Systems Science & Control Engineering: An Open Access Journal

ISSN: (Print) 2164-2583 (Online) Journal homepage: https://www.tandfonline.com/loi/tssc20

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To cite this article: M. Kermani & A. Sakly (2014) Stability analysis for a class of switched nonlinear time-delay systems, Systems Science & Control Engineering: An Open Access Journal, 2:1, 80-89, DOI: 10.1080/21642583.2013.879543

To link to this article: https://doi.org/10.1080/21642583.2013.879543

Systems Science & Control Engineering: An Open Access Journal, 2014
Vol. 2, 80–89, http://dx.doi.org/10.1080/21642583.2013.879543

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Stability analysis for a class of switched nonlinear time-delay systems

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(*Received 16 November 2013; final version received 27 December 2013*)

This paper investigates the stability analysis for a class of discrete-time switched nonlinear time-delay systems. These systems are modelled by a set of delay difference equations, which are represented in the state form. Then, another transformation is made towards an arrow form. Therefore, by applying the Kotelyanski conditions combined to the $M-$ matrix properties, new delay-independent sufficient stability conditions under arbitrary switching which correspond to a Lyapunov function vector are established. The obtained results are explicit and easy to use. A numerical example is provided to show the effectiveness of the developed results.

Keywords: discrete-time switched nonlinear time-delay systems; arbitrary switching; global asymptotic stability; M-matrix; Kotelyanski lemma; arrow matrix

1. Introduction

A switched system is a type of hybrid dynamical system that consists of a number of subsystems which are described by differential or difference equations and a switching signal selecting a subsystem to be active during a certain time interval. As a special class of hybrid systems, many dynamical systems can be modelled as switched systems (Branicky, 1998; Darouach & Chadli, 2013; Dong, Liu, Mei, & Li, 2011; Liberzon, 2003; Liberzon & Morse, 1999; Mahmoud, 2010; Phat & Ratchagit, 2011; Sun & Ge, 2011; Zhang, Abate, Hu, & Vitus, 2009; Zhang & Yu, 2009).

Recently, switched systems have strong engineering background in various areas and are often used as a unified modelling tool for a great number of real-world systems such as mechanical systems, automotive industry, power electronics, chemical engineering processing, communication networks, aircraft and air traffic control, chemical and electrical engineering and models for epidemiology (Pellanda, Apkarian, & Tuan, 2002; Tse & Bernardo, 2002; Wu, Shi, Su, & Chu, 2011; Zhang, Liu, & Huang, 2010).

The last decade has witnessed increasing research activities in the study of switched systems. Among of those research topics, stability analysis has attracted most of attention. Hence, several methods have been proposed for this matter (Arunkumar, Sakthivel, Mathiyalagan, & Marshal Anthoni, 2012; Branicky, 1998; Hespanha & Morse, 1999; Ishii & Francis, 2002; Jianyin & Kai, 2010; Liberzon, 2003; Lien, Yu, Chang, Chung, & Chen, 2012; Lien et al., 2011; Shim, Noh, & Seo, 2001; Sun, Wang, Liu, & Zhao, 2008; Vu & Liberzon, 2005; Zhai, Hu, Yasuda, & Michel, 2000; Zhang, Shi, & Basin, 2008; Zhang & Yu, 2009). In fact, stability under arbitrary switching is a fundamental and challenging research issue in the design and analysis of these systems (Mathiyalagan, Sakthivel, Marshal, & Anthoni, 2012; Zhang et al., 2008; Zhang & Yu, 2009).

Within this context, it is well known that a switched system is asymptotically stable if all the individual systems are stable and the switching is sufficiently slow, so as to allow the transient effects to dissipate after each switch. In this case, the existence of a common Lyapunov function for all the subsystems is a sufficient condition to guarantee the stability of the switched system under arbitrary switching law (Liberzon, 2003). Therefore, this method is usually very difficult to apply even for discrete-time switched linear systems; however, it becomes more complicated for the nonlinear case. However, some attempts are presented to construct a common Lyapunov function for nonlinear exponentially stable systems in Shim et al. (2001), and for general nonlinear asymptotically stable systems in Vu & Liberzon (2005).

To avoid the problem of existence of the common Lyapunov function, frequently, we are required to seek conditions to guarantee the stability of the switched systems for any admissible switched law. For this, many methodologies' efficient approaches have been developed, such as the multiple Lyapunov function approach (Branicky, 1998) and the average dwell time method (Hespanha & Morse, 1999; Ishii & Francis, 2002; Zhai et al., 2000; Zhang & Gao, 2010). Whereas, stability analysis under arbitrary switching

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remains the most essential issue in practical switched systems.

Furthermore, time-delay phenomenon also cannot be avoided in practical systems, for example, chemical processes, long-distance transportation systems, hydraulic pressure systems, hybrid procedure, electron network, network control systems and so on. The problem of time-delay may cause instability and poor performance of practical systems (Gao & Chen, 2007; Gao, Lam, & Wang, 2007). Therefore, the stability analysis of switched time-delay systems is very worthy to be researched.

Basically, current efforts to achieve stability in time-delay systems can be divided into two categories, namely, delay-independent criteria and delay-dependent criteria. In this paper, in view of delay-independent analysis, we expect to aid in studying stability analysis under arbitrary switching law.

Up to date, most of the previous results on stability analysis for discrete-time switched time-delay systems were interested in the linear case (Arunkumar et al., 2013; Branicky, 1998; Gao & Chen, 2007; Gao et al., 2007; Liberzon & Morse, 1999; Phat & Ratchagit, 2011; Sun & Ge, 2011; Wu et al., 2011; Zhang & Yu, 2009). Therefore, few results are developed for discrete-time switched nonlinear time-delay systems (Xu, 2002).

Motivated by these gaps, the objective of this paper is to present a novel approach for asymptotic stability of a class of discrete-time switched nonlinear time-delay systems. In fact, based on transforming the representation of the system under consideration into the arrow form matrix representation (Benrejeb & Borne, 1978; Benrejeb, Borne, & Laurent, 1982; Benrejeb & Gasmi, 2001; Benrejeb, Gasmi, & Borne, 2005; Benrejeb, Soudani, Sakly, & Borne, 2006; Elmadssia, Saadaoui, & Benrejeb, 2011, 2013; Filali, Hammami, Benrejeb, & Borne, 2012; Jabbali, Kermani, & Sakly, 2013; Kermani, Sakly, & M'sahli, 2012; Sfaihi & Benrejeb, 2013), the use of an appropriate Lyapunov function associated with the Kotelyanski conditions (Benrejeb et al., 2006; Borne, 1987; Borne, Gentina, & Laurent, 1972; Borne, Vanheeghe, & Duflos, 2007; Kotelyanski, 1952) and the $M-$ matrix proprieties (Gantmacher, 1966; Robert, 1966), new sufficient stability conditions are derived under arbitrary switching. The obtained stability conditions which correspond to a Lyapunov function vector are simple to employ, explicit and allow us to avoid the search for a common Lyapunov function which appears a difficult matter in this case.

Within the frame of studying the stability analysis, the said approach has already been introduced in Elmadssia et al. (2011, 2013) for continuous-time delay systems and in our previous work (Jabbali et al., 2013) for discrete-time switched linear time-delay systems, in a field related to the study of convergence. This proposed approach could be further used as a constructive solution to the problems of state feedback stabilization.

The rest of this paper is organized as follows. The problem is formulated and some basic notations and definitions are given in Section 2. The main results of this paper are presented in Section 3. Section 4 is devoted to derive new delay-independent conditions for asymptotic stability of switched nonlinear systems defined by difference equations. Remarks and numerical example are presented in Section 5 to illustrate the theoretical results, and the conclusions are drawn in Section 6.

2. Problem statements and preliminaries

2.1. *Problem statements*

Consider the following discrete-time switched nonlinear time-delay system formed by N subsystems given in the state form:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{N} \zeta_i(k)(A_i(\cdot)x(k) + D_i(\cdot)x(k-d)), \\ x(l) &= \phi(l), \quad l = -d, \ldots, -1, 0, \end{aligned} \tag{1}$$

where $x(k) \in \Re^n$ is the system state, $d > 0$ is the time delay, $\phi(l) : \{-d, -d+1, \ldots, 0\} \to \Re^n$: is a vector-valued initial function, $A_i(\cdot) \quad (i = 1, \ldots, N)$ and $D_i(\cdot)$ $(i = 1, \ldots, N)$ are matrices that have nonlinear elements of appropriate dimensions denoting the subsystems and $N \geq 1$ denotes the number of subsystems.

The switching sequence is defined through a switching vector: $\zeta(k) = [\zeta_1(k), \ldots, \zeta_N(k)]^{\mathrm{T}}$ whose components $\zeta_i(k) \quad \Re_+ \to M = \{0, 1\}$ are exogenous functions that depend only on the time and not on the state, they are defined through:

$$\zeta_i(k) = \begin{cases} 1 & \text{when } A_i(\cdot) \text{ and } D_i(\cdot) \text{ are active,} \\ 0 & \text{otherwise,} \end{cases} \quad i \in N, \tag{2}$$

it is obvious that $\sum_{i=1}^{N} \zeta_i(k) = 1$.

2.2. *Notations and definitions*

Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. I_n is an identity matrix with appropriate dimension. Let $\Re^n$ denote an n dimensional linear vector space over the reals $\|\cdot\|$ which stands for the Euclidean norm of vectors. For any $u = (u_i)_{1 \leq i \leq n}$, $v = (v_i)_{1 \leq i \leq n} \in \Re^n$, we define the scalar product of the vectors u and v as:$\langle u, v \rangle = \sum_{i=1}^{n} u_i v_i$.

We are here interested to establish delay-independent stability conditions for system (1) by using the $M-$ matrix properties combined to the Kotelyanski conditions. In order to prepare for a precise formulation of our results, we introduce the following definitions and lemma that will play a key role in deriving our main results.

Definition 1 *The hybrid system* (1) *is said to be uniformly asymptotically stable if for any* $\varepsilon > 0$, *there is a* $\delta(\varepsilon) > 0$

such that $\sup_{-d\leq l\leq 0} \parallel \phi(l) \parallel < \delta$ *implies* $\parallel x(k,\phi) \parallel \leq \varepsilon$, $k \geq 0$. *For arbitrary switching signal* $\zeta(k)$ *and there is also a* $\delta' > 0$ *such that* $\sup_{-d\leq l\leq 0} \parallel \phi(k) \parallel < \delta'$ *implies* $\parallel x(k,\phi) \parallel \to 0$ *as* $k \to \infty$ *for arbitrary switching signal* (2).

Next, we introduce several useful tools, including Kotelyanski lemma and definition of an $M-$ matrix.

Kotelyanski Lemma (Borne, 1987) *The real parts of the eigenvalues of matrix* A*, with non-negative off-diagonal elements, are less than a real number* μ *if and only if all those of matrix* M, $M = \mu I_n - A$, *are positive, with* I_n *the n identity matrix.*

When the principal minors of matrix $(-A)$ are positive, the Kotelyanski lemma permits to conclude on stability property of the system characterized by A.

Definition 2 (Borne, 1987) *The matrix* $A \in \Re^{n\times n}$ *is called a Z-matrix if it has null or negative off-diagonal elements.*

Theorem 1 (Borne, 1987) *In order that a Z-matrix* $A(\cdot)$ *is said an M-matrix if the following properties are verified:*

- *All the eigenvalues of* $A(\cdot)$ *have a positive real part.*
- *The real eigenvalues are positive.*
- *The principal minors of* $A(\cdot)$ *are positive*

$$(A(\cdot))\begin{pmatrix}1 & 2 & \dots & j\\ 1 & 2 & \dots & j\end{pmatrix} > 0 \quad \forall j \in 1,\dots,n. \tag{3}$$

- *For any positive vector* $x = (x_1,\dots,x_n)^{\mathrm{T}}$*, the algebraic equations* $A(\cdot)x$ *have a positive solution* $w = (w_1,\dots,w_n)^{\mathrm{T}}$.

Remark 1 A discrete-time system characterized by a matrix $A(\cdot)$ is stable if the matrix $(I_n - A(\cdot))$ verifies the Kotelyanski conditions, in this case $(I_n - A(\cdot))$ is an M-matrix.

3. Main results

In this part, after previous formulation, we can now state the main result of this paper summarized in the following theorem which gives stability conditions for the discrete-time nonlinear switched time-delay systems (1).

Theorem 2 *The discrete-time switched nonlinear time-delay system* (1) *is asymptotically stable under arbitrary switching rule* (2) *if the matrix* $(I_n - T_c(\cdot))$ *is an M-matrix where*

$$T_c(\cdot) = \max_{1\leq i\leq N}(T_{\zeta(k)}(\cdot)) \tag{4}$$

and

$$T_{\zeta(k)}(\cdot) = (|A_{\zeta(k)}(\cdot)| + |D_{\zeta(k)}(\cdot)|). \tag{5}$$

$$A_{\zeta(k)}(\cdot) = \begin{bmatrix} \sum_{i=1}^{N}\zeta_i(k)(a_i^{11}(\cdot)) & \dots & \dots & \sum_{i=1}^{N}\zeta_i(k)(a_i^{1n}(\cdot)) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{N}\zeta_i(k)(a_i^{n1}(\cdot)) & \dots & \dots & \sum_{i=1}^{N}\zeta_i(k)(a_i^{nn}(\cdot)) \end{bmatrix}, \tag{6}$$

$$D_{\zeta(k)}(\cdot) = \begin{bmatrix} \sum_{i=1}^{N}\zeta_i(k)(d_i^{11}(\cdot)) & \dots & \dots & \sum_{i=1}^{N}\zeta_i(k)(d_i^{1n}(\cdot)) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{N}\zeta_i(k)(d_i^{n1}(\cdot)) & \dots & \dots & \sum_{i=1}^{N}\zeta_i(k)(d_i^{nn}(\cdot)) \end{bmatrix}. \tag{7}$$

So, we obtain sufficient conditions for asymptotic stability of system (1).

Proof Let us consider the system (1) of any switching signal (2), let $(w_l > 0, \forall l = 1,\dots,n)$ next, we choose the following Lyapunov function candidate:

$$v(k) = v_0(k) + \sum_{j=1}^{r} v_j(k), \tag{8}$$

where

$$\begin{aligned} v_0(k) &= \langle |x(k)|, w\rangle,\\ v_j(k) &= \langle |D_{\zeta(k)}(\cdot)||x(k-j)|, w\rangle, \quad (j = 1,\dots,r). \end{aligned} \tag{9}$$

It suffices to show that

$$\Delta v(k) = v(k+1) - v(k) < \langle (T_c(\cdot))|x(k)|, w\rangle, \quad r > 0, \tag{10}$$

where

$$\Delta v(k) = \Delta v_0(k) + \sum_{j=1}^{r} \Delta v_j(k) \tag{11}$$

and

$$\Delta v_0 = \langle |x(k+1)|, w\rangle - \langle |x(k)|, w\rangle. \tag{12}$$

For any $r > 0$, we obtain from Equation (9) that

$$\begin{aligned} \Delta v_j =& \langle |D_{\zeta(k)}(\cdot)||x(k-j+1)|, w\rangle \\ &- \langle |D_{\zeta(k)}(\cdot)||x(k-j)|, w\rangle, \quad j = 1,\dots,r. \end{aligned} \tag{13}$$

Knowing that

$$\begin{aligned}\langle||x(k+1)|,w\rangle &= \langle|A_{\zeta(k)}(\cdot)x(k)+D_{\zeta(k)}(\cdot)x(k-r)|,w\rangle\\ &< \langle|A_{\zeta(k)}(\cdot)||x(k)|+|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle\\ &= \langle|A_{\zeta(k)}(\cdot)||x(k)|,w\rangle+\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle. \end{aligned} \tag{14}$$

On the other hand, we have

$$\sum_{j=1}^{r}\Delta v_j(k) = \Delta v_1(k)+\Delta v_2(k)+\cdots+\Delta v_{r-1}(k)+\Delta v_r(k). \tag{15}$$

Therefore, by Equations (9) and (15), we have that

$$\begin{aligned}\sum_{j=1}^{r}\Delta v_j(k) &= (\langle|D_{\zeta(k)}(\cdot)||x(k)|,w\rangle-\langle|D_{\zeta(k)}(\cdot)||x(k-1)|,w\rangle)\\ &\quad+(\langle|D_{\zeta(k)}(\cdot)||x(k-1)|,w\rangle\\ &\quad-\langle|D_{\zeta(k)}(\cdot)||x(k-2)|,w\rangle)\\ &\quad+(\langle|D_{\zeta(k)}(\cdot)||x(k-r+1)|,w\rangle\\ &\quad-\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle)\\ &= (\langle|D_{\zeta(k)}(\cdot)||x(k)|,w\rangle-\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle).\end{aligned} \tag{16}$$

Thus, we can eventually obtain that

$$\begin{aligned}\Delta v(k) &= \Delta v_0(k)+(\langle|D_{\zeta(k)}(\cdot)||x(k)|,w\rangle\\ &\quad-\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle).\end{aligned} \tag{17}$$

Moreover, it is not difficult to remark that

$$\begin{aligned}\langle|x(k+1)|,w\rangle &< \langle|A_{\zeta(k)}(\cdot)||x(k)|,w\rangle\\ &\quad+\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle.\end{aligned} \tag{18}$$

By Equations (11)–(14), we have that

$$\begin{aligned}\Delta v(k) &< \langle|A_{\zeta(k)}(\cdot)||x(k)|,w\rangle+\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle\\ &\quad-\langle|x(k)|,w\rangle\\ &\quad+\langle|D_{\zeta(k)}(\cdot)||x(k)|,w\rangle-\langle|D_{\zeta(k)}(\cdot)||x(k-r)|,w\rangle.\end{aligned} \tag{19}$$

We have

$$\begin{aligned}\Delta v(k) &< \langle|A_{\zeta(k)}(\cdot)||x(k)|,w\rangle-\langle|x(k)|,w\rangle\\ &\quad+\langle|D_{\zeta(k)}(\cdot)||x(k)|,w\rangle,\end{aligned} \tag{20}$$

and finally we obtain

$$\Delta v(k) < \langle(|A_{\zeta(k)}(\cdot)|+|D_{\zeta(k)}(\cdot)|-I_n)|x(k)|,w\rangle, \tag{21}$$

it follows that

$$\Delta v(k) \le \langle(T_c(\cdot)-I_n)|x(k)|,w\rangle, \tag{22}$$

where the matrix $T_c(\cdot)$ is defined in Equation (4).

Now, suppose that $I_n - T_c(\cdot)$ is an $M-$matrix, according to the proprieties of the $M-$matrix, we can find a vector $\rho \in \Re_+^{*n}$ ($\rho_l \in \Re_+^*$, $l = 1,\dots,n$) satisfying the relation $(I_n - T_c(\cdot))^{\mathrm{T}}w = \rho$, $\forall w \in \Re_+^{*n}$, so we can write

$$\begin{aligned}\langle(I_n-T_c(\cdot))|x(k)|,w\rangle &= \langle(I_n-T_c(\cdot))^{\mathrm{T}}w,|x(k)|\rangle\\ &= \langle\rho,|x(k)|\rangle.\end{aligned} \tag{23}$$

Then, we have

$$\langle(T_c(\cdot)-I_n)|x(k)|,w\rangle = \langle-\rho,|x(k)|\rangle. \tag{24}$$

Finally, we obtain

$$\Delta v(k) \le \langle(T_c(\cdot)-I_n)|x(k)|,w\rangle \le -\sum_{l=1}^{n}\rho_l|x_l(k)| < 0. \tag{25}$$

This completes the proof of Theorem 2. ■

Then, the discrete-time switched nonlinear time-delay system given in Equation (1) is asymptotically stable.

Remark 2 By Theorem 2, the stability conditions of the switched time-delay system (1) are independent of time-delay.

4. Stability conditions for switched systems defined by difference equations

The purpose of this section consists in applying the established result to discrete-time switched nonlinear time-delay systems modelled by the following switched nonlinear difference equation (Jabbali et al., 2013):

$$\begin{aligned}y(k+n)+\sum_{i=1}^{N}\zeta_i(k)\left[\sum_{p=0}^{n-1}a_i^{n-p}(\cdot)y(k+p)\right.\\ \left.+\sum_{p=0}^{n-1}d_i^{n-p}(\cdot)y(k+p-\tau)\right]=0,\end{aligned} \tag{26}$$

where $\zeta_i(k)$ are the components of the switching function $\zeta(k)$, $i = 1,\dots,N$, given in Equation (2). Then, the presence of both delay-time terms and nonlinearities of the coefficients makes the stability analysis of the problem (26) difficult. To solve this matter as a solution, we will adopt the following change of variable:

$$x_{p+1}(k) = y(k+p), \quad p = 0,\dots,n-1, \tag{27}$$

in fact, Equation (26) becomes

$$\begin{aligned} x_p(k+1) &= x_{p+1}(k), \quad p = 1,\dots,n-1, \\ x_n(k+1) &= \sum_{i=1}^{N} \zeta_i(k)\left[-\sum_{p=0}^{n-1} a_i^{n-p}(\cdot)x_{p+1}(k) \right. \\ &\quad \left. -\sum_{p=0}^{n-1} d_i^{n-p}(\cdot)x_{p+1}(k-\tau) \right], \end{aligned} \tag{28}$$

or under matrix representation, we obtain the following state form:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{N} \zeta_i(A_i(\cdot)x(k) + D_i(\cdot)x(k-\tau)), \\ x(l) &= \phi(l), \quad l = -\tau,\dots,-1,0, \end{aligned} \tag{29}$$

where $x(k)$ is the state vector of components $x_p(k)$, $p = 0,\dots,n-1$.

$\zeta(k)$ is the switched function defined in Equation (2).

The matrices $A_i(\cdot)$ and $D_i(\cdot)$ are given as follows:

$$A_i(\cdot) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -a_i^n(\cdot) & -a_i^{n-1}(\cdot) & \cdots & -a_i^1(\cdot) \end{bmatrix}, \tag{30}$$

$$D_i(\cdot) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ -d_i^n(\cdot) & -d_i^{n-1}(\cdot) & \cdots & -d_i^1(\cdot) \end{bmatrix}, \tag{31}$$

where $d_i^j(\cdot)$ is a coefficient of the instantaneous characteristic polynomial $P_i(\lambda)$ of the matrix $A_i(\cdot)$ given by

$$P_i(\lambda) = \lambda^n + \sum_{q=0}^{n-1} a_i^{n-q}(\cdot)\lambda^q, \tag{32}$$

and $d_i^j(\cdot)$ is a coefficient of the instantaneous characteristic polynomial $Q_i(\lambda)$ of the matrix $D_i(\cdot)$ defined such as

$$Q_i(\lambda) = \sum_{q=0}^{n-1} d_i^{n-q}(\cdot)\lambda^q. \tag{33}$$

In Jabbali et al. (2013), a change of coordinate for the system given in Equation (29) under the arrow form allows the synthesis of sufficient stability conditions easy to test.

This leads to the following state-space description:

$$z(k+1) = \sum_{i=1}^{N} \zeta_i(k)(M_i(\cdot)z(k) + N_i(\cdot)z(k-\tau)), \tag{34}$$

where $z(k)$ is the new state vector so as $z(k) = Px(k)$, P is the corresponding passage matrix and $M_i(\cdot)$ and $N_i(\cdot)$ are matrices in the arrow form represented as following:

$$M_i(\cdot) = P^{-1}A_i(\cdot)P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_i^1(\cdot) & \cdots & \cdots & \gamma_i^{n-1}(\cdot) & \gamma_i^n(\cdot) \end{bmatrix}, \tag{35}$$

$$N_i(\cdot) = P^{-1}D_i(\cdot)P = \begin{bmatrix} 0_{n-1,n-1} \cdots & 0_{n-1,1} \\ \delta_i^1(\cdot)\cdots\delta_i^{n-1}(\cdot) & \delta_i^n(\cdot) \end{bmatrix}, \tag{36}$$

and P is the corresponding passage matrix defined as follows:

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \alpha_1 & \alpha_2 & \dots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \dots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \dots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \dots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix}. \tag{37}$$

For defined distinct arbitrary constant parameters α_j ($j = 1,\dots,n-1$) and $\alpha_j \neq \alpha_q\ \forall j \neq q$, $q = 1,\dots,n-1$.

Let us introduce the notation elements of the matrix $M_i(\cdot)$ ($i = 1,\dots,N$) which are defined as follows:

$$\begin{aligned} \beta_j &= \prod_{\substack{q=1 \\ q\neq j}}^{n-1} (\alpha_j - \alpha_q)^{-1} \quad \forall j = 1,\dots,n-1, \\ \gamma_i^j(\cdot) &= -P_i(\alpha_j) \quad \forall j = 1,\dots,n-1, \\ \gamma_i^n(\cdot) &= -a_i^1(\cdot) - \sum_{j=1}^{n-1} \alpha_j, \end{aligned} \tag{38}$$

and the elements of the matrices $N_i(\cdot)$ $(i = 1,\dots,N)$ are as follows:

$$\begin{aligned} \delta_i^j(\cdot) &= -Q_i(\alpha_j) \quad \forall j = 1,\dots,n-1, \\ \delta_i^n(\cdot) &= -d_i^1(\cdot). \end{aligned} \tag{39}$$

Taking into account the previous relations, the matrices $T_i(\cdot)$ ($i = 1,\dots,N$) will be defined as follows:

$$T_i(\cdot) = \begin{bmatrix} |\alpha_1| & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}| \\ t_i^1(\cdot) & \cdots & \cdots & t_i^{n-1}(\cdot) & t_i^n(\cdot) \end{bmatrix}, \tag{40}$$

and the matrix $T_{\zeta(k)}(\cdot)$ is given as follows:

$$T_{\zeta(k)}(\cdot) = \begin{bmatrix} |\alpha_1| & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}| \\ \sum_{i=1}^{N} \zeta_i(k) t_i^1(\cdot) & \cdots & \cdots & \sum_{i=1}^{N} \zeta_i(k) t_i^{n-1}(\cdot) & \sum_{i=1}^{N} \zeta_i(k) t_i^n(\cdot) \end{bmatrix}, \quad (41)$$

where

$$t_i^j(\cdot) = |\gamma_i^j(\cdot)| + |\delta_i^j(\cdot)|, \quad j = 1, \ldots, n-1, \qquad t_i^n(\cdot) = |\gamma_i^n(\cdot)| + |\delta_i^n(\cdot)|. \quad (42)$$

After this formulation, we can deduce the following theorem for the stability for the discrete-time switched time-delay systems (29).

Theorem 3 *The discrete-time switched nonlinear time-delay system (29) is globally asymptotically stable under arbitrary switching rule (2) if there exist α_j ($j = 1, \ldots, n-1$), $\alpha_j \neq \alpha_q$, $\forall j \neq q$, such as follows:*

$$\text{(i)} \quad 1 - |\alpha_j| > 0 \quad \forall j = 1, \ldots, n-1 \quad (43)$$

$$\text{(ii)} \quad 1 - (\bar{t}^n(\cdot)) - \sum_{j=1}^{n-1} (\bar{t}^j(\cdot))|\beta_j|(1 - |\alpha_j|)^{-1} > 0, \quad (44)$$

where

$$\bar{t}^n(\cdot) = \max_{1 \le i \le N} (t_i^n(\cdot)), \qquad \bar{t}^j(\cdot) = \max_{1 \le i \le N} (t_i^j(\cdot)), \quad j = 1, \ldots, n-1. \quad (45)$$

Proof It suffices to verify that the matrix $(I_n - T_c(\cdot))$ is an $M-$matrix:

$$T_c(\cdot) = \begin{bmatrix} |\alpha_1| & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}| \\ \bar{t}^1(\cdot) & \cdots & \cdots & \bar{t}^{n-1}(\cdot) & \bar{t}^n(\cdot) \end{bmatrix}. \quad (46)$$

Since the elements $\alpha_j \quad (j = 1, \ldots, n-1)$ can be arbitrarily selected, the choice $|\alpha_j| \in]0, 1[$ with $\alpha_j \neq \alpha_q$, $\forall j \neq q$ then, the matrix $T_c(\cdot)$ with all elements positive.

Thus, the conditions of Theorem 3 can be deduced from the Kotelyanski conditions in the discrete case applied to the matrix $(I_n - T_c(\cdot))$.

In these conditions $(I_n - T_c(\cdot))$ is an $M-$matrix, we determine sufficient stability conditions for the system (29). The $n-1$ first conditions are checked because $|\alpha_j| \in]0, 1[$ $j = 1, \ldots, n-1$, however, the last condition yields to

$$\det(I_n - T_c(\cdot)) = \begin{vmatrix} 1 - |\alpha_1| & 0 & \cdots & 0 & -|\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 - |\alpha_{n-1}| & -|\beta_{n-1}| \\ -\bar{t}^1(\cdot) & \cdots & \cdots & -\bar{t}^{n-1}(\cdot) & (1 - \bar{t}^n(\cdot)) \end{vmatrix} > 0. \quad (47)$$

Then, we obtain: $1 - (\bar{t}^n(\cdot)) - \sum_{j=1}^{n-1} (\bar{t}^j(\cdot))|\beta_j| (1 - |\alpha_j|)^{-1} > 0$. ■

To simplify the use of the stability conditions, Theorem 3 can be reduced to the following corollary.

Corollary 1 *If the system (29) is asymptotically stable under arbitrary switching rule (2), the following conditions are satisfied $\forall \alpha_j \in]0\ 1[(j = 1, \ldots, n-1)$, $\alpha_j \neq \alpha_q \quad \forall j \neq q \quad \forall i = 1, \ldots, N$:*

$$\text{(i)} \quad \beta_j(P_i(\cdot, \alpha_j) + Q_i(\cdot, \alpha_j)) < 0, \quad (48)$$

$$\text{(ii)} \quad (P_i(\cdot, \lambda = 1) + Q_i(\cdot, \lambda = 1)) > 0 \quad (49)$$

$$\text{(iii)} \quad (\gamma_i^n(\cdot) + \delta_i^n(\cdot)) > 0. \quad (50)$$

Proof It is easy to see that relation (44) gives us (Sfaihi and Benrejeb, 2013)

$$\max_{1 \le i \le N} (|(\gamma_i^n(\cdot) + \delta_i^n(\cdot))|) + \sum_{j=1}^{n-1} \max_{1 \le i \le N} (|(\gamma_i^j(\cdot) + \delta_i^j(\cdot))|)|\beta_j|(1 - |\alpha_j|)^{-1} < 1. \quad (51)$$

It implies that

$$1 - \max_{1 \le i \le N} (|(\gamma_i^n(\cdot) + \delta_i^n(\cdot))|) - \sum_{j=1}^{n-1} \max_{1 \le i \le N} (|(\gamma_i^j(\cdot) + \delta_i^j(\cdot))|)|\beta_j|(1 - |\alpha_j|)^{-1} > 0. \quad (52)$$

Moreover, it is clear to see that $\forall 1 \le i \le N$ and $\forall 1 \le j \le n-1$:

$$1 - (|(\gamma_i^n(\cdot) + \delta_i^n(\cdot))|) - \sum_{j=1}^{n-1} (|(\gamma_i^j(\cdot) + \delta_i^j(\cdot))|)|\beta_j|(1 - |\alpha_j|)^{-1} > 0 \quad (53)$$

is more restrictive than relation (52).

On the other hand, to prove the Corollary 1, it is simple to consider conditions (48) and (50), in order to find Equation (49). For this, substitute relations (38) and (39) in Equation (53).

Then, relation (53) becomes

$$1 - (\gamma_i^n(\cdot) + \delta_i^n(\cdot)) - \sum_{j=1}^{n-1} (\gamma_i^j(\cdot) + \delta_i^j(\cdot))\beta_j(1 - |\alpha_j|)^{-1} > 0. \quad (54)$$

By Equations (38) and (39), we have that

$$1 + (a_i^1(\cdot) + d_i^1(\cdot)) + \sum_{j=1}^{n-1} \alpha_j + \sum_{j=1}^{n-1} \left(\frac{1}{(1-\alpha_j)} \left(\frac{(\lambda - \alpha_j)(P_i + Q_i)}{F(\lambda)} \right) \right)_{\lambda=\alpha_j} > 0, \quad (55)$$

where

$$F(\lambda) = \prod_{j=1}^{n-1} (\lambda - \alpha_j). \quad (56)$$

Therefore, to deduce the stability conditions of the switched system given in Equation (29), let us first observe that

$$\frac{(P_i(\cdot,\lambda) + Q_i(\cdot,\lambda))}{F(\lambda)} = \lambda + (a_i^1(\cdot) + d_i^1(\cdot)) + \sum_{j=1}^{n-1} \alpha_j + \sum_{j=1}^{n-1} \left(\left(\frac{(\lambda - \alpha_j)(P_i(\cdot,\lambda) + Q_i(\cdot,\lambda))}{(1-\alpha_j)F(\lambda)} \right) \right)_{\lambda=\alpha_j}. \quad (57)$$

Then, following that the developed stability condition (54) is equivalent to

$$\left(\frac{(P_i(\cdot,\lambda) + Q_i(\cdot,\lambda))}{F(\lambda)} \right)_{\lambda=1} > 0, \quad (58)$$

where

$$F(\lambda = 1) = \prod_{j=1}^{n-1} (1 - \alpha_j) > 0 \quad \forall \alpha_j \in]0 \quad 1[,$$

that is: $(P_i(\cdot, \lambda = 1) + Q_i(\cdot, \lambda = 1)) > 0$.

Then, asymptotical stability of the system (29) is under switching law (2).

This completes the proof. ■

5. Illustrative example

In this section, we give a numerical example to illustrate the performance of the proposed approach.

Consider the discrete-time switched nonlinear time-delay system described by the following switched difference equation:

$$y(k+2) + \sum_{i=1}^{2} \zeta_i(k) \left(\sum_{j=0}^{1} a(\cdot)_i^{2-j} y(k+j) + \sum_{j=0}^{1} d(\cdot)_i^{2-j} y(k+j-\tau) \right) = 0.$$

For the time-delay τ, according to Equations (27)–(31), we can obtain the following state representation:

$$x(k+1) = \sum_{i=1}^{2} \zeta_i \left(\begin{pmatrix} 0 & 1 \\ -a(\cdot)_i^2 & -a(\cdot)_i^1 \end{pmatrix} x(k) + \begin{pmatrix} 0 & 0 \\ -d(\cdot)_i^2 & -d(\cdot)_i^1 \end{pmatrix} x(k-\tau) \right),$$

$$x(k+1) = \sum_{i=1}^{2} \zeta_i(k)(A_i(\cdot)x(k) + D_i(\cdot)x(k-\tau)),$$

$$x(l) = \phi(l), \quad l = -\tau, \dots, -1, 0,$$

where the matrices are defined as follows:

$$A_1(\cdot) = \begin{bmatrix} 0 & 1 \\ -0.5 + 0.8f(\cdot) & 1 - 0.4f(\cdot) \end{bmatrix} \text{ and }$$

$$A_2(\cdot) = \begin{bmatrix} 0 & 1 \\ -0.6 + 0.7f(\cdot) & 1.1 - 0.8f(\cdot) \end{bmatrix},$$

$$D_1(\cdot) = \begin{bmatrix} 0 & 0 \\ -0.4 + 0.25\Phi(\cdot) & 0.8 - 0.6\Phi(\cdot) \end{bmatrix} \text{ and }$$

$$D_2(\cdot) = \begin{bmatrix} 0 & 0 \\ -0.2 + 0.4\Phi(\cdot) & 0.8 - 0.5\Phi(\cdot) \end{bmatrix},$$

where $f(\cdot)$ and $\Phi(\cdot)$ are unknown nonlinear functions.

Then, according to Equations (34)–(39), a change of base under the arrow form gives us the following matrices:

$$M_1(\cdot) = \begin{bmatrix} \alpha & 1 \\ \gamma_1^1(\cdot) & \gamma_1^2(\cdot) \end{bmatrix} \text{ and } M_2(\cdot) = \begin{bmatrix} \alpha & 1 \\ \gamma_2^1(\cdot) & \gamma_2^2(\cdot) \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0 & 0 \\ \delta_1^1(\cdot) & \delta_1^2(\cdot) \end{bmatrix} \text{ and } N_2 = \begin{bmatrix} 0 & 0 \\ \delta_2^1(\cdot) & \delta_2^2(\cdot) \end{bmatrix},$$

where

$$\gamma_1^1(\cdot) = -P_1(\alpha) = -[\alpha^2 + (1 - 0.4f(\cdot))\alpha - 0.5 + 0.8f(\cdot)],$$
$$\gamma_1^2(\cdot) = -(-1 + 0.4f(\cdot) + \alpha),$$
$$\gamma_2^1(\cdot) = -P_2(\alpha) = -[\alpha^2 + (1.1 - 0.8f(\cdot))\alpha - 0.6 + 0.7f(\cdot)],$$
$$\gamma_2^2(\cdot) = -(-1.1 + 0.8f(\cdot) + \alpha),$$

and

$$\delta_1^1(\cdot) = -Q_1(\alpha) = -[(0.8 - 0.6\Phi(\cdot))\alpha - 0.4 + 0.25\Phi(\cdot)],$$
$$\delta_1^2(\cdot) = -(-0.8 + 0.6\Phi(\cdot)),$$
$$\delta_2^1(\cdot) = -Q_{12}(\alpha) = -[(0.8 - 0.5\Phi(\cdot))\alpha - 0.2 + 0.4\Phi(\cdot)],$$
$$\delta_2^2(\cdot) = -(0.8 - 0.5\Phi(\cdot)).$$

Then, the stability conditions for the example given by the Corollary 1 are the following:

(i) $0 < \alpha < 1$,
(ii) $(P_1(\alpha) + Q_1(\alpha)) < 0$,
(iii) $(P_2(\alpha) + Q_2(\alpha)) < 0$,
(iv) $(P_1(1) + Q_1(1)) > 0$,
(v) $(P_2(1) + Q_2(1)) > 0$,
(vi) $(\gamma_1^2(\cdot) + \delta_1^2) > 0$, and
(vii) $(\gamma_2^2(\cdot) + \delta_2^2) > 0$.

Therefore, in case $\alpha = 0.2$, $\beta = 1$, conditions (ii), (iii), (iv), (v), (vi) and (vii) allow for deducing the following stability conditions:

(i) $f(\cdot) > 0.8 - 0.18\Phi(\cdot)$,
(ii) $f(\cdot) > 0.85 - 0.55\Phi(\cdot)$,
(iii) $f(\cdot) < 0.25 + 0.875\Phi(\cdot)$,
(iv) $f(\cdot) > 1 - \Phi(\cdot)$,
(v) $f(\cdot) < 4 - 1.5\Phi(\cdot)$, and
(vi) $f(\cdot) < 2.125 - 0.625\Phi(\cdot)$.

In the following, we determine the stability domain for the chosen α. Figure 1 illustrates the stability domain given by the nonlinear $f(\cdot)$ relative to the nonlinear $\Phi(\cdot)$.

Then, according to the stability domain shown in Figure 1, for particular values of the nonlinearities $f(\cdot) = 1$ and $\Phi(\cdot) = 1.5$, by choosing the sampling time $T_e = 0.2\,s$, $t_f = kT_e = 10\,s$ the switched time $t_1 = k_1T_e = 5\,s$, the simulation results are shown in Figures 2 and 3, where $\phi(l) = [-1 \quad 2]^{\mathrm{T}}$ for all $l = -5, -4, -3, -2, -1, 0$. Figure 2 shows the state responses; the state trajectories are depicted in Figure 3, which shows the stability of the system given in the example. Then, for the same values

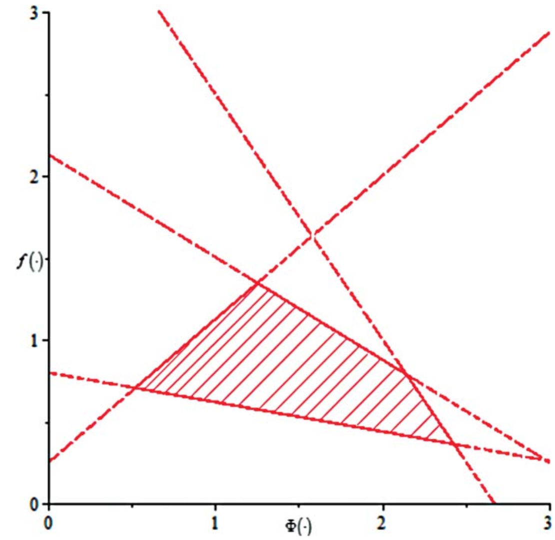

Figure 1. Stability domain given of the system illustrate in example obtained from Corollary 1.

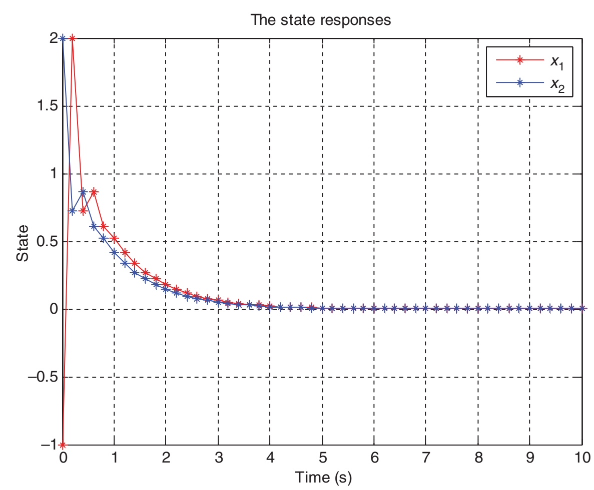

Figure 2. The state responses of the system given in example for $t_1 = k_1T_e = 5$ s.

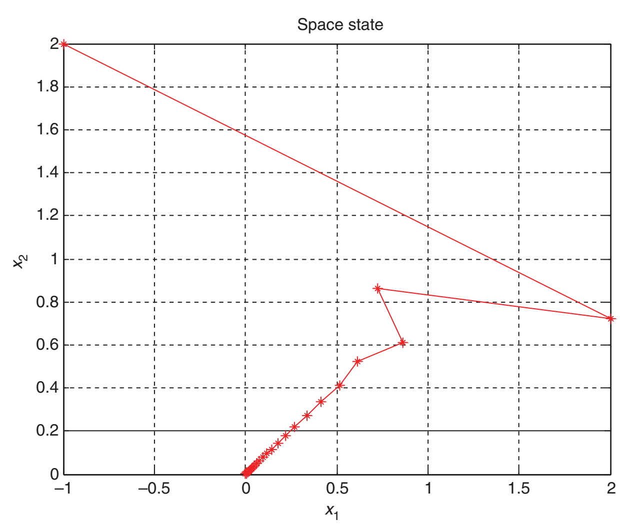

Figure 3. Trajectory response of the system given in example for $t_1 = k_1T_e = 5$ s.

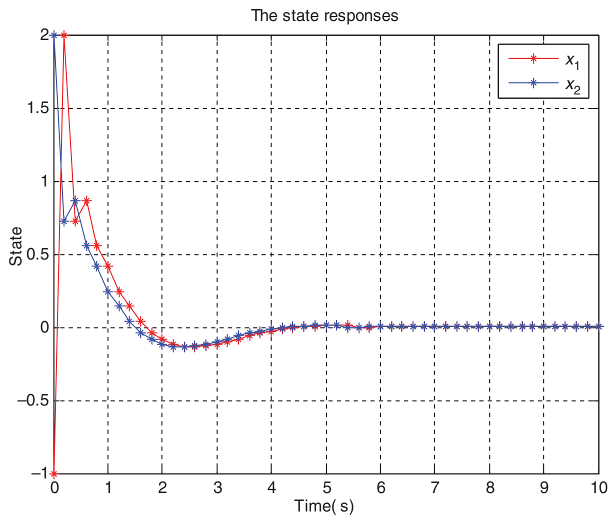

Figure 4. The state responses of the system given in example for $t_1 = k_1 T_e = 0.4\,s$.

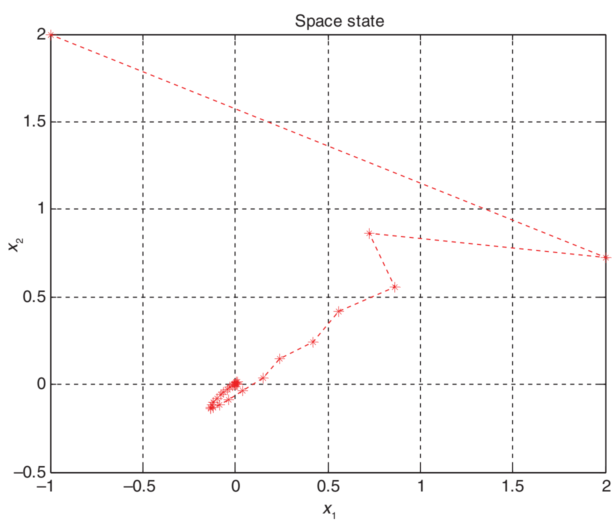

Figure 5. Trajectory response of the system given in example for $t_1 = k_1 T_e = 0.4\,s$.

of the nonlinearities $f(\cdot) = 1$ and $\Phi(\cdot) = 1.5$ and the sampling time $T_e = 0.2\ s$. The switched time $t_1 = k_1 T_e = 0.4\,s$ and $\phi(l) = [-1 \quad 2]^{\mathrm{T}}$ for all $l = -1, 0$ the evolution of the states and the state space are shown in Figures 4 and 5, respectively.

This example has been treated to show that these stability conditions are sufficient and very close to be necessary. On the other hand, it is uncertain to treat an example for which there exists a common Lyapunov function, in order to do a comparison with our approach.

6. Conclusions

In this paper, we have developed a new method for the stability analysis under arbitrary switching for a class of discrete-time switched nonlinear time-delay systems. These stability conditions were derived from an appropriate Lyapunov function associated with the application of the Kotelyanski stability conditions. The main benefit of this technique is that it is easy to use and it gives us an explicitly stability condition that guaranteed the stability under arbitrary switching law. Moreover, it can avoid the problem of existence of Lyapunov functions which are usually very difficult to apply, or even not possible. Simulation results have been presented to illustrate the effectiveness of the developed method.

The limits of this paper are that it has been confined to the boundaries of numerical examples. It would be beneficial to extend the research further so as to include real systems.

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