

2015

# Sensitivity-based robust feedback linearizing control of hydraulically actuated system

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**Sensitivity-based robust feedback linearizing control of hydraulically  
actuated system**

by

**Hui Zhou**

A thesis submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
MASTER OF SCIENCE

Major: Mechanical Engineering

Program of Study Committee:  
Atul Kelkar, Major Professor  
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Ames, Iowa

2015

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## ACKNOWLEDGEMENTS

I would like to take this opportunity to express my gratitude to those people who helped me throughout my master study with every aspect of my research and my life. First and foremost, my major professor, Dr. Atul Kelkar, for introducing me to this interesting topic and giving me many valuable instructions and guidance about my research and thesis writing. Besides, I would like to thank all my committee members for their willingness and help in serving my committee. Moreover, I would like to thank my senior Dr. Punit Tulpule. He gave me numerous suggestions, instructions and insights, which makes my research more smoothly. And I learned a lot from him about this field of research also about how to conduct a research. I would also like to thank Jonh Deere for sponsoring this project.

In the end, I would like to thank my family for giving me love and encouragement. Especially my brother, thank you for supporting my tuition which gives me a lot of courage to finish my research.



## ABSTRACT

Feedback linearization is an effective controller-design methodology for nonlinear systems where it is difficult to obtain a finite number of operating points to linearize the system for designing well-known linear robust controllers. Feedback linearization becomes one of very limited methodologies that can be used for control of such systems. Traditional implementations of feedback linearization technique are not robust, which means this control methodology does not account for system uncertainties. The reason being that the control law methodology assumes accurate knowledge of nonlinear dynamics of the system. Recently, in [1] a new methodology was proposed which adds robustness to feedback linearization. The methodology uses sensitivity dynamics-based control synthesis. The methodology was demonstrated on a simple proof-of-concept single actuator mass-spring-damper model.

This research is focused on application of robust feedback linearization technique to real life complex hydraulically actuated physical systems. In particular, the methodology is applied to the problem of controlling mechanical linkage configuration in excavator machines. The problem addressed is controlling of bucket angle of excavator such that the bucket is always kept parallel to ground irrespective of boom motion to avoid spilling of the load. The dynamics of systems such as excavator linkage actuated by hydraulic actuator are often complex and application of robust feedback linearization (RFL) methodology gets tedious and cumbersome. The work in this thesis is intended for demonstrating the applicability of RFL methodology for such complex systems and also to lay foundation for development of an automated user-friendly toolbox to enable easy use of such control technique in day-to-day practice. The uncertain parameter con-

sidered in the development in this thesis is the bulk modulus of the system as it is the most common uncertainty in the system. The modeling process also considers portability of models from some known commercial software tools such as SimHydraulics and SimMechanics.

The results presented show that the RFL methodology is very effective in achieving robust control of hydraulically actuated systems with uncertainties in hydraulic parameters.

## CHAPTER 1. INTRODUCTION

This thesis presents the application of a new nonlinear system control methodology to a complex real-life excavator system model. This control methodology namely, Robust Feedback Linearization (RFL) was developed recently by Tulpule as a part of his doctoral thesis work [1]. In his thesis [1], he proposed the RFL techniques and demonstrated this theory using a proof-of-concept nonlinear hydraulic system with a simple linear mass spring damper plant model. The results showed that RFL can improve the robustness of the system. However, because of some assumptions and approximations used during the derivation of equations of the motion of the system, the applicability of the methodology to complex systems needed further exploration. One of the reasons being that not only the hydraulic system models are highly complicated but also the methodology needs tedious complex computation, which can become very challenging to implement it on high fidelity physical models. This thesis uses an excavator system as an example to develop systematic process to implement RFL methodology on real-life systems with complex dynamics. With the advances in computer simulation technologies, several physical systems simulation software have emerged over last few decades. Some examples of software that are commonly used in industry for simulation of physical systems include Simscape, SimMechanics, AMESim, SimHydraulics, etc. Software such as Easy5 and SimHydraulics specialize in modeling of hydraulic systems in particular whereas others like SimMechanics are focused primarily on multibody (multi-link) systems modeling.

For designing control system and verifying control performance in simulation the software like Matlab/Simulink are widely used. Typically, user faces challenges in building

high fidelity simulation of complex system such as agriculture or construction machines which require use of multitude of different software which have to be combined to work in co-simulation environment. In addition, if one has to implement new modeling and/or control design strategies, compatibility of new methodology with existing simulation environment becomes a big hurdle.

In this work, an attempt is made to systematize the design of robust feedback linearizing control of hydraulically actuated systems based on sensitivity theory. A user friendly graphical user interface is also developed to facilitate the use of advanced control design methodology by relatively less experienced designer.

The rest of this chapter presents some prerequisite background information on sensitivity theory which is at the core of robust design methodology, its application to robust feedback linearization, excavator system under consideration, and automatic bucket leveling problem used for demonstrating control methodology developed.

## 1.1 Sensitivity

In the context of this research, sensitivity can be defined as the percentage change in the performance objective as a result of unit perturbation in design variables. The notion of sensitivity has been widely used in variety of engineering applications. Some example usage of sensitivity include: performance optimization, sensitivity optimization and robustness analysis [2]. Performance optimization using sensitivity is described in [3]. Another application of minimum sensitivity is quality control [4]. Sensitivity methods are also used in optimal structure design [5].

One of the seminal papers that concern with sensitivity in control systems is listed in reference [6]. A brief literature review about sensitivity research in the 1960s and 1970s is compiled in [1]. There are two types of optimization using sensitivity analysis approaches: deterministic approaches and probabilistic approaches [7]. These two approaches use

different methods to measure sensitivity of a design. Based on these two approaches, a handful of literature about single-objective optimization problem was reported [8]. In practice, most of the optimization problems tend to be multi-objective and a large body of research focused on multi-objective optimization [9].

It is to be noted that most of the optimization methodologies can help create robust design over a range of uncertain parameters, where designers have to estimate the upper bound of the uncertainty. A number of research studies provide robust optimal design considering estimation of uncertainty bounds [10] [11]. In Chapter 2, additional background and related discussion on sensitivity analysis is provided. Next section presents a prerequisite background material on robust feedback linearization.

## 1.2 Robust Feedback Linearization

Feedback linearization is a well known control design technique for nonlinear control systems. Essentially, this methodology involves first canceling the nonlinear dynamics of the system and rendering the system to be a set of pure integrators or in general a linear system, for which standard controller can be designed in the outer loop using linear techniques. For systems such as the one considered in this work, feedback linearization approach is most appealing as it is not possible to obtain a representative linearized model since there is no well defined operating point for servo controlled hydraulic actuators. Another big issue in such systems is presence of hard nonlinearity such as dead-band. Determining a practical operating point outside the dead band for hydraulic system is not possible [1]. Over the last two decades, feedback linearization has been used in many areas, including: power systems [12], helicopters [13], industry robots [14], [15], vehicles [16], [17], etc. The basic concept of feedback linearization theory is to eliminate the nonlinear part of the nonlinear system through inputs with proper structures and a transformation which can transform the system to an equivalent linear system. The

next step is to design a controller using linear techniques [18], like a PID controller or pole placement controller.

In spite of advantages of feedback linearization-based control law, there exists some shortcomings [19]: since feedback linearization methodology involves elimination of nonlinear dynamics using perfect known model of nonlinearity, it is inherently model dependent and lack robustness. The robustness analysis of feedback linearization therefore continues to be the topic of interest and presents interesting research challenges. There are two main categories of nonlinear plant control – conventional and soft computing methods. The conventional control theory uses the model-based method and the soft computing control theory, like fuzzy logic, uses the artificial intelligence methods [19]. Recently, a large body of research is concentrated on robustification of feedback linearization using techniques based on artificial neural networks or fuzzy logic [1]. The advantages of fuzzy uncertainty models and incorporating them into a feedback linearizing control design are discussed in [18], [20], [21]. Neural networks is another method used with feedback linearization [22], [23]. Some research also tries to combine feedback linearization with sliding mode control [24]. Another approach is min-max control which is in the sense of Lyapunov [25]. Adaptive feedback linearization is also a popular method to compensate for a nonlinear system [26]. Since adaptive control needs to use a linearized model for the plant, it can only provide local stability. In reviewing these literature above, some limitations are revealed, some methodologies need to estimate the uncertainty bound of systems, like fuzzy linearization and min-max approach [18], [20], [25].

The theory proposed in thesis [1], provides the least conservative design, because it does not require the knowledge of the uncertainty bound on the parameters [1], The theory developed uses sensitivity-based dynamics of the system to 'robustify' the feedback linearization. There are two feedback linearizing controller design methodologies: input-output linearization and state-space linearization [27]. In this thesis, an input-output framework is used as the hydraulic system models are primarily in input-output form.

Chapter 3 gives detailed development of robust feedback linearization for system under consideration.

### 1.3 Application to Hydraulic System

Electro-hydraulic systems have been widely used in industrial applications because of their high power-to-weight ratio, high stiffness, high payload capability, low cost, etc [28], [29]. These features are attractive for some high power requirements in agriculture machines and construction equipments, like combine harvester, tractor plough, hydraulic excavator, wheel loader, etc. Hydraulic systems are also extensively used in robot manipulators [15], [30]. Wind power plants are another critical application of hydraulic systems [31].

Although hydraulic systems are used extensively in practice, it is difficult to design an effective controller for hydraulic system, since the dynamic behavior of hydraulic systems is highly nonlinear [32]. These nonlinearities originate from two sources – the actuation subsystem and the loading subsystem [29]. The actuation subsystem’s nonlinearities are caused by nonlinear magnetic effects, chamber fluid compressibility [33], and nonlinear servo valve flow-pressure characteristics [28], [34], etc. Moreover, normally, loading systems are also with high nonlinearities, such as the application example in this thesis – hydraulic excavator, which consists of three main linkages. Also, there exist nonsmooth and discontinuous nonlinearities in the systems. These are caused by control input saturation, direction changes in the valve opening, and valve overlap [35]. In addition to these nonlinearities sources, hydraulic systems have a large body of model uncertainties caused by parameter uncertainties [35]; for example, bulk modulus, density, pump pressure, control volume between a servo valve and a cylinder etc. There is a wide variation in the effective bulk modulus [32], [36], which depends on the amount of air dissolved in the fluid. 2% dissolved air by volume can cut the bulk modulus by over 50%, which has

a significant impact on the system performance. Therefore, the excavator application considered here, uncertain parameter considered is the bulk modulus of the hydraulic oil and the control design objective is to ensure that the closed-loop system stability and performance is robust to bulk modulus variation.

As stated previously, linearization of hydraulic system dynamics is difficult due to continuum of operating points and one has to resort to effective nonlinear control design techniques such as feedback linearization. [17], [37], [38] present applications of feedback linearization to control of hydraulic systems. However, since feedback linearization is not inherently robust, the controller stability and performance is often affected by uncertain parameters in hydraulic systems nonlinear dynamics. As a result, there has been considerable research on intelligent control application to hydraulic system control. Back-stepping is another useful nonlinear control technique that is based on the Lyapunov theory and guarantees asymptotic tracking, but finding an appropriate candidate Lyapunov function can be challenging [15], [39]. Sliding mode control is another approach that can be considered for controlling uncertain hydraulic systems [40], but it is discontinuous. Another potential control approach is adaptive control [23], [28], [35], [41]. In the work of this thesis the focus is on development of robust feedback linearizing control strategy and its implementation on complex system - an excavator model. As stated before, this methodology does not need uncertainty estimates and therefore becomes attractive for excavator application. The computational platform used for this research is Matlab/Simulink. SimHydraulics and AMESim toolboxes were also used to validate the models and to determine the equations of motion needed for determining sensitivity dynamics.

### 1.3.1 Excavator Bucket Angle Control

The problem that is used in this work to demonstrate the application of robust feedback linearizing control approach is the bucket angle control in excavator machines.



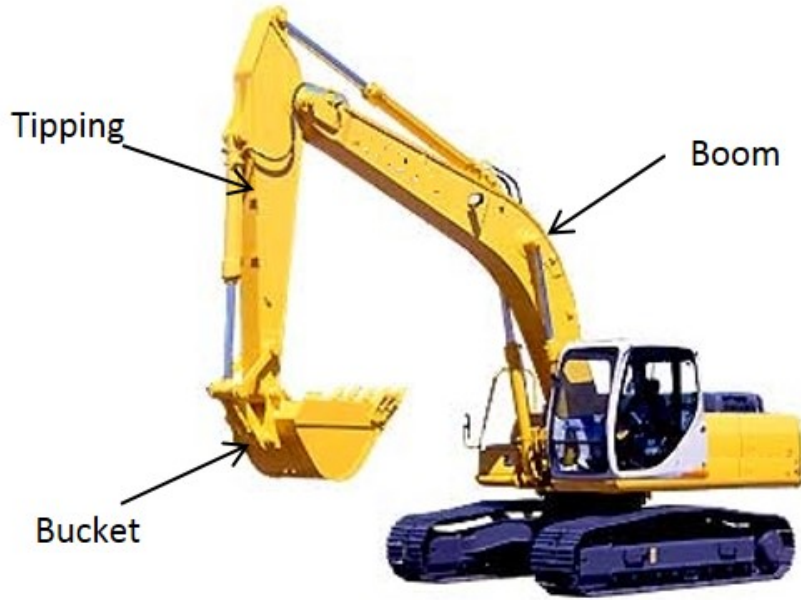


Figure 1.1 Excavator Linkage

The model used is the same as that in [1], which is obtained from AMESim excavator demo. A picture of a typical excavator is shown in Fig. (1.1). As shown in the figure, the excavator front linkage mechanism consists of three main linkages, which are boom, tipping stick and bucket, and a rotating platform. The hydraulic system model used in this work is for a double-acting cylinder and 4-ways directional valve. The hydraulic fluid used is Skydrol LD-4 with system temperature at 60 degrees Fahrenheit. There are several different sizes of excavators in practice. The smallest one produced by John Deere weighs 4173 pounds and has 14.8 hp. On the contrary, the largest one weighs in excess of 185876 pounds, has 532 hp.

Excavators are indispensable equipments in construction areas. Several tasks can be executed through excavators very efficiently, for example, earth removal, level digging work [42], [43] and straight line motion [44], [45]. Generally, skillful operators are required to operate fairly complex piece of machinery like excavators. Due to the time needed for training a skillful operator, the risks they have to run in various high dangerous environment and the increasing labor price, it is very important to make hydraulic

excavator more automatic. Another advantage of autonomous excavators is machine performance enhancement [46]. Level digging work and straight line motion are very common works performed by excavators. There are a large amount of literature about excavator dynamics and control.

In [47], fuzzy logic control is used for excavator system with the objective of better energy distribution and higher fuel economy. Teleoperated excavators have also been developed to reduce the risks of working in dangerous environments [48]. A brief literature review about excavator automation control is listed in [49]. In [1] integrated robust optimal design (IROD) methodology was developed for a design of robust optimal controller for planar mechanisms [1], [50], [51]. The result shows that IROD methodology performs better in many aspects than the state of the art control synthesis methods. The IROD methodology was demonstrated on nonlinear mechanical linkage systems and has been proven to be very effective in improving robustness. This thesis extends the IROD design methodology to include nonlinear hydraulic system by enabling sensitivity-based robust feedback linearization control of hydraulic actuator.

Because of inherent hard nonlinearities in hydraulic systems, the equations of motion are complex. Moreover, determining sensitivity dynamics for such systems becomes a very tedious computation. In particular, computing sensitivities in numerical form can lead to very slow computations and potential for errors. Even with good computers the computational efficiency can be a hurdle to overcome. In the excavator bucket leveling problem there are two hydraulic actuators. One pump was modeled which supplies fluid to both actuators. The higher fidelity models were developed using SimHydraulics and SimMechanics toolboxes in Matlab. The robust feedback linearization process developed in this work was implemented for these models.

## CHAPTER 2. SENSITIVITY

This chapter gives an overview of sensitivity and basic formulation of sensitivity dynamics for a general nonlinear system. The process of determining sensitivity equations is then demonstrated by a simple numerical example. A particular focus is on the sensitivity of system performance to change in parameter values.

### 2.1 Parametric Sensitivity

Parametric sensitivity describes how a system performance changes due to change in system parameter. Mathematically, it is defined as the percentage change in performance function caused by a unit change in the parameter. The basic idea behind robust control design is to minimize the parametric sensitivity function so that system performance variation due to change in parameter values is minimized. In order to achieve this objective the control design process uses the dynamics of system combined with its sensitivity dynamics and the performance function to be minimized contains some kind of energy terms in system states which include sensitivity states as well. Given below is basic formulation of sensitivity dynamics for a generic function.

Consider a generic function,

$$y = f(\vec{x}, \vec{b}) \tag{2.1}$$

where,  $\vec{x}$  is a vector of states, and  $\vec{b}$  is a vector of uncertain variables. According to the definition, the sensitivity  $y$ , with respect to the uncertain variables,  $\vec{b}$ , can be defined as

$$\frac{d\vec{y}}{d\vec{b}} = \frac{d\vec{f}}{d\vec{b}} + \frac{d\vec{f}}{d\vec{x}} \frac{d\vec{x}}{d\vec{b}} \quad (2.2)$$

if  $f$  has dimension of  $n_f$ ,  $b$  has dimension of  $n_b$ , and the length of  $x$  is  $n_x$ . Then Eq. (2.2) becomes

$$\frac{d\vec{y}}{d\vec{b}} = \left[ \frac{d\vec{f}}{d\vec{b}} \right]_{n_f \times n_b} + \left[ \frac{d\vec{f}}{d\vec{x}} \right]_{n_f \times n_x} \left[ \frac{d\vec{x}}{d\vec{b}} \right]_{n_x \times n_b} \quad (2.3)$$

and thus,  $\frac{d\vec{y}}{d\vec{b}}$  has dimension of  $n_f \times n_b$  and it is the sensitivity of  $y$  with respect to  $b$ . Also, we can express the derivative as  $\frac{d\vec{y}}{d\vec{b}} = \vec{y}_b$ .

Now, consider another third-order nonlinear dynamic system,

$$\begin{aligned} \dot{x}_1 &= -ax_1 \\ \dot{x}_2 &= -bx_2 + k - cx_1x_2 \\ \dot{x}_3 &= \theta x_1x_2 \\ y &= x_3 \end{aligned} \quad (2.4)$$

where,  $a, b, c, k$  are known parameters, and let  $\theta$  be an uncertain parameter. The sensitivity of output  $y$  with respect to  $\theta$ , is denoted as  $y_\theta$  and is given by

$$y_\theta = x_{3\theta} \quad (2.5)$$

As shown in the equation above, in order to get  $y_\theta$ , we have to solve the sensitivity of states, which is

$$\begin{aligned} \dot{x}_{1\theta} &= -ax_{1\theta} \\ \dot{x}_{2\theta} &= -bx_{2\theta} - cx_{1\theta}x_2 - cx_1x_{2\theta} \\ \dot{x}_{3\theta} &= x_1x_2 + \theta x_{1\theta}x_2 + \theta x_1x_{2\theta} \end{aligned} \quad (2.6)$$

where  $x_1, x_2, x_3$  are state solutions and the only unknowns are the sensitivity dynamics states  $x_{1\theta}, x_{2\theta}, x_{3\theta}$ .

Now, the system dynamics Eq. (2.4) can be augmented by sensitivity dynamics Eq. (2.6) to obtain the following set of state equations.

$$\begin{aligned}
\dot{x}_1 &= -ax_1 \\
\dot{x}_2 &= -bx_2 + k - cx_1x_2 \\
\dot{x}_3 &= \theta x_1x_2 \\
\dot{x}_{1\theta} &= -ax_{1\theta} \\
\dot{x}_{2\theta} &= -bx_{2\theta} - cx_{1\theta}x_2 - cx_1x_{2\theta} \\
\dot{x}_{3\theta} &= x_1x_2 + \theta x_{1\theta}x_2 + \theta x_1x_{2\theta} \\
y &= x_3 \\
y_\theta &= x_{3\theta}
\end{aligned} \tag{2.7}$$

The above set of equations can be solved simultaneously for  $x$  and  $x_\theta$ .

In this case, the number of system states is 3 and there is only 1 uncertain parameter, thus the number of sensitivity states is  $3 \times 1 = 3$ . In general, if the system has  $n$  states and  $m$  uncertain parameters, the number of total states of the system augmented by sensitivity states is  $n + n \times m$ .

## 2.2 Initial Condition of Sensitivity States

To solve the state equations for sensitivity dynamics, one needs initial condition for sensitivity states. But question is how to define initial conditions of sensitivity states?

For example, for the system of Eq. (2.4), the initial conditions denoted as  $[x_1(0), x_2(0), x_3(0)]$  would be some constants. The initial conditions for the corresponding sensitivity states are derivatives of  $x(0)$  with respect to  $\theta$ , which is  $[\frac{dx_1(0)}{d\theta}, \frac{dx_2(0)}{d\theta}, \frac{dx_3(0)}{d\theta}]$ . Since  $x(0)$  is con-

stant, its derivative equals zero. And thus, the initial conditions of sensitivity states are zeros.

Intuitively, it seems logical that initial condition for sensitivity states would be zero since prior to start of any motion of the system change in the value of any parameter of the system would be not have any relevance and hence the sensitivity will be always zero at  $t=0$ .

## CHAPTER 3. SENSITIVITY BASED ROBUST FEEDBACK LINEARIZATION

The sensitivity theory that was briefly reviewed in the previous chapter can be used to design feedback linearizing control, that is robust to uncertainties and/or modeling errors. The basic theoretical foundation for this extension was laid in [1]. This chapter reviews the development of sensitivity-based robust feedback linearization formulation and control system architecture for implementing such control design. Section 3.1 presents traditional feedback linearization formulation followed by sensitivity dynamics equations and combined augmented system dynamics in Section 3.2. Following augmented system dynamics selected relevant theorems related to system properties and stability are also presented for the purpose of completeness. The robust control problem structure is then discussed and the control system block diagram is provided.

### 3.1 Feedback Linearization

Consider a single-input single-output nonlinear system affine in control as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{3.1}$$

where,  $x \in \mathfrak{R}^n$  is the state vector,  $u \in \mathfrak{R}^p$  is the vector of inputs, and  $y \in \mathfrak{R}^m$  is the vector of output. If the system Eq. (3.1) can be transformed into the structure :

$$\dot{x} = Ax + B\gamma(x)[u - \alpha(x)] \quad (3.2)$$

where  $A$  is  $n \times n$ ,  $B$  is  $n \times p$ , the pair of  $(A, B)$  is controllable, the functions  $\alpha : \mathfrak{R}^n \rightarrow \mathfrak{R}^p$  and  $\gamma : \mathfrak{R}^n \rightarrow \mathfrak{R}^{p \times p}$  are defined in a domain  $D \subset \mathfrak{R}^n$  that contains the origin, and the matrix  $\gamma(x)$  is nonsingular for every  $x \in D$  [27]. The system is defined as feedback linearizable. And the feedback linearizing control law is given by

$$u = \alpha(x) + \beta(x)v \quad (3.3)$$

where,  $\beta(x) = \gamma^{-1}(x)$  and then the resulting linear system is

$$\dot{x} = Ax + Bv \quad (3.4)$$

Once the system is rendered linear by feedback linearizing control law, one can use variety of control law designs available for linear systems. Note that, since state space model of a system is not unique, the transformation that transfers the original system model into Eq. (3.2) is not distinct. However, the transformation has to be invertible and continuously differentiable. Some more details about transformation requirements are introduced in [27].

Feedback linearization approach can be classified as *full-state linearization* and *input-output linearization*. Full-state linearization results in the system that has relative degree of  $n$ , which is same as the order of the system. On the contrary, input-output linearization results in a system that has relative degree smaller than the order of the system  $n$ . Since for the application of interest in this work input-output linearization is more suited, the next section presents formulation of input-output linearization.



### 3.1.1 Input-Output Linearization

Consider the system 3.1, and differentiate the output  $y$  with respect to time to yield

$$\dot{y} = \frac{dh(x)}{dt} = \frac{dh(x)}{dx} \frac{dx}{dt} = \frac{dh(x)}{dx} \times [f(x) + g(x)u] = L_f h(x) + L_g h(x)u \quad (3.5)$$

where,  $L_f h(x) = \frac{dh(x)}{dx} f(x)$ , is called the *Lie Derivative* of  $h$  with respect to  $f$ , and  $L_g h(x) = \frac{dh(x)}{dx} g(x)$ . If  $L_g h(x)$  is zero, the output time derivative,  $\dot{y}$ , will not depend on input  $u$ . Taking time derivative again one gets,

$$\ddot{y} = \frac{d(L_f h(x))}{dx} \times [f(x) + g(x)u] = L_f^2 h(x) + L_g L_f h(x)u \quad (3.6)$$

If  $\ddot{y}$  is still independent of  $u$ , one has to keep differentiating the output until the coefficient of  $u$  is not zero. That is,

$$y^{(\rho)} = L_f^{(\rho)} h(x) + L_g L_f^{(\rho-1)} h(x)u \quad (3.7)$$

Let us define  $\nu = y^{(\rho)}$ , then the linearizing feedback control  $u$  can be calculated as follows:

$$u = \frac{1}{L_g L_f^{\rho-1} h(x)} [-L_f^{\rho} h(x) + \nu] \quad (3.8)$$

The control law of Eq. (3.8) reduces the closed-loop system into a series of  $\rho$  integrators:

$$y^{(\rho)} = v \quad (3.9)$$

Thus, the original nonlinear system is transformed into a simple linear system. Once linearized as in Eq. (3.9), one can use any linear control techniques. The choice of controller could be as simple as classical PID type controller or pole placement controller. For the hydraulic system application under consideration a simple proportional controller is used to minimize the tracking error which is the performance of interest for a linearized bucket leveling excavator system. The closed loop system dynamics becomes:

$$\begin{aligned}\dot{x} &= f(x) + g(x)\left[\frac{1}{L_g L_f^{\rho-1} h(x)}[-L_f^\rho h(x) + K(w - y)]\right] \\ y &= h(x)\end{aligned}\tag{3.10}$$

$$v = K(w - y)\tag{3.11}$$

where,  $w$  is the reference signal and  $K$  is a high gain. The control system block diagram is shown below:

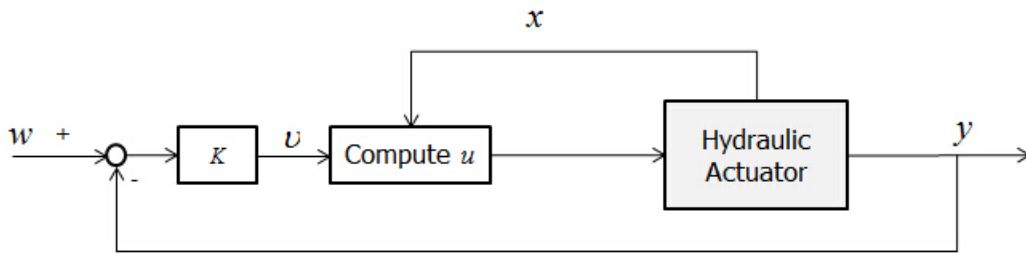


Figure 3.1 Feedback Linearization with Controller Block Diagram

### 3.2 Sensitivity Dynamics of Nonlinear System

As stated earlier, the feedback linearizing control law presented in previous section lacks in robustness as the control law is dependent on the accurate knowledge of the system for linearization to be valid. This section presents an approach developed in [1] to use sensitivity dynamics to improve the robustness of traditional feedback linearization.

In the development presented next an uncertain parameter considered is a scalar but the methodology is not restricted to only scalar parameter. The approach can be used for a vector of uncertain parameter as well, however, for the sake of algebraic simplicity a scalar example is used. Consider a system of Eq. (3.1) with uncertain parameter  $b$  and assume that the system is minimum phase:

$$\begin{aligned}\dot{x} &= f(x, b) + g(x, b)u \\ y &= h(x, b)\end{aligned}\tag{3.12}$$

According to the definition of sensitivity dynamics which is given in previous chapter, the sensitivity dynamics of system Eq. (3.12) is given by,

$$\begin{aligned}\dot{x}_b &= f_b(x, x_b, b) + g_b(x, x_b, b)u \\ y_b &= h_b(x, x_b, b)\end{aligned}\tag{3.13}$$

where,  $x_b \subset \mathfrak{R}^n$  is sensitivity dynamics states and  $y_b$  is output sensitivity.

Note that the feedback linearizing control law is dependent on system parameters. The original system augmented by sensitivity dynamics with parameter dependent control input becomes:

$$\begin{aligned}\dot{x} &= f(x, b) + g(x, b)u \\ \dot{x}_b &= f_b(x, x_b, b) + g_b(x, x_b, b)u_b \\ y &= h(x, b) \\ y_b &= h_b(x, x_b, b)\end{aligned}\tag{3.14}$$

where,  $u_b$  is sensitivity of control input. Now, the augmented system has states  $[x^T, x_b^T]^T$ , outputs  $[y, y_b]^T$  and two inputs  $[u, u_b]^T$ .

Some of the characteristics of the sensitivity dynamics and related stability properties presented in [1] are reviewed next in selected few theorems.

**Theorem 1.** *If the nonlinear system in Eq. (3.1) has relative degree  $r$  then the relative degree of sensitivity augmented system in Eq. (3.14) is  $[r, r]$ .*

**Theorem 2.** Consider the system in Eq. (3.14) and it has relative degree  $[r, r]$ , where  $r < n$  in  $D \in \mathfrak{R}^{2n}$ , then  $\forall \mathcal{X} \in D$ , there exists a neighborhood  $\mathcal{N} = N \oplus N_b$  of  $\mathcal{X}_0$  and smooth functions  $\phi_1(x, b), \phi_2(x, b), \phi_3(x, b), \dots, \phi_{n-r}(x, b)$  such that

$$L_g \phi_i(x, b) = 0, \quad \forall 1 \leq i \leq n-r \exists [x, x_b] \in \mathcal{N}$$

and the mapping

$$T = \begin{bmatrix} T_N \\ T_b \end{bmatrix} \quad (3.15)$$

restricted to  $\mathcal{N}$  is a diffeomorphism on  $\mathcal{N}$  where,

$$T_N(x, x_b, b) = \begin{bmatrix} \phi_1(x, b) \\ \phi_2(x, b) \\ \vdots \\ \phi_{n-r}(x, b) \\ \hline h \\ \vdots \\ L_f^{r-1} h \end{bmatrix} = \begin{bmatrix} \eta \\ \zeta \end{bmatrix} \quad (3.16)$$

and

$$T_b(x, x_b, b) = \begin{bmatrix} \phi_{1_b}(x, b) \\ \phi_{2_b}(x, b) \\ \vdots \\ \phi_{(n-r)_b}(x, b) \\ \hline h_b \\ \vdots \\ \frac{dL_f^{r-1} h}{db} \end{bmatrix} = \begin{bmatrix} \eta_b \\ \zeta_b \end{bmatrix} \quad (3.17)$$

**Theorem 3.** *If the zero dynamics given by  $f_0(\eta, 0)$  is asymptotically stable in a neighborhood  $D_\eta$  of operating point  $\eta_0$  then the sensitivity zero dynamics  $f_{0_b}(\eta, 0, \eta_b, 0)$  is also asymptotically stable at in a neighborhood of operating point  $[\eta^T, \eta_b^T]^T = [\eta_0^T, \eta_{b_0}^T]^T$  where*

$$\eta_{b_0} = \frac{\partial f_0(\eta_0, 0)}{\partial b} \left[ \frac{\partial f_0(\eta, 0)}{\partial \eta} \Big|_{\eta=\eta_0} \right]^{-1} \quad (3.18)$$

*In other words if system in Eq. (3.1) is minimum phase in domain  $D_\eta$  then the sensitivity augmented system in Eq. (3.14) is also minimum phase in domain  $D_\eta \oplus D_{\eta_b}$ .*

According to these 3 theorems, the sensitivity dynamics is linearized:

$$\nu_b = \frac{d}{db} L_f^\rho h(x) + \frac{d}{db} (L_g L_f^{\rho-1} h(x) u_b) \quad (3.19)$$

Thus,  $u_b$  can be calculated by Eq. (3.19), which is essentially same as equation Eq. (3.8). The same gain  $K$  can track the desired sensitivity dynamics efficiently, since as shown in above mentioned those 3 theorems, linearized sensitivity dynamics is also a set of  $\rho$  integrators. The desired sensitivity is zero, thus:

$$\nu_b = K(0 - y_b) = -K y_b \quad (3.20)$$

### 3.3 Robust Control Design

The last section discussed sensitivity augmented nonlinear system with feedback linearizing control and the objective is to minimize the system sensitivity so that the system is robust, i. e., and less sensitive to variation in parameters. Thus, the problem becomes an optimization problem. Unlike traditional optimization problem, the objective function in this case would be

$$J(u) = \frac{1}{2} \frac{(u(b_0) - u(b))^2}{b_0 - b} \quad (3.21)$$

The square term in the equation above keeps the function continuous and convex and also makes sure it has a global minimum. From Eq. (3.8) and Eq. (3.11), we can conclude that input  $u$  depends on output  $y$ . Therefore, when the input error is minimized, the output error is also minimized.

In the equation above, if  $b$  goes to  $b_0$ , then the objective function becomes

$$\lim_{(b_0-b) \rightarrow 0} \nabla_u J(u) = -u_b \quad (3.22)$$

The control input of sensitivity dynamics  $u_b$  is in the direction towards minimum sensitivity and a proper fixed step size  $K_2$  can be designed to stabilize the close system. Now, the term  $K_2 u_b$  becomes a control input correction and the sensitivity dynamics converges. The block diagram for the whole system with sensitivity dynamics and feedback linearizing control is shown in Fig. (3.2):

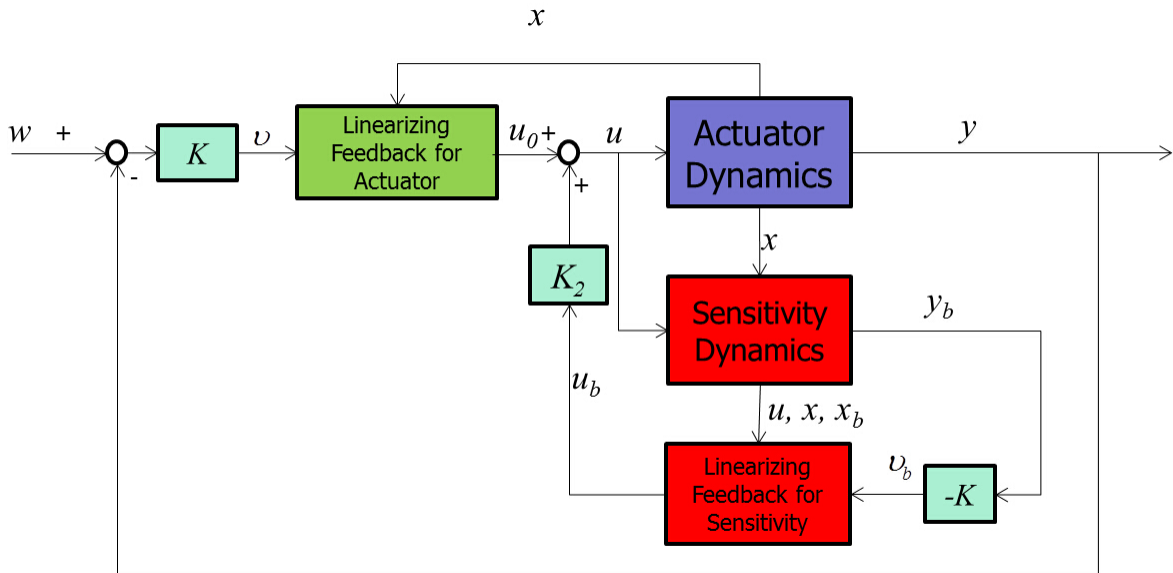


Figure 3.2 Robust Feedback Linearizing Control Structure

## CHAPTER 4. APPLICATION TO HYDRAULIC ACTUATORS

This chapter deals with the application of robust feedback linearization methodology to excavator bucket leveling control problem. The design objective is to keep the bucket leveled to the ground when the boom is moved up and down, so that the load in the bucket does not spill. The model of the hydraulic system is built using SimHydraulics toolbox in Matlab to be compatible with industry practice. For complex dynamic system under consideration a methodology is presented to circumvent the tedious algebraic computation and also describes how to connect the hydraulic actuator system dynamics with linkage system and design the hydraulic control system.

### 4.1 Excavator Robust Design Process

The robust design process flow is shown in Fig. (4.1). The first step is to build the excavator mechanical linkage system in SimMechanics. The next step is to accomplish integrated design with IROD, which generates optimal forces needed to be produced by the hydraulic actuators. IROD provides the reference for robust feedback linearizing control. Then, hydraulic system is built in SimHydraulics for the excavator. The next step in the design is the design of controller for hydraulic servo. In order to accomplish that a traditional feedback linearization is done and system is augmented with the sensitivity dynamics to enable robust design. The objective is to design a controller such that the control input follows closely the optimal input resulted from IROD design of linkage.

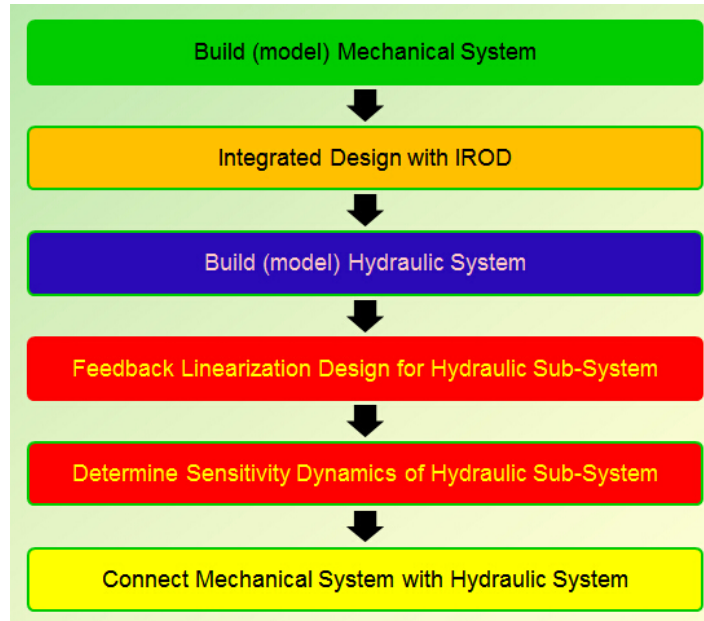


Figure 4.1 Robust Design Process

## 4.2 Excavator Mechanical Linkage System Design with IROD

The mathematical model excavator linkage mechanism used for integrated robust design was taken from AMESim. AMESim is another mechanical system modeling software widely used by industry. While AMESim and SimMechanics are competing software they have their own strengths and weaknesses and both are used in industry. In AMESim's excavator model there is no actuator model. This chapter focuses on development of hydraulic actuator models followed by robust feedback linearizing controller design and integration of linkage control system with hydraulic actuator control system to yield a complete closed-loop system. Furthermore, the process of robust feedback linearization design is systematized by developing a matlab based toolbox. Toolbox is described in more detail in the next chapter.



### 4.2.1 IROD

IROD combines the traditional sensitivity theory with relatively new advancements in Bilinear Matrix Inequality constrained optimization problems [1]. The problem considered is to obtain optimal control input concurrently with the optimal length of one of the links of the linkage to maintain the bucket levelled during commanded maneuver of the linkage. The IROD toolbox was used to accomplish this integrated design. The outcome of this result was determination of the optimal control input, i.e., hydraulic cylinder force profiles.

The optimal force profiles obtained from IROD are then used as the reference for the design of robust feedback linearizing controller. The high control system level block diagram is shown in Fig. (4.2). IROD's linearization utility was used to obtain linearized model of the excavator system.

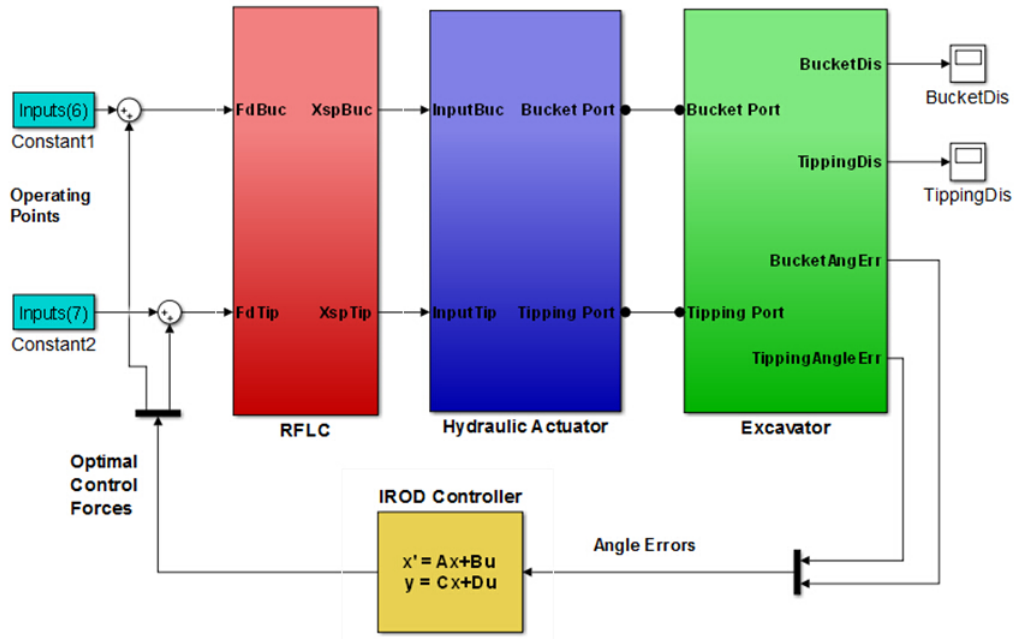


Figure 4.2 Excavator System Block Diagram

### 4.2.2 Mechanical Linkage System

The schematic diagram of the excavator model is shown in Fig. (4.3). Typically, in most excavator systems the operation of loading or unloading the bucket and digging are performed in manual mode. If one considers a typical task sequence, the operator digs and loads the bucket as much as possible then tilts the bucket following by lifting the boom, travelling to other location and then dumping the load. During all these motions operator tries to keep the bucket levelled to the ground to avoid spilling of the load. Intuitively, operator tends to not load the bucket fully to avoid spilling the load since the bucket tilt angle is controlled manually by the operator. This leads to loss of productivity longer times for the task and increased fuel cost.

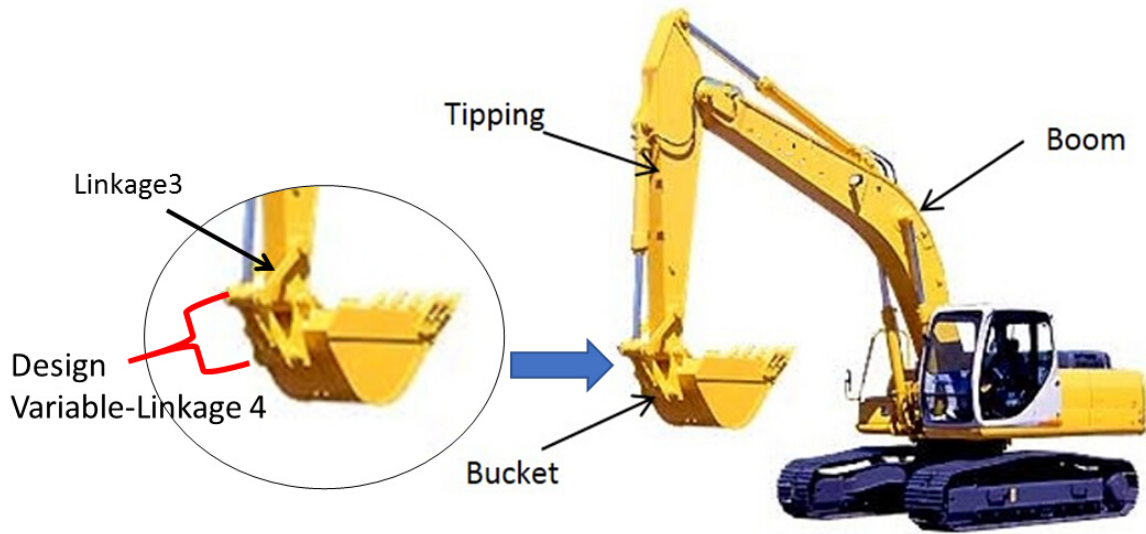


Figure 4.3 Excavator Schematic

The excavator parameters are shown in table 4.1. The length of linkage 4 is the design variable. Its original value was  $1.45m$  and its optimal length turned out to be  $1.97m$ . The control design challenge is how to design a controller that is robust to mass perturbations of the bucket as the load in the bucket is changing during operation. It was shown in [1] that IROD designs tend to perform better and are more robust over operating regime of the system than state-of-the-art  $\mathcal{H}_\infty$  controller.

Table 4.1 Excavator Parameters

Component	Variable	Description	Value
Boom	$m_{boom}$	Mass	8000 Kg
—	$I_{boom}$	Inertia	38500 Kg m <sup>2</sup>
Tipping	$m_{tip}$	Mass	2920 Kg
—	$I_{tip}$	Inertia	3600 Kg m <sup>2</sup>
Bucket	$m_{buc}$	Mass	4935 Kg
—	$I_{buc}$	Inertia	1900 Kg m <sup>2</sup>
Linkage3	$m_{lin3}$	Mass	168 Kg
—	$I_{lin3}$	Inertia	10 Kg m <sup>2</sup>
Linkage4	$m_{lin4}$	Mass	225 Kg
—	$I_{lin4}$	Inertia	13 Kg m <sup>2</sup>

### 4.3 Hydraulic Actuators

This section introduces hydraulic actuators applied on the excavator system mentioned in the previous section. The hydraulic actuator consists of a double-acting cylinder, one 4-way directional valve, one pressure relief valve and one pump, which are all very common components. Thus, hydraulic actuators in this thesis are very general. The schematic of hydraulic actuator is shown in Fig. (4.4). In this case, there are two controlled hydraulic actuators: the bucket and the tipping stick. They have different sizes of cylinders and cylinder parameters are provided in table 4.2. Both of these two 4-way directional valves have the same parameters, which are shown in table 4.3. Since the force capacity of the hydraulic actuator is based on diameters of piston and rod, and the pump pressure, designers can select feasible sizes of hydraulic actuators according to reference handbooks.

Table 4.2 Double-Acting Hydraulic Cylinder Parameters

	Piston Area A ( $A_i$ )	Piston Area B ( $A_o$ )	Piston Stroke ( $L$ )	Dead volume ( $V_0$ )	Contact Stiffness	Contact Damping
Bucket	0.0346m <sup>2</sup>	0.0213m <sup>2</sup>	1.33m	5 <sup>-5</sup> m <sup>3</sup>	10 <sup>9</sup> N/m	150N/(m/s)
Tipping	0.0254m <sup>2</sup>	0.0131m <sup>2</sup>	1.7m	5 <sup>-5</sup> m <sup>3</sup>	10 <sup>9</sup> N/m	150N/(m/s)

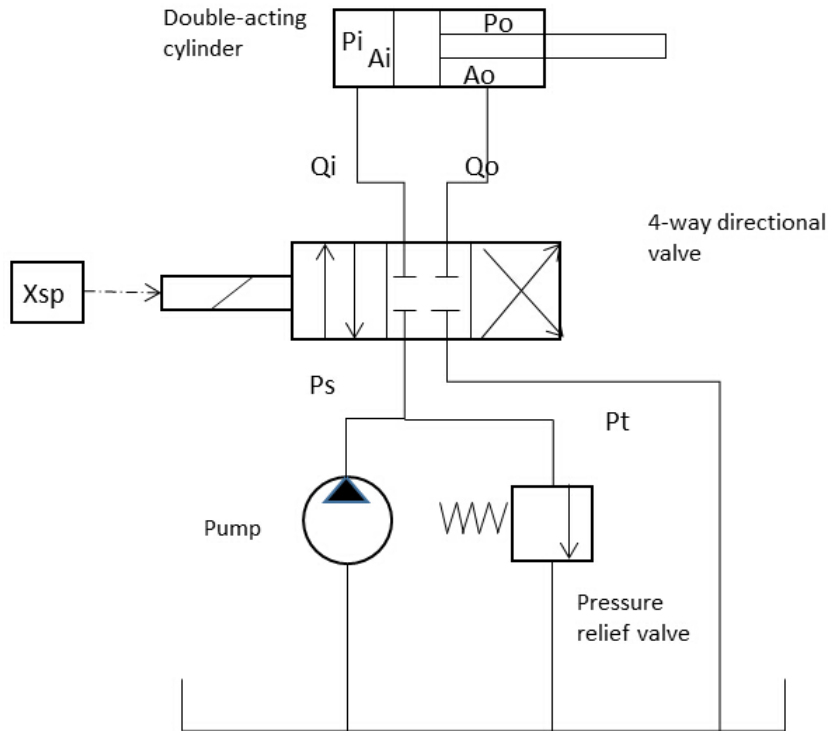


Figure 4.4 Hydraulic Actuator Schematic

## 4.4 Robust Feedback Linearization

After excavator mechanical system and hydraulic actuators are designed, the next step is to design robust feedback linearization for hydraulic sub-system. Firstly, we need to obtain the equation of motion for hydraulic systems. Then with the mathematical model, the next step is to design a traditional feedback linearizing control and test it on SimHydraulics model. Once the uncertain parameter of interest is chosen a sensitivity dynamics is derived and augmented plant is obtained. This section will go through a step-by-step process of the design.

### 4.4.1 Equation of Motion for Hydraulic System

The hydraulic actuator used in the excavator model consists of one 4-way directional valve and one double-acting cylinder, which are very widely used in industry. There

Table 4.3 Spool Valve and Fluid Parameters

Piston position ( $X$ )	---m
Piston velocity ( $\dot{X}$ )	---m/s
Spool valve position(control input)( $X_{sp}$ )	---m
Valve Passage Maximum Area ( $A_{max}$ )	0.0001m <sup>2</sup>
Valve Maximum Opening ( $h_{max}$ )	0.005m
Flow Discharge Coefficient( $C_d$ )	0.7
Critical Reynolds Number	12
Hydraulic Fluid	Skydrol LD-4
Density ( $\rho$ )	961.873Kg/m <sup>3</sup>
Pump Pressure ( $P_s$ )	3.2*10 <sup>7</sup> Pa
Tank Pressure ( $P_t$ )	0Pa
Bulk Modulus ( $\beta$ )	1.243×10 <sup>9</sup> Pa

are different types of combinations of valve and cylinder system used for actuation on agriculture and construction machines. One challenge with the commercially available modeling software like SimHydraulics or Easy5, etc. is that user has no access to dynamic model of the hydraulic components used in the system. As such, it is difficult to develop sensitivity dynamics without having analytical model. In this work analytical models of hydraulic system were developed along with its sensitivity dynamics.

For simplification, those terms that account for leakage are ignored. Given below are the equations describing the hydraulic actuators under consideration.

$$\begin{aligned}\dot{P}_i &= \frac{\beta}{(A_i X + V_0)} \times (q_i - A_i \dot{X}) \\ \dot{P}_o &= \frac{\beta}{(A_o(L - X) + V_0)} \times (-q_o + A_o \dot{X})\end{aligned}\quad (4.1)$$

and,

$$\begin{aligned}q_i &= C_d \frac{X_{sp}}{h_{max}} A_{max} \sqrt{\frac{2}{\rho}} (\sqrt{P_s - P_i} \times sg(X_{sp}) - \sqrt{P_i - P_t} \times sg(X_{sp})) \\ q_o &= C_d \frac{X_{sp}}{h_{max}} A_{max} \sqrt{\frac{2}{\rho}} (\sqrt{P_o - P_t} \times sg(X_{sp}) - \sqrt{P_s - P_o} \times sg(X_{sp}))\end{aligned}\quad (4.2)$$

where,

$$sg(X_{sp}) = \begin{cases} 1, & X_{sp} > 0 \\ 0, & X_{sp} \leq 0 \end{cases} \quad (4.3)$$

The equations above are normal format for hydraulic actuators. Eq. (4.2) is about spool-valve dynamics and cylinder dynamics is Eq. (4.1). After plug Eq. (4.2) into Eq. (4.1), the state equations are obtained:

For,  $X_{sp} > 0$

$$\begin{aligned} \dot{P}_i &= \frac{-\beta(A_i\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_s-P_i}}{h_{max}})}{V_0 + A_iX} \\ \dot{P}_o &= \frac{\beta(A_o\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_o-P_t}}{h_{max}})}{V_0 + A_o(L - X)} \end{aligned} \quad (4.4)$$

$X_{sp} \leq 0$

$$\begin{aligned} \dot{P}_i &= \frac{-\beta(A_i\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_i-P_t}}{h_{max}})}{V_0 + A_iX} \\ \dot{P}_o &= \frac{\beta(A_o\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_s-P_o}}{h_{max}})}{V_0 + A_o(L - X)} \end{aligned} \quad (4.5)$$

In order to validate the accuracy of the analytical models developed these models were compared with SimHydraulics in simulation. For validation exercise a simple mass-spring-damper system was connected with the hydraulic actuator and test signals were given to compare the output of the system. The parameters of mass-spring-damper system is shown in 4.4.

The force applied on mechanical system is given by Eq. (4.6), which is also the system output equation. In this application, we assume there is no friction.

$$F = P_i \times A_i - P_o \times A_o \quad (4.6)$$

and,

$$ma = F - kx - b\dot{x} \quad (4.7)$$

Table 4.4 Mass-Spring-Damper Parameters

	Mass( $m$ )	Spring Constant ( $k$ )	Damping Ratio( $d$ )
Value	100 kg	100 N/m	100 N/(m/s)

As shown in Fig. (4.5) and Fig. (4.6), the plots, indicating piston displacement and forces applied on mechanical system, obtained from the derived analytical model are and those obtained from SimHydraulics model are almost identical. This verifies the accuracy of the analytical models developed.

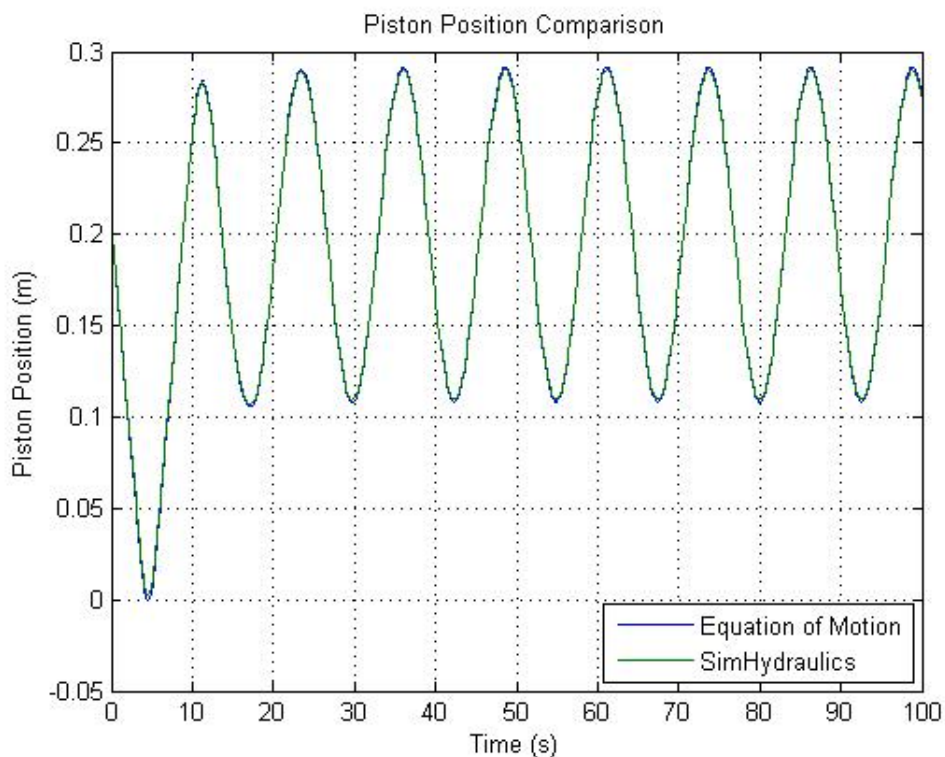


Figure 4.5 Piston Displacement Plots of EOM and SimHydraulics

#### 4.4.2 Design of Feedback Linearizing Control for Hydraulic Actuators

After developing the equations of motion for the hydraulic system, the next step is to design a traditional feedback linearizing control. Following the process of Chapter 3, the first step is to get the derivative of the output with respect to time:

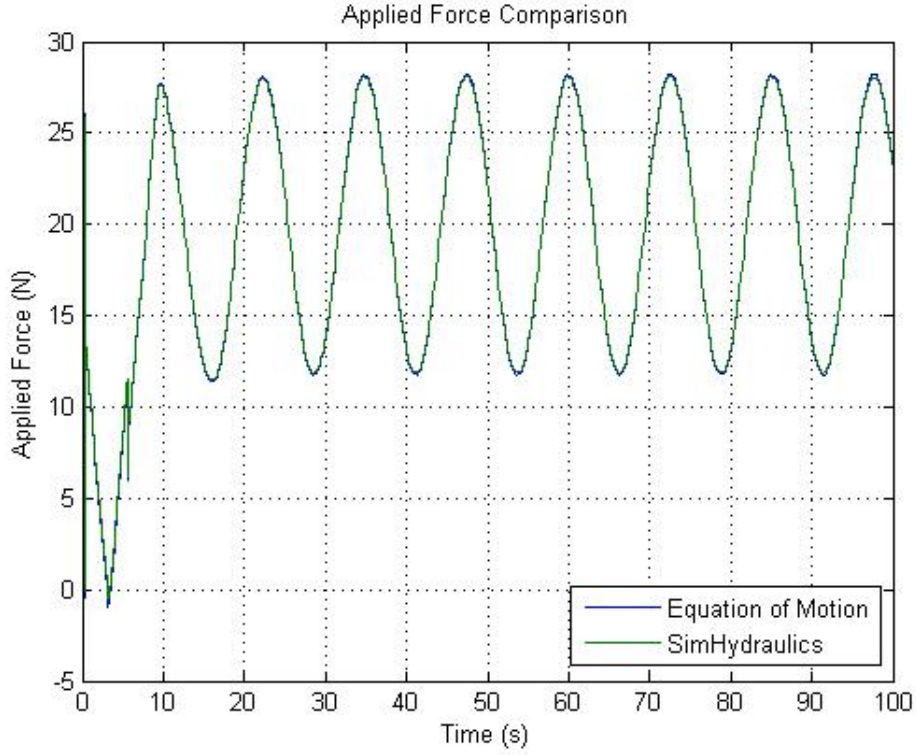


Figure 4.6 Force Plots of EOM and SimHydraulics

$$\dot{F} = \dot{P}_i \times A_i - \dot{P}_o \times A_o \quad (4.8)$$

If we plug state equations Eq. (4.4) or Eq. (4.5) (based on the sign of  $X_{sp}$ ) into Eq. (4.8), we can get:

for,  $X_{sp} > 0$

$$\begin{aligned} \nu = \dot{F} = & \frac{-A_i \beta (A_i \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_s - P_i}}{h_{max}})}{V_0 + A_i X} - \frac{A_o \beta (A_o \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_o - P_t}}{h_{max}})}{V_0 + A_o (L - X)} \\ = & \underbrace{-\frac{\beta ((A_i^2 + A_o^2) V_0 + A_i A_o X (A_i - A_o) + A_i^2 A_o L)}{(V_0 + A_i X)(V_0 + A_o L - A_o X)}}_{L_f h(x)} \\ & + \underbrace{\left( \frac{\sqrt{\frac{2}{\rho}} A_i A_{max} \beta C_d \sqrt{P_s - P_i}}{h_{max} (V_0 + A_i X)} + \frac{\sqrt{\frac{2}{\rho}} A_o A_{max} \beta C_d \sqrt{P_o - P_t}}{h_{max} (V_0 + A_o (L - X))} \right) X_{sp}}_{L_g h(x) Q} \end{aligned} \quad (4.9)$$



and,  $X_{sp} \leq 0$

$$\begin{aligned}
\nu = \dot{F} = & \\
& \frac{-A_i\beta(A_i\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_i-P_t}}{h_{max}})}{V_0 + A_iX} - \frac{A_o\beta(A_o\dot{X} - \frac{\sqrt{\frac{2}{\rho}}A_{max}C_dX_{sp}\sqrt{P_s-P_o}}{h_{max}})}{V_0 + A_o(L-X)} \\
= & \underbrace{-\frac{\beta((A_i^2 + A_o^2)V_0 + A_iA_oX(A_i - A_o) + A_i^2A_oL)}{(V_0 + A_iX)(V_0 + A_oL - A_oX)}}_{L_f h(x)} \\
& + \underbrace{\left(\frac{\sqrt{\frac{2}{\rho}}A_iA_{max}\beta C_d\sqrt{P_i-P_t}}{h_{max}(V_0 + A_iX)} + \frac{\sqrt{\frac{2}{\rho}}A_oA_{max}\beta C_d\sqrt{P_s-P_o}}{h_{max}(V_0 + A_o(L-X))}\right)X_{sp}}_{L_g h(x)Q}
\end{aligned} \tag{4.10}$$

In Eq. (4.9) and Eq. (4.10), we can see that the coefficient of input  $X_{sp}$  is not zero, which means the derivative of the output is dependent on the input  $X_{sp}$ . The relative degree of this system is 1 and the system is an input-output feedback system. According to Eq. (3.8), the feedback linearizing control in standard format is obtained,

$$X_{sp} = \frac{\nu - L_f h(x)}{L_g h(x)} \tag{4.11}$$

and for the specific excavator system, it would be

$$\begin{aligned}
X_{sp} = & \\
& \frac{\nu + \left(\frac{A_i^2\beta}{A_iX+V_0} + \frac{A_o^2\beta}{A_o(L-X)+V_0}\right)\dot{X}}{\beta C_d \frac{X_{sp}}{h_{max}} A_{max} \sqrt{\frac{2}{\rho} \left(\frac{A_i}{A_iX+V_0} \sqrt{P_s-P_i} + \frac{A_o}{A_o(L-X)+V_0} \sqrt{P_o-P_t}\right)}}, X_{sp} > 0
\end{aligned} \tag{4.12}$$

or,

$$\begin{aligned}
& \frac{\nu + \left(\frac{A_i^2\beta}{A_iX+V_0} + \frac{A_o^2\beta}{A_o(L-X)+V_0}\right)\dot{X}}{\beta C_d \frac{X_{sp}}{h_{max}} A_{max} \sqrt{\frac{2}{\rho} \left(\frac{A_i}{A_iX+V_0} \sqrt{P_i-P_t} + \frac{A_o}{A_o(L-X)+V_0} \sqrt{P_s-P_o}\right)}}, X_{sp} \leq 0
\end{aligned}$$

From Eq. (4.12), we can see that there is discontinuity caused by the sign of  $X_{sp}$ . In Simulink, the switch block is able to make this discontinuity a continuity and thus we need to set one criterion for the switch block. The denominators of Eq. (4.12) are always positive. Therefore,  $X_{sp}$  has the same sign with the numerator  $\nu + \left(\frac{A_i^2\beta}{A_iX+V_0} + \frac{A_o^2\beta}{A_o(L-X)+V_0}\right)\dot{X}$ , which can be the criterion for the switch block.

The feedback linearizing control linearized the original nonlinear system as one integrator. Since one integrator is simple, we can design one proportional controller for it. Theoretically, the higher value of the controller gain, the faster controller can track the reference. But hydraulic actuators, especially large ones, can not work at high frequency. Therefore, the controller gain should match the capacity of the hydraulic system. The gain selected was  $K = 10^5$ , in this case.

Controller designed using robust feedback linearization was tested in Simulink and Simscape. Simulink has a limitation on placing a Simscape physical network within a Simulink algebraic loop. Documentation about this issue is available in [52]. Since the control input  $X_{sp}$  is directly connected to the input of SimHydraulic model, there is an internal algebraic loop. Simulink's iterative algorithm to solve for constraint equations fails to find initial conditions with acceptable tolerance.

There are 3 possible ways to solve this problem according to [52]. The first approach is to replace the controller block by one Simscape network. But the control may not always be exactly represented by an exact physical system. Another solution is to add a unit delay block between the input signal and the SimHydraulics system. However, this method slows down the program significantly. Instead of adding a unit delay block, the most preferred way is to add one first order transfer function with a very slow time constant. The transfer function is in essence a continuous time delay which lets the variable solver take variable time steps unlike the discrete delay which has a sample time. Besides, it is able to filter out some noise. The transfer function is set up as

$$\frac{1}{0.0001s + 1} \tag{4.13}$$

with time constant as 0.0001s.

The desired control forces for tipping and bucket are shown in Fig. (4.7). Figures (4.8 and 4.9) show tracking error between desired forces and the actual forces. The tracking error is 3 orders of magnitude smaller than the force.

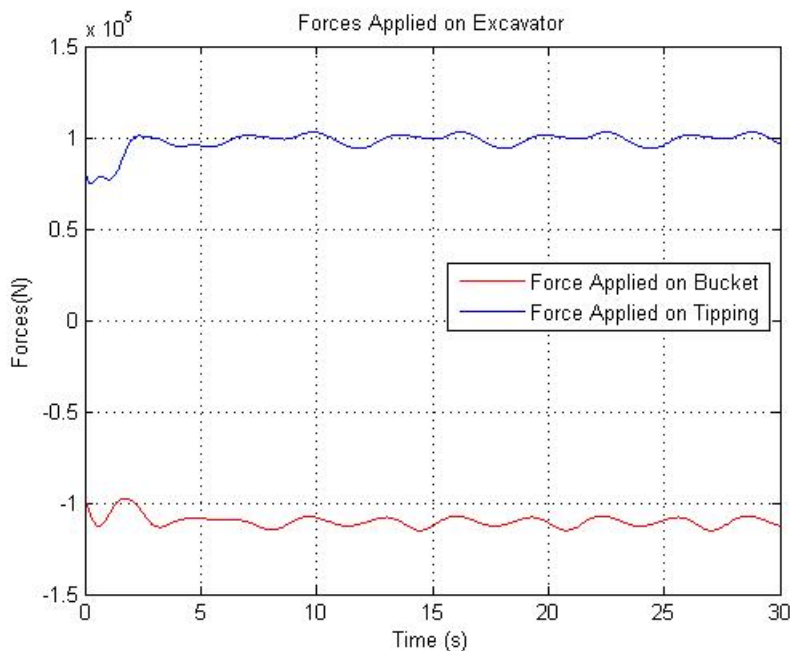


Figure 4.7 Forces Applied on Excavator

#### 4.4.3 Sensitivity Dynamics of Feedback Linearizing Control

Although, feedback linearization has good tracking performance, it is known that feedback linearization does not have robustness to parametric variations. In the following subsections, the sensitivity equations are derived for infinitesimal variations in bulk modulus of the hydraulic fluid. These equations are subsequently used to add robustness to the controller designed in previous sections.

Robustness to bulk modulus is important for hydraulic systems because bulk modulus is affected by many factors and has a wide range of variation. Bulk modulus is most sensitive to dissolved air. A small change of entrapped air can lead to a huge reduction in bulk modulus [32]. Besides dissolved air, liquid pressure, temperature and pipe rigidity can change bulk modulus significantly [53]. The variation of bulk modulus can reduce the system performance efficiency, increase disturbance and reduce machine life span [54]. In light of these adverse effects, it is significant to consider bulk modulus as the uncertain parameter in the design of robust feedback linearizing control. In present

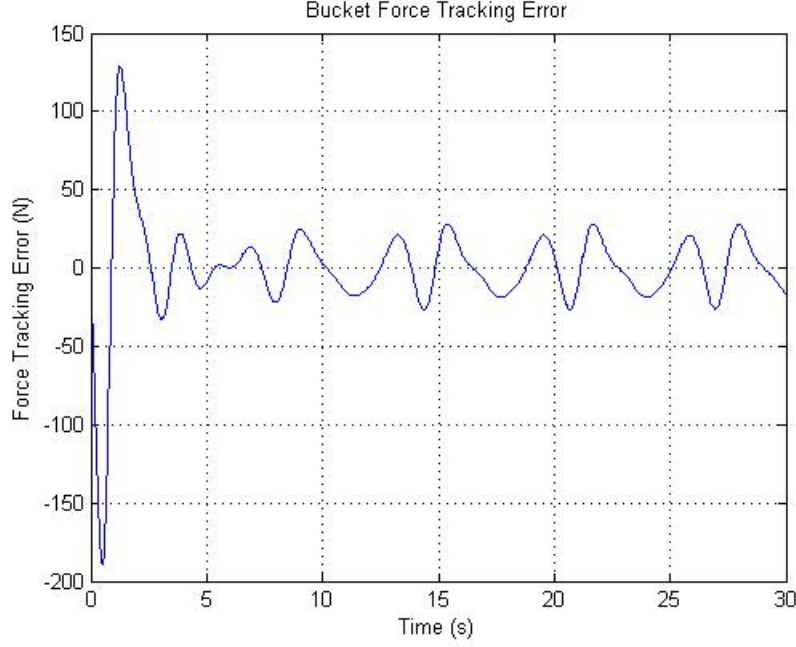


Figure 4.8 Bucket Force Tracking Error

study we have assumed single uncertain parameter, but the method can be extended for multiple uncertain parameters.

Sensitivity dynamics is obtained by differentiating state equations Eq. (4.4) and Eq. (4.5) with respect to bulk modulus. From hereon subscript  $b$  represents full derivative with respect to bulk modulus.

when  $X_{sp} > 0$

$$\begin{aligned}
 \dot{P}_{i_b} = & \frac{A_i \beta X_b (A_i \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_s - P_i}}{h_{max}})}{(V_0 + A_i X)^2} - \frac{A_i \beta \dot{X}_b}{V_0 + A_i X} \\
 & - \frac{A_i \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_s - P_i}}{h_{max}}}{V_0 + A_i X} + \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d X_{sp_b} \sqrt{P_s - P_i}}{h_{max} (V_0 + A_i X)} \\
 & - \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d P_{i_b} X_{sp}}{2 h_{max} \sqrt{P_s - P_i} (V_0 + A_i X)}
 \end{aligned} \tag{4.14}$$

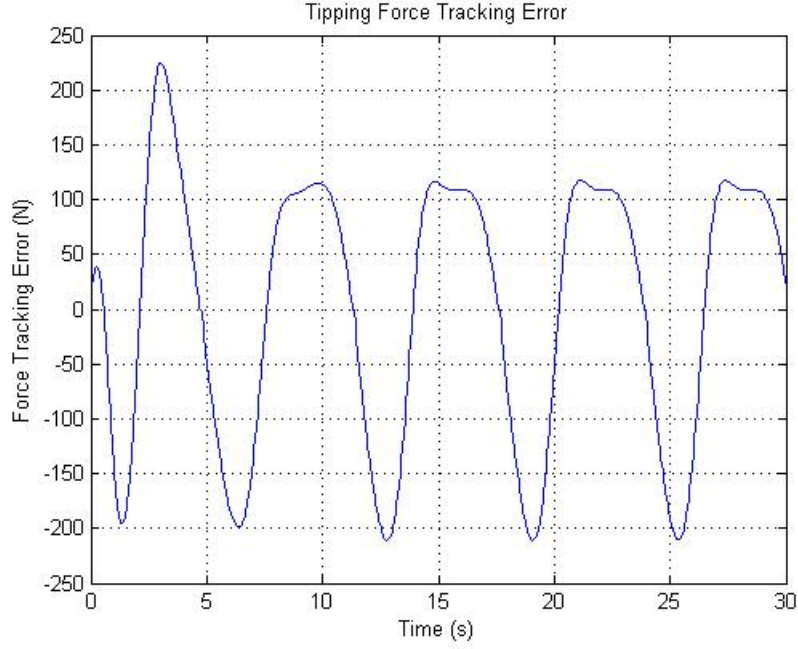


Figure 4.9 Tipping Force Tracking Error

$$\begin{aligned}
 \dot{P}_{o_b} = & \frac{A_o \beta X_b (A_o \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_o - P_t}}{h_{max}})}{(V_0 + A_o(L - X))^2} + \frac{A_o \beta \dot{X}_b}{V_0 + A_o(L - X)} \\
 & + \frac{A_o \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_o - P_t}}{h_{max}}}{V_0 + A_o(L - X)} - \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d X_{sp_b} \sqrt{P_o - P_t}}{h_{max}(V_0 + A_o(L - X))} \\
 & - \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d P_{i_b} X_{sp}}{2h_{max} \sqrt{P_o - P_t} (V_0 + A_o(L - X))}
 \end{aligned} \tag{4.15}$$

$$X_{sp} \leq 0,$$

$$\begin{aligned}
 \dot{P}_{i_b} = & \frac{A_i \beta X_b (A_i \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_i - P_t}}{h_{max}})}{(V_0 + A_i X)^2} - \frac{A_i \beta \dot{X}_b}{V_0 + A_i X} \\
 & - \frac{A_i \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_i - P_t}}{h_{max}}}{V_0 + A_i X} + \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d X_{sp_b} \sqrt{P_i - P_t}}{h_{max}(V_0 + A_i X)} \\
 & - \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d P_{i_b} X_{sp}}{2h_{max} \sqrt{P_i - P_t} (V_0 + A_i X)}
 \end{aligned} \tag{4.16}$$

$$\begin{aligned}
\dot{P}_{o_b} &= \frac{A_o \beta X_b (A_o \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_s - P_o}}{h_{max}})}{(V_0 + A_o(L - X))^2} + \frac{A_o \beta \dot{X}_b}{V_0 + A_o(L - X)} \\
&+ \frac{A_o \dot{X} - \frac{\sqrt{\frac{2}{\rho}} A_{max} C_d X_{sp} \sqrt{P_s - P_o}}{h_{max}}}{V_0 + A_o(L - X)} - \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d X_{sp_b} \sqrt{P_s - P_o}}{h_{max}(V_0 + A_o(L - X))} \\
&- \frac{\sqrt{\frac{2}{\rho}} A_{max} \beta C_d P_{i_b} X_{sp}}{2h_{max} \sqrt{P_s - P_o} (V_0 + A_o(L - X))}
\end{aligned} \tag{4.17}$$

The sensitivity of force  $F_b$  is obtained by differentiating the output Eq. (4.6) with respect to bulk modulus.

$$F_b = P_{i_b} \times A_i - P_{o_b} \times A_o \tag{4.18}$$

Note here, because the objective is to reduce the sensitivity to the variation of bulk modulus, the desired output sensitivity  $F_b$  is zero.

Eq. (4.14) to Eq. (4.18) are sensitivity dynamics of the system. The method explained in Chapter 3 Eq. (3.14) is used to obtain sensitivity of spool position  $X_{sp_b}$ .

The derivative of force sensitivity given in Eq. (4.18) with respect to time is:

$$\dot{F}_b = \dot{P}_{i_b} \times A_i - \dot{P}_{o_b} \times A_o \tag{4.19}$$

We need to plug Eq. (4.14) and Eq. (4.15) (or Eq. (4.16) and Eq. (4.17)) into Eq. (4.19) and define  $\nu = \dot{F}_b$ . According to Eq. (3.19), we can get:

for  $X_{sp} > 0$ ,

$$\begin{aligned}
\nu_b = \dot{F}_b = & \\
& \left\{ \frac{A_{max}A_oC_d\beta\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_d\beta\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \right\} X_{sp_b} \\
& - \left\{ \frac{A_o^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_o(L - X)} + \frac{A_i^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_iX} + \frac{A_o^3\dot{X}X_b\beta}{(V_0 + A_o(L - X))^2} \right. \\
& - \frac{A_i^3\dot{X}X_b\beta}{(V_0 + A_iX)^2} - \frac{A_{max}A_oC_dX_{sp}\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_dX_{sp}\beta P_{i_b}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_iX)\sqrt{P_s - P_i}} \\
& - \frac{A_{max}A_oC_dX_{sp}P_{o_b}\beta\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))\sqrt{P_o - P_t}} - \frac{A_{max}A_iC_dX_{sp}\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \\
& \left. - \frac{A_{max}A_o^2C_dX_bX_{sp}\beta\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))^2} + \frac{A_{max}A_i^2C_dX_bX_{sp}\beta\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)^2} \right\} \quad (4.20)
\end{aligned}$$

for  $X_{sp} \leq 0$ ,

$$\begin{aligned}
\nu_b = \dot{F}_b = & \\
& \left\{ \frac{A_{max}A_oC_d\beta\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_d\beta\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \right\} X_{sp_b} \\
& - \left\{ \frac{A_o^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_o(L - X)} + \frac{A_i^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_iX} + \frac{A_o^3\dot{X}X_b\beta}{(V_0 + A_o(L - X))^2} \right. \\
& - \frac{A_i^3\dot{X}X_b\beta}{(V_0 + A_iX)^2} - \frac{A_{max}A_oC_dX_{sp}\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} - \frac{A_{max}A_iC_dX_{sp}\beta P_{i_b}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_iX)\sqrt{P_i - P_t}} \\
& + \frac{A_{max}A_oC_dX_{sp}\beta P_{o_b}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_o(L - X))\sqrt{P_s - P_o}} - \frac{A_{max}A_iC_dX_{sp}\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \\
& \left. - \frac{A_{max}A_o^2C_dX_bX_{sp}\beta\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))^2} + \frac{A_{max}A_i^2C_dX_bX_{sp}\beta\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)^2} \right\} \quad (4.21)
\end{aligned}$$

If we rearrange Eq. (4.20) and Eq. (4.21), the expression of  $X_{sp_b}$  can be obtained as follows,

for  $X_{sp} > 0$ ,

$$\begin{aligned}
X_{sp_b} &= \frac{num}{den} \\
num &= \nu_b + \frac{A_o^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_o(L - X)} + \frac{A_i^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_iX} + \frac{A_o^3\dot{X}X_b\beta}{(V_0 + A_o(L - X))^2} \\
&\quad - \frac{A_i^3\dot{X}X_b\beta}{(V_0 + A_iX)^2} - \frac{A_{max}A_oC_dX_{sp}\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_dX_{sp}\beta P_{ib}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_iX)\sqrt{P_s - P_i}} \\
&\quad - \frac{A_{max}A_oC_dX_{sp}P_{ob}\beta\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))\sqrt{P_o - P_t}} - \frac{A_{max}A_iC_dX_{sp}\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \\
&\quad - \frac{A_{max}A_o^2C_dX_bX_{sp}\beta\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))^2} + \frac{A_{max}A_i^2C_dX_bX_{sp}\beta\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)^2} \\
den &= \frac{A_{max}A_oC_d\beta\sqrt{P_o - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_d\beta\sqrt{P_s - P_i}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)}
\end{aligned} \tag{4.22}$$

for  $X_{sp} \leq 0$ ,

$$\begin{aligned}
X_{sp_b} &= \frac{num}{den} \\
num &= \nu_b + \frac{A_o^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_o(L - X)} + \frac{A_i^2(\dot{X} + \dot{X}_b\beta)}{V_0 + A_iX} + \frac{A_o^3\dot{X}X_b\beta}{(V_0 + A_o(L - X))^2} \\
&\quad - \frac{A_i^3\dot{X}X_b\beta}{(V_0 + A_iX)^2} - \frac{A_{max}A_oC_dX_{sp}\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} - \frac{A_{max}A_iC_dX_{sp}\beta P_{ib}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_iX)\sqrt{P_i - P_t}} \\
&\quad + \frac{A_{max}A_oC_dX_{sp}\beta P_{ob}\sqrt{\frac{2}{\rho}}}{2h_{max}(V_0 + A_o(L - X))\sqrt{P_s - P_o}} - \frac{A_{max}A_iC_dX_{sp}\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)} \\
&\quad - \frac{A_{max}A_o^2C_dX_bX_{sp}\beta\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))^2} + \frac{A_{max}A_i^2C_dX_bX_{sp}\beta\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)^2} \\
den &= \frac{A_{max}A_oC_d\beta\sqrt{P_s - P_o}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_o(L - X))} + \frac{A_{max}A_iC_d\beta\sqrt{P_i - P_t}\sqrt{\frac{2}{\rho}}}{h_{max}(V_0 + A_iX)}
\end{aligned} \tag{4.23}$$

Since the desired  $F_b$  is zero and based on Eq. (3.20),

$$\nu_b = K(0 - F_{actual_b}) = -KF_{actual_b} \tag{4.24}$$

Here,  $K$  is the control gain used in feedback linearization. Note that sign of  $X - sp$  (not  $x_{sp_b}$ ) determines the switching in the spool valve equations.



In Eq. (4.22) and Eq. (4.23), the equation states are  $P_i, P_{ib}, P_o, P_{ob}, X_{sp}, X_{sp_b}$  which can be obtained by solving system and sensitivity equation.  $X, \dot{X}$  can be measured on-board from the excavator multibody system. In order to design controller for excavator multibody linkage (without hydraulics) we need to linearize the system at operating point. IROD toolbox was used to calculate the operating point for one set of angles of boom bucket and tipping. With this operating point, we can use Simulink linearization tool to obtain linear system. The linearized system is then differentiated with respect to the parameter  $\beta$  to get sensitivity augmented linear system and get  $X_b, \dot{X}_b$ .

As discussed in the previous chapter,  $X_{sp_b}$  gives direction towards minimum sensitivity. The step size  $K_2$  is selected such that the closed loop system remains stable, and the system converges to minimum sensitivity fast enough. Generally, larger  $K_2$  results in faster convergence, but if  $K_2$  is greater than a certain threshold the closed loop system becomes unstable. For these reasons  $K_2$  was chosen to be 100. This value was chosen based on experience because, as of now, there is no method to find optimal value of  $K_2$ .

In order to implement the sensitivity equations into Simulink environment, first the equations were derived using symbolic math toolbox. This toolbox facilitates automatic differentiation which removes the necessity of hand derived equations. Another advantage of symbolic computation is the analytic expression of equations reduces numerical errors.

MATLAB provides one command '*matlabFunctionBlock*', which converts the symbolic expression or function into a MATLAB function block in Simulink model. The equations are derived keeping symbolic variables for all parameters as well as states and inputs, and then numerical values for known parameters are substituted. Now, the equations have only state and input variables in symbolic form. These equations are then converted into MATLAB function block in Simulink model. This approach reduces human errors and facilitates automation of robust feedback linearization process.

Now, we need to connect SimHydraulics systems with the SimMechanical system of the excavator. The Fig. (4.10) shows the control signal flow diagram and Fig. (4.11) is

a Simulink layout about the hydraulic systems with robust feedback linearizing control, which we need to follow.

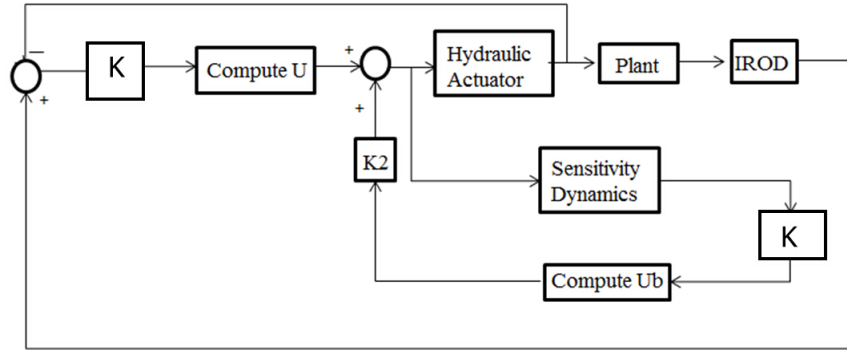


Figure 4.10 Excavator with Hydraulic Actuator Block Diagram

As shown in the excavator schematic diagram Fig. (4.3), 2 hydraulic actuators need to be controlled simultaneously. Both these actuators get power from one pump. The pump has to be able to satisfy the pressure requirements for both the actuators. In this study, a constant pressure source ( $P = 3.2 * 10^7 Pa$ ) is used as pump. In the future, pressure variations can also be considered as uncertain parameter

#### 4.4.4 Results

The input signal of the excavator system is the angle of boom, and it is selected as a sinusoidal one with frequency as 1 rad/sec and amplitude as 0.2m, which is very close to the actual conditions. The robust feedback linearizing control can track the optimized forces which are obtained from IROD, and also minimize sensitivity with respect bulk modulus.

Fig. (4.12) and Fig. (4.13) show the tracking errors comparison between robust design and nominal design for the bucket and the tipping stick respectively. The sensitivity with respect to bulk modulus for nominal design and robust feedback design are shown in

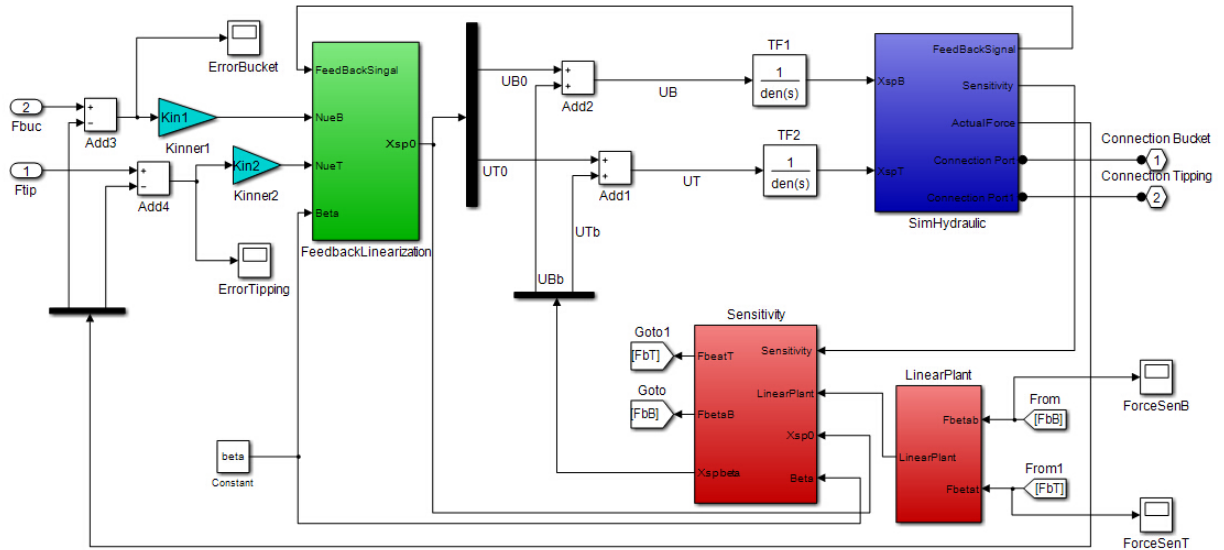


Figure 4.11 Hydraulic Systems with Robust Feedback Linearizing Control Simulink Diagram

Fig. (4.14) and Fig. (4.15) respectively. The plots show that robust feedback linearizing control makes the system less sensitive than normal feedback linearizing control. More importantly, higher robustness is achieved without any sacrifice in performance.

The RMS error in tracking performance and RMS force sensitivity to variation in bulk modulus are shown in Table 4.5 for both, nominal and robust design.

Table 4.5 Comparison between Robust and Nominal Feedback Linearization

	Robust	Nominal	Improvement
Bucket RMS Force Tracking Error	27.3892 $N$	27.3690 $N$	-0.05%
Bucket RMS Force Sensitivity	4.0750e <sup>-6</sup>	200.2318	99.9%
Tipping RMS Force Tracking Error	96.51 $N$	96.42 $N$	-0.02%
Tipping RMS Force Sensitivity	3.4599e <sup>-6</sup>	229.3474	99.9%

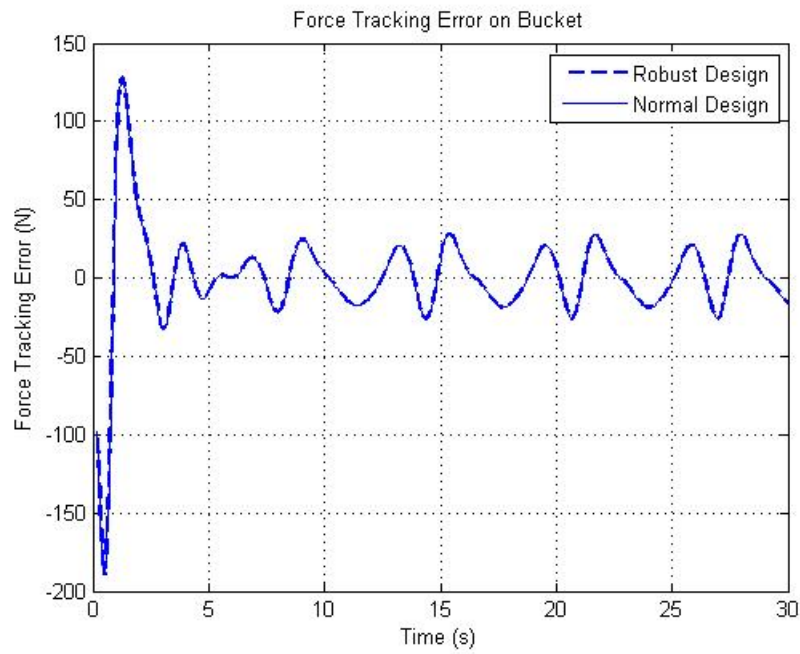


Figure 4.12 Tracking Error Comparison for Bucket Hydraulic Actuator

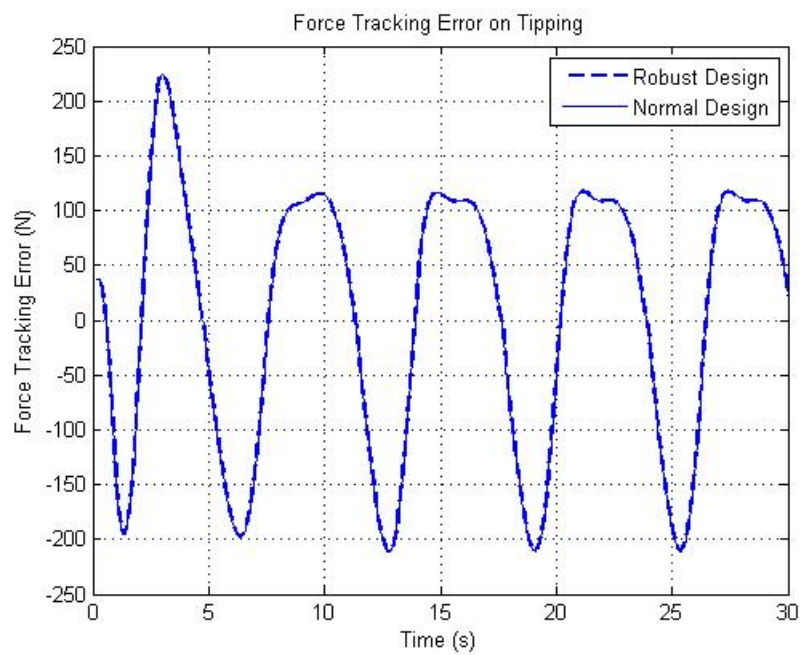


Figure 4.13 Tracking Error Comparison for Tipping Hydraulic Actuator

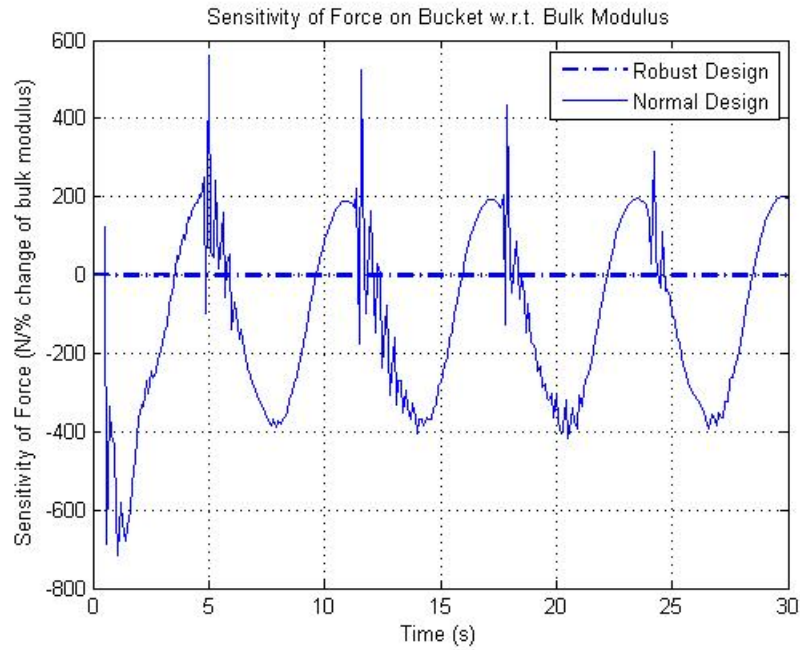


Figure 4.14 Force Sensitivity to Change in Bulk Modulus for Bucket Hydraulic Actuator

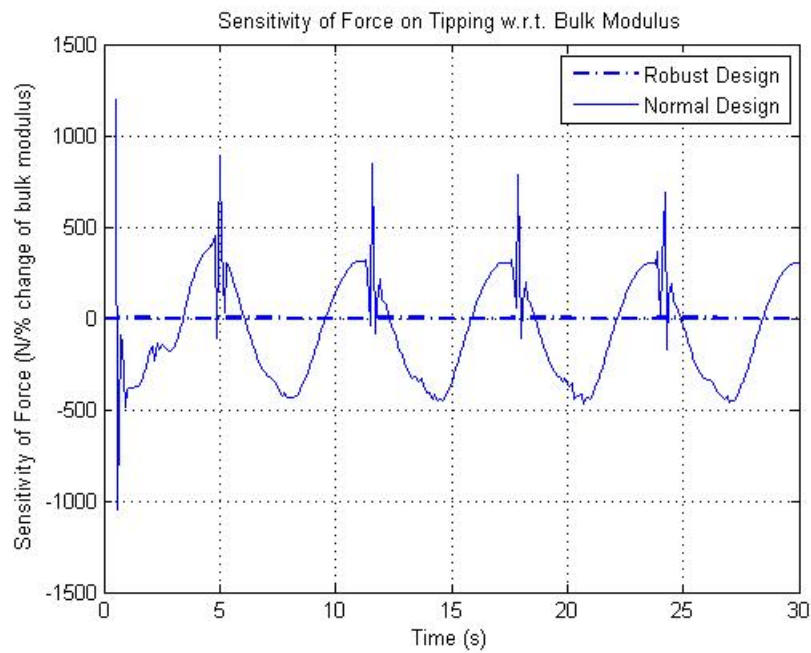


Figure 4.15 Force Sensitivity to Change in Bulk Modulus for Tipping Hydraulic Actuator

## CHAPTER 5. ROBUST FEEDBACK LINEARIZING CONTROL TOOLBOX

The process of designing robust feedback linearization requires thorough understanding of non-linear systems theory and investment of man hours to derive equations. In industrial applications, this may not be efficient method and hence a MATLAB based toolbox with user interface is developed to facilitate robust and efficient implementation of this approach. Robust feedback linearizing control toolbox (RFLC in short) is compiled into one MATLAB app to simplify the design process. The inputs from user are – equations of motion for the nonlinear system, definitions of inputs and outputs, values of parameters and uncertain parameter. The toolbox provides Simulink blocks which can be put into feedback linearization framework if same model is developed in Simulink.

In this chapter features of this toolbox are discussed and a tutorial is provided. The toolbox is demonstrated using excavator bucket level control problem. The sake of simplicity, only the case of  $X_{sp} > 0$  is shown here, but the process is repeated similarly for other case.

### 5.1 RFLC Tutorial

#### 5.1.1 Installation

Users can install the RFLC package into a Matlab app by following matlab user guide. After successful installation, RFLC app will appear in the apps gallery Fig. (5.1).

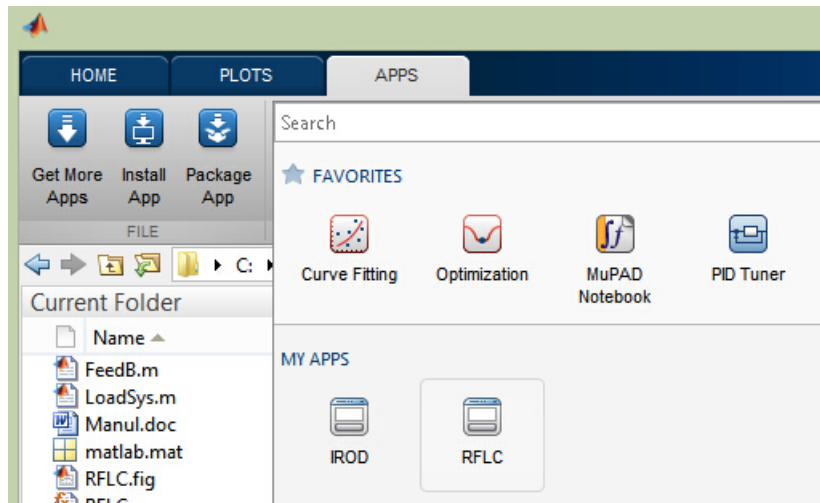


Figure 5.1 The MATLAB Apps Gallery with RFLC Installed in "My Apps"

### 5.1.2 RFLC Layout

The user interface developed for this toolbox is shown in Fig. (5.2). The layout consists of three main parts: system information, equation of motion and controller design.

- System Information: this section is used to input system data, for example input, output and parameters.
  - Enter the number of states: order of the system
  - Enter the name of inputs: input name
  - Enter the name of outputs: output name
  - Enter the known parameter names separated by commas: parameter names.
  - Enter the corresponding parameter values separated by commas: parameter values. The values should be listed in the same order as parameter names.
  - Clear button: clear all information of all of these 3 sections from memory.

- Equation of Motion: users need to type in the system state equations for this section.
  - Enter the name of state 1: do not type in the derivative of the state name, like if the state equation is  $\dot{X}_1 = A * X + B * U$ , the state name is  $X_1$  rather than  $\dot{X}_1$ .
  - Enter the state equation of state 1: for the example  $\dot{X}_1 = A * X + B * U$ , the equation should be  $A * X + B * U$ .
  - Add State button: Let's the users add new state variables and equations until the number of state variables is equation to the order of the system provided into 'System Information'.
  - Clear State Equations button: clear all state names and equations from memory.
  
- Controller Design: this section is used to design controllers. Users need to define output equations, uncertain parameters and block names and paths.
  - Enter the output equation: the format is the same as 'Enter the state equation of state 1' in last section
  - Enter the new FLC block name: users need to define a name for the Simulink block generated by the program. This block is used to calculate feedback linearizing control.
  - Enter the new FLC block path: define the directory for the Simulink block.
  - Run FLC button: It starts the control design process. First, System's relative degree is displayed in the command window. Also, the program saves the controller in symbolic format and the controller's default name is 'inputComp', in case that users want to check if the calculation is correct. An additional block is generated and saved into the user defined directory and its default



name is 'SiglTest', which is used to check the sign of the input signal. If the sign of input does not cause discontinuity, the 'SigTest' block can be ignored.

- Enter the uncertain parameter: the name of uncertain parameter
- Enter sensitivity dynamics block name: this block is the sensitivity dynamics of the system.
- Enter sensitivity dynamics block path: define the directory for the sensitivity dynamics block.
- Enter input sensitivity block name: the sensitivity of the input with respect to the uncertain parameter.
- Enter input sensitivity block path: define the directory for the input sensitivity block.
- Run SenDynmaics button: get the system sensitivity dynamics block and also the input sensitivity block. Like the button 'Run FLC', it saves the equations in these blocks in symbolic format and the default names are 'equStateSen' and 'InCompSen' respectively. 'equStateSen' has the same dimension with the system dynamics.

Some level of error handling is included in the program. So that relevant errors are fired when unexpected inputs are entered. For example, error message is fired when 'path' is not valid or does not exist.

### 5.1.3 Example: Excavator Bucket Hydraulic Actuator

The excavator bucket level control problem with hydraulic actuators is taken as an example to illustrate the effectiveness of the app in robust feedback linearization control design. The state equations for Excavator Bucket Hydraulic Actuator are given in Eq. (4.4) and Eq. (4.5). For simplification, we only consider Eq. (4.4).

- Import System Basic Information

In Eq. (4.4), there are 2 states:  $P_i$  and  $P_o$ . The system input is  $X_{sp}$  and output is  $F$ . The known parameters are:  $A_i, A_o, L, P_s, P_t, V_0, Cd, rho, A_{max}, h_{max}$  and the corresponding known parameter values are: 0.0346, 0.0213, 1.33,  $3.2 * 10^7$ , 0,  $5 * 10^{-5}$ , 0.7, 961.87,  $2 * 10^{-4}$ , 0.005. Fig. (5.3) shows the user interface with values entered in respected fields.

- Import System State Equations

Since the order of the system is 2, first step is to enter first state and corresponding state expression. Then, press the 'Add state' button. This clears the text typed in 'State name' and 'equations' fields, (but the equation is stored in the memory). In second step enter the second state and corresponding expression. Click add state 2 button to save the equation. The program detects that number of equations are same as order of the system defined previously, and does not allow entering further states. The 'Add states' button is now changed to 'Over'.

In case of any typos, the user has an option to clear the equations and start entering equations again.

- Feedback Linearization

Based on Eq. (4.6), the output equation is  $P_i * A_i - P_o * A_o$ . I named the new FLC block name as 'CompU0' and the block path is 'Temp/Feedback Linearization'. Note that Simulink model path has to be in matlabs path. Then RFLC generates two blocks 'CompU0' and 'SigTest' and also saves the equations that these two blocks represent.

- Obtain Sensitivity Dynamics of the system

The uncertain parameter is 'Beta' which represents bulk modulus. The block names and paths are shown in Fig. (5.6). After run the program, as shown in Fig. (5.7),

there are 2 states in the sensitivity dynamics block – 'SenDy1' and 'SenDy2'. Also, the program saves the equations of these 3 blocks. After RFLC generates all of these blocks, users need to connect them together according to Fig. (4.11).

## 5.2 Discussion

As shown in the previous excavator bucket hydraulic actuator example, RFLC app makes the controller design process easier and more accurate. Users do not have to have much background about feedback linearization and sensitivity analysis. also RFLC can design robust nonlinear controllers for any nonlinear system, that is, this program is not restricted to hydraulic systems.

RFLC can only work on the systems which have the specific structure Eq. (3.2), because the system transformation is not unique. Mean while, the system has to be SISO and have only one uncertain parameter. In the future, RFLC app will be used for more general systems.

RFLC

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**Robust Feedback Linearization Control**

**System Information**

Enter the number of states:

Enter the name of inputs:

Enter the name of outputs:

Enter the known parameter names separated by commas (for example: a,b,c,...)

Enter the corresponding parameter values separated by commas (for example: 1,2,3,...)

---

**Equation of Motion**

Enter the name of state 1

Enter the state equation of state 1

---

**Controller Design**

Enter the output equation

Enter the new FLC block name

Enter the new FLC block path

Enter the uncertain parameter

Enter sensitivity dynamics block name

Enter sensitivity dynamics block path

Enter input sensitivity block name

Enter input sensitivity block path

Figure 5.2 RFLC App Layout

**System Information**

Enter the number of states:

Enter the name of inputs:

Enter the name of outputs:

Enter the known parameter names separated by commas (for example: a,b,c....)

Enter the corresponding parameter values separated by commas (for example: 1,2,3.....)

Figure 5.3 System Information of Excavator Bucket Hydraulic System

**Equation of Motion**

Enter the name of state 2

Enter the state equation of state 2

Figure 5.4 Equation of Motion of Excavator Bucket Hydraulic System

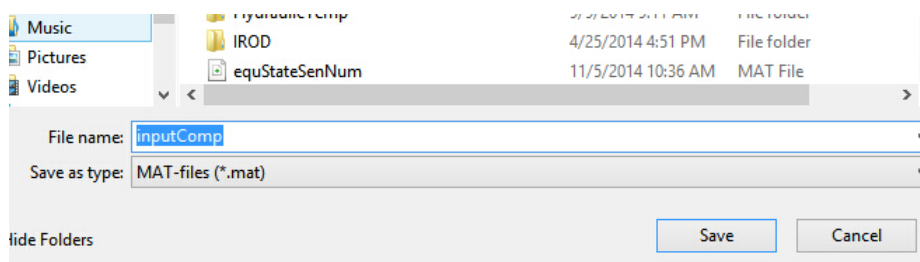


Figure 5.5 Save the Equation of Blocks Dialog

Enter the uncertain parameter	Beta	Run SenDynamics
Enter sensitivity dynamics block name	SenDy	
Enter sensitivity dynamics block path	Temp/SenDynamics	
Enter input sensitivity block name	CompUb	
Enter input sensitivity block path	Temp/SenDynamics	

Figure 5.6 Sensitivity Dynamics of Excavator Bucket Hydraulic System

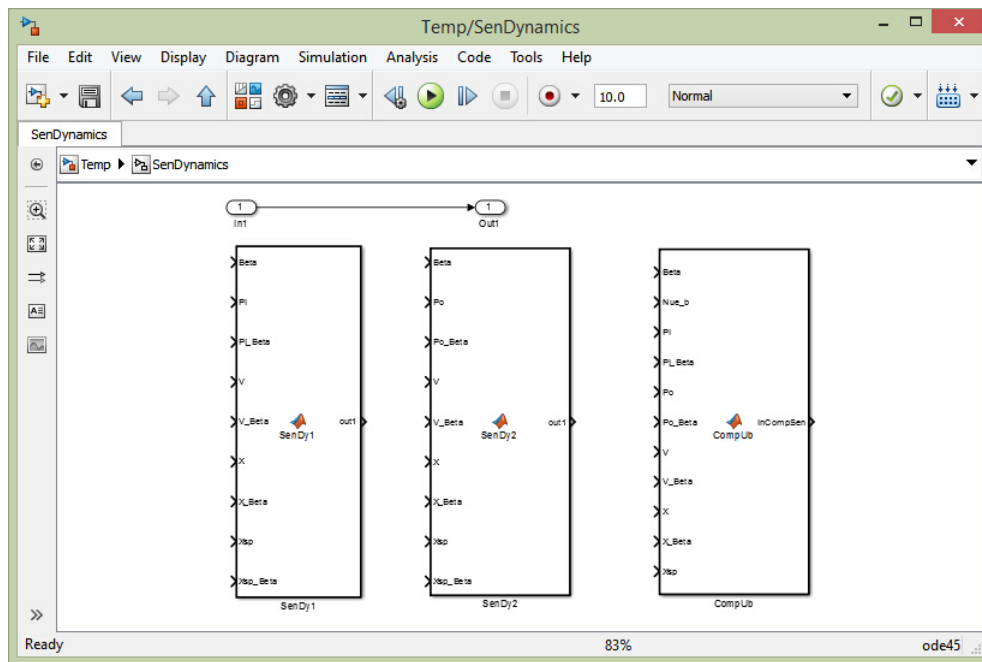


Figure 5.7 Sensitivity Dynamics Block of Excavator Bucket Hydraulic System

## CHAPTER 6. CONCLUSIONS AND FUTURE WORK

This chapter summarizes the contribution of this thesis and discusses possible future work.

### 6.1 Conclusions

The primary focus of the work in this thesis has been on: (a) application of robust feedback linearization methodology based on sensitivity dynamics of the system; (b) development of system process for integrating robust feedback linearizing control design of hydraulic actuator subsystem with robustly designed linkage system using IROD methodology; and (c) development of Matlab-based user friendly toolbox to enable efficient design of robust feedback linearizing control for a class of systems driven by hydraulic actuators and having multi-link mechanisms. An example system used throughout this work is the excavator system and the specific problem considered is that of automatic bucket leveling. The Matlab toolbox has been developed with friendly user interface so that complex tedious computational details are handled automatically behind the scene and user merely has to input the basic data.

While through out the thesis the example of excavator has been presented the methodology and toolbox developed is applicable for any generic system in a similar class of dynamic systems. The results presented in previous chapters clearly demonstrate that the robust feedback linearizing control methodology can be effectively used with high-fidelity models obtained from SimHydraulic, AMESim, and SimMechanics software. It is shown

that the novel method can improve the system robustness and performance efficiently. The work also presented how to integrate RFLC with IROD which is critical for dealing with complex hydraulically driven systems such as ag and construction machinery.

The key contribution of this thesis include:

1. Demonstration of a methodology to achieve robust control of hydraulically driven nonlinear multilink system.
2. Process and architecture to integrate RFLC with IROD to achieve integrated robust optimal design of complex system, and
3. Development of user-friendly Matlab based toolbox for designing of RFLC.

## 6.2 Future Work

The work presented here has laid a good foundation for integrated robust design and control of complex nonlinear multibody systems. This work has also opened several new doors for enhancement and/or improvement of methodology for efficient adoption into industrial community. Specifically, following are some key research directions that can be pursued in advancing the state-of-the-art:

1. Extension of validation effort of RFLC methodology to multiple uncertain parameters which will increase the computational complexity. However, the methodology presented in this work will remain the same. In this thesis, only bulk modulus was considered as an uncertain parameter, which is the most commonly uncertain in hydraulic systems. However, the methodology can be used for multitude of uncertain parameters and future work can validate this generalization.
2. Extend applicability of RFLC by developing interfaces to AMESim, ADAMS, etc. so that one can achieve robust design of systems already modeled inside these environment. In industry, different companies have different preferences in software.



The benefits of RFLC are demonstrated in this work however for its wide acceptance its integration with already existing commercial codes is necessary.

3. Implement on actual hardware, rather than software simulation. The RFLC methodology was demonstrated on high fidelity dynamic model but for complete validation the implementation on actual physical hardware is important. Future work should address the practical implementation aspect.
4. Develop interfaces in collaboration with hydraulic modeling software companies to enable generation of analytical representation of hydraulic dynamics so that RFLC toolbox can directly import these equations and perform the design automatically.

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