

---

# Dark matter : experimental evidence, relic density, and the supersymmetric candidate

---

*Author : Benjamin Audren*  
*Advisor : Konstantin Zarembo*

## ABSTRACT

The nature of the dark matter is still a mystery even seventy years after the first mention of it. Designing the invisible mass we probe by looking carefully at the galaxies and clusters of galaxies, this new sort of matter is still subject to many speculations. In this paper, we will first present the evidence that have lead to the hypothesis of dark matter. We will then show a way to explain its present energy density via the freeze-out of the interactions, leading to an estimate of the cross-section and the mass of the hypothetic dark matter particle. Then, we will describe how such a massive particle can be predicted to be stable by looking at the supersymmetric candidate for dark matter. We will finally conclude on how the experiment might help constraining the model and refining it.

July 3, 2009

# Contents

<b>Introduction</b>	<b>1</b>
<b>1 The evidence for dark matter</b>	<b>3</b>
1.1 Rotation curves of galaxies . . . . .	3
1.2 Gravitational lensing, virial theorem and other evidence . . . .	4
1.3 The baryonic abundance . . . . .	5
<b>2 Dark matter particle</b>	<b>7</b>
2.1 Desired properties <i>a priori</i> . . . . .	7
2.2 Freeze-out, equilibrium departure . . . . .	7
2.2.1 Statistical description . . . . .	7
2.2.2 Cold relics . . . . .	14
2.2.3 Dirac heavy neutrinos . . . . .	17
2.3 Short presentation of coannihilation . . . . .	20
<b>3 The Supersymmetric candidate</b>	<b>25</b>
3.1 Basic Ideas about supersymmetry, MSSM . . . . .	25
3.2 Soft breaking of supersymmetry . . . . .	26
3.3 Experimental detection, constraint, the end of the MSSM? . . .	28
3.4 Discussion . . . . .	28
<b>Conclusion</b>	<b>30</b>
<b>References</b>	<b>31</b>

## Introduction

Since the Einstein's equations of general relativity in 1915 and the proposed model of the universe by Friedmann, Robertson, Walker and Lemaitre, we know that it exists a strong link between how the universe expands and how much matter there is inside. If we consider a flat universe, the link defines a critical energy density  $\rho_c$  that must have the universe if this theory is correct.

$$H^2 = \frac{8\pi G}{3}\rho_c, \quad (1)$$

where H is the Hubble constant and G the gravitation constant. One can introduce the notation  $\Omega = \rho/\rho_c$  which is the contribution of a species to the critical density. When the curvature of our 4-dimensional universe is taken into account, the Friedmann's equation transforms in a inequality. As a result, an universe denser than the critical density will have a positive curvature, so a shape like a sphere and will be closed - *ie* finite. *A contrario*, with less than the critical density, the curvature will be negative and the universe open, *ie* infinitely extended in space - as for the intermediate case, a flat and also infinite universe. One possibility consists in measuring the density of our universe - a not so easy task. For decades, this has lead people to think that our universe was negatively curved, since the density was much less than the critical one.

However, one can also think the other way around. If we instead measure the curvature of the universe at a sufficiently large scale, we will be able to know its density. The Wilkinson Microwave Anisotropy Probe - WMAP for short - has confirmed with a high precision what previous results had already pin pointed out : it is very close to be flat, that is to say that there is a density of energy almost equal to the critical density predicted by Friedmann's equation. But this mission also confirmed that all our visible universe, all the 'light' matter - what we can see - contributes only up to  $\Omega_{LM} \simeq 0.4\%$  of this density so that the previous conclusion about the open universe, though wrong, was due to a real problem : we do not see the majority of the energy content of the universe. In a word, what constitutes the bulk of the universe is yet to be understood.

On the other hand, some observations raised first by Zwicky [14] in 1937, and that will be discussed in the section 1 tend to prove that we are making a mistake by estimating the mass of the galaxies and the clusters of galaxies

by just measuring the emitted light. This underestimation could lead to almost a factor of one hundred, huge discrepancy but still not enough to produce all the critical density. It will be discussed here the reason why this unseen dark matter is thought to be non baryonic, that is to say different from the matter we are accustomed to see around us, and thus the need to consider a yet-to-be observed dark matter particle  $\psi$  that would account for the missing matter part of the energy density of the universe.

To explain how this particle would have reached our times without being detected and completely annihilated, we will discuss in the section 2 the mechanism of freeze-out in detail and produce a natural constraint on the mass and cross-section values of this hypothetic particle. This calculation has lead to build up the new generation of detectors in order to check these regions. We will then detail the mechanism of coannihilation that can change considerably the relic density in some particular cases that will eventually show up in the case of supersymmetry.

The Standard Model being short of the prediction of such a particle, one has to search elsewhere a theory to predict it. There are several possibles models that produce a potential dark matter candidate, but I have chosen here to present only the supersymmetric candidate. In the section 3, I will thus present the ideas behind the supersymmetry, as well as the actual candidate, and try to show how the going-on experiments are constraining the model, and how recent results might lead to a new formulation of the breaking of supersymmetry. Due to the limited time of the internship, I have decided not to go into the mathematical details of supersymmetry but rather try to uncover the main ideas and the general line of thought under it, in order to present a general picture.

# 1 The evidence for dark matter

The problem that arises when we calculate the energy density of the light matter is that we assume a ratio mass/luminosity being the same than the one we observe from nearby stars. This issue has first been pointed-out by Zwicky in his paper from 1937 where he remarks that this assumption might be inconsistent. Indeed, there could be some unseen matter - therefore dark - that could either absorb the radiation from the galaxies, either not emit any light. In any case, this additional term would lead us to underestimate by a large factor the actual mass of this humongous objects.

Here follows the various features that Zwicky shown at the time to illustrate his point, and that now have been verified with higher accuracy to be correct.

## 1.1 Rotation curves of galaxies

Let us consider a typical spiral galaxy. If we assume that the bulb in the center is roughly spherical, we have that

$$\mathbf{F} = -\frac{GM(r)m}{r^2}\mathbf{e}_r = m\mathbf{a} = -m\frac{v^2(r)}{r}\mathbf{e}_r \quad (2)$$

$$v(r) = \sqrt{\frac{1}{r}\sqrt{GM(r)}} \quad (3)$$

In the expression of the gravitational force, only the mass enclosed by the orbit is considered. For a spherical distribution, one can expect the mass of the inside bulge to scale as  $r^3$ , hence to give the speed a linear behaviour by the eq. (3). When one leaves the bulge, as the matter distribution depart from spherical one, the mass will not increase as fast as  $r^3$ , so the dependence will become flatter and flatter until, at the border of the galaxy, the inside mass being a constant, one could expect a  $r^{-1/2}$  decay.

However, observations of orthoradial speeds of isolated stars and gas clouds outside the border of the visible galaxy [12] show a departure from that prediction (see fig. (1.1)). The flatness of the curve indicates that the mass is still increasing outside the visible arms with a  $r^2$  power law, indicating the presence of an unseen matter. This additional amount of mass might lead us to underestimate the total mass of a galaxy by a factor

of 5 or 10.

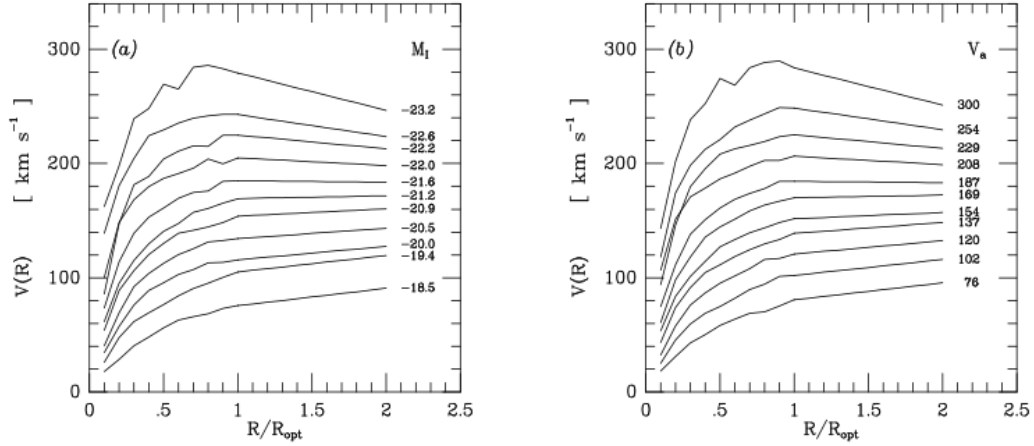


Figure 1: Rotation speed of galaxy, in units of luminous radius

One possible interpretation for that phenomenon is to claim that the laws of physics are no more valid at the large galactic scale, and that for large distances, the gravitational force scale as  $r^{-1}$  instead of the  $r^{-2}$ . This claim made by the Modified Newtonian Dynamic - MOND for short - is a way to describe these rotation curves. Though, this static model does not explain the other problems raised by the observations, namely the same problems of invisible mass but at the cluster scale.

## 1.2 Gravitational lensing, virial theorem and other evidence

In the theory of general relativity, we know that the light coming from behind a massive object will be bended towards the object. In a word, the location of a background of very distant stars will be distorted when for instance a cluster of galaxies will pass in front of it. We are able with that data to go back to the mass of the object, and here the result is even more critical : the underestimate of the total mass of a cluster would be of order one hundred !

It exist another evidence of this discrepancy when one looks in the kinematic motion of the galaxies inside a cluster. If one applies the Virial theorem, assuming that the cluster is in statistical equilibrium, with the various

speeds of the elements one can determine the mass of the total cluster. Here, the result is the same, meaning that the actual mass is far greater than what we collect from the light only.

Yet another way to determine the amount of dark matter in the clusters that have not been raised by Zwicky is to look at very large scale motion. Our local cluster is indeed falling in our neighbor Virgo cluster. By characterising this infall, we are able to determine the masses involved, and this result also leads to a higher value than expected.

The interesting point with all these different methods is that they use various independant physical principles and yet lead to the same result. This allows us to be confident enough on our current interpretation.

Moreover, the observation of the bullet cluster has clearly shown the disparity between the light matter barycenter and the total mass barycenter via gravitational lensing. This can be seen as a strong proof for dark matter hypothesis, but it is actually possible to explain this feature without making the assumption of an unknow matter. In [4] one of the dynamical development of the MOND theory is developed and simulated and they succeed in getting a different barycenter than the visible one. However, in this new development of the theory, known as generalised Einstein-Aether theories, they modify general relativity law by adding a vector field of non-zero time-like component. The point is then that the fluctuations of this field can be the seeds of baryonic matter collapse, but that a high value of this field can still remain where there is no baryonic matter around. In a nutshell, the unknown dark matter particles are replaced by an unknow vector field. To discriminate between the two models seems for now very difficult, so one has to make one hypothesis to choose between the two. Here, I have decided to focus on the dark matter hypothesis.

### 1.3 The baryonic abundance

If the dark matter hypothesis is picked up, it follows that we do not see an important part of the matter that constitutes the universe. But that does not imply *a priori* that it is some new kind of matter. One can think of all the normal baryonic matter that would escape our detection by being too faint, such as neutron stars, black holes or intergalactic medium - the fermionic matter, due to its very low mass, does not contribute significantly to the

energy density of the universe. Is all this would be enough to explain these nearly two orders of magnitude discrepancy ? That is what was thought before someone proposed a way to calculate independantly the baryon density in the universe and compare it with the total amount of mass we measure with these new methods. One possibility is to look at the nucleosynthesis.

During the first minutes after the Big Bang, the temperature was so high that nuclear fusion happened. During this short time, the protons merged to create stable atoms of Deuterium, Helium 3 and 4, Lithium 6 . In order to calculate theoretically the abundances of these elements at the end of the Big Bang Nucleosynthesis - BBN - one has to introduce the number of baryons per photons. Since we are able to calculate the number density of photons via general relativity, one can go back to the actual mass density of baryons. The point is that the abundances of the species after this process will depend more or less strongly on this parameter. It appears that the Deuterium has the strongest dependancy : if one is able to measure its abundance in the very early universe, so before anything happened to it, one can deduce the value of the free parameter and conclude about the mass density of the total baryons in the universe.

This work has been done and the outcome is that  $\Omega_{\text{baryons}} = 4.4 \pm 0.4\%$ . This definitively rules out the baryons for being the sole responsible for the missing mass of the universe, and even for being enough to explain the mass of the cluster of galaxies. Indeed, the WMAP experiment has estimated the total amount of matter in the universe to be  $\Omega_{\text{matter}} = 27\%$ . One thus clearly need to look for something else, another particle, to explain this missing 23%.



## 2 Dark matter particle

I have described so far the evidence that prove that something is amiss. What is this something ? If we assume that all the matter and energy we are seeing today was here since the beginning, after the inflation, then this particle that constitutes the dark matter should have also been there.

### 2.1 Desired properties *a priori*

Let us consider a hypothetical particle  $\psi$  and its antipartner  $\bar{\psi}$ , of mass  $m_\psi$ . First of all, this particle should be quite massive to account for this huge missing mass and despite its low density. We will see in the section 3 how such a massive particle can be stable and not decay into normal matter. It also should not interact directly with light, so being neutral, and colorless as well. We also want it to interact weakly with the normal matter, otherwise we would already have detected it, and that leads with the previous condition of the interaction via the exchange of a  $Z^0$  neutral weak boson.

Concerning the reactions involved, we want to take the easiest hypothesis possible. Thus we will consider only the processes of annihilation and pair creation  $\psi + \bar{\psi} \Leftrightarrow i + \bar{i}$  and state that they will be predominant, with  $i$  being *a priori* anything, photons, electrons, neutrinos, etc...

### 2.2 Freeze-out, equilibrium departure

#### 2.2.1 Statistical description

Once all this assumed, the next step is to place the calculation in the context. This dark matter particle should have been in thermal equilibrium in the very beginning of the universe, at least, as all the rest of the particles. The problem is the following : if a species stays in thermal equilibrium, when the universe cools down, the pair  $i\bar{i}$  will not have enough thermal energy to annihilate and pair create  $\psi\bar{\psi}$ . But if the dark matter particle still continue to annihilate, its density will drop drastically with time and its today's density will be negligible. Somehow, one has to think of a mechanism to drop out of this equilibrium and to keep the particle in a constant density.

This is called the decoupling. For different species the precise mechanism can vary whether there a particle-antiparticle number density symmetry or not, but we will here consider the simple case where there is such a thing.

In this case, we can get a visual picture of how it works by considering the fact that the universe expands as it cools. Hence, if the universe expands more rapidly than the particle/antiparticle annihilate themselves, there is a moment where annihilation will be very unlikely to happen since the partners will be so far away from each other. It is the freeze-out of interactions - in other terms, the reaction goes out of equilibrium. The particles remain at the same density that they had at that moment and from that point are only diluted in the universe, but no more annihilated. These relics from the past are a way to explain the presence of dark matter today, even if it tells nothing yet about the nature of that particle.

This simplified picture is not sufficient to explain for instance all the remaining baryons in the universe. For that, one has to claim an original unbalance between matter and antimatter. However, for dark matter it is not compulsory to add this original asymmetry, I will thus consider for the rest of the calculation that  $n_\psi = n_{\bar{\psi}}$ . In order to treat properly this problem, one has to wonder how to describe the evolution of the density of such species.

The density being linked with the phase space distribution (or measure for the mathematicians)  $f(x^\mu, p^\mu)$ , with  $x^\mu$  and  $p^\mu$  the four position and the four momentum respectively, the important equation that one will have to solve is the Boltzmann equation,

$$\hat{L}[f_\psi] = \hat{C}[f_\psi], \quad (4)$$

where  $\hat{L}$  is the Liouville operator and  $\hat{C}$  is the collision operator. Basically, this equation states that the evolution of a particle distribution is a function of its interaction and of the environnement. In covariant formalism, the Liouville operator can be expressed :

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma_{\beta\gamma}^\alpha p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}. \quad (5)$$

The Christoffel's symbols that appear in the expression illustrate the fact that how particles evolve in the space depends on how the space is measured. One has thus to assume a metric to continue the calculation, in order to obtain an expression for the Christoffel's symbols. From now on,

the greek indices will run from 0 to 3 and the latin letters will only cover 1 to 3. Since we have settled the problem after the inflation, we can reasonably assume that the variations of density are weak, and so that the Robertson Walker (RW) metric is a good description of the geometry of the universe from that moment until today. Moreover, since this is happening during the radiation dominated era, the primordial fluctuations of density are frozen-in and do not evolve or contribute to these events. The RW metric writes :

$$ds^2 = dt^2 - R(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\}, \quad (6)$$

where the convention  $c = 1$  has been picked, and  $r$  is dimensionless so  $R(t)$  is the cosmic scale factor and has a dimension of length. This last factor actually describes the stretching of the universe, the actual distance between two points. The choice of this metric implies that the distribution function will also be homogeneous and isotropic - because the spacial term has a common factor in front - that is to say that the only dependance of  $f$  in terms of  $x^\mu$  is with respect to time, and in terms of  $p^\mu$  is with respect to the energy. From now on, the calculation will be in the natural system of units imposing  $c = \hbar = k_B = 1$ , implying the equivalence between mass, energy, temperature, inverse time and inverse length. We can then compute the only Christoffel's symbols that will be relevant :

$$\Gamma^0_{ij} = -\frac{\dot{R}}{R} g_{ij}, \quad (7)$$

where the dot denotes the derivation with respect to time. With this expression one can get :

$$\hat{L} = E \frac{\partial}{\partial t} - \frac{\dot{R}}{R} |\mathbf{p}|^2 \frac{\partial}{\partial E}. \quad (8)$$

The next step is to find the evolution of the number density of our particle, where it has been defined as usually as :

$$n_\psi(t) = \frac{g}{(2\pi)^3} \int d^3\mathbf{p}_\psi f_\psi(E, t) = \frac{g}{(2\pi)^3} \int 4\pi|\mathbf{p}_\psi|^2 dp_\psi f_\psi(E, t), \quad (9)$$

with  $g$  denoting the number of internal degrees of freedom. After an integration by part that brings the factor 3 and the sign change and by using  $E dE = p dp$  and  $E^2 = p^2 + m^2$ , the Boltzmann equation (4) rewrites as follows,

$$\frac{\partial n_\psi(t)}{\partial t} + 3H n_\psi(t) = \frac{g}{(2\pi)^3} \int \hat{C}[f_\psi] \frac{d^3\mathbf{p}_\psi}{E_\psi} \quad (10)$$

Where  $H = \frac{\dot{R}}{R}$ , the Hubble parameter has been introduced. One has now to deal with the collision part. In a very general way, one should consider all the possible collisions and scattering processes. However, our supposed particle interacts weakly with its environnement, so one can limit the study only to the annihilation - pair creation channel with other particle, and neglect all the possible scattering processes. Even with this drastic - though justified - simplification the equation will need a numerical solving. The collisionnal term becomes

$$\begin{aligned} \frac{g}{(2\pi)^3} \int \hat{C}[f_\psi] \frac{d^3\mathbf{p}_\psi}{E_\psi} &= - \int d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_i d\Pi_{\bar{i}} (2\pi)^4 \delta^4(p_\psi + p_{\bar{\psi}} - p_i - p_{\bar{i}}) \quad (11) \\ &\dots \left\{ |\mathcal{M}|_{\psi+\bar{\psi} \rightarrow i+\bar{i}}^2 f_\psi f_{\bar{\psi}} (1 \pm f_i)(1 \pm f_{\bar{i}}) - \dots \right. \\ &\dots \left. |\mathcal{M}|_{i+\bar{i} \rightarrow \psi+\bar{\psi}}^2 f_i f_{\bar{i}} (1 \pm f_\psi)(1 \pm f_{\bar{\psi}}) \right\}, \end{aligned}$$

with a plus sign if the considered species is a boson, and a minus sign if it is a fermion, and where  $d\Pi_a = \frac{g_a}{(2\pi)^3} \frac{d^3\mathbf{p}_a}{2E_a}$ .

The other basic assumptions one can make are the T invariance - invariance with respect to time reversal - and the Maxwell-Boltzmann distribution for any species, regardless of their spin statistic. The first one is motivated despite the fact that for some cases the T invariance is broken in weak interactions because it allows us to equalize the matrix elements of the annihilation and pair creation. This is, to be honest, a weak argument, but the actual description of the difference between the two matrix elements is,

as far as I know, not clearly established. Again, this is a toy calculation, and we will see in the following parts that there are cases where it does not apply perfectly. To be correct, every time one predicts a potential candidate, one should check each hypothesis made along the calculation to see if it breaks somewhere. The second approximation provides us a formula for the distribution function of the species if in equilibrium, that is to say :

$$f_i(E_i) = \exp\left(-\frac{E_i - \mu_i}{T}\right) \text{ and } 1 \pm f_i \simeq 1, \quad (12)$$

where the Boltzmann constant is taken equal to 1,  $\mu_i$  and  $E_i$  respectively the chemical potential and the energy of the  $i$  particle. It is a well accepted fact that in the beginning, the universe was very close to be in thermal equilibrium. What we search to do here is to precisely characterize the moment where one of the species went out of that equilibrium. This ideal distribution will thus be used as an initial condition and as the actual distribution of the other species, assumed lighter and more interacting, so still in thermal equilibrium when  $\psi$  decoupled. The Boltzmann equation (10) becomes

$$\dot{n}_\psi + 3Hn_\psi = -\int d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_i d\Pi_{\bar{i}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_\psi + p_{\bar{\psi}} - p_i - p_{\bar{i}}) \left( f_\psi f_{\bar{\psi}} - f_i f_{\bar{i}} \right). \quad (13)$$

The equation looks at last like something we can talk about. The evolution of the time variation of the number density of a species is driven by the expansion of the universe - a dilution term, characterized by the Hubble factor  $H$  - on one side, and on the other side of the interaction with the other particles. If the collision term was strictly zero, we would find straightforwardly the  $R^{-3}$  dependence of  $n_\psi$ . To interpret the second term in a simple way, it will need again some manipulations.

As said previously, the  $i$  species are considered to be in thermal equilibrium all the time during the phenomenon we describe here. It makes sense because of the more numerous interactions : it is easier for it to achieve thermal equilibrium. This assumption will let us replace the  $f_i$  in the collisional term by (12). We can assume furthermore than the chemical potential of the  $i$  species is 0. Indeed, for the photons it is rigorously equals to zero, and for

other species, during the phase we are studying, the energy is much greater than the chemical potential. It also reflects the fact that they are in thermal equilibrium, so that their number density is conserved.

By looking at the RHS of (13), one can see that the integration of the energy part of the dirac term will lead to  $E_i + E_{\bar{i}} = E_\psi + E_{\bar{\psi}}$ . Hence, the  $f_i f_{\bar{i}}$  will transform after integration into  $\exp(-\frac{E_\psi + E_{\bar{\psi}}}{T})$ . This term corresponds to a  $f_\psi^{eq} f_{\bar{\psi}}^{eq}$ , 'eq' designating the equilibrium value. Indeed, this would be the phase space distribution of  $\psi$  if it would stay in equilibrium. From that, it is straightforward to see that these distribution terms will lead after integration over  $d\Pi_\psi d\Pi_{\bar{\psi}}$  to number density terms. Remembering that it has been assumed a symmetry between  $\psi$  and  $\bar{\psi}$ , so that  $n_\psi = n_{\bar{\psi}}$ , one finds:

$$\dot{n}_\psi + 3Hn_\psi = - \langle \sigma_{\psi+\bar{\psi}\leftrightarrow i+\bar{i}}|v| \rangle \left( n_\psi^2 - (n_\psi^{eq})^2 \right), \quad (14)$$

Where  $\langle \sigma_{\psi+\bar{\psi}\leftrightarrow i+\bar{i}}|v| \rangle$  designs the thermally averaged annihilation cross section times velocity and is given by (15) which can be rewritten as (16) to keep the symmetry in the expression. One has however to note the assumption made here concerning the matrix element  $|\mathcal{M}|^2$ . Indeed, it depends *a priori* on all the four-impulsions of both the initial and final state. It could thus appear wrong to think that it will be the same for the part in equilibrium and the part out of equilibrium, and so that the factorization in (14) comes a bit too early. However, one has also to remember that far from equilibrium, the collision term does not play a great role any more. In conclusion, this approximation is good as long as the species stay close to their equilibrium value, and becomes bad when this collision term becomes negligible. It is thus a quite reasonable approximation.

$$\langle \sigma_{\psi+\bar{\psi}\leftrightarrow i+\bar{i}}|v| \rangle = \frac{1}{2E_\psi 2E_{\bar{\psi}}} \int d\Pi_i d\Pi_{\bar{i}} (2\pi)^4 |\mathcal{M}|^2 \quad (15)$$

$$= \frac{1}{(n_\psi^{eq})^2} \int d\Pi_\psi \dots d\Pi_{\bar{i}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_\psi + p_{\bar{\psi}} - p_i - p_{\bar{i}}) e^{-E_\psi/T} e^{-E_{\bar{\psi}}/T}. \quad (16)$$

If we, at last, make the summation over all the different annihilation channels - ie for all the different i's - we have the final expression

$$\dot{n}_\psi + 3Hn_\psi = - \langle \sigma | v | \rangle \left( n_\psi^2 - (n_\psi^{eq})^2 \right). \quad (17)$$

At that time, one thing that can be done to see more clearly the effect of interaction is to suppress the effect of dilution, by placing our analysis in a comoving volume. One can thus introduce the entropy density  $s$  and define the variable  $Y \equiv \frac{n_\psi}{s}$ . Considering that the entropy of a comoving volume is conserved -  $sR^3 = \text{constant}$  - one can see that the LHS of (13) rewrites

$$\dot{n}_\psi + 3Hn_\psi = s\dot{Y}. \quad (18)$$

Moreover, it is convenient to make another variable change. So far, the evolution of the density was given with respect to time. To continue, one will have to assume an analytic shape for the cross section, and it is usually given as a function of temperature. It is thus useful to use the variable  $x = m/T$  where  $T$  is the temperature of the universe and  $m$  any mass scale, which for simplification purposes we will take it equals to  $m_\psi$ . The relation between the time and the temperature varies according to the era. Since our study takes place in the radiation dominated era - that is to say before the decoupling of the CMB and the beginning of the matter dominated era - the relation straightforwardly given by Friedmann's equation is the following :

$$t = 0.301g_*^{-1/2} \frac{m_{Pl}}{T^2} = 0.301g_*^{-1/2} \frac{m_{Pl}}{m_\psi^2} x^2 = \frac{x^2}{2 \cdot H(m)} \quad (19)$$

Where the Hubble parameter - with a dimension of inverse time - has been introduced to simplify the following expressions.  $m_{Pl}$  refers to the Planck mass, and  $g_*$  is the total number of effectively massless degrees of freedom (species with mass  $m_i \ll T$ ) given by :

$$g_* = \sum_{i \text{ bosons}} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{i \text{ fermions}} g_i \left( \frac{T_i}{T} \right)^4 \quad (20)$$

This quantity is thus calculated thanks to the standard model, for running the summation over all species still in equilibrium at the given time.

Since it is only the equilibrium particles that participate in that, it does not matter if we discover more massive particles, it will not change that value. It is fairly constant over large range of values, since it will vary only when a species decouples. One has also to note here that even though we are interested in the present day density of the particle, we do not take into account the matter dominated era to compensate the expression of time in terms of temperature. We do so because the decoupling happens during the radiation dominated era, and from then is completely decoupled from the rest of the universe, as the radiation. One can thus still use the relation for the temperature from the radiation phase to determine the evolution of the dark matter.

We eventually end with the rewriting of (17) as follows

$$\frac{dY}{dx} = \frac{-x \langle \sigma|v| \rangle s}{H(m)} (Y^2 - Y_{eq}^2) \quad (21)$$

Introducing  $H(x) \equiv x^{-2}H(m)$ , the proper Hubble constant, and  $\Gamma \equiv n_{eq} \langle \sigma|v| \rangle$  the inverse mean free time of flight between two collisions, both being of inverse time dimension, we can even cast this equation into a meaningful shape :

$$\frac{x}{Y_{eq}} \frac{dY}{dx} = -\frac{\Gamma}{H(x)} \left( \left( \frac{Y}{Y_{eq}} \right)^2 - 1 \right) \quad (22)$$

This shape allows us to make a consistency check of the equation. Let us assume the decoupling has occurred, so that  $\frac{\Gamma}{H(T)}$  is small, even with respect to the departure from equilibrium. Hence the RHS of (22) will be less than order unity. Then, we can see that  $\frac{x}{\Delta x} \frac{\Delta Y}{Y} \sim \frac{\Gamma}{H(T)}$ , so that  $\frac{\Delta Y}{Y} \sim \frac{\Delta x}{x} \frac{\Gamma}{H(T)}$ . Namely, even for large variation of  $x$  - ie of time - the relative variation of the number of particles in a comoving volume is low : this is the freeze out.

### 2.2.2 Cold relics

The more widely accepted model today is to say that the dark matter particles decoupled when they were already non relativistic - ie cold. This means that the particle is massive enough, hence the name of Weakly Interacting



Massive Particle. It exist also a description through warm relics, of lesser mass, that could also explain the dark matter, but I have decided here to treat only the first case since it corresponds to the future calculation for the supersymmetric candidate. To determine the remaining density of this cold dark matter particle, one has to assume a temperature dependency of the cross section.

The relation between the annihilation cross section and the speed is a function of the type of interaction.  $\langle \sigma|v| \rangle \propto v^p$  with  $p = 0, 2, \dots$  for respectively a S-wave, P-wave ... The equipartition of energy gives that  $v \propto T^{1/2}$  hence the chosen parametrization of the cross section

$$\langle \sigma|v| \rangle = \sigma_0 \left( \frac{T}{m_\psi} \right)^n = \sigma_0 x^{-n} \quad (23)$$

With  $n = 0, 1, \dots$  for respectively a S-wave, P-wave ... The equation can thus be rewritten in the form

$$\frac{dY}{dx} = -\lambda x^{-n-2}(Y^2 - Y_{eq}^2), \quad (24)$$

where  $\lambda = \left[ \frac{x \langle \sigma|v| \rangle s}{H(m)} \right]_{x=1}$   
and  $Y_{eq} = 0.145 \left( \frac{g}{g_{*S}} \right) x^{3/2} e^{-x}$ ,

and where it has been used that  $s$  scales as  $R^{-3}$  so as  $x^{-3}$  since  $T$  scales as  $R^{-1}$ .

From that point one can solve the equation numerically - since  $Y_{eq}$  is a function of  $x$ , this equation does not accept any exact solution. The results are shown on fig. 2 . The point, however, is to find a constraint on the value of the cross section or the mass of the particle according to the density it implies. With such a numerical solving, it will not be possible to reach this constraint. Luckily enough, one can also make a further assumption that leads to a rather good estimate [9] of the ending value. In the paper, they claim a precision of order 5%. I have checked the validity for meaningful values of  $\lambda$ , *i.e.* around  $10^{10}$ . One has to note however that for bigger and bigger  $\lambda$ , the convergence of the differential equation is becoming harder and

harder. I have used ode113, a numerical solver implemented in Matlab using the Runge-Kutta method. To improve the convergence, one has to take a huge number of points, and the time becomes very important. With 10000 points on a linear scale from 1 to 200, with  $\lambda = 1e8$  the precision is of order 1%. For smaller values of  $\lambda$ , the error lies within 10%, and further on the computation takes more than a dozen of minutes to proceed, I thus did not check. But it appears that this approximation is a very good one for the domain we are interested in. Let us just rewrite once more the equation introducing  $\Delta = Y - Y_{eq}$ :

$$\Delta' = -Y'_{eq} - \lambda x^{-n-2} \Delta (2Y_{eq} + \Delta). \quad (25)$$

Let us introduce the value  $x_F$ , which corresponds to the temperature, hence the time, where the particle decouples. For late times, that is for  $x \gg x_F$ , we know that the departure from equilibrium will be great, thus that  $Y \gg Y_{eq}$ , or  $\Delta \simeq Y$ . The equation can now be solved

$$\Delta' = -\lambda x^{-n-2} \Delta^2 \quad (26)$$

$$\int_{\Delta=0}^{\Delta=Y_\infty} \frac{d\Delta}{\Delta^2} = - \int_{x=x_F}^{x=+\infty} \lambda x^{-n-2} dx \quad (27)$$

$$\Delta_\infty = Y_\infty = \frac{n+1}{\lambda} x_F^{n+1} \quad (28)$$

The point consists now in determining  $x_F$ . In order to do this, one can recall the definition we took for the decoupling, which was the time where the expansion rate of the space becomes of the same order that the inverse mean free time for a collision (see eq. (22) and discussion). Namely, the freeze-out criterion becomes  $H(x_F) \simeq \Gamma(x_F)$ . Given that  $s = \frac{2\pi^2}{45} g_{*S} T^3 = \frac{2\pi^2}{45} g_{*S} x^{-3} m_\psi^3$  we end up with

$$x_F \simeq \ln \left( 0.0873 \frac{g}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_0 \right) - (n-1/2) \ln(x_F) \quad (29)$$

$$\Rightarrow x_F \simeq \ln \left( 0.0873 \frac{g}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_0 \right) - (n-1/2) \ln \left( \ln \left( 0.0873 \frac{g}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_0 \right) \right) \quad (30)$$

The passage from eq. (29) to (30) being justified by the fact that every-

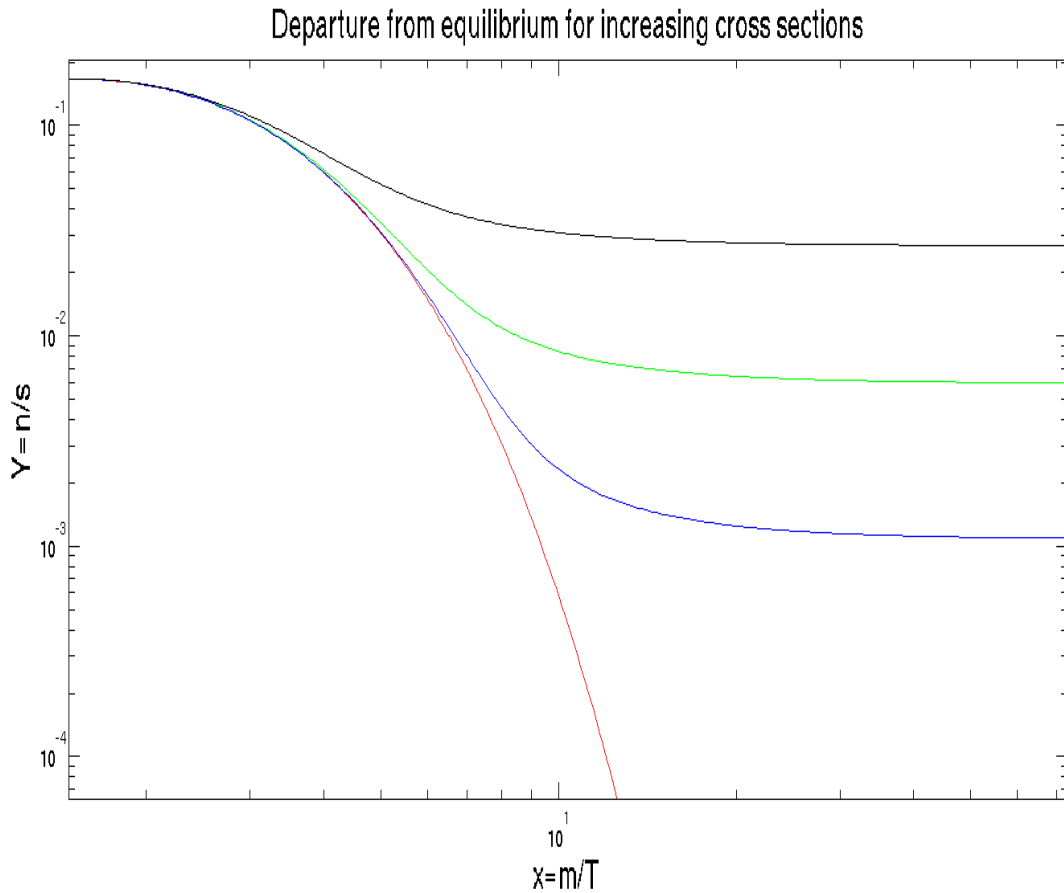


Figure 2: Numerical solution of the eq.(24) for values of  $\lambda$  of  $10^3, 10^4$  and  $10^5$ .

thing is expressed with units where  $c = \hbar = 1$ . This way, the logarithm of the big parenthesis is a big positive number, and since  $x_F$  is of order ten, one can in first approximation take  $x_F$  equal to the first logarithm and end up with eq. (30).

The last step consists now in giving a value to  $\sigma_0$  still mysterious so far. To do this, one must now assume something about the nature of these particles.

### 2.2.3 Dirac heavy neutrinos

One possibility is for instance to consider the case of Dirac heavy stable neutrino. The interaction process is thus through the exchange of a  $Z^0$

weak boson. The annihilation proceeds only through the s-wave channel, thus  $n = 0$  and  $\sigma_0 \simeq c_2 G_F^2 m_\psi^2 / 2\pi$  for a mass lower than  $m_Z/2$  and  $\sigma_0 \propto 1/m_\psi^2$  for a mass greater than  $m_Z/2$ , with  $c_2 \simeq 14$  - it corresponds to the number of annihilation channels open, here the three other neutrino families,  $e^- e^+$ ,  $\mu^- \mu^+$ ,  $u\bar{u}$ ,  $d\bar{d}$ ,  $s\bar{s}$ , with all the fermions (anti fermions) of helicity  $-\frac{1}{2}$  ( $+\frac{1}{2}$ ), and three colors for each of the quarks u, d and s- and  $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$  the Fermi coupling constant [9]. Indeed the variation of behaviour comes from the fact that for a lower mass, the scale of energy is such that the Fermi coupling constant will appear, so one has to find this  $m^2$  dependancy to keep the homogeneity. After that scale of energy, the only relevant scale becomes the mass of the particle, and thus the  $m^{-2}$  dependancy.

The fact that the dependency in mass changes over a certain value will lead to two possible values for the mass with only one value for the cross section. Taking  $g = 2$  for a massive spin 1/2 particle and  $g_{*S} \simeq g_* \simeq 60$  (at that time few particles had already decoupled), the Planck mass  $m_{Pl} = 1.2209 \times 10^{19} \text{ GeV}$ , we find for  $x_F$

$$x_F \simeq 18 + 3 \ln(m_\psi \text{ in GeV}) \quad (31)$$

$$Y_\infty = \frac{n+1}{\lambda} x_F^{n+1} = \frac{1}{\lambda} x_F \quad (32)$$

$$= 4 \times 10^{-11} (m_\psi \text{ in GeV})^{-3} \left[ 1 + \frac{3}{18} \ln(m_\psi \text{ in GeV}) \right] \quad (33)$$

One can notice here that the cold relics approximation is verified since  $x_f \sim \mathcal{O}(10)$ , we have that  $T_F/m_\psi \simeq \mathcal{O}(0.1)$  - in other terms, the thermal agitation at the time of decoupling was lower than the rest energy. Now we can change back the variable to the number density and the mass density, and at last divide by the critical density

$$\rho_0 = n_0 \times m_\psi = Y_\infty \times s_0 \times m_\psi = Y_\infty \times 2970 \times (10^2)^3 \times m_\psi \quad (34)$$

$$\rho_0 = 2.09 \times 10^{-28} \cdot (m_\psi \text{ in GeV})^{-2} \left[ 1 + \frac{3}{18} \ln(m_\psi \text{ in GeV}) \right] \quad (35)$$

$$\Omega_{\psi\bar{\psi}} = 2 \times \frac{\rho_0}{\rho_c} = \frac{2\rho_0}{3H^2/8\pi G} = \frac{2\rho_0 8\pi G}{3(100 \text{ km/s/Mpc})^2 h^2} \quad (36)$$

$$\Omega_{\psi\bar{\psi}} h^2 \simeq 3.7 \times 10^{-42} \frac{1}{\sigma_0 \text{ in } m^2} \quad (37)$$

The important point to notice here is the presence in the denominator of  $\sigma_0$ . That is to say, the weaker the particle interacts, the denser it will be today. Indeed, the small interaction implies an earlier decoupling, an earlier departure from the equilibrium, at a time where the equilibrium value was high. We know today with the precise measurements that this density should be  $0.23 \pm 0.04$ . This leads to a lower and an upper bound for the cross section in general, and the mass for this model. The wanted particle must thus lie within this limit to be still a good candidate for dark matter. All calculation done, this give the following boundaries for the candidate for dark matter :

$$1.4 \times 10^{-41} \text{ m}^2 \leq \sigma_0 \leq 1.9 \times 10^{-41} \text{ m}^2 \quad (38)$$

$$1.8 \text{ GeV} \leq m_\psi \leq 2.1 \text{ GeV} \quad (39)$$

$$\text{or } 1.9 \text{ TeV} \leq m_\psi \leq 2.3 \text{ TeV} \quad (40)$$

This particularly low value of cross-section justifies the hypothesis *a posteriori* of a weakly interacting particle. Indeed this value lies typically in the range of values of weak process cross-sections. It actually fits so well that it has been called the WIMP miracle : it is too beautiful to be true. And, as a matter of fact, latest experiments have already ruled out the hypothesis of a 4<sup>th</sup> generation neutrino. Concerning the first case, a neutrino lighter than the Z boson, that would lead to a spontaneous desintegration into a pair  $\psi\bar{\psi}$ . But the latest measurements of the width of the boson have ruled out this possibility. And for the particle with a mass larger than the Z mass, it has been ruled out by the bounds set by direct detection [7].

### 2.3 Short presentation of coannihilation

To conclude on this calculation, one has to talk of the possibility of coannihilation. Indeed, there is a way to modify the present day mass density of the hypothetic dark matter particle by taking into account another partner with a quite similar mass. By doing so, one can explain the presence of the correct mass density without constraining so directly the mass of the particles involved. There are two other cases where the classical calculation does not apply (see [6]) - namely the case where the relic particle could annihilate into slightly heavier particle, or when the annihilation takes place near a pole in the cross-section - but I will only treat here the problem of coannihilation since it will become in a handy later. The point to consider this deviation from normal computation is to be sure not to make a big mistake when calculating the relic density of dark matter when taking into account models that predicts close to degenerate masses, as it is the case for supersymmetry. The only difference is now that we will consider not only one possible  $\psi$  particle, but several  $\psi_k$  with masses  $m_k$ , that we will choose conventionnaly to be  $m_k < m_l$  for  $k < l$ . Our previous particle will now be called  $\psi_1$ .

Before going into the details, one has to remember two things. First of all, this  $\psi_1$  is still considered to be stable at cosmological scale, in opposition to  $\psi_k$  for  $k \neq 1$  which can decay. However, we shall make another assumption that I will justify later, namely that these new particles can not decay spontaneously into Standard Model particle, because it would violate a conserved quantum number. Hence, you have to respect this implicit symmetry when taking into account the possible reactions - this will be discussed more precisely in the section on supersymmetric candidate. Let us see now more precisely how this modify the previous calculation. The relative abundance of these species at freeze-out can be estimated in a first way by the usual ratio :

$$\frac{n_{\psi_k}}{n_{\psi_1}} = \frac{e^{-m_k/T_F}}{e^{-m_1/T_F}} \quad (41)$$

Hence, if it exist a  $\psi_k$ ,  $k > 1$  particle with a mass close enough to  $m_{\psi_1}$ , this particle could play a non negligible role in the previous calculation.

Indeed, if we are to take into account all processes that would change the density of  $\psi_k$ , we would have to consider the following reactions, for all  $l$  :



Once again, other reactions are forbidden by symmetry argument. Concerning the nature of  $i'$  in eq (43), it will be imposed by the choice of  $i$ , they are clearly not independent if one has to conserve the quantum numbers. For  $l = 1$ , the reaction (44) will not occur, and if we consider that it occurs for all other particles, we can assume that today the only remaining particle is  $\psi_1$  since all other have eventually decayed into it. We have now a set of  $N$  Boltzmann equations, one for each  $\psi_k$  :

$$\begin{aligned} \dot{n}_k + 3Hn_k = & - \sum_{i,l} [ \langle \sigma_{kl}|v \rangle (n_k n_l - n_k^{eq} n_l^{eq}) \\ & - ( \langle \sigma'_{kl}|v \rangle (n_k n_i) - \langle \sigma'_{lk}|v \rangle (n_l n_{i'}) ) \\ & - \Gamma_{kl}(n_k - n_k^{eq}) ], \end{aligned} \quad (45)$$

where the cross-sections and reaction rates  $\sigma_{kl}$ ,  $\sigma'_{kl}$ ,  $\sigma'_{lk}$  and  $\Gamma_{kl}$  are respectively the one of reactions (42), (43), (43) in reverse direction, and (44). If we say that all  $\psi_k$  particles will eventually decay into  $\psi_1$ , to compare the result with the first part, one has to take into account the total number density of all the particles,  $n = \sum_k n_k$ . After adding all the Boltzmann equation, one finds

$$\dot{n} + 3Hn = - \sum_{k,l} \langle \sigma_{kl}|v \rangle (n_k n_l - n_k^{eq} n_l^{eq}) \quad (46)$$

Indeed, the terms corresponding to reactions (43) compensate two by two, and for the  $\Gamma$  terms, we will now show that their contribution to this equilibrium is negligible since these rate of reaction are much faster. Indeed, one can estimate the difference between the rates of reaction of type (42)

compared with those of type (43) and (44). The behaviour of the particles will be different since at the temperature of freeze-out, the  $\psi_k$  will be non relativistic whereas the  $i$  will be relativistic. If one defines

$$n(p) = \frac{g}{e^{\frac{\epsilon(p)}{T}} \pm 1} \text{ with } \epsilon^2(p) = p^2 + m^2 \quad (47)$$

$$n(T) = \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{g}{e^{\frac{\epsilon(p)}{T}}}. \quad (48)$$

For non relativistic particles, we get

$$\begin{aligned} \epsilon &\simeq m \text{ and } n(p) \simeq g e^{-m/T} \\ \boxed{n &\sim (mT)^{3/2} e^{-m/T}}, \end{aligned} \quad (49)$$

and for relativistic particles,

$$\begin{aligned} \epsilon &\simeq p \text{ and } n(p) \simeq g/(e^{-p/T} \pm 1) \\ \boxed{n &\sim T^3}. \end{aligned} \quad (50)$$

Hence, we can derive the different reaction rate of equation of type (42) and (43) :

$$n_k n_l \sigma_{kl} \sim T^3 m_k^{3/2} m_l^{3/2} \sigma_{kl} \exp(-(m_k + m_l)/T) \quad (51)$$

$$n_k n_i \sigma'_{kl} \sim T^{9/2} m_k^{3/2} \sigma'_{kl} \exp(-m_k/T). \quad (52)$$

Hence the rate of the second type of reaction over the first one is

$$\frac{n_i}{n_l} = \left( \frac{T}{m_l} \right)^{3/2} \exp(m_l/T), \quad (53)$$

which for a freeze-out value of  $x_F \simeq 30$ , has a value of  $\mathcal{O}(10^{11})$ , if we assume that the cross-section for the different processus are roughly of the same magnitude. This result allows us to make a further hypothesis : since



the freeze-out is determined by reactions of type (42), and that the other ones occur more rapidly, we can assume that the distribution of particles in  $\psi_k$  remains in equilibrium during the whole process, namely  $n_k/n \simeq n_k^{eq}/n^{eq}$ . This explains how the  $\Gamma$  terms are cancelled in (46). Introducing

$$r_k = \frac{n_k^{eq}}{n^{eq}} = \frac{g_k(1 + \Delta_k)^{3/2} \exp(-x\Delta_k)}{g_{\text{eff}}}, \quad (54)$$

$$\text{with : } g_{\text{eff}} = \sum_{k=1}^N g_k(1 + \Delta_k)^{3/2} \exp(-x\Delta_k), \quad (55)$$

one can rewrite eq. (46) into

$$\dot{n} + 3Hn = - \langle \sigma_{\text{eff}} |v| \rangle (n^2 - n_{eq}^2), \quad (56)$$

$$\text{with : } \sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} r_i r_j. \quad (57)$$

One can see now the similarity with the previous calculation, the cross-section being a different expression of the several involved particles. So it is possible to do exactly the same computation than before, the ending estimate being modified by replacing  $\sigma_0$  by  $\sigma_{\text{eff}}$  and  $g$  by  $g_{\text{eff}}$ . Namely :

$$x_F \simeq \ln \left( 0.0873 \frac{g_{\text{eff}}}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_{\text{eff}} \right) - (n - 1/2) \ln(x_F) \quad (58)$$

$$\Rightarrow x_F \simeq \ln \left( 0.0873 \frac{g_{\text{eff}}}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_{\text{eff}} \right) - (n - 1/2) \ln \left( \ln \left( 0.0873 \frac{g}{g_{*S}^{1/2}} m_{Pl} m_\psi \sigma_{\text{eff}} \right) \right) \quad (59)$$

At this point, once again, one need to separate cases to express  $\sigma_{\text{eff}}$ . To start with the most striking example, let us consider that a squark ( $\tilde{q}$ ) has a mass very close to the one of the LSP. In that case, the cross-section  $\sigma_{kl}$  will not be all the same. Indeed one expects by looking to the Feynman diagrams that

$$\sigma_{22}(\tilde{q}\tilde{q} \longrightarrow gg) \simeq \left(\frac{\alpha_s}{\alpha}\right) \sigma_{12}(\psi_1\tilde{q} \longrightarrow qg) \quad (60)$$

$$\simeq \left(\frac{\alpha_s}{\alpha}\right)^2 \sigma_{11}(\psi_1\bar{\psi}_1 \longrightarrow q\bar{q}) \quad (61)$$

where  $g$  denotes the gluon,  $\alpha_s$  and  $\alpha$  respectively the strong and electroweak coupling. Let us call  $A = \alpha_s/\alpha$ . If we keep our simple model where the cross-section is independent of the temperature -  $n=0$  - we have that  $\sigma_{22} = A\sigma_{12} = A^2\sigma_{11}$  with  $A \simeq 20$ . The effective cross-section can now be rewritten as :

$$\sigma_{\text{eff}} = \sigma_{11} \left( \frac{1 + A\omega}{1 + \omega} \right)^2 \quad (62)$$

$$\text{with : } \omega = (1 + \Delta)^{3/2} \exp(-x\Delta)g_2/g_1 \quad (63)$$

$$\text{and : } g_{\text{eff}} = g_1(1 + \omega) \quad (64)$$

For the degenerate case,  $\Delta = 0$ ,  $\sigma_{\text{eff}} = \sigma_{11}(1 + Ag_2/g_1)^2/(1 + g_2/g_1) \simeq \sigma_{11}(Ag_2/g_1)^2/(1 + g_2/g_1)^2$ . If a single squark is degenerate in mass,  $g_2/g_1 = 3$  and it leads to a factor 200 less in relic density. We see that this behaviour can have a significant impact on the ending value for the constraint, it is thus crucial to take it into account.

### 3 The Supersymmetric candidate

We have seen in the previous part that the mass that should have this hypothetical dark matter particle is of order of several TeV. There is thus a big theoretical problem if we only consider the standard model, namely how such a massive particle can be stable ? We know that the only stable particles are the electron and the proton, everything heavier will eventually decay. To explain this, one must think of a way to protect this dark matter particle, to prevent it from decaying into normal matter. All the models that want to predict such a particle will thus have to find a way to explain this stability at such a high mass.

It should be noted that this problem of dark matter particle is not the only theoretical issue scientists are facing in this field. The hierarchy problem, the low vacuum energy expectation value, all these are being thought right now in the different optic of the various theories. Concerning supersymmetry, one of the most appreciated feature is usually considered to be the prediction of the right amount of dark matter, and the proper interaction cross-section values. Some recent results have however shown that our current understanding of supersymmetry breaking might not be correct and seems ruled out by some observations, and I will try to explain quickly why in the last part of this section.

Once again, I will not go in the mathematical details of supersymmetry, for various reasons, and I have just tried here to extract the important ideas and way of thinking of this theory to be able to discuss and think about it. Thus in the first parts I will present supersymmetry and their candidate for dark matter. I will then conclude this section with a discussion on the hypothesis of this model, and by presenting some arguments against this idea, in order to keep our awareness on the fact that this is not the only possibility.

#### 3.1 Basic Ideas about supersymmetry, MSSM

Supersymmetry is one of the possible way to explain how a massive particle of order  $TeV$  can be stable. The idea behind this theory is to state that there should be a symmetry between fermions and bosons. The supersymmetric transformation will thus take the standard model particles and change their spin number to 0 for a fermion superpartner (a scalar boson) and to 1/2

for a boson superpartner (a spin 1/2 fermion). If one defines the parity  $R = (-1)^{2S+L+3B}$ , with S, L and B respectively the spin, leptonic and baryonic quantum number, one can see that all standard model particles have a parity of +1 while all supersymmetric particles have a parity of -1. If the R-parity is conserved throughout the interactions - *i.e.* if we impose this conservation by forcing a symmetry on the lagrangian, then the Lightest Supersymmetric Particle - LSP for short - will be stable and will not decay into lighter normal matter. This is a good way to ensure the stability at cosmological scales of dark matter.

There are two 'however'. First of all, the symmetry between fermions and bosons is clearly broken at our scale of energy, we do not see a unique fermion spontaneously transforming into a unique boson, hence the mass of the superpartners will be higher than standard model particles. Thus one has to think of a way to break this supersymmetry. The second point is : what is this LSP ? Luckily enough, these are only two sides of the same coin, because the way the supersymmetry is broken will sort out the different candidates and tell us which one is the lightest, so which one can be our dark matter particle.

### 3.2 Soft breaking of supersymmetry

Before going any further, one can simplify things as much as possible by making several assumptions. First of all, one has to understand that if the supersymmetry was perfect, this model would not add *a priori* any new parameter, since the masses of the superpartners will be the same than their Standard Model equivalents. There is however one new parameter added even if the supersymmetry is unbroken. Indeed, for this theory to explain mass, it needs not one but two Higgs fields, because you can not produce the masses with only one due to the holomorphy [1]. It thus adds a coupling parameter  $\mu$  between the two Higgs fields. The name given to these new partners are a bit more exotic than usual names, with the higgsino, wino, bino, gravitino, etc... They are denoted with a  $\sim$  over the name of the Standard Model partners.

But, as said previously, the supersymmetry is broken at our scale, so one should have in principle to consider in the most general case a complete breaking of symmetry, introducing a huge number of free parameters. Here

comes the soft breaking idea, stating that the supersymmetry is not broken for any-dimensional operator, but only for the ones with dimension lower than 3. The number of parameter is drastically reduced, becoming of around one hundred, and making the theory renormalizable - nice property that would tend to prove that we are in the good way by doing this. This is still way too much, though, and to simplify more, one calls in the naturalness. Some of this new coefficients have been bounded by experiment to be less than  $\mathcal{O}(10^{-5})$ , and by invoking the naturalness, one claims that they are exactly zero, otherwise it produces unnaturally low values. Whether this argument is correct or not will be discussed in the last part, but the fact is that it brings the number of non-zero new parameters to six plus the first one,  $\mu$ , for a total of seven. If we assume in addition that we consider only standard model particles and not more exotic ones, then supersymmetry implies that there is only one supersymmetric partner. This simplest model is called the MSSM, Minimal SuperSymmetric Model.

One has also to note that the list of the possible candidates is not that large. Indeed, there are only seven particles that are electrically neutral and colorless in the MSSM, the three sneutrinos, the gravitino and the four neutralino - these last are a mixing of the superpartners of the neutral gauge and two neutral Higgs bosons. The lightest sneutrino has already been ruled out since if it exist it should interact in a very similar way than a 4<sup>th</sup> generation Dirac neutrino, and hence should have already been observed. The gravitino interact via gravitation, so is not within the same orders of magnitude than the weak process and so is ruled out as a WIMP. We will thus discuss about the neutralinos in the following. Since these neutralinos are a mixing from different particles, the mixing mass matrix will give the proportion of each sort of particle in the LSP. The point is, depending on the fraction of certain particles - one speaks of wino and bino fraction - the reactions will not be the same, and thus the relic density will change. According also to the mixing terms, some particle might have a mass close to the one of the LSP and thus change the result in the sense of the section 2.3.

The whole calculation is long and requires to take into account all the Feynman diagrams possible. To ease this process, a public available tool named DarkSUSY has been released. This calculate directly the relic density of the LSP given the 7 free parameters in the beginning, and take into account all the recent results on the constraints given by the detection of dark matter.

By doing so, the models are still possible or discarded, and one can adjust the free parameters to keep the theory right.

### 3.3 Experimental detection, constraint, the end of the MSSM?

There are different ways to look for dark matter and try to constraint the values of the mass. One is the direct detection : once a candidate picked up, one can compute the scattering cross-section with normal particles, set up huge detector and wait for a collision. This type of experiment has already ruled-out the heavy Dirac neutrino. One other way is to look indirectly to the traces of annihilation of dark matter - which is expected to be a very faint signature since their low density, but hopefully discernable. This is one of the mission of the Fermi Space Telescope for instance. By looking at these annihilation rate, or rather by not seeing them under a certain amount, one can progressively, as the precision increase, rule out a larger part of the MSSM phase space.

Recent data from the Fermi Space Telescope tend to show [1] that the MSSM is no more able to explain the relic density. To be more precise, the phase space is thus constrained that one can not adjust this theory anymore to predict a model not ruled out by the experimentation : with the detection constraint, it is no more possible to have the proper relic density. The next idea is then to change the way the supersymmetry is broken, to add a new free parameter in the theory and to be able to tune it to match experiment. This is called the Beyond Minimal SuperSymmetry Model, BMSSM, and is being developed by considering a five order term in the Lagrangian that has a broken supersymmetry. By doing so, the part of the phase space available is restored and some set of values of the BMSSM are candidates to verify both detection and cosmological constraints.

### 3.4 Discussion

As we have seen along this section, the supersymmetric model adopts a top-down approach. It has potentially a huge number of free parameter, and try to reduce this number to a meaningful one by using naturalness as a criterion. What is really naturalness ? It is apparently hard to define, and it more or less states that physical constants should not be too large or too small, that if you compare two physical terms and that the ratio is

constraint to be under a very small value, then it is probable that its exact value is actually zero. This is far from being a rigorous definition, and the question is how can we decide under which ratio this works ? How can we say it is not natural to have a very low value ? The fact is that this sort of intuition on how the nature really is can help to solve problems, but to rely on it too much might well lead to a wrong orientation. What if nature is no more 'natural' over a certain scale ? How can we be sure the principles we got from our experience of physics at our energy scale will still be valid at higher energy ? The history of science is rich in misunderstanding of unknown processes, and to try to constraint our theories with this strong *a priori* about how the nature should be seems a bit dangerous. One has thus to be careful when using these not very precise tools in order to keep doing science (see [13] for a more complete discussion on that subject).

The good point if we finally observe a supersymmetric particle is that it will fix the parameters and thus give us an idea of how supersymmetry is broken, and how are the more massive particles. The details of the Lagrangian might vary but it will not change things since higher order terms are suppressed by dimension analysis. The set of supersymmetric particles will thus be fixed and we will be able to make predictions about it.

## Conclusion

We have first seen in this work that if we are to explain properly the matter density of the universe, one can exhibit a yet-to-be-observed particle that is non-baryonic. Indeed, through Big-Bang nucleosynthesis, it has been proven that the baryons could explain only up to 4% of the total critical density, while the matter density is around 27%. Several other evidence are also pointing out the presence of some invisible mass in galaxies and galaxy clusters. It has been derived afterwards that the presence in a significative proportion of dark matter today can be explained by claiming the existence of a new particle, stable at cosmological scales, that interacts weakly with the Standard Model particles. The precise calculation of its relic density is to be followed carefully since the presence of particles of approximatively the same mass can make a huge difference in the result, in both raising and lowering it down.

In the second part, we have had a first insight in the way of predicting such a stable particle via looking at one the most supported candidate : supersymmetry. By imposing the conservation of a certain parity, one can find the way to 'protect' the lightest supersymmetric particle and actually prevent it from decaying into a Standard Model one, and thus open the room for a dark matter heavy lightly interacting particle. The details of the calculations as well as the precise theory of supersymmetry have not been investigated here, but we have shown how by varying the free parameter of the theory one can predict the relic density and thus have a check of the validity of this realisation. The other informations given by the public available tool DarkSUSY are most useful to confront the prediction to the on going experimentations. Finally, we have seen that our current understanding of the soft breaking of supersymmetry is questioned by the latest results of the Fermi Space Telescope, and that the MSSM might have to be extended into a BMSSM to survive this new result. The subject is thus still in need of further experiment before one could tell whether this theory or another is right or wrong. And it might be that the LHC will find or not the supersymmetric particles, allowing us to have a clearer view on this 'dark' subject.



## References

- [1] M. Berg. Naturalness and MSSM Dark Matter. In *Uppsala, seminar*, 08/06/2009.
- [2] G. Bertone, D. Hooper, and J. Silk. Particle dark matter: Evidence, candidates and constraints. *Phys. Rept.*, 405:279–390, 2005.
- [3] T.-P. Cheng. *Relativity, Gravitation and Cosmology*. Oxford University Press, 2005.
- [4] D.-C. Dai, R. Matsuo, and G. Starkman. Gravitational Lenses in Generalized Einstein-Aether theory: the Bullet Cluster. *Phys. Rev.*, D78:104004, 2008.
- [5] M. Dine. *Supersymmetry and String Theory*. Cambridge University Press, 2007.
- [6] K. Griest and D. Seckel. Three exceptions in the calculation of relic abundances. *Phys. Rev.*, D43:3191–3203, 1991.
- [7] D. Hooper. TASI 2008 Lectures on Dark Matter. 2009.
- [8] E. W. Kolb and T. M. S. *The Early Universe*. Wylde, Allan M., 1990.
- [9] B. W. Lee and S. Weinberg. Cosmological lower bound on heavy-neutrino masses. *Phys. Rev. Lett.*, 39:165–168, 1977.
- [10] R. N. Mohapatra. *Unification and Supersymmetry: The Frontiers of Quark-Lepton Physics, third Edition*. Springer, 2003.
- [11] A. Pais. *Inward Bound*. Oxford University Press, 1986.
- [12] M. Persic, P. Salucci, and F. Stel. The Universal rotation curve of spiral galaxies: 1. The Dark matter connection. *Mon. Not. Roy. Astron. Soc.*, 281:27, 1996.
- [13] B. Richter. Is 'naturalness' natural ? SUSY06. Invited talk.
- [14] F. Zwicky. On the Masses of Nebulae and of Clusters of Nebulae. *Astrophys. J.*, 86:217–246, 1937.