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Design of a nonlinear observer for mechanical systems with unknown inputs

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This paper deals with the design of nonlinear observers for a class of nonlinear mechanical systems in presence of uncertainties (or unknown inputs). Based on sliding-mode technique, a novel observer structure is developed in order to reconstruct the unmeasured velocity variable. The proposed observer guarantees an asymptotic velocity observation also in presence of uncertainties. Simulation results are included to show the effectiveness of our method.

Keywords: observer; sliding-mode technique; nonlinear mechanical systems; uncertainties

1. Introduction

In modern control theory, the observer design for mechanical systems has been extensively studied. In fact, most of control strategies of mechanical systems incorporate knowledge of the all state variables. However, in practice, velocity signals are always corrupted by noise. It is therefore necessary to use a state observer in order to estimate the state vector and then construct the control law.

In the literature, several types of observers have been developed (Marino & Tomei, 1995; Resendiz, Wen, & Fridman, 2008; Wu, Shi, Su, & Chu, 2012; Xian, Queiroz, Dawson, & McIntyre, 2004). In Fradkov, Niki-forov, and Andrievsky (2002), Zhang (2002), and Xu and Zhang (2004), adaptive observers were designed for systems with parametric uncertainties but these observers may lack robustness against sensors noise and unmodelled dynamics. By assuming that the dynamic model of the system is completely unknown, high-gain differentiators (filters) were used by Besançon (2003) and Khalil (2005). However, these observers are not exact with any fixed finite gain (as cited in Davila, Fridman, & Levant, 2005).

Recently, sliding-mode observers have been shown to be efficient in many analytical and experimental studies such as in robotic manipulator as presented by Canudas De Wit and Slotine (1991), induction motors given by Tursini and Petrella (2000), fault detection as cited by Edwards, Spurgeon, and Patton (2000), and so on. The problem of estimating motion from a sequence of images was studied by Unel, Sabanovic, Burak, and Dogan (2008). A robust exact differentiator was successfully applied in Bartolini, Pisano, Punta, and Usai (2003) and Pisano and Usai (2004)

with finite-time convergence and without requirement of any model information. More recently, Davila et al. (2005) developed a sliding-mode observer for a mechanical system based on the super twisting algorithm. In Saadaoui, Leon, Djemai, Manamanni, and Barbot (2006), a second-order sliding-mode observer is implemented to reconstruct state vector of the mechanical system in presence of uncertainties. In Davila, Pisano, and Usai (2011), the authors studied the problem of continuous and discrete state reconstruction for nonlinear systems via sliding-mode observers. In Su, Muller, and Zheng (2007), a simple sliding-mode observer is proposed to reconstruct velocity signal for mechanical systems with uncertainties. However, the design of this observer requires the knowledge of uncertainties modelling to determine the observer gains.

The purpose of this paper is to design a new sliding-mode observer for a class of nonlinear mechanical systems with uncertainties that ensure an asymptotic velocity observation in order to reconstruct the velocity signal. The results are based on some physical properties of mechanical systems. This observer can be separately designed from a controller; therefore, the separation principle theorem is satisfied.

The remainder of this paper is structured as follows. Section 2 displays the class of nonlinear mechanical systems and underlines important physical properties of these systems. Section 3 presents the observer design and the asymptotically convergence analysis. In Section 4, the main results are discussed by an academic simulation example to illustrate our observer performance. Finally, some conclusions are included in Section 5.

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2. Problem statement

Consider a class of general mechanical systems represented by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \Delta(t, q, \dot{q}) = \tau, \quad (1)$$

where $q \in R^n$ is the vector of generalized coordinates, $M(q)$ is the inertia matrix, $C(q, \dot{q})$ is the Coriolis and centrifugal forces matrix, $G(q)$ is the vector of gravitational forces, $\Delta(t, q, \dot{q})$ is an additional term Lebesgue measurable, uniformly bounded, and which represents an uncertainty disturbance, and τ is the torque delivered by the actuators. The control input τ is assumed to be given by some known feedback function. Let $x_1 = q$, $x_2 = \dot{q}$, and $u = \tau$ and the measured output $y = x_1$. So, the model (1) can be rewritten in the state-space form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -(M(y))^{-1}[C(y, x_2)x_2 + G(y) + \Delta(t, y, x_2) - u]. \end{aligned} \quad (2)$$

By assuming that only angular positions are available, the task is to design a nonlinear observer for the system (1) such that the velocity estimation error tends to zero in finite time. For our purpose, some physical properties of system dynamics (1) are required. These properties are given as follows (as cited in Canudas & Slotine, 1991):

- (1) $M(q)$ is a positive-definite symmetric matrix,
- (2) there exists a parameterization for matrix $C(\cdot, \cdot)$ such that

$$z^T \cdot \left[\frac{\dot{M}(x_1)}{2} - C(x_1, x_2) \right] \cdot z = 0 \quad \forall z \in R^n,$$

- (3) the Coriolis and centrifugal forces matrix $C(\cdot, \cdot)$ verify $C(x_1, x)y = C(x_1, y)x \quad \forall (x, y) \in R^n \times R^n$.

3. Observer design

Let $\hat{x}_1, \hat{x}_2 \in \mathfrak{N}^n$ denote the estimated position and velocity of system (1), and the estimation errors $e_1(t), \dot{e}_1(t), e_2(t) \in \mathfrak{N}^n$ be defined, respectively, by

$$e_1 = \hat{x}_1 - x_1, \quad (3)$$

$$\dot{e}_1 = \dot{\hat{x}}_1 - \dot{x}_1, \quad (4)$$

$$e_2 = \hat{x}_2 - x_2. \quad (5)$$

Let the signal $r(t) \in \mathfrak{N}^n$ be defined as

$$r(t) = \alpha e_1(t) + \dot{e}_1(t), \quad (6)$$

where α is a positive scalar chosen, under Assumption 1, so that $\text{sgn}(r(t)) = \text{sgn}(e_1(t))$, $\forall t \geq 0$, and $\text{sgn}(\cdot)$ is the

standard signum function defined as follows:

$$\begin{aligned} \text{sgn}(x) &:= [\text{sgn}(x_1) \quad \text{sgn}(x_2) \quad \cdots \quad \text{sgn}(x_n)]^T, \\ \forall x &= [x_1 \quad x_2 \quad \cdots \quad x_n]^T. \end{aligned} \quad (7)$$

Our objective is to ensure an asymptotic convergence of $e_1(t)$ and $e_2(t)$ to zero as $t \rightarrow \infty$. To this end, we propose the following nonlinear observer:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - (\lambda_1 + \alpha)e_1, \\ \dot{\hat{x}}_2 &= -(M(y))^{-1}[C(y, \hat{x}_2)\hat{x}_2 + G(y) - u] \\ &\quad - \lambda_2 e_1 - \lambda_3 \text{sgn}(e_1), \end{aligned} \quad (8)$$

where $\lambda_1, \lambda_2, \lambda_3$ are positive reals which represent the observer gains to be given later by Theorem 1.

To demonstrate the asymptotic convergence of the error dynamics to zero (i.e. the convergence of $e_1(t)$, $\dot{e}_1(t)$, and therefore the convergence of $e_2(t)$ to zero as $t \rightarrow \infty$), we define here a positive-definite Lyapunov function that regroups the dynamics of $e_1(t)$ and $\dot{e}_1(t)$. The proposed function is given by

$$V = \frac{1}{2}r^T r + \frac{1}{2}e_1^T e_1. \quad (9)$$

The objective is to find sufficient conditions on $\lambda_1, \lambda_2, \lambda_3$ so that the time derivative of V is negative definite which make the Lyapunov function V continually decreasing. The time derivate of Equation (9) gives

$$\dot{V} = r^T \dot{r} + e_1^T \dot{e}_1. \quad (10)$$

Using Equation (6), the first derivate of the signal r gives

$$\dot{r}(t) = \alpha \dot{e}_1(t) + \ddot{e}_1(t). \quad (11)$$

The first derivate of the velocity error between system (2) and observer (8) gives

$$\begin{aligned} \dot{e}_2 &= -(M(y))^{-1}[C(y, \hat{x}_2)\hat{x}_2 - C(y, x_2)x_2 - \Delta(t, y, x_2)] \\ &\quad - \lambda_2 e_1 - \lambda_3 \text{sgn}(e_1). \end{aligned} \quad (12)$$

Due to the third physical property (iii) of mechanical systems mentioned in Section 2, we have $C(y, x_2)\hat{x}_2 = C(y, \hat{x}_2)x_2$ and Equation (12) can be rewritten as follows:

$$\begin{aligned} \dot{e}_2 &= -(M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]e_2 \\ &\quad + (M(y))^{-1}\Delta(t, y, x_2) \\ &\quad - \lambda_2 e_1 - \lambda_3 \text{sgn}(e_1). \end{aligned} \quad (13)$$

Using Equations (4)–(6) and (8), we have

$$e_2 = \dot{e}_1 + (\lambda_1 + \alpha)e_1. \quad (14)$$

Using Equation (6), the velocity estimation error e_2 can be substituted by the term $r + \lambda_1 e_1$. Then, Equation (13) can be rewritten as follows:

$$\begin{aligned} \dot{e}_2 &= -(M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]\lambda_1 e_1 \\ &\quad + (M(y))^{-1}\Delta(t, y, x_2) - \lambda_2 e_1 - \lambda_3 \operatorname{sgn}(e_1). \end{aligned} \quad (15)$$

After taking the time derivate of Equation (4) and using Equation (15), the second derivate of the output error between system (2) and observer (8) leads to

$$\begin{aligned} \ddot{e}_1 &= \dot{e}_2 - (\lambda_1 + \alpha)\dot{e}_1 \\ &= -(M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]\lambda_1 e_1 \\ &\quad + (M(y))^{-1}\Delta(t, y, x_2) - \lambda_2 e_1 \\ &\quad - \lambda_3 \operatorname{sgn}(e_1) - (\lambda_1 + \alpha)\dot{e}_1. \end{aligned} \quad (16)$$

Using Equation (16) and substituting from Equation (11), the first derivate of the signal r leads to

$$\begin{aligned} \dot{r} &= \ddot{e}_1 + \alpha\dot{e}_1 \\ &= -(M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]\lambda_1 e_1 \\ &\quad + (M(y))^{-1}\Delta(t, y, x_2) - \lambda_2 e_1 \\ &\quad - \lambda_3 \operatorname{sgn}(e_1) - \lambda_1 \dot{e}_1. \end{aligned} \quad (17)$$

Now, considering Equation (17), the first time derivate of the Lyapunov function given by Equation (9) leads to

$$\begin{aligned} \dot{V} &= r^T \dot{r} + e_1^T \dot{e}_1 \\ &= -r^T (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - r^T (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]\lambda_1 e_1 \\ &\quad + r^T (M(y))^{-1}\Delta(t, y, x_2) - r^T \lambda_2 e_1 \\ &\quad - r^T \lambda_3 \operatorname{sgn}(e_1) - r^T \lambda_1 \dot{e}_1 + e_1^T \dot{e}_1. \end{aligned} \quad (18)$$

From Equation (6), we substitute \dot{e}_1 by $r - \alpha e_1$. So, dynamics given by Equation (16) can be rewritten as

$$\begin{aligned} \dot{V} &= r^T \dot{r} + e_1^T \dot{e}_1 \\ &= -r^T (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - r^T (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]\lambda_1 e_1 \\ &\quad + r^T (M(y))^{-1}\Delta(t, y, x_2) - r^T \lambda_2 e_1 \\ &\quad - r^T \lambda_3 \operatorname{sgn}(e_1) - r^T \lambda_1 r + r^T \lambda_1 \alpha e_1 + e_1^T r - e_1^T \alpha e_1. \end{aligned} \quad (19)$$

The following assumptions given by Menini and Tornambè (2002) are required for our analysis.

ASSUMPTION 1 *The initial conditions of the state vector of the mechanical system (1) $[q^T(t_0) \quad \dot{q}^T(t_0)]^T$ and the control force $u(t)$ are chosen so that the position and the velocity vector are bounded functions of time, that is, there exist two constants $k_q, k_v > 0$ such that $\|q(t)\| < k_q$ and $\|\dot{q}(t)\| < k_v$, for all times $t \geq t_0$.*

By this assumption, the continuity of $M(\cdot)$ and $C(\cdot, \cdot)$, the invertibility of $M(\cdot)$, and the linearity of $C(q, \dot{q})$ with respect to \dot{q} , there exist two constants $k_1, k_2 > 0$ such that, for each $t \geq t_0$,

$$\|M^{-1}(y(t))C(y(t), x_2(t))\| \leq k_1, \quad (20)$$

$$\|M^{-1}(y(t))C(y(t), \hat{x}_2(t))\| \leq k_2 k_v + k_2 \|\hat{x}_2(t) - x_2(t)\|. \quad (21)$$

ASSUMPTION 2 *The term $\Delta(t, y, x_2)$, representing uncertainties, is bounded by a positive constant Δ_{\max} .*

Since the position vector is a bounded function of time and by the continuity of $M(\cdot)$, we assume then that the term $M^{-1}(y) \cdot \Delta(t, y, x_2)$ is bounded by a positive constant μ , that is,

$$\|M^{-1}(y(t))\Delta(t, y, x_2)\| < \mu, \quad (22)$$

where $\|\cdot\|$ denotes the matrix norm induced by the Cartesian norm for vectors.

Note that by Assumption 1, and in order to guarantee that $\operatorname{sgn}(r(t)) = \operatorname{sgn}(e_1(t))$, $\forall t \geq 0$, one can choose the positive scalar α such that $\alpha > \rho_2/\rho_1$, where ρ_1 and ρ_2 are two positive constants given by $\|e_1\|_{-\infty} = \min_{1 \leq i \leq n} |e_{1,i}| > \rho_1$ and $\|\dot{e}_1\|_{+\infty} = \max_{1 \leq i \leq n} |\dot{e}_{1,i}| < \rho_2$. Indeed, using the definition of the signal r given by Equation (6), we have $r(t) = \alpha e_1(t) + \dot{e}_1(t)$. Then, to make $\operatorname{sgn}(r(t)) = \operatorname{sgn}(e_1(t))$, $\forall t \geq 0$, we have to choose α as the following:

- if $e_1 > 0 \Rightarrow r(t)$ must be $> 0 \Rightarrow \alpha e_1 + \dot{e}_1 > 0 \Rightarrow \alpha > \|\dot{e}_1/e_1\|$
- if $e_1 < 0 \Rightarrow r(t)$ must be $< 0 \Rightarrow \alpha e_1 + \dot{e}_1 < 0 \Rightarrow \alpha e_1 < -\dot{e}_1 \Rightarrow \alpha > \|\dot{e}_1/e_1\|$ (because e_1 is negative).

Finally, α must be chosen such that $\alpha > \|\dot{e}_1/e_1\|_{\max}$.

So, under Assumptions 1 and 2, using the proprieties of $M(y)$, and substituting $\operatorname{sgn}(e_1)$ by $\operatorname{sgn}(r)$ in Equation (19), we can write

$$\begin{aligned} \dot{V} &= r^T \dot{r} + e_1^T \dot{e}_1 \\ &= -r^T (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]r \\ &\quad - r^T \lambda_1 (M(y))^{-1}[C(y, \hat{x}_2) + C(y, x_2)]e_1 \\ &\quad + r^T (M(y))^{-1}\Delta(t, y, x_2) - \lambda_2 r^T e_1 \\ &\quad - \lambda_3 r^T \operatorname{sgn}(r) - \lambda_1 r^T r + \lambda_1 \alpha r^T e_1 + e_1^T r - \alpha e_1^T e_1. \end{aligned} \quad (23)$$

Using the property $x \cdot \operatorname{sgn}(x) = |x|$ and taking into account that the signals r and e_1 have the same sign (i.e. $r e_1 = |r||e_1|$), we can now upper bound the right-hand side of

Equation (23) as follows:

Let $\gamma = k_1 + k_2 \cdot k_v$. So, we can write

$$\begin{aligned} \dot{V} \leq & -\lambda_1 \|r\|^2 + \|r\|^2(k_1 + k_2 \cdot k_v + k_2 \|e_2\|) \\ & - \lambda_2 \|r\| \|e_1\| + \lambda_1(k_1 + k_2 k_v + k_2 \|e_2\|) \|r\| \|e_1\| \\ & + \lambda_1 \alpha \|r\| \|e_1\| + \|r\| \|e_1\| - \lambda_3 \|r\| \\ & + \mu \|r\| - \alpha \|e_1\|^2. \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{V} \leq & -\|r\|^2[\lambda_1 - \gamma - k_2 \|e_2\|] \\ & - \|r\| \|e_1\| [\lambda_2 - (1 + \lambda_1(\alpha + \gamma + k_2 \|e_2\|))] \\ & - \|r\| [\lambda_3 - \mu] - \alpha \|e_1\|^2. \end{aligned} \tag{25}$$

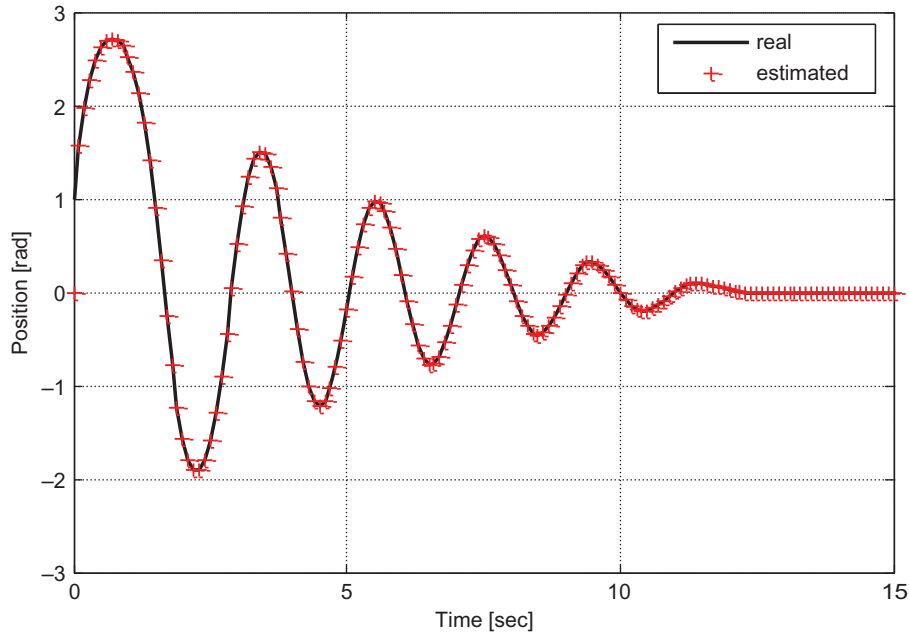


Figure 1. Real and estimated pendulum position in the uncontrolled case.

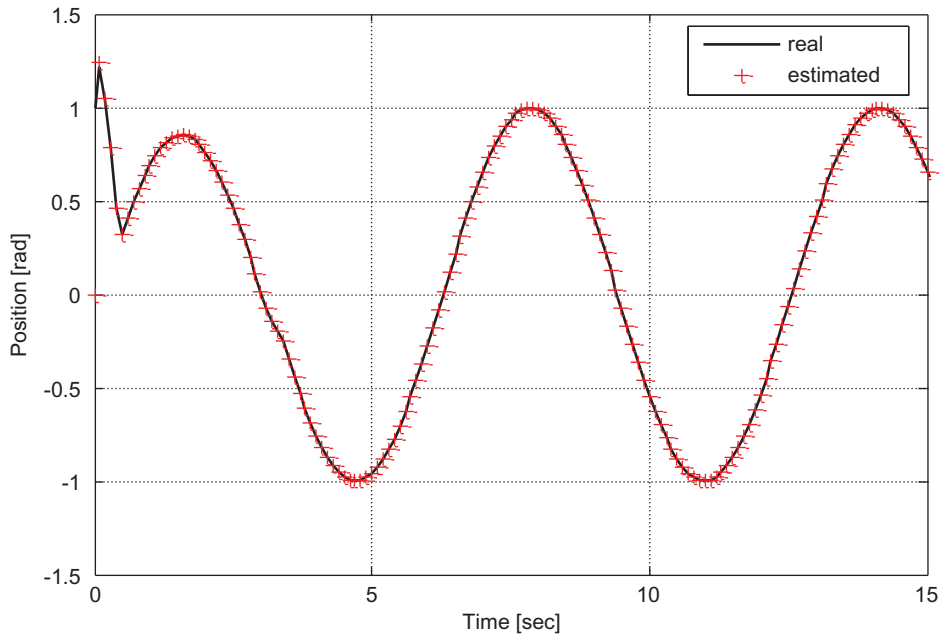


Figure 2. Real and estimated pendulum position in the controlled case.

Finally, if the following conditions are satisfied

$$\begin{aligned} \lambda_1 &> \gamma, \\ \lambda_2 &> 1 + \lambda_1(\alpha + \gamma), \\ \lambda_3 &> \mu, \end{aligned} \tag{26}$$

we can easily obtain a negative semi-definite function in a neighbourhood of $\|e_2\| = 0$. So, under conditions given by system (26) and physical properties mentioned in Section

II, V is a positive-definite Lyapunov function whose time derivative is negative semi-definite. From Equation (25) and conditions of Equation (26), since $\dot{V} \equiv 0$ means $r \equiv 0$ and $e_1 \equiv 0$, from Equation (6), we have $\dot{e}_1 \equiv 0$. By LaSalle's invariance theorem, we have $e_1(t) \rightarrow 0$ and $r(t) \rightarrow 0$ as $t \rightarrow \infty$, that is, there exist a time $t_c > 0$ such that $r(t) = e_1(t) = 0, \forall t \geq t_c$; from Equation (6), we have $\dot{e}_1 = 0$; and from Equation (14), $e_2(t) = 0, \forall t \geq t_c$: the asymptotic convergence of the estimation error system is then guaranteed. Now, the main result of this note is given in Theorem 1.

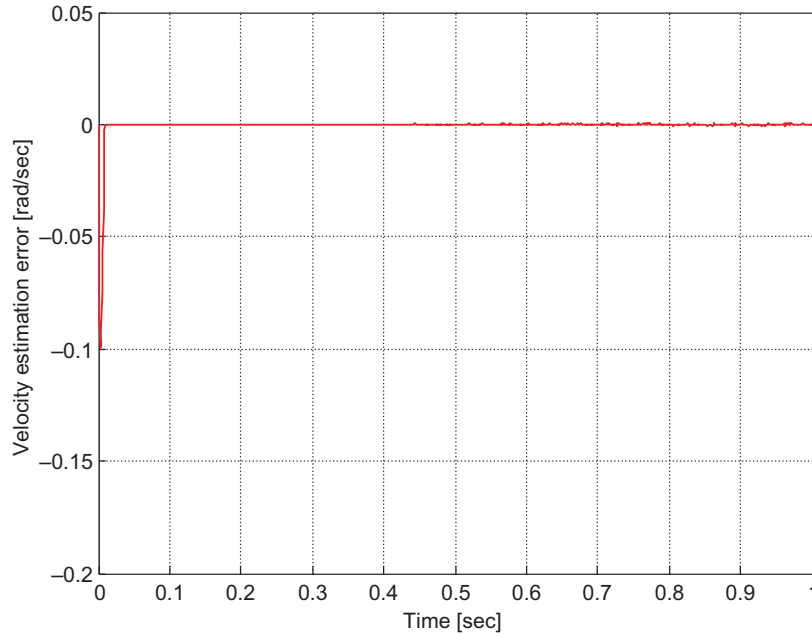


Figure 3. Finite-time velocity estimation error of the pendulum system.

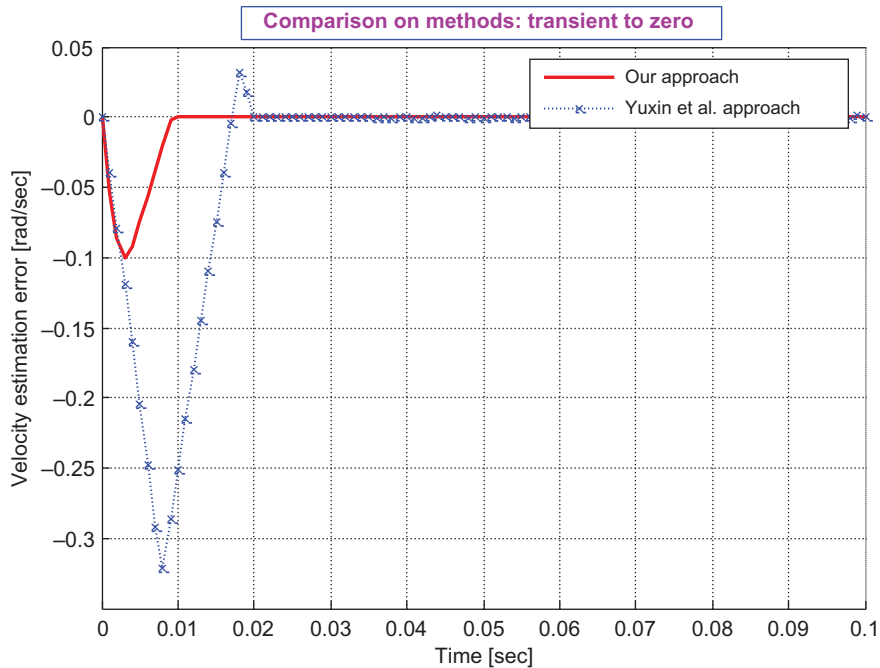


Figure 4. Velocity estimation error. A comparison on methods.

THEOREM 1 *Provided the conditions given by system (26), the observer (8) ensures a finite-time asymptotical convergence of estimated states to real states of system (2), that is, $(\hat{x}_1, \hat{x}_2) \rightarrow (x_1, x_2)$ in a finite time.*

$$\begin{aligned} \lambda'_1 &> \gamma + k_2\varepsilon, \\ \lambda'_2 &> 1 + \lambda_1(\alpha + \gamma + k_2\varepsilon), \\ \lambda'_3 &> \mu \end{aligned} \tag{27}$$

Remark 1 For each $\varepsilon > 0$, define a compact set $\Omega_\varepsilon = \{e_2 \in R^n / \|e_2\| < \varepsilon\}$. Then, the following conditions

imply the semi-global asymptotical convergence of the velocity estimation error system, the basin of attraction of

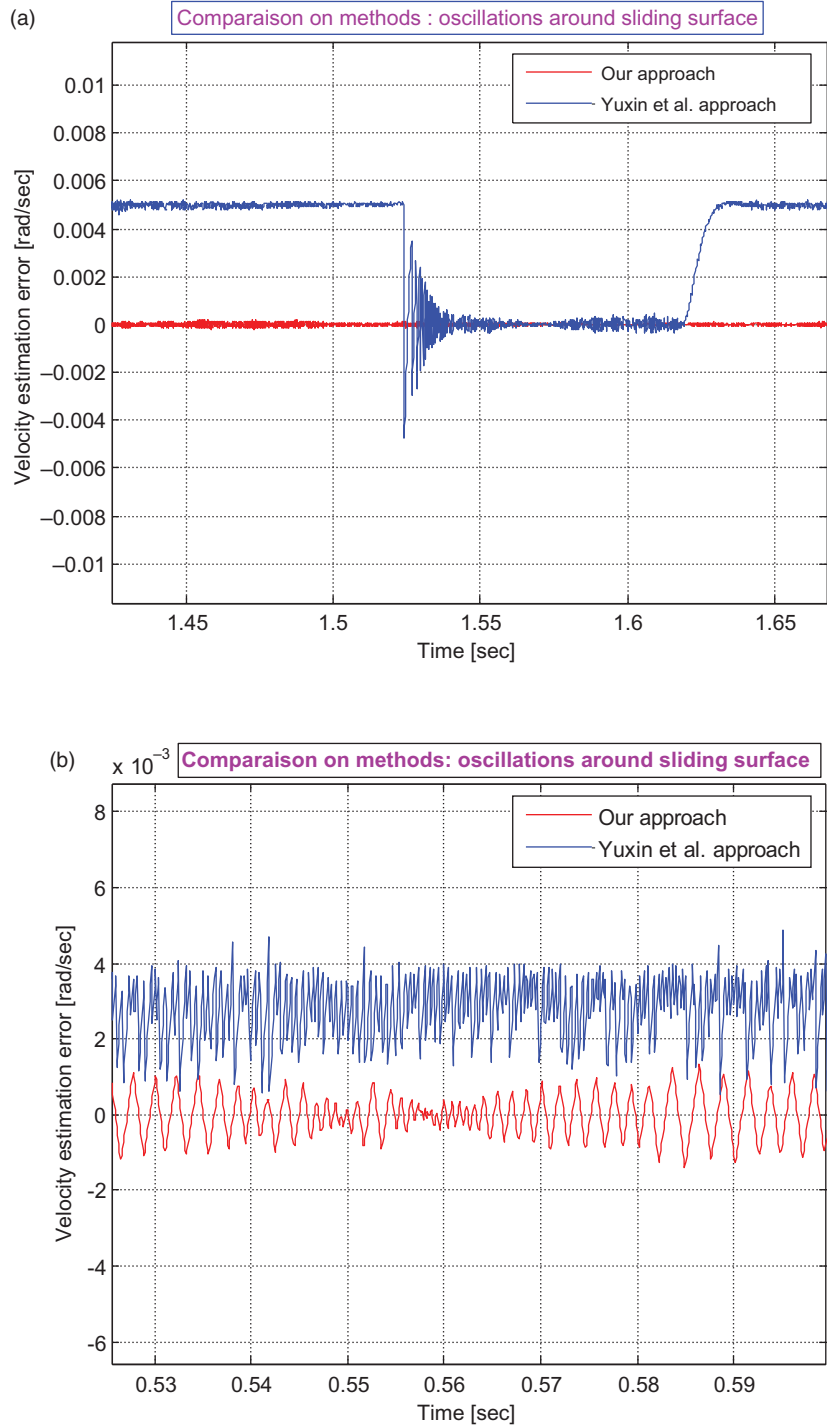


Figure 5. (a, b) Oscillations around sliding surface. A comparison on methods.

which is given by Ω_e . This estimate depends on the positive constant and can be chosen arbitrarily large.

4. Illustrative example

This section evaluates through simulations the performance of the proposed approach applied to uncertain mechanical systems.

Consider a pendulum system with Coulomb friction and external perturbation as given in Su et al. (2007) and Davila et al. (2005):

$$\ddot{\theta} = \frac{1}{J}u - \frac{g}{L} \sin(\theta) - \frac{V_s}{J}\dot{\theta} - \frac{P_s}{J}\text{sgn}(\dot{\theta}) + v, \quad (28)$$

where $J = 0.891$, $g = 9.815$, $L = 0.9$, $V_s = 0.18$, $P_s = 0.45$, and v is an uncertain external perturbation with $|v| \leq 1$. We assume that the term representing uncertainties is bounded. For simulation purposes, it was taken as $v(t) = 0, 5[\sin(2t) + \cos(5t)]$. As cited in Su et al. (2007), the controller u was taken as $u = -30 \text{sgn}(\theta - \theta_d) - 30 \text{sgn}(\dot{\theta} - \dot{\theta}_d)$, where $\theta_d = \sin(t)$ and $\dot{\theta}_d = \cos(t)$ are, respectively, the reference position signal and its derivate. Our proposed observer gains are chosen as $\lambda_1 = 12$, $\lambda_2 = 740$, and $\lambda_3 = 2$.

The model (28) can be rewritten in the state-space form as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{J}u - \frac{g}{L} \sin(x_1) - \frac{V_s}{J}x_2 - \frac{P_s}{J}\text{sgn}(x_2) + v. \end{aligned} \quad (29)$$

Our proposed observer is then given by

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - (\lambda_1 + \alpha)e_1, \\ \dot{\hat{x}}_2 &= \frac{1}{J}u - \frac{g}{L} \sin(\hat{x}_1) - \frac{V_s}{J}\hat{x}_2 - \lambda_2 e_1 - \lambda_3 \text{sgn}(e_1), \end{aligned} \quad (30)$$

where $e_1 = \hat{x}_1 - x_1$.

Simulation results given by Figures 1 and 2 show the efficiency of the proposed method in presence of uncertainties. It can be clearly seen that the proposed observer provides an excellent estimation of the pendulum position in both controlled and uncontrolled cases. Figure 3 shows the velocity estimation error of the system under consideration. This error converges asymptotically to zero in finite time.

To show the effectiveness of our approach, a comparison of the proposed observer over those proposed by Davila et al. (2005) and Su et al. (2007) is obtained. The observer proposed by Davila et al. and applied to the same system has the form

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + 1, 5(f^+)^{1/2}|x_1 - \hat{x}_1|^{1/2}\text{sgn}(x_1 - \hat{x}_1), \\ \dot{\hat{x}}_2 &= \frac{1}{J_n}u - \frac{g}{L_n} \sin(x_1) - \frac{V_{sn}}{J_n}\hat{x}_2 + 1, 1(f^+)\text{sgn}(x_1 - \hat{x}_1), \end{aligned} \quad (31)$$

where $M_n = 1.0$, $L_n = 1.0$, $J_n = M_n \cdot L_n^2 = 1.0$, and $V_{sn} = 0.2$ are the estimated nominal parameters and f^+ is a design parameter which is taken equal to 6 in Davila et al. (2005). In Su et al. (2007), the authors applied a sliding-mode observer to the model (24). According to them, their

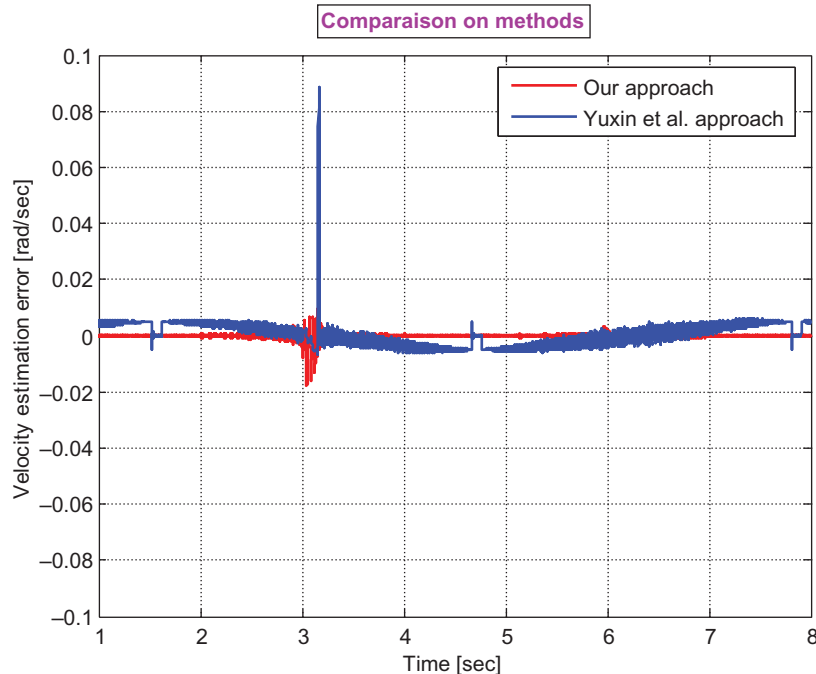


Figure 6. Robustness against uncertainties. A comparison on methods.

proposed observer “obtains much better velocity estimation” in comparison with the observer proposed by Davila et al. So, our comparison is reduced to the approach proposed in Su et al. (2007). The observer developed in this approach has the form

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2, \\ \dot{\hat{x}}_2 &= -K_0 \operatorname{sgn}(e) - (K_1 + I_N)\hat{x}_2 - K_2 e,\end{aligned}\quad (32)$$

where $e = \hat{x}_1 - x_1$ is the position estimation error, K_0 , K_1 , and K_2 are diagonal constant positive real definite matrices with $K_1 < K_2$, I_N denotes the $n \times n$ identity matrix and $\operatorname{sgn}(\cdot)$ being the standard signum function. In Su et al. (2007), simulation results are obtained with the following observer gains $K_0 = 25$, $K_1 = 500$, and $K_2 = 10^5$.

Firstly, from convergence rapidity point of view, it is clear from Figure 4 that our observer gives better transient than the Su et al. observer. Secondly, when zooming around the sliding surface in different times, we will remark from Figure 5(a) and 5(b) that the velocity estimation error converges asymptotically with the minimum rate of oscillations around zero when using our approach which is not the case for the Yuxin et al. approach. Finally, when extending the axis time to 8 s, it can be clearly seen from Figure 6 the presence of significant noises in the velocity estimation error by applying the observer given by Su et al. and therefore we can conclude about robustness of our observer against uncertainties.

From the comparison, we can conclude that our observer obtains a much better velocity estimation error.

5. Conclusions

A sliding-mode observer has been proposed to reconstruct the velocity signal for a nonlinear uncertain mechanical system, from only position measurements. The observer has been designed with the concept of simple gains without requiring uncertainties modelling. The separation principle theorem is trivial since the observer can be designed from a controller. Theoretical results have been supported by numerical simulations applied to a pendulum system with friction and external perturbation. These results are compared with those given by Su et al. (2007) and applied to the same system showing then the effectiveness of our approach. Further works will be done on designing a nonlinear observer for mechanical systems subject to non-smooth impacts.

References

Bartolini, G., Pisano, A., Punta, E., & Usai, E. (2003). A survey of applications of second-order sliding mode control to mechanical systems. *International Journal of Control*, 76(9), 875–892.

Besaçon, G. (2003). High-gain observation with disturbance attenuation and application to robust fault detection. *Automatica*, 39, 1095–1102.

Canudas De Wit, C., & Slotine, J. J. E. (1991). Sliding observers for robot manipulators. *Automatica*, 27, 859–864.

Davila, J., Fridman, L., & Levant, A. (2005). Second-order sliding-mode observer for mechanical systems. *IEEE Transactions on Automatic Control*, 50(11), 1785–1789.

Davila, J., Pisano, A., & Usai, E. (2011). Continuous and discrete state reconstruction for nonlinear switched systems via high-order sliding-mode observers. *International Journal of Systems Science*, 42(5), 725–735.

Edwards, C., Spurgeon, S. K., & Patton, R. J. (2000). Sliding mode observers for fault detection and isolation. *Automatica*, 36, 541–553.

Fradkov, A. L., Nikiforov, V. O., & Andrievsky, B. R. (2002). Adaptive observers for nonlinear nonpassifiable systems with application to signal transmission. In *Proceedings of the 41st IEEE Conference Decision and Control*, Las Vegas, NV, pp. 4706–4711. IEEE Xplore.

Khalil, H. K. (2005). A note on the robustness of high-gain-observer based controllers to unmodeled actuator and sensor dynamics. *Automatica*, 41, 1821–1824.

Marino, R., & Tomei, P. (1995). Adaptive observer with arbitrary exponential rate of convergence for nonlinear system. *IEEE Transactions on Automatic Control*, 40(7), 1300–1304.

Menini, L., & Tornambè, A. (2002). Velocity observers for nonlinear mechanical systems subject to non-smooth impacts. *Automatica*, 38, 2169–2175.

Pisano, A., & Usai, E. (2004). Output-feedback control of an underwater vehicle prototype by higher-order sliding modes. *Automatica*, 40(9), 1525–1531.

Resendiz, J., Wen Y., & Fridman, L. (2008). Two-stage neural observer for mechanical systems. *IEEE Transactions on Circuits and Systems*, 55(10), 1076–1080.

Saadaoui, H., Leon, J. D., Djemai, M., Manamanni, N., & Barbot, J. (2006). High order sliding mode and adaptive observers for a class of switched systems with unknown parameter: A comparative study. In *Proceedings of the 45th IEEE Conference Decision Control*, San Diego, USA, pp. 5555–5560. IEEE Xplore.

Su, Y., Muller, P. C., & Zheng, C. H. (2007). A simple nonlinear observer for a class of uncertain mechanical systems. *IEEE Transactions on Automatic Control*, 52(7), 1340–1345.

Tursini, M., & Petrella, R. (2000). Adaptive sliding-mode observer for speed-sensorless control of induction motors. *IEEE Transactions on Industry Applications*, 36(5), 1380–1387.

Unel, M., Sabanovic, A., Burak, Y., & Dogan, E. (2008). Visual motion and structure estimation using sliding mode observers. *International Journal of Systems Science*, 39(2), 149–161.

Wu, Z., Shi, P., Su, H., & Chu, J. (2012). State estimation for discrete-time neural networks with time-varying delay. *International Journal of Systems Science*, 43(4), 647–655.

Xian, B., Queiroz, M. S., Dawson, D. M., & McIntyre, M. L. (2004). A discontinuous output feedback controller and velocity observer for nonlinear mechanical systems. *Automatica*, 40(4), 695–700.

Xu, A., & Zhang, Q. (2004). Nonlinear system fault diagnosis based on adaptive estimation. *Automatica*, 40, 1181–1193.

Zhang, Q. (2002). Adaptive observer for multiple-input-multiple output (MIMO) linear time varying systems. *IEEE Transactions on Automatic Control*, 47(3), 525–529.