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A Remanufacturing News-vendor with Pricing and Take-back Pricing

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Abstract

This paper analyzes the problem of a remanufacturing news-vendor with selling and take-back price decision. In our model, the remanufacturer decides selling price, take-back price, and order quantity for new materials. She then uses the stochastic take-back quantity and the new material to meet the stochastic demand comparably to a news vendor setting. We allow demand and take-back supply to be correlated.

In this thesis, we study a production problem with dual input sources: raw materials and recycled or remanufactured take-back items. To answer when mixed-sourcing is best, we analyze the model under deterministic setting first, provide criteria for different sourcing strategies, and give corresponding joint optimal solutions. Assuming that a mixed strategy is optimal, we then analyze the stochastic case, and find the optimal joint decision for raw-material order quantity, selling product price and take-back price.

We find that, when the selling price remains fixed, the optimal take-back price and thus the expected take-back quantity does not change with increased demand and take-back supply variance. Also, the take-back price can exceed the net savings achieved by remanufacturing if consumers take this price into account when purchasing new products. And, the adding of randomness of demand and take-back supply will lower the optimal selling price and thus lower the take-back price.

In future research, we will provide numerical analysis to report the impact and performance if a required recycling level is imposed in the problem; study the remanufacture problems with multiplicative demand function; multiple customer classes, such as the trade-in consideration; or multiple order opportunities, such as postponing the raw material procurement.

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Chapter 1

Introduction

Remanufacturing is an industrial process to manufacture "as good as new" products from used products. The potential environmental benefits of remanufacturing are obvious. Remanufacturing a product can save about 85% of the energy that would otherwise have been used in the production process. Since less new raw materials and energy are consumed, remanufacturing reduces air pollution and lowers greenhouse gas emissions. And the reuse of items reduces water pollution and other environmental impacts by reducing the need for "conventional" waste disposal.

There also exists a big market for remanufacturing. In the United Kingdom, the remanufacturing industry employs more than 50,000 people and contributed around £5 billion to GDP in 2008. In the United States, 25% of plastic beverage containers, 52% of aluminum cans, and 55% of major appliances were recycled in 2008. Shaw Industry, Kodak, and Xerox etc. have shown that remanufacturing can be profitable.

Shaw Industries, in 2008, re-launched the Evergreen plant in Georgia, USA, the world's largest Type 6 Nylon recycling facility. Used carpet is collected across the United States, brought to the Evergreen Nylon Recycling facility and then recycled back to caprolactam, the main precursor for Nylon . This material can be used to make new nylon products that are as soft, aesthetic, and durable as before. More than 95 percent of all material entering Evergreen is now recovered ([Shaw industries(2008)]). Kodak also uses a recycling strategy for their single-use-cameras. Kodak pays photo-finishers a fee to return used single-use-cameras to a collection center. The cameras are then sent to a Kodak factory in Guadalajara, Mexico for recycling and reuse. The rate of recycling for Kodak Single Use Cameras was 84% in 2008 ([Kodak(2008)]). In general, closed-loop remanufacturing or closed-loop recycling seems to be a viable strategy in industries and companies where the useful life of a single product is much shorter than the life-cycle of the recyclable component.

When companies consider a closed-loop remanufacturing strategy they face a range of questions. One such question that we will address in this paper regards the take-back price: should it reflect only potential cost savings or does the take-back price have wider implications? Another question regards the impact of demand and supply uncertainty on the remanufacturing strategy. How does it affect prices, raw material ordering and profits?

To answer these questions we formulate an extension of price-setting news-vendor models [Petruzzi and Dada(1999)] in a closed-loop supply chain setting. According to the recent survey paper by [Guide and Van Wassenhove(2009)], the joint decision for pricing and news-vendor framework with remanufacturing consideration has not been addressed in the literature,

In our model, the remanufacturer decides selling price, take-back price, and order quantity for new materials. She then uses the stochastic take-back quantity and the new material to meet the stochastic demand comparably to a news vendor setting. We assume supply and demand are sensitive to both selling and take-back price, and we allow demand and take-back supply to be correlated.

We then provide the criteria for different sourcing strategies under the deterministic case, and find the optimal joint decision for stochastic demand and supply. We find that, when the selling price remains fixed, the optimal take-back price and thus the expected take-back quantity does not change with increased demand and take-back supply variance. And when the selling price and the impact of the take-back price on demand, and thus revenues, is large enough, it is possible that the optimal take-back price exceeds the difference between the cost of raw material and cleaning/recycling cost. We also find our model can be translated to a special version of classical news-vendor model plus a quadratic term, and the appearance of randomness of demand and take-back supply lowers the optimal selling price, thus the take-back price.

We briefly review related literature in chapter 2. We describe and formulate our model in chapter 3, and give the optimal solution in chapter 4. In chapter 5, we *report* on the solution algorithm and its use on a constructed example for Kodak's single-use cameras. In chapter 6, we summarize the contribution and limitation of this work and possible directions for future.

Chapter 2

Literature Review

[Guide and Van Wassenhove(2009)] formulate the five phases of CLSC research as: (i) Remanufacturing as a technical problem (ii) From remanufacturing to valuing the reverse-logistics process (iii) Coordinating the reverse supply chain (iv) Closing the loop (v) Prices and markets . Our paper, is located at phase iv and phase v. We consider an integrated closed-loop supply chain, and focus on the joint decision of price, take-back price, and virgin material inventory, in a newsvendor setting. To capture all pricing effects, we let supply and demand be sensitive to both selling and take-back price in a linear fashion.

The remanufacturing literature has been expanding since the 1990's and describes various business models.

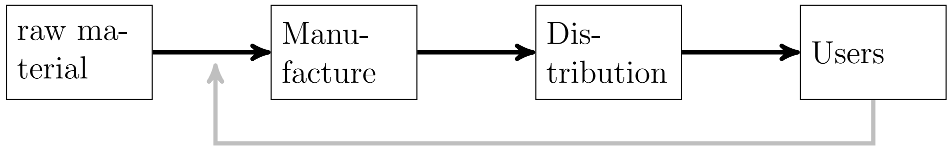


Figure 2.1: Remanufacturing Model 01

[Martijn Thierry(1995)] and [Guide et al.(2000)] describe an ideal remanufacturing System (Figure 2.1). In this framework, used item are collected, processed and combined with new raw material to create inputs to manufacturing process. Later, at the distribution stage, the demand is satisfied. In our paper, we follow this model.

Few models in the literature use this closed-loop recycling framework. [Atasu and Çetinkaya(2006)] discuss the inventory decision for new and remanufactured product, where the remanufactured products are perfectly substitutable for the new product, by consider the impact of shipping delay time and quantity of used items in a deterministic setting over a finite life-cycle. [Guide et al.(1998)] examine the impact of different delay buffers on a closed-supply-chain. [R. Teunter, E. van der Laan(2002)] show that an average cost model may not be appropriate for reverse logistics inventory. [Toktay, Wein, and Zenios(2000)] study the ordering policy using a queueing network model in the context of Kodak's single-use camera. However, these papers are based on the continuous time review model, and only focus on inventory management and not on pricing decisions.

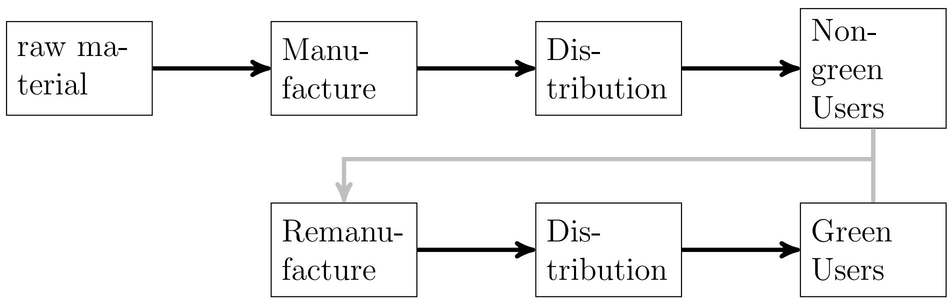


Figure 2.2: Remanufacturing Model 02

Based on the periodic review model, the literature can further be split into two main streams.

One stream splits the market into two parts: Non-green customers and green customers (Figure 2.2). The non-green customer only buys the item made from virgin raw materials, and the green customer will buy the item made from take-back items. Game theoretic models are used to analyze the competition between the two kinds of product and the competition with other manufacturers. E.g. [Ovchinnikov(2009)] studies the joint pricing and remanufacturing strategy of a firm that offers both new and remanufactured products in a deterministic setting.

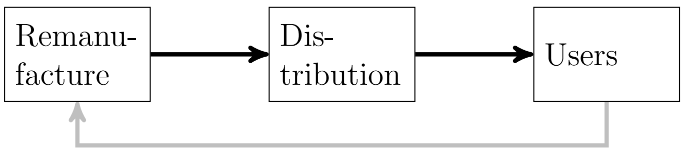


Figure 2.3: Remanufacture Model 03

The third stream considers the case that the inventory only comes from used-items (Figure 2.3). They focus more on how to set the sorting policy based on the condition of take-back item. [Guide et al.(2001)] consider a price sensitive multi-grade model in a deterministic setting. In their model, they split the take back item into n grades by quality, with the supply of each grade independent and increased by acquisition price. There is only one grade for sale, and the demand is a decreasing function of selling price, independent of supply. [Galbreth and Blackburn(2006)] analyze used products quantity and recovery rate to meet both deterministic and stochastic demand.

[Bakal and Akcali(2006)] investigate the effects of recovery yield rate on pricing decisions under deterministic demand and supply.

There are two papers related closely to our paper. [Ferrer and Swaminathan(2006)] consider a two/multi period remanufacturing problem under deterministic setting with demand sensitive to price. In their model, the manufacturer sells new products in the first period, and sells both new and remanufactured items in the second period. The available remanufacturing quantity is a fixed rate of last period's demand. And they decide the selling price and inventory level. Our model is different from theirs as (i) we consider acquisition(take-back) price, (ii) remanufacturing quantity is sensitive to both selling and take back price, and (iii) our demand and supply are random. [Ray, Boyaci, and Aras(2005)] study the optimal price and trade-in rebates to meet deterministic demand. Our model is different from theirs as (i) [Ray, Boyaci, and Aras(2005)] only take back used items when the customer buys a new one (our model do not have this constraint), (ii) in our model the demand is also affected by the take-back price, and (iii) we consider stochastic demand and supply.

Table 2.1 provides an overview of this literature. Our paper contributes to literature since we are the first to consider the pricing and inventory issue with remanufacturing under stochastic setting.

The model we present clearly is also an extension of pricing in the newsvendor problem. [Petruzzi and Dada(1999)] review pricing in the newsvendor problem. They point out that under certain assumptions, a unique optimal solution exists. Our paper adds the remanufacturing factor into the model and also shows that the unique optimal solution

	-review model	Frame†	Selling Price	Take-back Price	Demand
[Atasu and Çetinkaya(2006)]	Continuous time	Frame 01	Fixed	Fixed	Deterministic
[Guide et al.(1998)]	Continuous time	Frame 01	Fixed	Fixed	Deterministic
[R. Teunter, E. van der Laan(2002)]	Continuous time	Frame 01	Fixed	Fixed	Deterministic
[Toktay, Wein, and Zenios(2000)]	Continuous time	Frame 01	Fixed	Fixed	Stochastic
[Ovchinnikov(2009)]	Periodic	Frame 02	DV	None	Deterministic
[Guide et al.(2001)]	Periodic	Frame 03	DV	DV	Deterministic
[Galbreth and Blackburn(2006)]	Periodic	Frame 03	Fixed	DV	Stochastic
[Bakal and Akcali(2006)]	Periodic	Frame 03	DV	DV	Deterministic
[Ferrer and Swaminathan(2006)]	Periodic	Frame 01	DV	Fixed	Deterministic
[Ray, Boyaci, and Aras(2005)]	Periodic	Frame 01	DV	DV	Deterministic
Our paper	Periodic	Frame 01	DV	DV	Stochastic

Table 2.1: Overview of remanufacture model

†Frame 01 represents Figure 2.1, Frame 02 represents Figure 2.2, Frame 03 represents Figure 2.3.

††DV here is short for ‘Decision Variable’.

exists when certain conditions are satisfied.

Chapter 3

Model

We consider the problem faced by a remanufacturer that must determine the selling price of its product p_N, the take-back price for used items p_R, and raw material order quantity q in a one period news-vendor environment. At the beginning of the period, we decide p_N, p_R and q. Later, we collect take-back items. Let R be the quantity of take-back items and R is a random variable. After cleaning and refining these take-back items at a cost c_R per unit, we use them to add to our inventory of raw materials that cost c per unit, thus we have $R + q$ items in our inventory. In the end, we use these items to meet our demand D that is also a random variable. We sell all the inventory at a price p_N if the demand exceeds inventory, otherwise, we sell the leftover inventory at a salvage price s. The flow of the model can be viewed in figure 3.1.

Our objective is to maximize the expected profit. To build the inventory, we pay $c \cdot q$ for raw materials and $(p_R + c_R) \cdot R$ for take-back items. After the demand is realized, we earn $p_N \times \min\{D, q + R\}$ for selling products, and we obtain $s \times \max\{q + R - D, 0\}$ salvage revenue

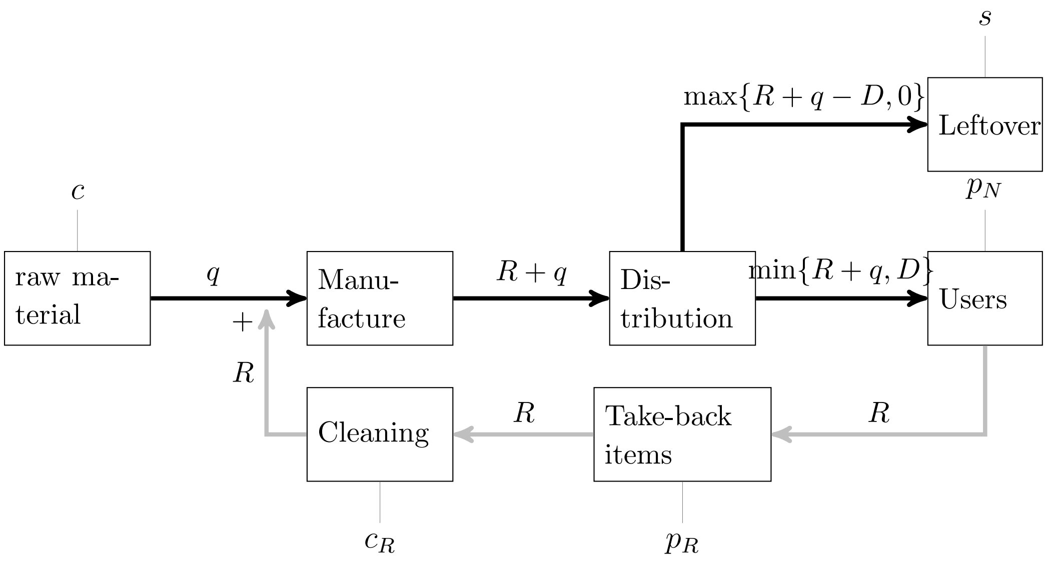


Figure 3.1: Remanufacture Model with Control Flow

p_N	Selling price of a new unit	Decision variable
p_R	take-back price of a used unit	Decision variable
c	Collecting price of a raw material	
c_R	Remanufacture cost of a used unit	
s	Salvage value of a unit, $s < c$	
D	number of new-item-Demand	Random variable
μ_D	the mean of Demand D	Dependent variable
R	quantity of take-back item	Random variable
μ_R	the mean of supply R	Dependent variable
q	Order quantity of raw materials from other supplier	Decision variable
ε_D	Random variable represent the variance of demand. Assume $E[\varepsilon_D] = 0$	Random variable
$f_1(\cdot)$	Probability density function of ε_D	
$F_1(\cdot)$	Cumulative distribution function of ε_D	
ε_R	Random variable represent the variance of supply. Assume $E[\varepsilon_R] = 0$	Random variable
$f_2(\cdot)$	Probability density function of ε_R	
$F_2(\cdot)$	Cumulative distribution function of ε_R	

Table 3.1: Parameters and Function Used

for leftover inventory.

The objective function can be formulated as equation (3.1).

$$
\begin{aligned}
\max \quad E[\Pi(p_N, p_R, q)] &= E\big[p_N \cdot \min\{D, q+R\} + s \cdot \max\{q+R-D, 0\} \\
&\quad -(p_R + c_R) \cdot R - c \cdot q\big] \qquad (3.1)
\end{aligned}
$$

We express demand D and take-back supply R in an additive form of selling price p_N and take price p_R. That is, we let demand be a linear decreasing function of selling price, and take-back supply a linear increasing function of take-back price.

$$
\begin{aligned}
D &= \alpha_D^0 - \beta_D^0 \cdot p_N \\
R &= \alpha_R^0 + \gamma_R^0 \cdot p_R
\end{aligned}
$$

Consider further that a customer who wants to buy a new product, will view a high take-back price as a positive factor in their buy decision. First, the take-back price can be viewed as a future bonus; second, a customer who wants to replace their used product, could think of the take back price as a discount. So the take-back price can affect the demand for new products. A reciprocal effects holds as well. A higher selling price p_N will stop customers from replacing their unit, and thus shrink the take-back supply market. We refine the demand and take-back supply as a linear function of both selling price and take-back price. By adding the random effect ε_D and ε_R, we have the final version of demand and take-back supply functions.

$$D = \alpha_D - \beta_D \cdot p_N + \gamma_D \cdot p_R + \varepsilon_D \qquad (3.2)$$

$$R = \alpha_R - \beta_R \cdot p_N + \gamma_R \cdot p_R + \varepsilon_R \qquad (3.3)$$

We also assume that:

$$\beta_D > \gamma_D, \qquad \beta_D > \beta_R$$
$$\gamma_R > \gamma_D, \qquad \gamma_D > \beta_D$$

This dominant assumption is very common in literature ([Talluri and Ryzin(2005)], [Maglaras and Meissner(2006)]). [1] In our model, it means the demand is more sensitive to the selling price than the take back price and that the selling price has more effect on demand than supply, and vice versa.

Now we have three decision variables(p_N, p_R, q), and two dependent variables (D and R). We analyze the constraints on them one by one.

- Order quantity, q In the classical news vendor problem, q should be greater than zero. However, in our case, things change. We have two input resources: raw materials and take back items. In some situations, we will have too many take-back items, thus besides satisfy the demand, we also have extra units. To simplify our model, we assume the remanufacturer can clean the remaining items and turn them into raw materials, then sell them to the market at the same price of the raw material cost, c. And we can use a negative q to denote this situation. Thus, we do not have constraint on order quantity.

- Selling price, p_N Customers pay p_N to acquire the product. The

[1] [Talluri and Ryzin(2005)] and [Maglaras and Meissner(2006)] use it to describe the relationship between multiple price and demand, then they can grantee the matrix is invertible and that its eigenvalues have positive real parts.

selling price should not be smaller than salvage value, s, otherwise, the remanufacturer would like to keep the product to the end of the period instead of satisfying demand. On the other hand, since we can sold things back to raw material market, we would prefer to sold thing to the raw material market if p_N is lower than c. And we have $c > s > 0$, the constraint on selling price is $p_N > c$

- Take-back price, p_R In our model, to acquire a used item we need to pay p_R to customers. However, we note that in the real life, people need to pay a recycling fee to deal with their used items sometimes (in Europe, car, IT product, etc.). To contain this situation, we allow p_R be smaller than zero. A negative p_R denote the customer will pay a recycling fee to the remanufacturer to deal with their used items instead of that the remanufacturer give the customer a take-back fee. Thus, we do not have constraint on take-back price.

- Demand quantity, D & Take-back quantity, R Clearly, the negative demand and supply are meaningless. Thus, both quantity should be greater than zero, that is $D > 0$ and $R > 0$.

We summarize this chapter by given the mathematic version of our model.

$$\max_{p_N, p_R, q} \quad E[\Pi(p_N, p_R, q)] \tag{3.4}$$

Subject To

$$\begin{aligned}
\Pi(p_N, p_R, q) &= p_N \cdot \min\{D, q+R\} + s \cdot \max\{q+R-D, 0\} \\
&\quad -(p_R + c_R) \cdot R - c \cdot q \\
D &= \alpha_D - \beta_D \cdot p_N + \gamma_D \cdot p_R + \varepsilon_D \\
R &= \alpha_R - \beta_R \cdot p_N + \gamma_R \cdot p_R + \varepsilon_R \\
D &\geq 0 \\
R &\geq 0 \\
p_N &\geq c
\end{aligned}$$

Chapter 4

Solution

As we have mentioned in chapter 3, our model has dual input sources: (i) raw materials, and (ii) take-back items. It will be really useful to know when to use both sources and when to use only one of them. We use the deterministic case to answer this question. Once we have ensured that we would like to use both sources, we move to the stochastic case, which allows us to study closed-loop supply chains with a higher degree of realism.

4.1 Deterministic Case

For deterministic case, we let

$$\epsilon_D \equiv 0$$
$$\epsilon_R \equiv 0$$

As we have noted in chapter 3, there is no constraint on order quantity, q, which follows Lemma 1.

Lemma 1. *For fixed p_N and p_R, the optimal order quantity q^* equal to $D - R$.*

By using Lemma 1, we can eliminate our problem to fix two variables, p_N and p_R. And to find the optimal solution of p_R and p_N, we first need to know the feasible space.

We draw an example of feasible space in Figure 4.1. The feasible

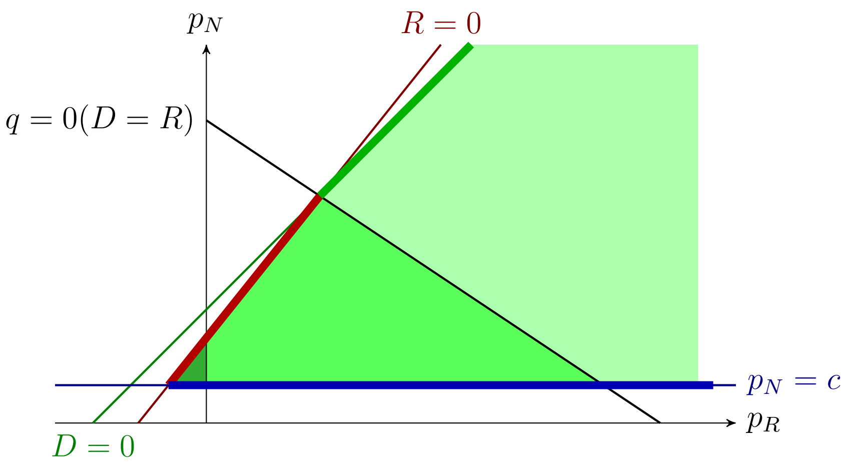


Figure 4.1: An example of feasible space in the deterministic setting

space of (p_N, p_R) is constrained by $R = 0$ (red line), $D = 0$ (green line), and $p_N = c$ (blue line). Furthermore, we split the feasible space into three parts by $p_R = 0$ and $q = 0$. Different part of the feasible space represents different strategy. We explain the strategies one by one.

- **Green part.** The solution within green part satisfies $R \geq 0$, $q \geq 0$, $p_N \geq c$, $p_R \geq 0$, and $D \geq 0$. Under this situation, the best strategy is to buy used-items from users and buy raw material from the market at the same time , then use both resources to build inventory, thus satisfying the demand.

- **Dark green part.** The solution within dark green part satisfies

$R \geq 0$, $q \geq 0$, $p_N \geq c$, and $D \geq 0$, but with a negative take-back price, p_R. The negative take-back price should correspond to the situation that the customer pay a disposal fee to the manufacturer. The existence of recycling fees charged by some companies suggest the validity of this solution.

- **Light green part.** The solution within the light green part satisfies $R \geq 0$, $p_N \geq c$, $p_R \geq 0$, and $D \geq 0$. However, the order quantity q is negative. Thus, instead of buying raw material from the market, we recycle the used item back to raw material, then sell it to the market. Shaw Industry, for example, recycles used carpets back to caprolactam in their Evergreen Nylon Recycling facility. They have the choice to sell caprolactam back to the market, if they have higher inventory.

- **Red Line.** The red line represent the line that there is no take-back items. This can be driven by two sides. From the customer side, a high price for new items or a low price for take-back items would lead to a reduced willingness for people to sell their used item. From the manufacture side, a high cleaning / transportation fee would lead to lower incentives for remanufacturing. Under this situation, we should only want to produce new product from raw material. The behavior under this strategy is well-studied. The solution of this line can be found in Appendix B.1

- **Green Line.** The green line represents the line that there is no demand. Still, this can be driven by two sides. A high price for the new item will stop people from buying, and a high cost of raw material will make the manufacture had no willing to produce. Under this situation, the remanufacturer should only recycle take back items back to raw material and sell them to the raw-material-market. The solution of this line can be found in Appendix B.2

- **Blue Line.** Blue line represent the line that the optimal sale price is equal to the raw material cost. This could caused by low cost (c or c_R) or small demand market size (α_D). Under this situation, the manufacturer has no interest in satisfying customers' demand. They could either lower their raw-material-order-quantity, or sold their refined take-back item to the market. This would lead to a negative raw-material-order quantity. The strategy should be taken here is clearly. We collect take-back items, recycle them back to raw material, and then sold all of them to raw-material-market. The solution of this line can be found in Appendix B.3

Now we focus on the solution of light green, green and dark green area. And we have theorem 1.

Theorem 1. *By letting order quantity q equal to $D - R$, profit Π is jointly concave in p_N and p_R under the dominant assumption.*

Thus, we know the ideal optimal solution is

$$\begin{aligned} p_N &= 2\gamma_R \cdot \mathbb{A} - (\gamma_D + \beta_R) \cdot \mathbb{B} + \gamma_R \cdot (\gamma_D - \beta_R) \cdot \mathbb{C} + c \\ p_R &= (\beta_R + \gamma_D) \cdot \mathbb{A} - 2\beta_D \cdot \mathbb{B} + [2\beta_D \cdot \gamma_R - (\beta_R + \gamma_D) \cdot \beta_R] \cdot \mathbb{C} \end{aligned}$$

where

$$\begin{aligned}
\mathbb{A} &= \frac{\alpha_D - c \cdot \beta_D}{4\beta_D \cdot \gamma_R - (\beta_R + \gamma_D)^2} \\
\mathbb{B} &= \frac{\alpha_R - c \cdot \beta_R}{4\beta_D \cdot \gamma_R - (\beta_R + \gamma_D)^2} \\
\mathbb{C} &= \frac{c - c_R}{4\beta_D \cdot \gamma_R - (\beta_R + \gamma_D)^2}
\end{aligned}$$

By comparing the ideal optimal take-back price p_R^* with the difference of raw material cost c and remanufacture cost c_R, we have Properties 1.

Properties 1. *If*

$$\begin{aligned}
& \beta_R + \gamma_D) \cdot \alpha_D + [(\beta_R + \gamma_D) \cdot \gamma_D - 2\beta_D \cdot \gamma_R] \cdot (c - c_R) \\
\geq \; & 2\beta_D \cdot \alpha_R + (\gamma_D - \beta_R) \cdot \beta_D \cdot c
\end{aligned}$$

holds, then the optimal take-back price p_R exceeds the difference between the cost of raw material c and cleaning/recycling cost c_R.

4.2 Stochastic case

Moving to the stochastic case, we assume the manufacturer has already decided to use both sources, raw material and take-back items, to produce. And we do not consider the boundary constraints here.

4.2.1 Rearrange and Simplify

Rethink the profit. Before, we assumed every unit in our inventory could be sold. The profit is $(p_N - c)$ per unit for raw material, and

$(p_N - c_R - p_R)$ per unit for take back item. We let the summation be our revenue function, $\Psi(p_N, p_R, q)$.

$$\Psi(p_N, p_R, q) = (p_N - c) \cdot q + (p_N - c_R - p_R) \cdot \mu_R \tag{4.1}$$

However, we cannot always sell every unit we have. When the demand is smaller than our inventory, we have $(q+R-D)^+$ in our storage. Instead of selling them at p_N per unit, we only get salvage value, s, and it "costs" us $(p_N - s)$ per unit. We use leftover function, $L(p_N, p_R, q)$, represent this part.

$$L(p_N, p_R, q) = (p_N - s) \cdot E[(q + R - D)^+] \tag{4.2}$$

Now, we can rewrite our objective function as revenue function minus leftover function.

$$\begin{aligned} E[\Pi(p_N, p_R, q)] &= p_N \cdot E[\min\{D, q + R\}] + s \cdot E[\max\{0, q + R - D\}] \\ &\quad -c \cdot q - (c_R + p_R) \cdot E[R] \\ &= \Psi(p_N, p_R, q) - L(p_N, p_R, q) \end{aligned} \tag{4.3}$$

We also can view the revenue function, Ψ, as the deterministic part of our objective function, and the leftover function L represents the stochastic part.

Notice that for the stochastic part, the leftover function (equation 4.2), we only care about the difference between D and R instead of the exact value of D and R. In that way, we define a new random variable to represent the difference of ϵ_D and ϵ_R.

$$\epsilon = \epsilon_D - \epsilon_R$$

and the corresponding p.d.f and c.d.f are $f(\cdot)$ and $F(\cdot)$. The difference between D and R should be $\mu_D - \mu_R + \epsilon$. The leftover function should be:

$$\begin{aligned} L(p_N, p_R, q) &= (p_N - s) \cdot E[(q + \mu_R - \mu_D - \epsilon)^+] \\ &= (p_N - s) \cdot \int_{-\infty}^{q+\mu_R-\mu_D} (q + \mu_R - \mu_D - x) \cdot f(x)\, dx \end{aligned}$$

From technical side, replacing ϵ_D and ϵ_R with ϵ helps us to simplify our computation since one random variables is always simpler than two. Another benefit is we avoid the difficulty of considering the connection between demand D and take-back supply R. Such a correlation can be expected since a portion of the people who sold their used items will buy a new one immediately as a replacement. So, a high volume of supply will be related to a high volume of demand. We allow this correlation to exist in our model.

We can now also simplify the remainder of our exposition by transforming the problem into an equivalent problem in which only the demand is uncertain. We do so by letting

$$\begin{aligned} \epsilon_D^N &\equiv \epsilon_D - \epsilon_R \\ \epsilon_R^N &\equiv 0 \end{aligned}$$

4.2.2 Result

Back to equation 4.3, where we have split the total profit into revenue part (Ψ) and leftover part (L). We can find the first partial derivatives of profit function by analyzing the marginal effort on both part.

Order Quantity, q We have more units to sell, and ideally, the revenue will increase by $(p_N - c)$. However, the possibility that this item becomes leftover is $F(q+\mu_R-\mu_D)$, and cost $(p_N - s)\cdot F(q+\mu_R-\mu_D)$.

$$\frac{\partial E[\Pi(p_N, p_R, q)]}{\partial q} = p_N - c - (p_N - s)\cdot F(q+\mu_R-\mu_D) \quad (4.4)$$

Take back price, p_R When the take back price, p_R, increases by 1 unit, we need to pay more for every take back item, at a total cost of μ_R. And this increment also spurs people to sell their used items, so we will have γ_R more take-back units, and increase our revenue by $\gamma_R\cdot(p_N - c_R - p_R)$. On the leftover side, we have γ_R more units to sell, and the demand is also increased by γ_D. Then the $(\gamma_R - \gamma_D)$ units have probability $F(q+\mu_R-\mu_D)$ to become leftover, and incur the leftover cost.

$$\begin{aligned}\frac{\partial E[\Pi(p_N, p_R, q)]}{\partial p_R} &= (p_N - c_R - p_R)\cdot\gamma_R - \mu_R \\ &\quad -(p_N - s)\cdot(\gamma_R-\gamma_D)\cdot F(q+\mu_R-\mu_D) \quad (4.5)\end{aligned}$$

Selling price, p_N When the selling price, p_N, increases by 1 unit, we can earn more from every sold item, thus, we earn $q+\mu_R$ more. And since fewer people want to sell their used items, we collect β_R items less, and lose $(p_N - p_R - c_R)\cdot\beta_R$. On the leftover side, the increment in p_N leads to more cost for every unsold unit, $E[(q+\mu_R-\mu_D-\epsilon)^+]$ in all. A change in p_N also changes the quantity of demand and the take back quantity. Then the expectation of leftover will increased by $(\beta_D-\beta_R)\cdot F(q+\mu_R-\mu_D)$.

$$\begin{aligned}\frac{\partial E[\Pi(p_N,p_R,q)]}{\partial p_N} &= q+\mu_R-(p_N-c_R-p_R)\cdot\beta_R \\ &\quad -E[(q+\mu_R-\mu_D-\epsilon)^+] \\ &\quad -(p_N-s)\cdot(\beta_D-\beta_R)\cdot F(q+\mu_R-\mu_D) \end{aligned} \tag{4.6}$$

The analysis above leads to Lemma Theorem 2.

Lemma 2. *The expected profit $E[\Pi]$ is concave in q and p_R for a fixed p_N. And the optimal solution is* [1]

$$q^*(p_N) = F^{-1}\left(\frac{p_N-c}{p_N-s}\right)+\mu_D^*(p_N)-\mu_R^*(p_N) \tag{4.7}$$

$$p_R^*(p_N) = p_N\cdot\frac{\beta_R+\gamma_D}{2\gamma_R}-\frac{\alpha_R+c_R\cdot\gamma_R-c\cdot(\gamma_R-\gamma_D)}{2\gamma_R} \tag{4.8}$$

Proof. See Appendix. □

Theorem 2. *When random variable ϵ satisfies $2r^2(\cdot)+r'(\cdot)>0$, where $r(\cdot)=f(\cdot)/[1-F(\cdot)]$, the optimal solution is the maximum p_N which satisfy*

$$\Delta E[\Pi(p_N,p_R^*(p_N),q^*(p_N))]/\Delta p_N = 0$$

Proof. See Appendix. □

[1] Here, $\mu_D^*(p_N)=\alpha_D-\beta_D\cdot p_N+\gamma_D\cdot p_R^*(p_N)$, $\mu_R^*(p_N)=\alpha_R-\beta_R\cdot p_N+\gamma_R\cdot p_R^*(p_N)$

4.2.3 Discussion

We have three interesting insights here. First, in Lemma 2, we note that when the selling price p_N fixed, the take-back price p_R does not change with increased demand and take-back supply variance. Second, it seems that the sum of take-back price and cleaning/recycling cost is not necessary lower than the cost of raw material. Third, by comparing Theorem 2 with the result of the classical newsvendor model ([Petruzzi and Dada(1999)]), we see a same assumption and conclusion. We try to explain these in this paragraph.

In the classical news-vendor model, the optimal order quantity should be

$$q^*_{Newsvendor} = F^{-1}\left(\frac{p_N - c}{p_N - s}\right) + \mu_D$$

compare with equation 4.7, we see our raw-material order quantity is shrunk by μ_R, the expected quantity of used-item, this can be explained by using inventory level. In the classical news-vendor model, inventory level is equal to the quantity of raw material. However, in our model, we use both raw material and take-back items to build our inventory level. The inventory level of these two models are exactly the same [2]. In other words, we use raw material order quantity to adjust our inventory level to fit the randomness in demand. We can view the raw material order quantity as a buffer to absorb the undesirable affection from randomness. And under this protection, the optimal take-back price p_R is independent of the randomness for a fixed selling price p_N.

[2]assume that we ignore that $F(\cdot)$ now also captures the randomness of tack-back supply.

To find the relationship between the cost of raw material and the sum of take-back price and cleaning/recycling cost, we first focus on the marginal benefit of take-back price p_R follow the analysis above. By increasing p_N by 1 unit, we need to pay more for one take back item, and this cost us μ_R; we have γ_R more take back item in our inventory and replace raw material, this will save us $(c - p_R - c_R) \cdot \gamma_R$; and the demand also increased by γ_D, so did our inventory level, we use raw material to adjust our inventory level correspondingly, and we can earn $(p_N - s) \cdot \gamma_D$. Thus, besides rely on the difference raw material cost c and cleaning/recycling cost c_R, the optimal take-back price should also depend on the selling price and the impact on demand and supply. When the selling price is high enough, and the impact of take-back price on demand is large enough, it is possible that the sum of take-back price and cleaning/recycling cost higher than the cost of raw material. For example, if we let

$$
\begin{aligned}
D &= 10000 - p_N + 0.15 \cdot p_R \\
R &= 100 - 0.2 \cdot p_N + 0.5 \cdot p_R
\end{aligned}
$$

$c = 400$, $c_R = 250$, $s = 250$, and $\epsilon \sim Norm(0, 20)$, the optimal solution is $(p_N, p_R, q) = (5507, 1842.6, 4887.3)$. Clearly, $c_R + p_R > c$.

Since $E[\Pi]$ is concave in q and p_R for a fixed p_N, we can translate our model to an equivalent one and reducing the optimal take-back price p_R.

Lemma 3. *Our model, is equivalent to*

$$\max_{p_N,\mathbb{Q}} E[\Pi'(p_N,\mathbb{Q})] = E[p_N \cdot \min\{D,\mathbb{Q}\} + s \cdot \max\{\mathbb{Q} - D, 0\} - c \cdot \mathbb{Q}] + \mathbb{X} \cdot p_N^2 + \mathbb{Y} \cdot p_N + \mathbb{Z}$$

by letting,

$$\begin{aligned}
\mathbb{Q} &= q + \alpha_R - \beta_R \cdot p_N + \gamma_R \cdot p_R^*(p_N) \\
\mathbb{X} &= \frac{\beta_R^2 - \gamma_D^2}{2\gamma_R} \\
\mathbb{Y} &= \frac{c \cdot \gamma_D^2 - \beta_R \cdot (\alpha_R + \gamma_R \cdot (c - c_R))}{2\gamma_R} \\
\mathbb{Z} &= (\frac{(\alpha_R - c_R \cdot \gamma_R + c \cdot \gamma_R)^2 - c^2 \cdot \gamma_D^2}{4\gamma_R})
\end{aligned}$$

$E[\Pi'(p_N,\mathbb{Q})]$ is a traditional news-vendor objective function plus a quadratic term $g(p_N)$. In our proof of Theorem 2, [3] the key point is $(d^3E[\Pi]/dp_N^3 < 0)$. In that way, $(dE[\Pi]/dp_N)$ is a concave function, and have at most two zero points. The quadratic term $g(p_N)$ do not change this property since $(d^3g(p_N)/dp_N^3 = 0)$. This is the reason why our model has the same result as the traditional news-vendor model.

We know that in the traditional news-vendor model, the optimal selling price is smaller or equal to the optimal riskless selling price [Mills(1959)]. We can find the same result in our model.

Properties 2. *The optimal selling price is always less or equal to* $\mathbb{M}/\mathbb{N}$*,*

[3] Also in the proof of classical news vendor model.

where

$$\begin{aligned}
\mathbb{M} &= 2\beta_D - \beta_R \cdot \frac{\beta_R + \gamma_D}{\gamma_R} > 0 \\
\mathbb{N} &= \alpha_D + c \cdot \beta_D \\
&\quad - \frac{(\beta_R + \gamma_D) \cdot (\alpha_R + c \cdot \gamma_D)}{2\gamma_R} - \frac{(\beta_R - \gamma_D) \cdot (c - c_R)}{2}
\end{aligned}$$

and p_N^* *is equal to* $\mathbb{M}/\mathbb{N}$ *if and only if the demand if fixed (the riskless case).*

Chapter 5

Numerical Example

Algorithm We have developed an algorithm (Appendix C.1) to find the best strategy and optimal solution in O(1) time for the deterministic case; and another algorithm (Appendix C.2) to find the optimal solution in $O(\log M)$ [1] time for the stochastic case.

Deterministic Case A typical profit graph under different α_D and α_R should be look as figure 5.1. The corresponding demand and supply of take-back items is shown in figure 5.2. [2] From figure 5.1, we can easily see that the basement of α_D and α_R can change our choice of

[1] M denote the range of possible value of selling price, p_N, and we assume to compute an integer take us O(1) time.

[2] In these figures, different color denote different strategies.

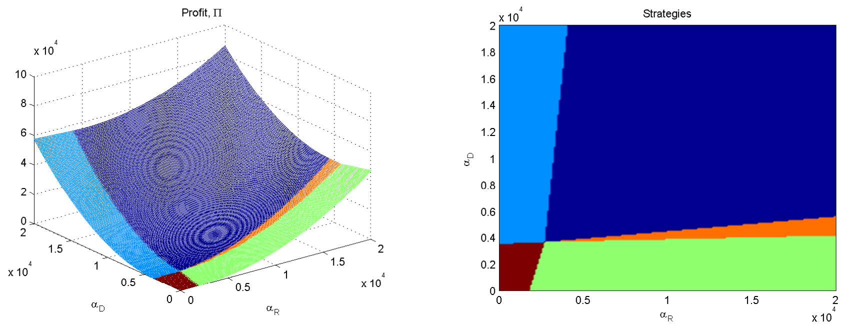
Blue. Take both sources, raw material (can be negative) and take-back item, to produce items.

Light blue. Only use raw material to produce items.

Green. Recycle back take-back item and sold them to raw material market, since there is no demand.

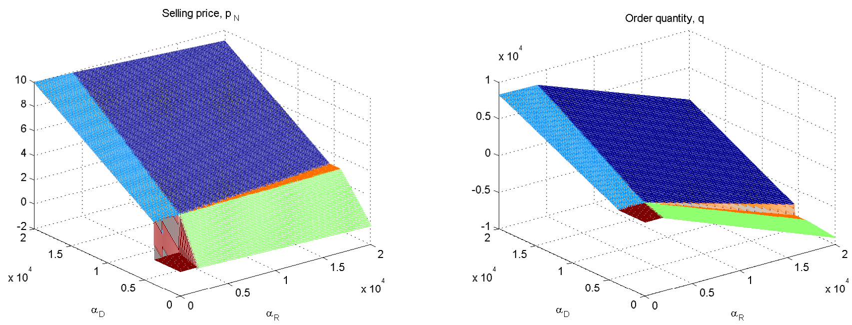
Orange. Recycle back take-back item and sold them to raw material market, since the optimal selling price is too low, and we ignore the demand.

Dark red. Do nothing.



(a) Price (b) Strategy

Figure 5.1: Profit & Strategies under different α_D and α_R
$c = 3$, $c_R = 4$, $s = 1$, $D = \alpha_D - 1200p_N + 300p_R$,
$R = \alpha_R - 100p_N + 1800p_R$.



(a) Selling price (b) Raw

Figure 5.2: Other variables under different α_D and α_R

strategies. We first consider the situation that α_R is low. Since α_R is low, the acquirement of take-back items will be hard. And when both α_D and α_R are low, there is limiting room for the selling price p_N grows, to sell something is profitless, then we would like to do nothing. As the α_D grows, we can increase p_N, and now we would like to produce new product from raw materials, and sold them. Now we move to the situation that α_R is high. Since α_R is high, we can acquire take-back items at a very reasonable price. And we α_D is low, we should just recycle the take-back items back to raw material, then sold them. As α_D grows, the demand begins to appear. However, at the very beginning, the corresponding selling price is still low. Thus, we choose to ignore the demand, and still serve the raw-material-market only. Later, as α_D grows and p_N is high enough, we would like to take both sources to produce items. [3]

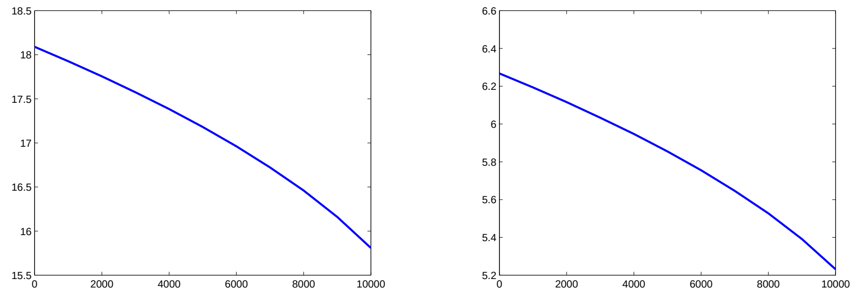
From figure 5.1(a), we see that without remanufacturing strategy, the company will do nothing when demand is low, and produce items when demand is high enough; with the adding of recycling, even at low level of demand, the company can have revenue by recycle take-back items, which is showed by green and orange part. And when demand is relative higher, the adding of remanufacturing can increase the total revenue significantly. In this experiment, it can reach 30%. Figure 5.2(a) and 5.2(b) also showed that the adding of recycling can reduce the selling price and shorten the use of raw material.

Stochastic Case As we have said in chapter 4, the introduce of randomness in demand and supply, will decrease the optimal selling price, and thus decrease the take-back price. Figure 5.3 examine this property.

Figure 5.4 shows the expected profit versus randomness. Here, we use dash-dot line to show the expected profit by using the price as the deterministic case. We see that by using the optimal price, the profit is increased significantly.

An Example We try to set the parameters to represent the Kodak single-use camera environment. We focus on those small, one-time-use

[3]Here, if order quantity still smaller than 0, means we use the take-back items to serve both demand and raw-material markets.



(a) Selling price (b) Take-back price

Figure 5.3: Price under different randomness
$c = 12$, $c_R = 5$, $s = 5$, $D = 30000 - 1500p_N + 1000p_R$, $R = 1100p_R$

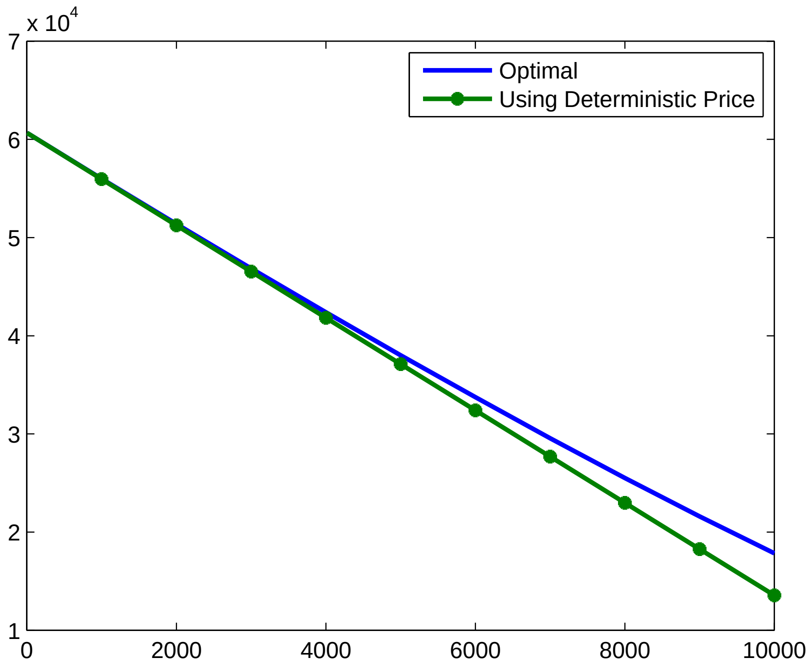


Figure 5.4: Expected profit under different randomness

cameras that sell for \$4 to \$10, an area Kodak and rival Fuji dominate. We simple set the cost of raw material, c, equal to 3 (a slightly lower than the lowest selling price); the cost of remanufacture, c_R, equal to 1 (we only need to replace the cover and the film), and the salvage value is 1 per unit. According to the work of [Toktay, Wein, and Zenios(2000)], the demand of Kodak single-use camera is around 18,000 per week in the United State in 2000. Since the small, one-time-use cameras is dominate by Kodak and Fuji, simple assume the size of market α_D is 36,000 ($18,000 \times 2$). The target selling price will around \$7 (the middle number between \$4 and \$10), thus we let $\beta_D = 3200$ ($36000/(2*7-3) = 3272.727$). γ_D should be smaller than β_D, let it equal to 2000. On the supply side, Kodak is recycled the camera regardless of the brand, thus we let the quantity of supply is only sensitive to take-back price. We assume γ_R should be more than twice (the market of Kodak and Fuji) of β_D, and let $\gamma_R = 8000$. The demand and supply function should be:

$$
\begin{aligned}
D &= 36000 - 3200 \cdot p_N + 2000 \cdot p_R \\
R &= 8000 \cdot p_R
\end{aligned}
$$

We apply our algorithm for both deterministic and stochastic case. Table 5.1 shows the expect profit under different setting. Here, Take-back ignored means we set the selling price as the no remanufacturing case, but count the optimal take-back price and raw material order quantity. And uncertainty ignored means we still use the price and order quantities as the deterministic case.

By comparing the first two cases (deterministic) in Table 5.1, we can see that by adding remanufacturing part, the profit is increased by 35%. And under stochastic circumstance, by adding remanufacturing part, the expected profit is increased by 37.8%.

To given an intuitive explanation on the effort of take-back price, we try to compare the no-remanufacturing case and the take-back ig-

	Order quantity	**Selling price**	**Take-back price**	**Expected sale**	**Expected salvage**	**Expected profit**
Deterministic						
No remanufacturing	13200	7.125	N/A	13200	0	54450
Take-back ignored	4106.25	7.125	1.5156	16231.25	0	72826.95
Remanufacturing	2159.3	7.6179	1.5772	14777	0	73574
Stochastic						
No remanufacturing	14295	7.0575	N/A	12982	1313	50047
Take-back ignored	5251.8	7.0575	1.5072	15996	1313.2	68220
Uncertainty ignored	3195.6	7.6179	1.5772	14393	1420.7	68957
Optimal	3452.9	7.5481	1.5685	14593	1407.9	68969

Table 5.1: Numerical Result

nored case. Under deterministic setting, by adding the take-back part, the demand is increased by 3031.25, create 12504 ($\Delta D \times (p_N - c)$) revenue; and the recycling part is saving 5873 ($(c - c_R - p_R) \times R$). Under stochastic setting, the expected demand is increased by 3014, create 12229 revenue; and the recycling part is saving 5942. Thus, we think the take-back price should reflect the potential demand increase as well as the potential cost saving.

Chapter 6

Conclusion

In this paper, we developed and analyzed a pricing and news-vendor framework for products with recycling or remanufacturing as a sourcing option. Our model helps managers to decide whether or not to start remanufacturing, and then determine the optimal selling price, take-back price and raw material order quantity, for quite a general class of demand and take-back supply distributions.

We contribute to the literature since we are the first to consider the pricing and inventory issue with remanufacturing under stochastic demand and potentially correlated stochastic take-back supply.

We provide the method to find the optimal solution. We find that, when the selling price remains fixed, the optimal take-back price and thus the expected take-back quantity does not change with increased demand and take-back supply variance. It seems that the raw material order quantity has adjusted our inventory level to fit the randomness in demand. Also, when the impact of take-back on demand, and thus revenues, is large enough, it is possible that the optimal take-back price exceeds the difference between the cost of raw material and cleaning/recycling cost. Later, by replacing the optimal take-back price, we find our model can be viewed as a special version of classical news-vendor model plus a quadratic term. Also, as the classical news-vendor model, we find the appearance of randomness would lower the optimal selling price, and thus the take-back price.

We offer a numerical example that is loosely modeled on the situation of Kodak's single-use cameras. We find remanufacturing can increase the expected profit by 5%.

Although we believe our model is simple, useful and applicable for single-period problem, we should also see the limitations of this model. The linear demand/supply function will not be suitable in extreme case, eg., when demand is near zero. To fix this, we can use a piece-wise linear function to fit the demand/supply function. Also, our model views all the take-back items as the same. However, for some kind of product, the condition of take-back items is an important factor in remanufacturing.

There are plenty extensions beyond this model. One major extension is to consider a multi-period setting. In that setting, we can incorporate the delay time of remanufacturing, and we can investigate the impact of earlier price setting. Another worthwhile research direction to consider is multiple customer classes and multiple conditions of take-back items. This would naturally lead to a multi-price strategy, maybe dependent on the years of usage.

Appendix A

Proof

Proof of Theorem 1.

Proof. Our objective function is,

$$\begin{aligned}\Pi(p_N, p_R, q) &= p_N \cdot \min\{D, q+R\} + s \cdot \max\{q+R-D, 0\} \\ &\quad -(p_R + c_R) \cdot R - c \cdot q \\ &= p_N \cdot (q+R) - (p_N - s) \cdot \max\{q+R-D, 0\} \\ &\quad -(p_R + c_R) \cdot R - c \cdot q\end{aligned}$$

That is, we assume everything in the inventory can be sold first, then for leftover, we lose $(p_N - s)$ per unit. The marginal benefit of increment in q can be expressed as follow.

$$\text{Marginal}_q = \begin{cases} s - c & , \text{ if } q + R \geq D \\ p_N - c & , \text{ if } q + R < D \end{cases}$$

Since we have assumed that $c > s$, we should keep $q + R \leq D$. On the other hand, since we have constrained selling price p_N greater than c, we would like to make q as high as possible. Thus, keep $D = q + R$ is the best way to maximize our profit. And we have the optimal order quantity $q^*(p_R, p_N)$ equal to $D - R$. We then apply $q^*(p_R, p_N)$ into our objective function.

$$\Pi = (p_N - c) \cdot D + (c - p_R - c_R) \cdot R$$

Consider the first partial derivatives of Π taken with respect to p_N and p_R,

$$\begin{aligned} \frac{\partial \Pi}{\partial p_N} &= D - (p_N - c) \cdot \beta_D - (c - p_R - c_R) \cdot \beta_R \\ \frac{\partial \Pi}{\partial p_R} &= (p_N - c) \cdot \gamma_D - R + (c - p_R - c_R) \cdot \gamma_R \end{aligned}$$

and the Hessian matrix

$$\begin{vmatrix} \frac{\partial^2 \Pi}{\partial p_N^2} & \frac{\partial^2 \Pi}{\partial p_N \partial p_R} \\ \frac{\partial^2 \Pi}{\partial p_R \partial p_N} & \frac{\partial^2 \Pi}{\partial p_R^2} \end{vmatrix} = \begin{vmatrix} -2\beta_D & \gamma_D + \beta_R \\ \gamma_D + \beta_R & -2\gamma_R \end{vmatrix}$$

Clearly, that both first principal minor is smaller than zero, and the second principal minor is greater than zero can be proved by using the dominant assumption as follows.

$$\begin{aligned} \det \begin{vmatrix} \frac{\partial^2 \Pi}{\partial p_N^2} & \frac{\partial^2 \Pi}{\partial p_N \partial p_R} \\ \frac{\partial^2 \Pi}{\partial p_R \partial p_N} & \frac{\partial^2 \Pi}{\partial p_R^2} \end{vmatrix} &= 4\beta_D \cdot \gamma_R - (\gamma_D + \beta_R)^2 \\ &\geq 4 \cdot (\min\{\beta_D, \gamma_R\}) - (\gamma_D + \beta_R)^2 \\ &> 4 \cdot (\min\{\beta_D, \gamma_R\}) - (2 \cdot \min\{\beta_D, \gamma_R\})^2 \\ &= 0 \end{aligned}$$

Thus, profit Π is jointly concave in p_N and p_R.

The optimal solution can be expressed as follows.

$$\begin{bmatrix} p_N^* \\ p_R^* \end{bmatrix} = \begin{bmatrix} \gamma_D + \beta_R & -2\gamma_R \\ -2\beta_D & \gamma_D + \beta_R \end{bmatrix}^{-1} \cdot \begin{bmatrix} c \cdot \gamma_D + \alpha_R - (c - c_R) \cdot \gamma_R \\ -\alpha_D - c \cdot \beta_D + (c - c_R) \cdot \beta_R \end{bmatrix}$$

$$q^* = D(p_N^*, p_R^*) - R(p_N^*, p_R^*)$$

We also provide the direct expression of the optimal solution here.

$$p_R = (\beta_R + \gamma_D) \cdot \mathbb{A} - 2\beta_D \cdot \mathbb{B} + [2\beta_D \cdot \gamma_R - (\beta_R + \gamma_D) \cdot \beta_R] \cdot \mathbb{C}$$

$$p_N = 2\gamma_R \cdot \mathbb{A} - (\gamma_D + \beta_R) \cdot \mathbb{B} + \gamma_R \cdot (\gamma_D - \beta_R) \cdot \mathbb{C} + c$$

$$D = [2\beta_D \cdot \gamma_R - \beta_R \cdot (\beta_R + \gamma_D)] \cdot \mathbb{A} - \beta_D \cdot (\gamma_D - \beta_R) \cdot \mathbb{B} + (\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D) \cdot (\beta_R + \gamma_D) \cdot \mathbb{C} \tag{A.1}$$

$$R = \gamma_R \cdot (\gamma_D - \beta_R) \cdot \mathbb{A} + [2\beta_D \cdot \gamma_R - \gamma_D(\beta_R + \gamma_D)] \cdot \mathbb{B} + 2\gamma_R \cdot (\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R) \cdot \mathbb{C} \tag{A.2}$$

$$q = [2\beta_D \cdot \gamma_R - \beta_R \cdot (\beta_R + \gamma_D) - \gamma_R \cdot (\gamma_D - \beta_R)] \cdot \mathbb{A} - [2\beta_D \cdot \gamma_R - \gamma_D(\beta_R + \gamma_D) + \beta_D \cdot (\gamma_D - \beta_R)] \cdot \mathbb{B} + (\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D) \cdot (\beta_R + \gamma_D - 2\gamma_R) \cdot \mathbb{C} \tag{A.3}$$

Here,

$$\begin{aligned}\mathbb{A} &= \frac{\alpha_D - c\cdot\beta_D}{4\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)^2}\\ \mathbb{B} &= \frac{\alpha_R - c\cdot\beta_R}{4\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)^2}\\ \mathbb{C} &= \frac{c - c_R}{4\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)^2}\\ \mathbb{D} &= \frac{1}{4\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)^2}\end{aligned}$$

$$\begin{aligned}\Pi &= \mathbb{D}\cdot\{\gamma_R\cdot(\alpha_D - c\cdot\beta_D)^2 + \beta_D\cdot(\alpha_R - c\cdot\beta_R)^2\\ &\quad +\gamma_R\cdot(\beta_D\cdot\gamma_R - \beta_R\cdot\gamma_D)\cdot(c-c_R)^2\\ &\quad -(\gamma_D+\beta_R)\cdot(\alpha_D - c\cdot\beta_D)\cdot(\alpha_R - c\cdot\beta_R)\\ &\quad +\gamma_R\cdot(\gamma_D-\beta_R)\cdot(\alpha_D - c\cdot\beta_D)\cdot(c-c_R)\\ &\quad +[2\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)\cdot\gamma_D]\cdot(\alpha_R - c\cdot\beta_R)\cdot(c-c_R)\}\end{aligned}$$

□

Proof of Properties 1

Proof.

$$\begin{aligned}&[4\beta_D\cdot\gamma_R - (\beta_R+\gamma_D)^2]\cdot[p_R^* - (c-c_R)]\\ =\ &(\beta_R+\gamma_D)\cdot\alpha_D + [(\beta_R+\gamma_D)\cdot\gamma_D - 2\beta_D\cdot\gamma_R]\cdot(c-c_R)\\ &-2\beta_D\cdot\alpha_R - (\gamma_D-\beta_R)\cdot\beta_D\cdot c\end{aligned}$$

□

Proof of Lemma 2.

Proof. Consider the first and second partial derivatives of $E[\Pi]$ taken with respect to q and p_R,

$$\begin{aligned}
\frac{\partial E[\Pi(p_N, p_R, q)]}{\partial q} &= p_N - c - (p_N - s) \cdot F(q + \mu_R - \mu_D) \\
\frac{\partial^2 E[\Pi(p_N, p_R, q)]}{\partial q^2} &= -(p_N - s) \cdot f(q + \mu_R - \mu_D) < 0 \\
\frac{\partial E[\Pi(p_N, p_R, q)]}{\partial p_R} &= (p_N - c_R - p_R) \cdot \gamma_R - \mu_R \\
&\quad -(p_N - s) \cdot (\gamma_R - \gamma_D) \cdot F(q + \mu_R - \mu_D) \\
\frac{\partial^2 E[\Pi(p_N, p_R, q)]}{\partial p_R^2} &= -2\gamma_R - (p_N - s) \cdot (\gamma_R - \gamma_D)^2 \cdot f(q + \mu_R - \mu_D) < 0 \\
\frac{\partial^2 E[\Pi(p_N, p_R, q)]}{\partial q \partial p_R} &= -(p_N - s) \cdot (\gamma_R - \gamma_D) \cdot f(q + \mu_R - \mu_D)
\end{aligned}$$

and the determinant of Hessian matrix:

$$\begin{aligned}
\det \begin{vmatrix} \frac{\partial^2 E[\Pi]}{\partial q^2} & \frac{\partial^2 E[\Pi]}{\partial q \partial p_R} \\ \frac{\partial^2 E[\Pi]}{\partial q \partial p_R} & \frac{\partial^2 E[\Pi]}{\partial p_R^2} \end{vmatrix} &= (p_N - s) \cdot f(q + \mu_R - \mu_D) \\
&\quad \times (2\gamma_R + (p_N - s) \cdot (\gamma_R - \gamma_D)^2 \cdot f(q + \mu_R - \mu_D)) \\
&\quad -(p_N - s)^2 \cdot (\gamma_R - \gamma_D)^2 \cdot f(q + \mu_R - \mu_D)^2 \\
&= 2\gamma_R \cdot (p_N - s) \cdot f(q + \mu_R - \mu_D) > 0
\end{aligned}$$

Thus, the expected profit $E[\Pi]$ is concave in q and p_R for a fixed p_N.

The optimal $(q^*(p_N), p_R^*(p_N))$ should satisfy

$$\begin{aligned}
0 = \frac{\partial E[\Pi(p_N, p_R, q)]}{\partial q} &= p_N - c - (p_N - s) \cdot F(q + \mu_R - \mu_D) \\
0 = \frac{\partial E[\Pi(p_N, p_R, q)]}{\partial p_R} &= (p_N - c_R - p_R) \cdot \gamma_R - \mu_R \\
&\quad -(p_N - s) \cdot (\gamma_R - \gamma_D) \cdot F(q + \mu_R - \mu_D)
\end{aligned}$$

Thus,

$$\begin{aligned} q^*(p_N) &= F^{-1}\left(\frac{p_N - c}{p_N - s}\right) - \mu_D^*(p_N) + \mu_R^*(p_N) \\ p_R^*(p_N) &= p_N \cdot \frac{\beta_R + \gamma_D}{2\gamma_R} - \frac{\alpha_R + c_R \cdot \gamma_R - c \cdot (\gamma_R - \gamma_D)}{2\gamma_R} \end{aligned}$$

Here,

$$\begin{aligned} \mu_D^*(p_N) &= \alpha_D - \beta_D \cdot p_N + \gamma_D \cdot p_R^*(p_N) \\ \mu_R^*(p_N) &= \alpha_R - \beta_R \cdot p_N + \gamma_R \cdot p_R^*(p_N) \end{aligned}$$

□

Proof of Theorem 2.

Proof. Let

$$\mathbb{A}(p_N) = \frac{\Delta E[\Pi(p_N, p_R^*(p_N), q^*(p_N))]}{\Delta p_N}$$

From chain rule

$$\mathbb{A}(p_N) = \left.\frac{\partial E[\Pi(p_N, p_R, q)]}{\partial p_N}\right|_{p_R = p_R^*(p_N), q = q^*(p_N)}$$

Apply equation 4.6, 4.8 and 4.7:

$$\begin{aligned} \mathbb{A}(p_N) &= \mathbb{B}(p_N) + \alpha_D - \beta_D \cdot p_N + \gamma_D \cdot p_R^*(p_N) \\ &\quad - (p_N - c_R - p_R^*(p_N)) \cdot \beta_R \\ &\quad - E\left[(\mathbb{B}(p_N) - \epsilon)^+\right] - (p_N - c) \cdot (\beta_D - \beta_R) \end{aligned}$$

where

$$\mathbb{B}(p_N) = F^{-1}\left(\frac{p_N - c}{p_N - s}\right)$$

and

$$\frac{\Delta \mathbb{B}(p_N)}{\Delta p_N} = \frac{1}{f(\mathbb{B}(p_N))} \cdot \frac{c-s}{(p_N - s)^2}$$

Consider finding the zeros of $\mathbb{A}(p_N)$:

$$\begin{aligned}
\frac{\Delta \mathbb{A}(p_N)}{\Delta p_N} &= \frac{1}{f(\mathbb{B}(p_N))} \cdot \frac{c-s}{(p_N - s)^2} - \beta_D + \gamma_D \cdot \frac{\Delta p_R^*(p_N)}{\Delta p_N} \\
&\quad - \left(1 - \frac{\Delta p_R^*(p_N)}{\Delta p_N}\right) \cdot \beta_R \\
&\quad - \frac{1}{f(\mathbb{B}(p_N))} \cdot \frac{c-s}{(p_N - s)^2} \cdot \frac{p_N - c}{p_N - s} - (\beta_D - \beta_R) \\
&= \frac{1}{f(\mathbb{B}(p_N))} \cdot \frac{(c-s)^2}{(p_N - s)^3} \\
&\quad + \frac{\Delta p_R^*(p_N)}{\Delta p_N} \cdot (\beta_R + \gamma_D) - 2\beta_D \\
&= \frac{1}{r(\mathbb{B}(p_N))} \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c-s} \\
&\quad + \frac{\Delta p_R^*(p_N)}{\Delta p_N} \cdot (\beta_R + \gamma_D) - 2\beta_D
\end{aligned}$$

where $r(\cdot) = f(\cdot)/[1 - F(\cdot)]$

$$
\begin{aligned}
\frac{\Delta^2 \mathbb{A}(p_N)}{\Delta p_N^2} &= \frac{\Delta \frac{1}{r(\mathbb{B}(p_N))}}{\Delta p_N} \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c - s} \\
&\quad -2 \cdot \frac{1}{r(\mathbb{B}(p_N))} \cdot \frac{(1 - F(\mathbb{B}(p_N)))}{c - s} \cdot f(\mathbb{B}(p_N)) \cdot \frac{\Delta \mathbb{B}(p_N)}{\Delta p_N} \\
&= \frac{\Delta \frac{1}{r(\mathbb{B}(p_N))}}{\Delta \mathbb{B}(p_N)} \cdot \frac{\Delta \mathbb{B}(p_N)}{\Delta p_N} \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c - s} \\
&\quad -2 \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c - s} \cdot \frac{\Delta \mathbb{B}(p_N)}{\Delta p_N} \\
&= -\left[\frac{\Delta r(\mathbb{B}(p_N))}{\Delta \mathbb{B}(p_N)} + 2 \cdot r^2(\mathbb{B}(p_N)) \right] \\
&\quad \times \frac{1}{r^2(\mathbb{B}(p_N))} \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c - s} \cdot \frac{\Delta \mathbb{B}(p_N)}{\Delta p_N}
\end{aligned}
$$

That is, if $2 \cdot r^2(\cdot) + r'(\cdot) > 0$ always hold, $\mathbb{A}(p_N)$ is concave in p_N, the optimal solution is the largest p_N that satisfies $\mathbb{A}(p_N) = 0$ □

Proof of Lemma 3.

Proof. From Lemma 2, $E[\Pi]$ is concave in q and p_R for a fixed p_N, our model is equivalent to

$$
\begin{aligned}
\max_{p_N, q} E[\Pi'(p_N, q)] &= E[p_N \cdot \min\{D, q + R\} + s \cdot \max\{q + R - D, 0\} \\
&\quad -(p_R + c_R) \cdot R - c \cdot q] \\
\text{s.t. } p_R &= p_N \cdot \mathbb{U} - \mathbb{V}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{U} &= \frac{\beta_R + \gamma_D}{2\gamma_R} \\
\mathbb{V} &= \frac{\alpha_R + c_R \cdot \gamma_R - c \cdot (\gamma_R - \gamma_D)}{2\gamma_R}
\end{aligned}
$$

Let $\mathbb{Q} = q + R$,

$$\begin{aligned}
\Pi'(p_N, \mathbb{Q}) &= p_N \cdot \min\{D, \mathbb{Q}\} + s \cdot \max\{\mathbb{Q} - D, 0\} \\
&\quad -c \cdot \mathbb{Q} + (c - p_R - c_R) \cdot R \\
&= p_N \cdot \min\{D, \mathbb{Q}\} + s \cdot \max\{\mathbb{Q} - D, 0\} - c \cdot \mathbb{Q} \\
&\quad +(c - p_N \cdot \mathbb{U} + \mathbb{V} - c_R) \cdot (\alpha_R - \beta_R \cdot p_N + \gamma_R \cdot (p_N \cdot \mathbb{U} - \mathbb{V})) \\
&= p_N \cdot \min\{D, \mathbb{Q}\} + s \cdot \max\{\mathbb{Q} - D, 0\} - c \cdot \mathbb{Q} \\
&\quad +(c + \mathbb{V} - c_R - p_N \cdot \mathbb{U}) \cdot (\alpha_R - \gamma_R \cdot \mathbb{V} + (\gamma_R \cdot \mathbb{U} - \beta_R) \cdot p_N) \\
&= p_N \cdot \min\{D, \mathbb{Q}\} + s \cdot \max\{\mathbb{Q} - D, 0\} - c \cdot \mathbb{Q} \\
&\quad +\mathbb{X} \cdot p_N^2 + \mathbb{Y} \cdot p_N + \mathbb{Z}
\end{aligned}$$

Here, $\mathbb{X}$, $\mathbb{Y}$ and $\mathbb{Z}$ are constants.

$$\begin{aligned}
\mathbb{X} &= -\mathbb{U} \cdot (\gamma_R \cdot \mathbb{U} - \beta_R) \\
&= \frac{\beta_R^2 - \gamma_D^2}{2\gamma_R} \\
\mathbb{Y} &= (c + \mathbb{V} - c_R) \cdot (\gamma_R \cdot \mathbb{U} - \beta_R) - \mathbb{U} \cdot (\alpha_R - \gamma_R \cdot \mathbb{V}) \\
&= \frac{c \cdot \gamma_D^2 - \beta_R \cdot (\alpha_R + \gamma_R \cdot (c - c_R))}{2\gamma_R} \\
\mathbb{Z} &= (c + \mathbb{V} - c_R) \cdot (\alpha_R - \gamma_R \cdot \mathbb{V}) \\
&= (\frac{(\alpha_R - c_R \cdot \gamma_R + c \cdot \gamma_R)^2 - c^2 \cdot \gamma_D^2}{4\gamma_R})
\end{aligned}$$

□

Proof of Properties 2.

Proof. In Lemma 3, we have prove our model is equivalent to [1]

$$\begin{aligned}\max_{p_N,\mathbb{Q}} E[\Pi'(p_N,\mathbb{Q})] &= E[p_N \cdot \min\{D,\mathbb{Q}\} + s\cdot\max\{\mathbb{Q}-D,0\} - c\cdot\mathbb{Q}] \\ &\quad +\mathbb{X}\cdot p_N^2 + \mathbb{Y}\cdot p_N + \mathbb{Z}\end{aligned}$$

and in both models the optimal selling prices should be the same. Define $z = \mathbb{Q} - \mu_D$, and replace $\mathbb{Q}$ by $z+\mu_D$, D by $\mu_D+\epsilon$

$$\begin{aligned}E[\Pi'(p_N,z)] &= E[p_N\cdot\min\{\mu_D+\epsilon, z+\mu_D\} + s\cdot\max\{z-\epsilon,0\} - c\cdot(z+\mu_D)] \\ &\quad +\mathbb{X}\cdot p_N^2 + \mathbb{Y}\cdot p_N + \mathbb{Z} \\ &= (p_N-c)\cdot(\mu_D+z) - (p_N-s)\cdot E[(z-\epsilon)^+] \\ &\quad +\mathbb{X}\cdot p_N^2 + \mathbb{Y}\cdot p_N + \mathbb{Z}\end{aligned}$$

Consider the first and second partial derivatives of $E[\Pi']$ taken with respect to p_N,

$$\begin{aligned}\frac{\partial E[\Pi'(p_N,z)]}{\partial p_N} &= \mu_D + z + (p_N-c)\cdot\frac{\partial\mu_D}{\partial p_N} - E[(z-\epsilon)^+] \\ &\quad +2\mathbb{X}\cdot p_N + \mathbb{Y} \\ \frac{\partial^2 E[\Pi'(p_N,z)]}{\partial p_N^2} &= 2\frac{\partial\mu_D}{\partial p_N} + 2\mathbb{X} \\ &= -2\beta_D + (\gamma_D+\beta_R)\cdot\frac{\beta_R}{\gamma_R}\end{aligned}$$

By using dominant assumption, we can see that the second partial derivatives of $E[\Pi']$ taken with respect to p_N is smaller than zero, thus, $E[\Pi']$ is a concave function in p_N for a fixed z. And the optimal selling

[1] The definition of $\mathbb{X}$, $\mathbb{Y}$, and $\mathbb{Z}$ can be found in proof of lemma 3.

price p_N satisfy

$$
\begin{aligned}
0 = \frac{\partial E[\Pi'(p_N, z)]}{\partial p_N} &= \mu_D + z + (p_N - c) \cdot \frac{\partial \mu_D}{\partial p_N} - E[(z-\epsilon)^+] \\
&\quad + 2\mathbb{X} \cdot p_N + \mathbb{Y} \\
p_N^*(z) &= \frac{\mathbb{N} + z - E[(z-\epsilon)^+]}{\mathbb{M}} \\
\mathbb{M} &= 2\beta_D - \beta_R \cdot \frac{\beta_R + \gamma_D}{\gamma_R} > 0 \\
\mathbb{N} &= \alpha_D + c \cdot \beta_D \\
&\quad - \frac{(\beta_R + \gamma_D) \cdot (\alpha_R + c \cdot \gamma_D)}{2\gamma_R} - \frac{(\beta_R - \gamma_D) \cdot (c - c_R)}{2}
\end{aligned}
$$

Clearly, $z - E[(z-\epsilon)^+] \leq 0$, and $z - E[(z-\epsilon)^+] = 0$ if and only if $\epsilon \equiv 0$. Thus, the optimal selling price is smaller or equal to the optimal riskless selling price. □

Appendix B

Boundaries in Deterministic Case

B.1 No take-back line, $R = 0$

The scenario that without remanufacture has been well-studied by literature.

B.1.1 Boundary

$R = 0$ starts at point (p_N^s, p_R^s)

$$
\begin{aligned}
p_N^s &= c \\
p_R^s &= \frac{\beta_R \cdot c - \alpha_R}{\gamma_R}
\end{aligned}
$$

ends at point (p_N^e, p_R^e) satisfies

$$\begin{aligned} p_N^e &= \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D} \\ p_R^e &= \frac{\beta_R \cdot \alpha_D - \beta_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D} \end{aligned}$$

And this line will existence as a boundary of feasible space if and only if

$$c \leq \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D}$$

B.1.2 Solution

The profit should be

$$\Pi_{NR} = D \cdot (p_N - c)$$

The first and second derivatives of Π_{NR} in p_N should be

$$\begin{aligned} \frac{\Delta \Pi_{NR}}{\Delta p_N} &= D + \frac{\Delta D}{\Delta p_N} \cdot (p_N - c) \\ \frac{\Delta^2 \Pi_{NR}}{\Delta p_N^2} &= 2 \frac{\Delta D}{\Delta p_N} \end{aligned}$$

Since $R = 0$, we have

$$\begin{aligned} -\beta_R + \gamma_R \cdot \frac{\Delta p_R}{\Delta p_N} &= 0 \\ \frac{\Delta p_R}{\Delta p_N} &= \frac{\beta_R}{\gamma_R} \\ \frac{\Delta D}{\Delta p_N} &= -\beta_D + \gamma_D \cdot \frac{\beta_R}{\gamma_R} \end{aligned}$$

Apply dominant assumption, $\gamma_R > \gamma_D > 0$ and $\beta_D > \beta_R > 0$, we have $\gamma_R \cdot \beta_D > \gamma_D \cdot \beta_R$, thus,

$$\frac{\Delta D}{\Delta p_N} = \frac{-\gamma_R \cdot \beta_D + \gamma_D \cdot \beta_R}{\gamma_R} < 0$$

Now, we know $\Delta^2 \Pi_{NR}/\Delta p_N^2$ is always smaller than zero, and the profit Π_{NR} should be a concave function in p_N. The optimal p_N can be found by letting $\Delta \Pi_{NR}/\Delta p_N = 0$.

$$\begin{aligned} p_N^* &= \frac{\alpha_D \cdot \gamma_R - \alpha_R \cdot \gamma_D}{2(\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)} + \frac{c}{2} \\ p_R^* &= \frac{\alpha_D \cdot \beta_R \cdot \gamma_R - \alpha_R \cdot \beta_D \cdot \gamma_R}{2\gamma_R \cdot (\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)} - \frac{\alpha_R - c \cdot \beta_R}{2\gamma_R} \end{aligned}$$

And the corresponding profit is

$$\Pi_{NR} = \frac{[(\alpha_D - c \cdot \beta_D) \cdot \gamma_R - (\alpha_R - c \cdot \beta_R) \cdot \gamma_D]^2}{4\gamma_R \cdot (\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)}$$

B.1.3 Conclusion

- If

$$c \leq \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D}$$

  then the optimal solution of p_N and p_R is

$$\begin{aligned} p_N^* &= \frac{\alpha_D \cdot \gamma_R - \alpha_R \cdot \gamma_D}{2(\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)} + \frac{c}{2} \\ p_R^* &= \frac{\alpha_D \cdot \beta_R \cdot \gamma_R - \alpha_R \cdot \beta_D \cdot \gamma_R}{2\gamma_R \cdot (\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)} - \frac{\alpha_R - c \cdot \beta_R}{2\gamma_R} \end{aligned}$$

And the corresponding profit is

$$\Pi_1(p_R) = \frac{[(\alpha_D - c \cdot \beta_D) \cdot \gamma_R - (\alpha_R - c \cdot \beta_R) \cdot \gamma_D]^2}{4\gamma_R \cdot (\beta_D \cdot \gamma_R - \gamma_D \cdot \beta_R)}$$

- Otherwise, this line will do not exist as the feasible boundary.

B.2 No demand line, $D = 0$

Under this scenario, the profit should be

$$\Pi_{ND} = R \cdot (c - p_R - c_R)$$

B.2.1 Boundary

No demand line start at point (p_N^e, p_R^e)

$$p_N^e = \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D}$$
$$p_R^e = \frac{\beta_R \cdot \alpha_D - \beta_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D}$$

B.2.2 Solution

The first and second derivatives of Π_{ND} in p_R should be

$$\frac{\Delta \Pi_{ND}}{\Delta p_R} = -R + \frac{\Delta R}{\Delta p_R} \cdot (c - p_R - c_R)$$
$$\frac{\Delta^2 \Pi_{ND}}{\Delta p_R^2} = -2\frac{\Delta R}{\Delta p_R}$$

Since $D = 0$, we have

$$\begin{aligned}
-\beta_D \cdot \frac{\Delta p_N}{\Delta p_R} + \gamma_D &= 0 \\
\frac{\Delta p_N}{\Delta p_R} &= \frac{\gamma_D}{\beta_D} \\
\frac{\Delta R}{\Delta p_N} &= -\beta_R \cdot \frac{\gamma_D}{\beta_D} + \gamma_R
\end{aligned}$$

Apply dominant assumption, $\gamma_R > \gamma_D > 0$ and $\beta_D > \beta_R > 0$, we have $\gamma_R \cdot \beta_D > \gamma_D \cdot \beta_R$, thus,

$$\frac{\Delta R}{\Delta p_N} = \frac{\gamma_R \cdot \beta_D - \gamma_D \cdot \beta_R}{\beta_D} > 0$$

Now, we know $\Delta^2 \Pi_{ND}/\Delta p_R^2$ is always smaller than zero, and the profit Π_{ND} should be a concave function in p_R. The optimal p_N can be found by letting $\Delta \Pi_{ND}/\Delta p_R = 0$.

$$\begin{aligned}
p_N^* &= \frac{\alpha_D + \gamma_D \cdot (c - c_R)}{2\beta_D} + \frac{\alpha_D \cdot \gamma_R - \alpha_R \cdot \gamma_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} \\
p_R^* &= \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} + \frac{c - c_R}{2}
\end{aligned}$$

and the corresponding profit is

$$\begin{aligned}
\Pi_{ND} &= \left(\frac{c - c_R}{2} - \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} \right) \\
&\quad \cdot (\gamma_R \cdot \frac{c - c_R}{2\beta_D} \cdot (\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D) - \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2\beta_R})
\end{aligned}$$

B.2.3 Conclusion

- If

$$\frac{\alpha_D + \gamma_D \cdot (c - c_R)}{\beta_D} \geq \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D}$$

  the optimal solution is

$$\begin{aligned} p_N^* &= \frac{\alpha_D + \gamma_D \cdot (c - c_R)}{2\beta_D} + \frac{\alpha_D \cdot \gamma_R - \alpha_R \cdot \gamma_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} \\ p_R^* &= \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} + \frac{c - c_R}{2} \end{aligned}$$

  the corresponding profit is

$$\begin{aligned} \Pi_{ND} &= \left(\frac{c - c_R}{2} - \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2(\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D)} \right) \\ &\quad \cdot (\gamma_R \cdot \frac{c - c_R}{2\beta_D} \cdot (\gamma_R \cdot \beta_D - \beta_R \cdot \gamma_D) - \frac{\alpha_D \cdot \beta_R - \alpha_R \cdot \beta_D}{2\beta_R}) \end{aligned}$$

- Otherwise, the optimal solution is

$$\begin{aligned} p_N^* &= \frac{\gamma_R \cdot \alpha_D - \gamma_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D} \\ p_R^* &= \frac{\beta_R \cdot \alpha_D - \beta_D \cdot \alpha_R}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D} \end{aligned}$$

  the corresponding profit is 0.

B.3 Low selling price line, $p_N = c$

Under this scenario, the profit should be

$$\Pi_{LS} = R \cdot (c - p_R - c_R)$$

B.3.1 Boundary

$p_N = c$ starts at point (p_N^s, p_R^s)

$$\begin{aligned} p_N^s &= c \\ p_R^s &= \frac{\beta_R \cdot c - \alpha_R}{\gamma_R} \end{aligned}$$

B.3.2 Solution

The first and second derivatives of Π_{LS} in p_R should be

$$\begin{aligned} \frac{\Delta \Pi_{LS}}{\Delta p_R} &= -R + \gamma_R \cdot (c - p_R - c_R) \\ \frac{\Delta^2 \Pi_{LS}}{\Delta p_R^2} &= -2\gamma_R < 0 \end{aligned}$$

Clearly, the profit Π_{LS} should be a concave function in p_R. The optimal p_N can be found by letting $\Delta \Pi_{LS} / \Delta p_R = 0$.

$$\begin{aligned} p_N^* &= c \\ p_R^* &= \frac{c - c_R}{2} - \frac{\alpha_R - \beta_R \cdot c}{2\gamma_R} \end{aligned}$$

The corresponding profit is

$$\Pi = \gamma_R \cdot \left(\frac{\alpha_R - \beta_R \cdot c}{2\gamma_R} + \frac{c - c_R}{2} \right)^2$$

B.3.3 Conclusion

- If

$$c - c_R \geq \frac{\beta_R \cdot c - \alpha_R}{\gamma_R}$$

  the optimal solution is

$$p_N^* = c$$
$$p_R^* = \frac{c - c_R}{2} - \frac{\alpha_R - \beta_R \cdot c}{2\gamma_R}$$

  The corresponding profit is

$$\Pi = \gamma_R \cdot \left(\frac{\alpha_R - \beta_R \cdot c}{2\gamma_R} + \frac{c - c_R}{2} \right)^2$$

- Otherwise, the optimal solution is

$$p_N^s = c$$
$$p_R^s = \frac{\beta_R \cdot c - \alpha_R}{\gamma_R}$$

  The corresponding profit is 0.

Appendix C

Algorithm

C.1 Deterministic Optimal Solution Algorithm

This algorithm is based on Theorem 1.

Step 0. Initialize.

We first need to find the optimal solution under each case

- Unconstrained solution

$$
\begin{aligned}
\begin{bmatrix} p_N^1 \\ p_R^1 \end{bmatrix} &= \begin{bmatrix} \gamma_D + \beta_R & -2\gamma_R \\ -2\beta_D & \gamma_D + \beta_R \end{bmatrix}^{-1} \cdot \begin{bmatrix} c \cdot \gamma_D + \alpha_R - (c - c_R) \cdot \gamma_R \\ -\alpha_D - c \cdot \beta_D + (c - c_R) \cdot \beta_R \end{bmatrix} \\
D^1 &= \alpha_D - \beta_D \cdot p_N^I + \gamma_D \cdot p_R^I \\
R^1 &= \alpha_R - \beta_D \cdot p_N^I + \gamma_R \cdot p_R^I \\
q^1 &= D^I - R^I
\end{aligned}
$$

- no recycling ($R = 0$)

$$
\begin{aligned}
p_N^2 &= \frac{1}{2} \cdot \left(\frac{\alpha_D \cdot \gamma_R - \alpha_R \cdot \gamma_D}{\beta_D \cdot \gamma_R - \beta_R \cdot \gamma_D} + c \right) \\
q^2 &= \frac{1}{2} \cdot \left(\alpha_D - \beta_D \cdot c - \frac{\alpha_R \cdot \gamma_D}{\gamma_R} + \frac{\beta_R \cdot \gamma_D}{\gamma_R} \cdot c \right)
\end{aligned}
$$

- no demand ($D = 0$)

$$
\begin{aligned}
p_R^3 &= \frac{1}{2} \left(c - c_R - \frac{\beta_D \cdot \alpha_R - \beta_R \cdot \alpha_D}{\gamma_R \cdot \beta_D - \gamma_D \cdot \beta_R} \right) \\
q^3 &= \frac{1}{2} \cdot \left(\alpha_R - \frac{\beta_R}{\beta_D} \cdot \alpha_D + \frac{\gamma_R \cdot \beta_D - \gamma_D \cdot \beta_R}{\beta_D} \cdot (c - c_R) \right)
\end{aligned}
$$

- too low selling price ($p_N = c$)

$$
\begin{aligned}
p_R^4 &= \frac{\beta_R \cdot c + \gamma_R \cdot (c - c_R) - \alpha_R}{2\gamma_R} \\
q^4 &= \frac{\gamma_R \cdot (c - c_R) + \alpha_R - \beta_R \cdot c}{2}
\end{aligned}
$$

Step 1. Judge Boundary Condition Compute the corresponding demand, supply and profit under each case, then check the boundary condition individually. If the boundary condition cannot be satisfied, let the corresponding profit equal to -1.

Step 2. Compare Find the maximum profit among all cases, and the corresponding strategy is our best choice. If the maximum profit is -1, means we would better do nothing under these parameters setting.

C.2 Stochastic Optimal Solution Algorithm

In this algorithm, we first determine the optimal selling price p_N^*, then compute corresponding collecting price p_R and order quantity q, in this way we can find our optimal profit.

The procedure to find the optimal p_N is based on the proof of theorem 2. We will use following formulations from that proof.

$$\begin{aligned}
\mathbb{A}(p_N) &= \mathbb{B}(p_N) + \alpha_D - \beta_D \cdot p_N + \gamma_D \cdot p_R^*(p_N) \\
&\quad - (p_N - c_R - p_R^*(p_N)) \cdot \beta_R \\
&\quad - E\left[(\mathbb{B}(p_N) - \epsilon)^+\right] - (p_N - c) \cdot (\beta_D - \beta_R) \\
\mathbb{C}(p_N) &= \frac{\Delta \mathbb{A}(p_N)}{\Delta p_N} \\
&= \frac{1}{r(\mathbb{B}(p_N))} \cdot \frac{(1 - F(\mathbb{B}(p_N)))^2}{c - s} \\
&\quad + \frac{\Delta p_R^*(p_N)}{\Delta p_N} \cdot (\beta_R + \gamma_D) - 2\beta_D
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{B}(p_N) &= F^{-1}\left(\frac{p_N - c}{p_N - s}\right) \\
r(\cdot) &= \frac{f(\cdot)}{1 - F(\cdot)}
\end{aligned}$$

Please refer the Appendix for the detailed meaning.

Step 0. Initialize.

Let $lower = c$, $upper = 1000$, $i = 1$, $\Pi^{(1)} = \Pi^{(2)} = -\infty$.

Step 1. Finding the Middle point.

a. If $\mathbb{C}(upper) \geq 0$ or $\mathbb{A}(upper) \geq 0$, then $upper = upper \times 2$, jump to step 1.a..

b. if $\mathbb{A}(lower) < 0$ and $\mathbb{C}(lower) < 0$, jump to step 3.

c. $p_N^U = upper$, if $\mathbb{A}(lower) > 0$, $p_N^L = lower$, jump to step 2.

d. Let $l = lower$, $u = upper$.

e. $middle = (l + u)/2$.

f. if $\mathbb{C}(middle) > 0$, then $l = middle$, jump to step 1.e..

g. if $\mathbb{C}(middle) < 0$, then $u = middle$, jump to step 1.e..

h. $i = 1$, $ind = 1$, $p_N^L = middle$.

Step 2. Binary Search.

a. $p_N^M = (p_N^L + p_N^U)/2$.

b. If $\mathbb{A}(p_N^M) < 0$, then $p_N^U = p_N^M$, jump to step 2.a..

c. If $\mathbb{A}(p_N^M) > 0$, then $p_N^L = p_N^M$, jump to step 2.a..

d. Let $p_N^* = p_N^M$, and compute corresponding p_R^*, q^* and Π^*.

e. Jump to step 4.

Step 3. No result.

We would better do nothing. STOP

Step 4. Optimal solution.

We have found our optimal solution. STOP.

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