Admissibility and control of switched discrete-time singular systems

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This paper studies the admissibility of switched discrete-time singular systems. Sufficient conditions for this admissibility are derived in a form of a set of strict linear matrix inequalities. Design of both state feedback and static output feedback controllers is given. Numerical examples are presented to illustrate the proposed approaches.

Keywords: singular systems; switched systems; admissibility; state feedback control; static output control; strict $\mathcal{LMIs}$

1. Introduction

Switched systems have received a lot of attention during the last decade. They are dynamical and consist of finite number of subsystems and a logical rule that governs the switching between these subsystems. Mathematically, these subsystems are generally described by a collection of induced differential or difference equations. The motivation for studying switched systems comes from the fact that switched systems have many applications in control of mechanical systems, power systems, aircraft and traffic control (Liberzon, 2003). Many works dealing with the stability analysis and the stabilization of regular or standard switched systems have been developed recently (Daafouz, Riedinger, & Iung, 2002; Ge, Sun, & Lee, 2001; Gelig & Churilov, 1998; Liberzon, 2003; Mancilla-Aguilar, 2003; Stevens & Lewis, 1991).

On the other hand, singular systems (known also as generalized, descriptor or differential algebraic systems) describe a large class of systems which are of great importance in theoretical and practical points of view (Boukas, 2008; Dai, 1989; Mills & Goldenberg, 1989; Newcomb & Dziurla, 1989; Xu & Lam, 2006). They are encountered in chemical, mineral, electronic and economic systems (Luenberger, 1979). In recent years a great deal of works has been devoted to the analysis and design techniques for singular systems (Chadli et al., 2008; Dai, 1989; Masubuchi, Akiyama, & Saeki, 2003; Masubuchi, Kato, Saeki, & Ohara, 2004; Yao, Guan, Chenb, & Hoc, 2006). However, if interesting results on controllers and observers design are developed for singular linear systems (Boukas, 2008; Chadli & Darouach, 2012, 2013; Darouach & Boutayeb, 1995; Xu & Lam, 2006 and references therein), only few results exist for the switched singular systems compared to the standard systems (Daafouz et al., 2002; Ge et al.,

2001; Liberzon, 2003; Mancilla-Aguilar, 2003). Then a lot of efforts are still necessary to investigate new approaches and to improve existing results in the standard systems and to extend them to switched singular systems. Recently switched singular systems have been considered in Boukas (2008), this work treats the case of the stability and control design in the framework of systems with Markovian jumps. The systems considered are modeled with mode independent singular matrix E.

In this paper we consider the switched singular systems whose subsystems are discrete-time linear-invariant systems, where the matrix E is mode dependent. First, the stability for switched singular systems is introduced via a switched quadratic Lyapunov function, then the admissibility is defined and extends the existing admissibility notion for singular systems. Sufficient conditions for switched singular systems to be admissible are given in strict linear matrix inequalities ($\mathcal{LMIs}$) terms (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). The obtained results are then applied to the stabilization of switched discrete-time singular systems using both state feedback controller and static output feedback controller. The results of this work extend the existing results of switched standard systems presented in Daafouz et al. (2002), where the less conservative approach based on the switched Lyapunov function is introduced.

The paper is organized as follows. In Section 2, the considered class of a discrete-time singular switched systems and the corresponding admissibility concepts are presented. In Section 3, the connection between our results and the existing ones is presented. In Section 4, a static state and output feedback controllers are designed in $\mathcal{LMIs}$ formulation. Finally, two numerical examples are provided to illustrate the effectiveness of the obtained results.

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Notation. Throughout this paper, R^n and $R^{n \times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\top$" denotes matrix transposition, the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite), the symbol $(*)$ denotes the transpose elements in the symmetric positions, I is the identity matrices with compatible dimensions and $I_N = \{1, 2, \ldots, N\}$.

2. Admissibility of switched discrete-time singular systems

Let us consider the following autonomous switched discrete-time singular system given by

$$E_{\sigma(t+1)}x(t+1) = A_{\sigma(t)}x(t), \qquad (1)$$

where $x(t) \in R^n$ is the state vector and $A_{\sigma(t)} \in R^{n \times n}$. The matrix $E_{\sigma(t)} \in R^{n \times n}$ and $\sigma(t)$ is a switching signal taking values in the finite set I_N and assumed to be available in real time.

Let $\xi(t) = [\xi_1(t), \xi_2(t), \xi_3(t), \ldots, \xi_N(t)]^\top$ be the indication function such that

$$\xi_i(t) = \begin{cases} 1 & \text{if } \sigma(t) = i; \\ 0 & \text{otherwise.} \end{cases}$$

Then the switched system (1) can be written as

$$\sum_{i=1}^{N} \xi_i(t+1) E_i x(t+1) = \sum_{i=1}^{N} \xi_i(t) A_i x(t). \qquad (2)$$

In the sequel of this paper we assume that matrices E_i are of constant and equal rank, i.e. $\text{rank} E_i = r \leqslant n$. This condition permits to study the singular switched systems with states running in the manifolds with the same dimensions.

For system (1) or (2) we can give the following definitions.

DEFINITION 1 *System (2) or the pair (E_i, A_i) is said to be regular if $\det(zE_i - A_i)$ is not identically zero.*

DEFINITION 2 *System (2) or the pair (E_i, A_i) is said to be causal if $\deg(\det(zE_i - A_i)) = \text{rank} E_i$.*

DEFINITION 3 *The pair (E_i, A_i) is said to be stable if it is regular and all of $\lambda(E_i, A_i)$ are within $D(0, 1)$, where $\lambda(E_i, A_i) = \{z | \det(zE - A) = 0\}$ and $D(0, 1)$ represents a disk with center at 0 and radius 1.*

Remark 1 As in the standard switched systems, the stability of the pairs (E_i, A_i) for all $i \in I_N$ does not guarantee the stability of system (2). In fact switching between two stable systems can produce an unstable trajectory.

DEFINITION 4 *System (2) is said to be stable, if it is regular and if there exists a switched quadratic Lyapunov function*

$$V(x(t), t) = x(t)^\top E_{\sigma(t)}^\top \mathcal{P}_{\sigma(t)} E_{\sigma(t)} x(t), \quad \mathcal{P}_{\sigma(t)} = \sum_{i=1}^{N} \xi_i(t) P_i \tag{3}$$

with

$$E_i^\top P_i E_i \geq 0, \quad i \in I_N.$$

and whose difference $\Delta V = V(x(t+1), t+1) - V(x(t), t)$ is negative.

DEFINITION 5 *System (2) is said to be admissible if it is regular, causal and stable.*

Remark 2

- When $E_i = I$, the singular system (1) or (2) becomes a standard switched one presented in Daafouz et al. (2002) and the switched Lyapunov function (3) becomes exactly the one given in Daafouz et al. (2002), where $P_i > 0$.
- When E_i and A_i are constant matrices $E_i = E$, and $A_i = A$ in this case the system becomes a non-switched singular system and the Lyapunov function (3) becomes the classical quadratic Lyapunov function (Owens & Debeljkovic, 1985).

Now let the pair (E_i, A_i) be given, then it is always possible to find nonsingular matrices M_i and N_i such that $E_i = M_i \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N_i$ and $A_i = M_i \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} N_i$. This decomposition can be obtained via singular value decomposition of matrix E_i followed by scaling of the bases. Then, we have the following lemmas (Dai, 1989).

LEMMA 1 *The pair (E_i, A_i) is causal if A_{i4} is nonsingular.*

LEMMA 2 *The pair (E_i, A_i) is regular if and only if there exist two nonsingular matrices M_i and N_i such that $E_i = M_i \begin{bmatrix} I_r & 0 \\ 0 & \mathcal{N}_i \end{bmatrix} N_i$ and $A_i = M_i \begin{bmatrix} A_i & 0 \\ 0 & I_{n-r} \end{bmatrix} N_i$, where $\mathcal{N}_i$ is a nilpotent matrix.*

LEMMA 3 *Let the pair (E_i, A_i) be regular, then it is causal if and only if $\mathcal{N}_i = 0$.*

Let the matrix $A^\dagger$ be any generalized inverse satisfying $AA^\dagger A = A$, then we have the following useful lemma, which can be used in the sequel of the paper (Harville, 1997).

LEMMA 4 *Let A and B be two matrices of dimensions $n \times p$ and $p \times m$, respectively, then we have*

(1) $\text{rank } AB = \text{rank } B$ *if and only if $p = \text{rank}\begin{bmatrix} A \\ I - BB^\dagger \end{bmatrix}$. In particular when B is of full row rank, the necessary and sufficient condition for $\text{rank} AB = \text{rank} B$ is that A must be of full column rank.*

(2) $\text{rank}AB = \text{rank}A$ if and only if $p = \text{rank}\,[\,B\ I - A^\dagger A\,]$. *In particular when A is of full column rank, the necessary and sufficient condition for* $\text{rank}AB = \text{rank}A$ *is that B must be of full row rank.*

The following theorem gives sufficient conditions for system (2) to be admissible.

THEOREM 5 *System (2) is admissible if one of the following assertions is satisfied*:

(1) *There exist positive-definite matrices* P_i, *symmetric matrices* Q_i *and matrices* F_i *and* G_i *such that the following* $\mathcal{LMIs}$ *hold* $\forall\,(i,j) \in I_N^2$.

$$\begin{bmatrix} -E_i^\top P_i E_i + A_i^\top F_i \\ +F_i^\top A_i & -F_i^\top + A_i^{\text{T}} G_i \\ & P_j - E_j^{\perp\top} Q_j E_j^\perp \\ (*) & -G_i - G_i^\top \end{bmatrix} < 0, \tag{4}$$

where $E_i^\perp$ *is any full row rank matrix such that* $E_i^\perp E_i = 0$.

(2) *There exist positive-definite matrices* X_i, *symmetric matrices* R_i *and matrices* F_i *and* G_i *such that the following* $\mathcal{LMIs}$ *hold* $\forall\,(i,j) \in I_N^2$:

$$\begin{bmatrix} -E_i(X_j - E_j^{\top\perp\top} R_j E_j^{\top\perp})E_i^\top \\ +A_i F_i + F_i^\top A_i^\top & -F_i^\top + A_i G_i \\ & X_i - E_i^{\top\perp\top} R_i E_i^{\top\perp} \\ (*) & -G_i - G_i^\top \end{bmatrix} < 0. \tag{5}$$

Proof First, we prove that the conditions of assertion 1 are sufficient for system (2) to be admissible.

Assume that there exist positive-definite matrices P_i, symmetric matrices Q_i and matrices F_i and G_i such that inequalities (4) are satisfied and let $Y_j = P_j - E_j^{\perp\top} Q_j E_j^\perp$ in this case we obtain

$$E_i^{\text{T}} Y_i E_i = E_i^{\text{T}} P_i E_i \geqslant 0 \tag{6}$$

since $P_i > 0$. Then inequalities (4) become $\forall(i,j) \in I_N^2$:

$$\begin{bmatrix} -E_i^\top Y_i E_i + A_i^\top F_i + F_i^\top A_i & -F_i^\top + A_i^\top G_i \\ (*) & Y_j - G_i - G_i^\top \end{bmatrix} < 0. \tag{7}$$

By pre-multiplying (7) by $[\,I\ A_i^{\text{T}}\,]$ and post-multiplying it by $\begin{bmatrix} I \\ A_i \end{bmatrix}$ we obtain

$$A_i^\top (P_j - E_j^{\perp\top} Q_j E_j^\perp) A_i - E_i^\top P_i E_i < 0 \tag{8}$$

or equivalently

$$A_i^\top Y_j A_i - E_i^\top Y_i E_i < 0. \tag{9}$$

Now, let M_i and N_i be two nonsingular matrices such that $E_i = M_i \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N_i$ and $A_i = M_i \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} N_i$, also let

$M_i^\top Y_j M_i = \begin{bmatrix} \bar{Y}_{ij} & Y_{ij2} \\ Y_{ij2}^\top & Y_{ij3} \end{bmatrix}$ and $M_i^\top Y_i M_i = \begin{bmatrix} \bar{Y}_i & Y_{i2} \\ Y_{i2}^\top & Y_{i3} \end{bmatrix}$, then from Equation (9) we obtain

$$N_i^\top \begin{bmatrix} \tau_1 & \tau_2 \\ * & A_{i2}^\top \bar{Y}_{ij} A_{i2} + \mathbf{R}_{ij} + \mathbf{R}_{ij}^\top \end{bmatrix} N_i < 0, \tag{10}$$

where τ_i represents a matrix without any importance and $\mathbf{R}_{ij} = A_{i2}^\top Y_{ij2} A_{i4} + \frac{1}{2} A_{i4}^\top Y_{ij3} A_{i4}$. Now, since $A_{i2}^\top \bar{Y}_{ij} A_{i2} \geqslant 0$ and by using Equation (10) we have $\mathbf{R}_{ij} + \mathbf{R}_{ij}^\top < 0$, or equivalently $\mathbf{R}_{ij}$ is nonsingular, i.e. $(A_{i2}^\top Y_{ij2} + \frac{1}{2} A_{i4}^\top Y_{ij3}) A_{i4}$ is nonsingular, which leads to A_{i4} nonsingular. From Lemma 1 the pair (E_i, A_i) is causal. On the other hand, since A_{i4} is nonsingular, define the following nonsingular matrices, $\bar{M}_i = \begin{bmatrix} I_r & -A_{i2} A_{i4}^{-1} \\ 0 & A_{i4}^{-1} \end{bmatrix} M_i^{-1}$ and $\bar{N}_i = N_i^{-1} \begin{bmatrix} I_r & 0 \\ -A_{i4}^{-1} A_{i3} & I_{n-r} \end{bmatrix}$, then we have $\bar{M}_i E_i \bar{N}_i = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and $\bar{M}_i A_i \bar{N}_i = \begin{bmatrix} \bar{A}_i & 0 \\ 0 & I_{n-r} \end{bmatrix}$, where $\bar{A}_i = A_{i1} - A_{i2} A_{i4}^{-1} A_{i3}$. From Lemmas 2 and 3 the pair (E_i, A_i) is regular.

Let $V(x(t), t) = x^\top(t) E_{\sigma(t)}^\top \mathcal{Y}_{\sigma(t)} E_{\sigma(t)} x(t)$, with $\mathcal{Y}_{\sigma(t)} = \sum_{i=1}^{N} \xi_i(t) Y_i$ and $E_i^\top Y_i E_i \geq 0, i \in I_N$, be a switched Lyapunov function candidate as defined in Definition 4. Then the difference of $V(x(t), t)$ along the solution of Equation (2) is given by

$$\begin{aligned} \Delta V &= V(x(t+1), t+1) - V(x(t), t) \\ &= x^\top(t+1) E_{\sigma(t+1)}^\top \mathcal{Y}_{\sigma(t+1)} E_{\sigma(t+1)} x(t+1) \\ &\quad - x(t)^\top E_{\sigma(t)}^\top \mathcal{Y}_{\sigma(t)} E_{\sigma(t)} x(t) \\ &= x^\top(t) \left(\sum_{i=1}^{N} \xi_i(t) A_i^\top \mathcal{Y}_{\sigma(t+1)} \sum_{i=1}^{N} \xi_i(t) A_i \right) x(t) \\ &\quad - x^\top(t) \sum_{i=1}^{N} \xi_i(t) E_i^\top \mathcal{Y}_{\sigma(t)} \sum_{i=1}^{N} \xi_i(t) E_i x(t) \\ &= x^\top(t) \sum_{i=1}^{N} \xi_i(t) \sum_{j=1}^{N} \xi_j(t+1)(A_i^\top Y_j A_i - E_i^\top Y_i E_i) x(t). \end{aligned} \tag{11}$$

Thus, from condition (9) we guarantee that $\Delta V < 0$. Consequently, the unforced singular system (2) is admissible since it is regular, causal and stable.

Now, since the pair (E_i, A_i) is regular and causal, there exist two nonsingular matrices S_i and N_i such that $E_i = S_i \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N_i$ and $A_i = M_i \begin{bmatrix} \bar{A}_i & 0 \\ 0 & I_{n-r} \end{bmatrix} N_i$. Let $Y_i = M_i^{-\top} \begin{bmatrix} \bar{Y}_i & Y_{i2} \\ Y_{i2}^\top & Y_{i3} \end{bmatrix} M_i^{-1}$ and $Y_j = M_i^{-\top} \begin{bmatrix} \bar{Y}_{ij} & Y_{ij2} \\ Y_{ij2}^\top & Y_{ij3} \end{bmatrix} M_i^{-1}$ then Equation (9) can be written as

$$N_i^\top \begin{bmatrix} \bar{A}_i^\top \bar{Y}_{ij} \bar{A}_i - \bar{Y}_i & \bar{A}_i^\top Y_{ij2} \\ Y_{ij2}^\top \bar{A}_i & Y_{ij3} \end{bmatrix} N_i < 0$$

or equivalently, since N_i is nonsingular matrix,

$$\begin{bmatrix} \bar{A}_i^\top \bar{Y}_{ij} \bar{A}_i - \bar{Y}_i & \bar{A}_i^\top Y_{ij2} \\ Y_{ij2}^\top \bar{A}_i & Y_{ij3} \end{bmatrix} < 0$$

and since $Y_{ij3} < 0$ (see Xu & Yang, 1999) we obtain

$$\bar{A}_i^\top \Phi_{ij} \bar{A}_i - \bar{Y}_i < 0. \tag{12}$$

Where $\Phi_{ij} = \bar{Y}_{ij} - Y_{ij2} Y_{ij3}^{-1} Y_{ij2}^\top > 0$. Now, the Schur complement applied to Equation (12) gives the following inequality:

$$\begin{bmatrix} -\bar{Y}_i & \bar{A}_i^\mathrm{T} \\ \bar{A}_i & -\Phi_{ij}^{-1} \end{bmatrix} < 0$$

which is equivalent to

$$\bar{A}_i \Phi_i \bar{A}_i^\top - \Psi_{ij} < 0, \tag{13}$$

where matrices $\Phi_i = \bar{Y}_i^{-1} > 0$ and $\Psi_{ij} = \Phi_{ij}^{-1} > 0$.

Now let us prove that if Equation (5) is satisfied then system (2) is admissible. Assume that there exist positive-definite matrices X_i and symmetric matrices R_i such that inequalities (5) are satisfied. and let $Z_i = X_i - E_i^{\top\perp\top} R_i E_i^{\top\perp}$ in this case inequalities (5) become

$$\begin{bmatrix} -E_i Z_j E_i^\top + A_i F_i + F_i^\top A_i^\top & -F_i^\top + A_i G_i \\ (*) & Z_i - G_i - G_i^\top \end{bmatrix} < 0 \tag{14}$$

$\forall (i,j) \in I_N^2$. By pre-multiplying Equation (14) by $[\, I \ A_i \,]$ and post-multiplying it by $\begin{bmatrix} I \\ A_i^\top \end{bmatrix}$ we obtain

$$A_i Z_i A_i^\top - E_i Z_j E_i^\top < 0. \tag{15}$$

Let M_i and N_i be two nonsingular matrices such that $E_i = M_i \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N_i$ and $A_i = M_i \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} N_i$. On the other hand, let $N_i Z_i N_i^\top = \begin{bmatrix} \bar{Z}_i & Z_{i2} \\ Z_{i2}^\top & Z_{i3} \end{bmatrix}$ and $N_i Z_j N_i^\top = \begin{bmatrix} \bar{Z}_{ij} & Z_{ij2} \\ Z_{ij2}^\top & Z_{ij3} \end{bmatrix}$, then inequalities (15) become

$$M_i \begin{bmatrix} \tau_3 & \tau_4 \\ * & A_{i3} \bar{Z}_i A_{i3}^\top + \mathbf{Q}_i + \mathbf{Q}_i^\top \end{bmatrix} M_i^\top < 0, \tag{16}$$

where τ_3 and τ_4 represent matrices without any importance and $\mathbf{Q}_i = (A_{i3} Z_{i2} + \frac{1}{2} A_{i4} Z_{i3}) A_{i4}^\top$. As in the above proof we have $\mathbf{Q}_i < 0$ which implies that matrix $A_{i4}^\top$ is nonsingular or equivalently A_{i4} is nonsingular, consequently the pair (E_i, A_i) is causal and regular. By using the same reasoning as above, we can show that Equation (15) is equivalent to Equations (13) or (9). This proves the theorem. ∎

Remark 3 Note that for $E_i = E$, conditions (5) are reduced to the existence of matrices $X_i > 0$, F_i and G_i such that the following $\mathcal{LMIs}$ hold

$$\begin{bmatrix} -EX_j E^\top + A_i F_i + F_i^\top A_i^\top & -F_i^\top + A_i G_i \\ (*) & X_i - E^{\top\perp\top} R_i E^{\top\perp} \\ & -G_i - G_i^\top \end{bmatrix} < 0 \tag{17}$$

which is only the dual conditions of (4) with $E_i = E$. This duality is not guaranteed for singular switched systems with matrix E mode dependent.

3. Connection with the existing results

3.1. Standard switched systems

For the standard case $E_i = I$, we have $E_i^\perp = 0$. Then conditions (4) are reduced to the existence of matrices $P_i > 0$, F_i and G_i such that the following $\mathcal{LMIs}$ hold $\forall (i,j) \in I_N^2$:

$$\begin{bmatrix} -P_i + A_i^\top F_i + F_i^\top A_i & -F_i^\top + A_i^\top G_i \\ (*) & P_j - G_i - G_i^\top \end{bmatrix} < 0 \tag{18}$$

and conditions (5) are reduced to the existence of matrices $X_i > 0$, F_i and G_i such that the following $\mathcal{LMIs}$ hold

$$\begin{bmatrix} -X_j + A_i F_i + F_i^\top A_i^\top & -F_i^\top + A_i G_i \\ (*) & X_i - G_i - G_i^\top \end{bmatrix} < 0. \tag{19}$$

Conditions (18)–(19) with $F_i = 0$ are exactly the stability conditions given in Daafouz et al. (2002) using poly-quadratic switched Lyapunov function. Note also that the conditions (18) or (19) with $P_j = P_i$ or $X_j = X_i$ correspond exactly to the stability condition for uncertain linear system presented in Peaucelle, Arzelier, Bachelier, and Bernussou (2000).

3.2. Singular systems

Singular linear systems. For $E_i = E$ and $A_i = A$, the singular system (2) becomes a time-invariant one, in this case condition (8) implies that

$$A^\top (P - E^{\perp\top} Q E^\perp) A - E^\top P E < 0. \tag{20}$$

Which is exactly the condition given in Xu and Lam (2006).

Singular switched systems. When $E_i = E$, the case where the matrix E is mode independent, we have $Q_i = Q$. For $P_i = P$, this case corresponds to the classical common Lyapunov function, which is less general than the poly-quadratic function presented in this note. In addition, the proposed conditions are in strict $\mathcal{LMIs}$.

4. Stabilization of discrete-time singular switched systems

Let us consider the following controlled discrete-time switched system

$$E_{\sigma(t+1)} x(t+1) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t) \tag{21a}$$

$$y(t) = C_{\sigma(t)} x(t) \tag{21b}$$

with $x(t) \in R^n$ being the state vector, $y(t) \in R^p$ the output vector, $u(t) \in R^m$ is the input vector, $A_{\sigma(t)} \in R^{n \times n}$, $B_{\sigma(t)} \in R^{n \times m}$ and $C_{\sigma(t)} \in R^{p \times n}$. The matrix $E_{\sigma(t)} \in R^{n \times n}$ and $\sigma(t)$ is a switching signal taking values in the finite set I_N and assumed available in real time.

In this section, we consider the stabilization of system (21) via state feedback controller and static output feedback control. Sufficient conditions for admissibility are given in a set of $\mathcal{LMIs}$ form.

4.1. State feedback controller

The closed loop system of (21) via state feedback controller

$$u(t) = \sum_{i=1}^{N} \xi_i(t) K_i x(t), \tag{22}$$

with $K_i \in R^{m \times n}$, can be written as

$$\sum_{i=1}^{N} \xi_i(t+1) E_i x(t+1) = \sum_{i=1}^{N} \xi_i(t)(A_i + B_i K_i) x(t). \tag{23}$$

DEFINITION 6   *The closed-loop system (23) is said to be admissible, if there exists a state feedback (22) which guarantees that system (23) is regular, causal and stable.*

To derive the admissibility conditions of (23), it suffices to substitute A_i by $A_i + B_i K_i$ in conditions (5), conditions more adapted to this problem than conditions (4), with $F_i = G_i$ and by putting $L_i = K_i G_i$, in this case we obtain the following theorem.

THEOREM 6   *The singular system (23) is admissible if there exist positive-definite matrices X_i, symmetric matrices R_i, matrices G_i and L_i such that the following set of $\mathcal{LMIs}$ hold $\forall\, i \in I_N$:*

$$\begin{bmatrix} -E_i(X_j - E_j^{\top\perp\top} R_j E_j^{\top\perp}) E_i^{\top} & -G_i^{\top} + \Xi_i \\ \quad + \Xi_i + \Xi_i^{\top} & \\ (*) & X_i - E_i^{\top\perp\top} R_i E_i^{\top\perp} \\ & -G_i - G_i^{\top} \end{bmatrix} < 0 \tag{24}$$

with $\Xi_i = A_i G_i + B_i L_i$. The controller gains are given by

$$K_i = L_i G_i^{-1}. \tag{25}$$

This result presents sufficient conditions for state feedback control of discrete-time singular switched systems (23). These conditions are linear in parameter matrices X_i, G_i and L_i, $i \in I_N$. They can be efficiently solved using existing numerical tools such as the LMITOOL software (El Ghaoui & Commeau, 1999; Vandenberghe & Boyd, 1996).

4.2. Static output feedback controller

In this section we consider the admissibility of system (21) via static output feedback control of the form

$$u(t) = \sum_{i=1}^{N} \xi_i(t) K_i y(t), \tag{26}$$

where $K_i \in R^{m \times p}$. The closed-loop system is given by

$$\sum_{i=1}^{N} \xi_i(t+1) E_i x(t+1) = \sum_{i=1}^{N} \xi_i(t) \mathbf{A}_i x(t) \tag{27}$$

with

$$\mathbf{A}_i = A_i + B_i K_i C_i. \tag{28}$$

For this closed-loop system we can give the following definition.

DEFINITION 7   *The closed-loop system (27) is said to be admissible, if there exists an output feedback given by Equation (26) which guarantees that system (27) is regular, causal and stable.*

Now, without loss of generality, we can make the following assumption which will be used in the sequel.

Assumption 1   The matrices C_i are of full row rank for all $i \in I_N$.

Remark 4   Assumption is generally used in the control, it means that the measurements are not redundant. If this condition is not satisfied we can always find a coordinates change which leads to this condition.

Based on this assumption, sufficient conditions for (27) to be admissible are given by the following lemma.

LEMMA 7   *Under Assumption 1, the closed-loop system (27) is admissible if there exist positive-definite matrices X_i, symmetric matrices R_i, matrices G_i, S_i and T_i such that the following linear constraints hold $\forall\, i \in I_N$:*

$$\begin{bmatrix} -E_i(X_j - E_j^{\top\perp\top} R_j E_j^{\top\perp}) E_i^{\top} & -G_i^{\top} + \Phi_i \\ \quad + \Phi_i + \Phi_i^{\top} & \\ (*) & X_i - E_i^{\top\perp\top} R_i E_i^{\top\perp} \\ & -G_i - G_i^{\top} \end{bmatrix} < 0 \tag{29}$$

$$C_i G_i = S_i C_i \tag{30}$$

with $\Phi_i = A_i G_i + B_i T_i C_i$. In this case the controller gains are given by

$$K_i = T_i S_i^{-1}. \tag{31}$$

Proof   Under Assumption 1, matrices C_i are of full row rank. Now assume that there exist positive-definite matrices X_i and matrices G_i, matrices S_i and T_i such that Equation (29) are satisfied, then we can see that matrices G_i are nonsingular and from Lemma 4 and (30) we can deduce that S_i is nonsingular for all $i \in I_N$. Now, consider Equation (5) with $F_i = G_i$ and A_i replaced by Equation (28). Let $T_i = K_i S_i$, then by using Equation (30) we obtain Equation (29).   ∎

Conditions of Lemma 7 are in $\mathcal{LMIs}$ form (29) under the linear equalities (30). They can be solved easily by LMI tools (such as the LMITOOL software El Ghaoui &

Commeau, 1999; Vandenberghe & Boyd, 1996) or by elimination of the equality constraints (30) to obtain only strict $\mathcal{LMIs}$ inequalities, this can be done as follows:

Assume that matrices C_i are of full row rank, this can be always obtained by a some regular transformation. In this case by singular value decomposition, there exist two unitary matrices U_i and V_i such that $U_iC_iV_i^\top = [\,\Sigma_i\ 0\,]$. Let $\bar{U}_i = \Sigma_i^{-1}U_i$ be a nonsingular matrix, then $\bar{U}_iC_iV_i^\top = [\,I_{p_i}\ 0\,]$, where $p_i = \text{rank}\,C_i$. Let $V_iG_iV_i^\top = \left[\begin{smallmatrix} G_{i1} & G_{i2} \\ G_{i3} & G_{i4} \end{smallmatrix}\right]$, in this case Equation (30) becomes

$$\bar{U}_iC_iV_i^\top V_iG_iV_i^\top = \bar{U}_iS_i\bar{U}_i^{-1}\bar{U}_iC_iV_i^\top$$

or equivalently

$$\begin{bmatrix} G_{i1} & G_{i2} \end{bmatrix} = \begin{bmatrix} \bar{U}_iS_i\bar{U}_i^{-1} & 0 \end{bmatrix}$$

which leads to $S_i = \bar{U}_i^{-1}G_{i1}\bar{U}_i$ and $G_{i2} = 0$. From the proof of Lemma 7, since G_i is nonsingular, we deduce that G_{i1} is nonsingular.

Now Let the partition of matrices V_i, V_iE_i, V_iB_i, $V_iE_i^{\top\perp\top}$ and $V_iA_iV_i^\top$ according to that of G_i be $V_i = \left[\begin{smallmatrix} V_{i1} \\ V_{i2} \end{smallmatrix}\right]$, $V_iE_i = \left[\begin{smallmatrix} E_{i1} \\ E_{i2} \end{smallmatrix}\right]$, $V_iE_i^{\top\perp\top} = \left[\begin{smallmatrix} \bar{E}_{i1} \\ \bar{E}_{i2} \end{smallmatrix}\right]$, $V_iB_i = \left[\begin{smallmatrix} B_{i1} \\ B_{i2} \end{smallmatrix}\right]$ and $V_iA_iV_i^\top = \left[\begin{smallmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{smallmatrix}\right]$. Let $\bar{T}_i = T_i\bar{U}_i^{-1}$, then by pre-multiplying (29) by $\left[\begin{smallmatrix} V_i & 0 \\ 0 & V_i \end{smallmatrix}\right]$ and post-multiplying it by $\left[\begin{smallmatrix} V_i^\top & 0 \\ 0 & V_i^\top \end{smallmatrix}\right]$ yields to the following theorem.

THEOREM 8   *Under Assumption 1, the closed-loop system (27) is admissible if there exist positive-definite matrices X_i, symmetric matrices R_i, matrices G_{i1}, G_{i3}, G_{i4} and $\bar{T}_i$ such that the following $\mathcal{LMIs}$ hold $\forall\ i \in I_N$:*

$$\begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} & \Sigma_{i13} & \Sigma_{i14} \\ (*) & \Sigma_{i22} & \Sigma_{i23} & \Sigma_{i24} \\ (*) & (*) & \Sigma_{i33} & \Sigma_{i34} \\ (*) & (*) & (*) & \Sigma_{i44} \end{bmatrix} < 0 \qquad (32)$$

with

$$\begin{aligned} \Sigma_{i11} =\ & -E_{i1}(X_j - E_j^{\top\perp\top}R_jE_j^{\top\perp})E_{i1}^\top + A_{i1}G_{i1} + A_{i2}G_{i3} \\ & + B_{i1}\bar{T}_i + G_{i1}^\top A_{i1}^\top + G_{i3}^\top A_{i2}^\top + \bar{T}_i^\top B_{i1}^\top, \end{aligned}$$

$$\begin{aligned} \Sigma_{i12} =\ & -E_{i1}(X_j - E_j^{\top\perp\top}R_jE_j^{\top\perp})E_{i2}^\top + A_{i2}G_{i4} + G_{i1}^\top A_{i3}^\top \\ & + G_{i3}^\top A_{i4}^\top + \bar{T}_i^\top B_{i2}^\top, \end{aligned}$$

$$\Sigma_{i13} = -G_{i1}^\top + A_{i1}G_{i1} + A_{i2}G_{i3} + B_{i1}\bar{T}_i,$$

$$\Sigma_{i22} = A_{i4}G_{i4} + G_{i4}^\top A_{i4}^\top - E_{i2}(X_j - E_j^{\top\perp\top}R_jE_j^{\top\perp})E_{i2}^\top,$$

$$\Sigma_{i14} = -G_{i3}^\top + A_{i2}G_{i4},$$

$$\Sigma_{i23} = A_{i3}G_{i1} + A_{i4}G_{i3} + B_{i2}\bar{T}_i,$$

$$\Sigma_{i24} = -G_{i4}^\top + A_{i4}G_{i4},$$

$$\Sigma_{i33} = V_{i1}X_iV_{i1}^\top - \bar{E}_{i1}R_i\bar{E}_{i1}^\top - G_{i1} - G_{i1}^\top,$$

$$\Sigma_{i34} = V_{i1}X_iV_{i2}^\top - \bar{E}_{i1}R_i\bar{E}_{i2}^\top - G_{i3}^\top,$$

$$\Sigma_{i44} = V_{i2}X_iV_{i2}^\top - \bar{E}_{i2}R_i\bar{E}_{i2}^\top - G_{i4} - G_{i4}^\top.$$

In this case the controller gains are given by

$$K_i = \bar{T}_iG_{i1}^{-1}\bar{U}_i. \qquad (33)$$

5.   Numerical examples

This section gives two illustrative numerical examples to show the effectiveness of the proposed approach. The first example deals with the state feedback control problem and the second one with the output feedback problem.

5.1.   Example 1

For the first example, we consider unstable model with two modes:

$$E_1 = \begin{bmatrix} 5.6 & 1.68 & 1.4 & 0 \\ 4.2 & 9.8 & 7.14 & 0.28 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} -4.34 & -9.52 & 5.74 & -8.40 \\ 11.34 & 5.60 & -7.14 & 10.08 \\ 7.00 & -3.92 & -1.40 & 1.68 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 27.5 & 6.0 & 13.5 & 11.5 \\ -7.0 & -12.5 & 32.5 & 23.5 \\ 18.5 & 25.5 & -19.5 & 18.0 \\ -26.5 & -7.5 & 11.0 & 23.0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -31.0 & 14.5 & 15.5 & -14.0 \\ -9.5 & 18.5 & -21.5 & -7.5 \\ -12.0 & -29.5 & 11.5 & -9.5 \\ -8.0 & -15.5 & -11.0 & -27.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.96 & 2.52 & 2.88 \\ 2.28 & -1.80 & 1.68 \\ -0.72 & 3.00 & 3.36 \\ 1.92 & 4.56 & 1.44 \end{bmatrix}$$

$$\text{and}\quad B_2 = \begin{bmatrix} 3.00 & 4.44 & 3.48 \\ -4.08 & 5.04 & -3.24 \\ 4.32 & 2.52 & 2.16 \\ 3.24 & 3.00 & -2.8800 \end{bmatrix}.$$

It is easy to see that the condition rank $E_1 = \text{rank}\,E_2$ is satisfied, also the regularity conditions for these subsystems are satisfied. By solving $\mathcal{LMIs}$ (24), we obtain the following results:

$$X_1 = \begin{bmatrix} 0.7901 & 0.2433 & -0.8078 & 1.7744 \\ 0.2433 & 6.9993 & -8.3014 & 3.3363 \\ -0.8078 & -8.3014 & 10.3857 & -4.7721 \\ 1.7744 & 3.3363 & -4.7721 & 14.7301 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 7.0915 & 5.6637 & 5.6332 & -6.2404 \\ 5.6637 & 5.0343 & 5.6973 & -4.4717 \\ 5.6332 & 5.6973 & 15.5431 & 2.3735 \\ -6.2404 & -4.4717 & 2.3735 & 10.5729 \end{bmatrix},$$
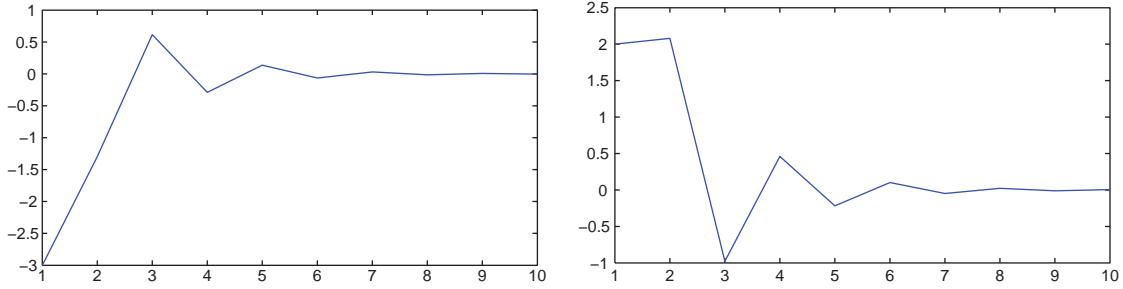
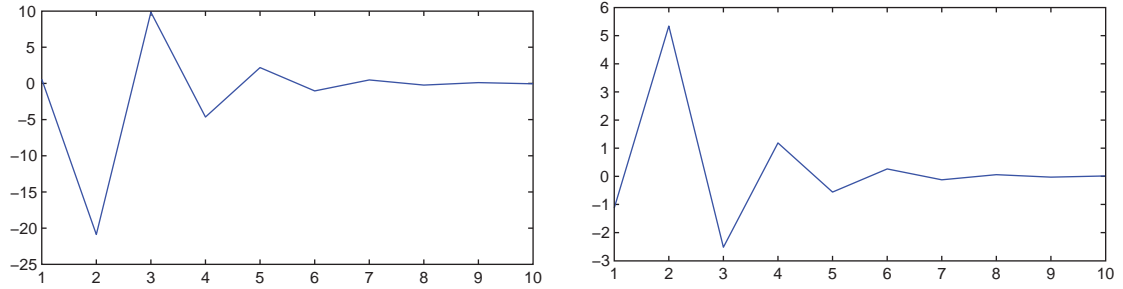Figure 1.   States $x_1(t)$ and $x_2(t)$ of the closed-loop system.

Figure 2.   States $x_3(t)$ and $x_4(t)$ of the closed-loop system.

$$G_1 = \begin{bmatrix} 1.9067 & 14.8773 & 42.7721 & 27.5002 \\ -14.1328 & 5.2717 & -95.2632 & -93.7725 \\ -43.8234 & 96.7773 & 0.4448 & -106.7421 \\ -24.4442 & 97.3287 & 103.2599 & -3.1477 \end{bmatrix},$$

$$G_2 = 10^3 \begin{bmatrix} 0.0037 & 0.3441 & -2.4533 & 1.4025 \\ -0.3444 & 0.0007 & -0.3425 & 0.3826 \\ 2.4575 & 0.3463 & 0.0046 & -1.3308 \\ -1.4010 & -0.3777 & 1.3289 & -0.0010 \end{bmatrix},$$

$$R_1 = 10^4 \begin{bmatrix} 0.0042 & -0.0566 & -0.3092 & 0.2218 \\ -0.0566 & -2.9013 & 3.1250 & 1.7915 \\ -0.3092 & 3.1250 & 5.9163 & 1.4588 \\ 0.2218 & 1.7915 & 1.4588 & 0.1190 \end{bmatrix},$$

$$R_2 = 10^4 \begin{bmatrix} -0.4056 & 0.1711 & -2.9062 & 0.0560 \\ 0.1711 & 0.5037 & 1.7560 & 0.1243 \\ -2.9062 & 1.7560 & 0.9914 & -0.5837 \\ 0.0560 & 0.1243 & -0.5837 & 0.0482 \end{bmatrix},$$

$$L_1 = 10^3 \begin{bmatrix} 0.5969 & -1.6163 & -1.0477 & 0.8133 \\ -0.0902 & 0.2432 & 0.1101 & -0.1612 \\ 0.1949 & -0.6736 & -0.3867 & 0.3068 \end{bmatrix},$$

$$L_2 = 10^3 \begin{bmatrix} -8.8616 & -1.3831 & 1.1373 & 4.1474 \\ -1.2309 & 0.6289 & -5.6387 & 3.9006 \\ -5.9036 & 0.2913 & -8.1805 & 7.8677 \end{bmatrix}.$$

In this case the controller gains obtained from Equation (25) are

$$K_1 = \begin{bmatrix} 7.3407 & 9.3057 & -13.7690 & -4.5421 \\ 81.0161 & -6.3068 & 29.0567 & -38.4358 \\ -71.7610 & -0.2671 & -21.8896 & 25.8279 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -1.0798 & 11.5894 & -0.9239 & 1.8529 \\ 6.1208 & -10.3406 & 0.5433 & 4.3894 \\ 5.2579 & -14.5859 & -4.5641 & -0.1930 \end{bmatrix}.$$

The simulation results under an arbitrary switching law, with the initial value $x^\top = [\,-3\ 2\ 0\ -1\,]$, shows the stability of the considered switched system; i.e. all the state trajectories converge to the origin (Figures 1 and 2).

Next, a second example is proposed to show the effectiveness of the derived static output controller.

5.2.   Example 2

This example consists of a switched singular system with three modes and described by

$$E_1 = \begin{bmatrix} 5.59 & 2.47 & 1.963 & 0.026 \\ 3.90 & 9.10 & 6.630 & 0.260 \\ 5.20 & 1.56 & 1.300 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 5.2 & 1.56 & 1.3 & 0 \\ 3.9 & 9.1 & 6.63 & 0.26 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 4.81 & 0.65 & 0.637 & -0.026 \\ 3.90 & 9.10 & 6.630 & 0.260 \\ 0 & 0 & 0 & 0 \\ 0.39 & 0.91 & 0.663 & 0.026 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 3.30 & 0.72 & 1.62 & 1.38 \\ -0.84 & -1.50 & 3.90 & 2.82 \\ 2.22 & 3.06 & -2.34 & 2.16 \\ -3.18 & -0.90 & 1.32 & 2.76 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2.48 & -1.16 & -1.24 & 1.12 \\ 0.76 & -1.48 & 1.72 & 0.60 \\ 0.96 & 2.36 & -0.92 & 0.76 \\ 0.64 & 1.24 & 0.88 & 2.20 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 2.80 & -2.24 & -1.36 & 0.72 \\ 1.84 & 1.28 & 3.60 & 3.76 \\ 1.36 & 5.20 & 2.56 & 1.92 \\ 0.88 & 1.92 & 2.96 & 3.12 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.9600 & 2.5200 & 2.8800 \\ 2.2800 & -1.8000 & 1.6800 \\ -0.7200 & 3.0000 & 3.3600 \\ 1.9200 & 4.5600 & 1.4400 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 3.0000 & 4.4400 & 3.4800 \\ -4.0800 & 5.0400 & -3.2400 \\ 4.3200 & 2.5200 & 2.1600 \\ 3.2400 & 3.0000 & -2.8800 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 2.3800 & 0.7000 & 1.1200 \\ 1.3300 & 0.7700 & 1.8900 \\ 1.8900 & 1.3300 & -2.2400 \\ 1.3300 & 2.3800 & 2.5200 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.4000 & -1.0000 & 1.2000 & 1.8000 \\ -14.0000 & -5.0000 & 18.0000 & 17.0000 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.6250 & 3.8750 & -0.8750 & -3.7500 \\ 0.3125 & -1.0625 & 0.5625 & 0.6250 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 0.4920 & 0.0900 & -0.3120 & -0.0300 \\ 0.0600 & -0.4500 & 0.8400 & -0.1500 \end{bmatrix}.$$

Solving Equations (29)–(30), we obtain the following results

$$X_1 = \begin{bmatrix} 83.3869 & -49.8576 & 3.3627 & 53.9172 \\ -49.8576 & 368.2777 & -422.9244 & -73.1192 \\ 3.3627 & -422.9244 & 554.9655 & 9.9328 \\ 53.9172 & -73.1192 & 9.9328 & 844.8909 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 90.6961 & -29.8137 & -28.4899 & 28.1298 \\ -29.8137 & 400.5853 & -459.7174 & -158.4733 \\ -28.4899 & -459.7174 & 600.5969 & 85.9891 \\ 28.1298 & -158.4733 & 85.9891 & 978.1242 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 91.0129 & -27.5422 & -33.7177 & 20.3228 \\ -27.5422 & 293.2606 & -372.7821 & -27.0162 \\ -33.7177 & -372.7821 & 564.1515 & -39.0869 \\ 20.3228 & -27.0162 & -39.0869 & 837.1100 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 218.4840 & -94.7511 & 77.5316 & 126.3899 \\ 53.4081 & 3.8116 & 169.1636 & 194.9970 \\ -122.6607 & 183.7677 & -62.2326 & -336.0752 \\ 318.0391 & 25.8317 & -260.8759 & -73.6182 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 395.3906 & -192.3779 & 188.5014 & -109.1839 \\ 32.5598 & 18.1982 & 12.8981 & -92.4736 \\ -244.3229 & 346.8400 & -142.0194 & -195.5994 \\ 77.9093 & 222.3521 & -80.0590 & -266.3606 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 416.8837 & -107.7716 & 88.7741 & -94.3066 \\ -183.9055 & -206.7481 & 584.1589 & -45.0112 \\ -22.5166 & 276.7853 & -466.4941 & 79.6081 \\ 48.4921 & -33.9213 & -57.6439 & -78.1847 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 381.5663 & 0.6047 \\ -11.6820 & -3.2195 \\ -204.5626 & 18.4734 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} -31.6377 & -84.8732 \\ -53.1620 & -206.6860 \\ -18.8201 & -248.1433 \end{bmatrix},$$

$$T_3 = 10^3 \begin{bmatrix} -1.0658 & -0.4133 \\ 1.2638 & -0.1727 \\ -0.7219 & 0.5238 \end{bmatrix},$$

$$S_1 = 10^3 \begin{bmatrix} -0.0536 & -0.0343 \\ -4.4623 & -0.1184 \end{bmatrix},$$

$$S_2 = 10^3 \begin{bmatrix} -0.0307 & 1.0052 \\ 0.0426 & -0.0844 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 343.6435 & 417.5517 \\ 247.6266 & -670.8262 \end{bmatrix}.$$

Then from Equation (31), we get the following output feedback controller gains

$$K_1 = \begin{bmatrix} 0.2893 & -0.0890 \\ 0.0884 & 0.0016 \\ -0.7262 & 0.0546 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.1563 & -0.8561 \\ -0.3305 & -1.4876 \\ -0.3023 & -0.6604 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -2.4477 & -0.9075 \\ 2.4108 & 1.7580 \\ -1.0617 & -1.4417 \end{bmatrix}.$$

The obtained results illustrate the effectiveness of the given sufficient $\mathcal{LMIs}$ admissibility conditions for a switched singular system.

6. Conclusion

In this paper we have presented a new approach to the stabilization of switched singular discrete-time systems. The properties of stability and admissibility for this class of systems are first introduced, then sufficient conditions for a switched singular system to be admissible are given in strict $\mathcal{LMIs}$ form. Both state feedback and static output feedback stabilization are presented. Two numerical examples were given to illustrate the obtained results.

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