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Incorporating stochastic analysis in wind turbine design: data-driven random temporal-spatial parameterization and uncertainty quantification

by

Qiang Guo

A thesis submitted to the graduate faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Major: Mechanical Engineering

Program of Study Committee:

Baskar Ganapathysubramanian, Co-major Professor

Jonathan Wickert, Co-major Professor

Eugene Takle

Frank Peters

Pranav Shrotriya

Iowa State University

Ames, Iowa

2013

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DEDICATION

I would like to dedicate this thesis to my beautiful and wonderful wife Yaqin Liu who always help me to become better and stronger. Without her unmitigated support in every possible way I would not have been able to accomplish this work. I would also like to thank my friends and family for their loving guidance and support during my study and research life.

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ABSTRACT

Wind turbines reliability is affected by stochastic factors in the turbulent inflow and wind turbine structures. On one hand, the variability in wind dynamics and the inherent stochastic structures result in random loads on wind turbine rotor and tower. On the other hand, the inherent structural uncertainties caused by imperfect control of manufacturing process introduce unpredictable failures and decreases wind generators availability. Therefore, to improve reliability, it is important to incorporate the variability in wind dynamics, and the inherent stochastic structures in analyzing and designing the next generation wind-turbines.

In order to perform stochastic analysis on wind turbine, there are several improvements need to be made. Current stochastic wind turbine analyses are mostly based on incomplete turbulence input models. These models either failed to account for temporal variation of the stochastic wind field or unable to preserve spatial coherence which is a very important property that describes turbulence structure. On the subject of modeling wind turbine, most commonly used wind turbine design code is based on stead, lumped component blade models which lack the ability to describe the complex 3D fluid-structure interaction (FSI); which is essential to provide precise blade stress distribution and deformation details. Finally, when it comes to analyzing simulation results, most of existing work are done by analyzing the time response of wind turbine, without looking at the stochastic nature of performance of wind turbines, and its relationship between stochastic sources in turbulent inflow and turbine structure.

In this work, we first develop a data driven temporal and spatial decomposition (TSD), which is capable of modeling any given large wind data set, to construct a low-dimensional yet realistic stochastic wind flow model. Results of several numerical examples on the TSD model show good consistency between given measured data and simulated synthetic turbulence. After that, a stochastic simulation based on TSD simulated full-field turbulence and a simplified wind turbine model is performed. The result of this analysis shows the adequacy of using TSD as

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turbulence simulation tool as well as the random nature of wind turbines' performance. Finally, a stochastic analysis on a full scale 3D rich-structural wind turbine model with stochastic composite material properties is performed. With a given steady wind load, the model gives the deformation and the stress distribution of the blades. Critical regions that are most likely to have stress larger than design strength of the material were identified. Failure analysis is then performed based Tsai-Wu failure criterion.

CHAPTER 1. OVERVIEW

Wind energy is a clean and sustainable energy resource that is a promising alternative to fossil fuel based energy. Great progresses have been made in wind power technology in the past three decades. Wind turbine capacity has developed from less than 100 kW in the 1980s to a current capacity of 5 MW. Wind turbine rotor diameter has also increased from 15 meters to 120 meters. Through improvements in the design of various subsystems of the wind turbine, the energy production cost has been significantly reduced. With these developments, large-scale application of wind power has become a reality [25]. As a pioneer in wind power technology, Denmark produced 22% of its electricity by means of wind power in 2010. In Asia, China added 16.5 GW of wind capacity in 2010 to reach 42.2 GW at the end of 2010 , which makes the country the largest wind power market in the world [24]. U.S. Department of Energy (DOE) has developed a scenario for supplying 20% of the nation's electricity by means of wind by 2030 [66].

However, there are significant challenges that have to be resolved in order to achieve these goals. In fact, during the twelve months from July 2011 to June 2012, only 3.2% of all electricity in the US was from wind [67]. One of the reasons for this sluggish use of wind is the current high cost of wind energy, which can be markedly reduced by improving the efficiency and reliability of wind turbines [41]. As part of the focus on improving wind turbine efficiency, both rotor diameter and turbine hub hight have been increased significantly. To minimize the vibration caused by gravity, thinner and lighter blades are needed for these large scale wind turbines, which will in turn affect their reliability. As a result, the need to decrease the weight of blades while ensuring reliability becomes an important issue.

The reliability of a wind turbine is affected by its inherent material and operating uncertainty/randomness. Examples of these random factors include turbulence inflow, random

1

property of composite material from which blades are made, and random location and severity of structural defects in blades. These uncertainties in wind turbine system inevitably affect the reliability of wind turbines. Therefore, the reliability analysis of wind turbines requires a rigorous incorporation of the effect of these randomness. This research focuses on developing and implementing new methods that incorporating stochastic analysis in wind turbine simulation.

1.1 Current Status of Wind Turbine Design

A wind turbine design work based on computational simulations could be done in three steps: representing the wind inflow as input of the wind turbine system, modeling the wind turbine, and solving time responses of variables of interest as the simulation result. Reviews on these three subjects are given in the following sections.

1.1.1 Status of wind modeling

The wind load on the turbine is a stochastic process whose direction and speed depends on location, and time. In other words, "... at any instant they are distributed irregularly in space, at any point in space they fluctuate chaotically in time, and at given point and a given time they vary randomly from realization to realization." [74] This randomness in wind conditions leads directly to the fluctuations in rotating speed of the rotor (for variable-speed wind turbine). Furthermore, randomness of wind direction causes random yaw motion of nacelle, which can further affect wind turbine's productivity. Turbulence introduces unsteady loads on the blades. All of these effects excite structural vibrations, introduce components failures, and further reduce the lifespan of wind turbine. Thus, stochastic wind loads have a critical impact on wind turbine performance.

Most of the past wind turbine design work was based on simplified wind load models that assume steady wind speed, constant wind speed gradient profile and constant turbulence intensity [51]. This kind of simplified wind model does not provide any insight into the effect of randomness in the wind load. They have, however, been used with some success in the context of *deterministic wind turbine* analysis [7, 8]. Subsequently, wind models that are capable of describing wind stochasticity have been developed. Gaussian and Weibull distributions have been used in approximating histograms of hourly mean wind speeds. These methods are based on parameter estimation techniques such as maximum likelihood method (MLM) [12]. While useful is representing randomness, these techniques lack the ability to describe *temporal and spatial* correlations of wind, or location-dependent characterization capabilities. This results in the development of methods that incorporate spatial variations, spatial correlation, and temporal correlation.

There have been several seminal works that deal with preserving either spatial correlation or temporal correlation. One such example is Veers/Sandia method [68]. The method first uses empirical coherence function and wind speed power spectral densities (PSDs) to describe the wind field. Choleski decomposition is then used to decompose the cross spectrum matrix. Velocity time-series at different locations are finally calculated with a inverse Fourier transform process. Another example is Mann's method [44], which starts with a spectral tensor that is achieved by taking incompressibility into account and solving the Navier-Stokes equations. The wind field is described in wave vector space and then transformed into spatial domain. Recently, proper orthogonal decomposition (POD) was used in [55] to simulate wind turbine inflow. In this method, stochastic wind flow is viewed as wind field snapshots at different instances. A spatial covariance matrix is decomposed to get multiple characteristic modes that are further used in constructing stochastic wind field. By performing one POD analysis at each time step, the wind flow is reproduced. Although the above three methods are easy to implement and efficient enough to get random wind flows, they do not account for both temporal correlation and spatial correlation of the wind flow at the same time. In software package TurbSim [34], a turbulence simulation code developed by National Renewable Energy Laboratory (NREL), Veers method is used. However, due to its inadequacy in describing temporal coherence, additional information about coherent structure has to be included in the model so that the turbulent structure in atmosphere could be more accurately represented [39].

most of existing stochastic wind models use assumptions of the certain stochastic properties of wind such as spatial coherence and temporal covariance, which may not be valid for all applications. For analysis that focus on general wind conditions, such methods are efficient and accurate enough. As the techniques of wind measurement developing, site specific high frequency data will become available. In this case, data driven stochastic wind models are desired for next generation wind turbine design. In fact, data driven wind models do exist. Messac et al. [47] developed a wind model that is customizable based on local geographical and climatic conditions. They used a nonparametric model to characterize the uncertainties in annual multivariate distribution of wind speed and orientation so that the model could be used in various locations. This model has been used to evaluate wind farm performance [79]. Although the model contains sufficient information for the purpose of annual wind farm performance evaluation, the diurnal variation of wind speed is not accounted for. Another method that accounts for diurnal variability in wind speed was developed by Negra et al., where 10 minute average wind speed data over 7 years was used [49]. This method generates synthetic wind speeds that are excellent statistical matches with the original data, but there is no easy way to extend the method to describe spatial variations. It is worth noting that none of above methods is able to model both spatial and temporal covariances at the same time.

Based on above discussion, there is a need of a more realistic parameterization of the wind that encodes location-, topography-, diurnal-, seasonal and stochastic affects, and at the same time be able to reproduce spatial and temporal covariances in the field measured wind data. However, such a comprehensive data driven parameterization is useful in practice only if it is relatively simple (low-dimensional), which is the bottleneck of data driven wind models.

1.1.2 Status of wind turbine modeling

There is a trade off between improving efficiency and increasing reliability of wind turbine. On one hand, in order to improve wind turbine efficiency, both rotor diameter and turbine hub height have to be increased significantly. On the other hand, to minimize the vibration caused by gravity, thinner and lighter blades are needed for these large scale wind turbines, which will in turn affect their reliabilities. A pressing challenge is to lighten the weight of blades and simultaneously achieve high system reliability.

The reliability of a wind turbine is affected by material and operating uncertainty/randomness. The stochasticity in the composite wind turbine blades depends on the type of composite material used, the placement of composite fiber pieces, the design of structural reinforcement members such as shear web and spar caps, and structure parameters such as ply thickness. Blades are made of composite laminate that has several plies. Each ply consists of a layer of unidirectional fibers, whose thickness and material properties can vary from ply to ply. The randomness in composite material is introduced by the manufacturing process where a large number of reinforcing elements need to be embedded into matrix. The resulting microscopic heterogeneity impacts random fiber positions, occurrence of defects in fiber, etc. and causes random behavior of the fiberglass ply under external loading [40]. Randomness in wind turbine structure and material causes non-uniform rotor deformation that further results in unbalanced load on the rotor. The random vibration introduced by this unbalanced load will decrease wind turbine's efficiency and reliability. Therefore, in order to improve reliability of large scale offshore wind turbines, a crucial first step is to have a better understanding of how they are affected by manufacturing uncertainties. To this end, a comprehensive wind turbine model that are able to incorporate structural and material uncertainties is required.

Present wind turbine modeling techniques can be basically classified into three types: finite element methods (FEM), multi-body-system (MBS), and the modal approach [43]. A good review of existing wind turbine design codes in these three classifications are given in [2, 50]. In the modal approach, modal analysis of beam-structured finite element wind turbine model is performed first. The first few natural modal shapes are then superimposed to get the displacement of turbine blade under certain give load. A good example is BLADED [21] which has been an industrial standard tool for several years. However, it can not describe the torsional blade deformations. This problem is solved in the MBS method, where the wind turbine system is divided into a finite number of rigid/flexural bodies connected with elastic joints. On each body 2-dimensional loads (lift, drag and aerodynamic moment) are applied at the aerodynamic center of the airfoil. A few ordinary/partial differential equations are used to describe the behavior of the system. One examples of such design codes is FAST[36]. MBS method is fast and reasonably accurate when general deflections of all rigid bodies are sufficient enough for simple analysis. However, by simplifying the 3D geometry to 2D airfoil cross sections, the method fails to capture the FSI between wind and rotor blades surface, which is essential for calculating the detailed deflection of the entire blade surface. In addition, for the purpose of structure design, MBS method is insufficient since it can not describe the effect of local structural changes on system outputs. To this end, FEM is introduced in wind turbine design. In FEM, the detailed blade geometry is discretized into finite elements, and the governing equations of the whole system is discretized into a set of differential equations. In contrast to the other two approaches, FEM calculates accurate results but at high computational cost.¹

Current wind turbines are upwind turbines whose rotors facing towards the wind ². In this case, blades may bend too much to hit the tower. To prevent such failure, the clearance between the rotor blades and turbine tower (tip-tower clearance) should not be less than the minimum of 30% for the rotor turning and 5% for load cases with the rotor standing still, in relation to the clearance in the unloaded state [18]. Three techniques are used to achieve this goal. One could move the rotor further from the tower or change the angle of the rotor so that the bottom of the rotor is further from the tower. However, the increased rotor overhang will increase load on the shaft which may further reduce the reliability of the gearbox. A better approach could be such that the blade is bent in the opposite direction of the blade bending moment when unloaded. This process is called pre-bent or pre-coning [22]. This makes simulating wind turbines more difficult because a pre-bent 3D wind turbine blade model is needed to fully assess the impact of the new design to the blade stress as well as its performance. Although FEM is very computational costly, it seems that it is the only choice among above methods when performing structure design of rotors with complex geometry.

Most of existing wind turbine design codes assume small deflections so that the aerodynamic loads can be applied to the undeformed structure. However, this assumption is less valid since modern wind turbine blades are more and more flexible as their size increases. Given stable wind load, the deformations of the flexible blade influences the aerodynamic force on blade surface and vice versa. Therefore, analysis of offshore wind turbine blade is actually an aeroelastic problem. To calculate the aeroelastic response, following aspects must be taken into account: aerodynamic forces, blade deformations and the coupling between them [71]. By running the analysis (with small displacement assumption) in a loop where the aerodynamic forces and

¹High computational cost is regarded as the major limitation of FEM.

 $^{^{2}}$ Rotor of downwind turbine would suffer strong turbulent which is caused by the fluid structure interaction between air flow and the tower, and generate large noise[53].

blade shape are updated in every iteration, the coupling problem can be solved. The only remaining issue is that the FEM result is approximation of the analytic solution, which means a small deviation from the beginning of the calculation could propagate to a very large one after several iterations. Therefore, a mesh update and refinement process is necessary in order to minimize the calculation error in each iteration. Using FEM to solve aeroelastic problem for flexible wind turbine becomes an very inefficient and cumbersome approach.

As an improvement of FEM, isogeometric analysis [28] provides much more accurate and efficient geometric modeling. It represents exact geometries at the coarsest level of discretization and eliminates geometrical errors from the beginning. The isogeometric method has been shown to get accurate simulation results for flexible wind turbine. A realistic 3D rotor model should have the ability to describe the complex 3D fluid-structure interaction (FSI) which will provide us precise blade deformation and stress distribution [7, 8]. In addition, to assess the affects of randomness in turbine structure to its performance, the wind turbine model must be able to incorporate randomness in its structure and material. The advantages of isogeometric analysis make it the ideal method for stochastic wind turbine analysis.

1.1.3 Status of stochastic analysis on wind turbine

Many turbulence models that are capable of describing long-term (10 minute average wind speed) and short-term (spatial or temporal correlations) wind stochasticity have been developed. They have been successfully utilized in many applications. For instance, [47] developed a wind model that is customizable based on local geographical and climatic conditions. They used a nonparametric model to characterize the uncertainties in annual multivariate distribution of wind speed and orientation so that the model could be used in various locations. This model has been used to evaluate wind farm performance [79]. Simulation of wind turbine aeroelastic response to wind turbine inflow turbulence is given in [1]. In [59], fatigue failure analysis is done with random wind load and 3-D finite element wind turbine model.

It must be mentioned here that almost all wind turbine simulation works based on stochastic wind inputs are done by generating time responses of certain quantities of interest of wind turbines. These results only describe how wind turbines operate given specific wind inputs. They provides no information about how the randomness in turbulence input affect the performance of wind turbines. To solve this problem, stochastic analysis/uncertainty quantification(UQ) has to be incorporated in wind turbine design.

Stochastic analysis provides strategies to evaluate the relationship between a stochastic input and various quantities of interest. In general, stochastic analysis is done by modeling the system of interest, incorporating uncertainties in system model or its inputs, and getting probability of the system output. For instance, to quantify the effect of the uncertainty that is introduced by the random wind conditions, stochastic analysis (like polynomial chaos [77, 78], stochastic collocation [16], or Monte Carlo analysis [20]) on a wind turbine model can be performed [42].

By including uncertainty quantification in the modeling process, system governing equations (mostly a set of differential equations) possess stochastic property as well. Such kind of equations are called stochastic differential equations (SDEs). Many methods have been developed to solve SDEs. Generally, these methods can be classified as statistical and non-statistical methods [16]. One of the most commonly used statistical methods is Monte Carlo sampling. In this method, realizations of random inputs are generated based on their probability distribution. For each realization the data is fixed and the problem is deterministic. A very large number of samples are needed to get accurate result. Monte Carlo method quickly becomes impractical when dealing with higher order complicated systems.

Non-statistical methods approximate the uncertainties in the system or its input first, which will be further used in system modeling. One example of this kind of methods is spectral stochastic finite element method (SSFEM) [20]. In this technique, the random field is expanded about its mean with a set of complete orthogonal polynomials whose coefficients are realizations of a set of random variables. Generalized polynomial chaos (gPC) [77] is another well developed non-statistical methods. With gPC, stochastic solutions are expressed as orthogonal polynomials of the input random parameters. Followed by Galerkin projection, the set of stochastic differential equations become deterministic decoupled equations which can be solved by using common numerical methods. Many orthogonal bases could be used to construct the polynomials. Hermite polynomial expansion is of most commonly used. In addition, a whole family of polynomials named Wiener-Askey polynomial chaos can be used according to different distribution functions of the random inputs [77]. In this way, the optimal convergence can be achieved. Similar to Monte Carlo method, non-statistical methods also convert the stochastic problem to a set of deterministic equations. However, the resulting equations generated by non-statistical methods are often coupled, which makes using this method to solve complicated system very difficult.

Most recently, stochastic collocation approach is used to solve SDEs [4]. This method is based on deterministic sampling method like Monte Carlo method. Instead of finding the solution of a huge amount of sampling points in the random field and then form a discrete solution of the random input, one can use less sampling points, get the deterministic solutions, and then use these solutions to form an approximate collocation solution. How to choose as less as sampling points to reduce the computational cost and, at the same time, meet the prescribed accuracy is the main concern of stochastic collocation method. One way is to use tensor products of one-dimensional Gaussian quadrature points as sampling points [4]. As the dimension of random fields increase, the number of sampling points grows exponentially fast. Sparse grid collocation (SGC) technique is introduced in order to reduce the number of sampling points in higher random dimensions [76]. Another issue of stochastic collocation is how to choose basis functions to interpolate the collocation solutions. Lagrange polynomials and, more general, gPC polynomials are used. Adaptive sparse grid collocation method is further developed by introducing the concept of adaptability in to the algorithm [16]. SPG method is a very efficient method that has its advantages in solving complicate, large stochastic dimensional problems.

1.2 Possible Improvements and Challenges

Fig. 1.1 shows different subroutines in wind turbine simulation work and possible improvements with respect to incorporating stochastic effects in the analysis. As the source of energy, inflow wind plays an important role in wind turbine simulation. Based on discussion in previous section, a data driven wind model that is able to reproduce temporal and spatial covariances of the source wind is in great need. However such model has to be low-complexity and easy to use.

The second type of improvements could be done in wind turbine modeling. A full-scale, 3D comprehensive wind turbine model is required. Such model would provide detailed deflections and accurate surface stress distributions, which are very important for successful wind turbine blade design. A developing work from [7, 8] is very promising. The difficulties of such model lies in incorporating stochastic parameters, such as random defects, material properties, and stochastic parameters, into the complicated wind turbine structure.

Improvements could also be done in interpreting simulation results. To improve wind turbine reliability, a crucial step is to have a better understanding of how they are affected by stochastic sources of the system such as wind input and random defects. This can be accomplished using stochastic analysis. Since wind turbine simulation is a very complicated and time consuming process, even getting one set of simulation results would require huge amount of computational effort. It becomes very difficult to perform stochastic analysis in wind turbine design, where a large number of simulations need to be done.



Figure 1.1 Possible improvements in stochastic wind turbine analysis.

1.3 My Contribution

The contributions of this work are listed as follows.

1. Stochastic wind modeling:

 (a) formulated a mathematical framework that is able to accurately represent wind flow using a low-complexity yet realistic stochastic model (see Chapter 2);

- (b) implemented a computational framework based on the mathematical framework that extracts statistical information from any given turbulence data and constructs an easy-to-use model (see Chapter 2);
- (c) applied the computational framework to three numerical examples (see Chapter 3).

2. Stochastic analysis of wind turbine:

- (a) used the stochastic wind flow that was developed in this work performed wind turbine simulation on a simplified wind turbine model (see Chapter 4);
- (b) performed stochastic analysis on the simplified wind turbine model using adaptive sparse grid collocation method (see Chapter 4);
- (c) Incorporated stochastic material properties into a full-scale, 3D, complex geometry, wind turbine rotor model (see Chapter 5);
- (d) performed stochastic analysis, stress analysis, and failure analysis on the 3D wind turbine model (see Chapter 5).

CHAPTER 2. TEMPORAL SPATIAL DECOMPOSITION: AN TURBULENCE SIMULATION METHOD

The motivation for this chapter is the fact that no single existing wind simulation method can provide a data-driven model that seamlessly accounts for spatial variations, diurnal variations, and temporal correlations. To this end, an efficient method that leverages the huge amount of meteorological data that is readily available will be very useful for various aspects of wind turbine analysis. In this chapter, we develop a data-driven space-time parameterization of any given large data set of wind to construct a low-dimensional yet realistic stochastic wind flow model. The framework is based on a two-stage model reduction: Bi-orthogonal Decomposition (BD) followed by Karhunen-Loève expansion (KLE). The resulting time-resolved stochastic model encodes most of the statistical properties in the given wind flow. Moreover, the temporal modes encode the variation of wind speed in the mean sense and resolve temporal correlation while the spatial modes provide deeper insight into spatial of the wind field - which is a key aspect in current wind turbine sizing, design and classification. In addition, several interesting ramifications of this low dimensional model are discussed. These include information about the energy cascade, which is computed as correlations between random energy of different spatial modes.

In the following part, section 2.1 focuses on constructing a low-complexity model of the random wind flow. Bi-orthogonal Decomposition (BD) [70, 69] and Karhunen-Loève expansion (KLE) [20] are used in constructing the low-dimensional wind model. The algorithmic details of the computational framework is give in section 2.1.4. Finally, section 2.2 concludes this chapter.

2.1 Mathematical Framework

Denote by $\mathbf{x} = (x_1, x_2, x_3) = (x, y, z)$ the three-dimensional coordinates with x, y, and z represents along-wind, transverse, and vertical axis respectively, Wind velocity $\mathbf{v} = (v_1, v_2, v_3) = (u, v, w)$ consists of three wind speed components along the three spatial axes. A stochastic wind can be defined as $\mathbf{v}(\mathbf{x}, t, \boldsymbol{\xi})$, where $\boldsymbol{\xi} = \{\xi_i, i = 1, \dots, n\}$ is a random vector associated with the random field. It is worth noting that the random field described above is actually a function $\mathbf{v} : \mathbb{X} \times \mathbb{T} \times \Sigma \to \mathbb{R}$ where $\mathbb{X} \subseteq \mathbb{R}^d (d = 1, 2, 3)$ denotes the spatial domain, $\mathbb{T} \subseteq \mathbb{R}$ denotes the temporal domain and Σ is the sample space of a set of random variables $\boldsymbol{\xi}$ that is related to the random field.

The analysis is clearer when the mean component $\bar{\boldsymbol{v}}(\boldsymbol{x})$ of the velocity data is removed so that only the fluctuation components $\boldsymbol{u}(\boldsymbol{x},t,\boldsymbol{\xi})$ are left, i.e.

$$\boldsymbol{u}(\boldsymbol{x},t,\boldsymbol{\xi}) = \boldsymbol{v}(\boldsymbol{x},t,\boldsymbol{\xi}) - \bar{\boldsymbol{v}}(\boldsymbol{x}),. \tag{2.1}$$

The mean component is defined as

$$\bar{\boldsymbol{v}}(\boldsymbol{x}) = \frac{1}{|\mathbb{T}|} \int_{\mathbb{T}} \langle \boldsymbol{v}(\boldsymbol{x}, t, \boldsymbol{\xi}) \rangle dt.$$
(2.2)

where $\langle \cdot \rangle$ denotes the average in the stochastic domain, $|\mathbb{T}|$ is the span of the temporal domain, \bar{v} is the ensemble average.

The goal of this work is to construct a simple stochastic model that encodes temporal and spatial covariance, and preserve all the statistical information, such as spatial coherence and wind speed power spectral density (PSD), present in the collected field measured data. We look for a model that has the form

$$\boldsymbol{u} = \sum K_{ijk} \boldsymbol{X}_i(\boldsymbol{x}) \ T_j(t) \ \boldsymbol{\xi}_k$$
(2.3)

where K_{ijk} are coefficients, the deterministic functions X_i track spatial correlations, T_j track temporal correlations and the random variables $\boldsymbol{\xi}_k$ encode the inherent variability. The goal becomes to find the simplest possible representation that still encodes all the required information. The main idea of the next section is to formulate a mathematical strategy of representing this meteorology data in terms of the smallest possible number of terms in Eqn. 2.3, by optimally designing \mathbf{X} , T, and $\boldsymbol{\xi}$. Noting that the data contains spatial, temporal and stochastic variabilities, we solve this problem in three stages. In the first stage, we decompose the data into temporal (T(t)) and coupled spatial-stochastic $(\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{\xi}))$ parts through the concept of Biorthogonal Decomposition; in the second part we decompose the spatial-stochastic part into spatial $(\boldsymbol{X}(\boldsymbol{x}))$ and stochastic $(\boldsymbol{\xi})$ components using the concept of the Karhunen-Loève decomposition; the third stage focuses on estimating probability density functions of TSD generated random variables.

2.1.1 Stage 1: a low-dimensional representation via bi-orthogonal decomposition

In the first stage, we are looking for a minimal representation of the data in the form

$$\boldsymbol{u}(\boldsymbol{x}, t, \boldsymbol{\xi}) = \sum_{i,j} K_{ij} \boldsymbol{\Phi}_i(\boldsymbol{x}, \boldsymbol{\xi}) T_j(t)$$
(2.4)

where *i* and *j* are independent indices. Consequently, the expression on the right hand side has an dramatically large number of terms. This is handled utilizing the Schmidt decomposition theorem [57], which states that – any representation of a tensor product space $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$ can be expressed as linear combination of tensor product of basis functions $\boldsymbol{\Phi}_i \otimes \Psi_i$, where $\boldsymbol{\Phi}_i \in \mathscr{H}_1, \Psi_i \in \mathscr{H}_2$. As a result, the representation can be reduced to

$$\boldsymbol{u}(\boldsymbol{x}, t, \boldsymbol{\xi}) \approx \sum_{i=1}^{M} K_i \boldsymbol{\Phi}_i(\boldsymbol{x}, \boldsymbol{\xi}) \ T_i(t).$$
(2.5)

where $\boldsymbol{\Phi}_i(\boldsymbol{x},\boldsymbol{\xi})$ are stochastic spatial modes and $T_i(t)$ are temporal modes.

Our goal becomes searching for the best choices for $\boldsymbol{\Phi}_i$ and T_i such that the decomposition uses the least number of terms M that will give us an accurate representation of $\boldsymbol{u}(\boldsymbol{x}, t, \boldsymbol{\xi})$. We pose this as an optimization problem. To do so, we define the error in this representation and design $\boldsymbol{\Phi}_i$ and T_i that minimize this error. The error, denoted by ε , is defined as the low-complexity stochastic model subtracted from the true data

$$\boldsymbol{\varepsilon}(\boldsymbol{x},t,\boldsymbol{\xi}) = \boldsymbol{u}(\boldsymbol{x},t,\boldsymbol{\xi}) - \sum_{i=1}^{M} K_i \boldsymbol{\Phi}_i(\boldsymbol{x},\boldsymbol{\xi}) \ T_i(t).$$
(2.6)

Approximation theory suggest that the best choice for the functions $\boldsymbol{\Phi}_i$ and T_i (we will also call them modes) is when they are orthogonal to each other [29]. Thus, we set T_i , $i = 1, \ldots, M$ to be orthogonal to each other in the time domain and $\boldsymbol{\Phi}_i$, $i = 1, \ldots, M$ to be weakly orthogonal in spatial domain. Mathematically, this is denoted in terms of the inner products:

$$\langle T_i, T_j \rangle_{\mathbb{T}} = \int_{\mathbb{T}} T_i(t) T_j(t) dt = \delta_{ij}$$
(2.7)

and

$$\langle \boldsymbol{\Phi}_i, \boldsymbol{\Phi}_j \rangle_{\mathbb{X}} = \int_{\mathbb{X}} \overline{\boldsymbol{\Phi}_i} \cdot \overline{\boldsymbol{\Phi}_j} \, d\boldsymbol{x} = \delta_{ij},$$
(2.8)

where $\overline{\boldsymbol{\Phi}_i}$ denotes the expectation of the spatial-stochastic mode, i.e.

$$\overline{\boldsymbol{\varPhi}_{i}(\boldsymbol{x})} = \int \boldsymbol{\varPhi}_{i}(\boldsymbol{x},\boldsymbol{\xi}) W(\boldsymbol{\xi}) \ d\boldsymbol{\xi}, \qquad (2.9)$$

and $W(\boldsymbol{\xi})$ is the multivariate joint probability density of random variables in the set $\boldsymbol{\xi}$.

Note that the error ε itself is an random field. We construct an associated scalar value with this random field to accomplish subsequent optimization. To this end, an error functional \mathscr{E} is defined as the norm of ε , i.e.

$$\mathscr{E} = \int_{\mathbb{T}} \langle \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \rangle_{\mathbb{X}} dt.$$
(2.10)

The error-functional is simply the inner product (i.e. an average) of the field over space, time and stochastic dimensions. Note that error functional depends on the choice of functions $T_i(t)$ and $\boldsymbol{\Phi}_i(\boldsymbol{x},\boldsymbol{\xi})$, i.e. $\mathscr{E}[T_1, \cdots, T_M, \boldsymbol{\Phi}_1, \cdots, \boldsymbol{\Phi}_M]$.

We now search for temporal functions that minimizes this error functional. This is accomplished by applying the calculus of variations and solving the associated Euler-Lagrange equations [15]. We provide full details of the derivation as follows.

Theorem 1. (Euler's equation) Let J[y] be a functional of the form

$$\int_{a}^{b} F(x, y, y') dx, \qquad (2.11)$$

defined on the set of functions y(x) which have continuous first derivatives in [a, b] and satisfy the boundary conditions y(a) = A, y(b) = B. Then a necessary condition for J[y] to have an extremum for a given function y(x) is that y(x) satisfy Euler's equation

$$F_y - \frac{d}{dx}F_{y'} = 0 \tag{2.12}$$

The goal of BD is to minimize the error functional

$$\mathscr{E}[T_1, \cdots, T_M] = \int_{\mathbb{T}} \langle \varepsilon, \varepsilon \rangle_{\mathbb{X}} dt.$$
(2.13)

Substituting representation error (Eqn. 2.6) in above equation yields

$$\mathscr{E}[T_1,\cdots,T_M] = \int_{\mathbb{T}} \left\langle u(\boldsymbol{x},t,\boldsymbol{\xi}) - \sum_{i=1}^M K_i \boldsymbol{\varPhi}_i(\boldsymbol{x},\boldsymbol{\xi}) T_i(t), \ u(\boldsymbol{x},t,\boldsymbol{\xi}) - \sum_{j=1}^M K_i \boldsymbol{\varPhi}_j(\boldsymbol{x},\boldsymbol{\xi}) T_j(t) \right\rangle_{\mathbb{X}} dt$$
(2.14)

According to the definition in Eqn. 2.8, by taking the spatial inner product, the randomness in Eqn. 2.14 is removed by the ensemble average operation, the error functional becomes

$$\begin{aligned} \mathscr{E}[T_1, \cdots, T_M] &= \int_{\mathbb{T}} \int_{\mathbb{X}} \left[u(\boldsymbol{x}, t) - \sum_{i=1}^M K_i \boldsymbol{\varPhi}_i(\boldsymbol{x}) T_i(t) \right] \left[u(\boldsymbol{x}, t) - \sum_{j=1}^M K_j \boldsymbol{\varPhi}_j(\boldsymbol{x}) T_j(t) \right] \, d\boldsymbol{x} \, dt \\ &= \int_{\mathbb{T}} \int_{\mathbb{X}} u^2(\boldsymbol{x}, t) - 2u(\boldsymbol{x}, t) \sum_{i=1}^M K_i \boldsymbol{\varPhi}_i(\boldsymbol{x}) T_i(t) + \sum_{i=1}^M K_i \boldsymbol{\varPhi}_i(\boldsymbol{x}) T_i(t) \sum_{j=1}^M K_j \boldsymbol{\varPhi}_j(\boldsymbol{x}) T_j(t) \, d\boldsymbol{x} \, dt \\ &= \int_{\mathbb{T}} \int_{\mathbb{X}} u^2(\boldsymbol{x}, t) - 2u(\boldsymbol{x}, t) \sum_{i=1}^M K_i \boldsymbol{\varPhi}_i(\boldsymbol{x}) T_i(t) + \sum_{i=1}^M K_i^2 \boldsymbol{\varPhi}_i^2(\boldsymbol{x}) T_i^2(t) \, d\boldsymbol{x} \, dt \\ &= \int_{\mathbb{T}} \left[\int_{\mathbb{X}} u^2(\boldsymbol{x}, t) d\boldsymbol{x} - 2\sum_{i=1}^M K_i T_i(t) \int_{\mathbb{X}} u(\boldsymbol{x}, t) \boldsymbol{\varPhi}_i(\boldsymbol{x}) d\boldsymbol{x} + \sum_{i=1}^M K_i^2 T_i^2(t) \right] \, dt \\ &= \int_{\mathbb{T}} F(t, T_1, \cdots, T_M, T_1', \cdots, T_M') dt \end{aligned}$$

where

$$F(t, T_1, \cdots, T_M, T'_1, \cdots, T'_M) = \int_{\mathbb{X}} u^2(\boldsymbol{x}, t) d\boldsymbol{x} - 2\sum_{i=1}^M K_i T_i(t) \int_{\mathbb{X}} u(\boldsymbol{x}, t) \boldsymbol{\Phi}_i(\boldsymbol{x}) d\boldsymbol{x} + \sum_{i=1}^M K_i^2 T_i^2(t).$$
(2.15)

In above derivation, the orthogonality of basis functions $\boldsymbol{\Phi}_i$ and T_i is applied. According to Theorem 1, the error functional $\mathscr{E}[T_1, \cdots, T_M]$ has extremum when T_i satisfy Euler's equations

$$F_{T_i} - \frac{d}{dx} F_{T'_i} = 0. (2.16)$$

That is

$$\int_{\mathbb{X}} u(\boldsymbol{x}, t) \boldsymbol{\Phi}_i(\boldsymbol{x}) d\boldsymbol{x} - K_i T_i(t) = 0, \qquad (2.17)$$

where

$$T_i(t) = \frac{1}{K_i} \langle u(\boldsymbol{x}, t, \boldsymbol{\xi}), \boldsymbol{\Phi}_i(\boldsymbol{x}, \boldsymbol{\xi}) \rangle_{\mathbb{X}}.$$
(2.18)

Applying temporal inner product $\langle \cdot, T_i \rangle_T$ to Bi-orthogonal Decomposition (Eqn. 2.5) and considering the orthogonality of the temporal basis functions yields

$$\boldsymbol{\Phi}_{i}(\boldsymbol{x},\boldsymbol{\xi}) = \frac{1}{K_{i}} \langle u(\boldsymbol{x},t,\boldsymbol{\xi}), T_{i}(t) \rangle_{\mathbb{T}}.$$
(2.19)

Above two equations define a coupled relationship between T_i and $\boldsymbol{\Phi}_i$. Now that we have two unknowns and two equations, they can be solved by substituting Eqn. 2.19 into Eqn. 2.18, which results in eigenvalue problem for temporal modes

$$K_i^2 T_i(t) = \int_{\mathbb{T}} C(t, t') T_i(t') dt', \qquad (2.20)$$

where C(t, t') is called temporal covariance that can be obtained by taking the inner product in spatial domain, i.e

$$C(t,t') = \langle u(\boldsymbol{x},t,\boldsymbol{\xi}), u(\boldsymbol{x},t',\boldsymbol{\xi}) \rangle_{\mathbb{X}}.$$
(2.21)

Setting $\mu_i = K_i^2$, Eqn. 2.20 can be transformed to

$$\mu_i T_i(t) = \int_T C(t, t') T_i(t') dt', \qquad (2.22)$$

where μ_i and $T_i(t)$ are eigenvalues and eigenfunctions of the covariance function C(t, t'). The optimal choice of temporal modes T_i and $\boldsymbol{\Phi}_i$ can be obtained by solving the eigenvalue problem.

Once μ_i and T_i are solved for, the spatial-stochastic functions are calculated using

$$\boldsymbol{\Phi}_{i}(\boldsymbol{x},\boldsymbol{\xi}) = \frac{1}{\sqrt{\mu_{i}}} \langle \boldsymbol{u}(\boldsymbol{x},t,\boldsymbol{\xi}), T_{i}(t) \rangle_{\mathbb{T}}.$$
(2.23)

It should be mentioned that the first M (usually $M \sim 3-6$) eigenvalues and eigenfunctions usually represent the data exceedingly well [16]. Thus, the first stage of the decomposition of the wind data results in representation involving temporal functions T_i and spatial-stochastic functions $\boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{\xi})$. Eqn. 2.5 now becomes

$$\boldsymbol{u}(\boldsymbol{x}, t, \boldsymbol{\xi}) = \sum_{i=1}^{M} \boldsymbol{a}_i(\boldsymbol{x}, \boldsymbol{\xi}) \ T_i(t), \qquad (2.24)$$

where

$$\boldsymbol{a}_i(\boldsymbol{x},\boldsymbol{\xi}) = K_i \boldsymbol{\Phi}_i(\boldsymbol{x},\boldsymbol{\xi}) = \sqrt{\mu_i} \; \boldsymbol{\Phi}_i(\boldsymbol{x},\boldsymbol{\xi}). \tag{2.25}$$

are spatial-stochastic modes.

Remark 1. We chose to decompose the data into spatial-stochastic and temporal parts in the first stage of the decomposition. This decomposition is one of three possible decomposition choices. These choices are enumerated in Table 2.1 where \mathbb{X} , \mathbb{T} , and Σ denote the spatial, temporal, and stochastic domains respectively. Such decompositions have been explored in other works. For instance, Venturi et al. [70, 69] investigated Type 1 decomposition. The decomposition suggested by Type 2 can be achieved by using generalized polynomial chaos [77, 78]. Type 3 decomposition is used in [46] to model uncertain cylinder wake. It can be shown that Type 1 and Type 3 decompositions will result in identical result.

 Table 2.1
 Choices for Bi-orthogonal Decomposition.

Type	Space 1	$Space \ 2$
1	$\mathbb{X}\times \varSigma$	T
$\mathcal{2}$	$\mathbb{X}\times\mathbb{T}$	Σ
3	$\mathbb{T}\times \varSigma$	X

Remark 2. The choice of the inner products (temporal Eqn. 2.7 and spatial-stochastic Eqn. 2.8) affect the properties of the decomposition. In particular, there are several different ways in which we can define an inner product over the spatial-stochastic modes (i.e. different ways to average over space and stochastic dimensions). These include the following possibilities:

$$egin{aligned} &\langle oldsymbol{\Phi}_i, oldsymbol{\Phi}_j
angle_0 = \int_{\mathbb{X}} \overline{oldsymbol{\Phi}_i} \cdot \overline{oldsymbol{\Phi}_j} \,\, doldsymbol{x}, \ &\langle oldsymbol{\Phi}_i, oldsymbol{\Phi}_j
angle_1 = \int_{\mathbb{X}} \overline{oldsymbol{\Phi}_i \cdot oldsymbol{\Phi}_j} \,\, doldsymbol{x}, \ &\langle oldsymbol{\Phi}_i, oldsymbol{\Phi}_j
angle_2 = \int_{\mathbb{X}} \overline{oldsymbol{\Phi}_i \cdot oldsymbol{\Phi}_j} - \overline{oldsymbol{\Phi}_i} \,\, \cdot \overline{oldsymbol{\Phi}_j} \,\, doldsymbol{x} \end{aligned}$$

The first inner-product (denoted by $\langle \cdot \rangle_0$) is a spatial integral of the product of expected values, while the second inner-product (denoted by $\langle \cdot \rangle_1$) is the expectation of the spatial integral. It can be shown that by taking inner products, $\langle \cdot \rangle_h$, of type h = 0, 1, 2, we obtain optimal representations with respect to mean, second-order moment, and standard deviation of the data, respectively. We have chosen to focus on representation that is optimal in the mean sense [70].

2.1.2 Stage 2: Karhunen-Loève expansion of spatial stochastic modes

In this stage, we decompose the spatial-stochastic functions $a_i(x, \xi)$ into a spatial part and a set of uncorrelated random variables. First, the mean of spatial-stochastic functions $\bar{a}(x)$ are removed so that only the fluctuation component is left for analysis.

$$\boldsymbol{\alpha}(\boldsymbol{x},\boldsymbol{\xi}) = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{\xi}) - \bar{\boldsymbol{a}}(\boldsymbol{x}). \tag{2.26}$$

Following the rational of Bi-orthogonal Decomposition (Eqn. 2.5), our goal is to decompose $\alpha(x, \xi)$ into a minimal set of linear combination of deterministic spatial functions and uncorrelated random variables, that is

$$\boldsymbol{\alpha}(\boldsymbol{x}, \boldsymbol{\xi}) \approx \sum_{i=1}^{N} C_i \, \xi_i \, \boldsymbol{X}_i(\boldsymbol{x}).$$
(2.27)

where C_i are coefficients of the expansion, ξ_i is a set of uncorrelated random variables, X_i are deterministic spatial functions. We pose this decomposition problem as an optimization problem, where the optimization problem is to minimize the error. The representation error is defined as

$$\boldsymbol{\varepsilon} = \boldsymbol{\alpha}(\boldsymbol{x}, \boldsymbol{\xi}) - \sum_{i=1}^{N} C_i \, \xi_i \, \boldsymbol{X}_i(\boldsymbol{x}) \tag{2.28}$$

This is a standard formulation of the Karhunen-Loève expansion. The goal of KLE is to find the optimal choice for functions X_i such that the representation error is minimized with a finite number (N) of expansion terms. We briefly describe the mathematical framework of KLE below [20]: The representation error is converted into a cost-functional for optimization by simply considering the mean-square error (i.e. the inner-product)

$$\mathscr{E}^2 = \int_{\mathbb{X}} \overline{\varepsilon}^2 \, d\boldsymbol{x}. \tag{2.29}$$

Minimization of the error-functional results in an eigenvalue problem, whose eigenfunctions are the desired spatial functions X_i

$$\int_{\mathbb{X}} R(\boldsymbol{x}_1, \boldsymbol{x}_2) \, \boldsymbol{X}_i(\boldsymbol{x}_2) \, d\boldsymbol{x}_1 = C_i^2 \, \boldsymbol{X}_i(\boldsymbol{x}_2), \qquad (2.30)$$

where $R(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the covariance kernel constructed from the spatial-stochastic functions $\boldsymbol{\alpha}(\boldsymbol{x}, \boldsymbol{\xi})$. We denote $C_i^2 = \lambda_i$ so that λ_i and $\boldsymbol{X}_i(\boldsymbol{x})$ are the eigenvalues and the eigenvectors of the covariance kernel. Eqn. 2.27 becomes

$$\boldsymbol{\alpha}(\boldsymbol{x},\boldsymbol{\xi}) \approx \sum_{i=1}^{N} \sqrt{\lambda_i} \, \xi_i \, \boldsymbol{X}_i(\boldsymbol{x}). \tag{2.31}$$

In order to solve the eigen-problem (Eqn. 2.30), the covariance kernel must be given or calculated from measured data. The Wiener-Khinchin theorem allows computing the covariance in a very efficient way.

Given realizations of the spatial-stochastic mode, i.e. $\alpha(x)$, the covariance, R, can be computed as:

$$R = \mathscr{F}^{-1}(\mathscr{F}(\boldsymbol{\alpha}) \times \mathscr{F}(\boldsymbol{\alpha})').$$
(2.32)

where $\mathscr{F}(\alpha)'$ is the complex conjugate of Fourier transform of α , and the diagonal entries of R contains the covariance.

To solve Eqn. 2.30 numerically, the equation should be discretized first. To this end, eigenfunction can be approximated by linear combination of N basis functions [20]

$$\boldsymbol{X}_{k}(\boldsymbol{x}) = \sum_{i=1}^{N} d_{i}^{(k)} h_{i}(\boldsymbol{x}).$$
(2.33)

Substitute above equation to the eigen-equation and set the error to be orthogonal to each basis function yields

$$\sum_{i=1}^{N} d_{i}^{(k)} \left[\int_{\mathbb{X}} \left[\int_{\mathbb{X}} R(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}) h_{i}(\boldsymbol{x}_{2}) d\boldsymbol{x}_{2} \right] h_{j}(\boldsymbol{x}_{1}) d\boldsymbol{x}_{1} - \lambda_{n} \int_{\mathbb{X}} h_{i}(\boldsymbol{x}) h_{j}(\boldsymbol{x}) d\boldsymbol{x} \right] = 0.$$
(2.34)

Above equation can be written in matrices form

$$AD = BD\Lambda. \tag{2.35}$$

$$A_{ij} = \int_{\mathbb{X}} \int_{\mathbb{X}} R(\boldsymbol{x}_1, \boldsymbol{x}_2) h_i(\boldsymbol{x}_1) h_j(\boldsymbol{x}_2) d\boldsymbol{x}_1 d\boldsymbol{x}_2.$$
(2.36)

$$B_{ij} = \int_{\mathbb{X}} h_i(\boldsymbol{x}) h_j(\boldsymbol{x}) d\boldsymbol{x} = \int_{\mathbb{X}} H^T(\boldsymbol{x}) H(\boldsymbol{x}) d\boldsymbol{x}.$$
 (2.37)

$$D_{ij} = d_i^{(j)}.$$
 (2.38)

$$\Lambda_{ij} = \delta_{ij}\lambda_i. \tag{2.39}$$

where $H(\boldsymbol{x}) = (h_1(\boldsymbol{x}), h_2(\boldsymbol{x}), \cdots, h_N(\boldsymbol{x}))$. Matrix A can be rewritten as

$$\begin{split} A &= \int_{\mathbb{X}} \int_{\mathbb{X}} H^{T}(\boldsymbol{x}_{1}) R(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}) H(\boldsymbol{x}_{2}) d\boldsymbol{x}_{1} d\boldsymbol{x}_{2} \\ &= \int_{\mathbb{X}} \int_{\mathbb{X}} H^{T}(\boldsymbol{x}_{1}) H(\boldsymbol{x}_{1}) R H^{T}(\boldsymbol{x}_{2}) H(\boldsymbol{x}_{2}) d\boldsymbol{x}_{1} d\boldsymbol{x}_{2} \\ &= \int_{\mathbb{X}} H^{T}(\boldsymbol{x}_{1}) H(\boldsymbol{x}_{1}) d\boldsymbol{x}_{1} R \int_{\mathbb{X}} H^{T}(\boldsymbol{x}_{2}) H(\boldsymbol{x}_{2}) d\boldsymbol{x}_{2} \\ &= BRB. \end{split}$$

and

$$\begin{aligned} R(\boldsymbol{x}_k, \boldsymbol{x}_l) &= \sum_{i=1}^N \sum_{j=1}^N h_i(\boldsymbol{x}_k) R_{ij} h_i(\boldsymbol{x}_l) \\ &= h_k(\boldsymbol{x}_k) R_{kl} h_l(\boldsymbol{x}_l) \\ &= R_{kl}. \end{aligned}$$

2.1.3 Stage 3: density estimation

The final step of the formulation is to identify the probability distributions of the uncorrelated random variables ξ_i . Note that we have same number of realizations of ξ_i as the number of random samples of the stochastic wind. Each realization is computed by inverting Eqn. 2.31 for ξ_i :

$$\xi_i = \frac{1}{\sqrt{\lambda_i}} \int_{\mathbb{X}} \boldsymbol{\alpha} \boldsymbol{X}_i(\boldsymbol{x}) d\boldsymbol{x}.$$
 (2.40)

Density estimation is a process of estimating the probability density function (PDF) of an undefined stochastic process based on observed data. Many approaches to density estimation have been developed, which can roughly be divided into two categories, namely parametric estimation and non-parametric estimation. Maximum likelihood estimation (MLE)[11] and Bayesian estimation[17] are two examples of parametric estimation. In MLE, the parameters of the targeting distribution are assumed as fixed but unknown. Then the parameters are found with certain optimization method such that the resulting PDF best describes the data observation. Bayesian estimation assumes the parameters are random variables with some known priori distributions. The distribution of the undefined process can be calculated using theorem of total probability and Bayes rule. To perform these methods, whether the targeting distribution or the prior density of parameter need to be known, which is usually not given or easily constructed. In this case, non-parametric density estimation can be used.

The simplest form of non-parametric density estimation is histogram which has drawbacks, such as discontinuity, dependence on the starting position, and the dependence of the number of samples. To avoid these shortcomings, more advanced approaches have been developed. The general expression for non-parametric density estimation is

$$p(x) \cong \frac{k}{NV},\tag{2.41}$$

where V is the volume surrounding x, N is the total number of samples, k is the number of examples inside V. Choosing a fixed value of k and determine the corresponding volume V from the data yields the k Nearest Neighbor (kNN) approach[14]. On the other hand, if choosing a fixed value of V and determine k from the data, Kernel Density Estimation (KDE) is resulted. A good introduction of KDE could be found in[58].

With respect to addressing stochasticity in wind, all wind simulation methods can be divided into two categories, i.e. parametric and nonparametric methods. All methods discussed in the introduction section use parametric methods. In most cases, the probability density function of the velocity fluctuations in homogeneous turbulence is assumed to be Gaussian or sub-Gaussian [31]. Although this assumption is often valid for most of the physical phenomena (including wind at a flat terrain) in nature, it is not necessarily true in the case of complicated terrain. On the other hand, although nonparametric methods are robust and easy to implement, its accuracy strongly depends on number of available data samples.

In this research, a nonparametric method, named kernel density estimation (KDE), is used to describe the stochasticity of the turbulence. It is worth noting that the applicability of the framework developed in this context should not depend on the choice of density estimation techniques. The choice of technique to construct the probability distribution of ξ_i given a finite number of observations of ξ_i is crucial. We utilize Kernel Density Estimation (KDE) methods [58] to construct the probability distributions of ξ_i in a non-parametric way.

In KDE, the PDF of a random variable ξ is estimated as

$$p(\xi) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{\xi - \xi_i}{h}\right), \qquad (2.42)$$

where $K(\cdot)$ is the kernel which is a symmetric function that integrates to one, and h > 0 is a smoothing parameter called the bandwidth. Common choices for $K(\cdot)$ are the multivariate Gaussian density function. The choice of the bandwidth is critical since a small values of h result in the estimated density with many 'wiggles', while large values of h result in very smooth estimations that do not represent the local distributions. Several approaches have been developed to chose proper h values. A good review on bandwidth selection can be found in [64]. We advocate using a simple formula based on Silverman's rule [60]. The optimal choice for haccording to Silverman's rule is given by

$$h = \left(\frac{4\hat{\sigma}}{3N}\right)^{\frac{1}{5}} \approx 1.06\hat{\sigma}n^{-\frac{1}{5}},$$
(2.43)

where $\hat{\sigma}$ is the standard deviation of the samples.

Based on above three techniques, we are now able to construct space-time decomposition of the wind filed snapshots and preserve the spatial and temporal correlations. In next chapter, we look at an numerical example that illustrate the power of this framework.

2.1.4 Algorithm & implementation

The algorithmic details of the framework is outlined in Table 2.2. The matrix of wind flow snapshots is first constructed. The temporal covariance matrix is calculated from the snapshot matrix and stored as a datafile. Next, the eigenvalue problem (corresponding to the Bi-orthogonal Decomposition) is solved to get temporal and spatial-stochastic modes. Following this, the spatial-stochastic modes are decomposed into spatial functions and uncorrelated random variables via the Karhunen-Loève expansion. This is accomplished by solving an eigenvalue problem. The eigenvectors computed are the spatial functions and they are used to compute realizations of the uncorrelated random variables (via Eqn. 2.40). Finally, a Kernel Density Estimator is used to construct the PDFs of random variables ξ_j^i . At this time, all the necessary elements for constructing synthetic wind flow were known. Finally, synthetic wind realizations are constructed by reversing above process. The complete framework is implemented in C++. SLEPc [26], which is based on PETSc [5], was used in solving eigenvalue problems. The complete framework can be downloaded from this URL [23].
Table 2.2 Steps of computational framework.

- 1. Data preparation
 - (a) Construct matrix of wind speed field snapshots from given data
- 2. Bi-orthogonal Decomposition
 - (a) Calculate temporal covariance from snapshots matrix
 - (b) Solve eigenvalue problem, get eigenvalues and eigenfunctions (temporal modes)
 - (c) Solve spatial stochastic modes
- 3. Karhunen-Loève expansion
 - (a) Calculate covariance kernel of each spatial mode (M spatial modes in total)
 - (b) Solve eigenvalue problem for each spatial mode, get basis spatial functions
 - (c) Calculate observations of each random variable ξ^i_j based on Eqn. 2.40
- 4. Kernel density estimation
 - (a) Estimate PDF for each random variable ξ^i_j based on observations
- 5. Construct synthetic wind flow
 - (a) Get random samples from random variables ξ_i^i
 - (b) Construct stochastic spatial modes according to Eqn. 2.31
 - (c) Construct synthetic wind flow using Eqn. 2.5

2.2 Discussion and Conclusion

Incorporating the effects of randomness in wind is critical for a variety of application involving wind energy. Continuous advances in data-sensing and meteorology has made possible the availability of large data-sets of location-, topography-, diurnal-, and seasonal sensitive meteorology data. While this data contains rich information, ease of use is bottlenecked by the unwieldy data-sizes. A pressing challenge is to utilize this data to construct a location-, topography-, diurnal-, and seasonal-, dependent low-complexity model that is easy to use and store. We formulate a data-driven mathematical framework that is capable of representing the spatial- and temporal- correlations as well as the inherent randomness of wind into a low-complexity parametrization. We leverage data-driven decomposition strategies like Biorthogonal Decomposition and Karhunen-Loève expansion for constructing the low-complexity model. We provide a software package that can be used to construct the low-complexity model to the community.

CHAPTER 3. NUMERICAL EXAMPLES OF TSD

Chapter 2 provides data-driven low-complexity turbulence simulation tool that is powerful in representing location- and topography specific wind given comprehensive wind field measurement. In this chapter, we test the framework by providing three numerical examples. Analysis on the examples include generating synthetic stochastic wind flow and comparing several wind turbulence statistics between synthetic and original flows.

3.1 Numerical Example 1: CWEX-11

First, we illustrate the methodology based on a recent meteorological experiment named CEWX-11. CWEX-11 is a collaborative experiment (Iowa State University (ISU) and the University of Colorado (UC), assisted by the National Center for Atmospheric Research (NCAR)). CWEX-11 and its 2010 counterpart (CWEX-10) in the Crop Wind-energy EXperiment, address observational evidence for the interaction between large wind farms, crop agriculture, and surface-layer, boundary-layer, and mesoscale meteorology [52].

3.1.1 Experiment setup

In the experiment, a surface flux station was installed to the south of a wind turbine, which makes the station measure upstream inflow of the wind turbine due to the fact that predominant summer winds in Iowa originate from south to slightly south-east. The surface station was equipped with a CSAT3 sonic anemometer that was located at height 4.5 m and an RMY propeller and vane anemometer that was located at height 10.0 m. The former measured wind speed in 3 directions at 20 Hz whereas the latter gave wind speed amplitude and its direction at 1 Hz. The experiment started on June 29 and lasted for 48 days.¹

A schematic of the experiment is shown in Fig. 3.1. Note that the figure is for illustrative purpose and is not drawn to scale. In this experiment, only wind speed magnitude was analyzed, but it is straightforward to perform analysis on any one of the three components of turbulence.



Figure 3.1 Setup of experiment

Ideally, we would like to have multiple wind field snapshots taken at regular intervals on a vertical plane that is perpendicular to the rotor. However, due to the constrain of the experiment, only two time-series were measured at heights 4.5 m and 10.0 m. Linear interpolation of the two time-series at 18 spatial points on z direction were performed to simulate more vertical measurements. The wind snapshots are constructed by using *Taylor's frozen turbulence hypothesis* [62]. Specifically, all measurements in the resulting 20 time-series over certain interval were treated as if they were taken at the same instant in the interval. This would provide us along-wind (x direction) measurements. Finally, one 2D snapshot taken at certain instant is constructed². Similarly, snapshots of the wind field at other instants were created to represent one full day of measurements. We have data for 28 such days. The full meteorology data

¹Excluding the days that mostly have opposite wind direction and the days with sudden rain, only data for 28 days that had the desired weather condition and wind direction were used in the analysis.

 $^{^{2}}$ Since no measurements were taken in transverse direction, only 2D wind field is analyzed in this analysis.

curated into this form can be represented as following matrix

In the matrix, each element $\{v_{ij}(x,z), i = 1, 2, \dots, m; j = 1, 2, \dots, n\}$ represents a snapshot of wind field. Each column contains snapshots that span an entire day. Each row represents data for a particular interval measured over different days. Thus, each row of data can be considered to be *realizations* of the stochastic wind field at a particular interval.

It is noteworthy that the length of time interval that was used to obtain wind field snapshots can be arbitrarily chosen. Although, most guidelines for wind turbine design and siting suggest considering wind variabilities over a period of 10 minutes for wind data analysis [18], in this example, analysis based on different choices of time intervals were performed and compared. In the results section, we will show that using 10 minutes time interval to construct wind field snapshots is a reasonable choice.

The meteorological data is provided in the common NetCDF (Network Common Data Form) format. The original meteorological data contains a variety of information, including wind speed and orientation, temperature, humidity, surface CO_2 flux. For this analysis we only consider the wind speed data. The wind speed data is extracted from the NetCDF file using MATLAB. TSD software package is then used to perform the decomposition and construct synthetic wind flow.

3.1.2 BD results

As discussed previously, CWEX-11 meteorological data consists of wind speed amplitude measured at 1 Hz over 28 days. Snapshots curated based on 10 minutes interval are used first and the reason of doing so is going to be explained in the following part. As a result, there are 144 snapshots in one day and we have 28 such days (samples). We will describe the stage wise construction of the low-dimensional model and its accuracy in the next few subsections. The mean component of the flow that is averaged across all 144 snapshots and 28 realizations according to Eqn. 2.2 is shown in Fig. 3.2. This plot shows us the mean wind profile such as vertical wind shear load. Note that the x-axis represents length of the snapshots that was calculated based on 10 minute interval and average wind speed.



Figure 3.2 10 minute period, averaged across all 144 time intervals and 28 realizations

The temporal covariance are calculated based on Eqn. 2.21. By taking the inner products across the spatial domain, we construct the covariance function C(t,t') which is shown in Fig. 3.3. Note that C has block structure, with regions of high covariance (marked by the solid boxes) along the diagonal and regions of large negative covariance along the off-diagonal. This structure of the covariance function follows the dynamics of stable, and unstable stratification of the atmospheric boundary layer seen in the US central plains. We discuss this by dividing the analysis into several distinct time periods (marked by the solid boxes). Following meteorological practice, the data starts at Coordinated Universal Time (UTC, or Greenwich time) 00:00. The first period is UTC 00:00-06:00 (CST 18:00-00:00) which corresponds to the time between sunset to midnight. In this period, insolation and, thus, heating is gradually cut off and the temperature of atmosphere cools down (from the ground up) due to rapid cooling of the ground. Because of the heavier density of the cooler air, the cooler air stays at the bottom (close to the surface). This generates a stably stratified boundary layer that does not change during the duration of the night. This results in the high covariance between adjacent time periods marked in the lower left box. The second period is the region of reduced covariance between UTC 06:00-13:00 which is basically the time from midnight to shortly after sunrise. During summer, the nocturnal Great Plains low-level jet (LLJ) exists in Iowa. For a good overview of LLJ, see [30]. The LLJ causes low level turbulence and enhanced mixing which reduces the covariance between adjacent-time wind fields. The third time period is between UTC 13:00-00:00 which corresponds to sunrise and day time. Because of the sunrise, the ground is rapidly heated. As a result, the warmer air near the ground becomes buoyant and rises rapidly with its place being taken by compensating flow of colder air from higher up in the atmospheric boundary layer. Eddies are thereby created, changing from smaller scale to larger scales, finally becoming the circulatory motion that crosses the entire boundary layer so that the high speed free stream flow is brought in the circulation. This phenomenon usually happens at noon, which can also be found in Fig. 3.5 at UTC 18:00. In addition, the covariance function exhibits periodic behavior that is caused by the periodical motion of eddies.



Figure 3.3 Covariance function C(t, t')

The temporal covariance function plays a very important role in this analysis since it provides almost all the needed information that describes the behavior of the wind flow. Compared to the original meteorological data, the temporal covariance is much easier to store and transmit, yet contains nearly the same amount of information as the original meteorological data. The eigenvalues of the temporal covariance function are shown in Fig 3.4. Fig 3.4(a) compares the relative magnitudes of eigenvalues; the magnitudes of the eigenvalues provide a notion of how much energy about the data is stored in each spatial mode [27]. Notice that the first eigenvalue is much larger than the other subsequent eigenvalues, which means the first mode contains largest portion of the energy in the turbulence field. In Fig 3.4(b) this is represented as the cumulative fraction of energy contained in the first k modes. This plot provides us with a precise notion of how many terms are needed to incorporate [27], say, 90% of the information available in the data into the low-complexity model. The first five eigenvalues cover about 90 % of the total energy of the turbulence field, which ensures that a five term decomposition will have a 90% accuracy of representation. This illustrates the advantage of Bi-orthogonal Decomposition. As a reduced order model, Bi-orthogonal Decomposition is able to represent a random flow with much fewer random modes compared to the number of original snapshots. In other words, by using BD, we reduced the terms in representing the given stochastic random flow from 144 snapshots to a simple 5-variable parametrization.



Figure 3.4 Spectrum of C(t, t')

Fig. 3.5 shows the first three eigenfunctions of C(t, t'). Note that the eigenfunctions are the temporal functions T_i . They track how the fluctuation component of the random flow varies during a day. Based on the plot, the largest mean (and standard deviation, which is not reported here) of wind speed occurs at UTC 18:00 because of the reason stated previously. From Fig. 3.4, we notice that the first mode carries about 85% of the total energy in the turbulence. Therefore, the trend of diurnal fluctuation of the turbulence field can be mostly seen in the first temporal mode, with the next two temporal modes look like white noise. Once



Figure 3.5 First three eigenvectors of temporal covariance function

eigenvalues and eigenvectors for temporal covariance function are solved, the spatial-stochastic modes $a_i(x, \xi)$ can be constructed (Eqn. 2.23 and Eqn. 2.25). Fig. 3.6 shows the expectation of the first three spatial modes. It is clear that the first spatial mode that carries the largest part of turbulence energy describes the vertical shear pattern, while the second mode describes the lateral shear pattern of the wind field. Higher modes that account for more complicated turbulence structures are insignificant since they contain limited turbulence energy.

Based on Remark 2, different choices of inner products result in optimal representation with respect to different statistical properties of turbulence. For the sake of illustration, BD results calculated by using inner product h = 2 are shown in Fig. 3.7, 3.8, 3.9, and 3.10. It is worth noting that by choosing inner product $\langle \cdot \rangle_2$, we are solving the eigenvalue problem with a different covariance kernel. The diagonal of Fig. 3.7 represents the (spatial) average of the flow standard deviation [70]. It can be seen that the maximum standard deviation occurs at UTC 18:00 which is the same time as maximum mean wind speed occurs (see in Fig 3.3). In addition, Fig. 3.8 shows that in order to get 90% of representation accuracy with respect to higher order statistics of turbulence, much more eigenvalues are needed. The first three eigenvectors (temporal modes) are shown in Fig. 3.9. Unlike Fig. 3.5 that corresponds to the case of h = 0, where the second and third temporal modes are "white noises", all three temporal modes have apparent trends of wind speed fluctuation. This is because the first



Figure 3.6 Stochastic spatial modes of C(t, t')

three eigenvalues only preserve 55% of total energy, which means none of them dominates the turbulence. Similar result is shown in Fig. 3.10 all three spatial modes contribute to the vertical sheer load.

Although analysis based on inner product type h = 2 shows many interesting results, when it comes to the resulted synthetic wind flow, we found that it is very similar to the case of h = 0. In fact, the two simulated flows only differ at the accuracies when representing the random flow for different applications. Therefore, in the following context, only the results using inner product h = 0 are shown.

3.1.3 KLE results

Using the technique introduced in section 2.1.2, the spatial-stochastic modes are decomposed into deterministic modes and random variables. KLE results of the first three spatial modes are shown in Table 3.1. Covariance matrices, eigenvalues, deterministic spatial modes, and probably distributions of random variables are shown in the table from top to bottom. As











Figure 3.9 First three eigenvectors of $C^{(2)}(t,t')$



Figure 3.10 Stochastic spatial modes of C(t, t')

discussed in Sec. 2.1.3, the distributions of ξ_j^i are estimated by using KDE.

From Fig. 3.6 we notice that as the mode order increases, the complexity of spatial mode also grows. As a result, higher order spatial modes require more KLE terms to be accurately represented. This statement can be verified by comparing results of the first three spatial modes. For instance, the smoothness of the covariance kernels, the complexity of deterministic modes, and the required numbers of KLE terms to achieve 90% of representation accuracies all increases as the mode number becomes higher.

It is worth to mention that the PDFs of ξ_j^i are not always Gaussian. Although Gaussian assumption is commonly used in turbulence analysis, it is seen from Fig. 3.1 that Gaussian assumption may not always be valid. Notice that each random sample corresponds to one day wind history, the randomness of this long term stochastic process may be different from the randomness of the more commonly used 10-minute average wind speed whose distribution is always regarded as Gaussian or Weibull.



3.1.4 The low-complexity wind model

The above two stages result in the calculation of the temporal functions, $T_i(t)$ (from Biorthogonal Decomposition), the spatial functions $(\mathbf{X}_j^i(\mathbf{x}))$ (from KLE decomposition), and the uncorrelated random variables, ξ_j^i (from the KDE fitting).³ Putting it all together gives us the low-complexity model for the wind

$$\boldsymbol{u}(\boldsymbol{x},t,\boldsymbol{\xi}) = \sum K_{ij} T_i(t) \boldsymbol{X}_j^i(\boldsymbol{x}) \boldsymbol{\xi}_j^i$$
(3.1)

Note that X_j^i and T_i are deterministic functions that encode spatial and temporal correlations of the wind. Different realizations (or stochasticity) of the wind is included into the low-complexity model via the uncorrelated random variables, ξ_j^i . Note that the probability distributions of ξ_j^i are constructed in a data-driven way from the meteorology data. As we will show in the results section, only a few terms (M = 3) are required to reconstruct a synthetic wind snapshot that **contains all the temporal and spatial correlations exhibited by the original data**. Fig. 3.11 gives a graphical description of this process. Synthetic wind snapshots exactly mimicking the meteorological data can be constructed by sampling from the distributions of the random variables, ξ_j^i . Interestingly, only 800 KB of storage space is needed to store all the necessary information of the reduced-order model, whereas more than 200 MB of storage space is needed to store all the wind flow snapshots that are used in the analysis! This demonstrates the advantage of the low-complexity model in terms of data size.

3.1.5 Statistical comparison

As discussed previously, to get 90% representation accuracy, five terms in Bi-orthogonal Decomposition and seven terms in Karhunen-Loève Expansion are needed. However, for the purpose of demonstration, 1-, 3-, and 10-term BD and three terms in KLE for each BD term were used in the analysis. In order to quantify the accuracy, 28 realizations (same as the number of samples of meteorological data set) of the synthetic wind flow are generated. Each realization is a 24-hour wind data consisting of 144 ten-minute snapshots (see Fig. 3.11 for one 24 hour synthetic dataset).

³The superscript in $X_j^i(\boldsymbol{x})$ and ξ_j^i represents the index of Bi-orthogonal Decomposition terms whereas the subscript denotes the index of KLE terms.



Figure 3.11 The low-complexity model: Process of constructing synthetic wind snapshots

In Fig. 3.12, realizations of the stochastic flow at height 10.0 m using one, three, and ten spatial modes are compared. Result shows that when smaller number of terms are used, the low frequency characteristics of the turbulence can be represented accurately. However, in order to describe high frequency behavior, higher modes in BD must be used.

One of the most important variables of turbulence is the power spectral density (PSD) that describes how the energy in turbulence is distributed to different frequency spectrum. To verify the similarity of the reconstructed wind flow and the meteorological data, their PSD functions are compared in Fig. 3.13 (a). The figure shows that the synthetic wind flow accurately reproduces energy in low frequency region. However, more terms need to be used in the representation to preserve energy in high frequency region. It is worth to mention that compare with POD method, even with only one BD mode, the synthetic wind mimics the true data in great detail.

Coherence spectrum is another important variable that describes the similarity of turbulence at two different spatial locations. Comparison of coherences between same set of time-series used in PSD analysis is given in Fig. 3.13 (b). Note that using only one BD mode, the synthetic wind can still preserve most of the spatial coherence information. This can be explained by the advantages of STD. Since STD performs orthogonal decompositions of both temporal and



Figure 3.12 Realizations of the stochastic flow

spatial covariances, the temporal variations and spatial correlations are preserved in an optimal way.

As previously mentioned, there are different choices of time intervals when constructing wind flow snapshots. In above content, only analysis on 10 minute snapshot was performed. The reason for choosing 10 minute snapshot can be demonstrated by comparing PSDs and coherences of the synthetic turbulence using different choices of time interval averaging processes. Fig. 3.14 shows results of such analysis. Since the accuracy of PSDs of synthetic wind only depends on how many total terms in the decomposition, there is no apparent difference in PSDs of different choices of time intervals. On the other hand, comparison of coherences tells us if performing analysis on snapshots that are constructed from less than 10 minute field, important information in turbulence coherence will be lost. Coherences correspond to 10, 15, and 20 minute snapshots are very close, which means the additional turbulence data provided by 15 and 20 minute snapshots, comparing to 10 minute snapshot, does not contain much more coherence information. This result is consistent with GL guideline that states the coherence structure in turbulence can be assumed to be unchanged in 10 minutes period of time [18].

According to Fig. 3.4, certain number of spatial modes are needed to achieve a desired



Figure 3.13 PSDs and coherences of the synthetic turbulence at 10 m using one, three, and ten modes compared with original flow

representation accuracy. On the other hand, the number of random inputs to the reduced order model equals the number of spatial points on each snapshot. By using different choices of time intervals, the resulting snapshots have different dimensions. Table 3.2 shows the relationship between the number of random inputs and the number of needed spatial modes for certain level of accuracy. Based on the table, when the number of random inputs increases by 20 times, the needed random spatial modes to achieve 94% of accuracy only increases from 3 to 8. In other words, the required number of spatial modes does not strongly depend on the number of random inputs, which makes TSD to be practical for problems with very large stochastic dimensions.

 Table 3.2
 Relationship between the number of random spatial modes (94% accuracy) and the number of random inputs.

 Number of random inputs.
 10000 10000 10000 04000

Number of random inputs	1200	6000	12000	18000	24000
Number of spatial modes	3	5	6	7	8

Finally, since the temporal covariance contains important temporal information, it is worth to compare the temporal covariances of the source and synthetic winds. The temporal covariance function of synthetic data $\hat{C}(t, t')$ is constructed by using the 28 simulated realizations of the synthetic random flow. Comparing it with the temporal covariance of the original data



Figure 3.14 Comparing PSDs and coherences of the synthetic turbulence using different choices of snapshot

reveals that they have almost identical pattern.

The temporal error (or the information loss)



Figure 3.15 Comparison of covariance functions of original (a) and synthetic (b) flows

can be defined as the L^2 -norm of the difference between the covariance functions:

$$\epsilon = \frac{\|C - \hat{C}\|_2}{\|C\|_2} = \left(\frac{\sum_{i=1}^n \sum_{j=1}^n |C(t_i, t_j) - \hat{C}(t_i, t_j)|^2}{\sum_{i=1}^n \sum_{j=1}^n |C(t_i, t_j)|^2}\right)^{1/2}.$$
(3.2)

where n = 144 is the number of snapshots in 24 hours. The temporal information-loss using a 9-term expansion is 2.25%. Thus, a 9-term data-driven expansion reproduces the temporal and spatial covariance of the original meteorological data to 97.75% accuracy. The spatial and temporal functions used in the construction of the low-complexity model provide significant insight into the structure of the wind field. For example, each spatialstochastic mode describes the wind field at different grades of detail [27]. The (stochastic) kinetic energy contained in each spatial-stochastic mode is defined as

$$\theta_i = \frac{1}{2} \int_{\mathbb{X}} [a_k(\boldsymbol{x}, \boldsymbol{\xi})]^2 \, d\boldsymbol{x}.$$
(3.3)

This allows us to calculate the correlation between the kinetic energy content across different spatial-stochastic modes

$$C_{KE}(i,j) = \frac{\overline{\theta_i \theta_j} - \overline{\theta_i} \ \overline{\theta_j}}{\sigma_{\theta_i} \sigma_{\theta_j}},\tag{3.4}$$

where σ_{θ_i} is the standard deviation of the random energy of *i*th spatial stochastic mode. The correlation provide information about how different spatial-stochastic modes exchange energy (i.e. how kinetic energy cascades across length scales). A high correlation between two modes means the energy transition between them is prominent. The mean value and standard deviation of the kinetic energy of different spatial modes are shown in Fig. 3.16.



Figure 3.16 Kinetic Energy of Spatial Modes.

Correlation between the different spatial-stochastic modes is shown in Fig. 3.17. Interestingly, the energy cascade occurs between consecutive and also some prominent non-consecutive modes. For example, it can be seen that the energy transfer from both the first and second modes mainly transfer to the fourth mode, which explains the reason that the kinetic energy of the fourth mode is larger than the third mode (see Fig. 3.16).



Figure 3.17 Correlation of random energy of spatial modes.

3.2 Numerical Example 2: CWEX-10

One of the challenges in wind farm commissioning is the social acceptance of onshore wind turbines. Wüstenhagen et al. defined three dimensions of social acceptance, namely sociopolitical acceptance, community acceptance and market acceptance [73]. Specifically, the community acceptance refers to the public opinion to wind energy project. Although people usually have positive attitudes about wind energy in general, they tend to resist wind projects during the actual on-site planning process with the so called NIMBY (Not In My Back Yard) argument. In fact, the public acceptance of wind projects is influenced by many factors. For example, the financial benefit to the community, the impact of wind farm to public activities, and the awareness of the local population to the benefits and impacts of wind facilities [32]. The impacts of wind farm on environment, wildlife, and people's ordinary life are among the most concerns of the community. The impact of wind turbines on wildlife (birds and bats) has been well studied in [6, 3]. Visual impact (disturbance) of wind turbine is investigated in [10]. A good review on wind turbine acoustic noise is given in [54].

On the other hand, despite the fact that most of the land-based wind resources are located in the states that contain a lot of farmlands, the effect of wind farm on crops is still unclear. It is important to investigate the upwind and downwind variation in air conditions to assess the impact of wind turbines on crops. This necessitates a realistic parameterization of air flow that encodes location-, topography-, diurnal-, seasonal and stochastic affects. However, such a parameterization is useful in practice only if it is relatively simple (low-dimensional). Interestingly, the data to construct such models are available at various resolutions from meteorological measurements. Such meteorology data contain rich information about location- and topography specific temperature, H_2O , and CO_2 concentration data which are important factors in air flow that would affect crop's growth. In this section, we utilized the data-driven framework developed previously to generate space-time parameterizations of the large scale meteorology data measured at upwind and downwind meteorology towers. Comparing the two parameterizations gave us deeper understanding of how wind turbine may variate CO_2 concentration in air flow.

3.2.1 Experiment setup

The data to construct the stochastic model of wind is based on meteorological data measured by the CWEX-10 that addresses observational evidence for the interaction between large wind farms, crop agriculture, and surface-layer, boundary-layer, and mesoscale meteorology (Rajewski et al. 2012) [52]. In the experiment, a surface concentration station (NLAE 1, where NLAE is the abbreviation of National Laboratory for Agriculture and the Environment) was installed 4.5 D (D denotes turbine's rotor diameter) to the south of a wind turbine, which makes the station measure upstream inflow of the wind turbine due to the fact that predominant summer winds in Iowa originate from south to slightly south-east. Three downwind concentration towers (NLAE 2, NLAE 3, and NLAE 4) were placed at 2.5 D, 17 D, and 35 D, respectively, north of the wind turbine. Since the NLAE 3 and NLAE 4 did not measure H_2O and CO_2 , measurements of surface concentrations at height 6.45m from the NLAE 1 and NLAE 2 are used in current analysis. The meteorological data consisted of high-speed (20 Hz) velocity, temperature, CO_2 and moisture measurements upwind and downwind of a working 1.5 MW turbine taken over two months.

Similar to the CWEX-11 example, the meteorology data is curated into a set of time-series vectors. To exclude the effects of environmental factors to the wind field and to make sure

NLAE 1 is upwind tower, days with rain, wrong wind direction, and invalid measurements are taken out from the original data set. As a result, 15 days of measurements are left for analysis. In this work, CO_2 surface concentrations data consisting of certain duration of measurements are treated as one snapshot of the time-series. The full meteorology data curated into this form can be represented as a matrix

In the matrix, each element $\{\vartheta_{ij}(\boldsymbol{x}), i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\}$ represents a snapshot of surface concentration time-series. Each column contains snapshots that span an entire day. Each row represents data for a particular time interval measured over different days. Thus, each row of data can be considered to be *realizations* of the stochastic wind flow at a particular time interval. This naturally motivates us to consider the wind flow as a one-dimensional random field with spatial and temporal variations, $\vartheta(\boldsymbol{x}, t; \boldsymbol{\xi})$, where \boldsymbol{x} , t, and $\boldsymbol{\xi}$ represents the spatial domain, temporal domain, and stochastic variability respectively.

The analysis is clearer when the mean component $\bar{\vartheta}(\boldsymbol{x})$ of the data is removed so that only the fluctuation components $\theta(\boldsymbol{x}, t, \boldsymbol{\xi})$ are left, i.e.

$$\theta(\boldsymbol{x}, t, \boldsymbol{\xi}) = \vartheta(\boldsymbol{x}, t, \boldsymbol{\xi}) - \bar{\vartheta}(\boldsymbol{x}), \qquad (3.5)$$

where the mean component is defined as

$$\bar{\vartheta}(\boldsymbol{x}) = \frac{1}{|\mathbb{T}|} \int_{\mathbb{T}} \overline{\vartheta(\boldsymbol{x}, t, \boldsymbol{\xi})} dt.$$
(3.6)

and |T| is the total time, i.e 24 hours, for one realization.

BD of ϑ is given in the following equation

$$\vartheta(\boldsymbol{x}, t, \boldsymbol{\xi}) \approx \bar{\vartheta}(\boldsymbol{x}) + \sum_{i=1}^{M} \sqrt{\mu_i} \Phi_i(\boldsymbol{x}, \boldsymbol{\xi}) \ T_i(t).$$
(3.7)

where $\Phi_i(\boldsymbol{x}, \boldsymbol{\xi})$, i = 1, ..., M are stochastic spatial modes that are weakly orthogonal in the time domain and $T_i(t)$ i = 1, ..., M are temporal modes that are orthogonal in spatial domain, and $\sqrt{\mu_i}$ are coefficients.

In Eqn. 3.7, $\bar{\vartheta}(\boldsymbol{x})$ represents the mean component and the second term on the right hand side represents the fluctuation part of the quantity of interest in the flow. The variance of the quantity of interest can be defined as

$$Var(\vartheta) = E[(\vartheta - \bar{\vartheta})^2]$$

= $E\left[\left(\sum_{i=1}^M \sqrt{\mu_i} \Phi_i(\boldsymbol{x}, \boldsymbol{\xi}) T_i(t)\right)^2\right]$
= $\sum_{i=1}^M \mu_i$ (3.8)

where we utilized the orthogonality of temporal and spatial modes.

3.2.2 Result: CO₂ uptake

Currently, most guidelines for wind turbine design and siting suggest considering wind variabilities over a period of 10 minutes for wind data analysis [18]. On the other hand, much longer averaging period is accepted for agricultural and atmospheric analysis. This is because changes in air flow conditions need about 20-30 minutes for the boundary layer to achieve a quasi-equilibrium. The process may take much longer at night, which implies even 30-minute average may not be long enough. Without loss of generality, 30-minute average is used in the analysis.

Based on Eqn. 3.6, the mean component of CO_2 concentration is calculated by taking the average on temporal and stochastic domain. Fig. 3.18 shows the comparison of mean components of the upwind and downwind flows. The increment of the mean CO_2 concentration is shown as 1.08% in the figure.

Fig. 3.19 shows the upwind temporal covariance with respect to 30-minute averaging period. Following meteorological practice, the plots start at Coordinated Universal Time (UTC, or Greenwich time) 00:00. This structure of the covariance matrix follows the behavior of metabolism of crops. The temporal covariance functions of upwind tower and downwind tower (not shown in the figure) are very similar, which makes drawing conclusion based on the comparison between them becomes difficult.



Figure 3.18 Mean components of the $\rm CO_2$ concentrations at upwind and downwind towers.



Figure 3.19 Temporal covariance of CO_2 surface concentration

Eigenvalues and cumulative fraction of the first several modes are plotted in Fig. 3.20. Based on Eqn. 3.8, the cumulative of the first M eigenvalues account for the variance of the CO_2 concentration field. It can be seen from the figure that the first eigenvalue accounts for about 99.6% of the total variance of the field, which could imply the CO_2 concentration field has a very simple structure so that it has only one dominating mode. By simply compare the first eigenvalue (since it is so prominent) of upwind and downwind data, we found that the *variance* of CO_2 concentration at downwind position decreases by 8.5%.



Figure 3.20 Eigenvalues (left) and cumulative fraction (right) of the first 20 modes.

The first temporal mode of the upwind and downwind field are given in Fig. 3.21. This figure shows the pattern of fluctuation of CO_2 concentration through time.

To sum up, following primary results are found in this analysis:

- 1. The mean CO_2 concentration at downwind tower increases by 1.08%
- 2. The variance of CO_2 concentration at downwind tower decreases by 8.5%
- 3. The CO_2 concentration field has only one dominating mode

These results suggest that wind turbine may have very limited effect on the crop's growth with respect to CO_2 concentration which is regarded as to be more crucial to crop's growth than CO_2 flux. On the other hand, CO_2 flux is more likely to be affected by wind turbine since



Figure 3.21 The first temporal mode of CO_2 surface concentration

surface flux consists information of turbulence that will certainly agitated by the wind turbine rotor. Therefore, analysis on CO_2 surface flux could show more interesting results.

3.3 Numerical Example 3: Full-field Stochastic Wind Simulation

The meteorological data used in previous two examples have limitations. The CWEX-11 data measures the wind profile at two locations: 4.5 meters and 10 meters. Obviously, this data do not describe the wind speed profile at hub height. Similarly, CWEX-10 only measures CO_2 concentration at height 6.45 m. Measurements at only one or two different heights are certainly not enough to capture all the spatial stochasticity in the wind. Third, the data dose not contain measurements on transverse direction, which makes getting wind flow snapshots on the plane of turbine rotor becomes impossible. To circumvent this insufficiency in measurement, wind filed snapshots on the plane that is perpendicular to the rotor is used, for which certain time interval has to be chose to construct snapshots. It is worth noting that the framework is generally applicable to a variety of meteorology data, and the its applicability should not be affected by choosing different snapshot constructing time intervals.

3.3.1 Experiment setup

Ideally, a good measurement should include high frequency data of wind speeds at different locations on the rotor plane. The accuracy of using LIDAR (LIght Detection And Ranging) to measure wind field has been investigated in [61]. Result of this research shows a promising application of LIDAR in wind turbine simulation. Given that more time is needed to put this technique into practice, a different source of data is used in this chapter to generate synthetic turbulence inflow of wind turbine. Specifically, a turbulence simulator that is developed by NREL, named TurbSim [34], is used to generate multiple realizations of a stochastic wind, which are further used as the source data of the temporal-spatial decomposition (TSD) framework developed in the research. By doing such analysis, we showcase the ability of TSD to reproduce the important turbulent statistic properties of given measured wind data.

TurbSim utilizes Veers/Sandia method [68] together with various spectral models, such as Kaimal [38] model and Von Karman Normal Turbulence Model (NTM) [72] that are recommended in the IEC guidelines [13], to simulate 3-D full-field turbulence. TurbSim also provides several spectral modes according to different terrain conditions and application scenarios. For the purpose of illustration, Kaimal spectral model and the IEC exponential coherence model are used in the simulation. The generated wind is designed for using as inflow of NREL 5MW offshore baseline wind turbine [35]⁴, which has 90 m hub height and 63 m blade tip to rotor center distance (this simulation is given in chapter 4). Therefore, the wind time-series are simulated on a 15×15 rectangular grid on the rotor plane that spans a $150 \text{ m} \times 150 \text{ m}$ area with the center of the grid located at the center of the rotor, which is shown in Fig. 3.22.

30 ten-minute simulations (realizations) of full field wind turbulence on this grid were generated with sampling frequency of 4 Hz. They serve as the original wind data of the TSD. The wind inflow can be looked as a time variant wind speed field on the rotor plane. Snapshots of this wind field are taken every 0.25 second (since the sampling frequency is 4 Hz). For ten-minute simulations, there are 2400 snapshots available. In the simulation, TurbSim used a referencing wind speed of 6.0 m/s at 10.0 m height, which results in about 8.3 m/s mean

⁴It is noteworthy that the NREL 5MW turbine is a conceptual design, thus no structural and material specification is provided.



Figure 3.22 15×15 grid on the 150 m $\times 150$ m rotor plane (yz plane) of the NREL 5MW wind turbine.

wind speed at hub-height.

Note that TurbSim simulates all three components of turbulence. The along-wind component u has the strongest correlation over different spatial scales among the three components [56]. For the purpose of demonstration, only the along-wind turbulence component is decomposed in this analysis. ⁵

3.3.2 Result: statistical comparison

The temporal covariance function of the TurbSim simulated along-wind turbulence field is illustrated in Fig. 3.23. Comparing to Fig. 3.3 in the first numerical example, this figure has a more uniform pattern. This is because a 10-minute turbulence is regarded as stationary whereas a 24-hour wind flow is apparently transient. Fig. 3.24 shows the eigenvalues and the cumulative fractions of the first several modes. Since the dimensions of temporal covariance function are 2400×2400 , which is much bigger than the case in numerical example one, much more number of BD modes are needed to get accurate parameterizations. In addition, the first

⁵It does not mean the other two turbulence components are not important in wind turbine simulation.

four eigenvalues are so close that the four corresponding modes are all crucial in contributing the randomness of the turbulence field. As seen in Fig. 3.25, the first spatial mode that accounts for the largest portion of total turbulence energy is mostly uniform, the second and third modes describe vertical and lateral shear pattern, and the fourth mode possess a more complicated pattern.



Figure 3.23 Covariance function of the TurbSim simulated turbulence.

Following the same process described in section 3.1.4, we simulated the synthetic wind that preserves temporal and spatial correlations of the TurbSim generated full-field wind. In Fig. 3.26, reconstructed time-series of the turbulence at the rotor center (using 2, 5, and 10 BD modes) are compared with the TurbSim full-field wind. Not surprisingly, the more modes used in the simulation, the greater detail in the original turbulence is preserved.

To determine how many modes to use in order to get reasonably accurate representation, we compare the PSDs of the synthetic turbulence (using 2, 5, and 10 BD modes) at rotor center with the PSD of the TurbSim full-field wind in Fig. 3.27.

In addition, the coherence spectra (calculated using 2, 5, and 10 BD modes) between hub center p_0 and three lateral positions p_1 , p_2 , and p_3 (see Fig. 3.22), are shown in Fig. 3.28 (a), (b), and (c) respectively. It is seen in the two figures that by using only 5 modes, the



Figure 3.24 Spectrum of covariance matrix, (a) eigenvalues, (b) cumulative fraction of energy.



Figure 3.25 Mean of the first four stochastic spatial modes.



Figure 3.26 Reconstructed time series of the along-wind turbulence at the rotor center compared with the original TurbSim simulated flow.

reconstructed turbulence closely reproduces the original wind. In addition, it seems that the coherence of further separated locations is preserved better by TSD.

3.4 Discussion and Conclusion

In this chapter, three numerical examples of TSD are given. The results of these examples show that TSD is promising in representing turbulence. However, it is worth to mention following issues. First, the meteorological data used in this analysis is measured by only one met tower. Because the lack of the measurement on transverse direction, getting wind flow snapshots on the plane of turbine rotor becomes impossible. To circumvent this insufficiency in measurement, wind filed snapshots on the plane that is perpendicular to the rotor is used, for which certain time interval has to be chose to construct snapshots. It is worth noting that the framework is generally applicable to a variety of meteorology data, and the its applicability should not be affected by choosing different snapshot constructing time intervals. In addition, the framework is able to incorporate both short-term (10-minute) and long-term (years) temporal coherences as long as corresponding data is available. Third, while the mathematical



Figure 3.27 PSDs of the synthetic turbulence at rotor center using 2, 5, and 10 modes



Figure 3.28 Coherence spectra (calculated using 2, 5, and 10 BD modes) between hub center p_0 and three lateral positions p_1 , p_2 , and p_3 (see Fig. 3.22).

CHAPTER 4. WIND TURBINE SIMULATION BASED ON STOCHASTIC WIND

In the previous chapter, three numerical examples are given to showcase the ability of TSD in wind field modeling. In this chapter, the result of the third numerical example 3.3 is used to perform stochastic analysis on the NREL 5MW offshore wind turbine model.

In general, stochastic analysis is done by representing the random input, modeling the system of interest, incorporating the randomness in system model or its inputs, and getting probability of system output. Therefore, this chapter is organized in the same fashion. In section 4.1, the wind turbine simulation software (FAST) and the NREL 5MW wind turbine model are introduced. Section 4.2 briefly describes the sparse grid collocation algorithm that is used in the stochastic analysis. The implementation details are introduced in 4.3. In section 4.4, simulation results are given and discussed. Section 4.5 finally concludes this chapter.

4.1 NREL 5MW Wind Turbine Model and FAST

FAST (Fatigue, Aerodynamics, Structures, and Turbulence) is a wind turbine simulator developed by NREL (National Renewable Energy Laboratory) [36]. It is able to predicting the responses of both two- and three-bladed horizontal-axis wind turbines with respect to both fatigue and extreme loads. In FAST, the wind turbine system is divided into a finite number of rigid/flexural bodies connected with elastic joints. The rigid bodies include the tower base, nacelle, and hub; the flexible bodies are blades, drive shaft, and tower. The behavior of the system is described by a few ordinary/partial differential equations.

FAST code contains a aerodynamic subroutine package, called AeroDyn [48], that is used to generate aerodynamic loads along the blade and tower. A schematic of how FAST operates with AeroDyn is shown in Fig. 4.1. Both FAST and AeroDyn need few input files to operate. Input files for FAST include a primary input file that contains the data for simulation control, turbine control, initial and environmental conditions, turbine configuration, and output definition; a tower input file that describes tower structure and tower mode shapes; and a blade input file contains blade structure parameters and blade mode shapes. Input files for AeroDyn contain a primary input file that specifies simulation configuration, directories of airfoil input files, and information of blade nodes. During the simulation process, FAST communicates with AeroDyn so that aerodynamic force along the blade at each step is calculated. Finally, FAST provides a summary output file about the entire simulation and an output file contains time-series of output variables that was defined in the FAST primary input file.



Figure 4.1 Schematic of how FAST operates with AeroDyn and their input/output files.

The wind turbine that is used in the following simulation is NREL 5 MW offshore baseline wind turbine [35]. According to the specification of NREL 5MW wind turbine, the blade length is 61 m. Three blades are connected with the hub whose radius is 2 m to form a rotor with radius equals 63 m. The blade surface is composed of a series of airfoil shapes stacked along axial direction. Starting from the cylinder blade root, the airfoil shape is smoothly shifted into a series of DU (Delft University) airfoils and NACA64 airfoils. Airfoil cross-sections that are used in the model are shown in Fig. 4.2. Table 4.1 shows the geometry definition of the blade model. Fig. 4.3 provides a graphical demonstration of the specifications of the blade geometry.



Figure 4.2 Airfoil cross-sections used in the design of the wind turbine rotor blades. [37]



Figure 4.3 Illustration of quantities from Table 4.1. [37]

Table 1.1 While fullome rotor geometry demittion. [91]							
RNodes (m)	AeroTwst (deg.)	Chord (m)	AeroCent (-)	AeroOrig (-)	Airfoil		
2.0000	0.000	3.542	0.2500	0.50	Cylinder		
2.8677	0.000	3.542	0.2500	0.50	Cylinder		
5.6000	0.000	3.854	0.2218	0.44	Cylinder		
8.3333	0.000	4.167	0.1883	0.38	Cylinder		
11.7500	13.308	4.557	0.1465	0.30	DU40		
15.8500	11.480	4.652	0.1250	0.25	DU35		
19.9500	10.162	4.458	0.1250	0.25	DU35		
24.0500	9.011	4.249	0.1250	0.25	DU30		
28.1500	7.795	4.007	0.1250	0.25	DU25		
32.2500	6.544	3.748	0.1250	0.25	DU25		
36.3500	5.361	3.502	0.1250	0.25	DU21		
40.4500	4.188	3.256	0.1250	0.25	DU21		
44.5500	3.125	3.010	0.1250	0.25	NACA64		
48.6500	2.310	2.764	0.1250	0.25	NACA64		
52.7500	1.526	2.518	0.1250	0.25	NACA64		
56.1667	0.863	2.313	0.1250	0.25	NACA64		
58.9000	0.370	2.086	0.1250	0.25	NACA64		
61.6333	0.106	1.419	0.1250	0.25	NACA64		
62.9000	0.000	0.700	0.1250	0.25	NACA64		

Table 4.1 Wind turbine rotor geometry definition. [37]
The above wind turbine blade model provides us a deterministic solver \mathcal{B} where an one-onone relationship between the wind speed input v time-series and the turbine response output time-series u exists, i.e.

$$\mathcal{B}(u: \boldsymbol{x}, \boldsymbol{v}) = 0. \tag{4.1}$$

where $x \in D$ are the coordinates in \mathbb{R}^3 that defines the location of the output time-series on the wind turbine structure.

For a stochastic turbulent input that is generated by TSD, the above equation becomes

$$\mathcal{B}(u: \boldsymbol{x}, \boldsymbol{v}(\boldsymbol{\xi})) = \mathcal{B}(u: \boldsymbol{x}, \boldsymbol{\xi}) = 0.$$
(4.2)

where $\boldsymbol{\xi}$ is a set of TSD generated random variables that represent the randomness in the synthetic turbulence. If *m* terms of decomposition are used in BD and *n* terms in KLE are used to describe each BD term, the resulting random variables $\boldsymbol{\xi} = \{\xi_i\}_{i=1}^p$ where $p = m \times n$.

Eqn. 4.2 defines a stochastic solver where the output of interest u is the stochastic response of p-dimensional random inputs $\boldsymbol{\xi}$ at location \boldsymbol{x} , i.e. $u(\boldsymbol{x}, \boldsymbol{\xi})$.

4.2 Uncertainty Quantification Using Adaptive Sparse Grid Collocation Method

In previous section, we introduced a stochastic wind turbine system whose wind input is full-field turbulence. By performing multiple simulations of the system, finite number of realizations of the stochastic output can be calculated. The problem now becomes how to solve Eqn. 4.2 and find the approximate solution $u(\boldsymbol{x}, \boldsymbol{\xi})$.

Let $\Theta_N = \{\boldsymbol{\xi}_i\}_{i=1}^M$ be a set of nodes in the N-dimensional random space Γ , where M is the number of nodes. Consider a smooth function $u : \mathbb{R}^N \to \mathbb{R}$, the Lagrange interpolation of u can be written as

$$\mathcal{I}u(\boldsymbol{x},\boldsymbol{\xi}) = \sum_{k=0}^{M} u(\boldsymbol{x},\,\boldsymbol{\xi}_k) \, L_k(\boldsymbol{\xi}), \qquad (4.3)$$

where $L_k(\boldsymbol{\xi})$ are Lagrange polynomials given by

$$L_k(\boldsymbol{\xi}) = \prod_{i=1, i \neq k}^M \frac{\boldsymbol{\xi} - \boldsymbol{\xi}_i}{\boldsymbol{\xi}_k - \boldsymbol{\xi}_i}.$$
(4.4)

 $u(\boldsymbol{x}, \boldsymbol{\xi}_k)$ is the value of u at the given node $\boldsymbol{\xi}_k \in \Theta_N$. Therefore, the value of u at any point $\boldsymbol{\xi} \in \Gamma$ can be approximated by $\mathcal{I}u(\boldsymbol{x}, \boldsymbol{\xi})$.

In the same fashion, $\boldsymbol{\xi}$ can be approximated by

$$\hat{\boldsymbol{\xi}} = \sum_{k=1}^{M} \boldsymbol{\xi}_k L_k(\boldsymbol{\xi}). \tag{4.5}$$

Substituting Eqn. 4.5 into the system governing equation yilds

$$\mathcal{B}\left(u:\boldsymbol{x},\sum_{k=1}^{M}\boldsymbol{\xi}_{k}\ L_{k}(\boldsymbol{\xi})\right)=0.$$
(4.6)

The collocation method converted the original stochastic problem to M deterministic problems, i.e.

$$\mathcal{B}(u: \boldsymbol{x}, \boldsymbol{\xi}_i) = 0, \qquad i = 1, \cdots, M.$$
(4.7)

The stochastic solution is approximated by

$$\hat{u}(\boldsymbol{x},\boldsymbol{\xi}) = \sum_{i=1}^{M} u(\boldsymbol{x},\boldsymbol{\xi}_k) L_k(\boldsymbol{\xi}).$$
(4.8)

Choosing appropriate sampling points for Lagrange interpolation is a crucial first step for collocation methods. For the simplest one-dimensional problem, the optimal selection is usually the Gauss quadrature. It is straight forward to use the tensor product of the one-dimensional nodes to chose nodes in multi-dimensional random spaces. Let $u(x, \xi)$ be a function in N-dimensional space that need to be approximated, the tensor product interpolation formula can be written as

$$(\mathcal{I}^{i_1} \otimes \cdots \otimes \mathcal{I}^{i_N})(u)(\boldsymbol{x}, \boldsymbol{\xi}) = \sum_{k_1=1}^{M_1} \cdots \sum_{k_1=1}^{M_N} u(\boldsymbol{x}, \boldsymbol{\xi}^{i_1}_{k_1}, \cdots, \boldsymbol{\xi}^{i_N}_{k_N}) (L^{i_1}_{k_1} \otimes \cdots \otimes L^{i_N}_{k_N}),$$
(4.9)

where M_k is the number of nodes used in the interpolation in the k^{th} dimension, \mathcal{I}^{i_k} is the interpolation function in the i_k direction, and $\boldsymbol{\xi}_{k_m}^{i_k}$ is the k_m^{th} point in the i_k direction. According to the equation, $M_1 \times \cdots \times M_N$ number of points are needed in the computation. This number grows quickly as the N increases. For the case that M points are chose at each direction, the total number of points in N-dimensional space is M^N . Full tensor product method will become impractical when dealing with high dimensional stochastic space.

To overcome this difficulty, one need to find a method that can both satisfy the interpolation accuracy and minimize the computational cost, i.e. use as less of points as possible. To this end, sparse grid method is used. The sparse grid method, based on Smolyak algorithm, reduces the number of nodes from full tensor product formula. In other words, it is a subset of full tensor product grids. It uses limited numbers of nodes while satisfy the prescribed computational accuracy.

Consider the one-dimensional interpolation function

$$\mathcal{U}^{M}(f) = \sum_{k=1}^{M} f(\xi_{k}) L_{k}.$$
(4.10)

Let Δ^i be the incremental interpolate defined as

$$\Delta^{i} = \mathcal{U}^{i} - \mathcal{U}^{i-1}, \qquad \mathcal{U}^{0} = 0.$$
(4.11)

Smolyak's interpolation $\mathcal{A}_{q,N}$ is given by

$$\mathcal{A}_{q,N}(f) = \mathcal{A}_{q-1,N}(f) + \sum_{|\mathbf{i}|=q} (\Delta^{i_1} \otimes \dots \otimes \Delta^{i_N})(f), \qquad (4.12)$$

where N is the stochastic dimension and q - N is the interpolation order. $\mathbf{i} = (i_1, \dots, i_N)$ and $|\mathbf{i}| = i_1 + \dots + i_N$. i_k is the level of interpolation along the k^{th} direction.

Let $\Theta^{(k)} = \{x_i\}_{i=1}^M$ be the set of nodes that is used to interpolate the one-dimensional function, where k denotes the order of polynomial that is used in the interpolation. To compute the sparse grid interpolation function, only the function values at the sparse grid are needed, i.e.

$$\mathcal{H}_{q,N} = \bigcup_{q-N+1 \le |\mathbf{i}| \le q} (\Theta^{(i_1)} \times \dots \times \Theta^{(i_N)})$$
(4.13)

Adaptive sparse grid collocation method is developed by following the dimension adaptive quadrature approach [19]. The basic idea of the adaptive procedure is to assess the stochastic dimensions according to the error of interpolation in that dimension. Details of this method can be found in [16].

4.3 Computational Implementation

The solution procedure of this chapter can be divided into three steps:

- A subroutine for computing deterministic solutions.
- A subroutine for allocating nodes and building interpolation functions.
- A subroutine for post-processing operations.

Deterministic solver: In section 3.3, a full-field turbulence representation is developed based on TSD and the along-wind turbulence component that was originally simulated by TurbSim. The newly simulated along-wind component (by TSD) together with the other two orthogonal components (lateral and vertical) are combined and then used as the input full-field turbulence of AeroDyn. This process is illustrated in the schematic in Fig. 4.4. By going through this detour, we seek to demonstrate the ability of TSD to reproduce given wind data (or field measurements when available). After incorporating all models as a compact solver, the input of the solver becomes realization of a set of TSD generated random variables, and the output becomes wind turbine response time-series. This provides us the deterministic solver.



Figure 4.4 Schematic of incorporating TSD into full-field wind simulation.

Sampling and interpolation: A package named FT-AdaGiO (Fault Tolerant ADAptive sparse GrId allOcator) in C++ developed by Xie et al. [75] was used to allocate the sampling points in the stochastic domain. In this analysis, 5 BD modes and 3 KLE modes for each BD mode were used, which makes a 15-dimensional stochastic domain. We used FT-AdaGiO to generate coordinates of nodes according to adaptive SGC algorithm and to construct the probabilistic output. The full-field turbulences were generated based on random samples and used as input of FAST.

Post-processing: Post-processing of current analysis includes extracting basic statistical solutions. Turbine responses include out-of-plane blade root bending moment, fore-aft tower bending moment, and power generation were calculated. Probability density functions of the maximum of these responses were estimated by using KDE.

4.4 Results

In this section, we show the results of NREL 5MW wind turbine simulation with TSD generated full-field turbulence inflow. The output of the simulation is defined (in the primary input file of FAST) as 39 variables of interest of the wind turbine. Examples of these variables include electrical generation, blade and tower deflections, and forces and moments acted on blade and tower. The results of a single simulation are given first followed by the results of stochastic analysis using SGC method.

4.4.1 Time responses

Time responses of power generation by using 2, 5, and 10 modes of BD are shown in Fig. 4.5. It is seen that all four responses have similar structure and the randomness in the wind has very limited influence on the power generation. Note that the rated wind speed for NREL 5MW wind turbine is 11.4 m/s, whereas the mean speed of TurbSim wind at hug height is 8.3 m/s. This explains the reason that the average power generation seen in Fig. 4.5 is much less than the rated 5 MW capacity.

Fig. 4.6 and 4.7 show the 200 seconds response time-series of flapwise and edgewise bending moment at the half span of blade. Fig. 4.8 shows time response of fore-aft bending moment at tower base. By visual inspection of these figures, it can be seen that a few TSD modes can accurately preserve the low frequency components of the turbine response. However, to reproduce the high frequency response seen in the full-simulation based on TurbSim wind, larger number of TSD terms are required.

To further investigate how well the PSD simulated wind describe the original TurbSim wind, comparisons between the PSDs of wind turbine time responses corresponding to reconstructed wind and original wind are given in Fig. 4.9 and 4.10. It is seen in the two figures that PSD



Figure 4.5 Time response of power generation by using 2, 5, and 10 modes and TurbSim wind (top to bottom).



Figure 4.6 Time response of flapwise bending moment at the half span of blade by using 2, 5, and 10 modes and TurbSim wind (top to bottom).



Figure 4.7 Time response of edgewise bending moment at the half span of blade by using 2, 5, and 10 modes and TurbSim wind (top to bottom).



Figure 4.8 Time response of fore-aft bending moment at tower base by using 2, 5, and 10 modes and TurbSim wind (top to bottom).

wind is able to locate frequency modes of wind turbine blade and tower vibrations as well as the TurbSim wind. As expected, as the number of terms in TSD model increases, its accuracy of estimating frequency responses of wind turbine increases as well.



Figure 4.9 PSD of flapwise bending moment at half span of the blade.



Figure 4.10 PSD of edgewise bending moment at half span of the blade.

4.4.2 Convergence analysis

Note that the analysis in last section are based on one simulation. With SGC method, multiple realizations of the stochastic output of the wind turbine system could be achieved, which allows us to investigate the system in the statistical point of view. Results of SGC simulation are shown in this section.

In the simulation, m modes of BD and 3 modes of KLE for each BD mode are used. In other words, to get one realization of the random wind flow, we need to sample from $m \times 3$ random variables. Therefore, the number of random dimensions of the wind input is $m \times 3$. It is comparatively hard for a system with large number of random input dimensions to converge. Thus, convergence checks on the computations of SGC by using $m = 1, \dots, 4$ BD modes are performed. Fig. 4.11 shows the result of one such convergence analysis where m = 2. It can be seen in the figure that as the interpolation level increases from 2 to 6, the differences between PDFs of succeeding interpolation levels become indistinguishable. To quantitatively check the rate of convergence, the mean square errors (MSEs) of the inverse cumulative distribution functions (ICDFs) of different levels with respect to level 6 are shown in Table 4.2. We conclude that for the case of m = 2, good convergence (MSE < 5%) is achieved at interpolation level 4.

Table 4.2 Normalized MSEs of different computational levels with respect to level 6, m = 2.

Computational Level	# or samples	MOL
2	85	1.0
3	230	0.42
4	434	0.033
5	672	0.015

The adaptive sampling procedure of SGC method is achieved by allocating new random samples in the next computation level based on the surplus of certain or all output variable(s) in current computation level. In this analysis, the out-of-plane bending moment at blade root is considered as the output that determines the sampling procedure. Although only the specified output has best convergence, the rest of the outputs should converge as well since they are coupled with each other.

According to Fig. 4.11, using the result of interpolation level 4 provides accurate estimations of the PDFs of out-of-plane bending moment. To check whether consistent convergences of other



Figure 4.11 PDFs of out-of-plane bending moment at blade root result from different SGC computation levels.

output variables have been achieved, similar analysis of convergences are done for edgewise and flapwise bending moments and shown in Fig. 4.12 and Fig. 4.13 respectively. The results of these analysis verify that the assumption on consistent convergence made in previous context is valid.

Follow the same procedure, convergence checks for cases m = 2, 3, 4 are performed and the results are shown in Table 4.3. It is worth to mention that the numbers of sampling points at the convergent level that are result from adaptive SGC method are much smaller than the result of non-adaptive approach. This fully illustrates the advantages and necessities of using adaptive SGC method.

Table 4.3 Number of random samples at convergent level for the case of m = 1, 2, 3, 4. # of BD modes convergent level # of non-adaptive samples # of adaptive samples 40 1 3 69 $\mathbf{2}$ 4 1457434 3 $\mathbf{5}$ 2601711499 4 593489 36158

In order to check how many BD modes are sufficient in the context of stochastic analysis,



Figure 4.12 PDFs of edgewise bending moment at half-span of blade, m = 2.



Figure 4.13 PDFs of flapwise bending moment at half-span of blade, m = 2.

another convergence analysis on the simulation results with respect to number of BD modes is performed. PDFs of the out-of-plane bending moment at blade root resulted from different values of m are shown in Fig. 4.14. It is seen that the curve of m = 4 is very close to the curve of m = 3, which means by using three spatial modes in BD (see the first three images in Fig. 3.25), the resulting synthetic wind is sufficient to be used as the stochastic input of this stochastic analysis. In other words, the fourth spatial mode is not prominent enough to affect the probabilistic output of wind turbine. This implies that the stochastic performance of wind turbine is only sensitive to larger turbulent structures that are described by the first three spatial modes (mean stream flow, vertical shear, and horizontal shear).



Figure 4.14 PDFs of out-of-plane bending moment at blade root based on the turbulent inflows that are constructed by 1, 2, 3, and 4 BD spatial modes.

4.5 Discussion and Conclusion

In this chapter, stochastic simulation of NREL 5 MW wind turbine based on full-field turbulence was performed. Results showed time and frequency responses of the wind turbine with TSD generated turbulence input are in accordance with the responses resulted from the original TurbSim wind. After that, stochastic analysis on the wind turbine system was performed. The result of this analysis provides valuable information on the probability distribution of structural vibration, forces, shearing, and bending of many flexible components (tower, blade, and main shaft) of the wind turbine system.

On the other hand, FAST models the wind turbine as rigid/flexible bodies connected with elastic joints. Though it is sufficient in simple applications that only require general deflections of bodies, it does not provide enough information on the local loads and detailed deformations on each body, which is very important information for blade design and fatigue analysis. In next chapter, we will perform such analysis on a full-scale 3D wind turbine model with rich structure details.

CHAPTER 5. WIND TURBINE SIMULATION BASED ON DETERMINISTIC WIND AND STOCHASTIC-ISOGEOMETRIC APPROACH

In previous chapter, we performed stochastic analysis on a simplified wind turbine model using full-filed turbulent inflow. As an attempt to achieve more accurate and detailed results on the deformation and surface stress distributions of wind turbine blades, this chapter focus on integrating stochastic analysis with a full-scale 3D pre-bent wind turbine rotor model, which is developed based on isogeometric analysis. An adaptive sparse grid collocation method is used to account for the stochastic input. Kernel density estimation is used to estimate the distribution of random variables that are related to the randomness of wind turbine model. This analysis gives insights into failure probability and critical regions of blade shell surface, which provide wind turbine developers a good reference to improve the design of wind turbine blades.

This chapter is organized as follows. The randomness in the composite material in wind turbine blades is first represented in section 5.1. In section 5.2, the comprehensive wind turbine blade model that is based on isogeometric analysis is introduced. Adaptive sparse grid collocation (SGC) algorithm is used in the stochastic analysis (see Chapter 3). The implementation details are introduced in 5.3. In section 5.4, simulation results are given and discussed. Section 5.5 finally concludes the paper.

5.1 Representing Randomness in Wind Turbine Blade Material

There is a trade-off between blade efficiency and strength in wind turbine blade design. From the aerodynamic point of view, the ideal blade should be as thin as possible. It also needs to be light to minimize loads on the nacelle and tower, and to minimize the manufacturing cost. On the other hand, blade must be strong enough to withstand the bending load that is caused by lift and gravity. As a result, existing wind turbine blades have a common structure which balances their performances with regards to these aspects. The cross section of a typical blade structure is shown in Fig. 5.1, where the blade has a hollow structure with thick root and thin tip. In the structure, spar caps carry the major flapwise bending moment; the blade shell maintains the aerodynamic shape, carries the edgewise bending load, and transfers this load to spars and other structure components; shear web joins the two spar caps and withstands the shearing load caused by tension and compression on both sides.



Figure 5.1 Wind turbine blade cross section.

The material used in wind turbine blades are usually fiber-reinforced composite material. This is because the material has larger strength-weight ratio than wood and metal and is also much cheaper than high quality carbon fiber. It is worth noting that fiberglass is mostly unidirectional and thus exhibits strength in one direction. Biaxial fabric $(\pm 45^{\circ})$ is used to make the shear web and some part of the shell so that it bears shear stress most effectively. When it comes to the blade shell, both fibers along the diagonal and fibers along axial and radial directions are needed to resist the torsion loads as well as both flapwise and edgewise bending loads. Therefore, the shell is made of laminated sandwich composites which consist of multiple plies in different directions. The choice of ply directions could be triaxial ($\pm 45^{\circ}, 0^{\circ}$) or quadraxial ($\pm 45^{\circ}, 0^{\circ}, 90^{\circ}$) where material in certain direction takes up to certain percentage of the space. For example, one such shell layup could be 16 plies with 90% of the fiber oriented in the $0^{\circ}/90^{\circ}$ direction and 10% in the $\pm 45^{\circ}$ direction. Manufacturers may have different ply layup procedures. It is noteworthy that the plies that are used in the layup process are usually smaller pieces compared to the length of the blade. To cover the entire blade geometry, multiple plies are needed on each layer of fabric, which results in different concentration of fiber toes at different locations. In addition, fiber alignments are subject to variabilities due to the prevalence of manual layup processes.

Resin infusion processes are performed after all plies are properly placed on the blade mold. In general, there are two types of resin infusion processes [22]. Resin transfer molding (RTM) is one of the commonly used methods. Under pressure, low viscosity resin is pushed or pulled (by pressure difference between atmosphere and vacuum) into the mold where several glass fiber cloths are placed. The other method is resin film infusion (RFI), where partially cured resin films are stacked with fiberglass textiles in a mold. Followed by a process that applies pressure, heat and vacuum, the stack is fully cured such that resin is infused throughout the mold. Both processes have to be controlled carefully to prevent occurrence of defects (void, waviness). Due to this random nature of wind turbine blade manufacturing process, uncertain material properties inevitably exist between blades. To represent this randomness, the properties that are required to fully describe the material need to be identified first.

Fiberglass laminate is an orthotropic material. Since wind turbine blade shell mainly carry loads on edgewise and flapwise directions, the material properties along the thickness are not significant. In this case, 4 properties are used to describe orthotropic blade shell: elastic modulus on flapwise (axial) direction E_1 , elastic modulus on edgewise (transverse) direction E_2 , shear modulus G_{12} , and major Poisson's ratio ν_{12} . Based on previous discussion, 4 properties are actually random variables that fully describe the stochastic behavior of composite material in wind turbine blades. We use the composite materials handbook (MIL-HDBK-17-2F) [65] as a guide in getting properties of composite materials. The probability density functions of all material properties are assumed to be *Gaussian*. In addition, the handbook suggests following relationship that defines ν_{12} ,

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1^t},\tag{5.1}$$

where ε_1^t and ε_2 denote axial tension strain and transverse strain respectively. The probability density function of random variable ν_{12} is estimated using an algorithm that is developed by Marsaglia [45]. Finally, the mean and standard deviation of all 4 properties of the fiberglass

Table 5	5.1 P	roperties of fib	perglass that	is used.
		$\mu({ m Pa})$	$\sigma({\rm Pa})$	
	E_1	46.54×10^9	2.03×10^9	
	E_2	12.69×10^9	0.26×10^9	
	G_{12}	4.69×10^9	0.25×10^9	
	ν_{12}	0.13	0.01	

that is used in this research are listed in Table 5.1 1 .

5.2 Wind Turbine Blade Model Based on Isogeometric Analysis

The primary advantage of isogeometric analysis is the solution space for dependent variables can be represented in terms of the same functions that represent the geometry. The isogeometric procedure that is used in our wind turbine rotor analysis is based on Non-Uniform Rational B-Splines(NURBS).

Just as its name implies, NURBS are developed from B-splines. Unlike FEM whose physical domain is divided into sub-domains, B-spline parametric space is divided into "patches". Define a set of coordinates in one dimension of the parametric space as a *knot vector*, denote by $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$, where $\xi_i \in \mathbb{R}, i = 1, 2, \ldots, n+p+1$ are the knots, p is the polynomial order, and n is the number of basis functions. Note that p = 0, 1, 2, 3 represents constant, linear, quadratic, and cubic piecewise polynomials respectively. If knots are equally spaced in the parametric space, they are said to be *uniform*. Correspondingly, *non-uniform* knots are unequally spaced.

B-spline basis functions are defined recursively as

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi).$$
(5.2)

where p = 1, 2, 3, ..., and

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1}, \\ 0 & \text{otherwise.} \end{cases}$$
(5.3)

¹The material is S2-449 17k/sp 381 unidirectional tape whose resin content is 28-29 wt%; fiber volume is 50.1-54.0 %; density is 1.85-1.92 g/cm^3 ; and ply thickness is 0.0033-0.0037 in.

With this definition, one may notice that for p = 0 and 1, B-spline basis functions are the same standard piecewise constant and linear basis functions in finite element method. However, this identity does not hold for $p \ge 2$. Repeated knots are allowed in knot vectors. A knot vector is said to be *open* if its first and last knots appear p + 1 times. Basis functions over open knot vectors are interpolatory at the ends of the parametric space interval, $[\xi_1, \xi_{n+p+1}]$, and generally not interpolatory at knots within the interval. In general, basis functions in order pare C^{p-1} -continuous. The continuity at a knot decreases by k if the knot is repeated k times. The basis function is interpolatory at a knot if it is repeated p times. Finally, it is worth noting that the support of $N_{i,p}$ is $[\xi_i, \xi_{i+p+1}]$ and $\sum_{i=1}^n N_{i,p}(\xi) = 1$ for any ξ .

Now that B-spline basis function is defined, B-spline curves in \mathbb{R}^d , denoted by $C(\xi)$, can be constructed by taking linear combination of B-spline basis functions

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) B_i.$$
(5.4)

where $B_i \in \mathbb{R}^d$, i = 1, 2, ..., n, are referred to as *control points*.

Consider a *control net* with control points $B_{i,j}$, i = 1, 2, ..., n, j = 1, 2, ..., m, and knot vectors $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, and $\mathcal{H} = \{\eta_1, \eta_2, ..., \eta_{m+q+1}\}$, a *B-spline surface* can be defined as

$$S(\xi,\eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta) B_{i,j},$$
(5.5)

where $N_{i,p}$ and $M_{j,q}$ are p^{th} order and q^{th} order B-spline basis functions respectively.

Similarly, given control net $\{B_{i,j,k}\}$, i = 1, 2, ..., n, j = 1, 2, ..., m, k = 1, 2, ..., l, and knot vectors $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, $\mathcal{H} = \{\eta_1, \eta_2, ..., \eta_{m+q+1}\}$, and $\mathcal{Z} = \{\zeta_1, \zeta_2, ..., \zeta_{l+r+1}\}$, a *B-spline solid* is defined by

$$S(\xi,\eta,\zeta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} N_{i,p}(\xi) M_{j,q}(\eta) L_{k,r}(\zeta) B_{i,j,k},$$
(5.6)

where $L_{k,r}$ are r^{th} order B-spline basis functions.

To obtain exact geometry in \mathbb{R}^d , projective transformation of B-spline entity in \mathbb{R}^{d+1} must be done. Let $\{B_i^w\}$ be a control net for a B-spline curve in \mathbb{R}^{d+1} with knot vector Ξ . Control points for desired NURBS curve in \mathbb{R}^d are derived from $\{B_i^w\}$ which are referred as "projective" control points for the desired curve.

$$(B_i)_j = (B_i^w)_j / w_i, \quad j = 1, \dots, d,$$

 $w_i = (B_i^w)_{d+1}.$ (5.7)

where $(B_i)_j$ is the j^{th} component of vector B_i , and w_i is referred to as the i^{th} weight. Non-Uniform Rational B-Splines(NURBS) curve is given by

$$C(\xi) = \sum_{i=1}^{n} R_i^p(\xi) B_i,$$
(5.8)

where $R_i^p(\xi)$ are rational basis functions given by

$$R_i^p(\xi) = \frac{N_{i,p}(\xi)w_i}{\sum_{k=1}^n N_{k,p}(\xi)w_k},$$
(5.9)

Rational surfaces and solids could be defined similarly

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}}}.$$
(5.10)

$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}\sum_{\hat{k}}^{l}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)L_{\hat{k},r}(\zeta)w_{\hat{i},\hat{j},\hat{k}}}.$$
(5.11)

Isogeometric analysis represents the geometry, motions of the model, properties such as strain states and etc. with the same basis functions. Based on the fact that NURBS forms almost the exact geometry of the model, resulting quantities of interest could be more accurate than the results by using FEA. Therefore, the rotor and surrounding fluid are modeled by NURBS in this work.

The NREL 5MW wind turbine blade model is constructed by using isogeometric analysis, which is shown in Fig. 5.2. The surrounding fluid domain is modeled with volumetric NURBS so that we can analyze the FSI between them. It is worth noting that the rotor can be divided into three equal portions, thus only one third of the rotor and the fluid domain is modeled for the purpose of increasing computational efficiency. The dimensions of the entire problem domain and the NURBS mesh are shown in Fig. 5.3. More details of modeling wind turbine rotor based on isogeometric analysis can be found in [7, 8].

The above wind turbine blade model provides us a deterministic solver where an one-to-one relationship between input and desired output exists. In our stochastic analysis, the inputs are



Figure 5.2 Surface meshes of the NREL 5MW wind turbine blade. . [9]



Figure 5.3 (a) Volumetric NURBS mesh of the computational domain and (b) A planar cut to illustrate mesh grading toward the rotor blade. [9]

the 4 wind turbine material parameters $(E_1, E_2, G_{12}, \text{ and } \nu_{12})$, and the outputs of interest are blade deformation, stress distribution, and etc. i.e.

$$\mathcal{B}(u: \boldsymbol{x}, \boldsymbol{q}) = 0. \tag{5.12}$$

where \mathcal{B} represents the deterministic solver, $\boldsymbol{x} \in \mathcal{D}$ are the coordinates in \mathbb{R}^3 , $\boldsymbol{q} = \{q_1, q_2, ...\}$ is a set that represents boundary conditions, material properties as well as any condition that defines the physical system, u is the output of interest at location \boldsymbol{x} . For a deterministic wind turbine model with no random material properties, one can find the value of u at any point \boldsymbol{x} with given \boldsymbol{q} . This results in a function $u: \mathcal{D} \to \mathbb{R}$.

Suppose one of the conditions, $q_i \in \mathbf{q}$, denote by α for the sake of convenience, is actually uncertain. Let Ω_{α} be the sample space of α which contains possible values ω that α can take, i.e. $\omega_{\alpha} \in \Omega_{\alpha}$. Define a σ -algebra, denote by \mathcal{F} , a non-empty set of events in Ω_{α} , such that it is closed under complements and countable unions. Let $\mathcal{P} : \mathcal{F} \to [0, 1]$ be a function returning the probability of particular event in \mathcal{F} happens. According to the properties of σ -algebra, \mathcal{P} should be countably additive and satisfy that $\mathcal{P}(\Omega_{\alpha}) = 1$. The three definitions introduced above can be composed as a probability space $(\Omega_{\alpha}, \mathcal{F}, \mathcal{P})$, where \mathcal{F} is the σ -algebra over sample space Ω_{α} , \mathcal{P} is the probability measure on \mathcal{F} . Based on above discussion, the deterministic system has been changed to stochastic system with random input $\alpha(\omega_{\alpha})$ and random output.

It is worth noting that $\alpha(\omega_{\alpha})$ is an abstract quantity that lies in abstract probability space $(\Omega_{\alpha}, \mathcal{F}, \mathcal{P})$. The problem becomes

$$\mathcal{B}(u: \boldsymbol{x}, \{\boldsymbol{q}_{-i}, \alpha(\omega_{\alpha})\}) = 0.$$
(5.13)

where $\boldsymbol{q}_{-i} = \{q_1, q_2, ..., q_{i-1}, q_{i+1}, ...\}$. The solution of above set of differential equations is actually a function $u : \mathcal{D} \times \Omega_{\alpha} \to \mathbb{R}$.

Since $\alpha(\omega_{\alpha})$ is just an abstract concept which could not be quantified. To solve equation (5.13), the randomness in input must be represented by numerical mean. To this end, define a measurable space $(\mathbb{R}, \varepsilon)$, on which lies a real-valued random variable $\theta_{\alpha} : \Omega_{\alpha} \to \mathbb{R}$. By measurable, it means for every subset $B \in \varepsilon$, its pre-image $\theta_{\alpha}^{-1}(B) \in \mathcal{F}$ where $\theta_{\alpha}^{-1}(B) = \{\omega_{\alpha} : \theta_{\alpha}(\omega_{\alpha}) \in B\}$. With this definition, we relate the abstract quantity $\alpha(\omega_{\alpha})$ to a real-valued quantity θ_{α} . The solver becomes

$$\mathcal{B}(u:\boldsymbol{x},\theta_{\alpha}) = 0. \tag{5.14}$$

where θ_{α} is the random variable which represents the randomness in the input parameter α . Furthermore, if multiple input parameters are actually random, the above equation is converted to

$$\mathcal{B}(u:\boldsymbol{x},\theta_1,\theta_2,...,\theta_k) = 0. \tag{5.15}$$

where θ_i , i = 1, 2, ..., k are random inputs. Since 4 material parameters are considered to be random as discussed previously, the wind turbine solver becomes

$$\mathcal{B}(u:\boldsymbol{x},\boldsymbol{\theta}) = 0, \tag{5.16}$$

where $\boldsymbol{\theta} = \{\theta_i\}_{i=1}^4$. As a result, the output of interest u is the stochastic response of 4 random inputs.

The next step of the analysis is to use SGC method to find the solution $u(\boldsymbol{x}, \boldsymbol{\theta})$ of the stochastic solver (Eqn. 5.15). Mathematical background can be found in section 4.2.

5.3 Computational Implementation

As discussed in section 1.1.3, SGC is used to perform stochastic analysis. It is noteworthy that SGC assumes all random variables are uncorrelated to each other. However, no analysis or experiment suggests that the four material properties satisfy this assumption. Yet for the purpose of demonstration, in this research we assume they are uncorrelated without loss of generality. In case of correlated random variables, SGC scheme can be used after principal component analysis (PCA), which is a technique to convert a set of possibly correlated variables into a set of uncorrelated variables (principal components) [33].

The solution procedure of this chapter can be divided into three steps:

- A subroutine for computing deterministic solutions.
- A subroutine for allocating nodes and building interpolation functions.
- A subroutine for post-processing operations.

Deterministic solver: In this research, we are interested in assessing the impact of the randomness which is caused by the random nature of wind turbine manufacturing process to rotor performance. To this end, a wind turbine blade model based on isogeometric analysis was used in the simulation. The blade shell was made of a symmetric fiberglass-epoxy composite with $[\pm 45/0/90_2/0_3]_s$ layup. The model was then simulated at prescribed steady wind velocity of 11.4 m/s and rotor angular velocity of 12.1 rpm². Fig. 5.4 shows the simulation setup. Results of the simulation including the blade wake velocity field, pressure contour on the blade surface, blade deformation, shell stress distribution, and etc. More details about the simulation could be found in [7, 8]. In our analysis, stress distribution on the blade was calculated and regarded as stochastic output of the system.



Figure 5.4 Simulation setup.

Sampling and interpolation: FT-AdaGiO was used to allocate the sampling points in the 4-dimensional random domain. The package generated coordinates of nodes $\theta_i = \{\theta_j \in [0, 1], j = 1 \cdots 4\}$ according to adaptive SGC algorithm and construct the probabilistic output. The random material properties of the blade were generated based on θ_i and incorporated in the deterministic solver. After that, outputs were calculated based on the random material properties. Although the number of samples greatly reduced by using adaptive SGC algorithm, solving the complicated deterministic solver so many times is still a time consuming task.

Post-processing: Post-processing of current analysis includes extracting basic statistical

²Ideally, stochastic inflow should have been used, however, the wind turbine model used in this experiment does not allow us to implement stochastic wind in the simulation. Thus, steady wind was used.

solutions. Mean and standard deviation of stress distribution on blade shell can be calculated with all realizations of the output of interest. Probability density function of the stress value at critical location on the blade can also be generated by KDE.

5.4 Results

The calculation of adaptive SGC procedure was carried out up to a level of interpolation (6) which has 168 sampling points. In order to make sure the selected samples are appropriate to get accurate solutions, the convergence property of the SGC framework was checked by comparing the PDFs at a critical location (where comparatively large stress occurs) of different SGC computation levels. The result of the convergence check is shown in Fig. 5.5. From the figure we notice that the difference between two consecutive levels decreases drastically as interpolation level increases, which implies good convergence. To have a better view of the convergence, the mean square error (MSE) of inverse cumulative distribution functions (ICDF) of different levels with respect to level 6 are shown in Table 5.2. Good convergence of the approximation error that is shown in Table 5.2 indicates using results of level 6 is sufficient.



Figure 5.5 PDFs at a critical location of different SGC computation levels.

Fig. 5.6 (a) shows one of the 3D scatter plots of the nodes that are allocated in computation

Table 5.2	Normalized	MSE of	different	computational	levels	with	respect	to	level	6.
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Computational Level	# of samples	MSE
2	8	1.0
3	40	0.28
4	88	0.064
5	136	0.0068

level 6. It can be seen from the figure that the nodes are sparsely distributed in the stochastic domain. As discussed previously, the number of samples used in computation level six is 168. The advantage of the adaptive procedure can be seen clearly by comparing Fig. 5.6 (a) with Fig. 5.6 (b) where 1824 samples are generated according to non-adaptive SGC algorithm. Although much less sampling points was used in the adaptive method, good convergence was achieved (see Fig. 5.5). Fig. 5.6 (a) also sheds light on the physical properties of the system. For example, the plot shows a inside-sparse structure which means taking more samples from the region where E_1 , E_2 , and G_{12} are in their intermediate values does not help a lot in refining the stochastic results.



Figure 5.6 Sampling points allocated by adaptive SGC algorithm (a) and non-adaptive SGC algorithm (b) at computation level 6.

In order to increase the clearance between blade tip and turbine tower, large turbine blades are pre-bent such that the clearance is still sufficient after wind load is applied and blades are deformed. Blade deformation is shown in Fig. 5.7. It can be seen in the figure that after the deformation, the blade is almost straight. Noting that the 11.4 m/s wind speed is close to the rated wind speed of most wind turbines, this result implies that pre-bent rotor will have the optimized power output at the rated wind speed. Similar analysis can be done to become a good reference of designing the optimal curvature of pre-bent blades.



Figure 5.7 Blade deformation under 11.4 m/s wind load and 12.1 rpm rotating speed.

5.4.1 Stress analysis

The deformation of the blade causes stress on the blade structure. A blade design has to be verified such that the stress introduced by structure deformation will not exceed the *design strength* of the material. According to the GL design code [18], the design strength is the characteristic value of the material divided by the *partial safety factor* of the material, where the characteristic value is calculated based on experiment results and corresponds to 95% survival probability and confidence level.

Since the flapwise bending load dominates the deformation of blades, mean contour plot of stress along flapwise direction on the 13^{th} ply, where maximum mean stress exists, is shown in Fig. 5.8. The red region on the Fig. 5.8 shows the area that suffers the largest average flapwise stress over the entire blade surface.

Although this figure gives us a general idea of the critical region, it does not provide all the information of the statistical behavior of the stress. A point with low mean and high variance of stress could be more critical than the point that has the maximum mean stress because high



Figure 5.8 Mean stress contour the 13^{th} ply.

variance may result in unexpected peak of stress that may damage the blade. Therefore, a plot of standard deviation of flapwise stress on the 13^{th} ply is shown in Fig. 5.9 (Noting that the 13^{th} ply is where maximum standard deviation of flapwise stress exists). Comparing with Fig. 5.8, the point with maximum standard deviation is almost identical to the point with maximum mean.



Figure 5.9 Contour of the standard deviation of stress on the 13^{th} ply.

We are interested in the probability density function at critical region. KDE method (see

Appendix 2.1.3) was used in estimating PDFs. Fig. 5.10 shows the PDF of the node that has the maximum standard deviation of stress values. It is interesting to note that the PDF is Gaussian-like that is commonly used for modeling physical phenomena.



Figure 5.10 PDF of the stress at the point that has the maximum standard deviation of flapwise stress.

According to the GL design code [18], the design strength $R_d = R_k / \gamma_{M_x}$. In this equation, R_k is the characteristic value given as

$$R_k(v,n) = \bar{x} \left[1 - v \left[1.645 + \frac{1.645}{\sqrt{n}} \right] \right],$$
 (5.17)

where \bar{x} denotes the mean of the test strength and n is the number of test values. γ_{M_x} can be calculated as follows

$$\gamma_{M_x} = 1.35 \prod_i C_{ix}, \tag{5.18}$$

where C_{ix} is the reduction factors decided by different types of analysis (for material produced by resin infusion method, and for short-term strength verification, $\gamma_{M_x} = 1.35 \times 1.1 = 1.485$). According to the test method in [65], the characteristic value of the material that is used in current analysis is 1.68 *GPa*. As a result, the design strength of the given material is $R_d = 1.68/1.485 = 1.13$ *GPa*, which means any stress on the blade that is higher than R_d would fail the stress verification. In Fig. 5.10, the flapwise stress takes values that are much smaller than the design strength of the material, which means current design is safe but either the blade material is unnecessarily strong or the structure design is redundant.

With probability density function at critical points, we can further investigate the probability of the stress at a critical point exceeding the design strength of the material. For the purpose of demonstration, consider the case whose design strength $R_d = 68MPa$. It is valuable to know where on the blade would break first. According to the definition of probability density function, the area for stress value $\sigma > R_d$ on the PDF equals the probability of the point has stress exceeds the design tensile strength of the material. Critical regions that have a large probability of failure can be calculated. As shown in Fig. 5.11, value at each point represents the probability of the stress at that point exceeds the design tensile strength. It can be seen in Fig. 5.11 that critical regions are located at the root transition area and half-length of the blade.



Figure 5.11 Critical region that could have stress higher than 68MPa with large probability.

Remark 3. The current example only shows the result of the laminate failure analysis in normal load case. However, the framework presented in this research can be used in many other analysis which includes analysis on laminate failure and stability failure with regard to the short-term strength, fatigue strength and stability in all load cases (normal/operational and extreme) [18].

5.4.2 Failure analysis

In the previous section, the analysis on flapwise stress was provided. However, failure in orthotropic material depends on stress components in all axis and the interaction between them. Tsai-Wu failure criterion [63] is widely used for identifying failures in the analysis of anisotropic composite materials. In Tsai-Wu criterion, failure is evaluated based on a combination of stress components. It is designed such that the combination of different stress components are considered for failure initiation in composite materials. The failure index can be calculated based on a multi-axial stress condition and regarded as an important indicator of failure. A failure index that is greater than 1.0 means failure occurs.³ Contour plot of the mean and standard deviation of failure index (calculated based on the actual design strength $R_d = 1.13 \ GPa$) on the blade surface was shown in Fig. 5.12. As seen in Fig. 5.12 (a), the maximum mean value of failure index (~ 0.4) is much smaller than 1.0. Fig. 5.12 (b) shows the standard deviations of failure index. In addition, it is noteworthy that maximum failure index is located at the 14^{th} ply rather than the 13^{th} ply that contains maximum flapwise stress. Although falpwise stress is a very important indicator in wind turbine structure analysis, it is not enough to predict failure.

To further investigate the probability that the maximum failure index exceeds 1.0, the PDF of failure index at the location that has maximum mean failure index on the blade was estimated and is given in Fig. 5.13. Result shows that the probability of failure (FI > 1.0) is nearly zero. This analysis provides us a unique method to check wether a wind turbine design is valid. We envision this analysis as a useful reference in wind turbine design.

5.5 Discussion and Conclusion

Current wind turbine simulations are mostly based on simplified blade models which could not meet the requirement for accurately analyzing the fluid structure interaction between blade and surrounding air domain. On the other hand, in spite of the fact that randomness inherent in blade manufacturing process may impact blade performance and turbine efficiency, the

³Tsai-Wu criterion does not provide the information about when and how the failure occurs.



Figure 5.12 Contour of failure index on the 14^{th} ply.



Figure 5.13 Probability density function of failure index at the point that has maximum mean failure index on the blade.

mechanism of randomness affecting the performance of wind turbines is still an open question. In this research, we try to solve these problems by integrating stochastic analysis, which is based on adaptive sparse grid collocation method, on a full scale pre-bent 3D wind turbine rotor model constructed with isogeometric analysis. With a given steady wind load, the model gives the deformation and the stress distribution of the blades. Four material properties regards to composite laminate were given as stochastic input of the system. Stochastic results were generated based on 168 sparsely allocated sampling points.

An example of stress analysis and failure analysis on NREL 5MW wind turbine blade design was given. The results identify the region that is most likely to have large stress or large variation of stress. Moreover, the probability density functions of the stress values at critical regions were calculated. Finally, the probability of the critical region to have stress values lager than given threshold was calculated. If we define the threshold according to the design strength of the material, this probability becomes the probability for the blade design to fail. All above results should give us a better understanding of how the randomness affect the performance of the blade.

CHAPTER 6. SUMMARY AND DISCUSSION

In this research, we explored the feasibility of incorporating stochastic analysis on wind turbine design. We first developed a low-complexity yet realistic data-driven turbulence simulation framework using temporal and spatial decomposition. A synthetic turbulent inflow was generated using this framework and used as the stochastic input of a simplified wind turbine model. We utilized adaptive sparse grid collocation method and performed stochastic analysis on this system, where primary results on stochastic wind turbines' performance were found. As an attempt to achieve more accurate and detailed results on the deformation and surface stress distributions of wind turbine blades, a 3D comprehensive wind turbine model that was developed based on isogeometric analysis and a deterministic wind input were used in simulations. Stochastic analysis on random composite material properties was done on this comprehensive wind turbine model, where stress and failure analysis were performed.

From the analysis performed in this research, we found that the temporal spatial decomposition framework is able to precisely reproduce the temporal and spatial statistics of any given large data set of wind. The random variables that are generated in the decomposition process showed that Gaussian randomness assumption may not be valid for turbulences in various environmental conditions. The temporal and spatial modes resulted from the decomposition also showed valuable information about the turbulence. Simulation results of using TSD synthetic inflow showed good consistency with the results of using original wind data. The results of stochastic analysis show that the stochastic performances of wind turbines are only sensitive to few largest spatial modes in the turbulent inflow. From the stochastic analysis on the 3D comprehensive wind turbine model, critical regions on blades were located, the probability of blades to have failures were calculated, and suggestions of blades design were made.

A data-driven low-complexity model that encodes the spatial- and temporal-correlations

and is location- and topography- sensitive has significant utility in a variety of wind energy applications. These include, wind farm layout optimization using realistic wind models; robust control of turbine operations based on the low-complexity stochastic wind model; stochastic analysis for robust (and lean) design of turbine-blades and other components; and using realistic wind models to analyze wind farm output integration with the power grid. While the mathematical framework developed here is used to analyze wind speed, it can also be used to represent other atmospheric data such as temperature and carbon dioxide flux. This framework can also be naturally extended to represent ocean waves, which is crucial for off-shore wind turbine siting, layout and design analysis. On the other hand, stochastic analysis of wind turbine provides us a better understanding of how the randomness affect the performance of wind turbine, which could be a good reference for the wind turbine design guideline.

It is worth noting that although TSD is capable of representing any given wind data, and its accuracy does not strongly depend on the number of modes used in the decomposition (see the discussion in Sec. 3.1.5), more modes are still desired for accurately representing the full-field wind. As a result, using it as the stochastic input of wind turbine model results in a very highdimensional stochastic problem whose convergence is difficult to achieve. In addition, we only showed the ability of TSD in modeling the along-wind turbulence component that has apparent temporal and spatial correlations, more work need to be done to show the usefulness of this framework on representing the transverse and vertical turbulence components. Moreover, the development TSD framework is still in the initial stage, thus more work need to be done for it to be used in industry. Furthermore, the availability of high frequency field measurements is the bottleneck of applying TSD in data-driven analysis. Finally, although the 3D wind turbine model that is used in this research has rich surface geometry, it does not have complete structure components such as shear webs and structural cores. To better estimate stress and predict failure, rotor models with complete blade structure are required.

Future works of this research include but not limit to the following topics. First, the computational framework of TSD needs to be tailored such that it is consistent with IEC industry standard, and easy to be used in existing wind simulation codes. Second, by comparing the TSD parameterizations of wind turbines inflow and wake, possible conclusions on how the

statistics structure of turbulence is changed by wind turbines could be drawn. Third, other properties of blades can be used as stochastic input. For example, random waviness defects and stochastic structure parameters. Fourth, incorporating TSD framework in complete wind turbine models, which would reveal more realistic simulation results.

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