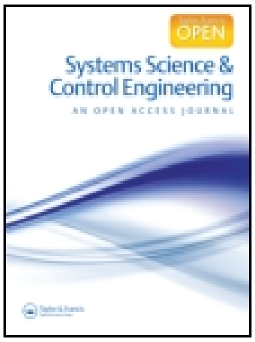

Systems Science & Control Engineering: An Open Access Journal

ISSN: (Print) 2164-2583 (Online) Journal homepage: https://www.tandfonline.com/loi/tssc20

Time-varying controller based on flatness for nonlinear anti-lock brake system

Mohamed Ben Abdallah, Mounir Ayadi, Frédéric Rotella & Mohamed Benrejeb

Systems Science & Control Engineering: An Open Access Journal, 2013
Vol. 1, No. 1, 91–104, http://dx.doi.org/10.1080/21642583.2013.857619

Time-varying controller based on flatness for nonlinear anti-lock brake system

Mohamed Ben Abdallah[a*], Mounir Ayadi[a], Frédéric Rotella[b] and Mohamed Benrejeb[a]

[a]*École Nationale d'Ingénieurs de Tunis, Université de Tunis El Manar, LR-11-ES18 Laboratoire Automatique, 1002, Tunis, Tunisia;* [b]*L'Institut National Polytechnique de Toulouse, École Nationale d'Ingénieurs de Tarbes, EA 1905 Laboratoire Genie de Production, 65016, Tarbes, France*

(*Received 28 May 2013; final version received 17 October 2013*)

It is shown that by the use of flatness the problem of pole placement, which consists in imposing closed-loop system dynamics, can be related to track desired trajectories in the finite-dimensional linear time-invariant case. Polynomial two-degree-of-freedom controller can then be designed with the use of an exact observer and without resolving the Bézout's equation. In this paper, an extension of these developments is proposed in the linear time-varying (LTV) framework. The proposed approach is illustrated with the control of nonlinear model of an anti-lock brake system. The time-varying controller obtained from the LTV model ensures the trajectory tracking of the nonlinear model.

Keywords: flatness; linear time-varying (LTV) systems; trajectory tracking; polynomial two-degrees-of-freedom controller

1. Introduction

In control theory, the study of linear time-varying (LTV) systems has been important since this situation is encountered not only when some parameters of the system vary with time, but also when the system to be controlled is nonlinear and the problem is approached by linearizing this system around a desired trajectory which leads to an LTV model.

For finite-dimensional and time-invariant linear systems, a well-known control design technique, named polynomial two-degree-of-freedom (2DOF) controllers (Aström & Wittenmark, 1997; Franklin, Powell, & Workman, 1998; Kučera, 1991), was introduced 50 years ago by Horowitz (1963). More details are given in the reference therein. This powerful method is based on pole placement and presents one drawback: it needs to know where to place all the poles of the closed-loop system at the outset.

Following Rotella, Carrillo, and Ayadi (2002), by the use of flatness design control principles, the problem of pole placement which consists in imposing closed-loop system dynamics can be related to track desired trajectories and a 2DOF controller is designed with very natural choices for high-level parameters design. In this design, we are led to a solution for the Bézout's equation which is independent of the closed-loop dynamics but depends only on the system model, and this solution is obtained without resolving Bézout's equation.

The 2DOF design controller problem is not easy to transcribe in the case of LTV systems due to the fact that the coefficients do not commute with the time derivative operator. Besides, the structure of the set of the poles of the closed-loop system is more complex. For this, the notions of poles of an LTV system, newly introduced in Marinescu and Bourlés (2009) along with a necessary and sufficient condition for exponential stability, are required to study the internal stability of the output closed-loop system.

In this case, the pole placement problem was solved recently by Marinescu (2010), who proposes some technical methods for the factorization of LTV transfer matrices. These key points lead to solve Bézout's equation written in the time-varying framework.

In order to overcome these two points, namely the choice of desired poles at the outset and the determination of solution for the Bézout's equation, we propose in this paper to extend the flatness-based control strategy developed in Rotella et al. (2002) to the case of time-varying systems. It will be seen that applying the guideline induced by a flatness-based control to an LTV system leads to express it in a natural 2DOF controller form.

The paper is organized as follows: Section 2 is devoted to showing a short survey on flatness. In Section 3, some background notions about single-input single-output (SISO) LTV systems are presented. In Section 4, we propose to design a polynomial controller based on flatness and exact observer of a state vector which is constituted by the flat output and its derivatives. In Section 5, the proposed strategy is illustrated with the control of the nonlinear anti-lock brake system (ABS).

*Corresponding author. Emails: mohamed.benabdallah@enit.rnu.tn, ben_abdallah_mohamed@yahoo.fr

2. Short survey on flatness

Flatness property, which was introduced by Fliess, Lévine, Martin, and Rouchon (1992), for continuous-time nonlinear systems, leads to interesting results for control design. The existence of a variable called a flat output permits to define all other system variables. The dynamic of such process can be then deduced without solving differential equations. Therefore, it is possible to express the state, as well as the input and the output of the system, as differential functions of the flat output (Fliess et al., 1992; Rotella & Zambettakis, 2007). Let us consider the nonlinear system described by the following differential equation:

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $x(t) \in \Re^n$ is the state vector and $u(t) \in \Re^m$ is the input vector. Roughly speaking, this system is called differentially flat if there exists a variable $z(t) \in \Re^m$ of the form:

$$z(t) = h(x(t), u(t), \dot{u}(t), \dots, u^{(r)}(t)) \tag{2}$$

such that the state and the input of the system are given by

$$x(t) = \mathbf{A}(z(t), \dot{z}(t), \dots, z^{(\alpha)}(t)), \tag{3}$$

$$u(t) = \mathbf{B}(z(t), \dot{z}(t), \dots, z^{(\alpha+1)}(t)), \tag{4}$$

where α is an integer. The variable $z(t)$ is called the flat output of the system or the endogenous variable. It makes possible to parameterize any variable of the system (Rotella & Zambettakis, 2007). The components of $z(t)$ must be differentially independent. The real output of the process is written as

$$y(t) = g(x(t), u(t)) \tag{5}$$

and from Equations (3) and (4), this output is written in function of the flat output as

$$y(t) = \mathbf{C}(z(t), \dot{z}(t), \dots, z^{(\sigma)}(t)), \tag{6}$$

where σ is an integer. In the linear case, the explicit expressions of the output $y(t)$ and the control $u(t)$ allow to relate the flat output to the partial state which was defined by Kailath (1980). The trajectories of the system are deduced from the definition of the flat output trajectory without integrating any differential equations. All these points, which have been formalized through the Lie–Bäcklund equivalence of systems in Fliess, Lévine, Martin, and Rouchon (1993, 1999), lead to propose a nonlinear feedback which ensures a stabilized tracking of a desired motion for the flat output. This methodology has been applied in many industrial processes as it was shown previously, for instance, on magnetics bearings (Lévine, Lottin, & Ponsart, 1996), chemical reactors (Rothfuss, Rudolph, & Zeitz, 1996), cranes or flight control (Lévine, 1999) or turning process (Rotella & Carrillo, 1998), among many other examples.

3. SISO LTV systems

Following Marinescu (2010), in the algebraic framework initiated by Malgrange (1962–1963) and popularized in systems theory by Fliess (1990) and related references, a linear system is a finitely presented module $\mathbf{M}$ over the ring $\mathbf{R} = \mathbf{K}[s]$ of differential operators in $s = \mathrm{d}/\mathrm{d}t$ with coefficients in an ordinary differential field $\mathbf{K}$ (i.e. a commutative field equipped with a unique derivative). If $\mathbf{K}$ does not exclusively contain constants (i.e. elements of derivative zero), $\mathbf{M}$ is an LTV system. In this paper, the following notations will be used: $u^{(n)}(t) = \mathrm{d}^n u(t)/\mathrm{d}t^n = s^n u(t)$.

When dealing with LTV systems, polynomials as function of s is *skew*, i.e. belong to the noncommutative ring $\mathbf{R} = \mathbf{K}[s]$ equipped with the commutation rule: $sa = as + \dot{a}$ (a is a time-varying function), which is the Leibniz rule of derivation of a product. Noting the integration operator by s^{-1} where:

$$s^{-1}h(t) = \int_{-\infty}^{t} h(\tau)\,\mathrm{d}\tau, \tag{7}$$

where $h(\tau) = 0$ for $(\tau \le \bar{\tau})$. This last hypothesis ensures commutativity between s and s^{-1}.

For finite-dimensional, several input–output descriptions have been introduced for LTV systems. Here, a time-varying linear system is described by the following state space model of dimension n:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \\ y(t) &= C(t)x(t). \end{aligned} \tag{8}$$

The matrices $A(t)$, $B(t)$ and $C(t)$ whose coefficients depend on the time are of dimensions $(n \times n)$, $(n \times 1)$ and $(1 \times n)$, respectively. Following Silverman (1966) and Silverman and Meadows (1967), the system (8) is uniformly controllable if there is a time interval $T = [t_1, t_2]$ such that the matrix:

$$\mathscr{K} = (K_0(t) \quad K_1(t) \quad \cdots \quad K_{n-1}(t))$$

has rank n for every t in T, with:

- $K_0(t) = B(t)$,
- for $i = 1$ to n: $K_i(t) = \dot{K}_{i-1}(t) - A(t)K_{i-1}(t)$.

If this condition is satisfied, the controllable form of (8) is given by

$$\begin{aligned} \dot{Z}(t) &= \bar{A}(t)Z(t) + \bar{B}(t)u(t), \\ y(t) &= \bar{C}(t)Z(t), \end{aligned} \tag{9}$$

with

$$\bar{A}(t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ -\psi_0(t) & -\psi_1(t) & \cdots & -\psi_{n-2}(t) & -\psi_{n-1}(t) \end{pmatrix},$$

$$\bar{B} = (0 \quad \cdots \quad 0 \quad 1)^{\mathrm{T}}, \quad \bar{C} = (\gamma_0(t) \quad \cdots \quad \gamma_{n-1}(t)).$$

To calculate the coefficients $\psi_i(t)$, $\eta(t)$ is first calculated using the following expression:

$$\eta(t) = (-1)^{n-1}\mathscr{K}^{-1}(t)K_n(t)$$

so we can deduce:

$$\Psi(t) = (\psi_0(t) \quad \psi_1(t) \quad \cdots \quad \psi_{n-1}(t))^{\mathrm{T}} = J_n[F_n(s)]^{-1}J_n\eta(t)$$

such that J_n is the $(n \times n)$ matrix which is given by

$$J_n = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & (-1)^{n-1} \end{pmatrix}$$

with $(J_n)^2 = I_n$.

The matrix $F_n(s)$ is given as follows:

$$F_n(s) = \begin{pmatrix} 1 & s & \cdots & s^{n-3} & s^{n-1} \\ 0 & 1 & \cdots & \cdots & (n-1)s^{n-2} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & \ddots & 1 & (n-1)s \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}. \quad (10)$$

Finally, this leads to:

$$\Psi(t) = J_n[F_n(s)]^{-1}J_n(-1)^{n-1}\mathscr{K}^{-1}(t)K_n(t).$$

To calculate $\bar{C}(t)$, we need to determine the change of variable $P(t)$, such that

$$Z(t) = P(t)x(t), \quad \bar{A}(t) = P(t)A(t)P^{-1}(t) + \dot{P}(t)P^{-1}(t),$$
$$\bar{C}(t) = C(t)P^{-1}(t) \quad \text{and} \quad \bar{B} = P(t)B.$$

This change of variable is given by

$$P(t) = \bar{\mathscr{K}}(t)\mathscr{K}^{-1}(t),$$

where $\bar{\mathscr{K}}(t)$ is the controllability matrix of the pair $(\bar{A}, \bar{B})$.

4. Controller flatness-based design for SISO LTV systems

4.1. *Flatness of SISO LTV systems*

Following Fliess, Lévine, Martin, and Rouchon (1995), a necessary and sufficient condition for the flatness of a linear system is its controllability. In this case, the first component of the Brunovsky-state from the controllable canonical form is considered as flat output. Let us consider the controllable state space equation (9) and let us denote by $z_i(t)$ the ith component of $Z(t)$:

$$\begin{aligned} \dot{z}_1(t) &= z_2(t) \\ &\vdots \\ \dot{z}_{n-1}(t) &= z_n(t), \\ \dot{z}_n(t) &= u(t) - \sum_{i=0}^{n-1}\psi_i(t)z_{i+1}(t), \\ y(t) &= \sum_{i=0}^{n-1}\gamma_i(t)z_{i+1}(t), \end{aligned} \quad (11)$$

which leads to:

$$\begin{aligned} u(t) &= z_1^{(n)}(t) + \sum_{i=0}^{n-1}\psi_i(t)z_1^{(i)}(t), \\ y(t) &= \sum_{i=0}^{n-1}\gamma_i(t)z_1^{(i)}(t). \end{aligned} \quad (12)$$

The variable $z_1(t)$, denoted as $z(t)$, can be considered for this system as a flat output (Fliess et al., 1995; Kailath, 1980). Then, the state vector of the controllable form $Z(t)$ is composed by the flat output and its derivatives.

4.2. *Tracking control and pole placement*

For a given planned trajectory of the flat output, $z_d(t)$, the control law based on flatness is as follows:

$$\begin{aligned} u(t) &= z_d^{(n)}(t) + \sum_{i=0}^{n-1}k_i(z_d^{(i)}(t) - z^{(i)}(t)) + \psi_i(t)z^{(i)}(t) \\ &= z_d^{(n)}(t) + \sum_{i=0}^{n-1}k_iz_d^{(i)}(t) + \sum_{i=0}^{n-1}(\psi_i(t) - k_i)z^{(i)}(t) \end{aligned} \quad (13)$$

and by introducing the polynomials:

$$K(s) = s^n + \sum_{i=0}^{n-1}k_is^i \quad (14)$$

where the k_i are chosen such that $K(s)$ is a Hurwitz polynomial. The control $u(t)$ can be written as

$$u(t) = K(s)z_d(t) + \sum_{i=0}^{n-1}(\psi_i(t) - k_i)z^{(i)}(t). \quad (15)$$

When this control is applied, the tracking error is verifying:

$$\lim_{t\to\infty}(z_d(t) - z(t)) = 0 \quad (16)$$

and the closed-loop dynamics are given by the roots of $K(s)$. This strategy differs from the usual pole placement for LTV systems obtained by a time-varying state feedback.

By denoting

$$\psi - k = \begin{pmatrix} \psi_0(t) - k_0 \\ \vdots \\ \psi_{n-1}(t) - k_{n-1} \end{pmatrix} \quad (17)$$

the previous control can be written as

$$u(t) = K(s)z_d(t) + (\psi - k)^{\mathrm{T}} Z(t), \quad (18)$$

where

$$Z(t) = \begin{pmatrix} z(t) \\ \dot{z}(t) \\ \vdots \\ z^{(n-1)}(t) \end{pmatrix} \quad (19)$$

is the state vector of the controllable form.

To implement the control (18), we need to estimate the vector $Z(t)$ with an observer. A full-order observer can be used, but in this solution, the difficulty appears in the choice of the observers' poles in the LTV framework. To overcome this point, an enlightening idea suggested in Fliess (2000) and applied in Marquez, Delaleau, and Fliess (2000) and Rotella et al. (2002) can be used. The realization of this controller, using the exact observer, will be the subject of the next part.

4.3. *The two-degree-of-freedom controller form*

Let us consider the model (9) where the first component of the state vector $Z(t)$ is the system flat output. By successive derivations of the output plant $y(t)$ until the order $(n-1)$, we obtain:

$$Y(t) = O(t)Z(t) + M(t)U(t), \quad (20)$$

where

- $Y(t) = (y(t) \ \cdots \ y^{(n-1)}(t))^{\mathrm{T}}$,
- $U(t) = (u(t) \ \cdots \ u^{(n-2)}(t))^{\mathrm{T}}$,
- $O(t)$ is the observability matrix of the pair $(\bar{A}(t), \bar{C}(t))$ and it is given by

  $$O(t) = (\bar{C}_0(t) \quad \bar{C}_1(t) \quad \cdots \quad \bar{C}_{n-1}(t))^{\mathrm{T}} \quad (21)$$

  such that:

  $$\bar{C}_0(t) = \bar{C}(t),$$

  $$\bar{C}_i(t) = \dot{\bar{C}}_{i-1}(t) + \bar{C}_{i-1}(t)\bar{A}(t) \quad \text{for } i = 1 \text{ to } n-1.$$

- The matrix $M(t)$ has the following expression:

$$M(t) = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ M_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & 0 & 0 \\ M_{n-2} & \cdots & M_1 & 0 \\ M_{n-1} & M_{n-1,2} & \cdots & M_1 \end{pmatrix} \quad (22)$$

with:

(1) $M_1(t) = \bar{C}_0(t)\bar{B}$,
(2) $M_i(t) = \dot{M}_{i-1}(t) + \bar{C}_{i-1}(t)\bar{B}$, for $i = 2$ to $n-1$,
(3) $M_{n-1,2}(t) = M_{n-2}(t) + \sum_{i=1}^{n-3} M_i^{(n-2-i)}(t)$,
(4) $M_{n-1,3}(t) = M_{n-3}(t) + \sum_{i=1}^{n-4} (n-i-2) M_i^{(n-3-i)}$,
(5) for $k > 3$, $M_{n-1,k}$ can be deducted by observing Equations (20) and (21).

As the pair $(\bar{A}(t), \bar{C}(t))$ is observable, the matrix $O(t)$ is of rank n and the state vector can be written as

$$Z(t) = O^{-1}(t)Y(t) - O^{-1}(t)M(t)U(t). \quad (23)$$

Taking into account the state-space equation (9) and avoiding variable derivations, we obtain

$$Z(t) = s^{-1}(\bar{A}(t)Z(t)) + s^{-1}(\bar{B}u(t)). \quad (24)$$

By rewriting this equation to the order $(n-1)$, Equation (24) becomes:

$$\begin{aligned} Z(t) &= s^{-1}(\bar{A}(t)s^{-1}(\bar{A}(t)\cdots s^{-1}(\bar{A}(t)Z(t))) \\ &\quad + s^{-1}(\bar{A}(t)\cdots s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t))) \\ &\quad + s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t)) + \bar{B}s^{-1}u(t). \end{aligned} \quad (25)$$

If the term $Z(t)$ is replaced in this equation by the one given in Equation (23), we obtain:

$$\begin{aligned} Z(t) &= s^{-1}(\bar{A}(t)\cdots s^{-1}(\bar{A}(t)O^{-1}(t)Y(t) \\ &\quad - \bar{A}(t)O^{-1}(t)M(t)U(t))) \\ &\quad + s^{-1}(\bar{A}(t)\cdots s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t))) \\ &\quad + s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t)) + \bar{B}s^{-1}u(t). \end{aligned} \quad (26)$$

To eliminate the terms containing the derivatives of the plant output $y(t)$ in $Y(t)$, we proceed by using successive integrations by parts leading to the following expression of the state vector:

$$\begin{aligned} Z(t) &= s^{-n+1}(\Theta_1(t)y(t)) + \cdots + s^{-1}(\Theta_{n-1}(t)y(t)) \\ &\quad + (\Theta_n(t)y(t)) + s^{-n+1}(\Delta_1(t)u(t)) + \cdots \\ &\quad + s^{-1}(\Delta_{n-1}(t)u(t)) + s^{-1}(\bar{A}(t)\cdots \\ &\quad \times s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t))) \\ &\quad + \cdots + s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t)) + \bar{B}s^{-1}u(t), \end{aligned} \quad (27)$$

where $\Theta_j(t) = (\theta_{1j}(t) \ \cdots \ \theta_{nj}(t))^{\mathrm{T}}$ and $\Delta_j(t) = (\delta_{1j}(t) \ \cdots \ \delta_{nj}(t))^{\mathrm{T}}$. The components $\theta_{ij}(t)$ and $\delta_{ij}(t)$ are function of the parameters $\psi_i(t)$ and their derivatives. The control law (18) becomes

$$u(t) = K(s)z_d(t) - S(s^{-1}, y(t)) - R^*(s^{-1}, u(t)), \quad (28)$$

where:

$$S(s^{-1}, y(t)) = (k - \psi)(s^{-n+1}(\Theta_1(t)y(t)) + \cdots + \Theta_n(t)y(t), \quad (29)$$

$$R^*(s^{-1}, u(t)) = (k - \psi)(s^{-n+1}(\Delta_1(t)u(t)) + \cdots + s^{-1}(\Delta_{n-1}(t)u(t)) + (k - \psi)(s^{-1}\bar{A}(t)\cdots s^{-1}(\bar{A}(t)\bar{B}s^{-1}u(t)) + \cdots + \bar{B}s^{-1}u(t)). \quad (30)$$

By denoting $R(s^{-1}, u(t)) = u(t) + R^*(s^{-1}, u(t))$, this control can be written in the 2DOF controller form as follows:

$$R(s^{-1}, u(t)) = K(s)z_d(t) - S(s^{-1}, y(t)). \quad (31)$$

The proposed control design can be seen as a 2DOF controller without resolving Bézout's equation. Now the design is focused on the choice of the trajectory of $z_d(t)$ to follow and the tracking dynamics with $K(s)$.

This regulator-observer permits to the system output to track a desired trajectory without using observer dynamics then the problem of pole placement, which consists in imposing closed-loop system dynamics, can be related to track desired trajectories. This design leads to a solution of Bézout's equation which is independent of the closed-loop dynamics but depends only on the system model.

Some remarks for the design:

(i) The obtained form of the control cannot be seen as a classical polynomial controller as in the case of linear-invariant parameter system (Rotella et al., 2002). The obtained result recovers the one in Rotella et al. (2002) when time-varying system is reduced to a time-invariant one.
(ii) In this design, it is difficult to give the expression of $R(s^{-1}, u(t))$ and $S(s^{-1}, y(t))$ in terms of the proper operator s^{-1}, then we need a numerical algorithm to determine these expressions for every t in T.
(iii) In order to reject a static perturbation, an integral action must be added to the model.
(iv) Here, we propose the case where we can calculate the controls and the states, which correspond with a trajectory for the flat output, only by using a numerical algorithm for local resolution of nonlinear equations. In this case, it is necessary to linearize the nonlinear model around the desired trajectory.
(v) The flatness-based nonlinear control has the advantage to overcome the problems generated by non-stable-zero dynamics (Isidori, 1989; Nijmeijer & Schaft, 1990). However, the information needed to determine this control may be obtained through nonlinear observers; this problem is avoided by the proposed 2DOF time-varying controller using the linearization around desired trajectories obtained from the flat output.

5. Application to ABS in vehicle

As an illustrative example of the proposed strategy, the control of the wheel slip in an ABS is studied. The considered process is an ABS, used to control the slip of each wheel of a vehicle to prevent it from locking so that a high friction is achieved and steerability is maintained (Johansen, Petersen, Kalkkuhl, & Ludemann, 2003).

5.1. Literature review

Several solutions for ABS based on different control algorithms have been proposed. A sliding mode approach in Drakunov, Özguñer, Dix, and Ashrafi (1995) applies a search for the optimum brake torque. This approach requires the tyre force; hence, a sliding observer is used to estimate it. The approach is tested in a simplified simulation environment. Another sliding mode approach is proposed in Unsal and Kachroo (1999). In this approach, the observability of the system is investigated. An extended Kalman filter and a sliding mode observer are compared via simulations. Another theoretical approach is presented by Freeman (1995). Freeman designs an adaptive Lyapunov-based nonlinear wheel slip controller, this controller has been extended in Yu (1997) by introducing speed dependence of the Lyapunov function. Neither of these two latter approaches has been tested in simulation or in an ABS equipment used in the experimental laboratory (Precup et al., 2011). The design of linear proportional-integral, gain-scheduling, and fuzzy control based on linearized ABS models is discussed in Precup et al. (2011). A robust proportional-integral-derivative (PID) controller based on loop-shaping and a nonlinear PID, where the nonlinear function gives a low/high gain for large/small errors respectively, is proposed in Jiang (2000). Other PID-type approaches to wheel slip control are considered in Jun (1998) and Solyom and Rantzer (2002). Also, there are several intelligent control schemes including fuzzy logic control, adaptive control, and neural network approach (Layne, Passino, & Urkovich, 1993; Lee & Zak, 2001; Mauer, 1995; Will, Hui, & Zak, 1998).

The main objective of this work is to design a control system which ensures the prevention of wheel-lock while braking and maintaining of the wheel slip the nearest possible to 0. In the literature, sliding mode control (SMC) is a preferable option to regulate wheel slip (Mitić et al., 2013), as it guarantees the robustness of system for changing working conditions. The main idea behind this control scheme is to restrict the motion of the system in a plane

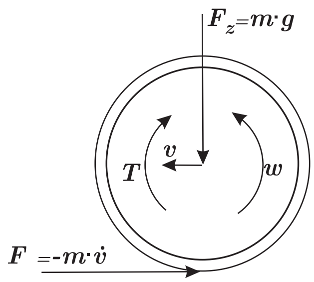

Figure 1. Quarter car forces and torques.

Table 1. Parameter sets for friction coefficient characteristics (Kiencke & Nielsen, 2005).

	c_1	c_2	c_3
Asphalt, dry	1.2801	23.99	0.52
Asphalt, wet	0.857	33.822	0.347
Concrete, dry	1.1973	25.168	0.5373
Cobblestones, dry	1.3713	6.4565	0.6691
Concrete, wet	0.4004	33.7080	0.1204
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0

referred to as "sliding surface," where the predefined function of error is zero. Sliding mode control is a nonlinear control method that alters the dynamics of a nonlinear system by the application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behaviour. In the case of SMC strategy, the controller lets to reach the desired slip and to continue to stay on the sliding surface. The wheel is not locked and we have a robust tracking of the desired wheel slip. The combination of SMC and fuzzy control (FC) is elaborated in Mitić, Antić, Perić, Milojković, and Nikolić (2012). A comparative analysis of several continuous-time SMCs of ABS is presented in Antić, Nikolić, Mitić, Milojković, and Perić (2010), both with a brief overview of existing SMC concepts in continuous-time domain.

5.2. *ABS model*

The problem of wheel slip control is better explained by looking at a quarter car model (see Figure 1). A mathematical model of the wheel slip dynamics is given by Freeman (1995), Drakunov et al. (1995) and Johansen et al. (2003):

$$\dot{\lambda}(t) = -\frac{1}{v(t)}\left[\frac{1}{m}(1-\lambda(t)) + \frac{r^2}{J}\right]F(\lambda) + \frac{1}{v(t)}\frac{r}{J}T(t),$$
$$\dot{v}(t) = -\frac{1}{m}F(\lambda), \tag{32}$$

where $\omega(t)$ is the angular speed of the wheel (rad/s), $v(t)$ the horizontal speed (m/s), $T(t)$ the brake-acceleration torque (N m), m the mass of the quarter car (450 kg), r the wheel radius (0.32 m), J the wheel inertia (1 kg m^2) and g the acceleration of gravity (9.81 m/s^2) and $\lambda(t)$ is the wheel slip given by

$$\lambda(t) = \frac{v(t) - r\omega(t)}{v(t)}. \tag{33}$$

The input signal $T(t)$ is a brake-acceleration torque applied to the wheel, it is expressed in (N m), and the output is the vehicle speed $v(t)$. The longitudinal slip $\lambda(t)$ is defined by the normalized difference between $v(t)$ and the speed of the wheel perimeter $\omega(t)r$. $F(\lambda)$ is the friction force, which depends on the normal force, steering angle, road surface, tyre characteristics and velocity of the car. The friction or adhesion coefficient $\mu(\lambda)$ is defined as the ratio of the frictional force acting in the wheel plane $F(\lambda)$ and the wheel ground contact force F_Z:

$$\mu(\lambda) = \frac{F(\lambda)}{F_Z}. \tag{34}$$

The calculation of friction force can be carried out using the Burckhardt method (Kiencke & Nielsen, 2005):

$$\mu(\lambda) = c_1 \cdot (1 - e^{-c_2\cdot\lambda(t)}) - c_3\lambda(t). \tag{35}$$

The parameters c_1, c_2 and c_3 are given for various types of road surfaces in Table 1.

In the case of asphalt and dry road, the friction force is given by

$$F(\lambda) = mg[1.28 \times (1 - \exp(-24\lambda(t))) - 0.52\lambda(t)]. \tag{36}$$

Figure 2 shows the friction force as a function of the wheel slip.

From the second equation of (32), we obtain:

$$\omega(t) = \left(\frac{v(t)}{r}\right)(1 + F^{-1}(m\dot{v}(t))), \tag{37}$$

where $F^{-1}()$ is the functional inverse of the friction force. Then, the first equation of (32) yields T as a function of $v(t)$, $\dot{v}(t)$ and $\ddot{v}(t)$. So the system (32) is flat with $v(t)$ which is a flat output of the nonlinear system. To design a control law which maintains the wheel slip the nearest possible to 0, we perform, in the next development, an approximation to a friction force $F(\lambda)$ by applying the Taylor series for this function at $\lambda = 0$ to obtain:

$$F(\lambda) = mg\left(1.28 \times \left(\frac{24\lambda}{1!} - \frac{(24\lambda)^2}{2!} - \frac{(-24\lambda)^3}{3!} + \cdots\right)\right.$$
$$\left. -0.52\lambda\right). \tag{38}$$

With a third-order approximation at 0 and due to the odd function of $\lambda(t)$, the expression of the friction force

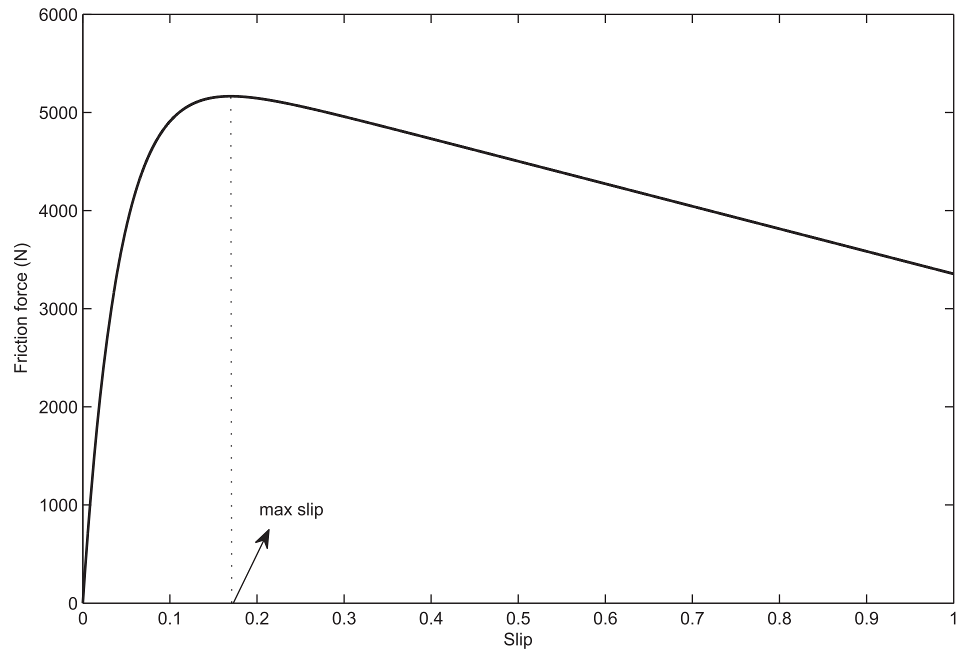

Figure 2. Friction force.

becomes:

$$F(\lambda) = a_1\lambda(t) + a_3\lambda^3(t), \quad (39)$$

where $a_1 = 30.2 \times mg$ and $a_3 = 2949.1 \times mg$. The equation of the system (32) becomes

$$\begin{aligned}\dot{\lambda}(t) &= -\frac{1}{v(t)}\left[\frac{1}{m}(1-\lambda(t)) + \frac{r^2}{J}\right](a_1\lambda(t) + a_3\lambda^3(t)) \\ &\quad + \frac{1}{v(t)}\frac{r}{J}T(t), \\ \dot{v}(t) &= -\frac{a_1\lambda(t) + a_3\lambda^3(t)}{m}.\end{aligned} \quad (40)$$

By analysing Equation (40), we remark that the input and the output of the system are function of a finite number of derivatives of the horizontal speed $v(t)$. By denoting $z(t) = v(t)$, we obtain:

$$\begin{aligned}\lambda(t) &= F^{-1}(m\dot{v}), \\ T(t) &= \frac{J}{r}\left(\dot{\lambda}(t)v(t) + \left[\frac{1}{m}(1-\lambda(t)) + \frac{r^2}{J}\right]\right. \\ &\quad \left.\times (a_1\lambda(t) + a_3\lambda^3(t))\right),\end{aligned} \quad (41)$$

where

$$\begin{aligned}&F^{-1}(m\dot{v}) \\ &= \left(\left(\frac{25m^2\dot{v}^2}{869719081g^2} + \frac{27543608}{692519906279817}\right)^{1/2}\right. \\ &\quad \left. - \frac{5m\dot{v}}{29491g}\right)^{1/3} - 302\Bigg/\left(88473\left(\left(\frac{25m^2\dot{v}^2}{869719081g^2}\right.\right.\right. \\ &\quad \left.\left.\left. + \frac{27543608}{692519906279817}\right)^{1/2} - \frac{5m\dot{v}}{29491g}\right)^{1/3}\right).\end{aligned} \quad (42)$$

Generally, for the nonlinear model (32) we must have $\lambda(t) < \lambda_{max}$, then the vehicle speed is a flat output of the considered nonlinear model. In the case of an asphalt and dry road, λ_{max} is equal to 0.17.

For the considered system, a desired trajectory $(T_d(t), \lambda_d(t), v_d(t))$ is defined and the following variables are given:

$$\begin{aligned}\delta T(t) &= T_d(t) - T(t), \\ \delta\lambda(t) &= \lambda_d(t) - \lambda(t), \\ \delta v(t) &= v_d(t) - v(t), \\ \delta\dot{\lambda}(t) &= \dot{\lambda}_d(t) - \dot{\lambda}(t), \\ \delta\dot{v}(t) &= \dot{v}_d(t) - \dot{v}(t).\end{aligned}$$

For the vehicle speed, let us define a desired trajectory sufficiently differentiable which takes the system from an initial state to an equilibrium final state:

$$v_d(t) = -11.8505 \cdot \left(\frac{-\sin(\pi t)}{\pi} + t\right) + v_{d0}, \quad (43)$$

where $v_{d0} = 23.74$ m/s is the initial condition for the horizontal speed. The trajectories for the brake-acceleration torque $T_d(t)$ and the tyre slip $\lambda_d(t)$ are deduced from Equation (41).

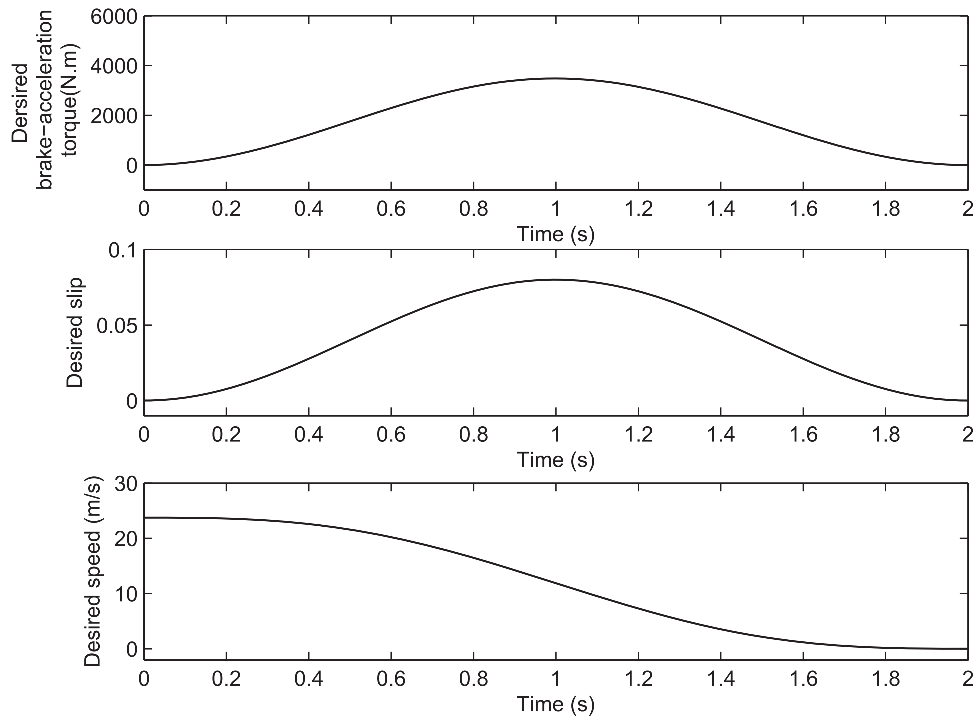

Figure 3. Desired trajectories for the input, the slip and the flat output of the nonlinear system.

Figure 3 shows the desired trajectories for the flat output, the input and tyre slip of the nonlinear system. The linearized model of Equation (40) around this desired trajectory is given by

$$\begin{aligned}\delta\dot{\lambda} &= \frac{-\dot{\lambda}_d}{v_d}\delta v - \left((a_1 + 3a_3\lambda_d^2)\left(\frac{1-\lambda_d}{m} + \frac{r^2}{J}\right)\right.\\&\qquad \left. - \frac{a_1\lambda_d + a_3\lambda_d^3}{m}\right)\frac{\delta\lambda}{v_d} + \frac{r}{Jv_d}\delta T,\\ \delta\dot{v} &= -\left((a_1 + 3a_3\lambda_d^2)\frac{\delta\lambda}{m}\right).\end{aligned} \tag{44}$$

To design the closed-loop control which allows to track variable reference trajectories, the following state space representation of the system is considered:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)\delta T(t),\\ \delta\lambda &= C(t)x(t),\end{aligned} \tag{45}$$

with $x(t) = (\delta\lambda\ \delta v)^{\mathrm{T}}$ is the state vector such that

$$\begin{aligned}A(t) &= \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t)\end{pmatrix},\\ B(t) &= \begin{pmatrix} \frac{r}{Jv_d} \\ 0 \end{pmatrix},\\ C(t) &= (0 \quad 1),\end{aligned} \tag{46}$$

where

$$\begin{aligned}A_{11}(t) &= -\frac{1}{v_d}\left((a_1 + 3a_3\lambda_d^2)\left(\frac{1-\lambda_d}{m} + \frac{r^2}{J}\right)\right.\\&\qquad\left. - \frac{a_1\lambda_d + a_3\lambda_d^3}{m}\right),\\ A_{12}(t) &= \frac{-\dot{\lambda}_d}{v_d},\\ A_{21}(t) &= \frac{-(a_1 + 3a_3\lambda_d^2)}{m},\\ A_{22}(t) &= 0.\end{aligned}$$

5.3. *Controller design*

For the model equation (44), it can be seen that δv is a flat output of the linearized system.

The linearization around a reference trajectory leads to an LTV system and its controllability matrix is given by

$$\mathscr{K}(t) = \begin{pmatrix} \mathscr{K}_{11}(t) & \mathscr{K}_{12}(t) \\ \mathscr{K}_{21}(t)\ 0 & \mathscr{K}_{22}(t)\end{pmatrix}, \tag{47}$$

where

$$\begin{aligned}\mathscr{K}_{11}(t) &= \frac{r}{Jv_d},\\ \mathscr{K}_{12}(t) &= \frac{r}{Jv_d^2}\left((a_1 + 3a_3\lambda_d^2)\left(\frac{1-\lambda_d}{m} + \frac{r^2}{J}\right)\right.\\&\qquad\left. - \frac{a_1\lambda_d + a_3\lambda_d^3}{m} - \dot{v}_d\right),\end{aligned}$$

$$\mathscr{K}_{21}(t) = 0,$$

$$\mathscr{K}_{22}(t) = \frac{r(a_1 + 3a_3\lambda_d^2)}{mJv_d}.$$

The controllability matrix $\mathscr{K}(t)$ has rank 2 because

$$\frac{r^2(a_1 + 3a_3\lambda_d^2)}{m(Jv_d)^2} \neq 0 \quad \forall t \geq 0.$$

Then, the system (45) is controllable and following Rotella and Zambettakis (2007), the time-varying linearized system (45) is flat. The observability matrix of the pair $(A(t), C(t))$ is given by

$$O_{(A(t),C(t))} = \begin{pmatrix} 0 & 1 \\ \dfrac{-(a_1 + 3a_3\lambda_d^2)}{m} & 0 \end{pmatrix},$$

which has rank 2 $\forall t \geq 0$. The system is then observable and its controllable canonical form is obtained by applying the algorithm presented in Section 2:

$$\begin{aligned} \delta\dot{Z}(t) &= \bar{A}(t)\delta Z(t) + \bar{B}\delta T(t), \\ \delta\lambda(t) &= \bar{C}\delta Z(t), \end{aligned} \tag{48}$$

with

$$\bar{A}(t) = \begin{pmatrix} 0 & 1 \\ -\psi_0(t) & -\psi_1(t) \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$\delta Z(t) = P(t)x(t)$ and $\delta Z(t) = (\delta z(t)\ \delta\dot{z}(t))^{\mathrm{T}}$. The previous control law (18) can be written as

$$\begin{aligned} T(t) &= \ddot{z}^d(t) + k_1(\dot{z}^d(t) - \dot{z}(t)) + k_0(z^d(t) - z(t)) \\ &\quad + \Psi_1(t)\dot{z}(t) + \Psi_0(t)z(t). \end{aligned} \tag{49}$$

This equation is transformed as follows:

$$T(t) = T_d(t) + (k_1 - \Psi_1(t))\delta\dot{z}(t) + (k_0 - \Psi_0(t))\delta z(t),$$

which leads to:

$$\delta T(t) = \Lambda(t)\delta Z(t), \tag{50}$$

with $\Lambda(t) = [(\Psi_0(t) - k_0)\ (\Psi_1(t) - k_1)]$.

From Equation (20), we deduce:

$$\begin{pmatrix} \delta v(t) \\ \delta\dot{v}(t) \end{pmatrix} = \begin{pmatrix} \bar{C}(t) \\ \dot{\bar{C}}(t) + \bar{C}(t)\bar{A}(t) \end{pmatrix} \delta Z(t) + \begin{pmatrix} 0 \\ \bar{C}(t)\bar{B} \end{pmatrix} \delta T(t), \tag{51}$$

which can be transformed in the following form:

$$\delta Y(t) = O(t)\delta Z(t) + M(t)\delta T(t), \tag{52}$$

with:

$$\delta Y(t) = (\delta v(t) \quad \delta\dot{v}(t))^{\mathrm{T}},$$

$$O(t) = \begin{pmatrix} \bar{C}(t) \\ \dot{\bar{C}}(t) + \bar{C}(t)\bar{A}(t) \end{pmatrix},$$

$$M(t) = \begin{pmatrix} 0 \\ \bar{C}(t)\bar{B} \end{pmatrix}.$$

Equation (48) can be written as

$$\delta Z(t) = s^{-1}[\bar{A}(t)\delta Z(t) + \bar{B}\delta T(t)]. \tag{53}$$

By replacing the expression of $\delta Z(t)$, deduced from Equation (52), in the right side of Equation (53), we obtain

$$\begin{aligned} \delta Z(t) &= s^{-1}[\bar{A}(t)O^{-1}(t)\delta Y(t)] \\ &\quad - s^{-1}[\bar{A}(t)O^{-1}(t)M(t)\delta T(t)] + \bar{B}s^{-1}\delta T(t), \end{aligned} \tag{54}$$

with:

$$\bar{A}(t)O^{-1}(t) = \begin{pmatrix} \alpha_1(t) & \alpha_2(t) \\ \alpha_3(t) & \alpha_4(t) \end{pmatrix}, \tag{55}$$

$$\bar{A}(t)O^{-1}(t)M(t) = \begin{pmatrix} \beta_1(t) \\ \beta_2(t) \end{pmatrix}. \tag{56}$$

By using integration by parts, it leads to the following expression of the state vector:

$$\begin{aligned} \delta Z(t) &= \begin{pmatrix} \alpha_2(t) \\ \alpha_4(t) \end{pmatrix} \delta v(t) + s^{-1}\left[\begin{pmatrix} \alpha_1(t) - \dot{\alpha}_2(t) \\ \alpha_3(t) - \dot{\alpha}_4(t) \end{pmatrix} \delta v(t)\right] \\ &\quad + s^{-1}\left[\begin{pmatrix} -\beta_1 \\ 1 - \beta_2 \end{pmatrix} \delta T(t)\right]. \end{aligned} \tag{57}$$

By rewriting the expression (50), the following form is obtained:

$$\begin{aligned} &\delta T(t) \\ &= \Lambda(t) \times \left[\begin{pmatrix} \alpha_2(t) \\ \alpha_4(t) \end{pmatrix} \delta v(t)\right. \\ &\quad + s^{-1}\left[\begin{pmatrix} \alpha_1(t) - \dot{\alpha}_2(t) \\ \alpha_3(t) - \dot{\alpha}_4(t) \end{pmatrix} \delta v(t)\right] \\ &\quad \left. + s^{-1}\left[\begin{pmatrix} -\beta_1 \\ 1 - \beta_2 \end{pmatrix} \delta T(t)\right]\right] \end{aligned} \tag{58}$$

and then:

$$\delta T(t) = S(s^{-1}, \delta v(t)) + R(s^{-1}, \delta T(t)) \tag{59}$$

with:

$$\begin{aligned} S(s^{-1}, \delta v(t)) &= \Lambda(t) \times \left[\begin{pmatrix} \alpha_2(t) \\ \alpha_4(t) \end{pmatrix} \delta v(t)\right. \\ &\quad \left. + s^{-1}\left[\begin{pmatrix} \alpha_1(t) - \dot{\alpha}_2(t) \\ \alpha_3(t) - \dot{\alpha}_4(t) \end{pmatrix} \delta v(t)\right]\right], \end{aligned} \tag{60}$$

$$R(s^{-1}, \delta T(t)) = \Lambda(t) \times s^{-1}\left[\begin{pmatrix} -\beta_1 \\ 1 - \beta_2 \end{pmatrix} \delta T(t)\right]. \tag{61}$$

Figure 4 illustrates the structure of the proposed method based on the flatness property with the use of an exact observer. The approach is tested in a simplified simulation environment. For the numerical simulations, the tracking model is set to be a second-order model with a time response

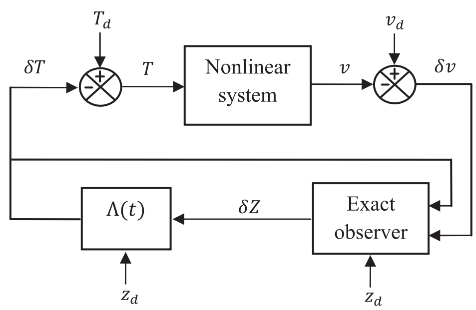

Figure 4. Tracking trajectories via state feedback controller using an exact observer.

of 0.005 s. Then, the polynomial $K(s)$ in Equation (14) is given by

$$K(s) = s^2 + 840s + 360000$$

Figure 5 shows that the trajectories of the nonlinear system follow the desired trajectories with good performances. The results show also that the control law obtained by the application of the flatness-based controller, allows to obtain high performances in terms of path tracking with an error which tends asymptotically to zero. These results point out the effectiveness of the use of the flatness-based approach for the LTV systems in a path tracking context.

This method with a direct calculation of the state vector which contains the flat output and its derivatives leads to a control law which can be seen as a 2DOF controller but without resolving Bézout's equation. This regulator-observer permits the output of the system to track a desired trajectory without using observer dynamics.

The robustness of the control scheme is investigated for different surface conditions. The parameters in Equation (35) are modified in accordance with surfaces, so that the parameters are changed in the model but not in the controller, the performances in tracking of speed still being correct (see Figure 6). We remark bad performances in terms of tracking of the wheel slip (see Figure 7) because in this design strategy, following Equation (13), the flat output of the system (speed) tracks the desired flat output (desired speed). Figure 8 shows simulation results when there is a very quick change from dry asphalt to wet road at a time of 1 s. Regarding the simulation results, it can be inferred that the speed can track the reference speed satisfactorily. Hence, a perfect tracking of the reference trajectory has been achieved. Furthermore, the control proposed in this paper leads to a robust tracking of the speed, which is the flat output, but not of the wheel slip. In addition, for various surface conditions, we remark an error on the behaviour of the wheel slip (see Figure 7) due to the bonded parametric variation.

In the following, the proposed strategy is compared with a classical observer-controller approach. The flatness-based control is designed by using a reduced order observer with a constant estimator error gain. The realization of this controller will be done with the calculation of the observable

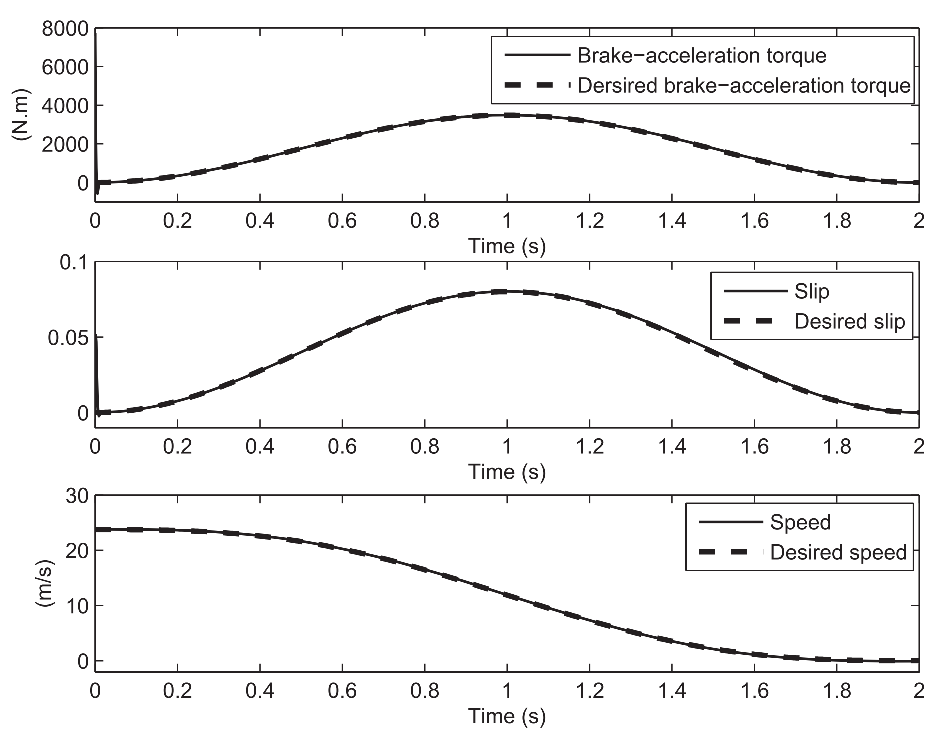

Figure 5. Input $T(t)$, slip $\lambda(t)$ and output $v(t)$ trajectories of the nonlinear system and the desired trajectories.

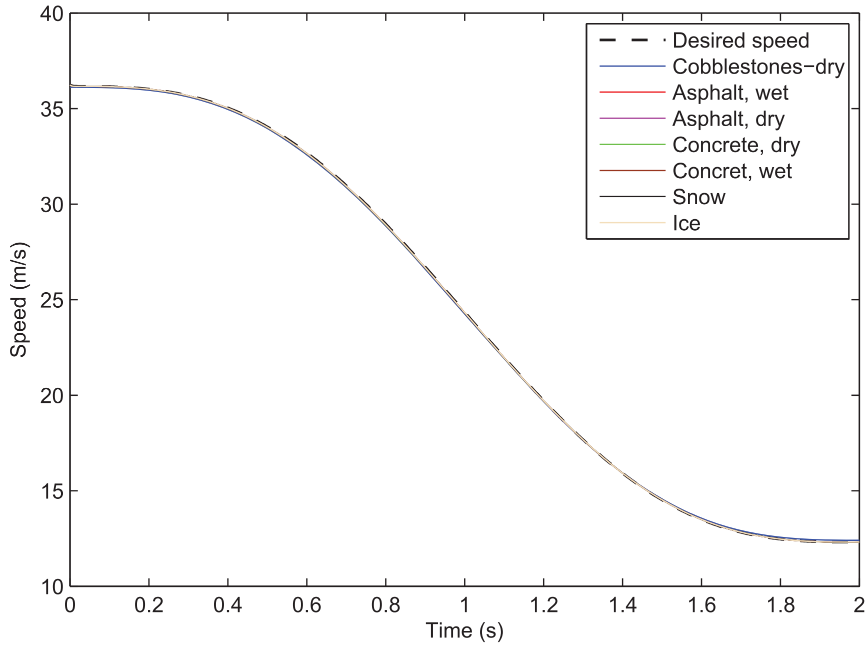

Figure 6. Output $v(t)$ trajectories of the nonlinear system on various surfaces and the desired trajectories.

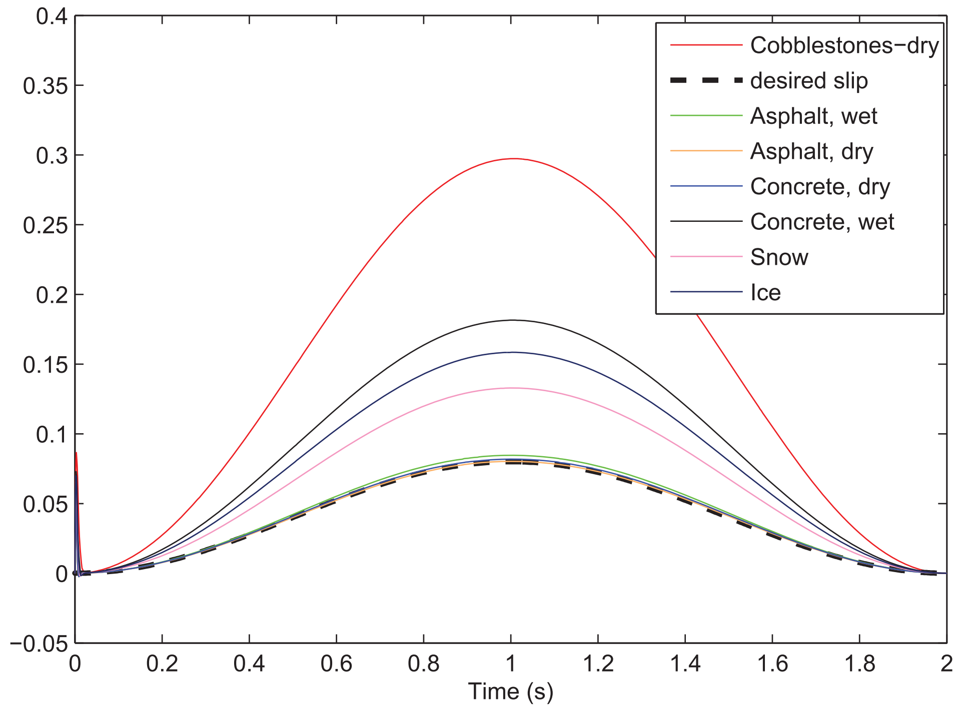

Figure 7. Slip $\lambda(t)$ trajectories of the nonlinear system on various surfaces and the desired trajectories.

form of the state equation (45), which is given by

$$\dot{x}_o(t) = \begin{pmatrix} 0 & -\tau_0(t) \\ 1 & -\tau_1(t) \end{pmatrix} x_o(t) + \begin{pmatrix} -\dfrac{r(a_1 + 3a_3\lambda_d^2)}{Jmv_d} \\ 0 \end{pmatrix} \delta T(t),$$
$$\delta v(t) = (0 \quad 1) x_o(t), \tag{62}$$

where

$$\tau_0(t) = \frac{(a_1 + 3a_3\lambda_d^2)}{mv_d^2} \left(\dot{\lambda}_d v_d + \left(1 - \lambda_d + \frac{mr^2}{J} - \left(\frac{a_1\lambda_d + a_3\lambda_d^3}{a_1 + 3a_3\lambda_d^2} \right) \right) \dot{v}_d \right), \tag{63}$$

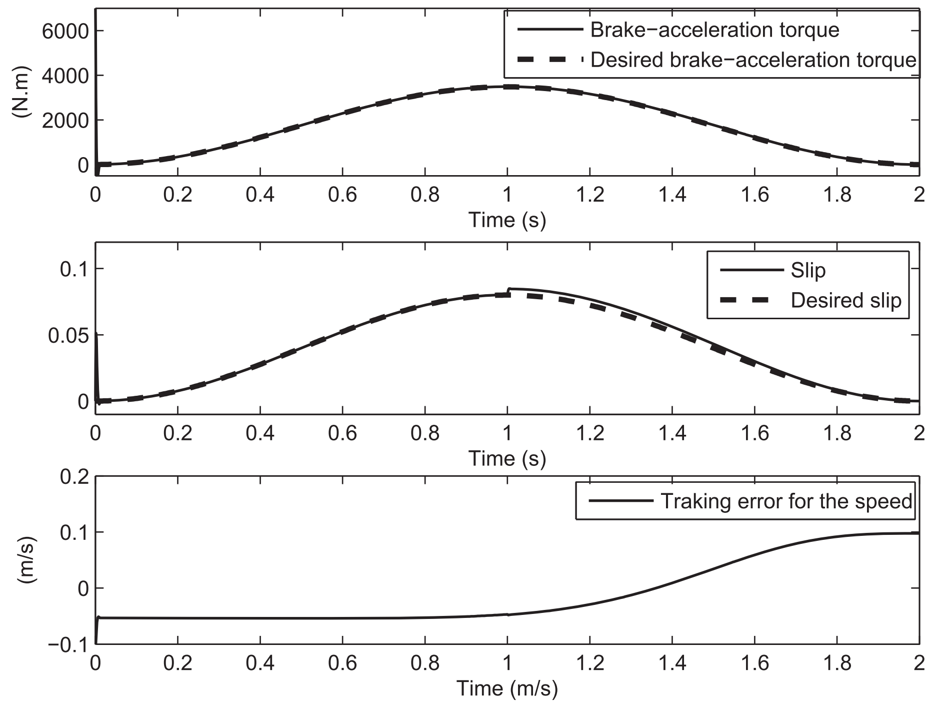

Figure 8. Tracking error for the speed, input $T(t)$ and slip $\lambda(t)$ trajectories of the nonlinear system and the desired trajectories when there is a change from dry asphalt to wet road at the time 1 s.

$$\tau_1(t) = \frac{(a_1 + 3a_3\lambda_d^2)}{mv_d}\left(1 - \lambda_d + \frac{mr^2}{J} - \frac{a_1\lambda_d + a_3\lambda_d^3}{a_1 + 3a_3\lambda_d^2}\right). \tag{64}$$

Following Rotella (2003), the estimated state vector of the observable form is given by

$$\hat{x}_o(t) = \begin{pmatrix} \zeta(t) + \lambda_0\delta v(t) \\ \delta v(t) \end{pmatrix}, \tag{65}$$

$$\dot{\zeta}(t) = \lambda_0\zeta(t) + \left(-\frac{r(a_1 + 3a_3\lambda_d^2)}{Jmv_d}\right)\delta T(t) + (\tau_1(t) - \lambda_0)\lambda_0. \tag{66}$$

By replacing $\delta\hat{x}_o(t)$ into the control law (50), we obtain

$$\delta T(t) = \Lambda(t)P(t)x_o(t), \tag{67}$$

where $P(t)$ is the change of variable from the observable form to the controllable form. With a constant dynamic observer $\lambda_0 = 10$ and by considering the tracking model set to be a second-order model with a time response of 0.005 s, the results are obtained in Figure 9.

These results point out the effectiveness of the use of the flatness-based approach for the LTV systems in a path tracking context. We have underlined the advantage of the use of a reduced order observer in order to design a flatness-based control for tracking a desired trajectory in the case of LTV systems. This advantage consists in the calculation of the estimator error gain which is found constant.

By observing the tracking errors of the various designs, the controller with a direct calculation of the state vector which contains the flat output and its derivatives was superior to the others in terms of path tracking. This design leads to a control law which can be seen as a 2DOF controller but without resolving Bézout's equation in a time-varying framework. This regulator-observer permits to the output of the system to track a desired trajectory without using observer dynamics. The error on the vehicle speed resulting from its inaccurate measurements perturbations used in the simulations is 0.11 m/s (the initial condition due to the inaccurate measurement). With this value, the tracking error of various designs tends asymptotically to zero. If this inaccurate measurement is important, the vehicle speeds will not track the desired trajectory in the case of the use of an exact observer. In fact, it should be clear from the previous developments that the relation linking the exact observer (integral reconstructor), $\delta\hat{Z}(t)$, and the actual value of the state, is given by

$$\delta Z(t) = \delta\hat{Z}(t) + \sum_{i=1}^{n-2}\left(\int_0^t \bar{A}^{i-1}(t)\delta Z_0(t)\,\mathrm{d}t\right)^{(i-1)}, \tag{68}$$

where $\delta Z_0(t)$ is the initial condition due to the inaccurate measurement. In a further development within the context of flatness and exact observer, our main concern is how to appropriately compensate the effects of the unknown initial conditions when the actual value of the state is replaced by its integral reconstructor in a given state-based feedback controller design.

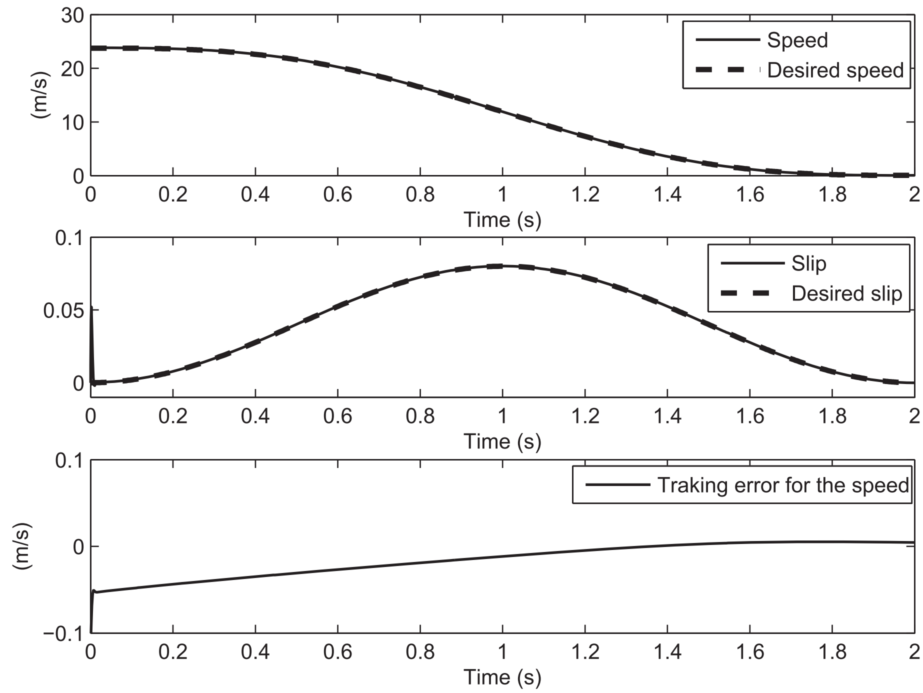

Figure 9. The output $v(t)$ and the slip $\lambda(t)$ trajectories of the nonlinear system and the tracking error for the speed by the use of a reduced order observer.

6. Conclusion

In this paper, a flatness-based control for tracking a desired trajectory in the case of LTV systems is proposed and developed. The proposed controller is based on an exact observer with a direct calculation of the state vector which contains the flat output and its derivatives. This regulator-observer permits to the system output to track a desired trajectory without using observer dynamics. The proposed method leads to a control design which can be seen as a 2DOF controller but without resolving Bézout's equation. The control law applied on an ABS gives a high level of performances in terms of the tracking of the wheel slip.

Beyond the framework of LTV systems, the result presented here opens the way to the control of nonlinear systems using their linearizations around a given trajectory.

References

Antić, D., Nikolić, V., Mitić, D., Milojković, M., & Perić, S. (2010). Sliding mode control of anti-lock braking system: An overview. *Facta Universitatis Series: Automatic Control and Robotics*, *9*(1), 41–58.

Aström, K., & Wittenmark, B. (1997). *Computer controlled systems. Theory and design* (3rd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Drakunov, S., Özguñer, Ü., Dix, P., & Ashrafi, B. (1995, March). ABS control using optimum search via sliding modes. *IEEE Transactions on Control Systems Technology*, *3*(1), 79–85.

Fliess, M. (1990). Une interprétation algébrique de la transformation de Laplace et des matrices de transfert. *Linear Algebra and its Applications*, *203*, 429–443.

Fliess, M. (2000). *Sur des pensers nouveaux faisons des vers anciens*. Conférence Internationale Francophone d'Automatique, CIFA2000, Lille, France, pp. 26–36.

Fliess, M., Lévine, J., Martin, Ph., & Rouchon, P. (1992). Sur les systèmes non linéaires différentiellement plats. *Comptes Rendus de l'Académie des Sciences Paris, I-315*, 619–624.

Fliess, M., Lévine, J., Martin, Ph., & Rouchon P. (1993). Linéarisation par bouclage dynamique et transformers de Lie-Bäcklund. *Comptes Rendus de l'Académie des Sciences Paris, I-517*, 981–986.

Fliess, M., Lévine, J., Martin, P., & Rouchon, P. (1995). Flatness and defect of nonlinear systems: Introductory theory and applications. *International Journal of Control*, *61*(6), 1327–1361.

Fliess, M., Lévine, J., Martin, Ph., & Rouchon, P. (1999). A Lie–Bäcklund approach to equivalence and flatness of nonlinear system. *IEEE Transactions Automatic Control*, *44*, 922–937.

Franklin, G. F., Powell, J. D., & Workman, M. (1998). *Digital control of dynamic systems*. Reading, MA: Addison-Wesley.

Freeman, R. (1995). *Robust slip control for a single wheel* (Tech. Rep. CCEC 95-0403). Santa Barbara, CA: University of California.

Horowitz, I. M. (1963). *Synthesis of feedback systems*. New York: Wiley.

Isidori, A. (1989). *Nonlinear control systems*. Berlin: Springer-Verlag.

Jiang, F. (2000). *A novel approach to a class of antilock brake problems* (PhD thesis). Cleveland State University, Cleveland.

Johansen, T. A., Petersen, I., Kalkkuhl, J., & Ludemann, J. (2003, November). Gain-scheduled wheel slip control in automotive brake systems. *IEEE Transactions on Control Systems Technology*, *11*(6), 799–811.

Jun, C. (1998). *The study of ABS control system with different control methods*. Proceedings of the 4th international symposium on advanced vehicle control (pp. 623–628), Nagoja, Japan.

Kailath, T. (1980). *Linear systems*. Englewood Cliffs, NJ: Prentice-Hall.

Kiencke, U., & Nielsen, L. (2005). *Automotive control systems for engine, driveline and vehicle*. New York: Springer.

Kučera, V. (1991). *Analysis and design of discrete linear control systems*. Englewood Cliffs, NJ: Prentice-Hall.

Layne, J. R., Passino, K. M., & Urkovich, S. Y. (1993, June). Fuzzy learning control for antiskid braking systems. *IEEE Transactions on Control System Technology*, *1*(2), 122–129.

Lee, Y., & Zak, H. S. (2001). *Genetic neural fuzzy control of anti-lock brake systems*. Proceedings of the 2001 American control conference, 25–27 June 2001 (Vol. 2, pp. 671–676), Arlington, VA.

Lévine, J. (1999). Are there new industrial perspectives in the control of mechanical systems? In P. M. Frank (Ed.), *Advances in control: Highlights of ECC'99* (pp. 197–226). London: Springer.

Lévine, J., Lottin, J., & Ponsart, J. C. (1996). A nonlinear approach to the control of magnetic bearings. *IEEE Transactions on Control Systems Technology*, *4*(5), 524–544.

Malgrange, B. (1962–1963). Systèmes différentiels à coefficients constants. *Séminaire Bourbaki*, *246*, 1–11.

Marinescu, B. (2010). Output feedback pole placement for linear time-varying systems with application to the control of nonlinear systems. *Automatica*, *46*(4), 1524–1530.

Marinescu, B., & Bourlès, H. (2009). An intrinsic algebraic setting for poles and zeros of linear time-varying systems. *Systems and Control Letters*, *58*, 248–253.

Marquez, R., Delaleau, E., & Fliess, M. (2000). *Commande par PID généralisé d'un moteur électrique sans capteur mécanique*. Conférence Internationale Francophone d'Automatique, CIFA 2000, Lille, France, pp. 453–458.

Mauer, G. F. (1995). A fuzzy logic controller for an ABS braking system. *IEEE Transactions on Fuzzy System*, *3*(4), 381–388.

Mitić, D. B., Antić, D., Perić, S., Milojković, M., & Nikolić, S. (2012, May). *Fuzzy sliding mode control for anti-lock braking systems*. Proceedings of the 7th international symposium on applied computational intelligence and informatics, SACI 2012 (pp. 217–222), Timisoara, Romania.

Mitić, D. B., Perić, S. L., Antić, D. S., Jovanović, Z. D., Milojković, M. T., & Nikolić, S. S. (2013). Digital sliding mode control of anti-lock braking system. *Advances in Electrical and Computer Engineering*, *13*(1), 33–40.

Nijmeijer, H., & Schaft, V. D. (1990). *Nonlinear dynamical control systems*. Berlin: Springer-Verlag.

Precup, R. E., Preitl, S., Radac, M. B., Petriu, E. M., Dragoş, C. A., & J. K., Tar (2011). Experiment-based teaching in advanced control engineering. *IEEE Transactions on Education*, *54*(3), 345–355.

Rotella, F. (2003). Systèmes linéaires non stationnaires. *Techniques de l'Ingénieur, tome Informatique Industrielle*, S-7185, 1–16.

Rotella, F., & Carrillo, F. J. (1998). *Flatness based control of a turning*. Proceedings process CESA'98, (Vol. 1, pp. 397–402), Hammamet, Tunisia.

Rotella, F., Carrillo, F. J., & Ayadi, M. (2002). Polynomial controller design based on flatness. *Kybernetika*, *38*(5), 571–584.

Rotella, F., & Zambettakis, I. (2007). Commande des systèmes par platitude. *Techniques de l'Ingénieur, Traité Informatique Industrielle, S-7450*, 1–18.

Rothfuss, R., Rudolph, J., & Zeitz, M. (1996). Flatness based control of a nonlinear chemical reactor. *Automatica*, *32*(10), 1433–1439.

Silverman, L. M. (1966). Transformation of time-variable systems to canonical (phase-variable) form. *IEEE Transactions on Automatic Control, AC-11*, 303–306.

Silverman, L. M., & Meadows, H. E. (1967). Controllability and observability in time-variable linear systems. *SIAM Journal Control and Optimization*, *5*, 64–73.

Solyom, S., & Rantzer, A. (2002). ABS control - A design model and control structure. In R. Johansson & A. Rantzer (Eds.), *Nonlinear and hybrid control in automotive applications* (pp. 85–96). New York: Springer-Verlag.

Unsal, C., & Kachroo, P. (1999). Sliding mode measurement feedback control for antilock braking systems. *IEEE Transactions on Control Systems Technology*, *7*(2), 271–281.

Will, A. B., Hui, S., & Zak, S. H. (1998). Sliding mode wheel slip controller for an antilock braking system. *International Journal of Vehicle Design*, *19*(4), 523–539.

Yu, J. S. (1997). *A robust adaptive wheel-slip controller for antilock brake system*. 36th IEEE conference on decision and control, San Diego, CA, pp. 2545–2546.