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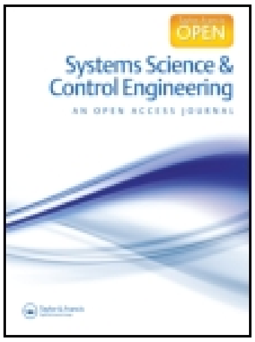

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Global stability in Lagrange sense for BAM-type Cohen–Grossberg neural networks with time-varying delays

Jigui Jian* and Zhihua Zhao

College of Science, China Three Gorges University, Yichang, Hubei 443002, People's Republic of China

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In this paper, we investigate the positive invariant sets and global exponential attractive sets for a class of bidirectional associative memory (BAM)-type Cohen–Grossberg neural networks with multiple time-varying delays. By applying inequality techniques, some easily verifiable delay-independent criteria for the ultimate boundedness and global exponential attractive sets of BAM-type Cohen–Grossberg neural networks are obtained by constructing appropriate Lyapunov functions. Finally, one example with numerical simulations is given to illustrate the results obtained in this paper.

Keywords: BAM-type Cohen–Grossberg neural networks; Lagrange stability; globally attractive set; inequality

1. Introduction

Recently, the stability of different types of Cohen–Grossberg neural networks and the bidirectional associative memory (BAM) neural networks has been widely studied by many researchers and various interesting results have been reported (Jiang & Cao, 2008; Li, 2009; Li, Fei, Tan, & Zhang, 2009; Li, Zhang, Zhang, & Li, 2010; Liu & Zong, 2009; Wang, Jian, & Guo, 2008; Zhang, Liu, & Zhou, 2012). In many applications, since BAM-type Cohen–Grossberg neural networks consider the interaction between two neural networks, the studies of the stability behavior of BAM-type Cohen–Grossberg neural networks are of greater interest than the studies of the stability of Cohen–Grossberg neural networks and BAM neural networks.

For BAM-type Cohen–Grossberg neural networks, the existence of periodic solution and an equilibrium point and their stability have been investigated in Jiang and Cao (2008), Li (2009), Li et al. (2010) and Zhang et al. (2012). But in many actual applications, these conclusions are no longer appropriate in the multistable dynamics which have multiple equilibrium and so many of them are unstable (Lu, Wang, & Chen, 2011; Wang & Chen, 2012). Such as the Cohen–Grossberg neural network, when applications are taken into account in biology, it is necessary and important to deal with multistable properties. In this context, it is worth mentioning that the Lagrange stability refers to the stability of the total system which does not require the information of equilibrium points, because the Lagrange stability is considered on the basis of the boundedness of solutions and the existence of global attractive sets (Liao, Luo, & Zeng, 2008). Just as verified in (Liao et al., 2008), outside the globally attracting set, there is no equilibrium point, chaos attractor, periodic state or almost periodic state in the neural networks. Therefore, the research on positive invariant sets and globally attractive sets of the neural networks have been done by many scholars (Liao et al., 2008; Luo, Zeng, & Liao, 2011; Song & Zhao, 2005; Tu, Jian, & Wang, 2011; Tu, Wang, Zha, & Jian, 2013; Wang, Jian, & Jiang, 2010). Song and Zhao (2005) investigated the dissipativity of neural networks with both variable and unbounded delays by constructing proper Lyapunov functions and using some analytic techniques. And the global stability in the Lagrange sense for a class of Cohen–Grossberg neural networks with time-varying delays and finite distributed delays was studied in Tu et al. (2011) and Wang et al. (2010). In Tu et al. (2013), the authors study the global dissipativity of a class of BAM neural networks with both time-varying and unbound delays. To our best knowledge, few authors have discussed the global attractive sets for BAM-type Cohen–Grossberg neural networks.

Motived by the above analysis, the aim of this paper is to study Lagrange stability and global exponential attractive sets for BAM-type Cohen–Grossberg neural networks with time-varying delays and some delay-independent criteria for the ultimate boundedness and global exponential attractive sets of BAM-type Cohen–Grossberg neural networks are obtained. And some results here obtained in this paper are more general than that of the existing reference on the globally exponentially attractive (GEA) set as special cases. The remaining paper is organized as follows: Section 2 describes some preliminaries including some necessary notations, definitions, assumptions and some lemmas. The main results are stated in Section 3. Section 4

*Corresponding author. Email: jiguijian@ctgu.du.cn

gives a numerical example to testify the theoretical analysis. Finally, conclusions are drawn in Section 5.

2. Problem statement

Consider the following BAM-type Cohen–Grossberg neural network model

$$\begin{cases} \dot{x}_i(t) = a_i(x_i(t)) \begin{bmatrix} -c_i(x_i(t)) + \sum_{j=1}^{m} a_{ij} f_j(y_j(t)) \\ + \sum_{j=1}^{m} b_{ij} f_j(y_j(t-\tau_j(t))) + I_i \end{bmatrix}, \\ \quad i = 1, 2, \ldots, n, \\ \dot{y}_j(t) = b_j(y_j(t)) \begin{bmatrix} -d_j(y_j(t)) + \sum_{i=1}^{n} m_{ji} g_i(x_i(t)) \\ + \sum_{i=1}^{n} n_{ji} g_i(x_i(t-\sigma_i(t))) + J_j \end{bmatrix}, \\ \quad j = 1, 2, \ldots, m, \end{cases} \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}}, y(t) = (y_1(t), \ldots, y_m(t))^{\mathrm{T}}$ are the neuron state vectors of the neural network (1), $a_i(x_i(t)) > 0$ for $i \in \Lambda = \{1, 2, \ldots, n\}$ and $b_j(y_j(t)) > 0$ for $j \in \Gamma = \{1, 2, \ldots, m\}$ represent amplification functions of the ith neurons from the neural field F_x and the jth neurons from the neural field F_y, respectively; $c_i(x_i), d_j(y_j)$ are appropriately behaved functions of the ith neurons from the neural field F_x and the jth neurons from the neural field F_y, respectively; f_j and g_i are the activation functions, I_i and J_j are the exogenous inputs. $A = (a_{ij})_{n\times m}$, $M = (m_{ji})_{m\times n}$; $B = (b_{ij})_{n\times m}$, $N = (n_{ji})_{m\times n}$ are the connection weight matrices and the delayed weight matrices, respectively, $I = (I_1, I_2, \ldots, I_n)^{\mathrm{T}}, J = (J_1, J_2, \ldots, J_m)^{\mathrm{T}}$ are external input vectors. The time-varying delays $\tau_j(t), \sigma_i(t)$ are non-negative and bounded, i.e. $0 \le \tau_j(t) \le \tau_j$, $0 \le \sigma_i(t) \le \sigma_i$. We define the vector functions f, g by

$$g(x(\cdot)) = (g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot)))^{\mathrm{T}} \in R^n,$$
$$f(y(\cdot)) = (f_1(y_1(\cdot), f_2(y_2(\cdot), \ldots, f_m(y_m(\cdot)))^{\mathrm{T}} \in R^m.$$

The initial conditions associated with Equation (1) are given by

$$\begin{cases} x_i(\theta) = \phi_i(\theta), & \theta \in [-\sigma, 0], i \in \Lambda, \\ y_j(\theta) = \psi_j(\theta), & \theta \in [-\tau, 0], j \in \Gamma, \end{cases} \tag{2}$$

where $\phi_i(\theta)$ and $\psi_j(\theta)$ are continuous real valued functions defined on their respective domains, $\sigma = \max_{1\le i\le n}\{\sigma_i\}$, $\tau = \max_{1\le j\le m}\{\tau_j\}$.

Remark 1 It is obvious that system (1) includes neural systems considered in Tu et al. (2013) as its special case. For example, $a_i(x_i(t)) = 1$, $b_j(y_j(t)) = 1$ for $i \in \Lambda$ and $j \in \Gamma$, $c_i(x_i(t)) = \tilde{a}_i x_i(t)$ and $d_j(y_j(t)) = \tilde{c}_j y_j(t)$ with the constants $\tilde{a}_i > 0$ and $\tilde{c}_j > 0$, system (1) reduces to the BAM neural network in Tu et al. (2013).

In order to establish the conditions of main results for the neural networks (1), we have the following assumptions:

- (H1) $a_i(u), b_j(u) \in C(R, R^+)$. Furthermore, there exist positive constants $\underline{a}_i$, $\bar{a}_i$, $\underline{b}_j$ and $\bar{b}_j (i \in \Lambda, j \in \Gamma)$ such that $0 < \underline{a}_i \le a_i(u) \le \bar{a}_i$, $0 < \underline{b}_j \le b_j(u) \le \bar{b}_j$, $u \in R$.
- (H2) $c_i(u), d_j(u) \in C(R, R^+)$. Moreover, there exist positive constants $\underline{c}_i$, $\bar{c}_i$, $\underline{d}_j$ and $\bar{d}_j (i \in \Lambda, j \in \Gamma)$ such that $\underline{c}_i u^2 \le u c_i(u) \le \bar{c}_i u^2$, $\underline{d}_j u^2 \le u d_j(u) \le \bar{d}_j u^2$, $u \in R$.

The set of bounded activation functions is defined as

$$B = \{p(x) | p_i(x_i) \in C(R, R), \exists k_i > 0, |p_i(x_i)| \le k_i, \forall x_i \in R\}$$

The sigmoid function is defined as

$$S = \left\{ p(x) \left| \begin{array}{c} p_i(0) = 0, p_i(x_i) \in C(R, R), \\ D^+ p_i(x_i) \ge 0, |p_i(x_i)| \le k_i, \forall x_i \in R \end{array} \right. \right\}.$$

Remark 2 In this paper, $f(\cdot), g(\cdot) \in B$ represent $|g_i(\cdot)| \le s_i$, $|f_j(\cdot)| \le r_j$ for $i \in \Lambda$ and $j \in \Gamma$, respectively, where s_i, r_j are all positive constants.

Let

$$\tilde{M} = \sum_{i=1}^{n} \bar{a}_i M_i s_i,$$

$$M_i = \bar{a}_i \left(\sum_{j=1}^{m} (|a_{ij}| + |b_{ij}|) r_j + |I_i| \right), \quad i \in \Lambda;$$

$$\tilde{N} = \sum_{j=1}^{m} \bar{b}_j N_j r_j,$$

$$N_j = \bar{b}_j \left(\sum_{i=1}^{n} (|m_{ji}| + |n_{ji}|) s_i + |J_j| \right), \quad j \in \Gamma.$$

Let $\Omega \subset R^{n+m}$ be a compact set in R^{n+m}. Denote the complement of Ω by $R^{n+m} \backslash \Omega$. For any

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in R^{n+m}, \ \rho\left(\begin{pmatrix} x \\ y \end{pmatrix}, \Omega \right) = \inf_{(x_1^T, y_1^T) \in \Omega} \left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \right\|$$

is the distance between $\binom{x}{y}$ and Ω. We call a compact set Ω as a global attractive set of networks (1), if for every solution

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in R^{n+m} \backslash \Omega$$

with initial condition (2), we have

$$\lim_{t\to+\infty} \rho\left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \Omega\right) = 0.$$

Obviously, if the network (1) has global attractive sets, then the solutions are ultimately bounded.

Definition 1 *A compact set $\Omega \in R^{n+m}$ is said to be a global exponential stable (GES) set of system (1), if there exists a constant α and a non-negative bounded continuous functional K such that for every solution $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in R^{n+m}\backslash\Omega$ with an initial condition (2), we have*

$$\rho\left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \Omega\right) \le K(\phi,\psi)\, e^{-\alpha(t-t_0)}.$$

Definition 2 *If there exists a radially unbounded and positive definite Lyapunov function $V(t) = V(x(t), y(t))$ and positive constants l and α such that for any solution $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in R^{n+m}\backslash\Omega$ of (1), $V(t) > l$ for $t \ge t_0$ implies $V(t) - l \le (V(t_0) - l)\, e^{-\alpha(t-t_0)}$, system (1) is said to be GEA. The compact set*

$$\Omega = \left\{\begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} | V(t) \le l.\right\}$$

is called a GEA set of Equation (1).

Definition 3 *Network (1) is called GES in the Lagrange sense, if it is both uniformly bounded and GEA.*

Lemma 1 *For $\forall x, y \in R$, $a > 0$, the inequality $-ax^2 + xy \le -\frac{1}{2}ax^2 + y^2/2a$ holds.*

Lemma 2 (Wang et al., 2010) *Let $a \ge 0, b \ge 0, p > 1, q > 1$ with $1/p + 1/q = 1$. Then the inequality $ab \le (1/p)a^p + (1/q)b^q$ holds, and the equality holds if and only if $a^p = b^q$.*

Lemma 3 (Luo et al., 2011) *Let $V(t) \in C[R^n, R^+]$ be a positive definite and radially unbounded function, and suppose there exist two constants $\alpha > 0$, $\beta > 0$ such that $D^+V(t) \le -\alpha V(t) + \beta$ for $t \ge t_0$, then $V(t) \ge \beta/\alpha$ implies*

$$V(t) - \frac{\beta}{\alpha} \le \left(V(t_0) - \frac{\beta}{\alpha}\right) e^{-\alpha(t-t_0)}.$$

3. Main results

Theorem 1 *If the activation functions $f(\cdot), g(\cdot) \in B$ and (H1), (H2) are also satisfied, then system (1) is globally exponentially stable in Lagrange sense and Ω_i for $i = 1, 2, 3, 4, 5$ are all GES set and the set $\Omega = \bigcap_{i=1}^{5} \Omega_i$ is a better GES set of Equation (1), where*

$$\Omega_1 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \middle| \begin{array}{l} \sum_{i=1}^{n} x_i^2(t) + \sum_{j=1}^{m} y_j^2(t) \\ \le \frac{\sum_{i=1}^{n} M_i^2/\underline{a}_i\underline{c}_i + \sum_{j=1}^{m} N_j^2/\underline{b}_j\underline{d}_j}{\min_{1\le i\le n,\ 1\le j\le m} \{\underline{a}_i\underline{c}_i, \underline{b}_j\underline{d}_j\}} \end{array} \right\}.$$

$$\Omega_2 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \middle| \begin{array}{l} \sum_{i=1}^{n} |x_i(t)| + \sum_{j=1}^{m} |y_j(t)| \\ \le \frac{\sum_{i=1}^{n} M_i + \sum_{j=1}^{m} N_j}{\min_{1\le i\le n,\ 1\le j\le m} \left\{\underline{a}_i\underline{c}_i, \underline{b}_j\underline{d}_j\right\}} \end{array} \right\}.$$

$$\Omega_3 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \middle| \begin{array}{l} |x_i(t)| \le \frac{M_i}{\underline{a}_i\underline{c}_i}, i \in \Lambda; \\ |y_j(t)| \le \frac{N_j}{\underline{b}_j\underline{d}_j}, j \in \Gamma \end{array} \right\}.$$

$$\Omega_4 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \middle| \begin{array}{l} \sum_{i=1}^{n} x_i^2(t) \le \frac{\sum_{i=1}^{n} M_i^2/\underline{a}_i\underline{c}_i}{\min_{1\le i\le n}\{\underline{a}_i\underline{c}_i\}}, \\ \sum_{j=1}^{m} y_j^2(t) \le \frac{\sum_{j=1}^{m} N_j^2/\underline{b}_j\underline{d}_j}{\min_{1\le i\le n}\{\underline{b}_j\underline{d}_j\}} \end{array} \right\}.$$

$$\Omega_5 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \middle| \begin{array}{l} \sum_{i=1}^{n} |x_i(t)| \le \frac{\sum_{i=1}^{n} M_i}{\min_{1\le i\le n}\{\underline{a}_i\underline{c}_i\}}, \\ \sum_{j=1}^{m} |y_j(t)| \le \frac{\sum_{j=1}^{m} N_j}{\min_{1\le j\le m}\{\underline{b}_j\underline{d}_j\}} \end{array} \right\}.$$

Proof (1) Employ a radially unbounded and positive definite Lyapunov function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{n} x_i^2(t) + \frac{1}{2}\sum_{j=1}^{m} y_j^2(t).$$

Calculating the Dini derivative of $V(t)$ along the positive semi-trajectory of Equation (1), and by virtue of Lemma 1, we obtain

$$\left.\frac{\mathrm{d}V(t)}{\mathrm{d}t}\right|_{(1)} \le -\sum_{i=1}^{n} \underline{a}_i\underline{c}_i x_i^2(t) + \sum_{i=1}^{n} \bar{a}_i \left(\sum_{j=1}^{m} (|a_{ij}| + |b_{ij}|)r_j + |I_i|\right) |x_i(t)| - \sum_{j=1}^{m} \underline{b}_j\underline{d}_j y_j^2(t)$$

$$
\begin{aligned}
&+\sum_{j=1}^{m}\bar{b}_j\left(\sum_{i=1}^{n}(|m_{ji}|+|n_{ji}|)s_i+|J_j|\right)|y_j(t)|\\
&=-\sum_{i=1}^{n}\underline{a}_i\underline{c}_ix_i^2(t)+\sum_{i=1}^{n}M_i|x_i(t)|-\sum_{j=1}^{m}\underline{b}_j\underline{d}_jy_j^2(t)\\
&+\sum_{j=1}^{m}N_j|y_j(t)|\\
&\le -\frac{1}{2}\sum_{i=1}^{n}\underline{a}_i\underline{c}_ix_i^2(t)+\frac{1}{2}\sum_{i=1}^{n}\frac{M_i^2}{\underline{a}_i\underline{c}_i}\\
&-\frac{1}{2}\sum_{j=1}^{m}\underline{b}_j\underline{d}_jy_j^2(t)+\frac{1}{2}\sum_{j=1}^{m}\frac{N_j^2}{\underline{b}_j\underline{d}_j}\\
&\le -\min_{\substack{1\le i\le n\\1\le j\le m}}\{\underline{a}_i\underline{c}_i,\underline{b}_j\underline{d}_j\}V(t)+\frac{1}{2}\sum_{i=1}^{n}\frac{M_i^2}{\underline{a}_i\underline{c}_i}+\frac{1}{2}\sum_{j=1}^{m}\frac{N_j^2}{\underline{b}_j\underline{d}_j}\\
&=-\alpha V(t)+\beta,
\end{aligned}
$$

where

$$
\alpha=\min_{\substack{1\le i\le n\\1\le j\le m}}\left\{\underline{a}_i\underline{c}_i,\underline{b}_j\underline{d}_j\right\},
$$

$$
\beta=\frac{1}{2}\sum_{i=1}^{n}\frac{M_i^2}{\underline{a}_i\underline{c}_i}+\frac{1}{2}\sum_{j=1}^{m}\frac{N_j^2}{\underline{b}_j\underline{d}_j}.
$$

According to Lemma 3, for $V(t)\ge\beta/\alpha$, $V(t_0)\ge\beta/\alpha$, we have

$$
V(t)-\frac{\beta}{\alpha}\le(V(t_0)-\frac{\beta}{\alpha})\mathrm{e}^{-\alpha(t-t_0)}.
$$

Hence Ω_1 is a GES set of Equation (1).

(2) Construct another positive definite and radially unbounded Lyapunov function as

$$
V(t)=\sum_{i=1}^{n}|x_i(t)|+\sum_{j=1}^{m}|y_j(t)|.
$$

So we can get

$$
\begin{aligned}
&D^+V(t)|_{(1)}\\
&\quad\le -\sum_{i=1}^{n}\underline{a}_i\underline{c}_i|x_i(t)|\\
&\qquad+\sum_{i=1}^{n}\left(\bar{a}_i\sum_{j=1}^{m}(|a_{ij}t|+|b_{ij}|)r_j+|I_i|\right)\\
&\qquad-\sum_{j=1}^{m}\underline{b}_j\underline{d}_j|y_j(t)|\\
&\qquad+\sum_{j=1}^{m}\bar{b}_j\left(\sum_{i=1}^{n}(|m_{ji}|+|n_{ji}|)s_i+|J_j|\right)\\
&\quad\le -\alpha V(t)+\beta,
\end{aligned}
$$

where

$$
\alpha=\min_{\substack{1\le i\le n\\1\le j\le m}}\{\underline{a}_i\underline{c}_i,\underline{b}_j\underline{d}_j\},\quad \beta=\sum_{i=1}^{n}M_i+\sum_{j=1}^{m}N_j.
$$

So we get

$$
V(t)-\frac{\beta}{\alpha}\le\left(V(t_0)-\frac{\beta}{\alpha}\right)\mathrm{e}^{-\alpha(t-t_0)}.
$$

And the set Ω_2 is a GES set of Equation (1).

(3) Choose another two positive definite and radially unbounded Lyapunov functions as

$$
V_i(t)=|x_i(t)|,\ i\in\Lambda;\quad V_j(t)=|y_j(t)|,\ j\in\Gamma.
$$

And we have

$$
\begin{aligned}
D^+V_i(t)|_{(1)}&\le -\underline{a}_i\underline{c}_iV_i(t)+M_i,\quad i\in\Lambda,\\
D^+V_j(t)|_{(1)}&\le -\underline{b}_j\underline{d}_jV_j(t)+N_j,\quad j\in\Gamma.
\end{aligned}
$$

So we have

$$
V_i(t)-\frac{M_i}{\underline{a}_i\underline{c}_i}\le\left(V_i(t_0)-\frac{M_i}{\underline{a}_i\underline{c}_i}\right)\mathrm{e}^{-\underline{a}_i\underline{c}_i(t-t_0)},\ i\in\Lambda,
$$

$$
V_j(t)-\frac{N_j}{\underline{b}_j\underline{d}_j}\le\left(V_j(t_0)-\frac{N_j}{\underline{b}_j\underline{d}_j}\right)\mathrm{e}^{-\underline{b}_j\underline{d}_j(t-t_0)},\ j\in\Gamma.
$$

So the Ω_3 is a GES set of Equation (1).

(4) Employ only the following two radially unbounded and positive definite Lyapunov functions as

$$
V_x(t)=\frac{1}{2}\sum_{i=1}^{n}x_i^2(t),\quad V_y(t)=\frac{1}{2}\sum_{j=1}^{m}y_j^2(t).
$$

The remaining proof is similar to the proof in the previous part (1). Meanwhile, consider only the following other two Lyapunov functions

$$
V_x(t)=\sum_{i=1}^{n}|x_i(t)|,\quad V_y(t)=\sum_{j=1}^{m}|y_j(t)|.
$$

The remaining proof is similar to that in the previous part (2). So the sets Ω_4 and Ω_5 are also GES sets of (1). According to the definition of intersection set, we know that the set $\Omega=\bigcap_{i=1}^{5}\Omega_i$ is a better GES set of NN (1). The proof of Theorem1 is completed. ■

Remark 3 When $a_i(x_i(t))=1, b_j(y_j(t))=1$ for $i\in\Lambda$ and $j\in\Gamma$, $c_i(x_i(t))=\tilde{a}_ix_i(t)$ and $d_j(y_j(t))=\tilde{c}_jy_j(t)$ with the constants $\tilde{a}_i > 0$ and $\tilde{c}_j > 0$, the set Ω_5 in Theorem 1 here is just the main result (I) of Theorem 3.2 in Tu et al. (2013).

THEOREM 2 *Let $p > 1$, $q > 1$ and $1/p + 1/q = 1$. Choose $\varepsilon_i > 0$, $\bar{\varepsilon}_j > 0 (i \in \Lambda, j \in \Gamma)$ such that $\mu_i = p\underline{a}_i\underline{c}_i - (p-1)\varepsilon_i > 0$, $\eta_j = q\underline{b}_j\underline{d}_j - (q-1)\bar{\varepsilon}_j > 0$. If the activation functions $f(\cdot), g(\cdot) \in B$ and (H1), (H2) are also satisfied, then NN (1) is globally exponentially stable in Lagrange sense and Ω_6 is a GES set, where*

$$\Omega_6 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \left| \begin{array}{c} \frac{1}{p}\sum_{i=1}^{n}|x_i(t)|^p + \frac{1}{q}\sum_{j=1}^{m}|y_j(t)|^q \\ \le \frac{\sum_{i=1}^{n} M_i^p/p\varepsilon_i^{p-1} + \sum_{j=1}^{m} N_j^q/q\bar{\varepsilon}_j^{q-1}}{\min_{\substack{1\le i\le n\\1\le j\le m}}\{\mu_i,\eta_j\}} \end{array} \right. \right\}.$$

Proof We introduce the following Lyapunov function

$$V(t) = \frac{1}{p}\sum_{i=1}^{n}|x_i(t)|^p + \frac{1}{q}\sum_{j=1}^{m}|y_j(t)|^q.$$

Calculating the Dini derivative of $V(t)$ along (1), and by virtue of Lemma 2, we can obtain

$$\begin{aligned} &D^+V(t)|_{(1)} \\ &\quad\le -\sum_{i=1}^{n}\underline{a}_i\underline{c}_i|x_i(t)|^p + \sum_{i=1}^{n}M_i|x_i(t)|^{p-1} \\ &\qquad - \sum_{j=1}^{m}\underline{b}_j\underline{d}_j|y_j(t)|^q + \sum_{j=1}^{m}N_j|y_j(t)|^{q-1} \\ &\quad\le -\sum_{i=1}^{n}\underline{a}_i\underline{c}_i|x_i(t)|^p + \sum_{i=1}^{n}\left(\frac{p-1}{p}\varepsilon_i|x_i(t)|^p + \frac{1}{p\varepsilon_i^{p-1}}M_i^p\right) \\ &\qquad - \sum_{j=1}^{m}\underline{b}_j\underline{d}_j|y_j(t)|^q + \sum_{j=1}^{m}\left(\frac{q-1}{q}\bar{\varepsilon}_j|y_j(t)|^q + \frac{1}{q\bar{\varepsilon}_j^{q-1}}N_j^q\right) \\ &\quad\le -\sum_{i=1}^{n}\left(\underline{a}_i\underline{c}_i - \frac{p-1}{p}\varepsilon_i\right)|x_i(t)|^p + \sum_{i=1}^{n}\frac{1}{p\varepsilon_i^{p-1}}M_i^p \\ &\qquad - \sum_{j=1}^{m}\left(\underline{b}_j\underline{d}_j - \frac{q-1}{q}\bar{\varepsilon}_j\right)|y_j(t)|^q + \sum_{j=1}^{m}\frac{1}{q\bar{\varepsilon}_j^{q-1}}N_j^q \\ &\quad\le -\alpha V(t) + \beta, \end{aligned}$$

where

$$\alpha = \min_{\substack{1\le i\le n\\1\le j\le m}}\{\mu_i,\eta_j\}, \quad \beta = \sum_{i=1}^{n}\frac{M_i^p}{p\varepsilon_i^{p-1}} + \sum_{j=1}^{m}\frac{N_j^q}{q\bar{\varepsilon}_j^{q-1}}.$$

And by Lemma 3, we get

$$V(t) - \frac{\beta}{\alpha} \le \left(V(t_0) - \frac{\beta}{\alpha}\right)e^{-\alpha(t-t_0)}.$$

So the set Ω_6 is a GES set of Equation (1). ■

Remark 4 When $a_i(x_i(t)) = 1, b_j(y_j(t)) = 1$ for $i \in \Lambda$ and $j \in \Gamma$, $c_i(x_i(t)) = \tilde{a}_i x_i(t)$ and $d_j(y_j(t)) = \tilde{c}_j y_j(t)$ with the constants $\tilde{a}_i > 0$ and $\tilde{c}_j > 0$ the sets Ω_6 in Theorem 2 here are just the main result of Theorem 3.1 in Tu et al. (2013).

THEOREM 3 *If the activation functions $f(\cdot), g(\cdot) \in S$ and (H1), (H2) are also satisfied, then NN (1) has positive invariant and globally exponential attractive sets*

$$\Omega_7 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \left| \begin{array}{c} \sum_{i=1}^{n}\int_0^{x_i(t)} g_i(s)\,ds \\ + \sum_{j=1}^{m}\int_0^{y_j(t)} f_j(\eta)\,d\eta \\ \le \frac{\tilde{M}+\tilde{N}}{\min_{\substack{1\le i\le n\\1\le j\le m}}\{\underline{a}_i\underline{c}_i, \underline{b}_j\underline{d}_j\}} \end{array} \right. \right\},$$

$$\Omega_8 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^{n+m} \left| \begin{array}{c} \sum_{i=1}^{n}\int_0^{x_i(t)} g_i(s)\,ds \\ \le \frac{\tilde{M}}{\min_{1\le i\le n}\{\underline{a}_i\underline{c}_i\}}, \\ \sum_{j=1}^{m}\int_0^{y_j(t)} f_j(\eta)\,d\eta \le \\ \frac{\tilde{N}}{\min_{1\le j\le m}\{\underline{b}_j\underline{d}_j\}} \end{array} \right. \right\}.$$

And $\Omega = \Omega_7 \cap \Omega_8$ is a better GES set of Equation (1).

Proof Firstly, employ the following Lyapunov function

$$V(t) = \sum_{i=1}^{n}\int_0^{x_i(t)} g_i(s)\,ds + \sum_{j=1}^{m}\int_0^{y_j(t)} f_j(\eta)\,d\eta.$$

Calculating the derivative of $V(t)$, we have

$$\begin{aligned} &\left.\frac{dV(t)}{dt}\right|_{(1)} \\ &\quad = \sum_{i=1}^{n} g_i(x_i(t))\dot{x}_i(t) + \sum_{j=1}^{m} f_j(y_j(t))\dot{y}_j(t) \\ &\quad \le -\sum_{i=1}^{n}\underline{a}_i\underline{c}_i x_i(t) g_i(x_i(t)) + \tilde{M} \\ &\qquad - \sum_{j=1}^{m}\underline{b}_j\underline{d}_j y_j(t) f_j(y_j(t)) + \tilde{N} \\ &\quad \le -\min_{\substack{1\le i\le n\\1\le j\le m}}\{\underline{a}_i\underline{c}_i, \underline{b}_j\underline{d}_j\}V(t) + \tilde{M} + \tilde{N} = -\alpha V(t) + \beta, \end{aligned}$$

where $\alpha = \min_{\substack{1\le i\le n\\ 1\le j\le m}} \{\underline{a}_i\underline{c}_i, \underline{b}_j\underline{d}_j\}$, $\beta = \tilde{M} + \tilde{N}$. In the light of Lemma 3, we get

$$V(t) - \frac{\beta}{\alpha} \le \left(V(t_0) - \frac{\beta}{\alpha}\right) \mathrm{e}^{-\alpha(t-t_0)}.$$

So the set Ω_7 is a GES set of Equation (1).

Secondly, consider the following Lyapunov functions

$$V_1(t) = \sum_{i=1}^{n} \int_0^{x_i(t)} g_i(s)\,\mathrm{d}s, \quad V_2(t) = \sum_{j=1}^{m} \int_0^{y_j(t)} f_j(\eta)\,\mathrm{d}\eta.$$

Similar to the proof in the previous part, we can obtain that the set Ω_8 is a GES set of Equation (1). Hence, $\Omega = \Omega_7 \cap \Omega_8$ is a better GES set of neural network (1). ■

4. Illustrative examples

In this section, we will give an example to verify our theoretical results.

Example 4.1 Consider the following example:

$$\begin{cases} \dot{x}_i(t) = a_i(x_i(t)) \left[\begin{array}{l} -c_i(x_i(t)) + \sum_{j=1}^{2} a_{ij} f_j(y_j(t)) \\ + \sum_{j=1}^{2} b_{ij} f_j(y_j(t-\tau_j(t))) + I_i \end{array} \right], \\ \qquad i = 1, 2, \\ \dot{y}_j(t) = b_j(y_j(t)) \left[\begin{array}{l} -d_j(y_j(t)) + \sum_{i=1}^{2} m_{ji} g_i(x_i(t)) \\ + \sum_{i=1}^{2} n_{ji} g_i(x_i(t-\sigma_i(t))) + J_j \end{array} \right], \\ \qquad j = 1, 2, \end{cases} \tag{3}$$

where $a_i(x_i(t)) = 2 + \cos x_i(t)$, $c_i(x_i(t)) = 3x_i(t)$, $g_i(x_i) = 2x_i/(1 + x_i^2)$; $b_j(y_j(t)) = 2 + \sin y_j(t)$, $d_j(y_j(t)) = 3y_j(t)$, $f(y_j) = \frac{1}{2}(|y_j + 1| - |y_j - 1|)$. Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $M = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, $N = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $I = (1\ 2)^{\mathrm{T}}$, $J = (2\ 1)^{\mathrm{T}}$. So $\underline{a}_i = \underline{b}_j = 1$, $\bar{a}_i = \bar{b}_j = 3$, $\underline{c}_i = \bar{c}_i = \underline{d}_j = \bar{d}_j = 3$, $s_i = r_j = 1$, $M_1 = 15, M_2 = 18, N_1 = 21, N_2 = 9$. Since $f(\cdot), g(\cdot) \in B$, according to Theorem 1, the neural network model (3) has positive invariant and globally exponential attractive sets as follows:

$$\Omega_1 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^4 \,\middle|\, x_1^2(t) + x_2^2(t) + y_1^2(t) + y_2^2(t) \le 119 \right\},$$

$$\Omega_2 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in R^4 \,\middle|\, |x_1(t)| + |x_2(t)| + |y_1(t)| + |y_2(t)| \le 21 \right\}.$$

Meanwhile, let the initial conditions $x_1(t) = 0.7 + y_2(t), x_2(t) = 1 + y_2(t), y_1(t) = 1.2 + y_2(t), y_2(t) = 0.9 + 0.5 \sin 2t$, and the delays $\tau_1 = \tau_2 = 100 - \sin t$,

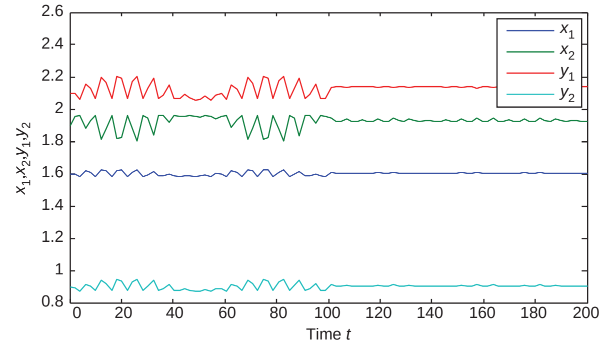

Figure 1. Time response of states $x_1(t), x_2(t), y_1(t)$ and $y_2(t)$ of Equation (3).

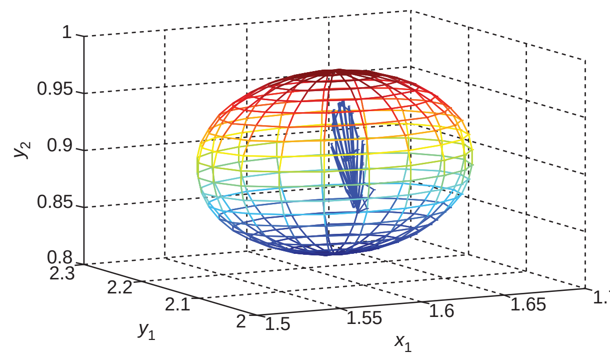

Figure 2. The ultimate bound of Equation (3) in coordinate system (x_1, y_1, y_2).

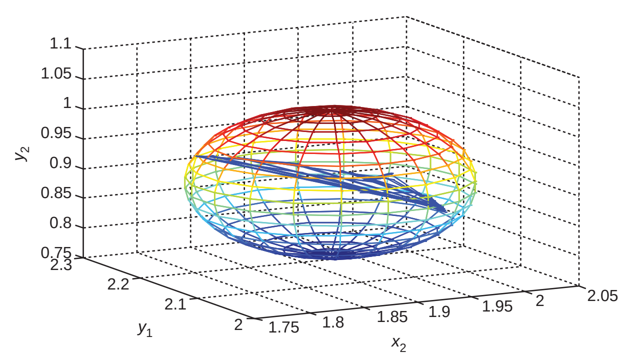

Figure 3. The ultimate bound of Equation (3) in coordinate system (x_2, y_1, y_2).

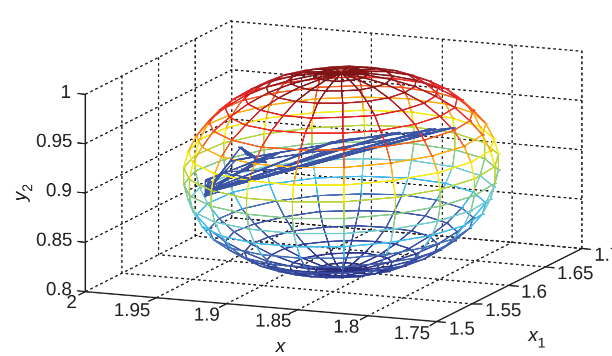

Figure 4. The ultimate bound of Equation (3) in coordinate system (x_1, x_2, y_2).

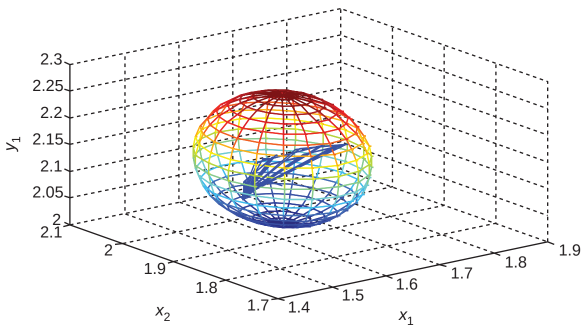

Figure 5. The ultimate bound of Equation (3) in coordinate system (x_1, x_2, y_1).

$\sigma_1 = \sigma_2 = 100 - \sin t$. Figure 1 shows time response of states $x_1(t)$, $x_2(t)$, $y_1(t)$ and $y_2(t)$. Figures 2–5 show the estimations of the ultimate bound of system (3) in the three-dimensional phase space, respectively.

5. Conclusions

Based on the Lyapunov stability theory and some inequalities, this paper has derived some sufficient delay-independent conditions of positive invariant set and globally exponential attractive set for the BAM-type Cohen–Grossberg neural networks with time-varying delays. According to the parameters, the detailed estimations for the positive invariant and globally attractive set of the BAM-type Cohen–Grossberg neural networks have been established without any hypothesis on the existence. Meanwhile, the results obtained in this paper are more general than that of the existing references (Tu et al., 2013) on the GEA set as special cases. Moreover, the proposed methods here can be also applied to nonlinear discrete-time systems with time-varying delays such as that in Dong, Wang, and Gao (2013) and Hu, Wang, Niu, and Stergioulas (2012). Finally, an illustrative example is shown to verify our results.

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Disclosure statement

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