# A new approach to order reduction using stability equation and big bang big crunch optimization 

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To cite this article: S. R. Desai \& Rajendra Prasad (2013) A new approach to order reduction using stability equation and big bang big crunch optimization, Systems Science \& Control Engineering: An Open Access Journal, 1:1, 20-27, DOI: 10.1080/21642583.2013.804463

To link to this article: https://doi.org/10.1080/21642583.2013.804463

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Published online: 08 Jul 2013.

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## REVIEW

# A new approach to order reduction using stability equation and big bang big crunch optimization 

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(Received 12 February 2013; final version received 8 May 2013)


#### Abstract

A new method of model order reduction is introduced by combining the merits of big bang big crunch (BBBC) optimization technique and stability equation (SE) method. A linear-continuous single-input single-output system of higher order is considered and reduced to a lower order system. The denominator polynomial of the reduced system is obtained by SE method, whereas the numerator terms are generated using BBBC optimization. Furthermore, step and frequency responses of the original reduced system are plotted. The superiority of the proposed method is justified by solving numerical examples from the available literature and comparing the reduced systems in terms of error indices.


Keywords: model order reduction; big bang big crunch; stability equation; single input single output

## 1. Introduction

The unending need for size reduction, driven by the demand for increased system complexity, necessitates the speeding up of simulation process in the design validation stage. Hence, model diminution has become a routine work in systems and control engineering and is under active research. This further ends up as a necessary procedure for simulating large complex systems. A plenty of order minimization techniques are available in the literature today (Antonio \& Viaro, 1983; Desai \& Prasad, 2011a; Genesio \& Milanese, 1976; Jamshidi, 1983; Mahmoud \& Singh, 1981; Prasad \& Pal, 1991; Prasad, Mittal, \& Sharma, 2005) and the best method is one which protects the vital dynamics of the system under consideration. Also, disentangle the best available model in light of the purpose for which the model is to be used, namely to design a control system to meet certain specifications that helps to find low-order approximated models, without incurring too much error. For such complex problems, nature inspired approaches are among the methods, that have proved to be useful. One such approach called big bang big crunch (BBBC) is used here for order reduction (Boby \& Pal, 2010).

During the last decades, the use of optimization methods has gained popularity not only in model reduction but in almost all fields (Zhang \& Shi, 2012; Zhang, Shi, \& Liu, 2013; Zhang, Shi, \& Mehr, 2011). Some of the optimization methods such as genetic algorithm (GA) and particle swarm optimization (PSO) have already proved to be effective in developing lower order approximations for systems having large dimension and controller design of the same (Desai \& Prasad, 2010, 2011b; Parmar, Prasad, \& Mukherjee, 2007; Sivanandam \& Deepa, 2009). This is because

GA and PSO helps in the elimination/optimizing some of the state variables from the original or a transformed system representation, a task which cannot be accomplished easily. This amounts to reduction in the storage and computation time requirements, without damaging the dynamical properties of the model making it viable to use. In spite of the current optimization methods, there is greater emphasis for the advancement of the so-called global optimization methods. Researchers are still striving to attain a universal optimization method that can be applied to all multifaceted problems with equal efficiency.

BBBC , a relatively recent method of evolutionary computation has been applied and proved to be successful in many areas, inclusive of target motion analysis, fuzzy model inversion, space trusses design, nonlinear controller design and airport gate assignment problem (Camp, 2007; Dogan \& Istefanopulos, 2007; Gen, Erol, \& Eksi, 2009; Gen \& Hocaoglu, 2008; Kumbasar, Eksin, Guzelkaya, \& Yesil, 2008; Kumbasar, Yesil, Eksin, \& Guzelkaya, 2008). This method is attractive because of its intuitiveness, fast convergence and simplicity; it requires no rigid first guess algorithms. Ease of implementation and exploring the majority of problem space (Pavel, 2011) are added advantages. Furthermore, it is unfussy to code and comprehend its most basic form. Hence, it is found to be useful in solving mixed integer optimization problems that are typical of complex engineering systems (Genç Eksin, \& Osman, 2010). BBBC being a numerical optimizer assists us in rationally searching the best values among the alternative ones to satisfy our needs. On the other hand, stability equation (SE) method basically being a stability preserving reduction technique (for stable original system) has been proved

[^0]to be successful. Furthermore, this method helps in good response matching during steady state between the original and the reduced system when subjected to impulse/step input. In this paper, a new composite method of reduction method combining the benefits of BBBC and biased SE method is introduced to meet the purpose. Numerical examples are solved by applying the proposed method. The integral square error (ISE) and integral relative error (IRE) values obtained justify that the proposed approach is better as compared to other conventional techniques available in the literature.

## 2. Big bang - big crunch algorithm

The BBBC method was developed and proposed as a novel optimization method by Erol and Eksin (2006). This method is derived from one of the evolution of the universe theories in physics and astronomy, describing how the universe was created, evolved and would end, namely the BBBC phase. The big bang phase comprising random energy dissipation over the entire search space or the transformation from an ordered state to a disordered or chaotic state. After the big bang phase, a contraction occurs in the big crunch phase. Here, the particles that are randomly distributed are drawn into an order. This aims to reduce the computational time and have quick convergence even in long, narrow parabolic shaped flat valleys or in the existence of several local minima.

The big bang phase is somewhat similar to creation of initial random population in GA. The designer should handle the impermissible candidates at this phase. Once the population is created, fitness values of the individuals are calculated (Genç, Eksin, \& Osman, 2010). The crunching phase is a convergence operator that has many inputs but only one output, which can be referred to as the center of "mass." The center of mass represents the weighted average of the candidate solution positions. Here, the term mass refers to the inverse of the merit function value (Singh \& Verma, 2011). After a number of sequential banging and crunching phases the algorithm converges to a solution. The point representing the center of mass " $X_{c}$ " of the population is calculated according to the formula

$$
\begin{equation*}
X_{c}=\frac{\sum_{k=0}^{N-1} \frac{X_{k}}{f_{k}}}{\sum_{k=0}^{N-1} \frac{1}{f_{k}}} \tag{1}
\end{equation*}
$$

where $X_{k}$ is a point within an $n$-dimensional search space generated, here it is related to the numerator polynomial coefficients, $f_{k}$ is a fitness function or objective value of the candidate $k$ and $N$ is the population size in banging phase. The convergence operator in the crunching phase is different from wild selection since the output term may contain additional information (new candidate or member having different parameters than others) than the participating ones. In the next cycle of the big bang phase, new solutions are created by using the previous knowledge (center of mass),
the fitness function $f_{k}$ (Erol \& Eksin, 2006) is

$$
\begin{equation*}
f_{k}=\sum_{i=0}^{M-1}\left[y(i \Delta t)-y_{r}(i \Delta t)\right]^{2} \tag{2}
\end{equation*}
$$

where

$$
M=\frac{T}{\Delta t}
$$

where $y(i \Delta t)$ and $y_{r}(i \Delta t)$ are the unit step responses of the higher-order and the reduced-order models at time $t=\Delta t$. Usually time $T$ is taken as 10 s and $\Delta t=0.1 \mathrm{~s}$.

The basic BBBC algorithm utilized here is as follows:
Step 1 [Start] The Big bang starts by generating the new population.
Step 2 [Evaluate Fitness value] For each iteration the algorithm will act such that each candidates will move in a direction to improve its fitness function. The action involves movement updating of individuals and evaluating the fitness function for the new position.
Step 3 [Compare Fitness Function] Compare the fitness function of the new position with the specified fitness function. Repeat the above steps for the whole set of candidates.
Step 4 [Maximum iteration] Stop and return the best solution if maximum iteration is reached or a specified termination criterion is met. Else, update and start generating new population at step 1.
Step 5 [Loop] Go to step 2 for fitness evaluation.

## 3. Statement of problem

Consider an $n$ th-order linear time invariant single-input single-output (SISO) system described by the transfer function

$$
\begin{equation*}
G(s)=\frac{N(s)}{D(s)}=\frac{\sum_{j=1}^{n} a_{2, j} s^{j-1}}{\sum_{j=1}^{(n+1)} a_{1, j} s^{j-1}} \tag{3}
\end{equation*}
$$

where $a_{2, j}$ 's and $a_{1, j}$ 's are scalar constants. The objective is to find the $r$ th $(r<n)$ order-reduced model $R(s)$, comprising scalar constants $b_{2, i}$ 's and $b_{1, i}$ 's represented in the form of

$$
\begin{equation*}
R(s)=\frac{N_{r}(s)}{D_{r}(s)}=\frac{\sum_{i=1}^{r} b_{2, i} s^{i-1}}{\sum_{i=1}^{r+1)} b_{1, i} s^{i-1}} \tag{4}
\end{equation*}
$$

## 4. SE method

The model order reduction (MOR) problem has been investigated in the literature extensively (Antoulas, Sorensen, \& Gugercin, 2001; Dia, Othman, \& Zaer, 2011). SE method is essentially a stability criteria-based reduction method and is one of the most popular frequency domain techniques available in the literature (Chen, Chang, \& Han, 1979; Rajendra, 1989). This method has the privilege of yielding stable-reduced order system, provided the original
system is stable. In other words, it retains the stability of the original system and also nullifies the steady state response matching issues. A composite method, making use of the advantages of SE method and the Pade approximation method for reducing high-order transfer functions of single-input/single-output systems and multivariable systems is presented in Chen, Chang, and Han (1980). Similarly, Panda, Tomar, Prasad, and Ardil (2009) and Vishwakarma and Prasad (2009) have dealt with performing MOR using heuristic techniques and are successful. Another way to minify the given SISO/multiple input multiple output (MIMO) system to its equivalent reduced form is being dealt with in this communication. A transfer function model of an original system is considered and its denominator polynomial is reduced to a lower order by using biased SE method. The reduction procedure by biased SE method is illustrated in the following algorithm.

## Algorithm for biased SE reduction method

1. Given a stable $G(s)=N(s) / D(s)$ of " $n$ th" order
2. Considering $D(s)$ and bifurcating into even $\left(D_{e}(s)\right)$ and odd $\left(D_{o}(s)\right)$ parts will provide the following SEs:
$D(s)=D_{e}(s)+D_{o}(s)$,
where
$D_{e}(s)=a_{11} \prod_{i=1}^{k_{1}}\left(1+s^{2} / z_{i}^{2}\right)$,
$D_{o}(s)=a_{12} s \prod_{i=1}^{k_{2}}\left(1+s^{2} / p_{i}^{2}\right)$,
where $k_{1}$ and $k_{2}$ are the integer part of $n / 2$ and $(n-1) / 2$, respectively, and $z_{1}^{2}<p_{1}^{2}<z_{2}^{2}<p_{2}^{2} \ldots$ By discarding the factors with larger magnitudes of $z_{i}$ and $p_{i}$, the reduced SEs of the desired order " $r$ " become (Pal, 1983; Rajendra, 1989)
$D_{r e}(s)=a_{11} \prod_{i=1}^{r_{1}}\left(1+s^{2} / z_{i}^{2}\right)$,
$D_{r o}(s)=a_{12} s \prod_{i=1}^{r_{2}}\left(1+s^{2} / p_{i}^{2}\right)$,
where $r_{1}$ and $r_{2}$ are the integer parts of $r / 2$ and $(r-1) / 2$, respectively. Thus, the reduced denominator is constructed as
$D_{r_{1}}(s)=D_{r e}(s)+D_{r o}(s)$.
3. Now, apply the reciprocal transformation to $D(s)$ to obtain
$\tilde{D}(s)=s^{n} D\left(\frac{1}{s}\right)=\tilde{D}_{e}(s)+\tilde{D}_{o}(s)$.
Reducing the denominator further gives
$\tilde{D}_{r_{2}}(s)=\tilde{D}_{r e}(s)+\tilde{D}_{r o}(s)$.
4. Compute
$\begin{aligned} D_{r}(s) & =D_{r_{1}}(s) \cdot D_{r_{2}}(s) \\ & =b_{11}+b_{12} s+b_{13} s^{2}+\cdots+b_{1 r+1} s^{r},\end{aligned}$
with $\left(\left(r=r_{1}+r_{2}\right)<n\right)$,
where $D_{r_{2}}(s)$ is reciprocal of $\tilde{D}_{r_{2}}(s)$.

## 5. Numerical examples

Example 1 Consider a fourth-order system (Boby \& Pal, 2010; Mukherjee \& Mishra, 1987) described by the transfer function as

$$
G(s)=\frac{s^{3}+7 s^{2}+24 s+24}{s^{4}+10 s^{3}+35 s^{2}+50 s+24}
$$

For $r_{1}=2, r_{2}=0$; the denominator polynomial of $G(s)$ is bifurcated into odd and even terms as

$$
\begin{aligned}
D(s) & =D_{e}(s)+D_{o}(s) \\
D_{e}(s) & =24+35 s^{2}+s^{4} \\
D_{e}(s) & =24\left(1+\frac{s^{2}}{0.6858}\right)\left(1+\frac{s^{2}}{34.71}\right) \\
D_{o}(s) & =50 s+10 s^{3} \\
& =50 s\left(1+\frac{s^{2}}{5}\right)
\end{aligned}
$$

Neglecting the factors with larger magnitudes of $z_{i}^{2}$ and $p_{i}^{2}$ in $D_{e}(s)$ and $D_{o}(s)$, respectively, the reduced second-order equation will be

$$
\begin{aligned}
D_{r_{1}}(s) & =D_{r e}(s)+D_{r o}(s) \\
D_{r e}(s) & =24\left(1+\frac{s^{2}}{0.6858}\right), \\
D_{r o}(s) & =50 s \\
D_{r_{1}}(s) & =24\left(1+\frac{s^{2}}{0.6858}\right)+50 s \\
& =s^{2}+1.428 s+0.6858
\end{aligned}
$$

For $r_{1}=0, r_{2}=2$; the reciprocal transformed $\tilde{D}(s)$ is expressed into the following SEs:

$$
\begin{aligned}
\tilde{D}(s) & =24 s^{4}+50 s^{3}+35 s^{2}+10 s+1 \\
& =\tilde{D}_{e}(s)+\tilde{D}_{o}(s) \\
\tilde{D}_{e}(s) & =24 s^{4}+35 s^{2}+1, \\
\tilde{D}_{o}(s) & =50 s^{3}+10 s
\end{aligned}
$$

Neglecting the factors with larger magnitudes of $z_{i}^{2}$ and $p_{i}^{2}$ in $\tilde{D}_{e}(s)$ and $\tilde{D}_{o}(s)$, respectively, the reduced second-order equation will be

$$
\begin{aligned}
\tilde{D}_{r_{2}}(s) & =\tilde{D}_{r e}(s)+\tilde{D}_{r o}(s) \\
& =34.3 s^{2}+10 s+1
\end{aligned}
$$

or

$$
D_{r_{2}}(s)=s^{2}+10 s+34.3
$$

Thus, the three second-order reduced denominators which are properly normalized are given as

$$
\begin{aligned}
& D_{r}(s)=D_{r_{1}}(s) \cdot D_{r_{2}}(s) \\
& D_{r}(s)=s^{2}+1.428 s+0.6858, \quad r_{1}=2, r_{2}=0 \\
& D_{r}(s)=s^{2}+10 s+34.3, \quad r_{1}=0, r_{2}=2 \\
& D_{r}(s)=s^{2}+3.913 s+1.6464, \quad r_{1}=1, r_{2}=1
\end{aligned}
$$

Using BBBC algorithm the numerator coefficients are generated by minimizing (2) for initial population size as 100 feasible solutions, the number of iterations is 50 and reduction rate is 0.8 .

$$
N_{r}(s)=0.6965 s+0.6858
$$

Thus, the reduced second-order system is given as

$$
R_{A}(s)=\frac{N_{r}(s)}{D_{r}(s)}=\frac{0.6965 s+0.6858}{s^{2}+1.428 s+0.6858}
$$

The second-order reduced system obtained by Boby and Pal (2010) is

$$
R_{B}(s)=\frac{0.9315 s+1.6092}{s^{2}+2.75612 s+1.6092}
$$

Figure 1 shows the step responses of the original system $G(s)$, the proposed reduced system $R_{\mathrm{SE}}(s)$ and the reduced system using dominant pole (Boby \& Pal, 2010) $R_{\mathrm{DP}}(s)$. It is seen that the responses are matching both in steady and transient states.

Figure 2 shows the bode plots of the original system $G(s)$, the proposed reduced system $R_{A}(s)$ and the reduced system using dominant pole (Boby \& Pal, 2010) $R_{B}(s)$. It is seen that the responses are comparable. Table 1 exhibits the superiority of the proposed method by comparing with reduced-order systems obtained by alternative methods available as a function of ISE calculated using

$$
\begin{equation*}
\operatorname{ISE}=\int_{0}^{\infty}\left[y(t)-y_{r}(t)\right]^{2} \mathrm{~d} t, \tag{5}
\end{equation*}
$$



Figure 1. Comparison of step responses for example 1.


Figure 2. Comparison of bode plots for example 1.
and IRE using

$$
\begin{equation*}
\operatorname{IRE}=\int_{0}^{\infty} g^{2}(t) \mathrm{d} t \tag{6}
\end{equation*}
$$

where $y(t), y_{r}(t)$ are the step responses of the higher- and reduced-order system, $g(t)$ is the impulse response of the system.

Example 2 Consider a ninth-order system having transfer function (Mukherjee, Satakshi, \& Mittal, 2005)

$$
\begin{gathered}
G(s)=\frac{s^{4}+35 s^{3}+291 s^{2}+1093 s+1700}{s^{9}+9 s^{8}+66 s^{7}+294 s^{6}+1029 s^{5}+2541 s^{4}} \\
+4684 s^{3}+5856 s^{2}+4620 s+1700
\end{gathered}
$$

Applying the algorithm for biased SE reduction method, the reduced denominators are found as
$D_{r}(s)=s^{3}+1.494 s^{2}+1.34 s+0.493, \quad r_{1}=3, r_{2}=0$,
$D_{r}(s)=s^{3}+9 s^{2}+46.54 s+187.43, \quad r_{1}=0, r_{2}=3$,
$D_{r}(s)=s^{3}+9.96 s^{2}+8.994 s+3.1813, \quad r_{1}=2, r_{2}=1$,
$D_{r}(s)=s^{3}+9.367 s^{2}+49.84 s+17.08, \quad r_{1}=1, r_{2}=2$.
The numerator coefficients are generated similarly using BBBC and is given below as

$$
N_{r}(s)=0.08717 s^{2}+0.3142 s+0.493
$$

Therefore, the reduced third-order model will be

$$
R_{A}(s)=\frac{N_{r}(s)}{D_{r}(s)}=\frac{0.08717 s^{2}+0.3142 s+0.493}{s^{3}+1.494 s^{2}+1.34 s+0.493}
$$

The third-order reduced system obtained by Boby and Pal (2010) is

$$
R_{B}(s)=\frac{0.5058 s^{2}-1.9848 s+3.5341}{s^{3}+3 s^{2}+5.5341 s+3.5341}
$$

Figure 3 shows the step responses of the original system $G(s)$, the proposed reduced system $R_{A}(s)$ and the reduced

Table 1. Comparison between various reduced-order for example 1.

| Method of order reduction | Reduced system | ISE | IRE |
| :--- | :--- | :---: | :---: |
| Proposed Method | $\frac{0.8 s+0.686}{s^{2}+1.47 s+0.686}$ | $3.5 \times 10^{-4}$ | 47.497 |
| Boby and Pal (2010) | $\frac{0.9315 s+1.6092}{s^{2}+2.75612 s+1.6092}$ | $2.78 \times 10^{-3}$ | 49.420 |
| Parmar et al. (2007) | $\frac{0.7442 s+0.699}{s^{2}+1.458 s+0.6997}$ | $2.85 \times 10^{-3}$ | 48.828 |
| Chen et al. (1980) | $\frac{0.6997(s+1)}{s^{2}+1.45771 s+0.6997}$ | $4.296 \times 10^{-3}$ | 43.276 |
|  | $\frac{96 s+288}{70 s^{2}+300 s+288}$ | $7.33 \times 10^{-2}$ | 79.75 |
|  | $\frac{20.57 s+24}{30 s^{2}+42 s+24}$ | $1.544 \times 10^{-2}$ | 47.723 |
| Pal (1983) | $\frac{16 s+24}{30 s^{2}+42 s+24}$ | $1.88 \times 10^{-2}$ | 40.14 |
| Prasad and Pal (1991) | $\frac{s+34.2465}{s^{2}+239.8082 s+34.2465}$ | 2.481 | 15.734 |
|  | $\frac{s+2.3014}{s^{2}+5.7946 s+2.3014}$ | $2.295 \times 10^{-1}$ | 33.968 |



Figure 3. Comparison of step responses for example 2.


Figure 4. Comparison of bode plots for example 2.


Figure 5. Comparison of step responses for example 3.


Figure 6. Comparison of bode plots for example 3.

Table 2. Comparison between various reduced-order for example 2.

| Method of order reduction | Reduced system | ISE | IRE |
| :--- | :---: | :---: | :---: |
| Proposed Method | $\frac{0.0789 s^{2}+0.3142 s+0.493}{s^{3}+1.3 s^{2}+1.34 s+0.493}$ | $2.52 \times 10^{-2}$ | 27.59 |
| Boby and Pal (2010) | $\frac{0.5058 s^{2}-1.985 s+3.534}{s^{3}+3 s^{2}+5.534 s+3.534}$ | $2.82 \times 10^{-2}$ | 29.42 |
| Chen et al. (1979) | $\frac{285 s^{2}+1093 s+1700}{3408 s^{3}+5031 s^{2}+4620 s+1700}$ | $2.96 \times 10^{-2}$ | 25.43 |
|  | $\frac{0.2882 s^{2}-2.159 s+3.99}{s^{3}+4.76 s^{2}+7.55 s+3.99}$ | $2.4 \times 10^{-1}$ | 35.01 |
| Mukherjee et al. (2005) | $\frac{0.2945 s^{2}-2.202 s+2.32}{s^{3}+2.501 s^{2}+4.77 s+2.32}$ | $8.77 \times 10^{-2}$ | 51.01 |
| Vishwakarma and Prasad (2009) | $\frac{1.0385 s^{2}-2.9906 s+4.686}{s^{3}+3 s^{2}+6.686 s+4.686}$ | $5.86 \times 10^{-2}$ | 73.05 |
|  | $\frac{-0.264 s^{2}+0.483 s+0.751}{s^{3}+2.195 s^{2}+2.046 s+0.751}$ | $2.61 \times 10^{-2}$ | 26.43 |

Table 3. Comparison between various reduced-order for example 3.

| Method of order reduction | Reduced system | ISE | IRE |
| :--- | :--- | :---: | :---: |
| Proposed Method | $\frac{16.91 s+5.255}{s^{2}+6.87 s+5.26}$ | $6.83 \times 10^{-4}$ | $23.17 \times 10^{2}$ |
| Dia et al. (2011) | $\frac{17.099 s+5.074}{s^{2}+6.972 s+5.151}$ | $3.01 \times 10^{-3}$ | $24.19 \times 10^{2}$ |
|  | $\frac{24.11 s+8}{s^{2}+9 s+8}$ | $4.8 \times 10^{-2}$ | $41.92 \times 10^{2}$ |
| Parmar et al. (2007) | $\frac{22.8212 s+8.01}{s^{2}+9 s+8}$ | $3.68 \times 10^{-2}$ | $37.42 \times 10^{2}$ |
| Mittal, Prasad, and Sharma (2004) | $\frac{7.091 s+1.9906}{s^{2}+3 s+2}$ | $2.72 \times 10^{-1}$ | $6.94 \times 10^{2}$ |
|  | $\frac{6.7786 s+2}{s^{2}+3 s+2}$ | $2.79 \times 10^{-1}$ | $6.297 \times 10^{2}$ |

system using dominant pole (Boby \& Pal, 2010) $R_{B}(s)$. It is seen that the responses are comparable. Similarly, Figure 4 shows the comparison of the bode plots. Table 2 compares various reduced-order systems in terms of ISE and IRE values.

Example 3 Consider an eighth-order system (Dia et al., 2011) described by the transfer function

$$
G(s)=\frac{18 s^{7}+514 s^{6}+5982 s^{5}+36380 s^{4}+122664 s^{3}}{+222088 s^{2}+185760 s+40320} \begin{gathered}
s^{8}+36 s^{7}+546 s^{6}+4536 s^{5}+22449 s^{4}+67284 s^{3} \\
+118124 s^{2}+109584 s+40320
\end{gathered}
$$

Using the algorithm for biased SE reduction method, the reduced denominators are found as

$$
\begin{aligned}
& D_{r}(s)=s^{2}+6.867 s+5.255, \quad r_{1}=2, r_{2}=0 \\
& D_{r}(s)=s^{2}+36 s+501.94, \quad r_{1}=0, r_{2}=2 \\
& D_{r}(s)=s^{2}+36.36 s+13.25, \quad r_{1}=1, r_{2}=1
\end{aligned}
$$

The BBBC algorithm generates the numerator coefficients to form the polynomial as

$$
N_{r}(s)=16.91 s+5.255
$$

Therefore, the reduced second-order model will be

$$
R_{A}(s)=\frac{N_{r}(s)}{D_{r}(s)}=\frac{16.91 s+5.255}{s^{2}+6.87 s+5.26}
$$

The second-order reduced system obtained by Dia et al. (2011) is

$$
R_{B}(s)=\frac{N_{r}(s)}{D_{r}(s)}=\frac{17.0989 s+5.0742}{s^{2}+6.9722 s+5.1514}
$$

Figure 5 shows the step responses of the original system $G(s)$, the proposed reduced system $R_{A}(s)$ and the reduced system using dominant pole (Dia et al., 2011) $R_{B}(s)$. It is seen that the responses are comparable. Similarly, Figure 6 shows the comparison of the bode plots. Table 3 compares the ISE and IRE values of various reduced-order systems.

## 6. Conclusions

A simple and effective method of obtaining the reducedorder model is presented using the BBBC and the SE method. The denominator of the reduced method is obtained by the SE method and the numerator of the reduced system is generated using BBBC optimization method. The ISE and IRE values obtained by the proposed reduced system indicate that there is an improvement in the consistency and computational efficiency. The worthiness of the proposed method is justified in the above examples. Systems with very large dimensions have been considered to explore the powerfulness of the method. Furthermore, the proposed method also works well for a general non square MIMO system. As an extension, the same method can also be applied for higher-order discrete systems by combining with other methods.

## References

Antonio, L., \& Viaro, U. (1983). A note on the model reduction problem. IEEE Transactions on Automatic Control, 28(4), 525-526.
Antoulas, A. C., Sorensen, D. C., \& Gugercin, S. (2001). A survey of model reduction methods for large-scale systems. Contemporary Mathematics, 280, 193-219.
Boby, P., \& Pal, J. (2010). An evolutionary computation based approach for reduced order modelling of linear systems. IEEE International Conference on Computational Intelligence and Computing Research (ICCIC), 2010, Coimbatore, India.
Camp, C. V. (2007). Design of space trusses using big bangbig crunch optimization. Journal of Structural Engineering, 133(7), 999-1008.
Chen, T. C., Chang, C. Y., \& Han, K. W. (1979). Reduction of transfer functions by the stability-equation method. Journal of the Franklin Institute, 308(4), 389-404.
Chen, T. C., Chang, C. Y., \& Han, K. W. (1980). Model reduction using the stability-equation method and the pade approximation method. Journal of the Franklin Institute, 309(6), 473-490.
Desai, S. R., \& Prasad, R. (2010). Genetically optimized model order reduction for PID controller. International Conference on System Dynamics and Control -ICSDC 2010, Manipal, India.
Desai, S. R., \& Prasad, R. (2011a). Design of PID controller using particle swarm optimized reduced order model. Eighth Control Instrumentation System Conference (An International Conference) CISCON-2011, Manipal, India.
Desai, S. R., \& Prasad, R. (2011b). Generating stable reduced order models via balanced truncation technique. National conference on Emerging trends in control, Communication and computer Science (ET3CS -2011), Rajasthan, India.
Dia, A., Othman, M. K. A., \& Zaer, S. A. (2011). Reduced order modeling of linear MIMO systems using particle swarm optimization. The seventh international conference on Autonomic and Autonomous Systems (ICAS 2011).
Dogan, M., \& Istefanopulos, Y. (2007). Optimal nonlinear controller design for flexible robot manipulators with adaptive internal model. IET Control Theory and Applications, l(3), 770-778.
Erol, O. K., \& Eksin, I. (2006). New optimization method: Big bang-Big crunch. Advances in Engineering Software, 37, 106-111.

Gen, H. M., Erol, O. K., \& Eksi, L. (2009). An application and solution to gate assignment problem for AtatOrk airport. DECOMM 2009.
Gen, H. M., \& Hocaoglu, A. K. (2008). Bearing-only target tracking based on Big bang-Big crunch algorithm. The 3rd international multi-conference on Computing in the Global Information Technology, (ICCGI 2008), pp. 229-233.
Genç, H. M., Eksin, I., \& Osman, K. E. (2010). Big bangBig crunch optimization algorithm hybridized with local directional moves and application to target motion analysis problem. IEEE international conference on Systems Man and Cybernetics (SMC) 2010, pp. 881-887.
Genesio, R., \& Milanese, M. (1976). A note on the derivation and use of reduced order models. IEEE Transactions on Automatic Control, 21(1), 118-122.
Jamshidi, M. (1983). Large scale systems modelling and control series (Vol. 9). New York: Prentice Hall.
Kumbasar, T., Eksin, I., Guzelkaya, M., \& Yesil, E. (2008). Big bang big crunch optimization method based fuzzy model inversion. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 5317 LNAI (pp. 732-740).
Kumbasar, T., Yesil, E., Eksin, I., \& Guzelkaya, M. (2008). Inverse fuzzy model control with online adaptation via big bang-big crunch optimization. 3rd International Symposium on Communications, Control, and Signal Processing, ISCCSP 2008, pp. 697-702.
Mahmoud, M. S., \& Singh, M. G. (1981). Large scale systems modelling (Vol. 1, 3rd ed.). International series on systems and control. Pergamon Press.
Mittal A. K., Prasad, R., \& Sharma S. P. (2004). Reduction of linear dynamic systems using an error minimization technique. Journal of Institution of Engineers IE(I) Journal - EL, 84, 201-206.
Mukherjee, S., \& Mishra, R. N. (1987). Order reduction of linear systems using an error minimization technique. Journal of Franklin Institute, 323(1), 23-32.
Mukherjee, S., Satakshi, \& Mittal, R. C. (2005). Model order reduction using response -matching technique. Journal of the Franklin Institute, 342, 503-519.
Pal, J. (1983). Improved Pade approximants using stability equation method. Electronic Letters, 19(11), 426-427.
Panda, S., Tomar, S. K., Prasad, R., \& Ardil, C. (2009). Model reduction of linear systems by conventional and evolutionary techniques. International Journal of Computational and Mathematical Sciences, 3, 28-34.
Parmar, G., Prasad, R., \& Mukherjee, S. (2007). Order reduction of linear dynamic systems using stability equation method and GA. International Journal of Computer and Information Engineering, 1(1), 26-32.
Pavel, Y. T. (2011). Big bang - Big crunch optimization method in optimum design of complex composite laminates. World Academy of Science, Engineering and Technology, p. 77.

Prasad, R., Mittal, A. K., \& Sharma, S. P. (2005). A mixed method for the reduction of multi-variable systems. Journal of Institution of Engineers, India, IE(I) Journal-EL, 85, 177-181.
Prasad, R., \& Pal, J. (1991). Use of continued fraction expansion for stable reduction of linear multivariable systems. Journal of Institution of Engineers, India, IE(I) Journal - EL, 72, 43-47.
Rajendra, P. (1989). Analysis and design of control Systems using reduced order models (PhD dissertation). University of Roorkee, Roorkee, India.

Singh, R., \& Verma, H. K. (2011). Big bang-Big crunch optimization algorithm for linear phase fir digital filter design. International Journal of Electronics Communication and Computer Engineering, 3.
Sivanandam, S. N., \& Deepa, S. N. (2009). A comparative study using genetic algorithm and particle swarm optimization for lower order system modelling. International Journal of the Computer, the Internet and Management, 17(3), 1-10.
Vishwakarma, C. B., \& Prasad, R. (2009). MIMO system reduction using modified pole clustering and genetic algorithm. Modelling and Simulation in Engineering, 2009, 1-5. doi:10.1155/2009/540895 (Article ID 540895).

Zhang, H., \& Shi, Y. (2012). Delay-dependent stabilization of discrete-time systems with time-varying delay via switching technique. ASME Journal of Dynamic Systems, Measurement, and Control, 134(4), 044503-5. doi:10.1115/1.4006218
Zhang, H., Shi, Y., \& Liu, M. (2013). H- $\infty$ step tracking control for networked discrete-time nonlinear systems with integral and predictive actions. IEEE Transactions on Industrial Informatics, 9(1), 337-345.
Zhang, H., Shi, Y., \& Mehr, A. S. (2011). Robust static output feedback control and remote PID design for networked motor systems. IEEE Transactions on Industrial Electronics, 58(12), 5396-5405.


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