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To cite this article: Sandeep Kumar, Shalu Rani & Mohit Kumar (2021) A New Order Function for Interval-Valued Intuitionistic Fuzzy Numbers and Its Application in Group Decision Making, Fuzzy Information and Engineering, 13:1, 111-126, DOI: [10.1080/16168658.2021.1936961](https://doi.org/10.1080/16168658.2021.1936961)

To link to this article: <https://doi.org/10.1080/16168658.2021.1936961>



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Published online: 23 Jun 2021.



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# A New Order Function for Interval-Valued Intuitionistic Fuzzy Numbers and Its Application in Group Decision Making

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## ABSTRACT

**Purpose:** We introduce a multiple attribute group decision making (MAGDM) method under interval-valued intuitionistic fuzzy (IVIF) domain. In these MAGDM problems, attribute values and attribute weights are represented in terms of IVIF sets (IVIFs).

**Methodology:** The present methodology is divided into two parts. Firstly, an appropriate order function has been developed for IVIFs. Secondly, an algorithm has been proposed for solving the MAGDM problems with IVIF decision information.

**Findings:** To show the effectiveness of the proposed method, a theoretical comparative study with respect to some existing literature is provided. A real-life based numerical example is given to demonstrate the proposed solution procedure. The advantage of this method is also described.

**Values:** Using proposed order function, interval-valued intuitionistic fuzzy hybrid geometric and ordered weighted averaging operators, the present method provides a most preferred alternative in six steps. On the basis of a comparative study, it can be observed that the present method simple and easy to implement for solving the IVIF MAGDM model occurrence in real-life decision making problems.

## ARTICLE HISTORY

Received 15 April 2019

Revised 12 May 2021

Accepted 26 May 2021

## KEYWORDS

Multiple attribute group decision making; interval-valued intuitionistic fuzzy number; IIFHG operator; order function; OWA operator

## 1. Introduction

In a pluralistic environment, multiple attribute decision making (MADM) refers to making the decisions to rank alternatives or select the best alternative(s) from a finite set of prespecified alternatives with regard to various attributes. There are many objective and subjective factors in real-life decision making process, such as the limitations of cognitive structure of decision makers (DMs), the cost concern, the complexity and fuzziness of the objects and the unpredictability of events. In such a case, DMs most often use natural languages or linguistic terms to describe or evaluate the vagueness of the considered objects. For solving such decision making problems, the fuzzy set theory [1] provides an effective way for expressing the decision information. This theory has been widely used in many research problems [2–6]. But, because of considering the membership function only, this theory is not capable for dealing the situations in which DMs face ‘neither this nor that’ situation to evaluate their preferences. This kind of uncertainty is usually known as the uncertainty with

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hesitation. The theory of intuitionistic fuzzy sets (IFSs) [7, 8] is very useful for handling such types of uncertainty and vagueness in the data of decision making problems. In fact, this theory can represent the preferences/information of DMs in terms of favour, unfavour and neutral. It is sometimes difficult for the DMs to obtain the exact values of the membership degree, the non-membership degree and the hesitancy degree. To cope with this situation, Atanassov and Gargov [9] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) that are expressed by an interval-valued membership function, an interval-valued non-membership function and an interval-valued hesitancy function.

Xu and Chen [10] presented an approach to solve group decision making (GDM) problems with interval-valued intuitionistic judgement matrices on the basis of the arithmetic aggregation operator and the hybrid aggregation operator. They defined the interval-valued intuitionistic judgement matrix with its score matrix and accuracy matrix. Park et al. [11] provided an extension of the technique for order preference by similarity to the ideal solution (TOPSIS) method for solving MAGDM problems with IVIF information in which attribute weights are partially known. They found the best alternative by using the different distance definitions. Wang et al. [12] proposed a mathematical programming approach for solving MADM problems in which both attribute values and attribute weights are characterised by IVIFNs. The TOPSIS method is used in this work to rank alternatives. Wan and Li [13] presented a fuzzy mathematical programming method for solving MADM problems with incomplete attribute weights. In this method, attribute values are represented in terms of IVIFSs, IFSs, trapezoidal fuzzy numbers, linguistic variables, intervals and real numbers. This method is based on the linear programming technique for multi dimensional analysis of preference (LINMAP). Chen and Huang [14] proposed a solution method for MADM problems in which attribute values as well as attribute weights are represented by IVIF values. This method is based on the linear programming methodology. Wang and Chen [15] developed a method for solving IVIF MADM problems based on the linear programming method and the extension of the TOPSIS method. The linear programming method is used to calculate optimal weights of attributes. Wang and Chen [16] proposed a new score function IVIF values and the linear programming method for solving IVIF MADM problems. In this method, the authors removed the drawback of an existing method. Yu et al. [17] studied a nonlinear programming method for solving MADM problems in which ratings of alternatives on attributes are represented by IVIFSs. The preference information on attributes in this method is incomplete. Kumar and Garg [18] solved an MADM problem under IVIFS environment by using set pair analysis. Tyagi [19] presented a new approach for solving intuitionistic fuzzy MADM problems. Safarzadeh and Rasti-Barzoki [20] developed a novel MADM method in which a lexicographic semi-order model is modified by using the best-worst method for the weights determination of the criteria. Safarzadeh et al. [21] extended a GDM method with the best-worst method. In this work, they proposed two mathematical models for evaluating the optimal weights of the criteria. Wan and Dong [22] launched several methods and theories for solving MADM/MAGDM problems under IVIF framework in form of a book 'Decision Making Theories and Methods Based on Interval-Valued Intuitionistic Fuzzy Sets' published in 2020.

From the above discussion, we mainly face the following issues in this article:

- (i) Due to the presence of complexity and uncertainty in problems, lack of knowledge, time restriction, a group of DMs is required in the decision process.

- (ii) How to calculate the reasonable and accurate knowledge about the attribute weights for a group of DMs in the decision making process under the IVIFS framework?
- (iii) To obtain the ordering relation between two IVIFNs, there needs an appropriate order function which satisfies some necessary properties of ordering.
- (iv) How to obtain the reasonable ranking of alternatives by a group of DMs on the basis of the OWA operator?

Motivated by these issues, therefore, the aim of this article is divided into two folds. Firstly, an appropriate order function has been developed by taking into account the degree of indeterminacy of IVIFSs. Secondly, based on this order function, an algorithm has been proposed for solving the MAGDM problems with IVIF decision information. In this method, all given decision matrices are fused into a collective IVIF decision matrix by using the IIFHG operator. After that the optimal attribute weights for a group of DMs are obtained by utilising the IIFHG operator, the proposed order function and the concept of normalisation. With the help of these weights, the weighted collective IVIF decision matrix is obtained. Utilising the proposed order function, this matrix is converted into a crisp matrix. Finally, the overall attribute value for each alternative is calculated from this crisp matrix. On the basis of these values, the alternatives are ranked.

This article is organised as follows. In Section 2, some preliminary concepts are given. A new order function for the IVIFNs is proposed in Section 3. In Section 4, an MAGDM problem is introduced with IVIF information. In this section, we develop a new method for solving such an MAGDM problem. In Section 5, a comparative study with respect to some existing literature is provided. In Section 6, a numerical example is given to demonstrate the proposed solution procedure. The work of this article is concluded in Section 7 with a future scope.

## 2. Preliminaries

In this section, we give a brief introduction of IVIFSs, IVIFNs and IIFHG and OWA operators.

**Definition 2.1 (Atanassov and Gargov [9]):** Let  $X = \{x_1, x_2, \dots, x_n\}$  be the finite universe of discourse. Mathematically, an IVIFS  $\tilde{A}$  in  $X$  is represented as

$$\tilde{A} = \{\langle x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\},$$

where  $\mu_{\tilde{A}}(x_i)$  and  $\nu_{\tilde{A}}(x_i)$  are interval-valued membership degree and interval-valued non-membership degree of element  $x_i$  belonging to the IVIFS  $\tilde{A}$ , respectively. The interval-valued intuitionistic fuzzy value (IVIFV)  $(\mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i))$  of element  $x_i$  belonging to the IVIFS  $\tilde{A}$  can be expressed as

$$([\mu_{\tilde{A}}^L(x_i), \mu_{\tilde{A}}^U(x_i)], [\nu_{\tilde{A}}^L(x_i), \nu_{\tilde{A}}^U(x_i)]),$$

where  $\mu_{\tilde{A}}(x_i) = [\mu_{\tilde{A}}^L(x_i), \mu_{\tilde{A}}^U(x_i)]$ ,  $\nu_{\tilde{A}}(x_i) = [\nu_{\tilde{A}}^L(x_i), \nu_{\tilde{A}}^U(x_i)]$ ,  $0 \leq \mu_{\tilde{A}}^L(x_i) \leq \mu_{\tilde{A}}^U(x_i) \leq 1$ ,  $0 \leq \nu_{\tilde{A}}^L(x_i) \leq \nu_{\tilde{A}}^U(x_i) \leq 1$ ,  $i = 1, 2, \dots, n$ .

If an IVIFS  $\tilde{A}$  contains only one element, i.e. if  $\mu_{\tilde{A}}^L(x_i) = \mu_{\tilde{A}}^U(x_i) = \mu_{\tilde{A}}(x_i)$  and  $\nu_{\tilde{A}}^L(x_i) = \nu_{\tilde{A}}^U(x_i) = \nu_{\tilde{A}}(x_i)$  for  $i = 1, 2, \dots, n$ , then the IVIFS  $\tilde{A}$  is reduced to an IFS =  $\{\langle x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ .

The following interval is called an intuitionistic fuzzy interval [23] of  $x$  in  $\tilde{A}$

$$\begin{aligned} \pi_{\tilde{A}}(x) &= 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \\ &= [1 - \mu_{\tilde{A}}^U(x) - \nu_{\tilde{A}}^U(x), 1 - \mu_{\tilde{A}}^L(x) - \nu_{\tilde{A}}^L(x)]. \end{aligned}$$

Let  $\tilde{a} = \langle [a, b], [c, d] \rangle$  be an IVIFN [24], where  $[a, b] \subset [0, 1], [c, d] \subset [0, 1]$  with  $b + d \leq 1$ . From [24, 25], three operational results of IVIFNs are as follows:

Let  $\tilde{a}^{(1)} = \langle [a_1, b_1], [c_1, d_1] \rangle, \tilde{a}^{(2)} = \langle [a_2, b_2], [c_2, d_2] \rangle$  and  $\tilde{a} = \langle [a, b], [c, d] \rangle$  be three IVIFNs, then

- (i)  $\tilde{a}^{(1)} \otimes \tilde{a}^{(2)} = \langle [a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2] \rangle;$
- (ii)  $\tilde{a}^\lambda = \langle [a^\lambda, b^\lambda], [1 - (1 - c)^\lambda, 1 - (1 - d)^\lambda] \rangle, \lambda > 0;$
- (iii)  $\lambda \tilde{a} = \langle [1 - (1 - a)^\lambda, 1 - (1 - b)^\lambda], [c^\lambda, d^\lambda] \rangle, \lambda > 0.$

It is notable that the above results are also IVIFNs.

In [11], the hesitancy degree of the IVIFN  $\tilde{a}$  is defined as the midpoint of the intuitionistic fuzzy interval of  $\tilde{a}$ , i.e.

$$\pi(\tilde{a}) = \frac{1}{2} ((1 - a - c) + (1 - b - d)).$$

For the measurement of an IVIFN  $\tilde{a}$ , the score function  $s$  [24] is defined as

$$s(\tilde{a}) = \frac{1}{2}(a - c + b - d),$$

where  $s(\tilde{a}) \in [-1, 1]$ .

**Definition 2.2 (Xu and Chen [25], Wei and Wang [26]):** Let  $\tilde{a}^{(k)} = \langle [a_k, b_k], [c_k, d_k] \rangle, (k = 1, 2, \dots, n)$  be  $n$  IVIFNs, then IIFHG operator  $\theta$  is defined as follows:

$$\begin{aligned} \theta_{\alpha, \lambda}(\tilde{a}^{(1)}, \tilde{a}^{(2)}, \dots, \tilde{a}^{(n)}) &= (\tilde{a}^{(\sigma(1))})^{\alpha_1} \otimes (\tilde{a}^{(\sigma(2))})^{\alpha_2} \otimes \dots \otimes (\tilde{a}^{(\sigma(n))})^{\alpha_n} \\ &= \left\langle \left[ \prod_{k=1}^n (\dot{a}^{(\sigma(k))})^{\alpha_k}, \prod_{k=1}^n (\dot{b}^{(\sigma(k))})^{\alpha_k} \right], \right. \\ &\quad \left. \left[ \left( 1 - \prod_{k=1}^n (1 - \dot{c}^{(\sigma(k))})^{\alpha_k} \right), \left( 1 - \prod_{k=1}^n (1 - \dot{d}^{(\sigma(k))})^{\alpha_k} \right) \right] \right\rangle \end{aligned}$$

where  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)^T$  is a weight vector of operator  $\theta$  with  $\alpha_k > 0 (k = 1, 2, \dots, n)$

and  $\sum_{k=1}^n \alpha_k = 1$  and  $\tilde{a}^{(\sigma(k))} = \langle [\dot{a}^{(\sigma(k))}, \dot{b}^{(\sigma(k))}], [\dot{c}^{(\sigma(k))}, \dot{d}^{(\sigma(k))}] \rangle$  is the  $k$ th largest of the

weighted IVIFNs  $\tilde{a}^{(k)}$  where  $\tilde{a}^{(k)} = \left( \tilde{a}^{(k)} \right)^{n \lambda_k}$ .

**Definition 2.3 (Yager [27]):** Let  $\phi : R^n \longrightarrow R$  be a function, if

$$\phi_{\omega}(g_1, g_2, \dots, g_n) = \sum_{j=1}^n \omega_j h_j,$$

then the function  $\phi$  is called an OWA operator, where  $h_j$  is the  $j$ th largest of a collection of the arguments  $g_j (j = 1, 2, \dots, n)$ ,  $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$  is the weighting vector associated with the function  $\phi$ ,  $\omega_j \geq 0$ ,  $\sum_{j=1}^n \omega_j = 1$ , and  $R$  is the set of real numbers.

### 3. Construction of New Order Function for IVIFNs

Consider an IVIFN  $\tilde{a}$  which is represented as

$$\tilde{a} = \langle [a, b], [c, d] \rangle.$$

The intuitionistic fuzzy interval of  $\tilde{a}$  is defined as in [23]

$$\pi_{\tilde{a}} = [1 - b - d, 1 - a - c].$$

The objective of an order function is to resolve the degree that shows how much a particular alternative satisfies the demand of the DM. The order function mainly tries to reduce the degree of hesitancy by reducing the intuitionistic fuzzy interval  $\pi_{\tilde{a}}$  in proportion of  $[a, b]/[c, d]$  in the favour of interval of membership/non-membership degrees.

By motivating the work [28, 29], the intuitionistic fuzzy interval  $\pi_{\tilde{a}}$  can be divided into three following parts:

- (i)  $[a, b]\pi_{\tilde{a}}$ ;
- (ii)  $[c, d]\pi_{\tilde{a}}$ ;
- (iii)  $[1 - b - d, 1 - a - c]\pi_{\tilde{a}}$ .

Taking the middle point of the intervals, the above three parts can be recasted as

- (i)'  $\left(\frac{a+b}{2}\right)\left(\frac{1-b-d+1-a-c}{2}\right)$ ;
- (ii)'  $\left(\frac{c+d}{2}\right)\left(\frac{1-b-d+1-a-c}{2}\right)$ ;
- (iii)'  $\left(\frac{1-b-d+1-a-c}{2}\right)\left(\frac{1-b-d+1-a-c}{2}\right)$ .

Now, the first-order function is defined as

$$\psi_{1,\tilde{a}} = \left(\frac{a+b}{2}\right) + \left(\frac{a+b}{2}\right)\left(\frac{1-b-d+1-a-c}{2}\right),$$

which represents favour degree relative to  $\tilde{a}$  for  $x \in X$ .

The function  $\psi_{2,\tilde{a}}$  is defined as

$$\psi_{2,\tilde{a}} = \left(\frac{c+d}{2}\right) + \left(\frac{c+d}{2}\right)\left(\frac{1-b-d+1-a-c}{2}\right),$$

which represents unfavour degree relative to  $\tilde{a}$  for  $x \in X$ .

From the function  $\psi_{2,\tilde{a}}$ , the second-order function  $\psi_{3,\tilde{a}}$  is defined as

$$\psi_{3,\tilde{a}} = 1 - \psi_{2,\tilde{a}},$$

which gives the upper value of favour degree relative to  $\tilde{a}$  for  $x \in X$ .

Here, we observe that  $\psi_{1,\tilde{a}} \leq \psi_{3,\tilde{a}}$  for every IVIFN  $\tilde{a}$ .

As is known that the IVIFNs  $\langle [1, 1], [0, 0] \rangle$  and  $\langle [0, 0], [1, 1] \rangle$  are the largest and the smallest IVIFN, respectively. For these IVIFNs, we get  $\psi_{1,\langle [1,1],[0,0] \rangle} = 1$  and  $\psi_{1,\langle [0,0],[1,1] \rangle} = 0$ . This gives  $\psi_{1,\tilde{a}} \in [0, 1]$ , for every  $\tilde{a}$ . Similarly, we can obtain the following results  $\psi_{2,\tilde{a}} \in [0, 1]$  and  $\psi_{3,\tilde{a}} \in [0, 1]$ .

Using the above concepts, we propose a new order function  $\psi_{\tilde{a}}$  which combines the order functions  $\psi_{1,\tilde{a}}$  and  $\psi_{3,\tilde{a}}$  as

$$\psi_{\tilde{a}} = \frac{\psi_{1,\tilde{a}} + \psi_{3,\tilde{a}}}{2},$$

where  $\psi_{\tilde{a}} \in [0, 1]$ .

Some properties of the proposed new order function  $\psi_{\tilde{a}}$  of IVIFNs are described as:

*Property 1.* For any IVIFN  $\tilde{a} = \langle [a, b], [c, d] \rangle$ ,  $\psi_{\tilde{a}} \in [0, 1]$ .

*Property 2.* If  $\tilde{a}$  is the largest IVIFN, i.e.  $\tilde{a} = \langle [1, 1], [0, 0] \rangle$ , then  $\psi_{\tilde{a}} = 1$ .

**Proof:** Let  $\tilde{a} = \langle [1, 1], [0, 0] \rangle$ , then  $\psi_{1,\tilde{a}} = 1$ ,  $\psi_{2,\tilde{a}} = 0$  and then  $\psi_{3,\tilde{a}} = 1$ .

Thus, we get

$$\psi_{\tilde{a}} = 1.$$

■

*Property 3.* If  $\tilde{a}$  is the smallest IVIFN, i.e.  $\tilde{a} = \langle [0, 0], [1, 1] \rangle$ , then  $\psi_{\tilde{a}} = 0$ .

**Proof:** Since  $\tilde{a} = \langle [0, 0], [1, 1] \rangle$ , so we can get  $\psi_{1,\tilde{a}} = 0$ ,  $\psi_{2,\tilde{a}} = 1$  and then  $\psi_{3,\tilde{a}} = 0$ .

Thus, we get

$$\psi_{\tilde{a}} = 0.$$

■

*Property 4.* If the IVIFN  $\tilde{a} = \langle a, 1 - a \rangle$ , where  $a \in [0, 1]$ , then  $\psi_{\tilde{a}} = a$ .

**Proof:** If  $\tilde{a} = \langle a, 1 - a \rangle$ , where  $a \in [0, 1]$ , then we obtain  $\psi_{1,\tilde{a}} = a$ ,  $\psi_{2,\tilde{a}} = 1 - a$  and then  $\psi_{3,\tilde{a}} = a$ .

Thus, we get

$$\psi_{\tilde{a}} = a.$$

■

**Theorem 3.1:** Let  $\tilde{a}^{(1)} = \langle [a_1, b_1], [c_1, d_1] \rangle$  and  $\tilde{a}^{(2)} = \langle [a_2, b_2], [c_2, d_2] \rangle$  be two IVIFNs, then

$$a_1 \leq a_2, b_1 \leq b_2, c_1 \geq c_2, d_1 \geq d_2 \Rightarrow \psi_{\tilde{a}^{(1)}} \leq \psi_{\tilde{a}^{(2)}}.$$

**Proof:** If  $a_1 \leq a_2, b_1 \leq b_2, c_1 \geq c_2$  and  $d_1 \geq d_2$ , then we get  $\psi_{1,\tilde{a}^{(1)}} \leq \psi_{1,\tilde{a}^{(2)}}$  and  $\psi_{2,\tilde{a}^{(1)}} \geq \psi_{2,\tilde{a}^{(2)}}$ . This gives

$$1 - \psi_{2,\tilde{a}^{(1)}} \leq 1 - \psi_{2,\tilde{a}^{(2)}},$$

which implies that

$$\psi_{3,\tilde{a}^{(1)}} \leq \psi_{3,\tilde{a}^{(2)}}.$$

We then obtain the following inequality:

$$\frac{\psi_{1,\tilde{a}^{(1)}} + \psi_{3,\tilde{a}^{(1)}}}{2} \leq \frac{\psi_{1,\tilde{a}^{(2)}} + \psi_{3,\tilde{a}^{(2)}}}{2}$$

i.e.  $\psi_{\tilde{a}^{(1)}} \leq \psi_{\tilde{a}^{(2)}}$ .

This completes the proof of Theorem 3.1. ■

**Remark 3.1:** For IVIFNs  $\tilde{a}^{(1)} = \langle [a_1, b_1], [c_1, d_1] \rangle$  and  $\tilde{a}^{(2)} = \langle [a_2, b_2], [c_2, d_2] \rangle$ , if

$$a_1 \geq a_2, b_1 \geq b_2, c_1 \leq c_2, d_1 \leq d_2 \quad \text{then } \psi_{\tilde{a}^{(1)}} \geq \psi_{\tilde{a}^{(2)}}.$$

#### 4. Formulation and Solution Procedure for MAGDM Problem

This MAGDM problem contains  $n$  alternatives  $o_1, o_2, \dots, o_n$  and  $m$  attributes  $u_1, u_2, \dots, u_m$ . The aim of this study is to select the most preferred alternative or rank of all alternatives under the IVIF environment. Each alternative is assessed on each of the  $m$  attributes and all these assessments, given by each DM are represented as IVIF decision matrices with the entries as IVIFNs. Assume that attribute weights, given by DMs are represented by IVIFNs. Let  $D = \{d_1, d_2, \dots, d_l\}$  be the set of  $l$  DMs, and  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_l)^T$  be the weight vector of  $d_i$ 's where  $\sum_{k=1}^l \lambda_k = 1, \lambda_k \geq 0$  for every  $k$ . For the  $k$ th DM  $d_k$ , the IVIF decision matrix is denoted as  $\tilde{D}^{(k)} = [\tilde{d}_{ij}^{(k)}]_{m \times n}$ , where  $\tilde{d}_{ij}^{(k)} = \langle [p_{ij}^{(k)}, q_{ij}^{(k)}], [s_{ij}^{(k)}, t_{ij}^{(k)}] \rangle$  is an evaluating IVIFN of attribute  $u_i$  for alternative  $o_j$ . The intervals  $[p_{ij}^{(k)}, q_{ij}^{(k)}]$  and  $[s_{ij}^{(k)}, t_{ij}^{(k)}]$  are the degree of satisfaction interval and the degree of dissatisfaction interval for alternative  $o_j$  with respect to attribute  $u_i$ , respectively.

In an MAGDM problem, there are two following important concepts [30]:

- (i) Aggregation of the opinions provided by DMs.
- (ii) Aggregation the collective values of attributes for each alternative.

On the basis of these concepts, we develop the solution procedure for the present MAGDM problem in the following steps:

*Step 1.* In this step, all individual decision opinions are fused into a group opinion. For this, all individual IVIF decision matrices  $\tilde{D}^{(k)} = [\tilde{d}_{ij}^{(k)}]_{m \times n} (k = 1, 2, \dots, l)$  are aggregated into a collective IVIF matrix  $\tilde{D} = [\tilde{d}_{ij}]_{m \times n}$  by utilising the IIFHG operator  $\theta$  (Definition 2.2). Then,



**Table 1.** The attribute weights.

Attributes(↓)/DMs(→)	$d_1$	$d_2$	...	$d_l$
$u_1$	$\tilde{w}_{11}$	$\tilde{w}_{12}$	...	$\tilde{w}_{1l}$
$u_2$	$\tilde{w}_{21}$	$\tilde{w}_{22}$	...	$\tilde{w}_{2l}$
⋮	⋮	⋮		⋮
$u_m$	$\tilde{w}_{m1}$	$\tilde{w}_{m2}$	...	$\tilde{w}_{ml}$

the  $ij$ th entry  $\tilde{d}_{ij}$  of the matrix  $\tilde{D}$  takes the following form:

$$\begin{aligned} \tilde{d}_{ij} &= \left\langle \left[ \prod_{k=1}^n (\dot{p}_{ij}^{(\sigma(k))})^{\alpha_k}, \prod_{k=1}^n (\dot{q}_{ij}^{(\sigma(k))})^{\alpha_k} \right], \left[ \left( 1 - \prod_{k=1}^n (1 - \dot{s}_{ij}^{(\sigma(k))})^{\alpha_k} \right), \right. \right. \\ &\quad \left. \left. \left( 1 - \prod_{k=1}^n (1 - \dot{t}_{ij}^{(\sigma(k))})^{\alpha_k} \right) \right] \right\rangle \\ &= \langle [p_{ij}, q_{ij}], [s_{ij}, t_{ij}] \rangle, \end{aligned}$$

where  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_l)^T$  is a weight vector of operator  $\theta$  and  $\langle [\dot{p}_{ij}^{(\sigma(k))}, \dot{q}_{ij}^{(\sigma(k))}], [s_{ij}^{(\sigma(k))}, t_{ij}^{(\sigma(k))}] \rangle$  is the  $k$ th largest of the weighted IVIFN  $\tilde{d}_{ij}^{(k)}$  where  $\tilde{d}_{ij}^{(k)} = \left( \tilde{d}_{ij}^{(k)} \right)^{l_{\lambda,k}}$ , ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ).

Since the entries in each decision matrix  $\tilde{D}^{(k)}$  are represented by IVIFNs, where the membership degree and the non-membership degree are contained in unit interval  $[0, 1]$ , so the normalisation of these entries is not necessary.

**Step 2.** The attribute weights provided by DMs can be represented in tabular form as in Table 1.

In Table 1, where  $\tilde{w}_{ik} = \langle [\mu_{\tilde{w}_{ik}}^L, \mu_{\tilde{w}_{ik}}^U], [v_{\tilde{w}_{ik}}^L, v_{\tilde{w}_{ik}}^U] \rangle$  is an IVIFN ( $i = 1, 2, \dots, m; k = 1, 2, \dots, l$ ).

In this process, all the weights information given by DMs  $d_1, d_2, \dots, d_l$  are aggregated into a single group weight information for attributes  $u_i$ , ( $i = 1, 2, \dots, m$ ) by utilising operator  $\theta$ . This grouping weight information for attributes will also be the IVIFNs. Suppose that these group weights are denoted by  $\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_m$ . The crisp value  $w_i$  for the  $i$ th IVIFN  $\tilde{w}_i$  ( $i = 1, 2, \dots, m$ ) has been evaluated by using the proposed order function. Afterwards, the values  $w_1, w_2, \dots, w_m$  have been normalised in order to determine the optimal attribute weights, say,  $w_1^*, w_2^*, \dots, w_m^*$ , respectively. The  $i$ th optimal attribute weight  $w_i^*$  is given by

$$w_i^* = \frac{w_i}{w_1 + w_2 + \dots + w_m},$$

where  $\sum w_i^* = 1$  and  $w_i^* > 0$  ( $i = 1, 2, \dots, m$ ).

**Step 3.** Using the calculated optimal attribute weights from Step 2, the entries of the weighted collective IVIF decision matrix  $\tilde{D}^* = [\tilde{d}_{ij}^*]_{m \times n}$  can be obtained as

$$\begin{aligned} \tilde{d}_{ij}^* &= w_i^* \tilde{d}_{ij} \\ &= \langle [1 - (1 - p_{ij})^{w_i^*}, 1 - (1 - q_{ij})^{w_i^*}], [s_{ij}^{w_i^*}, t_{ij}^{w_i^*}] \rangle \\ &= \langle [p_{ij}^*, q_{ij}^*], [s_{ij}^*, t_{ij}^*] \rangle. \end{aligned}$$

It is notable that the above  $ij$ th entry  $\tilde{d}_{ij}^*$  is an IVIFN by using the result (iii) in Definition 2.1.

*Step 4.* Utilise the proposed order function to calculate the entries of matrix  $\tilde{D}^*$ . The entries of calculated matrix  $D^* = [d_{ij}^*]_{m \times n}$  can be obtained as

$$\begin{aligned} d_{ij}^* &= \psi_{\tilde{d}_{ij}^*} \\ &= \frac{\psi_{1, \tilde{d}_{ij}^*} + \psi_{3, \tilde{d}_{ij}^*}}{2}, \end{aligned}$$

where

$$\begin{aligned} \psi_{1, \tilde{d}_{ij}^*} &= \frac{p_{ij}^* + q_{ij}^*}{2} + \left( \frac{p_{ij}^* + q_{ij}^*}{2} \right) \left( \frac{1 - q_{ij}^* - t_{ij}^* + 1 - p_{ij}^* - s_{ij}^*}{2} \right), \\ \psi_{3, \tilde{d}_{ij}^*} &= 1 - \left[ \frac{s_{ij}^* + t_{ij}^*}{2} + \frac{(s_{ij}^* + t_{ij}^*)}{2} \left( \frac{1 - q_{ij}^* - t_{ij}^* + 1 - p_{ij}^* - s_{ij}^*}{2} \right) \right]. \end{aligned}$$

*Step 5.* Utilise the OWA operator  $\phi$  to aggregate all the attribute values  $d_{ij}^*$  ( $j = 1, 2, \dots, n$ ) of the alternative  $o_j$ , and evaluate the overall attribute value  $\phi_\omega(o_j)$ .

$$\phi_\omega(o_j) = \sum_{i=1}^m \omega_i e_{ij}^*,$$

where  $e_{ij}^*$  is the  $i$ th largest of  $d_{ij}^*$  ( $i = 1, 2, \dots, m$ ) and  $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$  is the weight vector associated with operator  $\phi$  s.t.  $\omega_i \geq 0$ ,  $\sum_{i=1}^m \omega_i = 1$ . The value of  $\omega_i$  is determined by using the following formula [31]:

$$\omega_1 = \frac{1 - \alpha}{m} + \alpha, \omega_i = \frac{1 - \alpha}{m}, \quad i \neq 1, \alpha \in [0, 1].$$

*Step 6.* Rank all the alternatives  $o_j$  ( $j = 1, 2, \dots, n$ ) according to the values  $\phi_\omega(o_j)$  ( $j = 1, 2, \dots, n$ ) in descending order.

## 5. A Comparative Study

In order to compare the performance of the proposed approach with respect to some existing approaches, we describe a comprehensive study as follows:

### 5.1. Compare with Park et al.'s Method [11]

Park et al. [11] considered an MAGDM problem with IVIF decision matrices in which attribute weights are partially known. Using the IIFHG operator and the score function, a score matrix is constructed for a collective IVIF decision matrix. Then from this matrix and given incomplete information about the weights, the weights of attributes are determined through an optimisation model. Therefore, it is observed that the technique for determining the

weights is completely different from that of the present method. A numerical comparison is also provided with this approach in the next section.

### **5.2. Compare with Wan and Dong's Method ([22, pp. 139–177])**

First, Wan and Dong presented a method to aggregate different types of information such as real numbers, interval numbers, triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TrFNs) into IVIFN. The attribute weights are incompletely known. The optimal attribute weights are determined by using an intuitionistic fuzzy programming approach. Finally, a ranking order is obtained for the considered heterogeneous MAGDM problem with IVIF information. The present work is similar to this method in the sense that we also aggregated the decision information into IVIFN for a GDM problem and differs from this for the following aspects:

- (i) In the present work, the IIFHG operator is used for aggregating the IVIFNs into IVIFN. The advantage of this operator is that it can aggregate real numbers, interval numbers, TFNs, intuitionistic fuzzy numbers (IFNs) and IVIFNs into IVIFN. But, the operators in [22] are unable to aggregate IFNs/IVIFNs into IVIFN.
- (ii) In the present approach, each DM provides an attribute weight vector separately in which every component of each vector is represented by an IVIFN on the basis of his/her experience and knowledge. The set of attribute weights is incomplete in Wan and Dong's method. This description for the attribute weights seems more practical rather than the concept of incomplete weights.
- (iii) The IIFHG operator and the proposed order function are used to obtain the attribute weights in this work instead of the intuitionistic fuzzy programming approach.
- (iv) The OWA operator is used in order to find the ranking order of alternatives in place of the TOPSIS method.
- (v) The present approach is completed in six steps only whereas the approach in [22] takes 10 steps to complete. This indicates that the present algorithm is more computationally straightforward.

### **5.3. Compare with Wan and Dong's Method (22, pp. 243–270])**

In this method, a GDM with IVIF preference relations is discussed. The present work is similar to this method in the sense that both methods are provided for solving an MAGDM problem under the IVIFS environment and differs from this method because of using the IIFHG operator, the order function and the concept of normalisation for determining the weights of attributes in IVIF MAGDM problems.

### **5.4. Compare with the Other Studies [10, 12–19]**

The proposed approach is different from that of these studies because neither studies use the concept of the proposed order function, the IIFHG operator and the OWA operator simultaneously for the present MAGDM problem.

## 6. Numerical Example

In this section, we show the validity of the proposed procedure via a selection problem of suitable and ecofriendly air conditioning systems that can be installed in library of a university (based on [11]).

Suppose a four member committee is constituted in a university to study the contractor's offers and give the recommendations to the best alternative among the four feasible alternatives. These four alternatives might be adapted to the physical structure of the library. When making a decision, the attributes considered under which the alternatives are made, are as follows:

- $u_1$  : Performance
- $u_2$  : Maintainability
- $u_3$  : Flexibility
- $u_4$  : Cost
- $u_5$  : Safety

The IVIF decision matrices  $\tilde{D}^{(k)}$  ( $k = 1, 2, 3, 4$ ) are listed in Tables 2–5.

The information about the attribute weights, given by DMs  $d_1, d_2, d_3$  and  $d_4$ , is listed in Table 6.

Assume that the weight vector four DMs  $d_1, d_2, d_3$  and  $d_4$  is  $\lambda = (0.3, 0.2, 0.3, 0.2)^T$ .

**Table 2.** First interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}^{(1)}$ .

	$o_1$	$o_2$	$o_3$	$o_4$
$u_1$	$\langle [0.5, 0.6], [0.2, 0.3] \rangle$	$\langle [0.3, 0.4], [0.4, 0.6] \rangle$	$\langle [0.4, 0.5], [0.3, 0.5] \rangle$	$\langle [0.3, 0.5], [0.4, 0.5] \rangle$
$u_2$	$\langle [0.3, 0.5], [0.4, 0.5] \rangle$	$\langle [0.1, 0.3], [0.2, 0.4] \rangle$	$\langle [0.7, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.7, 0.8] \rangle$
$u_3$	$\langle [0.6, 0.7], [0.2, 0.3] \rangle$	$\langle [0.3, 0.4], [0.4, 0.5] \rangle$	$\langle [0.5, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.5, 0.8] \rangle$
$u_4$	$\langle [0.5, 0.7], [0.1, 0.2] \rangle$	$\langle [0.2, 0.4], [0.5, 0.6] \rangle$	$\langle [0.4, 0.6], [0.2, 0.3] \rangle$	$\langle [0.2, 0.3], [0.4, 0.6] \rangle$
$u_5$	$\langle [0.1, 0.4], [0.3, 0.5] \rangle$	$\langle [0.7, 0.8], [0.1, 0.2] \rangle$	$\langle [0.5, 0.6], [0.2, 0.3] \rangle$	$\langle [0.2, 0.3], [0.5, 0.6] \rangle$

**Table 3.** Second interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}^{(2)}$ .

	$o_1$	$o_2$	$o_3$	$o_4$
$u_1$	$\langle [0.4, 0.5], [0.2, 0.4] \rangle$	$\langle [0.3, 0.5], [0.4, 0.5] \rangle$	$\langle [0.4, 0.6], [0.3, 0.4] \rangle$	$\langle [0.3, 0.4], [0.4, 0.6] \rangle$
$u_2$	$\langle [0.3, 0.4], [0.4, 0.6] \rangle$	$\langle [0.1, 0.3], [0.3, 0.7] \rangle$	$\langle [0.6, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.6, 0.8] \rangle$
$u_3$	$\langle [0.6, 0.7], [0.1, 0.2] \rangle$	$\langle [0.3, 0.4], [0.4, 0.5] \rangle$	$\langle [0.7, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.7, 0.8] \rangle$
$u_4$	$\langle [0.5, 0.6], [0.1, 0.3] \rangle$	$\langle [0.2, 0.3], [0.6, 0.7] \rangle$	$\langle [0.4, 0.6], [0.3, 0.4] \rangle$	$\langle [0.3, 0.4], [0.4, 0.6] \rangle$
$u_5$	$\langle [0.1, 0.3], [0.3, 0.5] \rangle$	$\langle [0.6, 0.8], [0.1, 0.2] \rangle$	$\langle [0.5, 0.6], [0.2, 0.4] \rangle$	$\langle [0.2, 0.4], [0.5, 0.6] \rangle$

**Table 4.** Third interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}^{(3)}$ .

	$o_1$	$o_2$	$o_3$	$o_4$
$u_1$	$\langle [0.4, 0.7], [0.1, 0.2] \rangle$	$\langle [0.4, 0.5], [0.2, 0.4] \rangle$	$\langle [0.2, 0.4], [0.3, 0.4] \rangle$	$\langle [0.3, 0.4], [0.2, 0.4] \rangle$
$u_2$	$\langle [0.3, 0.5], [0.3, 0.4] \rangle$	$\langle [0.2, 0.4], [0.4, 0.5] \rangle$	$\langle [0.6, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.6, 0.8] \rangle$
$u_3$	$\langle [0.6, 0.7], [0.1, 0.2] \rangle$	$\langle [0.4, 0.5], [0.3, 0.4] \rangle$	$\langle [0.5, 0.7], [0.1, 0.3] \rangle$	$\langle [0.1, 0.3], [0.5, 0.7] \rangle$
$u_4$	$\langle [0.5, 0.6], [0.1, 0.3] \rangle$	$\langle [0.1, 0.2], [0.7, 0.8] \rangle$	$\langle [0.5, 0.7], [0.2, 0.3] \rangle$	$\langle [0.2, 0.3], [0.5, 0.7] \rangle$
$u_5$	$\langle [0.3, 0.5], [0.4, 0.5] \rangle$	$\langle [0.6, 0.7], [0.2, 0.3] \rangle$	$\langle [0.6, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.6, 0.8] \rangle$

**Table 5.** Fourth interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}^{(4)}$ .

	$o_1$	$o_2$	$o_3$	$o_4$
$u_1$	$\langle [0.6, 0.7], [0.2, 0.3] \rangle$	$\langle [0.4, 0.5], [0.4, 0.5] \rangle$	$\langle [0.4, 0.5], [0.3, 0.4] \rangle$	$\langle [0.3, 0.4], [0.4, 0.5] \rangle$
$u_2$	$\langle [0.3, 0.4], [0.3, 0.4] \rangle$	$\langle [0.1, 0.2], [0.2, 0.3] \rangle$	$\langle [0.6, 0.7], [0.1, 0.3] \rangle$	$\langle [0.1, 0.3], [0.6, 0.7] \rangle$
$u_3$	$\langle [0.7, 0.8], [0.1, 0.2] \rangle$	$\langle [0.3, 0.4], [0.5, 0.6] \rangle$	$\langle [0.5, 0.8], [0.1, 0.2] \rangle$	$\langle [0.1, 0.2], [0.5, 0.8] \rangle$
$u_4$	$\langle [0.5, 0.6], [0.1, 0.3] \rangle$	$\langle [0.2, 0.3], [0.4, 0.6] \rangle$	$\langle [0.4, 0.5], [0.2, 0.3] \rangle$	$\langle [0.2, 0.3], [0.4, 0.5] \rangle$
$u_5$	$\langle [0.1, 0.2], [0.5, 0.7] \rangle$	$\langle [0.6, 0.7], [0.1, 0.2] \rangle$	$\langle [0.5, 0.6], [0.3, 0.4] \rangle$	$\langle [0.3, 0.4], [0.5, 0.6] \rangle$

Step 1. Using the normal distribution based method [32], the weight vector  $\alpha = (0.155, 0.345, 0.345, 0.155)^T$  is obtained for operator  $\theta$ . After that all four IVIF decision matrices  $\tilde{D}^{(k)}$  ( $k = 1, 2, 3, 4$ ) are aggregated into a collective IVIF decision matrix  $\tilde{D}$  by using operator  $\theta$ . This matrix  $\tilde{D}$  is listed in Table 7.

Step 2. The information about attribute weights, provided by DMs  $d_1, d_2, d_3$  and  $d_4$ , is given in Table 6. To calculate the group weights for each attribute and these calculated weights are as follows:

$$\begin{aligned} \tilde{w}_1 &= \langle [0.1679, 0.3960], [0, 0.0001] \rangle, \tilde{w}_2 = \langle [0.1516, 0.4565], [0, 0] \rangle, \\ \tilde{w}_3 &= \langle [0.2802, 0.6581], [0, 0] \rangle, \tilde{w}_4 = \langle [0.2107, 0.5392], [0, 0.0001] \rangle, \\ \tilde{w}_5 &= \langle [0.1872, 0.4493], [0, 0.0002] \rangle. \end{aligned}$$

Using proposed order function  $\psi$ , the crisp values of  $\tilde{w}_i$  ( $i = 1, 2, 3, 4, 5$ ) are determined and these values are:

$$w_1 = 0.7422, w_2 = 0.7579, w_3 = 0.8591, w_4 = 0.8046, w_5 = 0.7676.$$

Normalised the value  $w_i$  for each  $i$ , to get the values of optimal weights  $w_i^*$  for  $i = 1, 2, 3, 4, 5$ . These values are as follows:

$$w_1^* = 0.1888, w_2^* = 0.1928, w_3^* = 0.2185, w_4^* = 0.2047, w_5^* = 0.1952.$$

Step 3. To calculate the entries of the weighted collective IVIF decision matrix  $\tilde{D}^*$  which is listed in following Table 8.

Step 4. Utilize the function  $\psi$  to calculate the entries of the decision matrix  $D^*$ . The calculated decision matrix  $D^*$  is listed in Table 9.

Step 5. Calculate the overall attribute value by using the operator  $\phi$  for each alternative  $o_j$  ( $j = 1, 2, 3, 4$ ). These values are as follows:

$$\phi_\omega(o_1) = 0.1714, \phi_\omega(o_2) = 0.1350, \phi_\omega(o_3) = 0.2011, \phi_\omega(o_4) = 0.06555.$$

where  $\omega = (0.36, 0.16, 0.16, 0.16, 0.16)^T$  is the weight vector to operator  $\phi$  for  $\alpha = 0.2$ .

Step 6. From step 5, we get

$$\phi_\omega(o_3) > \phi_\omega(o_1) > \phi_\omega(o_2) > \phi_\omega(o_4).$$

Therefore, the alternative  $o_3$  is best according to this method.

### 6.1. Compare with Park et al.'s Approach [11]

Park et al. [11] considered an MAGDM problem with IVIF decision matrices. In this problem, the weights of attributes are partially known. They used the TOPSIS approach to find the best alternative or ranking of all the alternatives. With respect to different measures (Hamming distance, normalised Hamming distance, Euclidean distance, normalised Euclidean distance), in this method, the relative closeness of each alternative to the IVIF positive ideal solution is calculated. In their work, they attempt the same numerical problem with incomplete information about the attribute weights. The obtained ranking by the present

**Table 6.** The attribute weights.

	$d_1$	$d_2$	$d_3$	$d_4$
$u_1$	$\langle [0.1032, 0.1669], [0.7032, 0.7889] \rangle$	$\langle [0.0782, 0.1160], [0.8023, 0.8665] \rangle$	$\langle [0.0785, 0.1196], [0.7936, 0.8495] \rangle$	$\langle [0.0651, 0.0969], [0.8023, 0.8736] \rangle$
$u_2$	$\langle [0.0665, 0.1112], [0.8124, 0.8649] \rangle$	$\langle [0.0230, 0.0667], [0.7661, 0.8671] \rangle$	$\langle [0.1786, 0.2474], [0.6390, 0.7509] \rangle$	$\langle [0.0201, 0.0445], [0.9066, 0.9503] \rangle$
$u_3$	$\langle [0.1867, 0.2380], [0.6160, 0.7098] \rangle$	$\langle [0.0862, 0.1185], [0.8128, 0.8552] \rangle$	$\langle [0.1487, 0.2820], [0.6020, 0.7098] \rangle$	$\langle [0.0228, 0.0573], [0.8802, 0.9410] \rangle$
$u_4$	$\langle [0.1323, 0.1885], [0.6216, 0.7570] \rangle$	$\langle [0.0388, 0.0741], [0.8783, 0.9155] \rangle$	$\langle [0.1115, 0.1820], [0.7377, 0.7951] \rangle$	$\langle [0.0472, 0.0734], [0.8311, 0.8885] \rangle$
$u_5$	$\langle [0.0273, 0.0841], [0.8256, 0.8893] \rangle$	$\langle [0.1806, 0.2383], [0.6487, 0.7362] \rangle$	$\langle [0.1426, 0.1854], [0.7132, 0.7908] \rangle$	$\langle [0.0391, 0.0704], [0.8745, 0.9085] \rangle$

**Table 7.** Collective interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}$ .

	$o_1$	$o_2$	$o_3$	$o_4$
$u_1$	$\langle [0.4385, 0.6199], [0.1549, 0.2848] \rangle$	$\langle [0.3502, 0.4797], [0.3114, 0.4681] \rangle$	$\langle [0.3516, 0.4906], [0.2940, 0.4214] \rangle$	$\langle [0.3000, 0.4170], [0.3114, 0.4887] \rangle$
$u_2$	$\langle [0.3000, 0.4573], [0.3404, 0.4710] \rangle$	$\langle [0.1138, 0.3010], [0.2511, 0.4773] \rangle$	$\langle [0.6395, 0.7711], [0.0980, 0.2263] \rangle$	$\langle [0.1000, 0.2103], [0.6012, 0.7678] \rangle$
$u_3$	$\langle [0.6116, 0.7117], [0.1089, 0.2083] \rangle$	$\langle [0.3379, 0.4387], [0.3872, 0.4887] \rangle$	$\langle [0.5213, 0.7804], [0.0980, 0.2083] \rangle$	$\langle [0.1000, 0.2366], [0.5577, 0.7569] \rangle$
$u_4$	$\langle [0.5000, 0.6395], [0.0980, 0.2567] \rangle$	$\langle [0.1758, 0.3134], [0.5305, 0.6496] \rangle$	$\langle [0.4387, 0.6252], [0.2263, 0.3262] \rangle$	$\langle [0.2103, 0.3109], [0.4050, 0.5613] \rangle$
$u_5$	$\langle [0.1323, 0.3623], [0.3747, 0.5482] \rangle$	$\langle [0.6395, 0.7521], [0.1089, 0.2083] \rangle$	$\langle [0.5452, 0.6502], [0.1770, 0.3005] \rangle$	$\langle [0.1849, 0.3121], [0.5031, 0.6118] \rangle$

**Table 8.** Weighted collective interval-valued intuitionistic fuzzy decision matrix  $\tilde{D}^*$ .

	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$
$D_1$	$\langle [0.2, 0.4], [0.3, 0.5] \rangle$	$\langle [0.2, 0.6], [0.1, 0.2] \rangle$	$\langle [0.4, 0.7], [0.1, 0.2] \rangle$	$\langle [0.3, 0.5], [0.1, 0.3] \rangle$	$\langle [0.1, 0.4], [0.2, 0.5] \rangle$
$D_2$	$\langle [0.2, 0.6], [0.1, 0.3] \rangle$	$\langle [0.1, 0.4], [0.2, 0.4] \rangle$	$\langle [0.3, 0.6], [0.2, 0.3] \rangle$	$\langle [0.2, 0.5], [0.1, 0.4] \rangle$	$\langle [0.4, 0.5], [0.3, 0.4] \rangle$
$D_3$	$\langle [0.1, 0.3], [0.2, 0.3] \rangle$	$\langle [0.2, 0.5], [0.1, 0.3] \rangle$	$\langle [0.2, 0.7], [0.1, 0.2] \rangle$	$\langle [0.3, 0.6], [0.1, 0.3] \rangle$	$\langle [0.3, 0.6], [0.1, 0.3] \rangle$
$D_4$	$\langle [0.3, 0.5], [0.2, 0.3] \rangle$	$\langle [0.1, 0.3], [0.2, 0.4] \rangle$	$\langle [0.2, 0.6], [0.1, 0.3] \rangle$	$\langle [0.1, 0.5], [0.2, 0.4] \rangle$	$\langle [0.1, 0.3], [0.2, 0.5] \rangle$

**Table 9.** Decision matrix  $D^*$ .

	$O_1$	$O_2$	$O_3$	$O_4$
$u_1$	0.1582	0.1061	0.1101	0.0908
$u_2$	0.0979	0.0607	0.02368	0.0343
$u_3$	0.2466	0.0883	0.2514	0.0433
$u_4$	0.1958	0.0602	0.1633	0.0683
$u_5$	0.0642	0.2348	0.1813	0.0591

approach is similar to that given by Park et al.’s approach [11] with only the alternatives  $o_1$  and  $o_2$  interchanged. Some advantages of the proposed approach are as follows:

- (i) This approach provides a pertinent tool especially for solving a more complicated IVIF MAGDM problem in which attribute values and attributes weights are characterised in terms of IVIFNs.
- (ii) The present approach is simpler and easy to implement.
- (iii) The best alternative is the same in both the approaches which shows the reliability of the proposed approach.

## 7. Conclusions

In some cases, determining ratings of alternatives on attributes and attribute weights usually depend on decision makers’s judgement and intuition, which are often vague and cannot be represented with crisp value and fuzzy numbers. In such a case, the concept of interval-valued intuitionistic fuzzy sets is a trustworthy tool for group decision making. In the present work, a multiple attribute group decision making problem is modelled in which ratings of alternatives on attributes with the attribute weights are represented by interval-valued intuitionistic fuzzy numbers. In this study, a method is introduced for solving such decision problems. In this way, a new order function is made for comparing two or more interval-valued intuitionistic fuzzy numbers. The technique for finding the attribute weights is based on this function.

Using the proposed order function, interval-valued intuitionistic fuzzy hybrid geometric and ordered weighted averaging operators, the present method provides a most preferred alternative in six steps. On the basis of a comparative study, it can be observed that the present method is simple and easy to implement for solving the interval-valued intuitionistic fuzzy multiple attribute group decision making model occurrence in real-life decision making problems. A numerical example is given to illustrate the proposed method.

In future work, the present approach can be extended to interval-valued Pythagorean fuzzy sets, interval-valued cubic intuitionistic fuzzy sets, linguistic interval-valued Atanassov intuitionistic fuzzy sets and other uncertain and fuzzy environments.

## Acknowledgments

The authors would like to thank Editor-in-Chief, associate editor and the anonymous referees for their suggestions which helped to improve the quality of the presentation.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

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