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Classroom discourse and Teacher talk influences on English language learner students' mathematics experiences

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Classroom Discourse and *Teacher Talk* Influences on English Language Learner Students' Mathematics Experiences

by

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Classroom Discourse and *Teacher Talk* Influences on English Language Learner Students' Mathematics Experiences

Mariana M. Petkova

ABSTRACT

This study examined the features of the classroom discourse in eight Algebra I classes from two urban high schools with diverse student populations. In particular, by using the discursive analysis perspective, the type of communication between teachers and students was examined. The study investigated to what extent teachers' patterns of discourse change as a result of the number of ELLs present or their particular teaching experiences and ESOL endorsement. Furthermore, the impact of teachers' cultural and linguistic backgrounds upon ELLs' mathematics experiences was explored, particularly the teachers' patterns of discourse and adjustments to their *teacher talk*, or modifications of instructions that contributed to ELLs' engagement in the mathematics classroom.

Data analysis from various sources (observations, video-recordings, frequency counts, interviews, the teachers' self-evaluations, and the researcher's and the ELLs' evaluations) indicated that to some extent all teachers changed their patterns of discourse simply due to the presence of ELLs, regardless of the total number in the class. Teachers with more teaching experience and with ESOL training had a smaller number of ELLs in

their classes, whereas in both schools the novice teachers were assigned to teach classes with the highest number of ELLs. The novice teachers frequently used almost the same strategies as their more experienced colleagues did. Yet the qualitative analysis of the *type of modifications to their speech they made*, the *type of questions they asked*, and the *provision of information of higher cognitive demand according to Bloom's Taxonomy* indicated that even though all teachers needed improvement in using these strategies, the more experienced teachers with ESOL training applied those strategies to a fuller extent. They more often *used slower and simpler speech* and *different questioning techniques* sensitive to the ELLs' level of English language acquisition (i.e., *pre-production, early-production, and speech emergence*) and *provided the students with content specific, enriched information.* However, they still did not ask enough questions that could provide the ELLs with opportunities to justify and explain their opinions, and rarely led the discussions to a point which could move the ELLs to the highest level of the subject-specific literacy – *intermediate speech* and fluency in mathematics in English.

CHAPTER I: INTRODUCTION, PURPOSE OF THE STUDY, RESEARCH QUESTIONS, DEFINITIONS AND TERMINOLOGY CLARIFICATION

The impending changes that accompany the United States' continued transition into the highly technological and specialized twenty-first century pose a unique challenge to its current educational system. The challenge arises from the need to provide equal access to a high quality education to a constantly increasing and diverse student population. According to the U. S. Census Bureau (2004), the number of people (five or more years old) in the United States who speak a language other than English at home increased from nearly 47 million in the year 2000 to 49.6 million in 2004. This accounts for nearly 18.5% of the total U. S. population. The changes in the general population of the country are inevitably reflected in the schools, with an increasing number of students categorized as English Language Learners (ELLs). The need to address the demographic shift in the schooling population poses serious questions for those involved with the educational system, including teachers, administrators, teacher educators, publishers, curriculum developers, politicians, researchers, as well as parents, and the students themselves.

The dilemma in the field of mathematics education, in essence, is that of providing each student with a quality and challenging mathematics education independent of his/her initial level of proficiency in the English language. At the same time, it is not sufficient for such students to attend the same schools, have the same teachers, same textbooks, and be exposed to the same curriculum as their fluently English speaking peers. ELLs are not provided with equal education and opportunities if they do not understand the material because of a lack of fluency in the language in which this material is presented (Lau versus Nichols, 1974). Thus, the ultimate goal must be to develop these students' knowledge both in mathematics and in the English language. To do so, schools must still provide *all* students with the knowledge and skills necessary to develop their abilities to creatively apply mathematics, to analyze problems and determine the most appropriate ways to solve them (Glenn Commission, 2000; U. S. Department of Education, 2001).

It is important to consider the statement by the Mathematics Learning Study Committee of the National Research Council (2002),

> Proficiency [in mathematics] is much more likely to develop when a mathematics classroom is a community of learners rather than a collection of isolated individuals. In such a classroom, students are encouraged to generate and share solution methods, mistakes are valued as opportunities for everyone to learn, and correctness is determined by the logic and structure of the problem, rather than by the teacher. (p. 26)

This notion is consistent with the National Council of Teachers of Mathematics (NCTM, 1991) recommendation that "the discourse of a classroom—ways of representing, thinking, agreeing and disagreeing—is central to what students learn about mathematics" (p. 34).

Several studies have investigated the influences of classroom discourse on students' learning in mathematics (Ben-Yehuda, Levy, Linchevski, & Sfard, 2005;

McNair, 2000; Sfard, 2002). Other studies have examined the specific nature of the mathematics classroom (Jacobson, & Lehrer, 2000; McClain, & Cobb, 1998; McNair, 1998, 2000) or small-group work (Blunk, 1998; Leonard, 2000; Zack, 1999) environment when discourse is a feature. Furthermore, a group of studies have investigated the role of mathematics teachers' beliefs and perceptions about discourse (Blanton, 2002; Blanton, Berenson, & Norwood, 2001; Branderfur, & Frykholm, 2000; Nathan, & Knuth, 2003; Renne, 1996). Another group of studies investigated teachers' instructional practices and employed strategies in promoting discourse (Patrick, Turner, Meyer, & Midgley, 2003; Sherin, 2002; Turner, Meyer, Midley, & Patrick, 2003; Wood, 1999). Some studies have focused specifically on mathematics teachers' interaction patterns (Forman, & Ansell, 2001, 2002; Kovalainen, Kumpulainen, & Vasama, 2001; Rittenhouse, 1998), teacher questioning techniques (Steele, 1999-2000), and error treatment and feedback (Weingrad, 1998). Other studies have examined the role of individual students' communication in relation to the mathematics classroom culture and discourse (Bills, 1999; Davidenko, 2000; Manouchehri, & Enderson, 1999).

Only a relatively small group of researchers have focused their efforts on investigating the nature of classroom discourse when ELL students with linguistically and culturally diverse backgrounds are present (Brenner, 1994, 1998; Davidenko, 2000; Moschkovich, 1999, 2002). However, questions such as whether mathematics teachers' patterns of discourse relate to the number of ELL students present in the classroom, how a mathematics teacher's experience and English Speakers of Other Languages (ESOL) endorsement relate to his or her patterns of discourse, and how teachers' own linguistic and cultural backgrounds affect their patterns of discourse when teaching mathematics in

English, and especially to classes with ELL students present, still remain open for investigation.

Purpose of the Study

It is important that pre- and in service teacher education programs provide information to assist teachers to reach *all* students and improve their instructional practices so “that all students have the opportunity to develop their mathematical potential, regardless of a lack of proficiency in the language of instruction” (NCTM, 1989, p. 142). Recent *Standards* documents reveal that current reform efforts “[demand] that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (NCTM, 2000, p. 12). To illustrate this need NCTM points out that ELLs “may need special attention to allow them to participate fully in classroom discussions” (p. 13). The aim of this study is to examine features of mathematics classroom discourse that may contribute to ELLs’ engagement in the mathematics classroom. In particular, the study examines the impact of students’ and teachers’ cultural and linguistic backgrounds on students’ experiences in mathematics.

Research Questions

Specifically, the study examines the type of communication that occurs between teachers and students in mathematics classrooms when ELLs are present. The study addresses the following research questions:

1. To what extent do teachers’ patterns of discourse in the mathematics classroom change as a result of the number of ELL student(s) present?

2. To what extent do mathematics teachers' experiences and teachers' ESOL endorsement (i.e., training) relate to their patterns of discourse when teaching mathematics to classes with ELLs present?
3. How do teachers' own linguistic and cultural backgrounds affect their patterns of discourse when teaching mathematics in English to classes with ELL students present?
4. What patterns of discourse do teachers use when ELLs are present in the mathematics classroom? What adjustments to *teacher talk* or modifications of instructions are observed?

Definitions and Terminology Clarification

Extant literature uses varying terminology to classify children who learn mathematics in their non-native tongue. As a result, it becomes necessary to clarify the intent of the terminology used in this manuscript. For example, Bradby (1992) provided the following definition for *Language Minority (LM)* and *Limited English Proficient (LEP)* students:

> *Language Minority* refers to children who come from homes in which a non-English language is spoken. The English language skills of language minority children range from not being able to speak English at all to being very fluent in English. Since [*sic*] those who study language acquisition are still debating about definitions, *Limited English Proficient* has several definitions; conceptually, however, LEP means that the children have sufficient difficulty with English that they are at a disadvantage in classes taught entirely in English. (p.1)

In the literature, LEP is used synonymously with *English Speakers of Other Languages (ESOL)* and *English Language Learners (ELLs).* In this manuscript, the term ELLs will be used to emphasize the process of acquiring language skills while learning the content of mathematics. The emphasis is then placed on teaching mathematics content while teaching language. However, because the use of the phrase ELLs is in transition in the setting in which this study takes place, the term ESOL will also be used to represent the normative educational practices. In Florida, the terminology LEP or ESOL students is still used to be consistent with the language used in the Florida Consent Decree.

ESOL Requirements

In 1990, as a result of a lawsuit filed by the League of United Latin American Citizens (LULAC), Farm Workers' Association of Central Florida, Haitian Refugee Center, and similar organizations against the Florida State Board of Education, The Florida Consent Decree was signed. The Consent Decree addresses issues regarding the right of access to all educational programs by students whose primary language is not English. The settlement agreement was developed in compliance with "federal and state law and regulations including the federal Equity Educational Opportunity Act, Title VI of the Federal Civil Rights Act, of 19964, and Florida Educational Equity Act, and related federal and state provisions regarding compensatory, migrant, and special education" (Florida Consent Decree, 1990). As a result, the United States District Court for the Southern District of Florida—Miami Division issued a ruling by which ESOL endorsement became a requirement for any teacher who is a primary provider of instruction or services to ELLs. Category I (Primary Language Arts/English) Teachers in the State of Florida are required to complete 15 credit hours or 300 in-service credit points in courses specifically designed to help ELLs in the mainstream classroom.

The courses should address areas such as: (a) Methods of teaching English to speakers of other languages (ESOL); (b) ESOL curriculum and materials development; (c) Cross-cultural communication and understanding; (d) Applied linguistics; and (e) Testing and evaluation of ESOL. Six years or more are allowed for the completion of the ESOL Endorsement, or 3 years for K-12 ESOL Coverage obtained by a passing score on an ESOL Subject Area Test.

To meet state ESOL requirements, teachers of Basic Subject Areas (Social Studies, Mathematics, Science, and Computer Literacy) are grouped in Category II, with its own set of specific timelines and requirements. For example, a mathematics teacher who provides instruction to any ELL student is required to complete 3 credit hours or 60 in-service credit points of ESOL quality training and instruction. The courses should address methods of teaching the subject matter paralleled with: (a) Methods of teaching ESOL; (b) ESOL curriculum and materials development; and (c) Testing and evaluation of ESOL. The timeline for a beginning teacher to complete these requirements is two years, while for an experienced teacher the timeline is a year. However, the completion of these courses only grants compliance with the Florida Consent Decree's minimum requirements for subject area teachers, while for an ESOL Endorsement 15 credit hours are still needed. Recently, many colleges and universities offering degrees in education for Category I teachers include ESOL Endorsement as part of their graduation requirements.

Teacher Talk (TT)

In the literature *teacher talk* refers to the language used by teachers in classrooms as opposed to their use of language in other settings (at home, at the store, at the doctor's office, etc.). In this study, I will use Ellis' (1994) definition of ***teacher talk*** as the process through which "teachers address classroom language learners differently from the way they address other kinds of classroom learners. They make adjustments to both language form and language function in order to facilitate communication. These adjustments are referred to as 'teacher talk' (Ellis, 1994, p. 726).

Teachers' Patterns of Discourse

The phrase, *patterns of discourse,* will refer to the different types of communication a teacher used with his or her students. Krussel et al. (2004) referred to patterns of discourse as "teachers' discourse moves" (deliberate actions taken by teachers) to facilitate the discourse in the mathematics classroom.

Discourse

"The term 'Discourse' with a capital 'D,' [refers to] ways of combining and integrating language, actions, interactions, ways of thinking, believing, valuing, and using various symbols, tools, and objects to enact a particular sort of socially recognizable identity" (Gee, 2005, p. 21).

Mathematical Discourse and Mathematical Discourse Communities

Before discussing research regarding discourse in mathematics classrooms, some clarification of the concept of such discourse must be provided. The definition for *mathematical discourse*, and *mathematical discourse communities* provided in Sherin (2002) is certainly applicable:

> *the process of mathematical discourse* refers to the way that the teacher and students participate in class discussions. This involves how questions and comments are elicited, and through what means the class comes to consensus. In contrast, *the content of mathematical discourse* refers to the mathematical substance of the comments, questions, and responses that arise. (p. 206)

By extension, the term mathematical discourse communities refer to classroom environments where “students are expected to state and explain ideas and to respond to the ideas of their classmates. Teachers are asked to facilitate these conversations and to elicit students’ ideas (p. 207). Thus, “becoming a member of a mathematical discourse community involves learning to talk about mathematics in ways that are mathematically productive” (p. 208).

CHAPTER II: LITERATURE REVIEW

This chapter will present the collected research knowledge and leading views about the influences of classroom discourse on students' learning of mathematics. Of particular interest are studies from classroom-based and second language acquisition (SLA) research that provide insights regarding the influences of classroom discourse on what and how ELL students learn in mathematics classrooms. A related area of interest is research that provides information about the process of developing ELLs' literacy and reading skills in conjunction with the development of conceptual understandings and skills in mathematics. Specifically, theoretical and empirical work that addresses the following will be discussed:

1. Difficulties that ELLs face in the mathematics classroom.
2. The nature of the mathematics classroom when discourse is a feature.
3. The role of the teacher in promoting discourse as a means of negotiating meaning in mathematics.
4. The relationship between interventions and ELLs' mathematics achievement.
5. Methods of assessment and their effect on ELLs' mathematics experiences.

Examination of findings from these studies will provide insights on the current state of knowledge regarding influence of classroom discourse on ELLs' learning of mathematics. In addition, research approaches used in these studies guided the formation of approaches utilized in the study that will be described later.

Difficulties that ELLs Face in the Mathematics Classroom

Students' Learning of Mathematics from the Discourse Perspective

Socio-cognitive theories provide an invaluable perspective on students' learning of mathematics in an environment that fosters classroom discourse. According to Vygotsky (1978), learning occurs when the individual internalizes external knowledge to supplement his/her knowledge. Such external knowledge can be accessible from interactions with other individuals possessing different or more knowledge in the domain under discussion. Bakhtin (1981) supports this notion and suggests that the process of learning is intrinsically both social and individual.

Based on these perspectives, one can conclude that discourse between a learner (the student) and an expert (the teacher or peer) contributes to the learner's cognitive development (and learning of mathematics in particular). As such, "the shift in perspective from looking at mathematics learning as an internal reasoning process to looking at what is *interactively* accomplished through talk is a critical one to note" (Hicks 1998, p. 243).

Research reveals how the external knowledge accessed in interactions with the teacher and peers is internalized, and how new meaning and understanding of mathematics is appropriated in order to form new knowledge. For example, Sfard (2002) reported on how students develop their thinking and learn new knowledge in mathematics by becoming skillful in the discursive use of new symbolic tools— specifically, a bar diagram and a dot plot (a visual representation of data similar to the bar diagram, where a series of dots are used instead of bars). The flow of discourse as students solved two word problems was examined. In particular, the study examined how the discourse was mediated by the graphic display of data, how a specific graphic display was accessed by the learners and used in initiating specific discourse, and finally, how much learning occurred as a result of such discourse (i.e., how skillful the students had become in participating in the mathematical discourse). Distinguishing between **pronounced** (a specific question), **attended** (the procedure involved in answering the question) , and **intended focus** (the answer), the analysis of the learning episodes revealed that the process by which students move from a pronounced focus to an attended and an intended focus is not straightforward, but is rather complex and happens gradually or in cycles. In order to solve the problem, students use ***intimations*** ("an association of the present situation with an experience of the past that enables a new discursive decision" (p. 331)) and ***implications*** (examining the applicability of their decisions). For example, if the **inducement** element (the element encouraging discussions) of the present situation asks for "better" batteries, the word "better" induces the association with the **source** "longer lasting" in the student's mind, which is translated into the **target** of "longer bar" on the graph, and then the **decision** is made to draw the upper limit line through the tip of the longest bar. In this tendency to draw presumed inferences, Sfard noted that sometimes discursive decisions are influenced by students' inherent assumptions about the discursive mechanism, or presumptions about the expected solution (metalevel intimations). Nevertheless, this study indicated that students in discourse-rich environments significantly improved their abilities to participate in mathematical discourse (thus learning occurred).

McNair (2000) reported on the characteristics of mathematics classroom discussions that result in better student learning. Two small group discussions were compared to investigate the factors that contribute to maximum or minimum learning outcomes. The research findings indicated that discussions that have three main characteristics — mathematical subject, purpose, and frame – provide maximum learning opportunities for students. The mathematical subjects are usually numbers, shapes, spaces, variables and the patterns and relationships between them; a mathematical purpose could be to solve a mathematical problem and must “add structure and understanding to mathematical systems of reasoning” (p. 206), and a mathematical frame is the system of organization of students’ experience in searching for patterns, generalizing and formalizing procedures, making connections, logical reasoning, proofs, and communicating their ideas.

Ben-Yehuda, Lavy, Linchevski, and Sfard (2005) provided additional information regarding how students’ learning of mathematics can be maximized. The unique aspect of this study is the methods used to investigate the mechanism of failure in mathematics of two students with learning difficulties. The results revealed that some students’ failure in mathematics is due to the instructor’s inability to use discourse to improve students’ comprehension and problem-solving skills in mathematics. The result is an inability to provide each individual student with a choice of tools to approach mathematics problems without the fear of exclusion. The researchers noted that each student’s learning potential can be significantly maximized with improvements in discourse that recognize individual needs and abilities and that exploit each student’s strengths.

Mathematics Classroom Discourse when ELLs are Present

The research discussed above reveals the mechanism by which external knowledge accessed in interactions between the teacher and the class is internalized and how new meanings and understandings of mathematics are appropriated in order to form new knowledge. However, there is a paucity of research (Brenner, 1994, 1998; Moschkovich 1999, 2002; Secada, 1996) that specifically focuses on the nature of the discourse in mathematics classrooms when the students present have linguistically and culturally diverse backgrounds. Although the recent reform agenda in education is oriented toward trying to involve *all* students in meaningful communication in the mathematics classroom (NCTM 1989, 1991; U.S. Department of Education, 2001), very little is known about what occurs in the classroom when a large population of ELL students is trying to learn mathematics at the same time as they are learning the language in which the subject is taught.

Second Language Acquisition (SLA) and Literacy Development in Connection with Mathematics

The process of learning mathematics cannot be examined in isolation from other aspects of learning such as cognitive development, general literacy development, language learning, writing and reading development not only for ELLs but for the student population in general. To that end, in this section I will discuss research that focuses on these issues. Specifically, research will be discussed that examines development of students' general literacy (listening, speaking, writing and reading abilities) simultaneously with the development of their mathematics conceptual understanding.

English Language Proficiency Influences on ELLs' Experiences in Mathematics

Several studies reveal a positive correlation between English proficiency and achievement in mathematics (Abedi and Lord, 2001; Bradby, 1992). Bradby (1992) examined a variety of factors to determine which of them were predominantly influencing the performance on achievement tests of ELL students from Asian and Hispanic backgrounds. The data from the National Educational Longitudinal Study of 1988 were analyzed and findings revealed a direct relationship between Hispanic ELL students' achievement both in mathematics and reading and their English language proficiency and family's socio economic status (SES) characteristics. In contrast, he found that for Asian ELL students, their SES was a more influential factor on their achievement in mathematics than their English language proficiency.

Abedi and Lord (2001) reported the results for 1,174 eighth-grade students from 11 schools in Los Angeles with diverse linguistic, ethnic, and SES backgrounds after they had been tested on two mathematics tests (one with and one without linguistic modifications) created with the use of the National Assessment of Educational Progress's (NAEP) released items. The purpose of this study was to determine the influence of students' language proficiency on their achievement on mathematics tests in which there was an emphasis on word problems. Results indicated lower performance for students who were in the process of learning English as a second language as compared to language proficient students. Results also indicated higher improvement in scores for the language-deficient, as well as language-proficient, students on test items with linguistic modifications.

Cultural Influences on ELLs' Experiences in Mathematics

The literature that examines culture and mathematics education focuses on two primary areas: students' views on learning mathematics and culturally relevant teaching. More recently, because of the diversity of the student population in classrooms, research has focused on the need to provide culturally responsive or relevant instruction to students. Regarding the former, several chapters were written in the late 1980s that discuss the influence of students' cultural background on their views about mathematics learning. For example, several studies indicate that there is a relationship between the students' cultural background and their mathematical achievement (Cocking & Chipman, 1988; MacCorquodale, 1988; Leap, 1988; Tsang, 1988). Primarily, these studies suggest that students' culture provides a lens with which they may examine their experiences in mathematics. In some cases, the importance the community and the family places on mathematics can enhance students' experiences in learning the subject by providing additional motivation (Tsang, 1988), whereas in other cases it may limit the students' (Cocking & Chipman, 1988) and parents' (MacCorquodale, 1988; Leap, 1988) involvement in education, particularly in mathematics. For example, Cocking and Chipman reported that Hispanic women in their study perceived mathematicians as sloppy, remote, obsessive, and calculating, and thus, because of their perceptions they and their children tend to shy away from mathematics. Other research indicates that Native Americans tend to view education as something reflective, visual, and more holistic/global, and they learn better when they work in cooperative small group settings (Eieife, 2002; Reyhner, Lee, & Gabbard, 1993). Thus, if teachers are not familiar with individual students' ways of learning and cultural values, the typical competitive western education with its use of traditional auditory teacher centered setting may lead Native American students to underachievement in mathematics. The possibility of such reactions is an important consideration for the mathematics education community, particularly when the emphasis is placed on "mathematics for all."

Discourse and SLA: Genesis

The first traces of the idea that second language acquisition develops in interaction with others and not in individuals in isolation can be seen in Wagner-Gough and Hatch (1975). In investigating the interactions between children from different backgrounds (Chinese and Iranian) throughout their process of learning English, the researchers presented the following picture of second language acquisition. First, language learners produce forms (i.e., words indicating verbs, nouns) without understanding their functions. Then, they start understanding the functions of these forms by referring to their native language knowledge, and by trying to group frequently occurring forms without yet knowing the target language rules. Thus, they move to a stage where they start to understand the semantic difference between variations of forms of a verb for example, and start to incorporate them in their speech. However, if the process of rule formation is placed in a social discourse where the language learners are provided with an input from native speakers (NS) of the target language, then the process of second language acquisition is more effective. Thus, "we should not neglect the relationship between language and communication if we are looking for explanations for the learning process" (p. 307).

Another significant contributor in the development of the interaction perspective (later to lead to the discursive perspective in research) is Long (1981, 1983, 1985). According to Long (1980), "*Input* refers to the linguistic forms used" (morphemes, words, utterances) and studies concentrating on input usually consider the forms the learner is exposed to; while "by *interaction* is meant the functions served by those forms, such as expansion, repetition and clarification" (p. 259) and studying interactions must concentrate on “describing the functions of those forms in (conversational) discourse” (Long, 1983, p. 127). Thus, to investigate the interactions of students and teachers in the classroom the researcher must take into account the participation in conversation of both the native speakers (NS) and nonnative speakers (NNS), taking turns, negotiation of meaning, etc.

Long (1981, 1983) analyzed the interactions between NS-NS and NS-NNS, where the NNS in his study were from various linguistic backgrounds. Long found that NS do use more modifications to the input when they interact with NNS in comparison to interaction with NS. These modifications include more frequent use of self- and other-repetitions, lower type-token ratio (i.e., slower speech patterns), comprehension and confirmation checks, and expansions. Modifications appeared to be used in order to avoid conversational difficulties or to repair the discourse when difficulties in conversation already had occurred. Later, Long (1996) formulated an updated version of the Interaction Hypothesis which relates the factors of importance in SLA: “ *negotiation for meaning*, and especially negotiation work that triggers *interactional* adjustment by the NS or more competent interlocutor, facilitates acquisition because it connects input, internal learner capacities, particularly selective attention, and output in productive ways” (pp. 451-452). Pica, Young, and Doughty’s (1987) study also revealed a positive impact of interactions and negotiation of meaning on comprehension. This study revealed that interactional modifications of input lead to more significant levels of comprehension than conventional ways of linguistically simplifying input. In Mackey (1995), ELLs participated in communicative tasks; some learners received a pre-modified input with no opportunities to interact, while other learners could use interaction in the process of input.

The study indicated that the learners who participated in interactions progressed more quickly in their SLA development.

Classroom discourse, SLA and learning in mathematics

Many researchers focused their attention on the interactions that occur in a classroom setting because a significant part of second language learning takes place in such an environment. Parallel to such research in the field of SLA, in the past decade, a similar trend has developed in classroom-oriented research by subject area specialists. This line of research adopts the discourse perspective in investigating the mechanisms by which ELL students learn in different content areas while faced with the obstacles of adjusting to a new culture and learning the language in which the different content areas are taught in school.

Regarding mathematics, Brenner (1998) and Moschkovich (1999, 2002) have investigated the nature of classroom discourse when the students involved have linguistically and culturally diverse backgrounds. Moschkovich (1999) observed discussions in a computer-based dynamic instructional environment, noting which teaching techniques improved ELLs' participation in the mathematics discussions about the geometric shapes and figures in a tangram puzzle. She found that teachers improved ELLs' participation in discussions by employing techniques such as utilizing objects to encourage students to talk about their properties and characteristics, giving sufficient time for group discussions (student-to-student discussions), asking students to repeat their statements using different expressions in order to clarify their statements, and using "revoicing" (reformulating the students' statements using formal mathematical terms) in order to show acceptance of the ELLs' responses and thus encourage their participation in discussions. Instead of correcting ELLs' linguistic mistakes and concentrating on language development, the teacher focused on whether students demonstrated conceptual understanding. Moschovich (2002) examined bilingual students' learning of mathematics with English as a second language (L2). Findings indicated that bilingual students may be capable of communicating meaning and competence in mathematics without learning the correct vocabulary— by using gestures, objects, or everyday examples as resources, or simply by using their first language (L1). Findings also indicated that involving bilingual students in classroom discourse provided them with practice leading not only to their L2 development, but also their mathematical development.

An examination of the mathematical communication in two algebra classes with large populations of ELLs (predominantly Hispanics) revealed that in classrooms in which small-group discussions were encouraged and computers were employed to stimulate discussions, more successful mathematical communication was exhibited, which later was spread to a large-group setting (Brenner, 1998). In contrast, in the class in which the teacher employed mostly whole-classroom instruction, the ELL students were more reluctant to speak aloud in front of a large group.

The Nature of the Mathematics Classroom Environment when Discourse is a Feature

Many researchers have studied the nature of the mathematics classroom environment when discourse is a feature. Although they did not specifically look at the effects of such an environment on ELL students' learning of mathematics, the findings that are reported are valid for all learners of mathematics and thus for ELLs as well (Blunk, 1998; Jacobson, & Lehrer, 2000; Leonard, 2000; McClain, & Cobb, 1998; McNair, 1998, 2000; Zack, 1999).

Tasks and Discourse

A characteristic of classroom discourse that fosters and maximizes students' learning potential of mathematics is that it must have a mathematical subject (McNair 1998, 2000). This usually requires that students be involved in a meaningful mathematical task. However, the completion of the same task by different students does not guarantee that they will absorb the lesson equally as well. Jacobson and Lehrer's (2000) research provides evidence that the difference among students' discourse while solving the same task chiefly determines what these students will learn and retain. They examined the difference in classroom discourse and students' learning in four 2^{nd} grade mathematics classrooms. The students were to design a quilt by performing transformations (slides, flips, reflections, turns, and rotations) of "core squares" composed of different shapes of triangles using computer software. The researchers attended to the actions of the teachers and the nature of the discourse promoted and how this affected students' learning of the geometry involved in the project. The findings indicated that the patterns of discourse promoted by the teachers were related to differences in teacher understanding and knowledge of students' reasoning and learning process regarding geometry and space. For example, Teacher A emphasized the new terminology focused on the core squares' transformations. The students were asked to clarify and elaborate their ideas about space and transformations in geometry. Teacher B's focus was on "why" and "how" questions, which required students to explain the process of making a quilt: "And how did that change the design?" Thus, she encouraged the students to reflect on their thinking process of why and how they used geometric motions to make the quilt design. Teacher C also encouraged discussions, but her focus was on the content and not the process of making the design. Her use of *revoicing* was mostly in the form of repetition and clarification of what the students said and rarely asking them to further elaborate on their ideas. Similarly, Teacher D did not ask the students to reflect on the transformations used to make the quilt. Even though the students were involved in discussions, they were only asked to recognize shapes and colors. Further investigation was performed on the students' learning and retaining of the knowledge in each of these classes to measure the effectiveness of the classroom discourse. The results indicated that the more knowledgeable the teachers were about students' thought and learning process of geometry (in classes A and B), the more students learned and retained the gained knowledge about geometry transformations and their applications. Thus, these studies show that the teacher plays an important role in the creation of an environment that facilitates discussions in mathematics by trying to involve all students in meaningful tasks.

McClain and Cobb (1998) reported on the role of imagery and discourse in students' conceptual understanding of mathematics. The students were involved in a year-long teaching experiment using the instructional theory *Realistic Mathematics Education* (RME). According to this theory, the instruction should start with the teacher's description of a problem situation using statements in a way that "students can evoke imagery of the situations described in the problem statements when solving tasks" (p. 61). For example, the teacher in the study used a narrative about a pumpkin seller whose pumpkins were carried in crates of ten; the goal was for students to use this imagery to make sense of the task—working with tens. "In this way, the students' construction of situation-specific imagery allows them to engage in personally meaningful mathematical activity and constitutes a basis for the students' subsequent mathematization of activity" (p. 61). The teacher then encouraged students to participate in subsequent activities by using this imagery to explain their thinking in terms of relationships rather than numerical patterns. McClain and Cobb introduced the two terms *folding back* and *dropping back* of discourse to describe observed changes in the discourse when difficulties in communication were encountered. In the first situation, communication was based on the prior discourse activity in which the imagery was "taken-as-shared" from all students since they were familiar with it. In contrast, in the second situation, the teacher needed to introduce new information because of the lack of shared imagery. The teacher played a central role in directing discourse to use both strategies when difficulties in discourse were encountered.

Small-Group Work and Discourse

While most research investigates the nature of discourse in the classroom, some researchers have examined the characteristics and nature of discourse elicited in small-group work (Blunk, 1998; Leonard, 2000; Zack, 1999). For example, Leonard examined the discourse in the small-group work of three diverse sixth grade mathematics classrooms during a lesson on making a hydrometer and measuring humidity. He investigated the effects of different discourse patterns on students' (and teachers') learning. The results of the study indicated that the students' personalities affected their behavior in small-group discourse much more than other factors such as gender. Usually, assertive students (male and female) were more involved in discourse. Also, during the whole-classroom discussions the teachers had more control and used more institutionalized discourse (giving clues to students of the changes in the lesson and expectations from them), in small-group discussions the discourse was more emergent and natural in nature. However, planning the task in advance, giving hands-on activities, and applying group pairing based on gender did not always guarantee that the emergent discourse was mathematical in nature. Furthermore, observed patterns revealed that students often used previous knowledge to initiate discussions. The teachers who took advantage of this and used both emergent and institutionalized discourse were more successful in facilitating discussions. The results also indicated that throughout their discourse, students exhibited improved knowledge about relative humidity. They also exhibited an understanding and application of important vocabulary in context.

Zack (1999) reported on an the argumentation of proofs of three members of a small group with the goal "to convey the sounds of mathematical talk in a classroom and school culture in which the children have been encouraged since their entry to the school (for many, at 6 years of age) to engage in conversation about ideas" (p. 134). More specifically, the focus was on the students' use of logical connectives—"culturally grounded elements of language." The results revealed that the children used logical connectives such as *because, but,* and *if … then …* in order to create a strong argument and connect their ideas. They also used parallel logical and syntactic structures such as *if… then …, but it doesn't so you can't*, which also contributed to the logical coherence of their argument. This also demonstrated the children's development toward use of more formal mathematical language.

However, while Leonald (2000) and Zack (1999) highlighted students' talk in small-group discussions, Blunk (1998) focused on the communication of the teacher involved in creating and maintaining small-group discussions. The subject of this study was another researcher, Magdalene Lampert, and her fifth-grade mathematics class. Findings indicated that in order to create and maintain small-group discussions, the teacher viewed her role not as a transmitter of information or merely assigning students to groups, but rather as facilitating students' social and cognitive skills of communication about mathematics. For example, early in the year, the teacher talked about the characteristics and the nature of the small groups and why the students will work in such groups. Later in the year, the teacher made explicit her expectations for student behavior during small group interactions and explained how she would evaluate their group work. This case study suggests that allowing students to engage in sophisticated, complex discussions about mathematics, creating meaningful tasks for students to discuss within group interaction, and maintaining a climate in which the students' spirit of curiosity is encouraged, are more important than finding the "right" answer (p. 210).

The Role of the Teacher in Promoting Discourse as a Means of Negotiating Meaning in Mathematics

The Multiple Roles of the Teacher in Mathematics Classroom Discourse

Teachers' Beliefs and Perceptions about Discourse

Many researchers have concentrated their efforts specifically on the effects of teachers' beliefs about discourse and their impact on the classroom environment (Blanton, Berenson, & Norwood, 2001; Brenderfur, & Frukholm, 2000; Nathan, & Knuth, 2003; Renne, 1996). For example, Renne investigated the factors that influenced a teacher's attempts to incorporate students' questions and initiatives in classroom discourse. Although the teacher attempted to shift to a more student-centered instructional approach and to incorporate students' questions and initiatives, some deviations from this pattern were observed. Often students' initiatives were converted by the teacher into teacher initiatives. That is, some of their questions were not directly answered or were ignored. Consequently, communication in the classroom was in the traditional initiation-reply-evaluation (IRE) sequence (Mehan, 1979) where the teacher initiates (with a question or statement), a student responds, and the teacher evaluates the students' response (verbally or by a gesture). Further investigations indicated that the teacher's detours to such teacher-centered instructions were influenced mainly by cultural beliefs and assumptions about teaching, learning and knowledge. Additionally, a lack of details about how to implement the reform, time constraints to complete the course, the number of students in the class, and struggle for control were also found to be influential factors in the observed teacher's behavior.

Brendefur and Frukholm (2000) reported on an investigation of two preservice teachers' beliefs and perceptions about discourse in relation to their mathematical understanding and internship practices. The findings revealed that even though both teachers were similar in age, attended the same mathematics methods class that promoted the reform-based perspective of discourse, and were assigned to intern in the same school with similar teachers, each teacher employed different instructional practices. One of the teachers encouraged communication and facilitated students in sharing ideas, while the other used a teacher-centered approach. Further investigation indicated that the observed differences in teaching practices were in accordance with the teachers' initial beliefs and dispositions toward mathematics and its teaching and learning.

Nathan and Knuth (2003) investigated the effects of a sixth-grade teacher's beliefs on her instructional practices when discourse and interactions were promoted over a period of two school years. They reported the "pivotal role" of the teacher's goals and beliefs in shaping her classroom practices (p. 178). Specifically, by analyzing *the flow of information*, they found that even though the teacher believed that students learn from their peers when they actively share ideas, little student-to-student (S-to-S) talk occurred during the first year when she attempted to apply a reformed curriculum to promote discourse. The vast majority of communication was vertical, teacher-to-class talk (T-to-C), which is very similar to the traditional IRE sequence. However, during the second year, S-to-S talk increased to 33%. By analyzing *the nature of scaffolding*, they found that while during the first year the T-to-C communication was predominantly analytic (addressed mathematical content), during the second year it dropped to 50% analytical and 50% social in nature. The S-to-S analytical and social talk also showed a similar pattern. Furthermore, by *analyzing the patterns of interaction at a global level*, they found that while the teacher had a central role in interactions during the first year, during the second year "a star pattern" emerged with a less evident teacher authority (p. 198).

Similar findings were reported in Blanton (2002) and Blanton et al. (2001). Blanton et al. thoroughly examined one preservice teacher's perceptions of discourse and her teaching approach in a seventh grade mathematics classroom. They noted a change of pattern in her methodology. Initially she primarily used the IRE pattern of classroom discourse and perceived the teacher as "a teller." Later, her pedagogy shifted to using questions that explored student solutions and strategies. At this point the student was perceived as "a teller." This shift in instructional approach was a crucial step in the teacher using a dialogue-based form of discourse and perceiving the student as an active participant in mathematics discourse. This study thus contributed to the notion that "a teacher's developing practice is inherently linked to the social dynamics of the classroom" (p. 228).

Blanton (2002) found that pre-service teachers' initial beliefs, despite being very influential in the beginning of the teaching practice, could be changed by a reflective study of their classrooms' discourse. Discursive reflections could provide teachers with information not only about students' learning in mathematics, but also about how teachers themselves could learn how to *teach* mathematics more successfully.

Teachers' Expectations and Methods of Teaching and Their Effect on ELLs' Mathematics Experiences

Several studies link teachers' expectations of ELLs' performance in mathematics, and understanding (or lack thereof) of their learning process of mathematics, to the way the teachers teach mathematics (Davidenko, 2000; Rhine, 1995a, 1995b, 1999). For example, Rhine analyzed the tutoring sessions of intermediate grade teachers of classes that include ELL students and examined the teachers' expectations of the ELLs' performance in mathematics. Rhine videotaped the interactions between the teachers and students during these tutoring sessions, used recall interviews with teachers, and performed quantitative and qualitative evaluations of their assessments. From the videotaped sessions, Rhine found that the teachers tended to teach differently when ELLs were present in a group. He reported that the teachers' limited understanding of ELLs' mathematics learning became apparent during the interview process. Teachers often linked the lack of English proficiency to a similar lack of mathematical knowledge or understanding. When asked to make predictions about students' achievement on tests, the teachers usually underestimated the ELLs' performance in comparison to their English speaking peers.

Other studies suggest that teachers limit their instructional approaches when teaching classes with ELLs. For example, Davidenko (2000) investigated the instructional practices and communication used in two algebra classes that included ELLs in order to evaluate the effects of teaching methods on students' learning practices. The data collected by classroom observations, videotaped interactions, and interviews with the algebra and English as Second Language (ESL) teachers, and with 9 students (both ELLs and English speakers) from the two algebra classes were analyzed. Because mathematics teachers were aware that ELLs were present in the classroom, they often reinforced computational skills and "instrumental learning" (learning experiences involving reinforcement of good behavior). Additionally, they usually assumed that ELLs could not handle higher-level mathematics involving word problems, mathematics communication, and discussions in English about mathematics concepts. Consequently, students taught in such a manner received only a limited conceptual understanding of mathematics and their knowledge was only at the procedural and computational level. Davidenko concluded that the ELLs' proficiency in English was not the sole factor that influenced their performance in mathematics. Other very influential factors are the teachers' expectations and methods of teaching which also contribute to the students' learning process.

Teacher Talk and Voices Used in Discourse

Some scholars focus their attention specifically on studying the nature of the teacher's communication and on finding patterns that provide insight into how teachers facilitate classroom interactions (Forman, & Ansell, 2001, 2002; Kovalainen, Kumpulainen, & Vasama, 2001; Rittenhouse, 1998). For example, Rittenhouse investigated how Magdalene Lampert (a teacher/researcher) enacted discursive norms and routines in the first month of the school year with her fifth-grade mathematics class. Rittenhouse noted that the teacher facilitated discourse by skillfully employing techniques described as *stepping in* and *stepping out* of discussions. When the teacher *stepped into* discussion her talk was rather conversational in nature, and she mainly participated in discussions by asking questions, providing additional information from her knowledge base of mathematics and thus contributing to the predominantly student-communication by presenting new ideas and introducing and explaining new vocabulary. In contrast, when *stepping out* of discussion, the teacher talked in a more didactic manner. Here, she commented on discussions ("talk about the discussion") or was formally teaching the rules and norms the students should employ in order to participate in a polite argument. Thus, the study demonstrated how the teacher's talk and "her dual role as participant and commentator provide us insight into one teacher's vision of what fostering students' understanding of mathematics looks like" (p. 187).

Kovalainen et al. (2001) examined how teachers' use of scaffolding strategies (involving the students in building on one another's ideas) facilitated classroom interactions. The investigation identified four complementary and partially overlapping strategies of scaffolding: *evocative* (asking stimulating questions), *facilitative* (relating culturally established knowledge, revoicing, modeling, monitoring), *collective* (enforcing the rules of discussions), and *appreciative* (expressing support, interest, pacing the tempo). The study demonstrated how, by using these four strategies, the teacher orchestrates classroom interactions into productive discourse about mathematics.

Forman and Ansell (2001, 2002) investigated the nature of the teacher's communication in relation to his/her personal experience in mathematics. They found that teachers often use different *voices* (different types of teacher's talk) when discussing theoretical versus standard strategies in mathematics. For example, one teacher used one *voice* when orchestrating discussions of student-invented strategies, and used another when a standard algorithm was demonstrated. When using the first *voice*, the teacher usually emphasized students' persistence as critical thinkers and risk-takers. She often used revoicing and encouraged students' thinking. However, when using the second *voice*, the teacher often talked about her own past mathematics experience, or about the experience of the students' parents or older siblings. Then, she talked about the confusing nature of standard algorithms and their limited use. Revoicing was rarely used and the algorithms were not explicitly explained.

Teacher Questioning

Some research reports on the particular effects of some specific parts of *teacher talk*, such as questioning techniques, error treatment and provision of (or lack of) feedback (informing the student if his/her responses or remarks are correct or are accepted) on ELL students' experiences in the mathematics classroom. For example, Steele (1999-2000) investigated how one teacher employed discourse and questioning techniques to develop students' algebraic reasoning while finding patterns. The activity of finding size, color, shape, and number patterns in a calendar was used as a tool to develop reasoning skills and vocabulary building in context. Steele found that the teacher was able to create an atmosphere of productive discourse in which students were facilitated in the development of their algebraic thinking. The teacher achieved that by employing challenging questioning techniques which stimulated students’ high-level thinking. For example, she asked students not only to make predictions for possible patterns, but also to support them with logical reasoning. Additionally, the teacher skillfully involved students to provide logical arguments and to correct themselves when necessary. Writing in mathematical logs was used in order for students to organize their thoughts in anticipating the teacher’s questions and their possible answers. The teacher not only asked students questions, she was also an active listener. She was always open to change her initial plan based on students’ predictions and ideas. Thus, this study demonstrated how the teacher successfully “used questions to probe, stimulate, and initiate students’ algebraic thinking” (p. 96).

Error Treatment and Feedback

Weingrad (1998) investigated what type of error treatment and feedback provided to students from the teacher cultivated polite mathematical argumentation. The study also provided insights into how teachers encourage students to take risks and participate in discussions about mathematics by overcoming the “face-threatening acts” (FTAs) when voicing their opinions or making public statements. Weingrad found that the teacher achieved this by balancing between politely requesting for all students to participate (requesting for bids) and nominating particular students to do so. The teacher also used polite criticism (when students violate the norms and rules of interaction) or provided challenges to elicit further elaboration of ideas. Moreover, without simplifying the request or without repeating it, the teacher used a “second nomination of challenge” and thus implied to a student that his or her response does not need fixing, but rather more elaboration. Weingrad also demonstrated how the teacher used politeness strategies to repair “breakdowns” in discourse (a situation when a student offered an incorrect answer or idea) and to return the discourse to its usual pattern. Furthermore, Weingrad demonstrated how the teacher may let students know that he/she is interested in their ideas. However, the study was limited in scope because it did not provide information on how the students perceive the teacher’s politeness and how they respond to it personally.

The Role of Students in Mathematics Classroom Discourse

Bills (1999) examined the role of individual students’ communication in relation to the classroom culture and discourse. He applied linguistic comparative analysis to study the speech patterns of two high school boys involved in one-to-one interaction with the teacher. As “a useful lens through which to review the relationship between social positioning and mathematical enculturation in teacher-pupil relationships” (p. 162), Bills used *modality markers*. One example of such modality markers was the speaker’s use of propositions in attachment to “private” verbs (verbs whose value cannot be measured and is known only in relation to its subject) such as *think*, *believe*, *suspect*. The way the students used these particular markers was considered to demonstrate their commitment or detachment from the mathematics classroom/community. A similar modality marker involved examining the use of *we* and *you* in mathematics talk. Moreover, the speaker’s addition of such adverbs as *obviously*, *actually*, *frankly*, and the use of tag questions such as *isn’t it?* was also considered an indicator of commitment. Bills found that one student’s communication mode was more impersonal in nature. The student often used *we* and *you* to show commitment to the mathematics community. The fist-person singular pronoun *I* was mostly used to express mathematical fact or action rather than personal opinion or statement. That speaker also exhibited confidence of his knowledge of mathematics and use of the technical terminology involved by frequent use of adverbs such as *just* and *obviously*. For example: “So obviously if the gradient of the normal is -1/2 the gradient of the tangent will be 2” (p. 166). In contrast, the other student exhibited a more personal aspect of mathematics learning. He often demonstrated insecurities, and used questions in order to receive affirmation from the teacher of his ideas or actions. The significance of this study is in its modeling of how researchers might use linguistic analysis to examine the role of individuals (students or the teacher) in the social environment of classroom discourse.

Manouchehri and Enderson (1999) also reported on the examination of mathematics classroom discourse to illuminate the role of students in shaping the ambiance of such discourse. Their findings indicated that the students mutually influenced each others’ learning of mathematics by engaging in small-group or whole-classroom discussions about mathematics. The mechanism by which this learning occurred involved the compilation of argumentation, collaboration, negotiation of meaning, and refinement of conclusions. By extension, the students were also involved in systematic group inquiry, where they were actively involved in idea sharing, finding patterns, and collaboration.

Relationships between Interventions and ELLs’ Mathematics Achievement

Several studies revealed improvement in ELLs’ mathematics achievement versus just positive differences in ELLs’ experiences in mathematics, by examining standardized test scores. Some linked this to the adoption of bilingual programs (Liberty, 1998), or summer training programs for both teachers and students (Lara-Alecio, Cmajdalka, Parker, Cuellar, and Irby, 1996). Other studies specifically examined what effect teachers' instructional practices that promote discourse (i.e., specific teacher "discursive moves") have on ELLs' mathematics achievement (Patrick, Turner, Meyer, & Midgley, 2003; Sherin, 2002; Turner, Meyer, Midley, & Patrick, 2003; Wood, 1999). However, several studies indicated that standardized tests are usually based on an English speaking population and thus are inherently biased against ELLs. They suggested that new assessment instruments need to be developed in order to more accurately measure ELLs' achievement in mathematics (Abedi, Lord, Hofstetter, & Baker, 2000; Gronna, Chin-Chance, & Abedi 2000; Liu, Anderson, & Thurlow 2000).

Mathematics Instruction in Bilingual Programs

Educators have considered different outlets to enhance the academic experience of ELLs. Regarding mathematics, for example, students may receive mathematics instruction in their native tongue as part of a bilingual education program. These classes allow students to develop their mathematical understanding while developing their literacy skills in their native language and English. Results from several programs revealed that ELL students were able to make achievement gains in mathematics while engaged in these programs. For example, Liberty (1998) conducted a study to examine the effects of a 2-year program in a school that employed an English-as-a-Second-Language (ESL)/Transitional Bilingual Education Program. The program addressed staff development, material adoption, and parental education. On a content knowledge of mathematics test written in Spanish, ELL students showed achievement levels near the national average. These positive results were attributed to rigorous teacher professional development programs, ESL certification, the acquisition of better materials, and parent education.

As another example, the study conducted by Lara-Alecio, Cmajdalka, Parker, Cuellar, and Irby (1996) reveals the influence of a summer program on both teachers and students. They conducted a 3 year-long study (from 1993 to 1995) of 200 fifth-grade ELL students (mostly Hispanic) from an urban Houston public school. Students, eight bilingual teachers, and eight bilingual aids participated in a 6-week voluntary summer program to improve students' English proficiency. The mathematics content was used as a means for teaching English. The researchers analyzed the results from pre- and post-tests (students were given the choice of the language on each test) and investigated four main indices of pedagogy: "(a) Activity structures, (b) Language content, (c) Language of instruction, and (d) Communication mode" (p. 4). The results of the assessments indicated that ELL students gained mathematical knowledge in four targeted areas—fractions, charts and graphs, measurement and geometry, and problem solving. Most of the gain was observed during the first year of the program. Additionally, data collected from interviewing teachers revealed general satisfaction with the program, curriculum materials, real world problems orientation, and instructional strategies learned for teaching mathematics concepts in both languages. Teachers' aides and small class size were also pointed out as positive facets of the program.

Another program that reported positive results in the mathematics achievement of ELL students is the QUASAR (Qualitative Understanding Amplifying Student Achievement and Reasoning) project (Lane, Silver, and Wang, 1995). Results from that project indicated that gains were evident in all groups of students, including bilingually educated Latino students. All students benefited almost equally from this reformed mathematics education and sufficiently developed their reasoning skills and critical thinking in mathematics.

Alternative Mathematics Programs for Migrant Students

In a descriptive report, Celedon-Pattichis (2004) discussed various programs designed to assist migrant students in learning mathematics—The University of Texas Migrant Student Program, Project SMART, ESTRELLA, and The Portable Assisted Study Sequence. Each program was designed in order to incorporate migrant students' linguistic and cultural considerations in delivering the mathematics content. Some of the programs used distance learning forms for delivering instruction and assessment via e-mails, interactive discussions, and lessons on video or TV, while others delivered information face-to-face by providing tutoring sessions for the students during a convenient time- after school or during the summer. The programs were developed using accumulated knowledge about second language acquisition and incorporating Cummins' (1992) distinctions between students' exhibition of basic interpersonal communication skills (BICS) and cognitive/academic language proficiency (CALP) needed in academic subjects such as mathematics.

The author discussed the source of difficulties and challenges of mathematics word problems for migrant students. From think-aloud protocols teachers can understand what words help or hinder students' understanding of the problem. Celedon-Pattichis found that, often, the language used in mathematics word problems could confuse students because it contains a mixture of everyday social language with academic language. Thus, students are often confused with words having a double meaning in different discourse. Additionally, some word problems might be difficult to understand due to a cultural problem because students' personal experiences do not match the linguistic expression used for stating the problem. For example, students might not have a schema of "Hall's planetarium" because they have never heard of or visited one. Thus, the programs used word problems and activities in mathematics that incorporate migrant students' experiences. For example, an activity is suggested that can be used to teach migrant students the concept of distance ($d = r \cdot t$). In this activity, the students can be asked to plan a trip, find the best route, how long each route is, and plan the budget for the trip. The success rate of the programs in the recent years has made them widely used to meet the needs not only of migrant, but also of any alternative, nontraditional, culturally, and linguistically diverse students.

Teacher Education and In-Service Programs

Educators have considered different approaches for helping teachers enhance the mathematics learning experiences of ELL students. In this section, I highlight several projects that reform traditional education in mathematics in an attempt to accommodate ELL students. For example, Cahnmann and Hornberger (2000) implemented a 3-day summer institute to address "language-based mathematics learning of ESOL students from low-income urban contexts" (p. 42). During the workshop they presented teachers, administrators, and resource specialists with samples of student work to raise their awareness about the link between mathematics, language, and assessment practices. Based on the analysis of students' work, the educators reflected on the complexities that are associated with the use of content specific vocabulary and grammar involved in mathematics.

Other researchers have examined the effects of innovative programs on the mathematical performance of ELLs in a classroom setting. For example, Halpen, Patkowski, and Brooks (1996) examined the effects of a pilot program with a class at the City University of New York Brooklyn College, which combined the teaching of ESL with Calculus I. The results of the program demonstrated how the students' language developed through their study of the concepts, reading, and vocabulary of Calculus. Their lingual development was thus dually enhanced, as they employed a glossary of English terms commonly used in Calculus and mathematics in general. Similarly, Brenner (1998) investigated the use of an innovative mathematics program—College Preparatory Mathematics—with Hispanic ELL students in order to evaluate classroom communication. The results revealed that students in the classroom in which techniques such as small-group discussions were encouraged and computers were employed to stimulate discussions, showed more successful mathematical communication, which later spread to a large-group setting.

Culturally Relevant Education

During the past decade, much attention has been given to the need for culturally relevant or responsive instruction. This notion was highlighted by Ladson-Billings (1994) who examined the effective teaching of African American Students. Culturally relevant instruction refers to pedagogy that recognizes the importance of students' culture and empowers students by using cultural references as a springboard for student learning. Currently, culturally relevant instruction is encouraged as a means to address the needs of a diverse student population (Cahnmann & Remillard, 2002; Gay 2000; Gustein, Lipman, Hernandez & de los Reyes, 1997; Mattews, 2003). To provide instruction that is culturally relevant, teachers need to understand how students' culture (i.e., values, beliefs, customs, social norms, and language) influences their expectation for learning, their preferred learning styles (e.g., independent vs. collaborative), their preferred communication, and preferred problem solving style. For example, Cahnmann and Remillard (2002) studied the issues and challenges two teachers experienced while teaching mathematics in culturally, linguistically, and socio-economically diverse classrooms. They focused on the role of the teachers in providing equal mathematical experiences to *all* students while exercising culturally relevant teaching. Cahnmann and Remillard found that even though both teachers were deeply committed to foster mathematics understanding in their students, they implemented the ideas of culturally relevant teaching in different manners. The teachers had different interpretations of the reformed ideas, received different support and professional development in their educational communities, and had different comfort levels and knowledge in mathematics. The researchers also indicated that even though it might be beneficial for the cultural and the linguistic background of the teacher to be similar to that of the students, *all* mathematics teachers could use some ideas from research and incorporate culturally relevant instruction in mathematics to diverse student populations.

Mattews (2003) also studied how teachers utilized students' prior knowledge in mathematics, along with their diverse cultural background, in their instruction in order to develop students' critical thinking and empower them with experiences in mathematics related to their culture. Research results revealed that mere "good intentions" for teaching mathematics by using a culturally relevant approach are not enough. Deeper changes in teaching methods, practices, beliefs, values, and expectations are needed. He provided an illuminating example of a successful application of culturally relevant teaching when he discussed a teacher who was able to involve students in critical discussions about the real-life application of large numbers and scientific notation. The teacher skillfully fostered the students' critical thinking and engaged them in an activity to compare the areas of parks in the community, thus relating students' informal knowledge about large numbers and cultural background.

Mattews (2003) also provided examples of the challenges teachers encountered when they tried to incorporate culturally relevant teaching in their mathematics instruction. They sometimes failed to involve their students in critical discussion because the discussion of social issues often interfered with teachers' comfort level about such issues or their knowledge in mathematics. Sometimes, the mathematics experiences that teachers provided to their students were based on the teachers' ideas of what might be relevant to students' culture, but often such relation was artificial and peripheral, because the teacher failed to understand the students' deep cultural and individual experiences, and thus failed to build an activity based on global cultural characteristics still related to the particular group of students. Teachers tended to build "to" instead of "on" students' cultural background and informal knowledge of mathematics. Another factor impeding on teachers' use of discussion as a critical source of knowledge that needs to be incorporated into their mathematics lesson is that some teachers viewed conversations about students' personal experiences and culture as a deviation from the lesson focus, and as providing too much "extra" information.

Teachers' Instructional Practices that Promote Discourse

Some scholars focused their attention on the successful instructional practices and techniques teachers use in order to promote discourse more effectively (Patrick, Turner, Meyer, & Midgley, 2003; Sherin, 2002; Turner, Meyer, Midley, & Patrick, 2003; Wood, 1999). For example, Sherin observed 78 eighth-grade classes, scrutinizing the dilemma teachers faced when they tried to promote a student-centered instruction approach that involves facilitation of the discussions, while at the same time ensuring that discussions are mathematical in nature and that learning occurs. Research results indicated that teachers who used instructional practices balanced in *process* and *content* of discourse, and shifted focus between both aspects, were actually more successful in the facilitation of classroom discourse.

Wood (1999) also investigated the role of the teacher in creating an environment in which students were engaged in mathematical discourse. He found that teachers who were able to first establish discursive norms and patterns of behavior in their students actually lifted the cognitive attention and focus off the social organization of the interactions (such as turn-taking and the use of courteous language) and shifted it to discourse about mathematical ideas. Thus, the students were able to follow each other's logic and ideas, and focused on the mathematical context rather than on its social form.

Turner et al. (2003) examined the effect of the teacher's discourse and instructional practices on students' motivation and performance in mathematics in two sixth grade classrooms with similar high-mastery/high-performance students. The researchers found that practices in the teacher's organizational and motivational discourse which were consistently positive and supportive resulted in academic self-regulation and higher mastery and performance in the mathematics classroom. On the other hand, the classroom in which the teacher was less consistent and sometimes used non-supportive discourse had a negative result that often consisted of failure in mastery and performance by the students. Similar findings were reported by Patrick et al. (2003). Patrick et al. observed analogous patterns in 6th grade teacher behavior elicited from the eight 6th grade classes observed. Thus, both studies indicate the decisive role of the teacher in providing a positive and supportive discursive environment in which students are encouraged to take risks, make mistakes, try out ideas, and collaborate with others. In such an environment, students become more motivated to pursue goals toward higher mastery and performance in mathematics.

Methods of Assessment and their Effect on ELLs' Mathematics Experience

Another area of research concentrates on studying the influences of the assessment structure on ELL students' achievement in mathematics. Several studies suggest that standardized tests are usually based on an English speaking population and thus are inherently biased against ELL students (Gronna, Chin-Chance, & Abedi 2000; Liu, Anderson, & Thurlow 2000). For example, Gronna, Chin-Chance, and Abedi (2000) investigated the performance of a large 3rd, 5th, and 7th grade Hawaii public school population on the Stanford Achievement Test (9th edition), administered during the 1998-1999 school year, in order to study the relationship between ELL students' achievement on such tests and their English language proficiency. They found that performance varied significantly between ELL and English speaking students. The scores of ELL students indicated a higher level of achievement in mathematics as opposed to reading when compared to the scores of non-ELL students. This higher level of achievement was specifically evident on calculation-type mathematics problems as opposed to word-type problems.

Kiplinger, Haug, and Abedi (2000) evaluated the effects of students' English reading ability on their success on mathematics tests. The chief goal of this study was to provide research data to guide future development and construction of new assessments with linguistic accommodations for ELLs and students with special needs. The researchers administered three versions of a mathematical test to 1,198 fourth-grade students. The first version of the test was written in English with no accommodations. The second version consisted of simplified phrasing in the problems' descriptive language. The third version was written in English without simplifications, but students were provided with a glossary to use during the exam. The researchers found that ELLs performed better when they used accommodations, most notably on the linguistically simplified version of the test. Furthermore, ELLs' performance on mathematics tests with a higher number of word problems was strongly related to their English reading abilities.

In a similar study, Abedi, Lord, Hofstetter, and Baker (2000) investigated the effects of specific accommodation strategies and aimed to determine their impact on ELLs', as well as on English proficient students', performance on mathematics tests containing word problems. They collected data from 946 eighth-grade students from five California middle schools who had been tested on five different tests: (a) with original NAEP items for comparison, (b) with linguistically modified items (simplified English version), (c) with a glossary, (d) with extra time provided, (e) with a combination of extra time and a glossary. They found that a majority of the ELLs improved their scores when extra time was given and when they had access to a glossary or were aided in understanding the mathematical concepts presented to them through the use of simpler language. Particular combinations of these methods increased performance even more.

Research Methods for Examining Classroom Discourse

In order for researchers to examine more closely and accurately ELLs' mathematics experiences and achievement in classrooms in which the dynamics involve teachers and students in rigorous discussions, new paradigms of research need to be adopted. This is supported by the fact that, as research already has implied, "there is a need of more interdisciplinary collaboration in research design, data collection, and analyses requiring close attention to talk" (Adler, 2001, p. 513). The new methodologies need to be able to properly evaluate the process and the content of mathematical discourse and assess the predominant factors that contribute to ELLs' membership in the mathematical discursive communities, in the sense that Sherin (2002) defines such membership. In order to organize and classify the main research traditions (main research approaches and adopted methods of research) of past and present research, and draw attention to the current trends in examining classroom discourse, the framework developed by Ellis (1994) is used in the present study. He applied Chaudron's four main categories to describe the traditions in research in the field of second language acquisition (SLA): (1) psychometric tradition, (2) interaction tradition, (3) discourse analysis, and (4) ethnographic tradition. An assumption is made here that research on discourse in the mathematics classroom has similar characteristics to research on discourse in the language classroom.

(1) **The psychometric tradition** usually examines mathematics achievement as an end product of the application of different methods of teaching, curricula, and use of

materials. Experimental methods of research are employed under this tradition and, usually, data of pre- and post-tests between control and experimental groups undergoing a specific treatment is analyzed. The impediment to the application of such methods of research is that usually little or no account is taken of the variety of the contributing factors to students' performance on post-tests. Usually, more positive results of the experimental group over the control group are automatically attributed solely to the treatment applied (Ellis, 1994). Due to these inherent flaws, no studies in the field of discourse in the mathematics classroom were identified as generally employing such a research methodology.

(2) Studies that investigate the relationship between students' behavior and performance as well as the teacher's interaction and methods usually fall under **the interaction analysis tradition**. Such research methods typically include counting the frequency of occurrences of events during interactions. Then, using coding schemes, the classroom interactions are categorized and analyzed. However, a problem with this method might be that in concentrating on different utterances in isolation (i.e., what the teacher said, what question was asked, and how often), the global picture of the discourse could be missed (i.e., why this was said, what were the teacher's intentions, what was the sequential flaw of the conversation), thus "casting doubt on the reliability and validity of the measurements" (Ellis, 1994, p. 567).

In the early 1980s Allen, Frohlich, and Spada developed a means of examining the interactions taking place in a classroom setting called the *Communicative Orientation of Language Teaching (COLT) Observation Scheme*. The *COLT Observation Scheme* was used to investigate the effects of communicative language teaching on second language acquisition in programs developing bilingual proficiency, or in second language (French, English, etc.) immersion programs. Most studies using the COLT observation scheme "provided evidence that a combination of form and meaning worked better than exclusive focus on either meaning or form" when adopting communicative methods of instruction (Spada, & Frohlich, 1995, p. 7). In addition, rationales for each of the categories used to code communication in a classroom are provided. However, the system is divided into two parts. Part A is used in real-time coding and emphasizes seven main categories to describe the events that take place in the language classroom: time, activities and episodes (drill, game, discussion), participant organization (whole class, individual or group work), content (topics-language, subject matter, management), content control (teacher/text, teacher/text/student, student), student modality (listening, speaking, reading, writing, other), and materials (text, audio, visual). As it is described, this part of the COLT coding scheme focuses on the instructional practices adopted in the classroom, while Part B is focused on coding the communicative features in the classroom. The coding is done from the audio (and/or video) recordings obtained during the classroom observations and focused on the verbal interactions that took place between students and teachers within activities and episodes. Seven main communicative categories are identified: use of target language, information gap (giving or requesting information), sustained speech (minimal or not), reaction to form/message, incorporation of utterances in discourse (correction, repetition, paraphrase, comment, etc.), and discourse initiation and form restriction (coded only for the students). However, there are important caveats to be considered before one uses the COLT observational scheme in the classroom:

> First, it is important to emphasize that the COLT scheme offers *one* way of looking at instructional practices and procedures in L2 classrooms and, depending on the user's purpose and needs, it may be more appropriately used in some contexts than in others. For example, if one is interested in undertaking a detailed discourse analysis of the conversational interactions between teachers and students, another method of coding analysis of classroom data would be more appropriate. Similarly, if one is interested in carrying out ethnographic research in classrooms, the COLT scheme (or any other scheme with a set of predetermined categories) would not be suitable given the difference in theoretical and methodological perspectives between ethnographic and interaction approaches to classroom observation. (Spada, & Frohlich, 1995, p. 10)

To avoid such impediments, some researchers investigating the discourse in mathematics classrooms have proposed the use of linguistic tools (Bills, 1999; Rowland, 2002) that aid in examining utterances not in isolation, but in relation to the global picture of discourse. For example, Rowland proposes the use of linguistic tools such as "hedges" (*maybe, probably*, *possibly*), "attribution shields" (so-and-so *says*…), and "shield-approximators" (*about*, *around*, *basically*) that focus on the "pragmatic meaning" of the mathematical discourse. The term "pragmatic meaning" is defined by Rowland as

> the means frequently (though not necessary consciously) used by speakers to convey affective messages to do with social relations, attitudes and beliefs, or to associate or distance themselves from the propositions they articulate. That is to say, pragmatic meaning is an important tool in fulfilling the interactional function of language. (p. 2)

This correlates to the already-addressed notion of Bills' modality markers that are used as linguistic tools to examine the role of individuals (students or teachers) in classroom discourse by measuring the status, reliability, and truth value of a statement.

(3) A more systematic description of the interactions that occur in mathematics classrooms may be gleaned from **the discourse analysis tradition**. Methods used in the discourse analysis tradition include analyzing classroom transcripts where account is taken of the nature of the mathematics classroom environment as a whole in addition to the role of both the teacher and students in contributing to the interactions in order to negotiate meaning and understanding in the process of teaching and learning mathematics. In such analysis, the functions of individual utterances are combined in a larger discourse unit (Brenner, 1994; Lobato, Clark, & Ellis, 2005; Knuth, & Peressini, 2001; Krussel, Edwards, & Springer, 2004; Sfard, 2002).

For example, Knuth and Peressini's (2001) framework for examining and classifying the teachers' discourse in mathematics classrooms used two such larger units for classification—*univocal* and *dialogic*. *Univocal discourse* refers to the teacher being the authority and, usually, any discrepancies in student answers from the teacher's expected responses are evaluated as mistakes. *Dialogic discourse* is when such discrepancies are used as "thinking devices" to generate further discussions and thus generate new mathematical understanding.

On the other hand Krussel et al. (2004) proposed their own framework for categorizing "teachers' moves" (deliberate actions they take) as facilitators of discourse in the mathematics classroom. According to this framework, the teacher's discourse: (1) has an intended purpose (to move the activity to reflections, justifications), (2) takes place in a setting (small-group or whole-class discourse), (3) has a particular form (verbal—questions, directions, statements, clarifications, challenge; or non-verbal—gestures, face expressions), and (4) results in consequences (immediate or long-term).

Enriching the discourse analysis tradition, Brenner (1994) developed a *Communication Framework for Mathematics* which he found very useful in classifying the communication in mathematics classrooms with a predominantly ELL population (for the application of this framework see Brenner, 1998). According to this framework, communication in mathematics classrooms or in small group-discussions falls into three main categories: 1) communication **about** mathematics—which reflects on cognition, reasoning, and metacognition; 2) communication **in** mathematics—math register, special vocabulary, symbolism, and representations; 3) communication **with** mathematics—problem-solving tools, investigations, alternative solutions, etc. The framework aids in finding patterns in the different types of mathematical communication and the languages (English, Spanish, or others) used in such interactions.

Lobato et al. (2005) categorized communicative actions, taken by teachers to facilitate students in their conceptual understanding of mathematics, in an *initiation-eliciting framework*. By analyzing three teaching episodes, the authors demonstrated how teachers used “telling” in a reformed way to promote conceptual understanding: (1) by focusing their communicative acts on function rather than on form, (2) by presenting new information in a conceptual rather than a procedural manner, and (3) by presenting each action in relation to other actions. Thus, the authors made a reformulation of teachers’ “telling” and demonstrated how they can use “initiation” (introducing new mathematical ideas that stimulate students’ thought in constructing conceptual understanding in mathematics) and “eliciting” (when the teacher uses an idea originating from a student in order for students to make further conjectures and employ new ways in viewing and conceptualizing mathematical situations) to foster students’ learning of mathematics.

(4) **The ethnographic tradition** in research involves naturalistic “uncontrolled” observations and detailed descriptions of the classroom discourse. Thus, it could provide more insight into teacher and student cognition, and a deeper account could be taken of the uncontrolled additional variables that affect the process of teaching and learning mathematics. However, one of the problems in this tradition of research is that its methods require a very experienced and well-trained ethnographer, independent of whether he/she is an active or non-active participant in the classroom discourse. Yet another problematic area with this type of research is the fact that it is time consuming to collect and analyze the data, and difficult to make generalizations and warrant the transferability of the study in other conditions and situations. However, despite the listed difficulties in applying such a line of research, naturalistic studies provide more insight into what and how things happen in the classroom (Ellis, 1994). Being mostly descriptive in nature, they contribute to the bank of knowledge in research, and as Lampert (1998) asserted “the purpose of such interpretive research is not to determine whether general propositions about learning and teaching are true or false, but to further our understanding of these particular kinds of human activity in the contexts where they occur” (p. 160) Following this idea, Lampert, along with teaching mathematics for seven years (from 1982 until 1989), conducted research on her own teaching. Later on, she built a research team that used ethnographic research methodology “in order to examine the practical dynamics elements of establishing and maintaining a culture, developing and

using tools for mathematical communication, and creating a curriculum in the context of work on problems" (p. 159). (Findings from this research team's investigations on discourse have been listed throughout this paper.)

Many studies cannot be simply classified into one distinctive category because they often employ a combination of research methods and span across several categories. Often, data from quantitative and qualitative studies of classroom discourse is used in order to create a better depiction of the nature of classroom discourse (Ellis, 1994). And as Adler (2001) stated in *The Handbook of Discourse Analysis*, even though discourse analysis and qualitative methods of research "are not widely accepted even within the educational establishment" (p. 513), if experts from different domains (i.e., first and second language acquisition, linguists, children's cognitive development specialists, psychologists, mathematics educators, and many more) work in collaboration in studying discourse, better results could be achieved.

As an example of such collaboration, Atweh, Bleicher, and Cooper (1998) used Hallidey and Hasan's (1989) socio-linguistic model to propose a framework for investigating the mathematics discourse in two ninth grade classes in two different schools—one a boys' school from high SES and the other a girls' school from low SES. The researchers focused on examining how gender and students' socioeconomic status (SES) affect the teacher's perception of their mathematics abilities and needs, and shape the discourse in the classroom. By the proposed framework, the comparison concentrated on finding different patterns in three areas: (1) differences in the ***field*** (social actions; what is happening in discourse), (2) differences in the ***tenor*** (the relations and role of the participants in discourse), and (3) differences in the ***mode*** (what part the language is playing—rhetorical mode; symbols in context).

Findings of differences in the ***field*** indicate that even though the discourse in both classes was teacher-centered, the two teachers used different approaches to present the curriculum. The teacher from the boys' school used more rigorous formal math language and related the topic under investigation to other subjects and real world practices. This was related to the teacher's perception that most of the students in his class are college-bound and need advanced mathematics skills. In contrast, the teacher from the girls' school perceived his students as consumers and thus needing skills in consumer math. He used less formal language and defined only useful concepts applying "rules-of-thumb." These differences in ***tenor*** revealed that in accordance to his perceptions that his students need to be self-regulated, independent learners, the teacher from the boys' school created a general atmosphere of competition and often used sarcasm to challenge his students. In contrast, the teacher from the girls' school assisted his students with their difficulties and corrected their mistakes in a very polite and courteous manner. Nevertheless, his language was personal and differed from impersonal mathematics formal talk. The comparison of the ***mode*** of discourse between the two classes revealed that the teacher from the boys' school used a voice of authority and stressed important terminology. At the same time, he often encouraged argumentation and applied sarcasm. In contrast, the teacher in the girls' school stressed little on key words and little or no argumentation was used. Thus, the authors conclude that "classroom interactions, being consistent with teacher perceptions, tend to have a self-fulfilling role for teacher expectations" (p. 82).

Chapter Summary

In this chapter I have provided insights about the theoretical frameworks, research approaches, main findings and discussions that have influenced my study's ideas. I have highlighted what is currently known about ELL students' learning of mathematics in classrooms in which discourse is a feature. Research has already indicated one encompassing idea: that involving ELLs in meaningful discussions about mathematics is a giant step in reaching the goal of providing membership in the mathematical discourse community and providing a quality mathematical education for *all* students.

CHAPTER III: METHODOLOGY AND PROCEDURES

Based on knowledge collected from the literature review, the goal of this research is to investigate the nature of the discourse in mathematics classrooms adopting approaches from the discourse analysis tradition. “In discourse analysis, the units of analysis are variable and may range from words, phrases, and sentences to paragraphs or even larger units” (Wood & Kroger, 2000, p. 28). Mathematics education researchers who have adopted the discourse analysis tradition use methods that consist of analyzing classroom transcripts in which account is taken of the nature of the mathematics classroom environment as a whole in addition to the role of both the teacher and the student. Thus, using discourse analysis, I examined the influences of *teacher talk* on the inclusion or exclusion of English Language Learners (ELLs) in classroom interactions. I investigated how teachers negotiate meaning and understanding in the process of teaching mathematics to classes with a very diverse linguistic and cultural student population. This includes an examination of whether teachers adjust or modify their patterns of discourse depending on the number of ELL students present. Furthermore, I investigated whether differences exist in teachers’ discourse methods based on their experience in teaching mathematics and their ESOL endorsement. The following research questions are addressed:

1. To what extent do teachers’ patterns of discourse in the mathematics classroom change as a result of the number of ELL student(s) present?

2. To what extent do mathematics teachers' experiences and teachers' ESOL endorsement relate to their patterns of discourse when teaching mathematics to classes with ELL students present?
3. How do teachers' own linguistic and cultural backgrounds affect their patterns of discourse when teaching mathematics in English, and especially to classes with ELL students present?
4. What patterns of discourse do teachers use when ELL students are present in the mathematics classroom? What adjustments to *teacher talk* or modifications of instructions are observed?

The Study

The study was conducted during the Fall Semester of the 2007-2008 school year and explored the patterns of discourse between high school mathematics teachers and their students, especially when ELL students are present.

Context

The participants were teachers and their mathematics classes from two urban U.S. public high schools in the Southeast with diverse student populations. The schools were deliberately chosen because they have a large population of ELL students from a variety of backgrounds (See Table 1).

Table 1 illustrates that the two schools' student populations are comparable in size and diversity. However, there are some discernible differences in the percentages with respect to their racial and ethnic groups. More specifically, while both schools have comparable percentages of Non-Hispanic Black, American Indian/Alaskan Native, and Multi-Racial students, there are differences in the proportion of Hispanic, White, and

Table 1
Schools' Demographics

Demographic characteristics	Green Bay High School[1]	Lincoln High School
Student population	1939 students	1872 students
American Indian / Alaskan Native	0.31% (6 students)	0.27% (5 students)
Asian / Pacific Islander	1.44% (28 students)	10.47% (196 students)
Non-Hispanic Black (African American)	34.61% (671 students)	39.58% (741 students)
Hispanic	46.73% (906 students)	14.64% (274 students)
Multi-Racial	3.51% (68 students)	4.97% (93 students)
Non-Hispanic White (Caucasian)	13.41% (260 students)	30.07% (563 students)
ELLs	10.93% (212 students)	6.20% (116 students)

[1] Pseudonyms are used for schools' names.

Asian/Pacific Islander students. Also, while in Green Bay High School the majority of the students are Hispanics, followed by Blacks and Whites, in Lincoln High School the majority are Blacks, followed by Whites, Hispanics, and Asians/Pacific Islanders. Furthermore, while in both schools the percentage of students from economically disadvantaged families constitutes approximately half of the population, the schools have different percentages of ELL students. Table 1 indicates that the percentage of ELLs in Green Bay High School is almost twice the one in Lincoln High School.

Both schools offer programs to aid the ELLs in their subject area classes. In addition to their core courses, most of the ELLs are encouraged to take intensive[2] reading and mathematics elective classes or are provided with after school tutoring programs. In some of the Algebra I or Intensive Mathematics classes, bilingual teacher aides – fluent in Spanish in Green Bay High School and fluent in Spanish, French, and Arabic in Lincoln High School – were available to assist ELLs.

Participants

The scope of the study was limited to eight teachers (four teachers per school) to allow focused attention on the discourse and to allow the researcher to examine the type of communication that occurs between teachers and students in mathematics classrooms when ELLs are present and provide answers to the study's research questions. Eight teachers participated in the study - there were two female teachers and two male teachers from Green Bay High School, and three female teachers and one male teacher from Lincoln High School. The teachers also varied by their linguistic and cultural

[2] Such intensive courses are Intensive Reading I, II, and III (wherein guided instructions are provided to improve students' vocabulary, comprehension, and critical reading skills) and Intensive Math I, II, and III (wherein instructions focus on helping students acquire the competencies necessary to pass the State's Comprehensive Test).

backgrounds – in each school there was one African American, one Hispanic, and two non-Hispanic White (one male and one female) teachers. Teachers were selected based on the following criteria:

1. Years of teaching experience—two teachers in each school with many years of teaching experience and two teachers in the beginning of their teaching careers were chosen to participate.
2. Teachers with/without ESOL endorsement—two of the teachers in each school with many years of teaching experience have an ESOL endorsement, and the inexperienced teachers did not yet have, or were currently working on, their ESOL endorsement. (Theoretically, a teacher who does not have their ESOL endorsement cannot teach an ESOL child, unless he or she is perhaps in compliance with the timeline set for a beginning teacher to complete these requirements in the first two years of teaching mathematics to ELLs; for an experienced teacher the timeline is a year.)
3. The teachers teach mathematics courses at the same level - Algebra 1 (with zero or a small number of ELLs or with a larger number of ELLs depending on the population of the classes).

Algebra I classes were selected because this is a *core* subject that is a graduation requirement for *all* students. Because of this, ELLs must also take and succeed in this course to graduate.

The study required finding teachers with certain years of experience and ESOL endorsement who are teaching Algebra I to classes with a varied number of ELL students, and who are willing to participate in the study. In both schools, three of the participants were easily identified, however the fourth participants were teaching a similar curricular course such as Intensive Mathematics in one school and Liberal Arts Mathematics in the other that include some algebra content. To be consistent across the curricula, I observed all teachers when they were teaching topics identical with the Algebra I curricula – linear equations. Additionally, in both schools, some of the Algebra I classes were taught using a computer-based instructional program, *I Can Learn Lab.*

Instruments

Several tools were used to collect data relating to the communication that occurs between teachers and students in mathematics classrooms when ELLs are present. A description of each instrument is provided below.

"Teacher Talk Test" (TTT) Forms 1 and 2

Each teacher was observed using the *TTT* protocol. They were observed while teaching similar topics to their mathematics class (with or without ELL students) – *Coordinates and Scatter Plots, Graphing Linear Equations, The Slope of a Line, Quick Graphs Using Slope-Intercept Form, Functions and Relations, Writing Linear Equations in Different Forms, Fitting a Line to Data,* and *Predicting with Linear Models.* The *TTT Form 1* was used to obtain information about the teacher's patterns of discourse and *teacher talk,* measuring the teachers' frequency of the below mentioned teaching techniques (see Appendix A).

The *TTT Form 1* is partially based on the *ELL Strategies Verification Form* provided to teachers by the local school district and used by the State Department of Education to perform "walk through" ELL Compliance Audits. The version used in this study, *TTT Form 1,* includes not only items that relate to second language acquisition

(SLA), but also items that relate to teaching mathematics as a content area, thus reflecting the idea that content area teachers should encourage ELL students to participate in classroom discourse and thus help such students develop their abilities in both mathematics and the English language (Brenner, 1994, 1998; Moschkovich, 1999, 2002). Guided by these ideas, items were included that account for teachers asking inferential and higher order questions according to Bloom's Taxonomy of Six Cognitive Levels and ELL students' four stages of language development—*pre-production, early production, speech emergence,* and *intermediate fluency*. The instrument also includes items that indicate the extent to which the teachers use modifications, or accommodations of their speech, when ELL students are present. For example, do they use synonyms for difficult mathematics terms, or any potentially difficult words in English? Additionally, guided by Gee's (2005) definition of "Discourse," with a capital "D", and the notion that "people build identities and activities not just through language but using language together with other 'stuff' that isn't language" (p. 20), I have also included items in the *TTT Form 1* that reflect not only *teachers' talk* and interactions with their students, but also teachers' actions and behaviors in general such as gestures, models or visual images, "hands on" activities and the like, that formulate communication with ELLs while they teach mathematics. The *TTT Form 1* is comprised of items that are deemed to be among the best practices by educational research, especially with regards to ELL students. The list does not suggest that all strategies should be used in each lesson. It rather encapsulates the most widely used strategies for teaching mathematics to ELL students according to the research found in the literature review (see Chapter Two).

After classroom observations, teachers were asked to complete a different version of the *TTT – Form 2* (see Appendix C). This instrument includes the same items as *Form 1*, however the teachers were asked to complete a checklist (yes/no/needs improvement) to evaluate their own patterns of discourse. Additionally, teachers were asked to rate their use of each strategy on a frequency scale from 1 to 5, with 5 as the most frequent. This provided an opportunity to collect data about each teacher's perceptions of his or her own teaching and on the classroom experiences they provide.

Pre-observation Teacher Questionnaire

To collect data about teachers' "way of thinking, believing, valuing" (Gee, 2005) not only about the subject they teach—mathematics, but also about the way they teach it to a linguistically and culturally diverse student population, a *Teacher Questionnaire* (see Appendix B) was developed. This instrument includes questions about teachers' years of teaching experience, ESOL certification, and their cultural and linguistic background.

To gather data about their perceptions about their ELL students, teachers were asked to identify the ELL students (if any) in the observed class and to comment on their perceived stage in SLA. (Each teacher was provided with a list of the definitions for each of the stages in SLA – *pre-production, early production, speech emergence,* and *intermediate fluency*.) The goal was to determine what understanding teachers have about their ELL students, linguistic and cultural differences, and whether they use this knowledge to modify their mathematics instruction. Teachers were provided opportunities to comment on their experiences with teaching mathematics to ELL students, and/or indicate if they have concerns or recommendations for improvement related to these experiences. To avoid the Hawthorne effect (the fact that the teachers might say what they want me to hear), data obtained from the teachers were compared to official data about the teachers and their ELL students collected from the school's personnel and guidance departments.

In order to ensure the study's trustworthiness and to eliminate any biases from influencing teachers' and ELL students' answers to the post-observation questionnaires, the study's participants were not provided access to any of the questionnaires ahead of time.

Student Questionnaire

This *Questionnaire* (see Appendix D) was used to collect data about how students perceive their own abilities and experiences in SLA and mathematics; thus, this allowed an opportunity to discern similarities or differences between students' and teachers' perceptions of their participation in classroom discourse. This instrument includes questions that address the students' ELL categorization according to their level of English language proficiency (each student was provided with a list of the definitions for each of the stages in SLA), and mathematics experience background—previous mathematics courses taken, and grades. Additionally, the instrument includes questions about students' family and personal attitudes about mathematics. Furthermore, during the interviews, students were given an opportunity to comment on their experiences with learning mathematics in English and to provide a self-evaluation about their participation in the observed lessons.

All students were read the same exact questions directly from the *Student Questionnaire* (see Appendix D for the specific questions) in order to limit the possibilities of asking a biased question and to minimize threats to the study's trustworthiness. To ensure that the questions were comprehensible to the ELL students of various levels of SLA, the complexity of the language used in the questionnaire was modified and simplified. The readability test of the *Student Questionnaire* indicated a reading ease of 83.4%, corresponding to a fourth-grade reading level according to the Flesh-Kincaid Grade Level Scale. The questions were translated for ELL students at the initial stages of English language acquisition. The *person translating* and assisting in *negotiating the meaning* of each question and answer in the dialogue between the researchers and the students read the questions directly from the Spanish-translated version of the questionnaire (see Appendix E), which was checked for simplicity by a language professional who speaks Spanish, or was asked to use simplified student-friendly language when translating in languages other than English and Spanish.

Data Collection Procedures

Teachers' Demographic Data

Demographic data about each teacher participant were obtained from their personnel files. This data was compared to the data they provided on the *Teacher Questionnaire*. This comparison allowed the detection of similarities or differences between teachers' perceptions of their ELL students and the information on file.

ELL Students' Demographic Data

Demographic data about the ELL students in Algebra I classes were obtained from the schools' guidance departments to verify students' ELL level and placement in the ELL program. This information was used to supplement and verify the information provided by ELL students during the post-observational interviews.

Classroom Observations

Each classroom was observed when the teacher introduced new material or reviewed an already taught topic (e.g., similar topics at the same level of mathematics – *Coordinates and Scatter Plots, Graphing Linear Equations, The Slope of a Line*, *Quick Graphs Using Slope-Intercept Form, Functions and Relations, Writing Linear Equations in Different Forms, Fitting a Line to Data*, and *Predicting with Linear Models)*. In order to ensure that teachers taught similar topics of the Algebra I curriculum in different schools, all observations were conducted within a three week period. Each teacher's Algebra I class (in one case Intensive Mathematics class and in the other Liberal Arts math class) were observed on at least three occasions in order to better detect teachers' instructional patterns of *talk* or behavior ("teacher discourse moves"). Each class was observed for approximately 20 minutes. The *TTT Form 1* Observational Protocol was used to document the frequency of different types of patterns of discourse and *teacher talk* and interactions (if any) with the ELL students. Multiple observations of the same teachers also provided evidence about the extent to which the observed patterns were robust and/or whether there were real differences in patterns based on content.

Videotaped Observations

Each observed instructional session was also videotaped. According to Wood and Kroger (2000), "a videotape is clearly required if one is concerned with the coordination of discourse with other activities, for example, with the performance of a (nonverbal) task or with features that are only available on video (e.g., facial expression)" (p. 70). The videotaped sessions were useful during the data transcription phase and when analyzing the teacher-student interactions, which included nonverbal communication.

Field Notes

During each observation, field notes were taken to capture classroom interactions emerging in the process of teaching/learning mathematics throughout the entire class period (i.e., for approximately 45 minutes). This allowed the researcher to capture additional information about the interactions between the teacher and the students and some specific characteristics of the nature of the classroom discourse including any unusual strategy that might not have been reflected in the *TTT Form 1* instrument.

Interviews

After the classroom observations, the teacher and group of ELL students in the class were interviewed. All interviews were conducted and videotaped on the day of the last observation to ensure that the discursive strategies used by the teacher are still vivid in the ELLs' and teachers' minds. The videotaped interviews allowed for a reliable and accurate account of participant comments.

Teacher Interviews

The teachers were asked to comment on their already completed pre-observation *Questionnaire for Teachers*. Teachers were also asked to self-evaluate *the talk* they had employed during the observed classroom session (with various numbers of ELL students present) using the *TTT Form 2*. They were asked to reflect and comment on their experience not only during the classroom sessions under investigation, but also on their general experiences in teaching mathematics to classes of a linguistically and culturally diverse student population.

Student Interviews

ELL students were asked to complete the *Student Questionnaire* during interviews. When needed, a translator (a teacher, a staff member, or student who spoke the same language) was present to ensure that the ELL student understood the nature of the interview. ELL students were asked to reflect on their participation in the particular lessons taught during the classroom sessions under observation. Emphasis was placed on what, in their opinion, was causing any problems in their participation in classroom discourse and comprehension of the mathematics lessons. Then, they were asked to comment on *the talk* their mathematics teacher had employed during the observed classroom sessions using the *TTT Form 3*. They were asked to reflect and comment on their experience not only during the classroom sessions under investigation, but also on their general experiences in learning mathematics in English as their non-native language. As mentioned before in the description of the *Student Questionnaire* instrument, in *TTT Form 3* the possible threats of the study's trustworthiness posed by the possibility of the researcher and/or the translator asking a biased question were also addressed. For this reason the questions were read directly from both the *Student Questionnaire* and *TTT Form 3* (see Appendices D and E for the specific questions asked). The person translating and assisting the researcher either read the questions directly from the versions translated in Spanish or used a simplified student-friendly language when translating in languages other than English and Spanish.

Data Analysis

The data analysis will be discussed in two sections. First, the frequency count of the used discursive strategies by using the *TTT Forms* instrument will be discussed.

Second, Krussel, Edwards, and Springer's (2004) framework and the method of analytic induction (i.e., "from the ground up"; Davidenko, 2000, p. 39) for analyzing the teacher's discourse will be described.

Data from TTT Forms 1, 2, and 3

During each 20-minute observation, tally marks were used to record each observed use of particular discursive strategy on the *TTT Form 1*. Then, I counted the frequencies for each box. Re-playing the video recordings of each observed session and checking its transcription permitted further refinement of the frequency count of the discursive strategies used by the teacher and allowed for additional qualitative analysis of the data. Additionally, two inter-rater reliability (e.g., *dependability*) tests were conducted. Initially, the researcher and a research associate, who is an expert in working with ELL students, both observed a classroom session of exactly 20 minutes of a mathematics teacher outside of the study sample and filled in *TTT Form 1*. Then, each separately re-played the video recording and read the transcription of the observed session and counted the total frequency for each box in *TTT Form 1.* The inter-rater reliability score was .75. After this, a training session was carried out, which permitted the researcher and research associate to clarify and reach consensus regarding the nature and meaning of the codes of the various discursive strategies. Then, a lesson that involved a teacher from the study sample was observed, coded and analyzed individually and the frequency counts from *TTT Form 1* were compared again. The inter-rater reliability score was .87.

Additionally, the data collected in *TTT Forms 1, 2 and 3* was illustrated using various visual displays to permit further analysis.

Characteristics of Krussel et al.'s (2004) Framework

The Krussel et al.'s framework was used to analyze the "teachers' discourse moves" (deliberate actions they take) as facilitators of discourse in the mathematics classroom. This framework provided a qualitative method of identifying patterns and themes to explain phenomena (i.e., to see if a trend existed in the teachers' discourse individually and across the sample). Categories were formed from the data collected during the observations and the video-recordings of the classroom sessions and this data was then matched with the data from the teachers' and ELL students' interviews, to see if a trend existed. The transcripts of the observations and interviews were read and coded for categories that were prevalent from the teachers' observed discourse moves and teachers' and ELLs' answers and were related to the research goals and questions.

The process of analyzing the "teachers' discourse moves" involved three phases: (a) an initial reading of the transcribed data for an overall sense of meaning, (b) detection of "meaning units" within the text, and (c) the formation of themes by grouping key phrases or actions for each teacher. These themes were then compared to establish which ones were more customary for each teacher and to determine when they could be considered a teacher's emergent pattern of discourse. Each theme was related to a category of teacher discourse as described in the Krussel et al. (2004) framework. According to this framework, the teacher's discourse: (a) has an intended *purpose* (to direct the activity to reflections, justifications, small-group or whole-class discourse); (b) takes place in a certain *setting* (assigned roles and norms in discourse); (c) has a particular *form* (verbal—questions, directions, statements, clarifications, challenge; or non-verbal—gestures, facial expressions), and (d) results in *consequences* (intended or unintended, immediate or long term). Each of these categories of teacher discourse is expounded upon below.

Purpose

The *purpose* of the teacher's discourse may be, for example, to shift discourse from a whole-class discussion to small-group work so as to initiate participation in activities requiring justifications and reflections, or simply to deal with discipline issues. The teacher's purpose can only be perceived by a researcher if he/she pays considerable attention to the flow of the discourse's text and the shift in its meaning. For example, if a teacher regularly asks the students questions such as, "And how did that change the problem? Correct. We switched the starting point," it is evident that the teacher is trying to direct the students toward reflections on their thinking, explanations, and justifications.

Setting

In observing the teacher's actions toward establishing a *setting* for classroom discourse, the researcher might infer not only from the present discourse, but also from previously-set norms of discourse. The researcher may become aware of such norms by looking at the classroom layout, or by observing that the students or the teacher already have established roles in discourse which naturally occur, without the teacher assigning them in front of the researcher. For example, if a teacher specifically states: "What is the slope in this equation? Please, raise your hands…," it is evident that the teacher is trying to establish certain norms for students in taking turns to participate in the mathematical discourse. And if a teacher simply asks "What is *m?"* and many students raise their hands to answer the question, it is then clear that the teacher has already established the norms of behavior for his/her students in participating in classroom discourse. Furthermore, if many students answer aloud the aforementioned questions posed by their teacher without raising their hands and being individually called upon, a researcher can conclude that the students comply with more liberal and less explicitly stated classroom behavior.

Form

The *form* of the teacher's discourse includes actual *teacher talk* (verbal) and *actions* (non verbal). For example a question: "How do you know that is true?" indicates that the teacher's discourse takes the form of a challenge. If the teacher says "I'm not sure I understand…," he/she is requesting clarification. (For more details see Krussel et al., 2004, p. 309). The teacher's non-verbal discourse also may be displayed in different forms—gestures, facial expressions, spatial proximity between the teacher and student, pausing after having posed a question, or the use of silence. For example, after collecting students' bell-work, if a teacher walks to the board in silence and writes the title of a lesson topic, he/she is switching the classroom discourse to instructional mode. In another situation, if a teacher walks around the students' seats and assists them in their individual or group work, a researcher can conclude that in this class, the teacher uses spatial proximity with his students as part of his instructional techniques.

Consequences

The teacher's discourse always has some *consequences* which may be intended (to shift the cognitive level of the task performed) or unintended (for example, lowering his/her expectations of ELLs), more immediate (shifting the dialogue from univocal to dialogic) or long-term (setting norms of politeness and turn-taking during classroom discussions, which may consequently accelerate future discourse toward meaning rather than form).

In conclusion, examine the classroom discourse by applying Krussel at al.'s framework provided valuable information about the studied sample. More specifically, the process of analyzing the teachers' discursive moves helped in answering the first research question (i.e., what patterns of discourse the teachers used when ELLs are present in the mathematics classroom and if there were any observed adjustments to *teacher talk* or modifications of instructions). The previously described frequency count of each teacher's use of different discursive strategies also permitted the detection of which strategies were most prevalent for a specific teacher. Furthermore, Krussel et al.'s framework permitted a qualitative view and consideration of not only prevalent themes (i.e., main patterns of discourse) typical for each teacher, but the consideration of secondary or less prevalent themes. These less prevalent themes were also important for this research because they provided information needed to answer the other research questions. The gathered information revealed the extent to which teachers' patterns of discourse changed as a result of the number of ELL students present and the extent to which teachers' experience, ESOL endorsement, and their own linguistic and cultural background, affected their pattern of discourse.

Method of Analytic Induction

To analyze teachers' discourse, the qualitative method of analytic induction was applied to the data. Lincoln and Guba (1985) stated that "inductive data analysis may be defined most simply as a process for "making sense" of field data" (p. 202). As quoted in Lincoln and Guba (1985), Reese asserted:

> The widespread distinction between induction as an inference moving from specific facts to general conclusions, and deduction as moving from general

> premises to specific conclusions is no longer respectable philosophically. This distinction distinguished one kind of induction from one kind of deduction. It is much more satisfactory to think of induction as *probable* inference and deduction as *necessary* inference. (p. 251)

According to Davidenko (2000), another researcher who adopted the method of *analytic induction* in her dissertation study:

> When we analyze data from a qualitative study, we look for *codes* that represent *instances of a concept that we are yet to define*. The larger categories *emerging from the codes* become our new concepts. We may use predefined categories to represent the concepts. In this case, through the analysis of the data, we attach *new meanings* to them. After I began to think inductively, I found the qualitative research process to be exciting and creative. (p. 40)

However, whereas Davidenko studied "how [ELL] students learn mathematics in English-taught mathematics classes" (p. 30), this study focuses on the teachers. Specifically, this study examines teachers' "discursive moves" and the influences of *teacher talk* on the inclusion or exclusion of ELL students in classroom interactions.

Validity, Reliability, and Objectivity Check of the Analysis Process

The framework of Lincoln and Guba (1985) dictates that when carrying out qualitative naturalistic methods of inquiry, rather than using the conventional terms *internal* and *external validity*, *reliability* and *objectivity*, four other terms are used instead: *credibility*, *transferability*, *dependability*, and *confirmability* (p. 300). Below I describe how this study's trustworthiness was attained by addressing each of the above criteria.

Credibility

For satisfying the credibility criteria of trustworthiness of the instruments some of the techniques suggested by Lincoln and Guba (1985) were used (i.e., prolonged engagement and persistent observation). The teachers were observed for the entire class period (typically 45 minutes) on three different occasions. The teacher-led instruction portion of the lesson (typically 20 minutes) was videotaped. This videotaped segment allowed the researcher to obtain frequency counts of the used discursive strategies. Additionally, I collected data from the teachers via pre- and post-observations interviews, and from post-observation interviews of their ELLs. Furthermore, triangulation of the data collection and analysis procedures was employed using different data collection models — observations, field notes, video-recordings, questionnaires and interviews, and school records' files. This data was analyzed according to Krussel et al.'s (2004) framework and frequency counts of the teachers' use of different discursive strategies were determined. The triangulation of sources, data, and methods facilitated the creation of a more holistic view of the discursive practices adopted by each teacher.

Furthermore, I engaged in peer debriefing to ensure the study's trustworthiness. In several of stages of the study, debriefing sessions were performed with two colleagues-researchers who are experts in the fields of teaching ELLs and research in mathematics education, and written records of these sessions were maintained. These experts helped me to improve the instruments and include teacher-appropriate and student-appropriate language and ensure the proper readability levels as per my audience. With the assistance of these sessions, the achieved readability levels as follows: 71.7% (corresponding to a 4.6-grade reading level) on the *Teacher Questionnaire* and *TTT Form 2* and 80.3%

(corresponding to a 4.3-grade reading level) on the *Student Questionnaire* and *TTT Form 3* which were respectively classified as Fairly Easy and Easy Readability Scores, according to the Flesh-Kincaid Grade Level Scale (for more information refer to www.plainlanguage.gov or www.plainlanguagenetwork.org). As described before, debriefing with a Ph. D. professional in the field of educating ELLs facilitated the achievement of an inter-rater compatibility rating of 87% in the frequency count of the teachers' use of the discursive strategies as per *TTT Form 1* instrument (see *Data from TTT Forms 1, 2, and 3,* pp. 65-66 in this manuscript).

After the analysis of the data from the video recordings, the transcriptions, *TTT Forms 1, 2, 3*, the questionnaires, and a more detailed description of the findings, another debriefing session was convened with the experts mentioned. In order to maximize the study's credibility, the research associate participated in a training session to learn to employ Krussel at al.'s framework and was given ample opportunity to code individually a sample of data using the same coding themes. The 22 discourse strategies reflected in *TTT Form 1* (refer to Appendix A) were used as comparative constants. Then, the constant-comparison of the data analysis between the researcher and research associate showed a reliability rating of .83 (e.g., 83%). This process gave the researcher an opportunity to compare findings to determine the consistency of interpretations and to resolve any discrepancies that were found. To confirm drawn conclusions, the research associate used the data from *TTT Forms 1, 2, and 3* and compared his findings with the ones made by the researcher to determine the researcher's consistency of interpretations. In cases of disagreement, the researcher and research associate discussed any discrepancies until consensus could be reached. Furthermore, after the frequency count of the teacher's used discursive strategies and the qualitative analysis, any deficiency in used strategies was indicated (e.g., negative case analysis was performed which "requires the researcher to look for disconfirming data in both past and future observations" (Lincoln & Guba, 1985, p. 310) in order to "refine a hypothesis until it *accounts for all known cases without exception*" (Lincoln & Guba, 1985, p. 309). Moreover, member checking was carried out with the participating teachers to ensure that "data, analytic categories, interpretations, and conclusions" correspond to an "adequate representation" of reality, and to provide continuous possibilities for them to react to such representations. A summary of some of the case studies' descriptions were also given to the study's participants to read and comment on, as part of this member checking.

Transferability

According to Lincoln and Guba, it is *not* the researcher's "task to provide an *index* of transferability"; however, it is the researcher's task to "provide only the thick description necessary to enable someone interested in making the transfer to reach a conclusion about whether transfer can be contemplated as a possibility" (p. 316). Thus in establishing the study's *transferability*, a thick description of how different stages of the study were carried out is provided. In order to ensure a high standard of transferability, a thick description of the observations and the subsequent processes of transcribing and analyzing the data, as well as its various representational formats (e.g., tables, charts, histograms) and analyses thereof is provided. Additionally, the instruments used in the study are provided in the appendices. This ensures that a "*data base* [has been provided] that makes transferability judgments possible on the part of potential appliers" (Lincoln, & Guba, 1985, p. 316).

Dependability and Confirmability

According to Lincoln and Guba, “a single audit, properly managed, can be used to determine the dependability and confirmability simultaneously” (p. 318):

> The auditor should see him- or herself as acting on behalf of the general readership of the inquiry report, a readership that may not have the time or inclination (or accessibility to the data) to undertake a detailed assessment of trustworthiness. (p. 326)

To comply with these criteria of trustworthiness, an auditor (an expert, who is not a member of my dissertation committee) inspected, verified, and examined drawn conclusions by examining the supporting documentation for accuracy and fairness from the onset of the study. The auditor was introduced to the study at its inception, as well as the development and testing of its various instruments, raw data collection, data reduction and analysis, study findings and final report, and further methodological notes and trustworthiness notes. Audit trial notes followed the suggestions made by Lincoln and Guba (1985). (For further details regarding this process refer to the Appendices A and B in Lincoln and Guba (1985), pp. 382-392).

CHAPTER IV: RESULTS AND INTERPRETATIONS

The results of the study are presented in two sections. The first section reports the results of the data analysis of the teacher questionnaires and interviews in relation to years of teaching experience, number of ELL students present, and the teachers' and ELL students' linguistic backgrounds. The second section presents each teacher and his or her classes as cases to be examined. The description of each case includes sample classroom excerpts and the results from analysis that applied the framework developed by Krussel et al. (2004). Furthermore, each case study provides the results of comparing the data from the researcher's preliminary evaluation (i.e., before an actual count of the frequencies with which the teachers use different discourse strategies), the teachers' self-evaluations, and the evaluations of the ELL students. Additionally, the frequencies with which different discursive strategies were used by each teacher are provided. Bar graphs are used to visually represent the findings and provide the reader with "a quick glance [of] an overall pattern of results" as prescribed by the American Psychological Association (APA), 2001, p. 176. In sum, section two will report the results of the data analysis in relation to research question one and will provide the results of the data analysis in relation to the research questions that are illustrated with detailed examples and specific evidence.

Characteristics of the Sample

Table 2 presents the demographic information about the study's participants that reflect the criteria used to identify the study's participants:

1. Years of teaching experience—at least two teachers in each school have many years of teaching experience and two teachers are in the beginning of their teaching careers.
2. Teachers with/without ESOL endorsement—at least two of the teachers in each school have many years of teaching experience and have an ESOL endorsement, and the inexperienced teachers either did not yet have their ESOL endorsements, or had just obtained them.
3. The teachers teach mathematics courses at the same level - Algebra I (with zero or a small number of ELLs or with a larger number of ELLs, depending on the population of the classes).

Information about the third criteria will be summarized and visually represented using graphs.

In both schools, three of the participants taught Algebra I, however the fourth participants of both schools taught a similar curriculum course such as Intensive Mathematics (Ms. Brown in Lincoln High School) and Liberal Arts Mathematics (Mr. Davison in Green Bay High School). In order to be consistent across the curricula, I observed all teachers when they were teaching a common topic – linear equations. Additionally, in both schools, some of the Algebra I classes were taught using an individualized computer assisted learning program called *I Can Learn Lab.* Furthermore, from the eight teachers who participated in the study, there were two female teachers and

Table 2

Overall Sample Information

Name	Years of teaching experience (teaching Algebra)	Languages spoken by teacher	ESOL endorsement (years)	Number of ELLs in class	Languages spoken by students
		Green Bay High School			
Mr. Able[3]	34 (34)	English	Yes (3)	2	Spanish
Ms. Barrera	2 (2)	English, Spanish	Just obtained (0.5)	9	Spanish
Ms. Chandler	9 (8)	English	Yes (5)	8	Spanish
Mr. Davison	16 (8)	English	Yes (7)	4	Spanish
		Lincoln High School			
Ms. Andersen	23 (23)	English, French	Yes (11)	4	Spanish, French, Creole, Arabic
Ms. Brown	0 (0)	English, Yoruba, limited French	Just obtained (0.5)	5	Spanish, Arabic
Ms. Cortez	10 (5)	English, Spanish	No (0)	4	Spanish, French, Creole, Arabic
Mr. Daniels	12 (12)	English	Yes (9)	3	Spanish

[3] Pseudonyms are used for teachers' names.

two male teachers from Green Bay High School, and three female teachers and one male teacher from Lincoln High School. The teachers also varied in their linguistic and cultural backgrounds – in each school there was one African American, one Hispanic, and two non-Hispanic White (one male and one female) teachers (see Table 2).

Years of Teaching Experience

The teachers in the sample varied greatly in their years of experience in teaching (see Table 2). In both schools, there were at least two teachers with many years of experience – Mr. Able and Mr. Davison in Green Bay High School, and Ms. Andersen and Mr. Daniels in Lincoln High School. In both schools, there also was at least one teacher who had recently begun his/her teaching career or at least was in the beginning of teaching Algebra I to classes with ELLs present – for example, Ms. Barrera in Green Bay High School and Ms. Brown in Lincoln High School had just started their teaching careers, and Ms. Chandler (Green Bay High School) and Ms. Cortez (Lincoln High School) had taught Algebra 1 to ELLs for about five to seven years.

ESOL Endorsement

Only one teacher in the sample – Ms. Cortez from Lincoln High School, had not fulfilled the requirement for content area teachers of 60 hours of training toward ESOL endorsement. Two other teachers from each school, Ms. Barrera and Ms. Brown, had recently completed their training and had little experience as teachers of Algebra I to ELL students (see Table 2).

Number of ELL Students Present

The number of ELL students in each classroom is more evenly distributed in Lincoln High School, whereas it is very unequally spread in Green Bay High School (see Table 2). This may be due to a trend in Green Bay High School, wherein most of the ELLs are assigned to Algebra I classes that employ computer labs, tutorials, tests and quizzes (as was indicated by the guidance department chair in the school).

Years of Teaching Experience, ESOL Endorsement, and Number of ELL Students Combined

An interesting observation is that the more years of teaching experience a teacher has, the smaller the numbers of ELLs present in his/her class. This is the case with Mr. Able and Mr. Davison in Green Bay High School; the situation is similar with Ms. Andersen and Mr. Daniels in Lincoln High School. In both schools, the teachers just beginning their teaching careers (Ms. Barrera in Green Bay High School and Ms. Brown in Lincoln High School) are assigned to teach classes with the highest number of ELLs. Even though these two teachers had recently completed their ESOL endorsement's requirement, they lack the practical experience of teaching Algebra I to classes with diverse student populations involving a high number of ELLs present.

Teachers' and ELL Students' Linguistic Backgrounds Combined

Table 2 depicts the number of languages spoken by each teacher, the number of ELLs in his/her class, and how many ELLs spoke the same language(s) as the teacher. Only three of the teachers spoke the same language as their ELL students—Ms. Barrera in Green Bay High School, Ms. Andersen and Ms. Cortez in Lincoln High School. Of these three teachers, only two of them — Ms. Barrera and Ms. Cortez also had similar cultural backgrounds as some of their ELL students. Ms. Barrera is from the Dominican Republic and Ms. Cortez is from Puerto Rico.

In addition, four of the teachers in the sample – Mr. Able, Ms. Chandler, Mr. Davison, and Mr. Daniels – do not speak any languages other than English. One teacher, Ms. Brown, speaks three languages (English, Yoruba, and limited French), yet none of her ELLs is from the same linguistic background (4 of them speak Spanish, and one speaks Arabic). However, even though these teachers did not speak their ELLs' native languages, an analysis of the interview data, as well as that collected from the observations and videotaped sessions, revealed the various ways in which these teachers dealt with the issue.

Case Study Analysis

In this section, a detailed description of the eight case studies will provide information about how each teacher exhibited some strategic modifications of his/her "discursive moves" in order to accommodate the ELLs present in his/her mathematics classroom. Each teacher will be discussed as a separate case as a member of a faculty of each school. Each case represents the synthesis of data obtained from video-recorded, transcribed and analyzed observation field notes, responses to the *Teacher* and *Student Questionnaires* (see Appendices B and D) , and the *TTT Forms 1, 2,* and *3* (see Appendices A, C, and E). Treating each teacher as a case allows for the examination of normal practices that constitute the classroom atmosphere and the mathematics classroom discourse. Finally, some of the similarities and differences in the teachers' observed discursive patterns will be presented.

Green Bay High School

Mr. Able[4]

Mr. Able, an African American in his late fifties, has over 30 years of teaching experience. He has a Master's Degree in Mathematics Education, is certified to teach secondary mathematics, and has completed the required 60 hours of training toward his ESOL endorsement three years prior to his involvement in this study. In addition to teaching Algebra I, he also teaches College Preparation classes and is one of the coaches for the school's track team. Mr. Able only speaks English. His Algebra I class consisted of 21 students, with only two officially indicated on his roster as ELL students of Hispanic background. However, the class appeared diverse, with an almost equal number of Hispanic, African-American and Caucasian students.

Mr. Able's classroom was very organized, clean, and laid out in traditional rows. The two ELL students were seated beside each other on the right side of the classroom. Bilingual students who spoke both English and Spanish were seated nearby to provide translation assistance if needed. During the interview, Mr. Able confirmed this observation by stating that he often, "teams a person who speak[s] Spanish and English with the ELL student to help him." He also pointed out that he is conscious of the presence of ELL students in his classroom and often "try[ies] to speak slow[ly] when lecturing."

Mr. Able's classroom was well-decorated. On the rear wall were colored posters in creative shapes. The posters contained his students' answers to some autobiographical questions and information on their hobbies and interests. The rest of the walls were

[4] Pseudonyms are used for both teachers' and students' names.

decorated with posters on which the students wrote “Math is like…., because it …”, where each student had completed the sentence in his/her own original way. Around the white board Mr. Able had posted mathematics vocabulary words in Spanish (which he said he did in order to assist his ELL students).

Prior to his first period class, students would come by to ask for help with the homework, to ask for recommendations, or to ask for his signature on a field trip form. The student/teacher interactions were all impressively carried out with mutual respect. The students seemed to respect him not only as their Algebra I teacher but also as a coach and as a person.

Typical classroom discourse. During each of the three observed classroom sessions, Mr. Able used the same basic class sequence that will be described in the examples that follow. First, he began the lesson with five-minutes of bell-work. He used an alarm clock to time the bell-work and placed a prepared transparency form that consisted of mathematics questions that the students had studied a couple of lessons previously. For example:

Solve the following equations for the corresponding variable:

1. $20 = 6x + 8$ 2. $-10 – k = -3$ 3. $2y – 7 = 15$

4. $15 – 4g = -33$ 5. $2.1 = 0.8 – z$

Mr. Able’s students complied with the following rules of behavior. All students worked on the assignment. After the allotted five minutes were over and they heard the sound of the alarm, they quietly got up (including the ELL students) and placed their bell-work on his desk in the left corner of the room.

Then, Mr. Able explained the bell-work and demonstrated the correct solutions, which were already written on another transparency. Before starting a new lesson, he usually informed the class of any upcoming events. For example, during the first observed session, he informed the students that there would be a test on Friday and that he would check their notebooks while they completed the test.

Following this initial stage of the class, Mr. Able wrote the title of the new lesson onto the white board: *Lesson 4.6 Quick Graphs Using Slope-Intercept Form.* He also wrote the equation: $y = mx + b$, and asked: "Can anybody tell me what *m* represents?" As I analyzed all of the video-recorded sessions, a similar pattern of questioning emerged. Most of the questions throughout the lessons had a similar purpose of involving the students in the mathematical discussions, and usually required one-word responses or a list of words. Thus, to this first question, a couple of students answered aloud: "the slope" and the teacher, nodding, said "Good" and simply continued:

> And, whenever you see the *b*, it is the y-intercept. What we mean by the y-intercept is the way it crosses the y-axis. When we talk about *m* today, the top is either you go up or down. When you go up, it is positive, when it is down it is negative. The bottom, you can go to the left or to the right, when you go to the left it is…?

Here Mr. Able changed the pitch of his voice, to indicate that he is asking a question, and looked toward the class, nodding toward a student (not an ELL) who raised his hand. The student (along with a couple of others) answered: "negative." Then, Mr. Able continued, "when you go to the right it is?" and a few students answered aloud, "positive."

Mr. Able also drew a small diagram,

$$m = \frac{\uparrow\downarrow}{\rightleftarrows} = \frac{rise}{run}$$

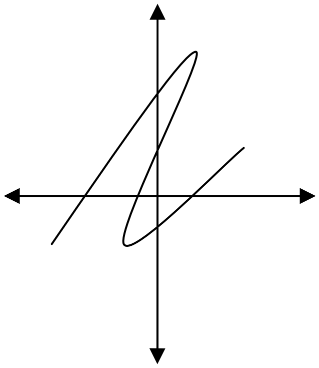
and continued explaining the graphing of lines by providing examples. The examples started with *Slope-intercept form*, but later on moved to examples in which the students were expected to perform some algebraic operations in order to transform the equations into $y=mx+b$ form and then graph them. With each example, Mr. Able asked the students questions such as, "Someone tell me what is *m?*" and if they answered with just one number "three", he would continue, "it is always the number before the *x*; don't write [it] as an integer" and thus the students who answered corrected themselves by saying "three over one." Then Mr. Able continued asking the whole class: "what is *b*" and, nodding to a student, wrote the answer "(0, 5)." After this, he simply continued:

> "then you graph this one. Start with (0,5). Always start with the y-intercept, go up 5, put your dot on 0, go up 5. Now, someone tell me how you do 3 over 1?"

These excerpts from this lesson exemplify Mr. Able's classroom discourse and climate. In all three observed twenty-minute sessions and video-recordings, Mr. Able always began his lessons with bell-work and then explained the lessons on the board, oftentimes using the overhead projector to place pre-prepared grids with the coordinate system on them, which he used to demonstrate plotting points and graphing linear equations by using the slope-intercept form, or point-slope form.

For example, for the second lesson, *Lesson 4.8 Functions and Relations,* he used various examples to review previously studied concepts. In this case, he used multiple representations to discuss functions (i.e., graphs, tables, Venn Diagrams, etc.):

1.



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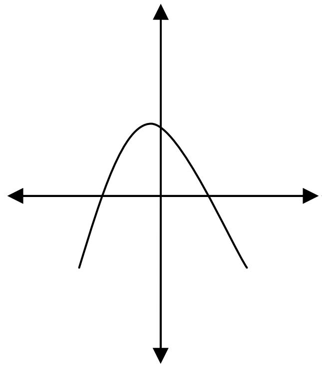

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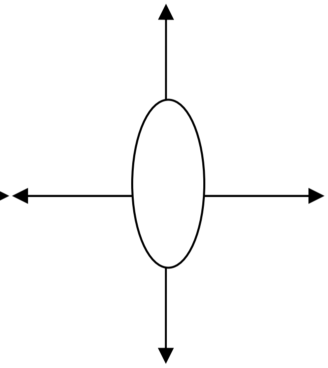


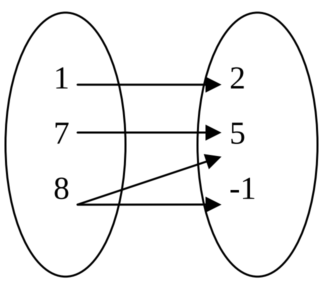
4.

Input	Output
-3	1
5	0
8	1
-2	0

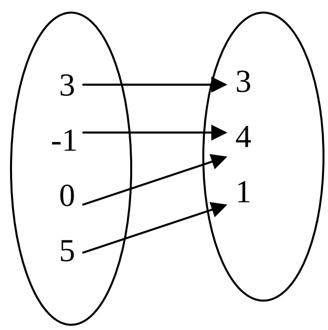
5.

Input	Output
10	1
0	3
-10	5
0	4

6. Input Output



7. Input Output



Later in the lesson, linear functions were represented using *Point-slope* equations of lines. In the third lesson, *5.2 Writing Linear Equations Given the Slope and a Point*, Mr. Able made connections to the first lesson by asking the students to find the slope (*m*) between two points and then demonstrated how this slope and either of the points could be used to write a *Point-slope* equation of the line containing them.

Krussel et al. framework. Applying the Krussel et al. (2004) framework to analyze Mr. Able's "discourse moves" as a facilitator of discourse in his Algebra I classroom, the following was determined. The ***purpose*** of Mr. Able's discourse was to encourage participation in whole-class discussions and activities. For example, he catered to his diverse student mix, including the ELLs, by using frequent questions such as, "What to do now, *m* or b?," "what is *m*?," and "what is b?" (e.g., either/or and one-word-response questions). Thus, he purposefully used simpler talk, synonyms and various visual representations to ensure that his students attained a better grasp of the mathematical concept being presented. For example, in order to better explain the representation of slope as $m = \frac{rise}{run}$, he used the words "the top" and "the bottom" when referring to the numerator and the denominator of the fraction representing the slope. At the same time, he drew arrows $\frac{\uparrow\downarrow}{\rightleftarrows}$ to indicate that each positive or negative integer represents the number of units they have to move up or down the grid.

However, Mr. Able did not try to direct the students toward reflecting on their thinking, or to provide explanations and justifications. Most of the questions were of the type "what is…" or "tell me…" and did not move the students to higher levels of cognitive demand according to Bloom's Taxonomy: *knowledge, comprehension, application, and analysis*. He asked only one "how…" question: "How do we now graph this equation?" which, had he waited, would have initiated a longer response than the one garnered. Moreover, had Mr. Able called upon an ELL student for the question, it would have encouraged that student to a higher level of SLA such as *speech emergence* or *intermediate speech*. However, Mr. Able missed this opportunity by immediately following that initial question with another, “What is this point here?” and by assisting the students by pointing to the y-intercept in the equation.

The ***setting*** for classroom discourse appeared to be established early in the school year as it was possible to discern certain norms of classroom behavior in each of the observed lessons. These normative practices were exemplified via the classroom layout and the well-established roles of the teacher and student in the classroom discourse that required no clarification during any of the observed lessons. For example, after the alarm clock went off to indicate the end of the bell work, all the students stopped writing and turned in their bell work according to demonstrated pre-established practices. Additionally, when Mr. Able asked general questions, despite the fact that a few students answered aloud, only one student took the next turn talking (by receiving an encouraging nod from Mr. Able), and then the same student continued talking, extending or correcting his/her answer if such corrections were needed.

The ***form*** of the teacher’s discourse includes both *teacher talk* (verbal) and *actions* (non verbal). For example, Mr. Able’s questions, “What would you have to do to get b by itself?” or “How do we now graph this equation?” indicated that Mr. Able’s discourse took the form of a challenge. However, he was satisfied with short responses and easily provided assistance in subsequent steps. He did not move the discourse to the higher levels of cognitive demand such as *synthesis* and *evaluation,* according to Bloom’s Taxonomy. He did not ask students to categorize, justify, or perform more critical analyses or to further explain some steps of their problem-solving process.

Close examination of Mr. Able’s lessons revealed the various forms through which Mr. Able’s non-verbal discourse was displayed. He often used gestures to demonstrate the slope of the line as first going up or down (rise) and the left or right (run); or he faced the class and used eye contact after posing a question, and with just a nod indicated which student may answer. He also walked between the rows when students performed individual work and assisted them or answered questions (if asked), thus establishing spatial proximity between teacher and student.

According to this Krussell et al.'s (2004) framework, the teacher's discourse always has some ***consequences,*** which may be *intended* or *unintended*, *immediate* or *long term*. For example, Mr. Able *intended* to shift the cognitive level of the task performed (graphing a line) by asking the students to explain how to do this, but he *unintentionally* assisted them in this task, thus lowering his expectations of their abilities to complete the task on their own. More important, whenever he asked ELL students to answer a question, after the ELLs provided a one-word response, Mr. Able did not challenge them to further explain the steps needed, but instead pointed out each consecutive step. Consequently, he *unintentionally* demonstrated lower expectations for ELLs and did not provide them with opportunities to practice their mathematics vocabulary in English. Some of the *immediate* or *long-term* consequences of the teacher discourse were apparent when Mr. Able wanted to shift the dialogue from *univocal* to *dialogic* and to involve the class. He usually faced the students and asked questions such as, "can someone tell me…," thus indicating that each student could participate in the discourse. Mr. Able also had set long-term norms of formality, such as taking turns speaking, during classroom discussions. This, in turn, allowed the students (and the ELLs in particular) to focus their attention on the meaning of the mathematical discourse rather than on its form, and set norms for general communication in English.

Perceptions of classroom discourse. On the graph below, I have represented the results of comparing the data from three sources of evaluation (i.e. *TTT Form 1, 2, and 3*) of teacher talk (see Figure 1). Figure 1 represents the researcher's preliminary evaluation of the teacher talk (i.e., before analyzing the teacher talk and counting of the frequency with which each discursive category is used), Mr. Able's self evaluation of the frequency with which he used the pre-determined categories of teacher talk and discourse characteristics, and his ELLs' evaluations of his use of the pre-determined teaching strategies and categories of teacher talk. An average ELL student score was determined by adding the evaluations of his ELLs for each discursive category and then dividing this by the number of ELL students. The numbers 1 to 22 on the y-axis correspond to the pre-determined major categories of the teacher talk that are described in greater detail in *TTT Forms 1, 2,* and *3* (see Appendices A, C, and E). On the x-axis is the frequency that each evaluator attributed to Mr. Able's talk or strategies, on a scale from 1 to 5 (with 5 representing the highest frequency).

The general level of agreement amongst the teacher, the researcher, and the ELLs in their evaluations of the strategies used by the teacher are presented by the computed pair-wise correlations (Pearson product-moment correlation coefficients), which show whether the teacher's perceptions of his own use of strategies matches those of the researcher and the ELLs. For this particular case, the correlation between the teacher and researcher is .62; between the teacher and ELLs it is .25, and between the researcher and ELLs it is .65.

As Figure 1 indicates, there are four strategies where there is complete consensus between the evaluations of the researcher, the teacher self-evaluation, and the ELLs'

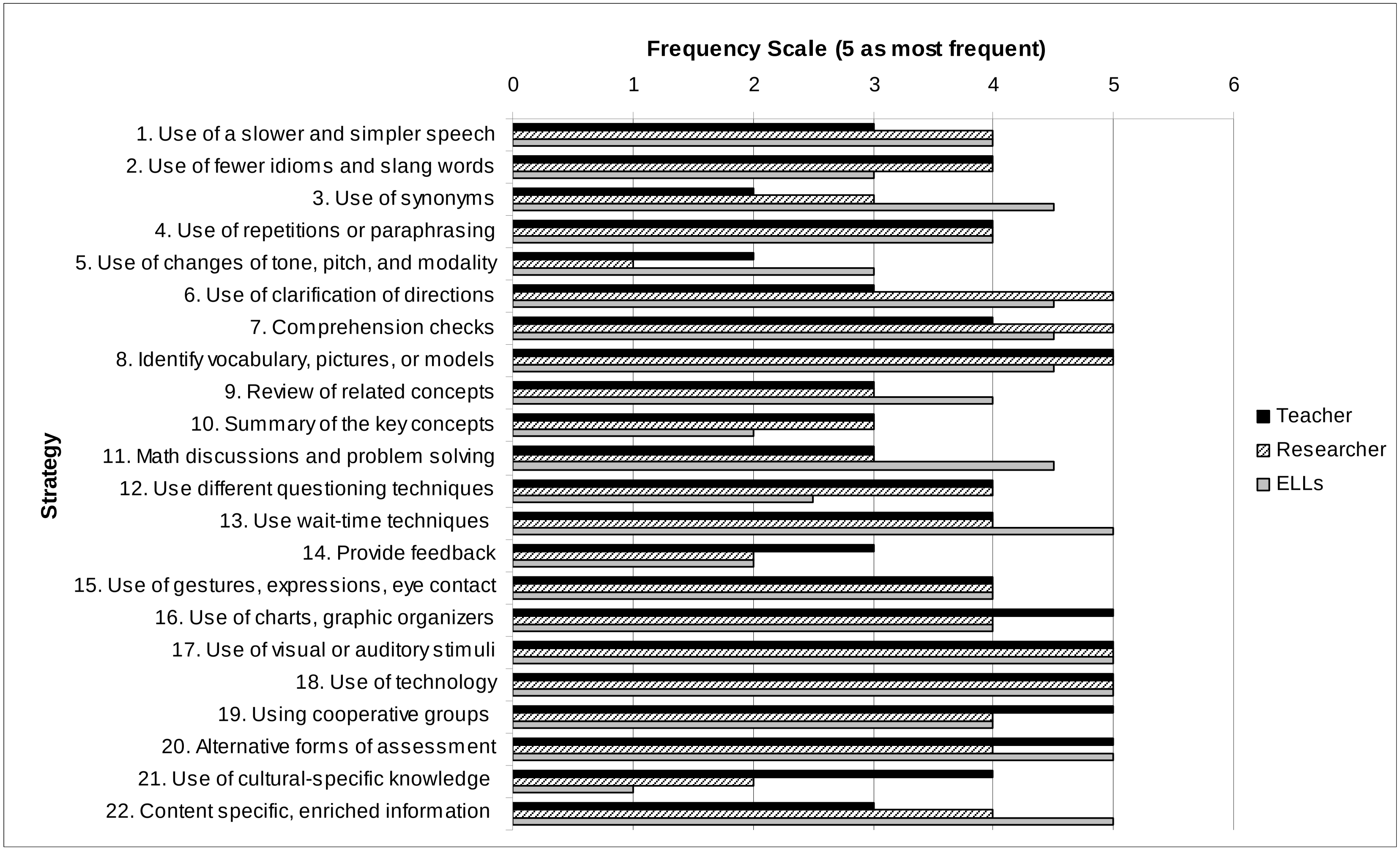


Figure 1. Teacher's, reasearcher's, and ELLs' evaluations of Mr. Able's frequency of use of various discursive strategies.

evaluations—*Use of a variety of visual or auditory stimuli: transparencies, pictures, flashcards, models, etc*. (strategy 17), *Use of technology to enrich a concept presentation* (18), *Use of repetitions* (4), and *Use of gestures, facial expressions, eye contact or demonstrations to enhance comprehension* (15). The first two strategies (17 and 18) were evaluated as most frequently used. The next two strategies (4 and 15) were also evaluated as ones traditionally employed by Mr. Able, but with a slightly lesser frequency.

The video-recordings of Mr. Able's classroom sessions also reveal that he consistently used an overhead projector, calculators, or pre-prepared spreadsheets with data so as to enhance his presentation of concepts. Mr. Able varied his presentation modes between transparencies containing pre-prepared bell work, lesson outlines, and the use of grids to graph linear equations. Mr. Able also often repeated or paraphrased his statements or asked students to repeat or restate them, especially when important mathematical concepts were formulated. For example, he used the phrase "always start with the y-intercept" and repeated it three times in three consecutive examples which the students were given to graph. At the same time, he asked them to recognize that b in the equation $y = mx+b$ represents the y-intercept and to write it as a coordinate pair such as (0, 5) and then to plot that point on the y-axis. Mr. Able also used facial expressions, gestures, and eye contact that exhibited awareness of culture-specific acumen.

Figure 1 also reveals some differences between the evaluations of the researcher, the teacher self-evaluation, and the ELLs' evaluations of the teacher's *use of change of tone, pitch, and modality* (strategy 5) and *providing opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics* (21). I evaluated Mr. Able as having used strategy 5 least frequently, whereas he judged that he used this strategy one to two times a month and the ELL students evaluated that Mr. Able used this strategy at least one to two times a week. Strategy 21, on the other hand, was evaluated by the ELL students as almost never having been used by Mr. Able, used rarely according to the researcher, and used three to four times a week according to the teacher's self-evaluation. Such anomalies in the results could be attributed to the fact that the conclusions of the researcher were elicited only from the observations of the three classroom sessions and the interviews with the teacher and students. However, since the focus of this investigation is on the classroom discourse and *teacher talk* influences on ELL students' mathematics experiences, these students' opinions were placed under special scrutiny.

Thus, with regards to how ELLs feel in Mr. Able's classroom, it is evident from Figure 1 that some strategies are not as often incorporated in his teaching style (look at the lowest bars from amongst the ELLs' evaluations of Mr. Able's discourse)—*Conclude a lesson with a summary of key concepts* (strategy 10), *Provide feedback* (14) and *Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics* (21). The ELLs' evaluations reveal that Mr. Able did not encourage the students to reflect on the concepts they had just learned by concluding lessons with a summary of important points. The ELL students in Mr. Able's class shared that they were not immediately provided with clear *feedback* if they had answered a question correctly or not. They also indicated that Mr. Able did not ask them to *talk or give examples from their country or family when solving mathematical problems*.

Summary of the frequency count of the teacher's discursive strategies. Figure 2 provides the frequency with which Mr. Able implemented each of the strategies found on the *TTT Form 1* on the observed lessons. The strategies most frequently employed are: *Use of different questioning techniques sensitive to the level of SLA* (strategy 12), *Use of gestures, facial expressions, eye contact, or demonstrations to enhance comprehension* (15), and *Use drawing of charts and graphics organizers to enhance comprehension* (16). This information provides additional evidence to support conclusions drawn from the previous graph (Figure 1), where most of the evaluators also indicated that Mr. Able used strategies 12, 15 and 16 fairly frequently. However, additional analysis of the types of questions posed by Mr. Able reveals that they usually elicited only one-word responses, or were general and thus elicited only a short list of words in response. This suggests that although Mr. Able is aware of the level of his ELLs – *early production* –he did not challenge them with questions that could lead them to move to the next levels of subject-specific literacy – *speech emergence* and *intermediate speech* in mathematics and in English. In order for ELLs (and all students in general) to become more active participants in the mathematics classroom discourse, they need to be given opportunities to share their opinions, to explore different methods of solving mathematical problems, or simply to be encouraged to participate more and thus to reinforce their learning of what is for them new and unfamiliar mathematics terminology.

The lack of valuations in category 2 (*Use of idioms and slang words from the mathematics vocabulary which if used, are accompanied by a proper explanation)* is likely due to the fact that Mr. Able used shorter sentences, gestures, or drawings to provide visual representation of idioms or slang words from the mathematics vocabulary.

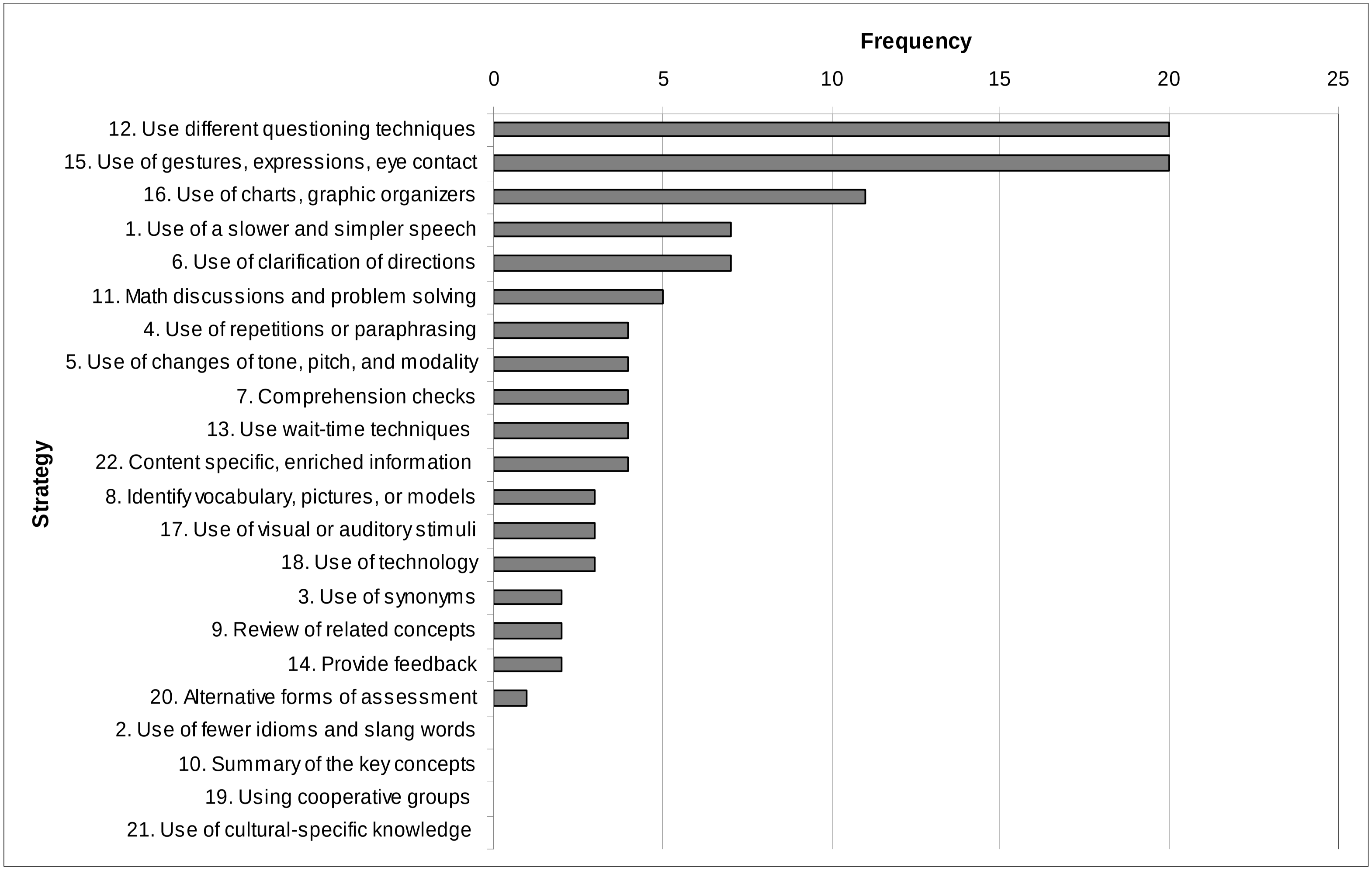


Figure 2. Frequency count of Mr. Able’s use of various discursive strategies during the three 20-minute video-recorded sessions.

For example, he used the drawing $\frac{\uparrow\downarrow}{\rightleftarrows}$ in order to assist his ELLs (and all students for that matter) in associating the words *rise* and *run* in the formula for slope $m = \frac{rise}{run}$ with the directions of movement to which they refer. Figure 2 also reveals that Mr. Able utilized only one style of teaching and did not *expose his students to different classroom work arrangements, such as cooperative groups or partner discussions* (i.e., lack of use of strategy 19).

Additionally, the two ELL students indicated in the interview that they found the observed lessons easy and that this was the reason they did not have any difficulties understanding the concepts. Because of that they did not feel the need to ask questions and thus participated just once, if at all. They were able to simply listen to the teacher's explanations and could, without many difficulties, execute the task of graphing equations.

Ms. Barrera

Ms. Barrera, a woman in her mid 40s originally from Central America, is fluent in both Spanish and English. She worked in Green Bay High School for five years: three years as a Teacher Assistant (aiding the Mathematics teachers with translations in Spanish), and the last two years as a teacher of Algebra I and Intensive Math. She earned an engineering degree in her country. Ms. Barrera initially obtained a temporary teaching certificate in Physics, and is currently working on her certification in Secondary Mathematics. She recently completed the requirement for ESOL training for subject area teachers by attending night classes.

Ms. Barrera taught her Algebra I class with the aid of a Computer Aided Instructional Program, *I Can Learn Lab,* which has tutorials, tests, and quizzes aligned with the Algebra I curriculum. The class consisted of 24 students, of which 9 were officially listed on the roster as ELL students, all of them with Hispanic backgrounds. Thirteen of the 24 students were also of Hispanic origin, five were African-American and 6 students were White. Considering the teacher's bilingual abilities and educational background, the school's guidance department enrolled more Hispanic ELLs in Ms. Barrera's Algebra I class so that they could be better assisted with both mathematics and language issues.

During an interview, Ms. Barrera shared that because she speaks English with an accent, she often uses comprehension checks in order to ensure that she is understood by her students. Additionally, these checks allowed her to determine whether she needs to modify her talk or use synonyms in order to negotiate with her students the meaning of the particular idea she was trying to convey.

Typical classroom discourse. The following excerpts provide specific examples of these tactics. Each line (a sentence or fragments with the same idea) is numbered so as to allow for clearer references to it thereafter:

Ms. Barrera: [1] If you remember yesterday I wrote the steps how you can determine if two lines are parallel or perpendicular.
[2] I am going to remember to you step one is: write the equations in slope-intercept form, which is $y = mx + b$.
[3] Do you remember guys, what is m and what is b?
[4] Raise your hands if you remember.
[5] Who's m?

Here the teacher, being ELL herself, used the words "remember" (line 2) instead of "remind" and "who" (line 5) instead of "what". The words were pronounced with a Spanish accent, but it appeared the students had no difficulties understanding, and if they did, they were negotiating the meaning that the teacher was trying to convey. The strategies used were numbers 9 (*Start with a review of related concepts*) (lines 1 and 2) and 12b (*Questioning technique* sensitive to ELL level of *early production* and requiring a one-word response) (lines 3 and 5).

ELL Student: [6] point-slope
Ms. Barrera: [7] Just, uh, very good, slope.
[8] That is going to be that coefficient together with x.
[9] And what is b?
[10] You remember?

The answer was not exact (line 6), but the teacher used strategies number 14 (*Provide feedback*) (line 7), strategy 3 (*Use of synonyms*) for "slope" as "that coefficient together with x", and strategy 12b (another *Use of questions requiring a one-word response*) to encourage and help the student better understand where the slope is located in the equation.

ELL Student: [11] y
Ms. Barrera: [12] y-intercept.
[13] Correct, y- intercept is b. Correct.
[14] Then now, I want to ask you guys.
[15] I am going to call equation one and equation two.
[16] Uh, Melissa, is equation one written in slope-intercept form?

Here again the answer was not exact (line 11), but the teacher used strategies number 14 (*Provide feedback*) (line 13) and 4 (*Use of repetitions or paraphrasing)* to paraphrase the student's answer into the more precise, correct answer (line 12), thus emphasizing subject-specific *lesson vocabulary* (strategy 8) – "slope" (line 7) and "y-intercept" (line 13). Furthermore, by calling on an ELL for participation (line 16), the teacher tried to *involve ELL students in mathematical discussions* (strategy 11) by using a *yes/no question* (strategy 12 b) (line 16) which showed sensitivity to the student's level of SLA (*early production*).

ELL Student: [17] No
Ms. Barrera: [18] Why?

With this *higher level question* (strategy 12c) (line 18), the teacher initiated a more thorough response and encouraged the ELL student to try to evolve to the next stages of SLA *speech emergence* or *intermediate fluency*. Furthermore, Ms. Barrera thus tried to involve the ELL student in justifications and explanations of answers and thus exhibited her higher expectations of the ELL students that they could handle discussions requiring higher levels of cognitive demand (*evaluations, justification,* and *explanations*), as per Bloom's Taxonomy (strategy 22). At the same time, she encouraged the development of their linguistic abilities by involving them in mathematics discussions.

ELL Student: [19] Because y is not isolated.
Ms. Barrera: [20] Correct.
[21] Because y is not isolated.
[22] You have to write or leave y alone.
[23] Then I am going to write equation one in slope-intercept form.
[24] That means I have to isolate y.
[25] What do I have to do in order to isolate my variable y?

Here the teacher *provided feedback* (strategy 14) (line 20), then *used repetition* (strategy 4) (line 21) to emphasize the importance of isolating y, and finally used simple language to explain that y must be written or left alone (strategy 1c- *use of slower and simpler speech*) (lines 21-24), yet at the same time she focused her talk on key concepts. With the last question (strategy 12 d) (line 25) she moved the discussion to higher linguistic and cognitive levels by asking the student to recommend the next step, thus encouraging the development of *intermediate speech* level of SLA in her ELL students.

Melissa: [26] You can subtract.
Ms. Barrera: [27] You have to subtract 5x.
[28] Okay, you have to move this term.
[29] In order to move this term, you have to do the opposite.

> [30] That is adding, you having to subtract minus 5x and minus 5x.
> [31] I have to do the same thing in both sides of my equation in order to keep the balance.
> [32] Okay guys?

Here the teacher *used repetitions and paraphrasing* of the student's answer (strategy 4) (line 27) and thus *provided feedback* (strategy 14) (lines 27-29) by extending upon the student's answer and explanations. She then *used various synonymous expressions* (such as "move this term" (line 28); "do the opposite" (line 29); "minus 5x" (line 30), etc., in order to help the students better grasp the concept of opposite operations in transforming the equation into Slope-Intercept form and maintaining the balance in the equation (line 31). Thus, she demonstrated *use of synonymous expressions* to teach the mathematical concept of performing "opposite operations" to transform equations (strategy 3). At the end, she performed a *comprehension check* (strategy 7), by asking the question "Okay guys?" (line 32). In the interview Ms. Barrera indicated that, by often asking her students this question, she was actually reassuring herself that her students were following her explanations and understood the consequences of the performed steps. As she said "Okay guys", she faced her students, and was indicating that she expected a response by providing some wait time. Observing the students' facial expressions or looking to see if they were shaking their heads (in agreement or disagreement) was an indication to her if they understood the operation she was performing on the board. Thus, Ms. Barrera was actually performing a *comprehension check* (strategy 7).

Krussel et al. framework. An application of Krussel et al. (2004) framework revealed that the ***purpose*** of Ms. Barrera's discourse was to involve more ELLs in mathematical discussions and to encourage justifications and explanations of their answers by moving from “What is this…” types of questions, initiating usually one-word or short responses, to higher order questions (“Why?” or “Because…?” [changing the pitch of her voice at the end to indicate that the student must provide a justification]). Ms. Barrera thus guided her students (both ELLs and fluent English speakers) to reflect on their thinking and to provide explanations and justifications. Moreover, she regularly called upon an almost equal number of ELL and non-ELL students. On average, throughout the three observed sessions, Mrs. Barrera called on at least three to four different ELLs and the same number of non-ELLs, and then individually helped at least three ELLs and any other students (one or two) if they requested help.

What was interesting with regards to Ms. Barrera’s teaching style was that even though she taught this Algebra I class in a computer lab, she often varied her instruction strategies. From a class-wide lecture using the overhead projector and involving the students in discussions, she often switched to individual work on the computers and provided individual help to particular students.

Ms. Barrera’s actions towards establishing a ***setting*** for classroom discourse can be inferred from conversations with other teachers and from my own observations that she had strict rules in order to ensure that the computers in the room are used properly. For example, she had explicitly written on the board her rules stating in essence that no food or drinks are permitted and that students should always bring their materials to school. Additionally, the students had their binders on a shelf and neat note-taking was encouraged.

The ***form*** of Ms. Barrera’s discourse also included both verbal (*teacher talk*) and non-verbal (actions) forms. Even though Ms. Barrera tried to emphasize both the meaning and form of mathematical sentences, being an ELL herself, she sometimes struggled with the pronunciation or proper usage of the mathematics terminology in English. In some instances, students were confused by her accent, improper sentence organization, or improper use of English grammar. For example, while reading a particular problem from the computer screen, Ms. Barrera was actually stating the following to a non-Spanish speaker who asked for her help: "The problem is determined the equation of the line contends that giving points and write answered in the standard form." Obviously she was trying to direct the student by paraphrasing what the problem asks for (i.e., to determine the equation of the line that contains the given points (to which she points), and to write the answer in standard form), but she was using a long sentence in improper English. Next however, Ms. Barrera negotiated the meaning of the text she just read, by breaking it into simpler sentences, using gestures to point to the given information, and explaining what steps should be taken to solve the problem:

> Okay, the first thing they are giving to you is two points, this and this.
> (She points to the points on the computer screen.)
> The first thing you have to find a slope.
> Okay, after that solve for b.
> And after that solve that equation.
> (She wrote the equation $y=mx+b$ in the student's notebook.)
> And then write that equation in standard form.

This short excerpt illustrates some instances of Ms. Barrera's verbal form of discourse (*teacher talk),* as well as the fact that she is a teacher who is an ELL herself. By being aware of that, she often tried to negotiate the meaning of whatever she was saying by using simpler and shorter sentences and by using non-verbal forms of discourse.

Ms. Barrera's non-verbal discourse was displayed in different forms—she often used gestures to demonstrate the slope of the line and also used colored markers when writing on the overhead: she marked important formulas in red, each consecutive step in blue, and all other information in black. She also often sat next to a student to help him/her on the computer and once encouragingly patted on the shoulder an ELL student who exhibited improvement in both his mathematics and English language skills.

The ***consequences*** of Ms. Barrera's discourse fell into the following categories - *intended* or *unintended*, *immediate* or *long term* as described in the Krussel et al. (2004) framework. Ms. Barrera *intentionally* shifted the cognitive level of the task performed by asking the students to explain how to perform certain steps. She demonstrated equally high expectations of ELL and non ELL students. However, she *unintentionally* made occasional mistakes in her proper use of the mathematical terminology and tried to compensate for this by further explanations and use of synonyms (e.g., she asked her students to "move" a term from one side of the equation to the other, and further added "do the opposite…when addition, subtract the term on both sides" (lines 28-31)). Nevertheless, some of the positive *immediate* or *long-term* consequences of the teacher discourse were also transparent. For example, when Ms. Barrera wanted to encourage ELL students (and all students for that matter) to start using proper reasoning techniques and justifications of the answers they proposed, she would sometimes ask a question: "can you add these two terms?" and would point to $3 + 5x$ for example. After the student answered "No", she would encourage him/her to properly state the reasons for that by simply stating "because…" and changing the pitch of her voice at the end. Thus she indicated that she expected a complete statement of the sort "because they are not like terms," *immediately* indicating to the student that in mathematics justifications, proper responses are needed. She thus set *long-term* norms of classroom discussions. This in turn, ensured that the students (and the ELLs in particular) focused on both the mathematical discourse's meaning (to justify) and its form (using "because…"), and set the norm for the proper usage of content-specific vocabulary in English.

Perceptions of classroom discourse. Figure 3 represents in the form of bar graphs the researcher's preliminary evaluation (i.e., before counting of the frequencies with which Ms. Barrera uses different discourse strategies), Ms. Barrera's self-evaluation, and the evaluations of her ELL students who volunteered to participate in the study. The pair-wise correlations (Pearson product-moment correlation coefficients) for Ms. Barrera's case study are as follows: the correlation between the teacher and researcher is -.07; between the teacher and ELLs it is -.23, and between the researcher and ELLs it is .27. The negative results could be attributed to an unrealistic self-evaluation and a lack of understanding of the ELL students, which is due to less years of teaching experience and a lack of ESOL training.

There is agreement on the use of strategies 7 (*Use of Comprehension Checks*) and 14 (*Provide Feedback*) as the most frequently used, followed by (slightly less frequently) strategies 3 (*Use of Synonyms*) and 18 (*Use of Technology*). Ms. Barrera used various strategies to ensure that she is understood by her students and encouraged their further participation in the discourse by often providing feedback. Throughout the observed lessons, she frequently asked her students whether they understood the content of her talk. If further clarification was necessary, she often modified her talk by using synonyms, explained ideas more thoroughly, and helped students visualize the concept under discussion by using the overhead projector or the computer screen.

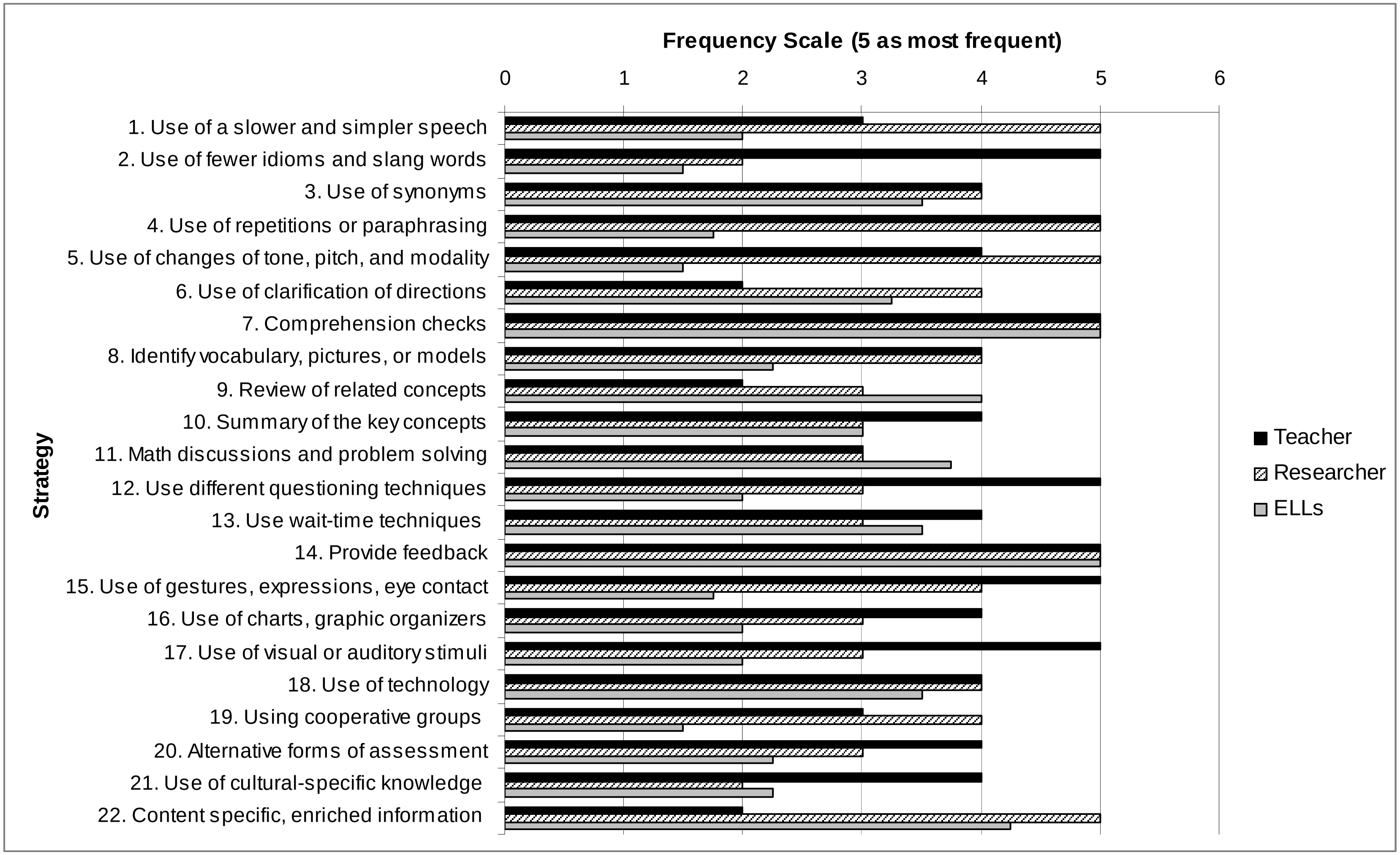


Figure 3. Teacher's , researcher's, and ELLs' evaluations of Ms. Barrera's frequency of use of various discursive strategies.

Figure 3 illustrates interesting differences between the researcher's evaluation, the teacher's self-evaluation, and the ELLs' evaluations of Ms. Barrera's frequency of use in the following strategies—strategies 1 (*Use of a slower and simpler speech),* 2 (*Use of fewer idioms and slang words*), 5 (*Use of change of tone, pitch, and modality*), 15 (*Use of gestures, facial expressions, and eye contact*), and 22 (*Provides students with content specific, enriched information, thus exhibiting equally high expectations from both ELL and English speaking students)*. With respect to strategy 22, while the ELL students and the researcher indicated that Ms. Barrera frequently used this strategy, her own reflections on her teaching style indicated that she thinks she is not applying the strategy frequently enough. Even though in the interview she revealed that this is something she strongly believes to help her ELLs, "For ESOL students, I like to use examples of the real life, and also do different activities that they can utilize the topic that I am teaching," she clearly thinks that she needs to improve upon her use of the strategy. On the other hand, strategies 1, 2, 5, and 15 were evaluated by the ELL students as rarely being used by Ms. Barrera (one to two times a month). Ms. Barrera and the researcher both felt that she used these strategies more frequently.

Summary of the frequency count of the teacher's discursive strategies. The actual frequency count of Ms. Barrera's use of each strategy, shown in Figure 4 below, reveals the frequency with which each category was used during the three observed lessons, and is in better consensus with the ELLs' evaluations, rather than with those of the researcher and the teacher's self-evaluation.

Figure 4 indicates the frequency with which Ms. Barrera implements the strategies from *TTT Form 1*. At first glance, we can immediately notice that the most typical strategies for Ms. Barrera are strategy 12 (*Use of Different Questioning Techniques*) and strategy 14 (*Provide Feedback*). Next in use are strategies 7 (*Use of comprehension checks*), 4 (U*se of Repetitions*) and 6 (*Use of Clarifications of Directions).* This data is further supported by the interview with an ELL student who confirms Ms. Barrera's use of the above strategies:

> Uh, I like my mathematics class because the teachers know how to explain, you know, like for all to understand. [If] she got to explain like twenty times for you to understand, she will do, and she always a good person. She…, if you don't understand English she will talk Spanish. She make[s] it easy to like mathematics that way. And she uh, no the easy way of the class is where you are the focus, if she see you. If you fail some quiz, she will help you for next time [so that] you can pass it, and she's a great teacher.

However, Figure 4 also reveals that Ms. Barrera less frequently applied strategies 15 (*Use of gestures, facial expressions or eye contact*), 2 (*Use of fewer idioms and slang words from the mathematics vocabulary, or if used a proper explanation was provided*), 13 (*Use of wait-time techniques after posing a question*), 16 (*Use drawings of charts and visual organizers*) and 20 (*Providing the students with alternative forms of assessment*). As the excerpts demonstrate, Ms. Barrera often asked the students questions to check their knowledge of previously-presented concepts, to finish subsequent steps of a problem, or to check their comprehension. However, she expected immediate responses and did not provide the students with enough time to think and process information and subsequently provided the correct answers herself. When students' answers were only partially correct, Ms. Barrera usually provided the correct statement or paraphrased

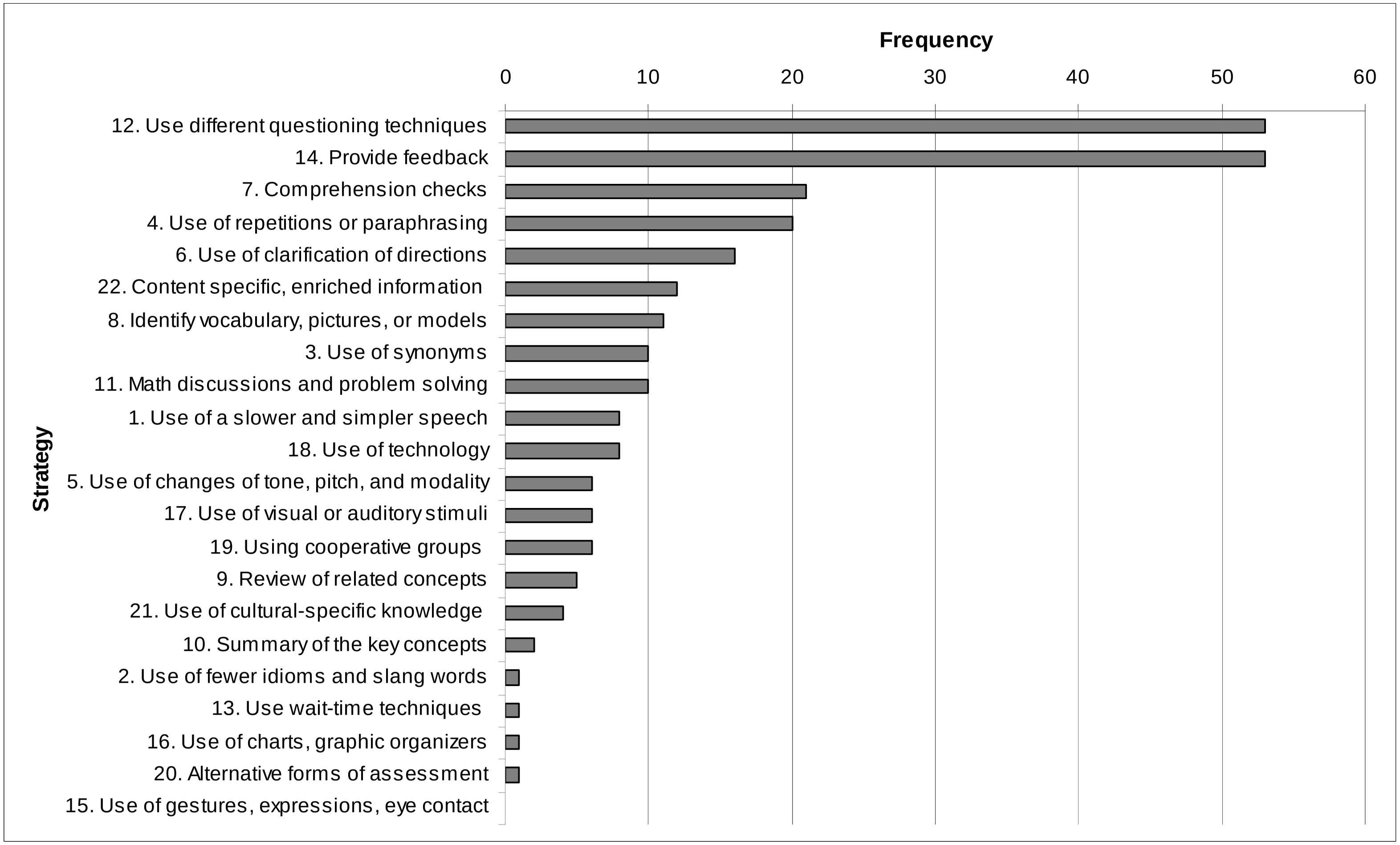


Figure 4. Frequency count of Ms. Barrera's use of various discursive strategies during the three 20-minute video-recorded sessions.

students' responses so that they were correct without asking students to do so themselves. Although she exhibited sensitivity to her ELLs' levels of English fluency and encouraged them to talk using mathematical terms, she did not allow time for them to think after she posed questions.

When ELL students were asked to elaborate on their experiences in their mathematics classroom, they expressed their preferences: "…like maybe work in groups, that would be better 'cause other people would know how to do it too." The ELL students were aware that Ms. Barrera was limited by the setting of their Algebra I class in a computer lab. They all indicated that she usually assisted them by explaining things to the class as a whole by demonstrating examples on the overhead projector. They also indicated in the interviews that Ms. Barrera helped them also individually by circulating between the desks and the computers and often sat next to them and provided additional assistance if needed. However, they still felt that Ms. Barrera could allow them to work in cooperative groups or with partners (strategy 20), because this would also be beneficial if they needed help at a particular moment when she was assisting another student. Figure 3 also demonstrates that, according to the ELLs, strategy 20 is not utilized frequently enough by Ms. Barrera, and the frequency count of the use of strategy 20 throughout the three observed sessions confirms that (Refer to Figures 3 and 4).

Ms. Chandler

Ms. Chandler is Caucasian, in her early 40s, and speaks only English. She is certified to teach secondary mathematics and has previous higher level mathematics and technology application experience because she worked as an analyst for a software company prior to becoming a high school mathematics teacher. She has taught in her current school for eight years, and feels very comfortable teaching Algebra I in a computer lab. She completed her ESOL endorsement requirement of 60 hours through course work six years ago.

Ms. Chandler's Algebra I class consists of 26 students, eight of whom are ELLs of a Hispanic background. In the interview, she shared that even though "it is challenging to teach ESOL students," she finds them "to be mostly motivated and capable mathematically." She also added that she had equally high expectations in mathematics from her ELL and fluent English-speaking students.

Typical classroom discourse. The following pattern emerged from the three observed lessons — Ms. Chandler never used whole-class lecturing as a teaching technique, but continually circulated amongst her students while they worked on the computers and assisted them if they asked for help or if she decided they needed any. The excerpt below demonstrates which strategies in particular Ms. Chandler utilized when a female ELL student-Narissa - asked her for help:

Ms. Chandler: [1] So when you're writing the notes, when you're writing the notes, it wasn't making sense?
Narissa: [2] No because...
Ms. Chandler: [3] I see… go on. [Ms. Chandler acknowledged that the student had a problem, and encouraged her to continue.]
Narissa: [4] It was number five on the um sheet.
Ms. Chandler: [5] Yeah? [Again: *Encouragement to continue*]
Narissa: [6] It was one of them where you had the fractions.
Ms. Chandler: [7] Oh right, the fractions are more difficult, but, so here's the deal.
[8] You got y, so you were getting that problem because you have to know the slope and you also have to know b.
[9] So you're doing m and now you put the two in there.
[10] That's exactly right.

In line 7, strategy 14 *(provide feedback)* is used, as Ms. Chandler acknowledged that fractions are more complicated. Furthermore, she provided *assistance and the* *clarification of the steps* that the student needs to execute next in order to solve the problem (strategy 6) – that is, to write the equation in slope-intercept form when two points are given (lines 8 and 9). The teacher also provided the student with *feedback* that she had found the slope correctly (strategy 14 again in line 10). During the interview, Narissa indicated that only after Ms. Chandler provided her with the *feedback* (strategy 14) that she had found the slope correctly and *assisted* her in the next steps (strategy 6) was she able to complete the problem she was struggling with. The remainder of the excerpt will demonstrate other specific strategies (besides strategy 14 and 6) that Ms. Chandler used, as well as the examples that she provided, in order to guide the student to the task's completion and complete the rest of the examples from that lesson on her own:

Ms. Chandler: [11] So the only other thing you have to do is… you have to use one of the points.
[12] So they are your points, right? (*x, y*) (*x*, *y*).
[13] So just you can pick either point.
[14] It doesn't matter which.
[15] So use this one because it has the one, so it seems easier.
[17] So we put that in place in *y*.
[18] Here's your *m* you already did.
[19] Put that in for *x* because your whole what you're trying to do is find *b*.

Here the teacher pointed to the given points (-1, 1) and (1, 5) (strategy 15 - *Use of gesture and demonstrations to enhance comprehension,* lines 11 and 12), demonstrated why and how the student should substitute one of the points (lines 13, 14, and 15), and wrote out:

$$1=2(-1) + b.$$

The language the teacher used to explain the solution was *simplified* (i.e., she used simple commands and shorter sentences when explaining concepts – strategy 1a) and thus was adapted to the ELL student's level of SLA-*pre-production.* However, the use of "put that in" instead of "substitute" (lines 17 and 19) indicates that even though Ms. Chandler demonstrated for the ELL student the solution in writing, she did not model the correct vocabulary in order for the ELL to move to the next level of English development – *early production* (i.e., lack of use of strategy 1b, wherein teachers need to model/demonstrate correct responses both in mathematics and in English). The next excerpt further demonstrates Ms. Chandler's use of strategy 1a (*Use of a slower and simpler speech*) when demonstrating to the ELL student how to solve the linear equation for *b.* However, Ms. Chandler continued to use "put in" instead of "substitute", and thus missed the opportunity to expose the ELL to the appropriate mathematics terminology for the performed mathematical operation (i.e., lack of use of strategy 1b):

Ms. Chandler: [20] So now you're just gonna solve this for *b*.
[21] So you multiply of course, trying to get *b* alone so you add two.
[22] It's just like solving an equation.
[23] So now you know *m* and you know *b*, and so then you can write the equation *y* equals …and you put in the *m* two *x* plus *b*.
[24] So you're half done with all of these.
[25] You already got the 2 but you just have to find the *b*.
[26] So for every one.
[27] Go ahead.
Narissa: [28] Oh, I thought that you was supposed to stop at… see?
Mrs. Chandler: [29] …yeah, so you had one more step.
[30] So like here.
[31] Put in zero one.
[32] So zero equals one plus *b*…and then you just solve it.

Here Ms. Chandler provided the student with *feedback* to indicate that she understood where she was experiencing difficulty (strategy 14, line 29). Initially she performed a *comprehension check* by encouraging the student to continue on her own (strategy 7, line 27) but, upon observing that the student could not complete the necessary operations on her own, Ms. Chandler decided to demonstrate how to find *b* with one more example (lines 30 to 32). However, while she continued *using simplified speech in her demonstration* (strategy 1a), she again used “put in” instead of “substitute (lines 23 and 31), and “get *b* alone” instead of “isolate *b* on one side of the equation” (line 21) when explaining how to solve for or find *b* (i.e., lack of strategy 1b). Next, Ms. Chandler decided to show yet another example in order to make sure the ELL student would be able to complete the exercise set from this lesson on her own:

Ms. Chandler: [33] I’ll do one more so you can see it.
[34] Zero equals two plus *b*…
[35] So minus two.
[36] *b* equals negative two.
[37] So you just had one more step.
[38] So *y* equals negative two *x* minus two.
[39] Skyla, be quiet.

Here the teacher interrupted her instructions and made a remark to a (non-ELL) student causing a disciplinary problem by *changing her tone, pitch and mode of talking* (strategy 5, line 39). This demonstrated that the teacher was able to handle a discipline problem and, as the following excerpt will demonstrate, continued to maintain the instructional “momentum” without much disturbance:

Ms. Chandler: [40] Minus two.
[41] So what you should do is take this one.
[42] I mean you can finish this one in class today.
[43] Because…just finish out all the b’s and write all the equations.
[44] And you’re done with that one.

[Then, Ms. Chandler continued to *assist* the student with one more example (a fourth example) and *clarified her directions* (strategy 6) in order to make sure the student understood which tasks to complete in class and which to complete at home in order to learn the lesson]:

[45] This one is actually even easier because what I can do, you can write it on the paper if you want or I can put you on the quiz on the computer.

[46] Because on these, they tell you the slope so you don't even have to figure it out.
[47] They tell you the slope and they tell you the point.
[48] And so then you have to write y equals mx plus b.

[She writes *y=mx+b*].

[49] You put the point, you put the slope, and you find b.
[50] That's all you have to do.
[51] So then you'll have those two done and you'll be totally caught up.
[52] So this one let me know after you finish that one if you want to do it on the computer or if you want to take it home and do it.
[53] You don't even have to write the notes because they're so much like those notes.
[54] And then you gotta get both of those done, and these I'll take back and you can do both of those later.

In this longer excerpt, Ms. Chandler modeled for the ELL student (Narissa) the solutions of four sample problems. Then, she *summarized* what Narissa needed to do in order to complete the assignment on her own (strategy 10, lines 41 to 50). Ms. Chandler also provided her with choices between *alternative forms of assessment* – a quiz on the computer or take-home completion of the task by modifying (i.e., shortening) the notes at parts (strategy 20, lines 52 to 54).

The excerpts above also demonstrate the atmosphere "typical" of Ms. Chandler's classroom – i.e., how the teacher managed the discipline in her mathematics classroom and facilitated students in their individual work on computers by providing assistance (to both ELLs and students fluent in English) when needed.

Krussel et al. framework. The ***purpose*** of Ms. Chandler's discourse was to individually assist her students and attend to their individual needs. She regularly called upon and helped an almost equal number of ELL and non-ELL students. On average throughout the three observed sessions, Mrs. Chandler helped at least four to five different students. By frequently circulating around the room, she not only helped in mathematics, but she was helping them fix problems with their computers, and managed discipline problems (as was demonstrated in the excerpts above). As the excerpts above also illustrated, she often gave ELL students the option of choosing between writing all the work in their notebooks and then taking the quiz again, or finishing their notes on two lessons at home and taking more quizzes the following day. Ms. Chandler also graded their notebooks and thus evaluated their progress in writing and reading in mathematics (i.e., utilization of strategy 20 — providing the ELL students with alternative forms of assessments). In the interviews, Ms. Chandler indicated that she was grading all students' mathematics notebooks in the middle and the end of each semester. However, with the ELL students she carried out this check daily or weekly, which gave her a better idea of their progress in her class and thus allowed her to determine which of them needed her immediate assistance.

To classify Ms. Chandler's actions towards establishing a ***setting*** for mathematics classroom discourse, inferences were made from the interviews with Ms. Chandler and with her ELLs, and also from the observations. For example, during the interview, Ms. Chandler indicated that she enjoys teaching Algebra I in a Computer Lab setting. However, she indicated that being unable to speak Spanish, which was the native language of eight of her ELL students, increased her challenges. On the other hand, her experience with them thus far into the school year (already 3 months have passed) indicated that they generally had good background knowledge in mathematics and were very motivated to learn more. In my interviews with them, her ELL students indicated that they also enjoyed the class' computer lab setting. They also said that they liked the class because they worked at their own pace and knew that they could always ask Ms. Chandler to assist them if they encountered difficulties while solving a problem. The observations confirmed that such a classroom setting facilitated "stress-free" student-teacher interactions in mathematics. However, in such a setting, the ELL students lacked exposure to group or partner discussions about mathematics. As was demonstrated in the previous case study, Ms. Barrera, who also taught Algebra I in a computer lab, often switched between a class-wide lecture and individual work. She was involving her students in discussion even if it was just to explain something from the bell work or in order to present a concept to the whole class before assigning them to individual work on the computers.

The ***form*** of Ms. Chandler's discourse also included both verbal (*teacher talk*) and non verbal (actions) forms. As was demonstrated in the above excerpt of her interaction with an ELL student (Narissa), Ms. Chandler's *teacher talk* included shorter sentences and simple commands (i.e., she utilized strategy 1a). Thus, she demonstrated awareness that she was explaining mathematics to an ELL in a very early stage of English language acquisition – *pre-production* – and as a result used slower and simpler speech. She also used non-verbal actions such as gestures and demonstrations to enhance her ELL students' comprehension of her explanations (i.e., she utilized strategy 15). For example, she moved her hand up to show that a line with a positive slope goes up; then she moved her hand down, horizontally, and vertically to demonstrate lines with negative, zero, and undefined slopes, respectively. She also demonstrated to her ELLs how to show their work in their notebooks. For example, during her explanations to Narissa in the excerpt above, Ms. Chandler first wrote the equation of a line ($y=mx+b$), then she pointed to the given points (-1, 1) and (1, 5) and demonstrated how the student should substitute one of the points into the equation, and then she wrote out: $1=2(-1) + b$. However, even though Ms. Chandler demonstrated for Narissa the correct solution in writing, the use of “put that in” instead of “substitute” indicates that she did not model for the ELL student the correct mathematics vocabulary in English. This indicates omission of utilization of strategy 1b, wherein teachers need to model/demonstrate correct responses both in mathematics and in English in order for the ELLs to be able to move to the next level of English development – *early production.*

The ***consequences*** of Ms. Chandler’s discourse fell into the following categories - *intended* or *unintended*, *immediate* or *long term* according to Krussel et al.’s (2004) framework. For example, Ms. Chandler *intentionally* simplified her talk when talking to ELL students, as was demonstrated in the excerpts of her discussion with Narissa (the ELL from a *pre-production* stage of SLA) above. Thus, she demonstrated awareness of the level of SLA of her ELLs and their need to still develop their conceptual understanding of mathematics in English. However, even though she stated in the interview that she holds equally high expectations of ELL and non-ELL students, she *unintentionally* often “took the floor” and was the main speaker, as exhibited in the above excerpts. Thus, she was not providing the ELL students with many chances to be equal participants in mathematics discussions. On a different note, some of the *immediate* or *long-term* consequences of her discourse were also apparent. For example, Ms. Chandler often checked ELL students’ comprehension of her explanations by asking them to complete the assignment on their own after her assistance, but mainly she was doing this by just watching them quietly if they were writing the solution of subsequent problems correctly or, as she indicated in the interview, by checking their written homework on the following day. As was demonstrated in the excerpts above, she encouraged her students to show all the work that they performed in solving a mathematics problem, but she did not encourage them to reason, analyze, or simply discuss the solution with her. Thus she set *long-term* norms of writing in mathematics in English, but did not target the development of her ELLs' oral linguistic abilities.

Perceptions of classroom discourse. Figure 5 represents in the form of bar graphs the researcher's preliminary evaluation (i.e., before an actual count of the frequencies with which Ms. Chandler uses different discursive strategies), Ms. Chandler's self-evaluation, and the evaluations by her ELL students that volunteered to participate in the study. The pair-wise correlations for Ms. Chandler's case study are as follows: the correlation between the teacher and researcher is .77; between the teacher and ELLs it is .53, and between the researcher and ELLs it is .70.

As Figure 5 indicates, according to ELL students, Ms. Chandler most frequently employed the following strategies: 14 (*Provide feedback),* 6 (*Use of clarification of directions)*, 13 (*Use of wait-time after posing a question)*, and 18 (*Use of technology to enrich a concept presentation)* (see Figure 5). The students in Ms. Chandler's class learn Algebra I using the *I Can Learn Lab* (i.e., computers were utilized as an inherent part of instruction), but the video-recorded sessions also reveal that the teacher assisted her students (and the ELLs in particular) in utilizing the technology in their problem solving processes (strategy 18). As one could infer from the excerpts provided above, Ms. Chandler certainly utilized strategies 14 and 6. Additionally, her "laid back" style of explaining concepts and also telling students she would come to check on their work

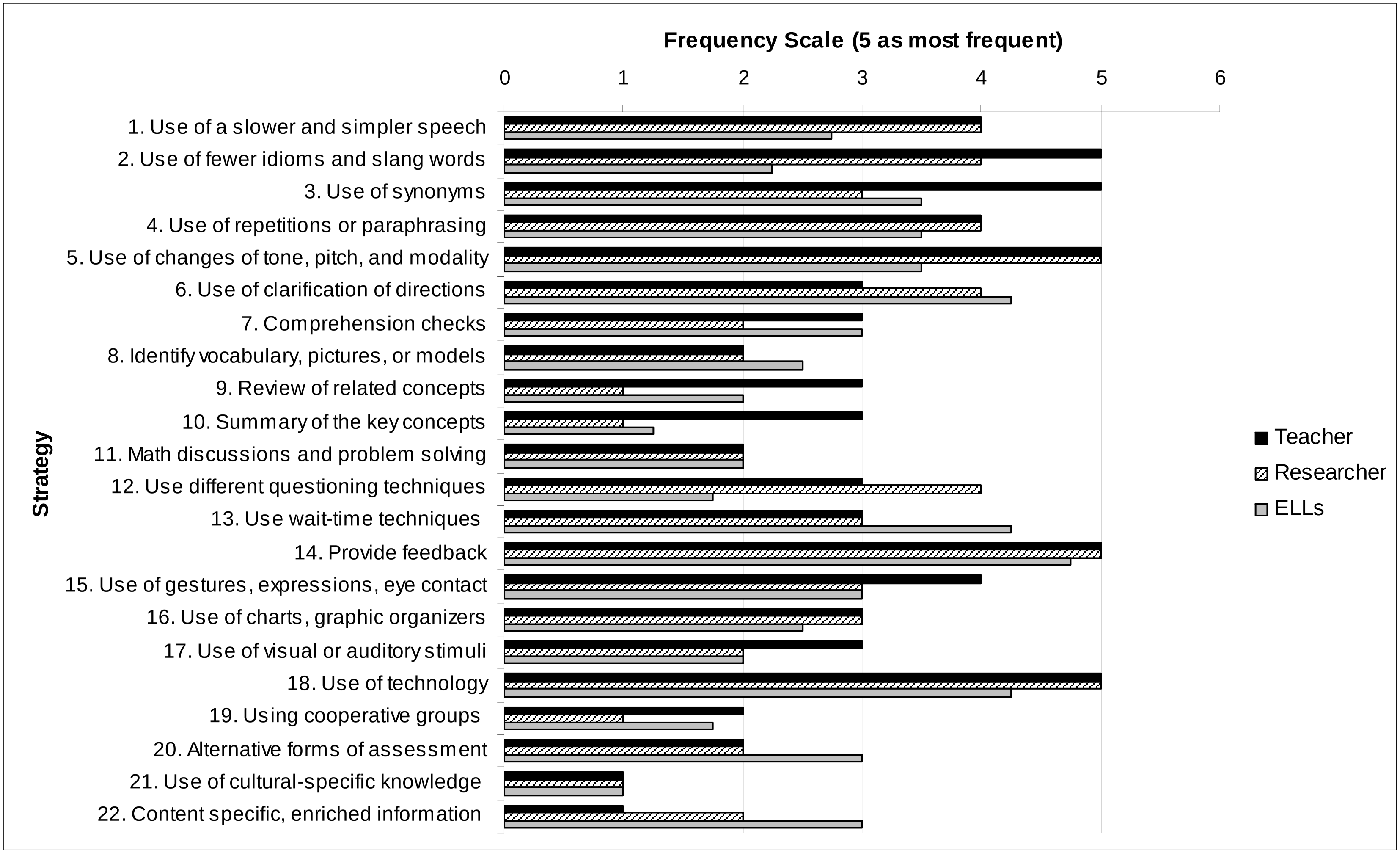


Figure 5. Teacher’s, researcher’s, and ELLs’ efvaluations of Ms. Chandler’s freqeuency of use of various discursive strategies.

again ensured that she was *providing them with enough time* to grasp the new or difficult for them concept (strategy 13).

However, Figure 5 also reveals that while the teacher evaluated herself as most frequently using strategies 2 (*Use of fewer idioms and slang words),* 3 (*Use of synonyms),* and 5 (*Use of change of tone, pitch, and modality),* according to her ELLs (and the researcher), she used these strategies only once or twice a week. Such disparities between the teacher, ELL and researcher evaluations could be attributed to the difference between the teacher's self-perception of how frequently she used the same strategies and the results elicited from the three observed sessions and the interviews. From Figure 5, it is evident that Ms. Chandler evaluated that she frequently used idioms and slang words from the mathematics vocabulary when explaining concepts to ELL students. She also reflected in her self-evaluation of her talk that she used more synonyms and often changed her tone, pitch, and modality (strategies 2, 3, and 5 respectively) so as to better present the concepts behind the mathematical terms used. The ELL students indicated that their teacher frequently was *giving directions and providing assistance* when a specific task was posed to them (strategy 6). They also indicated that Ms. Chandler provided to them *feedback* and *extra wait-time* (strategies 14 and 13, respectively) on a regular basis.

However, from Figure 5, it is apparent that all evaluators agree that Ms. Chandler did not provide many opportunities for students to *share cultural background experiences* when solving mathematical problems (strategy 21). She also did not *conclude the lesson by summarizing the key concepts* (strategy 10).

Summary of the frequency count of the teacher's discursive strategies. Figure 6 below indicates the frequency with which Ms. Chandler implemented the strategies identified in *TTT Form 1*. Figure 6 indicates that Ms. Chandler most frequently used strategies 14 (*Provide feedback*) and 1 (*Use of slower and simpler speech*), followed by strategies 5 (*Use of change of tone, pitch, and modality*) and 6 (*Use of clarification of directions and assistance*). However, Ms. Chandler did not utilize strategies 11 (*Involve students in mathematical discussions and problem solving*) and 19 (*Expose students to different classroom work arrangements*).

These results further confirm the results found by analyzing Ms. Chandler's discourse by applying Krussel et al.'s (2004) framework and the researcher, the teacher, and the ELLs evaluations of Mrs. Chandlers's discourse reported in Figure 5.

Mr. Davison

Mr. Davison is a 40 year old Caucasian, and speaks only English. He has been a teacher for 16 years and has taught Algebra I and Liberal Arts for eight of those years. He completed his 60 hours ESOL endorsement requirement through in-service points and by taking additional evening courses. During the interview, he shared that he often uses bilingual students to peer-tutor ELLs. He also said that he is aware of the presence of ELL students in his class and the fact that he does not speak their language:

> I try to slow my teaching to give students a chance to ask questions. It also gives me a chance to read the expressions of the students. I can usually tell if they understand or not. Also it gives me a chance to change the way I present the material.

The class was diverse, with an almost equal number of Hispanic (9), African

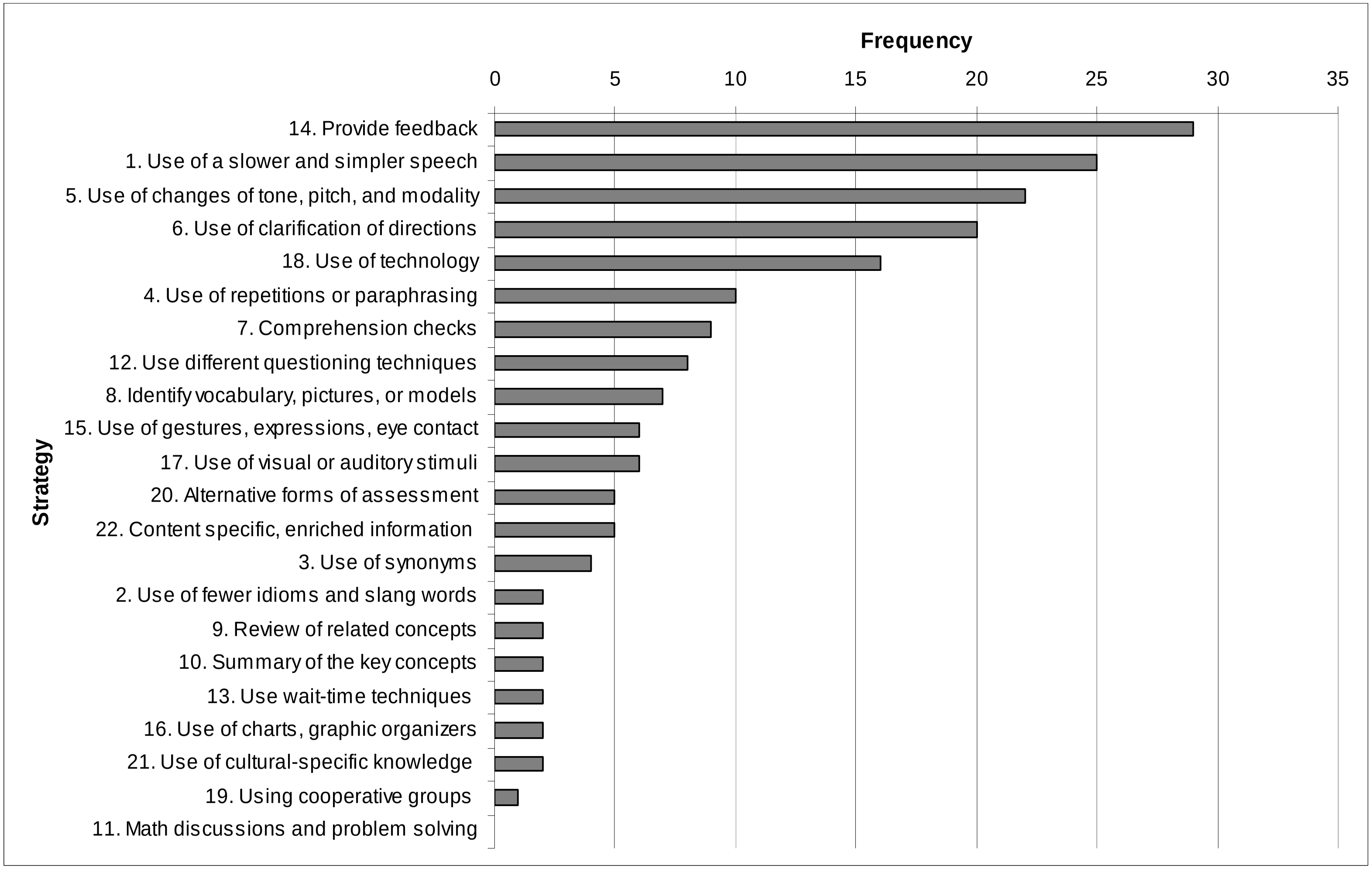


Figure 6. Frequency count of Ms. Chandler’s use of various discursive strategies during the three 20-minute video-recorded sessions.

American (8), and Caucasian (5) students. Mr. Davison's classroom had student desks lined on both side-walls, facing the center. His desk was in the back and, after taking attendance on the computer, he usually walked to the center of the room, where the overhead projector was located, facing the white board on the front wall. Mr. Davison usually taught by using both the white board and the overhead projector, often switching between the two.

Typical classroom discourse. During all three observed classroom sessions, Mr. Davison used either the overhead projector or the white board to write solutions to problems the students were solving collectively under his guidance. During this time, the students referred to the book to read the lesson-specific vocabulary and examples. He created a "stress free" environment where students freely asked questions or readily provided answers to individual or general questions. From the discussions that took place and the responses students provided during each observed lesson, it was evident that the students were becoming more active in the classroom mathematics discourse, as the following excerpts will demonstrate. For example, the third session was a review of previously taught concepts, and when approaching an application problem, the teacher asked the students to recall the difference between *exact interest* and *ordinary interest*. After posing the question twice and seeing that the students still did not respond, Mr. Davison began leading the class in the following dialogue:

Mr. Davison: [1] How many days are in the year exactly?
Ashley (not an ELL): [2] 365
Mr. Davison: [3] 365. How many days are in a banker's year?
Joshua (not an ELL): [4] 360.
Mr. Davison: [5] That's the difference.
[6] Exact interest, and that's the way to remember it, exact interest is exactly 365 days, OK?

[7] Ordinary interest or banker's interest is 360 days.
[8] Because 360 is an easier number to work with.
[9] That's what they tell you.
[10] The real reason is…which will give the bankers more money?

Here the teacher, even though he was talking to the whole class, was applying *questioning techniques* sensitive to the fact that there were ELL students present in his mathematics classroom (strategy 12b, lines 1, 3, and 10). More specifically, Mr. Davison was asking questions usually eliciting a one-word response which are appropriate for ELLs in the *early production* stage of SLA. From the following two excerpts, it becomes evident that some ELL students began to participate, and that Mr. Davison called on them by name if they did not raise their hands:

Maria (an ELL): [11] 360.
Mr. Davison: [12] 360, because look, let's say you have a six months loan.
[13] That is 180 days.
Mr. Davison: [14] Right?
[15] So, 180 over 365 is…somebody with a calculator…?
[16] Trevor, what is it?

The teacher *used a specific example* from the real world (line 12, strategy 22c) and *initiated participation* of students in the calculations (strategy 11, line 15) leading them to understanding the difference in value of both interests. Here the teacher involved another ELL student in the mathematical discussion by calling on him by name (strategy 11, line 16) and asking him to provide the answer by *using a calculator* (strategy 18, line 15).

Trevor (an ELL): [17] 0.4893149506.
Mr. Davison: [18] OK, now, what's 180 over 360? It's just 0.5, it's just half.
[19] So what 's gonna make them more money?
[20] It is not much of a difference but what is bigger?
[21] The 0.5.
[22] That's the banker's interest.
Maria (the previous ELL): [23] But couldn't you round that up?
Mr. Davison: [24] Well, but that's the thing!
[25] They don't round it up.

[26] I mean, if I am paying interest, I ain't rounding up, I…, It's gonna cost me money.
[27] You see what I am saying?

Maria again: [28] That is why the bankers want 360.

Mr. Davison: [29] Right, that's why they want 360.
[30] It's easier to work with because 360 is an even number, but they also use it because it's a little bit more.

This excerpt illustrates how Mr. Davison's attempts to involve all his students by particularly calling on some ELLs when noticing that they do not participate in the classroom mathematics discourse (strategy 11).

Krussel et al. framework. The ***purpose*** of Mr. Davison's discourse was to *involve his students in discussions* (strategies 11 and 12; lines 1, 3, 10, 15, 18, 19, 20 – involvement of all students, and 16 – a specific ELL student is called) which in this example makes them realize for themselves the difference between the two definitions of interest (exact and ordinary). By using *synonymous words* "banker's interest" for "ordinary interest" (lines 7and 22) and "more," "not much of a difference but…bigger" for representing the difference between the two types of interest (exact and ordinary interest) in *simplified sentences*: "So what's gonna make them more money? It is not much of a difference but what is bigger?" (lines 19 and 20), Mr. Davison tried to aid his ELL students (and all students for that matter) in understanding the concepts behind the terminology "exact interest" and "ordinary interest" (and its synonym "banker's interest, line 22) (strategies 1a and 3).

After an analysis of the three observed classroom sessions, a pattern unique to Mr. Davison emerged. He *never directly corrected his students* when their answers were incorrect (strategy 1b). He used the strategy of *repeating* the question (strategy 4), whereby the students seemed to perceive the "*unspoken feedback*" that the answer is incorrect and they should try again (strategy 14) until the right answer were provided. Whenever the right answer was provided, Mr. Davison usually re-stated and elaborated the response (strategy 4 again). For example, when first asking: "…quarterly is how many times a year?" and receiving the answer "1.25," he repeated "how many times a year?" and when another student said "3" he asked again until someone answered "four". Then, Mr. Davison indicated that this is the correct answer by repeating: "four times a year which is three months." This demonstrated that he was satisfied with short responses and easily provided the explanations as to why this is the correct response. Here the researcher is not stating that this is an appropriate strategy for use with ELLs, but is instead reporting on the observed pattern in Mr. Davison's mathematical discourse. The reader is thus provided with a glimpse into the actual discourse that took place during the classroom observations. However, the example demonstrates how Mr. Davison usually missed opportunities to move the discourse to higher levels of cognitive demand (*synthesis* and *evaluation)* as per Bloom's Taxonomy (strategy 22 e and f). Furthermore, by not asking the ELL students in particular (strategy 12d) to further explain some steps while problem solving (strategy 12 c and d), he did not provide them with opportunities to expand their level of English language acquisition to the more advanced levels of *speech emergence* and *intermediate fluency*.

Mr. Davison had created a relaxed classroom ***setting***. Regardless of the classroom work arrangement – lectures, cooperative groups or whole-classroom discussions, his students (ELLs and non-ELLs) naturally participated by asking questions or readily providing answers. Even though some of the calculations they provided were incorrect, the students demonstrated active interest and involvement in classroom activities. Furthermore, they exhibited mutual respect toward each other.

The ***form*** of the teacher's discourse included both *teacher talk* (verbal) and *actions* (non verbal). During the observations, Mr. Davison exhibited focus on the mathematical concept discussed (and explaining the concept by using more informal/conversational English), rather than on the form of presenting it (i.e., stating a formal definition of the concept in English). For example, this is demonstrated in the dialogue that took place when an ELL student asked, "But couldn't you round that up?" (line 23 in the excerpt provided above). Mr. Davison answered: "Well, but that's the thing! They don't round it up. I mean, if I am paying interest, I ain't rounding up, it is gonna cost me money. You see what I am saying?" (lines 24 to 27), which indicates that he tried not to simply provide the answer to the question. He asked his students to critically think and realize the difference that occurred if rounding was indeed performed (here we do see an attempt of applying strategy 22 – providing the students with content specific information). However, even though Mr. Davison demonstrated that he was trying to provide opportunities for his students (both ELLs and non-ELLs) to build upon their prior knowledge and be able to solve real world problems, he did not ask them to further explain, criticize, or justify their thinking while problem solving that could expand on ELLs' language skills and all students' critical thinking skills.

Mr. Davison's non-verbal discourse was displayed in different forms—he often used his hands to gesture when talking, or used eye contact after posing a question, and with a nod or calling on a student indicated who may speak. He also walked between the rows while students worked in pairs or groups and assisted them or answered questions (when called).

According to Krussel et al.'s (2004) framework, Mr. Davison demonstrated *intended* efforts to make all his students (both ELLs and non-ELLs) feel as equal partners in the discourse that took place in his classroom. However, he *unintentionally* neglected to shift the cognitive level of the tasks performed (simple interests, deposits, etc.) by not asking the students to further explain how to carry out the particular steps, or to critically evaluate their answers or further check and expand upon them. He assisted them by asking mostly questions requiring one–word or short responses and thus demonstrated his lower expectations that they would not be able to complete the task on their own. Only on a few occasions throughout all three observed sessions did Mr. Davison ask questions of the type "Well, if we're depositing all this, but she's getting cash back…so what would we do now?" which encouraged the ELL student answering to reply: "So we subtract." "Right, so we're going to subtract," Mr. Davison reassured him by providing feedback and re-stating the sentence, upon which the student completed the sentence "thirty 20's". Such instances of challenging the students to explain the strategies used to completely solve problems were rare and Mr. Davison usually assisted them by asking questions leading to the next step.

Additionally, some of the *immediate* and *long-term* consequences of Mr. Davison's discourse were apparent. For example, when he wanted to shift the dialogue from *univocal* to *dialogic* and involve the class in figuring out how to find the annual interest rate, he faced the students and said "…well, then you gotta figure out how much per year. Let's put it like this: If I paid hundred dollars worth of interest in 6 months, how much would I pay in a whole year," it was not surprising that many students immediately answered "200." Then, Mr. Davison continued giving further examples such as the following: "What if I paid 50 dollars of interest in 3 months?" and thus *was* involving all students to participate in the discourse. However, in assisting them with more and more specific questions, which as an *immediate consequence* involved many of the students, Mr. Davidson inadvertently prevented them from reasoning and justifying their responses. The apparent protocol was to simply supply a short response and move on. An *immediate consequence* of Mr. Davison's talk and questioning techniques was thus the encouragement of students' participation in classroom discourse. The *long-term consequences* were student involvement in tasks requiring mental acuity up to the fourth level of Bloom's Taxonomy (knowledge, comprehension, application, and analysis), and also the encouragement of ELLs to attain the *speech emergence* level of SLA. This, in turn, indicated that while Mr. Davison provided the students (and the ELLs in particular) with equal opportunities to focus on the mathematical discourse's meaning rather than on its form, he was not directing them to critically reflect on the results and to explain their thinking, and he was not providing the ELLs with opportunities to develop to the next stage of SLA – *intermediate speech.*

Perceptions of classroom discourse. On Figure 7 below are represented the results of comparing the data from three sources of evaluation (i.e., *TTT Form 1, 2, and 3)* of teacher talk. The pair-wise correlations (Pearson product-moment correlation coefficients) for Mr. Davison's case study are as follows: the correlation between the teacher and researcher is .68; between the teacher and ELLs it is .17, and between the researcher and ELLs it is .43.

As Figure 7 indicates, there are three strategies where there is almost complete consensus between the evaluations of the researcher, the teacher self-evaluation, and the ELLs' evaluations—*Use of a slower and simpler speech* (strategy 1), *Provide feedback* (14), and *Use of gestures, facial expressions, eye contact, or demonstrations to enhance comprehension* (15). Another strategy that all agreed that Mr. Able used in a consistent manner but in a smaller frequency is: *Provides students with content specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students* (22). The observations, as well as the video recordings, provide evidence that Mr. Davison employed the above-listed strategies very often. He habitually used shorter sentences and adapted his speech to his audience (a diverse group of students, including ELLs). To foster his ELLs' development of mathematics communication in English, he focused his *teacher-talk* during whole-classroom discussions on key concepts and then provided opportunities for his students to engage in small group-work and partner discussions, where they could apply these concepts in problem solving. (Refer to the excerpt at the beginning of the description of Mr. Davison's classroom discourse for an example.)

However, in reference to which strategies were least frequently used by Mr. Davison according to his ELLs, from Figure 7 it is evident that these are strategies 2 (*Use of fewer idioms and slang words)* and 20 (*Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics*). During the interviews, Mr. Davison's ELL students (three students) shared a similar opinion that the observed lessons were rather easy for them

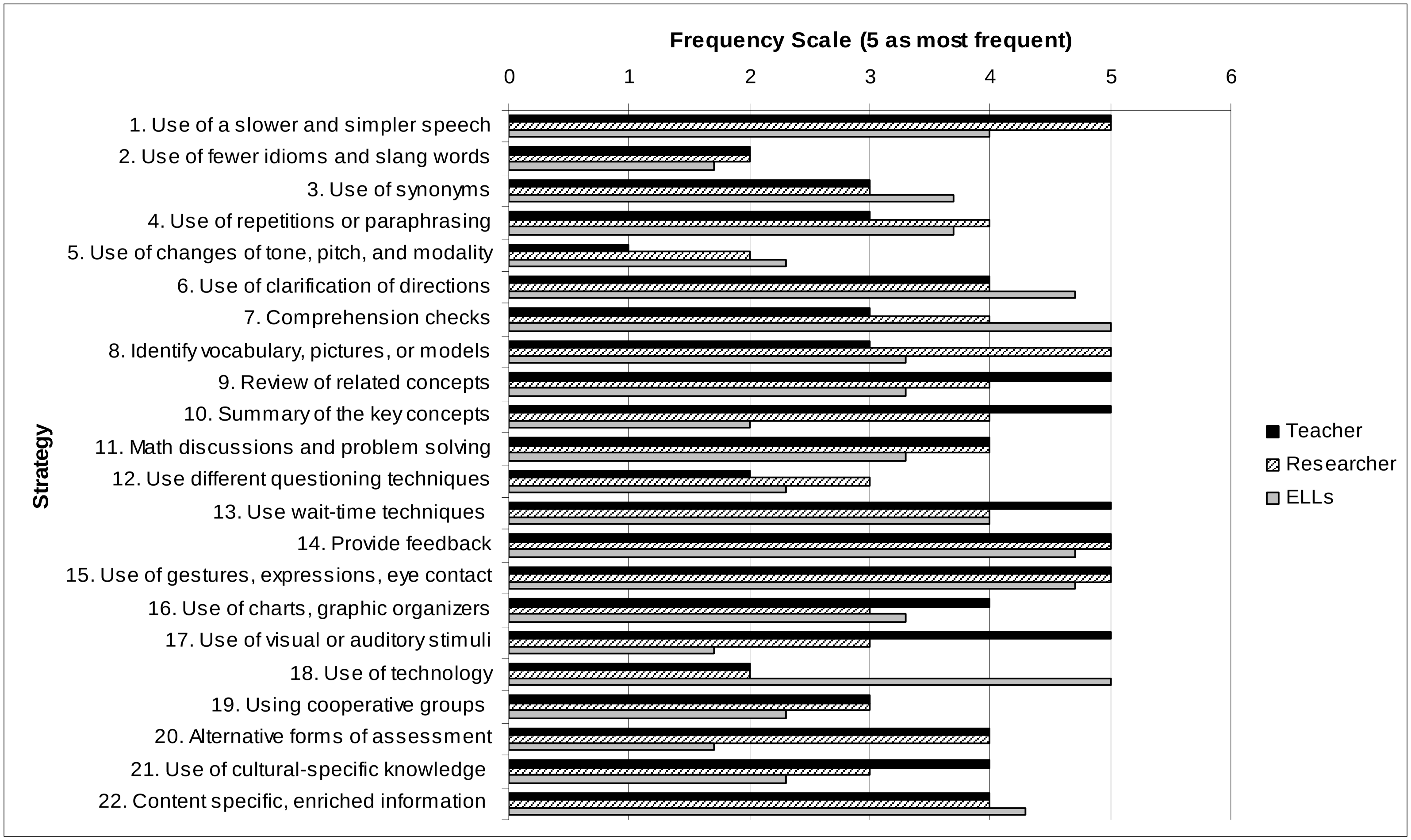


Figure 7. Teacher's, researcher's, and ELLs' evaluations of Mr. Davison's freqeuncy of use of various discursive strategies.

because, as one of the ELL students said: “the teacher gave a very good explanation of [them]”[referring to the lessons].

Summary of the frequency count of the teacher’s discursive strategies. Figure 8 indicates the frequency with which Mr. Davison implemented each category of teacher talk and teaching strategies found on the *TTT Form 1*. As Figure 8 indicates, the strategies most frequently employed by Mr. Davison are: *Use of different questioning techniques sensitive to the ELLs’ level of SLA (strategy 12)* and *providing feedback (strategy 14).* This pattern additionally supports the data reflected in the previous figure (see Figure 7), which included the researcher’s evaluation, the teacher self-evaluation, and the ELLs’ evaluation of Mr. Davison’s style of teaching mathematics to classes with ELLs.

However, further examinations of the questions with which Mr. Davison addressed his ELLs indicate that most of the questions required only a one-word response or a short list of words. Thus, the data indicates that by using questions appropriate for ELLs from initial stages of ELL (English) language development (i.e., questions that initiate simple responses), Mr. Davison was aware that his ELLs were in the stage of *production* of English. However, as the excerpts above also demonstrated, he was satisfied with his ELLs’ short responses and did not challenge them with questions that could lead them to move to the highest levels of the subject-specific literacy – *intermediate speech* and fluency in mathematics in English. Moving the mathematics discussions to higher levels of cognitive demand (i.e., *analysis, synthesis,* and *evaluation*) on Bloom’s taxonomy creates more opportunities for all students (and ELLs in particular) to become critical mathematics thinkers.

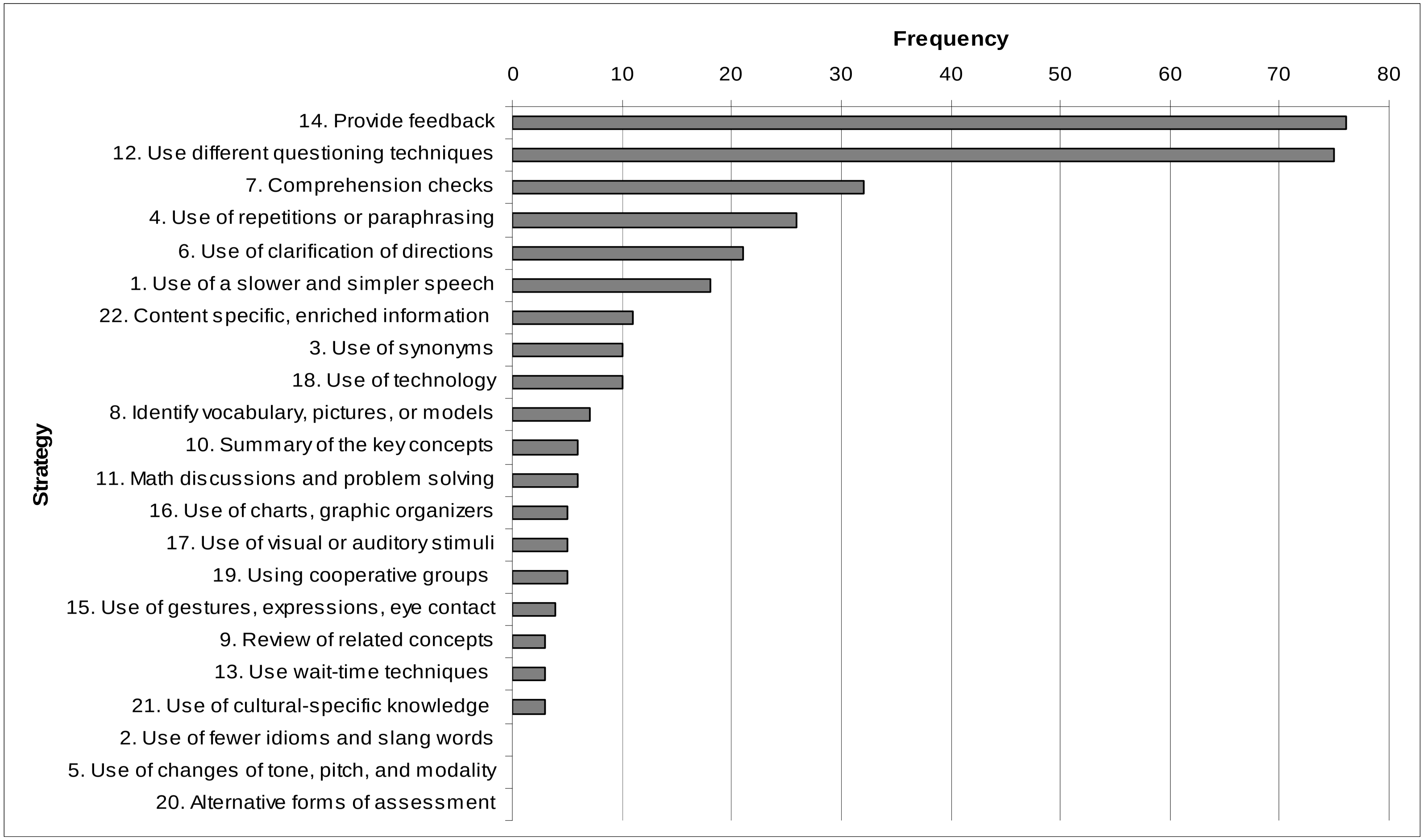


Figure 8. Frequency count of Mr. Davison’s use of various discursive strategies during the three 20-minute video-recorded sessions.

Despite the fact that Mr. Davison created opportunities for his ELL students to participate in the mathematics discourse, he did not ask enough higher order questions such as “Why?”, “How?, “What is your opinion?”, or “Compare/contrast ideas,” and did not provide them with opportunities to justify and explain, to drawn conclusions and, consequently, expand on their learning of mathematics and English. Figure 8 also reveals the aforementioned omission of teaching strategies 2 (U*sing of fewer idioms and slang words)* and 21 (*Providing opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge).*

Lincoln High School

Ms. Andersen

Ms. Andersen is Caucasian and in her mid 40s. She has a professional teaching certificate in secondary mathematics and is presently working on her National Board Certification (NBC). She has 23 years of teaching experience, the entire duration of which she has taught Algebra I, Geometry, and Intensive Mathematics classes. She completed her requirement of 60 hours of ESOL training through coursework 10 years ago. However, during the interview, she shared: “I am still learning to incorporate more strategies that I find via classes – CRISS …or NBC [National Board Certification] classes as well.”

Ms. Andersen’s Algebra I class was very small; it consisted of only 11 students, eight of whom were Black or African American, two were Hispanic, and two were White. Four of the students in the class were ELLs. Two of them (one male and one female) were Haitians and spoke Creole, French, and very limited English. They were repeating their Algebra I class. During the interview, Ms. Andersen indicated that she has a minor in French and can speak it fluently, even though her reading and writing skills were becoming more limited from not practicing the language more often. She also indicated her "love [to] practice French with Haitian students." One of the other ELL students in her class (a male student) was of Hispanic background. Even though the teacher was aware that at home his family speaks mostly in Spanish, she indicated that he was the most comfortable with English of the ELLs in her class. She classified him as having an *intermediate* level of fluency in English. The other ELL student spoke Arabic and was in *pre-production* to *early production* level in his English proficiency. He was repeating his Algebra I class as well. In connection to this, Ms. Andersen said:

> Since we are receiving an influx of Bosnians, Palestinians, and Muslims [from other countries], they are trying to learn English with mostly poor American school habits. Because of their limited schooling due to political and religious issues, it is of utmost importance they are screened, tested, and placed in small learning communities with pairing/sharing grouping.

Typical classroom discourse. From the three observed sessions in Ms. Andersen's Algebra I class, the following pattern of classroom organization became evident. First, she usually assigned bell work which she either wrote directly on the overhead projector, or had pre-written on transparencies. Then, the students were expected to take notes while she presented the new lesson by again using the overhead projector. Often, she asked the students review questions, thus involving them in classroom discourse and building upon their prior knowledge in her explanation of the new mathematical concept, evidence of which will be provided throughout the excerpts below. The class often used the McDougal Littell Publishing Company's *Algebra 1* textbook by Larson, Boswell, Kanold, and Stiff (2004). Ms. Andersen usually pointed out which information the students needed to outline in their notes or which exercises they must do by specifically asking them to "dog ear" (an idiom she used in reference to marking the pages by bending) the corner of the page containing the information she wanted them to pay specific attention to. Her efforts to teach her students good note-taking skills are demonstrated in the following excerpt:

Ms. Andersen: [1] I want you to "dog ear" two pages that we keep looking at in this book all the time.

[Here in order for her ELLs to also understand what she meant by "dog ear", Ms. Andersen demonstrated the bent corners of the pages from her book to which she was referring].

[2] I want you to just memorize perhaps one of the graphs, because you make mistakes like you did last year.

[3] Just keep in mind an equation that's just x equals, just goes like this…

[While repeating this, Ms. Andersen was also writing on the overhear projector $x=a$, and drew out a coordinate system and with her marker showed the direction of the graph of the equation $x=a$ (parallel to the y-axes). She also paused so as to provide her students with enough time to start note-taking and draw the same graph that she was demonstrating, and which was also drawn on the page in the book she was pointing to her students].

Thus, Ms. Andersen utilized strategies 17 (*Use of variety of visual stimuli*: transparencies and pictures from the book) and 20 (*Provide opportunities for students to read and write in mathematics,* lines 1 to 3).

Ms. Andersen: [4] OK?

[Here the teacher looked up and checked if her students were done writing the equation $x=a$ and sketching the graph of the equation, and whether or not they were nodding confirmation that they understood the direction in which the graph goes].

[5] You know what that means and then, hopefully, your y you know goes the other way.

[Here the teacher wrote on the overhead the equation $y=b$ and drew its graph, and then pointed for the students to see how the graph of this equation is in a different direction from the first graph. And, again, after providing sufficient time for her students to take notes, she continued]:

[6] And I've pointed out numerous times I want you to "dog ear" your book…

Here the teacher *checked* if her students were following her and *comprehending* her drawings and explanations (strategy 7, lines 3 and 4). And because this observation was a review lesson, she wanted them to take notes of the important sections of the already completed chapter and she was also checking their *previous knowledge* (strategy 9, line 5) by observing if they nodded in agreement that the graph of $y=b$ should go in the opposite direction. Then, Ms. Andersen *repeated* her directions (strategy 4, line 6). At this moment, a student (the ELL student of Hispanic background who was most comfortable in English in comparison to the other ELLs in her class) interrupted:

Jennifer: [6] I've got a question.
[7] You said that the lines should be parallel, but should the axes and the lines should be parallel if you have *x* equals it would be, or *y* equals?
Ms. Andersen: [8] The *x*-equation is parallel to the *y*-axis.
Jennifer: [9] Right.
Ms. Andersen: [10] The y-equation would be parallel to the *x*-axis. If you would like to write that down as your personal note.
Jennifer: [11] Right, and also if it's on the *x*-axis there is no way it's going to be on horizontal?
Ms. Andersen: [12] Exactly. Good…good perception and good information for the rest of the students to pick up on.
[Then the teacher continued by facing the whole class.]
[13] The two pages I want us to go back to, and when I mean "dog ear", fold it over, this might be used next year.
[Here, Ms. Andersen demonstrated again what she meant by using the idiom "dog ear" by holding the corner of the page, thus clarifying to all students (especially to her ELLs) that she meant for them to mark the page as important].
[14] Let's go back to page 213…

On page 213 in the book, in a box entitled *Equations of horizontal and vertical lines,* are presented the graphs of two general equations: $y=b$ and $x=a$. On the same page, the graphs and the solutions of two model examples – example 5, which asks the students to “graph the equation $y=2$,” and example 6, asking them to “graph the equation $x= -3$” – are provided. The teacher now explained the examples in detail and then continued:

Ms. Andersen: [15]…The other page that I can really continue to push that I’d like you to “dog ear” is on page 2…, I think it’s page 228: c*lassification of lines by slope.*

On this page, in the box are presented four graphs of lines with slope $m>0$, $m<0$, $m=0$, and *m* as undefined. Here (line 15), Ms. Andersen utilized strategy 17 (*showing charts and graphic organizers to enhance teacher talk*) and thus directed the students to pages of the chapter with important information for them to mark and review in detail. Then, after some outside interruptions and once the students opened their books to this page, she continued explaining the visual representations provided on page 288 of the book. The excerpt that follows provides a more detailed glimpse at Ms. Andersen’s *teacher talk:*

Ms. Andersen: [16] You can see how the line is going down based on the slope being negative.
[17] Okay? We did those problems.
[18] When it’s going up, is in the case where it is undefined, which is the fourth one.
[19] And then if I wanted to include another line, this is crossing through the *y*-axis at negative four, therefore it would have a level slope.
[20] A level flat line which would mean it equals zero.
[21] Keep in mind zero, and does not exist or undefined, are not the same things.

Here, Ms. Andersen utilized strategy 3 (lines 19 to 21) — *use of synonyms* a level slope and a level flat line in the description of the mathematical term for slope $m=0$, and strategy 17—*showing charts and graphic organizers to enhance teacher talk,* and thus helped her students (and her ELL students in particular) better understand the underlying concepts in that particular mathematic vocabulary. She also utilized strategy 1c (*teacher* *talk focused on key concepts,* lines 16-21) and strategy 1d (*teacher talk fostering conceptual understanding through content*, line 21).

Ms. Andersen: [22] Make sure you are looking at the last two lines on page 228.
[23] Know we don't always go ahead.
[24] We go back as a reference.
[25] Haven't you read a novel and forgot something in a chapter and you wanted to go back a few pages?
Students (a few students said together): [26] Uhhuh.
Ms. Andersen: [27] That's what we do in this book.
[28] We condense the notes.
[29] We're already on page 244.
[30] And if you've taken notes by me every day, you've made your own personalized Algebra 1 study guide, bell work, examples.

This segment of the teacher's discourse (lines 30, and 33 below) displays not only how Ms. Andersen guided her students in note taking, but also that she was setting expectations for *all* of them (including her ELL students) to practice writing in mathematics in English and thus create their "own personalized Algebra I" journals with definitions, visual representations (graphs), and examples from class work, bell work, and homework:

Ms. Andersen: [31] Alright, so I mean we have not written 244 pages of notes, have we?
[32] Every thing has been condensed.
[33] You know what you need to know exam wise and Algebra, so it takes you to the next class.
[34] Okay, before we go to the last section, which is dealing with functions and identifying functions, some of you have your homework out.
[35] If you have any questions on page 244, let's go over it at this time.
[36] 13 to 45 and list every fourth problem.

The assigned homework from page 244 included practice and application problems asking the students to find the slope and the y-intercept given the equation of a line, and then to graph the line. Here, Ms. Andersen utilized strategy 22 (*application of the content specific information* the students were exposed to while learning this chapter).

Ms. Andersen: [40] Okay?
[41] Any questions from last night?
[42] I prefer to start that before we do the next section.

This excerpt from Ms. Andersen's discourse illustrates the *teacher talk* pattern that she naturally adhered to by utilizing the above mentioned strategies, and will be analyzed in the following paragraph using the Krussel et al. (2004) framework.

Krussel et al. framework. The ***purpose*** of Ms. Andersen's discourse was to foster conceptual understanding and expanded literacy through content (strategy 1d—*Use of pattern of speech appropriate to students with Intermediate fluency in SL,* lines 16, 18, 19, and 20; Refer to Appendix A and Figure 9). The fact that she regularly *used a variety of visual stimuli* (strategies 16 and 17) – transparency sheets on the overhead projector when giving the students bell work or teaching a new lesson, pictures from the textbook (line 22, 29, 35, and 36), or modeling solutions on the white board using different colors – indicated that Ms. Andersen catered to the needs of the ELLs present in her Algebra 1 classroom. Her focused *use of slower and simpler speech* (strategy 1, lines 16 to 22) and various visual representations were done in order to aid her ELL students to better grasp mathematical concepts. For example, she strategically pointed out to students *visual representations* (strategy 17) of the special cases of equations of vertical (line 18) or horizontal lines (lines 19 and 20), and she explicitly taught them "condensed" *note-taking skills* (strategy 16, lines 28 to 32) by pointing out which parts of the book are important to mark ("dog ear") and *read for future reference* (strategy 20, lines 22 to 25).

However, Ms. Andersen did not involve her students or her ELL students in particular, toward reflections on their thinking, further explanations, or justifications (strategy 22e and f). Most of her questions were of the type "what is…", "which…" or "do you remember…" (strategy 12a and b) which only tested students' knowledge and perhaps comprehension and application (strategy 22a, b, and c), but did not force them to perform analysis, synthesis, and evaluation (strategy 22d, e, and f). There were only a few isolated instances of such lines of questioning in each of the observed sessions.

The ***setting*** for classroom discourse appeared to have been established early in the school year, as the students conformed to certain pre-established norms of behavior during the observations. For example, in turn-taking, usually once a student provided an answer to a general question, the same student continued talking (lines 6, 9, and 11) until the small task was completed (in the particular example, the teacher answered and explained questions to this particular student) or another student would be asked to continue. However, during the classroom observations, Ms. Andersen did not provide different classroom settings for her students. She usually started with bell work on an overhead projector and then presented the new lesson (again on an overhead projector) while the students were expected to take notes. In this classroom setting, the students freely asked questions, as the excerpt above illustrated (lines 6, 9, and 11), but they were *not exposed to classroom arrangements that fostered cooperative group work, partner discussions, or games* (i.e., lack of utilizing strategy 19).

The ***form*** of Ms. Andersen's discourse included both actual *teacher talk* (verbal) and *actions* (non verbal). Even though the *teacher's* natural *talk* in her native English was not very simple, and she often included phrases such as "I believe firmly," "you may be assured," "increments of 1," she often accompanied this talk with actions such as *gestures* (strategy 15), drawings, or the use of colored markers while writing on the overhead or the white board (strategies 16 and 17). Thus, by enhancing her teacher talk via strategies such as gestures and visual stimuli, Ms. Andersen demonstrated attentiveness to the presence of ELL students in her mathematics classroom, and used demonstrations to enhance their comprehension of her talk. For example, while explaining the signs of numbers representing the coordinates of ordered pairs in the four different quadrants, Ms. Andersen said:

> …and what is so neat about this, I want you to see. Look at I and III, aren't they total opposites of each other" Two pluses and two minuses. And look at the reverse, of quadrants II and IV. Look at the signs, because if you could fold, there would be symmetry. Fold and see.

The previous excerpt illustrates that while Ms. Andersen's talk included words with precise meanings such as "reverse," "total opposites," "symmetry," her discourse often took the form of a challenge; yet at the same time she *modeled, or asked the students to model, the situation she was explaining* (strategy 1b). Hence through her verbal and non verbal discourse, she *demonstrated correct responses both in mathematics and English* that were appropriate for ELLs from the stage of *early production* of English language (also part of strategy 1b). However, she was usually satisfied with ELL students' short responses and did not challenge them to further *experiment with the English language* in *explaining their thinking* while problem solving (i.e., she did not apply strategies 1d, 12c or d, and 22f).

Some of the ***consequences*** (*intended* or *unintended*, *immediate* or *long term*) of Ms. Andersen's classroom discourse became clear after an analysis of the three observed sessions. For example, she *intended* to include her ELL students from lower levels of English language acquisition in various tasks (plotting points or graphing lines) by asking for their participation in the task; however, she *unintentionally* assisted them in the task's execution by asking them what the next step of the task would be and by posing questions that required only single-word answers or short responses. Thus, she exhibited her lower expectations that her ELL students would not be able to complete the task on their own. Consequently, she *unintentionally* did not provide her ELLs with enough opportunities to practice their mathematics vocabulary in English.

Despite the fact that the students could freely ask Ms. Andersen questions while she was explaining a lesson (as the excerpts above demonstrated), the ELL students with lower levels English language acquisition exhibited limited *immediate* participation in the mathematics discourse. However, in the *long-term*, Ms. Andersen still facilitated them in learning Algebra in English by expecting them to write in their mathematics notebooks in English. Under her directions, the ELL students were creating their "personalized Algebra I study guides" as were the rest of the students in the class, and later they could refer to their own notes and study from them (see the excerpts above, lines 30 and 33). Furthermore, by explicitly showing work, and often using a red colored marker to show important steps while solving mathematical problems, Ms. Andersen was modeling for her ELLs how to approach mathematical problems and how to describe their own work by using proper math notation and vocabulary. This, in turn, provided *all* her students (and the ELLs in particular) with opportunities to study from their self-created mathematics journals and in the *long-term* improve their knowledge of the proper mathematical notation and terminology used in the mathematical discourse.

Perceptions of classroom discourse. Figure 9 represents in the form of bar graphs the researcher's preliminary evaluation (i.e., before an actual count of the frequencies with which Ms. Andersen uses different discourse strategies), Ms. Andersen's self-evaluation, and the evaluations by her ELL students who volunteered to participate in the study. The pair-wise correlations for Ms. Andersen's case study are as follows: the correlation between the teacher and researcher is .61; between the teacher and ELLs it is .12, and between the researcher and ELLs it is .46.

Figure 9 displays complete agreement between the evaluations of the researcher, the teacher self-evaluation, and the ELLs' evaluations that Ms. Andersen most frequently *Provided feedback* (strategy 14). The other strategies for which there was almost complete agreement over their frequent and consistent use by Ms. Andersen were—*Use of a slower and simpler speech (1),Use of wait-time techniques after posing a question (13),* and *Providing students with content specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students (22)*. The excerpts from the video-recordings of Ms. Andersen's classroom also revealed that she readily provided her ELL students with feedback to their answers as she did, for example, in line 12: "Exactly. Good…good perception and good information for the rest of the students to pick up on" (strategy 14). In the excerpts above, it was also demonstrated that Ms. Andersen talked slowly and often paused between sentences (strategy 1, lines 3 and 4), thus providing enough time for her students to take notes (including her ELL students).

In the interviews, the ELL students indicated that Ms. Andersen provided enough

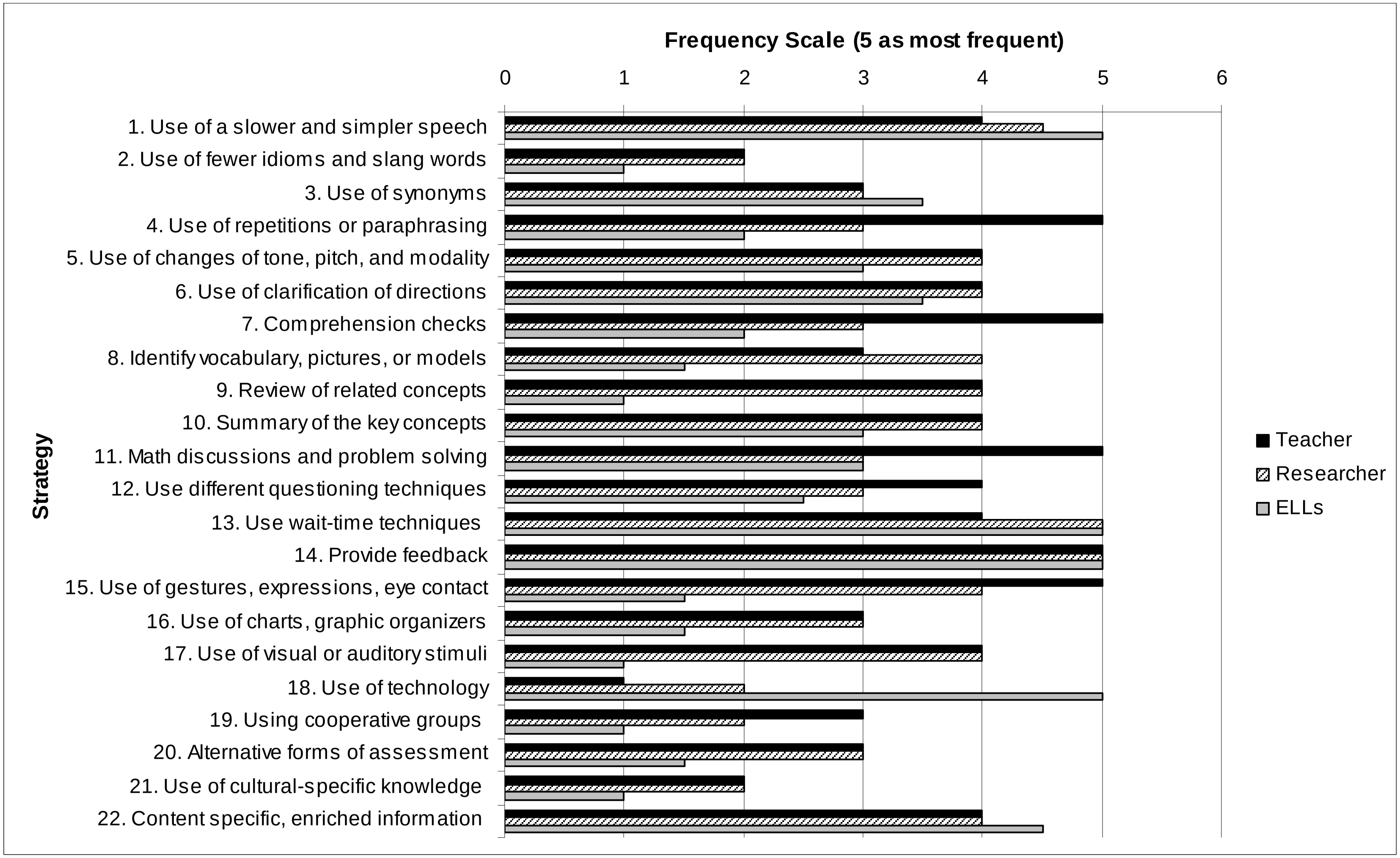


Figure 9. Teacher’s, researcher’s, and ELLs’ evaluations of Ms. Andersen’s frequency of use of various discursive strategies.

wait-time after asking a question (strategy 13), which gave them the opportunity to organize their answer to the question in English better. The video-recordings also confirmed that usually there was provided adequate wait-time after a question was posed, thus giving equal chances for both ELL and non-ELL students to participate in the classroom discourse. The excerpts above (lines 16 to 33), as well as from the rest of the video-recordings, also demonstrated that Ms. Andersen *provided her students with content specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students* (strategy 22).

Summary of the frequency count of the teacher's discursive strategies. Figure 10 represents the frequency count of the strategies employed by Ms. Andersen during the three observed classroom sessions. Figure 10 indicates that Ms. Andersen employed strategies 12 (*Use of different questioning techniques* sensitive to the ELLs' level of SLA), 14 (*Providing feedback*), 13 (*Use of wait-time techniques after posing a question),* and 1 (*Use of a slower and simpler speech*) most frequently. For example, Ms. Andersen often used expressions such as "Correct!" and "Very good!" which indicated frequent utilization of strategy 14 (*providing feedback)* not only to inform her students of the correctness of their answers, but also to encourage their participation in discourse. She even used expressions in French such as "Tres bien!" or "Bon!" when addressing her Haitian ELL students who also spoke French, which also indicated frequent use of the technique *providing feedback (14)* when addressing ELL students too, and as a result also encouraged their participation in discourse.

Both charts (Figures 9 and 10) indicate that Ms. Andersen frequently used *wait-*

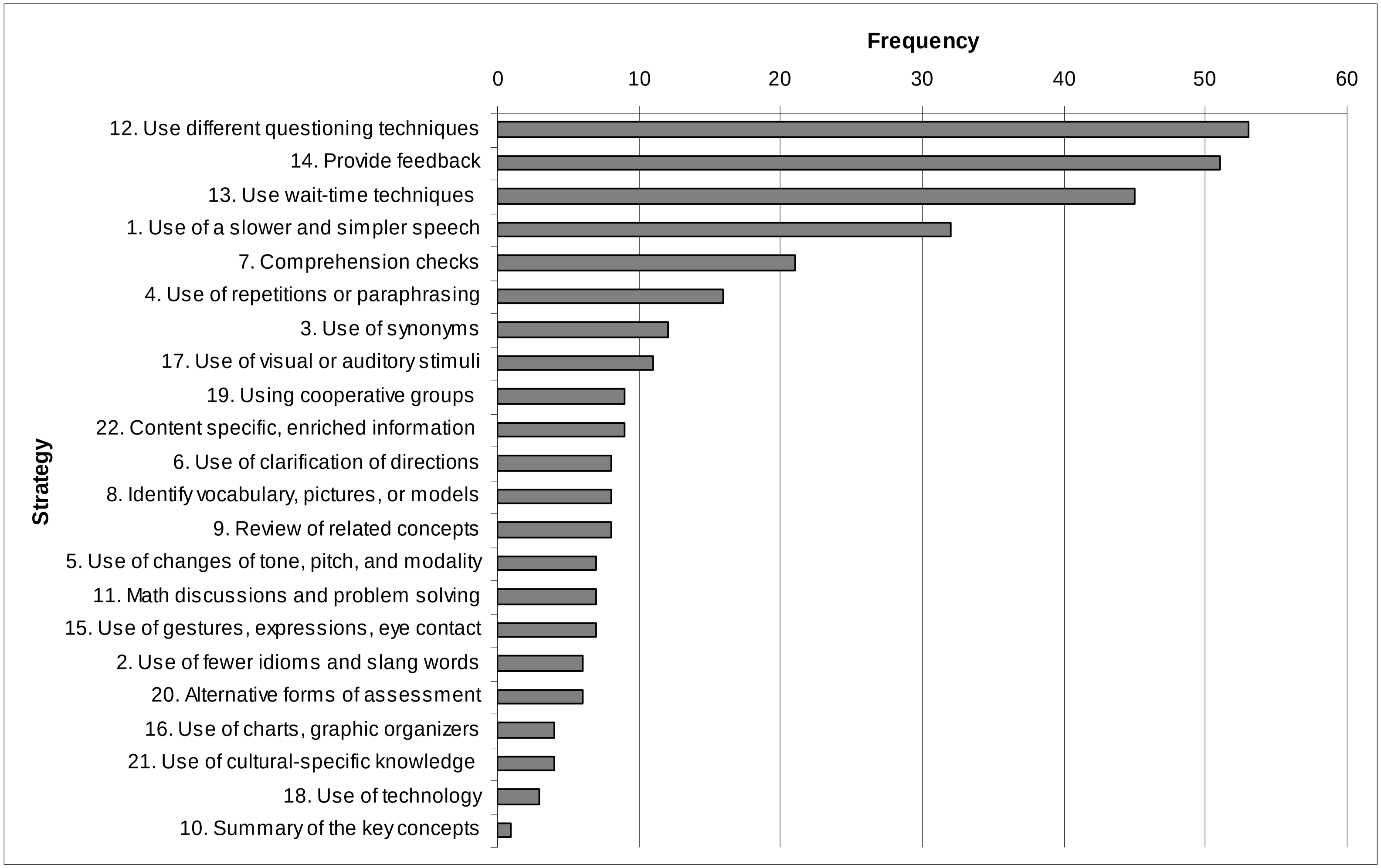


Figure 10. Frequency count of Ms. Andersen's use of various discursive strategies during the three 20-minute video-recorded sessions.

time techniques (strategy 13). By re-playing the video-recordings and measuring the wait-time after a question was posed, it was found that Ms. Andersen provided different *wait-times* after posing questions in relation to each question's difficulty and to whom the question was addressed (recordings revealed that she provided at least from three to five seconds of wait-time, and often more, with repetition of the questions, when addressing an ELL from a lower level of English language acquisition). She also used *simpler commands and shorter sentences* when explaining concepts (strategy 1). For example, in the excerpts above, Ms. Andersen used the simpler command "Let's go back to page 213…" (line 14), or modified her talk using shorter sentences when explaining the differences in graphs of the equations $x=a$ and $y=b$ to an ELL student: "The x-equation is parallel to the y-axis." (line 8), and "the y-equation would be parallel to the x-axis" (line 10). Such adaptation of her speech to her audience exhibited Mr. Andersen's awareness of the presence of ELL students at different stages of SLA. Further analysis of the *teacher talk* revealed the means by which she fostered her ELLs' *early production* of correct responses both in mathematics and English. For example, she demonstrated her solutions to a mathematical problem on the overhead projector by using different colors for definitions (green), solutions (blue) and important facts (red). Then, she focused her *teacher-talk* on key concepts and encouraged her ELL students to apply these concepts while explaining the steps of the problem's solution. However, she assisted them by asking leading questions which required only short responses, and often provided teacher-directed instructions and explained things, rather than expecting the students to finish more problems on their own – her discourse was usually similar to that in the sample excerpts at the beginning of this section on her discourse.

However, from both graphs (see Figures 9 and 10), it becomes evident that strategies 2 (*Use of fewer idioms and slang words from the mathematics vocabulary)* and 21 (*Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge)* were the most lacking in Ms. Andersen's discourse methods. Furthermore, the lack of use of strategy 2 indicates that by rarely using idioms such as "if and only if", "right-angle" etc., Ms. Andersen was exhibiting awareness that an ELL student in an early stage of English language acquisition might think that there is a "left-angle" or simply might misunderstand her. In the excerpts above, Ms. Andersen used the idiom "dog ear", but each time she used it, she also demonstrated and explained that she expected the students to mark the page by folding the corner of the page she was referring to. At the same time, by not providing opportunities for her ELL students to share their previous experiences while problem solving and also by not enhancing her instructions by building up on students' personal or cultural-specific knowledge (strategy 21), Ms. Andersen demonstrated a lack of awareness of the benefits of applying this strategy – benefits which are indicated in research in the field of teaching ELLs and were discussed in the review section of this manuscript.

Ms. Brown

Ms. Brown is in her early twenties. She has just graduated from college and is teaching at the high school from which she graduated. She was not a mathematics education major and did not take any courses on methods of teaching mathematics. She has a temporary teaching certificate and is currently working on her ESOL certification.

She is fluent in Yoruba, her native language, English, and has limited fluency in French. This is her first year of teaching Algebra I and Intensive Mathematics.

Ms. Brown's class consisted of 17 students, five of whom were ELLs. Four of these ELLs are Hispanics and one is Arabic. Three of the Hispanic ELLs are in their early stages of English language acquisition—between *early-production* and *speech-emergence.* The fourth of the Hispanic students, and the Arabic student, are in the *intermediate stage* of fluency in English. The class is diverse with three White, eight Hispanic, and nine Black students. In the interview, Ms. Brown indicated that she tries to aid her ELL students by using cooperative groups as part of her instructional techniques and always allows discussions in which students could share their previous personal and cultural-specific experiences in mathematics. When asked to comment on any concerns she has about teaching mathematics to ELL students, she said: "My greatest concern is the language barrier. Oftentimes [I] have to have students translate processes into Spanish. My students who are bilingual enable me to bridge the language gap." She nonetheless further commented that, "My experience with [ELL students] has been extremely positive. The students who are strong in English oftentimes translate for the students who are not as strong. This greatly helps me in the classroom." In the Interview, Ms. Brown also shared that for the Arabic student she occasionally used the help of a teacher assistant (TA) who knew Arabic. However, because the Arabic ELL student was relatively more comfortable in English and only occasionally needed help (usually in word problems), this TA was often busy helping in other classes. During the three observations in Ms. Brown's class, the TA was on other assignments, and the Arabic student was working well in the class on his own.

Typical classroom discourse. To better visualize the atmosphere in Ms. Brown's mathematics classroom, a detailed excerpt from an activity Ms. Brown used to teach her students *Scatter Plots* and the concept of data correlation will be provided. The following excerpt will demonstrate both the teacher-student interactions and sometimes student-to-student interactions that arose in Ms. Brown's classroom during that activity due to the particular blend of students (ELL and non-ELL students), as well as the teacher's discursive moves with the students:

Ms. Brown: [1] Alright, can you guys get in height order for me at the front of the room?
[2] (*After a while)* Alright, Okay.
[3] So, can you guys line up in reverse—I'm sorry, from—come up here, so all switch, so it would be…(*the students line up shortest to tallest from left to right):* Jasmine, Rosita, George, and Bryan *(here fictitious names are used for easier presentation; Jasmine, Rosita, and George were ELL students, and Bryan was a non-ELL).*
[4] Alright, so we have our class members organized from tallest, I mean, from shortest to tallest, right?
[5] So in part D it's asking us if "whether our height is correlated with the number of siblings we have" (*the teacher read this from a prepared worksheet).*
[6] So, if it is so, Jasmine would have the least amount of siblings, right?
[7] And Bryan would have the most amounts of siblings.
[8] So let's see if it works.
[9] Jasmine how many siblings do you have?
Jasmine: [10] Five-four, five-four.
Ms. Brown: [11] Five?
Jasmine: [12] Four.
Ms. Brown: [13] Five?
Jasmine: [14] Five-four.
Class: [15] What? Huh? What is she talking about?
Jasmine: [16] Me, I am five-four.
Ms. Brown: [17] No, no, no, how many siblings do you have, not how tall you are.

Lines 8 to 17 reveal that Jasmine (an ELL student from the *early-production* stage of English language acquisition) encountered difficulties in understanding the meaning of the word "siblings." However, rather that using a synonym for the word "siblings," Ms.

Brown corrected Jasmine by saying she didn't mean to ask her how tall she is (i.e., Ms. Brown exhibited lack of use of strategy 3-*use of synonyms*). Additionally, the incident revealed that even though Ms. Brown was aware that she had ELLs and tried to involve them in the activity, she was not aware that this particular ELL student was from the early production of English language and is just beginning to experiment with the language. In this case, the teacher *needs to model/demonstrate correct responses for her both in mathematics and English* or *use synonyms* in order to negotiate the meaning of her instructions and questions (i.e., needs to apply strategy 1b or 3).

Rosita (*next to Jasmine*):	[18] She got nine all together, she said five-four.
Jasmine:	[19] No, no, no (*thinks and holds up three fingers*), three.
Ms. Brown:	[20] Three?
Jasmine:	[21] (*nods*)
Student (*in background*):	[22] She got a twin sister…
Rosita (*to Jasmine*):	[23] I got more than you.
Ms. Brown:	[24] Okay, you have three.
	[25] Okay, Rosita, so for us to have a positive correlation, Rosita should have more.
	[26] Rosita, how many do you have?
Rosita:	[27] Thirteen.

Lines 19 to 27 reveal that Rosita (an ELL transitioning between the *speech-emergence* and *intermediate-fluency* level of English language acquisition) had initially assumed that Jasmine was claiming to have 9 siblings (line 18). Then, because she was turning around to grab her other classmates' attention, she did not take note of Jasmine's gesture (holding up three fingers, line 19), and Ms. Brown's repetitions that Jasmine has three siblings (lines 20 and 24). Rosita even teased Jasmine (interrupting her dialogue with Ms. Brown) that she has more siblings than her (line 23). Then, she simply heard the teacher's explanation of a positive correlation (line 25) and the question toward her (line 26), and thus answered with her pre-prepared number of 13 (line 27). That answer reveals that

Rosita had not understood that the point of the exercise was not to provide fictitious data that meets the criteria of positive correlation, but rather to answer with real data so that the class could determine whether there was in fact a correlation between height and number of siblings. This further reveals how some ELLs, despite developing good *basic interpersonal communication skills* (BISC), still need time to transition to the more advanced level of *cognitive academic language proficiency* (CALP) to fully understand the true context of the academic task (Cummings, 1983; Ellis, 2000). Although Ms. Brown questioned the number of siblings Rosita reported (in line 28 next), she did not voice any doubts (if she had any such doubts) in Rosita's understanding of the cognitive demands of the task:

Ms. Brown: [28] Thirteen? (*Rosita nods),* perfect.
[29] Alright, Rosita has thirteen.
[30] So for our correlation to work if this is a positive correlation, Jose should have more siblings than Rosita.
[31] Jose, how many siblings do you have?
Jose: [32] You mean like brothers and sisters?
Ms. Brown: [33] Yeah.

Here it was the third ELL—Jose (in the *speech-emergence* stage of English language acquisition) that finally initiated for Ms. Brown to negotiate the meaning of the term "siblings" and used the synonymous words "like brothers and sisters" (line 32). When he received *feedback* (strategy 14, line 33) that his assumption of the word "siblings" meaning is correct, he answered:

Jose: [34] I only got a sister.
Ms. Brown: [35] You only got one, hmm…alright.
Bryan: [36] (*mumbling a big number in the vicinity of 13*).
Jose: [37] Oh, twenty.
Ms. Brown: [38] No, uh-uh, that's fine, don't lie.
[39] You don't have to lie.
[40] It's alright, we are proving our point.

Here Ms. Brown realized that the students need assistance in order to clarify their misunderstanding of what answers are expected from them and *clarified the directions* (strategy 6) by stating that in order for the students to understand the point of the activity they need to provide truthful (real life) data:

Ms. Brown: [41] Bryan, how many brothers and sisters do you have?
Bryan: [42] Five.

Thus, Bryan who was not an ELL could clear his confusion of what answer to provide (his confusion was exhibited in line 36), and truthfully reported the actual number of his siblings (in line 42).

Ms. Brown: [43] Five?
[44] Okay.
[45] So we go from 3 to 13, to 1, to 5, right?
[46] Hmm, does that look like it has a relationship with the height?
Class: [47] No.
Ms. Brown: [48] ‘Cause we don’t have a positive.
[49] Is that, do we have a negative correlation there?
Class: [50] No.
Ms. Brown: [51] No, so we don’t have any correlation, alright.
[52] So, that was all I was trying to prove with that.
[53] Thank you guys.

As is evident, the point of the activity had become apparent to the class, and by analyzing the data provided from the students participating in the activity, they were able to reach the conclusion that there was no correlation between the students’ heights and the number of siblings they have (lines 45 to 53). Here Ms. Brown *summarized* the reported data (i.e., utilized strategy 10), and asked the students *to analyze* it (strategy 22d) in order to complete the activity.

Krussel et al. framework. According to Krussel et al.'s (2004) framework, the ***purpose*** of Ms. Brown's discourse was to initiate participation for *all* her students, including ELLs, in whole-class or group activities, leading to better understanding of the new concepts she is teaching (in the above excerpt – scatter plots and data correlation). Throughout the observations it became clear that Ms. Brown frequently *called on* her ELL students (i.e., utilized strategy 11) to complete problems she had just modeled for them how to solve, and tried to involve them in discussions. For example, three of the four students (Jasmine, Rosita and Jose) who participated in the activity from the excerpt above were ELLs. Ms. Brown also oftentimes used "hands on" activities or games such as the one described in the excerpt above, or group work so that *all* her students (and ELLs in particular) could better grasp a mathematics concept. This indicates that she utilized strategy 19 and, as a result, her ELL students were exposed to *different classroom work arrangements* such as group work, partner and whole-class discussions. During the observed lessons, she attended to the fact that she had a diverse student population and ELLs present in her mathematics classroom and often called on them, thus *involving them in the problem solving and math discussions* (strategy 11) or asked them *questions* (strategy 12).

However, as the excerpt above exhibits, Ms. Brown had a limited understanding that ELLs from different stages of English language acquisition have different needs and that she needed to adjust her talk in order to accommodate them in the classroom discourse. Thus, initially Ms. Brown omitted to start the activity with *clear directions* and did not utilize strategy 6 until later in the activity (lines 38-40), when it became apparent that the students need assistance in order to provide realistic data and to complete the activity. As the excerpt also demonstrated, initially Ms. Brown did not realize that using the word “siblings” was not familiar to her ELLs from the *early-production* to *speech-emergence* stages of English language acquisition. Thus, she did not utilize strategy 6 of *using the synonymous words* “brothers and sisters” until an ELL (Jose) was unable to negotiate the meaning of the word (lines 31-33). Thus, the example above demonstrates that even though Ms. Brown involved her ELLs in the classroom discourse, she did not adjust her talk to their level of English language development (i.e., lack of use of strategy 1b).

The ***setting*** for classroom discourse was evidently established early in the school year, as the students exhibited a familiarity with certain expectations and norms of classroom behavior in each of the observed lessons. For example, Ms. Brown regularly started her lessons with bell-work. Then, she usually collected the students’ bell-work, as well as their homework from the previous night. However, during the observed and video recorded sessions of classroom discourse, Ms. Brown’s students often did not follow the norms of turn-taking she was trying to establish. For example, when Ms. Brown asked questions (general or specific), even though a specific student might be asked to answer, many students either answered aloud or continued interrupting each other. But, as was demonstrated in the excerpt above, usually Ms. Brown repeated her questions to the student they were addressed to and kept most of her students focused on completing the mathematics task at hand. Furthermore, as illustrated in the excerpt above, Ms. Brown established a classroom *environment* that encouraged active learning by often asking students to perform “hands on” activities or to work in groups, thus *exposing her ELL students to different classroom work arrangements* (strategy 19).

The ***form*** of Ms. Brown's discourse included both *teacher talk* (verbal) and *actions* (non-verbal discourse). For example, the use of questions of the type "how," "tell me about," "compare/contrast," as the following excerpts illustrate: "How does this question look like that equation?" and "As *x* is going up, can anyone tell what's happening to *y*?", reveal Ms. Brown's efforts to encourage her ELL students to expand their literacy in both the English language and mathematics (i.e., utilization of strategies 12 c and d). In order to provide answers to such questions, the students (including ELLs) need to move to operations involving higher cognitive demand according to Bloom's taxonomy **–** *analyzing, distinguishing,* and *explaining* content-specific, enriched information (strategy 22d, e, and f).

Ms. Brown's non-verbal discourse was displayed in different forms. For example, she moved her hand up or down to help her ELL students understand positive or negative scatterplots' correlations, or she moved her hand up or down and left or right to demonstrate the slopes of lines as going up or down (rise) and the left or right (run). Thus, these examples demonstrate that Ms. Brown utilized strategy 15 (*use of gestures, facial expressions, eye contact or demonstrations*) so that her ELL students could better understand the concepts of scatter plot and slope of a line. Another display of Ms. Brown's non-verbal discourse is her use of her index finder in front of her lips whenever she wanted to indicate to the class that they need to quiet down and listen. Additionally, when switching between activities, she usually would raise her arm up and say "Alright, so today we're gonna be discussing scatter plots" or 'Alright, so, example one…" She also circulated between the rows when students performed group work and assisted them or answered questions.

The ***consequences*** of Ms. Brown's discourse fell into the following categories – *intended* or *unintended*, *immediate* or *long term* as described in the Krussel et al. (2004) framework. Ms. Brown demonstrated *intentions* to shift the cognitive level of the task performed (strategy 22 d and e) for both ELL and non ELL students. For example, by *intentionally* choosing three ELLs from the four students to participate in the activity described in the excerpt above, she *involved the ELLs* in the classroom discourses (strategy 11). Most notably, by frequently posing questions to ELL students (strategy 12), she was demonstrating an *intentional* goal to provide them with opportunities to practice their mathematics vocabulary in English and was demonstrating high expectations. She provided them with equal opportunities not only to share personal data (strategy 21), but also to use that data to help them better understand the concept of scatter plots (strategy 22c) and to identify whether they saw any correlation (strategy 22d). In the excerpt above, it was also demonstrated that Ms. Brown *unintentionally* caused confusion by using certain words (siblings) and by not clarifying the directions of the activity. Later, negotiation of the meaning of the word "siblings" was initiated by her ELL student Jose, who asked Ms. Brown for *feedback* (strategy 14) whether "siblings" is synonymous to "brothers and sisters" (lines 31-33). After this, Ms. Brown clarified the directions by explaining that there is a need to provide realistic data in order to discover whether there is indeed a correlation between students' height and the number of siblings they have (lines 38-40).

Some of the *immediate* or *long-term* consequences of Ms. Brown's discourse are also of interest. For example, she often shifted the dialogue from *univocal* to *dialogic* and exposed her students to different classroom work arrangements (strategy 19), such as using cooperative groups or partner discussions. This was demonstrated not only in the excerpt above, but also in the other observations. However, she had problems setting *long-term* norms of turn-taking and politeness during the discussions (as illustrated by the instance involving Rosita in the excerpt above (lines 18 and 23)). Thus, even though she provided her ELL students (and all students for that matter) with opportunities to participate in the classroom discourse by involving them in the activity of the above excerpt (strategy 11), setting norms of polite and orderly communication in English could further enrich the students' possibilities to become better team-players and equal partners in future work collectives.

Perceptions of classroom discourse. Figure 11 represents the researcher evaluation, the teacher's self-evaluation, and her ELL students' evaluations of Ms. Brown's *teacher talk* and discourse characteristics. The pair-wise correlations (Pearson product-moment correlation coefficients) for Ms. Brown's case study are as follows: the correlation between the teacher and researcher is -.14; between the teacher and ELLs it is -.26, and between the researcher and ELLs it is .59. This negative result is due to an unrealistic self-evaluation and perhaps also due to a lack of understanding of the ELLs, since Ms. Brown has no teaching experience and lacks ESOL certification.

Figure 11 reveals that all evaluators agreed that Ms. Brown uses two strategies frequently – *Use of wait-time techniques after posing a question* (strategy 13) and *Provide feedback* (14). For example, the above excerpt also demonstrates that Ms. Brown consistently provided her students with feedback (strategy 14) to indicate whether their responses were correct or needed further modification, by the use of such expressions as

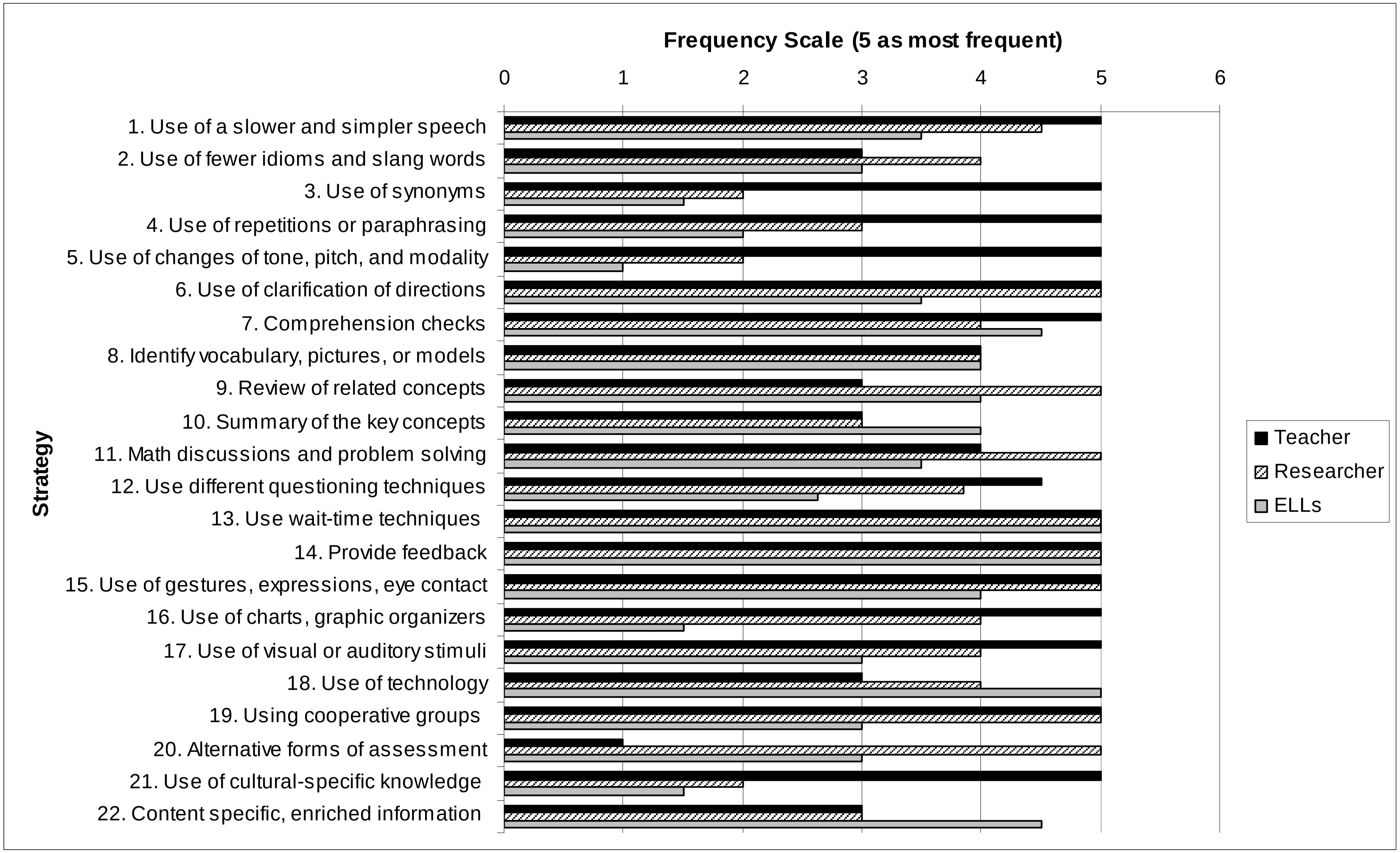


Figure 11. Teacher's, researcher's, and ELLs' evaluations of Ms. Brown's frequency of use of various discursive strategies.

“alright” (see lines 2, 4, 35), “perfect” (line 28) , “okay” (lines 2, 24, 25, 29, 44) , or “hmm” whenever she wanted them to reconsider their answers. During the observations (and measuring the time after lines 45, 46, and 49 in the excerpt above) and in the post-observation interview, Ms. Brown indicated that after asking a question she often paused and thus indicated to the students: “Think about it!” to provide them enough wait-time to rethink and/or correct their answers (strategy 13). Figure 11 also reveals that Ms. Brown consistently used the following strategies in her discourse: *Identify subject specific and important lesson vocabulary and provide context embedded examples, pictures, or models (*strategy 8) and *Use of gestures, facial expressions, eye contact, or demonstrations to enhance comprehension* (15). For example, when introducing the lesson about *Scatter Plots,* Ms. Brown began as follows:

> Today we’re gonna be discussing scatter plots…So what I have on the overhead, you also have on the bottom of your note sheet. But first we have a little vocab. A scatter plot is a graph that shows the relationship between two separate data, okay? And the way that the data can have a relationship. They can have a positive relationship, a negative relationship, or no correlation.

Furthermore, as we saw in the excerpt at the beginning describing Ms. Brown’s classroom discourse, she did not simply formally introduce the vocabulary to her students but rather used an activity to embed the definitions in a real-life context by looking for a correlation between the students’ heights and number of sibling (i.e., utilized strategy 8). ELL students also felt that they understood the vocabulary by the manner it was taught to them, and noted that Ms. Brown used this strategy very frequently (strategy 8). Figure 11 also makes it apparent that all agreed that Ms. Brown frequently used *gestures, facial* *expressions, eye contact, or demonstrations to enhance comprehension* (strategy 15) – a facet described and analyzed in the section regarding the *form* of Ms. Brown's discourse as per Krussel et al.'s (2004) framework.

However, with regards to how ELL students feel in Ms. Brown's classroom and how they evaluated the teacher's discourse, Figure 11 reveals some wide disparities between the teacher and ELL student evaluations. For example, for a few of the strategies, Ms. Brown and the ELL students disagreed on the frequency with which the strategies are used. The particular strategies where there is disagreement between Ms. Brown and her ELL students are: *Use of synonyms* (strategy 3), *Use of change of tone, pitch, and modality* (5) and *Provide opportunities for students to share experiences and build on personal or cultural-specific knowledge while problem-solving and thus build cross-cultural knowledge* (21). As revealed in the provided excerpt, some of Ms. Brown's ELL students who participated in the activity encountered difficulties in understanding the meaning of the word "siblings." Only later in the dialogue when Jose asked "You mean like brothers and sisters?" (line 32) did Ms. Brown realize that the student needed help with vocabulary, and in turning to Bryan she asked "Bryan, how many brothers and sisters do you have?" (line 41). Thus, as indicated by the ELLs' evaluations and verified by the excerpt provided, Ms. Brown did not make frequent *use of synonyms* or other expressions that could help her students better understand the concepts (strategy 4). The ELLs also indicated, contrary to Ms. Brown's opinion, that she does not *change her pitch or modality of talk* (strategy 5) and thus they are completely unaware if certain words or phases in her talk carry a greater degree of importance. According to ELL students, Ms. Brown did not differentiate her speaking tone and grammatical or instructional tone (e.g.

lack of use of strategy 5). As a result, ELLs could not discern whether something was important as part of the lesson instruction or if it was just said in a conversational mode.

The ELL students also indicated that Ms. Brown does not ask them to *give examples from their country or family when solving mathematical problems* (strategy 21). Even though the initial excerpt about the scatter plot activity is an instance of this strategy, it was an isolated one which occurred infrequently, as the ELLs indicated in the interview. The observed classroom sessions did not reveal another instance of the use of this strategy. However, Ms. Brown evaluated that she utilized her ELLs' cultural perspectives and backgrounds as insights to further modify and eventually improve her instructional approaches.

Summary of the frequency count of the teacher's discursive strategies. Figure 12 indicates the frequency with which Ms. Brown implemented each of the discursive strategies found in the *TTT Form 1*. The strategies most frequently employed by Ms. Brown are—*Provide feedback* (strategy 14), *Check for comprehension (7),* and *Use of different questioning techniques (12).* This provides additional evidence to support conclusions drawn from the previous graph (see Figure 11). Furthermore, in the text above were demonstrated many examples of Ms. Brown's implementation of strategies 12 (see the examples provided under the discussion of the *form* of Ms. Brown's discourse) and strategy 14 (see lines 2, 4, 25, 28, 29, 35, and 44 in the excerpt above). Examples of her use of strategy 7 – *Use of comprehension checks throughout the lesson* are the observed instances when she often stated definitions by finishing them with the question "…right?" thus eliciting the students' reactions (or nods) of agreement or

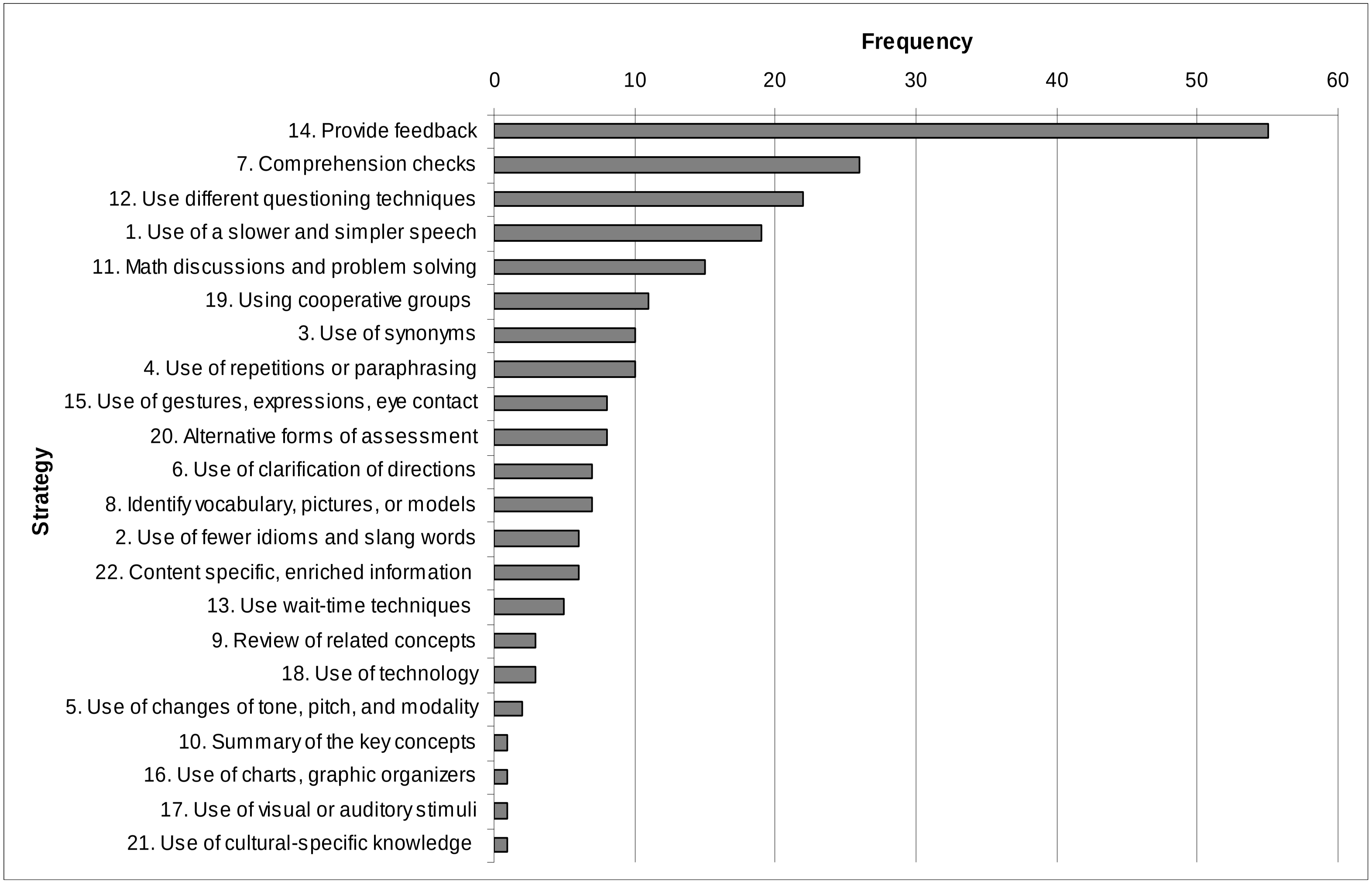


Figure 12. Frequency count of Ms. Brown’s use of various discursive strategies during the three 20-minute video-recorded sessions.

disagreement with, or understanding or not, her statements (see lines 4, 6, 45, 51 in the excerpt with the scatter plot activity).

Figure 12 also reveals the previously discussed omission by Ms. Brown of implementing the following characteristics of teacher talk and other discursive strategies: *Conclude a lesson with a summary of the key concepts* (strategy 10), *Use of charts, graphic organizers —Venn diagrams, tree diagrams, time lines, semantic maps, outlines, etc.* (16), *Use of a variety of visual or auditory stimuli: transparencies, pictures, flashcards, models, etc.* (17), and *Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge* (21). The observations also revealed that Ms. Brown rarely asked her students to *summarize the key concepts* that they have just learned (strategy 10). Moreover, even though she often used an overhead projector to write the bell work on it, she did not utilize the overhead projector to *draw charts or graphic organizers* on it, or *show pictures or visual models* (strategies 16 and 17). Figure 12 shows that Ms. Brown frequently omitted the use of strategy 21. This supports the conclusions drawn from the previous graph (see Figure 11) wherein the ELL students also indicated Ms. Brown’s less frequent use of strategies 16 and 21.

Ms. Cortez

Ms. Cortez, a Puerto Rican in her late 50s, has taught in the USA for 10 years, for five of which she has taught Algebra I classes. She is certified to teach middle school mathematics, but this year she accepted a position in a high school and is currently working on her high-school mathematics certification. She has not yet completed her content area teachers’ requirement of 60 hours of training toward ESOL endorsement.

However, in the interview she indicated that because she has worked with Hispanic students in Puerto Rico, she is aware of their educational needs. She finds her Central and South American ELL students to be very motivated, disciplined, and responsible. As an example, she showed some of their folders, which were very organized and complete. She commented that even though they have difficulties learning mathematics in English, they do their best so as to receive a better education. They always do their homework assignments, bring their materials to class, and are “excellent students.”

Ms. Cortez’ Algebra I class consisted of 17 students, nine of whom were Hispanics, six African Americans, and two multi-racial students. Four of the students were ELLs, three of whom were Spanish-speaking students with different levels of fluency in the English language. The fourth ELL student was from Central America and spoke French. Ms. Cortez allowed the Hispanic ELLs to work in a cooperative group with students fluent in both Spanish and English and they were helping each other by sometimes translating directions, or with problem solving. For the ELL student who spoke French, the school had assigned a French language specialist as a teacher assistant (TA) to assist Ms. Cortez with this ELL student. There was also another TA who assisted Ms. Cortez with the Hispanic ELLs. The ELLs (and the TAs) were seated in the front right corner of the room and were able to see the lesson on the overhead and participate in classroom discussions.

Typical classroom discourse. From the three observed classroom sessions, some patterns typical of Ms. Cortez emerged. For example, unless giving a test, she always *started her lessons with a review of the previously learned related concepts* (strategy 9) and thus tried to involve her students (including ELLs) in the mathematical discourse.

Next, she collected their homework assignments and gave them bell-work which was usually prepared and written on transparencies. After collecting the bell-work she usually began teaching the new lesson. And finally, from prepared worksheets she assigned problems for students to work on, individually or in cooperative groups (strategy 19—*Expose students to different classroom work arrangements*). During that time, she and her TAs assisted the students according to their individual needs.

The observations of Ms. Cortez discursive moves while teaching a new lesson revealed that her teacher-student interactions are strongly reminiscent of a model recognized in the review literature as IRF (Initiation-Response-Follow-up). According to this model, "[t]he element of structure that is most clearly defined, however, is that of 'teaching exchange', which typically has three phases, involving an 'initiating' move, a 'responding' move, and a 'follow-up' move" (Ellis, 2000, p. 574). According to this study, under the 'initiation move' could fall the teacher's discursive strategies 11 (*involving students in mathematical discussions* or problem solving by calling them by name) and 12 (*using different questioning techniques* sensitive to the ELLs' level of English acquisition). Under the "responding move" fall any answer the students provide, but because they are not part of the teacher discourse, they will not be placed under scrutiny in this study. And lastly, based on this study, under 'follow-up move' could fall the teacher's discursive strategy 14 (*provide feedback*) and 4 (*use of repetitions* or paraphrasing of students' answers). The following excerpt (lines 1 to 52) will be used to demonstrate and provide evidence of this and some of the other strategies utilized in Ms. Cortez discourse:

Ms. Cortez: [1] Uhh, who remembers the topic that we were working with?

Student 1 (not and ELL student) **:** [2] I remember.
Student 2 (an ELL): [3] Who remember what?
Ms. Cortez: [4] The topic, which one was yesterday's topic?
Student 1: [5] Coordinate…(*the students speaks the rest unclearly*)
Ms. Cortez: [6] Yes, coordinate-plane.

Here the questions in line 1 and 4 fall under the teacher's "initiation" move and demonstrated her use of questions sensitive to ELLs from *pre-production* and *early-production* levels of English language acquisition, requiring, correspondingly, a one-word response or a list of words (strategies 12 a and 12b). One student answered (lines 2 and 5) — a "responding" move, and Ms. Cortez immediately did a "follow-up" move by *providing feedback* (strategy 14) and repeating and completing the student's answer so that the others could hear it (strategy 4).

Ms. Cortez: [7] Now, I have a coordinate plane.

At this moment the teacher drew the coordinate plane (strategy 16-*Use of charts or drawings*) on a transparency on the overhead projector (strategy 18-*Use of technology*). Then, she pointed to the drawing (strategy 15-*Use of gestures*) and asked the students:

Ms. Cortez: [8] Now, what about…what about the quadrants?
[9] Who can say something about quadrants?
[10] Remember, you divide it…the graph is going to be divided in how many quadrants?
Students: [11] Four.
Ms. Cortez: [12] Okay.

Here again lines 10, 11, and 12 provide evidence that Ms. Cortez adhered to the IRF "teaching exchange" model. However, for the purpose of this study, it is more interesting for the reader to observe how she gradually decreased the type of her questioning from questions that required an extended response and could potentially involve ELLs from an *intermediate level* (strategy 12d), to *speech-emergence* (strategy 12c), and finally to a question that required a single-word response and thus potentially could involve ELL students from an *early production* stage of English language acquisition (strategy 12b). However, such a shift in the type of the questions, as well as the fact that the teacher expressed satisfaction with the one word response, demonstrate that she did not encourage her students to elaborate on their answers and thus to begin to experiment with the language which they are just beginning to acquire.

Ms. Cortez: [13] What is going to be this axis right here?
Ms. Cortez and Students: [14] *Y*.
Ms. Cortez: [15] What is going to be this?
Students: [16] *X*.
Ms. Cortez: [17] *X*, very good.
[18] And we said yesterday that we have four quadrants.
[19] We are going to start with one, two, three, and four.
[20] This is the fourth quadrant.
[21] The signs over here are going to be…?

Here Ms. Cortez raised the intonation of her voice (strategy 5—*Change of pitch of voice)* and thus indicated that this is a question and the students should finish her sentence (strategy 12b- *Use of different questioning techniques*, in this case, a question requiring a one-word response). As expected, a couple of students answered aloud:

Students: [22] Positive.
Ms. Cortez: [23] Positive and positive.
[24] And I said yesterday that one way it is easy for you to remember the signs…you go to the first one.
[25] Then go to the opposite.
[26] The opposite quadrant is going to have the opposite signs, too.
[27] This is going to be positive and positive, then you're going to have here…?
Students: [28] Negative and negative.
Ms. Cortez: [29] Then here, in the second quadrant, you have negative…?
Students: [30] Negative and positive.
Ms. Cortez: [31] Very, very good.
[32] Okay, then what are you going to have in the opposite quadrant…?

Here the teacher enthusiastically *provided feedback* (strategy 14, line 31) that the students are correct and they appropriately responded to the *change of the pitch* of her voice (strategy 5 again, line 32) and thus she encouraged them to continue answering her *questions* (strategy 12b, line 32):

Students: [33] Positive and negative.
Ms. Cortez: [34] Very good.
[35] Something else that you didn't get yesterday, that you have a question about yesterday's class.
[36] Okay, remember that I said that the first ordered pair is going to be the *x*,the second one is going to be your *y*.
[37] And if you have *x*, the first numbers you have to move to the…?

Here the teacher decided to test (strategy 20—*Use of an oral form of assessment)* a particular (ELL) student's knowledge (strategy 22a—*Lower level of cognitive demand*) of a previously explained concept (strategy 9—*Review of a related concept)*. She made eye contact and nodded to that student (strategy 15—*Use of eye contact and gestures*), thus indicating that he should continue her sentence (strategy 12b—*Use of a question requiring a list of words*):

Ricardo (an ELL) **:** [38] To the right, or to the left.
Ms. Cortez: [39] To the right, or to the left.
[40] Second number is going to be the *y*.
[41] The *y* you move it…?
Ricardo (the same ELL)**:** [42] Up or down.
Ms. Cortez: [43] Up or down.
[44] Up is going to be positive or negative?
Students (a few students say aloud): [45] Positive.
Ms. Cortez: [46] Very good.
[47] What about if I move to the left side?
Students (aloud) : [48] Negative.
Ms. Cortez: [49] It's going to be negative.
[50] You got it. Okay.
[51] Today's class we are going to continue working with graphs and the topic for today is going to be…scattered...plots.
[52] I am going to pass the paper for today's class.

At this moment, the teacher was satisfied with the review, and started instructions on the new topic — *Scattered plots.* She passed copies of the same prepared worksheet that Ms. Brown (in case study number 3) was using.

Krussel et al. framework. The ***purpose*** of Ms. Cortez' discourse was to involve all her students (and ELLs in particular) in whole-class discussions and individual or group-work, leading to better understanding of the concepts of *The Coordinate Plane, Scatter Plots,* transforming linear equations in *Slope-intercept* or *Point-Slope Forms* and then G*raphing Lines* by applying the concepts of intercepts and slope. For example, Ms. Cortez regularly *used repetitions or paraphrasing* of her or her students' statements (strategy 4, lines 4, 39, 43, and 49), or often *used charts and graphic organizers* (strategy 16, lines 24 to 33: which were used as a *semantic map* for her students to better remember the signs of ordered pairs in different quadrants), which indicates that Ms. Cortez was attending to the ELLs in her mathematical classroom and that she was trying to involve them in the mathematical discourse by "visualizing the lesson" (i.e., by providing variety of visual stimuli: transparencies, charts, and diagrams). Furthermore, she exhibited sensitivity to the level of SLA of her ELLs by trying to use different questioning techniques (strategy 12), thus encouraging her ELL students' transition from *pre-production* and *early production* of mathematics answers in English to *speech emergence* and *intermediate speech*. However, for example, most of the questions were of the type "what is…" (lines 13, 15, 21), general questions requiring one-word (lines 10, 21) or a list of words as responses (lines 27, 29, 32, 37, and 41), or either/or questions (lines 45, 47, all in strategy 12b), and did not move the students above the four levels of cognitive demand according to Bloom's Taxonomy (knowledge, comprehension, application, and analysis; strategy 22 a, b, c, d, and e). Ms. Cortez nonetheless also asked questions such as: “Who can say something about quadrants?” (line 9) or “What is the next step? To leave alone the b, what do you have to do?” which in turn encouraged the student to use longer sentences in English in their answers. Thus, in turn, she encouraged her ELL students to develop a higher level of English Language acquisition such as *speech emergence* or *intermediate speech*.

The students’ adherence to certain norms of classroom behavior revealed that the ***setting*** for classroom discourse was established early in the school year. Moreover, even though there were students enrolled in Intensive Math, and others taking Algebra I, they all demonstrated good responses to the pre-established procedures during the bell-work activity. For example, when the teacher gave the following directions, “For the Intensive Math class, if you finish, pass the paper right now. Pass your bell work for Intensive Math. Algebra I, please still work on your bell work…. Three more minutes,” the students seemed to be accustomed to the procedures and complied with the teacher. The two TAs present assisted the ELL students by translating the teacher’s directions in French and Spanish and collected the ELLs’ bell-work. Additionally, from the fact that some students occasionally raised their hands and said: “Miss, I don’t get this part...” and Ms. Cortez or some of the TAs immediately assisted them, it became obvious that the class was imbued with an atmosphere in which students felt free to ask questions or seek individual assistance if they did not understand something.

The ***form*** of Ms. Cortez’ discourse included both actual *teacher talk* (verbal) and *actions* (non verbal). For example, her questions: “Who can say something about quadrants?” or “What is the next step? To leave alone the b, what do you have to do?”

indicated that she encouraged ELLs (and all other students) to develop their communication in mathematics using English language. However, on very rare occasions she did ask students to reflect on their thinking or justify the steps necessary for reaching a solution (strategy 22, e and f). Even though she encouraged ELLs to communicate both in their native language and in English, often she missed the opportunities to move the discourse to the higher levels of cognitive demand that *synthesis* and *evaluation* require, according to Bloom's Taxonomy. The below excerpt provides an example of such an occasion in which discourse with an ELL was in fact moved to a higher level of cognitive demand, requiring the student to analyze and explain what type of a correlation exists between data:

Ms. Cortez: [1] (*points to paper)* E…the amount of ink remaining in a pen.
[2] Do you think it's a one relation?
[3] Remember, when you are writing…
[4] Okay, when you write a lot, what's going to happen with the ink?
Miguel: [5] It's going to decrease.
Ms. Cortez: [6] It's going to be…
Miguel: [7] Negative.
Ms. Cortez: [8] The more words...the more words that you have, it means this is going to increase.
[9] What is going to happen with the ink?
Miguel: [10] It's going to run out, it's going to run out.
Ms. Cortez: [11] Huh?
Miguel: [12] There isn't going to be any more ink in the pen.
Ms. Cortez: [13] Okay, then.
[14] The words are going to increase and the ink is going to…
Miguel: [15] Decrease.
Ms. Cortez: [16] Decrease, okay.
[17] Then, what type of relationship are we going to have here?
[18] Positive, negative, or…or it's going to be no relationship at all.
Miguel: [19] Negative.
Ms. Cortez: [20] Negative, very good. Excellent!

As the excerpt above demonstrates, Ms. Cortez' discourse did not exhibit flawless English grammar; however being aware of this, she *used repetitions* (strategy 4, lines 8, 16, and 20) or *paraphrasing* of her own (lines 3, 4, and 8) or her students' words (lines 6, and 14), and *simpler talk* and shorter sentences (strategy 1) so as to be better understood by her students. Requesting that her student paraphrase his sentence (lines 10 and 12), and then also paraphrasing his answers herself (in lines 14 and 15), demonstrated that Ms. Cortez was encouraging the ELL student to use specific mathematics vocabulary – for example, the words "increase/decrease" or "positive/negative" when talking about the relationship between the number of words and the amount to ink.

Ms. Cortez also used different forms of non-verbal discourse (strategy 15—*Use of gestures, eye contact or demonstration to enhance comprehension*). For example, she often pointed on the overhead to the sections of the prepared transparencies where she wanted her students to focus their attention. She also walked between the students' seats and assisted them or answered questions (if asked) and checked on ELL students' progress with the tasks.

Some of the ***consequences*** (*intended* or *unintended*, *immediate* or *long term)* of Ms. Cortez' discourse were classified according to Krussel et al.'s (2004) framework, as follows below. For example, Ms. Cortez *intended* to shift the cognitive level of the task performed (for example when determining if there is a correlation between the number of words written and the amount of ink in a pen) by asking the students to explain what they think, but *unintentionally* she assisted them in doing this and thereby lowered her expectations of their abilities to complete the task on their own. From the other point of view, some of the *immediate* and *long term* consequences of Ms. Cortez' discourse can be gleaned from the following instance: in one of the observed classroom sessions, Ms.

Cortez performed a folder check and said the following to an ELL student who had been recently assigned to her class:

> You're going to put all graded papers here, and on the right side the ones that I haven't graded yet. That way when I open the folder, because you're new, when I open the folder I will look to the right side. Okay? Thank you.

This example illustrates how Ms. Cortez' *used alternative forms of assessment* (strategy 20) in order to monitor the progress of all her students in her class, and especially that of the ELL students. She evaluated their work not only on paper-and-pencil tests, but also collected their homework and bell-work for grading, and checked their folders and oftentimes examined them orally by asking specific students certain questions. Additionally, by walking around the students' desks while they performed individual or group work on pre-prepared worksheets, she (and her TAs) not only assisted the students in better understanding and completing the task at hand (*immediate consequence* of her discourse), but also monitored their progress in her mathematics class in general. This, as she indicated in the interview, provided her with opportunities to modify her instruction and explain a concept again or use more examples to model how the concept can be applied in solving a mathematical problem. Therefore, as a *long term* consequence from her modified discourse to assist her students better, she increased her ELLs' chances to become more active participants in mathematical activities and to improve their fluency in expressing their questions or thoughts in English.

Perceptions of classroom discourse. Figure 13 represents the researcher's evaluation, the teacher's self-evaluation, and the ELLs' evaluations of Ms. Cortez' use of each of the strategies found in *TTT Form 1*. From Figure 13 it is noticeable that Ms.

Cortez evaluated herself as always using all of the strategies. On the form with which she was provided (*TTT Form 2*, see Appendix C), next to the printed words *5-Always*, she added with her own handwriting: *"that* [meaning *when*] *they need."* As a result, the pair-wise correlations involving the teacher cannot be calculated because it results in a division by zero, as there is no deviation from the mean of 5. Nevertheless, the pair-wise correlation between the researcher and ELLs it is .36. Figure 13 also indicates that there are four strategies where there is agreement among all of the evaluators as the most frequently used by Ms. Cortez – *Use of repetitions* (strategy 4), *Use of clarifications of directions* (6), *Start a lesson with a review of a related concept* (9), and *Provide feedback* (14). The frequent use of these strategies was demonstrated throughout the excerpts provided above.

For example, in the first excerpt, Ms. Cortez asked students to reflect on the lesson she taught previously by asking them *review* questions (strategy 9, lines 1, 4, 18, 24, 35, and 36). She also frequently *repeated* her or her students' statements (Strategy 4, lines 17, 23, 39, 43, and 49), and often provided them with feedback, thus indicating to them the validity of the answers they provided (strategy 14, lines 6, 12, 17, 31, 34, and 46). Throughout the observations it was also observed that after assigning the students to work individually or in cooperative groups, Ms. Cortez often circulated between their seats and clarified the directions or provided assistance if the students asked for help (strategy 6).

However, the researcher and Ms. Cortez' ELL students both evaluated that she did not incorporate two of the strategies as often as she thought — *Use of fewer idioms*

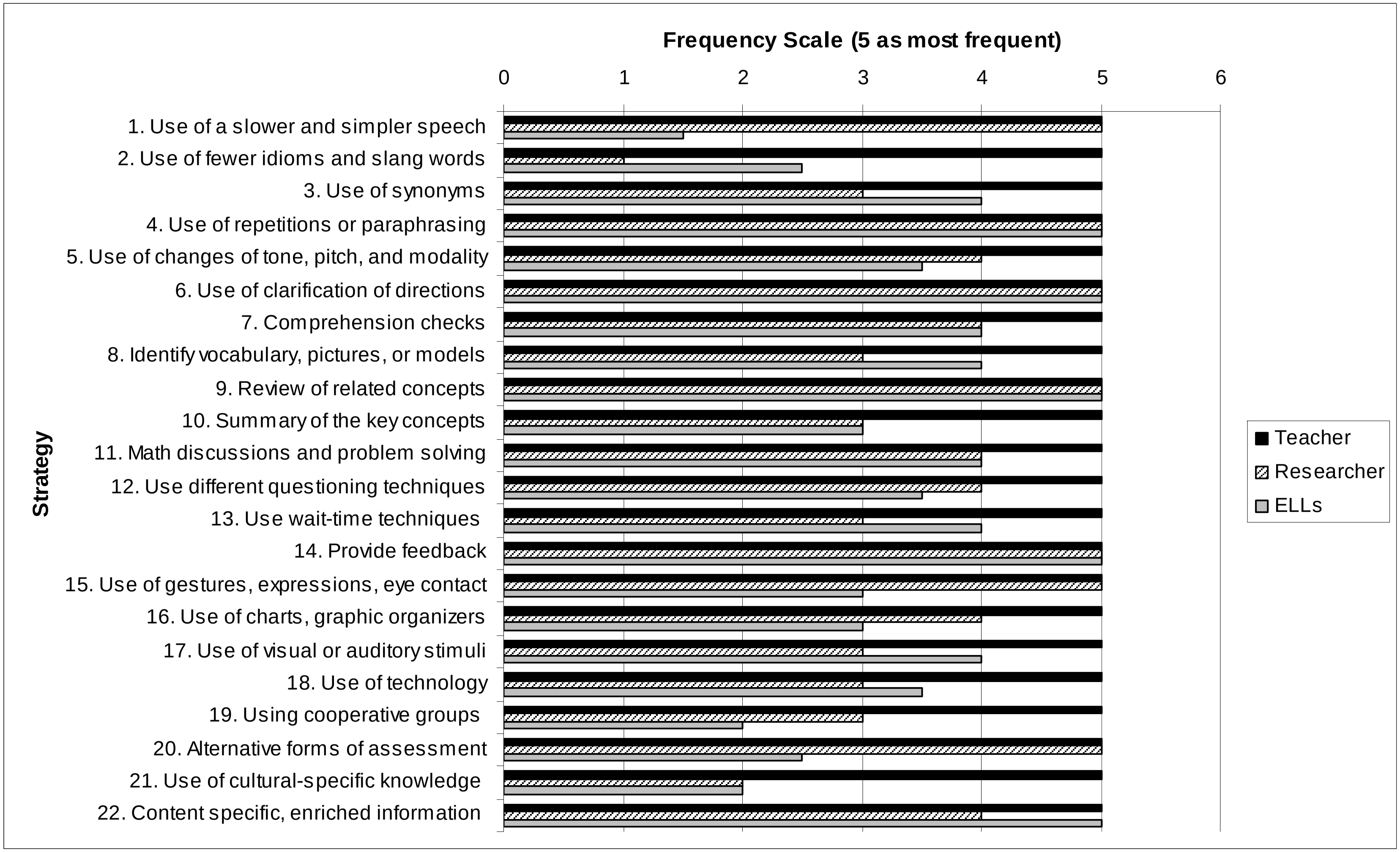


Figure 13. Teacher’s, researcher’s, and ELLs’ evaluations of Ms. Cotrez’ frequency of use of various discursive strategies.

and slang words from the mathematics vocabulary (strategy 2) and *Provide opportunities for students to share experiences and build up on personal or cultural specific knowledge while problem solving in mathematics and thus building on cross cultural knowledge* (21). In relation to how ELLs feel in Ms. Cortez' classroom, Figure 13 indicates that her ELL students evaluated that two other strategies were not as often incorporated in her teaching style either — *Use of a slower and simpler speech* (1) and *Expose students to different classroom work arrangements, such as using cooperative groups or partner discussions* (19). The difference in ELLs' opinions of Ms. Cortez' less frequent use of strategy 1 might be due to the fact that most often they (being in very initial stages of English language acquisition) actually heard the French and Spanish/Portuguese translated versions of Ms. Cortez' talk and perhaps it was the translators who did not employ slower or simpler speech. For example, during the interviews, two of the ELLs indicated that usually they worked in a group and helped each other and/or were aided by the TA who was translating. However, for the more fluent ELL (a girl), it was not hard to understand Ms. Cortez, but it was hard to translate Ms. Cortez' speech in Portuguese to her peer (a boy in the stage of English *pre-production*) because she did not know the mathematics vocabulary in Portuguese. Additionally, the ELLs indicated that even though they worked in groups, these groups usually consisted not only of them, but also the TAs. Thus, they indicated that they were not provided with opportunities for cooperative work or partner discussions with their English speaking peers.

Summary of the frequency count of the teacher's discursive strategies. Figure 14 indicates the frequency with which Ms. Cortez implemented each of the discursive strategies found in the *TTT Form 1*. Figure 14 indicates that the strategies most frequently employed by Ms. Cortez were: *Use of repetitions* (4), *Use of different questioning techniques*, sensitive to the ELLs' level of SLA (12), *Provide feedback* (14), and *Use of a slower and simpler speech* (1). This information supports two of the conclusions reached by the researcher and the ELLs, as reflected in the previous graph (see Figure 13). In particular, Ms. Cortez often used *repetitions* or paraphrasing of her statements (strategy 4) or asked students to repeat or restate them, especially when important concepts in mathematics were formulated. Also, Ms. Cortez frequently provided all her students, and especially the ELLs, with *feedback* as to whether their answers were correct, in both the mathematics and English language contexts (strategy 14).

The excerpts above, and the thorough qualitative analysis of the type of questions Ms. Cortez employed (strategy 12) during the observations indicated that she frequently switched between questions that initiated one-word responses, general questions that encouraged lists of words, and either/or questions (strategy 12b, lines 1, 4, 10, 13, 15, 21, 27, 29, 32, 37, 41, and 44 in the first excerpt). She also used questions that encouraged ELLs' *speech emergence* and *intermediate speech* development, but not very frequently (strategy 12 c and d; line 8 and line 9 in excerpt 1; and lines 4, 9, and 17 in excerpt 2). This indicated that Ms. Cortez was aware of the level of SLA of her ELLs – *pre-production* and *early production*, and provided them with questions that led them to the next levels of the subject-specific literacy – *speech emergence* and *intermediate speech* in mathematics in English. However, she did not challenge them to share their opinions and explore different methods of solving mathematical problems. She rarely asked her ELL students to justify, criticize, or explain their solutions.

However, while in Figure 13 there was disagreement between the researcher's

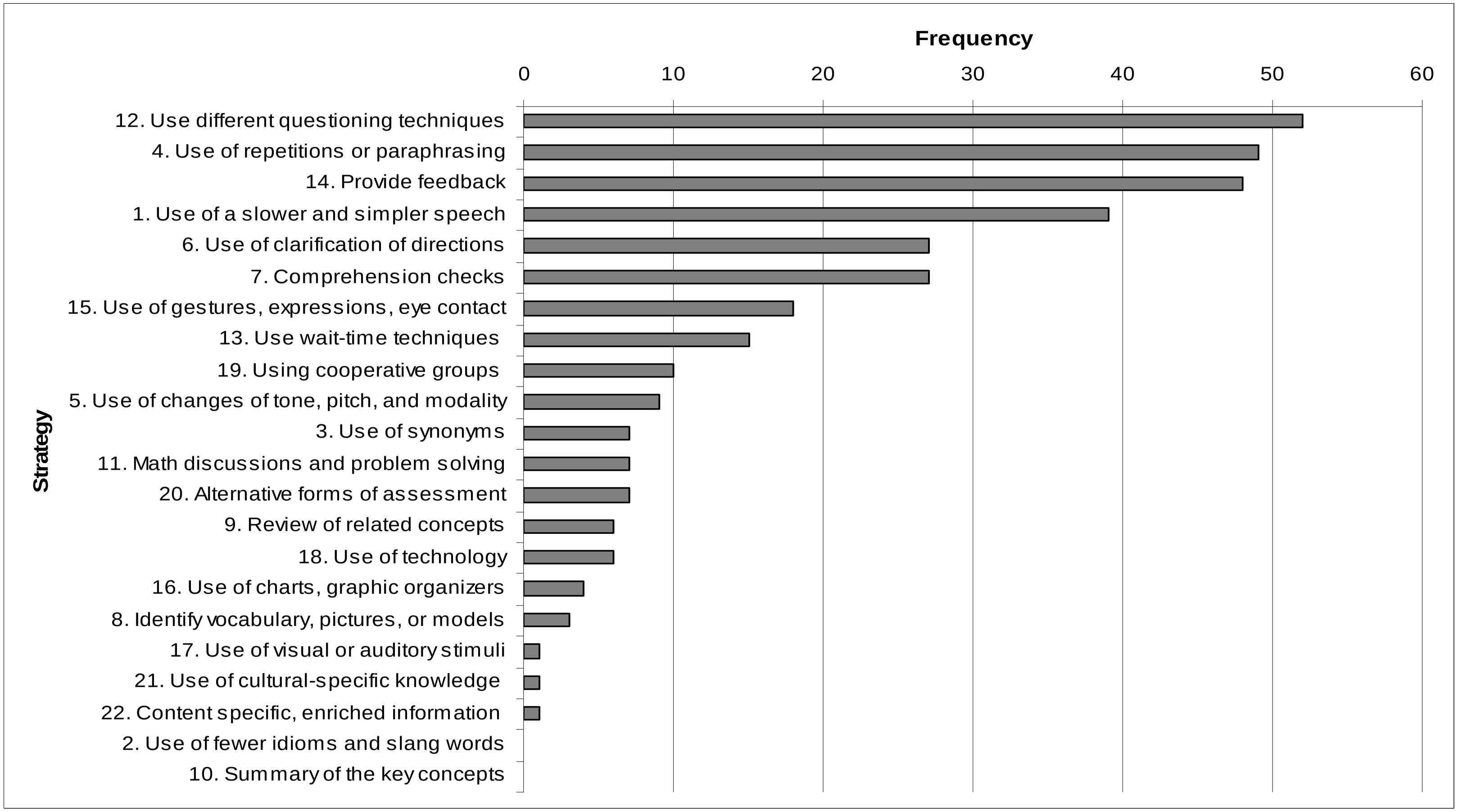


Figure 14. Frequency count of Ms. Cortez' use of various discursive strategies during the three 20-minute video-recorded sessions.

evaluation and ELLs' evaluation in strategy 1 (*Use of a slower and simple speech*), the actual frequency count of Ms. Cortez' employment of this strategy during the observations indicates (see Figure 14) that she indeed applied this strategy very often. As was conjectured above, this difference in ELLs' opinions might be due to the fact that most often they (being in very initial stages of English language acquisition) actually heard the French and Spanish/Portuguese translated versions of Ms. Cortez' talk and possibly it was the translators who did not employ slower or simpler speech.

Figure 14 also reveals the previously discussed omission by Ms. Cortez to *Conclude a lesson with a summary of the key concepts* (strategy 10), *Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge* (21), and to *Provide the students with context specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students* (22). The chart also reveals a lack in Ms. Cortez' talk in her *Use of fewer idioms and slang words from the mathematics vocabulary which if used, were not accompanied by a proper explanation or visual representation* (strategy 2), and an omission to *Use visual or auditory stimuli-- pictures, flashcards, models, etc.* (strategy 17).

Mr. Daniels

Mr. Daniels, a 60 year old Caucasian, has a 12-year teaching experience. He is certified to teach secondary mathematics and has completed the required 60 hours of training toward his ESOL endorsement nine years prior to the date of this study. In his teaching career, he has always taught Algebra I classes together with Geometry and college preparatory classes. Mr. Daniels is presently teaching Algebra I with the aid of the school's computer lab tutorials, tests, and quizzes developed by the program *I Can Learn Lab*.

His Algebra I class consisted of 20 students, 12 of which were African American, five Hispanic, and two White students. Initially there were four ELLs (two Hispanics and two African American students), but one of the African American ELLs withdrew from school and one of the Hispanic ELLs was suspended out of school for 10 days. In the interview Mr. Daniels commented on his experience of teaching mathematics to ELL students as follows: "Having worked with ELL students in the past, the math vocabulary is critical so I emphasize this especially in their notebooks. Most [ELLs] have good basic skills, but they have problems when answering 'word' based problems". Mr. Daniels reported that his opinion is based on his observations that usually when his ELL students call him for help; it is usually when they encounter a word problem. In his opinion, it is because his current ELL students were in more advanced stages of SLA (*speech emergence and intermediate fluency)* that he is more successful in helping them. He negotiated with them the meanings of word problems and provided them with context-embedded examples, pictures, or models and thus helped them solve the problems. However, he continued:

> My primary concern is the level I ESOL student who cannot communicate in English at any level. In 12 years of teaching, I have had only one succeed. We need to make speaking basic English the first priority before we put them in Math, Science, etc. All we do is set them up to fail.

Typical classroom discourse. The excerpt below illustrates some of the natural discourse that took place while Mr. Daniels circulated around his Computer Lab room.

During such sessions, the students used tutorial programs that facilitated their individual progress in Algebra I. Usually, Mr. Daniels assisted the students who asked for help, but he also monitored *all* his students and directed his attention more towards the ELLs or those students struggling in mathematics. In the excerpt below, Mr. Daniels assisted an ELL student in understanding the concept of the slope of a line:

Mr. Daniels: [1] Okay let's have you do this one up here on this sheet here (*pointed to the computer screen and the students' notebook).*
[2] Okay you do the same thing.
[3] Okay, remember your *y*'s are on the tops.
[4] So you put your *y*'s on the bottom *(pointed to the y's*).
[5] It's got to be rise over run.
[6] That's the most common mistake people make is what you did right there.
[7] These *y*'s, that has to be first, okay *(pointed again*)?

Mr. Daniels observed that the student was making an erroneous substitution in the formula and switched the places of rise and run. In his explanation, the teacher *used simplified speech and shorter sentences* (strategy 1a, lines 2 to 5, and 7). He also *used gestures* (strategy 15, lines 1, 4, and 7) to better articulate the meaning of his talk. Mr. Daniels also used simpler *synonymous words* (strategy 3, lines 3 and 4) such as "top" and "bottom" instead of "numerator" and "denominator," so as to be better understood by the ELL student. He also used the more informal statement of the slope formula as "rise over run" and focused his talk on the procedures of finding the slope, thus fostering ELL students' *early production* in English (strategy 1b, line 5).

(The student tried again by substituting the point's coordinates in the formula for slope)

Mr. Daniels: [8] Right.
[9] Okay six, then there's a minus six though, right.
[10] It's a negative six minus a negative four.
[11] And usually, like I said, you've got two signs side by side.
[12] Then you got negative two minus four.

[13] Okay you only have one of these sign combinations, right?
Lester (an ELL): (*nods*)
Mr. Daniels: [14] So you replace that negative and negative with a positive.
[15] So you've got positive four there.
Mr. Daniels: [16] What's negative six plus four? *(the teacher wrote -6+4 while talking)*

Here is exemplified a situation in which, while teaching the concept of slope, Mr. Daniels found that his ELL student had problems with operations involving integers of different signs. Thus, based on his students' needs, he modified his instructions and *provided the needed assistance* (strategy 6, lines 10, 12, 14 and 15).

Lester: [17] Um
Mr. Daniels: [18] Negative six plus four.
[19] What's the difference between six and four?
Lester: [20] Six and four?
[21] Four.
Mr. Daniels: [22] You have four dollars you spend six dollars how much are you short?
Lester: [23] Two
Mr. Daniels: [24] So its going to be a negative two.
[25] You're two dollars short, right?
Lester: (*nods and writes -2 on his sheet of paper)*
Mr. Daniels: [26] What's negative two minus four?
Lester: [27] Two

As the student's answer in line 21 demonstrated, he really needed assistance with operations with positive and negative integers. Rather than directly correcting him (strategy 1b—the teacher needs to *model/demonstrate* correct responses both in Math and English with students from the *early production* stages of their SLA, line 22), Mr. Daniels decided to explain the problem by using money (a concrete object) instead of numbers (abstract). Thus, he related the problem to a real-life situation that the ELL student more likely encountered in his daily life (strategy 22 c, line 22 and 25). The student responded correctly to the thus-presented problem (line 23).Then, when he asked a similar question with abstract numbers again (line 26), and received an erroneous response (line 27), Mr. Daniels made an instructional decision to continue presenting the mathematical operations with positive and negative integers via operations with money:

Mr. Daniels: [28] Okay I take two dollars from you, then I take four more dollars from you.
[29] How much have I taken?
Lester: [30] Two.
Mr. Daniels: [31] I took two bucks from you.
[32] I got it in my hand.
[33] I take four more from you, how much do I have in my hand?
Lester: [34] Six.
Mr. Daniels: [35] I've taken six dollars from you, right?
[36] The signs are the same.
[37] You add and keep the signs.
[38] Okay a negative divided by a negative is a?
Lester: [39] Um, positive
Mr. Daniels: [40] When you simplify two sixth (*points to 2/6)*, what's that the same as?
Lester: [41] Three…uh…one third (*writes 1/3)*
Mr. Daniels: [42] One over three.
[43] Okay so is that answer up there? (*points to screen)*
Mr. Daniels: [44] Did that help you, Lester?
Lester: [45] (*nods)*
Mr. Daniels: [46] I know it was kind of a math break down, but at least you got something.

Throughout this excerpt, Mr. Daniels also occasionally *checked the student's comprehension* (strategy 7, lines 13, 16, 25, 35, and 44), *repeated or paraphrased* his or his student's statements (strategy 4, lines 3, 5, 7, 18, 19, 31, 32, and 33), and provided him with *feedback* of whether an operation was performed correctly (strategy 14, line 8) or, by asking the student to perform the same operation again (by using an example with money), he indicated to the student that his answer was incorrect. Thus, he used a more subtle form of providing feedback without directly correcting the student's errors.

Krussel et al. framework. The analysis of Mr. Daniels' "discourse moves" using Krussel et al.'s (2004) framework reveal that the ***purpose*** of Mr. Daniels' discourse was to assist his ELL students in improving their mathematics and language abilities by modeling/demonstrating correct responses, both in mathematics and English language, as was demonstrated in the excerpt above (strategy 1b, lines 5, 10, 14, 36, 37 and 38). He assisted his students, including ELLs, in executing computer adapted activities, thus improving their understanding of the new concepts in the particular lessons (in the excerpt above, it was the concept of slope and related operations involving integers). By analyzing all of the observed sessions, it became apparent that Mr. Daniels was aware of the presence of ELLs in his Algebra I class and consciously catered to their specific needs. For example, when formally teaching the topic of slope to the ELL student from the excerpt above, he purposefully *used simpler talk and shorter sentences* (strategy 1a and b, lines 3, 4, 7, 19, 22, 25, 28, 31 to 33) and *paraphrased his sentences and questions* (strategy 4, lines 7, 19, 22, 25, 28) to negotiate the meaning of the concept under scrutiny. Frequently, after *assisting individual students* with 3 to 4 examples, he asked them to complete other sample problems (including some word problems as well) similar to the one whose solution he had just modeled (strategies 1a to 1c, and 22a to 22c). He oftentimes "broke down" the steps of solving a problem (strategy 6) in a manner similar to that exemplified by the excerpt above so that the ELL students or those who were struggling with a certain mathematical concept to better grasp it.

Mr. Daniels established a ***setting*** for classroom discourse by instituting certain long-standing expectations and norms of classroom behavior. Specifically, he expected that his students work in their notebooks simultaneously while working on the computer, and he monitored their progress on both as was demonstrated in the excerpt above (strategy 20, line 1). Students seated in neighboring computers were allowed to talk, but only on task-related topics. However, some of them often misbehaved and involved themselves in out-of-task conversations and moved from their assigned seats. For example, in one of the observations, while Mr. Daniels assisted one student, some of the other students talked aloud and thus interrupted their peers' individual work on the computers. Generally, Mr. Daniels managed to keep his students focused on the mathematical task at hand, but in order to maintain such order he often had to interrupt his instructions to deal with a particular student or discipline issue. In the interviews, the students indicated that because they were not exposed to different classroom work arrangement, such as cooperative groups or partner discussions (i.e., lack of utilizing strategy 19), after starting diligently on their work they soon experienced boredom and as a result involved themselves in non-mathematics oriented activities.

The ***form*** of Mr. Daniels' discourse included both *teacher talk* (verbal) and *actions* (non-verbal discourse). For example, after modeling or demonstrating the solution of a mathematical problem (strategy 1b, as demonstrated in the excerpt above), he often encouraged his students (especially ELLs) to try to talk mathematically in English: "Ok, so you talk me through the next one" or "Ok, so try this one." This indicated that Mr. Daniels' discourse took the form of a challenge by encouraging his students (all the while not differentiating between non-ELL and ELL students) to move to operations of higher cognitive demand according to Bloom's taxonomy – *application, analysis* ("…just kind of sketch the points and you'll see…Which scatter plot represents a non-linear relationship?"), *synthesis* ("predict best price estimate" or "Now stop for a second. What happens if they ask for one that's between 2 and 4? What would we expect the value at 3 to be?"; strategy 22d and 22e). Furthermore, questions such as the latter revealed Mr. Daniels' efforts to encourage his ELLs to expand their fluency in both the English language and mathematics (strategy 12b to 12d — *Use of different questioning techniques, sensitive to the ELLs' level of SLA)* and participate in the teacher-student discourse by explicitly voicing the operations they perform. However, even though he encouraged his students to draw diagrams or write in mathematics, he did not expose them to "hands on" activities or work in groups (i.e., he did not use strategy 19—*Expose students to different classroom work arrangement)*. Furthermore, he did not ask them to justify and perform more critical analyses or further explanations of more complicated steps while problem solving (strategy 22f—*Move the discourse to the highest level of cognitive demand according to Bloom's taxonomy*).

The ***consequences*** of Mr. Daniels' discourse may be classified according to Krussel et al.'s (2004) framework as *intended* or *unintended*, *immediate* or *long term* as follows. For example, as exemplified in the excerpt above relating his explanation of the concept of slope to the ELL student, he *intentionally* used *simplified speech and shorter sentences* (strategy 1a) and *synonymous words* (strategy 3) such as "top" and "bottom" instead of "numerator" and "denominator, and "y's" for "rise". He thus demonstrated sensitivity to the level of SLA of his ELL student —transitioning between *early production* and *speech emergence*. However, he *unintentionally* was "taking the floor" and did not provide many opportunities for his student to articulate where his problems in operations of integers arise. For example, when Mr. Daniels understood that the ELL student was more successful in performing operations with positive and negative integers once the mathematical operations were transferred to operations with money, he continued using this technique in subsequent examples. An *immediate consequence* of this type of discourse was that the student performed the operations more correctly. However, due to the general fact that Mr. Daniels was mostly using one type of classroom discourse organization – students working individually on the computers and being assisted by Mr. Daniels whenever the need arose – demonstrated that his ELL students were exposed to only teacher-student mathematical interactions. Even though the students were allowed to talk with others in their vicinity, these conversation were rarely mathematics-oriented in nature. As a result, Mr. Daniels did not provide ample opportunities for ELL students to use new mathematics vocabulary in dialogue. Thus, a close examination of Mr. Daniels' discourse indicated that as *long term consequences* of his manner of facilitating a single type of classroom discourse, his ELL students were assisted in developing their conceptual understanding of mathematics, but were not given opportunities to be equal partners in cooperative group discussions.

Perceptions of classroom discourse. Figure 15 below represents the researcher's evaluation, the teacher's self-evaluation, and the ELLs' evaluations of Mr. Daniels' *teacher talk* and use of different discursive strategies identified in *TTT Form 1*. The pair-wise correlations (Pearson product-moment correlation coefficients) for Mr. Daniels' case study are as follows: the correlation between the teacher and researcher is .71; between the teacher and ELLs it is .83, and between the researcher and ELLs it is .58.

There is general agreement that Mr. Daniels most frequently employed strategies 1 (*Use of a slower and simpler speech*), 4 (*Use of repetitions*), 7 (*Use of Comprehension*

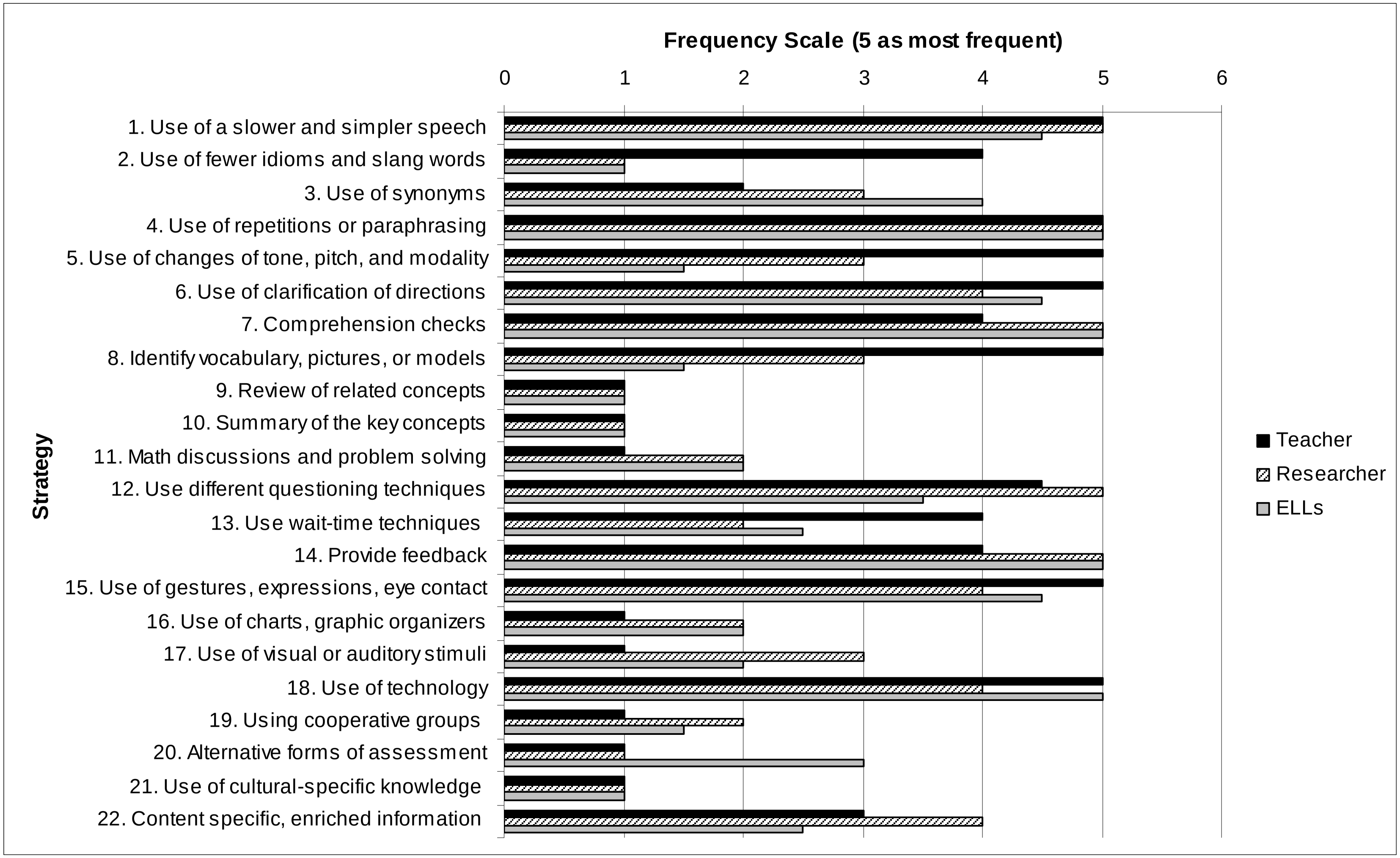


Figure 15. Teacher's, researcher's, and ELLs' evaluations of Mr. Daniels' frequency of use of various discursive strtegies.

Checks), and 14 (*Provide Feedback*), followed by (slightly less frequently) strategies 6 (*Use of Clarification of directions*), 15 (*Use of gestures, facial expressions, eye contact, or demonstrations to enhance comprehension*), and 18 (*Use of technology*). Mr. Daniels' frequent use of these strategies was already demonstrated in the excerpt above and discussed in the previous paragraphs.

Figure 15 reveals agreement that Mr. Daniels used least frequently the following discursive strategies: *Start a lesson with a review of a related concept* (9), *Conclude the lesson with a summary of the key concepts* (10), and *Provide the students with opportunities to share experiences and build upon personal or cultural specific knowledge while problem solving in mathematics* (21). Interesting disagreement in the evaluations is observed in the following two strategies: *Use of fewer idioms and slang words from the mathematics vocabulary* (2) and *Provide students with alternative forms of assessment—portfolios, vocabulary banks, oral presentations, and writing or reading in mathematics* (20). Mr. Daniels evaluated himself as using few idioms and that if they were used, were accompanied by a proper explanation or visual representation. However, the ELL students and the researcher indicated that even though on occasion Mr. Daniels explained some idioms of the mathematics vocabulary, this was not done frequently enough. On the other hand, for category 20, the ELLs indicated that Mr. Daniels provided them with alternative forms of assessment such as writing or reading in mathematics, whereas the researcher and Mr. Daniels felt that he should have used this strategy more often and included oral presentations, or portfolios and vocabulary banks as other forms to assess his students' progress in mathematics (and especially that of his ELL students).

In the interviews, the ELL students indicated that they had difficulties with word problems in mathematics and that it was usually when presented with such problems that they sought assistance from Mr. Daniels. He usually performed *clarifications of the directions* (strategy 6) for them and helped by *modeling* a variety of examples so that they could see the solution process (strategy 1b). He also asked his ELLs to apply the explained concepts to solve new problems and make predictions as to what would happen in different situations. Thus, he *provided them with content specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students* (strategy 22). Figure 16 below also confirms that Mr. Daniels applied this strategy frequently.

Summary of the frequency count of the teacher's discursive strategies. Figure 16 indicates the frequency with which Mr. Daniels implemented each of the discursive strategies found in the *TTT Form 1*. The strategies most frequently employed are: 1 (*Use of a slower and simpler speech)* and 14 (*Provide feedback*), as corroborated by the evaluations chart (see Figure 10). It also shows that Mr. Daniels' most frequent strategy was 12 (*Use of different questioning techniques*). Additionally, a qualitative analysis of the types of questions employed by Mr. Daniels reveals that they usually elicited one-word responses, or were general questions that encouraged a short list of words as a response. This indicates that Mr. Daniels was aware of the level of his ELLs – *early production* or in transition to *speech emergence* or *intermediate fluency*. Furthermore, the qualitative analysis indicated that Mr. Daniels used questions that challenged his ELLs and could potentially lead them to move to higher levels of subject-specific literacy – *speech emergence* and *intermediate speech* in mathematics in English – but such

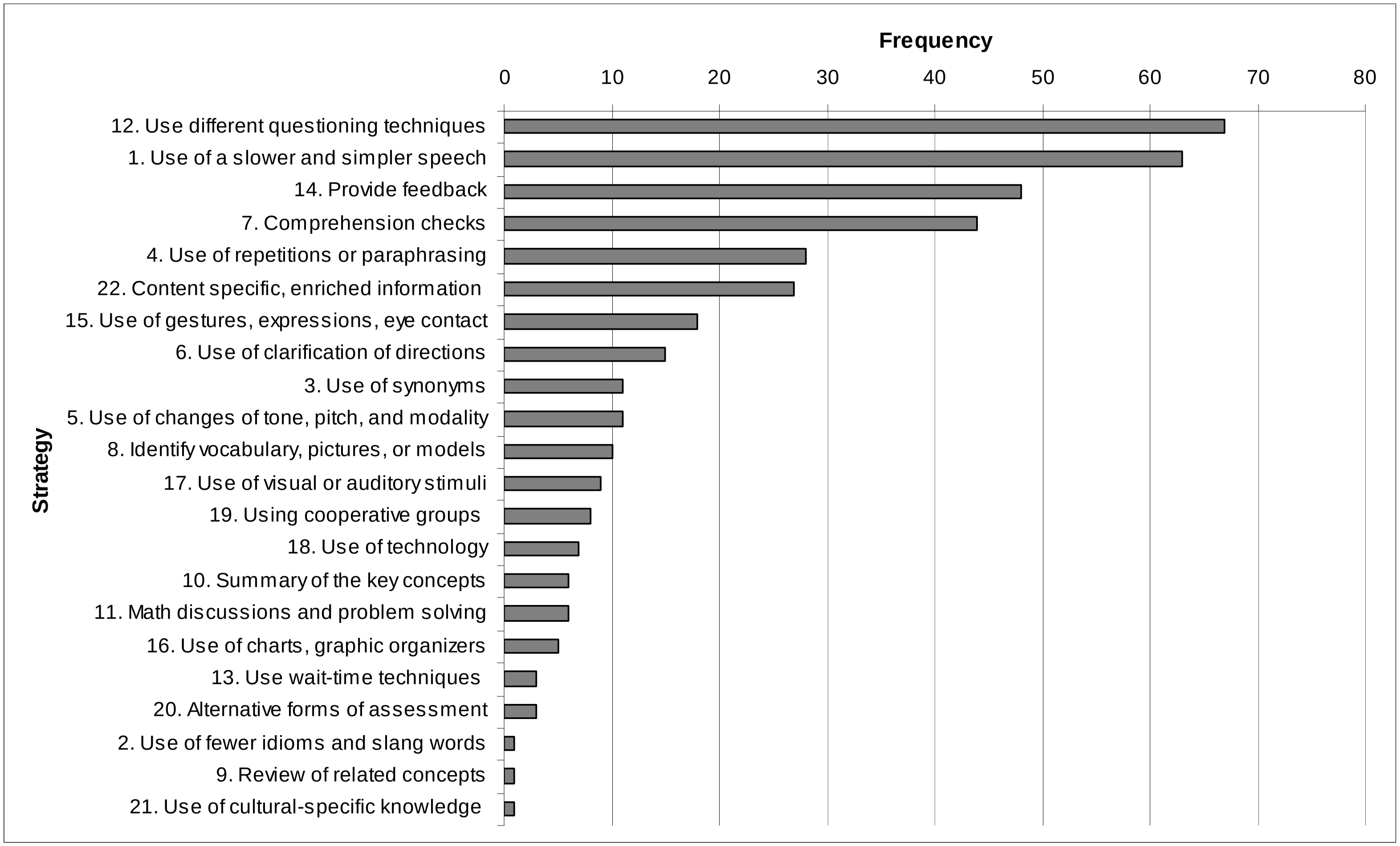


Figure 16. Frequency count of Mr. Daniels' use of various discursive strategies during the three 20-minute video-recorded sessions.

questions were not very frequent. Such findings are with agreement with those reported when examining Mr. Daniels' ***form*** of *teacher talk* according to Krussel et al.'s (2004) framework. Figure 16 also confirms the previously discussed omission by Mr. Daniels to use the following discursive strategies: *Use of fewer idioms and slang words from the mathematics vocabulary, or if used a proper explanation or visual representation is provided* (strategy 2), *Start a lesson with a review of related concepts* (9), *Conclude a lesson with a summary of the key concepts* (10), and *Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge* (21).

Summary of Results

To summarize the results from the detailed description and analysis of each case study, and to allow comparison between teachers, Table 3 and Figure 17 were devised (see Table 3 and Figure 17 below). Table 3 presents the general level of agreement between the teacher, the researcher, and the ELLs in their evaluations of the strategies used by each teacher. More specifically, Table 3 presents the computed pair-wise correlations (Pearson product-moment correlation coefficients), that show whether a given teacher's perceptions of his/her own use of strategies match those of the researcher and the ELLs. As can be discerned from the table, negative correlation coefficients were observed for the novice teachers — Ms. Barrera from Green Bay High School and Ms. Brown from Lincoln High School, who were also recently enrolled in the ESOL certification process. The negative pair-wise coefficients observed between the teacher

Table 3

Pearson Product-Moment Correlation Coefficients

Teacher Name	Teacher (X)	Researcher (Y)	ELLs (Z)	R (X, Y) (Tcr.,Res.)	R (X, Z) (Tcr.,ELLs)	R (Y, Z) (Res.,ELLs)
			Green Bay High School			
Mr. Able	Xm=3.77	Ym=3.73	Zm=3.86	**Rxy = .62**	**Rxz = .25**	**Ryz = .65**
	Sx=0.972	Sy=1.08	Sz=1.14			
Ms. Barrera	Xm=3.91	Ym=3.73	Zm=2.80	**Rxy = -.07**	**Rxz = -.23**	**Ryz = .27**
	Sx=1.02	Sy=0.98	Sz=1.13			
Ms. Chandler	Xm=3.23	Ym=2.86	Zm=2.81	**Rxy = .77**	**Rxz = .53**	**Ryz = .70**
	Sx=1.27	Sy=1.36	Sz=1.02			
Mr. Davison	Xm=3.68	Ym=3.64	Zm=3.33	**Rxy = .68**	**Rxz = .17**	**Ryz = .43**
	Sx=1.21	Sy=0.95	Sz=1.14			
			Lincoln High School			
Ms. Andersen	Xm=3.68	Ym=3.43	Zm=2.64	**Rxy = .61**	**Rxz = .12**	**Ryz = .46**
	Sx=1.09	Sy=0.93	Sz=1.50			
Ms. Brown	Xm=4.25	Ym=3.97	Zm=3.30	**Rxy = -.14**	**Rxz = -.26**	**Ryz = .59**
	Sx=1.11	Sy=1.05	Sz=1.22			
Ms. Cortez	Xm=5	Ym=3.77	Zm=3.59	**Rxy =**	**Rxz =**	**Ryz = .36**
	Sx=0	Sy=1.11	Sz=1.06	**undef.**	**undef.**	
Mr. Daniels	Xm=3	Ym=3.11	Zm=2.89	**Rxy = .71**	**Rxz = .83**	**Ryz = .58**
	Sx=1.51	Sy=1.79	Sz=1.54			

self-evaluation and the researcher evaluation indicate a lack of realistic vision of the classroom approaches. In a similar manner, the negative correlation coefficients between the teacher self-evaluation and the ELLs' evaluations indicate that teachers who had not completed the ESOL certification process lacked or had not yet developed an understanding of their ELL students and their educational needs in the mathematics classroom.

In reporting the pair-wise correlation coefficients an extreme example was Ms. Cortez. In her case, the correlation coefficients between the evaluations of the teacher and the researcher, and the teacher and ELLs, could not even be calculated. The formula involves division by the standard deviations from the mean, and because Ms. Cortez had evaluated herself as having used all discursive strategies from TTT Form 2 with a frequency of 5 (i.e., always/most frequently), the standard deviation from the mean was zero, thus it was not possible to obtain a result. This unrealistic self-evaluation of the used discursive strategies and lack of understanding of her ELL students can be attributed to the fact that Ms. Cortez, despite having previous teaching experience, had obtained this experience while teaching in a middle school in Puerto Rico. She had a middle school mathematics certification and no ESOL certification. Since this was her first year of teaching in a high school in the USA, she was working on her high school mathematics certification and was not yet enrolled in ESOL certification classes. For the other teachers, the higher positive correlation coefficients indicate that teachers with more teaching experience had developed a better sense of the teaching practices they routinely employed and could more realistically evaluate where they needed improvement. For example, in the cases of Ms. Chandler (Green Bay High School) and Mr. Daniels

(Lincoln High School), the highest positive correlation coefficients were observed across the three evaluators. The more accurate self-evaluation and better understanding of their ELLs could be attributed to the fact that both teachers were experienced and had their with ESOL certification for longer period of time.

Figure 17 combines the data from the frequency count of each teacher's use of different strategies during the observed lessons (see Figures 2, 4, 6, 8, 10, 12, 14, and 16). On the x-axis, the numbers from 1 to 22 correspond to the categories of *teacher talk* and other discursive strategies that are described in greater detail in *TTT Form 1* (See Appendix A). On the y-axis are placed the frequencies with which each strategy was used by each teacher during the observed classroom sessions. Above the numbers of each discursive strategy, the different-shaded bar graphs (8 bars corresponding to 8 teachers) represent each teacher's frequency of use of each category. For clarity, the legend provided below the graph allows the reader to connect each teacher's name with the assigned shading. (The eight bars above each category represent each teacher in the same order they were described in the study or in the order as presented in Table 2). Thus, as the detailed description and analysis of each case study has shed light on the specific patterns of strategies typically used by each teacher, Figure 17 allows for comparisons between the teachers in answering the research questions of this study. As Figure 17 indicates, the most frequently used strategies by all teachers (with minor variations) were: 12 (*Use of different questioning techniques, sensitive to the ELLs' level of SLA*) and 14 (*Provide feedback*). This conclusion is well grounded and is based on all the data that was triangulated by using different sources and methods of analysis. However, the additional qualitative analysis revealed differences in the types of questions the teachers asked their

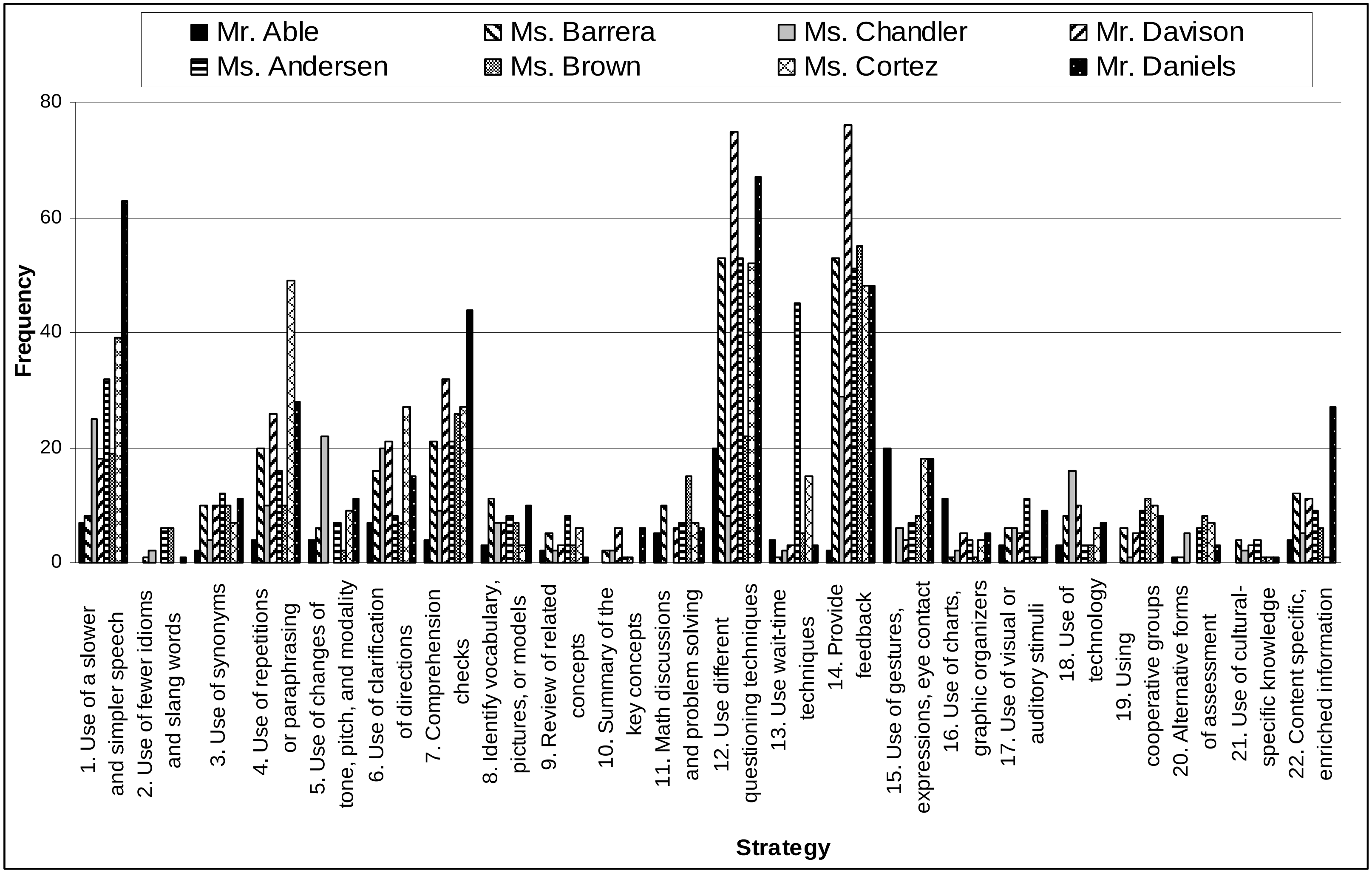


Figure 17. Teachers' frequencies of used strategies during the three 20-minute video-recorded sessions.

ELLs. These differences in questioning techniques, as well as other differences and similarities between the teachers, will be summarized in relation to the study's research questions.

Question 1

In investigating the extent to which teachers' patterns of discourse in the mathematics classroom change as a result of the number of ELL student(s) present, the following findings emerged. As indicated in the beginning of Chapter Four (see Table 2), most of the ELL students in Green Bay High School were assigned to Algebra I classes with computer labs, tutorials, tests and quizzes (i.e., Ms. Barrera's and Ms. Chandler's classrooms), and were more evenly distributed in Lincoln High School. However, as the analysis of data from different sources (observations, video-recordings and frequency counts, interviews and the researcher evaluation, teachers' self-evaluations and ELLs' evaluations) indicated, to some extent all teachers changed their patterns of discourse in the mathematics classroom as a result of simply the presence of ELL student(s), regardless of their number.

For example, even in the case of teachers who did not share their ELLs' linguistic and cultural backgrounds, and even if there was only a single ELL student or a couple of ELLs from certain cultural or linguistic groups, there were changes in the classroom setting which subsequently influenced changes in the classroom discourse. As described in the case analyses, for example, Mr. Able and Mr. Davison in Green Bay High School and Ms. Brown in Lincoln High School, both of whom did not speak their ELLs' native languages, tried to seat ELLs from similar linguistic backgrounds in close proximity to each other and thus make sure that they were also seated next to students that spoke both

English and these ELLs' native language. Thus, these teachers exposed their ELL students to *classroom arrangements* facilitating *peer or group discussions* in mathematics in both English and their native language (i.e., utilization of strategy 19). On the other hand, Mr. Daniels in Lincoln High School, who also spoke only English, despite not pairing his students in groups (they worked individually on computers), was observed to utilize other strategies from *TTT Form 1*: strategy 12 (using different *questioning techniques* sensitive to the ELLs' levels of SLA), 1 (*adapting his speech to the level of ELLs present*), and 14 (*providing feedback*). He also often performed *comprehension checks* (strategy 7) to see if his ELL students understood him, and *used gestures, facial expressions, eye contact or demonstrations to enhance comprehension* (strategy 15). Strategies 12, 14, and 1 were used often by Ms. Andersen (Lincoln High School) as well, especially when addressing the ELLs who did not speak French (the language she sometimes used to improve the communication with two of her ELLs). To be better understood by her ELLs who did not share her linguistic background, she used wait-time techniques after posing a question (strategy 13). Ms. Chandler (Green Bay High School), who also spoke only English and her ELLs did not work in groups because they worked individually on computers (as in Mr. Daniel's class), also often used strategies 14 and 1, but she also often *used clarifications of directions and individual assistance when her ELLs were executing specific mathematical tasks* on the computers (strategy 6) and was *using the technology* (strategy 18) to enhance her ELLs' comprehension (just as Mr. Daniels did).

The teachers who spoke the language of their ELLs - Ms. Barrera (from Green Bay High Schhol) and Ms. Cortez (from Lincoln High School) - with 9 and 4 ELLs respectively, both also used most frequently strategies 12, 14, and 7. Additionally, Ms. Cortez also had French-speaking ELLs, besides the Spanish ELLs with whom she shared similar cultural and linguistic background. She also seated her ELLs in close proximity to other students or teacher assistants who shared their linguistic background and also spoke English fluently.

Question 2

In analyzing the data to answer question 2 – i.e., to what extent do mathematics teachers' experiences and teachers' ESOL endorsement relate to their patterns of discourse when teaching mathematics to classes with ELL students present – the following findings emerged. As the combined data in Table 2 demonstrates, the teachers with more years of teaching experience and having an ESOL endorsement for a long period of time had a smaller number of ELLs present in their classes (Mr. Able and Mr. Davison in Green Bay High School, and Ms. Andersen and Mr. Daniels in Lincoln High School). Moreover, in both schools, the teachers who had just begun their teaching careers and just completed or were in the process of completing their ESOL requirement (Ms. Barrera in Green Bay High School and Ms. Brown in Lincoln High School) were assigned to teach classes with the highest number of ELLs. In relation to what extent the teachers' experiences and ESOL endorsement related to their patterns of discourse when teaching mathematics to classes with ELL students present, the following patterns emerged: combined data from the frequency count of the strategies used during the 20-minute recorded sessions (see Figure 17) revealed that the teachers who just started their teaching careers and lacked practical experience of teaching Algebra I to classes with diverse student populations involving a high number of ELLs (Ms. Brown and Ms.

Barrera) frequently used almost the same strategies as their more experienced colleagues did.

Yet additional qualitative analysis of the *type of modifications to their speech they made* (strategy 1a to d), of the *type of questions they asked* (strategy 12a to d), and the *provision of information of higher cognitive demand according to Bloom's Taxonomy* (strategy 22a to f) indicated that even though all teachers generally needed improvement in using these strategies, the more experienced teachers (such as Mr. Able and Mr. Davison from Green Bay High School, and Ms. Andersen and Ms. Daniels from Lincoln High School) who had completed their ESOL endorsement's requirement a long time prior to the observations were applying those strategies to a fuller extent. That is, at the least, they more often utilized strategies 1 and 12 c, if not d; and 22 c, and d, if not f. Evidence to support this claim was provided when analyzing the cases of Mr. Able, Mr. Davison, Ms. Andersen, and Mr. Daniels.

For example, Mr. Able's questions "What would you have to do to get b by itself?" or "How do we now graph this equation?" indicated that his discourse too often took the form of a challenge. However, he readily provided assistance in subsequent steps and thus missed opportunities to move the discourse to the higher levels of cognitive demand such as *synthesis* and *evaluation*, as per Bloom's Taxonomy. As another example, Mr. Daniels, after modeling the solution of a mathematical problem (strategy 1b, as demonstrated in the excerpt from his case study), often encouraged his ELL students to try to explain their solutions in English: "Ok, so you talk me through the next one" or "Ok, so try this one." This indicated that he challenged his ELL students to move to operations of higher cognitive demand according to Bloom's taxonomy –

application, analysis ("…just kind of sketch the points and you'll see…Which scatter plot represents a non-linear relationship?"), and *synthesis* ("predict best price estimate" or "Now stop for a second. What happens if they ask for one that's between 2 and 4? What would we expect the value at 3 to be?" – exemplifying strategies 22d and 22e). Furthermore, with the *use of such different types of questions* (strategies 12b to 12d), Mr. Daniels encouraged his ELLs to expand their fluency in both the English language and mathematics, and participate in the teacher-student discourse by explaining the operations they performed. However, he still did not ask them to justify and perform more critical analyses or to provide further explanations of more complicated steps while problem solving (strategy 22f—*Move the discourse to the highest level of cognitive demand according to Bloom's taxonomy*). Moving the mathematics discussions to higher levels of cognitive demand (i.e., *analysis, synthesis,* and *evaluation*) on Bloom's taxonomy creates more opportunities for all students (and ELLs in particular) to become critical mathematics thinkers.

However, despite the fact that the teachers with more teaching experience and ESOL endorsement such as Mr. Able, Mr. Daniels, Ms. Andersen, and Mr. Davison created opportunities for their ELL students to participate in the mathematics discourse, they still did not ask enough questions which could provide the ELLs with opportunities to justify and explain their opinions and, consequently, expand on their learning of mathematics and English. They still rarely lead the discussions to a point which could move the ELLs to the highest level of the subject-specific literacy – *intermediate speech* and fluency in mathematics in English.

Moreover, it was observed that novice teachers (such as Ms. Brown in Lincoln High School and Ms. Barrera in Green Bay High School) often had problems maintaining discipline in their classrooms, and they confirmed this in their interviews (for example, Ms. Barrera in Green Bay High School). As a result, the teachers were switching to discourse that fostered primarily teacher-centered activities and avoided "hands-on" activities, or scaffolding activities involving group discussions. Even though case study 6 revealed how Ms. Brown used an activity to teach the concept of scatter plot and data correlation, such instances were scarce and generally avoided by the novice teachers because they had problems with their students' behavior and maintaining the focus of the discussions on the mathematical task at hand (as was observed in the next two sessions in Ms. Brown's class).

Question 3

In reference to how teachers' own linguistic and cultural backgrounds affect their patterns of discourse when teaching mathematics in English to classes with ELL students present, the following findings are of particular relevance. Even though in general it is beneficial for the teachers to have a similar linguistic or cultural background as their ELLs (as in the case of Ms. Barrera in School 1 and Ms. Cortez in School 2), this is not a determining factor for successfully involving their ELLs in the classroom discourse (Cahnmann and Remillard, 2002). Research in the field of teaching mathematics to ELLs indicates that more essential factors in involving *all* students and fostering their active interest and learning would be to incorporate culturally responsive instruction by utilizing their own backgrounds and culture to best suit the specific needs of their students (Cahnmann, & Remillard, 2002; Kersaint, Thompson, & Petkova, 2009, p. 65). For example, Ms. Andersen demonstrated good rapport with her ELLs that was achieved through her own education and teacher development classes. She took advantage of the fact that she speaks French (she has a minor in French) and thus engaged her Haitian students in the classroom discourse more directly, for example. In the interviews, she indicated that she "is still learning to incorporate more strategies" in her teaching practices, and that this is an ongoing process for her.

The teachers who shared the cultural and linguistic background of the majority of their ELLs - Ms. Barrera with 9, and Ms. Cortez, with 3 Spanish-speaking students (out of 4 ELLs) - both also utilized most frequently strategies 12, 14, 7, and 4 from amongst the teachers in the sample (see Figure 17). Additionally, Ms. Cortez, who had one French-speaking ELL besides the Spanish ELLs, also seated her ELLs in close proximity to others students (who spoke French) or teacher assistants who shared their linguistic background and also spoke English fluently. However, both teachers exhibited the same lack of providing opportunities for ELL students to share experiences and build on personal cultural specific knowledge while problem solving (i.e., lack of implementing strategy 21) as did the rest of the teachers in the sample (see Figure 17).

Question 4

In investigating what patterns of discourse the teachers used when ELL students were present in the mathematics classroom, and what adjustments to *teacher talk* or modifications of instructions the teachers made, the following findings emerged. For example, Figure 17 shows that the most frequently used strategies by all teachers (with minor variations) were: 12 (*use of different questioning techniques, sensitive to the ELLs' level of SLA*), 14 (*provide feedback*), 1 (*use of slower and simpler speech*), 4 (*use of* *repetitions or paraphrasing when important mathematics concepts are formulated*), and 7 (*use of comprehension checks*). This finding is well grounded and is based on all the analysis of triangulated data.

However, additional qualitative analysis revealed that most of the questions the teachers asked their ELLs were of a type that required usually one-word or a short list of words in response, or were yes/no or either/or questions. Further analysis also revealed that the most attempts to move the questioning techniques to a higher level were made by Mr. Daniels, Ms. Cortez, and Mr. Davison with questions such as “Why?”, “What do you recommend?”, or questions that elicited their ELLs to expand not only their literacy in mathematics but also to develop to *speech emergence* and *intermediate speech* in English language. Throughout the observations, most teachers, after receiving responses to their questions, usually *provided students with feedback* (strategy 14) in most cases indicating whether the answer was correct or not. The teachers also adapted their speech to their audience and, being aware that there are ELLs in the classroom, *used simple commands and shorter sentences, and modeled the correct responses both in mathematics and English* (strategy 1 a, b, and c). However, the qualitative analysis of their *teacher talk* also revealed that on more rare occasions when presenting a new concept, the teachers used advanced organizers and at the same time used their talk to lead the students to small group work or “hands-on” activities.

Further, Figure 17 reveals that the teachers least frequently used strategies 10 (*Conclude a lesson with a summary of the key concepts*), 2 (*Use of fewer idioms and slang words*), 21 (*Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge*), and 9 (*Start a lesson with a review of a related* *concept*). However, while the chart in Figure 17 reveals the frequency count of strategies that were utilized only during the 20 minutes of the observed classroom sessions, the other sources of data collection (e.g., ELL and teacher interviews, and the researcher's observations and field notes throughout the entire classroom session) revealed that strategy 9, and to some degree strategy 10, were in fact also more frequently utilized by some of the teachers.

For example, Mr. Able and Ms. Barrera in Green Bay High School, and Ms. Andersen and Ms. Cortez in Lincoln High School, traditionally used bell-work in which they included review questions of previously learned concepts and thus employed strategy 9. Additionally, the classroom observations for the duration of the entire sessions, as well as the interviews with the teachers and their ELLs also confirmed that some of the teachers conclude the lessons with a summary of the important concepts the students just learned. For example, Ms. Barrera and Mr. Davison in Green Bay High School, and Ms. Andersen and Ms. Cortez in Lincoln High School, were evaluated by their ELLs as using strategy 10 at least a couple of times a week. However, all data collected from different sources (observations, video-recordings and frequency counts, interviews and evaluations of the researcher, teachers' self-evaluations, and ELLs' evaluations) revealed a consistent lack of use of strategy 21 (*Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge*).

CHAPTER V: DISCUSSION AND CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS

The purpose of this study was to examine the discourse created between a teacher and students in eight mathematics classrooms with ELLs present. The study was an attempt to shed light on current practices established in these classrooms and some of the areas where improvements need to be made to increase the mathematics learning potential of ELL students. Furthermore, this research aimed to provide some information about the impact of students' and teachers' cultural and linguistic backgrounds upon students' experiences in learning mathematics.

The participants of the study were eight teachers and their mathematics classes from two urban U.S. public high schools in the Southeast, with diverse student populations with ELLs from various backgrounds.

Discussion and Conclusions

In analyzing the data to answer question 1, the results of this study indicated that the teachers changed their patterns of discourse due to the mere presence of ELL student(s) in the classroom, irrelevant of the number of such students present. These observations are consistent with Rhine (1995a, 1995b, 1999) and Davidenko's (2000) findings, who also reported that teachers tend to teach differently when ELLs are present in a group. However, Rhine further reported that teachers often linked the lack of English proficiency to a similar lack of mathematical knowledge or understanding, and they tended to underestimate the ELLs' performance. Such teachers' behavior Rhine related to teachers' limited understanding of ELLs' mathematics learning. Davidenko also reported that the teachers often assumed that the ELL students could not handle word problems or discussions in mathematics in English because of their limited proficiency in English. Thus, according to Davidenko, the teachers tended to reinforce computational skills and instrumental learning (learning experiences involving reinforcement of good behavior). In this study, teachers were observed whose expectations of their ELLs were both similar and different, in some respects, from those reported in Rhine and Davidenko's study. Some commented on the limited English abilities of their students and related that to similarly limited mathematical abilities, while other teachers clearly stated that while their ELL students might not be very fluent in English yet, they are very motivated students and have good prior knowledge in mathematics.

In analyzing the data to answer question 2, i.e., to understand to what extent mathematics teachers' experiences and ESOL endorsement relate to their patterns of discourse when teaching mathematics to classes with ELL students present, inconclusive results were observed. More specifically, the results of this study did not establish an "optimal" learning experience for the ELLs in the classes of teachers with the most years of teaching experience or having an ESOL endorsement for a longer time. Actually, data from the frequency count of the strategies used during the 20-minute recorded sessions (refer to Figure 17) revealed that the novice teachers frequently used almost the same strategies as their more experienced colleagues did—more specifically, Figure 17 indicates that the teachers (with slight differences) utilized most often strategies 12 (*use* *of different questioning techniques*), 14 (*provide feedback*), and 1 (*use of slower and simpler speech*).

However, additional qualitative analysis of the *type of modifications to their speech* (strategy 1a to d), of the *type of questions they asked* (strategy 12a to d), and the *provision of information of higher cognitive demand according to Bloom's Taxonomy* (strategy 22a to f) indicated that even though all teachers generally needed improvement in using these strategies, the more experienced teachers who also had completed their ESOL endorsement's requirement a long time prior to the observations were applying those strategies to a fuller extent. On the other hand, despite the fact that they created opportunities for their ELL students to participate in the mathematics discourse, they still did not ask enough questions which could provide the ELLs with opportunities to justify and explain their conclusions and, consequently, expand on their learning of mathematics and English.

These observations seem to be consistent with the observations in other studies investigating classroom discourse (Blanton, Berenson, & Norwood, 2001; Brenderfur, & Frukholm, 2000; Nathan, & Knuth, 2003; Renne, 1996). For example, in Brenderfur and Frukholm's study the two teachers subject to investigation were similar in age, attended the same mathematics methods class that promoted discourse, but when assigned to teach in the same school employed different teaching practices—one encouraged communication while the other used a teacher-centered approach. In Renne's (1996) study, the teacher initially attempted to shift the discussions towards one that is more student-centered and to incorporate students' questions and initiatives. However, the teacher often converted the student initiatives to teacher initiatives and consequently detoured the communications to the traditional initiation-reply-evaluation (IRE) sequence wherein the teacher initiates (with a question or statement), a student responds, and the teacher evaluates the students' response (verbally or by a gesture). Further investigations in both studies revealed that the observed differences in teaching patterns could be attributed to the teachers' initial beliefs and disposition toward mathematics and its teaching and learning.

On the other hand, Nathan and Knuth (2003), who analyzed one teacher's patterns of discourse on a more general level over a period of two school years, reported the following observed change. During the first year, the teacher facilitated teacher-central interactions, but during the second year the teacher's authority was less evident, and "a star pattern" emerged. Blanton et al.'s (2001) study contributes to the notion that "a teacher's developing practice is inherently linked to the social dynamics of the classroom" (p. 228). However, as Renne (1996) also indicated, a lack of details about how to implement discussions, time constraints to complete the course or prepare the students for standardized state tests, the number of students in the class, and the struggle for maintaining discipline were also found to be influential factors in the observed teacher's behaviors. The multiple factors presented in these studies offer a glimpse as to why a direct correlation between the teachers' patterns of discourse and their years of teaching experience and years from completion of their ESOL endorsement was not found in this study.

In analyzing the data to answer question 3, i.e. how teachers' own linguistic and cultural backgrounds affect their patterns of discourse when teaching mathematics in English to classes with ELL students, the results of this study are consistent with those of

Cahnmann and Remillard’s (2002) study. In this study, the researchers indicated that even though it might be beneficial for the teachers to have a similar cultural or linguistic background to that of their students, this is not a decisive factor in providing equal mathematics experiences to *all* students. Their study also indicated that all mathematics teachers could use some ideas from research and incorporate culturally relevant instruction in mathematics to diverse student populations. The eight teachers in this study did not utilize strategy 21 (i.e., provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem-solving in mathematics and thus build cross-cultural knowledge). Data from the frequency count of the strategies used during the 20-minute recorded sessions (refer to Figure 17) also reveals that some teachers utilized relatively more frequently strategy 22 (i.e., provided their students with content specific, enriched information, thus exhibiting equally high expectations from ELL and non-ELL students).

In analyzing the data to answer question 4 — what patterns of discourse teachers use when ELL students are present in the mathematics classroom and/or what adjustments to *teacher talk* or modifications of instructions are observed, the present study has reported that besides above-discussed frequent use of strategies 12 (*use of different questioning techniques*), 14 (*provide feedback*), and 1 (*use of a slower and simpler speech*), the next strategies more often utilized by the eight teachers in the sample (with small exceptions) were strategies 4 (*use of repetitions or paraphrasing of teachers’ or students’ statements*), 7 (*performing comprehension checks throughout the lesson*), and 6 (*use of clarification of directions or assistance in executing a mathematical task*) (See Figure 17). The reported findings are consistent with those of Long (1981, 1983), who also found that native speakers (NS) do use more modifications to the input when they interact with nonnative speakers (NNS), as opposed to when they interact with native speakers. Such modifications, according to Long's studies, include more frequent use of self- and other-repetitions, slower speech patterns, comprehension and confirmation checks, and explanations. According to Long, the purpose of such modifications is to improve the dialog and repair the discourse when troubles in conversations have already occurred. Research in the field of mathematics classroom discourse indicates that teachers improved ELLs' participation in discussions by using "revoicing" (reformulation of students' statements using formal mathematical terms) and by asking the students to paraphrase their statements in order to clarify their meanings (Moschkovich 1999, 2002), or by facilitating a computer-based dynamic instructional environment in which small-group discussions are encouraged (Brenner 1998; and Moschkovich, 2002). Although six of the teachers from the sample frequently utilized the strategy of "revoicing" (in the study referred as strategy 4), and three of the teachers taught Algebra I employing computer-assisted instruction, most of the teachers rarely used small group work. Only two or three of the teachers (Refer to the cases of Mr. Davison, Ms. Cortez, and occasionally Ms. Brown) more often exposed their students to classroom arrangements that *facilitated small group work or partner discussions* (strategy 19).

Research in the field of teaching ELLs pointed out the importance of "understanding students' cultural perspectives and backgrounds [because it] might provide insights about behaviors and reactions to instructional approaches" (Kersaint, Thompson, & Petkova, 2008, p. 64). Furthermore, research indicated that "when students experience the mathematics in a classroom as not relating to them or their culture, they might feel invisible and unconnected with the content" (Davidson & Kramer, 1997, p. 139). Moreover,

> knowing what a student knows about a topic also helps teachers deal with misconceptions. Frequently students have incorrect background knowledge that can become a powerful impediment to learning. Eliciting students' prior knowledge about a topic helps bring to light misunderstandings, simplistic knowledge, or flawed interpretations. Once brought to light, we can help students repair misconceptions with accurate information. (Santa, Havens, & Valdes, 2004, p. 7)

Thus, in light of previous research, this study furnishes key insights into what improvements in the current teaching practices could be implemented in order to encourage ELL students to become active learners and participants in mathematics classroom discourse by illuminating, for example, that in practice many teachers do not p*rovide enough opportunities for students to share experiences and build up on personal or cultural-specific knowledge* (i.e. lack of utilizing strategy 21).

Research (Goodell, & Parker, 2001) also pointed out that in order for ELLs to construct their own knowledge in both English and mathematics "the teacher must be the facilitator, helping students to construct their own knowledge by establishing learning situations in which this is possible, for example, through the use of hands-on manipulatives, whole-class discussion, group discussion, or presentation of project work" (p. 419). Research (Campbell & Rowan, 1997) also indicated that in order for ELL students to move to a more advanced level of English language fluency (*speech* *emergence* or *intermediate speech*) and cognitive development, they need to be asked more often higher order questions and thus become more equal partners in the classroom discourse. Santa, Haves, and Valdes (2004) visualized very graphically the following situation:

> Most of us will remember how it feels to be a student in a classroom dominated by teacher talk and interrogation. The teacher asks the questions. One-by-one, students reel off answers until someone hits the correct one. The teacher remains the sole evaluator and controller of comprehension. Gazden (1988) calls this model of discourse IRE: the teacher initiates (I) talk by asking a question; a student responds (R); and the teacher evaluates (E) the response. (p. 55)

This study reveals that even though the teachers often asked their ELL students questions and thus involved them in classroom discussions, they did not utilize the full range of questioning techniques available. Most of the questions that they asked were “yes/no”, “either/or” questions or “required one word or list of word responses” (strategy 12a and b). Teachers were found to not p*rovide enough opportunities for students to enhance both their linguistic and mathematics development by being asked to categorize, predict, explain, justify, or criticize approaches to solving mathematical problems* (i.e., lack of use of strategies 12c and d, and 22e and f) and *did not expose the students to different classroom arrangements such as using group discussions, and hands on activities* (i.e., not very often utilizing strategy 19).

Research in classroom discourse (Santa, Havens, & Valdes, 2004; Tomlinson, 2001) underscores the necessity for creating a mathematical classroom environment in which *student talk* (including ELL students’ talk), rather than *teacher talk* becomes central. Such student-centered discussions enhance comprehension, facilitate higher-level thinking and problem solving, and improve communication skills (Santa, Havens, & Valdes, 2004). Furthermore, based on previous research and current findings, the study indicates that whereas some ELL students can be challenged by what seems to be a "simple" question according to a non-ELL teacher, "all students need to be accountable for information and thinking at high levels" (Tomlinson, 2001, p. 104) and be asked various types of questions. Teachers can vary their questions to ensure that they are more open-ended and require explanations and justifications of answers. By encouraging the students to build upon one another's answers and varying their questions appropriately, teachers can "nurture motivation though success" (Tomlinson, 2001, p. 104) and in turn become more successful in accommodating their ELL students in the mathematical classroom discourse.

Limitations

There are two limitations to this study. First, the small sample of high school teachers/participants in the study elicited a qualitative non-relational analysis of the collected data and prohibited the use of significance tests such as chi-square. In effect, the generalizations from this study are limited in scope.

However, as Wood and Kroger (2000) pointed out, "because the focus of discourse analysis is language use rather than language users, the critical issue concerns the size of the sample of discourse (rather than the number of people) to be analyzed" (p. 80). In this study the "discursive moves" of eight teachers were analyzed during three 20-minute video-recorded classroom sessions, which actually amasses to analyzing very large samples of discourse during the total of 24 video-recorded sessions. As a result, this study involved the analysis of much larger samples of language use than the sample of language users might otherwise indicate. Thus, as Wood and Kroger state "[t]he most likely problem for the analysis is that the sample is too large rather than too small" (p. 80). They continue: "the question about number comes down to having sufficient number of arguments of sufficient quality and having sufficient data for those arguments to be well grounded" (p. 81). Therefore, by providing thick descriptions of each case study and thus giving the reader opportunities to judge for him/her-self, the study satisfies its main aim: to shed light on current practices established in the mathematics classrooms under scrutiny, and it illuminates the areas where improvements need to be made.

However, even though this study provides thick descriptions of the data collection and analysis procedures, its claims are still subject to the facets described by Lincoln and Guba:

> While generalizations are constrained by facts (especially if the facts are the particulars from which the generalization is induced), there is no single necessary generalization that *must* emerge to account for them. There are always (logically) multiple possible generalizations to account for any set of particulars, however extensive and inclusive they might be. (p. 114)

This leads to noting the second limitation of this research. As Lincoln and Guba (1985) pointed out, "naturalistic inquiry operates as an *open* system; no amount of member checking, triangulation, persistent observation, auditing, or whatever can ever compel; it can at best *persuade*" (p. 329). Thus, this study's criteria for trustworthiness are also open-ended.

However, by abiding to the five major techniques proposed by Lincoln and Guba (1985, p. 301) the study is made more *persuasive.* More specifically, this study is developed, carried out, and described with consistently taking into account the naturalistic inquiry's criteria for trustworthiness, as expressed in Guba's new terms: *credibility* (as an alternative to *internal validity*), *transferability* (as an alternative to *external validity*), *dependability* (as an alternative of *reliability*), *and confirmability* (as an alternative of *objectivity*). (For more details of how exactly the criteria for trustworthiness were satisfied, refer to the end of the methodology section of this study – Chapter III)

Recommendations for Further Research

This study furnished valuable insights into the classroom discourse and *teacher talk* influences on ELLs' mathematics experiences. The findings lead to further questions that future research can seek the answers to:

1. What changes (if any) in the patterns of mathematics teacher's discourse would be observed if the study is carried over longer periods of time and with a larger teacher and ELL samples?
2. What effects do the cultural and linguistic backgrounds of the ELLs present in the mathematical classroom have on the teacher's choice of adopted discursive strategies (i.e., do teachers adopt strategies that differ in accordance with the ELLs' backgrounds)?
3. Which teaching strategies are most effective in teaching mathematics to ELLs from specific cultural and linguistic backgrounds?

4. What are the effects of long-term intervention programs offered by teacher development programs to aid teachers in teaching mathematics to classes where ELL students are present?
5. Would the results be different with a different age group sample (such as elementary and middle school students)?

This inquiry and any future research as suggested here could contribute to the collected knowledge in the field of teaching mathematics to diverse classrooms with many ELLs present – as is the current and emerging situation in U.S classrooms. Findings from such research and recommendations for improvement can directly assist decision-makers to implement the necessary changes through criteria changes for teachers' certification programs and/or improving opportunities for teacher education and teacher development programs. Furthermore, the interventions suggested by such research can be elaborated in the daily practices of the mathematics teachers and can help them function effectively in diverse mathematics classroom settings, particularly when ELL students are involved.

REFERENCES

Abedi, J, & Lord, C. (2001). The language factor in mathematics test. *Applied Measurement in Education*, 14(3), 219-34.

American Psychological Association. (2001). Publication manual of the American Psychological Association. Washington, DC: Author.

Bakhtin, M. M. (1986). *Speech genres and other late essays.* Austin, TX: The University of Texas Press.

Ben-Yehuda, M., Lavy, I., Linchevski, L, & Sfard, A. (2005). Doing wrong with words: What bars students' access to arithmetical discourses. *Journal for Research in Mathematics education, 36(3),* 176-247.

Bills, L. (1999). Students talking: an analysis of how students convey attitude in maths talk. *Educational Review, 51,* 161-171.

Blanton, M. L., Berenson, S. B., & Norwood, K. S. (2001). Using classroom discourse to understand a prospective mathematics teacher's developing practice. *Teaching and Teacher Education, 17,* 227-242.

Blanton, M. (2002). Using an undergraduate geometry course to challenge pre-service teachers' notions of discourse. *Journal of Mathematics Teacher Education, 5,* 117-152.

Blunk, M. L. (1998). Teacher talk about how to talk in small groups. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 190-212). Cambridge, UK: Cambridge University Press.

Bradby, D. (1992). *Language characteristics and academics achievement: A look at Asian and Hispanic eight graders in NELS:88.* Washington, DC: U. S. Department of Education.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3,*125-153.

Brenner, M. E. (1994). A communication framework for mathematics: Exemplary instruction for culturally and linguistically different students. In B. McLeold (Ed.), *Language and learning: Educating linguistically diverse students* (pp.233-267). Albany: SUNY Press.

Brenner, M. E. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal, 22,* 149-174.

Campbell, P. F, & Rowan, T. E. (1997). Teacher questions + student language + diversity = mathematical power. In J. Trentacosta, (1997 Yearbook Ed.) & M. J. Kenney (General Yearbook Ed.), Multicultural and gender equity in the mathematics classroom. The gift of diversity. 1997 yearbook (pp. 60-70). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Cocking, R. R., & Chipman, S. (1988). Conceptual issues related to mathematics achievement of language minority children. In R. R. Cocking, & J. P. Mestre (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 17-46). Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Davidenko, S. (2000). Learning mathematics in English: ESL and non-ESL students' perspectives. *Dissertation Abstracts International.* (UMI No. 9977964)

Davidson, E. & Kramer, L. (1997). Integrating with integrity curriculum, instruction, and culture in the mathematics classroom. In J. Trentacosta, (1997 Yearbook Ed.) & M. J. Kenney (General Yearbook Ed.), *Multicultural and gender equity in the mathematics classroom. The gift of diversity. 1997 yearbook* (pp. 131-141). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Ellis, R. (1994). The study of second language acquisition. Oxford, MA: Oxford University Press.

Forman, E. & Ansell, E. (2001). The multiple voices of a mathematics classroom community. *Educational Studies in Mathematics, 46,* 115-142.

Forman, E. & Ansell, E. (2002). Orchestrating the multiple voices and inscriptions of a mathematics classroom. *Journal of Learning Sciences, 11(2&3),* 251-274.

Gee, J. P. (2005). *An introduction to discourse analysis: Theory and method* (2nd ed.). New York: Routledge.

Glen Commission (2000). *Before it's too late: A report to the nation from the national commission on mathematics and science teaching for the 21st century.* Washington, DC: U. S. Department of Education.

Goodell, J. E., & Parker, L. H. (2001). Creating a connected, equitable mathematics classroom: Facilitating gender equity. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 411-431). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Gronna, S., Chin-Chance, S., & Abedi, J. (2000). *Differences between the performances of limited English proficient students and students who are labeled proficient in English of different content areas: reading and mathematics.* New Orleans, LA: American Educational Research Association. (ERIC Document Reproduction Service No. ED440551)

Halpern, N., Patkowski, M., & Brooks, E. (1996). Mathematics and the ESL student. (ERIC Document Reproduction Service No. ED436100)

Hicks, D. (1998). Closing reflections on mathematical talk and mathematics teaching. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 241-252). Cambridge, UK: Cambridge University Press.

Jacobson, C., & Lehrer, R. (2000). Teacher appropriation and student learning of geometry through design. *Journal for Research in Mathematics Education, 31(1),* 71-88.

Johnson, M. (2004). *A philosophy of second language acquisition.* New Haven, MA: Yale University Press.

Kovalainen, M., Kumpulainen, K., & Vasama, S. (2001). Orchestrating classroom interaction a community of inquiry: Models of teacher participation. *Journal of Classroom Interaction, 36(2),* 17-28.

Krussel, L., Edwards, B., & Springer, G. T. (2004). Teacher's discourse moves: A framework for analyzing discourse in mathematics classrooms. *School Science and Mathematics, 104,* 307-312.

Lane, Silver, & Wang (1995). An examination of the performance gains of culturally and linguistically diverse students on a mathematics performance within the Quasar Project. (ERIC Document Reproduction Service No. ED390927)

Leap, W. H. (1988). Assumptions and Strategies guiding mathematics problem solving by Ute Indian Students. In R. R. Cocking, & J. P. Mestre (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 161-186). Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Leonard, J. (2000). Let's talk about the weather. *Mathematics Teaching in the Middle School, 5(8),* 518-523.

Liberty, P. (1998). *Title VII REFORMS: Rethinking education for minority students. Evaluation report 1997-98* (Publication No. 97.19.). Austin, TX: Office of Program Evaluation. (ERIC Document Reproduction Service No. ED428104)

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Newbury Park, CA: Sage Publications, Inc.

Liu, K., Anderson, M., & Thurlow, M. (2000). *Report on the participation and performance of limited English proficient students on Minesota's basic standards tests, 1999.* St. Paul, MI: Minnesota State Department of Children, Families, and Learning. (ERIC Document Reproduction Service No. ED447718)

MacCorquodale, P. (1988). Mexican-American women and mathematics: Participation, aspirations, and achievement. In R. R. Cocking & J. P. Mestre (Eds.), *Linguistic and Cultural differences on learning mathematics* (pp. 1137-160). Hillsdate, NJ: Erlbaum.

Maouchehri, A., & Enderson, M. C. (1999). Promoting mathematical discourse: Learning from classroom examples. *Mathematics Teaching in the Middle School, 4,* 216-222.

Mattews, L. E. (2003). Babies overboard! The complexities of incorporating culturally relevant teaching into mathematics education. *Educational Studies in Mathematics, 53,* 61-82.

McClain, K., & Cobb, P. (1998). The role of imagery and discourse in supporting students' mathematical development. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 56-81). Cambridge, UK: Cambridge University Press.

McNair, R. E. (1998). Building a context for mathematical discussion. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 82-106). Cambridge, UK: Cambridge University Press.

McNair, R. E. (2000). Working in the mathematics frame: maximizing the potential to learn from students' classroom discussions. *Educational Studies in Mathematics*, *42*, 197-209.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematics discussions. *For the Learning of Mathematics, 19(1),* 11-19.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4(2&3),* 189-212.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991) *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principle and standards for school mathematics.* Reston, VA: Author.

Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21(2),* 175-207.

Patrick, H., Turner, J. C., Meyer, D. C., & Midgley, C. (2003). How teachers establish psychological environments during the first days of school: Association with avoidance in mathematics. *Teachers College Record, 105(8),* 1521-1558.

Renne, C. G. (1996, Spring). Structuring classroom lessons: Attempts to incorporate student questions and initiatives during math lessons. *Teacher Education Quarterly*, 5-18.

Rhine, S. (1995a). *Students' language proficiency effects upon teachers' assessment of students' mathematical understanding.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco: CA.

Rhine, S. (1995b). The challenge of effectively preparing teachers of limited-English-proficient students. *Journal of Teacher Education*, 46(5), 381-389.

Rhine, S. (1999). *Mathematics reform, language proficiency, and teachers' assessment of students' understanding.* OR: Willamette University. (ERIC Document Reproduction Service No.ED429121)

Rittenhouse, P. S. (1998). The teacher's role in mathematics conversation: Stepping in and stepping out. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school:Studies of teaching and learning* (pp. 163-189). Cambridge, UK: Cambridge University Press.

Secada, W. G., & Carey, D. (1990). *Teaching mathematics with understanding to limited English proficient students* (Urban Diversity Series N. 101). New York: ERIC Clearinghouse on Urban Education. (ERIC Document Reproduction Service No. ED322284)

Sfard, A. (2002). The interplay of intimations and implementations: Generating new discourse with new symbolic tools. *The Journal of the Learning Sciences*, 11, 319-357.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in mathematics classroom. *Journal of Mathematics Teacher Education, 5*, 205-233.

Steele, D. F. (1999-2000, Winter). Observing 4th-grade students as they develop algebraic reasoning through discourse. *Childhood Education, 76(2)*, 92-96.

Stevens, J. P. (1999). *Intermediate statistics: A modern approach* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Tsang, S-L. (1988). The mathematics achievement characteristics of Asian-American Students. In R. R. Cocking, & J. P. Mestre (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 123-136). Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Turner, J. C., Meyer, D. K., Midley, C., & Patrick, H. (2003). Teacher discourse and sixth graders' reported affect and achievement behaviors in two high-mastery/ high-performance mathematics classrooms. *The Elementary School Journal, 103(4),* 357-389.

U. S. Department of Education (2001). No Child Left Behind Act of 2001. Retrieved January 17, 2003, from http://www.ed.gov/policy/elsec/leg/esea02/index.html.

U. S. Census Bureau American Community Survey (2004). Washington, D. C.: U. S. Bureau of the Census. Retrieved April 30, 2006 from http://factfinder.census.gov.

Ventriglia, L. (1982). *Conversations of Miguel and María.* Reading: Addison Wesley.

Vygotsky, L. S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Weingrad, P. (1998). Teaching and learning politeness for mathematical argument in school. In Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 213-240). Cambridge, UK: Cambridge University Press.

Wood, L. A., & Kroger, R. O. (2000). *Doing discourse analysis. Methods for studying action in talk and text.* Thousand Oaks, CA: Sage Publications, Inc.

Zack, V. (1999). Everyday and mathematical language in children's argumentation about proof. *Educational Review, 51(2),* 129-146.

APPENDICES

Appendix A: Teacher Talk Test (TTT) Form 1

Strategies	Sample statements	Frequency	Total	Sample teacher statements
I. "Vocal" Strategies:				
1. Use of a slower and simpler speech—shorter sentences (caregiver speech) to adapt her/his speech to the appropriate level of ELL students present (pre-production, early production, speech emergence, and intermediate fluency):				
a) Pre-production	Since this is so- called "silence period," the teacher should use simplified speech, i.e. simple commands and shorter sentences when explaining things.			
b) Early Production	At this stage the students are just beginning to experiment with the language, and thus at this stage it is inappropriate to correct errors in grammar and pronunciation. Teachers need to model/demonstrate correct responses both in mathematics and English.			
c) Speech Emergence	At this stage teachers should begin the presentation of new concepts using advance organizers and at the same time focus the *teacher-talk* on key concepts and use their talk to lead the students to small group work and hands on activities			
d) Intermediate Fluency	The *teacher talk* should foster conceptual understanding and expanded literacy through content			
2. **Use of (fewer) idioms and slang words** from the mathematics vocabulary, or if used a proper explanation (or visual representation) is provided	Right-angled triangle (Unaware that the word *right* here refers to a particular type of triangle a student might think that there are left-angled triangles) absolute value GCD (greatest common divisor) If and only if			
3. **Use of synonyms** that can be used in the description of mathematical terms and that will help the student better understand the concept behind them	greater (bigger) less (smaller) addition (plus) Subtraction (minus) Congruent (equal)			
4. **Use of repetitions** or paraphrasing of his/her statements or asking students to repeat or restate them, especially when important concepts in mathematics are formulated	This figure is a parallelogram… The opposite sides in this figure are parallel …Juan, would you repeat, a parallelogram is what? In other words…			
5. **Use of change of tone, pitch, and modality** to convey better comprehension	***Change of pitch*** When a word or phrase that carries the greatest degree of stress in a sentence is said with increased loudness. ***Change of modality*** (speaking mode, grammatical mode, instructional mode)			
6. **Use of clarification of directions** and assistance when specific mathematical task or activity is posed for execution	Here is what you need to do… This is another way to do this…			
7. **Check for comprehension** throughout the lesson	Trung, do you understand what the next step is?			

Appendix A (Continued)

8. Identify subject specific and important **lesson vocabulary** and provide context embedded examples, pictures, or models.	Exponent Radical Etc.			
9. Start a lesson with a **review** of related concepts	Let's see what we have learned about…yesterday			
10. Conclude a lesson with a **summary** of the key concepts	Who would summarize …			
11. **Involve** students in mathematical discussions and problem solving	What do you think? What would you suggest? How do you know this is true? Tell me more about…? Consider this… Who would explain…? (or just call a student by name)			
II. Questioning Strategies:				
12. **Use different questioning** techniques, sensitive to the level of ESOL of the students, or their stages of Second Language Acqusition (as summarized by Linda Ventriglia (1982):				
a) Pre-production	point to…; find the…; is this a/an…; etc. Who wants the…?			
b) Early Production	yes/no questions (Is this a square?) Either/Or questions One-word response (What variable is this?) General questions that encourage lists of words (What signs of operations do we use?)			
c) Speech Emergence	Why? How? Tell me about…? Describe…			
d) Intermediate Speech	What do you recommend? What is your opinion....? What would happen if…? Compare/contrast How are these …similar or different? Create…			
13. Use **wait-time** techniques after posing a question	(measured in sec) Provide at least three seconds of thinking time			
14. Provide **feedback**	Well done; Hm-m; I see; I agree			
III. Enhancement to teacher talk's strategies:				
15. Use of gestures, facial expressions, eye contact, (at the same time showing awareness of their culture-specific appropriateness), or demonstrations to enhance comprehension	Gestures Facial expressions Eye contact Special proximity			
16. Use charts, graphic organizers—(draw)	Venn diagrams, tree diagrams, time lines, semantic maps, outlines, etc.			
17. Use of a variety of visual or auditory stimuli—(show)	Transparencies, pictures, flashcards, models, etc.			
[The following strategies might be lesson dependent] 18. Use of technology to enrich a concept presentation	calculators, computers, Internet, videos, overhead projectors, Power Point presentations, Mathematics application software—Geometers' Sketchpad, spread sheets, etc.			

Appendix A (Continued)

19. Expose students to different classroom work arrangements, such as using cooperative groups or partner discussions	Small group work Dyads (pair work and discussions) Collective discussions (scaffolding) Games			
20. Provide students with alternative forms of assessment—portfolios, vocabulary banks, oral presentations, writing or reading in mathematics, etc.	Portfolios Vocabulary Banks Oral Presentations Journal writing Research			
21. Provide opportunities for students to share experiences and build up on personal or cultural-specific knowledge while problem solving in mathematics and thus building cross-cultural knowledge	Tell me what you know about…			
22. Provide students with content specific, enriched information, thus exhibiting equally high expectations from LEP and non-LEP students.	Moves to the higher level of cognitive demand according to Blooms' Taxonomy:			
a) Knowledge *b) Comprehension* *c) Application* *d) Analysis* *e) Synthesis* *f) Evaluation*	Define, describe, match… Explain, give example, paraphrase… Modify, prepare, relate, … Distinguish, outline, identify… Categorize, predict, design … Justify, criticize, explain…			

Appendix B: Pre-observation Teacher Questionnaire

1. Name:____
2. Gender: M_____ F_____
3. Age:_____
4. Years of teaching experience:_________
5. Length of time teaching Algebra I :___________________________
6. Type of teaching certification: Temporary:____ Permanent: _______
7. Are you Math certified?
   a. Yes_____ What level? ____
   b. No _____
8. Have you ever had a Math Method course:
   a. Yes______ What type? ____
   b. No ______
9. ESOL endorsement:
   a. Yes______ Year of completion_________ What type? _________________________
      How was it obtained (coursework, inservice points, additional courses, etc.)
      ____
   b. No______
10. List how many ESOL students you have in each of your mathematics classes and their level of ESOL (if known). To what extent can you connect the students' level of ESOL with the stages in Second Language Acquisition —pre-production, early production, speech emergence, and intermediate fluency (See the list of definitions for each category):

ID	**Student Name**	**In which class/period**	**ESOL level (if known)**	**Stage in SLA**	**Comments**

11. Is English your native language:
    a. Yes______
    b. No______
    c. Native Language____
12. Do you speak any other languages other than your native language?
    a. Yes______
    If yes, specify which language(s) and grade your ability in each:
    Language: ___________ Reading: ___fluent ___ limited ___ not fluent
    Writing: ___fluent ___limited ___not fluent
    Speaking: ___fluent ___limited ___not fluent
    Language: ___________ Reading: ___fluent ___limited ___not fluent
    Writing: ___fluent ___limited ___not fluent
    Speaking: ___fluent ___limited ___not fluent
    b. No_______

Comment on any concerns you have in your experience with teaching mathematics to ESOL students; any positive or negative experiences; recommendations for improvement; etc. (Write on additional paper if needed)

____
____
____

Thank you for completing this questionnaire!

Appendix C: Post-observation Teacher Questionnaire

Teacher Talk Test (TTT) Form 2					
No:	**Strategies:**	Evaluate the extent to which you use the following strategies when ESOL students are in your classroom: **(use a checkmark)**			**How Often This Strategy is Used?—** Rate Using a Frequency Scale from 1 to 5, with 5 as most frequent 1—Never 2—Rarely (1 or 2 times a month) 3—Sometimes (1 or 2 times a week) 4—Usually (3 or 4 times a week) 5—Always
		Yes	**No**	**Needs Improvement**	
I.	**"Vocal" Strategies:**				
1.	Use of a slower and simpler speech				
2.	Use of fewer idioms and slang words				
3.	Use of synonyms				
4.	Use of repetitions or paraphrasing				
5.	Use of changes in tone, pitch, and modality				
6.	Use of clarification of directions				
7.	Comprehension checks				
8.	Identify subject-specific vocabulary and provide context-embedded examples, pictures, or models				
9.	Start a lesson with a review of related concepts				
10.	Conclude a lesson with a summary of the key concepts				
11.	Involve students in mathematical discussions and problem solving				

Appendix C (Continued)

II.	**Questioning Strategies:**				
12.	Use different questioning techniques that are sensitive to the level of ESOL of the students, or their stages of Second Language Acquisition				
	a) *pre-production*—point to…; find the…; is this a/an…; etc.				
	b) *early production*—yes/no questions; either/or questions; one-word or two-word responses; general questions that require a lengthy response;				
	c) *speech emergence*—Why? How? Tell me about…? Describe…;				
	d) *intermediate speech*—What do you recommend? What is your opinion....? What would happen if…? Compare/contrast…; Create…				
13.	Use wait-time techniques after posing a question				
14.	Provide feedback				
III.	**Enhancement to *teacher talk* strategies:**				
15.	Use of gestures, facial expressions, eye contact, or demonstrations				
16.	Use of charts, graphic organizers—Venn diagrams, tree diagrams, time lines, semantic maps, outlines, etc.				
17.	Use of a variety of visual or auditory stimuli: transparencies, pictures, flashcards, models, etc.				

18.	Use of technology				
19.	Expose students to different classroom work arrangements, such as using cooperative groups or partner discussions				
20.	Provide students with alternative forms of assessment				
21.	Provide opportunities for students to share experiences and expand on personal or cultural-specific knowledge while solving problems in math				
22.	Provide students with content specific, enriched information				

Please comment on why you chose to use the teaching practices that you identified.
____

Thank you for completing this questionnaire!

Appendix D: Questionnaire for ELL Students *

[* This questionnaire could be modified in a version which is more student-friendly, or in the ELLs' native languages, if possible (or if ELL specialists or native speakers of that language are available and could contribute as translators, the interview could be only oral as the interview session is audio and/or video-recorded).]

1. Name:____
2. Boy_____ Girl_______
3. How old are you? ______
4. Where were you born? ____
5. What is your first language?
   ____
6. What is your mom's first language?
   ____
7. What is your father's first language? ____
8. What languages do you speak at home? About how much of the time do you use each language at home?
   1. ___________-_________%
   2. ___________-_________%
   3. ___________-_________%
9. How do you describe your knowledge of English in speaking, reading, and writing? (Do you only speak English? Or can you also read English? Can you write in English? Tell me more, please.)

____

10. Tell me about your previous mathematics classes and grades.

____

11. How well do you like Math?

____

12. How well does your mother like math?

____

13. How well does your father like math?

____

14. Think about the mathematics classes where I came to visit or where the video camera was being used. How much did you participate in class? How often did you ask questions? If you didn't ask questions, why not?

____

15. Was the lesson where I came to visit or where the video camera was being used easy or hard for you? [Easy/Hard] Why? What part(s)? Tell me more, please.

____

16. Now, think about your mathematics class this year. For all the lessons, even the ones I did not observe, please fill out the *TTT Form 3* for the things your teacher might have done. Your teacher might have used some, but not all of the things that are listed.

Thank you!

Appendix E: Post-observation Student Questionnaire

Teacher Talk Test (TTT) Form 3				
No:	**Strategies:**	Comment on your mathematics teacher's use of the following strategies (use **a checkmark)**		**How Often Is This Strategy Used?—**Rate using a Frequency Scale from 1 to 5, with 5 as most frequent: **1**—Never, **2—**Rarely (1 or 2 times a month), **3**—Sometimes (1 or 2 times a week), **4**—Usually (3 or 4 times a week), **5**—Always
		Yes	**No**	
I.	**"Vocal" Strategies:**			
1.	Use of slow and simple talk; short sentences			
2.	Use of few slang (jargon) words (words connected in sentences or groups typical for the country, region, or technical terms)			
3.	Use of similar words			
4.	Use of repetitions in same or almost the same words			
5.	Use of changes in her/his voice to louder, higher, faster, etc.			
6.	Use of explanations what you need to do more than once so you would understand			
7.	Does the teacher ask/check if you understand?			
8.	Does the teacher write lesson vocabulary words, give examples, or show pictures?			
9.	Does the teacher start a lesson with a review of (related) similar ideas?			
10.	Does the teacher ask students to tell what they learned today?			
11.	Does the teacher ask students to talk and explain their solutions?			

Appendix E (Continued)

II.	**Questioning Strategies:**			
12.	Does the teacher use any of these types of questions? How often?			
	a) *pre-production*—point to…; find the…; is this a/an…; etc.			
	b) *early production*—yes/no questions; either/or questions; one-word or two-word responses; general questions that require a lengthy response;			
	c) *speech emergence*—Why? How? Tell me about…? Describe…;			
	d) *intermediate speech*—What do you recommend? What is your opinion....? What would happen if…? Compare/contrast…; Create…			
13.	Does the teacher give you time to think before you need to answer a question?			
14	Provide feedback			
III.	**Enhancement to teacher talk's strategies:**			
15	Does the teacher use her hands or face, or look at students when talking?			
16.	Did the teacher draw pictures to explain or group ideas?			
17.	Does the teacher show pictures, cards, or small models to explain words or how to do math problems?)			

18.	Are calculators, projectors, or computers used? Or other technology?			
19.	Does the teacher let you work in different ways—in groups with other students or by 2?			
20.	When given a grade, is it only from a test written on paper, or you are asked to do different things? If yes, give some examples, please.			
21.	Does your teacher ask you to talk and give examples from your country or family when solving math problems?			
22	Does your teacher explain most of the difficult parts of the lesson so that you can do most of the homework on your own?			

Thank you!

ABOUT THE AUTHOR

Mariana Petkova earned her Bachelor of Science degree in Mathematics and Computer Science from the University of Plovdiv, Bulgaria. Following this, Mrs. Petkova completed her graduate level research and qualifying exams in pursuit of a Candidate of Science Degree in Computer Science at the Bulgarian Academy of Sciences, Institute of Mathematics, Sofia, Bulgaria.

Currently, she is an International Baccalaureate (IB) Mathematics Higher Level (HL) teacher at King High School in Tampa, Florida. Prior to this, Mrs. Petkova was a Math Magnet Teacher at Jefferson High School, where she taught students in AP Calculus, Algebra I Honors. Mrs. Petkova has also worked for the Bulgarian Academy of Sciences, the Huntington Learning Center, and Youth Environmental Services. A supporter of a student-centered instructional approach, she strives to provide time for discussions, problem solving, and activities in every class period.

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