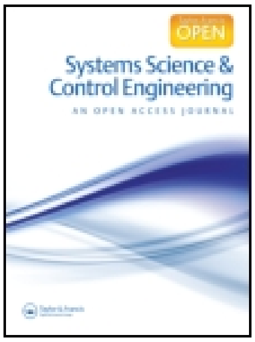

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A novel approach in the finite-time controller design

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This paper deals with the finite-time stabilization of a class of nonlinear systems. Based on the backstepping technique, a new recursive procedure is proposed which entwines the choice of the Lyapunov function with the design of the feedback control laws. The main efficiency of the proposed technique is due to adding a dummy state variable to the state vector. The dynamic equation of this state variable has a special structure which makes the design procedure of the finite-time controller more feasible. The designed controller guarantees the stabilization of the closed-loop system in a finite time. Computer simulations reveal the efficiency of the proposed technique and also verify the theoretical results.

Keywords: backstepping technique; finite-time stabilization; strict-feedback form; Lyapunov function

1. Introduction

The classical stability concepts, such as Lyapunov stability, asymptotical stability and bounded input–bounded output (BIBO) stability, study the stability of systems in an infinite time-interval. The concept of finite-time stabilization naturally arises from finite-time optimal control problems (Bhat & Bernstein, 2000) and deals with the dynamical systems whose operation time is limited to a fixed finite time-interval. From practical considerations, for such systems which should operate only over a finite time-interval, finite-time stability is the only meaningful description of stability. Additionally, when the classical concepts of stability require that system states be bounded, the bound values are not prescribed while the finite-time stability requires prescribed bounds on system states (Dorato, 2006). It also should be noted that the term finite-time stability has been used with different meanings in the literature. In this paper, this definition is used to describe the dynamical systems whose states approach to zero in a finite time (Hong, Wang, & Xi, 2005).

Recently, finite-time control of nonlinear systems has received increasing attentions (Amato, Ariola, & Dorato, 2001; Guo & Vincent, 2010; Honga, Xub, & Huangb, 2002; Zhu, Shen, & Li, 2009). Finite-time stabilization of higher-order systems (Hong, 2002), lower-triangular systems (Hong, Wang, & Cheng, 2006; Huang, Lin, & Yang, 2005; Pongvuthithum, 2009; Zhanga, Fengb, & Sunb, 2012), switched systems (Orlov, 2005), non-autonomous systems (Moulay & Perruquetti, 2008), time-delay system (Moulay, Dambrine, Yeganefar, & Perruquetti, 2008) and finite-time stabilization using output feedback (Hong, Huang, & Xu, 2001), dynamic gain (Praly & Jiang, 2004), backstepping (Reichhartinger & Horn, 2011) and control vector Lyapunov function (Nersesov, Haddad, & Hui, 2008) have been developed in the literature. Also, finite-time control via the terminal sliding mode (Chen, Wu, & Cui, 2013; Chuan-Kai, 2006; Feng, Yu, & Man, 2002; Lu, Chiu, & Chen, 2010) and fast terminal sliding mode (Hao, Lihua, & Zhong, 2013) has been studied, extensively. In all of these methods, in addition to structural limitations in each approach, the design procedure of the finite-time controller is almost complicated.

This paper presents a simple design method for the finite-time stabilization of a class of nonlinear systems. In the proposed method, first a dummy state variable is augmented to the state vector. The dynamic equation of this state variable has a special structure which makes the design procedure of the finite-time control law more feasible. Then, based on the backstepping technique, a recursive procedure that entwines the choice of a Lyapunov function with the design of the feedback control law is proposed. In this procedure, the controller design for the whole system breaks into a sequence of design problems for some lower order systems. In order to show the great positive effects of adding the dummy state variable, a design example is considered. The finite-time stabilizing controller is designed with and without adding the dummy state variable to the equations of the design example. Finally, simulation results of the closed-loop system verify the theoretical result and also reveal the great improvements, due to adding the dummy

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state variable, on the stability of the closed-loop system and also the transient responses of the state variables and the control input.

The remainder of the paper is arranged as follows. First, the preliminaries about the finite-time stability are given in Section 2. In Section 3, the design procedure of the finite-time stabilizing control law is explained in detail. Next, in Section 4, the proposed approach is applied to a design example. Finally, conclusions are made in Section 5.

2. Preliminaries

Consider the following nonlinear system:

$$\dot{x} = f(x), \quad x(t_0) = x_0, \tag{1}$$

where $x \in U \subset R^n (0 \in U)$ is the state vector, $f : U \to R^n$ is a continuous vector function and $f(0) = 0$.

Definition 1 *The equilibrium point ($x = 0$) of system (1) is finite-time stable, if it is the Lyapunov stable and finite-time convergent. In other words, for every initial condition $x(0) \in U/\{0\}$, there is a settling time $T > 0$ such that $\lim_{t\to T} x(t) = 0$ and $x(t) = 0$ for all $t \geq T$ (Hong et al., 2005).*

Lemma 1 *Suppose that there exists a continuously differentiable function $V(x) : U \to R$, and real numbers $c > 0$ and $0 < \alpha < 1$, such that,*

$$\begin{aligned} &V(0) = 0,\\ &V(x) \text{ is positive definite on } U,\\ &\dot{V}(x) \leq -cV^{\alpha}(x), \quad \forall x \in U. \end{aligned} \tag{2}$$

Then the settling time T (refer to Definition 1) exists and satisfies,

$$T(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}, \tag{3}$$

where $V(x_0)$ is the initial value of $V(x)$.

Proof See (Bhat & Bernstein, 2000) ■

Lemma 2 *For real numbers $l_i, i = 1, 2, \ldots, n$ and every $\alpha \in (0, 1)$, the following inequality holds:*

$$(|l_1| + \cdots + |l_n|)^{\alpha} \leq |l_1|^{\alpha} + \cdots + |l_n|^{\alpha}. \tag{4}$$

Proof See (Yu, Yu, Shirinzadeh, & Man, 2005) ■

3. Design of the finite-time controller

Consider the following nonlinear system which is given in the strict-feedback form:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2,\\ &\vdots\\ \dot{x}_{n-1} &= f_{n-1}(x_1, x_2, \ldots, x_{n-1}) + g_{n-1}(x_1, x_2, \ldots, x_{n-1})x_n,\\ \dot{x}_n &= f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u, \end{aligned} \tag{5}$$

where $x_i \in R (i = 1, \ldots, n)$, $u \in R$ and f_1 to f_n are continuous functions which vanish at origin, and also over the domain of interest, $g_i \neq 0$ for $i = 1, 2, \ldots, n$.

Moreover, consider a dummy state variable with the following dynamical equation:

$$\dot{x}_0 = -x_0^{\beta} + g_0(x_0)x_1, \tag{6}$$

where $g_0 \neq 0$ and $\beta = (2q - p)/p$. (where p and q are positive odd integers and $q < p < 2q$).

The approach of considering a new state variable x_0 increases the dimension of state vector; however, the structure of Equation (6) is such that x_0 is finite-time stable (in the absence of x_1) and adding this equation to state-space equations (5) makes the design procedure of the finite-time controller more feasible. Therefore, the augmented state-space equations are as follows:

$$\begin{aligned} \dot{x}_0 &= -x_0^{\beta} + g_0(x_0)x_1,\\ \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2,\\ &\vdots\\ \dot{x}_{n-1} &= f_{n-1}(x_1, x_2, \ldots, x_{n-1}) + g_{n-1}(x_1, x_2, \ldots, x_{n-1})x_n,\\ \dot{x}_n &= f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u. \end{aligned} \tag{7}$$

The goal is to design a finite-time stabilizing control law u for system (7). In order to show the design procedure, let us start with the following special case of Equations (7) and then gradually complete it:

$$\dot{x}_0 = -x_0^{\beta} + g_0(x_0)x_1, \tag{8a}$$
$$\dot{x}_1 = u. \tag{8b}$$

Equation (8a) is finite-time stable for $x_1 = \varphi_1(x_0) = 0$. To show this point, consider $V_0(x_0) = 0.5x_0^2$ as a Lyapunov function candidate for equation $\dot{x}_0 = -x_0^{\beta}$. Then,

$$\begin{aligned} \dot{V}_0 &= \frac{\partial V_0}{\partial x_0}\dot{x}_0\\ &= -x_0^{\beta+1} = -2^{(\beta+1)/2}(\frac{1}{2}x_0^2)^{(\beta+1)/2}\\ &= -2^{(\beta+1)/2}V_0^{(\beta+1)/2} = -cV_0^{\alpha}, \end{aligned} \tag{9}$$

where $\alpha = (\beta + 1)/2 = q/p \in (0, 1)$ and $c = 2^{(\beta+1)/2} = 2^{\alpha} > 0$. Therefore, according to Lemma 1, Equation (8a) is finite-time stable with $x_1 = 0$. Now, the finite-time stabilizing control law u will be designed for system (8). For this purpose, consider the following Lyapunov function:

$$V_1(x_0, x_1) = V_0(x_0) + |x_1|. \tag{10}$$

Therefore,

$$\begin{aligned} \dot{V}_1 &= \frac{\partial V_0}{\partial x_0}(-x_0^\beta + g_0 x_1) + \text{sgn}(x_1)u \\ &= -cV_0^\alpha + \frac{\partial V_0}{\partial x_0} g_0 x_1 + \text{sgn}(x_1)u \\ &= -cV_0^\alpha + g_0 x_0 x_1 + \text{sgn}(x_1)u. \end{aligned} \tag{11}$$

Now, choosing,

$$u = -g_0 |x_1| x_0 - c x_1^\alpha \tag{12}$$

and considering that $\text{sgn}(x_1)x_1^\alpha = |x_1|^\alpha$ (because $\alpha = q/p$ and q and p are odd integers), thus,

$$\dot{V}_1 = -cV_0^\alpha - c|x_1|^\alpha. \tag{13}$$

According to Lemma 2, $V_1^\alpha = (V_0 + |x_1|)^\alpha \leq V_0^\alpha + |x_1|^\alpha$, thus,

$$\dot{V}_1 = -cV_0^\alpha - c|x_1|^\alpha \leq -cV_1^\alpha. \tag{14}$$

Therefore, according to Lemma 1, the closed-loop system (8) is finite-time stable with the proposed control law (12). Now, if the following more general structure is considered:

$$\begin{aligned} \dot{x}_0 &= -x_0^\beta + g_0(x_0)x_1 \\ \dot{x}_1 &= f_1(x_1) + g_1(x_1)u \end{aligned} \tag{15}$$

Then the input transformation $u = (w_1 - f_1)/g_1$ will reduce Equation (15) to the following equations:

$$\begin{aligned} \dot{x}_0 &= -x_0^\beta + g_0(x_0)x_1, \\ \dot{x}_1 &= w_1, \end{aligned} \tag{16}$$

where according to above discussion, Equations (16) can be stabilized in a finite time by $w_1 = -g_0|x_1|x_0 - cx_1^\alpha$. Therefore, the finite-time stabilizing controller u for system (15) is as follows:

$$\begin{aligned} u &= \varphi_2(x_0, x_1) \\ &= \frac{1}{g_1}(\underbrace{-g_0|x_1|x_0 - cx_1^\alpha}_{w_1} - f_1). \end{aligned} \tag{17}$$

The above discussions are summarized in the following Lemma.

Lemma 3 *Considering system (15), the state feedback control law (17) stabilizes the origin of Equation (15) in a finite time and the corresponding Lyapunov function is* $V_1(x_0, x_1) = 0.5x_0^2 + |x_1|$.

Now, consider the following third-order system:

$$\begin{aligned} \dot{x}_0 &= -x_0^\beta + g_0(x_0)x_1, \\ \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2, \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)u. \end{aligned} \tag{18}$$

After one step of backstepping, the first two equations in Equation (18), with x_2 as the control input, can be globally stabilized in a finite time by $x_2 = \varphi_2(x_0, x_1)$ (where $\varphi_2(x_0, x_1)$ is given in Equation (17)) and $V_1(x_0, x_1) = 0.5x_0^2 + |x_1|$ is the corresponding Lyapunov function. To backstep, apply the following change of variables $z_2 = x_2 - \varphi_2$. Thus, state-space equations (18) are transformed to the following equations:

$$\begin{aligned} \dot{x}_0 &= -x_0^\beta + g_0 x_1, \\ \dot{x}_1 &= f_1 + g_1\varphi_2 + g_1 z_2, \\ \dot{z}_2 &= f_2 + g_2 u \\ &\quad \underbrace{- \frac{\partial \varphi_2}{\partial x_0}(-x_0^\beta + g_0 x_1) - \frac{\partial \varphi_2}{\partial x_1}(f_1 + g_1\varphi_2 + g_1 z_2)}_{-\dot{\varphi}_2}. \end{aligned} \tag{19}$$

By the input transformation $u = (w_2 - f_2 + \dot{\varphi}_2)/g_2$, one has

$$\begin{aligned} \dot{x}_0 &= -x_0^\beta + g_0 x_1, \\ \dot{x}_1 &= f_1 + g_1\varphi_2 + g_1 z_2, \\ \dot{z}_2 &= w_2. \end{aligned} \tag{20}$$

Consider the following Lyapunov function for system (20),

$$V_2(x_0, x_1, z_2) = V_1(x_0, x_1) + |z_2|. \tag{21}$$

Thus, $\dot{V}_2$ can be easily calculated as follows:

$$\begin{aligned} \dot{V}_2 &= \frac{\partial V_1}{\partial x_0}(-x_0^\beta + g_0 x_1) + \frac{\partial V_1}{\partial x_1}(f_1 + g_1\varphi_2) \\ &\quad + \frac{\partial V_1}{\partial x_1} g_1 z_2 + \text{sgn}(z_2)w_2. \end{aligned} \tag{22}$$

Since φ_2 is previously designed such that $((\partial V_1/\partial x_0)(-x_0^\beta + g_0 x_1) + (\partial V_1/\partial x_1)(f_1 + g_1\varphi_2) \leq -cV_1^\alpha)$, thus

$$\dot{V}_2 \leq -cV_1^\alpha + \frac{\partial V_1}{\partial x_1} g_1 z_2 + \text{sgn}(z_2)w_2, \tag{23}$$

Choosing,

$$\begin{aligned} w_2 &= -|z_2|\frac{\partial V_1}{\partial x_1} g_1 - cz_2^\alpha \\ &= -|z_2|\text{sgn}(x_1)g_1 - cz_2^\alpha, \end{aligned} \tag{24}$$

results in,

$$\dot{V}_2 \leq -cV_1^\alpha - c|z_2|^\alpha. \tag{25}$$

Using Lemma 2 and the inequality (25) (similar to the case of V_1), it can be obtained that $\dot{V}_2 \le -cV_2^{\alpha}$. Thus, the following control law guarantees finite-time stabilizing of system (18):

$$u = \varphi_3(x_0, x_1, x_2) = \frac{1}{g_2}(\underbrace{-|z_2|\mathrm{sgn}(x_1)g_1 - cz_2^{\alpha}}_{w_2} + \dot{\varphi}_2 - f_2), \quad (26)$$

with the following Lyapunov function:

$$V_2(x_0, x_1, x_2) = \frac{1}{2}x_0^2 + |x_1| + |x_2 - \varphi_2|. \quad (27)$$

Now, consider the following forth-order system:

$$\begin{aligned}\dot{x}_0 &= -x_0^{\beta} + g_0(x_0)x_1,\\ \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2,\\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3,\\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u. \end{aligned} \quad (28)$$

After two step of backstepping, the first three equations in Equation (28) can be globally stabilized in a finite time by $x_3 = \varphi_3(x_0, x_1, x_2)$ (where $\varphi_3(x_0, x_1, x_2)$ is given in Equation (26)) and $V_2(x_0, x_1, x_2) = \frac{1}{2}x_0^2 + |x_1| + |x_2 - \varphi_2|$ is the corresponding Lyapunov function. To backstep, apply the change of variables $z_3 = x_3 - \varphi_3$. Similar to the previous case, the process can be repeated to obtain the finite-time stabilizing control law for the system (28).

Ultimately, the backstepping method may be applied in a systematic way to design the finite-time stabilizing control law, $u = \varphi_n(x_0, x_1, x_2, \ldots, x_n)$ for state-space equations (7). If the proposed process be repeated n times, then the corresponding Lyapunov function $V_n(x_0, x_1, x_2, \ldots, x_n)$ for system (7) will be obtained as

$$V_n(x_0, x_1, x_2, \ldots, x_n) = \frac{1}{2}x_0^2 + |x_1 - \varphi_1| + |x_2 - \varphi_2| + \cdots |x_n - \varphi_n|,$$

where φ_1 is zero function and the scalar functions φ_is $(i = 2, 3, \ldots, n)$ should be computed in a backstepping procedure (similar to the proposed procedure in the design of φ_2 and φ_3). Finally, the finite-time stabilizing control law $u = \varphi_n(x_0, x_1, x_2, \ldots, x_n)$ will be achieved.

4. Design example

In this section, a design example is considered to show the efficiency of the proposed method in the design of a finite-time stabilizing controller and also to show the positive effects of adding a dummy state variable to the system equations. For this purpose, the design of a finite-time stabilizing control law is done by the proposed method once without adding a dummy state variable and another time by adding this state variable.

Consider the following state-space equations:

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_2,\\ \dot{x}_2 &= -x_1 + u.\end{aligned} \quad (29)$$

4.1. *First approach (without adding the dummy state variable x_0)*

Consider the first state equation $\dot{x}_1 = x_1^2 + x_2$. Choosing $x_2 = \varphi(x_1) = -x_1^2 - x_1^{\beta}$, leads to the finite-time stabilization of this equation with the Lyapunov function $V_1(x_1) = 0.5x_1^2$, where $\dot{V}_1(x_1) = -x_1^{\beta+1} = -cV_1^{\alpha}$. Set $z = x_2 - \varphi$, then,

$$\begin{aligned}\dot{x}_1 &= x_1^2 + \varphi + z,\\ \dot{z} &= -x_1 + u - \dot{\varphi}.\end{aligned} \quad (30)$$

By input transformation $u = w + \dot{\varphi} + x_1$, one has

$$\begin{aligned}\dot{x}_1 &= x_1^2 + \varphi + z,\\ \dot{z} &= w.\end{aligned} \quad (31)$$

Choosing $V_2(x_1, z) = V_1(x_1) + |z|$ as the Lyapunov function for Equation (31), then,

$$\dot{V}_2 = \frac{\partial V_1}{\partial x_1}(x_1^2 + \varphi) + \frac{\partial V_1}{\partial x_1}z + \mathrm{sgn}(z)w. \quad (32)$$

Putting

$$\begin{aligned} w &= -\frac{\partial V_1}{\partial x_1}|z| - cz^{\alpha}\\ &= -x_1|z| - cz^{\alpha}, \end{aligned} \quad (33)$$

then $\dot{V}_2 \le -cV_2^{\alpha}$. Therefore, the following control law guarantees the finite-time stabilization of system (29):

$$\begin{aligned} u &= w + \dot{\varphi} + x_1\\ &= x_1 - x_1|z| - cz^{\alpha} + \frac{\partial \varphi}{\partial x_1}(x_1^2 + x_2), \end{aligned} \quad (34)$$

where $\varphi = -x_1^2 - x_1^{\beta}$ and $z = x_2 - \varphi = x_2 + x_1^2 + x_1^{\beta}$.

4.2. *Second approach (adding the dummy state variable x_0)*

After adding a dummy state variable with state equation (6) to the equations (29), one has

$$\begin{aligned}\dot{x}_0 &= -x_0^{\beta} + x_1,\\ \dot{x}_1 &= x_1^2 + x_2,\\ \dot{x}_2 &= -x_1 + u,\end{aligned} \quad (35)$$

which has the same structure as state-space equations (18). By substituting

$$g_0 = 1, \quad f_1 = x_1^2, \quad g_1 = 1$$
$$f_2 = -x_1, \quad g_2 = 1. \tag{36}$$

And according to the proposed procedure in the previous section and considering Equations (26) and (36), the finite-time controller is as follows:

$$u = (-|z_2|\mathrm{sgn}(x_1) - cz_2^{\alpha} + \frac{\partial \varphi_2}{\partial x_0}(-x_0^{\beta} + x_1) + \frac{\partial \varphi_2}{\partial x_1}(x_1^2 + x_2) + x_1), \tag{37}$$

where $\varphi_2 = -|x_1|x_0 - cx_1^{\alpha} - x_1^2$ (refer to Equation (17)) and $z_2 = x_2 - \varphi_2$.

4.3. Computer simulations

The time responses of the state variables related to designed control laws (34) and (37) (in the first and second approaches) are shown in Figures 1 and 2. As it can be seen, adding the dummy state variable has great improvements on the transient response and settling time of the closed-loop system. The time responses of control signals (34) and (37), which are related to the first and second approaches, respectively, are shown in Figure 3. As it is seen in Figure 3, the time response of control law (34) is highly oscillating. In fact, computer simulations show that the proposed idea in adding the dummy state variable leads to a better transient response, less settling time and less control effort.

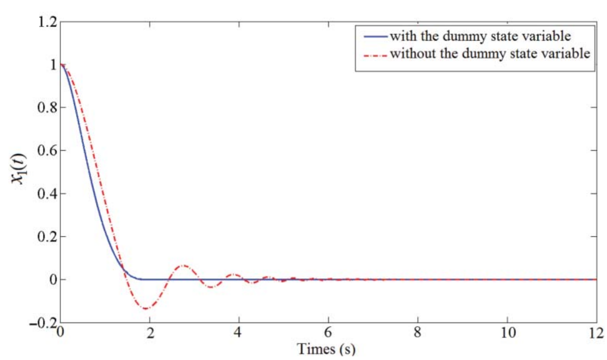

Figure 1. Time responses of $x_1(t)$ in the first and second approaches.

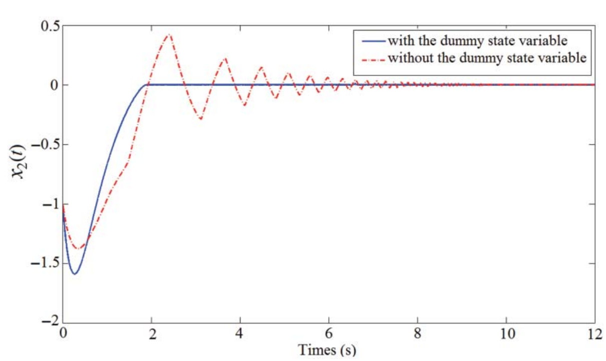

Figure 2. Time responses of $x_2(t)$ in the first and second approaches.

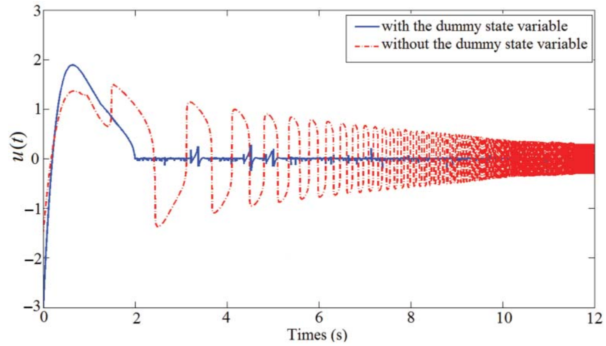

Figure 3. Time responses of $u(t)$ in the first and second approaches.

5. Conclusions

In this paper, a new procedure was presented to design a finite-time stabilizing controller. In the proposed method, a dummy state variable with a finite-time stable dynamic equation was added to the state vector. Then, based on backstepping idea, a recursive procedure that interlaced the choice of a Lyapunov function with the design of the finite-time control law was proposed. Finally, by means of computer simulations, the theoretical results were verified and the great improvements due to adding the dummy state variable on the stability of the closed-loop system and transient responses of state variables and the control input were shown.

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