^{k} n_i^2)/4$ and variance $[N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)]/72$ under the null hypothesis, where N is the combined sample size for all treatments, and n_i is the sample size of the i^{th} treatment. Therefore, the test statistic J could be standardized to Z_{JT} by (5).

$$Z_{JT} = \frac{J - [(N^2 - \sum_{i=1}^{k} n_i^2)/4]}{\sqrt{[N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)]/72}} \tag{5}$$

Odeh (1971) and Tryon and Hettmansperger (1973) further discuss the JT test. A modified JT test proposed by Tryon and Hettmansperger (1973) is described in the next section. Barlow, Bartholomew, Bremner, and Brunk (1972) and Robertson, Wright and Dykstra (1988) discuss ordered alternatives in more detail.

2.4. Modified Jonckheere-Terpstra Test

Tryon and Hettmansperger (1973) illustrated a modified version of the JT test (MJT). The JT test and MJT test are both used for testing the nondecreasing ordered alternatives, but the MJT test gives a weight for each Mann-Whitney statistic based on the order of the populations. The MJT test statistic can be written as follows:

$$T_{MJT} = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j - i) U_{ij}, \tag{6}$$

where U_{ij} is the same as defined in the JT test.

Under the null hypothesis, the distribution of T_{MJT} can also be approximated by a normal distribution. The mean of T_{MJT} is

$$E_0(T_{MJT}) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)(n_i n_j / 2) \tag{7}$$

The variance of T_{MJT} is

$$\begin{aligned} \mathrm{var}_0(T_{MJT}) &= \mathrm{var}\{\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i) u_{ij}\} \\ &= \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 \, \mathrm{var}(u_{ij}) + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{s=1}^{k-1} \sum_{t=s+1}^{k} (j-i)(t-s) \, \mathrm{cov}(u_{ij}, u_{st}) \end{aligned} \tag{8}$$

where $\mathrm{var}_0(u_{ij}) = \frac{n_i n_j (n_i + n_j + 1)}{12}$ for $1 \le i \le j \le k$

$\mathrm{Cov}_0(u_{ij}, u_{it}) = \frac{n_i n_j n_t}{12}$ for $1 \le i \le j, t \le k, j \ne t$

$\mathrm{Cov}_0(u_{ij}, u_{st}) = 0$ for all distinct i, j, s, t in $\{1, \dots, k\}$

$$\mathrm{Cov_0}(u_{ij}, u_{si}) = -\frac{n_s n_i n_j}{12} \text{ for } 1 \leq s \leq i \leq j \leq k$$

$$\mathrm{Cov_0}(u_{ij}, u_{jt}) = -\frac{n_i n_j n_t}{12} \text{ for } 1 \leq i \leq j \leq t \leq k$$

$$\mathrm{Cov_0}(u_{ij}, u_{sj}) = \frac{n_i n_j n_s}{12} \text{ for } 1 \leq i, s \leq j \leq k, i \neq s$$

According to Neuhauser, Liu, and Hothorn (1998), the MJT test generally has higher powers than the JT test.

2.5. Dietz Test

Unlike the tests listed previously which consider univariate data, the Dietz's test (1989) is designed for multivariate uncensored data. In her paper, Dietz proposes a multivariate generalization of Jonckheere's test for ordered alternatives. The test statistic is a function of coordinate-wise Jonckheere-Terpstra statistics. The null and alternative hypotheses for the Dietz test are: H_0: $F_1(x) =\ldots=F_k(x)$ for all x versus H_1: $F_1^{(g)}(x) \geq \ldots\geq F_k^{(g)}(x)$ for all x and g=1,…,p, with at least one strict inequality for at least one g.

Here $F_j(x)$ and $F_j^{(g)}(x)$ are continuous multivariate distribution functions and marginal distribution functions of x_{ij}'s which are $p\times 1$ vector of observations on p variables for the i^{th} subject in treatment j, separately, with j=1,…,k; i=1,…,n_j. In this work, we consider bivariate data such that p=2. The corresponding test statistic for bivariate data is

$$J^{(1)} = (J_1 + J_2)/\sqrt{\mathrm{var}J_1 + \mathrm{var}J_2 + 2\mathrm{cov}(J_1, J_2)}, \quad (9)$$

where J_1 and J_2 are the mean-centered Jonckheere test statistics for coordinates 1 and 2, $\mathrm{var}J_1, \mathrm{var}J_2, \mathrm{cov}(J_1, J_2)$ are variance of J_1, variance of J_2, and covariance between J_1 and J_2, respectively. The formulas to calculate J_1, J_2, $\mathrm{var}J_1, \mathrm{var}J_2, \mathrm{cov}(J_1, J_2)$ are as follows:

$$J_g = \sum_{u<v}^{k}(U_{uv}^{(g)} - n_u n_v/2), g = 1,2$$

$$U_{uv}^{(g)} = \sum_{i=1}^{n_u}\sum_{i'=1}^{n_v} \emptyset(x_{iug}, x_{i'vg})$$

$$\emptyset(a,b) = 1 \text{ if } a < b;$$

$$= \frac{1}{2} \text{ if } a = b;$$

$$= 0 \text{ if } a > b.$$

$$Var_0(J_g) = \left[N^2(2N+3) - \sum_{j=1}^{k} n_j^2\left(2n_j + 3\right)\right]/72$$

$$Cov_0(J_1, J_2) = (N+1)[N^3 - \sum_{j=1}^{k} n_j^3) - 3(N^2 - \sum_{j=1}^{k} n_j^2)]r_{12}/36(N-2) +$$

$$[3N(N^2 - \sum_{j=1}^{k} n_j^2) - 2(N^3 - \sum_{j=1}^{k} n_j^3)]\hat{\tau}_{12}/24(N-2)$$

where r_{12} is the Spearman correlation coefficient for the 1st and 2nd coordinates and $\hat{\tau}_{12}$ is the Kendall correlation coefficient for the 1st and 2nd coordinates, and the formulas for calculating them are as follows (Lehmann 1975, p.370):

$$r_{12} = 3\sum_{i,j,k}^{N} \text{sign}[(R_{j1} - R_{i1})(R_{j2} - R_{k2})]/(N^3 - N)$$

$$\hat{\tau}_{12} = 2\sum_{i<j}^{N} \text{sign}[(R_{j1} - R_{i1})(R_{j2} - R_{i2})]/N(N-1)$$

$$\text{Sign}(a) = 1 \text{ if } a > 0$$

$$= 0 \text{ if } a=0$$

$$=-1 \text{ if } a<0$$

where $\mathbf{R}_L = (R_{L1}, R_{L2})$, L=1,..., N; $N = \sum_j^k n_j$, are 2-vectors of ranks corresponding to the $\mathbf{x}_{ij}$, where the N observations for each coordinate are ranked among themselves.

Under the null hypothesis H_0, the test statistic $J^{(1)}$ is asymptotically standard normal .

2.6. Two Approaches for Multivariate Censored Data

Leconte, Moreau and Lellouch (1994) proposed a new rank test family for two samples with bivariate censored data. In order to test the global hypothesis, H_0, that the joint distribution of two event times is identical in two groups, they used transformations to reduce the two-dimensional problem to a one-dimensional problem and applied a weighted logrank test statistic for the univariate censored data. The transformations they considered were the following: the minimum function, the maximum function and the sum function.

Krogen and Magel(2000) extended the idea proposed by Leconte, Moreau and Lellouch (1994) of reducing censored bivariate data to censored univariate data and developed nonparametric tests to test for a nondecreasing ordering among k populations.

This research extends this idea in testing for nondecreasing treatment effects over k bivariate populations based on k bivariate uncensored samples. The proposed tests are introduced in Chapter 3.

3. PROPOSED K-SAMPLE TESTS FOR BIVARIATE NORMAL DATA

Continuing the example given in Chapter 1, researchers want to know if there are nondecreasing effects between the control group, the group given instructions on exercising and the group given instructions on diet and exercising. In general, to test nondecreasing treatment effects for k populations based on bivariate uncensored data, the hypotheses are as follows:

H_0: $F_1(x) = F_2(x) = \ldots = F_k(x)$ (10)

H_1: $F_1^{(g)}(x) \geq F_2^{(g)}(x) \geq \ldots \geq F_k^{(g)}(x)$, with at least one strict inequality for at least one g. where g=1, 2.

Dietz's test given in (9) of Chapter 2 is designed to test the hypotheses in (10).

The tests proposed in this chapter are based on transforming bivariate uncensored data to univariate data. The JT test and MJT test can then be performed on the univariate data. The transformations are similar to the ones by Leconte, Moreau, and Lellouch (1994) which used three pseudo-rank functions (sum, maximum, and minimum). Our proposed transformations are based on bivariate uncensored data instead of censored data.

Let $\mathbf{x}_{ij}=(x_{ij}^{(1)}, x_{ij}^{(2)})$ denote a 2×1 Vector representing the values of the variables for the i^{th} subject receiving the j^{th} treatment, j=1,2,…,k; i=1,2,…,n_i. The x_{ij}'s are assumed to be independent. Let $\mathbf{R}_{ij}= (R_{ij}^{(1)}, R_{ij}^{(2)})$ denote the rank vector corresponding to $\mathbf{x}_{ij}$. $R_{ij}^{(1)}$ will denote the rank of $x_{ij}^{(1)}$ in relation to all first component observations and $R_{ij}^{(2)}$ will denote the rank of $x_{ij}^{(2)}$ in relation to all the second component observations. Average ranks would be assigned in case of ties.

The sum, maximum and minimum functions are based on ranks of the original observations and are defined as follows:

- Bivariate sum function:

  $\text{Sum}\,(R_{ij}^{(1)}, R_{ij}^{(2)}) = R_{ij}^{(1)} + R_{ij}^{(2)}$,

- Bivariate maximum function:

  $\text{Max}\left(R_{ij}^{(1)}, R_{ij}^{(2)}\right) = R_{ij}^{(1)}, \text{if } R_{ij}^{(1)} \geq R_{ij}^{(2)} \text{ or } R_{ij}^{(2)} \text{ if } R_{ij}^{(1)} \leq R_{ij}^{(2)}$,

- Bivariate minimum function:

  $\text{Min}\left(R_{ij}^{(1)}, R_{ij}^{(2)}\right) = R_{ij}^{(1)}, \text{if } R_{ij}^{(1)} \leq R_{ij}^{(2)} \text{ or } R_{ij}^{(2)} \text{ if } R_{ij}^{(1)} \geq R_{ij}^{(2)}$,

By using the given functions, we reduce the bivariate data to univariate data. Let $G_i(x)$ i=1, 2,…, k, denote the CDF's based on the transformed data. We are now testing the following set of hypotheses:

$$H_0: G_1(x) = G_2(x) = \ldots = G_k(x) \qquad (11)$$

H_1: $G_1(x) \geq G_2(x) \geq \ldots \geq G_k(x)$, with at least one inequality is strict.

After reducing the data to univariate data, the JT and MJT tests are applied. Their test statistics are defined by (4) and (6) in Chapter 2, respectively. Since both of these test statistics have an asymptotic normal distribution if H_0 is true, their standardized versions are used. The standardized version of the JT test is given in (5) in Chapter 2, the MJT test may be standardized by subtracting the mean in (7) and dividing by the square root of the variance in (8). Let JT_{sum}, JT_{max}, JT_{min} denote the JT test statistics obtained by transforming the data using sum, maximum, and minimum functions, and MJT_{sum}, MJT_{max}, MJT_{min} the counterparts for corresponding MJT tests. The comparisons on these two tests and Dietz's test are investigated by a simulation study which is described Chapter 4.

4. DESCRIPTION OF SIMULATION STUDY

A Monte Carlo simulation study is conducted to compare the estimated powers of the Dietz's test, the Jonckheere-Terpstra test (JT test), and the modified Jonckheere-Terpstra (MJT test) test. All three tests are applied to the same samples from underlying bivariate distributions. However, the JT test and the MJT test are applied on the univariate data reduced from the bivariate data. Significance levels of the tests and powers are estimated by counting the number of times each tests rejects the null hypothesis divided by the number of simulations. In this study, 10,000 simulations were conducted for every situation considered.

For this simulation study, the only type of underlying distribution considered was the bivariate normal. The probability density function of a random vector Y′= [Y1, Y2] which has a bivariate normal distribution is given by $f(y_1, y_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\,(-1/2(1-\rho^2)[\frac{(y_1-\mu_1)^2}{\sigma_1^2} + \frac{(y_2-\mu_2)^2}{\sigma_2^2} - 2\rho(y_1-\mu_1)(y_2-\mu_2)/\sigma_1\sigma_2\,]$, where ρ is the correlation coefficient between y_1 and y_2 ,$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim (\mu, \sum)$ where the mean vector μ is $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and the variance-covariance matrix $\sum$ is $\sum = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$. Five parameters are needed in determining a bivariate normal distribution. In generating random samples from a bivariate normal distribution, we used a method given by Han (2006). For this method, we need to obtain the formula for the Cholesky square root of the variance-covariance matrix $\sum$=LL′, where $L = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{\sigma_2^2\,(1-\rho^2)} \end{pmatrix}$.

The procedure for generating a bivariate normal random variate is as follows:

1. Generate a pair of independent standard normal variates, $\mathbf{z}' = (z_1, z_2)$.
2. The desired bivariate random variate is obtained by taking $\boldsymbol{\mu}+\mathbf{Lz}$. In other words, y_1 and y_2 can be obtained from these two transformations of z: $y_1 = \mu_1 + \sigma_1 z_1$ and $y_2 = u_2 + (\rho\sigma_2 \times z_1) + \left(\sqrt{\sigma_2^2\ (1-\rho^2)} \times z_2\right)$. Therefore, in addition to these five parameters μ_1, μ_2, σ_1, σ_2, ρ, we need to generate a pair of independent standard normal variates z_1 and z_2. In the SAS code, we use "rannor" function to generate these two variables, and a macro function to generate k populations simultaneously. Next "proc rank" procedure is used to obtain the ranks of these two bivariate variables, separately, and then apply "sum", "max" and "min" functions on them to obtain univariate data. Part of the SAS codes is attached in the Appendix. All codes are implemented using SAS 9.2®.

In this study, two values of ρ are considered: ρ=0.4 and ρ=0.8.

The number of populations, k, considered for comparison at one time in this study is 3, 4, and 5, with equal variances imposed within the k populations. In comparing JT_{sum}, JT_{max}, JT_{min}, MJT_{sum}, MJT_{max}, MJT_{min} and the Dietz test statistics, various location shifts and both equal and unequal sample sizes are considered. The significance level of 0.05 is used throughout all simulations. Significance levels are estimated in each case. The following types of location shifts considered for each number of populations are given below.

The following mean shift combinations are considered when k=3:

a) The first (k-1) populations are equal, and the location shifts from the (k-1) population to the k population are identical for both parameters: for example, (1, 1), (1, 1), and (2.5, 2.5).

b) There is equal spacing for both parameters between each shift and the location shifts for the first and second parameters are equal: for example, (1, 1), (2, 2), and (3, 3).

c) There is equal spacing for both parameters shift, but the location shifts are unequal between the first parameter and the second parameter: for example, (1, 1), (2, 1.5), and (3, 2).

d) The first parameters are equal for all the populations, and there is equal spacing between the second parameters: for example, (1, 1), (1, 3), and (1, 5).

e) The first parameters are equal for all the populations, and there is unequal spacing between the second parameters: for example,(1,1),(1,2), and (1,4).

f) There is equal spacing between the first parameters and unequal spacing between the second parameters: for example, (1, 1), (2, 1.5), and (3, 2.5).

g) There is unequal spacing between the first parameters and unequal spacing between the second parameters: for example, (1, 1), (2, 2.5), and (2.5, 2.5).

h) There is equal spacing between the first parameters and the second parameters are equal: for example, (1, 1), (2, 1), and (3, 1).

i) There is unequal spacing between the first parameters and the second parameters are equal: for example, (1, 1), (1.5, 1), and (3, 1).

For each of above simulation scenarios, both equal and unequal sample sizes (n_1, n_2, n_3) are taken into account. To be specific, we consider two combinations of equal sample sizes: for example, (5, 5, 5) and (10, 10, 10), and four combinations of unequal sample sizes: for example, (10, 5, 5), (5, 5, 10), (5, 10, 10), and (5, 10, 5).

The following types of mean shift combinations are considered when k=4:

a) There is equal spacing between the first and second parameters and this spacing is equal: for example, (1, 1), (1.5, 1.5), (2, 2), and (2.5, 2.5).

b) The first parameters are equal, and there is equal spacing between the second parameters: for example, (1, 1), (1, 2), (1, 3), and (1, 4).

c) There is equal spacing between the first parameters and between the second parameters, but this spacing is not the same: for example, (1, 1), (1.5, 2), (2, 3), and (2.5, 4).

d) The first (k-1) populations are equal, and the location shift from the (k-1) population to the k population is identical for both parameters: for example, (1, 1), (1, 1), (1, 1), and (3, 3).

e) The first (k-2) populations and the last two populations are equal, and the location shift from the (k-2) population to the (k-1) population is identical for both parameters: for example, (1, 1), (1, 1), (3, 3), and (3, 3).

f) The last three populations are equal, and the location shift from the first population to the second population is identical for both parameters: for example, (1, 1), (2, 2), (2, 2), and (2, 2).

g) There is unequal spacing between the first parameters and the second parameters: for example,(1, 1), (1.5, 3),(2, 3), and (2, 3.5).

h) There is unequal spacing between the first parameters and second parameters are equal: for example, (1, 2), (1.5, 2), (2.5, 2), and (3, 2).

i) The first and second parameters are equal, but there is unequal spacing: for example, (1, 1), (1.5, 1.5), (2.5, 2.5), and (3, 3).

Similar to k=3, for each scenario, equal and unequal sample sizes (n_1, n_2, n_3, n_4) are taken into account. Examples of equal sample sizes are (5, 5, 5, 5) and (10, 10, 10, 10), and combinations of unequal sample sizes include (5, 10, 10, 5), (5, 5, 10, 10), (5, 5, 5, 10), and (5, 5, 10, 5).

The following types of mean shift combinations are considered when k=5:

a) The first (k-2) populations are equal and the last two populations are equal, and the location shift from the (k-2) population to the (k-1) population is identical for both parameters: for example, (1, 1), (1, 1), (1, 1), (2, 2), and (2, 2).

b) The first parameters are equal, and there is equal spacing between the second parameters: for example, (1, 1), (1, 1.5), (1, 2), (1, 2.5), and (1, 3).

c) The first (k-1) populations are equal, and the location shift from the (k-1) population to the k population is identical for both parameters: for example, (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), and (2, 2).

d) There is equal spacing between the first parameters and between the second parameters, but the spacing is not the same for the first parameters and the second parameters: for example, (1, 1), (1.25, 1.5), (1.5, 2), (1.75, 2.5), and (2, 3).

e) There is unequal spacing between the first parameters and equal spacing between the second parameters: for example, (1, 1), (1, 1.5), (1, 2), (2, 2.5), and (2, 3).

f) The first three populations have unequal spacing between the parameters and the last two populations have equal spacing between the parameters: for example, (1, 1), (1.2, 1.7), (1.7, 1.7), (2.3, 2.9), and (3, 3.6).

g) There is unequal spacing between the first parameters and the second parameters: for example, (1, 1), (1.5, 1.2), (1.8, 2.4), (2.4, 2.5), and (3, 3).

h) There is unequal spacing among the first parameters and equal spacing among the second parameters: for example, (1, 1), (1.75, 1), (2, 1), (2.25, 1), and (3, 1).

i) The first parameters are equal and there is unequal spacing between the second parameters: for example, (1, 1), (1, 1.4), (1, 2), (1, 3), and (1, 3)

j) The last four populations are equal, and the location shift from the first population to the second population is identical for both parameters: for example, (1, 1), (2, 2), (2, 2), (2, 2), and (2, 2).

Again, for each case, equal and unequal sample sizes (n_1, n_2, n_3, n_4, n_5) are taken into account: For example, two combinations of equal sample are (5, 5, 5, 5, 5) and (10, 10, 10, 10, 10), and four combinations of unequal sample sizes are (5, 5, 5, 10, 10), (5, 5, 10, 10, 10) , (5, 10, 10, 10, 10) , and (10, 10, 10, 10, 5).

Results of the simulation study may be found in Chapter 5. Conclusions are given in Chapter 6.

5. SIMULATION RESULTS

In this chapter, the simulation results for all cases are presented. Powers of these test statistics, which we denote by JT_{sum}, JT_{max}, JT_{min}, MJT_{sum}, MJT_{max}, and MJT_{min} and DIETZ, are estimated for different location shifts and both equal and unequal sample sizes as mentioned in Chapter 4. The k populations are always bivariate normal distributions with equal variances. We considered correlations of ρ=0.4 and ρ=0.8.Significance levels are estimated in all cases to make sure the significance levels are maintained. The powers are then estimated and compared for all of the tests.

In each of the following tables which show simulation results, "case" in the first column means different means combinations which are described below. The remaining columns contain the estimated powers of the seven test statistics.

For the k=3, cases 1 to 10 represent the following ten different means combinations:

Case 1=(1,1)(1,1)(1,1);

Case 2=(1,1)(1,1)(2.5,2.5);

Case 3=(1,1)(2,2)(3,3);

Case 4=(1,1)(2,1.5)(3,2);

Case 5=(1,1)(1,3)(1,5) ;

Case 6=(1,1)(1,2)(1,4);

Case 7=(1,1)(2,1.5)(3,2.5);

Case 8=(1,1)(2,2.5)(2.5,2.5);

Case 9=(1,1)(2,1)(3,1);

Case 10=(1,1)(1.5,1)(3,1).

For the k=4, cases 1 to 10 represent the following ten different means combinations:

Case 1=(1,1)(1,1)(1,1)(1,1);

Case 2=(1, 1) (1.5,1.5)(2,2) (2.5,2.5);

Case 3=(1,1)(1,2)(1,3)(1,4) ;

Case 4=(1,1)(1.5,2)(2,3) (2.5,4);

Case 5=(1,1)(1,1)(1,1)(3,3);

Case 6=(1,1)(1,1)(3,3)(3,3);

Case 7=(1,1)(2,2)(2,2)(2,2) ;

Case 8=(1,1)(1.5,3)(2,3)(2,3.5);

Case 9=(1,2)(1.5,2)(2.5,2)(3,2);

Case 10=(1,1)(1.5,1.5)(2.5,2.5)(3,3).

For the k=5, cases 1 to 11 represent the following eleven different means combinations:

Case 1=(1,1)(1,1)(1,1)(1,1)(1,1);

Case 2=(1,1)(1,1)(1,1)(2,2)(2,2);

Case 3=(1,1)(1,1.5)(1,2)(1,2.5)(1,3);

Case 4=(1,1)(1,1)(1,1)(1,1)(1,1)(2,2);

Case 5=(1,1)(1.25,1.5)(1.5,2)(1.75,2.5)(2,3);

Case 6=(1,1)(1,1.5)(1,2)(2,2.5)(2,3);

Case 7=(1,1)(1.2,1.7)(1.7,1.7)(2.3,2.9)(3,3.6);

Case 8=(1,1)(1.5,1.2)(1.8,2.4)(2.4,2.5)(3,3);

Case 9=(1,1)(1.75,1)(2,1)(2.25,1)(3,1);

Case 10=(1,1)(1,1.4)(1,2)(1,3)(1,3);

Case 11=(1,1)(2,2) (2,2) (2,2) (2,2).

We start with looking at the estimated significant levels. This corresponds to the estimated powers for case 1 in each table. As we expected, all significant levels of the three tests hold at 5% when the number of populations, k, is 3, 4, 5, no matter what the sample size and the correlation are.

For three populations and equal sample size 5 and 10, Dietz's test has the highest power among these tests no matter the correlation ρ equal to 0.4 or 0.8 except for two special cases, and MJT_{sum} has higher power than others in most cases, sum transformation has higher power than other two transformations in most cases. It is also noted that the estimated powers of the MJT_{sum} test are close to the powers of Dietz's test (Tables 1-4).

In the case of three populations and different unequal sample size cases, Dietz's test still has the highest power. The MJT_{sum} generally has highest powers among the univariate test with the sum transformation having higher power than other two transformations in most cases (Tables 5-12).

For four populations and equal sample sizes 5 and 10, Dietz's test has the highest power among these tests no matter the correlation ρ equal to 0.4 or 0.8 except for five special cases. The MJT_{sum} generally has the second highest power among the test statistics (Tables 13-16).

In the case of four populations and unequal sample size combinations, the Dietz's test still has the highest power among them in most of these cases. The MJT_{sum} has higher power than others (besides Dietz's test) in most all cases. The sum transformation is the best transformations to use (Tables 17- 24).

For five populations and equal sample sizes of 5 and 10, Dietz's test has the highest power among these tests no matter whether the correlation ρ is equal to 0.4 or 0.8 in most cases. MJT_{sum} generally has the second highest powers (Tables 25-28).The same thing was found for the unequal sample size combinations (Tables 29-36).

Table 1. Estimated Powers for Equal Sample Size Case: k=3; Bivariate Normal with ρ=0.4, n=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0514	0.0492	0.0512	0.0484	0.0474	0.0486	0.0506
2	0.3426	0.3116	0.3226	0.3264	0.2985	0.3065	0.3500
3	0.5259	0.4814	0.4800	0.5114	0.4700	0.4709	0.5373
4	0.3440	0.3183	0.3183	0.3350	0.3091	0.3051	0.3568
5	0.4282	0.3615	0.3562	0.4143	0.3588	0.3534	0.4791
6	0.3034	0.2859	0.2403	0.2908	0.2746	0.2332	0.3353
7	0.4346	0.3947	0.4030	0.4217	0.3813	0.3913	0.4487
8	0.5199	0.4816	0.4807	0.5022	0.4672	0.4681	0.5379
9	0.1949	0.1838	0.1765	0.1879	0.1766	0.1704	0.2066
10	0.2016	0.1919	0.1681	0.1913	0.1830	0.1601	0.2123

Table 2. Estimated Powers for Equal Sample Size Case: k=3; Bivariate Normal with ρ=0.4, n=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0484	0.0480	0.0492	0.0487	0.0493	0.0493	0.0505
2	0.5732	0.5346	0.5323	0.5684	0.5279	0.5248	0.5823
3	0.8049	0.7634	0.7652	0.8061	0.7632	0.7640	0.8190
4	0.5865	0.5392	0.5411	0.5840	0.5406	0.5401	0.6014
5	0.7356	0.6451	0.6461	0.7305	0.6498	0.6520	0.7752
6	0.5455	0.5164	0.4396	0.5404	0.5157	0.4395	0.5736
7	0.6997	0.6528	0.6542	0.6967	0.6508	0.6562	0.7139
8	0.7996	0.7537	0.7473	0.7971	0.7511	0.7473	0.8139
9	0.3231	0.2870	0.2869	0.3204	0.2854	0.2872	0.3379
10	0.3296	0.3086	0.2728	0.3262	0.3076	0.2731	0.3411

Table 3. Estimated Powers for Equal Sample Size Case: k=3; Bivariate Normal with ρ=0.8, n=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0469	0.0485	0.0480	0.0447	0.0464	0.0471	0.0492
2	0.2989	0.2925	0.2930	0.2872	0.2815	0.2787	0.3021
3	0.4485	0.4383	0.4360	0.4370	0.4246	0.4238	0.4583
4	0.3090	0.3008	0.3010	0.2978	0.2900	0.2895	0.3165
5	0.3629	0.3252	0.3285	0.3470	0.3193	0.3263	0.4080
6	0.2618	0.2687	0.2176	0.2510	0.2577	0.2115	0.2851
7	0.3811	0.3661	0.3739	0.3686	0.3540	0.3613	0.3892
8	0.4381	0.4271	0.4273	0.4257	0.4136	0.4129	0.4491
9	0.1743	0.1643	0.1677	0.1651	0.1597	0.1605	0.1839
10	0.1741	0.1805	0.1516	0.1636	0.1719	0.1450	0.1816

Table 4. Estimated Powers for Equal Sample Size Case: k=3; Bivariate Normal with ρ=0.8, n=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0473	0.0479	0.0457	0.0483	0.0468	0.0462	0.0481
2	0.5022	0.4884	0.4847	0.4953	0.4811	0.4792	0.5019
3	0.7114	0.7011	0.6982	0.7125	0.6978	0.6996	0.7195
4	0.5056	0.4930	0.4913	0.5039	0.4913	0.4874	0.5146
5	0.6407	0.5870	0.5839	0.6337	0.5895	0.5900	0.6781
6	0.4529	0.4738	0.3717	0.4472	0.4749	0.3738	0.4765
7	0.6187	0.5973	0.6134	0.6172	0.5955	0.6117	0.6248
8	0.7186	0.6983	0.7035	0.7158	0.6947	0.7024	0.7249
9	0.2841	0.2662	0.2682	0.2815	0.2662	0.2644	0.2964
10	0.2768	0.2960	0.2252	0.2729	0.2961	0.2266	0.2823

Table 5. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ=0.4, n_1=5, n_2=n_3=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0480	0.0497	0.0513	0.0476	0.0508	0.0485	0.0492
2	0.5477	0.5040	0.5098	0.5729	0.5200	0.5280	0.5908
3	0.6621	0.6130	0.6087	0.6520	0.6057	0.6007	0.6722
4	0.4619	0.4197	0.4216	0.4521	0.4132	0.4142	0.4751
5	0.5793	0.4568	0.5621	0.5654	0.4628	0.5402	0.6236
6	0.4534	0.3932	0.4028	0.4546	0.4007	0.3967	0.4905
7	0.5811	0.5313	0.5395	0.5773	0.5274	0.5368	0.5936
8	0.6321	0.5752	0.5855	0.6088	0.5541	0.5637	0.6306
9	0.2542	0.2242	0.2410	0.2497	0.2223	0.2319	0.2679
10	0.2795	0.2573	0.2482	0.2820	0.2606	0.2490	0.3000

Table 6. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.4, n_1=5, n_2=10, n_3=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0504	0.0496	0.0503	0.0485	0.0477	0.0487	0.0507
2	0.3495	0.3211	0.3255	0.3355	0.3108	0.3172	0.3570
3	0.5309	0.4887	0.4892	0.5240	0.4817	0.4840	0.5465
4	0.3556	0.3189	0.3285	0.3503	0.3171	0.3239	0.3694
5	0.4595	0.3972	0.4127	0.4483	0.3971	0.4126	0.5007
6	0.3215	0.3120	0.2591	0.3148	0.3079	0.2563	0.3469
7	0.4423	0.4078	0.4075	0.4371	0.4012	0.4002	0.4566
8	0.5243	0.4774	0.4813	0.5186	0.4734	0.4761	0.5409
9	0.2093	0.1960	0.1914	0.2044	0.1921	0.1864	0.2188
10	0.1978	0.2022	0.1642	0.1925	0.1992	0.1614	0.2192

Table 7. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.4, n_1=10, n_2=n_3=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0518	0.0544	0.0525	0.0509	0.0540	0.0527	0.0531
2	0.3791	0.3434	0.3470	0.3348	0.3090	0.3079	0.3542
3	0.6531	0.5995	0.6024	0.6449	0.5904	0.5934	0.6616
4	0.4379	0.4066	0.3997	0.4330	0.4005	0.3954	0.4534
5	0.5496	0.5664	0.3827	0.5363	0.5485	0.3910	0.5947
6	0.3710	0.3976	0.2634	0.3548	0.3766	0.2604	0.3895
7	0.5408	0.4985	0.4968	0.5257	0.4861	0.4823	0.5487
8	0.6665	0.6220	0.6065	0.6663	0.6211	0.6086	0.6894
9	0.2507	0.2447	0.2063	0.2491	0.2396	0.2080	0.2675
10	0.2314	0.2345	0.1849	0.2190	0.2242	0.1790	0.2355

Table 8. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.4, n_1=n_2=5, n_3=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0472	0.0448	0.0496	0.0458	0.0457	0.0508	0.0487
2	0.5033	0.4595	0.4593	0.5221	0.4790	0.4808	0.5417
3	0.6433	0.5989	0.5937	0.6372	0.5949	0.5890	0.6603
4	0.4452	0.3976	0.4142	0.4394	0.3923	0.4067	0.4622
5	0.5519	0.3842	0.5699	0.5446	0.3956	0.5583	0.6013
6	0.4156	0.3259	0.3849	0.4187	0.3345	0.3893	0.4549
7	0.5587	0.5089	0.5152	0.5571	0.5056	0.5161	0.5779
8	0.6198	0.5645	0.5744	0.6001	0.5496	0.5544	0.6261
9	0.2378	0.2001	0.2337	0.2371	0.2005	0.2294	0.2540
10	0.2535	0.2191	0.2381	0.2586	0.2239	0.2422	0.2806

Table 9. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.8, n_1=5, n_2=10, n_3=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0511	0.0509	0.0499	0.0493	0.0494	0.0478	0.0505
2	0.3041	0.2980	0.3000	0.2940	0.2903	0.2918	0.3060
3	0.4685	0.4583	0.4537	0.4621	0.4516	0.4510	0.4759
4	0.3150	0.3072	0.3089	0.3080	0.3012	0.3032	0.3224
5	0.3853	0.3581	0.3612	0.3745	0.3545	0.3620	0.4214
6	0.2776	0.3104	0.2231	0.2707	0.3049	0.2214	0.2987
7	0.3897	0.369	0.3853	0.3819	0.3639	0.3782	0.3932
8	0.4518	0.4413	0.4397	0.4419	0.4354	0.4317	0.4608
9	0.1811	0.1738	0.1743	0.1756	0.1715	0.1737	0.1887
10	0.1775	0.1979	0.1457	0.1734	0.1947	0.1439	0.1836

Table 10. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.8, n_1=n_2=5, n_3=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0505	0.0495	0.0491	0.0498	0.0506	0.0498	0.0520
2	0.3254	0.3139	0.3161	0.2913	0.2834	0.2820	0.3009
3	0.5598	0.5494	0.5457	0.5547	0.5445	0.5421	0.5717
4	0.3672	0.3679	0.3520	0.3633	0.3623	0.3485	0.3778
5	0.4742	0.5561	0.3223	0.4679	0.5337	0.3364	0.5209
6	0.3206	0.3972	0.2176	0.3068	0.3753	0.2234	0.3360
7	0.4585	0.4496	0.4528	0.4490	0.4380	0.4400	0.4647
8	0.5893	0.5777	0.5641	0.5932	0.5789	0.5699	0.6076
9	0.2106	0.2242	0.1782	0.2079	0.2196	0.1788	0.2199
10	0.1914	0.2247	0.1483	0.1803	0.2128	0.1459	0.1942

Table 11. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.8, n_1=10, n_2=n_3=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0523	0.0489	0.0516	0.0518	0.0491	0.0501	0.0516
2	0.4276	0.4168	0.4202	0.4492	0.4375	0.4400	0.4637
3	0.5751	0.5627	0.5603	0.5668	0.5555	0.5539	0.5825
4	0.3869	0.3683	0.3851	0.3790	0.3650	0.3794	0.3927
5	0.4621	0.3155	0.5447	0.4559	0.3269	0.5290	0.5094
6	0.3544	0.2811	0.3651	0.3587	0.2906	0.3649	0.3874
7	0.4794	0.4632	0.4730	0.4770	0.4639	0.4715	0.4899
8	0.5452	0.5216	0.5448	0.5295	0.5109	0.5254	0.5495
9	0.2127	0.1776	0.2337	0.2096	0.1797	0.2290	0.2263
10	0.2278	0.2015	0.2264	0.2304	0.2059	0.2286	0.2445

Table 12. Estimated Powers for Unequal Sample Size Case: k=3; Bivariate Normal with ρ= 0.8, n_1=5, n_2=n_3=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0466	0.0473	0.0474	0.0454	0.0454	0.0470	0.0468
2	0.4741	0.4680	0.4569	0.4923	0.4869	0.4783	0.5067
3	0.5829	0.5706	0.5688	0.5748	0.5612	0.5573	0.5867
4	0.3984	0.3793	0.3920	0.3909	0.3749	0.3809	0.4045
5	0.5018	0.3954	0.5395	0.4894	0.4069	0.5134	0.5397
6	0.3822	0.3521	0.3525	0.3785	0.3636	0.3469	0.4125
7	0.5042	0.4880	0.4994	0.5006	0.4838	0.4927	0.5113
8	0.5414	0.5222	0.5326	0.5199	0.5062	0.5092	0.5374
9	0.2186	0.1922	0.2251	0.2138	0.1930	0.2160	0.2274
10	0.2415	0.2317	0.2170	0.2440	0.2377	0.2154	0.2588

Table 13. Estimated Powers for Equal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, n_1=n_2=n_3=n_4=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0537	0.0511	0.0539	0.0548	0.0528	0.0546	0.0543
2	0.3927	0.3585	0.3630	0.3963	0.3612	0.3650	0.4064
3	0.3529	0.3098	0.3172	0.3533	0.3161	0.3265	0.3830
4	0.6448	0.5857	0.5935	0.6480	0.5881	0.5956	0.6645
5	0.4451	0.4126	0.4166	0.4380	0.4110	0.4125	0.4541
6	0.6900	0.6467	0.6452	0.6819	0.6400	0.6381	0.6986
7	0.1983	0.1867	0.1833	0.2002	0.1896	0.1859	0.2045
8	0.4160	0.3632	0.4014	0.4218	0.3702	0.4062	0.4354
9	0.2332	0.2172	0.2056	0.2311	0.2190	0.2093	0.2464
10	0.6064	0.5623	0.5616	0.6082	0.5672	0.5615	0.6261

Table 14. Estimated Powers for Equal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, n_1=n_2=n_3=n_4=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0493	0.0513	0.0497	0.0491	0.0503	0.0488	0.0489
2	0.6367	0.5944	0.5866	0.6320	0.5891	0.5826	0.6464
3	0.5971	0.5361	0.5258	0.5897	0.5343	0.5267	0.6228
4	0.9083	0.8734	0.8718	0.9079	0.8711	0.8711	0.9204
5	0.7396	0.6956	0.6951	0.7263	0.6819	0.6839	0.7473
6	0.9361	0.9042	0.9092	0.9303	0.8971	0.9014	0.9365
7	0.3184	0.2934	0.2952	0.3135	0.2884	0.2908	0.3220
8	0.6969	0.6130	0.6730	0.6901	0.6081	0.6645	0.7094
9	0.3823	0.3436	0.3421	0.3785	0.3420	0.3400	0.3967
10	0.8756	0.8405	0.8342	0.8706	0.8372	0.8331	0.8848

Table 15. Estimated Powers for Equal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, $n_1=n_2=n_3=n_4=5$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0459	0.0476	0.0493	0.0484	0.0482	0.0515	0.0465
2	0.3283	0.3219	0.3200	0.3314	0.3282	0.3258	0.3342
3	0.2961	0.2800	0.2721	0.2981	0.2876	0.2809	0.3224
4	0.5643	0.5430	0.5451	0.5657	0.5499	0.5496	0.5772
5	0.4018	0.3926	0.3947	0.4002	0.3900	0.3920	0.4057
6	0.6053	0.5940	0.5934	0.5972	0.5869	0.5873	0.6032
7	0.1738	0.1679	0.1721	0.1743	0.1713	0.1747	0.1766
8	0.3634	0.3212	0.3870	0.3677	0.3306	0.3922	0.3749
9	0.2022	0.1936	0.1891	0.2070	0.1991	0.1946	0.2132
10	0.5245	0.5096	0.5121	0.5238	0.5118	0.5094	0.5303

Table 16. Estimated Powers for Equal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, $n_1=n_2=n_3=n_4=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0494	0.0508	0.0468	0.0486	0.0489	0.0468	0.0475
2	0.5548	0.5444	0.5453	0.5502	0.5391	0.5393	0.5596
3	0.5169	0.4832	0.4837	0.5093	0.4811	0.4831	0.5399
4	0.8400	0.8248	0.8282	0.8391	0.8226	0.8288	0.8503
5	0.6543	0.6347	0.6415	0.6390	0.6232	0.6291	0.6488
6	0.8801	0.8688	0.8694	0.8729	0.8594	0.8622	0.8784
7	0.2860	0.2816	0.2804	0.2808	0.2762	0.2773	0.2853
8	0.6127	0.5491	0.6353	0.6022	0.5431	0.6302	0.6146
9	0.3349	0.3175	0.3202	0.3304	0.3174	0.3172	0.3428
10	0.8047	0.7904	0.7937	0.8008	0.7866	0.7866	0.8090

Table 17. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, $n_1=5$, $n_2=n_3=10$, $n_4=5$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0507	0.0501	0.0519	0.0506	0.0515	0.0512	0.0492
2	0.4397	0.4090	0.4032	0.4386	0.4068	0.4005	0.4488
3	0.4019	0.3639	0.3624	0.4011	0.3620	0.3617	0.4273
4	0.6995	0.6454	0.6465	0.6942	0.6415	0.6436	0.7118
5	0.4280	0.4003	0.3908	0.3980	0.3719	0.3636	0.4120
6	0.8420	0.7961	0.7994	0.8602	0.8171	0.8203	0.8711
7	0.1891	0.1767	0.1800	0.1829	0.1696	0.1743	0.1871
8	0.4311	0.3696	0.4179	0.4175	0.3612	0.4033	0.4259
9	0.2693	0.2493	0.2402	0.2720	0.2510	0.2438	0.2813
10	0.6938	0.6471	0.6425	0.6995	0.6544	0.6481	0.7130

Table 18. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, $n_1=n_2=5$, $n_3=n_4=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0499	0.0486	0.0497	0.0507	0.0510	0.0507	0.0490
2	0.4934	0.4556	0.4577	0.4882	0.4486	0.4502	0.4999
3	0.4500	0.3545	0.4521	0.4434	0.3637	0.4307	0.4721
4	0.7783	0.7154	0.7351	0.7709	0.7087	0.7256	0.7875
5	0.6933	0.6416	0.6455	0.7707	0.7210	0.7239	0.7798
6	0.8039	0.7615	0.7601	0.7625	0.7225	0.7208	0.7801
7	0.2137	0.1984	0.2023	0.1850	0.1735	0.1809	0.1898
8	0.4925	0.4288	0.4815	0.4493	0.3936	0.4323	0.4637
9	0.2873	0.2414	0.2831	0.2820	0.2415	0.2757	0.2938
10	0.7336	0.6859	0.6879	0.7167	0.6733	0.6680	0.7329

Table 19. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, $n_1=n_2=n_3=5$, $n_4=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0534	0.0530	0.0538	0.0557	0.0532	0.0568	0.0539
2	0.4878	0.4499	0.4443	0.4895	0.4512	0.4427	0.5001
3	0.4491	0.3558	0.4311	0.4490	0.3672	0.4287	0.4799
4	0.7707	0.7176	0.7327	0.7716	0.7188	0.7276	0.7871
5	0.7164	0.6686	0.6644	0.7565	0.7108	0.7111	0.7694
6	0.7809	0.7393	0.7335	0.7535	0.7106	0.7080	0.7659
7	0.2031	0.1924	0.1932	0.1950	0.1839	0.1858	0.1982
8	0.4833	0.4152	0.4671	0.4625	0.4038	0.4519	0.4748
9	0.2817	0.2331	0.2714	0.2809	0.2369	0.2698	0.2915
10	0.7224	0.6757	0.6731	0.7142	0.6676	0.6631	0.7217

Table 20. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.4, $n_1=n_2=5$, $n_3=10$, $n_4=5$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0564	0.0536	0.0534	0.0578	0.0553	0.0536	0.0555
2	0.4099	0.3809	0.3821	0.4104	0.3798	0.3853	0.4212
3	0.3702	0.3092	0.3486	0.3691	0.3182	0.3470	0.3975
4	0.6693	0.6109	0.6264	0.6686	0.6147	0.6253	0.6866
5	0.3975	0.3664	0.3701	0.4380	0.3983	0.4000	0.4477
6	0.7584	0.7214	0.7147	0.7559	0.7178	0.7081	0.7685
7	0.2208	0.2063	0.2077	0.2033	0.1902	0.1889	0.2072
8	0.4483	0.3923	0.4292	0.4203	0.3742	0.4027	0.4305
9	0.2489	0.2110	0.2333	0.2492	0.2166	0.2372	0.2562
10	0.6399	0.5943	0.5940	0.6382	0.5903	0.5958	0.6526

Table 21. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, n_1=5, n_2=n_3=10, n_4=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0523	0.0530	0.0502	0.0533	0.0526	0.0505	0.0523
2	0.3587	0.3547	0.3498	0.3588	0.3530	0.3491	0.3622
3	0.3395	0.3251	0.3247	0.3396	0.3253	0.3266	0.3574
4	0.6231	0.6014	0.6053	0.6173	0.5977	0.6028	0.6289
5	0.3721	0.3650	0.3638	0.3503	0.3441	0.3442	0.3572
6	0.7588	0.7466	0.7492	0.7829	0.7689	0.7696	0.7903
7	0.1658	0.1638	0.1647	0.1590	0.1573	0.1595	0.1619
8	0.3745	0.3258	0.4022	0.3602	0.3191	0.3901	0.3677
9	0.2275	0.2171	0.2196	0.2282	0.2222	0.2238	0.2373
10	0.5984	0.5879	0.5830	0.6021	0.5903	0.5917	0.6101

Table 22. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, n_1=n_2=5, n_3=10, n_4=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0507	0.0518	0.0478	0.0520	0.0532	0.0484	0.0499
2	0.3521	0.3495	0.3396	0.3564	0.3479	0.3420	0.3554
3	0.3184	0.2711	0.3333	0.3197	0.2837	0.3291	0.3396
4	0.5953	0.5669	0.5887	0.5923	0.5698	0.5859	0.6019
5	0.3401	0.3341	0.3336	0.3688	0.3641	0.3635	0.3716
6	0.6777	0.6665	0.6643	0.6765	0.6651	0.6634	0.6829
7	0.1796	0.1806	0.1794	0.1679	0.1654	0.1671	0.1679
8	0.3884	0.3435	0.4079	0.3662	0.3276	0.3836	0.3712
9	0.2167	0.1947	0.2221	0.2189	0.1981	0.2225	0.2251
10	0.5592	0.5454	0.5439	0.5580	0.5465	0.5444	0.5605

Table 23. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, n_1=n_2=5, n_3=n_4=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0517	0.0523	0.0516	0.0534	0.0522	0.0525	0.0515
2	0.4234	0.4162	0.4159	0.4184	0.4109	0.4094	0.4211
3	0.3842	0.3085	0.4286	0.3785	0.3211	0.4042	0.4014
4	0.6885	0.6544	0.6904	0.6827	0.6542	0.6806	0.6986
5	0.6076	0.5903	0.5978	0.6820	0.6684	0.6691	0.6872
6	0.7210	0.7086	0.7055	0.6833	0.6704	0.6631	0.6902
7	0.1863	0.1831	0.1824	0.1656	0.1660	0.1629	0.1643
8	0.4275	0.3769	0.4530	0.3872	0.3510	0.4060	0.3953
9	0.2502	0.2056	0.2788	0.2464	0.2073	0.2686	0.2556
10	0.6497	0.6369	0.6388	0.6357	0.6250	0.6259	0.6392

Table 24. Estimated Powers for Unequal Sample Size Case: k=4; Bivariate Normal with ρ= 0.8, $n_1=n_2=n_3=5$, $n_4=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0537	0.0542	0.0506	0.0527	0.0539	0.0533	0.0523
2	0.4180	0.4078	0.4036	0.4204	0.4090	0.4085	0.4228
3	0.3714	0.2938	0.4012	0.3729	0.3090	0.3940	0.3934
4	0.6811	0.6469	0.6771	0.6804	0.6504	0.6757	0.6901
5	0.6277	0.6135	0.6162	0.6716	0.6590	0.6625	0.6756
6	0.6983	0.6857	0.6898	0.6724	0.6602	0.6612	0.6791
7	0.1809	0.1790	0.1787	0.1715	0.1716	0.1694	0.1711
8	0.4127	0.3616	0.4419	0.3962	0.3497	0.4261	0.4036
9	0.2394	0.2059	0.2529	0.2397	0.2096	0.2495	0.2489
10	0.6360	0.6205	0.6237	0.6313	0.6175	0.6194	0.6364

Table 25. Estimated Powers for Equal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, $n_1=n_2=n_3=n_4=n_5=5$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0520	0.0521	0.0522	0.0517	0.0527	0.0533	0.0515
2	0.3121	0.2898	0.2881	0.3111	0.2874	0.2869	0.3175
3	0.2317	0.2085	0.2091	0.2302	0.2105	0.2102	0.2445
4	0.1822	0.1746	0.1710	0.1840	0.1757	0.1727	0.1852
5	0.4273	0.3889	0.3902	0.4292	0.3898	0.3917	0.4410
6	0.4759	0.4262	0.4384	0.4718	0.4280	0.4365	0.4889
7	0.7276	0.6723	0.6740	0.7232	0.6743	0.6737	0.7391
8	0.6340	0.5863	0.5866	0.6355	0.5899	0.5892	0.6470
9	0.2094	0.1925	0.1920	0.2116	0.1926	0.1938	0.2196
10	0.4347	0.3943	0.3950	0.4355	0.3955	0.3987	0.4467
11	0.1850	0.1754	0.1747	0.1847	0.1752	0.1756	0.1885

Table 26. Estimated Powers for Equal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, n_1=n_2=n_3=n_4=n_5=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0500	0.0544	0.0485	0.0493	0.0517	0.0489	0.0505
2	0.5126	0.4677	0.4759	0.5056	0.4622	0.4696	0.5176
3	0.3958	0.3558	0.3514	0.3911	0.3539	0.3491	0.4114
4	0.2910	0.2708	0.2760	0.2876	0.2683	0.2761	0.2967
5	0.6872	0.6328	0.6439	0.6835	0.6317	0.6423	0.7007
6	0.7264	0.6702	0.6864	0.7246	0.6646	0.6827	0.7412
7	0.9460	0.9226	0.9203	0.9450	0.9224	0.9202	0.9515
8	0.8908	0.8535	0.8578	0.8900	0.8518	0.8568	0.9004
9	0.3422	0.3099	0.3050	0.3388	0.3099	0.3037	0.3557
10	0.6852	0.6272	0.6394	0.6823	0.6247	0.6372	0.6979
11	0.2933	0.2712	0.2655	0.2886	0.2713	0.2629	0.2988

Table 27. Estimated Powers for Equal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, n_1=n_2=n_3=n_4=n_5=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0498	0.0494	0.0512	0.0518	0.0512	0.0506	0.0499
2	0.2751	0.2697	0.2685	0.2754	0.2682	0.2687	0.2755
3	0.2044	0.1934	0.1955	0.2038	0.1983	0.1988	0.2145
4	0.1576	0.1574	0.1566	0.1605	0.1588	0.1593	0.1636
5	0.3649	0.3543	0.3552	0.3670	0.3571	0.3564	0.3727
6	0.3914	0.3724	0.3858	0.3896	0.3697	0.3871	0.3959
7	0.6407	0.6305	0.6314	0.6406	0.6323	0.6288	0.6509
8	0.5437	0.5239	0.5392	0.5460	0.5266	0.5362	0.5492
9	0.1845	0.1780	0.1786	0.1851	0.1816	0.1814	0.1942
10	0.3707	0.3450	0.3630	0.3707	0.3476	0.3669	0.3792
11	0.1639	0.1592	0.1624	0.1661	0.1601	0.1660	0.1662

Table 28. Estimated Powers for Equal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, n_1=n_2=n_3=n_4=n_5=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0514	0.0488	0.0526	0.0511	0.0487	0.0514	0.0517
2	0.4480	0.4372	0.4390	0.4447	0.4333	0.4348	0.4530
3	0.3431	0.3310	0.3289	0.3379	0.3282	0.3264	0.3537
4	0.2597	0.2551	0.2574	0.2595	0.2554	0.2543	0.2619
5	0.5917	0.5765	0.5748	0.5883	0.5736	0.5748	0.6014
6	0.6468	0.6168	0.6395	0.6421	0.6138	0.6376	0.6528
7	0.8936	0.8806	0.8831	0.8915	0.8792	0.8812	0.8959
8	0.8253	0.8120	0.8134	0.8227	0.8095	0.8128	0.8298
9	0.2957	0.2851	0.2846	0.2940	0.2833	0.2852	0.3027
10	0.6035	0.5716	0.6024	0.6008	0.5714	0.6019	0.6122
11	0.2540	0.2468	0.2505	0.2522	0.2465	0.2475	0.2564

Table 29. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, n_1=n_2=n_3=5, n_4=n_5=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0514	0.0502	0.0504	0.0503	0.0500	0.0504	0.0502
2	0.4234	0.3898	0.3879	0.4146	0.3803	0.3808	0.4264
3	0.3017	0.2528	0.2889	0.2934	0.2536	0.2787	0.3122
4	0.2790	0.2546	0.2513	0.3226	0.2963	0.2931	0.3334
5	0.5233	0.4804	0.4922	0.5089	0.4715	0.4746	0.5301
6	0.5875	0.5386	0.5480	0.5791	0.5304	0.5376	0.5958
7	0.8637	0.8224	0.8267	0.8694	0.8282	0.8321	0.8800
8	0.7600	0.7084	0.7098	0.7444	0.6922	0.6968	0.7592
9	0.2612	0.2277	0.2536	0.2574	0.2339	0.2468	0.2762
10	0.5661	0.4969	0.5441	0.5720	0.5009	0.5519	0.5885
11	0.1903	0.1791	0.1818	0.1662	0.1574	0.1559	0.1683

Table 30. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, $n_1=n_2=n_3=5$, $n_4=n_5=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0526	0.0510	0.0512	0.0529	0.0509	0.0514	0.0522
2	0.3617	0.3540	0.3562	0.3533	0.3438	0.3496	0.3626
3	0.2585	0.2166	0.2779	0.2522	0.2228	0.2633	0.2635
4	0.2379	0.2328	0.2330	0.2757	0.2699	0.2688	0.2820
5	0.4536	0.4327	0.4526	0.4427	0.4209	0.4356	0.4535
6	0.5079	0.4853	0.5072	0.5004	0.4807	0.4981	0.5115
7	0.7885	0.7733	0.7807	0.7944	0.7762	0.7884	0.8043
8	0.6761	0.6559	0.6629	0.6603	0.6415	0.6487	0.6701
9	0.2309	0.2025	0.2404	0.2263	0.2078	0.2294	0.2376
10	0.4889	0.4349	0.5101	0.4970	0.4406	0.5169	0.5089
11	0.1684	0.1648	0.1657	0.1478	0.1442	0.1425	0.1490

Table 31. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, $n_1=n_2=5$, $n_3=n_4=n_5=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0498	0.0499	0.0503	0.0490	0.0503	0.0521	0.0516
2	0.4562	0.4189	0.4181	0.4712	0.4341	0.4345	0.4845
3	0.3123	0.2666	0.2949	0.3035	0.2662	0.2856	0.3189
4	0.2812	0.2663	0.2631	0.3119	0.2911	0.2896	0.3226
5	0.5413	0.4909	0.5032	0.5299	0.4808	0.4914	0.5479
6	0.6073	0.5512	0.5760	0.6127	0.5539	0.5750	0.6278
7	0.8800	0.8381	0.8423	0.8875	0.8488	0.8473	0.8960
8	0.7642	0.7143	0.7204	0.7511	0.6971	0.7031	0.7693
9	0.2751	0.2419	0.2600	0.2710	0.2440	0.2520	0.2844
10	0.5643	0.5017	0.5417	0.5734	0.5122	0.5487	0.5902
11	0.1890	0.1758	0.1773	0.1638	0.1511	0.1525	0.1663

Table 32. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, $n_1=n_2=5$, $n_3=n_4=n_5=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0485	0.0472	0.0500	0.0474	0.0468	0.0487	0.0477
2	0.3883	0.3750	0.3780	0.4036	0.3904	0.3947	0.4081
3	0.2621	0.2319	0.2737	0.2551	0.2361	0.2602	0.2700
4	0.2528	0.2449	0.2437	0.2743	0.2667	0.2653	0.2775
5	0.4619	0.4395	0.4559	0.4530	0.4351	0.4424	0.4604
6	0.5278	0.4972	0.5289	0.5304	0.5047	0.5308	0.5415
7	0.8057	0.7859	0.7979	0.8160	0.7975	0.8071	0.8218
8	0.6772	0.6556	0.6698	0.6622	0.6373	0.6577	0.6733
9	0.2265	0.2084	0.2336	0.2221	0.2100	0.2193	0.2316
10	0.4952	0.4476	0.5153	0.5023	0.4573	0.5183	0.5161
11	0.1651	0.1654	0.1640	0.1438	0.1406	0.1413	0.1462

Table 33. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, $n_1=5$, $n_2=n_3=n_4=n_5=10$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0465	0.0470	0.0475	0.0493	0.0486	0.0476	0.0489
2	0.4922	0.4520	0.4513	0.4994	0.4604	0.4589	0.5127
3	0.3299	0.2920	0.3047	0.3298	0.2932	0.3055	0.3421
4	0.2901	0.2760	0.2713	0.2996	0.2838	0.2752	0.3044
5	0.5859	0.5376	0.5420	0.5878	0.5371	0.5422	0.5999
6	0.6654	0.6075	0.6188	0.6649	0.6120	0.6227	0.6759
7	0.9056	0.8712	0.8705	0.9075	0.8729	0.8715	0.9163
8	0.8174	0.7693	0.7768	0.8177	0.7682	0.7755	0.8284
9	0.2816	0.2590	0.2567	0.2809	0.2601	0.2563	0.2902
10	0.6322	0.5630	0.5918	0.6339	0.5682	0.5966	0.6485
11	0.1639	0.1580	0.1558	0.1547	0.1477	0.1485	0.1581

Table 34. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, n_1=5, n_2=n_3=n_4=n_5=10

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0487	0.0492	0.0485	0.0505	0.0498	0.0500	0.0497
2	0.4264	0.4150	0.4116	0.4317	0.4173	0.4189	0.4346
3	0.2855	0.2656	0.2821	0.2873	0.2697	0.2812	0.2933
4	0.2638	0.2568	0.2594	0.2705	0.2656	0.2652	0.2707
5	0.5060	0.4922	0.4971	0.5063	0.4898	0.4975	0.5130
6	0.5758	0.5495	0.5736	0.5771	0.5530	0.5762	0.5860
7	0.8347	0.8199	0.8234	0.8369	0.8227	0.8249	0.8435
8	0.7386	0.7253	0.7264	0.7415	0.7239	0.7278	0.7444
9	0.2468	0.2334	0.2411	0.2447	0.2361	0.2383	0.2519
10	0.5400	0.4988	0.5469	0.5452	0.5042	0.5475	0.5504
11	0.1484	0.1469	0.1481	0.1414	0.1374	0.1401	0.1431

Table 35. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.4, n_1=n_2= n_3=n_4=10, n_5=5

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0523	0.0489	0.0502	0.0530	0.0509	0.0523	0.0521
2	0.4333	0.3997	0.3989	0.4292	0.3963	0.3963	0.4424
3	0.3382	0.3057	0.2965	0.3370	0.3069	0.2994	0.3513
4	0.1646	0.1544	0.1522	0.1541	0.1464	0.1451	0.1609
5	0.5733	0.5308	0.5240	0.5729	0.5299	0.5251	0.5836
6	0.6227	0.5715	0.5812	0.6218	0.5685	0.5790	0.6378
7	0.8704	0.8288	0.8290	0.8633	0.8232	0.8226	0.8759
8	0.8175	0.7703	0.7764	0.8173	0.7688	0.7738	0.8299
9	0.2732	0.2539	0.2471	0.2695	0.2523	0.2482	0.2806
10	0.5628	0.5145	0.5069	0.5581	0.5106	0.5004	0.5710
11	0.2926	0.2667	0.2734	0.2983	0.2735	0.2806	0.3056

Table 36. Estimated Powers for Unequal Sample Size Case: k=5; Bivariate Normal with ρ= 0.8, $n_1=n_2=n_3=n_4=10$, $n_5=5$

case	MJT_{sum}	MJT_{max}	MJT_{min}	JT_{sum}	JT_{max}	JT_{min}	DIETZ
1	0.0492	0.0500	0.0516	0.0504	0.0508	0.0516	0.0510
2	0.3697	0.3594	0.3648	0.3665	0.3600	0.3620	0.3693
3	0.2925	0.2870	0.2749	0.2919	0.2840	0.2808	0.3030
4	0.1449	0.1435	0.1449	0.1379	0.1357	0.1374	0.1400
5	0.4988	0.4911	0.4819	0.4997	0.4906	0.4828	0.5054
6	0.5445	0.5243	0.5410	0.5459	0.5261	0.5428	0.5542
7	0.7848	0.7741	0.7704	0.7779	0.7679	0.7647	0.7868
8	0.7289	0.7141	0.7181	0.7323	0.7150	0.7199	0.7401
9	0.2310	0.2254	0.2181	0.2289	0.2243	0.2217	0.2344
10	0.4868	0.4662	0.4624	0.4830	0.4653	0.4565	0.4900
11	0.2551	0.2524	0.2545	0.2634	0.2560	0.2604	0.2643

6. CONCLUSIONS

In general, the Dietz test performs the best for the situations considered with the underlying distributions being the bivariate normal distributions. The estimated powers of MJT_{sum} and JT_{sum} are often close with the MJT_{sum} generally having a little higher power. The sum transformation was the best of the three transformations to use for bivariate normal data.

This study only compared Dietz's test with the other tests for bivariate normal populations. Future work will include comparing the estimated powers of the tests when the underlying populations are bivariate exponential and bivariate skewed normal.

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APPENDIX

SAS codes for the simulation

SAS code A: bivariate normal with k=3 equal sample size and ρ=0.4

```
%macro bivariate_Normal( mean11, mean12, mean21, mean22,mean31, mean32,sig11,

sig12,sig21, sig22,sig31, sig32,rho,samples,n);

data bivariate_Normal (keep=sample group i y1 y2);

 array seeds{2} seed1-seed2;

 do u=1 to 2;

 seeds{u}=int(ranuni(0)*1e9);

end;

do sample=1 to &samples;

   do group=1 to &grp;

     do i=1 to &n;

     r1 = rannor(seed1);

     r2 = rannor(seed2);

 if group=1 then do;

     y1 = &mean11 + &sig11*r1;

     y2 = &mean12 + &rho*&sig12*r1+sqrt(&sig12**2-&sig12**2*&rho**2)*r2;

 end;

  if group=2 then do;

          y1 = &mean21 + &sig21*r1;

          y2 = &mean22 + &rho*&sig22*r1+sqrt(&sig22**2-&sig22**2*&rho**2)*r2;
```

```
end;
 if group=3 then do;
         y1 = &mean31 + &sig31*r1;
         y2 = &mean32 + &rho*&sig32*r1+sqrt(&sig32**2-&sig32**2*&rho**2)*r2;
  end;
output;
    end;
   end;
 end;
  run;
title1 "rank y1 y2 by sample";
proc sort data=bivariate_Normal;
  by sample;
run;
proc rank data=bivariate_Normal out=one ties=mean;
   by sample;
   var y1 y2;
   ranks rankedy1 rankedy2;
 run;
 title1 "get univariate  data by using sum max min";
 data new1;
   set one;
   sum1=sum(rankedy1, rankedy2);
```

```
  max1=max(rankedy1, rankedy2);
  min1=min(rankedy1, rankedy2);
run;
data table1;
/*need separate the data in order to use sql, otherwise just do one-one, not one-five*/
 set new1;
 by sample;
 if group=1 ;
 rename sum1=group1 max1=group11 min1=group111;
 keep sample group i  sum1 group1 max1 group11 min1 group111;
run;
data table2;
 set new1;
 by sample;
 if group=2 ;
 rename  sum1=group2 max1=group22 min1=group222;
 keep sample group i  sum1 group2 max1 group22 min1 group222;
run;
data table3;
 set new1;
 by sample;
 if group=3;
 rename sum1=group3 max1=group33 min1=group333;
```

```
keep sample group i  sum1 group3 max1 group33 min1 group333;
run;
title1  'mjt---MJT statistic T=sumsum(j-i)Uij';
proc sql;
 create table t4 as
 select *
 from table1
 right join  table2
 on table1.sample = table2.sample;
quit;
data x1x2 ;
   set t4;
   flag=group1-group2;
   if flag<0 then val=1*1;          /*Ttmj=sumsum(j-i)Uij*/
     else if flag>0 then val=0;
     else if flag=0 then val=1*0.5;
run;
proc means noprint data=x1x2;
   var val;
   by sample;
   output out=result1 sum=sum1;
run;
proc sql;
```

```
create table t5 as
select *
from table1
right join table3
on table1.sample = table3.sample;
quit;
data x1x3;
  set t5;
  flag=group1-group3;
  if flag<0 then val=2*1;          /*Tmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=2*0.5;
run;
proc means noprint data=x1x3;
  var val;
  by sample;
  output out=result2 sum=sum1;
run;
proc sql;
create table t6 as
select *
from table2
right join table3
```

```
on table2.sample = table3.sample;
quit;
data x2x3;
  set t6;
  flag=group2-group3;
  if flag<0 then val=1*1;          *Tmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=1*0.5;
run;
 proc means noprint data=x2x3;
  var val;
  by sample;
  output out=result3 sum=sum1;
run;
data com;
  set result1 result2 result3;
  by sample;
run;
proc sort;
  by sample;
run;
proc means noprint data=com;
   var sum1;
```

```
  by sample;
  output out=result4 sum=sum1;
run;
title1 "power---MJT---max---sql";
data x11x22 ;
  set t4;
  flag=group11-group22;
  if flag<0 then val=1*1;         /*Ttmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=1*0.5;
  run;
proc means noprint data=x11x22;
  var val;
  by sample;
  output out=result11 sum=max1;
run;
data x11x33;
  set t5;
  flag=group11-group33;
  if flag<0 then val=2*1;         /*Ttmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=2*0.5;
 run;
```

```
proc means noprint data=x11x33;
  var val;
  by sample;
  output out=result22 sum=max1;
run;
data x22x33;
  set t6;
  flag=group22-group33;
  if flag<0 then val=1*1;          /*Tmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=1*0.5;
run;
proc means noprint data=x22x33;
  var val;
  by sample;
  output out=result33 sum=max1;
run;
data com11;
  set result11 result22 result33;
  by sample;
run;
proc sort;
  by sample;
```

```
run;
proc means noprint data=com11;
  var max1;
  by sample;
  output out=result44 sum=max1;
run;
title1 "power---MJT---min---sql";
data x111x222 ;
  set t4;
  flag=group111-group222;
  if flag<0 then val=1*1;
     else if flag>0 then val=0;
     else if flag=0 then val=1*0.5;
  run;
proc means noprint data=x111x222;
  var val;
  by sample;
  output out=result111 sum=min1;
run;
data x111x333;
  set t5;
  flag=group111-group333;
  if flag<0 then val=2*1;
```

```
    else if flag>0 then val=0;
    else if flag=0 then val=2*0.5;
run;
proc means noprint data=x111x333;
  var val;
  by sample;
  output out=result222 sum=min1;
run;
data x222x333;
  set t6;
  flag=group222-group333;
  if flag<0 then val=1*1;
    else if flag>0 then val=0;
    else if flag=0 then val=1*0.5;
run;
proc means noprint data=x222x333;
  var val;
  by sample;
  output out=result333 sum=min1;
run;
data com111;
  set result111 result222 result333;
  by sample;
```

```
run;
proc sort;
 by sample;
run;
proc means noprint data=com111;
  var min1;
  by sample;
  output out=result444 sum=min1;
run;
data  resultcom;
  merge result4 result44 result444;
  by sample;
run;
title1 "mean for MJT E=sumsum(j-i)(ninj/2)";
data ET;
  set resultcom;
  by sample;
  mean=0;
  do i=1 to (&grp-1);
   do j=(i+1) to &grp;
  mean=mean+(1/2)*(j-i)*&n*&n;
   end;
  end;
```

```
run;
title1 "MJT--variance---secondpart=2*sumsumsumsum(j-i)(t-s)cov(Uij,Ust)";
data varsecondpart;
 set resultcom;
 by  sample;
 secondpart=0;
 do i=1 to (&grp-1);
  do j=(i+1) to &grp;
   do s=1 to (&grp-1);
    do t=(s+1) to &grp;
     if (i<j and i<t) and j^=t and i=s then tempcov=(1/12)*&n*&n*&n;
       else if s<i  and i<j and i=t then tempcov=-(1/12)*&n*&n*&n;
       else if i<j and j<t and j=s then tempcov=-(1/12)*&n*&n*&n;
       else if (i<j and s<j) and i^=s and j=t then
       tempcov=(1/12)*&n*&n*&n;
      else tempcov=0;
     secondpart=secondpart+ (j-i)*(t-s)*tempcov;   /*note: I did not multipy 2 by
 secondpart since the covariance part already double it*/
        end;
      end;
    end;
  end;
 run;
```

```
title1 "MJT--variance---firstpart=sumsum(j-i)**2*var(Uij)";
data varfirstpart;
 set resultcom;
 by  sample;
 firstpart=0;
 do i=1 to (&grp-1);
  do j=(i+1) to &grp;
    if i<j then tempvar=(1/12)*&n*&n*( &n+&n+1);
     else if i>=j then tempvar=0;
     firstpart=firstpart+(j-i)**2*tempvar;
    end;
  end;
 run;
 data MJTCOM;
   merge resultcom ET varsecondpart varfirstpart;
   by sample;
   varmjt=firstpart+secondpart;
   Zsum=(sum1-mean)/sqrt(varmjt);
   Zmax=(max1-mean)/sqrt(varmjt);
   Zmin=(min1-mean)/sqrt(varmjt);
   keep sample sum1 max1 min1 mean firstpart secondpart varmjt Zsum Zmax     Zmin;
  run;
  data totalpower;
```

```
set MJTCOM;
if  Zsum>=1.645 then p1=1;
  else if Zsum<1.645 then p1=0;
if  Zmax>=1.645 then p2=1;
  else if Zmax<1.645 then p2=0;
if  Zmin>=1.645 then p3=1;
  else if Zmin<1.645 then p3=0;
run;
title1 "jt test for univariate data";
proc sort data=new1;
   by sample;
run;
ods listing close;
ods select none;
ods select JTTest;
proc freq data=new1 ;
   by sample;
   tables group*(sum1 max1 min1 rankedy1 rankedy2)/jt; /*compare jt by using sum max
min*//*j1 j2  for calcute dietz  j test*/
ods output JTTest=new2;
run;
ods select all;
ods listing;
```

```
proc means noprint data=bivariate_Normal;
  by sample group;
  var y1;
  output out=cnt n=ny1;
run;
data cnt;
  set cnt;
    tmp_y1=(2*ny1+3)*(ny1**2);
    tmpy1=ny1**2;
    tmpy11=ny1**3;
 run;
 proc sort data=cnt;
    by sample;
          run;
 proc means noprint data=cnt;
 by sample;
   var tmp_y1 tmpy1 ;
   output out=temp1 sum=tmp_y1 tmpy1;
   title2 ' 2nd component of var_j_g on page 3766 of Dietz';
 run;
 proc means noprint data=cnt;
   by sample;
   var ny1;
```

```
output out=temp2 sum=N;
title2 ' N for var_j_g on page 3766 of Dietz and jt_Z page237';
run;
data jtvalue;
  set new2;
  if Name1="_JT_";
run;
data combinedtemp;
  merge temp1 temp2 jtvalue;
  by sample;
run;
data new3;
  set combinedtemp;
  by sample;
  mean=(N**2-tmpy1)/4;
  variance=N**2*(2*N+3)/72-tmp_y1/72;
  sd=sqrt(variance);
  z=(cValue1-mean)/sd;
  keep sample Table cValue1 mean variance sd z;
run;
title2 "power for sum1 max1 min1";
proc format;
  value resultfmt 1='reject'
```

```
0='not reject';
run;
data new4;
set new3;
if z>=1.645 then p=1;
else if z<1.645 then p=0;
run;
proc transpose data=new4 out=mv_jt(drop=_name_) prefix=table_;
by sample;
var p;
run;
title1 "get var(j1),var(j2)";
data combined;
merge temp1 temp2;
by sample;
a=(N**2)*(2*N+3)-tmp_y1;
varj1=a/72;
b=(N**2)*(2*N+3)-tmp_y1;
varj2=b/72;
run;
title1 "get cov(j1,j2)";
proc means noprint data=cnt;
by sample;
```

```
var tmpy1 tmpy11;
output out=temp3 sum=tmpy1 tmpy11;
run;
title2 "get spearman kendall correlation coeffition with outpout";
proc corr data=bivariate_Normal spearman kendall outs=spmancc  outk=kdallcc noprint;
  by sample;
  var y1 y2;
run;
proc sort data=spmancc;
  by sample;
run;
data spmancc;
  set spmancc;
  by sample;
  if _NAME_='y2';
  keep sample _NAME_ y1;
run;
proc sort data=kdallcc;
  by sample;
run;
data kdallcc;
  set kdallcc;
  by sample;
```

```
  if _NAME_='y1';
  keep sample _NAME_ y2;
run;
data temp4;
  merge temp2 temp3 spmancc kdallcc;
  by sample;
  keep sample tmpy1  tmpy11  N y1 y2 a1 a2 b1 b2 cov; /*y1 is the spearman cc and y2 is
the kendall cc*/
  a1=(N**3-tmpy11)-3*(N**2-tmpy1);
  a2=3*N*(N**2-tmpy1)-2*(N**3-tmpy11);
  b1=(N+1)*a1*y1/(36*N-72);
  b2=a2*y2/(24*N-48);
  cov=b1+b2;
 run;
 data new5;
   set new2;
    if name1='_JT_';        /*prob2 need use jt statisites for j1 j2,not z  jt*/
 run;
 proc sort data=new5;
    by sample;
 run;
 proc transpose data=new5 out=mv_j1j2(drop=_name_) prefix=table_;
    by sample;
```

```
  var nvalue1;
run;
data j1j2;
   set mv_j1j2;
         tmp=comb(&grp, 2);
         j1=table_4-&n*&n*tmp/2;   /*j=sum(u-nunv/2 dietz page3765  if k=4,5 the
number need change ,not only multiply group k (from k take 2)*/
         j2=table_5-&n*&n*tmp/2;
         keep  sample j1 j2;
 run;
title1 "get dietz j (sum statistics)";
data temp5;
   merge  combined j1j2 temp4;
   by sample;
   j=(j1+j2)/(varj1+varj2+2*cov)**0.5;
   keep sample varj1  varj2 cov  j;
 run;
 data totalpower;
    set MJTCOM;
    if  Zsum>=1.645 then p1=1;
      else if Zsum<1.645 then p1=0;
    if  Zmax>=1.645 then p2=1;
      else if Zmax<1.645 then p2=0;
```

```
if Zmin>=1.645 then p3=1;
  else if Zmin<1.645 then p3=0;
run;
data temp6;
  set temp5;
  if j>=1.645 then p=1;
    else if j<1.645 then p=0;
run;
data all;
  merge totalpower mv_jt temp6 end=eof;
   by sample;
  mjt_sum+p1;
  mjt_max+p2;
  mjt_min+p3;
  jt_sum+table_1;
  jt_max+table_2;
  jt_min+table_3;
  dietz+p;
  if eof then do;
   mjt_sum= mjt_sum/&samples;
   mjt_max= mjt_max/&samples;
   mjt_min= mjt_min/&samples;
   jt_sum= jt_sum/&samples;
```

```
jt_max= jt_max/&samples;
jt_min= jt_min/&samples;
dietz=dietz/&samples;
file 'power.txt' mod;
put @1 "&n" @4 "&samples" @13 mjt_sum @21 mjt_max @29 mjt_min @37 jt_sum
@45 jt_max @53 jt_min @61 dietz ;
end;
run;
%mend bivariate_Normal;
%let grp=3;
/*%let n=10; */
%macro looper(samples,firstn,lastn,inc);
%do n=&firstn %to &lastn %by &inc;
*%bivariate_Normal (1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 0.4, &samples,&n);
%bivariate_Normal (1, 1, 1, 1, 2.5, 2.5, 2, 2, 2, 2, 2, 2, 0.4,&samples,&n);
*%bivariate_Normal (1, 1, 2, 2, 3, 3,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 2, 1.5, 3, 2,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 1, 3, 1, 5,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 1, 2, 1, 4,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 2, 1.5, 3, 2.5,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 2, 2.5, 2.5, 3.5,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 2, 1, 3, 1,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
*%bivariate_Normal (1, 1, 1.5, 1, 3, 1,2, 2, 2, 2,  2, 2, 0.4,&samples,&n );
```

```
%end;
%mend looper;
%looper(10000,5,10,5);
```

SAS code B: bivariate normal with k=3, unequal sample size $n_1=n_2=5$ $n_3=10$ and $\rho=0.4$

```
%macro bivariate_Normal( mean11, mean12, mean21, mean22,mean31, mean32,sig11,
sig12,sig21, sig22,sig31, sig32,rho,samples,case);
data bivariate_Normal (keep=sample group i y1 y2);
array seeds{2} seed1-seed2;
 do u=1 to 2;
 seeds{u}=int(ranuni(0)*1e9);
end;
do sample=1 to &samples;
   do group=1 to &grp;
   if group=1 then n=&n1;
   if group=2 then n=&n2;
   if group=3 then n=&n3;
     do i=1 to n;
     r1 = rannor(seed1);
     r2 = rannor(seed2);
       if group=1 then do;
       y1 = &mean11 + &sig11*r1;
```

```
y2 = &mean12 + &rho*&sig12*r1+sqrt(&sig12**2-&sig12**2*&rho**2)*r2;
end;
if group=2 then do;
y1 = &mean21 + &sig21*r1;
y2 = &mean22 + &rho*&sig22*r1+sqrt(&sig22**2-&sig22**2*&rho**2)*r2;
end;
if group=3 then do;
y1 = &mean31 + &sig31*r1;
y2 = &mean32 + &rho*&sig32*r1+sqrt(&sig32**2-&sig32**2*&rho**2)*r2;
end;
output;
end;
end;
end;
run;
title1 "rank y1 y2 by sample";
proc sort data=bivariate_Normal;
by sample;
run;
proc rank data=bivariate_Normal out=one ties=mean;
by sample;
var y1 y2;
ranks rankedy1 rankedy2;
```

```
run;
title1 "get univariate  data by using sum max min";
data new1;
  set one;
  sum1=sum(rankedy1, rankedy2);
  max1=max(rankedy1, rankedy2);
  min1=min(rankedy1, rankedy2);
run;
data table1;    /*need separate the data in order to use sql, otherwise just do one-one,not
one-five*/
  set new1;
  by sample;
  if group=1 ;
  rename sum1=group1 max1=group11 min1=group111;
  keep sample group i  sum1 group1 max1 group11 min1 group111;
run;
data table2;
  set new1;
  by sample;
  if group=2 ;
  rename  sum1=group2 max1=group22 min1=group222;
  keep sample group i  sum1 group2 max1 group22 min1 group222;
run;
```

```
data table3;
 set new1;
 by sample;
 if group=3;
 rename  sum1=group3 max1=group33 min1=group333;
 keep sample group i  sum1 group3 max1 group33 min1 group333;
run;
title1  'mjt---MJT statistic T=sumsum(j-i)Uij';
proc sql;
create table t4 as
select *
from table1
right join  table2
on table1.sample = table2.sample;
quit;
data x1x2 ;
   set t4;
   flag=group1-group2;
   if flag<0 then val=1*1;          /*Tmjt=sumsum(j-i)Uij*/
     else if flag>0 then val=0;
     else if flag=0 then val=1*0.5;
 run;
 proc means noprint data=x1x2;
```

```
  var val;
  by sample;
  output out=result1 sum=sum1;
run;
proc sql;
create table t5 as
select *
from table1
right join  table3
on table1.sample = table3.sample;
quit;
data x1x3;
   set t5;
   flag=group1-group3;
   if flag<0 then val=2*1;           /*Ttmj- sumsum(j-i)Uij*/
      else if flag>0 then val=0;
      else if flag=0 then val=2*0.5;
  run;
  proc means noprint data=x1x3;
    var val;
    by sample;
    output out=result2 sum=sum1;
  run;
```

```
proc sql;
create table t6 as
select *
from table2
right join  table3
on table2.sample = table3.sample;
quit;
data x2x3;
   set t6;
   flag=group2-group3;
   if flag<0 then val=1*1;          *Ttmj=sumsum(j-i)Uij*/
      else if flag>0 then val=0;
      else if flag=0 then val=1*0.5;
run;
proc means noprint data=x2x3;
   var val;
   by sample;
   output out=result3 sum=sum1;
run;
data com;
   set result1 result2 result3;
   by sample;
run;
```

```
proc sort;
 by sample;
run;
proc means noprint data=com;
  var sum1;
  by sample;
  output out=result4 sum=sum1;
 run;
 title1 "power---MJT---max---sql";
 data x11x22 ;
   set t4;
   flag=group11-group22;
   if flag<0 then val=1*1;          /*Tmj  sumsum(j-i)Uij*/
     else if flag>0 then val=0;
     else if flag=0 then val=1*0.5;
   run;
  proc means noprint data=x11x22;
    var val;
    by sample;
    output out=result11 sum=max1;
  run;
  data x11x33;
    set t5;
```

```
 flag=group11-group33;
 if flag<0 then val=2*1;
   else if flag>0 then val=0;
   else if flag=0 then val=2*0.5;
run;
proc means noprint data=x11x33;
  var val;
  by sample;
  output out=result22 sum=max1;
run;
data x22x33;
  set t6;
  flag=group22-group33;
  if flag<0 then val=1*1;
  else if flag>0 then val=0;
  else if flag=0 then val=1*0.5;
run;
proc means noprint data=x22x33;
  var val;
  by sample;
  output out=result33 sum=max1;
run;
data com11;
```

```
set result11 result22 result33;
by sample;
run;
proc sort;
by sample;
run;
proc means noprint data=com11;
var max1;
by sample;
output out=result44 sum=max1;
run;
title1 "power---MJT---min---sql";
data x111x222 ;
set t4;
flag=group111-group222;
if flag<0 then val=1*1;          /*Tmj=sumsum(j-i)Uij*/
else if flag>0 then val=0;
else if flag=0 then val=1*0.5;
run;
proc means noprint data=x111x222;
var val;
by sample;
output out=result111 sum=min1;
```

```
run;
data x111x333;
  set t5;
  flag=group111-group333;
  if flag<0 then val=2*1;          /*Ttmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=2*0.5;
run;
proc means noprint data=x111x333;
  var val;
  by sample;
  output out=result222 sum=min1;
run;
data x222x333;
  set t6;
  flag=group222-group333;
  if flag<0 then val=1*1;          /*Ttmj=sumsum(j-i)Uij*/
    else if flag>0 then val=0;
    else if flag=0 then val=1*0.5;
run;
proc means noprint data=x222x333;
  var val;
  by sample;
```

```
  output out=result333 sum=min1;
run;
data com111;
  set result111 result222 result333;
  by sample;
run;
proc sort;
  by sample;
run;
proc means noprint data=com111;
   var min1;
   by sample;
   output out=result444 sum=min1;
run;
data  resultcom;
   merge result4 result44 result444;
   by sample;
run;
title1 "mean for MJT E=sumsum(j-i)(ninj/2)";
data ET;
   set resultcom;
   by sample;
   mean=(1/2)*(2-1)*&n1*&n2+(1/2)*(3-1)*&n1*&n3+(1/2)*(3-2)*&n2*&n3;
```

```
run;

title1 "MJT--variance---secondpart=2*sumsumsumsum(j-i)(t-s)cov(Uij,Ust)";

data varsecondpart;

  set resultcom;

  by  sample;

  secondpart=0;

  do i=1 to (&grp-1);

    do j=(i+1) to &grp;

      do s=1 to (&grp-1);

        do t=(s+1) to &grp;

   if (i<j and i<t) and j^=t and i=s then tempcov=(1/12)*&n1*&n2*&n3;

     else if s<i  and i<j and i=t then tempcov=-(1/12)*&n1*&n2*&n3;

     else if i<j and j<t and j=s then  tempcov=-(1/12)*&n1*&n2*&n3;

     else if (i<j and s<j) and i^=s and j=t then tempcov=(1/12)*&n1*&n2*&n3;

     else

      tempcov=0;

    secondpart=secondpart+ (j-i)*(t-s)*tempcov;          /*note: I did not multipy 2 by

   secondpart since the covariance part already double it*/

         end;

            end;

     end;

   end;

   run;
```

```
title1 "MJT--variance---firstpart=sumsum(j-i)**2*var(Uij)";
data varfirstpart;
 set resultcom;
 by  sample;
 firstpart=(2-1)**2*(1/12)*&n1*&n2*( &n1+&n2+1)+(3-1)**2*(1/12)*&n1*&n3*(
&n1+&n3+1)+(3-2)**2*(1/12)*&n3*&n2*( &n3+&n2+1);
run;
data MJTCOM;
  merge resultcom ET varsecondpart varfirstpart;
  by sample;
  varmjt=firstpart+secondpart;
  Zsum=(sum1-mean)/sqrt(varmjt);
  Zmax=(max1-mean)/sqrt(varmjt);
  Zmin=(min1-mean)/sqrt(varmjt);
  keep sample sum1 max1 min1 mean firstpart secondpart varmjt Zsum Zmax Zmin;
run;
data totalpower;
   set MJTCOM;
   if  Zsum>=1.645 then p1=1;
     else if Zsum<1.645 then p1=0;
   if  Zmax>=1.645 then p2=1;
     else if Zmax<1.645 then p2=0;
   if  Zmin>=1.645 then p3=1;
```

```
else if Zmin<1.645 then p3=0;
run;
title1 "jt test for univariate data";
proc sort data=new1;
   by sample;
run;
ods listing close;
ods select none;
ods select JTTest;
proc freq data=new1 ;
   by sample;
   tables group*(sum1 max1 min1 rankedy1 rankedy2)/jt; /*compare jt by using sum max
min*//*j1 j2  for calcute dietz  j test*/
ods output JTTest=new2;
run;
ods select all;
ods listing;
proc means noprint data=bivariate_Normal;
   by sample group;
   var y1;
   output out=cnt n=ny1;
   run;
data cnt;
```

```
set cnt;
  tmp_y1=(2*ny1+3)*(ny1**2);
  tmpy1=ny1**2;
  tmpy11=ny1**3;
run;
proc sort data=cnt;
  by sample;
        run;
proc means noprint data=cnt;
by sample;
  var tmp_y1 tmpy1 ;
  output out=temp1 sum=tmp_y1 tmpy1;
  title2 ' 2nd component of var_j_g on page 3766 of Dietz';
run;
proc means noprint data=cnt;
   by sample;
  var ny1;
  output out=temp2 sum=N;
title2 ' N for var_j_g on page 3766 of Dietz and jt_Z page237';
run;
data jtvalue;
  set new2;
  if Name1="_JT_";
```

```
run;
data combinedtemp;
 merge temp1 temp2 jtvalue;
 by sample; run;
data new3;
  set combinedtemp;
  by sample;
  mean=(N**2-tmpy1)/4;
  variance=N**2*(2*N+3)/72-tmp_y1/72;
  sd=sqrt(variance);
  z=(cValue1-mean)/sd;
 keep sample Table cValue1 mean variance sd z;
 run;
 title2 "power for sum1 max1 min1";
 proc format;
   value resultfmt 1='reject'
              0='not reject';
  run;
  data new4;
   set new3;
    if  z>=1.645 then p=1;
      else if  z<1.645 then p=0;
  run;
```

```
proc transpose data=new4 out=mv_jt(drop=_name_) prefix=table_;
   by sample;
   var p;
run;
title1 "get var(j1),var(j2)";
data combined;
   merge temp1 temp2;
   by sample;
   a=(N**2)*(2*N+3)-tmp_y1;
   varj1=a/72;
   b=(N**2)*(2*N+3)-tmp_y1;
   varj2=b/72;
run;
title1 "get cov(j1,j2)";
proc means noprint data=cnt;
    by sample;
    var tmpy1 tmpy11;
   output out=temp3 sum=tmpy1 tmpy11;
   run;
title2 "get spearman kendall correlation coeffition with outpout";
proc corr data=bivariate_Normal spearman kendall outs=spmancc  outk=kdallcc noprint;
    by sample;
    var y1 y2;
```

```
run;
proc sort data=spmancc;
   by sample;
run;
data spmancc;
   set spmancc;
   by sample;
   if _NAME_='y2';
         keep sample _NAME_ y1;
run;
proc sort data=kdallcc;
   by sample;
run;
data kdallcc;
   set kdallcc;
   by sample;
   if _NAME_='y1';
         keep sample _NAME_ y2;
run;
data temp4;
   merge temp2 temp3 spmancc kdallcc;
   by sample;
```

```
keep sample tmpy1  tmpy11  N y1 y2 a1 a2 b1 b2 cov;  /*y1 is the spearman cc and y2 is
the kendall cc*/
 a1=(N**3-tmpy11)-3*(N**2-tmpy1);
 a2=3*N*(N**2-tmpy1)-2*(N**3-tmpy11);
 b1=(N+1)*a1*y1/(36*N-72);
 b2=a2*y2/(24*N-48);
 cov=b1+b2;
run;
data new5;
  set new2;
   if name1='_JT_';        /*prob2 need use jt statisites for j1 j2,not z  jt*/
run;
proc sort data=new5;
   by sample;
run;
proc transpose data=new5 out=mv_j1j2(drop=_name_) prefix=table_;
   by sample;
   var nvalue1;
run;
data j1j2;
   set mv_j1j2;
        j1=table_4-&n1*&n2/2-&n1*&n3/2-&n2*&n3/2;    /*j  sum(u-nunv/2 dietz
 page3765  if k=4.5 the number need change *
```

```
        j2=table_5-&n1*&n2/2-&n1*&n3/2-&n2*&n3/2;
        keep  sample j1 j2;
run;
title1 "get dietz j (sum statistics)";
data temp5;
  merge  combined j1j2 temp4;
  by sample;
  j=(j1+j2)/(varj1+varj2+2*cov)**0.5;
  keep sample varj1 varj2 cov  j;
run;
data temp6;
   set temp5;
    if  j>=1.645 then p=1;
      else if j<1.645 then p=0;
run;
data all;
  merge totalpower mv_jt temp6 end=eof;
  by sample;
  mjt_sum+p1;
  mjt_max+p2;
  mjt_min+p3;
  jt_sum+table_1;
  jt_max+table_2;
```

```
jt_min+table_3;

dietz+p;

if eof then do;

 mjt_sum= mjt_sum/&samples;

 mjt_max= mjt_max/&samples;

 mjt_min= mjt_min/&samples;

 jt_sum= jt_sum/&samples;

 jt_max= jt_max/&samples;

 jt_min= jt_min/&samples;

 dietz=dietz/&samples;

file 'power2.txt' mod;

put @1 "&case" @4 "&samples" @13 mjt_sum @21 mjt_max @29 mjt_min @37 jt_sum

@45 jt_max @53 jt_min @61 dietz ;

end;

run;

%mend bivariate_Normal;

%let grp=3;

%let n1=5;

%let n2=5;

%let n3=10;

*%bivariate_Normal (1, 1, 1, 1, 1, 1,2, 2, 2, 2,  2, 2, 0.4,10000, 1);

*%bivariate_Normal (1, 1, 1, 1, 2.5, 2.5, 2, 2, 2, 2, 2,2, 0.4,10000,2);

*%bivariate_Normal (1, 1, 2, 2, 3, 3,2, 2, 2, 2,  2, 2, 0.4,10000,3);
```

```
*%bivariate_Normal (1, 1, 2, 1.5, 3, 2,2, 2, 2, 2,  2, 2, 0.4,10000,4 );

*%bivariate_Normal (1, 1, 1, 3, 1. 5,2, 2, 2, 2,  2, 2, 0.4,10000,5 );

*%bivariate_Normal (1, 1, 1, 2, 1, 4,2, 2, 2, 2,  2, 2, 0.4,10000,6 );

*%bivariate_Normal (1, 1, 2, 1.5, 3, 2.5,2, 2, 2, 2,  2, 2, 0.4,10000,7 );

*%bivariate_Normal (1, 1, 2, 2.5, 2.5, 3.5,2, 2, 2, 2,  2, 2, 0.4,10000,8 );

*%bivariate_Normal (1, 1, 2, 1, 3, 1,2, 2, 2, 2,  2, 2, 0.4,10000,9 );

%bivariate_Normal (1, 1, 1.5, 1, 3, 1,2, 2, 2, 2,  2, 2, 0.4,10000,10 );
```