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Smart Autonomous Grain Carts Towards a Solution to Harvesting-on-Demand

Yan Tian
Iowa State University

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Smart autonomous grain carts towards a solution to harvesting-on-demand

by

Yan Tian

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Yan-bin Jia

James Oliver

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content of this thesis. The Graduate College will ensure this thesis is globally accessible

and will not permit alterations after degree is conferred

Iowa State University

Ames, Iowa

2017

DEDICATION

I would like to dedicate this thesis to my parents, without whom I would have never gone so far. I also want to dedicate this work to my girlfriend, whom I have not met yet.

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ABSTRACT

We address the harvesting-on-demand problem in which a group of grain carts unload multiple combine harvesters in an agricultural harvesting operation. In a general harvesting operation, combine harvesters are used to collect the grains from the field. Once the on board tank of any combine is filled, a grain cart is needed to unload the load from the combine so it could keep working. Currently, combines are served individually by a grain cart. In this work, we investigate the case when there are fewer grain carts than combine harvesters.

Initially we introduce the formulation of the problem and required parameters of the combine harvesters, the grain carts and the field. We start the analysis of the problem with a simple case when there are multiple combines and a single grain cart. A scheduling strategy for the grain cart to serve all the combines without interrupting their work is proposed. Based on the analysis of the single grain cart operation, we discuss the position of the depot to minimize the traveling distance of the grain cart to unload itself in a harvesting operation. Both cases when the field is and is not rectangular are considered. The path planning of the grain carts moving between combines is based on the numerical technique proposed by Mengzhe. In this work, we present a primitive based path planning method for the grain cart to obtain a suboptimal path for the grain cart moving between any two combine harvesters. Then we generalize the scheduling strategy to a group of grain carts serving multiple combines when the number of grain carts is fewer than that of the combines. Two techniques are proposed, which are round-robin scheme and load balance scheme. In each of the techniques, two different approaches are presented. The performance of round-robin and load balance strategy is compared so that the farmers can select the correct strategy to apply in their harvesting operation easily. Finally, we extend the strategies in scenarios when the grain carts are big enough so that no depot is required. An application of the proposed scheduling schemes with BOE-bots in Vicon environment is shown.

CHAPTER 1. INTRODUCTION

1.1 Overview

In my thesis, we address the problem of a group of grain carts unloading multiple combine harvesters in a harvesting operation. In a general harvesting operation, combine harvesters are used to collect the grains from the field. Once the on board tank of any combine is filled, a grain cart is needed to unload the load from the combine so it could keep working. Currently, combines are served individually by a grain cart. In this work, we investigate the case when there are fewer grain carts than combine harvesters.

Initially we introduce the formulation of the problem and required parameters of the combine harvesters, the grain carts and the field. We start the analysis of the problem with a simple case when there are multiple combines and a single grain cart. A scheduling strategy for the grain cart to serve all the combines without interrupting their work is proposed. Based on the analysis of the single grain cart operation, we discuss the position of the depot to minimize the traveling distance of the grain cart to unload itself in a harvesting operation. Both cases when the field is and is not rectangular are considered. The path planning of the grain carts moving between combines is based on the numerical technique proposed by Mengzhe. In this work, we present a primitive based path planning method for the grain cart to obtain a suboptimal path for the grain cart moving between any two combine harvesters. Then we generalize the scheduling strategy to a group of grain carts serving multiple combines when the number of grain carts is fewer than that of the combines. Two techniques are proposed, which are round-robin scheme and load balance scheme. In each of the techniques, two different approaches are presented. The performance of round-robin and load balance strategy is compared so that the farmers can select the correct strategy to apply in their harvesting operation easily. Finally, we extend the strategies in scenarios when the grain carts are big enough so that no depot is required. A simulation video is presented

to validate the feasibility of the proposed techniques.

Next, we provide the outline of the thesis.

1.2   Thesis Outline

The thesis is outlined as follows. In Chapter 2, a complete literature review is provided in the area of auto-guidance vehicles, vehicle scheduling and path planning, and task allocation. In Chapter 3, the work on the scheduling and path planning strategy of single grain cart sering multiple combine harvesters is presented. The optimal depot position and motion planning for the grain carts moving between combines are analyzed. In Chapter 4, we generalize the problem to multiple grain carts. Two difference techniques are presented. In Chapter 5, we extend the proposed schemes in a non-rectangular field. In Chapter 6, the summary of the work is presented as well as the discussion of future work.

1.3   Publications

1. Rui Zou, Yan Tian and Sourabh Bhattacharya (2015). Path Planning Algorithms for Source Seeking Agents. In 27th Chinese Control and Decision Conference. pp.4865-4870.

2. Yan Tian and Sourabh Bhattacharya (2016). Smart Autonomous Grain Carts Towards a Solution to Harvesting-on-Demand. In ICRA 2017. (Under review)

3. Yan Tian and Sourabh Bhattacharya (2016). Smart Autonomous Grain Carts Towards a Solution to Harvesting-on-Demand. International Journal of Robotics Research. (Under preparation)

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

In the last two decades, there has been a significant rise in the world food price index Brown (2012). An ever growing world population and variability in warming, drought, flooding, and precipitation Field et al. (2014) will only add to the cause in the future. A significant component of the food price is the cost of production which involves the cost for labor and agricultural machinery. With large-sized farms dominating agriculture in the United States MacDonald et al. (2013), there is a compelling need to automate several farming practices. Traditionally, large-scale harvesting operations engage multiple combines individually served by a single grain cart. In this work, we investigate the harvesting-on-demand problem, which deals with the scheduling and motion planning for a team of grain carts that serve multiple combine harvesters when the number of grain carts is fewer than the number of combines.

2.1.1 Vehicle Guidance

Past research in vehicle guidance for agricultural vehicles mainly focus on four aspects of navigation, namely, sensors, navigation planner, vehicle motion models and steering controllers Reid et al. (2000). Navigation sensors, which include image processing system Burgos-Artizzu et al. (2011),Han et al. (2004), RTK-DGPS Luo et al. (2009),Nagasaka et al. (2004), are the core of the technology since they play the role of detecting the environment. In Zhang et al. (1999), the authors presented an on-field navigation system, which combined vision sensor, fiber optic gyroscope (FOG) and RTK-DGPS, for agricultural vehicles. In Noguchi et al. (2004), two motion control algorithms were proposed and applied on a master-slave robot system which was designed for in farm operations. An extensive survey of automatic vehicle and its application was proposed in Reid et al. (2000). In this work, the proposed techniques can be applied on autonomous vehicles to build an system to minimize manual operation.

2.1.2 Scheduling and Path Planning

There has been a significant amount of work on scheduling and vehicle routing for manned as well as unmanned vehicles in numerous applications, for example, persistent surveillance Bullo et al. (2011); Smith et al. (2011); Ulusoy et al. (2013), transportation networks Bräysy and Gendreau (2005a,b), mobility-on-demand problems Zhang and Pavone (2016), just to name a few. However, its application to agricultural vehicles has rather been limited. In Foulds and Wilson (2005); Basnet et al. (2006), the authors addressed the scheduling problem of farm-to-farm harvesting operations. In Orfanou et al. (2013), a planning approach for scheduling sequential tasks involved in biomass harvesting performed by machinery teams was presented. A scheduling method based on a two-phase meta-heuristic for a longterm cropping schedule was presented in Guan et al. (2009). In Ali and Van Oudheusden (2009), the authors addressed the motion planning of one combine by using integer linear programming formulation. In contrast to the aforementioned work, we deal with a dynamic scenario in which the routing schedule of the grain carts has to match with the harvesting schedule of the combines.

In the past decade, there have been some works related to path-planning for agricultural vehicles. In Oksanen and Visala (2009), the authors proposed methods to solve field coverage problem in addition to computing an efficient route. In Hameed et al. (2013), the authors proposed a coverage planning that took the presence of obstacles into account. In Bochtis and Vougioukas (2008); Spekken and de Bruin (2013), the authors presented techniques to generate paths that minimized the time spent in non-working activities. An elaborate survey of various techniques for coverage path planning in autonomous agricultural vehicles was presented in Jin and Tang (2010) and Jin and Tang (2011). In this work, our objective is to plan feasible paths for a non-holonomic vehicle when the final destination is a function of time.

2.1.3 Task Allocation

An important component of our proposed scheme involves allocating tasks to individual grain carts. There has been a significant amount of work related to task allocation for multi-robot systems. In Korsah et al. (2013), the authors addressed the problem of multi-robot coordination, and presented a comprehensive taxonomy that explicitly took into consideration the issues of interrelated utilities and

constraints. In Nieto-Granda et al. (2014), the authors proposed coordination strategies for multi-robot exploration and mapping. An extensive review of task allocation and decision making problems was presented in Cap et al. (2015) and Kernbach et al. (2013).

CHAPTER 3. SINGLE GRAIN CART HARVESTING-ON-DEMAND PROBLEM

3.1 Introduction

Harvesting-on-demand problem can be described as the management of the resources to meet specific requirement. The resources to be dealt with include cooperation of the vehicles, as well as the parameters of an environment. In this chapter, we investigate the harvesting-on-demand problem in which a group of grain carts serving multiple combine harvesters.

3.2 Combine Harvester

Combine harvester is a machine for harvesting crops, for example, wheat, oats, rye, barley corn, soybeans and flax. Figure 3.1 shows a combine at work. In this active mode, the header cuts the crop and feeds it into threshing cylinder. Grain and chaff are separated from the straw when the crop goes through the concave grates. After being sieved, the waste straw is ejected and the grain stored temporarily in tank. Let C denote the maximum capacity of on-board tank.
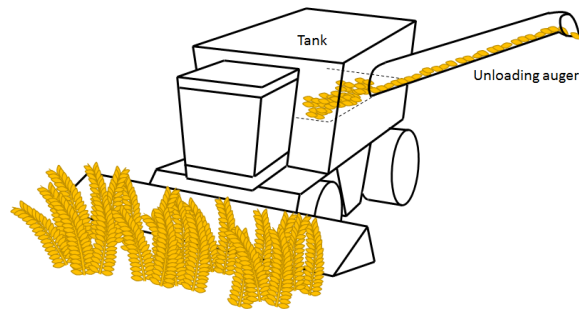
Figure 3.1: Combine harvester

Threshing grain loss is an important issue in a combine harvester. For any combine, the quantity of threshing grain loss depends on the forward speed of the harvester. Flufy and Stone (1983) show that

automatic control is better than manual control for minimizing the threshing grain loss. The forward speed of the combine is controlled to achieve a level of crop feed that minimizes the threshing grain loss. Let r_f denote the rate at which the grain fills the storage tank. To begin with, we assume that the combine moves on the field with constant speed leading to a constant r_f.

Since the tank has a limited capacity, modern combines have an unloading auger for removing the grains from the tank to other vehicles. Let r_u denote the unloading rate of the tank using the auger. Therefore, when a combine harvests and unloads simultaneously, the net rate at which the tank is being emptied is $r_u - r_f$ ($r_u > r_f$).

3.3 Grain Cart

A grain cart, also known as a chaser bin, is a trailer towed by a tractor. In this paper, we use the term grain cart to represent a tractor-trailor system. Figure 3.2 (a) shows the appearance of a grain cart. Because of the larger capacity, one can use it to collect grains from multiple combines and transport them to a nearby truck or depot. In this paper, we denote the maximum capacity of a grain cart by C_g.
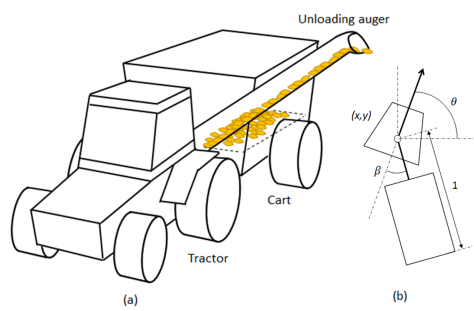
Figure 3.2: Grain cart

Figure 3.2 (b) shows a grain cart. We model the grain cart as a trailer attached to a car-like robot. The robot is hitched by the trailer at the center. The length of the rotation arm for the trailer is assumed to be 1. The equations of motion for the grain cart are as follows:

$$\dot{q} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ -v\sin\beta + \omega \end{pmatrix},$$

,where $q = (x, y, \theta, \beta) \in \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1$ is the configuration and $u = (v, \omega) \in U = [-1, 1]^2$ is the control. In the configuration q, (x, y) is the coordinate of robot's center, θ is the robot's orientation, β is the angle between tractor and the trailer. v and ω denote the speed and angular velocity of the robot, respectively (Chyba and Sekhavat (1999) Chitsaz (2013)).

3.4   Problem Formulation
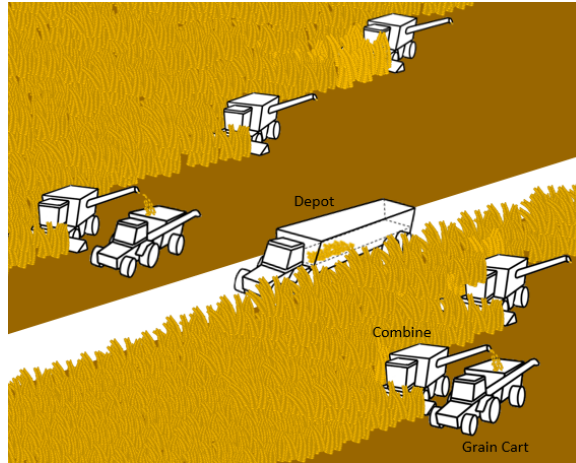
Figure 3.3: Harvesting Operation

At the beginning of the harvesting operation, the combine that harvests the row nearest to the depot enters the field (blue combine in Figure 3.4). Subsequently, the rest of the combines enter the field sequentially. While serving a combine, the grain cart is always on the side which is closer to the depot to minimize the distance traveled. To avoid collision with the combines, when a grain cart meets a combine on its way to the depot, it waits for the combine to move first. When a grain cart is filled, it travels to the depot to unload itself. As shown in Figure 3.4, we number the combines and the grain carts in the order in which they enter the field. We assume that all the combines have the same tank capacity C, filling rate r_f and unloading rate r_u. In the beginning, the combines enter the field along adjacent rows sequentially with constant velocity V_c and a time gap ΔT. Each grain cart serves a combine till it is completely empty. Then it moves to the next combine. We introduce the following notations to denote the pertinent time intervals during this operation:

1. T: Time for each grain cart to move between two adjacent combines.

2. $T_f(= \frac{C}{r_f})$: Time for a combine to fill its tank.

3. $T_u(= \frac{C}{r_u - r_f})$: Time for a grain cart to empty the tank of a fully filled combine.
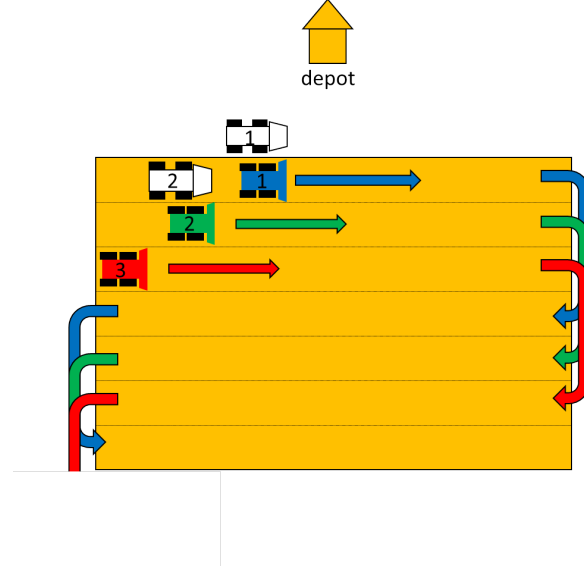
Figure 3.4: Motion of grain cart between combines

When the harvesters reach the end of the field, they will stop moving and a grain cart will empty all the combines. Figure 3.4 shows the sequence of operations for the combines. After harvesting a row, the combines move to the next row. Each row of harvesting starts with a new serving cycle. The turning trajectory at the end of each row is based on the edge condition which has been analyzed in Jin and Tang (2010). The authors present trajectories for a combine to travel from one row to another with the smallest overlap or shortest path distance based on different edge conditions.

In the rest of the paper, we make the following assumptions. (i) The farmland has a constant crop density. Initially, the analysis is for the simple case of a rectangular field which is subsequently generalized to the case of non-rectangular fields in Section 5.1 (ii) All the combine harvesters have the same tank capacity and forward velocity. (iii) The traveling time of the grain cart between combines i and j is $|i - j|T$.

3.5 Unloading Scheduling Strategy with Single Grain Cart

In this section, we first consider the problem of scheduling a single grain cart to serve N combines. Once the grain cart gets filled, it goes to the depot to unload the grains. We assume that the capacity of

the grain cart is not sufficient to hold the grains in the entire field.

The combines enter the field sequentially with a constant time gap ΔT. We number the combines $1, 2, ..., N$ in the order they enter the field. The grain cart serves the combines in the same order. After all the combines are served once, the grain cart goes to the depot and unloads the grains. The capacity of the grain cart is $C_g = NC\frac{r_u}{r_u - r_f}$. Let T_d denote the time required by the grain cart to make a trip to the depot before it arrives at the next combine. In general, T_d varies due to the change in the relative distance of the combine from the depot during the harvesting process.

Lemma 1. T_d *is a constant for the strategy that minimizes the distance traveled by the grain cart during its trip to the depot.* T_d *is equal to the longest time taken by the grain cart to travel to the depot after serving a combine.*

Proof. For a static depot, the distance between the grain cart and the depot changes during the harvesting operation. As the distance between the combines and the depot reduces during the harvesting operation, the grain cart can visit the next combine earlier. Let $C' \leq C$ be the volume of the load in the tank of the combine when the grain cart arrives. The time required for the grain cart to fill its tank to C_g is : $T_2 = \frac{C'}{r_f} + \frac{C'}{r_u - r_f} + \frac{C_g - C'_g}{r_f} = \frac{C}{r_f} + \frac{C}{r_u - r_f}$, where $C'_g = \frac{r_u}{r_u - r_f} C'$. Therefore, the total time spent by a combine to unload an amount C_g is independent of C'.
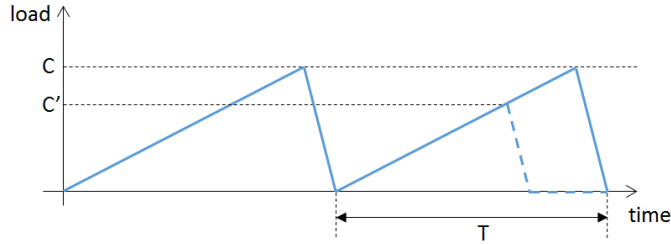
Figure 3.5: Load variation

Figure 3.6 shows the path followed by the grain cart between the time it leaves the depot and the time at which it unloads the combine to fill its own tank. If it arrives at the combine when its tank is filled at a level $C' < C_g$, it follows path L_2 to reach the combine and then travels along path L_3 to fill its tank by unloading the combine. The other option for the grain cart is to arrive at the combine just when its tank is completely full following path L_1. This may require it to wait at the depot after it finishes unloading itself. Since $L_1 < L_2 + L_3$, the second option is better for the grain cart to minimize

the distance traveled. Therefore, the grain cart will wait at the depot in case it finishes unloading early in order to arrive at the combine just when its tank is completely filled. .   □
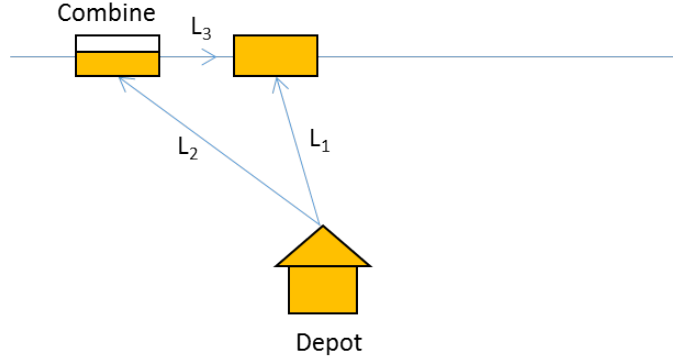
Figure 3.6: Traveling path of grain cart

Since T_d is a constant, we assume $T_d = \lambda T$ (λ is a scaling parameter). The time during which a single combine fills its tank equals to the time for a grain cart to serve all the other combines and unload itself. Therefore, we can obtain the expression:

$$(N-1)(T + \frac{C}{r_u - r_f}) + \lambda T = \frac{C}{r_f} \tag{3.1}$$

$$\Rightarrow C = \frac{(N-1+\lambda)(r_u - r_f)r_f T}{r_u - Nr_f}$$

From Equation 3.1, we can obtain the minimum required capacity C of the combines to carry out the harvesting operation with the given number of vehicles. This equation gives us the relationship between all the parameters involved in a harvesting operation when there are multiple combines and one grain cart. In addition to the minimum capacity of the combines, we can also obtain other parameters from 3.1. For example, given C, r_f and r_u, we can calculate N.

Figure 3.7 shows the load variation of the vehicles when $N = 2$ and $M = 1$.

3.6   Optimal Depot Location

In the scheduling strategy proposed in Section 3.5, one question may be asked is that where the depot should be located to minimize the travel distance of the grain cart. In this section, we address the problem of optimal position of the depot.
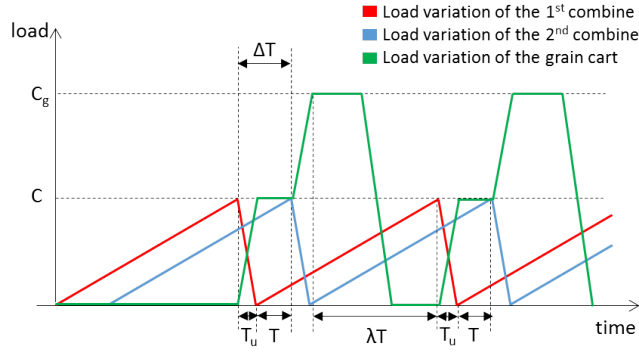
Figure 3.7: Load variations of 2 combines and 1 grain cart
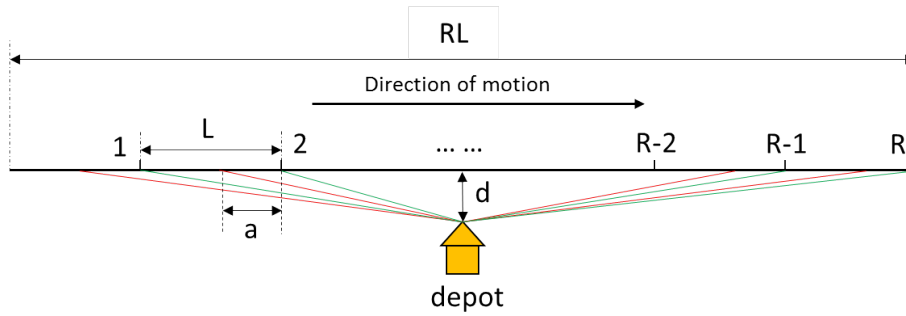
Figure 3.8: Working path of the vehicles

We assume that the grain cart unloads N combines and visits the depot located next to the field. Then it returns to the first combine to repeat the process. We call this a cycle. During each cycle, the combines travel a constant distance L since the field is assumed to have a constant crop density. We assume R complete cycles are performed while harvesting a complete row which implies that the length of the row is RL. During the time the grain cart visits the depot, the combines travel for a distance a $(a < L)$. As shown in Fig.3.8, the grain cart follows the red path to go to the depot and comes back along the green one in each cycle. We can see that the length of each path is not the same which leads to a changing traveling time. We assume that the grain cart follows an optimal strategy to minimize the distance traveled during its visit to the grain cart. From Lemma 1, this leads $T_d(T_d = \lambda T)$ to be a constant. The grain cart reaches the first combine exactly at the time it just gets filled.

To reduce the fuel and time spent during the harvesting operation, we need to find the optimal position of depot so that the total distance traveled by the grain cart is minimized. First, we consider the scenario of a single row in the next subsection. In this case, each combine harvests from one end of the field to the other only once to finish the harvesting operation.

3.6.1 Single Row

Before the combines reach the end of row, the grain cart goes to the depot N times which means the total length of the field is RL. Assume that the horizontal distance between the depot and the beginning of the row is xL, where $1 \leq x \leq R, x \in \mathbb{R}$ and the vertical distance between the depot and field is $d, 0 \leq d, d \in \mathbb{R}$. As shown in Fig 3.9, before the combines reach the depot, for any period n, the distance traveled by the grain cart is $\sqrt{((x-n)L)^2 + d^2} + \sqrt{((x-n)L+a)^2 + d^2}$. After the combines pass the depot, it becomes $\sqrt{((n-x)L)^2 + d^2} + \sqrt{((n-x)L-a)^2 + d^2}$, where a is the distance traveled by the grain cart during the time when the grain cart is leaving. Therefore, the total distance traveled by the grain cart L_{total} can be expressed as follows:

$$L_{total} = \sum_{n=1}^{\lfloor x \rfloor} \sqrt{((x-n)L)^2 + d^2} + \sqrt{((x-n)L+a)^2 + d^2} + \sum_{n=\lfloor x \rfloor+1}^{R} \sqrt{((n-x)L)^2 + d^2} + \sqrt{((n-x)L-a)^2 + d^2}$$

$$(3.2)$$

The problem is to find the optimal position of the depot so that the total distance traveled by the grain cart is minimized.
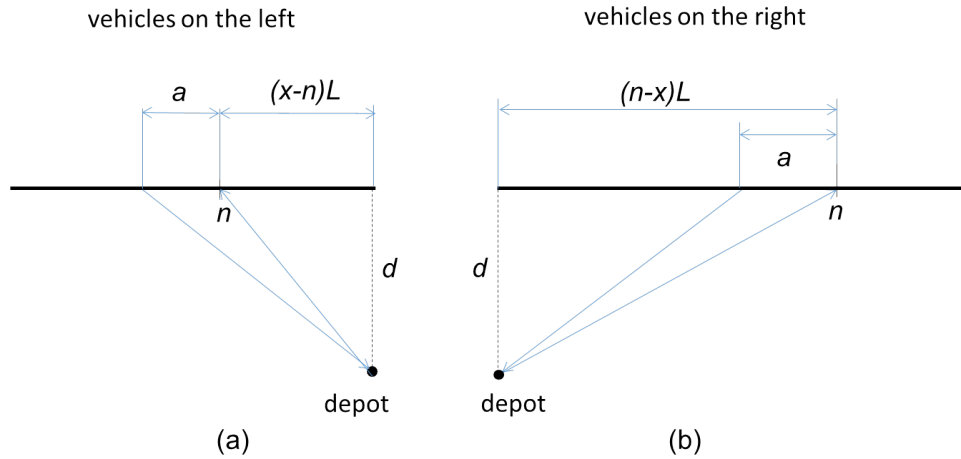
Figure 3.9: Travel Condition

Proposition 1: The optimal position of the depot is $x^* = \frac{(R+1)L-a}{2}$.

Proof. As shown in Figure 3.8, the grain cart visits the depot N times while harvesting the first row. We define the path followed by the grain cart to visit the depot for the nth time and come back, as the nth path. We divide the n paths into pairs. We let the nth path and the $(R+1-n)^{th}$ path be a pair. If R is

even, there are $\frac{R}{2}$ pairs. When R is odd, there are $\frac{R-1}{2}$ pairs and the one in the middle, which is $(\frac{R+1}{2})^{th}$ path is unpaired. The length of each pair of paths is given by the following expression:

$$L_{pair} = \sqrt{((x-n)L)^2 + d^2} + \sqrt{((x-n)L+a)^2 + d^2} + \sqrt{((R+1-n-x)L)^2 + d^2} + \sqrt{((R+1-n-x)L-a)^2 + d^2}$$

(3.3)

The second derivative L''_{pair} is:

$$L''_{pair} = \frac{L^2 d^2}{(((x-n)L+a)^2 + d^2)^{\frac{3}{2}}} + \frac{L^2 d^2}{(((x-n)L)^2 + d^2)^{\frac{3}{2}}} + \frac{L^2 d^2}{(((x-1)L)^2 + d^2)^{\frac{3}{2}}} + \frac{L^2 d^2}{(((x-1)L+a)^2 + d^2)^{\frac{3}{2}}} \geq 0$$

(3.4)

Since $L''_{pair} \geq 0$, L_{pair} is a convex function Boyd and Vandenberghe (2004). Therefore, the minimum exists and the solution is unique. $L'_{pair} = 0$ implies $x^* = \frac{(R+1)L-a}{2}$. This is true for all the pairs of trajectories, so we know L_{total} is also convex and reaches its minimum at $x^* = \frac{(R+1)L-a}{2}$. When R is odd, the $\frac{R+1}{2}th$ path has no other group to pair with. Its length is $L_{mid} = \sqrt{((x-\frac{R+1}{2})L)^2 + d^2} + \sqrt{((x-\frac{R+1}{2})L-a)^2 + d^2}$. It can be shown that L_{mid} is also convex and reaches minimum at $x^* = \frac{(R+1)L-a}{2}$. The proposition is proved. $\square$

3.6.2 Multiple Rows
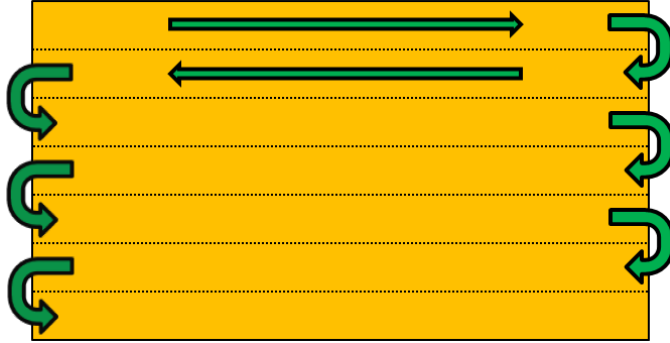
Figure 3.10: Order of harvesting operation

When the field contains r rows, the combines harvest from one end of the field to the other. After finishing the current row, the combines turn to the nearest unharvested row. Fig.3.4 shows the order of harvesting operation. We notice that the direction of motion of the combines in the current row is always opposite to the previous one. We assume all the rows have the same width w. The distance traveled by the grain cart in any row i can be expressed as follows:

$$when\ i\ is\ odd: L_i = \sum_{n=1}^{\lfloor x \rfloor} \sqrt{((x-n)L)^2 + (d+(i-1)w)^2} + \sqrt{((x-n)L+a)^2 + (d+(i-1)w)^2} +$$

$$(3.5)$$

$$\sum_{n=\lceil x \rceil}^{R} \sqrt{((n-x)L)^2 + (d+(i-1)w)^2} + \sqrt{((n-x)L-a)^2 + (d+(i-1)w)^2}$$

$$when\ i\ is\ even: L_i = \sum_{n=1}^{\lfloor R-x \rfloor} \sqrt{((R-x-n)L)^2 + (d+(i-1)w)^2} + \sqrt{((R-x-n)L+a)^2 + (d+(i-1)w)^2} +$$

$$(3.6)$$

$$\sum_{n=\lceil R-x \rceil}^{R} \sqrt{((n-(R-x))L)^2 + (d+(i-1)w)^2} + \sqrt{((n-(R-x))L-a)^2 + (d+(i-1)w)^2}$$

$$L_{total}^r = \sum_{i=1}^{r} L_i \qquad (3.7)$$

L_{total}^r is the total distance traveled by the grain cart when there are r rows. We know that $L_{total}'' \geq 0$, so the minimum of L_{total}^r exists and is unique. By solving $(L_{total}^r)' = 0$, we can find the optimal location x^* which minimizes the total distance traveled by the grain cart. The expression of L_{total}' is as follows:

$$when\ i\ is\ odd: L_i' = \sum_{n=1}^{\lfloor x \rfloor} \left(\frac{L^2(x-n)}{\sqrt{((x-n)L)^2 + (d+(i-1)w)^2}} + \frac{L(L(x-n)+a)}{\sqrt{((x-n)L+a)^2 + (d+(i-1)w)^2}} \right)$$

$$(3.8)$$

$$+ \sum_{n=\lceil x \rceil}^{R} \left(-\frac{L^2(n-x)}{\sqrt{((n-x)L)^2 + (d+(i-1)w)^2}} - \frac{L(L(n-x)-a)}{\sqrt{((n-x)L-a)^2 + (d+(i-1)w)^2}} \right)$$

$$when\ i\ is\ even: L_i' = \sum_{n=1}^{\lfloor R-x \rfloor} -\frac{L^2(R-x-n)}{\sqrt{((R-x-n)L)^2 + (d+(i-1)w)^2}} - \frac{L(L(R-x-n)+a)}{\sqrt{((R-x-n)L+a)^2 + (d+(i-1)w)^2}}$$

$$(3.9)$$

$$+ \sum_{n=\lceil R-x \rceil}^{R} \frac{L^2(n+x-R)}{\sqrt{((n-(R-x))L)^2 + (d+(i-1)w)^2}} + \frac{L(L(n+x-R)-a)}{\sqrt{((n-(R-x))L-a)^2 + (d+(i-1)w)^2}}$$

$$L_{total}' = \sum_{i=1}^{r} L_i' \qquad (3.10)$$

To obtain an estimate of x^*, we first obtain x_i^* for each row individually using the method presented in the previous section. Then we can obtain the estimated optimal depot position $\bar{x}^*$ by calculating the

mean value of the optimal location in each row, $\bar{x^*} = \frac{\sum_{i=1}^{r} x_i^*}{r}$. When i is odd, $x_i^* = \frac{(R+1)L-a}{2}$. When i is even, $x_i^* = \frac{(R-1)L+a}{2}$. Therefore, we can approximate the optimal depot position for r rows. When r is even, $\bar{x^*} = \frac{L-a}{2r} + \frac{RL}{2}$. When r is even, $\bar{x^*} = \frac{RL}{2}$. Next, we present a bound on the maximum error incurred in the approximation.

First, we consider the case when there are two rows. As before, we divide the paths into groups. We let the nth path and the $(R+1-n)^{th}$ path, $(n \leq \frac{R}{2})$ of each row be in group n. Each group contains 4, or 2 pairs of paths, one pair from each row. The length of paths in group n is given by the following expression:

$$L_{group} = \sqrt{((x-n)L)^2 + d^2} + \sqrt{((x-n)L+a)^2 + d^2} + \sqrt{((n-x)L)^2 + d^2} + \sqrt{((n-x)L-a)^2 + d^2}$$

$$\text{(3.11)}$$

$$+ \sqrt{((R-x-n)L)^2 + (d+w)^2} + \sqrt{((R-x-n)L+a)^2 + (d+w)^2}$$
$$+ \sqrt{((n-(R-x))L)^2 + (d+w)^2} + \sqrt{((n-(R-x))L-a)^2 + (d+w)^2}$$

$$L'_{group} = \frac{L^2(x-n)}{\sqrt{((x-n)L)^2 + d^2}} + \frac{L(L(x-n)+a)}{\sqrt{((x-n)L+a)^2 + d^2}} - \frac{L^2(n-x)}{\sqrt{((n-x)L)^2 + d^2}} - \frac{L(L(n-x)-a)}{\sqrt{((n-x)L-a)^2 + d^2}}$$

$$\text{(3.12)}$$

$$- \frac{L^2(R-x-n)}{\sqrt{((R-x-n)L)^2 + (d+w)^2}} - \frac{L(L(R-x-n)+a)}{\sqrt{((R-x-n)L+a)^2 + (d+w)^2}}$$
$$+ \frac{L^2(n+x-R)}{\sqrt{((n-(R-x))L)^2 + (d+w)^2}} + \frac{L(L(n+x-R)-a)}{\sqrt{((n-(R-x))L-a)^2 + (d+w)^2}}$$

It was proved in the previous section that L_{pair} is convex. Since L''_{group} is the summation of L''_{pair} of each row, $L''_{group} \geq 0$. Therefore, for 2 rows, the total travel distance L_{total}, which is the summation of the length of paths in all the groups, is convex as well. By substituting $x = \frac{RL}{2}$ we can get that $L'_{group} > 0$, which indicates the optimal position of the depot is to the left of $x = \frac{RL}{2}$ when there are two rows. When there are more rows, as long as r is even, it can be generalized that L'_{total} increases. So $x^* < \frac{RL}{2}$. For any row i individually, the optimal depot position x_i^* is located at $\frac{(R+1)L-a}{2}$ or $\frac{(R-1)L+a}{2}$. When there are multiple rows, it is obvious that $\frac{(R-1)L+a}{2} < x^* < \frac{(R+1)L-a}{2}$. So when there are even rows, we know that $\frac{(R-1)L+a}{2} < x^* < \frac{RL}{2}$.

When r is odd, the only difference is that there is always an additional row starting from $r = 3$. By substituting $x = \frac{L-a}{2r} + \frac{RL}{2}$ to equation 3.10 we obtain $L'_{group} > 0$. As r increases, L'_{group} increases. As a

result, x^* is to the left of $\frac{L-a}{2r} + \frac{RL}{2}$ when r is odd. In this case, $\frac{(R-1)L+a}{2} < x^* < \frac{L-a}{2r} + \frac{RL}{2}$.

Therefore, we reach a conclusion that when r is even, the optimal location of depot is in $\left(\frac{(R-1)L+a}{2}, \frac{RL}{2}\right)$ while it is in $\left(\frac{(R-1)L+a}{2}, \frac{L-a}{2r} + \frac{RL}{2}\right)$ when r is odd. The maximum error is $\frac{(L-a)(r+1)}{2r}$.

3.7 Path Planning of the Grain Cart

3.7.1 Numerical Approach

The numerical approach is used to obtain the time-optimal lane change maneuver in section 3.7.2. In this section we would like to give a description of the numerical approach to obtain the time-optimal solution for the grain cart to move between combines. We use q_i and q_g to denote the initial and goal configuration of the path. Here q_i is set to be the state when grain cart leaves the combine, and q_g is the configuration of the next combine. In the rest of this section, we will elaborate this path planning approach.

Denote the set of admissible path from the configuration q_i as $\mathscr{A}_{x_i,y_i,\theta_i,\beta_i}$. Given a goal configuration q_g, we define the corresponding value function $u : q \to \mathbb{R}^+ \cup \{0\}$ Takei et al. (2010):

$$u(q(T)) = \inf\{T : q(t) \in \mathscr{A}_{x_i,y_i,\theta_i,\beta_i}, q(T) = q_g\} \tag{3.13}$$

The value function can be regarded as the optimal cost-to-go for the tractor-trailer model with given constraints, an initial configuration and a final configuration. By applying dynamic programming principle for the Eqn. (3.13), we have

$$u(q(t)) = \inf\{u(q(t+\Delta t)) + \Delta t : q(t) \in \mathscr{A}_{x_i,y_i,\theta_i,\beta_i}\} \tag{3.14}$$

Dividing the terms by Δt and taking $\Delta t \to 0$, we are able to derive

$$-1 = \inf\{\nabla u \cdot \dot{q} : |v| = 1, |\omega| \le 1\} \tag{3.15}$$

With the equations of motion of the grain cart, *Hamilton-Jacobi-Bellman equation* is obtained as follows

$$-1 = \cos\theta \frac{\partial u}{\partial x} + \sin\theta \frac{\partial u}{\partial y} - \sin\beta \frac{\partial u}{\partial \beta} + \inf_{|\omega| \le 1}\{\dot{\theta}(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \beta})\} \tag{3.16}$$

The last term in Eqn. (3.16) can be eliminated by applying bang-bang principle $w = \pm 1$. Since q_i is the goal configuration which has no cost-to-go, we have $u(q_g) = 0$. For the points located in the

obstacle or outside the space, we define the cost-to-go to be infinity. In the next section, we present the update scheme of the defined value function.

3.7.1.1 Update Scheme

In order to find the time-optimal path satisfying Eqn. (3.16), we apply fast sweeping method and propose an update scheme for the value function $u(q)$ for the entire space. The idea is to take advantage of the fact that the value function has zero cost-to-go at the goal configuration, and to compute the value function from the nodes close to the goal configuration, to the nodes at farther positions.

With this in mind, we first set up a four dimensional uniform Cartesian grid with refinement $(h_x, h_y, h_\theta, h_\beta)$. Let $u_{a,b,c,d} = u(q_{a,b,c,d}) = u(ah_x, bh_y, ch_\theta, dh_\beta)$ be the approximation of the solution on the grid nodes. Moreover, we discretize ω in the range of $[-1, 1]$ and further define $u^*_{a,b,c,d}$ as follows

$$u^*_{a,b,c,d} = \min_{\omega_i \in [-1,1]} \{u(q_{a,b,c,d} + \dot{q}\Delta t)\} + \Delta t \tag{3.17}$$

, where $\dot{q} = (\cos(ch_\theta), \sin(ch_\theta), \omega_i, -\sin(dh_\beta) + \omega_i))^T$, ω_i is the ith element in the discretization and Δt is the length of time step. The value of $u(q_{a,b,c,d} + \dot{q}\Delta t)$ is approximated by taking the average value of the adjacent nodes in the presented grid.

Finally, the update scheme can be described as follows

$$u^{n+1}_{a,b,c,d} = \min\{u^n_{a,b,c,d}, u^{*n}_{a,b,c,d}\} \tag{3.18}$$

, where the superscripts denote the iteration. We set up the termination condition of the computation as follows

$$(\|u^{n+1}_{a,b,c,d} - u^n_{a,b,c,d}\|_2)^2 < \varepsilon \tag{3.19}$$

, where $\varepsilon > 0$.

3.7.1.2 Computing Trajectory

By using the obtained value function $u(q)$, we are able to derive the time-optimal path from any initial configuration q_i to the goal configuration q_g. The control law can be summarized as follows

$$\dot{x} = \cos\theta \tag{3.20}$$

$$\dot{y} = \sin\theta \tag{3.21}$$

$$\dot{\theta} = -sgn(\frac{\partial u}{\partial\theta} + \frac{\partial u}{\partial\beta}) \tag{3.22}$$

$$\dot{\beta} = -\sin\beta + \dot{\theta}. \tag{3.23}$$

Note that the partial derivative in Eqn. (3.22) is obtained by applying centered difference approximation. The values of u which are not on the nodes are computed using a nearest-neighbor interpolation.

The numerical approach computes the time-optimal trajectory efficiently if the corresponding value function is provided. The main time consumption is in computing the value function of the final configuration. But in real implementation, one can compute the value function beforehand. Hence, the time cost of computing the value function will not influence the real operation on path planning. Figure 3.11 shows the time optimal path obtained with numerical approach of a grain cart moving between two points .
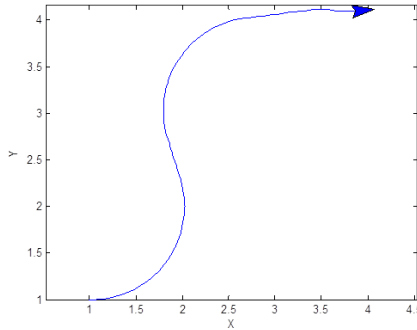
Figure 3.11: Path of grain cart with numerical approach

3.7.2 Primitive-Based Motion Maneuvre

The trajectory is generated by a numerical approach Zhang and Bhattacharya (2015) that is solution to find the time-optimal path between an initial and a final configurations for a tractor-trailer
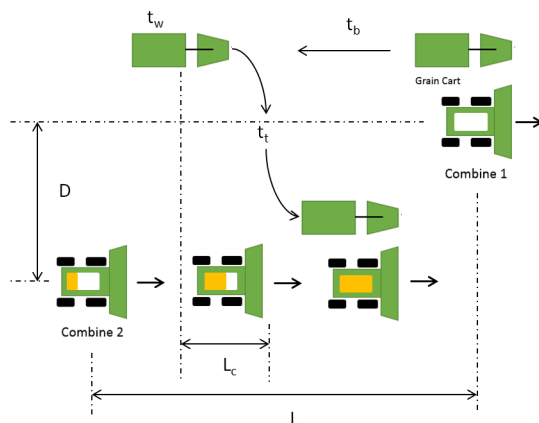
Figure 3.12: Path planning for the grain cart between two combines

The proposed maneuver resembles a parallel parking maneuver for a tractor-trailer with the exception that the objective here is to "park" a grain-cart alongside a moving combine. Figure 3.12 illustrates the position of the grain cart relative to the two combines as it implements the proposed maneuvers. Let L denote the horizontal distance maintained between two adjacent combines while they are moving on two adjacent lanes. The grain cart in Figure 3.12 moves from combine 1 to combine 2. The trajectory contains the following primitives:

1. Back-up: After serving the current combine, the grain cart moves backward in a straight line with a speed v_g while the combines keep moving forward with a speed v_c. Let t_b denote the duration of this phase.

2. Stall: The grain cart stops, and waits for a time t_w until the relative position of combine 2 with respect to the grain cart is L_c as shown in the figure.

3. Lane-change: The grain cart follows a time-optimal trajectory presented in Zhang and Bhattacharya (2015) to reach combine 2. This stage resembles a lane-change maneuver by vehicles observed frequently on highways. Subsequently, the grain cart moves parallel to the combine.

The only exception occurs when the grain cart returns to the initial combine after serving the combine at the other end. In this case, it simply performs the lane-change maneuver.

The maneuvre proposed in the previous section contains some decision variables, namely t_b and t_w. The aforementioned variables can be tuned to either minimize the distance traveled (D_{t_b}) by the grain cart during the maneuvre or the total time spent ($T(t_b, t_w)$) to complete the maneuvre.

Based on the user's preferences, the following cost function can be formulated:

$$J(t_b, t_w) = \lambda T(t_b, t_w) + (1 - \lambda)D(t_b) \quad \lambda \in [0, 1], \tag{3.24}$$

where the parameter λ is decided by the user.

For a given distance D and L_c, let t_{lc} denote the time required to complete the lane-change maneuvre. This is computed off-line by the numerical approach described in the previous section. Based on the geometric constraints imposed by the maneuvre, we obtain the following equation:

$$L + L_c = (v_g + v_c)t_b + v_c t_w$$

$$\Rightarrow t_w = \frac{(L + L_c) - (v_g + v_c)t_b}{v_c}. \tag{3.25}$$

The expression of the total traveling time and distance is as follows:

$$T(t_b, t_w) = t_b + t_w + t_{lc}$$

$$\Rightarrow T(t_b) = t_b + \frac{(L + L_c) - (v_g + v_c)t_b}{v_c} + t_{lc} \tag{3.26}$$

$$D(t_b) = t_b v_g + t_{lc} v_{lc}.$$

where v_{lc} is the velocity of grain cart during lane-changing. Substituting (3.24) in (3.27) leads to the following expression for the cost function:

$$J(t_b) = (\lambda - \frac{v_g + v_c}{v_c}\lambda + (1 - \lambda)v_g)t_b + \lambda t_{lc} + \left[\frac{L + L_c}{v_c} + (1 - \lambda)t_{lc}v_{lc}\right]. \tag{3.27}$$

Since J_{t_b} is a linear function of t_b, we can conclude the following:

1. At $\lambda = \frac{v_c}{v_c + 1}$, $J(t_b)$ is independent of t_b.

2. For $\lambda < \frac{v_c}{v_c + 1}$, the minimum value of J_{t_b} occurs at $t_b = 0$. This implies that the grain cart should wait after serving combine 1, and subsequently perform the lane change maneuvre to serve combine 2.

3. For $\lambda > \frac{v_c}{v_c + 1}$, the minimum value of J_{t_b} occurs at $t_b = \frac{L + L_c}{v_g + v_c}$. This implies that the grain cart should back-up after serving combine 1 till it is at a horizontal distance of L_c behind combine 2, and subsequently perform the lane change maneuvre. There is no time spent in the stall maneuvre since $t_w = 0$.

Therefore, the optimal value of t_b lies at either end of the interval for $\lambda \neq \frac{v_c}{1 + v_c}$.

CHAPTER 4.   MULTIPLE GRAIN CARTS HARVESTING-ON-DEMAND PROBLEM

4.1   Introduction

In Chapter 3, we introduce the scheduling plan of the harvesting operation when where is single grain cart serving all the combines. In this chapter, we generalize the scheduling plan to general case when there are multiple grain carts and the number of grain carts is fewer than the number of the combines. Two classes of scheduling plans based on the way the combines are allocated with the grain carts are proposed.

4.2   Round-Robin Scheme

In a Round-Robin Scheme, the grain carts serve the combines following the FCFS(First Come Fist Served) rule. Based on the frequency at which the grain cart visits the depot, two variants of the scheme are proposed.

4.2.1   Cycle-based Round-Robin Scheme

The engagement between the grain carts and the combines is divided into several rounds. A round is defined as the time period in which M combines get served, or each grain cart serves a combine. In round 1, combine i is served by grain cart i for $i \leq M$. The grain cart starts to serve a combine only when the tank is completely filled. After emptying the tank, the grain cart travels to the combine $i + M$ in round 2, which also happens to be the combine that has not been served for the longest time. This continues till the grain cart serves the combine $(i + (\lceil \frac{N}{M} \rceil - 1)M)$. If N is not a multiple of M, the grain cart returns to the beginning of the group to serve combine $(i + \lceil \frac{N}{M} \rceil M) \bmod N$. If N is a multiple of M, then the grain cart returns to combine i.

Let the lowest common multiple of M and N be L. After serving $\frac{L}{M}$ combines, a grain cart is assigned to travel to the depot to unload itself ($C_g = \frac{L}{M}C\frac{r_u}{r_u-r_f}$).A grain cart is said to have completed a *cycle* when it returns to the combine to which it was assigned at round 1. In the round-robin method, a grain cart completes a cycle after every $\frac{L}{M}$ rounds. During that time, the grain cart serves $\frac{L}{M}$ combines, and every combine gets served $\frac{L}{N}$ times by different grain carts. Figure 4.1 shows the load variation of the first combine in a complete cycle when there are 2 grain carts serving N combines.
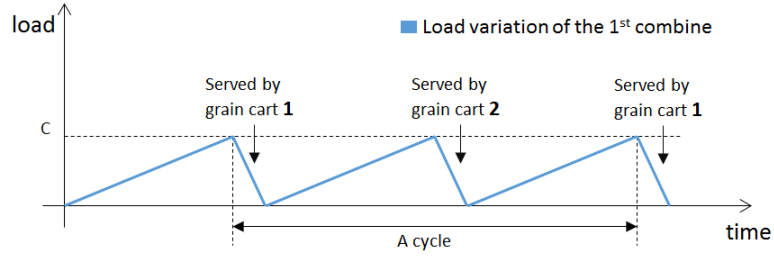
Figure 4.1: Travel time comparison between using numerical approach and primitive-based approach

Let $N(r,k)$ denotes the combine that is served by grain cart k at round r:

$$f(r,k) = [k+M(r-1)]\bmod N \quad r,k \in \mathbb{Z}^+, k \leq N$$

$$N(r,k) = \begin{cases} f(r,k) & \text{if } f(r,k) \neq 0 \\ N & \text{if } f(r,k) = 0 \end{cases}. \tag{4.1}$$

Let $T^k_{r,r+1}$ denote the time spent by the grain cart k in traveling between two consecutive rounds. Mathematically, it can be given by the following expression:

$$T^k_{r,r+1} = |N(r+1,k) - N(r,k)|T. \tag{4.2}$$

Each cycle contains $\frac{L}{M}$ rounds and a grain cart serves $\frac{L}{M}$ combines. A grain cart moves from the end of the combines group to the front $\frac{L}{N}$ times in each cycle . In between rest of the rounds it moves from combine k to combine $k+M$ ($1 \leq k \leq N-M$) ($\frac{L}{M} - \frac{L}{N}$) times. This leads to the following expression for the total time spent to travel between combines in a cycle:

$$\sum_{r=1}^{\frac{L}{M}} T_{r,r+1} = (\frac{L}{M} - \frac{L}{N})MT + (\frac{L}{N} - 1)(N-M)T$$

$$\Rightarrow \sum_{r=1}^{\frac{L}{M}} T_{r,r+1} = 2L(1 - \frac{M}{N})T. \tag{4.3}$$

Consider a combine-grain cart pair that engage in round 1. Before the next cycle, the combine gets served $(\frac{L}{N}-1)$ times by a different grain cart each time. Moreover, the combine allocated to the grain cart during round 1 gets filled $\frac{L}{N}$ times and gets served $\frac{L}{N}-1$ times by different grain carts in a cycle. During this time, the grain cart serves $\frac{L}{M}-1$ other combines and goes to the depot once. The travel time to the depot is $\lambda T (0 < \lambda \le N - M)$. Equating the quantities leads to the following equation:

$$2L(1-\frac{M}{N})T + (\frac{L}{M}-1)\frac{C_{RR}}{r_u - r_f} + \lambda T = \frac{L}{N}\frac{C_{RR}}{r_f} + (\frac{L}{N}-1)\frac{C_{RR}}{r_u-r_f}$$

$$\Rightarrow C_{RR} = (2(MN-M^2) + \frac{\lambda MN}{L})\frac{r_f(r_u-r_f)}{Mr_u - Nr_f}T \qquad (4.4)$$
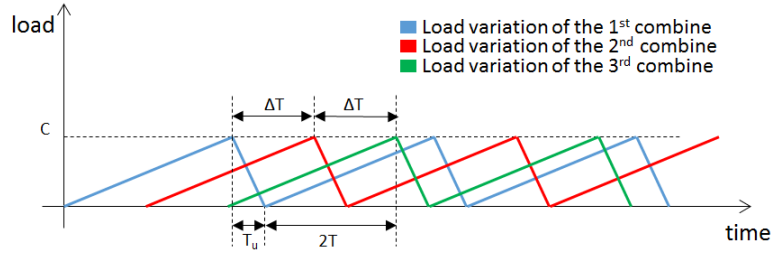
Figure 4.2: Load variation of 3 combines with round-robin

Figure 4.2 shows the load variation of three combines served by two grain carts. It shows that in the time of ΔT, a grain cart serves a combine completely and moves to the next one. When there are N combines and M grain carts, during ΔT, a grain cart serves the current combine i and goes to combine $M + i$. The expression for T can be obtained as follows:

$$M\Delta T = T_u + MT \qquad (4.5)$$

$$\rightarrow T = \Delta T - \frac{T_u}{M}$$

, where $T_u = \frac{C}{r_u - r_f}$ is the unloading time. (4.6) provides as expression for T in terms of the parameters of the problem. **Since T is the time taken by the grain cart to travel between two consecutive combines, (4.6) is a constraint that needs to be satified by the motion planner for the grain cart**.

4.2.2 Loop-based Round-Robin Scheme

In Round-Robin scheme presented above, we know that a grain cart unloads itself after serving $\frac{L}{M}$ grain carts. In a case when there are 5 combines and 2 grain carts, every grain carts needs to serve 5

combines before it goes to the depot, which implies $C_g \geq 5C\frac{r_u}{r_r - r_f}$. Considering the capacity of real agricultural vehicles, this may be difficult to guarantee. In this subsection, we present a variant of the proposed round-robin scheme.

As we know, the combines enter the field one after another with a constant time gap ΔT. Grain cart i initially serves combine i and follows the FCFS rule. In this strategy, instead of going to the depot after each complete cycle, a grain cart unloads itself before moving back from the end of the combines group to the front, which we call a *loop*. We assume the unloading time is λT. Figure 4.3 shows the load variation when there are 5 combines serves by 2 grain carts. In the first loop, grain cart 1 served combine 1,3,5 and grain cart 2 serves combine 2 and 4. After spending λT on unloading themselves to the depot, the two grain carts come back to the front of the group. However, in the second loop, the serving order will be different. Grain cart 2 serves the first combine since it comes back from the depot earlier and grain cart 1 serves the second combine. In a general case when there are M grain carts and N combines, the serving order will be back to the original after every M loops of serving.
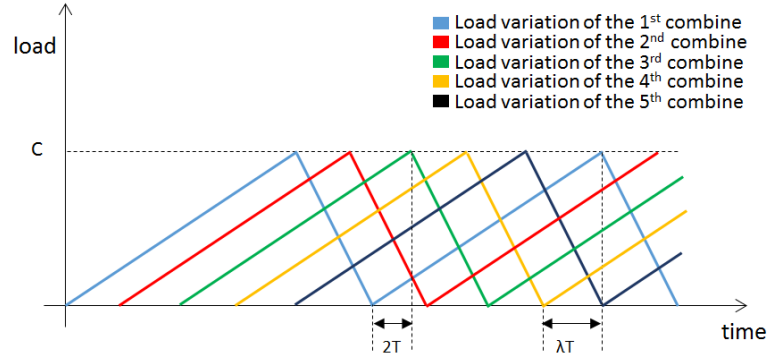
Figure 4.3: Load variation of 5 combines with round-robin 2

In this scheduling strategy, the load of each grain cart when they arrive at the depot maybe different. The maximum capacity required for the grain cart is $C_g \geq \lceil \frac{N}{M} \rceil C \frac{r_u}{r_u - r_f}$. In one loop, a grain cart serves either $\lceil \frac{N}{M} \rceil$ or $\lfloor \frac{N}{M} \rfloor$ combines then goes back to the front of the group.

For a combine $i, (i \leq M)$, when it needs to get served the second time, we assume grain cart k will take the job. We can find that before going to the depot and traveling to combine i, grain cart k was

working on combine $N+i-M$. So the following expression can be obtained:

$$((N+i-M)-i)\Delta T + \lambda T = \frac{C}{r_f} \tag{4.6}$$

$$\rightarrow C = ((N-M)\Delta T + \lambda T)r_f$$

In this case, during the time of ΔT, a grain cart serves combine i and goes to combine $M+i$ which indicates that $M\Delta T = T_u + MT$. The benefit of this scheduling plan is that the required capacity of the grain carts is lower. Since there is no traveling back to the front of group of the combines from the end, the possibility of collision during that time is minimized. However, the grain carts need to travel to the depot more frequently.

4.2.2.1 Collision Avoidance:

In round-robin scheme, we notice that the grain carts are always moving among the combine harvesters. There is possibility that the grain carts will collide with each others. To avoid the occurrence of collision, we introduce the priority method of the grain carts path planning. As we have discussed above, when a grain cart finishing serving the current combine, if the next combine is at the end of the combines group, it moves backwards and then follows a maneuver to the target. If the next combine is in the front of the group, it follows the time-optimal maneuver to it directly. We set different priorities to the grain carts' motion between combines:

1. When two grain carts meet, the one moving from the end of the group to the front of the group has lower priority.

2. When two grain carts meet and they both follow the same type of path (path from front to end of group), the one starts moving earlier has higher priority.

Following the proposed rules, when two grain carts meet, the one with higher priority moves while the other one stops and waits until their distance is safe. The safe distance is based on the dimension of the vehicles. In this case, the traveling time between adjacent combines T is the longest time a grain cart need to travel with the waiting time into consideration. Since there is a always a time difference between any two combines being served, each grain cart will wait one time the most during their current motion.

4.3 Partitioning Schemes

The minimum capacity of the combines is dictated by the frequency at which it can be served by the grain carts. As the distance traveled by the grain cart increases, the frequency at which the combines get served decreases. In order to alleviate this problem, we propose partitioning schemes that allocate a smaller group of combines to individual grain carts so that the distance traveled by the grain carts is minimized compared to the round-robin scheme proposed in the previous subsection. The benefit of the partitioning strategy is that the minimum capacity of the combines depends on the ratio between N and M instead of their actual values which is the case with the round-robin scheme.

As in the previous case, all the combines start moving with a time gap ΔT. However, instead of serving all the combines together, each grain cart will be allocated a specific group of combines to serve. Let $n = \frac{N}{M}$. If n is an integer, then the combines are divided in M groups, $G_1, \ldots, G_M$. G_i contains all the combines from combine $(i-1)n+1$ till combine ni. In each group, the grain cart will follow the strategy proposed in Section 3.5. After serving every vehicle in the group, the grain cart goes to the depot to unload itself before starting a new round. Therefore, the minimum value of C is obtained by solving the minimum capacity of n combines that can be served by 1 grain cart. This can be obtained from (4.4) by substituting $M = 1$ and $N = n$.

$$C = \frac{(2(n-1)+\lambda)r_f(r_u - r_f)T}{r_u - nr_f} \tag{4.7}$$

The result is the same as that in Section 3.5. If n is not an integer, we propose the following two techniques.

4.3.1 Load Balancing with Integral Constraints (LBIC)

In this technique, the load is balanced keeping in mind that each combine is served only by one grain cart. However, since $\frac{N}{M}$ may not be an integer, the partitions will have unequal numbers of combines. In order to minimize the maximum time required to complete a cycle for any partition, the following procedure can be used. Let $n' = \lfloor \frac{N}{M} \rfloor$. First, assign n' combines to each partition. Then split the remaining $N \bmod M$ combines among $N \bmod M$ groups chosen arbitrarily. The maximum number of combines a group can have is $n' + 1$. From (4.4), the minimum capacity of the combines is given by the

following expression:

$$C_{LBIC} = \frac{(2n' + \lambda)r_f(r_u - r_f)T}{r_u - (n' + 1)r_f},$$

(4.8)

4.3.2 Load Balancing without Integral Constraints(LB/IC)

In this technique, each grain cart serves exactly the same amount of load. In other words, the amount of grain unloaded by each grain cart is the same, which is $\frac{N}{M}C$. Since $\frac{N}{M}$ is not an integer, some of the combines will be served by more than one grain-cart which is a major distinction from the previous technique. If a specific combine is served by a grain cart, we assume it belongs to the partition served by that grain cart. This implies that a single combine can contribute to more than one partition when it is being shared. Since the grain carts would minimize their travel distance while serving the combines in their partition, it can be seen that adjacent partitions will share combines located at the end of the partition. A natural question arises regarding the number of combines that should be shared between two adjacent partitions to minimize the capacity. The following proposition provides an answer.

Proposition 3: At most one combine is shared by two adjacent partitions.

Proof. Consider a specific grain cart, and its corresponding partition. Let m be the number of combines that are only served by the grain cart. Let k_1 and k_2 be the number of combines shared with the previous and the next partitions, respectively. Let C_i' and C_j'' denote the load served by the grain cart for combine $i(1 \leq i \leq k_1)$ in the previous group, and combine $j(1 \leq j \leq k_2)$ in the next group, respectively. The first combine gets served again after all the other combines get served once. The grain cart spends the same amount of time serving the remaining combines in addition to the waiting time for the first combine. Equating these two times leads to the following:

$$\frac{C}{r_f} + \frac{C - C_1'}{r_u - r_f} = \lambda T + \sum_{i=2}^{k_1} \frac{C_i'}{r_u - r_f} + (k_1 - 1)T + mT +$$

$$m\frac{C}{r_u - r_f} + \sum_{j=1}^{k_2} \frac{C_j''}{r_u - r_f} + k_2 T + (m + k_1 + k_2 - 1)T$$

(4.9)

Since each grain cart carries a load of $\frac{N}{M}C$, this leads to the following equality:

$$\frac{mC_1'}{r_u - r_f} + \sum_{i=1}^{k_1} \frac{C_i'}{r_u - r_f} + \sum_{j=1}^{k_2} \frac{C_j''}{r_u - r_f} = \frac{N}{M}C.$$

(4.10)

Substituting (4.10) into (4.9) leads to the following expression for the minimum capacity:

$$C = \frac{(m + k_1 + k_2 - 1 + \lambda)r_f(r_u - r_f)T}{r_u - \frac{N}{M}r_f}. \tag{4.11}$$

From (4.11), we can conclude that C is a decreasing function of $k_1 + k_2$. $1 \leq k_1, k_2$ leads to the proposition. $\square$

The minimum capacity of the combines depends on the maximum number of combines served by any single grain cart. In this partitioning technique, the minimum number of combines served by each grain cart is at least $n' + 1$, where $n' = \lfloor \frac{M}{N} \rfloor$. However, in some cases, there may be some groups with $n' + 2$ combines. For example, when $N = 5$ and $M = 3$, there will be 3 combines assigned to the second grain cart. Therefore, the number of combines in any partition is at least $n' + 1$, and at most $n' + 2$. The following proposition provides the conditions for M and N under which the maximum size of a partition is $n' + 2$.

Proposition 4: If $N - 1$ is not a multiple of M, there is at least one partition that has $n' + 2$ combines. Otherwise, all partitions have $n' + 1$ combines.

Proof. For a grain cart g_k, we can define a vector p_k, called the *load profile*, where $p_k[i]$ denotes the load served by the grain cart g_k for combine i. Let $\tilde{p}_k$ be a vector that denotes the elements of the vector p_k in reverse order.

We consider the case when N and M are coprime. Else, the combines and the grain carts can be divided into l similar subgroups containing $\frac{N}{l}$ combines served by $\frac{M}{l}$ grain carts, where l is the greatest common divisor of N and M. The necessary and sufficient conditions that lead to $(n + 2)$ combines in one partition will hold across all subgroups. With this assumption, we have the following cases.

From symmetry, $p_{\frac{M}{2}}[n' + 1] = \tilde{p}_{\frac{M}{2}+1}[1]$. If all partitions have $n' + 1$ combines, the profiles are given by the following. Let p_1 be given by the following:

$$p_1[i] = \begin{cases} C & i \leq n' \\ \beta_1 & i = n' + 1 \end{cases}, \tag{4.12}$$

where $\beta_1 = \frac{N}{M} - \lfloor \frac{N}{M} \rfloor$. For the grain carts serving the partitions in the middle, we assume the following

load profile:

$$p_k[i] = \begin{cases} \alpha_k & i = 1 \\ C & 1 < i \leq n' \qquad 1 < k < n, \\ \beta_k & i = n' + 1 \end{cases} \tag{4.13}$$

where $\alpha_k(0 \leq \alpha_k \leq 1)$ is the part of load of the combine, which shared with the previous partition, that served by g_k and $\beta_k(0 \leq \beta_k \leq 1)$ is the part shared with the following group. From the cumulative sum of the loads of the grain carts from g_1 to g_j, we obtain the following equations:

$$\begin{cases} (\alpha_j + \beta_j + (n' - 1))C = \frac{N}{M}C \\ \beta_{j-1} + \alpha_j = 1 \end{cases} \Rightarrow \begin{cases} \alpha_j = 1 - (j-1)(\frac{N}{M} - \lfloor \frac{N}{M} \rfloor) \\ \\ \beta_j = j(\frac{N}{M} - \lfloor \frac{N}{M} \rfloor) \end{cases} . \tag{4.14}$$

When M is even, $\beta_{\frac{M}{2}} = \frac{1}{2}$ from symmetry. This leads to the following relation:

$$\frac{N-1}{M} = \lfloor \frac{N}{M} \rfloor. \tag{4.15}$$

When M is odd, $\alpha_{\frac{M+1}{2}} = \beta_{\frac{M+1}{2}}$. Using this relation, leads to the following condition:

$$\begin{cases} (\alpha_{\frac{M+1}{2}} + \beta_{\frac{M+1}{2}} + (n' - 1))C = \frac{N}{M}C \\ \alpha_{\frac{M+1}{2}} = \beta_{\frac{M+1}{2}} = 1 - \frac{M-1}{2}(\frac{N}{M} - \lfloor \frac{N}{M} \rfloor) \end{cases}$$
$$\Rightarrow \frac{N}{M} - \lfloor \frac{N}{M} \rfloor = \frac{1}{M}. \tag{4.16}$$

$\square$

Based on (4.11), we obtain the following expressions for the minimum capacity of the grain carts when all the partitions contain $(n' + 1)$ grain carts:

$$C_{LB/IC} = \frac{(n' + \lambda)r_f(r_u - r_f)T}{r_u - \frac{N}{M}r_f}. \tag{4.17}$$

When $(n' + 2)$ combines are present in any group, the expression for the minimum capacity can be obtained by replacing n' with $n' + 1$ in the above expression.

4.4 Analysis

When $\frac{N}{M}$ is not an integer, one can either use the round-robin scheme or the partitioning scheme. By comparing the two schemes, we obtain the following:

$$\frac{C_{LBIC}}{C_{RR}} = \frac{r_n - \frac{N}{M}r_f}{r_u - (n'+1)r_f} \frac{\frac{N}{M} + \lambda}{N - M + \lambda}. \tag{4.18}$$

From the expression we can tell that $\frac{r_n - \frac{N}{M}r_f}{r_u - (n'+1)r_f} \cong 1$. For a constant $\frac{N}{M}$, $N - M$ increases as N increases thereby leading to the fact that $\frac{C_{LBIC}}{C_{RR}} << 1$. Therefore, the round-robin method has a much lower efficiency compared to the partitioning method for large N. Figure 4.4 shows the simulation results for M and N in the range 1 to 20 and $\lambda = 5$. We can see that in all cases the performance of LBIC is better than that of the round-robin method.
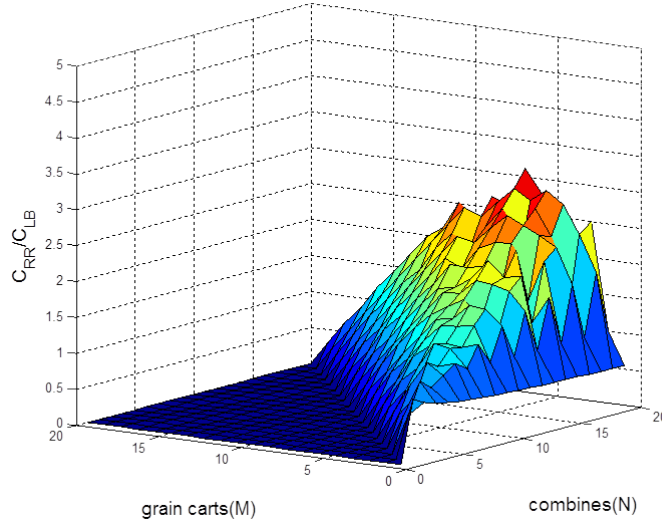

Figure 4.4: Capacity ratio between LBIC and round-robin

When the largest group size is $n' + 1$, the performance of LB/IC and LBIC can be compared as follows:

$$\frac{C_{LB/IC}}{C_{LBIC}} = \frac{r_u - \lceil \frac{N}{M} \rceil r_f}{r_u - \frac{N}{M}r_f} < 1. \tag{4.19}$$

Therefore, when all the groups have $n' + 1$ combines, LB/IC is better than LBIC.

When any partition contains $n' + 2$ combines with LB/IC, we need to decide between LB/IC and LBIC. By comparing the schemes, the following expression can be obtained:

$$\frac{C_{LB/IC}}{C_{LBIC}} = \frac{(n' + 1 + \lambda)(r_u - (n' + 1)r_f)}{(n' + \lambda)(r_u - \frac{N}{M}r_f)}. \tag{4.20}$$

It is obvious that $\frac{n' + 1 + \lambda}{n' + \lambda} > 1$ and $\frac{r_u - (n' + 1)r_f}{r_u - \frac{N}{M}} < 1$. So the performance of the two methods depends on the characteristics of the system. Figure 4.5 shows a comparison of performance between the different
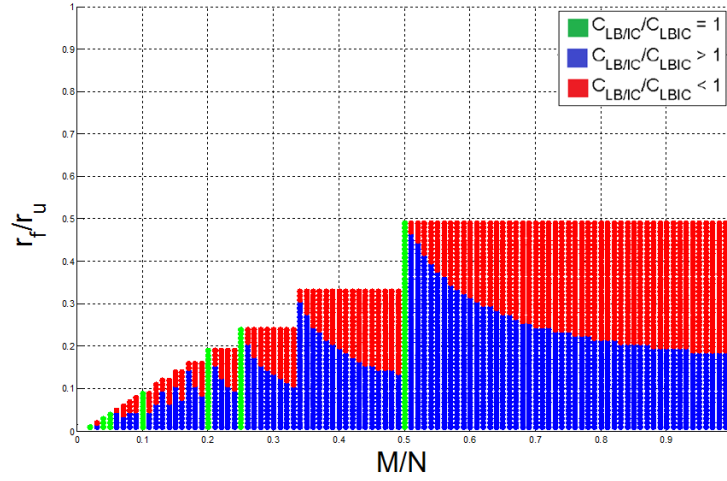
Figure 4.5: Capacity comparison between LBIC and LB/IC

strategies for different values of $\frac{r_f}{r_u}$ and $\frac{M}{N}$.

From Figure 4.5, we can conclude the following. For a given value of the parameters, if $\left(\frac{M}{N}, \frac{r_f}{r_u}\right)$ is in the red region, LB/IC has a better performance whereas LBIC is better if it lies in the blue region. The two methods have the same performance in the green region.

4.5   Scheduling Strategy without Depot

In above analysis, we have addressed the harvesting-on-demand problem in a large scale field. Multiple agricultural vehicles including combine harvesters, grain carts and a static depot are taking into consideration. However, for the harvesting operation in the cases when a depot is not required, such as small field operation, the capacity of the grain carts is large enough to handle the complete field. In this section, we will extend the proposed scheduling schemes to the case when there is no depot in the system.

4.5.1 Round-Robin Scheme

When the depot is removed from the system, the two variants of round-robin scheme turn to be the same. The grain carts just travel between the group of combines following FCFS policy until the end of the field. In this case, after each cycle of unloading, the grain carts will go back to the original combines that are they assigned at the first cycle and repeat. Figure 4.6 shows the load variation of three combines when there are two grain carts serving them. The expression of the parameters can be obtained as follows:

$$2L(1 - \frac{M}{N})T + (\frac{L}{M} - 1)\frac{C_{RR}}{r_u - r_f} + = \frac{L}{N}\frac{C_{RR}}{r_f} + (\frac{L}{N} - 1)\frac{C_{RR}}{r_u - r_f}$$

$$\Rightarrow C_{RR} = 2(MN - M^2)\frac{r_f(r_u - r_f)}{Mr_u - Nr_f}T$$
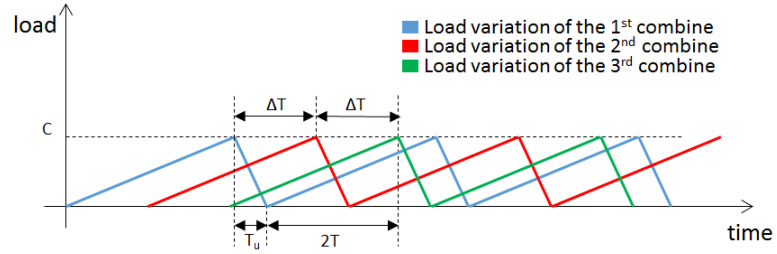
Figure 4.6: Load variation of 3 combines with round-robin

4.5.2 Partitioning Scheme

When there is no depot in the system, partitioning scheme is still divided into two strategies: LBIC and LB/IC. The difference is that in this case the traveling time from the combines to the depot is not considered in the expression.

For load balancing with integral constraint, each group have integer number of combines. Let $n' = \lfloor \frac{N}{M} \rfloor$. If $\frac{N}{M}$ is integer, then the expression of the parameter is as follows:

$$2(n' - 1)\Delta T + (n' - 1)(\frac{C}{r_u - r_f}) = \frac{C}{r_f}$$

$$\Rightarrow C = \frac{2(n' - 1)r_f(r_u - r_f)}{r_u - n'r_f}\Delta T \tag{4.21}$$

If $\frac{N}{M}$ is not integer, then the result can be obtained by replacing n' in equation 4.21 with $n' + 1$.

Figure 4.7 shows the load variation of one of the groups of combines served by a grain cart. α is the fraction of the combine shared with the previous group that served by the grain cart in this group. β is the fraction of the combine shared with the next group that served by the grain cart. In this case, instead of going to the depot, the grain cart goes back to the front of the group of the combine after finishing one cycle of serving.
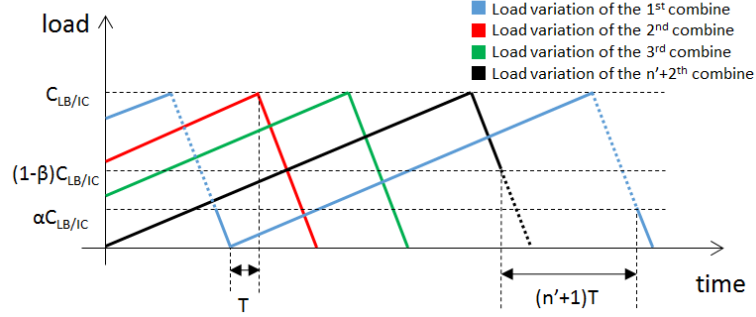
Figure 4.7: Load variation of n'+2 combines with LB/IC without depot

By equating the traveling time of the grain cart in one cycle and the filling time of a combine, the expression of the parameters can be obtained as follows:

$$C = \frac{2n'r_f(r_u - r_f)}{r_u - \frac{N}{M}r_f}\Delta T. \tag{4.22}$$

By comparing the performance of partitioning and round-robin schemes based on the required combine capacity, we can get the result as shown in Figure 4.8. It is obvious that partitioning scheme
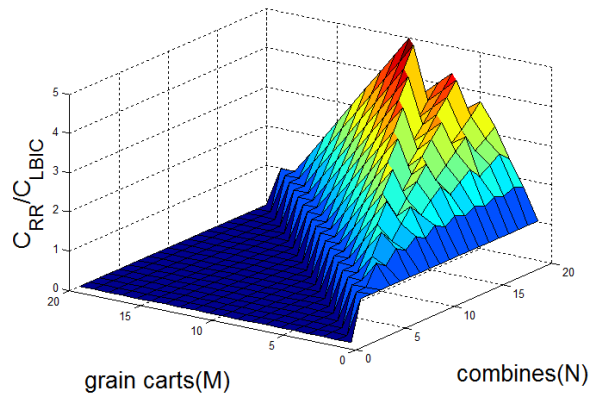
Figure 4.8: Capacity ratio between LBIC and round-robin without depot

has a much better performance than round-robin scheme in most cases. The comparison of LBIC and

LB/IC is shown in Figure 4.9. The result is similar as that when the depot is considered, but different in details. The customer can make decision easily based on the number of vehicles they have.
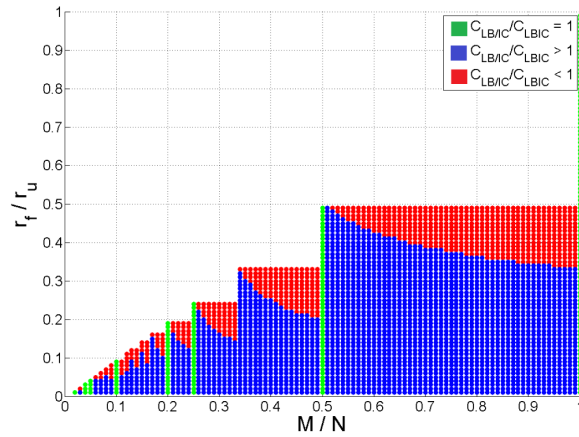
Figure 4.9: Capacity comparison between LBIC and LB/IC without depot

4.6 Implementation

In this section, we introduce a experimental testbed used to implement the proposed scheduling techniques.

4.6.1 Experiment Setup

Figure 4.10: Ground robot

As shown in Figure, the environment is a $2000mm \times 4000mm$ plane, which represent a rectangle corn field that needs to be harvested. Vicon tracking system is used to collect real-time position information of all the vehicles moving inside the field. Figure 4.11 shows the experiment environment with

Vicon system. We use BOE-bot to simulate the agricultural vehicles as shown in Figure 4.10. Each robot is mounted with a raspberry pi 2 board as the controller, an a wifi adapter as the communication unit. Five BOE-bots are used in the experiment. Three of them are used to simulate the combine harvester while two others are used to simulate the grain carts. The motion of the robots are controlled by the software APMplanner 2.0.

Figure 4.11: Vicon system

Figure 4.12: Testbed

With the testbed, we implement the three proposed scheduling schemes: loop-based round-robin scheme, LBIC and LB/IC. As shown in Figure 4.12, three BOE-bots start moving one after another while two BOE-bots travel between them following the proposed techniques. From the implementation, the feasibility of the scheduling schemes is proved.

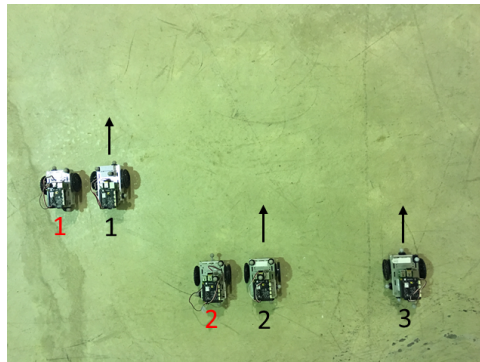# CHAPTER 5. NON-RECTANGULAR FIELD ANALYSIS

In the previous sections, we considered the case when the field was rectangular. In this section, we will extend the proposed techniques to the case of non-rectangular fields.

5.1   Scheduling Strategy in Non-rectangular Fields

In a real field, usually there is no small obstacles like a tree or stone. However, when there is irremovable obstacle like a lake, the field needs to be divided into non-rectangular parts. In this section, we will extend the proposed techniques to the case of non-rectangular fields.

For any non-rectangular field, we assume that N combines harvest the grains in the field. First we divide the field into multiple strips parallel to the longest edges as shown in Figure 5.3. Each white block is a strip, which is the area the group of combines cover before all of them reach the end of the field, we call it a *row*. In a rectangular field, each row is a rectangle which means every combine has the same distance to travel. However, the combines may travel unequal distances in a non-rectangular row as shown in Figure 5.1. The orange part are the paths for the combines in a row which have different lengths.
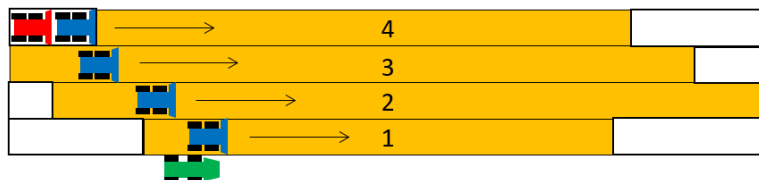

Figure 5.1: Scheduling strategy with complement method

To apply the proposed scheduling strategies in this case, we introduce a technique called the *complement method*. In this method, we extend the non-rectangular row to a rectangle, and apply the schemes proposed in the previous section. The details regarding the technique are as follows:

1. Each row consists of N rectangular paths with different lengths. First, we extend the row to form a complete rectangle. Let L_{ext} denote the horizontal distance between the left and the right end of the row. We extend each path so that the length of all the paths after completing the procedure is equal to L_{ext}. In Figure 5.1, the orange part is the original paths which we call *real paths* and the additional white part is called *virtual paths*.

2. The N combines are located at the beginning of each real path. We assume there are N *virtual combines* are parked at the beginning of the completed paths and they enter the field sequentially. When a virtual combine passes a real combine, the real one starts working until reaching the end of its real path. In Figure 5.1, the blue vehicles are the real combines and the red one is the virtual combine. Real combines 1, 2 and 3 have already started to move in the forward direction. Combine 4 will start its motion once the virtual combine passes it.

3. The grain cart which is the green vehicle shown in Figure 5.1 serves the harvesters following a round-robin strategy. The grain cart only serves the real combines. If its next target is virtual, it stays with the current combine and waits until a real combine is ready to be served.

The complement method can be applied to any non-rectangular fields for all proposed scheduling strategies. The required capacity of the combines is the same as that in a rectangular row . But in the operation, some of the combine capacity may not be used efficiently because in non-rectangular field, when then combines get served, they may not get filled yet. The efficiency of the method is $\frac{A_{real}}{A_{rectangle}}$ where A_{real} is the area of the non-rectangular field and $A_{rectangle}$ is the area of the rectangular field after complement method. Complement method provides a way for us to apply the proposed scheduling schemes in non-rectangular fields while the optimality is not maintained.

5.1.1 Optimal Depot Position in Non-rectangular Field

In the complement method, a general non-rectangular field can be divided to a stack of r rectangular rows with different lengths. We define the length of the field L_{field} as the distance between the left and right ends of the field. For any row i, the horizontal distance between the beginning of the row and the left end of the field is Lb_i. The distance between the end of the row and right end of the field is Le_i.

During the harvesting operation in each row, the grain carts go to the depot n_i times before reaching the end. We assume the horizontal distance is x as shown in Figure 5.2.
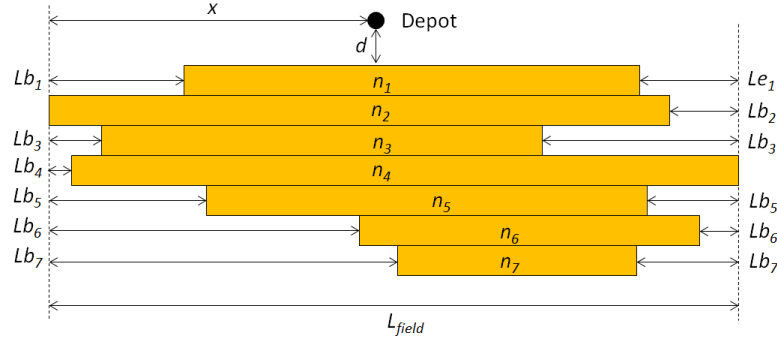
Figure 5.2: Non-rectangular field depot position

In this case, the travel distance of the grain cart to the depot for any single row i can be expressed as follows:

when i is odd :

$$
L_i = \begin{cases}
\sum_{k=1}^{n_i} \sqrt{(Lb_i - x + kL)^2 + (d + (i-1)w)^2} + \sqrt{(Lb_i - x + kL - a)^2 + (d + (i-1)w)^2} \\
(Lb_i + L \geq x) \\
\\
\sum_{k=1}^{\lfloor \frac{x - Lb_i}{L} \rfloor} \sqrt{(x - Lb_i - kL)^2 + (d + (i-1)w)^2} + \sqrt{(x - Lb_i - kL + a)^2 + (d + (i-1)w)^2} \\
+ \sum_{k=\lfloor \frac{x - Lb_i}{L} \rfloor + 1}^{n_i} \sqrt{(Lb_i - x + kL)^2 + (d + (i-1)w)^2} + \sqrt{(Lb_i - x + kL - a)^2 + (d + (i-1)w)^2} \\
(Lb_i + L < x < L_i - Le_i) \\
\\
\sum_{k=1}^{n_i} \sqrt{(x - Lb_i - kL)^2 + (d + (i-1)w)^2} + \sqrt{(x - Lb_i - kL + a)^2 + (d + (i-1)w)^2} \\
(x \geq L_i - Le_i)
\end{cases}
$$

when i is even :

$$
L_i = \begin{cases}
\sum_{k=1}^{n_i} \sqrt{(Lb_i - x + (n_i - k)L)^2 + (d + (i-1)w)^2} + \sqrt{(Lb_i - x + (n_i - k)L + a)^2 + (d + (i-1)w)^2} \\
(Lb_i + L \geq x) \\
\\
\sum_{k=1}^{\lfloor n_i - \frac{x - Lb_i}{L} \rfloor} \sqrt{(Lb_i - x + (n_i - k)L)^2 + (d + (i-1)w)^2} \\
+ \sqrt{(Lb_i - x + (n_i - k)L + a)^2 + (d + (i-1)w)^2} \\
+ \sum_{k=\lfloor n_i - \frac{x - Lb_i}{L} \rfloor + 1}^{n_i} \sqrt{(x - Lb_i - (n_i - k)L)^2 + (d + (i-1)w)^2} \\
+ \sqrt{(x - Lb_i - (n_i - k)L - a)^2 + (d + (i-1)w)^2} \\
(Lb_i + L < x < L_i - Le_i) \\
\\
\sum_{k=1}^{n_i} \sqrt{(x - Lb_i - (n_i - k)L)^2 + (d + (i-1)w)^2} + \sqrt{(x - Lb_i - (n_i - k)L - a)^2 + (d + (i-1)w)^2} \\
(x \geq L_i - Le_i)
\end{cases}
$$

Where w is the width of each row. L is the distance combines travel during one period they get filled. The total travel distance of grain cart for multiple rows is:

$$
L_{total} = \sum_{i=1}^{r} Li \tag{5.1}
$$

The type of the function highly depends on the shape of the field. The optimal position of depot can be found by solving:

$$
minimize\ L_{total}(x) \tag{5.2}
$$

$$
subject\ to\ 0 \leq x \leq L_{field} \tag{5.3}
$$

Figure 5.3: Sample field from google map

Figure 5.3 shows the top view of a corn field in Iowa. We assume that one Balzer's grain cart (1200 bushels capacity) and four John Deere's S650 combine harvesters (200 bushels capacity) are used to

execute the harvesting operation from the north edge. The header width of the combine is 12m which means the average width of a row is around 50m ($w = 50$). Based on USDA's report, the average corn production in U.S. is 171 bushels per acre in 2014. So each combine needs to travel 400m to fill its tank($L = 400$). By solving $L'_{total}(x) = 0$, we can find the optimal depot position is 890 meters from the west edge of the field, which is located at the cross shown in Figure 5.3.

CHAPTER 6.   SUMMARY AND FUTURE WORK

In this section, we summarize the problems we have addressed in this work as well as future research direction.

6.1   Summary

In this work, we focused on the scheduling and path planning problem of agricultural vehicles engaged in harvesting operation. Initially, we give a detailed analysis of the scenario when there is a single serving all the combines. The parameters of the operation including the number of the vehicles, the harvesting and unloading rate and the load capacity as well as the traveling time to the depot are taken into consideration. An expression that shows the relationship of all the parameters is presented. With the function, we can plan the harvesting operation by varying any parameter while keep others as constant. Furthermore, we analyze the depot position in both rectangular field and non-rectangular field. When there is only one row, the optimal position of the depot is obtained so that the total traveling distance of the grain cart to the depot is minimized. When there are multiple rows, the optimal position of the depot varies based on the number of rows. We find the region in which the depot will locate and propose the method to solve the location numerically based on the convexity of the function of the total travel distance. After that, we present the primitive-based path planning for the grain cart to move between combines based on the numerical approach proposed by Mengzhe in his previous work.

In Chapter 4, we generalize the scheduling strategy to the case when there are multiple grain carts serving all the combine harvesters. In this case, we assume the number of grain carts is fewer than the number of the combines. Two classes of strategy are presented: round-robin scheme and partitioning scheme. In round-robin scheme, all the grain carts work together to serve the combines following FCFS policy. Two variants of the scheme are presented based on the frequency of the grain carts unloading

themselves. A priority based collision avoidance method is proposed so that the grain carts will not collide with each other during the motion. In partitioning scheme, the combines are divided into small groups so that each grain cart takes care of one group on combines. The scheme is divided into two variants based on if the number of vehicles in each group is integer or not. In load balancing with integral constraint (LBIC), each group of combines are served individually by a grain cart. In load balancing without integral constraint (LB/IC), all the grain carts have exactly the same amount of load to serve which means that there are combines shared by groups. Two important propositions are presented which indicate that only 1 combine is shared between two adjacent groups and the groups will have different number of combine only if $\frac{N-1}{M}$ is not an integer. The performance of the two schemes are analyzed and compared. Then We extend the scheduling scheme to non-rectangular field with complement method as well as the case when the grain cart is big enough so that a depot is not necessary. An application of the proposed seduling schemes with BOE-bots in the Vicon environment is shown in the end which indicates the feasibility of the strategies.

6.2   Future Work

6.2.1   Uncertainty of the Field

In this work we assume that the combine harvesters harvest with a constant rate r_f. However, in real operation the harvesting rate can be affected by many factors such as varying moving speed and grain density of the field. In this section, we discuss the system performance with uncertainties.

In this case, we consider there is only one grain cart starting from a depot to serve a group of combines. We can model the serving system as a queue. The grain cart is a station and the combines are customers who enter the station with an arriving rate to get served. To analyze the system with queuing theory, we need to determine the customer arriving rate and the serving rate of the station.

Instead of a determinate constant r_f, we assume the harvesting rate follows a continuous uniform distribution on the interval $[r_{fmin}, r_{fmax}]$ with an expectation value $\bar{r}_f$. This is a reasonable assumption since a combines can always fill its tank in a finite time for sure. The range of the interval depends on the characteristics of the field and the vehicles. We know in a given field, a combine needs to travel a constant distance L to fill its tank once since the capacity of the tank C is a constant. For each unit

of time, the combine moves a distance $\frac{\bar{r}_f}{C}L$. So the expected time for it to fill the tank $\bar{T}_f = \frac{C}{\bar{r}_f}$ is also a uniform distribution on the interval $[\frac{C}{r_{fmax}}, \frac{C}{r_{fmin}}]$. Once a combine is filled, we say a customer arrives. Then the customer arriving rate can be defined as the reciprocal of the average time between two consecutive customers. For a group of harvesters, there is a constant time gap ΔT between any two adjacent combines start moving. The times any two consecutive combines, combine n and $n+1$, get filled are $(n-1)\Delta T + \bar{T}_f$ and $n\Delta T + \bar{T}_f$. We let the two time intervals are $[t_{1min}, t_{1max}]$ and $[t_{2min}, t_{2max}]$. So the time difference between customer arriving T_{gap} can be found as a standard triangular distribution with the probability density function:

$$P(T_{gap}) = \begin{cases} \frac{T_{gap}}{(T_{max}-T_{min})^2} - \frac{T_{2min}-T_{1max}}{(T_{max}-T_{min})^2} \\[2mm] (T_{2min} - T_{1max} \leq T_{gap} < T_{2min} - T_{1min}) \\[2mm] -\frac{T_{gap}}{(T_{max}-T_{min})^2} + \frac{T_{2max}-T_{1min}}{(T_{max}-T_{min})^2} \\[2mm] (T_{2min} - T_{1min} \leq T_{gap} \leq T_{2max} - T_{1min}) \end{cases} \tag{6.1}$$

where $T_{1max} - T_{1min} = T_{2max} - T_{2min} = T_{max} - T_{min}$. The customer arriving rate $\lambda = \frac{1}{T_{gap}}$ also follows the same standard triangular distribution.

Proof. Let T_1 and T_2 be independent random variables following uniform distribution on interval $[T_{1min}, T_{1max}]$ and $[T_{2min}, T_{2max}]$. Let $T_{gap} = T_2 - T_1$. Then joint probability density function of T_1 and T_2 is:

$f_{T_1,T_2}(t_1, t_2) = \frac{1}{(T_{max}-T_{min})^2}$

Using the cumulative distribution function technique, the cumulative distribution function of T_{gap} is:

$$F_{T_{gap}}(t) = P(T_{gap} \leq t)$$

$$= P(T_2 - T_1 \leq t)$$

$$= \begin{cases} \int_{T_{2min}}^{T_{1max}+t} \int_{T_2-t}^{T_{1max}} \frac{1}{(T_{max}-T_{min})^2} dT_1 dT_2 \\[2mm] (T_{2min} - T_{1max} \leq t < T_{2min} - T_{1min}) \\[2mm] 1 - \int_{T_{1min}+t}^{T_{2max}} \int_{T_{1min}}^{T_2-t} \frac{1}{(T_{max}-T_{min})^2} dT_1 dT_2 \\[2mm] (T_{2min} - T_{1min} \leq t \leq T_{2max} - T_{1min}) \end{cases}$$

$$
= \begin{cases}
\frac{(T_{1max}+t)(T_{1max}+t-T_{2min})-\frac{1}{2}(T_{1max}+t-T_{2min})^2}{(T_{max}-T_{min})^2} \\
(T_{2min}-T_{1max} \leq t < T_{2min}-T_{1min}) \\
1 - \frac{\frac{1}{2}(T_{2max}-T_{1min}-t)^2-(t-T_{1min})(T_{2max}-T_{1min}-t)}{(T_{max}-T_{min})^2} \\
(T_{2min}-T_{1min} \leq t \leq T_{2max}-T_{1min})
\end{cases}
$$

Differentiating with respect to t yields the probability density function:

$$
P(t) = \begin{cases}
\frac{t}{(T_{max}-T_{min})^2} - \frac{T_{2min}-T_{1max}}{(T_{max}-T_{min})^2} \\
(T_{2min}-T_{1max} \leq t < T_{2min}-T_{1min}) \\
-\frac{t}{(T_{max}-T_{min})^2} + \frac{T_{2max}-T_{1min}}{(T_{max}-T_{min})^2} \\
(T_{2min}-T_{1min} \leq t \leq T_{2max}-T_{1min})
\end{cases}
$$

which is the probability density function of the standard triangular distribution. $\square$

In our future works, we plan to extend the results in this work to build a framework to address the problem of optimal placement and path-planning of farm vehicles in harvesting operations. The theoretical foundations of the framework will be based on ideas from queuing theory and game theory. The problem of harvesting multiple fields will also be addressed in the future for large scale farming operations. Based on the theoretical results, a planner will be developed for vehicles involved in automated harvesting operation.

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