INFORMATION ASYMMETRY IN BUDGET ALLOCATION: A ANALYSIS OF THE TRUTH-INDUCING INCENTIVE SCHEME

A Thesis
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agriculture and Applied Science

By

Yun Zhou

In Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

Major Department:
Statistics

May 2016

Fargo, North Dakota

North Dakota State University
Graduate School

Title

INFORMATION ASYMMETRY IN BUDGET ALLOCATION: A ANALYSIS OF THE TRUTH-INDUCING INCENTIVE SCHEME

By

Yun Zhou

The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

SUPERVISORY COMMITTEE:

Dr. Rhonda Magel
Co-Chair

Dr. Joseph Szmerekovsky
Co-Chair

Dr. Ruilin Tian

Approved:

Oct. 21, 2016 — Date

Dr. Rhonda Magel — Department Chair

ABSTRACT

Truth-inducing incentive schemes are used to motivate project managers to provide unbiased project information to portfolio manager to reduce information asymmetry between portfolio manager and project managers. To improve the scheme, we identify the proper value of penalty coefficients in the truth-inducing incentive scheme when information asymmetry is present. We first describe the allocation method that achieves budget optimization under certain assumptions and identify the proper coefficients while accounting for the differing perceptions of both portfolio manager and project managers. We report a bound on the ratio between the two penalty coefficients in the truth-inducing incentive scheme and then we conduct a simulation study to narrow down the bound. We conclude that the penalty coefficient for being over budget should be reduced when the portfolio budget is tight and the penalty coefficients should be equivalent to the organizational opportunity costs when the portfolio budget is sufficient.

ACKNOWLEDGMENTS

This thesis was supported by Dr. Joseph Szmerekovsky, Dr. Yarong Yang and Dr. Rhonda Magel who provided insight and expertise that greatly assisted the thesis.

I would also like to show my gratitude to Dr. Ruilin Tian for her time and feedback.

I thank Matthew Warner, Andrew Taylor and Phillip Bode for comments and language support that greatly improved the manuscript.

This thesis would not have been done without my family behind me. I am indebted for the mental and physical support they have provided that has spurred me on through tough times during not only my thesis but also my entire graduate study. I am thankful for their love and prayers.

TABLE OF CONTENTS

LIST OF TABLES

Table | Page

LIST OF FIGURES

<u>Figure</u> <u>Page</u>

LIST OF APPENDIX TABLES

Table Page

1. INTRODUCTION

In budget allocation, information asymmetry occurs when the project managers have more accurate information than their supervisor (the portfolio manager) regarding the project cost (Dadbeh and Mogharebi, 2013). Information asymmetry between the project managers and the portfolio manager also creates an opportunity for the project managers to build biased budget proposals, mainly budgetary slack, to improve their performance when their evaluation considers budget attainment. This results in a false evaluation of managers' performance, which betrays the basic purpose of budgeting (Antle and Eppen, 1985; Kren and Liao, 1988).

Studies addressing budgetary slack attributed to information asymmetry mainly focus on economic incentives such as compensation schemes to reduce budgetary slack. An effective compensation scheme is the truth-inducing incentive scheme. Prior experimental studies (Chow et al., 1988; Chow et al., 1991; Steven, 2000; Hobson et al., 2011) have analyzed the effectiveness of truth-inducing incentive schemes at reducing budgetary slack by arbitrarily setting the key coefficients in the scheme. However, no study has analyzed the appropriate value of key coefficients in truth-inducing incentive schemes, despite the fact that the value of these coefficients may affect the overall efficacy of the schemes.

In our study, we examine the effects of the values of these coefficients in the truth-inducing incentive scheme on budgetary slack and then provide guidelines for portfolio managers to set appropriate coefficients. The use of proper coefficients is important for portfolio managers to optimize their budget allocation. We first illustrate the allocation method that achieves budget optimization by assuming that the uncertain costs follow a normal distribution and then determine the allocated budget depending on the total budget and the variance of the project costs. Next we identify the proper penalty coefficients that account for both the portfolio manager's and the project managers' perceptions of the budget optimization. Finally, to validate the results from the identification process, we conduct a simulation of the budget allocation problem. Our results show that a small over budget penalty coefficient would motivate the project manager to report unbiased project cost to the portfolio manager, although the actual allocated budget may have a large variation when the portfolio budget is tight and the number of projects is small.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes the process to identify the values of penalty coefficients. Section 4 presents a simulation study of the budget allocation problem. Section 5 summarizes the results from the simulation study. Section 6 provides conclusions and future research directions.

2. LITERATURE REVIEW

As previously stated, our work is related to three topics: project portfolio management with an emphasis on budget allocation, budgetary slack attributed to information asymmetry, and incentive schemes. We review each topic in this section.

2.1. Project Portfolio Management Concentrating on Budget Allocation

The management of a portfolio of projects is important to maximizing the return on investments. Budget allocation, as a large portion of resource allocation, is important for successful project portfolio management. This subsection reviews the literature on project portfolio management concentrating on budget allocation.

In the project portfolio management literature, Engwall and Jerbrant (2002) analyze the resource allocation syndrome and summarize that allocation of resources to a project portfolio is a complicated process that includes politics, horse-trading and interpretation. Labrosse (2010) presents the benefits of project portfolio management, including the optimal allocation of insufficient resources. A review of project portfolio management can be found in Meskendahl (2010) who indicates that optimal resource allocation is a fundamental aspect of project portfolio success. He also proposes an overall comprehensive conceptual model regarding project portfolio management, strategy implementation, and business success.

To resolve the problem of allocating a fixed budget across a portfolio of projects, Hu and Szmerekovsky (2016) develop a budget allocation methodology. They assume that the uncertain costs of a project follow a normal distribution and determine the allocated budget to that project based on the portfolio budget and the variance of the uncertain costs. Basso and Peccati (2001) propose a dynamic programming algorithm to solve the project-financing problem. With a limited budget, a project-financing firm has to decide which projects to undertake as well as the amount to invest. The latter part is similar to our project budget allocation problem. However, to our knowledge we are the first to consider information asymmetry when solving the budget allocation problem in the context of project portfolio management.

2.2. Budgetary Slack Attributed to Information Asymmetry

Information asymmetry between the portfolio manager and the project managers in the allocation process is a complex interaction with many aspects to consider. In fact, the existence of information asymmetry creates an opportunity for project managers who are directly in charge of the project implementation to negotiate for a biased budget, mainly budgetary slack. This subsection reviews the literature regarding budgetary slack attributed to information asymmetry.

Kren and Liao (1988) demonstrate the influence of information asymmetry from the accounting information perspective. They indicate that when project managers are aware of the existence of information asymmetry they are not motivated to report their unbiased information unless it improves their subsequent performance, resulting in suboptimal allocation of the portfolio budget if this bias occurs in the budget. Dunk and Nouri (1998) review the literature regarding the factors that impact the creation of budgetary slack and indicate that the information asymmetry between superior and subordinate is one of the factors that contributes to the slack. Fisher et al. (2002) examine the effect of the subordinate-superior information asymmetry on negotiated budgets and show the positive relationship between budgetary slack and information asymmetry on negotiation results. Kren and Maiga (2007) examine the subordinate-superior information asymmetry’s effect on budget participation and budgetary slack. They analyze sample data of 49 project managers in S&P 500 firms and conclude that there is a significantly indirect negative relationship between participation and budgetary slack when information asymmetry is present. This work highlights the importance of accounting for information asymmetry when performing budget allocation for a portfolio of projects, an aspect of portfolio management which has been neglected.

2.3. Incentive Scheme

The literature addressing the effect of information asymmetry mainly focuses on economic incentives. Kren and Liao (1988) propose that an economic incentive that promotes the project managers to provide unbiased information can be used as a motivation method.

In the economic incentive literature, our work focuses on the truth-inducing incentive scheme. The linear form of a truth-inducing incentive scheme first appeared in Weitzman (1976) as the New Soviet Incentive Model aiming to motivate the subordinates for truthfully reporting their productive capability. He reforms an economic incentive plan by determining the bonus size based on the difference between the plan target and the portion of attainment. Kren and Liao (1988) summarize the New Soviet Incentive Model as the truth-inducing incentive scheme, and this is where our original truth-inducing incentive scheme comes from. Waller (1988) conducts an economic experiment regarding the joint effect of the truth-inducing incentive scheme and risk preference on budgetary slack. His experiment illustrates that the truth-inducing incentive scheme significantly reduces more slack with the risk neutral subjects but has little to no impact on the risk averse subjects. Chow et al. (1988) examine the joint effect of the truth-inducing incentive scheme and information asymmetry on budgetary slack through a similar experiment and demonstrate that the truth-inducing incentive scheme significantly reduces budgetary slack when information asymmetry is present compared to when it is absent. They also explain that social pressure is another factor that affects the degree of budgetary slack. Chow et al. (1991) extend the experiment into multiple periods. By manipulating the initial superior-subordinate information asymmetry, results show that the truth-inducing incentive scheme significantly reduces budgetary slack when the information regarding subordinate's past performance is not available (information asymmetry is present), but it is just as effective as the alternative incentive scheme (slack-inducing incentive scheme) when past performance becomes available (information asymmetry is absent). Brown et al. (2009) provide a review of participative budgeting experiments and indicate that there is an anomaly in that no studies explain the reason why the subordinates create slack under truth-inducing incentive schemes. Rasmuben (2015) also suggests the same anomaly as Brown et al. (2009) after reviewing a series of experimental studies on information asymmetry and compensation schemes. Our research addresses this anomaly. In our analysis we show that a truth inducing incentive scheme with proper coefficients will not result in any budgetary slack, but one with the wrong coefficients will still result in budgetary slack.

As mentioned in section 2.2, information asymmetry is a complex interaction, hence, a few studies have incorporated non-pecuniary factors into consideration. Steven (2000) conducts an experiment by adding the effects of reputation and ethical factors into the experiment to test the effects of information asymmetry, the incentive scheme, reputation, ethics, and their interaction. Results indicate that there are interactions between the incentive scheme and the non-pecuniary factors. Steven (2000) also indicates that the truth-inducing incentive scheme is the strongest and most consistent factor in reducing budgetary slack. In addition, Steven indicates that the slack under the truth-induce incentive scheme was generated by risk aversion and the difficulty of understanding the scheme. Based on previous findings, Hobson et al. (2011) design an experiment that measures moral values before conducting similar experiments on the pay scheme and budgetary slack. They conclude that the truth-inducing incentive scheme is effective for reducing budgetary slack (39% on average) when the subordinates have insufficient moral value. However, the scheme appears ineffective for those who have sufficient moral value since the budgetary slack rate stays consistently low (2% difference). Similar results are found in Chong and Eggleton (2007)'s analysis of the joint effect of reliance on economic incentive, information asymmetry, and organizational commitment using 109 managers' performances. Their results show that a manager's performance is consistent with the degree of information asymmetry and reliance on economic incentives when they have sufficient organizational commitment.

On the other hand, although these experimental studies (Chow et al., 1988; Chow et al., 1991; Steven, 2000; Hobson et al., 2011) investigate the effectiveness of the truth-inducing incentive schemes, little research has analyzed the value of key coefficients in the truth-inducing incentive schemes. In fact, the value of these coefficients may affect the overall efficacy of the truth-inducing incentive schemes. Without sufficient research, the difficulty to set appropriate coefficients might be a reason why the truth-inducing incentive schemes are rarely put into practice (Fisher et al., 2002)

In our research, we simplify the original truth-inducing incentive scheme in Kren and Liao (1988) and identify the penalty coefficients by the allocation method in Hu and Szmerekovsky (2016). To our knowledge, we are the first to optimally identify the coefficients to use in the truth-inducing incentive scheme.

3. PROCESS

The process to identify the value of coefficients in the truth-inducing incentive scheme is discussed in three parts: the basic form of the truth-inducing incentive scheme, the budget allocation method, and identifying the penalty coefficients.

3.1. The Basic Form of the Truth-Inducing Incentive Scheme

As previously mentioned, the truth-inducing incentive scheme that promotes a project manager's accurate prediction of project costs can be used as an economic incentive to reduce the effect of information asymmetry (Kren and Liao, 1988). Suppose that a project manager (the subordinate) proposes a budget needed as X_B and that the actual cost is X. The compensation to the project manager, R, can be determined as follows:

$$R = \begin{cases} W + k_0 X_B + k_1(X - X_B), \; if \; X \geq X_B \\ W + l_0 X_B - l_1(X_B - X), \; if \; X < X_B \end{cases}, \tag{3.1}$$

where W is a certain wage, and k_0, l_0, k_1, and l_1 are bonus coefficients set by the portfolio manager (Kren and Liao, 1988).

However, since the portfolio manager and the project managers work for the same company, a project manager's compensation is likely independent of the proposed budget. Thus, k_0 and l_0 are set to zero in this case. We adjust the incentive scheme to be an indirect reward to each project manager after the project is coleted as well as one of the performance evaluation criteria for his/her promotion. Our truth-inducing incentive scheme is described as follows:

$$R_i = \begin{cases} W_i - k_i(C_i - b_i), \; if \; C_i \geq b_i \\ W_i - l_i(b_i - C_i), \; if \; C_i < b_i \end{cases}, \tag{3.2}$$

where R_i is the net reward for the project manager for project i, W_i is the wage of the project manager for project i, k_i is the per unit over budget penalty for project i, l_i is the per unit under budget penalty for project i, b_i is the allocated project budget for project i, and C_i is the uncertain cost of project i.

3.2. Budget Allocation Method

This study focuses on identifying the values of the coefficients k_i and l_i, as previous experimental studies (Chow et al., 1988; Chow et al., 1991; Steven, 2000; Hobson et al., 2011) demonstrated that the truth-inducing incentive scheme significantly reduces budgetary slack when information asymmetry exists between the portfolio manager and the project managers. Therefore, this scheme reduces the effect of information asymmetry and helps to optimize the budget allocation.

Hu and Szmerekovsky (2016) developed a new method to allocate a budget to each project. They introduced β and γ as per unit over budget and under budget costs to optimize the budget allocation. A penalty of β will be incurred for each dollar over budget when the project runs over budget and a penalty of γ will be incurred for each dollar under budget when the project runs under budget. The penalties β and γ are the opportunity costs of the organization. In Hu and Szmerekovsky (2016), B is the fixed budget for the entire portfolio, b_i is the project budget for project i, and C_i is the uncertain cost which follows a normal distribution and has a cumulative distribution function $F_i(c_i)$ with mean μ_i and standard deviation σ_i. Then, based on Proposition 1 in Hu and Szmerekovsky (2016), the portfolio manager's ideal budget b_i^* satisfies $b_i^* = \mu_i + \sigma_i Z_0$, where $Z_0 = \Phi^{-1}\left(\frac{\beta}{\beta+\gamma}\right)$, the optimal standard score for minimizing the over and under budget cost of project i. According to Proposition 2 in Hu and Szmerekovsky (2016), if $B \geq \sum_{i=1}^{n} b_i^*$, then project i receives a budget b_i that is equivalent to the portfolio manager's ideal b_i^*; if $B < \sum_{i=1}^{n} b_i^*$, then project i receives a budget as follows:

$$b_i = \mu_i + \frac{\sigma_i}{\sum_{j=1}^{n} \sigma_j}\left(B - \sum_{i=1}^{n} \mu_i\right). \tag{3.3}$$

Note that Expression (3.3) becomes the portfolio manager's ideal budget b_i^* when fixed budget B is binding ($B < \sum_{i=1}^{n} b_i^*$).

We use the same parameters as Hu and Szmerekovsky (2016), B is the fixed budget for the entire portfolio, b_i is the project budget that the portfolio manager allocates to project i, C_i is the uncertain cost, and b_i^* is the portfolio manager's ideal budget. In addition, b_i' is the project managers' ideal budget which satisfies $b_i' = \mu_i + \sigma_i Z_i$, where $Z_i = \Phi^{-1}(\frac{k_i}{k_i+l_i})$ as in Proposition 1 in Hu and Szmerekovsky (2016); b_i'' is the project managers' expected budget; X_i is the proposed mean project cost from the project manager.

There are three basic assumptions in our process that help us derive the value of the coefficients: (1) The uncertain project cost C_i follows a normal distribution with cumulative density function $F_i(c_i)$, mean μ_i, and standard deviation σ_i; (2) The portfolio manager has perfect knowledge about the value of the standard deviation of project cost for each project, but has no knowledge about the mean; (3) Each project manager reports his/her mean cost X_i to the portfolio manager assuming other managers report their real mean. We make Assumptions (2) and (3) due to two facts. First, in most cases, due to the difficulty of understanding the range of a project cost distribution (standard deviation), project managers are more likely to mispresent the central tendency (mean) of the project cost distribution. Therefore, the portfolio manager trusts the proposed standard deviation but questions the proposed mean. Based on this fact, we make Assumption (2). Second, projects within the same portfolio are likely to make use of similar technical expertise allowing project managers to have shared knowledge and understanding of each other's projects. Based on this fact, we make Assumption (3).

If B is large enough for all project managers to receive ideal budgets ($B \geq \sum_{i=1}^{n} b_i'$) with all reported costs ($B \geq \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$), then each project receives the budget b_i that is equal to the project manager's expected budget b_i'' and equals $X_i + \sigma_i Z_0$ ($b_i = b_i'' = X_i + \sigma_i Z_0$), the ideal budget from the portfolio manager's perspective. As stated earlier, project managers may build budgetary slack to improve their performance regarding their budget attainment, so they may propose a higher mean X_i than the real mean μ_i. As a result, the portfolio manager has to estimate the allocated budget based on the proposed mean of the uncertain cost X_i instead of the real mean μ_i. If B is binding for reported costs ($B < \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$) but enough for all project managers' ideal budgets ($B \geq \sum_{i=1}^{n} b_i'$), then theoretically, the portfolio manager allocates the budget $b_i = X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}(B - \sum_{i=1}^{n} X_i)$ as in Proposition 2 in Hu and Szmerekovsky (2016) while the project managers' expected budgets $b_i'' = X_i + \sigma_i Z_0$. However, this case rarely happens in reality due to a shortage of funds. If B is binding for all project managers' ideal budgets ($B < \sum_{i=1}^{n} b_i'$) but enough for reported costs ($B \geq \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$), then the project managers expect to receive the budget $b_i'' = X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}(B - X_i - \sum_{j \neq i} \mu_j)$ due to Assumption (2) while the portfolio manager allocates the budget $b_i = X_i + \sigma_i Z_0$. This is because the project managers expect the portfolio manager to make budget allocation by using X_i as the mean for themselves and μ_j ($j \neq i$) for all other projects. Nonetheless, this case is rather rare as well, because the project managers are more likely to propose higher budgets to acquire their ideal budget when they are aware of the scarcity of funds. If B is binding for all project managers' ideal budgets ($B < \sum_{i=1}^{n} b_i'$) and reported costs ($B < \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$), the project managers' expected budgets are $b_i'' = X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}(B - X_i - \sum_{j \neq i} \mu_j)$ while the portfolio manager allocates the budgets $b_i = X_i + \frac{\sigma_i}{\sum_{i=1}^{n}\sigma_i}(B - \sum_{i=1}^{n} X_i)$. Thus, we skip the second and third situations because they are uncommon. The results of the first and fourth situations are summarized as follows.

Proposition 1: Given uncertain cost C_i follows a normal distribution, there are two cases: (1) if $B \geq \sum_{i=1}^{n} b_i'$ and $B \geq \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$, then $b_i = b_i'' = X_i + \sigma_i Z_0$; (2) otherwise if $B < \sum_{i=1}^{n} b_i'$ and $B < \sum_{i=1}^{n}(X_i + \sigma_i Z_0)$, then $b_i = X_i + \frac{\sigma_i}{\sum_{i=1}^{n}\sigma_i}(B - \sum_{i=1}^{n} X_i)$ while $b_i'' = X_i + \frac{\sigma_i}{\sum_{i=1}^{n}\sigma_i}(B - X_i - \sum_{j\neq i}\mu_j)$.

The project managers perceive the allocated budget as the optimal budget when the allocated budget is equivalent to the project manager's ideal budget. Now consider Case (2) in Proposition 1 above where the project managers attempt to receive their ideal budget b_i' by proposing an X_i greater than the actual mean, even though they know the total budget is binding ($B < \sum_{i=1}^{n} b_i'$). However, it is unrealistic that all project managers receive their ideal budget when the total budget is binding. Therefore, in this extreme case in which all project managers expect to receive their ideal budget, their expected budget b_i'' is equivalent to their ideal budget b_i', which results in

$$X_i + \frac{\sigma_i}{\sum_{i=1}^{n}\sigma_i}\left(B - X_i - \sum_{j\neq i}\mu_j\right) = \mu_i + \sigma_i Z_i. \tag{3.4}$$

Based on this equation, X_i is calculated as (derivation is shown in Lemma 1 of Appendix A)

$$X_i = \frac{\sum_{i=1}^{n}\sigma_i}{\sum_{j\neq i}\sigma_j}(\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j\neq i}\sigma_j}\left(B - \sum_{j\neq i}\mu_j\right). \tag{3.5}$$

After solving X_i in our model, the actual allocated budget b_i is then calculated with X_i as (derivation is shown in Lemma 2 of Appendix A)

$$b_i = \left(1 - \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\right)\mu_i + \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right) + \sigma_i Z_i$$

$$- \frac{\rho_i}{\rho_i + 1}\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j, \tag{3.6}$$

where $\rho_i = \frac{\sigma_i}{\sum_{j \neq i} \sigma_j}$.

Note that the budget allocation in Case (1) in Proposition 1 is skipped here, because it is more relevant to the next section. It will be discussed in Section 3.3 when identifying coefficients k_i and l_i.

3.3. Identifying the Penalty Coefficients

This section shows the process of identifying the values of k_i and l_i. Each subsection starts with determining the value of Z_i since $Z_i = \Phi^{-1}(\frac{k_i}{k_i + l_i})$ is the only parameter that contains k_i and l_i, and then identifies the ratio between k_i and l_i instead of the specific values for k_i and l_i since Z_i is determined by the value of $\frac{k_i}{k_i + l_i}$.

This section consists of four subsections. The portfolio manager and the project managers may have divergent preferences or goals when asymmetric information is present, which results in divergent judgments of the optimal budget allocation (Antle and Eppen, 1985). Thus, subsections 3.3.1 and 3.3.2 describe the specific judgments of the two parties regarding the optimal budget allocation. Subsection 3.3.3 demonstrates the process to identify the value of Z_i that makes project managers propose the true mean ($X_i = \mu_i$). Subsection 3.3.4 summarizes the results in the first three subsections.

3.3.1. Portfolio manager

The portfolio manager assumes the budget allocation is optimal when the actual allocated budget b_i is equivalent to the portfolio manager's ideal budget b_i^*.

Now consider a special case where total budget B is not binding. Based on Proposition 1, if B is large enough, then the allocated budget $b_i = X_i + \sigma_i Z_0$, while the portfolio manager's ideal budget $b_i^* = \mu_i + \sigma_i Z_0$. The project managers propose X_i to the portfolio manager, and the project managers' expected budgets $b_i'' = X_i + \sigma_i Z_0$, while their ideal budgets $b_i' = \mu_i + \sigma_i Z_i$, where b_i' is the optimal budget that minimize the over and under budget cost. In order to receive their ideal budgets, the project managers propose $X_i = \mu_i + \sigma_i(Z_i - Z_0)$. Calculating the actual allocated budgets b_i using this X_i, the portfolio manager uses $b_i = \mu_i + \sigma_i Z_i$. Z_i is equivalent to Z_0 when the actual allocated budget b_i is equivalent to the portfolio manager's ideal budget b_i^*. Therefore, the ratio between k_i and l_i is equivalent to the ratio between β and γ ($\frac{k_i}{l_i} = \frac{\beta}{\gamma}$), where β and γ are the per unit over budget and under budget costs for the organization (portfolio manager), respectively (Hu and Szmerekovsky, 2016).

Next consider the general case where B is binding for all project managers' ideal budgets and all reported costs. The portfolio manager considers the allocated budget b_i to be optimal when b_i is equivalent to the portfolio manager's ideal budget b_i^*. Hence, setting b_i equal to b_i^* and calculating the value of Z_i that satisfies this equation determines the ratio of k_i and l_i.

In this case, the actual allocated budget b_i is given by (3.6) and the portfolio manager's ideal budget b_i^* is given by (3.3). (3.3) can be rewritten using ρ_i as

$$b_i^* = \frac{1}{\rho_i + 1}\mu_i + \frac{\rho_i}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right). \tag{3.7}$$

When b_i equals b_i^*, we obtain (derivation is shown in Lemma 3 of Appendix A)

$$Z_i = \sum_{j \neq i} \frac{\sigma_j Z_j}{\sum_{h \neq i} \sigma_h} + \frac{B - \sum_{i=1}^{n} \mu_i}{\sum_{i=1}^{n} \sigma_i} \left(1 - \sum_{j \neq i} \frac{\sigma_j}{\sum_{h \neq i} \sigma_h} \right). \tag{3.8}$$

Hence, the value of Z_i satisfies (3.8), which has an infinite number of solutions. As a result, the ratio between k_i and l_i has an infinite number of solutions as well. In this case, any ratio between k_i and l_i that makes the value of Z_i that satisfy (3.8) would make the actual allocated budget equivalent to the portfolio manager's ideal budget, which satisfies the portfolio manager's optimal budget allocation.

3.3.2. Project manager

The project managers want the actual allocated budget b_i to be equivalent to their ideal budgets b_i', so they attempt to receive their ideal budget b_i', even though they know that the total budget B is binding. This subsection describes the derivation for the ratio between k_i and l_i that optimizes the budget allocation from the project managers' perspectives. Since the special case where B is not binding has already been discussed in the previous subsection, this subsection focuses on the case where the project managers are aware of the shortage of the total budget B.

The actual allocated budget b_i is given by (3.6) and the project manager's ideal budget b_i' is

$$b_i' = \mu_i + Z_i \sigma_i. \tag{3.9}$$

When b_i equals b_i', we have

$$\left(1 - \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\right) \mu_i + \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1} \left(B - \sum_{j \neq i} \mu_j \right) + \sigma_i Z_i - \frac{\rho_i}{\rho_i + 1} \sum_{j \neq i} (\rho_j + 1) \sigma_j Z_j = \mu_i + Z_i \sigma_i. \tag{3.10}$$

The value of Z_i can be determined using the Gaussian Elimination Method in Shores (2007) and then substituting $\rho_i = \frac{\sigma_i}{\sum_{j \neq i}^{n} \sigma_j}$ to obtain (derivation is shown in Lemma 4 of Appendix A)

$$Z_i = \frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \sigma_j}. \tag{3.11}$$

Since $Z_i = \Phi^{-1}(\frac{k_i}{k_i + l_i})$, the ratio between k_i and l_i is

$$\frac{k_1}{l_1} = \frac{k_2}{l_2} = \cdots = \frac{k_n}{l_n} = \frac{\Phi\left(\frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \sigma_j}\right)}{1 - \Phi\left(\frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \sigma_j}\right)}. \tag{3.12}$$

In this case, the ratio between k_i and l_i that satisfies Expression (3.12) would make the actual allocated budget equivalent to the project manager's ideal budget.

3.3.3. Analysis of the value of Z_i regarding truth-telling

As previously stated, the value of Z_i in (3.11) would make the allocated budget satisfy the project managers' optimal budget allocations. In this subsection, we are interested in analyzing a value of Z_i that makes the project managers report the real expected cost (mean) to the portfolio manager. In other words, we are interested in determining the value of Z_i that makes the proposed mean X_i equal to μ_i, which requires

$$\begin{aligned} X_i &= \frac{\sum_{j=1}^{n} \sigma_j}{\sum_{j \neq i} \sigma_j} (\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j \neq i} \sigma_j} \left(B - \sum_{j \neq i} \mu_j \right) \\ &= \mu_i. \end{aligned} \tag{3.13}$$

Based on this equation, the value of Z_i is as in (3.11) (derivation is shown in Lemma 5 of Appendix A).

This result is not surprising because it is reasonable that the value of Z_i that makes project managers receive their ideal budget also incentives them to propose the real expected cost to their superior.

3.3.4. Discussion

For the special case where the total budget B is not binding, the budget allocation is optimal when the ratio between the over budget penalty k_i and the under budget penalty l_i is equal to the ratio between the organizational opportunity costs β and γ ($\frac{k_i}{l_i} = \frac{\beta}{\gamma}$). For the two cases when the total budget is binding, any value of Z_i that satisfies (3.8) will make the budget allocation optimal for the portfolio manager, while the value of Z_i that satisfies (3.11) is the only option to optimize the budget allocation for the project managers. Additionally, the value of Z_i in (3.11) is also the optimal value to motivate the project managers to report real project information (truth telling). Therefore, solving for the value of Z_i which satisfies (3.8) and (3.11), we obtain the ratio between k_i and l_i that satisfies (3.12).

However, it is impossible that each project manager receives their ideal budget when the total budget is binding. Hence, in practice if the portfolio manager experiences a budget which is binding it must be the case that the ratio between k_i and l_i exceeds the ideal in (3.12) for at least one project manager. That is, for some project manager h

$$\frac{k_h}{l_h} > \frac{\Phi(\frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \sigma_j})}{1 - \Phi(\frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \sigma_j})}. \tag{3.14}$$

The ratio between the over budget penalty k_h and the under budget penalty l_h is larger than 1 when the total budget B is moderately tight ($B > \sum_{i=1}^{n} \mu_i$), so k_h is larger than l_h, which gives the project managers an incentive to create budgetary slack. On the other hand, the lower bound is smaller than 1 when the total budget B is extremely tight ($B < \sum_{i=1}^{n} \mu_i$), so the ratio between k_h and l_h can be smaller than 1. Thus, k_h is smaller than l_h. This encourages the project managers to propose a smaller mean, therefore, increasing the likelihood of going over budget. In either case, to induce the project manager to tell the truth, the portfolio manager needs to reduce the ratio $\frac{k_h}{l_h}$ by either reducing k_h or increasing l_h. This result is counter intuitive as it suggests reducing over budget penalties and/or increasing under budget penalties when budgets are tight. Intuitively, one would expect that being over (under) budget would be a greater (lesser) concern when budgets are tight, but our results show that using coefficients which reflect this will only encourage dishonesty between project managers and portfolio managers, whereas doing the reverse will reduce the incentives for dishonesty.

This leads to our primary recommendation for portfolio managers. Initially, coefficients should be set to reflect the interests of the organization the portfolio manager represents, $k_i = \beta$ and $l_i = \gamma$. If this does not result in tight budgets with project managers misrepresenting their budgetary needs to gain budgetary slack, then the coefficients can be left unchanged. Otherwise, over budget penalties should be reduced and/or under budget penalties increased until the project managers are induced to tell the truth. The resulting truth-inducing inventive scheme for the general case of *n* projects is

$$R_i = \begin{cases} W_i - k_i(C_i - b_i), & if\ C_i \geq b_i \\ W_i - l_i(b_i - C_i), & if\ C_i < b_i \end{cases}, \tag{3.15}$$

where $\frac{k_i}{l_i} = \frac{\Phi(\frac{B-\sum_{i=1}^{n}\mu_i}{\sum_{i=1}^{n}\sigma_i})}{1-\Phi(\frac{B-\sum_{i=1}^{n}\mu_i}{\sum_{i=1}^{n}\sigma_i})}$ or $\frac{k_i}{l_i} = \frac{\beta}{\gamma}$.

4. SIMULATION STUDY

To further understand the impact of the ratio between k_i and l_i, we conducted a simulation study using the statistical software R. The purpose of this simulation study is to investigate the influence of both the coefficients' ratio and the amount of the total budget on the project managers' proposed budget and the budget allocation.

The exact parameter values we use are described in Table 1. Expected project costs (mean) μ_i and the standard deviations of project cost σ_i are randomly generated for six different sets with varying numbers of projects under the given conditions, each including five thousand problem instances. To manipulate the ratio between the two coefficients, the under budget penalty l_i is fixed at 1, while the over budget penalty k_i varies over eight values. Four values of k_i are smaller than l_i which indicates the extremely tight budget case ($B < \sum_{i=1}^{n} \mu_i$) and other four values of k_i are larger than l_i which indicates the moderately tight budget case ($B > \sum_{i=1}^{n} \mu_i$). Values of Z_i are calculated using k_i and l_i. To examine the effect of the amount of the total budget on the budget allocation, we set the total budget to three different amounts. In addition, as mentioned in Section 3.3, the relationship between the total budget and the sum of the real means has direct impact on the ratio between the over and under budget penalty coefficients. Thus, we consider the two situations separately. When the total budget is extremely tight ($B < \sum_{i=1}^{n} \mu_i$), the total budget is 30%, 50%, and 70% of the amount needed to fund all projects at the ideal level based on the standard score Z_i ($B = a * \sum_{i=1}^{n} b_i'$, $a = 0.3, 0.5, 0.7$). In contrast, when the total budget is moderately tight ($B > \sum_{i=1}^{n} \mu_i$), the total budget is calculated as 30%, 50%, and 70% of the amount over the expected cost needed to fund all projects at the ideal level ($B = \sum_{i=1}^{n} \mu_i + c * Z * \sum_{i=1}^{n} \sigma_i$, $c = 0.3, 0.5, 0.7$). The proposed mean X_i, the actual allocated budget b_i, and the portfolio manager's ideal budget b_i^* are then calculated according to the generated expected project costs and the standard deviations, the values of Z_i, and the total budget. Two slack rates are used to interpret the impact of the value of the coefficients' ratio and the amount of the total budget on the project managers' proposed mean and the budget allocation. Slack rate 1 (SR1) is calculated by the difference between the proposed mean X_i and the real mean μ_i over the real mean ($\frac{X_i - \mu_i}{\mu_i}$). Slack rate 2 (SR2) is the difference between the actual allocated budget b_i and the portfolio manager's ideal budget b_i^* over the portfolio manager's ideal budget b_i^* ($\frac{b_i - b_i^*}{b_i^*}$). SR1 tests the impact on the project managers' behavior while SR2 tests the impact on the actual budget allocation.

Table 1

Parameter Values Used for Simulation Study

Parameter	Values
Number of projects (n)	2, 3, 5, 10, 15, and 20
Penalty per dollar over budget (k_i)	0.1, 0.25, 0.5, and 0.8, when $B < \sum_{i=1}^{n} \mu_i$; 1.25, 2, 4, and 10, when $B > \sum_{i=1}^{n} \mu_i$.
Penalty per dollar under budget (l_i)	Fixed l_i as 1, to control the ratio.
Expected project cost (μ_i)	Randomly generated from a continuous uniform [20, 25] distribution.
Standard deviation of project cost (σ_i)	Randomly generated from a continuous uniform [$0.2\mu_i$, $0.3\mu_i$] distribution.
Total budget (B)	30%, 50%, and 70% of the amount needed to fund all projects at the ideal level when $B < \sum_{i=1}^{n} \mu_i$; 30%, 50%, and 70% of the amount of the standard deviation to fund all projects at the ideal level when $B > \sum_{i=1}^{n} \mu_i$.

5. RESULTS

This section discusses the results from our simulation study. Complete results are shown in Tables B1 and B2 in Appendix B.

For both moderately tight and extremely tight budgets the overall trend for SR1 is the same and can be seen in Figures 1 and 2. As suggested by our analysis, smaller over budget penalties lead to less misrepresentation of budgetary needs by project managers. Also of note is the degree to which project managers exaggerate their budgetary needs. Though the SR1 values are unsurprisingly higher with extremely tight budgets, that project managers can request more than double the actual project needs on average is surprising. Even with moderately tight budgets inflating expected costs by nearly 50% on average was observed. These effects are strongest for smaller portfolio with fewer projects.

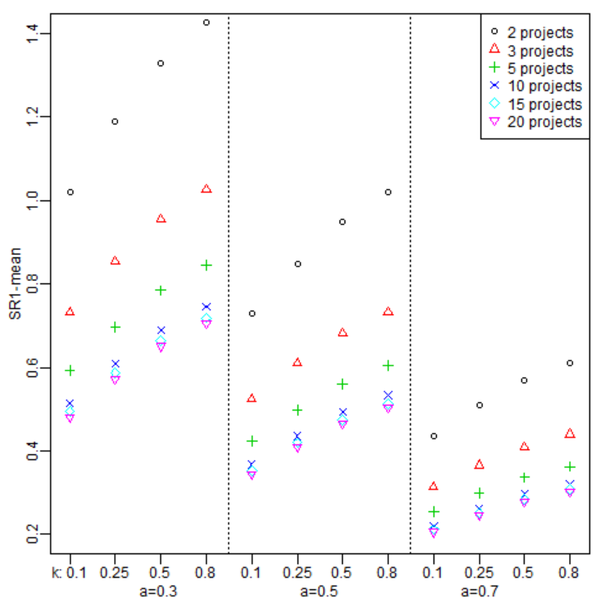

Figure 1. SR1 Mean with Extremely Tight Budget ($B < \sum_{i=1}^{n} \mu_i$)

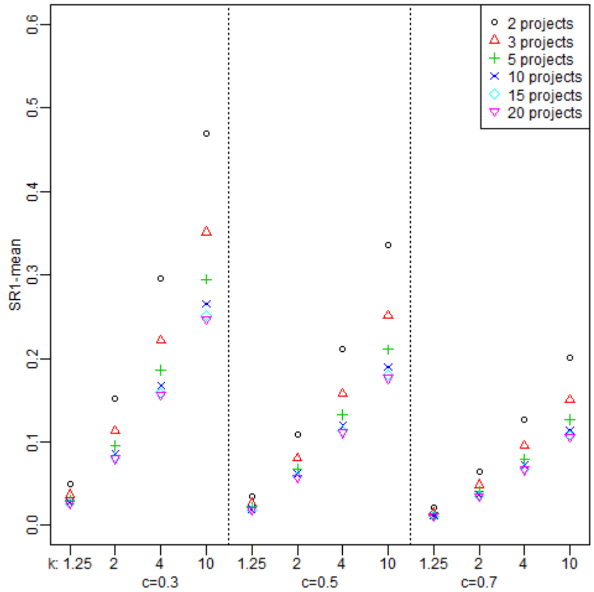

Figure 2. SR1 Mean with Moderately Tight Budget ($B > \sum_{i=1}^{n} \mu_i$)

For SR2 we see the trend reverse as can be seen in Figures 3 and 4. That is, smaller over budget penalties lead to greater budget misallocations even though the project managers exaggerate the budgetary needs less. This effect occurs because large exaggerations of budgetary needs lead to an overall inflated budgetary need. Relative to the overall inflated budgetary needs the individual projects inflated mean costs have less substantial impact on the budget allocation. Again, the degree to which the allocated budget can deviate from the ideal is worth noting. Though smaller than the SR1 values, substantial average slack rates close to 17% were observed for SR2. As with SR1, the effects are strongest with smaller portfolios.

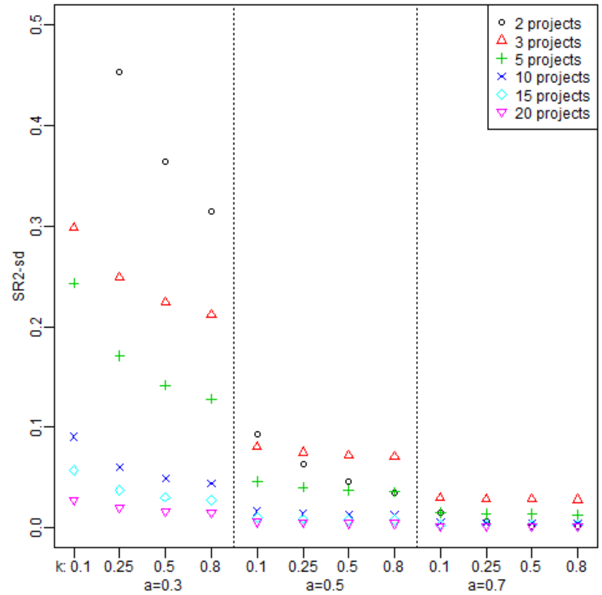

Figure 3. SR2 Standard Deviation with Extremely Tight Budget ($B < \sum_{i=1}^{n} \mu_i$)

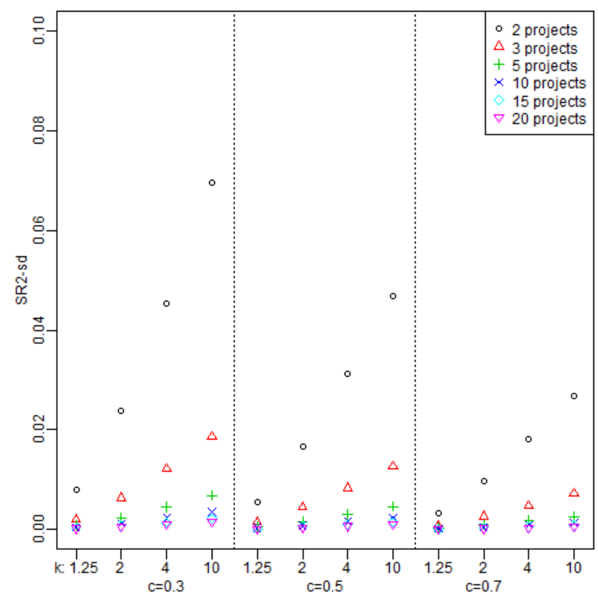

Figure 4. SR2 Standard Deviation with Moderately Tight Budget ($B > \sum_{i=1}^{n} \mu_i$)

In conclusion, a smaller over budget would stimulate project managers to report the accurate expected costs to the portfolio manager.

6. CONCLUSION AND FUTURE RESEARCH

The central problem we investigate in this study is to identify the value of key coefficients in a truth-inducing incentive scheme for portfolio managers to use when allocating budgets across projects. We have shown that these coefficients should initially be set equal to organizational costs for being over or under budget. However, if this results in project managers misrepresenting project budgetary needs to inflate their budgets, over budget penalties should be reduced and/or under budget penalties increased until the project managers are induced to tell the truth. A simulation study confirmed our results and highlighted the large amounts of slack that can result from using the wrong coefficients.

Future research could explore relaxing the assumptions in our model. Specifically, the portfolio manager's knowledge of standard deviations and the project managers' knowledge of each other's costs can be relaxed to add more information asymmetry to the model. In addition, the assumption that project managers assume other project managers report their true expected costs can be relaxed to create a more complex and realistic dynamic.

REFERENCES

[1] Antle, R. & Eppen, G. D. (1985). Capital rationing and organizational slack in capital budgeting. *Management Science, 31*(2), 163.

[2] Brown, J. L., Evans J. H. III & Moser, D. V. (2009). Agency theory and participative budgeting experiments. *Journal of Management Accounting Research, 21*(1), 317-345.

[3] Basso, A. & Peccati, L.A. (2001). Optimal resource allocation with minimum activation levels and fixed costs. *European Journal of Operational Research, 131*(3), 536-549.

[4] Chong, V. K. & Eggleton, I. R. C. (2007). The impact of reliance on incentive-based compensation schemes, information asymmetry and organizational commitment. *Management Account Research*, *18*(3), 312-342.

[5] Chow, C. W., Cooper, J. C., & Haddad, K. (1991). The effects of pay schemes and ratchets on budgetary slack and performance: A multi-period experiment. *Accounting, Organizations and Society, 16*(1), 47.

[5] Chow, C. W., Cooper, J. C., & Waller, W. S. (1988). Participative budgeting: Effects of a truth-inducing pay scheme and information asymmetry on slack and performance. *The Accounting Review*, *63*(1), 111-122.

[6] Dadbeh, F. & Mogharebi, N. (2013). A study on effect of information asymmetry on earning management: Evidence from Tehran stock exchange. *Management Science Letter*, *3*(7), 2161-2166

[7] Dunk, A.S. & Nouri, H. (1998). Antecedents of budgetary slack: A literature review and synthesis. *Journal of Accounting Literature, 17*, 72-96.

[8] Engwall, M. & Jerbrant, A. (2003). The resource allocation syndrome: The prime challenge of multi-project management? *International Journal of Project Management, 21*(6), 401-409.

[9] Fisher, J. G., Frederickson, J. R. & Peffer, S. A. (2002). The effect of information asymmetry on negotiated budgets: An empirical investigation. *Accounting, Organizations and Society*, *27*(1), 27-43

[10] Fisher, J. G., Maines, L. A., Peffer, S. A. & Sprinkle, G. B. (2002). Using budgets for performance evaluation: Effects of resource allocation and horizontal information asymmetry on budget proposals, budget slack, and performance. *The Accounting Review*, *77*(4), 847-865.

[11] Hobson, J. L., Mellon, M. J., & Stevens, D. E. (2011). Determinants of moral judgments regarding budgetary slack: An experimental examination of pay scheme and personal values. *Behavioral Research in Accounting*, *23*(1), 87-107.

[12] Hu, Q., & Szmerekovsky, J. (2016). Project Portfolio Selection: A newsvendor Approach. *Decision Sciences*, to appear.

[13] Kren, L. & Maiga, A. (2007). The intervening effect of information asymmetry on budget participation and segment slack. *Advances in Management Accounting 16*, 141-157, Emerald Group Publishing Limited.

[14] Kren, L., & Liao, W. M. (1988). Role of accounting information in the control of organizations: A review of the evidence. *Journal of Accounting Literature*, *7*, 280-309.

[15] LaBrosse, M. (2010). Project-portfolio management. *Employment Relation Today*, *37*(2), 75-79.

[16] Meskendahl, S. (2010). The influence of business strategy on project portfolio management and its success- a conceptual framework. *International Journal of Project Management*, *28*(8), 807-817.

[17] Rasmußen, A. (2015). Reporting behavior: A literature review of experimental studies. *Central European Journal of Operations Research*, *23*(2), 283-311.

[18] Shores, T. S. (2007). Applied linear algebra and matrix analysis. Springer Science & Business Media.

[18] Stevens, E. D. (2000). Determinants of budgetary slack in the laboratory: An Investigation of controls for self-interested behavior (March 2000). Available at SSRN: http://ssrn.com/abstract=219131 orhttp://dx.doi.org/10.2139/ssrn.219131

[20] Weitzman, M. (1976). The new soviet incentive model, *Bell Journal of Economics*, *7*(1), 458-476.

[21] Waller, W. S. (1988). Slack in participative budgeting: The joint effect of a truth-inducing pay scheme and risk preferences. *Accounting, Organizations and Society, 13*(1) 87-98.

APPENDIX A. PROOFS OF LEMMAS

In Appendix A, we show the derivation of key parameters in our work.

Lemma 1. The derivation for the proposed mean X_i. From (3.4) we have the following:

$$X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - X_i - \sum_{j\neq i} \mu_j\right) = \mu_i + \sigma_i Z_i,$$

$$\frac{\sum_{j\neq i} \sigma_i}{\sum_{i=1}^{n} \sigma_i} X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - \sum_{j\neq i} \mu_j\right) = \mu_i + \sigma_i Z_i,$$

$$\frac{\sum_{j\neq i} \sigma_i}{\sum_{i=1}^{n} \sigma_i} X_i = \mu_i + \sigma_i Z_i - \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - \sum_{j\neq i} \mu_j\right),$$

$$X_i = \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j\neq i} \sigma_i}(\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j\neq i} \sigma_i}\left(B - \sum_{j\neq i} \mu_j\right).$$

Lemma 2. The derivation for actual allocated budget b_i in (3.6). As indicated in Proposition 1,

$$b_i = X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - \sum_{i=1}^{n} X_i\right),$$

where $X_i = \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j\neq i} \sigma_i}(\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j\neq i} \sigma_i}(B - \sum_{j\neq i} \mu_j)$ as proved in Lemma 1.

Replace $\frac{\sigma_i}{\sum_{j\neq i} \sigma_i}$ by using ρ_i,

$$X_i = (\mu_i + \sigma_i Z_i)(\rho_i + 1) - \rho_i(B - \sum_{j\neq i} \mu_j) = (\mu_i + \sigma_i Z_i)(\rho_i + 1) - \rho_i[B - (\sum_{j=1}^{n} \mu_j - \mu_i)]$$

$$= \mu_i + (\rho_i + 1)\sigma_i Z_i - \rho_i\left(B - \sum_{i=1}^{n} \mu_i\right).$$

Replace the X_i to calculate b_i,

$$b_i = X_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - \sum_{i=1}^{n} X_i\right) = X_i + \frac{\rho_i}{\rho_i + 1}\left(B - \sum_{i=1}^{n} X_i\right)$$

$$= \mu_i + (\rho_i + 1)\sigma_i Z_i - \rho_i(B - \sum_{i=1}^{n} \mu_i) + \frac{\rho_i}{\rho_i + 1}[B - \sum_{i=1}^{n}[\mu_i + (\rho_i + 1)\sigma_i Z_i - \rho_i(B - \sum_{i=1}^{n} \mu_i)]]$$

$$= \mu_i + (\rho_i + 1)\sigma_i Z_i + \frac{\rho_i}{\rho_i + 1}(\sum_{j=1}^{n} \rho_j - \rho_i)(B - \sum_{i=1}^{n} \mu_i) - \frac{\rho_i}{\rho_i + 1}\sum_{i=1}^{n}(\rho_i + 1)\sigma_i Z_i$$

$$= \mu_i + \sigma_i Z_i + \frac{\rho_i(\sum_{j=1}^{n} \rho_j - \rho_i)}{\rho_i \ + 1}(B - \sum_{i=1}^{n} \mu_i) - \frac{\rho_i}{\rho_i + 1}[\sum_{j=1}^{n}(\rho_j + 1)\sigma_j Z_j - (\rho_i + 1)\sigma_i Z_i]$$

$$= \mu_i + \sigma_i Z_i + \frac{\rho_i(\sum_{j \neq i} \rho_j)}{\rho_i \ + 1}(B - \sum_{i=1}^{n} \mu_i) - \frac{\rho_i}{\rho_i + 1}[\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j]$$

$$= \left(1 - \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\right)\mu_i + \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right) + \sigma_i Z_i - \frac{\rho_i}{\rho_i + 1}\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j$$

Lemma 3. Derivation of Z_i in (3.8) when $b_i = b_i^*$.

Replace $\frac{\sigma_i}{\sum_{j \neq i} \sigma_i}$ by using ρ_i in b_i^*,

$$b_i^* = \mu_i + \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}\left(B - \sum\nolimits_{i=1}^{n} \mu_i\right) = \frac{1}{\rho_i + 1}\mu_i + \frac{\rho_i}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right),$$

$$b_i = \left(1 - \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\right)\mu_i + \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right) + \sigma_i Z_i - \frac{\rho_i}{\rho_i + 1}\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j$$

$$= b_i^* = \frac{1}{\rho_i + 1}\mu_i + \frac{\rho_i}{\rho_i + 1}\left(B - \sum_{j \neq i} \mu_j\right)\left[1 - \frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1} - \frac{1}{\rho_i + 1}\right]\mu_i +$$

$$\left[\frac{\rho_i \sum_{j \neq i} \rho_j}{\rho_i + 1} - \frac{\rho_i}{\rho_i + 1}\right]\left(B - \sum_{j \neq i} \mu_j\right) + \sigma_i Z_i - \frac{\rho_i}{\rho_i + 1}\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j = 0,$$

$$Z_i = \frac{\rho_i}{(\rho_i + 1)\sigma_i}\sum_{j \neq i}(\rho_j + 1)\sigma_j Z_j + \frac{1}{\sigma_i}\left[\frac{\rho_i\left(\sum_{j \neq i} \rho_j - 1\right)}{\rho_i + 1}\right]\left[B - \sum\nolimits_{i=1}^{n} \mu_i\right]$$

Replace ρ_i by using $\frac{\sigma_i}{\sum_{j \neq i} \sigma_i}$ in Z_i,

$$= \frac{1}{\sum_{i=1}^{n}\sigma_i}\left[\sum_{j\neq i}\frac{\sum_{j=1}^{n}\sigma_j}{\sum_{h\neq j}\sigma_h}\sigma_h Z_h\right] + \frac{1}{\sum_{i=1}^{n}\sigma_i}\left[1-\left(\sum_{j\neq i}\frac{\sigma_j}{\sum_{h\neq j}\sigma_h}\right)\right]\left(B-\sum\nolimits_{i=1}^{n}\mu_i\right)$$

$$\Longrightarrow Z_i = \sum_{j\neq i}\frac{\sigma_j Z_j}{\sum_{h\neq j}\sigma_h} + \frac{B-\sum_{i=1}^{n}\mu_i}{\sum_{i=1}^{n}\sigma_i}\left(1-\sum_{j\neq i}\frac{\sigma_j}{\sum_{h\neq j}\sigma_h}\right).$$

Lemma 4. Derivation of Z_i in (3.11) when $b_i = b_i'$.

$$b_i = \left(1-\frac{\rho_i\sum_{j\neq i}\rho_j}{\rho_i+1}\right)\mu_i + \frac{\rho_i\sum_{j\neq i}\rho_j}{\rho_i+1}\left(B-\sum_{j\neq i}\mu_j\right) + \sigma_i Z_i - \frac{\rho_i}{\rho_i+1}\sum_{j\neq i}(\rho_j+1)\sigma_j Z_j = b_i'$$

$$= \mu_i + Z_i\sigma\ ,$$

$$\Longrightarrow \left(\frac{\rho_i\sum_{j\neq i}\rho_j}{\rho_i+1}\right)\left(B-\sum\nolimits_{i=1}^{n}\mu_i\right) - \frac{\rho_i}{\rho_i+1}\sum_{j\neq i}(\rho_j+1)\sigma_j Z_j = 0.$$

Rewrite the equation using matrices:

$$C = \begin{pmatrix} \left(\frac{\rho_1(\rho_2+\rho_3+\cdots\rho_n)}{\rho_1+1}\right)\left(B-\sum\nolimits_{i=1}^{n}\mu_i\right) \\ \left(\frac{\rho_2(\rho_1+\rho_3+\cdots\rho_n)}{\rho_2+1}\right)\left(B-\sum\nolimits_{i=1}^{n}\mu_i\right) \\ \vdots \\ \left(\frac{\rho_n(\rho_1+\rho_2+\cdots\rho_{n-1})}{\rho_n+1}\right)\left(B-\sum\nolimits_{i=1}^{n}\mu_i\right) \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & \frac{\rho_1}{\rho_1+1}(\rho_2+1)\sigma_2 & \cdots & \frac{\rho_1}{\rho_1+1}(\rho_n+1)\sigma_n \\ \frac{\rho_2}{\rho_2+1}(\rho_1+1)\sigma_1 & 0 & \cdots & \frac{\rho_2}{\rho_2+1}(\rho_n+1)\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\rho_n}{\rho_n+1}(\rho_1+1)\sigma_1 & \frac{\rho_n}{\rho_n+1}(\rho_2+1)\sigma_2 & \cdots & 0 \end{pmatrix},$$

$$Z_i = \begin{pmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{pmatrix},$$

$$C - A * Z_i = 0.$$

Using Gaussian Elimination in Shores (2007).

$$Z_1 = Z_2 = \cdots = Z_n = \frac{B - \sum_{i=1}^{n} \mu_i}{\sum_{i=1}^{n} \sigma_i}.$$

Lemma 5. Derivation of Z_i in (3.13) when $X_i = \mu_i$.

$$X_i = \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j \neq i} \sigma_i} (\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j \neq i} \sigma_i} \left(B - \sum_{j \neq i} \mu_j \right) = \mu_i.$$

Simplify X_i as follows,

$$X_i = \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j \neq i} \sigma_i} (\mu_i + \sigma_i Z_i) - \frac{\sigma_i}{\sum_{j \neq i} \sigma_i} \left(B - \sum_{j \neq i} \mu_j \right)$$

$$= \mu_i + \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j \neq i} \sigma_i} \sigma_i Z_i - \frac{\sigma_i}{\sum_{j \neq i} \sigma_i} \left(B - \sum_{j=1}^{n} \mu_j \right).$$

When $X_i = \mu_i$, $\mu_i + \frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j \neq i} \sigma_i} \sigma_i Z_i - \frac{\sigma_i}{\sum_{j \neq i} \sigma_i} \left(B - \sum_{j=1}^{n} \mu_j \right) = \mu_i$.

$$\frac{\sum_{i=1}^{n} \sigma_i}{\sum_{j \neq i} \sigma_i} \sigma_i Z_i - \frac{\sigma_i}{\sum_{j \neq i} \sigma_i} (B - \sum_{j=1}^{n} \mu_j) = 0.$$

$$Z_i = \frac{B - \sum_{j=1}^{n} \mu_j}{\sum_{i=1}^{n} \sigma_i}.$$

APPENDIX B. SIMULATION RESULTS

Table B1

Simulation Results for $B < \sum_{i=1}^{n} \mu_i$

n	a	k_i	l_i	SR1-mean	SR1-sd	SR1-min	SR1-max	SR2-mean	SR2-sd	SR2-min	SR2-max
2	0.3	0.1	1	0.9486	0.1876	0.8159	1.0813	0.1641	0.6277	-0.2798	0.6079
2	0.3	0.25	1	1.2120	0.2185	0.9675	1.2765	0.1091	0.5582	-0.2856	0.5039
2	0.3	0.5	1	1.2691	0.2499	1.0924	1.4457	0.0884	0.5476	-0.2989	0.4756
2	0.3	0.8	1	1.3731	0.2747	1.1789	1.5673	0.0762	0.5501	-0.3128	0.4652
2	0.5	0.1	1	0.6763	0.1295	0.5847	0.7679	0.0232	0.2199	-0.1323	0.1787
2	0.5	0.25	1	0.8020	0.1598	0.6889	0.9150	0.0175	0.2229	-0.1401	0.1751
2	0.5	0.5	1	0.9063	0.1787	0.7799	1.0326	0.0129	0.2185	-0.1416	0.1674
2	0.5	0.8	1	0.9805	0.1935	0.8436	1.1173	0.0108	0.2181	-0.1435	0.1650
2	0.7	0.1	1	0.4058	0.0785	0.3504	0.4613	0.0048	0.0918	-0.0601	0.0697
2	0.7	0.25	1	0.4812	0.0952	0.4139	0.5485	0.0030	0.0934	-0.0631	0.0690
2	0.7	0.5	1	0.5441	0.1075	0.4681	0.6201	0.0018	0.0928	-0.0639	0.0674
2	0.7	0.8	1	0.5879	0.1134	0.5077	0.6681	0.0010	0.0908	-0.0632	0.0652
3	0.3	0.1	1	0.7047	0.1132	0.6002	0.8152	0.0813	0.4585	-0.2816	0.5456
3	0.3	0.25	1	0.8352	0.1346	0.7108	0.9664	0.0661	0.2769	-0.1465	0.3663
3	0.3	0.5	1	0.9439	0.1518	0.8033	1.0915	0.0488	0.2536	-0.1522	0.3201
3	0.3	0.8	1	1.0208	0.1641	0.8691	1.1807	0.0416	0.2449	-0.1542	0.3028
3	0.5	0.1	1	0.5031	0.0807	0.4285	0.5819	0.0129	0.0990	-0.0684	0.1172
3	0.5	0.25	1	0.5967	0.0958	0.5083	0.6903	0.0089	0.0957	-0.0710	0.1092
3	0.5	0.5	1	0.6742	0.1080	0.5739	0.7791	0.0066	0.0933	-0.0726	0.1035
3	0.5	0.8	1	0.7292	0.1171	0.6209	0.8434	0.0053	0.0938	-0.0744	0.1027
3	0.7	0.1	1	0.3019	0.0486	0.2570	0.3493	0.0025	0.0400	-0.0313	0.0441
3	0.7	0.25	1	0.3582	0.0573	0.3052	0.4141	0.0014	0.0393	-0.0323	0.0421
3	0.7	0.5	1	0.4045	0.0650	0.3445	0.4679	0.0009	0.0391	-0.0330	0.0409
3	0.7	0.8	1	0.4375	0.0701	0.3728	0.5059	0.0005	0.0389	-0.0333	0.0402
5	0.3	0.1	1	0.5849	0.0818	0.4897	0.6863	0.0383	0.3408	-0.3020	0.4808
5	0.3	0.25	1	0.6932	0.0968	0.5798	0.8125	0.0354	0.1393	-0.0775	0.2533
5	0.3	0.5	1	0.7836	0.1096	0.6551	0.9185	0.0257	0.1215	-0.0787	0.2113
5	0.3	0.8	1	0.8474	0.1180	0.7083	0.9928	0.0210	0.1137	-0.0800	0.1924
5	0.5	0.1	1	0.4177	0.0584	0.3494	0.4900	0.0065	0.0456	-0.0356	0.0742
5	0.5	0.25	1	0.4954	0.0692	0.4147	0.5812	0.0045	0.0435	-0.0373	0.0679
5	0.5	0.5	1	0.5597	0.0783	0.4684	0.6566	0.0033	0.0422	-0.0381	0.0637
5	0.5	0.8	1	0.6054	0.0852	0.5061	0.7106	0.0027	0.0417	-0.0390	0.0621
5	0.7	0.1	1	0.2507	0.0352	0.2094	0.2940	0.0013	0.0179	-0.0166	0.0267
5	0.7	0.25	1	0.2971	0.0419	0.2482	0.3488	0.0007	0.0177	-0.0173	0.0256
5	0.7	0.5	1	0.3358	0.0472	0.2808	0.3939	0.0004	0.0174	-0.0176	0.0245
5	0.7	0.8	1	0.3632	0.0512	0.3029	0.4262	0.0003	0.0174	-0.0180	0.0242
10	0.3	0.1	1	0.5191	0.0660	0.4263	0.6160	0.0134	0.1747	-0.2467	0.3000
10	0.3	0.25	1	0.6152	0.0780	0.5051	0.7296	0.0158	0.0600	-0.0353	0.1483

（Continued）

Table B1. *Simulation Results for* $B < \sum_{i=1}^{n} \mu_i$ *(continued)*

n	c	k_i	l_i	SR1-mean	SR1-sd	SR1-min	SR1-max	SR2-mean	SR2-sd	SR2-min	SR2-max
10	0.3	0.5	1	0.6951	0.0878	0.5708	0.8236	0.0111	0.0507	-0.0366	0.1179
10	0.3	0.8	1	0.7518	0.0953	0.6174	0.8917	0.0094	0.0480	-0.0373	0.1089
10	0.5	0.1	1	0.3707	0.0471	0.3041	0.4396	0.0029	0.0188	-0.0169	0.0404
10	0.5	0.25	1	0.4393	0.0556	0.3605	0.5209	0.0020	0.0178	-0.0176	0.0362
10	0.5	0.5	1	0.4965	0.0633	0.4069	0.5891	0.0015	0.0173	-0.0183	0.0342
10	0.5	0.8	1	0.5370	0.0680	0.4409	0.6367	0.0012	0.0170	-0.0186	0.0328
10	0.7	0.1	1	0.2224	0.0282	0.1826	0.2639	0.0006	0.0073	-0.0079	0.0143
10	0.7	0.25	1	0.2636	0.0335	0.2162	0.3125	0.0003	0.0071	-0.0082	0.0134
10	0.7	0.5	1	0.2979	0.0379	0.2445	0.3534	0.0002	0.0071	-0.0085	0.0130
10	0.7	0.8	1	0.3222	0.0408	0.2644	0.3821	0.0001	0.0070	-0.0086	0.0127
15	0.3	0.1	1	0.5002	0.0614	0.4076	0.5959	0.0164	0.0752	-0.0694	0.2069
15	0.3	0.25	1	0.5929	0.0728	0.4828	0.7058	0.0098	0.0371	-0.0231	0.1048
15	0.3	0.5	1	0.6699	0.0822	0.5457	0.7975	0.0070	0.0318	-0.0242	0.0842
15	0.3	0.8	1	0.7245	0.0890	0.5901	0.8628	0.0059	0.0300	-0.0248	0.0768
15	0.5	0.1	1	0.3573	0.0438	0.2913	0.4252	0.0018	0.0118	-0.0111	0.0284
15	0.5	0.25	1	0.4234	0.0520	0.3450	0.5042	0.0013	0.0111	-0.0117	0.0254
15	0.5	0.5	1	0.4785	0.0588	0.3896	0.5695	0.0009	0.0108	-0.0121	0.0238
15	0.5	0.8	1	0.5175	0.0637	0.4215	0.6164	0.0008	0.0106	-0.0124	0.0231
15	0.7	0.1	1	0.2144	0.0263	0.1747	0.2553	0.0004	0.0046	-0.0052	0.0100
15	0.7	0.25	1	0.2540	0.0312	0.2068	0.3022	0.0002	0.0045	-0.0055	0.0093
15	0.7	0.5	1	0.2871	0.0353	0.2338	0.3418	0.0001	0.0044	-0.0056	0.0090
15	0.7	0.8	1	0.3105	0.0381	0.2528	0.3692	0.0000	0.0044	-0.0057	0.0087
20	0.3	0.1	1	0.4913	0.0595	0.3987	0.5861	0.0135	0.0501	-0.0321	0.1686
20	0.3	0.25	1	0.5822	0.0704	0.4723	0.6944	0.0071	0.0267	-0.0173	0.0811
20	0.3	0.5	1	0.6580	0.0796	0.5337	0.7844	0.0052	0.0231	-0.0181	0.0656
20	0.3	0.8	1	0.7116	0.0860	0.5770	0.8485	0.0043	0.0217	-0.0186	0.0592
20	0.5	0.1	1	0.3509	0.0424	0.2847	0.4188	0.0014	0.0086	-0.0083	0.0222
20	0.5	0.25	1	0.4158	0.0502	0.3374	0.4959	0.0009	0.0081	-0.0088	0.0196
20	0.5	0.5	1	0.4700	0.0567	0.3813	0.5605	0.0007	0.0078	-0.0091	0.0183
20	0.5	0.8	1	0.5083	0.0614	0.4121	0.6061	0.0006	0.0077	-0.0093	0.0178
20	0.7	0.1	1	0.2105	0.0255	0.1707	0.2510	0.0003	0.0033	-0.0039	0.0077
20	0.7	0.25	1	0.2495	0.0302	0.2023	0.2977	0.0002	0.0032	-0.0041	0.0072
20	0.7	0.5	1	0.2820	0.0341	0.2286	0.3364	0.0000	0.0032	-0.0042	0.0069
20	0.7	0.8	1	0.3050	0.0368	0.2474	0.3636	0.0000	0.0032	-0.0043	0.0068

Table B2

Simulation Results for $B > \sum_{i=1}^{n} \mu_i$

n	c	k_i	l_i	SR1-mean	SR1-sd	SR1-min	SR1-max	SR2-mean	SR2-sd	SR2-min	SR2-max
2	0.3	1.25	1	0.0495	0.0097	0.0427	0.0564	-0.0001	0.0052	-0.0038	0.0004
2	0.3	2	1	0.1534	0.0302	0.1320	0.1747	-0.0003	0.0158	-0.0115	0.0108
2	0.3	4	1	0.2992	0.0575	0.2585	0.3399	-0.0007	0.0294	-0.0215	0.0201
2	0.3	10	1	0.4737	0.0919	0.4087	0.5386	-0.0011	0.0453	-0.0331	0.0309
2	0.5	1.25	1	0.0355	0.0069	0.0306	0.0404	-0.0001	0.0037	-0.0027	0.0025
2	0.5	2	1	0.1092	0.0211	0.0943	0.1242	-0.0003	0.0110	-0.0080	0.0075
2	0.5	4	1	0.2137	0.0416	0.1843	0.2432	-0.0005	0.0205	-0.0150	0.0139
2	0.5	10	1	0.3395	0.0659	0.2929	0.3861	-0.0009	0.0308	-0.0226	0.0209
2	0.7	1.25	1	0.0213	0.0041	0.0183	0.0242	-0.0000	0.0022	-0.0016	0.0015
2	0.7	2	1	0.0657	0.0131	0.0565	0.0750	-0.0001	0.0066	-0.0048	0.0045
2	0.7	4	1	0.1279	0.0251	0.1101	0.1456	-0.0003	0.0119	-0.0087	0.0081
2	0.7	10	1	0.2038	0.0396	0.1758	0.2318	-0.0005	0.0174	-0.0128	0.0117
3	0.3	1.25	1	0.0370	0.0059	0.0314	0.0427	-0.0000	0.0022	-0.0020	0.0021
3	0.3	2	1	0.1139	0.0182	0.0970	0.1316	-0.0002	0.0065	-0.0060	0.0064
3	0.3	4	1	0.2225	0.0358	0.1893	0.2573	-0.0003	0.0125	-0.0115	0.0122
3	0.3	10	1	0.3530	0.0566	0.3003	0.4079	-0.0006	0.0190	-0.0175	0.0185
3	0.5	1.25	1	0.0263	0.0043	0.0224	0.0305	-0.0000	0.0015	-0.0014	0.0015
3	0.5	2	1	0.0814	0.0131	0.0692	0.0941	-0.0001	0.0046	-0.0042	0.0046
3	0.5	4	1	0.1588	0.0256	0.1350	0.1836	-0.0003	0.0086	-0.0079	0.0084
3	0.5	10	1	0.2518	0.0402	0.2142	0.2907	-0.0004	0.0128	-0.0119	0.0124
3	0.7	1.25	1	0.0158	0.0025	0.0135	0.0183	-0.0000	0.0009	-0.0008	0.0009
3	0.7	2	1	0.0488	0.0078	0.0416	0.0563	-0.0001	0.0027	-0.0025	0.0026
3	0.7	4	1	0.0953	0.0154	0.0810	0.1103	-0.0002	0.0050	-0.0046	0.0049
3	0.7	10	1	0.1511	0.0242	0.1287	0.1747	-0.0003	0.0073	-0.0067	0.0071
5	0.3	1.25	1	0.0307	0.0043	0.0256	0.0359	-0.0000	0.0010	-0.0011	0.0013
5	0.3	2	1	0.0946	0.0131	0.0792	0.1107	-0.0001	0.0029	-0.0032	0.0038
5	0.3	4	1	0.1847	0.0258	0.1544	0.2164	-0.0002	0.0055	-0.0062	0.0073
5	0.3	10	1	0.2927	0.0411	0.2445	0.3433	-0.0003	0.0084	-0.0095	0.0111
5	0.5	1.25	1	0.0219	0.0031	0.0183	0.0257	-0.0000	0.0007	-0.0008	0.0009
5	0.5	2	1	0.0676	0.0095	0.0564	0.0792	-0.0001	0.0020	-0.0023	0.0027
5	0.5	4	1	0.1321	0.0185	0.1103	0.1547	-0.0001	0.0038	-0.0043	0.0049
5	0.5	10	1	0.2094	0.0293	0.1750	0.2553	-0.0002	0.0056	-0.0064	0.0073
5	0.7	1.25	1	0.0131	0.0018	0.0110	0.0154	-0.0000	0.0004	-0.0005	0.0005
5	0.7	2	1	0.0405	0.0057	0.0338	0.0475	-0.0000	0.0012	-0.0013	0.0015
5	0.7	4	1	0.0791	0.0111	0.0661	0.0927	-0.0001	0.0022	-0.0025	0.0028
5	0.7	10	1	0.1255	0.0175	0.1050	0.1470	-0.0001	0.0032	-0.0037	0.0041
10	0.3	1.25	1	0.0272	0.0034	0.0223	0.0322	-0.0000	0.0004	-0.0005	0.0007
10	0.3	2	1	0.0838	0.0106	0.0688	0.0993	-0.0000	0.0012	-0.0016	0.0020
10	0.3	4	1	0.1640	0.0208	0.1344	0.1942	-0.0001	0.0022	-0.0030	0.0037
10	0.3	10	1	0.2601	0.0329	0.2134	0.3081	-0.0001	0.0034	-0.0046	0.0057
10	0.5	1.25	1	0.0194	0.0025	0.0159	0.0230	-0.0000	0.0002	-0.0004	0.0005
10	0.5	2	1	0.0599	0.0076	0.0491	0.0709	-0.0000	0.0008	-0.0011	0.0014
10	0.5	4	1	0.1171	0.0149	0.0959	0.1387	-0.0001	0.0015	-0.0021	0.0025
10	0.5	10	1	0.1859	0.0234	0.1526	0.2201	-0.0001	0.0023	-0.0031	0.0038
10	0.7	1.25	1	0.0117	0.0015	0.0096	0.0138	-0.0000	0.0002	-0.0002	0.0003
10	0.7	2	1	0.0360	0.0046	0.0295	0.0426	-0.0000	0.0005	-0.0007	0.0008
10	0.7	4	1	0.0702	0.0089	0.0576	0.0832	-0.0000	0.0009	-0.0012	0.0015

（Continued）

Table B2. *Simulation Results for* $B > \sum_{i=1}^{n} \mu_i$*(continued)*

n	c	k_i	l_i	SR1-mean	SR1-sd	SR1-min	SR1-max	SR2-mean	SR2-sd	SR2-min	SR2-max
10	0.7	10	1	0.1115	0.0140	0.0915	0.1320	-0.0001	0.0013	-0.0018	0.0022
15	0.3	1.25	1	0.0262	0.0032	0.0213	0.0312	-0.0000	0.0002	-0.0003	0.0005
15	0.3	2	1	0.0809	0.0099	0.0658	0.0962	-0.0000	0.0007	-0.0011	0.0014
15	0.3	4	1	0.1579	0.0193	0.1286	0.1879	-0.0000	0.0014	-0.0020	0.0026
15	0.3	10	1	0.2507	0.0307	0.2039	0.2981	-0.0001	0.0021	-0.0031	0.0039
15	0.5	1.25	1	0.0187	0.0023	0.0152	0.0223	-0.0000	0.0002	-0.0002	0.0003
15	0.5	2	1	0.0578	0.0071	0.0470	0.0687	-0.0000	0.0005	-0.0007	0.0010
15	0.5	4	1	0.1128	0.0139	0.0918	0.1342	-0.0000	0.0010	-0.0014	0.0018
15	0.5	10	1	0.1790	0.0220	0.1456	0.2129	-0.0001	0.0014	-0.0021	0.0026
15	0.7	1.25	1	0.0112	0.0014	0.0092	0.0134	-0.0000	0.0001	-0.0001	0.0002
15	0.7	2	1	0.0347	0.0043	0.0282	0.0412	-0.0000	0.0003	-0.0004	0.0006
15	0.7	4	1	0.0677	0.0083	0.0551	0.0805	-0.0000	0.0005	-0.0008	0.0010
15	0.7	10	1	0.1074	0.0132	0.0874	0.1278	-0.0000	0.0008	-0.0012	0.0015
20	0.3	1.25	1	0.0258	0.0031	0.0209	0.0307	-0.0000	0.0002	-0.0003	0.0004
20	0.3	2	1	0.0794	0.0096	0.0644	0.0946	-0.0000	0.0005	-0.0008	0.0011
20	0.3	4	1	0.1552	0.0187	0.1258	0.1849	-0.0000	0.0010	-0.0015	0.0020
20	0.3	10	1	0.2462	0.0298	0.1996	0.2936	-0.0001	0.0015	-0.0023	0.0030
20	0.5	1.25	1	0.0184	0.0022	0.0149	0.0220	-0.0000	0.0001	-0.0002	0.0002
20	0.5	2	1	0.0567	0.0068	0.0460	0.0676	-0.0000	0.0004	-0.0006	0.0007
20	0.5	4	1	0.1108	0.0134	0.0899	0.1321	-0.0000	0.0007	-0.0010	0.0014
20	0.5	10	1	0.1758	0.0213	0.1425	0.2096	-0.0000	0.0010	-0.0016	0.0020
20	0.7	1.25	1	0.0110	0.0013	0.0089	0.0132	-0.0000	0.0001	-0.0001	0.0001
20	0.7	2	1	0.0340	0.0041	0.0276	0.0406	-0.0000	0.0002	-0.0003	0.0004
20	0.7	4	1	0.0665	0.0080	0.0539	0.0792	-0.0000	0.0004	-0.0006	0.0008
20	0.7	10	1	0.1056	0.0127	0.0855	0.1257	-0.0000	0.0006	-0.0009	0.0011