State Estimation and Parameter Identification of Continuous-time Nonlinear Systems

by

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Abstract

The problem of parameter and state estimation of a class of nonlinear systems is addressed. An adaptive identifier and observer are used to estimate the parameters and the state variables simultaneously. The proposed method is derived using a new formulation. Uncertainty sets are defined for the parameters and a set of auxiliary variables for the state variables. An algorithm is developed to update these sets using the available information. The algorithm proposed guarantees the convergence of parameters and the state variables to their true value. In addition to its application in difficult estimation problems, the algorithm has also been adapted to handle fault detection problems.

The technique of estimation is applied to two broad classes of systems. The first involves a class of continuous time nonlinear systems subject to bounded unknown exogenous disturbance with constant parameters. Using the proposed set-based adaptive estimation, the parameters are updated only when an improvement in the precision of the parameter estimates can be guaranteed. The formulation provides robustness to parameter estimation error and bounded disturbance. The parameter uncertainty set and the uncertainty associated with an auxiliary variable is updated such that the set is guaranteed to contain the unknown true values.

The second class of system considered is a class of nonlinear systems with time-varying parameters. Using a generalization of the set-based adaptive estimation technique proposed, the estimates of the parameters and state are updated to guarantee convergence to a neighborhood of their true value. The algorithm proposed can also be extended to detect the fault in the system, injected by drastic change in the time-varying parameter values. To study the practical applicability of the developed method, the estimation of state variables and time-varying parameters of salt in a stirred tank process has been performed. The results of the experimental application demonstrate the ability of the proposed techniques to estimate the state variables and time-varying parameters of an uncertain practical system.

For my father, Gurdev Dhaliwal...

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Chapter 1

Introduction

1.1 Motivation

Effective monitoring of a process is possible only when accurate information on the state variables and parameters of the process are available. Example of *process state variables* are concentrations of the reacting species in a reactor, temperature and molecular weight distribution in a polymerization process. These variables uniquely define the states of the process and in many cases may directly/indirectly define the final product quality. Rate of heat production in a reactor, overall heat coefficient in jacketed reactors and specific growth rate in bioreactors are the examples of *process parameters*. Information on the parameters of a process provides a better understanding of the process dynamics and also allow for the development of an accurate and representative models of process.

In practice, due to inadequacy of available sensors or operational limitations, some of the essential process state variables cannot be measured frequently. In addition important process parameters may have to be estimated from available measurements.

In such cases, estimates of the inaccessible, but essential, state variables and parameters of the process are usually obtained by employing state and parameter estimation methods. Many techniques exist for the estimation of states for a variety of classes of dynamical systems that can achieve accurate state estimates in a variety of conditions. However, these techniques rely on the knowledge of the system parameters. Uncertainty in the model parameters for instance can generate (possibly large) bias in the estimation of the unmeasured state variables. In cases where large uncertainties of the process parameters exist, it is imperative to use techniques that are able to combine state observation with parameter estimation.

The motivation for this research arises from the need to develop reliable state and parameter estimation methods that are capable of providing continuous and accurate estimates of inaccessible state variables and parameters of a nonlinear process in a presence of (a) exogenous disturbance, (b) time-varying parameters and (c) random fault occurrences in the system, all of which are frequently encountered in practice.

1.2 Organization of the Dissertation

Chapter 2: Chapter 2 is divided into two parts. First, the technical preliminaries required to develop the parameter and state estimation methodology proposed in Chapter 3 are introduced. The topics include Persistence of Excitation (PE), Lyapunov Stability, Projection Algorithm, Observability, State Observers and Adaptive identifiers. The second section contains a review of the past and recent works in the field of parameter and state estimation of nonlinear systems.

CHAPTER 3: In this chapter, we consider the problem of parameter identification and state estimation of a continuous-time nonlinear system subject to exogenous disturbance. The formulation is developed to provide robustness to parameter estimation error and bounded disturbance. The uncertainty associated with an auxiliary variable defined for state estimation is updated such that the set is guaranteed to contain the unknown true values. A simulation example is used to illustrate the developed procedure and ascertain the theoretical results.

CHAPTER 4: In this chapter, the adaptive observer is used to solve the problem of simultaneous state estimation and time-varying parameter estimation of a continuous-time nonlinear system. Using a set-based adaptive estimation, the estimates for the parameters and the state variables are updated to guarantee convergence. The algorithm is proposed to detect a fault in the system triggered by a drastic change in the time-varying parameters. A simulation example is used to illustrate the developed procedure and ascertain the theoretical results.

CHAPTER 5: Based on the results in Chapter 4, the estimation technique is applied to a mixing tank problem with two inlet streams of different concentration, mixing to give a product steam of a particular concentration. The developed method is used to estimate state and time varying parameters of the experimental process. The estimation routine employed guarantees convergence of state and parameters to their true values.

CHAPTER 6: A summary of the design procedure given in Chapter 3 and 4 is provided, and conclusions are drawn based on the investigations of Chapters 3, 4 and 5. Suggestions for directions of future work are given.

Chapter 2

Literature Review

The proposed design methodology for simultaneous parameter and state estimation of class of a nonlinear systems is largely developed from the concepts of linear system theory, parameter identifiers, projection algorithm and adaptive observers. In this chapter, these concepts are briefly introduced for the understanding of this thesis work. The detailed discussion regarding the relationships between the concepts are discussed in Chapter 3. This chapter also summarizes the recent and early works by researchers active in robust adaptive estimation techniques that are of importance in relation to this thesis..

2.1 Technical Preliminaries

2.1.1 Persistence of Excitation

The concept of persistent excitation (PE), when it arose in the 1960s in the context of system identification. The term PE was coined to express the property of the input signal to the plant that guarantees that all the modes of the plant are excited. In the late 1970s, it became clear that the concept of PE also played an important role in the convergence of the controller parameters to their desired values. Recent work on robustness of the adaptive systems in the presence of bounded disturbance, time-varying parameters, and unmodeled dynamics of the plant revealed that the concept of PE is also intimately related to speed of convergence on the parameters to their final values, as well as the bounds on the magnitudes of the parameter errors. In both linear and nonlinear adaptive systems, parameter convergence is related to the satisfaction of persistence of excitation condition, which can be defined in the continuous time as follows.

Definition 2.1.1. *[Ioannau and Sun, 1996], [Khalil, 1992]: A vector function $\phi :$ is said to be persistently exciting if there exist positive constants α_1, α_2 and T_0 such that*

$$\alpha_1 I \geq \int_t^{t+T_0} \phi(\tau)\phi(\tau)^T d\tau \geq \alpha_2 I, \qquad \forall t \geq 0 \tag{2.1}$$

Although the matrix $\phi(\tau)\phi(\tau)^T$ may be singular at every instant τ, the PE condition requires that ϕ span a entire n_θ dimensional space as τ varies from t to $t + T_0$, that is, integral of matrix $\phi(\tau)\phi(\tau)^T$ should attain full rank over any interval of some length T_0 or in other words, (2.1) requires that $\phi(t)$ varies such that the integral of the matrix $\phi(\tau)\phi(\tau)^T$ is uniformly positive definite over any time interval $[t, t + T_0]$. The properties of PE signals as well as various other equivalent definitions and interpretations are given in the literature [Sastry and Bodson, 1989; Eykhoff, 1974; Anderson, 1977; Narendra and Annaswamy, 1989].

In adaptive linear systems, the PE condition is converted to the sufficient richness (SR) condition on the reference input signal. Necessary and sufficient conditions for parameter convergence are then developed in terms of the reference signal. A popular result implies that exponential convergence is achieved whenever the reference signal contains enough frequencies, i.e., whenever the spectral density of the signal is nonzero in at least n_θ points, where n_θ is the number of unknown parameters in the adaptive scheme. Otherwise, convergence to a characterizable subspace of the parameter space is achieved [Boyd and Sastry, 1986].

Despite the fact that the theory of parameter convergence for linear systems is well established, very few results are available for nonlinear systems. This is mainly because the familiar tools in linear adaptive control cannot be directly extended to nonlinear systems. In most of the available results, stability and performance properties are proved by assuming that a vector function, which depends on closed-loop signals is persistently exciting. However, the means of verifying this PE condition *a priori* for a given nonlinear system remains an open problem, in general. In [Lin and Kanellakopoulos, 1998], a procedure is provided for determining *a priori* whether or not a specific reference signal is sufficiently rich for a specific output feedback nonlinear system, and hence whether or not parameter estimates will converge. Nevertheless, the main result in [Lin and Kanellakopoulos, 1998] is that the presence of nonlinearities in the plant usually reduces the SR condition requirement on the reference signal and thus enhances parameter convergence.

2.1.2 Lyapunov Stability

Lyapunov stability analysis plays an important role in the stability analysis of dynamical systems described by ordinary differential equations. This technique is very useful and convenient in practice because the stability of the system can be determined directly from the differential equations describing the system. In other words, the Lyapunov method enables one to determine the nature of stability of an equilibrium point of the system without explicitly integrating the ordinary differential equations. In addition, the Lyapunov analysis is applicable to continuous-time and discrete-time systems, linear and nonlinear systems, time-invariant and time-varying systems.

From the classical theory of mechanics, a vibratory system is stable if its total energy is continually decreasing until an equilibrium state is reached. A physical example that illustrates this concept is a simple pendulum in which the equations of motion described by the forces acting on the system, vanish at steady state [Khalil, 2002]. The method of Lyapunov, is based on the following behavior. If the system has an asymptotically stable equilibrium state, then the stored energy of the system decays with increasing time until it finally reaches its minimum value at the equilibrium state. For a general system, however it is not simple to describe its dynamics through an "energy function". To overcome this difficulty, the "Lyapunov function" which acts as a fictitious energy function, was introduced [Ogata, 1987].

The Lyapunov function, denoted by $V(.)$, is a scalar, positive definite function. It is generally assumed to be continuous with continuous partial derivatives. When taken along the system's trajectory, the time derivative of the Lyapunov function is negative definite or negative semidefinite. These desired properties of the Lyapunov function can be formally stated in the stability theorem described by [Khalil, 2002] for a non-autonomous system.

Theorem 2.1.1. *[Khalil, 1992] Consider the non-autonomous system*

$$\dot{x}(t) = f(t, x(t)) \tag{2.2}$$

where $f : [0, \infty) \times D \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in $x(t)$ on $[0, \infty) \times D$, and $D \subset \mathbb{R}^n$ is a neighborhood of origin $x(t) = 0$. Let $x(t) = 0$ be an equilibrium point for the system (2.2) at $t = 0$ and $D = \{x(t) \in \mathbb{R}^n \;\Big|\; ||x(t)|| < r\}$. Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continuously differentiable function such that

$$\alpha_1(||x(t)||) \leq V(t, x(t)) \leq \alpha_2(||x(t)||)$$

$$\dot{V}(t, x(t)) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x(t)) \leq 0$$

$$\int_t^{t+\epsilon} \dot{V}\Big(\tau, \varphi(\tau, t, x(t))\Big) d\tau \leq -\lambda V(t, x(t)), \qquad 0 < \lambda < 1$$

$\forall t \leq 0, \forall x(t) \in D$, for some $\epsilon > 0$, where $\alpha_1(.)$ and $\alpha_2(.)$ are class $\mathcal{K}$ functions defined in $[0, r)$ and $\varphi(\tau, t, x(t))$ is the solution of the system that starts at $(t, x(t))$. Then, the origin is uniformly asymptotically stable.
If all the assumptions hold globally and $\alpha_1(.)$ belongs to class $\mathcal{K}_\infty$, then the origin is globally uniformly asymptotically stable.
If

$$\alpha_j(r) = K_j r^{\varsigma}, \quad K_j > 0, \quad \varsigma > 0, \quad j = 1, 2$$

then the origin is exponentially stable.

Now that the stability considerations based on Lyapunov theory are defined, the next step consists of finding a convenient Lyapunov function to design the adaptive updating laws, such that Theorem 2.1.1 is satisfied.

2.1.3 Projection Algorithm

It is important to mention that, in general, the parameters that characterize a system, have a physical meaning and are bounded above and/or below. For this reason, it is desired to constrain the parameter estimates to lie inside a bounded set. An effective method for keeping the parameter estimates within some defined bounds is to use a projection algorithm.

In many practical problems where θ represents the parameters of a physical plant, we may have some *a priori* knowledge as to where θ is located in $\mathbb{R}^n$. This knowledge usually comes in terms of upper or lower bounds for the elements of θ or in terms of a well defined subset of $\mathbb{R}^n$, etc. Using this *a priori* information, adaptive laws can be designed that are constrained to search for estimates of θ in the set where θ is located. Intuitively such a procedure may improve the convergence and reduce the time taken in convergence when initial values of the parameter is chosen to be far away from the unknown θ.

In [Krstic et al., 1995], a projection operator is defined for the general convex parameter set Π. Consider a convex set $\Pi_\epsilon = \left\{\hat{\theta} \in \mathbb{R}^p \middle| \mathcal{P}(\hat{\theta}) \leq \epsilon\right\}$, where the convex function $\mathcal{P} : \mathbb{R}^p \to \mathbb{R}$ is assumed to be smooth. The set Π_ϵ is the union of the set $\Pi = \left\{\hat{\theta} \in \mathbb{R}^p \middle| \mathcal{P}(\hat{\theta}) \leq 0\right\}$ and a boundary around it. The interior of Π is denoted by $\mathring{\Pi}$, and $\nabla_{\hat{\theta}}\mathcal{P}$ represents an outward normal vector at $\hat{\theta} \in \partial\Pi$. The projection operator is defined as follows

$$Proj(\tau) = \begin{cases} \tau, & \hat{\theta} \in \mathring{\Pi} \quad \text{or} \quad \nabla_{\hat{\theta}}\mathcal{P}^T\tau \leq 0 \\ \left(I - c(\hat{\theta})\Gamma\frac{\nabla_{\hat{\theta}}\mathcal{P}\nabla_{\hat{\theta}}\mathcal{P}^T}{\nabla_{\hat{\theta}}\mathcal{P}^T\nabla_{\hat{\theta}}\mathcal{P}}\right)\tau, & \hat{\theta} \in \Pi_\epsilon \setminus \mathring{\Pi} \quad \text{and} \quad \nabla_{\hat{\theta}}\mathcal{P}^T\tau > 0 \end{cases} \tag{2.3}$$

$$c(\hat{\theta}) = \min\left\{1, \frac{\mathcal{P}(\hat{\theta})}{\epsilon}\right\}$$

Here, Γ belongs to $\mathcal{G}$ of all positive definite symmetric $p \times p$ matrices and τ is the vector of nominal update laws that is, in the absence of the projection algorithm the update law would be $\dot{\hat{\theta}} = \tau$.
The properties of the projection operator, $\mathit{Proj}\{\tau, \hat{\theta}, \Gamma\}$, are given by

1. The mapping $\mathit{Proj} : \mathbb{R}^p \times \Pi_\epsilon \times \mathcal{G} \to \mathbb{R}^p$ is locally lipschitz in its arguments $\tau, \hat{\theta}, \Gamma$.

2. $\mathit{Proj}\{\tau\}^T \Gamma^{-1} \mathit{Proj}\{\tau\} \leq \tau^T \Gamma^{-1} \tau, \qquad \forall \hat{\theta} \in \Pi_\epsilon$.

3. Let $\Gamma(t), \tau(t)$ be continuously differentiable and

$$\hat{\theta} = \mathit{Proj}\{\tau\}, \qquad \hat{\theta}(t)(0) \in \Pi_\epsilon.$$

   Then, on its domain of the definition, the solution $\hat{\theta}(t)$ remains in Π_ϵ.

4. $\tilde{\theta}^T \Gamma^{-1} \mathit{Proj}\{\tau\} \leq \tilde{\theta}^T \Gamma^{-1} \tau, \qquad \forall \hat{\theta} \in \Pi_\epsilon, \theta \in \Pi$.

The adaptive laws with the projection modification given by (2.3) retain all the properties established in the absence of the projection and guarantee that $\hat{\theta} \in \Pi_\epsilon \quad \forall \quad t \geq 0$ provided $\hat{\theta}(0) = \hat{\theta}_0 \in \Pi_\epsilon$ and $\theta \in \Pi_\epsilon$.

2.1.4 Observability

Consider a continuous time linear system of the form

$$\dot{x} = Ax + Bu, \tag{2.4a}$$

$$y = Cx, \tag{2.4b}$$

where $x \in \mathbb{R}^n$ is a state vector, $u \in \mathbb{R}^n$ is the control input, $y \in \mathbb{R}^n$ are the outputs, and matrices, A, B and C are of appropriate dimensions. Observability is a property of dynamical system, first introduced by [Kalman, 1960]. This property is meant to express the availability of measurement data with respect to one's ability to reconstruct or make inferences regarding the values of unmeasured state variables.

Definition 2.1.2. *A linear continuous time system given by* (2.4) *is "observable" if for any initial state x_0 and some final time t, the initial state x_0 can be uniquely determined by knowledge of the inputs u and outputs y for all time t.*

In other words, observability is related to the problem of determining the value of the state vector knowing only the output y over some interval of time. This is a question of determining when the mapping of the state into the output associates a unique state with every output that can occur. If a system is observable, then its initial state can be determined. If the initial state is known, then values of the states at any time can be calculated. Hence, observability implies that values of the state at any time are fully reconstructible as long as the inputs and outputs are known exactly.

Observability can be checked by a matrix rank test performed on the system's *observability matrix*.

Theorem 2.1.2. *The continuous time LTI system* (2.4) *is observable if and only if the observability matrix is defined by*

$$\mathcal{O}(C, A) \triangleq [C^T, (CA)^T, \cdots, (CA^{(n-1)})^T]^T$$

is of rank n.

The concept of observability is central to the design of *state observers* and *state estimators*, which are discussed in the next section.

2.1.5 State Observers

Many nonlinear control design and adaptive system techniques assume state feedback; this implies that all the state variables are measured and are available for feedback. In practice, this is not always true, either for economic or technical reasons, such as sensor failures. In most cases,only a subset of the state variables are available for measurement. Intuitively, we want to use the measured states or outputs of the system and extend the state-dependent techniques to output-dependent techniques for system design. The idea is similar to what has been widely applied in LTI systems, i.e., build an observer that yields asymptotic estimates of the system state based on the output of the system, and then update the control/ adaptation law using on-line estimation of the unmeasured states.

In control theory, a state observer is a dynamical system whose outputs are the estimates of the state variables of the system [Ioannau and Sun, 1996]. The main criterion that observers must satisfy is that the estimation error $\tilde{x}(t) = (x(t) - \hat{x}(t))$ tends to zero in the limit as $t \to \infty$ where $\hat{x}(t)$ is the estimate of the state $x(t)$ at time t. If the dynamics of the plant give rise to a linear time-invariant system, then there exists an estimator of the form

$$\hat{x}(t) = A\hat{x}(t) + L(y - \hat{y}) + Bu, \tag{2.5a}$$

$$\hat{y} = C\hat{x} + Du, \tag{2.5b}$$

which guarantees convergence of the state estimation error to zero, provided that the plant is observable. The observer given by Eqs. (2.5a) and (2.5b) is referred to as a *Luenberger observer* [Ioannau and Sun, 1996]. The matrix L is designed so that the matrix $(A-LC)$ is stable, which ensures the stability of the observer's error dynamics. In fact, the eigenvalues of $(A - LC)$, and, therefore, the rate of convergence of $\tilde{x}(t)$ to zero can be arbitrarily chosen by designing L appropriately. Therefore, it follows that $\hat{x}(t) \to x(t)$ exponentially fast as $t \to \infty$, with a rate that depends on the matrix $(A - LC)$. This result is valid for any matrix A and any initial condition $x(0)$ as long as (C, A) is an observable pair.

The problem of combined state and parameters estimation is considered in this thesis. The general structure of the adaptive observer is shown in the Figure 2.1. Throughout, it is assumed that the plant (2.5) is observable. The observability of

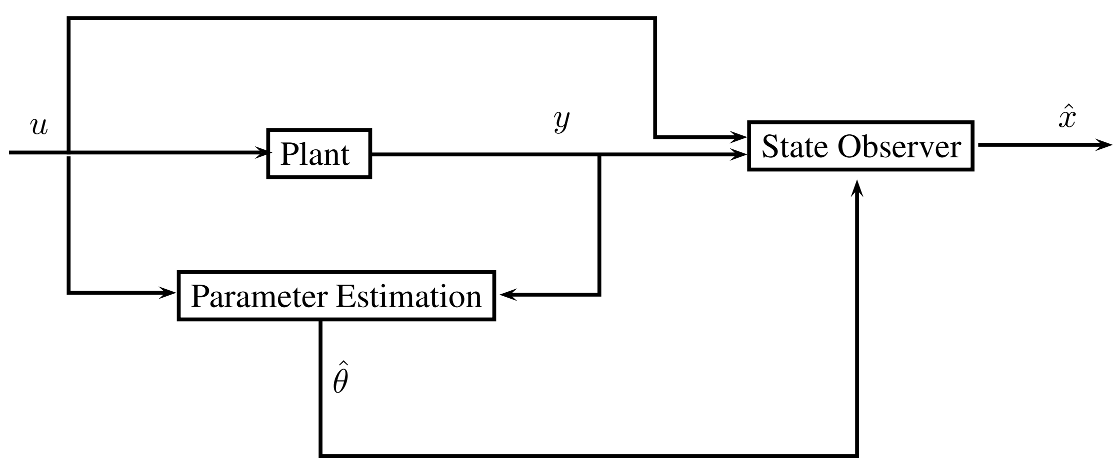

Figure 2.1: General structure of the adaptive Luenberger observer.

(C, A) is used to guarantee the existence of the state space representation of the plant in the observer form that in turn enables the design of a stable adaptive observer. Moreover, the observability of (C, A) establishes the PE condition from the properties of the input u.

2.1.6 Adaptive Identifiers

The adaptive identifiers represent a class of real time parameter estimation schemes that are used to estimates (typically) slow time-varying parameters of dynamical systems. Under suitable conditions, these identifiers can guarantee convergence of the estimated parameters to the unknown parameter values. The design of such scheme includes the selection of plant input so that a certain signal vector, is PE. Adaptive identifier designs are natural extension of observer design for linear time invariant (LTI) systems with unknown parameters. When the parameters of the system are unknown, an adaptive identifier is designed to estimate the parameters of the dynamical system. This was first accomplished in [Kreisselmeier, 1977; Kudva and Narendra, 1973]. Traditionally, an adaptive identifiers consists of a state prediction subject to parameter estimations and a parameter update law. Different representations have been discussed in detail for LTI systems [Ioannau and Sun, 1996; Narendra and Annaswamy, 1989; Sastry and Bodson, 1989]. Basic methods used to design adaptive laws include Lyapunov-based design, gradient methods, and recursive least squares methods. Subsequently alternative techniques have been generalized to the design of adaptive observers for nonlinear systems, linear time-varying systems and systems with disturbances. Adaptive laws only become parameter identifiers if the input signal u has to be chosen to be sufficiently rich so that the regressor vector ϕ is PE.

2.2 Parameter Estimation in Nonlinear Systems

Parameter estimation is the process of attributing a parametric description to an object, a physical process or an event based on measurements that are obtained from that object (or process, or event). The measurements are made available by a sensory system. Parameter estimation plays an important role in many disciplines. In a dynamical system, if all parameters and all state variables at an initial time are known, a prediction can be made as to the future state of the system. This is called the forward problem. Unfortunately, for most experimental systems of interest, only a subset of state variables and parameters can be measured simultaneously, making prediction of the future state of the system difficult or impossible. The field of parameter and state estimation, also known as inverse problem theory, is a mature discipline [Tarantola, 2005; Levy, 2008; Beck and Arnold, 1974] concerned with determining unmeasured states and parameters in an experimental system. This is important since measurement of some of the parameters and states may not be possible, yet knowledge of these unmeasured quantities is necessary for predictions of the future state of the system. This field is important across a broad range of scientific disciplines, including geosciences, biosciences, nanoscience, and many others.

2.3 State Estimation

State Estimators are deterministic/stochastic dynamic systems that are used to reconstruct the inaccessible but important process state variables, from available measured variables. The problem of state estimation in chemical processes has been studied extensively since the mid 1970s. In particular, the extended Kalman Filter (EKF) has been used widely for state estimation [Bastin and Dochain, 1991]. The design of an EKF is based on the linear approximation of a nonlinear process model.

It is generally recognized that, the linearization at each time step can introduce large errors and even cause divergence of the filter [Wan and Van Der Merwe, 2000]. These concerns are especially acute in complex industrial set-ups [Wilson et al., 1998]. Although higher order Kalman filters exist, they are more difficult to implement and prone to instability. Due to the complex nonlinear behavior of many chemical and biochemical processes, reliable state estimation should be based on nonlinear models that can capture the complex nonlinear behavior. Furthermore, several studies have found that linear state estimators are inadequate for many nonlinear processes [Valluri and Soroush, 1996] and [Tatiraju and Soroush, 1997], motivating the use of nonlinear observers/estimators.

The Luenberger observer is well established method of estimating the state variables of a known observable system using input-output data,that can be adjusted to handle to estimate the state of a linear time-invariant system with unknown parameters as well. The structure of the observer as the adaptive laws for updating its parameters has to be chosen judiciously for this purpose. This was accomplished in [Carroll and Lindorff, 1973; Kudva and Narendra, 1973; Luders and Narendra, 1974; Narendra and Annaswamy, 1989]. In 1977, an alternate method of generating the estimates of the states and the parameters of the plant was suggested [Kreisselmeier, 1977] where the adaptive algorithms ensured faster rate of convergence of the parameters estimates under certain conditions.

When the system further depends on some unknown parameters, the observer design has to be modified so that both state variables and parameters can be estimated, leading to so-called *adaptive observers.* Various results in that respect can be found, going back to ([Luders and Narendra, 1974], [Carroll and Lindorff, 1973] and [Kreisselmeier, 1977]) for linear systems, or ([Bastin and Gevers, 1988], [Marino, 1990]) for nonlinear ones, but nonlinearities depending only on input/output.

Recently an alternative result on adaptive observation for linear time-varying systems [Zhang, 2002], an adaptive observer has been designed which guarantees global exponential convergence for noise-free systems. The adaptive observer proposed provides robustness in the presence of modeling and measurement noises. In the paper [Adetola and Guay, 2009], the authors considered a system with exogeneous disturbances and showed that parameter convergence can be guaranteed under certain conditions of persistency of excitation condition. The authors proposed a novel set-based adaptive estimation with an appropriate adaptation law for the unknown parameters. The proof of the convergence of the estimates to their true values is achieved using Lyapunov theories.

The problem of parameter estimation has been of considerable interest during the last two decades [Niethammer et al., 2001] and [Xu and Hashimoto, 1993]. In most applications, it is generally assumed that the parameters are essentially constant over the identification process [Marino et al., 2000] and [Stephan et al., 1994]. A number of situations arise where the time-varying behavior of unknown parameters cannot be neglected. Control algorithm needs to be updated on-line to increase their performance. Motivated by this interest, several approaches have been proposed to simultaneously estimate the state and identify the parameters [Zhu and Pagilla, 2003], [Kreisselmeier, 1986].

2.4 Summary

Concepts and principles of parameter and state estimation are reviewed in this chapter. An overview of recent developments in parameter and state estimation of systems has been presented.

An interesting problem is presented when, in addition to unknown parameters, states of the system are also unknown. For this problem, application of set based adaptive observer technique is possible with simultaneous state estimation using Luenberger observer. The convergence to true parameter and state values can be guaranteed by Lyapunov theories. Another motivating problem is the estimation of time varying parameters and unknown state. The similar techniques can be applied to this problem.

The following chapters present approaches, which are applicable to a class of nonlinear system. Furthermore, develop an algorithm for the fault detection, in case of sudden change of the parameters. Moreover, techniques suggested can be used to estimate the new parameters and states of the system.

Chapter 3

Adaptive observers for nonlinear systems

The main objective of this chapter is parameter identification and state estimation of a class of continuous-time nonlinear system subject to exogenous disturbances. Using a set-based adaptive estimation, the parameters are updated only when an improvement in the precision of the parameter estimates can be guaranteed. The formulation provides robustness to parameter estimation error and bounded disturbance. A simulation example is used to illustrate the developed procedure and ascertain the theoretical results.

3.1 Introduction

Parameter identification is an important problem in the theory of control systems. The problem of simultaneous parameter identification and state estimation problem has attracted the attention of various research groups. It is very useful in treating many practical problems such as fault detection, signal transmission or control, and, more recently, for synchronization of chaotic systems. A Luenberger observer [Luenberger, 1964] allows asymptotic reconstruction of the state of a linear system from measurements of input and output, provided that system parameters are known. For the case where no *a priori* knowledge of the system parameters is available, adaptive observers have been proposed [Kudva and Narendra, 1973]. Motivated by this interest, several approaches have been proposed to simultaneously estimate the state and identify the parameters [Zhu and Pagilla, 2003] and [Kreisselmeier, 1977].

This chapter is mainly influenced from the identification scheme presented in [Adetola and Guay, 2010] and [Adetola, 2008], but is motivated from the state observation viewpoint, thus providing further insight into the interrelations of such schemes. Following earlier works [Adetola and Guay, 2009] and [Adetola and Guay, 2010], for a class of nonlinear systems, a set-based adaptive identifier for parameters is used. This method ensures convergence of the parameter to its true value provided the true parameters fall within the initial uncertainty set. For state estimation, a Luenberger-like observer is chosen to make the continuous-time error dynamics converge to the origin, *i.e* $e(t) \to 0$ as $t \to \infty$. An identifier is also designed for state estimation that ensures the convergence to true state with a condition that the initial estimate of the variables lie inside the initial uncertainty set. The algorithm in [Adetola and Guay, 2009] is applied for state estimation, which ensures non-exclusion of the true state. The notation adopted in [Adetola, 2008] is used throughout this chapter.

3.2 Problem description

Consider a nonlinear system of the form

$$\begin{aligned} \dot{x} =& Ax + b(y)\theta + \omega(t) \\ y =& Hx \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ is the vector state variables, $y \in \mathbb{R}^r$ is the vector output variables, $\theta \in \mathbb{R}^p$ is vector of unknown parameter. It is assumed that θ to be uniquely identifiable lies within a known compact set $\Theta^0 = B(\theta_0, z_\theta)$, the ball centered at θ_0 is a nominal parameter value, with radius z_θ. The exogenous variable $\omega(t)$ represents a bounded time-varying uncertainty, such that $|\omega(t)| < \bar{\omega}$. The vector-valued function $b(y)$ is sufficiently smooth. The following assumptions are made about (3.1).

Assumption 2.1: The state variables $x(t) \in \mathbb{X}$ evolve on a compact subset of $\mathbb{R}^n$.

Assumption 2.2: The system is observable.

The aim of this work is to provide the true estimates of plant parameters and estimation of the state in the presence of unknown bounded disturbances.

3.3 State and uncertainty set estimation

3.3.1 State estimation

Let the estimator model for (3.1) be chosen as

$$\dot{\hat{x}} = A\hat{x} + b(y)\hat{\theta} + KHe + c^T\dot{\hat{\theta}}, \qquad K > 0, \tag{3.2}$$

$$\dot{c}^T = (A - KH)c^T + b(y), \qquad c(t_0) = 0. \tag{3.3}$$

Define the state prediction error $e = x - \hat{x}$ and the auxiliary variable $\eta = e - c^T\tilde{\theta}.$, where $\tilde{\theta} = \theta - t\hat{h}eta$. And error dynamics is given by:

$$\dot{e} =(A - KH)e + b(y)\tilde{\theta} - c^T\dot{\hat{\theta}} + \omega(t). \tag{3.4}$$

where $e(t_0) = x(t_0) - \hat{x}(t_0)$. The η dynamics are given by:

$$\dot{\eta} = (A - KH)\eta + \omega(t), \qquad \eta(t_0) = e(t_0) \tag{3.5}$$

As $\omega(t)$ is not known, an estimate of η is generated by matrix differential equation

$$\dot{\hat{\eta}} = (A - KH)\hat{\eta}, \qquad \hat{\eta}(t_0) = e(t_0). \tag{3.6}$$

with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics

$$\dot{\tilde{\eta}} = (A - KH)\tilde{\eta} + \omega(t), \qquad \tilde{\eta}(t_0) = 0. \tag{3.7}$$

As $w(t)$ is not known, an estimate of η is generated from (3.6) with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics given by (3.7),$\tilde{\eta}(t_0) = \tilde{\eta}^0 \in \chi^0$, where $\chi \triangleq B(0, z_\eta)$, and z_η is set radius found at the latest set update.

Lemma 3.3.1. *[Desoer and Vidyasagar, 1975] Consider the system*

$$\dot{x}(t) = Ax(t) + u(t)$$

Suppose the equilibrium state $x_e = 0$ of the homogeneous equation is exponentially stable. Then,

1. *if $u \in L_p$ for $1 < p < \infty$, then $x \in L_p$*

2. *if $u \in L_p$ for p= 1 or 2, then $x \to 0$ as $t \to \infty$.*

Consider a Lyapunov function

$$V_\eta = \frac{1}{2}\tilde{\eta}^T P \tilde{\eta} \tag{3.8}$$

it follows from (3.7) that

$$\dot{V}_\eta = \frac{1}{2}\tilde{\eta}^T P((A - KH)\tilde{\eta} + \omega(t)) + \frac{1}{2}((A - KH)\tilde{\eta} + \omega(t))^T P\tilde{\eta} \tag{3.9}$$

Using the following Ricatti equation

$$P(A - KH) + (A - KH)^T P = -Q \tag{3.10}$$

$$\dot{V}_\eta \leq -\frac{1}{2}\tilde{\eta}^T Q\tilde{\eta} + \tilde{\eta}^T P\omega(t) \tag{3.11}$$

and

$$\tilde{\eta}^T Q\tilde{\eta} \leq \lambda_{max}(Q)\tilde{\eta}^T\tilde{\eta} \leq 2\frac{\lambda_{max}(Q)}{\lambda_{min}(P)}V_\eta \tag{3.12}$$

By Young's Inequality

$$\begin{aligned} \tilde{\eta}^T P\omega(t) &= \frac{1}{2}\tilde{\eta}^T\tilde{\eta} + \frac{1}{2}\omega(t)^T P^T P\omega(t) \\ &\leq \frac{1}{2\lambda_{min}(P)}V_\eta + \frac{\parallel P^T P \parallel}{2}\bar{\omega} \end{aligned} \tag{3.13}$$

from (3.11), (3.12) and (3.13)

$$\dot{V}_\eta \leq -\frac{1}{2}\bigg(\frac{2\lambda_{max}(Q)}{\lambda_{min}(P)} - \frac{1}{2\lambda_{min}(P)}\bigg)V_\eta + \frac{\parallel P^T P \parallel}{2}\bar{\omega} \tag{3.14}$$

Considering (3.7), if $\omega(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 3.3.1). Hence, the right hand side of (3.14) is finite.

3.3.2 Set adaptation for η

An update law for the worst-case progress of the state in the presence of disturbance is given by

$$z_\eta = \sqrt{\frac{V_{z\eta}}{4\lambda_{min}(P)}} \tag{3.15}$$

$$V_{z\eta}(t_0) = 4\lambda_{max}\bigg(P(t_0)\bigg)(z_\eta^0)^2 \tag{3.16}$$

$$\dot{V}_{z\eta} = -\frac{1}{2}\bigg(\frac{2\lambda_{max}(Q)}{\lambda_{min}(P)} - \frac{1}{2\lambda_{min}(P)}\bigg)V_\eta + \frac{\parallel P^T P \parallel}{2}\bar{\omega} \tag{3.17}$$

where $V_{z\eta}(t)$ represents the solution of the ordinary differential equation (3.17) with initial condition (3.16). The state uncertainty set, defined by the ball $\chi(0, z_\eta)$ is updated using (3.7) and the error bound (3.15) according to the following algorithm:

Algorithm 3.3.1. *Error bound z_η, the uncertain ball $\chi \triangleq B(0, z_\eta)$ is adapted on-line with algorithm:*

1. *Initialize* $z_\eta(t_{i-1}) = z_\eta^0, \tilde{\eta}(t_{i-1}) = 0$

2. At time t_i, update

$$\chi = \begin{cases} \Big(0, \chi(t_i)\Big), if z_\eta(t_i) \leq z_\eta(t_{i-1}) - \parallel \hat{\eta}(t_i) - \hat{\eta}(t_{i-1}) \parallel - \frac{\bar{\omega}}{\lambda_{min}(A-KH)} \\ \Big(0, \chi(t_{i-1})\Big), otherwise \end{cases}$$

3. Iterate back to step 2, incrementing $i = i + 1$.

Algorithm 3.3.1 ensures that χ is only updated when z_η value has decreased by an amount which guarantees a contraction of the set. Moreover z_η evolution given as in (3.15) ensures non-exclusion of $\tilde{\eta}$ as given below.

Lemma 3.3.2. *The evolution of $\chi = B(0, z_\eta)$ under* (3.6),(3.15) *and Algorithm 3.3.1 is such that*

1. $\chi(t_2) \subseteq \chi(t_1), \qquad t_0 \leq t_1 \leq t_2$

2. $\tilde{\eta} \in \chi(t_0) \implies \tilde{\eta} \in \chi(t) \quad \forall t \geq t_0$

Proof. 1. If $\chi(t_{i+1}) \nsubseteq \chi(t_i)$, then

$$\sup_{\tilde{\eta} \in \chi(t_{i+1})} \parallel \tilde{\eta}(t_i) \parallel \geq z_\eta(t_i) \tag{3.18}$$

However, it follows from triangle inequality and Algorithm 3.3.1 that χ, at the time of update, obeys

$$
\begin{aligned}
\sup_{\tilde{\eta}\in\chi(t_{i+1})} \|\tilde{\eta}(t_i)\| \leq & \sup_{\tilde{\eta}\in\chi(t_{i+1})} \| \tilde{\eta}(t_{i+1}) \| + \| \tilde{\eta}(t_{i+1}) - \tilde{\eta}(t_i) \| \\
& \leq z_\eta(t_{i+1}) + \| \eta(t_{i+1}) - \eta(t_i) \| + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| \\
& \leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| \\
& \quad + \| e^{(A-KH)(t_{i+1}-t_i)}\eta(t_i) + \int_{t_i}^{t_{i+1}} e^{(A-KH)(t_{i+1}-\tau)}\omega(\tau)d\tau - \eta(t_i) \| \\
& \leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| + \| I - e^{(A-KH)(t_{i+1}-t_i)} \| \eta(t_i) \\
& \quad + \int_{t_i}^{t_{i+1}} e^{(A-KH)(t_{i+1}-\tau)}\omega(\tau)d\tau \\
& \leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| + \frac{\bar{\omega}}{\lambda_{min}(A-KH)} \\
& \leq z_\eta(t_i).
\end{aligned}
$$

which contradicts (3.18). Hence, χ update guarantees $\chi(t_{i+1}) \subseteq \chi(t_i)$ and the strict contraction claim follows from the fact that χ is held constant over the update intervals $\tau \in (t_i, t_{i+1})$.

2. We know $V_\eta(t_0) \leq V_{z\eta}(t_0)$ (by definition) and it follows from (3.14) and (3.17) that $\dot{V}_\eta(t) \leq \dot{V}_{z\eta}(t)$. Hence, we have

$$V_\eta(t) \leq V_{z\eta}(t) \qquad \forall t \geq t_0 \tag{3.19}$$

and since $V_\eta = \frac{1}{2}\tilde{\eta}^T P \tilde{\eta}$, it follows that

$$\| \tilde{\eta}^T P \tilde{\eta}(t) \|^2 \leq \frac{V_{z\eta}(t)}{4\lambda_{min}(P(t))} = z_\eta^2(t) \quad \forall t \geq t_0. \tag{3.20}$$

Hence, if $\tilde{\eta} \in \chi(t_0)$, then $\tilde{\eta} \in B(0, z_\eta(t)), \forall t \geq t_0$.

□

3.4 Parameter and uncertainty set estimation

Following [Adetola, 2008], the parameter estimation scheme has been generated for the above mentioned system.

3.4.1 Parameter adaptation

Let $\Sigma \in \mathbb{R}^{n_\theta \times n_\theta}$ be generated from

$$\dot{\Sigma} = cH^T H c^T, \qquad \Sigma(t_0) = \alpha I \succ 0, \tag{3.21}$$

The preferred parameter update law, based on Equations (3.2),(3.3) and (3.6), as proposed in [Adetola and Guay, 2009] is given by

$$\begin{aligned} \dot{\Sigma}^{-1} =& - \Sigma^{-1} cH^T H c^T \Sigma^{-1}, \\ \Sigma^{-1}(t_0) =& \frac{1}{\alpha} I, \end{aligned} \tag{3.22}$$

$$\begin{aligned} \dot{\hat{\theta}} =& \text{proj}\left\{ \gamma \Sigma^{-1} cH^T H(e - \hat{\eta}), \hat{\theta} \right\}, \\ \hat{\theta}(t_0) =& \theta^0 \in \Theta^0, \end{aligned} \tag{3.23}$$

where $\text{Proj}\{\phi, \hat{\theta}\}$ denotes a Lipschitz projection operator [Krstic et al., 1995] such that

$$- \text{Proj}\{\phi, \hat{\theta}\}^T \tilde{\theta} \leq -\phi^T \tilde{\theta}, \tag{3.24}$$

$$\hat{\theta}(t_0) \in \Theta^0 \implies \hat{\theta}(t) \in \Theta, \forall t \geq t_0 \tag{3.25}$$

where Θ^0 is initial uncertainty set. $\Theta \triangleq B(\hat{\theta}, z_\theta)$, where $\hat{\theta}$ and z_θ are the parameter estimate and set radius found at the latest set update respectively. The following Lemma will prove useful in the analysis of the estimation scheme proposed above.

Lemma 3.4.1. *[Adetola and Guay, 2009] The identifier law* (3.22) *and parameter update law* (3.23) *is such that the estimation error* $\tilde{\theta} = \theta - \hat{\theta}$ *is bounded. Moreover, if*

$$\int_{t_0}^{\infty} \left[\| \tilde{\eta} \|^2 - \| e - \hat{\eta} \|^2 \right] d\tau < +\infty \tag{3.26}$$

and

$$\lim_{t \to \infty} \lambda_{min}(\Sigma) = \infty \tag{3.27}$$

are satisfied, then $\tilde{\theta}$ *converges to zero asymptotically.*

Proof. Let $V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T \Sigma \tilde{\theta}$, it follows from (3.22), (3.23) and $c^T\tilde{\theta} = e - \tilde{\eta} - \hat{\eta}$ that

$$\dot{V}_{\tilde{\theta}} = -\gamma \tilde{\theta}^T c H^T H(e - \hat{\eta}) + \frac{1}{2}\tilde{\theta}^T c H^T H c^T \tilde{\theta} \tag{3.28}$$

By Young's Inequality

$$\begin{aligned} \dot{V}_{\tilde{\theta}} \leq -\gamma(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \gamma\tilde{\eta}^T H^T H(e-\hat{\eta}) + \frac{1}{2}(e-\hat{\eta})^T H^T H(e-\hat{\eta}-\tilde{\eta}) \\ - \frac{1}{2}\tilde{\eta}^T H^T H(e-\hat{\eta}-\tilde{\eta}) \end{aligned} \tag{3.29}$$

$$\dot{V}_{\tilde{\theta}} \leq -(e-\hat{\eta})^T H^T H(e-\hat{\eta})\left(\gamma - \frac{1}{2}\right) + \frac{1}{2}\tilde{\eta}^T H^T H\tilde{\eta} + (\gamma - 1)\tilde{\eta}^T H^T H(e-\hat{\eta})$$

$$\dot{V}_{\tilde{\theta}} \leq \tilde{\eta}^T H^T H\tilde{\eta} \underbrace{\left[\frac{1}{2} + \frac{(\gamma-1)}{2}\right]}_{L} - (e-\hat{\eta})^T H^T H(e-\hat{\eta}) \underbrace{\left[\left(\gamma - \frac{(\gamma-1)}{2}\right) - \frac{1}{2}\right]}_{M}$$

$$\dot{V}_{\tilde{\theta}} \leq -\mathbf{M}(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \mathbf{L}\tilde{\eta}^T H^T H\tilde{\eta} \tag{3.30}$$

where $\mathbf{M}$ and $\mathbf{L}$ are positive constants, implying that $\tilde{\theta}$ is bounded. Moreover, it follows from (3.30) that

$$V_\theta(t) = V_{\tilde{\theta}}(t_0) + \int_{t_0}^{t} \dot{V}_{\tilde{\theta}}(\tau) d\tau \tag{3.31}$$

$$\leq V_{\tilde{\theta}}(t_0) - \underline{M} \int_{t_0}^{t} \| \, e - \hat{\eta} \, \|^2 \, d\tau + \underline{L} \int_{t_0}^{t} \| \, \tilde{\eta} \, \|^2 \, d\tau \tag{3.32}$$

Considering the dynamics of (3.7), if $\omega(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 3.3.1). Hence, the right hand side of (3.32) is finite in view of (3.26), and by (3.27) we have $\lim_{t\to\infty} \tilde{\theta}(t) = 0$ □

3.4.2 Parameter set adaptation

An update law that measures the worst-case progress of the parameter identifier in the presence of a disturbance is given by

$$z_\theta = \sqrt{\frac{V_{z\theta}}{4\lambda_{min}(\Sigma)}} \tag{3.33}$$

$$V_{z\theta}(t_0) = 4\lambda_{max}\Big(\Sigma(t_0)\Big)(z_\theta^0)^2 \tag{3.34}$$

$$\dot{V}_{z\theta} = -M(e - \hat{\eta})^T H^T H(e - \hat{\eta}) + LV_{z\eta} \tag{3.35}$$

where $V_{z\theta}(t)$ represents the solution of the ordinary differential equation (3.35) with the initial condition (3.34). The parameter uncertainty set, defined by the ball $B(\hat{\theta}_c, z_c)$ is updated using the parameter update law (3.23) and the error bound (3.33) according to the following algorithm:

Algorithm 3.4.1. *1. Initialize* $z_\theta(t_{i-1}) = z_\theta^0, \hat{\theta}(t_{i-1}) = \hat{\theta}^0$

2. At time t_i, update

$$\left(\hat{\theta}, \Theta\right) = \begin{cases} \left(\hat{\theta}(t_i), \Theta(t_i)\right), & \textit{if} \quad z_{\theta(t_i)} \leq z_\theta(t_{i-1}) - \parallel \hat{\theta}_i - \hat{\theta}(t_{i-1}) \parallel \\ \left(\hat{\theta}(t_{i-1}), \Theta(t_{i-1})\right), & \textit{otherwise} \end{cases}$$

3. Iterate back to step 2, incrementing $i = i + 1$.

Algorithm 3.4.1 ensures that Θ is only updated when the value of z_θ has decreased by an amount which guarantees a contraction of the set. Moreover z_θ evolution as given in (3.33) ensures non-exclusion of θ as given below.

Lemma 3.4.2. *The evolution of $\Theta = B(\hat{\theta}, z_\theta)$ under* (3.22), (3.33) *and Algorithm 3.4.1 is such that*

1. $\Theta(t_2) \subseteq \Theta(t_1), \qquad t_0 \leq t_1 \leq t_2$

2. $\theta \in \Theta(t_0) \implies \theta \in \Theta(t) \quad \forall t \geq t_0$

Proof. 1. If $\Theta(t_{i+1}) \nsubseteq \Theta(t_i)$, then

$$\sup_{s \in \Theta(t_{i+1})} \parallel s - \theta(t_i) \parallel \geq z_\theta(t_i) \tag{3.36}$$

However, it follows from triangle inequality and Algorithm 3.1 that Θ, at the time of update, obeys

$$\begin{aligned}\sup_{s\in\Theta(t_{i+1})} \parallel s-\hat{\theta}(t_i)\parallel &\leq \sup_{s\in\Theta(t_{i+1})} \parallel s-\hat{\theta}(t_{i+1})\parallel + \parallel \hat{\theta}(t_{i+1})-\hat{\theta}(t_i)\parallel \\ &\leq z_\theta(t_{i+1}) + \parallel \hat{\theta}(t_{i+1})-\hat{\theta}(t_i)\parallel \\ &\leq z_\theta(t_i),\end{aligned}$$

which contradicts (3.36). Hence, Θ update guarantees $\Theta(t_{i+1}) \subseteq \Theta(t_i)$. And Θ is held constant over update intervals $\tau \in (t_i, t_{i+1})$.

2. We know that $V_{\tilde{\theta}}(t_0) \leq V_{z\theta}(t_0)$ (by definition) and it follows from (3.30) and (3.35) that $\dot{V}_{\tilde{\theta}}(t) \leq \dot{V}_{z\theta}(t)$. Hence, by the comparison lemma, we have

$$V_{\tilde{\theta}}(t) \leq V_{z\theta}(t) \qquad \forall t \geq t_0 \tag{3.37}$$

and since $V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T\Sigma\tilde{\theta}$, it follows that

$$\parallel \tilde{\theta}^T\Sigma\tilde{\theta}(t)\parallel^2 \leq \frac{V_{z\theta}(t)}{4\lambda_{min}(\Sigma(t))} = z_\theta^2(t) \quad \forall t \geq t_0. \tag{3.38}$$

Hence, if $\theta \in \Theta(t_0)$, then $\theta \in B(\hat{\theta}(t), z_\theta(t)), \forall t \geq t_0$.

□

3.5 Simulation Example

To illustrate the effectiveness of the proposed method, we consider the following system subject to an additional disturbance

$$\dot{x_1} = (x_2 + x_3^2\theta_1 - x_3\theta_3) + u_1 + \omega_1$$

$$\dot{x_2} = (-x_1 + x_3 + x_3\theta_2 + x_3\theta_3) + u_2 + \omega_2$$

$$\dot{x_3} = (-x_1 - 2x_2 - x_3 + x_3\theta_3) + u_3 + \omega_3$$

$$y = Hx$$

where $\theta^T = [\theta_1, \theta_2, \theta_3]$, the input is taken as constant, $u = [-0.001 \quad 0.001 \quad 0.002]^T$. The true parameter values are $\theta = [2.9 \quad 3.1 \quad 0.7]^T$. The bounded noise term is $\omega(t) = sin(0.01t)[1 \quad 1 \quad 1]^T$. The initial radius of the uncertainty set for θ is $z_\theta^0 = 19$. The initial radius of the uncertainty set for η is $z_\eta^0 = 10$. Initial conditions for state are $x(0) = [0.1 \quad 0.03 \quad 0.04]$. Initial estimates of the states are $\hat{x}(0) = [0.4 \quad 0.12 \quad 0.16]^T$.The center of the parameter uncertainty set is assumed to be $\hat{\theta}_c^0 = [2.74 \quad 3.18 \quad 0.86]^T$ at time $t = 0$. For this example, $H = [0 \quad 0 \quad 0.2]$.

The set-based adaptive identifier is used to estimate the parameters and along with an uncertainty that is guaranteed to contain the true value of the parameters. Figure 3.1 shows the estimates of the parameters converging asymptotically to their true values. Simultaneously an auxiliary variable is used to estimate the unmeasured state variables. The estimation of the unknown states follows the true state value as shown in Figure 3.3 and the error associated with state prediction is shown in Figure 3.2. The proposed technique updates the estimates only when estimation improvement is guaranteed. The proposed uncertainty set update for parameter identification and state estimation, guarantees to contain the true values at all time instants. As depicted in Figure 3.4, the uncertainty bound z_θ reduces over time and the true parameter always lies within the uncertainty set as the distance between true and estimated parameters, δ_θ is always less than z_θ. The radius of uncertainty set for $\tilde{\eta}$ i.e z_η is also decreasing with time as shown in Figure 3.5. It also shows the non-exclusion of the $\tilde{\eta}$, $\delta_\eta < z_\eta$, which ensures that the true state of the system is estimated.

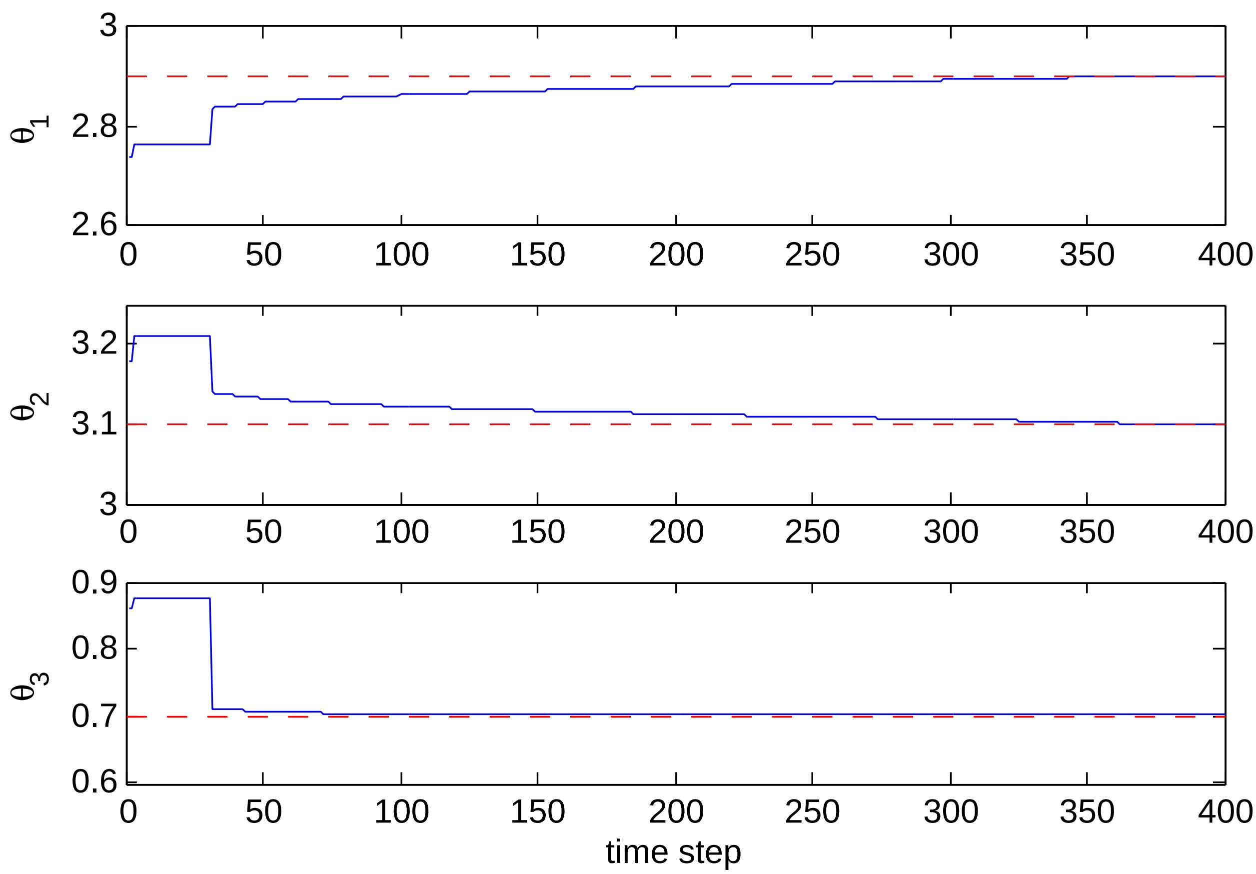

Figure 3.1: Time course plot of the true parameter: θ dashed lines($--$) and parameter estimates: $\hat{\theta}$ solid lines ($-$).

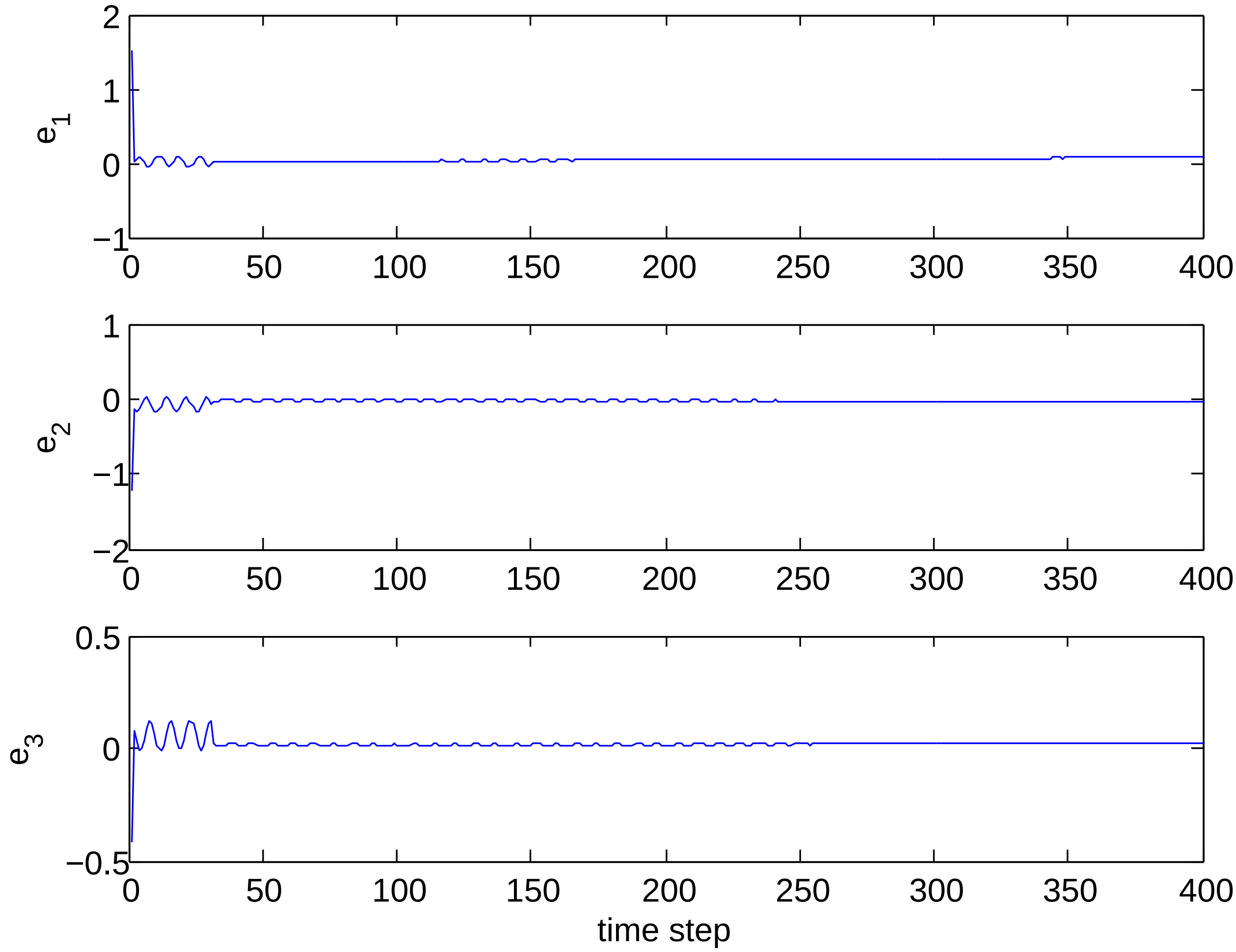

Figure 3.2: Time course plot of the state estimation error $e = x - \hat{x}$.

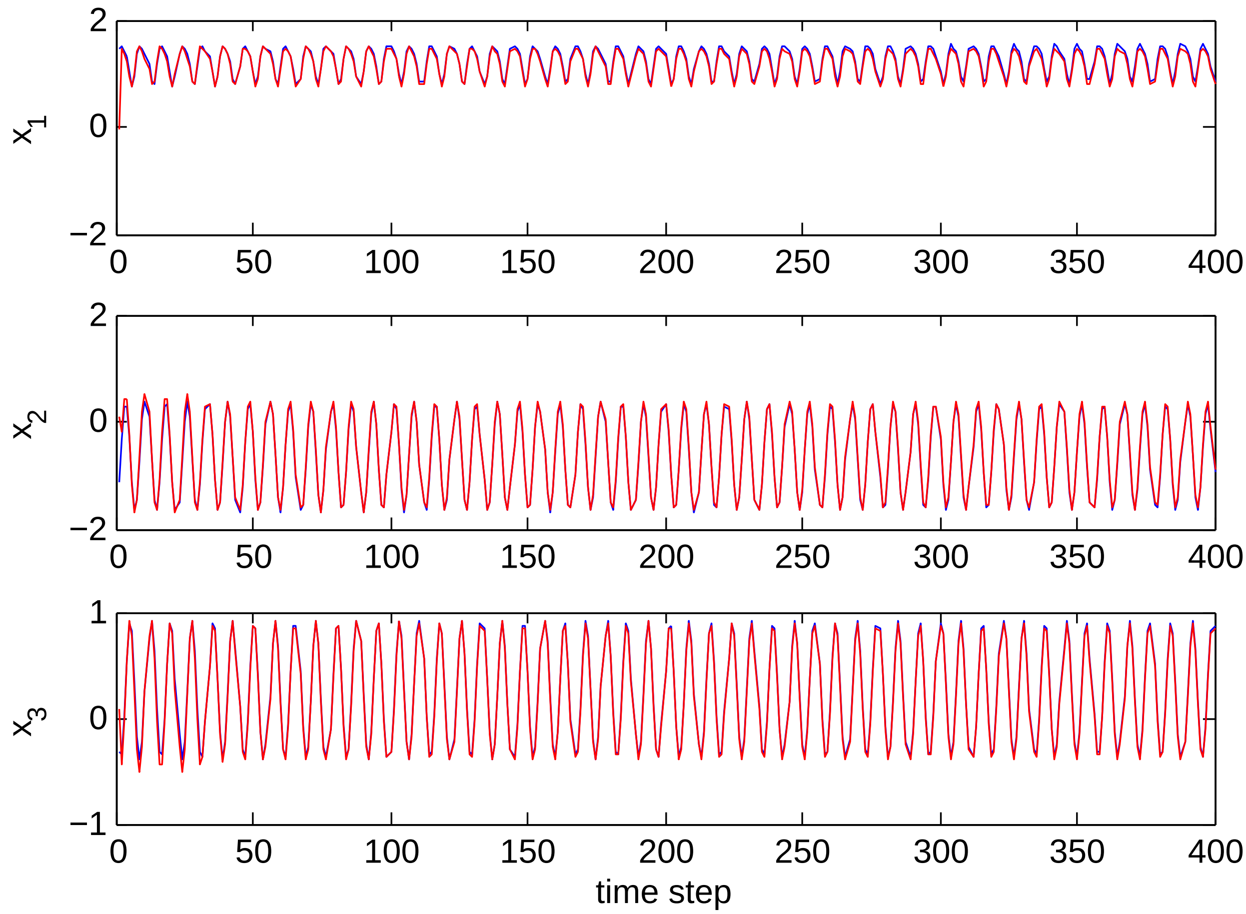

Figure 3.3: Time course plot of the estimated state: $\hat{x}$ dashed lines($--$) and true state: x solid lines ($-$).

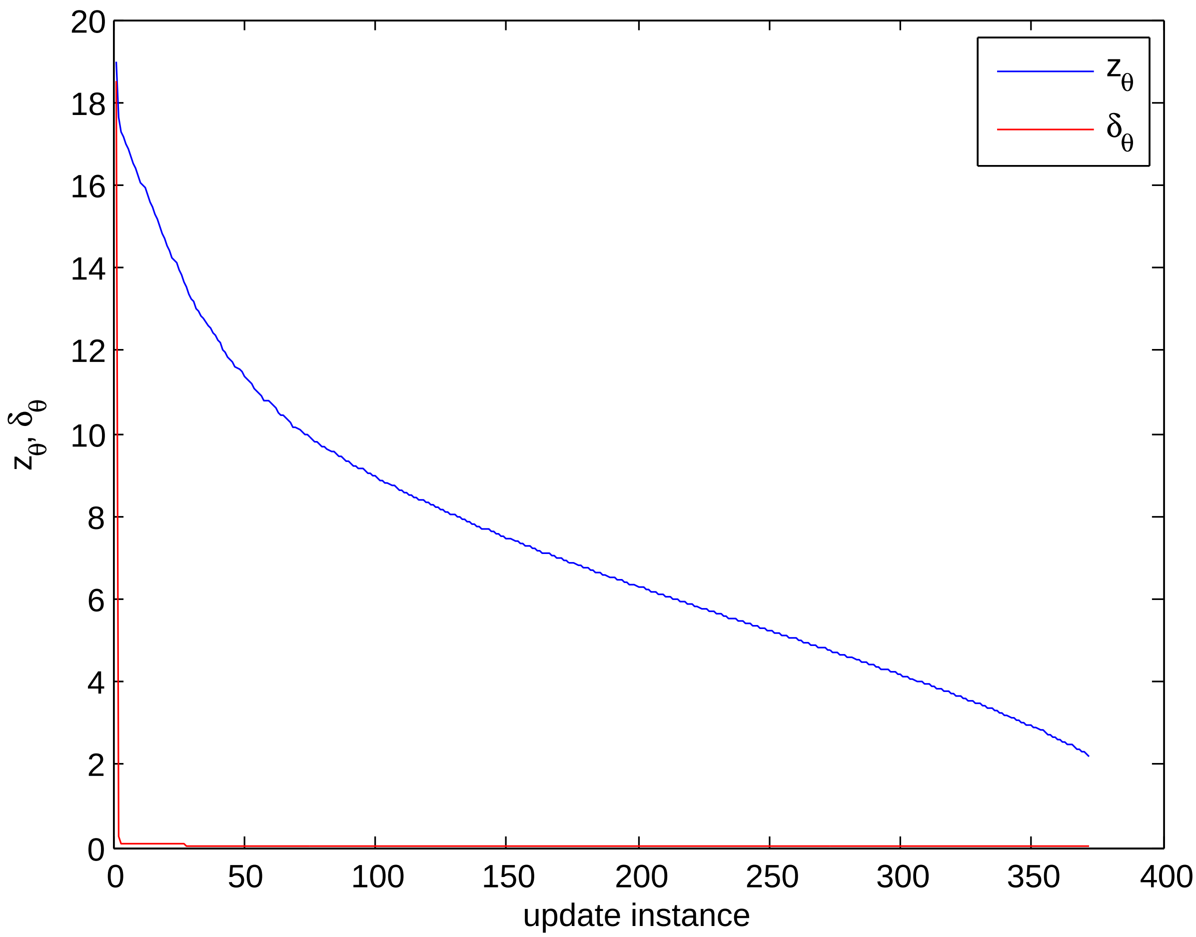

Figure 3.4: The progression of the radius of parameter uncertainty set at time steps when set is updated.

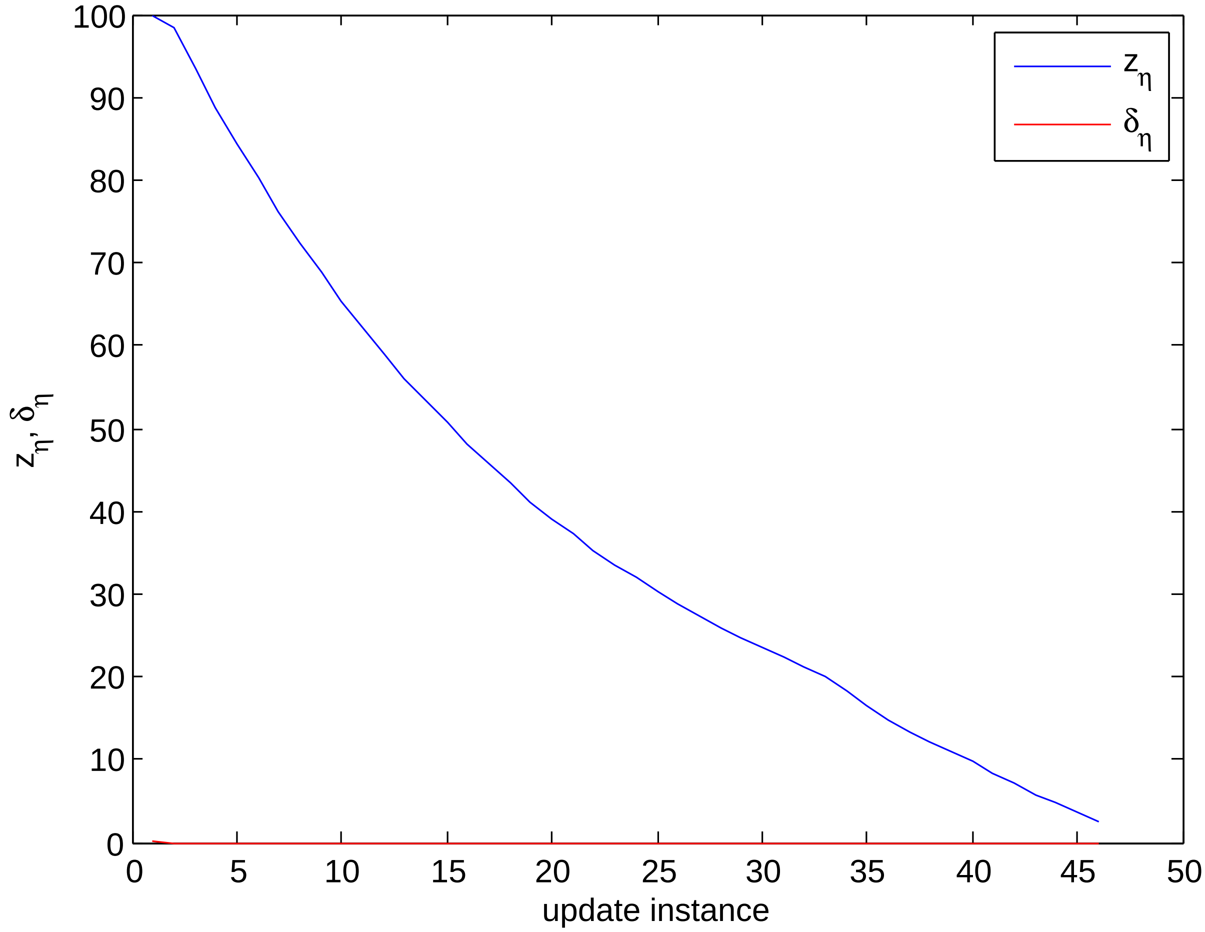

Figure 3.5: The progression of the radius of uncertainty set for η at time steps when set is updated.

Chapter 4

Systems with Time-varying Parameters

Most of the methods for identification of parameters assume constant parameters. However the parameters of practical plants are often time-varying and control systems must rely on the online estimation of the uncertain parameters to ensure a certain degree of closed-loop performance. In this chapter, the set-based technique developed in Chapter 3 is modified for time varying parameters. Apart from estimation, algorithm is developed for fault detection in the system. This method is applied to a non-linear system to demonstrate the effectiveness.

4.1 Introduction

In most practical situations, process parameters cannot be assumed to be constant. Some time-varying behaviour of the parameters must be considered. In [Adetola and Guay, 2010], the authors proposed a novel set-based adaptive estimation with an appropriate adaptation law for the unknown parameters. Following the same concepts developed in Chapter 3 for nonlinear systems identification and constant parameter estimation, the proposed method is derived using a new formulation. The proof of the convergence of the estimates to their true values is achieved using Lyapunov theories.

Apart from parameter identification and state estimation, another important application area of the proposed design is the problem of fault detection. In particular, if the value of the parameter is outside some acceptable bounds, then a fault should be identified and hence this detection can be used to apply a different control in physical plant.

In this chapter, following earlier works [Adetola and Guay, 2009] and [Adetola and Guay, 2010], a set-based adaptive identifier for time-varying parameters is proposed. This method ensures convergence of the parameter to its mean value provided the true parameters fall within the initial uncertainty set. For state estimation, a Luenberger-like observer is chosen to ensure that the continuous-time error dynamics converge to zero asymtotically, i.e $e(t) \to 0$ when $t \to \infty$. The algorithm in [Adetola and Guay, 2009] is modified for both parameter and state estimation to detect the abrupt change in the parameters, which ensures non-exclusion of the true parameter and true state respectively.

This chapter is organized as follows. The problem description is given in Section 4.2. The parameter estimation routine is presented in Section 4.4. State estimation and uncertainty set adaptation are detailed in Section 4.3. This is followed by a simulation example in Section 4.5.

4.2 Problem description

Consider a nonlinear system

$$\begin{aligned}\dot{x} =& Ax + b(y)\theta(t) \\ y =& Hx\end{aligned} \tag{4.1}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^r$ is the output, $\theta(t) \in \mathbb{R}^p$ is an unknown time varying bounded parameter vector assumed to be uniquely identifiable lying within a known compact set $\Theta^0 = B(\theta_0, z_\theta)$, where θ_0 is a nominal parameter value, z_θ is the radius of the parameter uncertainty set. The vector-valued function $b(y)$ is sufficiently smooth. The following assumptions are made about (4.1).

Assumption 2.1: The state variables $x(t) \in \mathbb{X}$ a compact subset of $\mathbb{R}^n$.

Assumption 2.2: The system is observable.

Assumption 2.3: The time varying parameters is such that it satisfies $\int_0^\infty \theta(\tau)d\tau = constant$.

The aim of this work is to provide estimates of the time varying parameters and the state variables, simultaneously.

4.3 State and uncertainty set estimation

4.3.1 State estimation

Let the estimator model for (4.1) be chosen as

$$\dot{\hat{x}} = A\hat{x} + b(y)\hat{\theta}(t) + KHe + w^T\dot{\hat{\theta}}(t), \qquad K > 0, \tag{4.2}$$

Let

$$\theta(t) = \theta_0 + \mu(t),$$

where θ_0 is the average value and $\mu(t)$ is a time varying component of the parameters, such that $\| \mu(t) \| \leq c_1$.

$$\dot{w}^T = (A - KH)w^T + b(y), \qquad w(t_0) = 0. \tag{4.3}$$

resulting in the state prediction error $e = x - \hat{x}$ and an auxiliary variable $\eta = e - w^T\tilde{\theta}(t)$ dynamics:

$$\dot{e} = (A - KH)e + b(y)\tilde{\theta}(t) + b(y)\mu(t) - w^T\dot{\hat{\theta}}(t) \tag{4.4}$$

where $e(t_0) = x(t_0) - \hat{x}(t_0)$,

$$\dot{\eta} = (A - KH)\eta + b(y)\mu(t), \qquad \eta(t_0) = e(t_0) \tag{4.5}$$

An estimate of η is generated from

$$\dot{\hat{\eta}} = (A - KH)\hat{\eta}, \qquad \hat{\eta}(t_0) = e(t_0) \tag{4.6}$$

with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics

$$\dot{\tilde{\eta}} = (A - KH)\tilde{\eta} + b(y)\mu(t), \qquad \tilde{\eta}(t_0) = 0. \tag{4.7}$$

An estimate of η is generated from (4.6) with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics given by (4.7), $\tilde{\eta}(t_0) = \tilde{\eta}^0 \in \chi^0$, where $\chi \triangleq B(0, z_\eta)$, and z_η is set radius found at the latest set update.

Lemma 4.3.1. *[Desoer and Vidyasagar, 1975] Consider the system*

$$\dot{x}(t) = Ax(t) + u(t)$$

Suppose the equilibrium state $x_e = 0$ of the homogeneous equation is exponentially stable. Then,

1. *if $u \in L_p$ for $1 < p < \infty$, then $x \in L_p$*
2. *if $u \in L_p$ for p= 1 or 2, then $x \to 0$ as $t \to \infty$.*

Consider a Lyapunov function

$$V_\eta = \frac{1}{2}\tilde{\eta}^T P \tilde{\eta} \tag{4.8}$$

it follows from (4.7) that

$$\dot{V}_\eta = \frac{1}{2}\tilde{\eta}^T P\Big[(A-KH)\tilde{\eta} + b(y)\mu(t)\Big] + \frac{1}{2}\Big[(A-KH)\tilde{\eta} + b(y)\mu(t)\Big]^T P\tilde{\eta} \tag{4.9}$$

$$\dot{V}_\eta = \frac{1}{2}\tilde{\eta}^T\Big[P(A-KH) + (A-KH)^T P\Big]\tilde{\eta} + \frac{1}{2}\tilde{\eta}^T Pb(y)\mu(t) + \frac{1}{2}\mu(t)^T b(y)^T P\tilde{\eta} \tag{4.10}$$

Using the following Ricatti equation

$$P(A-KH) + (A-KH)^T P = -Q \tag{4.11}$$

$$\dot{V}_\eta = -\frac{1}{2}\tilde{\eta}^T Q\tilde{\eta} + \tilde{\eta}^T Pb(y)\mu(t) \tag{4.12}$$

and

$$\tilde{\eta}^T Q\tilde{\eta} \leq 2\frac{\lambda_{max}(Q)}{\lambda_{min}(P)}V_\eta \tag{4.13}$$

By Young's Inequality

$$\tilde{\eta}^T Pb(y)\mu(t) \leq \frac{c_1 V_\eta}{\lambda_{min}(P)} + \frac{c_1 b(y)^T b(y)}{2\lambda_{max}(P^T P)} \tag{4.14}$$

from (4.12), (4.13) and (4.14)

$$\dot{V}_\eta \leq -\left[\frac{2\lambda_{max}(Q)}{\lambda_{min}(P)}V_\eta\right] + \left[\frac{c_1 V_\eta}{\lambda_{min}(P)} + \frac{c_1 b(y)^T b(y) \parallel P^T P \parallel}{2}\right] \tag{4.15}$$

$$\dot{V}_\eta \leq -\left[\frac{2\lambda_{max}(Q)}{\lambda_{min}(P)} - \frac{c_1}{\lambda_{min}(P)}\right]V_\eta + \frac{c_1 b(y)^T b(y) \parallel P^T P \parallel}{2} \tag{4.16}$$

Considering (4.7), if $\mu(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 4.3.1). Hence, the right hand side of (4.16) is finite.

4.3.2 Set adaptation for η

An update law for the worst-case progress of the η in the presence of time varying parameters is given by

$$z_\eta = \sqrt{\frac{V_{z\eta}}{4\lambda_{min}(P)}} \tag{4.17}$$

$$V_{z\eta}(t_0) = 4\lambda_{max}\Big(P(t_0)\Big)(z_\eta^0)^2 \tag{4.18}$$

$$\dot{V}_{z\eta} = -\left[\frac{2\lambda_{max}(Q)}{\lambda_{min}(P)} - \frac{c_1}{\lambda_{min}(P)}\right]V_\eta + \frac{c_1 b(y)^T b(y) \parallel P^T P \parallel}{2} \tag{4.19}$$

where $V_{z\eta}(t)$ represents the solution of the ordinary differential equation (4.19) with initial condition (4.18). The state uncertainty set, defined by the ball $B(0, z_\eta)$ is updated using (4.6) and the error bound (4.17) according to the following algorithm:

Algorithm 4.3.1. *Error bound z_η, the uncertain ball $\chi \triangleq B(0, z_\eta)$ is adapted online with algorithm:*

1. *Initialize $z_\eta(t_{i-1}) = z_\eta^0, \tilde{\eta}(t_{i-1}) = 0$*
2. *At time t_i, update*

$$\chi = \begin{cases} \Big(0, \chi(t_i)\Big), if z_\eta(t_i) \leq z_\eta(t_{i-1}) - \parallel \hat{\eta}(t_i) - \hat{\eta}(t_{i-1}) \parallel - \frac{\|b(y)\mu(t)\|}{\lambda_{min}(A-KH)} \\ \Big(0, \chi(t_{i-1})\Big), otherwise \end{cases}$$

3. *Iterate back to step 2, incrementing $i = i + 1$.*

Algorithm 4.3.1 ensures that χ is only updated when z_η value has decreased by an amount which guarantees a contraction of the set. Moreover z_η evolution given as in (3.15) ensures non-exclusion of $\tilde{\eta}$ as given below.

Lemma 4.3.2. *The evolution of* $\chi = B(0, z_\eta)$ *under* (4.7),(4.17) *and Algorithm 4.3.1 is such that*

1. $\chi(t_2) \subseteq \chi(t_1), \qquad t_0 \leq t_1 \leq t_2$
2. $\tilde{\eta} \in \chi(t_0) \implies \tilde{\eta} \in \chi(t) \quad \forall t \geq t_0$

Proof. 1. If $\chi(t_{i+1}) \nsubseteq \chi(t_i)$, then

$$\sup_{\tilde{\eta} \in \chi(t_{i+1})} \| \tilde{\eta}(t_i) \| \geq z_\eta(t_i) \tag{4.20}$$

However, it follows from triangle inequality and Algorithm 4.3.1 that χ, at the time of update, obeys

$$\begin{aligned}
\sup_{\tilde{\eta} \in \chi(t_{i+1})} \|\tilde{\eta}(t_i) \| &\leq \sup_{\tilde{\eta} \in \chi(t_{i+1})} \| \tilde{\eta}(t_{i+1}) \| + \| \tilde{\eta}(t_{i+1}) - \tilde{\eta}(t_i) \| \\
&\leq z_\eta(t_{i+1}) + \| \eta(t_{i+1}) - \eta(t_i) \| + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| \\
&\leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| \\
&\quad + \| e^{(A-KH)(t_{i+1}-t_i)} \eta(t_i) + \int_{t_i}^{t_{i+1}} e^{(A-KH)(t_{i+1}-\tau)} b(y)\mu(\tau) d\tau - \eta(t_i) \| \\
&\leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| + \| I - e^{(A-KH)(t_{i+1}-t_i)} \| \eta(t_i) \\
&\quad + \int_{t_i}^{t_{i+1}} e^{(A-KH)(t_{i+1}-\tau)} b(y)\mu(\tau) d\tau \\
&\leq z_\eta(t_{i+1}) + \| \hat{\eta}(t_{i+1}) - \hat{\eta}(t_i) \| + \frac{\| b(y) c_1 \|}{\lambda_{min}(A - KH)} \\
&\leq z_\eta(t_i),
\end{aligned}$$

which contradicts (4.20). Hence, χ update guarantees $\chi(t_{i+1}) \subseteq \chi(t_i)$ and the strict contraction claim follows from the fact that χ is held constant over the update intervals $\tau \in (t_i, t_{i+1})$.

2. We know $V_\eta(t_0) \leq V_{z\eta}(t_0)$ (by definition) and it follows from (4.16) and (4.19) that $\dot{V}_\eta(t) \leq \dot{V}_{z\eta}(t)$. Hence, by the comparison lemma, we have

$$V_\eta(t) \leq V_{z\eta}(t) \qquad \forall t \geq t_0 \tag{4.21}$$

and since $V_\eta = \frac{1}{2}\tilde{\eta}^T P \tilde{\eta}$, it follows that

$$\| \tilde{\eta}^T P \tilde{\eta}(t) \|^2 \leq \frac{V_{z\eta}(t)}{4\lambda_{min}(P(t))} = z_\eta^2(t) \quad \forall t \geq t_0. \tag{4.22}$$

Hence, if $\tilde{\eta} \in \chi(t_0)$, then $\tilde{\eta} \in B(0, z_\eta(t)), \forall t \geq t_0$.

□

4.4 Parameter and uncertainty set estimation

Following [Adetola, 2008], the parameter estimation scheme has been generated for the above mentioned system.

4.4.1 Parameter adaptation

Let $\Sigma \in \mathbb{R}^{n_\theta \times n_\theta}$ be generated from

$$\dot{\Sigma} = wH^T H w^T, \qquad \Sigma(t_0) = \alpha I \succ 0, \tag{4.23}$$

based on Equations (4.2),(4.3) and (4.6), the prefered parameter update law as proposed in [Adetola and Guay, 2009] is given by

$$\begin{aligned} \dot{\Sigma}^{-1} &= -\Sigma^{-1} w H^T H w^T \Sigma^{-1}, \\ \Sigma^{-1}(t_0) &= \frac{1}{\alpha} I, \end{aligned} \tag{4.24}$$

$$\begin{aligned} \dot{\hat{\theta}}(t) =& \text{proj}\left\{\gamma \Sigma^{-1} w H^T H(e-\hat{\eta}), \hat{\theta}(t)\right\}, \\ \hat{\theta}(t_0) =& \theta^0 \in \Theta^0, \end{aligned} \tag{4.25}$$

where $\text{Proj}\{\phi, \hat{\theta}(t)\}$ denotes a Lipschitz projection operator [Krstic et al., 1995] such that

$$-\text{Proj}\{\phi, \hat{\theta}(t)\}^T \tilde{\theta}(t) \le -\phi^T \tilde{\theta}(t), \tag{4.26}$$

$$\hat{\theta}(t_0) \in \Theta^0 \implies \hat{\theta}(t) \in \Theta, \forall t \ge t_0 \tag{4.27}$$

where $\Theta \triangleq B(\hat{\theta}(t), z_\theta)$, where $\hat{\theta}(t)$ and z_θ are the parameter estimate and set radius found at the latest set update respectively.

Theorem 4.4.1. *[Adetola and Guay, 2009] The identifier law* (4.24) *and parameter update law* (4.25) *is such that the estimation error $\tilde{\theta}(t) = \theta(t) - \hat{\theta}(t)$ is bounded. Moreover, if*

$$\int_{t_0}^{\infty} \left[\| \tilde{\eta} \|^2 - \| e - \hat{\eta} \|^2\right] d\tau < +\infty \tag{4.28}$$

where, $c \in \mathbb{R}$ and

$$\lim_{t \to \infty} \lambda_{min}(\Sigma) = \infty \tag{4.29}$$

are satisfied, then $\tilde{\theta}$ converges to zero asymptotically.

Proof. Consider a Lyapunov function,

$$V_{\tilde{\theta}(t)} = \frac{1}{2}\tilde{\theta}(t)^T \Sigma \tilde{\theta}(t)$$

it follows from (4.24), (4.25) and $w^T\tilde{\theta}(t) = e - \tilde{\eta} - \hat{\eta}$ that

$$\dot{V}_{\tilde{\theta}(t)} = \tilde{\theta}(t)^T \Sigma(t) \dot{\tilde{\theta}}(t) + \frac{1}{2}\tilde{\theta}(t)^T w H^T H w^T \tilde{\theta}(t) \tag{4.30}$$

If we assume convergence of $\hat{\theta}$ to average value of parameters, then $\tilde{\theta} = \theta_0 - \hat{\theta}$, where θ_0 is the mean value of the parameter and by Young's Inequality

$$\begin{aligned}\dot{V}_{\tilde{\theta}(t)} \leq & -\gamma(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \gamma\tilde{\eta}^T H^T H(e-\hat{\eta}) \\ & + \frac{1}{2}(e-\hat{\eta})^T H^T H(e-\hat{\eta}-\tilde{\eta}) - \frac{1}{2}\tilde{\eta}^T H^T H(e-\hat{\eta}-\tilde{\eta})\end{aligned} \tag{4.31}$$

$$\begin{aligned}\dot{V}_{\tilde{\theta}(t)} \leq & -(e-\hat{\eta})^T H^T H(e-\hat{\eta})\left(\gamma - \frac{1}{2}\right) + \frac{1}{2}\tilde{\eta}^T H^T H\tilde{\eta} \\ & + (\gamma-1)\tilde{\eta}^T H^T H(e-\hat{\eta})\end{aligned}$$

$$\dot{V}_{\tilde{\theta}(t)} \leq \tilde{\eta}^T H^T H\tilde{\eta} \underbrace{\left[\frac{1}{2} + \frac{(\gamma-1)}{2}\right]}_{L} - (e-\hat{\eta})^T H^T H(e-\hat{\eta}) \underbrace{\left[\left(\gamma - \frac{(\gamma-1)}{2}\right) - \frac{1}{2}\right]}_{M}$$

$$\dot{V}_{\tilde{\theta}(t)} \leq -\mathbf{M}(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \mathbf{L}\tilde{\eta}^T H^T H\tilde{\eta} \tag{4.32}$$

where $\mathbf{M}$ and $\mathbf{L}$ are positive constants, implying that $\tilde{\theta}(t)$ is bounded. Moreover, it follows from (4.32) that

$$V_{\theta(t)}(t) = V_{\tilde{\theta}(t)}(t_0) + \int_{t_0}^{t} \dot{V}_{\tilde{\theta}(t)}(\tau)d\tau \tag{4.33}$$

$$V_{\theta(t)}(t) \leq -\mathbf{M}\int_{t_0}^{t}(e-\hat{\eta})^T H^T H(e-\hat{\eta})d\tau + \mathbf{L}\int_{t_0}^{t}\tilde{\eta}^T H^T H\tilde{\eta}d\tau \tag{4.34}$$

Considering the dynamics of (4.7), if $\dot{\theta}(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 4.3.1). Hence, the right hand side of (4.34) is finite in view of (4.28), and by (4.29) we have $\lim_{t\to\infty}\tilde{\theta}(t) = 0$ □

4.4.2 Parameter set adaptation

An update law that measures the worst-case progress of the parameter identifier in the presence of a disturbance is given by

$$z_{\theta(t)} = \sqrt{\frac{V_{z\theta(t)}}{4\lambda_{min}(\Sigma)}} \tag{4.35}$$

$$V_{z\theta(t)}(t_0) = 4\lambda_{max}\Big(\Sigma(t_0)\Big)(z_\theta^0)^2 \tag{4.36}$$

$$\dot{V}_{z\theta(t)} = -M(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + LV_{z\eta} \tag{4.37}$$

where $V_{z\theta}(t)$ represents the solution of the ordinary differential equation (4.37) with the initial condition (4.36). Moreover from $\eta = e - w^T\tilde{\theta}(t)$

$$|e| \leq |\eta| + |w||\tilde{\theta}|$$

$$|e| \leq z_\eta + 2|w|z_\theta \tag{4.38}$$

From (4.38) and Algorithm proposed below, it is ensured that time-varying parameters are inside the uncertainty set. The parameter uncertainty set, defined by the ball $B(\hat{\theta}_c, z_c)$ is updated using the parameter update law (4.25) and the error bound (4.35) according to the following algorithm:

Algorithm 4.4.1. *1. Intialize* $z_\theta(t_{i-1}) = z_\theta^0, \hat{\theta}(t_{i-1}) = \hat{\theta}^0$

2. *If* $|e| > z_\eta + 2|w|z_\theta$, *increase* z_θ *to arbitrarily large value to keep the true parameter inside the uncertainity set.*

3. *At time* t_i, *update*

$$\left(\hat{\theta}, \Theta\right) = \begin{cases} \left(\hat{\theta}(t_i), \Theta(t_i)\right), & \text{if } \; z_{\theta(t_i)} \leq z_\theta(t_{i-1}) - \parallel \hat{\theta}_i - \hat{\theta}(t_{i-1}) \parallel \\ \left(\hat{\theta}(t_{i-1}), \Theta(t_{i-1})\right), & \text{otherwise} \end{cases}$$

4. *Iterate back to step 2, incrementing* $i = i + 1$.

Algorithm 4.4.1 ensures that Θ is only updated when the value of z_θ has decreased by an amount which guarantees a contraction of the set. Moreover z_θ evolution as given in (4.35) ensures non-exclusion of $\theta(t)$ as given below.

Lemma 4.4.1. *The evolution of* $\Theta = B(\hat{\theta}, z_\theta)$ *under* (4.24), (4.35) *and Algorithm 4.4.1 is such that*

1. $\Theta(t_2) \subseteq \Theta(t_1), \qquad t_0 \leq t_1 \leq t_2$, *excluding the update of set radius by step 2 of algorithm.*

2. $\theta \in \Theta(t_0) \implies \theta \in \Theta(t) \quad \forall t \geq t_0$

Proof. 1. If $\Theta(t_{i+1}) \nsubseteq \Theta(t_i)$, then

$$\sup_{s \in \Theta(t_{i+1})} \parallel s - \theta(t_i) \parallel \geq z_\theta(t_i) \tag{4.39}$$

However, it follows from triangle inequality and Algorithm 3.1 that Θ, at the time of update, obeys

$$\begin{aligned}\sup_{s\in\Theta(t_{i+1})} \parallel s-\hat{\theta}(t_i) \parallel &\leq \sup_{s\in\Theta(t_{i+1})} \parallel s-\hat{\theta}(t_{i+1}) \parallel + \parallel \hat{\theta}(t_{i+1})-\hat{\theta}(t_i) \parallel \\ &\leq z_\theta(t_{i+1})+ \parallel \hat{\theta}(t_{i+1})-\hat{\theta}(t_i) \parallel \\ &\leq z_\theta(t_i),\end{aligned}$$

which contradicts (4.39). Hence, Θ update guarantees $\Theta(t_{i+1}) \subseteq \Theta(t_i)$. And Θ is held constant over update intervals $\tau \in (t_i, t_{i+1})$.

2. We know that $V_{\tilde{\theta}}(t_0) \leq V_{z\theta}(t_0)$ (by definition) and it follows from (4.32) and (4.37) that $\dot{V}_{\tilde{\theta}}(t) \leq \dot{V}_{z\theta}(t)$. Hence, by the comparison lemma, we have

$$V_{\tilde{\theta}(t)}(t) \leq V_{z\theta(t)}(t) \qquad \forall t \geq t_0 \tag{4.40}$$

and since $V_{\tilde{\theta}(t)} = \frac{1}{2}\tilde{\theta}(t)^T\Sigma\tilde{\theta}(t)$, it follows that

$$\parallel \tilde{\theta}(t)^T\Sigma\tilde{\theta}(t) \parallel^2 \leq \frac{V_{z\theta}(t)}{4\lambda_{min}(\Sigma(t))} = z_\theta^2(t) \quad \forall t \geq t_0. \tag{4.41}$$

Hence, if $\theta(t) \in \Theta(t_0)$, then $\theta(t) \in B(\hat{\theta}(t), z_\theta(t)), \forall t \geq t_0$.

□

4.5 Simulation Example

To illustrate the effectiveness of the proposed method, we consider the following system

$$\dot{x_1} = (x_2 + x_3\theta_1(t) - x_3\theta_3) + u_1$$

$$\dot{x_2} = (-x_1 + x_3 + x_3\theta_2 + x_3\theta_3) + u_2$$

$$\dot{x_3} = (-x_1 - 2x_2 - x_3 + {x_3}^2\theta_3) + u_3$$

$$y = Hx$$

where $\theta(t)^T = [\theta_1(t), \theta_2, \theta_3]$, the input is taken as constant, $u = [-0.001 \quad 0.001 \quad 0.002]^T$. The true parameter values are $\theta(t) = [(sin(0.1t)+1.9) \quad 3.1 \quad 0.7]^T$. The initial radius of the uncertainty set for θ is $z_\theta^0 = 100$. The initial radius of the uncertainty set for η is $z_\eta^0 = 100$. Initial conditions for state are $x_0 = [1 \quad 0.3 \quad 0.4]^T$. Initial estimates of the state are $\hat{x}_0 = [5 \quad 1.5 \quad 2]^T$.The center of the parameter uncertainty set is assumed to be $\hat{\theta}_c^0 = [3.5 \quad 4 \quad 2]^T$ at time $t = 0$. For this example, $H = [0 \quad 0 \quad 4]$.

The set-based technique developed in this chapter is applied to the above mentioned simulation example. The adaptive identifier is used to estimate the time-varying parameters and an uncertainty set is defined such that it guarantees to contain the true value of the parameters. Figure 4.1 shows the estimates of the time varying parameters converging asymptotically to the mean values. If the value of the parameter moves outside some acceptable bounds, the algorithm is capable of fault detection. To demonstrate this feature, the mean value of the parameters is forcibly changed to new value. The algorithm detects the fault and the estimates converge to the new mean value of the parameters. Simultaneously an auxiliary variable is used to estimate the unmeasured state variables. The proposed technique updates the estimates only when estimation improvement is guaranteed. The estimates of the unknown states follows the true state value as shown in Figure 4.3. The straight line shows that the error is in the neighborhood of zero. The error associated with state prediction is shown in Figure 4.2. As the uncertainty in the system is injected forcibly by changing the parameters, the algorithm detects the fault and starts updating the state estimates such that the error associated with the state variables converge to zero. The proposed uncertainty set update for parameter identification and state estimation, guarantees to contain the true values at all time instants. As depicted in Figure 4.4, the uncertainty bound z_θ reduces over time and the true parameter always lies within the uncertainty set. Similarly, the distance between the true and estimated parameters, δ_θ is always less than z_θ. As a result of fault detection, the value of z_θ is reset to a large value to ensure the non-exclusion of true parameter values. Similarly, the radius of uncertainty set for η i.e z_η is also decreasing with time as shown in Figure 4.5. It also shows the non-exclusion of the true values of the auxiliary variables η, $\delta_\eta < z_\eta$, which ensures that the true state of the system is estimated. The algorithm detects the forced uncertainty, and ensures the non-exclusion of state from uncertainty set. In this case, the size of the uncertainty, z_η, is set to 100 when a fault due to the parameters is detected.

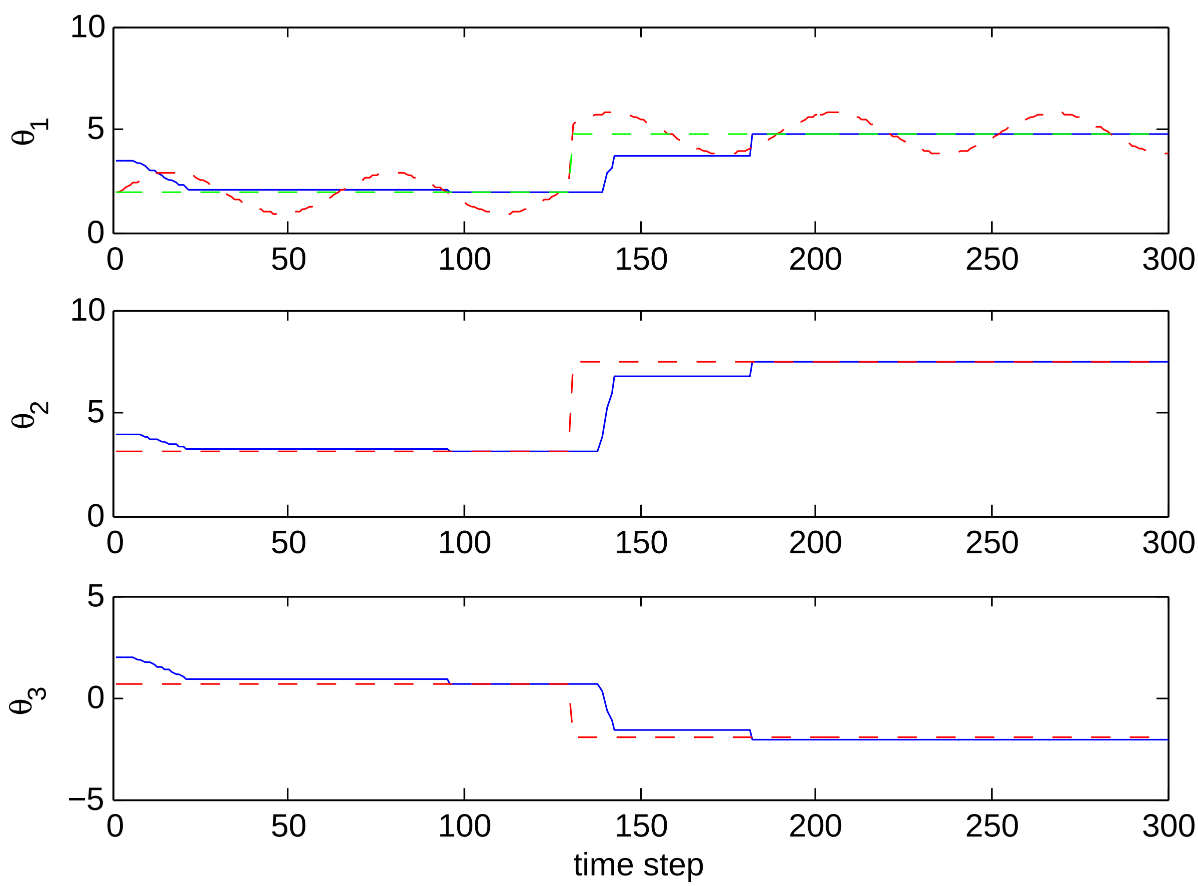

Figure 4.1: Time course plot of the true parameter: θ dashed lines(−−) and parameter estimates: $\hat{\theta}$ solid lines (−).

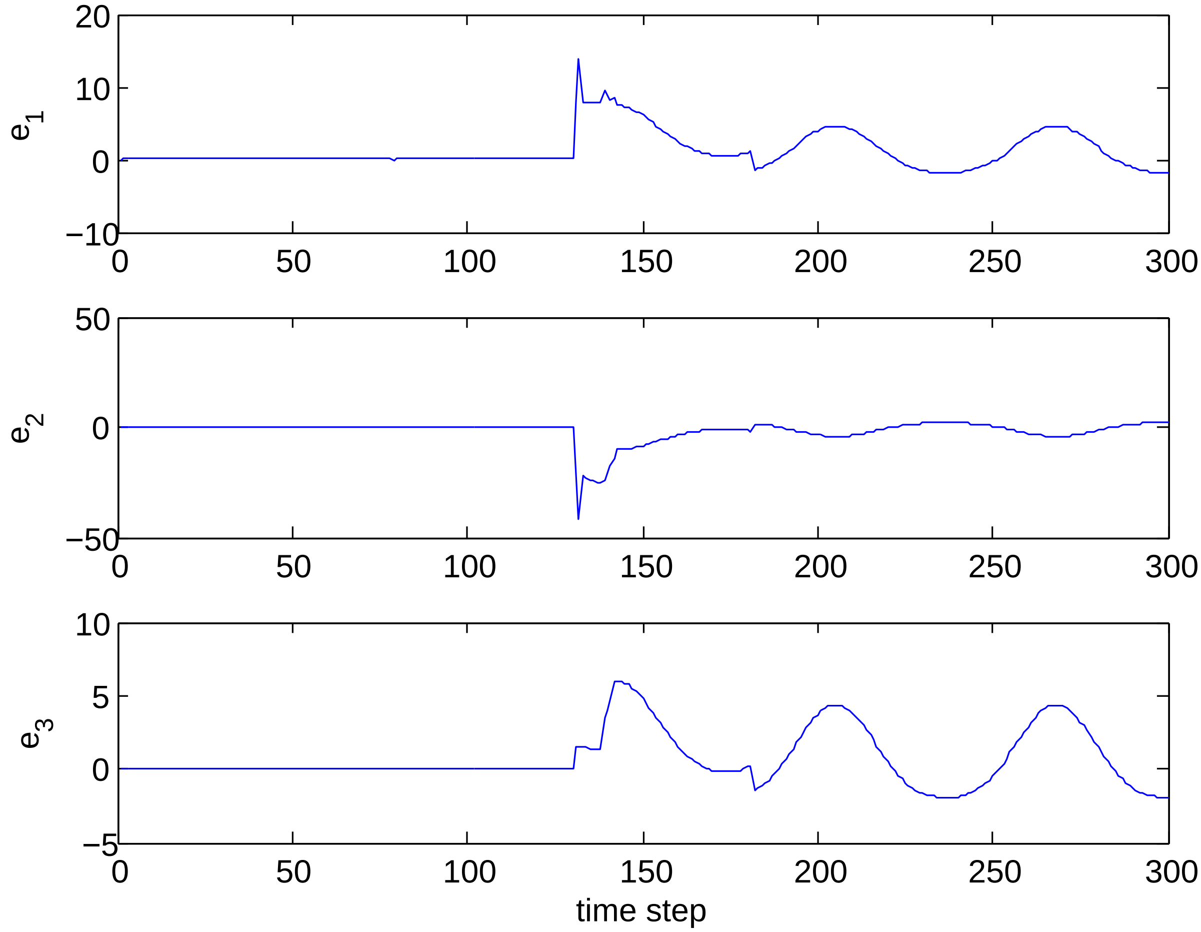

Figure 4.2: Time course plot of the state estimation error $e = x - \hat{x}$.

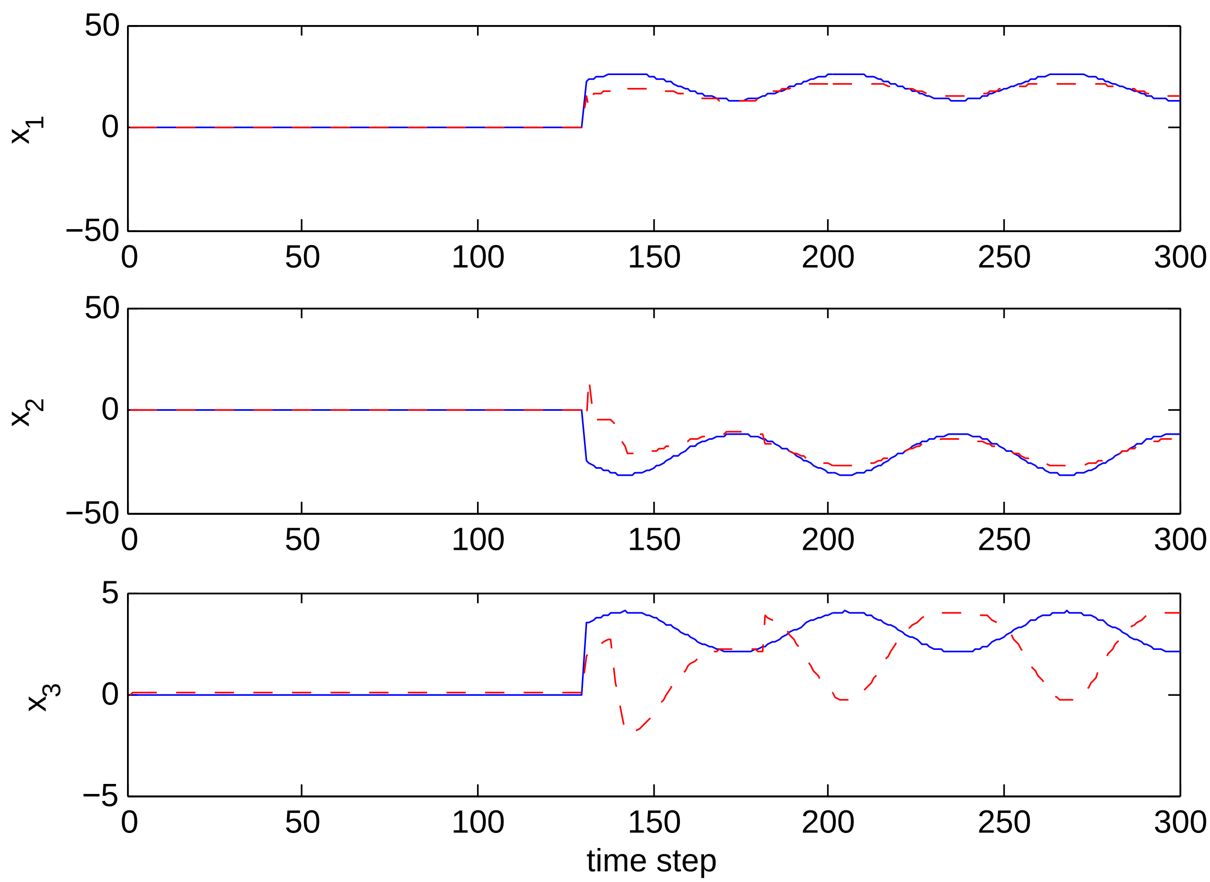

Figure 4.3: Time course plot of the estimated state: $\hat{x}$ dashed lines$(--)$ and true state: x solid lines $(-)$.

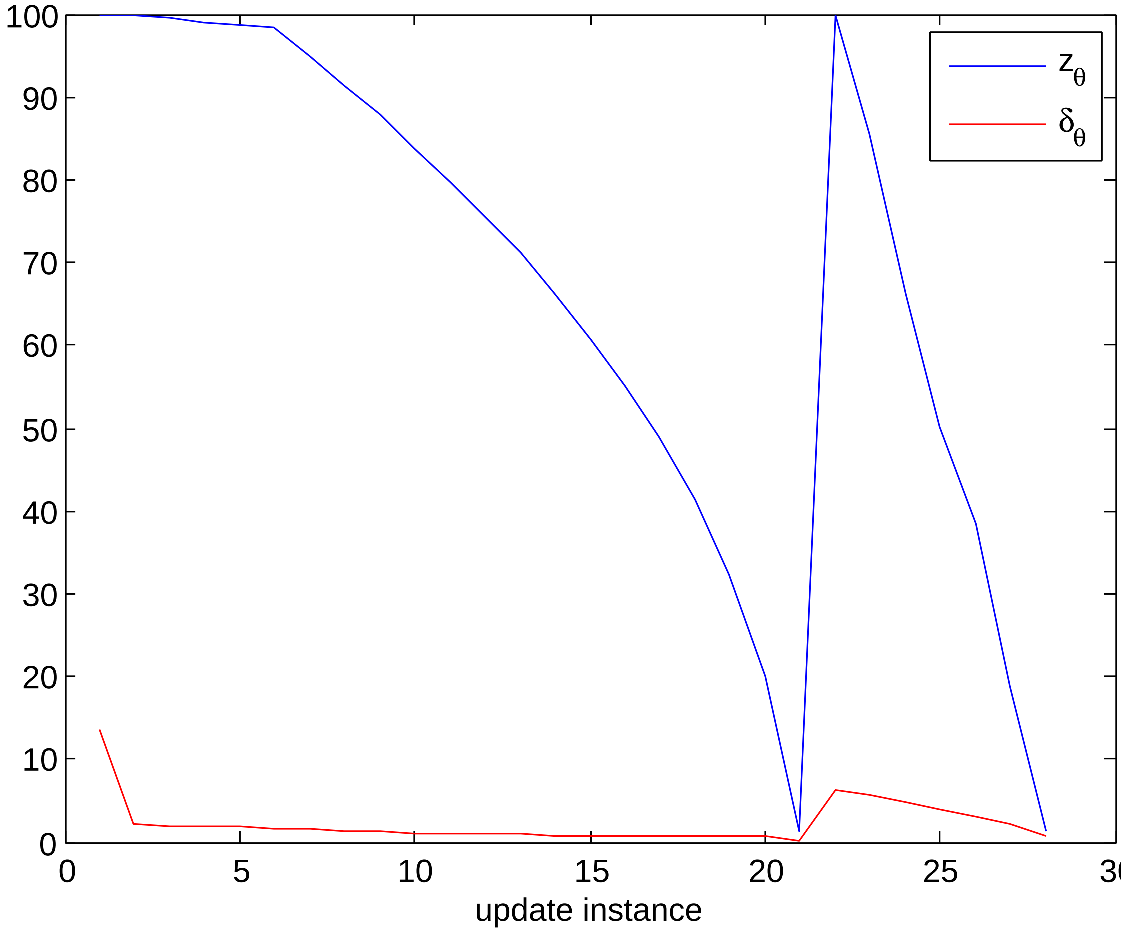

Figure 4.4: The progression of the radius of parameter uncertainty set at time steps when set is updated.

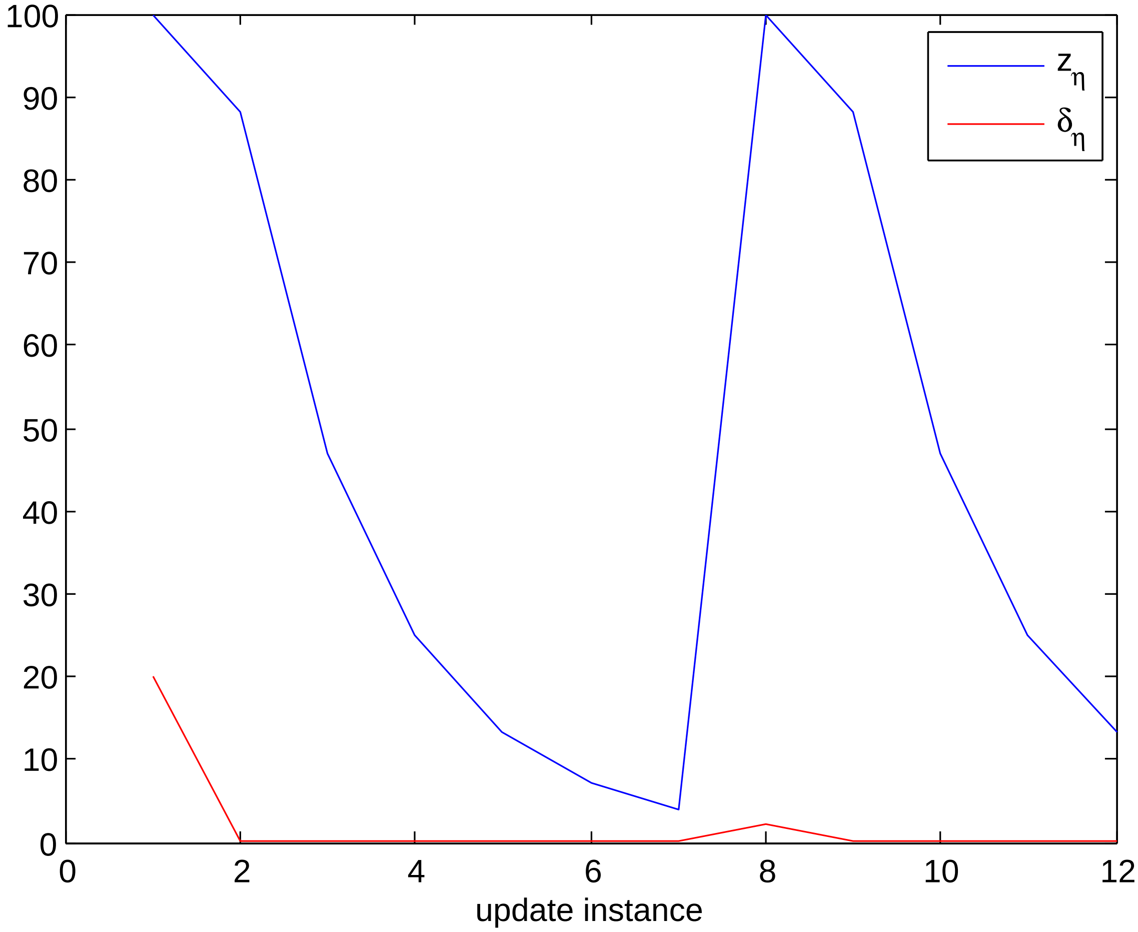

Figure 4.5: The progression of the radius of uncertainty set for η at time steps when set is updated.

Chapter 5

Application: Mixing Tank Problem

The purpose of this chapter is to investigate the performance of the method proposed in Chapter 4 on a simple mixing tank experimental problem. It is a low dimensional problem, and meets all the necessary assumptions from Chapter 4. The model structure of the experimental set up is shown to fit the class of the systems considered in the earlier chapters. The performance of the estimators is tested by comparing the estimations generated by the method with the experimental data.

5.1 System Description

5.1.1 Model development

The flow diagram for this example is shown in Figure 5.1. The experimental set up consists of two inlet streams and one outlet stream. The flow rate of inlet deionized water stream is represented by F_1 (lit/min) and concentration of salt is denoted by C_1 (g/lit). The salt stream is pumping the salt solution in the mixing tank at a flow rate of F_2 (lit/min) and the concentration of salt in the stream is C_2 (g/lit). A constant rpm stirrer is used for mixing the streams in the tank, C_4 (g/lit) is the concentration of the salt present in the tank. The flow rate of the outlet stream is represented by F_3(lit/min) and the concentration of salt in the product stream is C_3 (g/lit). The cross-sectional area of the mixing tank is A (cm^2). h (cm) represents the level of solution in the mixing tank.

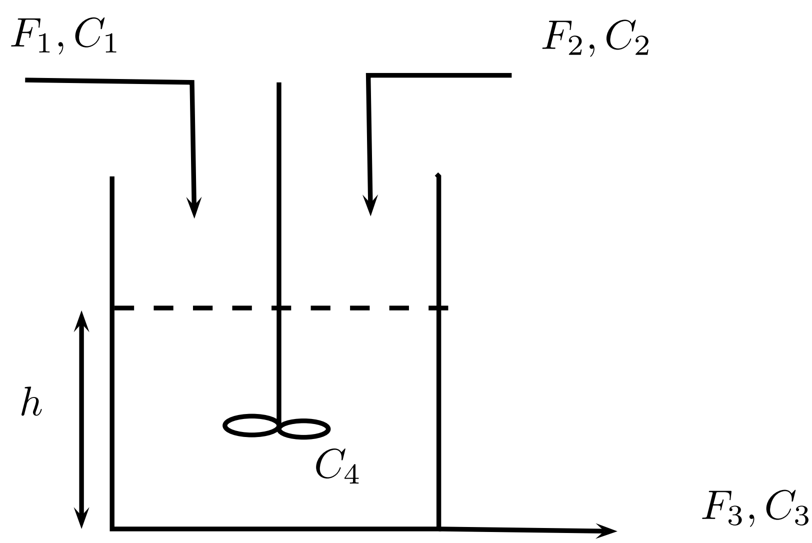

Figure 5.1: Experiment notations

A mass balance yields:

$$F_1 + F_2 = F_3 + \frac{dh}{dt}A$$

and the tank height dynamics can be described by the ordinary differential equation

$$\dot{h} = \frac{1}{A}[F_1 + F_2 - F_3]$$

A salt balance on the system is given by

$$F_2C_2 = F_3C_3 + hA\frac{dC_4}{dt}$$

that yields the salt concentration dynamics:

$$\dot{C}_4 = \frac{F_2C_2}{Ah} - \frac{F_3C_3}{Ah}$$

The following assumptions have been made to simplify the model of the system.

1. The feed has a uniform composition throughout the operation.
2. Mixing is perfect which implies that the exit stream has the same composition as in the tank, $C_3 = C_4$.
3. There is no salt content in stream 1, which implies $C_1 = 0$.

The model can be restated as follows:

$$\begin{pmatrix} \dot{h} \\ \dot{C}_3 \end{pmatrix} = \begin{bmatrix} \frac{1}{A} & \frac{1}{A} & -\frac{1}{A} \\ 0 & \frac{C_2}{Ah} & -\frac{C_3}{Ah} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

The above structure can be written as

$$\dot{x} = b(y)\theta \tag{5.1a}$$

$$y = Hx \tag{5.1b}$$

where

$$\dot{x} = \begin{pmatrix} \dot{h} \\ \dot{C}_3 \end{pmatrix},$$

$$b(y) = \begin{bmatrix} \frac{1}{A} & \frac{1}{A} & -\frac{1}{A} \\ 0 & \frac{C_2}{Ah} & -\frac{C_3}{Ah} \end{bmatrix}$$

and

$$\theta = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

5.1.2 Process Flow Diagram

Figure 5.2 depicts the process flow diagram of the experimental set up along with the position of sensors. The water pump 1 is an inlet pump. The flow rate of this pump can be regulated manually using a rotameter. Pump 2 is pumping salt solution of known concentration from a storage tank to the mixing tank. The solution is well mixed with the help of a stirrer and the mixed solution is pumped out with Pump 3. The flow rates of pumps 2 and 3 are regulated automatically with the help of inputs generated in the *Simulink* and transmitted through MCC (Multiple Control Circuit) as shown in the Figure 5.2.

The height in the mixing tank is measured by a pressure transmitter 'a', located at the bottom of the mixing tank. The outlet flow salt concentration is measured by conductivity meter 'b', located at the outlet stream. The information of these two attributes is transmitted to *Simulink* through MCC.

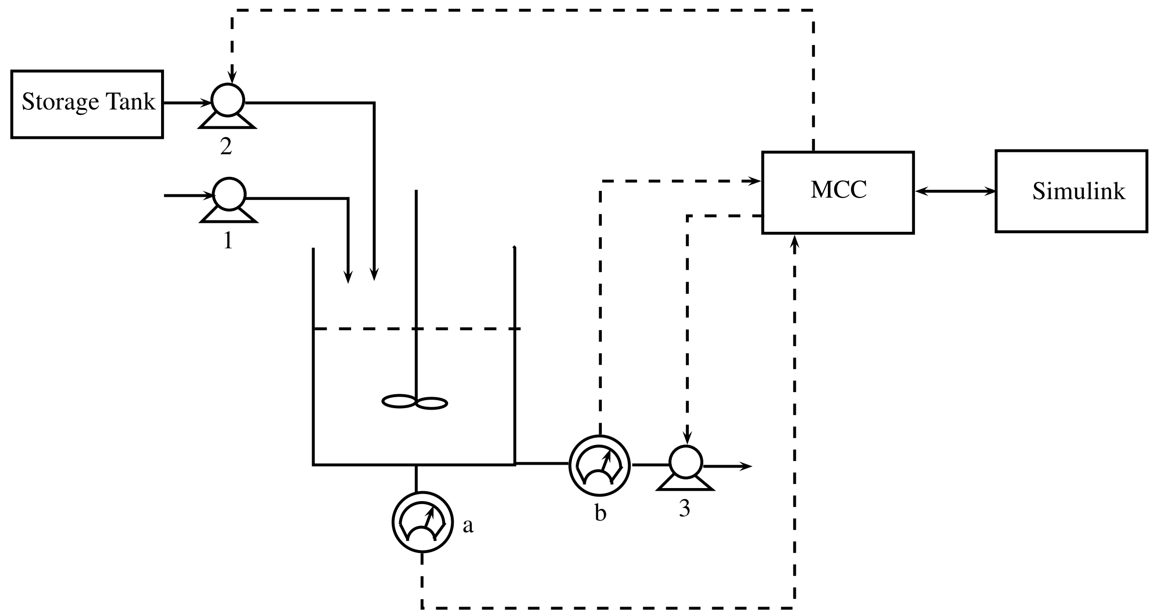

Figure 5.2: Process flow diagram of experimental set up.

5.2 Procedure

5.2.1 Calibration

All the pumps and sensors in the set up are first calibrated. The outputs from the system are recorded in mA(milli Amperes). The relation of the physical attributes to the recorded mA is assumed to be linear. Calibration curve are generated for height, concentration of output stream and flow rates of different pumps. They are given by:

$$\text{height (cm)} = 19.865 \times (\text{Value recorded in Simulink}) - 20.038, \tag{5.2}$$

$$\text{concentration(g/lit)} = 2.5922 \times (\text{Conductivity recorded in Simulink}) - 7.1697, \tag{5.3}$$

for outlet pump,

$$\text{flow rate(lit/min)} = 0.3019 \times (\text{Setting on the pump}) + 0.0121, \quad (5.4)$$

for salt inlet pump,

$$\text{flow rate(lit/min)} = 0.1549 \times (\text{Setting on the pump}) + 0.0067. \quad (5.5)$$

Equations (5.2), (5.3), (5.4) and (5.5) are the relation of height, concentration, outlet pump and intlet pump respectively to the respective signals recorded by the sensors. These relations had been used to calculate the height of solution in the mixing tank, concentration of salt in the outlet stream and flow rates of the pumps, from the corresponding signals generated by the sensors.

5.2.2 Experimental run

A 15 g/l salt solution is prepared in the storage tank. The initial conditions for the experiments are chosen to be 10 g/l and 12 cm for the salt concentration and the height, respectively. A solution of 10 g/l is prepared and is filled in the mixing tank to a level of 12 cm. The initial conditions for the pumps are chosen such that the level of the salt solution remains within reasonable limits of the initial conditions. The deionized water from pump 1 is set to a constant flow rate of 0.4 l/min. The salt solution is pumped by pump 2 at a flow rate of 0.6 l/min. F_2 is set to $F_2 = 0.6 + 0.1sin(0.1t)$. The outlet flow rate is fixed at 1 l/min.

With the above mentioned initial conditions, the experiment is performed and the data is recorded using the *Simulink*. The height and concentration of salt in the mixing tank is calculated using (5.2) and (5.3). This data is used as a reference for comparing the estimated values with the true values of parameters and the state variables. For this example $H = [400 \quad 0; 0 \quad 100]$.

5.2.3 Estimation

The primary aim of the experiment is to investigate the performance of the proposed method for estimation of state and the time varying parameters of the system. The method developed in Chapter 4 is used to generate the estimates of the plant parameter and state variables and is compared with the data from the experiment.

Let the estimator model for (5.1) be chosen as

$$\dot{\hat{x}} = b(y)\hat{\theta}(t) + KHe + w^T\dot{\hat{\theta}}(t), \qquad K > 0, \tag{5.6}$$

Let

$$\theta(t) = \theta_0 + \mu(t) \tag{5.7}$$

$$\dot{w}^T = -KHw^T + b(y), \qquad w(t_0) = 0. \tag{5.8}$$

resulting in the state prediction error $e = x - \hat{x}$ and an auxiliary variable $\eta = e - w^T\tilde{\theta}(t)$ dynamics:

$$\dot{e} = -KHe + b(y)\tilde{\theta}(t) + b(y)\mu(t) - w^T\dot{\hat{\theta}}(t) \tag{5.9}$$

where $e(t_0) = x(t_0) - \hat{x}(t_0)$,

$$\dot{\eta} = -KH\eta + b(y)\mu(t), \qquad \eta(t_0) = e(t_0) \tag{5.10}$$

An estimate of η is generated from

$$\dot{\hat{\eta}} = -KH\hat{\eta}, \qquad \hat{\eta}(t_0) = e(t_0) \tag{5.11}$$

with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics

$$\dot{\tilde{\eta}} = -KH\tilde{\eta} + b(y)\mu(t), \qquad \tilde{\eta}(t_0) = 0. \tag{5.12}$$

Let $\Sigma \in \mathbb{R}^{n_\theta \times n_\theta}$ be generated from

$$\dot{\Sigma} = wH^T Hw^T, \qquad \Sigma(t_0) = \alpha I \succ 0, \tag{5.13}$$

based on Equations (5.6),(5.8) and (5.11), the preferred parameter update law as proposed is given by

$$\begin{aligned} \dot{\Sigma}^{-1} &= -\,\Sigma^{-1} wH^T Hw^T \Sigma^{-1}, \\ \Sigma^{-1}(t_0) &= \frac{1}{\alpha} I, \end{aligned} \tag{5.14}$$

$$\begin{aligned} \dot{\hat{\theta}}(t) =& \text{proj}\left\{\gamma \Sigma^{-1} wH^T H(e - \hat{\eta}), \hat{\theta}(t)\right\}, \\ \hat{\theta}(t_0) =& \theta^0 \in \Theta^0, \end{aligned} \tag{5.15}$$

where $\text{Proj}\{\phi, \hat{\theta}(t)\}$ denotes a Lipschitz projection operator.

The parameter set is adapted at every step according to Section 4.4.2 and Algorithm 4.4.1 is used to guarantee that the parameter estimates converge to the true values.

Similarly, the estimates for auxiliary variable η is generated by (5.11) and the worst case progress of η is given by Section 4.3.2. Convergence of the estimates to the true state variables is ensured by Algorithm 4.3.1.

The simulation is performed using the above mentioned technique. The initial conditions mentioned in Section 5.2.2 are used.

5.3 Results and Discussion

The results obtained by the simulation are shown in the Figures 5.3 to 5.7. Figure 5.3 shows that the estimated flow rates are converging to the mean value of the true flow rates. The estimate of F_2 converges to its true mean value as expected, but the difference between true mean value and estimated value is slightly greater than that for F_1 and F_3. However, it is likely that the time varying behaviour of F_2 contributes to some unmodeled dynamics or delays, present in the pump. The parameter update law and algorithm is successful in finding a very good estimate of the flow rates of the system. Figure 5.4 depicts that the state estimates, i.e. height and concentration of salt in the outlet stream, are in a small neighbourhood of their true values. The height of the solution is well estimated by the method. Although, the estimate for concentration follows the true state, there is a persistent error as shown in Figure 5.5. The possible reason for this behaviour is the location of the salt solution inlet at the bottom of the tank. Moreover, outlet of the tank is very close to the salt solution inlet which may cause some turbulence and nonideal mixing near the outlet. As shown in Figure 5.5, the estimation error for height is very small, and is decreasing with time. As shown in Figure 5.6, the uncertainty bound z_θ reduces over time and the true parameter always lies within the uncertainty set as the distance between true and estimated parameters, δ_θ is always less than z_θ. Similarly, the radius of uncertainty set for η i.e z_η is also decreasing with time as shown in Figure 5.7. It also shows the non-exclusion of the true values of the auxiliary variables η, $\delta_\eta < z_\eta$, which ensures that the true state of the system is estimated.

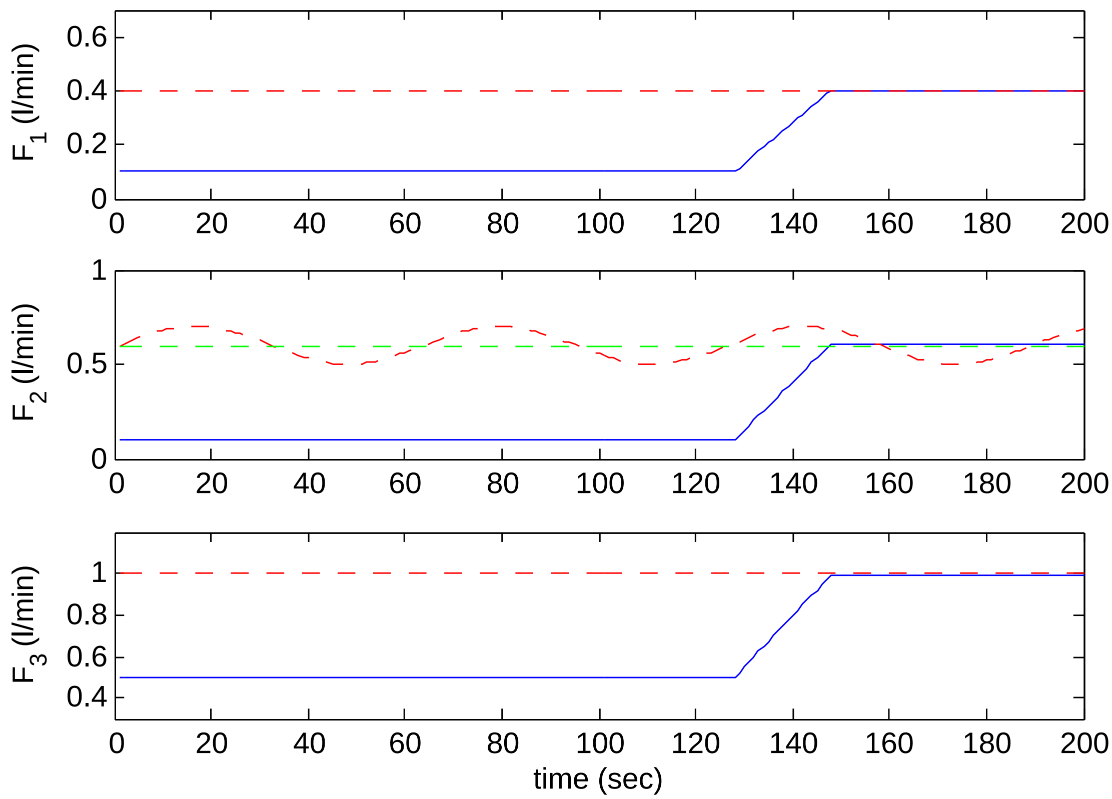

Figure 5.3: Time course plot of the estimates of flow rates.

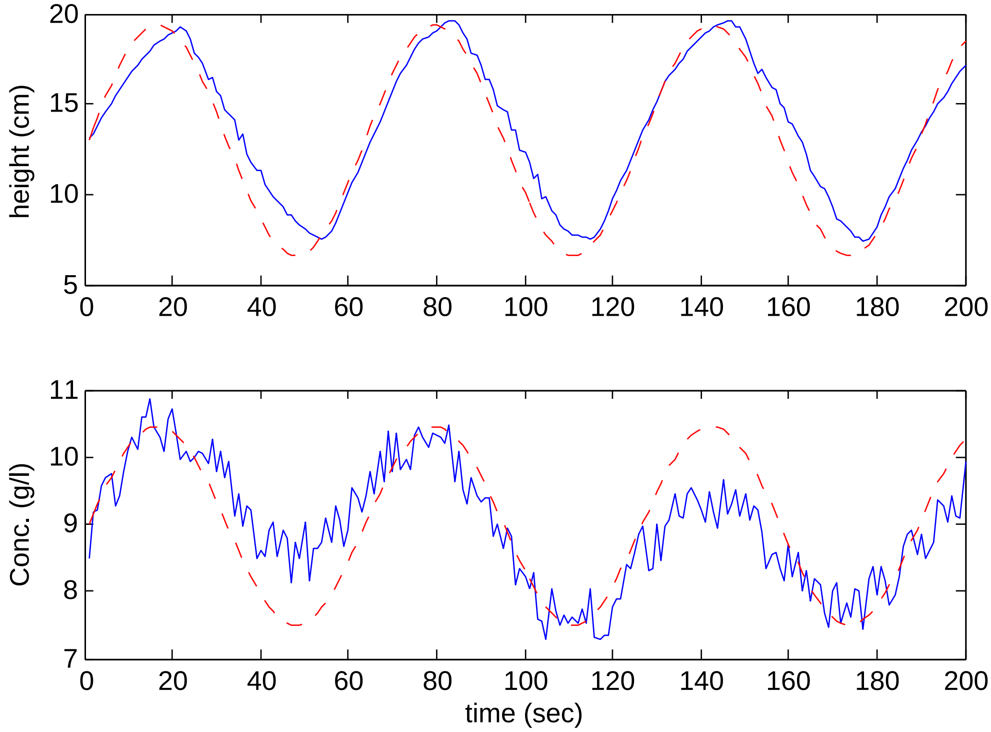

Figure 5.4: Time course plot of the estimated state: $\hat{x}$ dashed lines($--$) and true state: x solid lines ($-$).

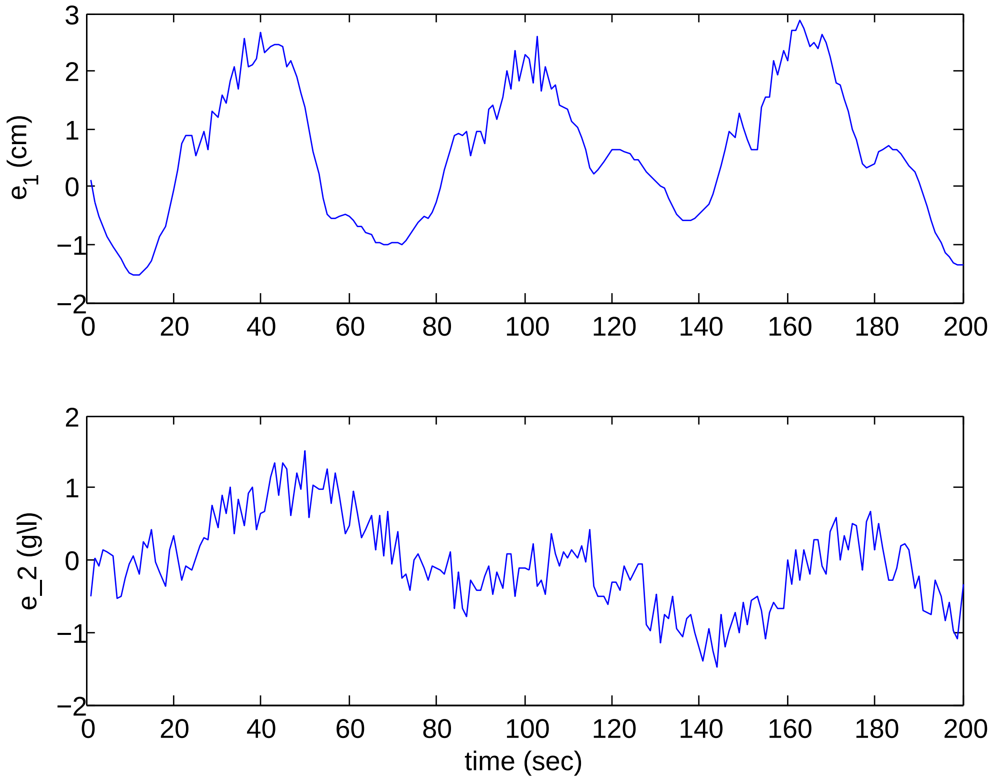

Figure 5.5: Time course plot of the state estimation error $e = x - \hat{x}$.

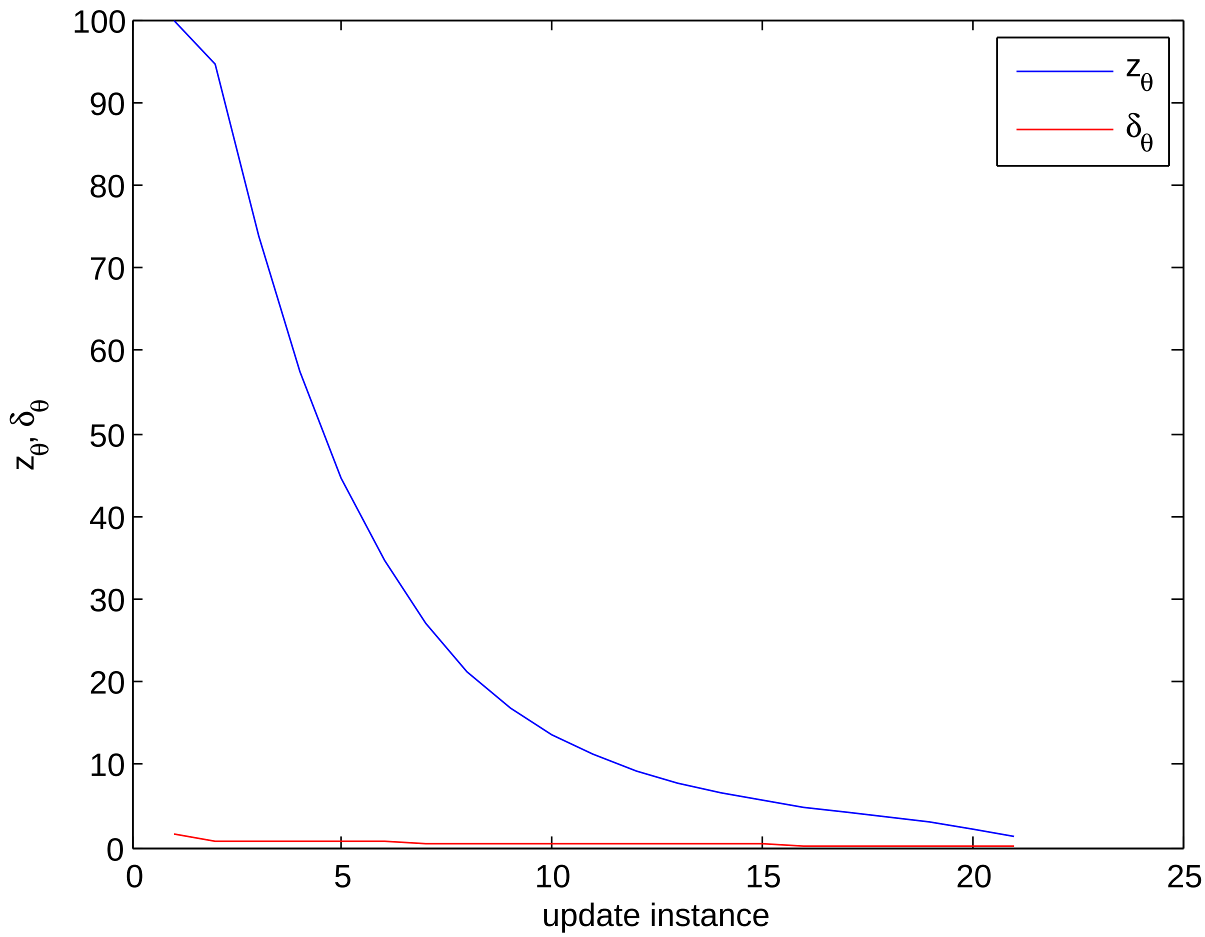

Figure 5.6: The progression of the radius of parameter uncertainty set at time steps when set is updated.

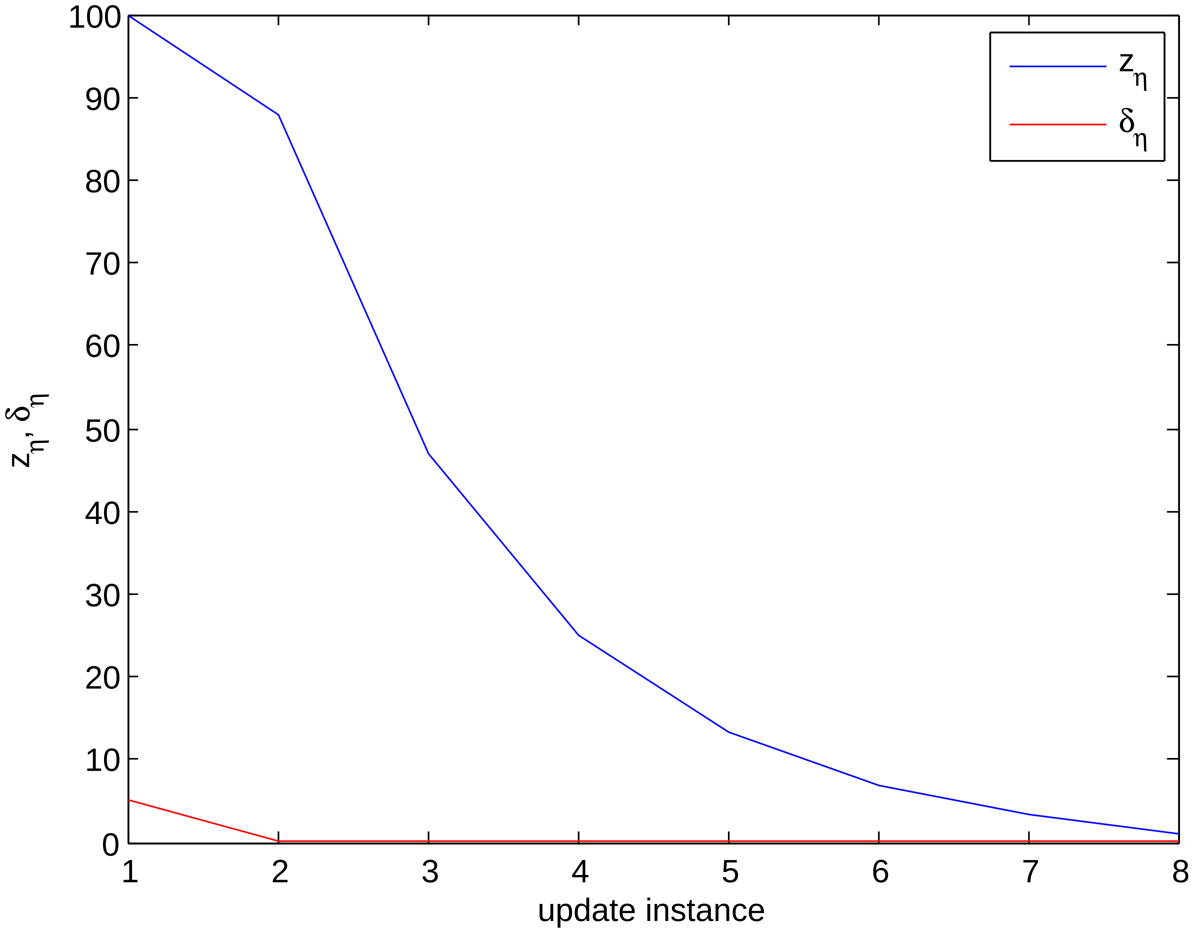

Figure 5.7: The progression of the radius of uncertainty set for η at time steps when set is updated.

Chapter 6

Conclusion and Future Work

A method is proposed for the simultaneous parameter estimation and state estimation of a class of nonlinear systems with constant and/or time-varying parameters. The problem of estimation has been divided into three broader steps. The first step is the state and parameter estimation step. Estimation is performed by using: 1) adaptive law for estimating the parameters and 2) Luenberger observer for estimating the state variables. The second step consists in developing techniques and conditions under which one can guarantee convergence of the state and parameter estimates to their unknown true value. The techniques proposed in this thesis exploits a Lyapunov stability criterion to guarantee boundedness of the estimates and a set-update algorithm to guarantee containment of the unknown parameter values in a computable uncertainty set. The third step is fault detection. employs the state estimation and parameter estimation uncertainty sets, to detect faults. The main contributions of the work are: 1) A set-based technique for estimating unknown state variables in the presence of unknown bounded disturbance. 2) Estimation of unknown parameters (constant and time-varying) using set-based technique. 3) An Algorithm has been developed to detect the faults in the system. 4) Application of the proposed methodology to a practical problem of mixing tank.

A set-based adaptive estimation technique is proposed for simultaneous state estimation and parameter identification of a class of continuous-time nonlinear systems subject to time-varying disturbances. The set-based adaptive identifier for parameters is used to estimate the parameters along with an uncertainty set that is guaranteed to contain the true value of the parameters. Simultaneously an auxiliary variable is used to estimate the unmeasured state variables. Sufficient conditions are given that ensure the convergence of the adaptive observer. The proposed technique updates the uncertainty sets only when estimation improvement is guaranteed. The proposed uncertainty set update for parameter identification and state estimation, guarantees to contain the true values at all time instants. The method guarantees convergence of the parameter estimation error to zero and determines the unknown state of the system in the presence of unknown bounded disturbances. The estimation and identification algorithms have been implemented to a simulation example.

The time-varying parameter problem has been addressed with a modification of the adaptive estimation technique proposed for simultaneous state estimation and parameter identification of a class of continuous-time nonlinear systems with constant parameters. The adaptive identifier along with an uncertainty set update algorithm is defined such that it guarantees the convergence of the parameter estimates and state estimates to true value. Sufficient conditions are given that guarantee the convergence of the adaptive observers. The proposed uncertainty set update for parameter identification and state estimation, guarantees containment of the true values at all time instants. The method guarantees convergence of the parameter estimation error to zero and significantly determines the unknown state of the system. The algorithm detects faults in the system, induced by changing the true parameter value. The algorithm resets the radius of uncertainty set for the parameters as well as the auxiliary variable η to an arbitrarily large value to ensure non-exclusion of true values. As soon as the algorithm detects the fault, it updates the estimates until the convergence to the new parameter and state values has been achieved. The estimation and identification algorithms have been implemented to a simulation example to demonstrate its effectiveness.

The estimation of simple mixing tank with time varying flow rate. The system consists of two inlet streams of different flow rate and concentration of salt. The flow rate of salt stream is time-varying. The parameter and state estimation scheme for time-varying parameters is applied to this practical example. The data obtained from the experimental run is used to compare the estimates with the true value. It is demonstrated that the proposed method estimates the state of the system as well as the time varying parameters accurately, despite imperfect mixing effects of the solutions.

Future research work should be dedicated to the analysis of the robustness of the adaptive identifier and observer for time-varying parameters. It would be desirable to study the stability properties of the method with respect to the bounded unknown disturbances and unmodeled dynamics. The method proposed in this work guarantees the time-varying parameter convergence to the mean value. Another interesting problem would be to find a systematic method for estimating the true value of the time-varying parameter. This could prove useful to relax the assumptions on the magnitude of the rate of change of parameters. As the algorithm developed is capable of detecting faults, future work can be directed in investigating its application to practical fault detection and isolation problems.

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