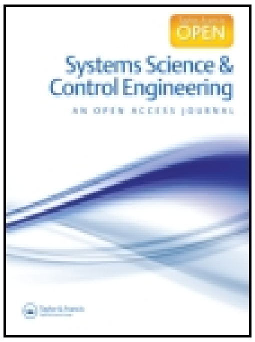

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Systems Science & Control Engineering: An Open Access Journal

ISSN: (Print) 2164-2583 (Online) Journal homepage: https://www.tandfonline.com/loi/tssc20

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To cite this article: Guoliang Wei, Licheng Wang & Fei Han (2013) A gain-scheduled approach to fault-tolerant control for discrete-time stochastic delayed systems with randomly occurring actuator faults, Systems Science & Control Engineering: An Open Access Journal, 1:1, 82-90, DOI: 10.1080/21642583.2013.842509

To link to this article: https://doi.org/10.1080/21642583.2013.842509

Systems Science & Control Engineering: An Open Access Journal, 2013
Vol. 1, No. 1, 82–90, http://dx.doi.org/10.1080/21642583.2013.842509

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A gain-scheduled approach to fault-tolerant control for discrete-time stochastic delayed systems with randomly occurring actuator faults

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(*Received 17 August 2013; final version received 5 September 2013*)

This paper is concerned with the probability-dependent gain-scheduled fault-tolerant control problem for a class of discrete-time stochastic nonlinear delayed systems with randomly occurring actuator faults (ROAFs) by utilizing parameter-based Lyapunov functional. The occurrence of the possible actuator faults is modeled by a random sequence in terms of a time-varying Bernoulli distribution with measurable probability in real time. The nonlinear functions are assumed to satisfy the sector nonlinearities. The purpose of the addressed fault-tolerant control problem is to design a controller with scheduled gains such that, for the admissible ROAFs, nonlinearities, time delays and noises, the closed-loop system is exponentially mean-square stable while preserving a guaranteed H_∞ performance. By using the semi-definite programme method, the time-varying fault-tolerant controller is derived which is dependent on the occurrence probability of the actuator faults. Therefore, the main results lead to less conservatism than those obtained by conventional methods with fixed controller gains only. A simulation example is exploited to demonstrate the effectiveness of the proposed design procedures.

Keywords: fault-tolerant control; randomly occurring actuator fault; time-varying Bernoulli distribution; probability-dependent Lyapunov function; gain-scheduled controller; discrete-time stochastic systems

1. Introduction

In modern practical systems, the increasing operational conditions inevitably magnify the possibility of faults, which may potentially cause a reduction of performance and/or launch a threat to the safety and reliability of the plant. Therefore, fault detection and fault-tolerant control problems have been intensively studied in the past decades, see Gao and Ding (2007a), Gao and Ding (2007b), Ding, Guo, and Jeinsch (1999), Ye and Yang (2009), Yang and Ye (2008), Blanke, Kinnaert, Lunze, and Staroswiecki (2003), Niemann and Stoustrup (2005), Zhang and Jiang (2008), and Zhou and Ren (2001). In view of the application domains, a host of fault-tolerant controller design techniques has been developed to keep the system stable and maintain acceptable performance in the presence of faults, see e.g. Yang and Ye (2008), Patton (1997), Gao and Ding (2007b), Ye and Yang (2009), Blanke et al. (2003), Niemann and Stoustrup (2005), Zhang and Jiang (2008), and Zhou and Ren (2001). In general, the controller design methods can be classified into 'passive' and 'active' approaches according to the structure of the designed controllers. The passive one is a simple method appropriate to the slight changes of parameters and signals. On the other hand, the controllers designed by the active approach can be scheduled on-line according to the different situations, and are therefore more suitable to significant changes of system parameters caused by faults.

It is well known that stochastic perturbation exists pervasively in reality and is frequently a source of system performance degradation. Over the past few decades, stochastic systems have been extensively investigated by many researchers and considerable results have been reported in the literature, see e.g. Boukas and Liu (2002), Chen, Guan, and Liu (2005), Gao, Lam, and Wang (2006), Wei and Wang (2009), Shen, Wang, Shu, and Wei (2009), Shen, Wang, Liang, and Liu (2011), and Ding, Wang, Dong, and Shu (2012). Time delays also serve as one of the main causes for system performance reduction or even instability. Consequently, the time-delay systems with stochastic perturbations have drawn a lot of research attention, see Chen et al. (2005), Gao et al. (2006), Hu, Wang, and Gao (2011), Hu, Wang, and Gao (2012), and Dong, Wang, Ho, and Gao (2010). Moreover, in real-world applications, nonlinearity is an inevitable feature that has long been the main stream of research topics in the control community. In the past years, the analysis and synthesis problems of

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nonlinear stochastic systems have been intensively studied, see e.g. Yue and Han (2005), Wang, Wang, and Liang (2009), and Wei and Wang (2009). Among others, the so-called sector-nonlinearity has attracted particular attention, because such kind of nonlinearity is quite general that covers the widely used Lipschitz condition as a special case. So far, the control, filtering and model reduction problems for systems with sector-nonlinearities have stirred recurring research interests, see e.g. Han (2005), Lam, Gao, Xu, and Wang (2005), and Lin and Hu (2001) for some recent publications.

The randomly occurring phenomenon is a newly emerged research topic that has drawn initial research attention. It refers to those phenomenon that appears intermittently in a random way based on certain probabilistic law. If not properly catered, the randomly occurring phenomenon may potentially cause a lot of undesired problems that would seriously degrade the operating efficiency of the plant. Therefore, the randomly occurring phenomenon has started to gain the focus from researchers and some important results have appeared, see e.g. Wang, Yang, Ho, and Liu (2006), Shu, Lam, and Xiong (2009), Yang, Wang, Ho, and Gani (2007), Wang et al. (2009), Gao and Chen (2007), and Wei and Wang (2009). Based on the Bernoulli distribution, a very flexible and effective model has been built and extensively applied to characterize some kinds of randomly occurring phenomena including the missing measurements (Dong, Wang, & Gao 2010; Dong et al. 2010; Shu et al. 2009; Wei & Wang 2009), the randomly varying sensor delays (He, Wang, & Zhou 2007; Shen et al. 2009; Wang, Ho, & Liu 2004) and the randomly occurring nonlinearity (Ding, Wang, Hu, & Shu 2013; Hu et al. 2011; Hu et al. 2012; Wang et al. 2009). Similarly, the system faults may occur in a random way especially in a networked environment. Rather than assuming that the faults occur definitely, the probabilistic faults occur due probably to random failures and repairs of the components, intermittently switching in the interconnections of subsystems, etc. As such, there is a great need to develop a new strategy to cope with such kind of system faults. The main purpose of this paper is to introduce a new randomly occurring actuator fault model based on the time-varying Bernoulli distribution.

Recently, the gain-scheduled control and filtering problems have become significant research topics in the control community, see e.g. de Souza and Trofino (2006), Rugh and Shamma (2000), and Cao, Lin, and Shamash (2002). For the gain-scheduling design, the gains of controllers/filters consist of not only the constant part but also the time-varying parameters of systems that are securable in real time. Consequently, the gain-scheduled controllers and filters can be scheduled online according to the time-varying parameters. Apparently, this kind of controllers/filters have less conservatism than the conventional ones with constant (fixed) gains only. The gain-scheduled control/filtering problems have been thoroughly dealt with for both the continuous- and discrete-time systems in the past decade, see e.g. de Souza and Trofino (2006), Hoang, Tuan, Apkarian, and Hosoe (2004), and Rugh and Shamma (2000). On the other hand, the parameter-dependent Lyapunov function approach has recently been exploited to design controllers and filters for uncertain time-varying systems (see e.g. Gao, Shi, & Wang 2007) and applied in the gain-scheduling control/filtering problems with hope to achieve better control/filter performance requirements (Apkarian, Pellanda, & Tuan 2000; de Souza & Trofino 2006). Motivated by the above discussion, in this paper, the gain-scheduled approach and the parameter-dependent Lyapunov functional are utilized to design a fault-tolerant controller for discrete-time stochastic systems with randomly occurring actuator faults (ROAFs).

In this paper, the gain-scheduled fault-tolerant control problem is addressed for the discrete-time stochastic delayed systems with ROAFs. The main contributions of this paper can be described as follows: (1) a new fault-tolerant control problem is considered by a gain-scheduling approach for a class of discrete-time stochastic delayed systems with ROAFs; (2) a new actuator fault model is built by a stochastic variable sequence satisfying time-varying Bernoulli distribution; (3) the time-varying fault-tolerant controller gains are designed that consist of not only the constant part but also the time-varying probability parameters; and (4) an easy-to-implement algorithm is developed to design the controller. The desired fault-tolerant controller is designed by employing the gain-scheduling method which leads to less conservatism than the traditional one with constant gains only. In the simultaneous presence of ROAFs, time delays, nonlinearities and noise disturbances, the closed-loop system is guaranteed to be exponentially mean-square stable and satisfies a given H_∞ performance level. A simulation example is exploited to illustrate the effectiveness of the proposed design procedures.

Notation. In this paper, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$, $\mathbb{Z}^+$ denote, respectively, the n-dimensional Euclidean space, the set of all $n \times m$ real matrices, the set of all positive integers. $|\cdot|$ refers to the Euclidean norm in $\mathbb{R}^n$. $l_2[0, \infty)$ is the space of square-integrable vector functions over $[0, \infty)$. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). For a matrix M, M^{T} and M^{-1} represent its transpose and inverse, respectively. The shorthand $\mathrm{diag}\{M_1, M_2, \ldots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1, M_2, \ldots, M_n$. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions. In addition, $\mathbb{E}\{x\}$ and $\mathrm{Prob}\{y\}$ will, respectively, mean expectation of x and probability of y.

2. Problem formulation

Consider the following discrete-time stochastic delayed systems:

$$x(k+1) = Ax(k) + A_\tau x(k-\tau) + Bu(k) + E\phi(r(k)) + D_1 v(k) + D_2 x(k)\omega(k), \quad (1)$$

$$z(k) = Lx(k) + L_\tau x(k-\tau), \quad (2)$$

$$x(k) = \rho(k),\ k = -\tau, -\tau+1, \ldots, 0, \quad (3)$$

where $x(k) \in \mathbb{R}^{n_1}$ is the state, $z(k) \in \mathbb{R}^{n_2}$ is the controlled output, $\rho(k)$ is the initial state of the system, τ is a constant delay and $r(k) := Cx(k) + C_\tau x(k-\tau)$. $v(k) \in \mathbb{R}^{n_3}$ is the vector of unknown disturbance input, which is assumed to belong to $l_2[0,\infty)$. $\omega(k)$ is an one-dimensional Gaussian white noise sequence satisfying $\mathbb{E}\{\omega(k)\} = 0$ and $\mathbb{E}\{\omega^2(k)\} = 1$. A, A_τ, B, C, C_τ, D_1, D_2, E, L and L_τ are constant matrices with appropriate dimensions.

The nonlinear disturbance $\phi(\cdot)$ with ($\phi(0) = 0$) satisfies the following sector-bounded condition:

$$[\phi(r(k)) - \Phi_1 r(k)]^{\mathrm{T}}[\phi(r(k)) - \Phi_2 r(k)] \le 0, \quad (4)$$

where Φ_1 and Φ_2 are constant real matrices of appropriate dimensions with $\Phi_2 - \Phi_1 > 0$. In this case, the nonlinear function $\phi(\cdot)$ is said to belong to the sector $[\Phi_1, \Phi_2]$.

Setting $\Phi = \Phi_2 - \Phi_1 > 0$ and $\phi(r(k)) = \Phi_1 r(k) + \phi_s(r(k))$, the sector-bounded condition (4) can be transformed to the following form:

$$\phi_s^{\mathrm{T}}(r(k))(\phi_s(r(k)) - \Phi r(k)) \le 0. \quad (5)$$

Remark 1 The sector-bounded condition described in Equation (4) is a more general bounded condition of nonlinearity than the widely used Lipschitz condition. The sector nonlinearity has been extensively applied in neural networks and gene regulatory networks to describe nonlinear activation functions. In the past few years, a rich body of literature about the control, filtering and model reduction problems for systems with sector nonlinearities have appeared, see e.g., Han (2005), Lam et al. (2005), and Lin and Hu (2001).

When the actuators experience failures, we let $u_f(k)$ describe the control signal from the actuators and denote the faulty actuator model as

$$u_f(k) = \theta(k)u(k), \quad (6)$$

where $\theta(k)$ is a random variable sequence to depict the randomly occurring characteristic of the actuator fault and satisfies the following time-varying Bernoulli distribution:

$$\mathrm{Prob}\{\theta(k) = 1\} = \mathbb{E}\{\theta(k)\} = \delta(k),$$
$$\mathrm{Prob}\{\theta(k) = 0\} = 1 - \mathbb{E}\{\theta(k)\} = 1 - \delta(k), \quad (7)$$

where $\delta(k)$ is a time-varying positive scalar sequence that takes values on the interval $[\delta_1 \delta_2] \subseteq [01]$ with δ_1 and δ_2 being the lower and upper bounds of $\delta(k)$, respectively. In this paper, for simplicity, we assume that $\theta(k)$, $\omega(k)$ and $\rho(k)$ are uncorrelated.

Remark 2 Recently, the randomly occurring phenomenon has been initially discussed and some results can be found in e.g. Wang et al. (2004, 2006, 2009). The practical Bernoulli distribution model with time-invariant probability has been exploited to account for the randomly occurring phenomenon (Gao & Chen 2007; Wei & Wang 2009; Wang et al. 2006). In practice, the actuator fault may occur randomly due to some environmental changes which gives rise to a randomly occurring phenomenon. In this paper, the occurrence of actuator fault is described by a random variable sequence $\theta(k)$ satisfying a time-varying, rather than time-invariant Bernoulli distribution model, which certainly has less conservatism to deal with the time-varying systems with ROAFs according to time-varying probabilities.

Remark 3 In Equation (7), the time-varying parameter $1 - \delta(k)$ is called the failure rate of actuator, which represents the occurring probability of actuator failure. In practical system, the failure rate of system component rises due to the increase in the component's life and some environment causes. Even though, in engineering, the failure rate cannot be permitted to exceed some certain levels, the change of the failure rate brings serious effects on the performances of the system.

In this paper, the following probability-dependent gain-scheduled fault-tolerant controller is considered

$$u(k) = G(\delta)x(k), \quad (8)$$

where $G(\delta)$ is the fault-tolerant controller gain sequences to be designed that have the following structure:

$$G(\delta) = G_0 + \delta(k)G_u, \quad (9)$$

where, for every time step k, $\delta(k)$ is the time-varying probability of Bernoulli distribution taking value over $[\delta_1, \delta_2]$, and G_0 and G_u are the fixed parts of the controller gain to be designed.

Remark 4 The fault-tolerant controller gain proposed in Equation (9) consists of the time-varying occurrence probability of actuator faults in systems (1), which will be scheduled according to the time-varying probability. Such a controller would certainly lead to less conservatism than the conventional fault-tolerant controller with constant gains only for parameter-varying systems (1). Note that gain-scheduled control/filtering problems have recently been paid considerable research attention (Cao et al. 2002; de Souza & Trofino 2006; Rugh & Shamma 2000).

The closed-loop system of Equation (1) with the state-feedback gain-scheduled fault-tolerant controller (8) is given as follows:

$$x(k+1) = Ax(k) + A_\tau x(k-\tau) + \theta(k)BG(\delta)x(k) + E\phi(r(k)) + D_1 v(k) + D_2 x(k)\omega(k). \quad (10)$$

In the next section, the fault-tolerant controller proposed in Equation (8) is designed by developing a probability-dependent Lyapunov functional and the convex programming method such that, for all admissible time delays, nonlinearities, ROAFs and disturbance noises, the closed-loop system (10) is exponentially mean-square stable and, under zero initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{|z(k)|^2\} \le \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{|v(k)|^2\} \quad (11)$$

for all non-zero $v(k)$ and given attenuation level $\gamma > 0$.

3. Main results

In the following theorem, a sufficient condition is obtained to solve the desired parameter-dependent fault-tolerant control problem for a class of discrete-time stochastic delayed systems (1) with ROAFs by exploiting the Lyapunov theory and convex programming method. It is shown that the gains of the fault-tolerant controller can be derived by solving the convex optimization problem via the semi-definite programme method in terms of the securable time-varying probability.

THEOREM 1 *Consider the discrete-time stochastic delayed systems (10). Assume that there exist positive-definite matrices $\mathcal{P}(\delta(k)) > 0$, $\mathcal{P}_\tau > 0$, nonsingular slack matrix S and matrices $\bar{G}_0$ and $\bar{G}_u$ such that the following linear matrix inequalities (LMIs) hold*:

$$\begin{bmatrix} \mathcal{P}_\tau - \mathcal{P}(\delta(k)) & * & * & * & * & * & * & * \\ 0 & -\mathcal{P}_\tau & * & * & * & * & * & * \\ \Phi CS & \Phi C_\tau S & -2I & * & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * & * \\ \bar{A} & \bar{A}_\tau & E & D_1 & -T(k) & * & * & * \\ \Delta_\delta(k)B\bar{G}(\delta) & 0 & 0 & 0 & 0 & -\Delta_\delta(k)T(k) & * & * \\ D_2 S & 0 & 0 & 0 & 0 & 0 & -T(k) & * \\ LS & L_\tau S & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (12)$$

where

$$\bar{A} = AS + \delta(k)B\bar{G}(\delta) + E\Phi_1 CS,$$
$$\bar{G}(\delta) = \bar{G}_0 + \delta(k)\bar{G}_u, \Delta_\delta(k) = \delta(k)(1-\delta(k)),$$
$$\bar{A}_\tau = A_\tau S + E\Phi_1 C_\tau S, T(k) = -\mathcal{P}(\delta(k+1)) + S + S^{\mathrm{T}}. \quad (13)$$

In this case, the constant gains of the desired fault-tolerant controller can be obtained as follows:

$$G_0 = \bar{G}_0 S^{-1}, G_u = \bar{G}_u S^{-1} \quad (14)$$

and the closed-loop system (10) is then exponentially mean-square stable and satisfies Equation (11) with the prescribed H_∞ index $\gamma > 0$ for all $\delta(k) \in [\delta_1 \delta_2]$.

Proof Let $P(\delta(k)) = S^{-T}\mathcal{P}(\delta(k))S^{-1}$, $P_\tau = S^{-T}\mathcal{P}_\tau S^{-1}$ and define the following probability-dependent Lyapunov function:

$$V(k) := x^{\mathrm{T}}(k)P(\delta(k))x(k) + \sum_{l=k-\tau}^{k-1} x^{\mathrm{T}}(l)P_\tau x(l). \quad (15)$$

Then, noting $\mathbb{E}\{\theta(k) - \delta(k)\} = 0$ and $\mathbb{E}\{\omega(k) = 0\}$, we have from Equation (10) that

$$\begin{aligned} &\mathbb{E}\{\Delta V(k)\} \\ &= \mathbb{E}\{x^{\mathrm{T}}(k+1)P(\delta(k+1))x(k+1) - x^{\mathrm{T}}(k) \\ &\quad \times (P(\delta(k)) - P_\tau)x(k) - x^{\mathrm{T}}(k-\tau)P_\tau x(k-\tau)\} \\ &= \mathbb{E}\{[(A + \delta(k)BG(\delta))x(k) + A_\tau x(k-\tau) + E\phi(r(k)) \\ &\quad + D_1 v(k) + D_2 x(k)\omega(k) + (\theta(k) - \delta(k))BG(\delta))x(k)]^{\mathrm{T}} \\ &\quad \times P(\delta(k+1))[(A + \delta(k)BG(\delta))x(k) + A_\tau x(k-\tau) \\ &\quad + E\phi(r(k)) + D_1 v(k) + D_2 x(k)\omega(k) + (\theta(k) \\ &\quad - \delta(k))BG(\delta))x(k)] - x^{\mathrm{T}}(k)(P(\delta(k)) - P_\tau)x(k) \\ &\quad - x^{\mathrm{T}}(k-\tau)P_\tau x(k-\tau)\} \\ &= \mathbb{E}\{[(A + \delta(k)BG(\delta))x(k) + A_\tau x(k-\tau) + E\phi(r(k)) \\ &\quad + D_1 v(k)]^{\mathrm{T}} P(\delta(k+1))[(A + \delta(k)BG(\delta))x(k) \\ &\quad + A_\tau x(k-\tau) + E\phi(r(k)) + D_1 v(k)] - x^{\mathrm{T}}(k) \\ &\quad \times (P(\delta(k)) - P_\tau)x(k) - x^{\mathrm{T}}(k-\tau)P_\tau x(k-\tau) \\ &\quad + x^{\mathrm{T}}(k)D_2^{\mathrm{T}} P(\delta(k+1))D_2 x(k) + \delta(k)(1-\delta(k)) \\ &\quad \times x^{\mathrm{T}}(k)G^{\mathrm{T}}(\delta))B^{\mathrm{T}} P(\delta(k+1))BG(\delta))x(k)\}. \end{aligned} \quad (16)$$

From Equations (4)–(5) and (16), it can be seen that

$$\begin{aligned} &\mathbb{E}\{\Delta V(k)\} \\ &\le \mathbb{E}\{[(A + \delta(k)BG(\delta) + E\Phi_1 C)x(k) + (A_\tau + E\Phi_1 C_\tau) \\ &\quad \times x(k-\tau) + E\phi_s(r(k)) + D_1 v(k)]^{\mathrm{T}} P(\delta(k+1)) \\ &\quad \times [(A + \delta(k)BG(\delta) + E\Phi_1 C)x(k) + E\phi_s(r(k)) \\ &\quad + (A_\tau + E\Phi_1 C_\tau)x(k-\tau) + D_1 v(k)] + x^{\mathrm{T}}(k)D_2^{\mathrm{T}} \\ &\quad \times P(\delta(k+1))D_2 x(k) + \delta(k)(1-\delta(k))x^{\mathrm{T}}(k)G^{\mathrm{T}}(\delta)) \\ &\quad \times B^{\mathrm{T}} P(\delta(k+1))BG(\delta))x(k) - 2\phi_s^{\mathrm{T}}(r(k))\phi_s(r(k)) \\ &\quad + 2\phi_s^{\mathrm{T}}(r(k))\Phi(Cx(k) + C_\tau x(k-\tau)) - x^{\mathrm{T}}(k-\tau) \\ &\quad \times P_\tau x(k-\tau) - x^{\mathrm{T}}(k)(P(\delta(k)) - P_\tau)x(k)\}. \end{aligned} \quad (17)$$

We are now ready to prove the exponential stability of the system (10) with $v(k) = 0$. Obviously, Equation (17) with $v(k) = 0$ results in

$$\mathbb{E}\{\Delta V(k)\} \leq \mathbb{E}\{\xi^{\mathrm{T}}(k)\Xi\xi(k)\}, \tag{18}$$

where $\xi(k) = [x^{\mathrm{T}}(k)x^{\mathrm{T}}(k-\tau)\phi_s^{\mathrm{T}}(r(k))]^{\mathrm{T}}$ and

$$\Xi = \begin{bmatrix} \Xi_1 & * & * \\ \Xi_2 & \Xi_3 & * \\ \Xi_4 & \Xi_5 & \Xi_6 \end{bmatrix}, \tag{19}$$

with

$$\begin{aligned}
\Xi_1 &= (A + \delta(k)BG(\delta) + E\Phi_1 C)^{\mathrm{T}} P(\delta(k+1))(A + \delta(k) \\
&\quad \times BG(\delta) + E\Phi_1 C) + P_\tau - P(\delta(k)) \\
&\quad + D_2^{\mathrm{T}} P(\delta(k+1)) D_2 + \delta(k)(1-\delta(k)) G^{\mathrm{T}}(\delta) B^{\mathrm{T}} \\
&\quad \times P(\delta(k+1)) BG(\delta)), \\
\Xi_2 &= (A_\tau + E\Phi_1 C_\tau)^{\mathrm{T}} P(\delta(k+1))(A + \delta(k)BG(\delta) \\
&\quad + E\Phi_1 C), \\
\Xi_3 &= (A_\tau + E\Phi_1 C_\tau)^{\mathrm{T}} P(\delta(k+1))(A_\tau + E\Phi_1 C_\tau) - P_\tau, \\
\Xi_4 &= E^{\mathrm{T}} P(\delta(k+1))(A + \delta(k)BG(\delta) + E\Phi_1 C) + \Phi C, \\
\Xi_5 &= E^{\mathrm{T}} P(\delta(k+1))(A_\tau + E\Phi_1 C_\tau) + \Phi C_\tau, \\
\Xi_6 &= -2I + E^{\mathrm{T}} P(\delta(k+1)) E.
\end{aligned} \tag{20}$$

Now, we prove that $\Xi < 0$ follows from Equation (12). Pre- and post-multiply the LMIs in Equation (12) by $\mathrm{diag}\{S^{-T}, S^{-T}, I, I, I, I, I\}$ and its transpose, and, from the inequality $P^{-1}(\delta(k+1)) \geq -S^{\mathrm{T}} P(\delta(k+1)) S + S + S^{\mathrm{T}}$, we can obtain

$$\begin{bmatrix}
P_\tau - P(\delta(k)) & * & * & * & * & * & * & * \\
0 & -P_\tau & * & * & * & * & * & * \\
\Phi C & \Phi C_\tau & -2I & * & * & * & * & * \\
0 & 0 & 0 & -\gamma^2 I & * & * & * & * \\
\tilde{A} & \tilde{A}_\tau & E & D_1 & -\Pi(k) & * & * & * \\
\Delta_\delta(k) BG(\delta) & 0 & 0 & 0 & 0 & -\Delta_\delta(k)\Pi(k) & * & * \\
D_2 & 0 & 0 & 0 & 0 & 0 & -\Pi(k) & * \\
L & L_\tau & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \tag{21}$$

with $\Pi(k) = P^{-1}(\delta(k+1)), \tilde{A} = A + \delta(k)BG(\delta) + E\Phi_1 C$ and $\tilde{A}_\tau = A_\tau + E\Phi_1 C_\tau$. Furthermore, by the Schur complement, we can know from Equation (21) that $\Xi < 0$ and, subsequently,

$$\mathbb{E}\{\Delta V(k)\} < -\lambda_{\min}(\Xi)\mathbb{E}|\xi(k)|^2, \tag{22}$$

where $\lambda_{\min}(\Xi)$ is the minimum eigenvalue of Ξ. Finally, it can be confirmed from Lemma 1 of Wang et al. (2006) that the closed-loop system (10) is exponentially mean-square stable.

Let us now move to the proof of the H_∞ performance for the system (10). To do so, assume zero initial condition and consider the following index:

$$\begin{aligned}
J &:= \mathbb{E}\left\{\sum_{k=0}^{N}[z^{\mathrm{T}}(k)z(k) - \gamma^2 v^{\mathrm{T}}(k)v(k)]\right\} \\
&= \mathbb{E}\left\{\sum_{k=0}^{N}[z^{\mathrm{T}}(k)z(k) - \gamma^2 v^{\mathrm{T}}(k)v(k) + \Delta V(k)]\right\} \\
&\quad - \mathbb{E}V(N+1) \\
&\leq \mathbb{E}\left\{\sum_{k=0}^{N}[z^{\mathrm{T}}(k)z(k) - \gamma^2 v^{\mathrm{T}}(k)v(k) + \Delta V(k)]\right\} \\
&= \mathbb{E}\left\{\sum_{k=0}^{N}\eta^{\mathrm{T}}(k)\Gamma\eta(k)\right\},
\end{aligned} \tag{23}$$

where $\eta(k) = [x^{\mathrm{T}}(k)x^{\mathrm{T}}(k-\tau)\phi_s(r(k))v^{\mathrm{T}}(k)]^{\mathrm{T}}$ and

$$\Gamma = \begin{bmatrix} \Xi_1 + L^{\mathrm{T}}L & * & * & * \\ \Xi_2 + L_\tau^{\mathrm{T}}L & \Xi_3 + L_\tau^{\mathrm{T}}L_\tau & * & * \\ \Xi_4 & \Xi_5 & \Xi_6 & * \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 \end{bmatrix}, \tag{24}$$

with Ξ_i $(i = 1, \ldots, 6)$ being defined in Equation (20) and

$$\begin{aligned}
\Gamma_1 &= D_1^{\mathrm{T}} P(\delta(k+1))(A + \delta(k)BG(\delta) + E\Phi_1 C), \\
\Gamma_3 &= D_1^{\mathrm{T}} P(\delta(k+1))E, \\
\Gamma_2 &= D_1^{\mathrm{T}} P(\delta(k+1))(A_\tau + E\Phi_1 C_\tau), \\
\Gamma_4 &= -\gamma^2 I + D_1^{\mathrm{T}} P(\delta(k+1)) D_1.
\end{aligned} \tag{25}$$

Again, by the Schur Complement, it is easily known that $\Gamma < 0$ holds from Equation (21), which implies $J < 0$. Letting $N \to \infty$, we can have that Equation (11) is satisfied with the prescribed performance index $\gamma > 0$. The proof of this theorem is now complete. ■

Remark 5 In Theorem 1, a parameter-dependent Lyapunov functional has been developed to design the proposed gain-scheduled fault-tolerant controller gains and reduce the conservatism of the designed controllers. Such parameter-dependent Lyapunov functional technique has been extensively used to solve the control and filtering problems for uncertain systems and parameter-varying systems, see e.g. de Souza and Trofino (2006) and Gao et al. (2007). Unfortunately, the number of LMIs in Equation (12) is actually infinity due to the time-varying parameter $\delta(k) \in [\delta_1 \;\; \delta_2]$ and therefore constant gains of the fault-tolerant controller cannot be obtained directly by solving LMIs in Equation (12). In the following theorem, by transforming the description of $\delta(k)$, we attempt to convert LMIs in Equation (12) into computationally accessible ones.

Setting $P(\delta(k)) = P_0 + \delta(k)P_\delta$, we can easily know that $\mathcal{P}(\delta(k)) = \mathcal{P}_0 + \delta(k)\mathcal{P}_\delta$. Then, we have the following theorem by converting the expression of the time-varying parameter $\delta(k)$ in Theorem 1 into a new form.

THEOREM 2 *Consider the discrete-time nonlinear stochastic systems* (10). *If there exist positive-definite matrices $\mathcal{P}_0 > 0$, $\mathcal{P}_\delta > 0$, $\mathcal{P}_\tau > 0$, nonsingular matrices S, $\bar{G}_0$ and $\bar{G}_u$ such that the following LMIs hold*:

$$\mathbb{M}^{ijmn} = \begin{bmatrix} \mathcal{P}^{ijmn} & * & * & * & * & * & * & * & \\ 0 & -\mathcal{P}_\tau & * & * & * & * & * & * & \\ \Phi CS & \Phi C_\tau S & -2I & * & * & * & * & * & \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & \\ \bar{A}^{ijmn} & \bar{A}_\tau & E & D_1 & -T^{ijmn} & * & * & * & \\ \Delta_\delta^{ijmn}\bar{B}^{ijmn} & 0 & 0 & 0 & 0 & 0 & -\Delta_\delta^{ijmn}T^{ijmn} & * & * \\ D_2 S & 0 & 0 & 0 & 0 & 0 & 0 & -T^{ijmn} & * \\ LS & L_\tau S & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{26}$$

where $i,j,m,n = 1,2$ and

$$\begin{aligned} \mathcal{P}^{ijmn} &= \mathcal{P}_\tau - \mathcal{P}_0 - \delta_i\mathcal{P}_\delta, T^{ijmn} = -\mathcal{P}_0 - \delta_n\mathcal{P}_\delta + S + S^{\mathrm{T}}, \\ \bar{A}^{ijmn} &= AS + \delta_i B(\bar{G}_0 + \delta_m\bar{G}_u) + E\Phi_1 CS, \\ \Delta_\delta^{ijmn} &= \delta_i(1-\delta_j), \\ \bar{B}^{ijmn} &= B(\bar{G}_0 + \delta_m\bar{G}_u), \end{aligned} \tag{27}$$

then there exists a controller in the form of Equation (8) *(G_0 and G_u can be obtained according to Equation* (14)*) such that the closed-loop system* (10) *is exponentially mean-square stable and, under zero initial condition, satisfies Equation* (11) *with the prescribed index $\gamma > 0$.*

Proof First, for $\delta(k)$, letting

$$\alpha_1(k) = \frac{\delta_2 - \delta(k)}{\delta_2 - \delta_1}, \quad \alpha_2(k) = \frac{\delta(k) - \delta_1}{\delta_2 - \delta_1}, \tag{28}$$

it is easy to obtain that

$$\delta(k) = \alpha_1(k)\delta_1 + \alpha_2(k)\delta_2 \tag{29}$$

with $\alpha_i(k) \geq 0$ $(i = 1,2)$ and $\alpha_1(k) + \alpha_2(k) = 1$.

Similarly, for $\delta(k+1)$, setting

$$\beta_1(k) = \frac{\delta_2 - \delta(k+1)}{\delta_2 - \delta_1}, \quad \beta_2(k) = \frac{\delta(k+1) - \delta_1}{\delta_2 - \delta_1}, \tag{30}$$

we can have

$$\delta(k+1) = \beta_1(k)\delta_1 + \beta_2(k)\delta_2, \tag{31}$$

where $\beta_i(k) \geq 0$ $(i = 1,2)$ and $\beta_1(k) + \beta_2(k) = 1$.

From the above transformations, we can see that

$$\mathcal{P}(\delta(k)) = \sum_{i=1}^{2}\alpha_i(k)\{\mathcal{P}_0 + \delta_i\mathcal{P}_\delta\}, \tag{32}$$

$$\bar{G}(\delta) = \sum_{i=1}^{2}\alpha_i(k)\{\bar{G}_0 + \delta_i\bar{G}_u\}, \tag{33}$$

$$\mathcal{P}(\delta(k+1)) = \sum_{n=1}^{2}\beta_n(k)\{\mathcal{P}_0 + \delta_n\mathcal{P}_\delta\}. \tag{34}$$

Again, from Equation (26), it is easy to find that

$$\sum_{i,j,m,n=1}^{2}\alpha_i(k)\alpha_j(k)\alpha_m(k)\beta_n(k)\mathbb{M}^{ijmn} < 0. \tag{35}$$

Then, from Equations (28)–(34), it can be concluded that Equation (12) is true. The proof is now complete. ∎

Remark 6 In Theorem 2, we have changed the infinite LMIs that are dependent on the time-varying probability in Theorem 1 to finite ones that are dependent on the upper and lower bound of $\delta(k)$. By Theorem 2, the constant parameters of controller can easily be obtained by solving a set of LMIs via available softwares.

In the following, according to Theorem 2, a detailed design procedure is given to obtain the desired fault-tolerant controller.

ALGORITHM 1 *Probability-dependent fault-tolerant controller design algorithm.*

Step 1*: Initialize the positive integer N, the initial state $\rho(k)$, time delay τ, the bounds δ_1 and δ_2 of the time-varying parameters, the matrices A, A_d, B, C, C_τ, D_1, D_2, E, Φ_1, Φ_2, L and L_τ and set $k = 0$.*

Step 2*: Solve the LMIs in Equation* (26) *for $i,j,m,n = 1,2$ to obtain the positive-definite matrix $\mathcal{P}_0$, $\mathcal{P}_\delta$, $\mathcal{P}_\tau$, nonsingular slack matrices S, matrices $\bar{G}_0$, $\bar{G}_u$ and then the constant gains of fault-tolerant controller G_0 and G_u by Equation* (14).

Step 3*: Derive gain-scheduled controller gain $G(\delta)$ in Equation* (9) *according to measured time-varying probability $\delta(k)$ in real time and then set $k = k+1$.*

Step 4*: If $k < N$, then go to Step* 3*, otherwise go to Step* 5.

Step 5*: Stop.*

4. An illustrative example

In this section, a numerical example is given to illustrate the effectiveness of the proposed fault-tolerant controller design method.

Table 1. The measured time-varying probabilities $\delta(k)$.

k	1	2	3	4	5	6	7	8	9	10	...
$\delta(k)$	0.9691	0.9681	0.9672	0.9662	0.9651	0.9641	0.9630	0.9619	0.9607	0.9595	...

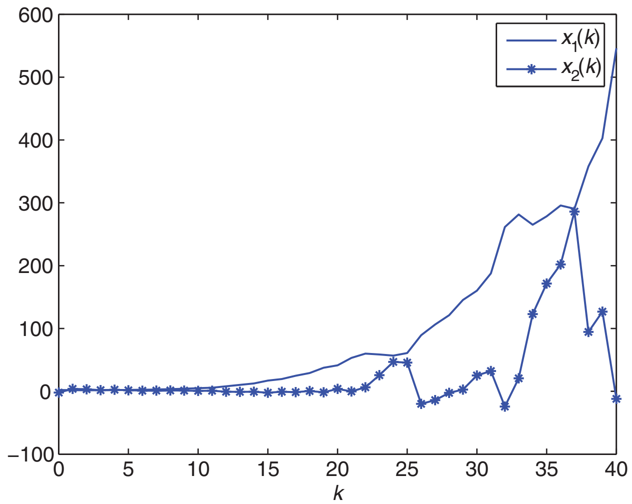

Figure 1. State evolution $x(k)$ of uncontrolled systems.

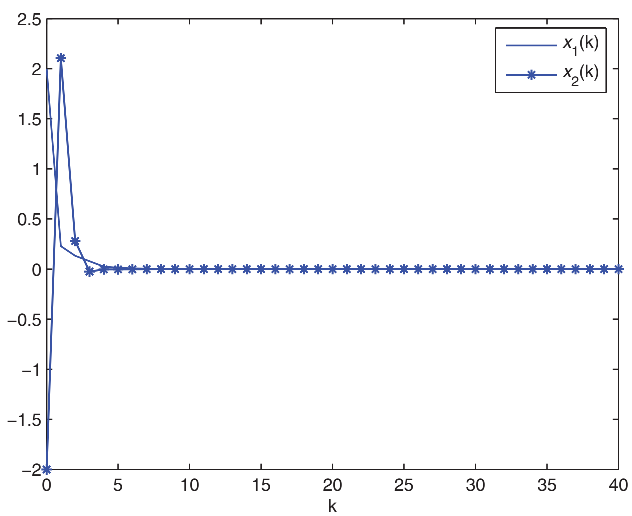

Figure 2. State evolution $x(k)$ of controlled systems.

The system parameters are given as follows:

$$A = \begin{bmatrix} 0.9 & -0.2 \\ 0 & 0.74 \end{bmatrix}, \quad A_\tau = \begin{bmatrix} 0.31 & 0 \\ 0.02 & 0.26 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.56 & 0.02 \\ 0 & 0.43 \end{bmatrix}, \quad C = \begin{bmatrix} 0.31 & 0 \\ 0 & 0.52 \end{bmatrix},$$
$$C_\tau = \begin{bmatrix} 0.21 & 0 \\ 0 & 0.32 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.14 & 0 \\ 0 & 0.11 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} -0.13 & 0 \\ 0.15 & 0.28 \end{bmatrix}, \quad E = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},$$

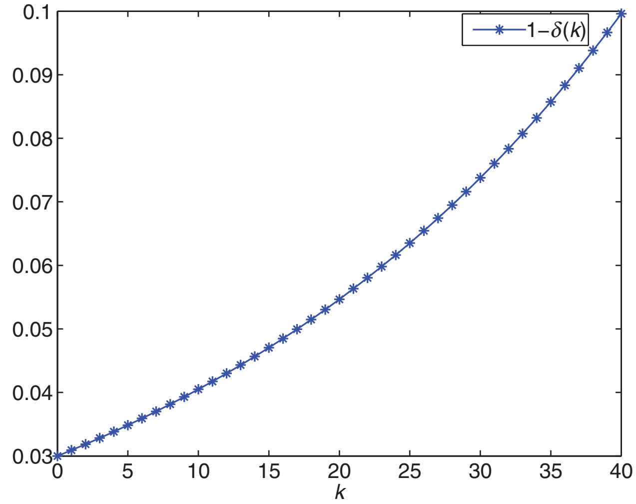

Figure 3. Time-varying failure rate $1 - \delta(k)$.

$$\Phi_1 = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.07 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} 0.51 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$L = \begin{bmatrix} 0.21 & 0 \\ 0 & 0.11 \end{bmatrix}, \quad L_\tau = [0.11 \ \ 0.12],$$
$$\delta_1 = 0.9, \delta_2 = 0.97, \gamma = 0.9, N = 40.$$

Set the initial state $\rho(k) = [2 \ \ -2]^{\mathrm{T}}$ $(k = -\tau, -\tau + 1, \ldots, 0)$ and assume the measured time-varying probability parameter as the data in Table 1.

The parameters of the sector-bounded nonlinearity are selected as

$$\phi(r(k)) = \frac{\Phi_1 + \Phi_2}{2} r(k) + \frac{\Phi_2 - \Phi_1}{2} \sin(r(k)).$$

According to Theorem 2 and Algorithm 1, the Lyapunov matrices P_0, P_δ and P_τ, slack matrix S and the constant controller parameters G_0 and G_u can be obtained as follows:

$$P_0 = \begin{bmatrix} 0.3473 & 0.0184 \\ 0.0184 & 0.3453 \end{bmatrix}, \quad P_\delta = \begin{bmatrix} 0.3338 & 0.0203 \\ 0.0203 & 0.3317 \end{bmatrix},$$
$$P_\tau = \begin{bmatrix} 0.3331 & 0.0202 \\ 0.0202 & 0.3310 \end{bmatrix}, \quad S = \begin{bmatrix} 1.9095 & -0.1455 \\ -0.1455 & 1.9249 \end{bmatrix},$$
$$G_0 = \begin{bmatrix} -1.7113 & 0.4454 \\ -0.0000 & -1.8350 \end{bmatrix},$$
$$G_u = \begin{bmatrix} 0.0025 & -0.0007 \\ 0.0000 & 0.0027 \end{bmatrix}.$$

Figure 1 gives the response curves of state $x(k)$ of uncontrolled systems. Figure 2 depicts the simulation results of state $x(k)$ of the controlled systems. Figure 3 shows the time-varying failure rate $1 - \delta(k)$. The simulation results have illustrated our theoretical analysis.

5. Conclusions

In this paper, the probability-dependent gain-scheduled fault-tolerant control problem has been dealt with for a class of discrete-time stochastic delayed systems with ROAFs. The actuator faults are assumed to occur in a random way in terms of a time-varying Bernoulli distribution. Two theorems and an algorithm have been established to obtain the sufficient condition for solving the addressed fault-tolerant control problem by employing probability-dependent Lyapunov functional. According to the obtained results, the fault-tolerant controller with the gain including the time-varying distribution probability has been designed such that, for the admissible ROAFs, time delays, nonlinearities and noise disturbances, the closed-loop system is exponentially mean-square stable and satisfies H_∞ performance with a prescribed index $\gamma > 0$. The effectiveness of the proposed design procedure has been illustrated via a numerical example.

Acknowledgements

This work was supported in part by the Alexander von Humboldt Foundation of Germany, and the National Natural Science Foundation of China under Grant 61074016.

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