Title:
Connecting Math Methods and Student Teaching through Practice-Based Strategies: A Study of Pre-Service Teachers' Math Instruction

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Acceptance Date:
2012

Series:
UCLA Electronic Theses and Dissertations

Degree:
Ph.D., Education 0249UCLA

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Permalink:
http://escholarship.org/uc/item/6hn1d3vc

Abstract:

UNIVERSITY OF CALIFORNIA

Los Angeles

Connecting Math Methods and Student Teaching through Practice-Based Strategies:

A Study of Pre-Service Teachers' Math Instruction

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Education

by

Mollie Helen Appelgate

2012

ABSTRACT OF THE DISSERATION

Connecting Math Methods and Student Teaching through Practice-Based Strategies:

A Study of Pre-Service Teachers' Math Instruction

by

Mollie Helen Appelgate

Doctor of Philosophy in Education

University of California, Los Angeles, 2012

Professor Megan Loef Franke, Chair

There have been many calls for greater research into the connection between what is taught to pre-service teachers and how those teachings emerge in teacher practice (Cochran-Smith & Zeichner, 2005; Grossman, 2008). Understanding this connection and strengthening it is vital to the increased effectiveness of not just teacher education programs but of teachers and the increased learning of students. In order to strengthen this connection, researchers have been pushing for pre-service teacher learning to become more practice-based (Ball et. al, 2009, Windschitl et al., 2009).

The teacher education program in this study used a practice-based framework to design a math methods course which articulated critical aspects for teaching and learning mathematics (i.e., ensuring mathematical rigor, creating mathematical student discourse, and using equitable practices), and taught high-leverage strategies to meet these critical aspects.

This study investigated how these practice-based, high-leverage strategies emerged in pre-service teacher practice in their student teaching classrooms. Focusing on secondary math in a large urban school district, this study sought to answer the questions 1) How do the practice-based strategies taught in a math methods class emerge in pre-service teachers' student teaching practice? 2) What supports the emergence of these strategies in a pre-service teacher's student teaching practice and what impedes it? The study followed six pre-service teachers through a yearlong methods course and into their student teaching classrooms, and used classroom observations, interviews, artifact collection and logs of teacher practice to answer the questions. The findings suggest that pre-service teachers can use high-leverage practices in a way that is rigorous, creates student mathematical discourse, and equitable participation. The study proposes the following additions to the design of future math methods courses: 1) pre-service teachers enacting the practices in environments with increasingly more independence and less support before trying it in their own classrooms and, 2) sharing with their math methods course peers their findings after the enactment of the strategies in their student teaching classroom. These findings have implications for how we may more effectively teach methods to bring about change in classroom practices.

The dissertation of Mollie Helen Appelgate is approved.

José Felipe Martínez

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DEDICATION

This dissertation is dedicated to

Faynessa Armand, Kathryn Stevens, Kate Beaudet and Cyrene St. Amant.

You have not only been supportive professionally and personally, you've been a huge part of my inspiration for working in education. My goal in being a teacher educator is to work to create an educational system that prepares and supports wonderful teachers like you to stay in the classroom, or the school, for their career. The thoughtfulness, dedication, resilience, and constant innovation and reflection that you show as you work in our public schools represent standards that I want to live up to each day. I admire you for always caring about your students, your work and the world around you. You began as my fellow teachers and became my role models and friends.

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ACKNOWLEDGMENTS

This dissertation would not be possible without the support of my family, my UCLA family, and my friends.

To my mother: Thank you for your steadfast support and for always wanting to actively pitch in to help, whether it be watching Levi or finding a misplaced reference. You are not only my biggest supporter, you are an amazing role model. You spoke to me from a very young age about inequality, showed me the pain it can cause and taught me how to stand up against it. Plus, my intense curiosity is a direct result of yours. For this support and these values, I am immeasurably grateful. To my father and Joyce: Thank you for giving me a goal to beat and being the supportive voice at the end of the line on my walks to the library.

To my extended but wonderfully close family: Grandpa – thanks for believing in me and loving me as both an engineer and an educator; Emily and Zach – thank you so much for the cheers, they meant the world to me; Steve, Cindy, and Nathan – I don't think you'll ever understand how important it is to me that you are in my life. To my entire family: Going home to Iowa is a joy because of each of you and in May I needed that support so badly and you were right there.

At UCLA I am thankful to everyone in Center X, especially Kyndall Brown, Annamarie François, Sheila Lane, and Jackie Honda. The people of Center X became my UCLA family and I would not have made it through this process without the opportunity to be involved in schools that Center X provided and the relationships I built with the amazing educators there.

To Jaime Park, Imelda Nava and Helen Davis: I hope to grow up to be like you someday very soon. Sharing an office with you for the past year has been a joy and a privilege which I have

appreciated everyday. Your thoughtfulness about how to approach issues in classrooms, as well as those of life and educational policies, will shape me always.

To Karen Hunter Quartz: Thank you for being so supportive of this study and for always taking the time to talk with me about the dissertation process and being understanding of the time required to work on it. I am so thankful for having had this opportunity to work with you, IMPACT and IMPACT research team.

To the members of my RAC, Rema Reynolds, Mel Bertrand, Cliff Lee, Antero Garcia, Danny Martinez, Nicole Mirra, Rhoda Freelon, and Cueponcaxochitl Moreno: I would not be graduating this year without your support. You are amazing writers and scholars (and just interesting people) and I so appreciated how your work and our discussions motivated me to keep going.

To the pre-service teachers in this study: Although I cannot name you, I want you to know how much I enjoyed spending time in your classrooms and getting to know you. I so appreciated your openness and honesty; It gave me great insight into your student teaching experiences and I learned so much that I hope to use with future pre-service teachers. It seems that only a tenth of what I learned from you went into this paper but I hope it honors the struggles and successes of being a pre-service teacher. I admire each of you for the hard work and thoughtfulness you put into teaching.

To Megan Franke: I survived this process because you believed in the ideas and stayed by my side telling me it was possible. Your advice, practicality and positivity at just the right moments were critical to the design and writing of this dissertation. I couldn't have had a better chair and advisor. From being an amazing dissertation chair to still working in elementary math classrooms, you are my ultimate professor role model.

To my dissertation committee, Felipe, John, Jody, Jim: Thank you for applying your thoughtfulness and expertise to my dissertation. I felt very privileged to have each of you on my committee and I hope to do you proud in the future.

To Erica Shelton, best friend and writer extraordinaire: I cannot thank you enough for talking me off the writing and personal problem ledge more times than I can count. Having a dear friend who I can call at 2 am in the morning to commiserate with about hating twenty pages I've worked on for a week or to help stop the unhelpful tape repeating in my head is one of the great gifts in life.

To the girls of the house on Hilgard: You got me through so many personal and academic rough patches. Whether it was making sure I ate, staying up late studying with me in the living room, or watching Levi so I could go to the library – I so valued this support. This camaraderie cannot be paralleled. In particular, thank you to Phoebe Musandu, Chloe Knudsen-Robbins and Jennifer Li for their support through this process. They helped me stay on track to finishing this but kept me connected to the real word at the same time.

To D. Joseph Armstrong: They say getting a PhD is really hard on a relationship and I have to agree with that. I could not have survived the qualifying exams or the dissertation proposal stages without your unfailing support and patience. You were both my escape and my anchor during this process. The value of this in a boyfriend is not quantifiable. Thank you.

To the people of the state of California: Thank you for subsidizing my degree. As a beneficiary of this UCLA education, an ardent support of public universities and a taxpayer myself, I not only hope and pray that future Californians will have this same opportunity, I pledge to actively work toward keeping public universities accessible and affordable.

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Appelgate, M. (June 2009 & 2010). *Pluto Platters and Flying Saucers: An Investigation of Circles*. UCLA Math Methods Institute.

Appelgate, M. (June 2011). *Integers: The Beginning.* UCLA Math Methods Institute.

Appelgate, M. (June 2011). *Understanding Fractions.* UCLA Math Methods Institute.

Appelgate, M. (April 2012). *Strengthening the Connection between Math Methods and Pre-service Teacher Practice*. National Council of Teachers of Mathematics Research Pre-Session. Philadelphia, Pennsylvania.

Appelgate, M. (December 2012). *Connecting Learning Methods to Emerging Practice: A Study of Pre-service Teachers' Math Instruction*. California Educational Research Association. Anaheim, California.

Appelgate, M. & Beaudet, K. (2011). Research, practice and policy brief: Multiple measures of good teaching. *UCLA Center X Change*. http://centerx.gseis.ucla.edu/xchange-repository/spring-2011/the-briefing-room/center-x-research-practice-and-policy-brief

Nava, I., Park, J. & **Appelgate, M.** (2011). Observation rubric for secondary science. *UCLA Center X Change*. http://centerx.gseis.ucla.edu/xchange-repository/spring-2011/teacher-workroom/observation-rubrics-for-secondary-mathematics-and-science

Park, J., Nava, I., & **Appelgate, M.** (2011). Observation rubric for secondary mathematics. *UCLA Center X Change*. http://centerx.gseis.ucla.edu/xchange-repository/spring-2011/teacher-workroom/observation-rubrics-for-secondary-mathematics-and-science

Schweig, J., **Appelgate, M.** & Quartz, K.H. (2011) UCLA IMPACT: Integrating research and practice using multiple measures. *UCLA Center X Change*. http://centerx.gseis.ucla.edu/xchange-repository/spring-2011/teacher-workroom/ucla-impact-integrating-research-and-practice-using-multiple-measures

Chapter One: Introduction and Background

> "…By almost any standard, many, if not most of the nation's 1,450 schools, colleges, and departments of education are doing a mediocre job of preparing teachers for the realities of the 21st century classroom."
> -Arne Duncan, United States Secretary of Education, October 22, 2010

Reading this quote struck conflicting chords in me: part of me rushed to defend schools of education, and the other part of me worried that this is true. Both of my reactions were based on my experiences with teacher education. Coming into teaching with an emergency credential, I attended a local university's teacher education program at night to earn my credential while teaching middle school math and science. The focus was on knowing how to do the math, i.e., math subject matter content knowledge. Encouraging discussions around our mathematical reasoning or different problem solving methods was not a focus of the course, nor was pedagogical content knowledge, which is the knowledge needed to teach the math. Because I already knew the math, I rarely found the information we learned in the university credential program courses to be useful to me or helpful in developing my teaching skills. Also, we rarely engaged in intellectual work – we almost never talked about theory or ideas that would have helped us frame what we were experiencing in the classroom.

In contrast to this is UCLA's Teacher Education Program (TEP). For the last three years I have had the opportunity to work both as a field supervisor and a seminar teacher for beginning secondary math teachers in this program. Through this work I observed the stark differences between the knowledge and teaching practices TEP students learn and those of the post-bachelor credential program I attended. Although the amount of credit hours required is almost identical, the programs' philosophy and curriculum are very different. UCLA's TEP program uses a social justice lens to approach both learning and teaching. There is discussion about theory, and students often read research articles in preparation for discussion in class. In the mathematics education courses offered by TEP, the focus is on developing knowledge about how to teach mathematics content, and developing equitable practices to assure all students are engaged throughout practice.

Inevitably, teacher education programs must make decisions about which types of knowledge and practices to emphasize as they prepare teachers for the classroom. The two examples described previously point to the differences that can exist in teacher education programs, and while one might agree or disagree with a particular approach, the reality is that we know little from research about the effects of particular approaches. My personal experiences in my math methods course, my work with TEP, and the recent scrutiny of schools of education, caused me to wonder about the connection between what is taught in teacher education courses and how this preparation is taken up by teachers in their math classrooms.

There is very little research that connects what is learned in educational methods courses to teacher practice (Cochran-Smith & Zeichner, 2005). Looking specifically at math methods, this paucity of research remains true. In fact, most of the studies in a recent meta-analysis of research investigating aspects of math methods courses showed that most studies did not examine what was taught in the math methods courses, and very few studies followed pre-service teachers into their student teaching classrooms to look at their practices (Clift & Brady, 2005). Instead, the studies recorded how teacher beliefs, reflective practices, or attitudes towards equity changed from the beginning to the end of the course.

Although we do not have a research base for effective practices in a math methods course, for the past 25 years researchers have been working to conceptualize the knowledge required for teaching (Ball & Bass, 2003; Grossman, 1990; Hill, Schilling, Ball, 2004; Shulman, 1986). Shulman (1986), in a seminal piece on teacher knowledge, frames the knowledge required for teaching as "1) subject matter content knowledge, 2) pedagogical content knowledge, and 3) curricular knowledge" (p. 9). Subject matter content knowledge refers to the knowledge of a subject – its facts, concepts, language, hows and whys, and the value of the concepts in the subject. Pedagogical content knowledge is the knowledge required for *teaching* the subject matter. It includes understanding which "forms of representation" (i.e., charts, graphs, examples, demonstrations) and tasks are the most meaningful for student understanding. It also means being aware of and preparing for student preconceptions and misconceptions. As Shulman (1986) describes it, pedagogical content knowledge "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (p. 9). Curricular knowledge is knowing "the full range" of the curricular materials for the subject you are teaching, including the alternative texts (those not prescribed for the course), software, visual materials, and how the subject relates to the content being taught in the other courses the student is taking (p. 10).

Educational researchers studying mathematics have been working to measure both subject matter content knowledge and pedagogical content knowledge of teachers, and have attempted to link this knowledge or other measures of teacher knowledge (for example, teacher SAT scores, or using the college major as a proxy for subject matter knowledge) to teacher quality (Darling-Hammond, 2000; Hill, Rowan, & Ball, 2005; Monk, 1994; Wilson, Floden, and Ferrini-Mundy, 2001). Recently, however, researchers have begun to call for greater investigation into the practice of teaching, arguing that instead of hypothesizing about the qualifications that may lead to good teaching, we need to look at teacher practice to see what teachers do, and how the knowledge they bring plays out as they engage students in learning.

Trying to get closer to the practice of teaching, Ball & Bass (2002) approach the question of what teachers need to know by looking at what teachers *do in the classroom*, instead of what they know, calling this a "practice-based theory of mathematical knowledge for teaching" (p. 5). Ball and Forzani (2009) reassert this need to make teacher *practice* the center of teacher education. Similar to Ball and Bass (2002), they argue that the teacher education curriculum needs to be based on what teachers *do* in the classroom, not their beliefs or qualifications. They summarize this potential teacher education curriculum as "a practice-focused curriculum for the learning teacher [that] would include significant attention not just to the knowledge demands of teaching but the actual tasks and activities involved in the work" (p. 503).

If the goal is to move the knowledge for teaching closer to the practice of teaching, then we must figure out how to do that for pre-service teachers. If the knowledge for teaching should be closer to practice, then the learning of teaching needs to move closer to practice. In order to do this, Ball et al. (2009) developed a framework for a practice-based approach to mathematics teacher education, which includes “articulating the work of teaching mathematics, identifying and choosing high-leverage practices, and teaching mathematical knowledge for teaching” (p. 461).

Similarly, Ball and Forzani (2009) recommend teacher education programs develop instruction around what effective teachers *do* and then help pre-service teachers unpack all the elements of that practice. Although they acknowledge that coming to a consensus on the core tasks and activities of effective teacher practice may be difficult for researchers and educators, they argue this is crucial work for bringing the learning of teaching closer to teacher practice.

From a socio-cultural perspective, this moving of teacher learning closer to practice makes sense if we want to strengthen the connection between pre-service teacher learning and their classroom practice. Learning is about participation in a community whether you are a student in a K-12 classroom or a pre-service teacher learning in a university classroom. And the context of this learning is important to how the learners participate and integrate new knowledge into their own practice.

Working to move closer to practice in ways that make explicit these areas of teacher practice, the professors in the teacher education program researched in this study have come to a consensus around practices important to the teaching of mathematics. Building on current research, they have created a set of foci for pre-service teacher learning which includes engaging students in mathematically rigorous tasks, engaging students in mathematical discourse, and using practices that promote equity in the math classroom.

Although the program investigated in this study has developed instruction around the mathematical practices of teachers they feel are most important, the issue of the emergence of this knowledge into pre-service teachers' student teaching classrooms and eventually into their own classrooms remains a question. Research has shown that often times, when pre-service teachers who were taught reform-based practices in university education classes enter the classroom for student teaching, they revert to their original conception of mathematics teaching, which is generally a traditional, teacher-centered model (Ebby, 2000; Steele, 2001).

In a teacher education math methods course centered on teacher practices, what can we learn about how pre-service teachers take up the practices, and make sense of them in relation to their own teaching? What practices or strategies do pre-service teachers easily make use of in student teaching? Which practices are more rare? Why? Are there practices that would be important to learn in pre-service methods courses, practices that would support pre-service teachers to get started on a different way of teaching mathematics? Are there practices that are difficult for pre-service teachers to use in student teaching or to use as a new teacher? Learning more about how pre-service teachers make sense of and use the practices that serve as the focus of the math methods course will help us better understand what is difficult for pre-service teachers to do in their student teaching and why. These questions led to this study's research questions.

Research questions

1. How do the practice-based strategies taught in a math methods class emerge in pre-service teachers' student teaching practice? Which strategies do student teachers use and why?

2. What supports the emergence of these strategies in a pre-service teacher's student teaching practice and what impedes it?

This research, which sits within a larger study of pre-service teacher preparation, followed pre-service teachers through an entire academic year, from the beginning of their math methods course to the end of their student teaching. Its goal was to investigate the connections between a university methods course and pre-service teacher practice. Using classroom observations, teacher practice logs, surveys, and interviews, this study examined pre-service teachers' practices to see how the strategies they learned in a mathematics methods course emerged in the pre-service students' teaching. The research recorded *which* strategies emerged, and *how* they emerged in the classroom. Interviews were conducted to understand the reasoning behind the choice of particular strategies, and to find out what supported, or possibly impeded, the emergence of particular strategies in their teaching practice. With a dearth of educational research literature on math methods and math method's connection to teacher practice, this study will help fill a gap in the literature. In their investigation of over 300 research articles on teacher education, Wilson, Floden and Ferrini-Mundy (2001) concluded, "We need more studies that relate specific parts of teachers' preparation (subject matter, pedagogy, clinical experiences) to the effects on their teaching, practice" (p. iv). This study, which relates a specific part of teacher preparation, math methods, to student teacher practice, helps answer this call.

Chapter Two: Literature Review

Making sense of what secondary mathematics pre-service teachers are learning about practice from their mathematics methods course requires understanding how students acquire mathematics knowledge, critical aspects of effective math teaching, and the research-based knowledge of math methods courses. Since this study is based on the intersection of these three ideas, the literature review is organized around these elements.

Practices and strategies used by teachers of mathematics should be based on how students acquire new mathematical knowledge, and what we know about effective mathematics teaching. Effective math teaching uses strategies that address how students learn. If effective math teaching uses strategies that address how students learn, then an effective math methods course must address how students learn. Research on the learning and teaching of mathematics has found certain aspects of practice particularly important, so it follows that these areas should drive or frame the teaching of math methods. However, we know very little about the content of math methods courses-- what is taught, how it is taught, and the rationale for these choices. Although there is some literature that tries to relate math methods to practice, it is very difficult to understand the relationship without insight into the math methods course. In answering my research questions, I hope to find and better understand the relationship between what is taught in the math methods course and the practice of pre-service teachers, while using the research on the learning and teaching of mathematics to frame the investigation. First, we will look at how students learn math, then we will look at the research on effective math teaching, and then the literature review will conclude by discussing what research says about math methods courses.

How Students Learn Math and Algebraic Concepts: Implications for Teaching

In the book *How Students Learn: Mathematics in the Classroom*, Donovan & Bransford (2005) share three of the most important implications for teachers of mathematics based upon the science of how students learn. In order for students to learn, new teachings must: 1) Build upon students' prior understandings and address students preconceptions.

2) Incorporate both conceptual understanding and procedural fluency, so that procedures are connected to meaning. 3) Incorporate meta-cognitive strategies so students can take ownership of their learning (p. 1).

1) Building on Prior Understandings and Addressing Student Preconceptions: What this Looks Like in the Classroom

Said another way, this means that for students, "new understandings are constructed on a foundation of existing understandings and experiences" (Donovan & Bransford, 2005, p. 4) and that a teacher must not only connect new knowledge to prior knowledge, but in order to help students learn, a teacher must also address common beliefs of students such as math is all about "learning to compute," or "following rules," or that only certain people are good at math (Fuson, Kalchman & Bransford, pg. 220).

Multiple Methods to Approach a Problem.

Fuson, Kalchman & Bransford (2005) take this work on how students learn and describe what engaging in these practices looks like in mathematics classrooms. It first means allowing and encouraging students to create and engage in multiple strategies of problem solving. This allows students to connect strategies they know with ones that are emerging. This is not about accepting all methods, even if they are not mathematically sound, but about helping students develop skills at analyzing different methods and by understanding there are multiple ways to approach a problem. As Fuson, Kalchman and Bransford (2005) say, "using and valuing multiple methods in math class keeps students' engagement in strategy development… alive" (p. 224).

Mathematical Discourse.

Engaging in mathematical discourse is another classroom technique that helps students learn mathematics and addresses how students learn. Talking about math, especially a discussion that has students talking about their mathematical thinking and reasoning, builds upon student understanding and has the potential to address student preconceptions (Fuson, Kalchman and Bransford, 2005). In such a classroom, the teacher and the students are actively engaged in listening to students explain why their method worked for a particular problem, and then engaging in a class discussion to understand, compare, and contrast the multiple approaches. This works for many reasons. First, students who explain their thinking help others to understand the concept by allowing them to hear other approaches and compare and contrast their own methods. Second, as students explain their thinking the teacher becomes a learner, and teachers "frequently discover [student] thinking that can provide a springboard to further instruction, enabling them to extend thinking more deeply or understand and correct errors" (p. 228).

While researching the teaching and learning practices of three high schools, Boaler (2003) found that students showed greater math achievement gains in classes where the teacher used more effective questioning strategies to encourage discourse and mathematical thinking, compared to students using the same curriculum where teachers used practices that quickly directed students towards the right answer, or gave students very little opportunity for discussion or learning.

Explaining mathematical thinking often leads teachers and students to represent their ideas with drawings. As Fucson, Kalchman, & Bransford (2005) discuss, drawings “support classroom math talk because they are a visual referent for all participants” and often help students connect their understandings to the formal language of school mathematics (p. 228). In addition, the pictures become most effective when they are shared using common academic language. Developing academic language is an evolving process requiring teacher modeling and teacher support, so that students begin to use it in their own descriptions of their own methods.

Making Explicit Connection to Students’ Understandings.

Having students share their multiple methods and creating student discourse help**s** students link learning to prior knowledge and address preconceptions, but sometimes teachers may need to create “bridges” between the formal math of schools and the informal mathematical experiences (Fucson, Kalchman & Bransford, 2005, p. 231). For example, explicitly drawing representations of real-world values and pictures, like $.74 as seven dimes and four pennies, and their formal math counterparts, such as seventy-four hundredths and seven tenths sticks with four hundredths pieces, in order to help students make connections between what they know and how it relates to the mathematical language and practices they are learning in school (p. 230).

Kalchman & Koedinger (2005) take these same principles of learning and investigate how they apply to learning functional relationships, which serve as the basis for algebra. As they explain, connecting functions to student experiences works to bridge the knowledge and ideas of functions to the everyday lives of students, which serves to increase understanding of the algebraic ideas being taught. In fact, contrary to what many teachers may believe, students understand algebraic ideas and have an easier time solving algebraic problems in the context of real-life events (Koedinger and Nathan, 2004). Findings like this are consistent with the research on how students learn and cause Kalchman & Koedinger (2005) to advocate that algebra teachers build on prior knowledge by using student experiences and "powerful instructional contexts" to "bridge" new developing content knowledge with what students already know (p. 359).

2) Procedural Fluency and Conceptual Understanding must be Intertwined:

What this looks like in the classroom

Procedural fluency, defined by Kilpatrick, Swafford, and Findell (2001) as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately" and conceptual understanding as "comprehension of mathematical concepts, operations and relations" (p. 116), must be linked for students to develop new and lasting understandings. This becomes particularly important when students enter a more abstract class, such as algebra. "By the time a student begins algebra... the network of knowledge must include many new concepts and procedures that must be effectively linked and available to support new algebraic understandings" (p. 232).

However, often times when students enter algebra they are missing the foundational knowledge necessary to be successful (Loveless, 2008). Some of this may be attributed to **the** elementary curriculum. Falkner, Levi & Carpenter (1999) (as cited in Carpenter, Franke and Levi, 2003) found that the majority of students in grades 1-6 had misconceptions of the meaning of the equal sign, a concept vital to the understanding of algebra. In order to address these issues before students come to algebra, Carpenter, Franke and Levi developed ways for teachers to integrate algebraic thinking into the elementary math curriculum "that are more consistent with the ways that students have to think to learn algebra successfully" (p. 1). One approach is using number sentences to get students to contemplate relationships between the numbers in order to get at ideas about the equal sign and what it means. Consistent with the math learning literature, their strategies also include getting students to discuss the math - making conjectures, explaining conjectures, and evaluating conjectures. In algebra, developing a procedural fluency and a conceptual understanding means students must be able to represent functions in multiple ways, and be comfortable using each of those representations, which can include pictures, words, graphs, equations, and tables (Kalchman & Koedinger, 2005).

3) Meta-cognitive activities must be supported:

What this looks like in the classroom

Metacognition needs to taught and supported through mathematical instruction that works towards "making student thinking visible" (Fucson, Kalchman & Bransford, 2005, p. 239). In the classroom this could, for example, include students analyzing errors – which involves finding the error, correcting the error, and explaining why it is an error. This helps students think about the same issues when analyzing their own work, and helps students focus on explaining the process of getting to a solution, as opposed to only considering the answer (Fucson, Kalchman &

Bransford). It can also include students explaining their methods to one another, comparing methods, and helping others through explanation.

Kalchman & Koedinger (2005), in their application of these principles to algebra, agree that all the previously explained metacognitive strategies work with algebra, but they also stress the importance of creating a classroom environment where students feel safe and supported enough to take risks – risks involved in trying new methods, exploring alternative approaches, sharing ideas, and asking for help.

Critical Aspects of Effective Mathematics Teaching

This section on effective math teaching is framed around three aspects of classroom practice that should be present for effective math teaching. These include mathematical rigor, mathematical discourse, and equitable access to content.

Mathematical Rigor

Adding It Up (2001), developed by the Mathematics Learning Study Committee of the National Research Council, presents a research-based look at the knowledge, learning, and teaching of mathematics. They describe effective teaching as "teaching that fosters the development of mathematical proficiency over time," with five factors of mathematical proficiency (Kilpatrick, Swafford, and Findell, 2001, p. 424):

> 1) *conceptual understanding* - comprehension of mathematical concepts, operations, and relations
>
> 2) *procedural fluency* - skill in carrying out procedures flexibly, accurately,

efficiently, and appropriately

3) *strategic competence* - ability to formulate, represent, and solve mathematical problems

4) *adaptive reasoning* - capacity for logical thought, reflection, explanation, and justification

5) *productive disposition* - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with belief in diligence and one's own efficacy (p. 5)

Through focusing on interwoven strands of mathematical proficiency, this concept of teaching requires skills and concepts that are intertwined throughout the math course, which has implications for teaching. Although they state, “we endorse no single approach” (which seems to say they wish to stay out of the “math wars”), they take definite stances on what effective and rigorous teaching includes.

Mathematical Discourse

Although the field of educational research is still trying to describe exactly what it means to be an effective math teacher, there is considerable research about effective ways to engage students in mathematical learning. Franke, Kazemi and Battery (2007) organize these ways to engage students into three broad categories: “a) creating mathematical classroom discourse, b) developing classroom norms that support opportunities for mathematical learning, and c) building relationships that support mathematical learning” (p. 226).

Discourse in a math classroom can be described as a conversation about mathematical ideas that develops mathematical understanding. It can take place among a whole class or in smaller groups, and can be a conversation between students or between the teacher and students. Supporting discourse for understanding goes beyond the traditional initiation-response-evaluation format traditionally followed in math classrooms. It involves drawing out student understandings in order to learn about, focus, and develop further understanding. To use discourse to increase learning, teachers focus discussions on mathematical ideas and develop cognitively engaging tasks that support discourse in the classroom. Teachers must pay close attention not only to the details of the discussion, but also to issues such as who is participating and why, in order to increase greater equity of opportunity (Franke et al., 2007).

If discourse as described above is to be developed, classroom norms must be developed to create the space for rich discussions. Sometimes these norms go against both commonly held student and teacher perceptions of a mathematics classroom, and must be developed from scratch. Norms developed around the teaching and learning of math are considered socio-mathematical norms. In a classroom focused on discourse, examples of socio-mathematical norms include an expectation that one will explain his/her thinking, identify connections between strategies and ideas, and treat mistakes as opportunities for learning. As Franke et al., (2007) conclude: "social and socio-mathematical norms that emerge in classrooms have consequences for what students learn about particular mathematical ideas as well as what it means to do mathematics" (p. 242). Franke et al. argue that in order to create the classroom norms required for such discourse, a teacher must build meaningful relationships with students. This includes understanding each student's "race, cultural histories, and previous experiences," which in turn "enables teachers to build relationships that challenge assumptions and open opportunities" (p. 248).

Student discourse in the math classroom is important for students to become mathematically proficient. Students need to investigate, analyze and justify mathematical methods and situations by asking questions, explaining reasoning, comparing methods, and answering questions, for example (Kilpatrick et al., 2001). And the teacher needs to create these opportunities with thoughtful tasks and questions. *Adding It Up* (2001) breaks down their recommendations for discourse as follows:

> - A significant amount of class time should be spent in developing mathematical ideas and methods rather than only practicing skills.
>
> - Questioning and discussion should elicit students' thinking and solution strategies and should build on them, leading to greater clarity and precision.
>
> - Discourse should not be confined to answers only but should include discussion of connection to other problems, alternative representations and solution methods, the nature of justification, and the like (p.426).

Equitable Access to the Content

Evidence of equitable practices is another key feature of a math classroom designed for learning. The National Assessment of Educational Progress (NAEP) results and the annual California Educational Opportunity Reports show that outcomes for math education are not equitable by race, socio-economic status, or gender (Rampey, Dion, and Donahue, 2009; Roger et al., 2007). Recognizing the need to include equity in effective math classrooms, the National Council of Teachers of Mathematics (NCTM), included equity as one of their six principles to guide the “content and character of school mathematics” (p. 11). They state more than once that “excellence in mathematics education requires equity - high expectations and strong support for all students,” and clarify that “equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (p.12). Bishop and Forgasz (2007) frame equity as equity of access. Access refers to not only whether students are admitted to key gatekeeping courses, but also concerns the content of those courses as it is actually realized in the classroom and experienced by students” (p. 1152).

Although Kilpatrick et al. (2001), in *Adding It Up*, may define and describe effective teaching, Hiebert and Grouws (2007) are much more cautious. In their review of the literature on how math teaching affects mathematical learning, they feel they cannot make a claim about what effective math teaching is because of the complexity of teaching, with its many contexts and variables, and the difficultly of linking teaching to student achievement. However, they find the strongest connection between what is taught and what is learned is based upon the opportunities students had to learn the material. This concept, aptly called “opportunity to learn,” is “ widely considered the single most important predictor of student achievement” (Kilpatrick et al. 2001). The opportunity to learn is influenced by many things; some are within a teacher’s control and some are not.

A short list of influences regarding the opportunity to learn includes time spent on a topic, how the topic is addressed, questions asked, activities planned, curriculum, learning goals, discussions, and classroom environment (Hiebert and Grouws, 2007). In addition, it also includes considering students' experiences, levels of knowledge, and needs. In looking at teaching and students' opportunity to learn, Hiebert and Grouws (2007) were able to investigate math instruction that specifically developed procedural fluency and those that developed conceptual understanding. They found that instruction that develops procedural fluency is teacher-centered, has clearly defined learning goals, "is rapidly paced, includes teacher modeling with many teacher-directed product-type questions and displays a smooth transition from demonstration to substantial amounts of error free practice" (p. 382). Instructional practices that developed conceptual understanding were those that made explicit connections between multiple representations, ideas, and methods, and those that caused students to put forth effort to understand the mathematical concepts (Hiebert and Grouws). These findings show how access to particular math practices may affect how or what a student learns, which has implications for equity or equitable access to the content.

Each of these things – rigor, discourse, and equitable access to content – helps to explain effective math teaching, and frames the strategies that are taught in the math methods course with the goal that they will be used in the pre-service teachers' student teaching classrooms and then, in the end, in the pre-service teachers' own classrooms.

Knowledge of the Math Methods Course

The study of mathematics teacher education has a small research base (Cochran-Smith & Zeichner, 2005). Within that, there are very few studies examining strategies for teaching math methods, and fewer yet that try to connect math methods to teacher practice. In their review of the research concerning math teacher education from 1995-2002, Clift and Brady (2005) found 20 studies for their meta-analysis of mathematics teacher education. Of those, only five examined the content of a math education course and followed the pre-service teachers into the field to investigate the connection, and three of those were investigating changes in beliefs or reflective thinking, not practices.

One that did look at teacher practice followed three pre-service teachers through one academic year by observing their one semester math methods course, taking notes, collecting documents, and interviewing students, and then following those students for one semester of student teaching, in order to see how teachers' ideas of the purpose of mathematics changed (Ebby, 2000). Although each of the pre-service teachers began the course with traditional notions of mathematics, all three teachers left with different notions of what it meant to teach mathematics. However, when they entered their classrooms as student teachers, only one was able to create a classroom where she could use these newly developed ideas about teaching math.

A study by Steele (2001) followed four elementary education students for four years (in combination with Steele and Widman (1997)) to see if the cognitively-based strategies taught in math methods persisted in both beliefs and practice. For two of the students this approach did persist in both beliefs and practice; however, for the other two, issues with curriculum, support and school environment may have prevented them from using these strategies or persisting in these beliefs.

More recent work in the connection of math methods courses to student teaching practices includes Ensor (2001) and Bednarz and Proulx (2005). These studies, which took place in South Africa and Canada, respectively, looked at case studies of students to investigate how what is learned in university math education classes relates to classroom practices when students are placed in student teaching assignments. Ensor (2001) frames this relationship between what is learned and how it is used in a classroom setting as "recontextualizing" knowledge. Following seven students for two years, the first of which they were attending a yearlong math methods course, and in the second they were full-time teachers, Ensor (2001) found that although all seven said they used tasks taught to them at the university, Ensor herself never witnessed any during her classroom observations. What each of the teachers appropriated most was a "professional argot from the math methods course" (p. 313). This included such features as, in a teacher's own words, a "desire to that they talk to each other while they are working" and a "desire to make math more concrete and real and experiential"(p. 312). Ensor (2001) briefly touched on possible constraints that could have limited the teachers' practice in the classroom, including other teachers, school setting, and lack of support, but comes to no decisive answer. She concluded that there is an "apparent inconsistency between what is offered in teacher education courses, and the manner in which beginning teachers perform in classrooms" (p. 317), and reiterates a conclusion of Ebby (2000) that "we need to reframe the goals of a methods course as begin about developing habits of mind to learn from the classroom. Through such habits, pre-service teachers can learn from fieldwork in ways that are generative instead of imitative and in ways that foster future growth" (Ebby, 2000, p. 93-94).

Bednarz and Proulx (2005) found something similar – that student teachers "appropriated principles on a general level and were not stuck on a particular way of presenting specific mathematics content" (p. 5). These principles included things such as "emphasis on students' work," encouraging student reasoning and putting math concepts into relevant contexts (p. 3). They concluded that the reason behind the variation in appropriation of principles is because teachers "have different visions and backgrounds and are interpreting things in different ways….Since different outcomes and interpretations are possible, meaning that we cannot control or predict them, it is now time to *educate* teachers and not *train* them" (p. 5).

There is also little research on how mathematics teacher education affects pre-service teachers of algebra. Kieran (2007), in her review of studies that focused on pre-service teachers of algebra, found that most studied the relationship between pre-service teachers' subject-matter knowledge and beliefs, their attitudes and beliefs, or measured their subject-matter knowledge (p. 745). One study that connected math methods to pre-service teaching focused on how students taught a particular algebraic concept, slope, after learning about it in math methods (Stump, 1999). Although Stump found pre-service teachers' understanding of slope expanded from teachings in the methods class, when she followed the pre-service teachers into their student teaching classrooms she observed that they had difficulty using the multiple representations to teach the concept of slope. She concluded methods classes needed to provide more opportunities to challenge students to use multiple representations of mathematical ideas, and that encouraging the use of non-traditional curricular materials may assist with this.

Conclusion

Examining the literature on the learning and teaching of mathematics, one can see comparable ideas concerning rigor, discourse and equitable access to the content. For example, students engaging with multiple approaches to a problem, multiple representations, and connecting procedures and conceptual understanding, are all ways in which students learn mathematics best. These ideas map on directly to the ideas presented as effective math teaching in the book *Adding it Up,* which stresses five interwoven strands of fluency which indicate rigorous math teaching, including knowing procedures, conceptual understanding, strategic confidence (which would be the strand that understanding multiple representations would fall under), and adaptive reasoning, which would be evident if a student was comparing different approaches or justifying one approach to a mathematical problem. A similar parallel between how students learn and effective teaching strategies is also visible with mathematical discourse. Although equitable access to the content is not evident in the literature on how students learn mathematics, it is clear that students cannot learn mathematics content without the opportunity to learn it. In the research on learning literature, it seems to go without saying that access to content is necessary for student learning. Fortunately, the research on teaching picks up the ideas on opportunity to learn, and emphasizes the need for equitable access to learning opportunities in math.

These bodies of literature on the learning and teaching of mathematics reflect one another, so it is notable that the math methods literature not only does not mirror back these ideas in the research, it barely takes notice of them. In contrast, this study used the concepts of mathematical rigor, mathematical discourse, and equitable access to content to frame how the teachings of the math methods course connect to student teacher practice.

Chapter Three: Methodology

Over the past 10 years there has been a tremendous press to move learning to teach closer to practice so that pre-service teachers have the opportunity to learn in, from, and for practice. The secondary mathematics methods course at a large research university investigated in this study was restructured with this purpose of moving learning to teach closer to practice. The goal of my study was to understand how the opportunities to learn in and from practice within a math methods course are taken up by pre-service teachers in their student teaching classrooms. Specifically, this study investigated how strategies[1] taught in a math methods education course emerged as pre-service teachers student taught.

As a part of a larger study, I documented the work within the math methods course which pre-service secondary math teachers attend for an entire academic year. The documentation, which I called, in this paper, the Preliminary Investigation of the Math Methods Course, included a close examination of the strategies that are taught, and how they are taught, in the math methods course. In the primary part of the dissertation study, I observed six of the pre-service teachers from the math methods course in their student teaching classrooms to investigate how and why they incorporated these strategies into their classroom practices. For Phase I of the dissertation study, I observed each pre-service teacher teach three full lessons in their student teaching classroom to understand which strategies emerged, how they emerged, why they emerged and what supported or impeded their emergence. In Phase II, I observed teachers for two lessons after they were given the directive from the math methods professor to try the identified high-leverage strategy of Think-Pair-Share in their student teaching classrooms. Then, I observed the teachers for two lessons in which they could implement either Think-Pair-Share or

[1] For this study of math teacher education and practice, a strategy is defined as a deliberate and planned method or approach chosen with the purpose of engaging students in math and increasing mathematical proficiency.

another high-leverage strategy from the math methods course. These two observation phases were designed to see how the strategies were emerging in the pre-service teachers' classrooms, and then to understand how the enacted strategies emerged in the student teaching classrooms. The goal of this study was to better understand the connection between the math methods course and student teacher practice so we can strengthen math teacher education programs with the goal of strengthening teaching and student outcomes. To move the learning of teaching closer to practice we must to better understand how to support that learning and provide opportunities to learn from practice.

Background: Pre-service Teacher Education Program

The teacher education program, housed in a large research university in southern California, is currently trying a new model of teacher education called urban teacher residency (UTR). This new teacher education design, based on an apprenticeship model, has pre-service teachers in a classroom in a large urban district from the first day of school until the last, working alongside one or two mentor teachers. This 18-month graduate teacher education program begins in the summer with foundational coursework. In the fall, the pre-service teachers begin their yearlong apprenticeship in an experienced mentor teacher's classroom. In addition, throughout the academic year the pre-service teachers attend university classes one full day and one afternoon each week. In the late spring, the pre-service teachers wrap up their coursework, get their teaching credential, and are hired by a school district (or charter school) to begin teaching in the fall in their own classrooms.

Preliminary Investigation of the Math Methods Course

Classroom context of the math methods course.

The math methods course is a three-quarter sequence required of all secondary math pre-service teachers enrolled in the urban teacher residency (UTR) Master's program. Each of the classes in the sequence is taught by the same professor, who also supervises the pre-service teachers in the field. In the fall quarter of 2010, this class was taught on the university campus for three hours each Friday. In the winter and spring quarters, the math methods class was taught for two hours each Tuesday in a high school classroom in the district. This math methods class was framed around the idea that the purpose of math teaching is to develop students' mathematical proficiency. The term "Mathematical proficiency," as used by the professor and defined by Kilpatrick, Swafford, & Findell, (2001) in *Adding It Up,* is five "interwoven and interdependent" strands, that include:

- *procedural fluency* - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- *productive disposition* - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with belief in diligence and one's own efficacy;
- *conceptual understanding* - comprehension of mathematical concepts, operations, and relations;
- *strategic competence* - ability to formulate, represent, and solve mathematical problems;

  adaptive reasoning - capacity for logical thought, reflection, explanation, and justification. (p.5)

The professor's reform-oriented intentions were made clear in the fall 2010 course syllabus. The opening quote on the syllabus said, "Developing mathematical understanding requires that students have the opportunities to present problem solutions, make conjectures, talk about a solution process, prove why solutions work, and make explicit generalizations" (Franke, Kazemi and Battey, 2007). Her goals for the fall quarter were explicitly spelled out in the syllabus and focused on eliciting student thinking, creating math discourse in the classroom, and supporting diverse learners in order to help students develop mathematical proficiency. Similarly, the assignments - one of which required pre-service teachers to videotape their teaching and analyze their strategies, student discourse and student learning - reflected these goals also.

In class, the professor focused on teaching pedagogical content knowledge, or *how* to teach the math, instead of teaching the math itself. In teaching *how* to teach the math, she regularly used strategies the pre-service teachers could use in their teaching. For the purposes of this study, I define *strategy* as a deliberate and planned method or approach chosen with the purpose of engaging students in math and increasing mathematical proficiency. The professor taught the strategies by integrating them into the math methods lesson, rather than only explaining how the strategy may work in the classroom. Although she incorporated the strategies into the lesson so that the pre-service teachers could experience them as their students would, she also made the strategies explicit by posting the protocol or directions for their use, describing her rationale for using them, and sharing alternative ways to use the strategies. Afterward, the students would often debrief with the professor, who asked how they, as students, were able to participate and engage with the lesson because of the use of the strategy.

Data collection overview of the math methods course.

Throughout the entire academic year, I observed each time the math methods course met. While observing and taking notes, I recorded the tasks, activities, questions and conversations of the professor and the pre-service teachers. I also kept logs of time spent on each activity and discussion, as well as collected classroom artifacts distributed by the professor. The goal of these observations and artifact collection was to get a fully informed view of the happenings in the math methods course.

Data sources of the math methods course.

Observations.

Observations were used to record what was taught, and how it was taught, in the math methods course. I chose this method because as Rowan and Correnti (2009) note, classroom observations are "often seen as the gold standard for data collection in research on teaching" (pg. 115). Pianta and Hamre (2009), also speaking about observations as a tool for capturing practice, conclude that observations "can be directly related to the investigation and experimentation of specific interventions aimed at teaching" (pg. 115). In the math methods course, the observations were conducted for each class by taking notes of the activities and the discourse. The strategies the professor used to get at the ideas of creating mathematical rigor, mathematical discourse, and equitable access to the mathematical content in the classroom, and the student and professor discourse around those strategies, was the focus of the observations.

Artifact collection.

Classroom artifacts are an important aspect to teaching because artifacts are a concrete way to understand how a teacher approaches a lesson and what the teacher thinks is important in designing a lesson. As Matsumura et al. (2006) report, classroom artifacts provide additional evidence of "the rigor and implementation of lesson activities and application of classroom assessments" (p. 7). I collected all the handouts that were distributed at each math methods class to gather additional evidence for which strategies were addressed, how they were addressed, and to what extent and depth they were addressed.

Data analysis of the math methods course.

One of the first artifacts collected from the math methods professor was the student teaching observation rubric for secondary math pre-service teachers. This student teaching observation rubric was developed in collaboration with the math methods professor, the science methods professor, and myself as part of a larger project, with the goal of using it to evaluate student teaching performance (see Appendix A). The observation rubric helps to quantify four overarching characteristics of mathematical teaching: 1) the mathematical rigor of the lesson, 2) the quality of math discourse, 3) the use of practices to encourage equitable participation in a math class and 4) classroom ecology or classroom management practices. The first three characteristics of mathematical teaching were chosen because they represent three aspects of classroom practice that have been deemed important to mathematical learning by the research literature (Franke, Kazemi, Battey, 2007; Fuson, Kalchman, Bransford, 2005; Kalchman & Koedinger, 2005; National Council of Teachers of Mathematics, 2000), and by the math methods professor, who was the primary developer of the observation rubric. The math methods professor, who also supervised her student teachers in the field, wanted to develop this tool to evaluate her pre-service teachers' student teaching in the field, and currently uses it for this purpose.

This student teaching observation rubric, which focused on three aspects of classroom practice vital to teaching mathematics, framed my approach to looking at the math methods course and student teacher practice. It also guided the development of the list of high-leverage practices and the log of teacher practice, which I designed to investigate pre-service teacher practice when I could not be in their classrooms to observe. This tool will be explained in greater detail in the student teaching observation section.

To gather information on the strategies taught in the math methods course to help me better determine what I would be looking for in student teaching practice, I reread the fall quarter math methods observational notes, coding for strategies the math methods professor taught and explicitly explained. As previously defined, a *strategy* is a deliberate and planned method or approach chosen with the purpose of engaging students in math and increasing mathematical proficiency. In this case I am using this definition, but because it is applied to the math methods class and the professor's choices, I am also including that the strategy was "explicitly" explained by the math methods professor. This means that the strategy, when used in the math methods class, was accompanied by the rationale for using it in the pre-service teachers' own math classrooms (student teaching classrooms and future classrooms), and often times the professor also posted directions or a protocol for using the strategy.

After creating a list of all the strategies the professor used, I returned to the list of strategies and coded each one, based on whether the strategy was designed and taught by the math methods professor to address aspects of mathematical rigor, mathematical discourse, and/or encourage equitable participation in the class. Most of the strategies fell in all three categories, but certain ones highlighted particular aspects of the rubric. For example, a teacher presenting a mathematical problem with an error and asking the class to analyze it, a strategy termed "error analysis," highlights mathematical rigor according to the rubric, but the strategy has a parallel goal of getting students to talk about what they see and how they would approach the problem--in other words, encouraging mathematical discourse. However, for example, if students discuss this and share in pairs, then this also encourages equitable participation in the class, so the strategy, as implemented, covers all three aspects of the rubric.

This coding of the observational notes led me to create a list of what I call high-leverage strategies. Windschitl et al. (2009) defines high-leverage practices as "those most likely to stimulate significant advancements in student thinking when executed with proficiency" (p. 4) or "planning or enactment practices that aim to engage learners in forms of discourse that lead to and embody learning" (p. 7). In his work, focused on science teaching, Windschitl chooses high-leverage practices that help pre-service teachers develop the planning goal of "constructing the big idea" of the unit and then, in the classroom, practices that elicit student thinking, help students make sense of the content, and press students for evidence-based explanations (p. 7). Ball et al. (2009) defined the criteria for choosing high-leverage practices (HLPs) as:

> Criteria for HLPs based on examinations of the work of mathematics teaching:
>
> - Helps to improve the learning and achievement of all students
> - Supports student work that is central to the discipline of the subject matter
> - Are used frequently when teaching
> - Applies to different approaches in teaching subject matter and to different topics in the subject matter

Criteria for HLPs necessitated by teacher education context:

- Can be articulated and taught
- Is accessible to learners of teaching
- Can be revisited in increasingly sophisticated and integrated acts of teaching.
- Is able to be practiced by beginners in their field-based settings. (p. 461)

In the case of this study, I label these high-leverage strategies because they are strategies based on the work of math teaching developed from research and chosen by the university's teacher education program - rigor, discourse, and equity- and they follow the criteria set forth in Ball et al. as being able to be articulated, taught, accessible, increasingly sophisticated, and able to be practiced by beginners.

Based on this coding scheme and the criteria developed by Ball et al., the list of high-leverage strategies observed in the math methods course were:

- Quick write
- Using interactive notebooks
- Whip around to share analysis and/or reasoning from reading/activity/class
- Teacher writing student thoughts/reasoning on the board
- Using big problem in context to get at mathematical idea
- Presenting problems in the context of real life – culturally relevant
- Board talk
- Think, Pair, Share
- Create graphic organizer
- Categorizing/Sorting items
- Say, Mean, Matter
- Pair, Share
- Describe to your partner (not show) - activity to develop academic language
- Structured group talks
- Sentence starters
- Summarizing ideas in groups
- Web – Concept map
- Compare mathematical methods used in a math problem
- Error analysis

A description of these strategies can be found in Appendix B.

Pre-Service Teacher Classroom Observations

Participants.

The six pre-service teachers followed in this study were selected from the secondary math methods course taught by a university professor in the urban teacher residency (UTR) teacher education program. Out of the twelve students in the math methods class, I chose to focus on the six pre-service teachers because they were all student teaching in Algebra I classrooms. Controlling for content was important because it allowed me to look in detail at practices in one particular subject, at one particular level, without concern that the content was the reason for the differences. The other primary benefit to focusing on algebra is that this subject is pivotal in the math class trajectories of students. Often called a gatekeeper course, algebra success is commonly linked to greater rates of college attendance and college graduation rates (Oakes, 1990; Moses & Cobb, 2001; Spielhagen, 2006). Secondarily, algebra is taught in a wide range of grade levels, with 7^{th} graders at the low extreme and 12^{th} graders at the top. Therefore, in Algebra I, pre-service teachers are teaching the same wide range of grade levels that presumably a secondary math methods class would address, and each of the strategies addressed in the methods course would make sense to draw upon.

All of the pre-service teachers in this study were also teaching in the same large, urban district and in the same general geographical area of that district. Each school also had similar demographics. The majority of students at each high school were Latino, ranging from 71-92% of the student population. Between 16 and 35% of the students were classified as English Language Learners (ELL) and between 48 and 62% of the students had already been reclassified as English proficient. The students' family incomes were generally very low, as 55-100% of the students at the high schools were eligible for free or reduced lunches.

Data collection overview of pre-service teacher classroom observations.

I conducted observations and recorded the observations in the student teaching classrooms by taking notes and audio recording the lesson, as well as collecting classroom artifacts created or used by the pre-service teachers.   After each observation I interviewed the teacher about the lesson to specifically find out what influenced the design of the lesson, the use of the particular strategies noted, and the ways the pre-service teacher felt that he or she encouraged discourse and equitable participation. In addition to my observations, pre-service teachers were asked to use a log of teacher practice every day for two non-consecutive weeks, in order to record the possible use of strategies in their classrooms and gain a better understanding of the emergence of these strategies when I was not able to observe them in person. Finally, to wrap up the observations, I conducted an end-of-the-year interview with each pre-service teacher to get their thoughts on the whole year of student teaching.

Data sources of pre-service teacher classroom observations.

Observations.

In the pre-service teacher's student teaching classrooms, I gathered data primarily using observational notes and audio recording. This method of data collection allowed me to see which strategies the student teachers were using, and how they were using them, to try to bring about rigor, student discourse, and equitable access. In my observational notes, I divided the paper for notes in half – writing on the left-hand side what the students said, and on the right hand side what the teacher said, along with times listed on the far left margin. This set-up allowed me to see the discourse happening in the math classrooms clearly, particularly in regards to who was doing the most speaking, and how those conversations developed and progressed. The audio recordings allowed me to delve into the wording and development of the strategies so that I could see exactly how high-leverage strategies from the math methods course emerged in the pre-service teachers' classrooms.

Logs of teacher practice.

Logs of teacher practice (see Appendix C) provided additional evidence to help me answer the research questions: 1) How often do the strategies taught in a math methods class emerge in pre-service teacher practice in their student teaching classrooms? 2) Which strategies do the pre-service teachers use? Logs of teacher practice helped document the answers to these questions because logs of instructional practice recorded strategies used in a greater number of lessons than an observer could see in a limited number of visits. Although I could find no previous research on pre-service teachers using logs, as Rowan and Correnti (2009) explain, logs are designed to capture frequency of teacher practices in the classroom. In their study, "Instruction is conceptualized as a series of repeated exposures to instruction, and the key measurement problem is to obtain an estimate of the overall amount or rate of exposure to particular elements of instruction occurring over some fixed interval of time" (p. 210). Their answer to this problem of capturing teacher practice, which they term "enacted curriculum," is logs. In my study I am trying to find out exactly this – a rate of exposure.

In my case, I want to know how often pre-service student teachers use strategies they have been taught in the math methods course. I needed to discover which strategies pre-service teachers used so I could begin to understand the strength of the connection between the strategies taught in a math methods course and pre-service teacher practice in the classroom. I recognize that the logs are self-report documents, and I designed them to provide an indication of strategy use as clearly as possible. With the goal of being clear to pre-service teachers, the strategy names listed in the log are the same as those used in the methods course. The pre-service teachers marked on the log if they used a strategy and how often it was used in the lesson. There is the possibility that filling out the logs encouraged the pre-service teachers to use the strategies more often. However, I also believe that there are many influences on what a pre-service teacher decides to do, including their field supervisor who is also associated with these strategies. Other studies have shown it is not been that easy to influence or push pre-service teachers' (or teacher's) to use reform-oriented practice (Ebby, 2000; Steele, 2001).

I used these teacher practice logs to capture classroom practice for two weeks, in separate sets that each covered one week of instruction, once in early May and once in early June. Using them over the course of a week allowed for the possibility of capturing a greater range of strategies, because often times teachers have math quizzes one day a week and plan instructional activities around those times. Because I administered them for a full week at a time, I increased the probability of capturing a range of instructional goals and therefore strategies. Also, by using the logs repeatedly day after day, the pre-service teachers became more familiar with the tool and more adept at filling it out. The pre-service teachers filled them out themselves after receiving a quick information session on the tool. Although this may bring up issues of reliability, Rowan and Correnti (2009) found that on items concerning frequency of activities, teachers matched professional raters 81%-90% of the time, and on other issues, like cognitive demand, they matched an average of 73% of the time. Also, these logs, designed primarily in checklist form, gathered information about items with which the pre-service teachers were familiar, as the logs included strategies from the math methods course.

Logs of teacher practice added to the data of the observations and allowed a view into the classrooms when observations did not occur. Logs are a useful tool for recording classroom activities, and compliment observation data because as Kennedy (1999) points out, logs are "concrete and potentially accurate descriptions of what teachers are actually teaching in their classrooms. In addition, because they require a clear form of reporting, it is easier for researchers to convey to one another what their database consists of than is often the case with observation data" (pg. 345).

Similar to the rubric for observing pre-service teachers designed by the math methods professor, the log used the three mathematical aspects of the rubric and the initial understandings from the math methods observations to frame the tool's design. Below is an example of how the teacher practice log related to the rubric and to the strategies gathered from math methods.

Teaching Dimensions	Contemplating Level 1	Emerging/Applying Level 2	Integrating Level 3	Innovating Level 4	Examples of instructional strategies
Mathematical Discourse					
Participation Structures	No participation learning structures for student participation and/or discourse.	Participation learning structures with limited structure for equitable student participation -Some seating arrangements allow for discourse in pairs/small groups.	Participation learning structures with some structure for equitable student participation. -Seating arrangements are in pairs/small groups -Some consideration for student needs.	Participation learning structures *with* structure for equitable student participation -Pair sharing -Small groups have individual roles and responsibilities. -Consideration for student needs.	• Pair-share, dyad, group

Figure 1. Student teaching observation rubric – the strand of mathematical discourse.

Which classroom structures of participation did you use today? (**Please check any that apply**.)

☐ Whole class
☐ Small groups, size: _____
☐ Pairs
☐ Roles and/or responsibilities within small groups

Figure 2. How the log of teacher practice operationalizes this strand of the rubric.

Strategies used today

Please make a check mark in the left hand column if you used this strategy today in class. If you used the strategy multiple times during the class period please circle the number of times used on the right.

☑	**Strategy**	**Tally**
	Warm-Up	2 3 4 5+
	Interactive notebooks	2 3 4 5+
	KWL Chart	2 3 4 5+
	T poses high-level question to get at student reasoning	2 3 4 5+
	Board talk	2 3 4 5+

Figure 3. How high-leverage strategies from the math methods course were recorded in the log of teacher practice. This is a partial list of the strategies provided as an example.

Classroom artifacts.

Classroom artifacts provided additional insight into pre-service teacher practice because they provided evidence of how a teacher approached a mathematical topic, and what the teacher considered important in designing a lesson. In analyzing the data from classroom observations, I used these artifacts to trace which strategies were used and how they were implemented. In addition, in the pre-service teacher classrooms I used the artifacts as a jumping-off point for the after-lesson interview with the pre-service teacher.

Interviews.

Both after-lesson and end-of-the-year interviews were vital to understanding why the pre-service student teachers used particular strategies, and what they felt supported the emergence of these strategies in their student teaching classrooms.

Through interviews after the student teaching observations, I found out why the pre-service student teacher approached the lesson the way s/he did. It is finding out the "why" that allowed me to understand the thinking behind their instructional choices, the sources of that thinking, and what supported or impeded making those choices.

After-lesson prompt*: Please walk me through the decision making process you used to design today's lesson and activities.* **Follow-up question***: I would like to understand why you made the choices you did, particularly in regards to student talk (math discourse), and any choices you made to address issues of equity.*

This open-ended after-lesson prompt was designed to keep the lines of communication open, and non-judgmental, so I could understand the influences, both positive and negative, on the emergence of strategies from the math methods course in the classroom. Initially, I did not prompt the pre-service teachers with questions about discourse and equitable structures. But as they talked, if they did not explain the reasons behind choices that dealt with these issues, I asked the follow-up question to gain a greater understanding of the reasons behind their choices regarding those particular aspects of their math teaching.

In the final end-of-the-year interview I asked the same two questions that were asked in the after-lesson interviews, but I asked the pre-service teachers to consider the questions for the entire year. For example, I asked them to walk me through how they generally planned out daily lessons and activities. Then I followed up with asking how in general, over the course of the year, they encouraged or addressed mathematical discourse and issues of equity and equitable participation. I also reviewed the strategies the pre-service teachers used over the course of my study, and asked why these particular strategies emerged, and what supported and/or impeded their emergence. Finally, I concluded the interview by asking directly what the pre-service teachers thought about their classroom practice in relationship to what they learned in their math methods course, and their ideas for making the connection between the math methods course and teacher practice stronger.

Plan for data collection and analysis.

In order to capture the complexity of teaching and learning, data was collected using a variety of measures. Using multiple measures allowed me to look at which strategies were taken up by pre-service teachers, and provided different sources for understanding the frequency and ways in which various strategies were used. In my quest to capture teaching practice in the math methods classes and pre-service teachers' classrooms, having multiple sources allowed me to triangulate the information for increased confidence in the findings.

Using methodology based on grounded theory, the data was collected, analyzed and developed into themes to understand how these high-leverage strategies were emerging in the pre-service teachers' classrooms, and to produce a hypothesis for how we may work to strengthen this connection between the math methods course and student teacher practice. Using multiple sources of data, I was able to triangulate my data in order to confirm its validity.

Table 1

Overview of the Data Sources by Research Question.

Research questions	Sources of data
How do the strategies taught in a math methods class emerge in pre-service teachers' student teaching practice? Which strategies do student teachers use and why?	• Observations of math methods classes • Observations of the student teaching of pre-service teachers • Logs of teacher practice • Interviews with pre-service teachers • Classroom artifacts from math methods • Classroom artifacts from the pre-service teachers' student teaching
What supports the emergence of these strategies in a pre-service teacher's student teaching practice and what impedes it?	• Interviews with pre-service teachers

Table 2

Data Collection Plan – An Overview of the Timeline for Data Collection.

Method	Timeline
Preliminary Investigation - Math Methods	
Observations	September 2010 - June 2011
Artifact Collection	September 2010 – June 2011
Secondary Math Pre-Service Teacher Classrooms	
Observations	March 2011-June 2011
Logs	March 2011 – June 2011
Artifact Collection	March 2011 – June 2011
Interviews	March 2011-June 2011

Data analysis.

Based on a grounded theory methodology, this study weaved together the processes of data collection and data analysis (Corbin & Strauss, 1990). As soon as my first piece of data was collected I began the analysis, so that the data collected would inform how I collected my next piece of data. Through the use of this iterative process and systematic review, I was able to uncover repeating or missing concepts, organize my data, and develop a grounded theory. While collecting information I began to look for organizing concepts - grouping, and comparing and contrasting them, to see how they related to one another. Although my codes and categories were grounded in my data, my research questions continually led me to evaluate how the information I gathered related to the ideas of mathematical rigor, student discourse, and how issues of equity are addressed in the classroom.

This organizing and defining concepts, and relating them to one another, is often referred to as constant comparison. This involves looking for data that are both a challenge to, and are consistent with, grouping definitions, as well as finding patterns and accounting for inconsistencies (Corbin & Strauss, 1990). For example, as I gathered observation data in pre-service teacher classrooms, I observed commonalities of practice, looked for patterns, and compared these practices to those taught in the math methods course. As Corbin and Strauss (1990) promote, as I collected more information I constantly reviewed the data in an iterative process to find those consistencies and inconsistencies. I used this on-going analysis to help formulate my end-of-the year interviews with the pre-service student teachers, and then I used the interviews to try to clarify and understand inconsistencies that came up in the data.

With multiple sources of data - observations, artifact collection, logs of teacher practice, and interviews - I was able to triangulate the information gathered so that I could generate a grounded theory from this investigation. Using multiple measures allowed me to gather sufficient data to capture teacher practice in two locations – the math methods course and the pre-service teacher classroom – and to better understand the connection between the two.

Chapter Four: Findings

Using data from the observations of pre-service teacher instruction, the logs of teacher practice, and the interviews with pre-service teachers after each lesson and at the end of the year, this chapter shares how the strategies taught in math methods emerged in the pre-service teachers' student teaching classrooms. It begins by sharing the findings from the initial three visits to the classroom, called Phase I. Next, the Phase II findings describe how strategy use changed after the directive was given by the math methods professor to try the high-leverage practices from the math methods course. The findings chapter concludes by looking at what the pre-service teachers felt supported and impeded their use of these high-leverage strategies from the math methods course.

A. Phase I: Initial Classroom Visits

The methodology of the investigation called for three initial observations of each of the six pre-service teachers to establish a baseline of pre-service teacher practice in their student teaching classrooms. The focus of these baseline observations was to see how the high-leverage strategies taught in the university math methods course were emerging in the student teaching classrooms.

Overview of findings from Phase I: Initial classroom visits.

There were four primary findings from these initial visits: 1) Five of the six pre-service teachers used the strategies in their student teaching classrooms. 2) Frequently, the pre-service student teachers used a strategy without planning for or fully developing one of the factors that made it high-leverage. 3) Most often the way the pre-service teachers chose to structure the high-leverage strategies did not create the student-to-student mathematical discourse intended. 4) The discourse created by the strategies rarely included students explaining processes or the rationale behind their math work.

The case of Christine.

The following example presents an account of the strategies and practices of one pre-service teacher during the three initial observations in her student teaching classroom. Her use of the high-leverage strategies from math methods is representative of common practices of the pre-service teachers in the study, and provides examples that illustrate the four primary findings.

Centered in the very middle of the Algebra I classroom, standing at a tall rolling cart on which the document reader is placed, is Christine. She writes clearly for her 15 students, often using different colors to distinguish between ideas and steps, her words and numbers projected behind her. "Can anyone add to that?" she asks after a student has shared an answer. The math methods professor often uses this same question to get more participation. In this lesson, my second observation, she incorporates two high-leverage strategies from the math methods class. The graphic organizer, designed to help students find the greatest common factor of the three terms from a trinomial, is technically a three circle Venn diagram. She writes "Step 1: Find the greatest common factor (GCF)" and writes the problem 55x^2y-15xy^2+10y. She draws three overlapping circles and writes one of the terms in each circle in purple ink. "The part that's over lapping is the part they have in common," she explains. "Now you find the GCF by factoring each term," she says as she goes on to work through the factoring of each term with the whole class. Christine directs the students to look at the term 55x^2y first (Christine, classroom observation, April 7, 2011).

Christine: What are the variables?

Student: x^2 and y

Christine: What are the factors of 55?

Student: 5 and 11.

Christine: Can I factor it further?

Student: No.

Christine: What are the factors of x^2?

Student: x and x.

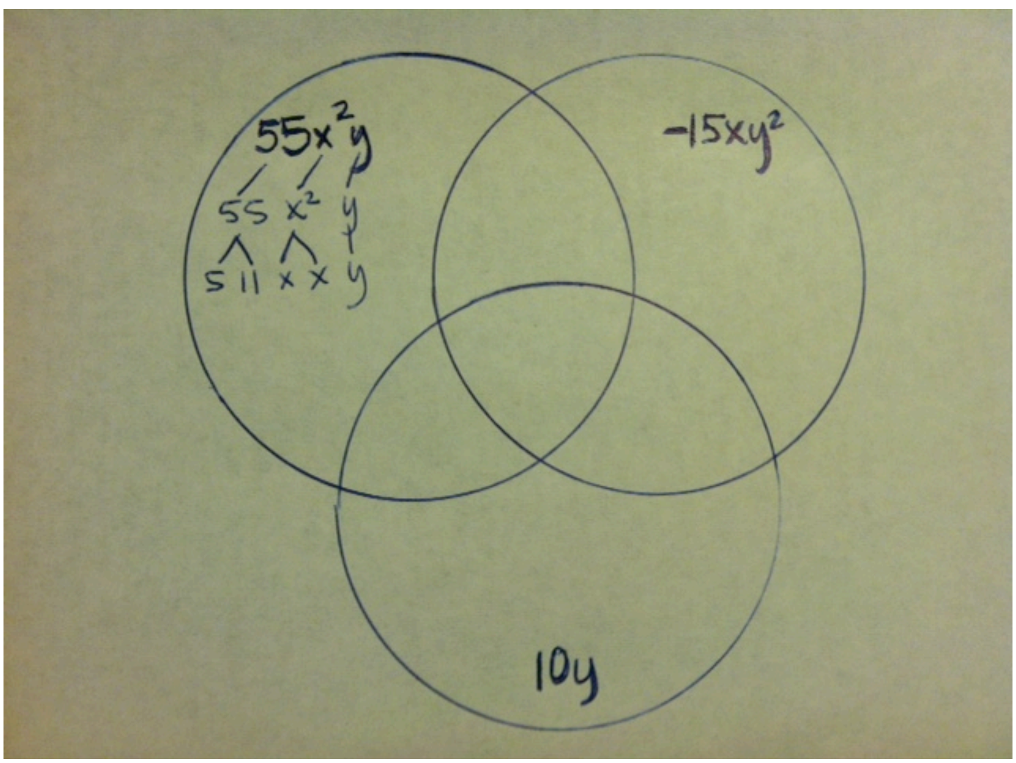

Figure 4: The three-circle Venn Diagram with one term in each circle from Christine's lesson.

The factor tree is complete for the first term, 55x^2y.

She writes their answers down in the circle, creating a factor tree going down from the term using red ink. They move on to the next term of -15xy^2. The pattern of Christine asking quick, factual questions and students responding continues until they get each term factored, get the common factors circled in each part of the Venn Diagram, and then get the common factor placed in the middle. About a third of the class participates in helping her fill in the Venn Diagram.

The next strategy she used during the lesson was Boards Up. This strategy can be used in a number of ways but primarily it uses small, individual white boards and dry-erase markers to motivate students and engage them in mathematical problems and explaining their thinking. After the students complete a question, they hold the boards up for the teacher or other students to see, and the students can share their reasoning. It can be done in groups, pairs or individually. In this instance, Christine had all students in pairs with the exception of one group of three. Their problems were on factoring polynomials (example: Factor 36xy^2+12x^2).

In general, the groups enthusiastically work and the classroom is quiet with concentration when she first presents a problem. Christine reminds them twice that the partner without the board must do the problem on the blank sheet she's passed out while the other partner works on the white board. There is not much student talk because each student is working individually and seems to use his partner to check his answers. Sometimes they ask Christine or her mentor for help, but the students do not seem to be turning to one another for assistance. When enough time has passed for students to do the problem, Christine says, "Boards Up!" and the students hold up their boards for her to see and she gives them points.

On the third problem a number of groups get the problem wrong and a student asks, "What's wrong miss? How's it 20?" Christine goes on to explain to the whole class by writing the factor trees down on the overhead for the two terms in the problem showing how the GCF was 20. They work on two more. "It better be right or I'll be mad at you," one student tells his partner. A few students clamor for more problems. I hear, "I want more" and "Miss, can we get one more – extra hard?" (Christine, classroom observation, April 7, 2011).

Christine's use of two high-leverage strategies is shared in detail here because the number of high-leverage strategies Christine uses is typical of the pre-service student teachers in this study (See Table 1). She uses three strategies during these first three lessons, which represents both the median and mode of the number of high-leverage strategies tried by the pre-service student teachers. In addition, the strategies she used were also used by the other pre-service teachers during these initial visits (See Table 1).

Christine also uses the strategies in a way that is representative of their use in other pre-service teacher classrooms. For example, Henry, another of the pre-service teachers, also used Boards Up for review. Similar to Christine's set up and students' responses, there is very little discussion created between students while doing the problems, nor is there student explanation of processes or reasoning. In his classroom, each student has a white board so there is possibly less incentive to talk, but when students answer a problem correctly he moves on to the next problem, just as

Christine did. Similarly, when students get stuck, he reviews the problem step by step using quick questions to students to lead them to the solution. Christine shared in her post-lesson interview that she used Boards Up to motivate and engage students. Henry shared that for him the strategy was also about motivating students and creating equitable participation during a review activity. “Boards Up [is about] equitable access – everyone gets the material and I get a much higher amount of participation. I tell them to talk to each other too before raising boards” (Henry, interview, March 30, 2011).

Findings of Phase I: Revisited, Connected and Explained.

Finding one: All but one teacher used the strategies.

Evidence from these three initial observations showed five of the six teachers used multiple high-leverage strategies from the math methods course. The frequency of use of high-leverage strategies was variable between the pre-service teachers’ student teaching classrooms. For example, during these three visits one pre-service teacher used zero high-leverage practices from math methods, while another pre-service teacher used six. (See Table 3.) The four other pre-service teachers used two to three strategies. However, by the third visit only one pre-service teacher used any high-leverage strategies from math methods. Christine, whose lesson number two was described in detail here, used a total of three strategies during the first two visits and then did not use any high-leverage strategies during the third of the initial visits.

Table 3

Frequency of Use of High-Leverage Strategies Listed by Pre-Service Teachers during the Initial Three Classroom Visits (Phase I).

Pre-service Teacher	1st visit	2nd visit	3rd visit	TOTAL # of high-leverage strategies tried during first 3 visits
Christine	1	2	0	3
Henry	3	2	1	6
Alex	2	1	0	3
Stephanie	0	0	0	0
Daniel	1	1	0	2
Maxine	1	1	1	3

Note: High-leverage strategies from the math methods class were observed a total of 17 times.

Table 4

Frequency of Use of High-Leverage Strategies Listed by Strategy during the Initial Three Classroom Visits (Phase I).

Strategy	# of times used in first three visits
Exit slip	2
Graphic organizer	2
Boards Up	2
Sentence starters	3
Structured group work	2
Whip Around	2
Think-Pair-Share	2
Project in context of real life	2

Note: There were eight different strategies used a total of 17 times by the pre-service teachers.

Finding two: Strategies used without development for high-leverage.

The findings show that frequently the pre-service teachers used the strategy without planning for or fully developing one of the factors that made it high-leverage. In Christine's use of the graphic organizer, she designed it to be grade-level appropriate by using the three terms with coefficients and variables in order to find the common factor, so it is considered rigorous. She encouraged students to share their answers and suggestions with her to fill in the graphic organizer, so she was encouraging, and creating a situation for student-to-teacher discourse. However, there was not any structure set up in the strategy or classroom practice to ensure all students could participate in the creation or filling out of the graphic organizer. In the her use of Boards Up, she again used grade-level content, so it is rigorous, and every partner group has a white-board, so it was set up for equitable participation. However, the structure of putting students in pairs did not actually get students talking. Talk was only witnessed if the student with the white-board wasn't sure how to proceed, and more than once a confused partner turned to ask Christine or the mentor teacher for help before asking the partner. After the answer was shared on the boards, Christine moved on to the next problem. Students were not asked to share their methods or ideas with the class. In this case, although the teacher suggestion was there, the routines or structures to help students talk about the math were not in place to make that happen yet.

Similarly, another pre-service teacher, Alex, who seemed intent on having all students participate, did not address rigor when he used the strategy, Whip-Around, on my first visit to his classroom. This strategy, used by Alex about 30 minutes into the period to understand what students had learned during a review lesson on quadratic formula, yielded participation from every student. His prompt was, "Take one minute to think about what you learned about quadratics. We'll do a quick run around the room about what you learned... We'll share one thing." After a student shared, Alex added the instruction to use the sentence starter, "I learned…" Here is a sample of the Whip-Around.

Student A: I learned the quadratic formula.

Student B: I learned how to use the quadratic formula.

Student C: Formula. That one (pointing to the board).

Alex: What's it called?

Student C: Quadratic formula.

Student D: How to solve for x.

Alex: I learned…

Student D: I learned how to solve for x. (Alex, classroom observation, March 30, 2011)

Reflecting on this strategy after teaching it, Alex said the goals of using Whip-Around were to "ease them into review," because they were taking a practice test right after the Whip-Around, and "making every student having to talk" (Alex, interview, March 30, 2011). Based on the ease at which students took up the idea of the Whip-Around, and shared one right after another during the Whip-Around, it was clear the students had done it before. As Alex shared, "There are some strategies [from the math methods course] I've tried to use. Whip-Around is one that I've found is easy to do. I don't know about engagement but it does get them to participate" (Alex, interview, March 30, 2011).

These examples demonstrate that it is difficult for pre-service teachers to set up the strategies in their initial uses of them so that they incorporate rigor, discourse and equitable participation. In fact, fifteen out of the total 17 strategies observed across all six pre-service teachers did not include planning for an aspect of the strategy that made it high-leverage (six out of 17 strategies) or not fully developing in implementation an aspect of the strategy (nine out of the 17 strategies).

Rigor seemed hard to integrate for strategies like Whip-Around and Reflection because the students were often being asked to share how they felt during these strategies, not to develop or share math content knowledge. Discourse was the hardest aspect to fully develop. In all the cases where one of the aspects was planned for but not fully developed, the underdeveloped aspect was discourse. Not fully developed meant that student discourse was intended and planned for by the pre-service teacher but either did not happen or happened unevenly (few kids talked) or the discourse was unclear (students shared something that did not make sense).

Finding Three: Struggle creating student-to-student discourse.

Most often the way the pre-service teachers chose to structure the high-leverage strategies in Phase 1 did not create the student-to-student mathematical discourse intended. Both the graphic organizer and Boards Up, strategies used by Christine, can be designed with student-to-student discourse. In the cases described previously the strategy created very little student-to-student discourse.

Alex also demonstrates these struggles to create student-to-student discourse. While doing the warm-up during my second visit to his classroom, Alex tried an informal Think-Pair-Share. Turning to his students he asked, "How do you think this – the equation – relates to this (he points to a graph on the white board)? Talk to you partner" (Alex, classroom observation, April 8, 2011). Students do not talk to one another. The question is rigorous and open-ended, and the idea behind talking to a neighbor was designed for equitable participation. However, since no one shares with his or her partner, it is missing a critical element of the high-leverage strategy Alex wanted to use. He continues the lesson by answering the question, finishing the warm-up, doing two more examples, and then working with the class to write down the two primary steps in simplifying rational expressions.

When Alex shared his thoughts after the lesson he said that it was "new content so I think they did not have the [information] to talk about it… because it was new, [the goal was to] go through notes, lecture and see if they can practice" (Alex, interview, April 8, 2011). Later. in the short after-lesson interview, he adds, "I just wanted them to talk amongst themselves [since] it was a lot of me lecturing… share ideas with each other and formulate their own knowledge – critically think about content as opposed to them just copying down stuff I tell them" (Alex, interview, April 8, 2011).

In each of these cases the pre-service teachers planned and designed the strategy for student-to-student discourse to occur, but it did not happen as intended. Of the 17 high-leverage strategies that pre-service teachers used during Phase I, 10 pre-service teachers intended for student-to-student discourse to occur. However, in only one of these 10 strategies used did students appear to be sharing with each other fairly consistently across the classroom.

Finding four: Discourse rarely included students sharing their process and reasoning. The final finding of the initial visits was that the discourse created by the strategies rarely included students explaining the processes or the reasoning behind the math work. As was previously shared, encouraging student-to-student discourse was difficult, but when pre-service teachers did include students sharing with the class or one another while using a high-leverage strategy, it most often included sharing answers, leaving out their thought processes or mathematical reasoning. In the examples from Christine and Henry's class, students were highly engaged in the work and shared answers, but they were not explaining – either the process of arriving at a solution, or the reasoning and rationale behind an approach or idea. This was common in all of the pre-service teachers' classrooms. In fact, of the 17 high-leverage strategies observed during these first three visits to each pre-service teacher's classroom, only five high-leverage strategies were implicitly designed by the pre-service teachers to have students explaining the process or the rationale behind the solutions[2].

The evidence from the first three initial visits shows that pre-service teachers were able to implement the high-leverage strategies from the math methods course, but they struggled to include rigor, equitable participation and student discourse in these high-leverage strategies. In addition, creating mathematical discourse, particularly student-to-student discourse, and getting students to explain processes and strategies was not easy for pre-service teachers to develop.

B. Phase II: Final Four Classroom Visits

In Phase II of the study, in the final four visits to each of the six classrooms[3], the protocol changed as planned. After a lesson on Think-Pair-Share in the math methods class, and a directive from the math methods professor to use these strategies in their student teaching

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[2] See Appendix D for two examples of pre-service teachers that planned and implemented strategies to encourage students sharing their mathematical processes and reasoning.

[3] Four of the six pre-service teachers had four visits and two had five final visits. The two that had five, had one more than the others because I wanted to see Think-Pair-Share or another high-leverage practice, and for multiple reasons I did not get to observe the intended strategy during a particular visit. So I went to the school site for a fifth visit to ensure I observed all teachers trying these high-leverage strategies four times.

classrooms, the six pre-service teachers tried the high-leverage practice of Think-Pair-Share[4] for the first two visits and chose Think-Pair-Share or another high-leverage practice for the last two visits. After I observed the pre-service teachers teach, they reflected on their planning process and instruction. I also shared feedback with the pre-service teacher, which included two things I liked about the lesson and one thing I wondered about. Usually this created a short conversation about the strategies used and the lesson.

Overview of the findings from Phase II: Final four classroom visits.

There were six primary findings from the second phase of the study: 1) After pre-service teachers were directed to try the strategies and support was provided for using these high-leverage strategies, the pattern showed that use of the strategies increased in the pre-service teachers' student teaching classrooms. 2) All the pre-service teachers used high-leverage strategies and used them in a way that addressed each aspect of the strategy that made it high-leverage. In addition, all six were able to design and carry out a Think-Pair-Share that encouraged and created student-to-student discourse. 3) The pre-service teachers were able to adapt the strategies in innovative ways to meet the needs of their student teaching classroom. 4) Figuring out how to encourage and create discourse, student-to-student talk, and having students talk about strategies and reasoning, was a learning process that required the pre-service teachers to try the strategies over time. The pre-service teachers had to grapple with many issues, including figuring out the questions to pose, how to present it to students, and making time for sharing. The more often they tried the strategy, the more they learned. 5) Although student-to-

[4] Think-Pair-Share generally involves students individually thinking about a question or idea (Think) and then pairing up with a partner to talk about his/her thinking (Pair) and then sharing their own and/or their partner's ideas with the class (Share). Discourse *between students* is at the center of this strategy and the structure of the pair provides the opportunity for equitable participation. The rigor of the problem or question posed is left up to the design of the teacher.

student discourse increased, pre-service teachers continued to find it challenging to provide outlets for students to share processes or reasoning with others. 6) Time for reflection and discussion with the outside observer was an important part of this process of learning.

These last four visits showed that all six pre-service teachers could use these strategies in a way that encouraged and created student-to-student discourse. Using Think-Pair-Share not only increased student-to-student discourse in the classroom, but it also increased the likelihood that the discourse included students sharing the processes and reasoning behind their thinking.

The case of Stephanie.

Stephanie's use of the high leverage strategies during Phase II was representative of the six pre-service teachers in that she was able to design and carry out Think-Pair-Share, and she was able to adapt and innovate the strategy in ways that met her classroom needs. However, she was also chosen because she is the one teacher who tried no high-leverage practices during the three initial visits, so the evolution of her thinking and practice is more evident. Part of this evolution was her continued struggle with getting students to talk and in using high-leverage strategies.

Stephanie's small high school of approximately 450 students was located in a modern, newly built campus that housed 1300 students in another small high school. Her Algebra I class was mostly boys; with between 22 and 29 students during the observations, there were never more than seven girls. And the boys were talkative. As Stephanie describes it, this was encouraged by her mentor's relationship with the students and his style of teaching, which Stephanie explains as structured in a way to increase positive disposition towards math, but also get through the material quickly by focusing on procedural fluency.

Phase I: Stephanie's initial classroom visits.

Stephanie was not described previously because she did not implement any high-leverage practices during the three initial observations. The structure of her lessons during those three visits began with a warm-up and continued with her modeling a problem, having her students try one, and then repeating that until the end of the period. She scaffolded these problems so each subsequent problem added a more complex component.

When explaining how she thought about discourse and equitable participation in her planning during the first lesson, she shared that "Equitable participation is a struggle with this class because they're always very vocal - or specific students are. Even if I called on one student by name, someone else would respond. I've tried using the popsicle sticks[5] but it's hard to remember to pull them out, and it takes extra time for me to wait for the kids. Those days [when she uses the popsicle sticks] it's a little better but kids still call out; even then it's a struggle. Student discourse is hard because if I ask them... Think-Pair-Share, they won't immediately talk. They'll be off topic. So it's more informal. 'Can you ask [another student] instead?' [she'd say to a student with questions.]" (Stephanie, interview, April 12, 2011) When asked how she took discourse and equitable participation into account in planning for observation three, Stephanie said, "Today was more focused on giving them more practice… not so much explicit strategies

[5] Popsicle sticks are a method for calling on students randomly. Each popsicle stick has a student name on one end and they are drawn from a container such as a can. After they have called on a student, often times teachers will place the stick in the container name-end down so they know which students they have already called upon.

for student discourse. It would have been nice to sort of have them articulate I guess the whole solution thing, like with the last problem, but yeah, time-wise, I knew it was going to crunch with tying it all together…" (Stephanie, interview, April 27, 2011).

Phase II: Stephanie's final classroom visits.

For the final visits, Stephanie used Think-Pair-Share during four of the five classroom observations. For the first visit she used the high-leverage strategy to activate students' prior knowledge about proportions – the lesson of the day. She was careful to make a seating arrangement that put students together in pairs. Prior to class she designed a chart for the students to fill out what they knew about proportions, what their partner knew, and what the class knew. In her conversation after the observation, Stephanie said "I was actually surprised that they [students] actually talked and filled it out… which for my class is a huge success. I think giving them something to fill out was definitely a good like thing I took from [the math methods professor's] Think-Pair-Share" (Stephanie, interview, May 6, 2011).

She also noted that using Think-Pair-Share resulted in positive changes in her students. She said, "Today I noticed that Oscar and Carlos were talking about their work which I found really nice because usually Carlos doesn't help other people, he just finishes and sits" (Stephanie, interview, May 6, 2011). She also found that students had more to say when called upon because they had a chance to discuss their answers with their partners. "I noticed they were a lot more willing to say something [when called upon], or they weren't so much, 'I don't know.'" (Stephanie, interview, May 6, 2011).

While Stephanie was positive about trying Think-Pair-Share with her students, she questioned aspects of its set-up. “I would like to change it [Think-Pair-Share] so that it’s more math related like content-wise, not just prior knowledge, because that’s still something that I’m struggling with. But if I did it at the end of the class perhaps it might be what they’ve learned, and that was, it would be more like justifying, like the adaptive reasoning part of the proficiencies” (Stephanie, interview, May 6, 2011). Nor did it necessarily mean she wanted to try it again. Even after saying she saw some successes, when asked if she was up for trying it again Stephanie said, “Maybe… I want to try it again at the end of class, sort of like the way I had it planned. The other thing I’m having trouble with is ending class and doing that [Think-Pair-Share] when I could be going over more content. Sort of like - do I introduce the quadratics or do I do the Think-Pair-Share?” (Stephanie, interview, May 6, 2011).

During the next visit Stephanie used Think-Pair-Share for completing-the-square (a method used to solve quadratic equations). Right after she wrote a problem on the board, she said, “Turn to your partner and see what you think.” It was unclear what they were supposed to be thinking about. She did not ask for their ideas and there seemed to be no sharing between the students. Stephanie did not follow through to find out their thoughts, but instead went on to explain the problem.

In the subsequent interview she talked about not being as explicit as she may have needed to be with Think-Pair-Share, and not planning enough. Stephanie was trying to reconcile how she was able to use the strategy in the classroom versus how the math methods professor used it in the math methods course. “I realized today I still had to go around and ask guiding questions. When [the math methods professor] does it in class, she never does that because she doesn't have to because we all know what to talk about. But then I always struggle with - am I giving them too much [help or information]" (Stephanie, interview, May 31, 2011). The thoughts she shared with me after her first two uses of Think-Pair-Share, and her mixed feelings about the success of the strategy, may have contributed to the fact that she didn't use any high-leverage practices in the next (sixth) visit.

When we met to reflect after this sixth observation she said, "The main strategy today was supposed to be the summary, the Quick-Write. I was going to have them Whip-Around or share with a partner depending on time, but then somehow I looked at the clock, and while I thought there was a lot of time left, there just wasn't" (Stephanie, interview, June 1, 2011).

We talked about her plans for the next day, which included a short warm-up with the rest of the period for a quiz. She shared that she would like to do an individual student reflection, and we discussed when she could do this during the lesson, and how the information would be used. I wondered how a high-leverage strategy could be used before the quiz to help students with understanding, and I prompted her to think about including student discourse in the strategy. She chose to do Think-Pair-Share again and designed the strategy in a way that allowed the students to review the content for the quiz. Because she was worried about accountability, we also talked about how she could use her stamp system to encourage the students to explain and talk about the process they used to solve one of the problems that Stephanie had written on the board, and not just give the answer. (She stamps students' work when they finish the warm-up problems. These translate into points for students' grades.). However, as the following lesson description and transcript show, Stephanie used her Think-Pair-Share plan with the class to get them thinking and talking student-to-student about the processes of solving both the completing-the-square and Pythagorean Theorem problems. Plus, she developed an innovative way to use Think-Pair-Share in a way that worked with her content, classroom, and accountability system.

Stephanie opened the lesson by pairing off the students and having each partner work on one of two problems that were similar, but with different numbers – one on completing the square and the other on the Pythagorean theorem. As the students worked on their two problems, they were almost completely silent except for a few chairs scraping across the floor. Someone sharpened a pencil. After about nine minutes Stephanie told the students, “Listen up. In your pairs now I want person one to explain what they did to person two” (Stephanie, classroom observation, June 3, 2011).

The students began explaining to each other how they solved their problem.

> Student: The A is 7, squared, plus b, but there’s no b, equals 9 squared. And 7 times 7 equals 49 and then I brought the b down. And then 9 times 9 is 81 and then I subtract both sides and then I got b squared equals 48. And then I brought that down and oh, I did that wrong. Oh, it’s cause I guessed on my hand. (Stephanie, classroom observation, June 3, 2011)

All five pairs in the row closest to me were talking math. A scan of the rest of the classroom looked like all pairs, with the exception of one, were engaged in quiet talk over their work during this time. I heard the person near me say things like, “That gave me -29. I put the x down and I got the two from…” and “This here… this is how you do it…”

After completing the warm-up with the Think-Pair-Shares, Stephanie gave them a quiz on the same topics. As the students were beginning to finish the quiz she wrote a reflection question (Quick-Write) on the board for students to answer on the back of their quizzes. The Quick-Write was:

> If you finish respond to:
>
> How did explaining your steps to your partner help you review for the quiz?
>
> "Explaining my steps helped me…"

Here are some of the students' responses:

> It helped me see another point of view – that they see it from.
>
> My steps explaining this to my partner helped me out because I learned in a few seconds what I didn't know.
>
> I had to explain in my own words, which helped me a lot.
>
> It helped me fix my mistakes and it also made me go over it a couple of time to get my answers correct.
>
> It was kind of like a first aid kit for help. (Stephanie, interview, June 3, 2011)

In our after-lesson interview, Stephanie said, "It went a lot better than I expected." There were a number of pleasant surprises and benefits that she noted, too. "It seems like most of them realized that it helped to have them like, explain, and the fact that they explained it and that they could either catch their mistakes or just figure out what to do or if things made sense or not" (Stephanie, interview, June 3, 2011).

In addition, none of the students with Individual Education Programs (IEPs) requested to leave the room to take the quiz with the resource teacher. Stephanie remarked that this was the first time that this had happened all year (Stephanie, interview, June 3, 2011). Most importantly, she seemed pleased by the students' focus on math, and the fact that they were honest and said when they were not focused on math. She also noted how using her stamp system to support math discourse seemed to help the students stay on task in their discussions. "I think giving them different problems sort of, helped them -- I don't know why but they were really quiet today when they were working and I was really surprised. I was going to give them only till like, 11:10 a.m., but then I gave them two minutes because I could see that they were actually working and they did need a little more time. I think the stamps were definitely necessary to get them going…and I found that they were actually very honest about if they had explained or not. There were like three people that didn't explain so they actually told me…" (Stephanie, interview, June 3, 2011).

There were a few issues as Stephanie told me later. A few students (25 students were present that day) did not complete the problems, so they had nothing to explain. One or two students had missed the days when she taught how to complete the square, so they had trouble explaining to their partner. She also realized that she never wrote the answers to the problems on the board, or had students explain or share out to the class. It was interesting -- everyone was so focused on how to get to the solution, that the solution became less important and no one asked for it. This time, when I asked if she would try this again, she said, "I might actually -- I liked how they did on this one." I don't know how well they would do, if they weren't very comfortable with it yet."

Stephanie is beginning to think of Think-Pair-Share as useful in the classroom for reviewing, but she questions how it would be used for other types of learning.

For the final observation, Stephanie once again used Think-Pair-Share with her students in a similar format to the previous visit, but the students worked on five quadratic equation problems individually, and then shared their process with their partner. For Stephanie, the Think-Pair-Share helped keep the pairs working together at a more similar pace, and it structured student-to-student assistance in a more formal way than had occurred before. Stephanie also felt that the additional time she gave them helped their conversations. "I think that giving them more time, and expecting them to take more time, sort of, helped them talk longer. I noticed a lot of them, when I walked by, they were much more explicit, as opposed to 'Oh, I did this, this and this.' When I walked by, sometimes I tried to push them towards academic language." Although in each after-lesson interview she had shared ideas for changing, this is the first time she mentioned trying to push them further in the Think-Pair-Share by encouraging academic language. Stephanie said: "They [her students] are starting to get used to it [Think-Pair-Share]." It seems she was getting more used to it as well. (Stephanie, interview, June 10, 2011).

Findings of Phase II: Revisited, Connected and Explained.

Finding one: The pattern showed strategy use increased with direction and support. After direction and support was provided for trying these high-leverage strategies, the pattern showed the pre-service teachers' increased their use of the strategies (See Table 3). Making the time and space to try these strategies in the classroom, and struggle with what it means to do them effectively, plus allowing time for reflection after the lesson, helped the pre-service teachers increase their use of the strategies, and this promoted more student discourse in their classes.

Table 5

Frequency of high-leverage strategies used during Phase I and II of this study.

Pre-service teacher	# of Visits PHASE I	#Strategies Used PHASE I	Strategies Used PHASE I	# of Visits PHASE II	# Strategies Used PHASE II	Strategies Used PHASE II
Christine	3	3	exit slip, graphic organizer, Boards Up	4	8	exit slip, TPS, TPS, Whip-Around, sentence frames, graphic organizer, reflection, sentence starters.
Henry	3	6	Boards Up, sentence starters, structured group talk, sentence starters, Whip-Around, exit slip	5	12	exit slip, TPS, exit slip, TPS, Boards Up, sentence starters, Whip-Around, interactive notebooks, TPS, graphic organizer, interactive notebooks, TPS
Alex	3	3	Whip-Around, sentence starters, TPS	4	7	TPS, TPS, TPS, Whip-Around, project in context of real life, graphic organizer, same project in context of real life
Stephanie	3	0	none	5	6	TPS, graphic organizer, TPS, TPS, Quick write, TPS,
Daniel	3	2	TPS, structured group work	4	4	TPS, TPS, TPS, TPS
Maxine	3	3	graphic organizer, project in context of real life, project in context of real life.	4	8	TPS, TPS, Whip-Around, project in context of real life, TPS, project in context of real like, project in context of real life, TPS

Note: TPS = Think-Pair-Share

Finding two: Pre-service teachers used high-leverage strategies.

All the pre-service teachers were able to use high-leverage strategies, and use them in a way that addressed each aspect that made the strategy high-leverage (i.e., rigor, discourse, and equitable participation). This included encouraging and creating student-to-student discourse. For example, Stephanie showed during Phase II that she could use multiple high-leverage strategies during a lesson, including graphic organizers, Think-Pair-Share, and reflection, when she was directed to try them.

All six pre-service teachers were able to design and carry out a Think-Pair-Share that encouraged and created student-to-student discourse. In Phase I, only two pre-service teachers tried this method of encouraging student-to-student discourse, and both had few, or no, students talk to one another, although the teachers both posed rigorous, thoughtful, and open-ended questions. In Phase II, five of the six were also able to design strategies that got students explaining their processes and strategies, some to different degrees than others. In addition, when the pre-service teachers used Think-Pair-Share, they were much more likely to include students' explanations of processes and reasoning in the lesson (52% of the time) than if they used other high-leverage strategies (13% of the time). See Table 4.

Table 6

Frequency of Use of High-Leverage Strategies, Listed by Strategy, during Phase II, with the Percentage of Times the Strategy Included Student-to-Student Discourse and Students Explaining the Process and Strategies.

Strategy	# of times	Student-to-student discourse included (%)	Students explaining processes or strategies (%)
Exit slip	3	33	33
Graphic organizer	4	25	0
Boards Up	1	100	100
Sentence starters	2	0	0
Structured group work	0	0	0
Whip Around	4	25	25
Think-Pair-Share	19	100	52
Project in context of real life	5	40	40
Reflection	4	0	0

Note: Pre-service teachers were observed using 42 total high-leverage strategies during Phase II.

Stephanie ended her exploration of Think-Pair-Share with her students getting more comfortable explaining processes and procedures to each other. Henry, who through his own anecdotes shared that he used Think-Pair-Share and other strategies to encourage student talk a lot, during the observations had multiple students share multiple strategies for approaching problems with each other and the class, some with more prompting than others (See Appendix E for a detailed example.)

Finding three: Pre-service teachers innovated.

The third finding of Phase II was that the pre-service teachers were able to adapt the strategies in innovative ways to meet the needs of their student teaching classroom. In Stephanie's class, she figured out a way to make her classroom setup, the regular routine of the warm-up, and her accountability system work to encourage discourse, as was described in detail earlier.

Henry provides another example of pre-service teacher innovation. When the math methods professor instructed the pre-service teachers to try Think-Pair-Share in their classrooms, Henry went back into his student teaching classroom and began using the strategy with his students almost immediately. When I visited his classroom a week later, it was clear this strategy was one his students were familiar with. He presented a problem to his students and asked them to share their answer, their work and their reasoning on a slip of paper (See Figure 2). After writing their own work and thoughts, they were asked to share these with their partner and compare, both aloud and on paper, their answers and reasoning.

What I thought + work	What my partner thought and why…
What we agreed on and why…	

Figure 5: The model Henry projected on the board for students to use with Think-Pair-Share.

Henry explained that he adapted Think-Pair-Share in this way because he felt his students were most comfortable working individually, so he pushed them to describe their thinking and rationale in writing, and then to talk to one another about their strategies, and then compare those strategies with a partner, aloud and in writing (For a more detailed account of this lesson see Appendix F).

Finding four: Figuring out how to get students to talk about math was a process that required trying strategies multiple times.

Figuring out how to encourage and create discourse, student-to-student talk, and having students talk about strategies and reasoning, was a learning process for pre-service teachers that required trying strategies over time. The pre-service teachers had to grapple with many issues, including figuring out the questions to pose, how to present the strategy to students, and how to make time for sharing. The more often they tried the strategy, the more they learned.

Stephanie began her exploration of using high-leverage strategies when she used Think-Pair-Share to get at students' prior understanding to begin a lesson. She asked, "What do you know about proportions?" They were asked to fill out a section of the chart, then talk to their neighbor about what they knew, and then Stephanie had members of the class share out their ideas. Stephanie built on these ideas to teach the lesson. It was a thorough Think-Pair-Share, although it did not have students share processes or reasoning; they were sharing their ideas. Her third time using Think-Pair-Share with students. however, the students were sharing processes, and the fourth time she started to push them to use academic language. Also, trying it multiple times helped Stephanie see that her students could use the strategies to learn and reflect on their learning. Therefore, trying strategies multiple times is important to learning, and pushing the strategies to their fullest potential.

This same type of work on the strategy occurred for the other pre-service teachers as well. Most of the pre-service teachers struggled with what question to ask, and debated how to set-up the strategy in a way that worked for their students and classroom context. For Maxine, grappling with how to use the strategies meant she had to integrate them into the problem-based learning curriculum of her school. The first time she tried Think-Pair-Share in the classroom, she was very explicit and put the directions in a PowerPoint presentation. However, the students were used to working in large groups and, as she mentioned, not used to being very accountable for sharing out. So the first time Maxine tried Think-Pair-Share she did not feel very successful because there was not a lot of talk when she asked them to share in their groups. But when the project they were working on switched to a pair format, and she was a little less formal in her presentation, it went better, she felt, and there was more consistent participation and a greater amount of student-to-student discourse.

Finding five: Student sharing of processes and reasoning still a challenge. Although student-to-student discourse increased, the pre-service teachers continued to be challenged to provide outlets for students to share processes or reasoning with others.
While Think-Pair-Share, as a strategy, was much better at helping pre-service teachers encourage students to share processes and reasoning than other strategies, only 14 of the 42 high-leverage strategies observed in Phase II included students explaining processes and/or strategies. For Stephanie, when she was last observed, she had begun to feel confident about using Think-Pair-Share as a way to get students to share procedures, but she had not yet pushed to get her students to explain reasoning or consider multiple strategies. This was similar to all the other pre-service teachers with the exception of Henry, who, in three of the last four visits, had students explaining multiple strategies and their reasoning.

When strategies such as Reflection or Whip-Around were used, the pre-service teachers rarely designed the strategy so that students were able to share explaining or reasoning. Part of this was that these strategies, while encouraging equitable participation, were most often used by the pre-service teachers to learn about students or students' learning, so the pre-service teacher could use the information either to get to know students or to design the next day's lesson.

Finding six: Reflection and discussion with outside observer were important. Time for reflection and discussion with the outside observer was an important part of the teachers' process of learning to use the high-leverage strategies. The outside observer nudged Stephanie to consider using Think-Pair-Share again, to see if Stephanie could get the strategy to a point where she herself felt it was successful for her and her students. Also, the outside observer acted as a sounding board as Stephanie thought through how a different adaption of Think-Pair-Share share might go. Reflection and discussion were vital to her trying the strategy of Think-Pair-Share again, and using it in a way that encouraged students to share the processes of solving problems with each other. This was the case for a number of the other pre-service teachers. And though not planned, while the pre-service teachers were working to figure out if and how to use the strategies, support ended up being a vital part of the process.

For Alex, the outside observer was more of a mirror to reflect back his practice. It showed him a different angle on the lesson, because as the pre-service teacher running the lesson, Alex was not able to focus on the things that the outside observer could see.

Alex, in his version of Think-Pair-Share, had students verbally explain the problems to each other without being able to see each other or the problem. He organized the individual desks to face each other and had students stand up a book between them. He repeated the directions and modeled how to do this with me as an example for his students. Then he gave each student a different problem on a post-it note. The problems were on simplifying rational expressions. All students worked on their math or talked with another person about the math during this work time. In my observation notebook I wrote that I had never seen them all work so consistently before to solve problems.

Although it took a good deal longer than the five allotted minutes, Alex asked the first partner to begin explaining their problem and solution. All the students began to talk. I heard " x squared over x + 2 and…" as students began to share their problems. I saw a student get up and move around to be beside his partner (as opposed to facing him with a book in between). Alex came over to redirect him. Another pair dropped their book and was talking, and clearly letting the other one see the work. The students began to explain their process of simplifying to each other and I wrote down snippets of what I heard, "So you make the diamond with the -6 on top and -1 on the bottom and you're going to make -6 with -3 times 2. So you know now you have x-3 then you'll divide it…" (Alex, classroom observation, April 28, 2011).

In my field observation notebook, the pre-service teacher talk was written on one half of the page and the student talk was written on the other half of the page. During every other observation with Alex, the pre-service teacher side was full and the student side had one or two word answers. On this day though, I had multiple pages that were heavy with words on the student side. When we sat down to discuss the lesson however, Alex was a little dismayed at how it went.

> Well, I was hoping that since they have seen these type of problems before and they've all had practice, that it would have been -- not that big of a stretch for them to explain it. Well it turns out it was a little more difficult than what I expected. I guess it's just because some of them are used to verbalizing it, like saying [it out loud] probably. Like

> some of them I see that they can go to the steps and write it down, but they have a hard time explaining it. (Alex, interview, April 28, 2011)

He was realizing that having students explaining how to do math, even procedural explaining, required much consideration and was going to take practice.

At this point, I jumped in and showed him my notebook of the student talk I recorded. As I reflected back, I shared about what I heard the groups saying. Even the groups he had been worried about were talking and explaining their math processes. I pointed out that because I have no responsibility in the classroom I could really concentrate on what students were saying and heard every student discussing (either talking or listening) math for over eight minutes.

The activity, as he said, hadn't gone like he'd envisioned it. Students wanted to see each other's work, and more than once he tried to get students back in their seats so that they could not compare their work visually. Regardless, there was a vast difference in the amount of student talk and participation recorded during this lesson than had occurred in all the previous lessons, and reflecting that back to Alex played a critical role in him seeing the student discourse and the difference compared to his previous lessons.

Logs of Teacher Practice.

The logs of teacher practice have not been integrated into these findings because they were not found to be a reliable source of data. Of the 41 lessons that the pre-service teachers recorded using logs of teacher practice, I observed 11 of them. The numbers of strategies I recorded while observing only matched the pre-service teachers' logs of teacher practice five of the 11 times.

Sometimes they reported too few strategies used, and sometimes too many. However, the overall trend in pre-service teacher strategy use reported in the logs and those observed were similar. I found that the pre-service teachers that recorded using few strategies were in fact using fewer strategies, and those reporting using many strategies were in fact using a greater number of strategies. Another reason to question the logs' reliability was that multiple pre-service teachers had trouble classifying the strategies they used in the classroom. For example, one pre-service teacher asked on the form if a worksheet counted as using manipulatives and another pre-service teacher said she posed a math problem in the context of real life on a day that I observed to be entirely focused on increasing procedural fluency. So although the general trends of which pre-service teachers were able to integrate fewer or more of the strategies in their classes is backed up by observations, the questions about the reliability of this data mean it could not be used to look closely at the details of pre-service teacher practice.

In Phase II of the study, it was shown that the pre-service teachers could use these strategies, could innovate with these strategies, and could use these strategies to create student-to-student discourse, as well as get students to share their thought processes and strategies. However, it was also true that trying the strategies repeatedly, and reflecting upon using the strategies, was an important part of the learning process as they continued to try to encourage student discourse and equitable participation in their student teaching classrooms. The direction from the methods course professor to try these strategies along with an outside observer in the classroom increased the pre-service teachers' use of these strategies.

C. Supports and Impediments for Using High-Leverage Strategies

The pre-service teachers used high-leverage strategies from the math methods course when they were observed. However, trend data from their own logs of teacher practice shows that when there is not an outside observer present, only two of the six pre-service teachers reported using these high-leverage strategies consistently.

In order to better understand this varied use of the strategies, the pre-service teachers were asked what fostered and what impeded their use of these strategies in their classrooms. Part of understanding the connection between the math methods course and pre-service teacher practice involves understanding what the pre-service teachers perceive as supports or obstacles to implementing high-leverage strategies. This chapter describes the supports and obstacles to implementing high-leverage strategies from all six pre-service teachers' perspectives.

Support for Using High-Leverage Strategies

This section delves into the ways that pre-service teachers felt supported, or motivated, to use high-leverage strategies from the math methods course in their student teaching classrooms. The supports are shared below beginning with those most commonly mentioned as supports by the pre-service teachers.

Peers as motivators.

The most commonly mentioned motivation for trying these high-leverage strategies was hearing that their peer pre-service teachers were using the strategies successfully in their classrooms. Three of the six pre-service teachers mentioned peers as a motivator.

As Maxine said, “Somebody like Henry would talk about his success with Board Talk. When I’d hear the success that people were having with different strategies, I thought, “Oh, that's another one that [I should try]” (Maxine, interview, June 17, 2011). This is interesting because Henry also cites a peer as being his motivator.

> The first step is always hard, like for me to try Board Talk. The reason why I tried [was that] I started seeing success from other [pre-service] teachers. …Because if [the math methods professor] does it, it’s not convincing to me because she can do it so well, but if the apprentice has some success with it, then I’ll try. (Henry, interview, June 13, 2011)

Therefore, one pre-service teacher trying a strategy and sharing his or her success seems to set off a chain reaction.

Math methods professor directive, modeling and list of strategies as motivators.

For others the support or motivator to try these strategies came from the math methods professor. Although only two pre-service teachers provided this as a motivator during a recorded interview, all the pre-service teachers mentioned in casual conversation as being motivated by her encouragement and appreciating her using the strategies in class to model how they could be used. “For us, really, honestly, it was [the math method professor] encouraging us to try new things and [her] wanting to see new things and new strategies being implemented,” Calvin said (interview, June 17, 2011).

This directive from the math methods professor led at least one pre-service to become more confident in the strategies themselves.

> I think doing it once myself, sort of made me see that, okay, like even though the kids aren't as like knowledgeable as we are in our methods class, it's still possible to make it work, which was something I wasn't completely convinced about. (Jessica, interview, June 16, 2011)

Before directing the pre-service teachers to try these strategies, the math methods professor modeled[6] these strategies multiple times. Jessica referenced this modeling when she shared what supported her to use these strategies. "I think what really worked for me was just every week [the math method professor] doing the strategies, showing a new strategy, so I really did like how she consistently modeled the strategies for us" (Jessica, interview, June 16, 2011).

As part of this study I created a list of high-leverage strategies taught in the math methods course, and included this list in the logs of teacher practice that were filled out by each of the six pre-service teachers (see Chapter III: Methodology for how this list was created). In the math methods course the general consensus among the pre-service teachers was that having this list of the strategies from the math methods course was helpful in their planning. "I see the list of strategies and I think oh, that would be so good to use with my class. I'm glad I have this list so I can go back through and really think about next year [too]" (Maxine, interview, June 17, 2011).

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[6] Modeling here means that the math methods professor provided an opportunity for the pre-service teachers to experience the strategy as a learner.

Each of these supports, from peers to the list of strategies, worked to encourage these pre-service teachers to try these high-leverage strategies focused on rigor, discourse and equitable participation.

Impediments to Using the High-Leverage Strategies

Pre-service teachers felt many factors impeded their opportunities to use the high-leverage strategies, including mentor's teaching style, limited time, unease with trying something different, and lack of encouragement and support. The following impediments are discussed in order, from the most frequently mentioned to the least frequently mentioned.

Mentor teaching style and their classroom norms.

The most commonly cited obstacles identified by pre-service teachers were elements of their student teaching classroom context. These included his or her mentor's teaching and planning style, classroom management and norms, student reactions and skill levels, and testing. Five of the six pre-service teachers identified their mentor's teaching style and the established classroom norms as a challenge to trying high-leverage strategies from the math methods course in their student teaching classrooms.

In the student teaching classroom, the mentor teacher is generally creating opportunities for the pre-service teacher to experience and participate in the daily instructional practices and norms set up by the mentor teacher. It is this mentor to whom pre-service teachers turned for guidance and direction on a daily basis. Therefore, the instructional practice of the mentors, which the pre-service teachers shared, did not include opportunities for engaging in the high-leverage strategies, and was perceived by the pre-service teachers as affecting their agency to implement the strategies. As Maxine said, “It…is like [my mentor] doesn’t do them. So I, you know, didn’t do them” (interview, June 17, 2011). Another pre-service teacher felt similarly:

> [My mentor] was supportive in the sense that s/he was open to trying all these things and seeing how it would carry out in practice… but I haven’t really seen a lot of that in his practice either so trying to, I don’t feel like I did get that support… (Calvin, interview, June 17, 2011)

As Calvin brings up, having a supportive mentor can mean many things. All of the pre-service teachers felt that their mentors were supportive in particular ways. However, the pre-service teachers said that their mentors did not use the high-leverage strategies from the math methods course in the classroom so the pre-service teachers did not have the opportunity to experience or engage in the high-leverage strategies with students in their student teaching classroom by participating in the strategy with the mentor teacher. And only one pre-service teacher said she worked together with her mentor to develop a strategy taught in the math methods course (the Exit Slip). Five of the pre-service teachers felt bound by the teaching styles of their mentors, and the norms and structures designed by their mentors to support that teaching style.

> To be honest I found [implementing discourse strategies] a little bit difficult, a little bit with [my mentor’s] structure for his classes and trying to figure out and wondering whether or not it was my place to kind of go outside of his normative structure, and try and find new ways to promote discourse. (Daniel, interview, June 16, 2011)

Another pre-service teacher felt discomfort in challenging the norms established in the classroom by the mentor teacher.

> "I think not being able to set our own norms and classroom management affects [the opportunity to use the strategies] a lot. Just because there are things that you want to try halfway through [the school year] but it's not really… It's really difficult because you feel like it started out as someone else's classroom, and it still isn't quite yours." (Jessica, interview, June 16, 2011)

It is particularly difficult to challenge these norms when the strategies from the math methods course are designed to encourage equitable participation and discourse, and this emphasis is not present in the class where the pre-service teacher is student teaching. For example, Jessica described normal classroom interactions when her mentor was leading the lesson:

> In our class if we have a whole class discussion, it's really just three kids calling out ideas. So it has a lot to do with like from the beginning how was it (the classroom) set up to make it happen… okay, whoever answers, answers, and then good, okay, we are moving on. It's not equitable access, as if everyone was expected to participate in the answer. (Jessica, interview, June 16, 2011)

She described her mentor as someone whose goals included going "beyond what the book has and preparing the kids for Algebra II." As she describes it, "[my mentor] focuses more on, "Okay, this is what you do…and in that way he can quickly get through a lot [of content] by telling the kids you do this, you do this." (Jessica, interview, June 16, 2011) Jessica found that this singular focus on covering maximum content dictated what Jessica felt was possible in her student teaching classroom.

> Every time I would come back from a meeting with [my math methods professor] "Oh yeah I want to try this, and then [my mentor] would be like, "Well remember that time wise…" and I am like, "Okay, okay." And in the end, of course, [my mentor] is there every day and [my math methods professor] is not, so [my mentor] sort of won a lot more than she [the math methods professor] did." (Jessica, June 16, 2011)

Existing norms that have already been set up in the classroom can also affect how students react to trying new strategies, and these existing norms can be an impediment to implementation of a new strategy.

> "Some of it is that the classroom environment wasn't set up to use it [particular high-leverage strategies]. Like posing a big problem – that, the kids sort of don't like it. The few times we've tried – Okay so let's try -- this they really just want us to tell them what to do" (Jessica, June 16, 2011).

In these examples, pre-service teachers have shared how their perceptions of their mentors and the established classroom norms make it uncomfortable for the pre-service teachers to use the strategies from the math methods course.

Transition from graduate student to teacher.

Three pre-service teachers mentioned the way they learned these strategies in the math methods course made it challenging to use the strategies with their students. Their concerns focused primarily on how to think about and plan for the strategies as a teacher when they had only participated in these strategies from a student perspective.

> "I felt like a lot of times in methods class there is this constant changing of teacher-student hat. But a lot of times, because we were being taught, I feel much more of the student. So it's like we were participating in the activities that we want our students to do, but of course the way that we think is very different than how our students think. So it was always this really, I don't know - it was an odd balance." (Aimee, interview, June 14, 2011)

Daniel brings up the same idea of student hat versus teacher hat and how not quite knowing how to shift between those perspectives made it difficult for him to see how to use the strategies in his classroom.

> "Yeah, so I struggle with that in terms of like oh this lesson [in math methods] was fun but then I… there's disconnect for me on how to translate that in to my classroom. And now that I was with the teacher hat on, the disconnect was more in her [the math methods professor] sample lessons, because she would do it where she would like actually model the teacher. I'm the teacher, you're the students, and here is how I would go through it. And because it was a specific subject, with a specific content, I was so easily able to just become a student and just enjoy the lesson that I wasn't taking the time to then reflect as a teacher how would I do a similar lesson with different content. (Daniel, interview, June 16, 2011)

Pre-service teachers' expectations of students.

Some of the pre-service teachers felt the strategies were difficult to implement because of their sense of student behavior, student knowledge, and student comfort level. The pre-service teachers' conception of disparity in students' mathematical knowledge made using some of these strategies difficult for at least two pre-service teachers.

> "Discourse is a little tricky because there are a number of students in there that are not proficient so it's hard for them to talk about the material. When I ask them questions... they can talk it out after you probe them with enough questions. (Calvin, interview, April 13, 2011)

This example seems to say that Calvin believed initially that the students could not talk about the math because they were not proficient, but then he says that with enough questions they are able to talk about it. Perhaps it was the newness of mathematical discourse that made it difficult, rather than lack of proficiency.

Aimee felt she had more success using the strategies with her older group of students in Algebra II, where the students are consistently more on the same level mathematically compared to her Algebra I class. "I wish I had implemented more of what was taught [in math methods]. It was easier with the older kids (referring to her students in a more advanced math course) but not so much with Algebra I. I think for Algebra I it really was based on dynamics of the classroom. Just noticing the huge gap between the stronger students and the lower skilled students… so I felt like I was constantly trying to compensate for that" (Aimee, June 14, 2011)

Classroom management and student behavior also played a role in the pre-service teachers' comfort with trying these strategies.

> "I think as soon as I had a pretty good grasp on classroom management and [was] getting to that comfortable spot where I knew that, "Oh, this is working for them." It [became] really hard for me to try new things with them [like the math methods strategies]. I was always afraid that if I did, it would be chaotic" (Aimee, June 14, 2011).

Henry believed his students were not comfortable with sharing ideas out loud in a mathematics class. He felt this was outside the students' mathematical norms, and they were impeded by peer opinion.

> They are not used to, maybe it's also my fault too, I should really build more community in Algebra I in the beginning and spend more time building community. Like when I'm doing structured group talk, when I'm doing Think-Pair-Share, when I'm doing anything that has to do with them talking to each other, it has not been very effective. Because maybe they are not used to it, they are not familiar with each other, they are scared like if they put their ideas out there they'll get, you know, they'll get put down, some people will put them down. So I've not done a good job creating that community in my classroom for them to really open and share their ideas and really be enthusiastic about it. That's why when they are reflecting and they are doing it, it's more individual and that's what they are used to. More or less they are used to doing individual work. (Henry, June 13, 2011)

Henry's thoughts raise the question whether the impediments to using high-leverage strategies and creating student discourse are really about teacher expectations and classroom norms, rather than an issue with student behavior and ability.

Testing.

In each of the six Algebra I classrooms the mentor and pre-service teachers spent weeks preparing for the California Standards Test. Some pre-service teachers felt the intensity of this and the potential results much more strongly, and it worried them.

> I can’t screw his test scores over. I mean it doesn't affect me but his test scores will affect him. The idea that trying all of these strategies - what if they don't work - because the days it doesn't work it really just it doesn't affect me as a teacher - the students loose a day or they lose time where they should be learning and we spend days fixing when like I tried something and it didn't work. (Jessica, interview, June 16, 2011).

Although by the end of Phase II all the pre-service teachers were using the strategies from the math methods course, we must understand the supports and impediments that the pre-service teachers feel in order to make the connection between the strategies taught in the math methods course and those the pre-service teachers use in their classrooms stronger and more consistent across all the student teaching classrooms. This section provided supports to build upon, such as peers and impediment we can work to mitigate, such as the addressing issues of changing from the student hat in the university classroom to teacher hat in the student teaching classroom.

Chapter Five: Discussion

Using These Findings to Move Teacher Education Forward

A Practice-based framework for math methods.

To move the work of the math methods course closer to practice I am going to return to the framework that began this study, a framework designed to push the teaching of university math methods courses closer to teacher practice. For over 10 years Ball et al. (2009) have been working to develop a practice-based approach in teacher education courses. By practice-based they mean helping pre-service teachers "do instruction, not just hear and talk about it" (p. 459). Their framework presents a way to approach the design of math teacher preparation that includes:

1) Articulating the work of teaching mathematics
2) Indentifying and choosing high-leverage practices
3) Mathematical knowledge for teaching (p. 461)

In this study the math methods professor followed Ball et al.'s (2009) framework to help design a math methods course by articulating the work by creating a rubric, identifying and choosing strategies to focus on, and incorporating mathematical knowledge for teaching into the math methods course. But following this framework was not enough to make math methods a fully practice-based methods course that allowed for these carefully chosen high-leverage practices to emerge in pre-service teachers' student teaching classrooms at a consistent, highly skilled level. We learned this from the findings of Phase I, when pre-service teachers were observed using from zero to six of the high-leverage strategies. When the strategies were used, the pre-service teachers often implemented them without planning for or creating one or more aspects that would make the strategy high-leverage. Therefore, in Phase II the professor required the pre-service teachers to use, or "enact," the strategies in their student teaching classroom. This enactment helped pre-service teachers see that the strategies could work, and the pre-service teachers adapted the practices for their own student teaching classrooms in innovative ways. Practicing the strategies over time, and having an outside observer, were also important in this learning through enactment process.

Proposed new practice-based framework for math methods.

In light of these findings I added to and adapted Ball et al.'s (2009) original framework to include enactment and time for sharing in the math methods course.

Proposed new framework for approaching the design of a math methods course:

1 - Articulating the work of teaching mathematics

2 - Identifying and choosing high-leverage practices

3 - Teaching the strategy, providing the rationale, and decomposing in the math methods course (all including mathematical knowledge for teaching)

4 - Enactment and inquiry (includes preparing for and implementing the strategy, as well as support in the student teaching classroom)

5 – Sharing in the Math Methods course (repeat 3-5)

1 - Articulating the work of teaching mathematics

In the mathematics education program in this study the professor, working with other professors and a graduate student, articulated core ideas behind teaching mathematics by creating a rubric designed to use for observing pre-service teachers. These core ideas included mathematical rigor, mathematical discourse, equitable access to the content, and classroom ecology, which were broken down into smaller, observable components in the rubric (See Appendix A).

This articulation is important because it makes explicit what it means to teach mathematics, and it provides the principles behind the strategies. In the case of the high-leverage strategies investigated for this study, the three aspects of math teaching (i.e., rigor, student mathematical discourse, and equitable participation) were what tied the strategies together and provided the rationale for using them. By the end of the year each of the pre-service teachers was discussing how he or she integrated these three aspects of teaching into their practice, and this was due to the teacher education program's articulation of what it means to teach math.

2 - Identifying and choosing high-leverage practices

In this study the high-leverage strategies were chosen after observing five months in the math methods course to determine which strategies, that were taught, could be used in a manner that was mathematically rigorous, could create mathematical discourse for students, and provide equitable access to the content. These aspects of math practice were chosen to code the math methods professor's strategies because these were the aspects of math teaching the professor and her colleagues in teacher education had articulated were important to math teaching, based on the literature. After coding the strategies, a number of high-leverage strategies arose. From these, the strategy of Think-Pair-Share was chosen to investigate and be used by the pre-service teachers in their teaching. Think-Pair-Share was chosen as a focus strategy during Phase II because of it's flexibility in setting and content, and because it had at its core student-to-student discourse and equitable participation, which are critical aspects of student learning, but which are challenging for pre-service teachers to integrate into their lessons.

In this study, identifying and choosing the high-leverage strategies allowed the pre-service teachers to work with particular strategies and to develop instructional practices around rigor, student discourse and equitable practices. It also helped to strengthen the connection between the math methods course and student teaching. By choosing and teaching high-leverage strategies, the math methods professor pushed the math methods course closer to being *practice-based*.

3 - Teaching the strategy, providing the rationale, and decomposing in the math methods course (all including mathematical knowledge for teaching)

In this study the math methods professor taught the high-leverage strategies, provided the rationales for their use, and broke down aspects of the strategies. In particular, she always talked about how *to set up the strategy for students*. In this new framework I propose that the decomposing go a bit further to make sure it addresses many of the questions the pre-service teachers had about how *the strategy would be taken up by students*. For example, with Think-Pair-Share, this decomposition would include discussion about time needed for each step, e.g., procedure (how to give directions), structure (how to set up pairs), accountability (how will you know the students learned?), how to structure students sharing with the class, and question posing (how do you know it is a good question?). It would also include addressing concerns such as what structures could be provided to encourage talk with students who may not be accustomed to talking in a math class, and how one can address classroom culture to make student discourse around math more of a norm.

The decomposition and the addressing of the pre-service teachers' concerns could be vital to pre-service teachers such as Stephanie, who really questioned whether the strategies would work in the context of her student teaching classroom. As socio-cultural theory explains, context is an integral part of learning (Gutierrez, 2012). And the urban contexts of the pre-service teachers' classrooms had particular pressures. For example, during the year of this study, three of the five schools the pre-service teachers taught in were labeled Program Improvement Schools by the district, based on No Child Left Behind mandates to label schools that did not meet test score goals for the previous year. This label can often lead to increased focus on procedural fluency in mathematics. If we want the ideas and strategies to take hold in pre-service teachers' thinking and practice, we must work to make their learning practice-based so that they have opportunities to experience using the high-leverage strategies in the contexts in which they teach. As teacher educators, we have to address not only the considerations of the strategy itself (e.g., which kinds of questions work best), but we also have to address the issues around how the strategy will be taken up by students in the context of the classroom (e.g., if they are not used to talking about math, how do you move towards this norm?). It is particularly important that these strategies attend to the vital aspects of classroom practice that increase student learning but are more difficult to implement, such as creating student mathematical discourse and equitable participation for students.

4 – Enactment and Inquiry (includes preparing for and implementing the strategy, as well as support in the student teaching classroom)

Part of the objective of this study was to understand the connection between the strategies taught in the math methods course and pre-service teacher practice. This study found that the connection is strengthened when pre-service teachers are directed to try strategies from the math methods course, and support is provided (having an outside observer, and having time to reflect on the lesson with that observer).

This directive, with support of an outside observer, increased pre-service teachers use of the strategies. However, the varying degree of depth of student-to-student discourse, and the limited encouragement of students to explain their processes or reasoning during this discourse, indicates that we need to strategize how to use math methods courses to make these strategies even more practice-based, and tied closer to the classrooms in which students will be teaching. Pre-service teachers themselves provided a lot of ideas for this. One idea was having pre-service teachers demonstrate the strategies in the math methods classroom. This way of teaching university education courses follows along the line of "pedagogies of enactment," which calls for pre-service teachers to be given "opportunities to practice elements of interactive teaching in settings of reduced complexity" during their teacher education program (Grossman & McDonald, 2008). The opportunities are considered "approximations of practice" (Grossman et al. 2009). In the case of this study, trying strategies such as Think-Pair-Share in the pre-service teachers' classrooms with an outside observer present could be considered an approximation of practice. However, there were no other opportunities that led up to this enactment with actual students. Enactment in the math methods course would provide a more supportive approximation of practice, rather than going directly to their student teaching classroom, and would be a bridge to trying the strategy in their own student teaching classroom.

It could be argued, based on the findings of this study, that if the pre-service teachers had more of a gradual introduction to the strategies, with steps to approximation of practice, it may facilitate their development before they try this with students in their own pre-service teaching classrooms, and thus improve their experiences using the strategies. Grossman and McDonald (2008) argue that teacher education programs need to focus more on developing skilled practice by adding "skill and enactment" to their curriculums (p. 460). We cannot leave enacting practice entirely up to mentor teachers and the field experience; otherwise we are left recreating the status quo in classrooms.

I would advocate that we should take this suggestion from the pre-service teachers even further and have pre-service teachers move along a "trajectory of participation" (Chan, 2010). In Chan's study with elementary math pre-service teachers, the pre-service teachers began by using the high-leverage practice in the math methods course for their fellow pre-service teachers; then they moved on to practicing in a university lab school where students are familiar with the practice; then they used it in student teaching; and finally in their own classrooms. Chan found this trajectory to support the development of greater skill in eliciting student thinking.

In secondary math we could follow a similar route to include trying chosen strategies in the math methods course, in classroom environments with support, and then in their own classrooms. Similar to the proposal of Grossman and McDonald (2008) and the Chan (2010) study, secondary math education professors could systematically experiment to find which opportunities for "approximation of practice" tend to lead pre-service teachers toward greater development of their practice of these high-leverage strategies. This could be combined with other practice-based pedagogies like video, or visits to exemplary teachers' classrooms, to provide a greater understanding how these strategies work in classrooms with students.

An important note here is to be careful about not essentializing the strategy. Ball et al. (2009) caution against teaching "tools for practice" rather than the practice itself (p. 459). This caution I think speaks to some of the concern I feel in advocating for particular strategies like Think-Pair-Share without more development, without the rationale constantly being reiterated, and without a discussion afterward about how well the strategy worked in the student teaching classroom. The teaching and practice of these strategies needs to be carefully developed so that teachers do not think of strategies taught in math methods as add-ons. This seems evident in Stephanie's deliberation, where she shared, "The other thing I'm having trouble with is like just really ending class and doing that [Think-Pair-Share] when I could be going over more content" (Stephanie, interview, May 31, 2011). From this and other discussions with pre-service teachers, doing strategies like Think-Pair-Share was perceived as an extra thing to do, and not fully integrated into their thinking as a way to teach and deepen understanding of the content. It really gets back to why the strategy matters. In this study, the professor focused on Think-Pair-Share because student discourse and equitable participation are critical to student learning, but also because these aspects of teacher practice require careful attention and intentional planning.

Part of this integration could come from following a cycle of inquiry into pre-service teacher practice as advocated by Heibert et al. (2007) in their framework for analyzing teaching. This framework is based on studying the effects of practice on student learning, and includes defining the learning goal, determining what students learned, creating hypotheses about how the teaching supported (or did not support) this learning, and ideas for improving this teaching practice (Heibert et al. 2007). Taking pre-service teachers through this approach (which I call "Inquiry") to analyze their practice using the high-leverage strategies would provide time and space for pre-service teachers to deepen the development of the strategy which they will try in their student teaching classroom. By investigating how students are learning through their teaching, it would also help pre-service teachers use the strategies in a more integrated way in their practice. This approach could be integrated into enactments of the high-leverage strategies. In each enactment there would be an outside observer (professor, field support provider, or mentor) to support and reflect back what s/he saw during the enactment. This is particularly important if we want pre-service teachers to move beyond just copying the strategies they've seen in the math methods course and move toward integrating student mathematical discourse and equitable practices into not only their daily planning but also into their disposition and beliefs about student learning.

5 – Sharing in the Math Methods Course

After enactment with inquiry, the pre-service teachers would come back to the math methods course and share their experiences and their inquiry findings. This enactment, inquiry, and sharing could be repeated multiple times for the same strategy or for different strategies.

For example, Stephanie felt she created an "ultimate disaster" by choosing to use the potentially high-leverage strategy of using manipulatives to help students conceptually understand solving quadratic equations by completing the square. Stephanie could have benefited from sharing her experiences using the strategy with her fellow students in math methods. If her use of manipulatives were set up as an enactment and inquiry, she may have seen the rough lesson with the manipulatives as part of the process of learning. It would have been a way to investigate her own practice and share her experience with her peers.

This enactment, inquiry, and sharing cycle could also be used to push classroom discourse in Think-Pair-Share, or any other strategy, with each enactment. Structuring the teaching and investigation of high-leverage practices in math methods as an inquiry into practice and student learning would allow pre-service teachers to bring back their experiences to the math methods class. There they could discuss their successes and struggles in practicing high-leverage strategies in the classroom, using a framework as described previously by Heibert et al. (2007). This would create a cycle of learning that could work to strengthen the connection between the math methods course and teacher practice – bringing us closer to practice-based teacher education.

This proposed new practice-based framework for the math methods course also addresses many of the most commonly cited supports and impediments to using the high-leverage strategies. It strengthens the supports and potentially minimizes the impediments. For example, the most commonly cited supports in this study were peers and the directive to try the practices. In this proposed framework, the gradual increase in independence with practicing the strategy (the trajectory of participation) would begin with pre-service teachers using a high-leverage strategy in the math methods course classroom. Peers would both experience and reflect on the use of the strategy. Near the end of their pre-service year the pre-service teachers would also be using the strategies in their student teaching classrooms, and returning to share their experiences and inquiry findings. With the directed enactment and peer sharing fully integrated into the math methods course in this new framework, the supports noted in this study are potentially strengthened.

This new framework also addresses impediments the pre-service teachers felt impacted their use of the strategies. The two most commonly cited impediments were the discomfort the pre-service teachers felt implementing strategies that were not part of the classroom norms, and their difficulty seeing how the high-leverage strategies may work because of switching from graduate student hat to teacher hat in contemplating the strategies. Although the math methods course and professor may have little ability to impact the classroom norms of the pre-service teachers' student teaching classrooms, this framework includes practice to get the pre-service teachers comfortable with the strategy in 'safer' settings before enacting it in the classroom. In addition, it provides space for pre-service teachers to bring up questions about how to implement the strategy in their own classroom context. Finally, the framework has them using the strategy with the teacher hat on before using the strategy in their own student teaching classroom. Therefore the framework works to remedy or minimize the impediments pre-service teachers felt they faced when considering using the high-leverage strategies from the math methods course.

Although this framework increases support and minimizes impediments, the concerns the pre-service teachers felt raises some questions about student teaching that should also be considered. For example, what needs to be present in a student teaching placement for pre-service teachers to feel completely supported to try these strategies? How could we support mentors to make sure these factors are present in the student teaching classrooms? Also, in light of the fact that individual placements so distinctly formed the pre-service teachers' learning experiences, how can we ensure every pre-service teacher has the opportunity to develop their practice with these strategies over time?

In conclusion, this practice-based teacher education study advances what it means for teachers to learn. The findings tell us that engaging in the practice of using the strategies is critical to learning and incorporating the strategies into pre-service teacher classroom practice. But this process of learning also requires feedback from an outside observer and self-reflection, so that the pre-service teachers have the time and space to see the results of their engagement with these strategies from the math methods course. Combining the Ball et al. (2009) framework for designing mathematics teacher education with the findings of this study resulted in a proposed new framework that also includes enactment of the strategies, inquiry into an aspect of the teaching of the strategy, sharing in the math methods course, and outside observer support.

Appendix A

Secondary Mathematics Pre-Service Teacher Observation Rubric (see next two pages)

mathematics observation rubric

Developed by Jaime Park, Imelda Nava and Mollie Appelgate — **UCLA** Teacher Education Program

MATHEMATICAL RIGOR

TEACHING DIMENSIONS	CONTEMPLATING LEVEL 1	EMERGING/APPLYING LEVEL 2	INTEGRATING LEVEL 3	INNOVATING LEVEL 4	EXAMPLES OF INSTRUCTIONAL STRATEGIES
INSTRUCTIONAL FOCUS OF MATHEMATICAL TASK	There is no instructional focus or limited one-dimensional focus (e.g., solely on memorizing facts, rules, formulas, or definitions.)	Instructional focus is primarily one-dimensional with vague or little connection between mathematical concepts and different representations. Low level, primarily procedural/ memorization of steps/facts that have little connection to understanding the mathematical concept. Focus on producing correct answer.	Instructional focus is clear and includes high-level tasks (doing mathematics, non-algorithmic thinking, procedures with connections) but focus is producing the right answer.	High level task(s) that include(s) complexity, presses for generalizations across cases, strong connections between multiple strategies/representations, conjectures with evidence and explanations for conclusions.	• Pressing for accuracy (AL terminology) • Prior knowledge • Pressing for reasoning • Getting students to link ideas
IMPLEMENTATION OF THE TASK	Teacher delivers inaccurate math OR Teacher teaches non-math content.	Teacher focuses primarily on the procedural knowledge of the problem regardless of the intentions of the original task OR Teacher does the complex thinking for the students.	Teacher engages some students in some complex thinking using high-level tasks, questions, strategies, and feedback.	Teacher engages students in complex thinking using high-level tasks, questions, strategies, and feedback.	• Evaluating strategies • Connecting ideas across methods/ • representations • Point to key info
ENGAGING STUDENTS IN LEARNING	Students have little or no opportunity to engage with content in ways likely to improve their understanding of mathematical concepts, procedures, and reasoning	Strategies for intellectual engagement offer opportunities for students to develop their own understanding of mathematical procedures.	Strategies for intellectual engagement offer structured opportunities for students to actively develop their own understanding of mathematical concepts, procedures, or reasoning.	Strategies for intellectual engagement offer structured opportunities for students to actively develop their own understanding of mathematical concepts, procedures, and reasoning.	• Think, Pair, Share • Testing conjectures • Practicing problems • Poster presentations
CHECKING FOR UNDERSTANDING	Teacher is not monitoring student progress in the lesson.	Teacher is monitoring student progress but does not use formative assessment to inform instruction	There are multiple opportunities using various strategies to monitor student progress throughout the lesson and attempts to use the information to make instructional decisions	There are multiple opportunities using various strategies to monitor student progress throughout the lesson and this information is used to make sound instructional decisions during the lesson to further students' mathematical understanding.	• Error analysis • Consensus • Justifications • White boards/ thumbs up

MATHEMATICAL DISCOURSE

TEACHERS DISCOURSE: QUESTIONING	Teacher asks no, or only non-math questions or provides no wait time, or questions lead learners to misunderstandings	Asks yes/no, recalling of fact questions.	1-2 efforts to ask students to explain their thinking using reasoning and appropriate evidence	3 or more efforts to ask students to explain their thinking using reasoning and appropriate evidence.	• Predictions, conjectures, evidence/ rationale • Wait time • Evaluating strategies/ ideas • Error analysis; counter examples, comparisons • Follow-up questions, feedback • Pair-share, dyad, group • Panel, presentations, • Whole-class discussion, seminar • Re-voicing, summarizing, modeling • Consensus, proof
TEACHER DISCOURSE: LINKING IDEAS	No linking in class discourse.	Teacher revoices or acknowledges student response.	Teacher revoices, acknowledges or questions student response to further the discussion 1-2 times.	Teacher revoices, acknowledges or questions student response to further the discussion 3 or more times.	
STUDENTS DISCOURSE: LINKING IDEAS	No student linking in discourse	Students link their answers or ideas to others but do not use the connection to compare strategies, generate ideas or build upon knowledge.	Students link their answers or ideas to others 1-2 times in ways that compare strategies, generate ideas, or build upon knowledge.	Students link their answers or ideas to others 3 or more times in ways that compare strategies, generate ideas, or build upon knowledge.	
STUDENT DISCOURSE: MATH RIGOR	Non-math student talk OR Math ideas not generated by students (i.e. repeating what Teacher said or only asking questions.)	Student talk that only conveys procedural knowledge (i.e., definitions, procedures, rules and/or correctness of answer, or providing an answer.)	Student talk that conveys procedural knowledge in relation to conceptual understanding or mathematical reasoning.	Student talk that conveys procedural knowledge in relation to conceptual understanding and mathematical reasoning.	
STUDENT PARTICIPATION	Zero, one or two students participate in the math discussion.	More than one or two but less than ¼ of students in class participate in discussion around the math topic.	About 1/2 of students in class participate in discussion around the math topic.	Majority of students in class participate in discussion around the math topic.	
PARTICIPATION STRUCTURES	No participation learning structures for student participation and/or discourse.	Participation learning structures with limited structure for equitable student participation -Some seating arrangements allow for discourse in pairs/small groups	Participation learning structures with some structure for equitable student participation. -Seating arrangements are in pairs/small groups -Some consideration for student needs.	Participation learning structures with structure for equitable student participation -Pair sharing -Small groups have individual roles and responsibilities. -Consideration for student needs.	

mathematics observation rubric

Developed by Jaime Park, Imelda Nava and Mollie Appelgate

UCLA Teacher Education Program

EQUITABLE ACCESS TO CONTENT

TEACHING DIMENSIONS	CONTEMPLATING LEVEL 1	EMERGING/APPLYING LEVEL 2	INTEGRATING LEVEL 3	INNOVATING LEVEL 4	EXAMPLES OF INSTRUCTIONAL STRATEGIES
SUPPORTING DEVELOPMENT OF ACADEMIC LANGUAGE	Little to no support of *learners'* language needs in instructional task. OR Content is oversimplified, limiting access to content	Teacher applies scaffolding and language development strategies to support the conceptual understanding of the learning task(s).	Level 2 plus Teacher provides explicit models and opportunities to practice, and feedback for *learners* to develop further language proficiency.	Level 3 plus Teacher differentiates language strategies to further develop language proficiency.	• Accessing prior knowledge • Multi-tiered, multicultural, application tasks • Academic language strategies: word bank, association, sentence frames, etc. • Use of multiple learning modalities – productive & receptive, visual, kinesthetic, auditory, etc. • SDAIE/sheltered strategies: group projects, choral reading, concept mapping, graphic organizers, prediction, quick write, quick draw, reflection, sentence starters/frames, verbalizing, vocabulary cards, etc. • Technology • Differentiated instruction though teacher input and student output.
SDAIE TO SUPPORT ELLS	Little to no evidence of SDAIE strategies used to support ELLs.	Some SDAIE strategies are evident.	Several SDAIE strategies are evident.	Several SDAIE strategies are used during instruction and 3 + SDAIE strategies are used as a form of formative assessment.	
MAKING CONTENT RELEVANT FOR LEARNERS	Limited to no evidence of connecting content to the real world (can include culturally relevant pedagogy or critical pedagogy).	Connection to the real world (can include culturally relevant pedagogy or critical pedagogy) is vague and not fully integrated into instructional focus.	Connection to the real world (can include culturally relevant pedagogy or critical pedagogy) is clear and integrated into the instructional focus and learning tasks.	Connection to the real world (can include culturally relevant pedagogy or critical pedagogy) and student lives is fully integrated into the learning tasks and instructional focus.	
DIFFERENTIATION	Little to no evidence of the use of diverse learning modalities to address student achievement needs.	Some evidence of the use of diverse learning modalities to address student achievement needs.	Evidence of diverse learning modalities that effectively address student learning and multi-tiered tasks (differentiated math tasks in relation to its level of difficulty), including tasks using multiple representations.	Level 3 plus: Effective use of diverse learning modalities, learning tasks that include multi-levels and multiple representations that effectively address most or all students' achievement needs.	

CLASSROOM ECOLOGY

CLASSROOM NORMS	Few (1-2) to no observable classroom routines (class running, lesson running, inter-action) exist causing student actions to interfere with learning.	Some (3-4) observable classroom routines exist (class running, lesson running, and interaction) that may or may not facilitate a positive learning environment.	Observable classroom routines exist (3-4) (class running, lesson running, and interaction) that facilitate a positive learning environment.	Observable routines (more than 4) (classroom running, lesson running, and interaction routines) exist that facilitate and maintain a positive, productive -- optimal learning environment.	• Instructional time is used wisely • Structure, transitions from activity to activity are planned • Classroom routines: a) class-running routines, b) lesson-running routines, c) interaction routines planned • Safety concerns are addressed appropriately • Teacher is professional in speech, dress, interactions with students, responsibilities to the profession
DEMOCRATIC CLASSROOM	Teacher demonstrates little to no respect of student input.	A few voices are respected but teacher's response to student input does not often enhance learning and does not often promote healthy student-student-teacher relationship.	Some voices are respected but teacher's response to student input enhances learning and can promote healthy student-student-teacher relationship.	Most to all voices are respected. Negotiations and student input enhances learning and promotes healthy student-student-teacher relationship without undermining teacher knowledge and responsibility.	
PROFESSIONALISM	Little to no evidence of professionalism pertaining to the following: Planning, dress, academic language/ professional talk, organization, timeliness.	Missing 2 or more: Planning, dress, academic language/ professional talk, organization, timeliness.	Missing 1: Planning, dress, academic language/professional talk, organization, timeliness.	Planning, dress, academic language/ professional talk, organization, and timeli-ness all evident.	TRANSFORMING PUBLIC SCHOOLS UCLA CENTER X XCHANGE PUBLICATIONS AND RESOURCES FOR PUBLIC SCHOOL PROFESSIONALS

Appendix B

High-Leverage Strategies From Math Methods

Board Talk

White boards and markers are given to individual students, pairs or groups so that they can solve problems given to them, one at a time, by the teacher. The white boards work as motivators and then students can hold up their work for the whole class to see while they explain. Sometimes used as a game with points given for correct answers and correct explanations for how to solve.

Graphic Organizers

Any visual representation of ideas, topics, main ideas. Some examples are brainstorming, T-charts, concept maps, K-W-L charts, circle maps, Venn Diagram, Open Mind, etc.

Interactive Notebooks

Interactive notebooks can be structured or unstructured depending on the topic or the context. Entries may include math activities and problems, reflection, questions, t-charts, explaining reasoning, etc. They are considered interactive when the teacher responds to the entries of the student and the student has the opportunity to respond back.

K-W-L Chart

K-W-L charts have three columns. The first column, “What I already know,” and the second column, “What I want to know,” are generally filled out individually, or as a class, at the beginning of a lesson or unit. The third column, “What I have learned?” is filled out at the end of a lesson or a unit to recap what students have learned.

Quick write

In response to a prompt, students are asked to write as much as they can in a very short time... 3-5 minutes.

Say, Mean, Matter

Using a visual or a text, a teacher asks students: What do you see, or what does it say? What does it mean? (making meaning of what they see or read); Why does it matter? (significance, interpretation, analysis, or how the text relates to what they are studying in class). Students can also fill in a chart with "Say, Mean, and Matter" heading up three columns. Students can choose passages from a text for the "Say" column, then they interpret it for the "Mean" column, and finally explain why it's important for the "Matter" column.

Think-Pair-Share

The teacher provides a question for students to consider. Each student thinks about, jots down or writes some thoughts about the question. When instructed, the students then turn to a partner and each member of the pair shares the thoughts on the question or prompt in turn. The teacher then organizes a debrief with partners from around the class sharing their ideas.

Whip-Around

This gives students, who might not usually be comfortable speaking out, an opportunity to respond. Going around the room, each student very quickly responds or shares a thought, idea, or writing in turn until the whole class has responded.

Source: California Reading and Literature Project, Annotated List of Strategies, August 2009. Originally compiled by Anne Sirota, edited by Mollie Appelgate.

Appendix C

Log of Teacher Practice (see next three pages)

Secondary Math Teacher Practice Log Pre-service Teacher Form Spring 2011

Directions: Thinking back over your Algebra I lesson for the day, please fill out the following form. While filling out the form, if you would like to clarify an answer or have a question, please feel free to write on the form, either next to the item or at the end of the form in the space for comments/thoughts/additions/suggestions.

Name: ____ Today's date: ______________________

Class name and period: ____

Please mark the day on which you taught the lesson for which you are filling out this form. Record the date and the number of minutes in that class period.

Monday	Tuesday	Wednesday	Thursday	Friday

The lesson objective (***or goals of the lesson):*** ____

____

Strands of math fluency addressed in the lesson (check any that applies):

☐ Procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
☐ Productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with belief in diligence and one's own efficacy
☐ Conceptual understanding - comprehension of mathematical concepts, operations, and relations
☐ Strategic competence - ability to formulate, represent, and solve mathematical problems
☐ Adaptive reasoning - capacity for logical thought, reflection, explanation, and justification

Which classroom structures of participation did you use today? (Please check any that applies.)

☐ Whole class
☐ Small groups, size: _____
☐ Pairs
☐ Roles and/or responsibilities within small groups

Which cues did you use today? (Please check ant that applies.)

☐ Verbal
☐ Written
☐ Pictures/representations
☐ Manipulatives/tools

Strategies used today
Please make a check mark in the left hand column if you used this strategy today in class. If you used the strategy multiple times during the class period please circle the number of times used on the right.

☑	Strategy	Tally
	Warm-Up	2 3 4 5+
	Interactive notebooks	2 3 4 5+
	KWL Chart	2 3 4 5+
	Brainstorm	2 3 4 5+
	Quick write	2 3 4 5+
	Whip around	2 3 4 5+
	Whole-class discussion	2 3 4 5+
	Think, Pair, Share	2 3 4 5+
	Dyad	2 3 4 5+
	T poses BIG overarching problem to get at mathematical idea	2 3 4 5+
	T poses math problem in the context of real-life	2 3 4 5+
	T poses high-level question to get at student reasoning	2 3 4 5+
	Board talk	2 3 4 5+
	Graphic organizer	2 3 4 5+
	Categorizing/Sorting items	2 3 4 5+
	4-Corners Vocabulary chart	2 3 4 5+
	Say, Mean, Matter	2 3 4 5+
	Describe, not Show	2 3 4 5+
	Structured group talks	2 3 4 5+
	Sentence starters or frames	2 3 4 5+
	Concept map or web	2 3 4 5+

☑	Strategy	Tally
	Group project	2 3 4 5+
	Group presentation	2 3 4 5+
	Error analysis	2 3 4 5+
	Ss present/share *different* methods	2 3 4 5+
	Explicit comparing of methods	2 3 4 5+
	Four-fold way	2 3 4 5+
	S Reflection/debrief on activity	2 3 4 5+
	Ss share summaries of lesson or activity	2 3 4 5+
	Visual directions given by T	2 3 4 5+
	Visual modeling by T	2 3 4 5+
	Ss use Manipulatives	2 3 4 5+
	Multiple representations used or shared by T	2 3 4 5+
	Multiple representations used or shared by Ss	2 3 4 5+
	Explicit connections made between representations	2 3 4 5+
	Other:	2 3 4 5+
	Other:	2 3 4 5+
	Other:	2 3 4 5+

*T= teacher, Ss=Students

Connections to the real word (including culturally relevant or critical activities) were made today in:
(Please check any that applies and if applicable, provide a brief explanation.)

❒ Warm-up ____

❒ Main activity/lesson ____

❒ Wrap-up ____

❒ Teacher's talk/rationale ____

Comments/Additions/Thoughts/Suggestions:

Purpose of the Log

The goal of this log is to find out which strategies and practices used in UCLA's Math Methods class emerge in your own classroom during your student teaching. The questions this log will help answer are, "To what extent can you use these strategies in your setting? Which ones are helpful or useful for you and which are not?" Eventually, we want to understand how and why those particular strategies emerge in your class (or do not), but for now this log gives us a window into how often you use them in your classroom.

This log was designed as a checklist in order to be quicker to fill out, compared to an open response survey. However, if you want to add commentary at the end of the form or elsewhere, you are always invited to do so.

Thank you so much for your time!

Appendix D

Example of Finding Four, Phase I: Two pre-service teachers who designed strategies for students to share processes and reasoning during Phase I, and the issues that were observed.

Daniel, during the first visit, set up a warm-up that transitioned the students from the idea of calculating area to multiplying polynomials. During the warm-up he used an informal version of the Think-Pair-Share. He asked his students, “Is that the only way to find the area of this? With partners I want you to find another way. Are there any other ways to find the area than this one method (He points to a larger rectangle divided into four different rectangles with the sides labeled with different numbers)” (Daniel, classroom observation, March 31, 2011)? This question could generate many different responses. The classroom with its half-hexagon tables was not set up explicitly in pairs, but Daniel walked around during the Think-Pair-Share listening to what groups were saying. The groups near the front, where he was located, seemed to be talking about something and the groups near me, at the back of the room, were not talking. After a very short time he said, “Let’s come back. I heard some good ideas. Miguel (calling on someone at the front), what did you do” (Daniel, classroom observation, March 31, 2011)?
Is your argument here about Daniel that he successfully engaged students in process explanations, but that not all students gave them? Not sure why we would expect this every day – every student – if in one day he had half the folks share processes? This seems pretty good.
Daniel’s design and implementation of Think-Pair-Share in this instance was informal (no typed or written directions), quick (less than four minutes for the question, thinking, and sharing), and he asked a question with many potential answers. According to cooperative group research, it

included many elements of effective cooperative learning (Cohen, 1994), and according to the rubric developed by his math methods professor, the approach was rigorous and included components to create student discourse and equitable participation. His reflection at the end also showed he was, through his planning, thinking about how to connect ideas and get students involved in the thinking and creating of these connections through discourse. "…my intent [with the Think-Pair-Share] was to offer some opportunities for discussion and engagement. Our plan, with [my mentor], was to go directly to multiplying polynomials, but I wanted them to… I specifically designed the warm-up so I could connect the warm-up to the box method (of multiplying polynomials)" (Daniel, interview, March 31, 2011). He is thinking about connecting topics and creating opportunities for students to make those connections. But less than half the class talked during the time he gave them to share with their partner. Although the student-to-student discourse planned for and intended was uneven across the classroom, Daniel implemented the strategy in a way that included students sharing their reasoning with their partner and the class.

Student teaching in a project-based school, Maxine, another pre-service teacher, had greater support for group and project-based work because of the nature of the curriculum in her student teaching classroom. On visit three, Maxine's students presented the results of their most recent project, which was to design a fundraiser for scholarships for undocumented students. Over the course of multiple weeks they were tasked with developing a plan for how to fundraise, graphing how much they would raise versus how much they would sell, and explaining it using Excel graphs, PowerPoint, and a quick advertising movie. During the mathematical portion of the presentation they were supposed to, according to the rubric, explain every number and variable in the equation, identify both the x and y-intercepts, and describe their relevance to the fundraiser. In each of the group presentations observed, the students did a thorough job explaining how they would raise the money. However, when it came to explaining connections and the relevance of the equations and intercepts related to the fundraising, the students sped by this part, either by not making the connections, or being very unclear about them. Although the sharing of student reasoning was planned for and intended, and Maxine and her mentor teacher set up the space and designed the project and the presentation rubric to ensure the math and its meaning was explained, students' mathematical reasoning remained unclear.

Appendix E

Example of Finding Two, Phase II: Pre-service teachers used high-leverage strategies

Specifically: Use of Think-Pair-Share to prompt students to share multiple strategies.

After putting the CST release question on the document reader, Henry asked the students to "independently, individually" work on it. The problem: Marcy has a total of 100 dimes and quarters. If the total value of the coins is $14.05, how many quarters does she have? (a. 27 b.40 c.56 d.73). After approximately five minutes of letting them work, Henry asked them: "Can you talk to your partner about what you got and why. We want to get at why, reasons." (Henry, classroom observation, May 12, 2011.) When they began talking he walked around and kept asking pairs "Why?" He continued stressing the reasoning aspect of the question through the share part of the Think-Pair-Share.

Henry: Based on what you learned from your neighbor and your own interpretation, who can give me a good strategy?

Student A: I got A.

Henry: How'd you get A?

Student A: I multiplied 27 times .25.

Henry: Why did you multiply 27 times .25?

Student A: Because there are four quarters in a dollar. And I got 6.75 plus dimes. And there are 73 dimes left.

Henry: Why?

Student A: Cause you multiply by 10 and you get 7.30. You add both of them together you get 14.05.

Henry: But what I'm saying is: how did you get 73 dimes?

Student B: Subtract 27 from 100.

Henry: Why did you subtract it from 100?

Student B: Because those are quarters and you said there are 100 dimes and quarters.

Henry: Who has a different strategy? (He calls on a student.) First, do you agree with Joseph?

Student C: Yes. I put .25 q plus .10d equals 14.05.

Henry: So you actually have an equation and then…

Student C: And you just plug in those numbers and put it into q and subtract from 100 and that would be d, and then you just multiply them, and if it equals 14.05 then it is correct.

Henry: (Henry repeats what the student said so the whole class could hear.) Someone else? What is another strategy?

Student D: Isn't there another equation on the bottom we could use, like dime plus quarter equals 100?

Henry asks how this would be helpful, and the students suggest substitution or elimination to solve the two equations, and Henry does this with them on the document reader. (Henry, classroom observation, May 12, 2011)

Appendix F

Example of Finding Three, Phase II: Pre-service teachers innovated.

Specifically: Innovation with Think-Pair-Share and reasoning.

Henry provides another example of pre-service teacher innovation. When the math methods professor instructed the pre-service teachers to try Think-Pair-Share in their classrooms, Henry went back into his student teaching classroom and began using the strategy with his students almost immediately. When I visited his classroom a week later, it was clear this strategy was one his students were familiar with.

> **Henry:** Your own name on the left, your partner's name on the right. And then you have a bottom section – "What we agree to because…" Let's do it quickly. Your name on the left, your partner's name on the right and the bottom section – "What we agreed on and why…"
>
> So just to remind you guys… If you are ready, please put your pencil down so I know you are ready [pencils drop on the desks]. Make sure you have this right now-- what I thought, what my partner thought, and what we agreed on and why. Are you guys ready? So we try to begin with a CST question and end with a CST question. I think you guys will have fun with this one.

This is the model projected on the front board:

What I thought + work	What my partner thought and why…
What we agreed on and why…	

A student says, “No.”

Another says, “This is boring.” But the class is paying attention and waiting for the problem.

Henry: For the first two minutes I want you guys to independently, individually and quietly work on this problem on your own, okay? I don’t want to hear any talking in the first two minutes. I want you to do it on your own, all right?

Here it is. Number 25. [He shows the problem on the projector.]

25. The opposite of a number is less than the original number. A. This statement is never true. B. This statement is always true. C. This statement is true for positive numbers. D. This statement is true for negative numbers.

Henry: I want you to spend a good two minutes and you need to write down your reasoning – what you think and why. This is on your own.

Henry: [42:07] Write down the answer and why do you think that is… It's really useful if you can try to come up with numbers or different examples, okay, from this statement. If you can generate several examples, just use some numbers, it will really help you out with this problem, okay? [Students are working and quiet].

Henry: [42:52] If you are done with your thinking and you think you have an answer, please put your pencil down so I know you're done. I want you to take a good few minutes to think about what you think it is, all right?

Henry: [43:33] All right, now talk to your partner about it, okay, and see if you guys agree. Make sure you write down what your partner says.

Henry: [44:01] Talk to your partner. Write it down on the right-hand side.

[Student discourse. I hear:

> **Student A:** What'd you get? I got C.
>
> **Student B:** Positive…If a negative is flipped to a positive… So how could that be working against the value of…. The opposite of a number is…
>
> **Student C**: What'd we agree on, April?

Henry: [45:01] Make sure you write on the bottom section as well. [students still talking quietly]

Henry: [46:17] All right, do I have a volunteer to tell me like, their case?

Student D: I said that the answer is C.

Henry: Why?

Student D: Because like the opposite of a negative number would be positive and that would be greater than the negative, obviously. And I'm looking for LESS than the original number, so the original number would have to be in the positive so opposite would be a negative. Okay, example, thank you. This is the original number 10 and the bars mean find the opposite. Please turn this in. (Henry, classroom observation, May 5, 2011)

References

Ball, D.L & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60,* 497-511.

Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Eds.), *Proceedings of the Annual Meeting of the Canadian Mathematics Education Study Group* (p. 3-14). Kingston, Canada: CMESG Program Committee.

Ball, D L., Sleep, L., Boerst, T., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *Elementary School Journal*, 109(5), 458- 474.

Bednarz, N., & Proulx, J. (2005). Practices in mathematics teacher education pro- grams and classroom practices of future teachers: From the educator's perspectives and rationales to the interpretation of them by the future teachers. Paper presented at the conference of the 15th ICMI Study on the Professional Education and Development of Teachers of Mathematics, Águas de Lindóia, Brazil.

Boaler, J. (2003). Studying and capturing the complexity of practice: The case of the dance of agency. Retrieved on February 16, 2011 from ERIC http://eric.ed.gov/ERICWebPortal/search/detailmini.jsp?_nfpb=true&_&ERICExtSearch_SearchValue_0=ED500873&ERICExtSearch_SearchType_0=no&accno=ED500873

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically : integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Chan, A. (2010). *Identity and Practice: Preservice Teacher Learning within a Practice-Based Mathematics Methods Course*. (Doctoral Dissertation). Retrieved from ProQuest Digital

Dissertations. (AAT 3441479)

Clift, R. T. & Brady, P. (2005). Research on methods courses and field experiences. In Studying teacher education: The report of the AERA panel on research and teacher education. Cochran-Smith, M. & Zeichner, K. (Eds.) Washington, DC: American Educational Research Association.

Cochran-Smith, M., & Zeichner, K. M. (Eds.) (2005). *Studying teacher education : the report of the AERA Panel on Research and Teacher Education*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Cohen, E G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, 64(1), 1-35.

Corbin, J., & Strauss, A. (1990). *Basics of qualitative research*. Newbury Park, CA: Sage.

Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Educational Policy Analysis Archives,* 8(1).

Donovan, M.S. & Bransford, J.D. (Eds.). (2005). *How students learn: Mathematics in the classroom.* Washington, DC: National Academies Press.

Duncan, Arne. (October 22, 2010) Teacher Preparation: Reforming the Uncertain Profession—Remarks of Secretary Arne Duncan at Teachers College, Columbia University. Retrieved from http://www2.ed.gov/news/speeches/2009/10/10222009.html

Ebby, C B. (2000). Learning to teach mathematics differently: The interaction between coursework and fieldwork for preservice teachers. *Journal of mathematics teacher education*, 3(1), 69-97.

Ensor, P. (2001). From preservice mathematics teacher education to beginning teaching: A study in recontextualizing. *Journal for Research in Mathematics Education, 32*(3), 296-320.

Franke, M.L., Kazemi, E., & Battey, D. (2007). Mathematics Teaching and Classroom Practice. In F.K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (Vol.1, pp. 225-256). Charlotte, NC: Information Age Publishing.

Fuson, K.C., Kalchman, M. & Bransford, J.D. (2005). Mathematical understanding: An introduction. In *How students learn: Mathematics in the classroom.* Washington, DC: National Academies Press.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American educational research journa*l, 38(4), 915-945.

Grossman, P. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College Press.

Grossman, P. (2008). Responding to our critics: From crisis to opportunity in research on teacher education. *Journal of Teacher Education, 59(1),* 10–23.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9).

Grossman, P. & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*, 184–205.

Gutierrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education? In *Equity in Discourse for Mathematics Education: Theories, Practices, and Policies.* Edited by Beth Herbel-Eisenmann, Jeffrey Choppin, David Wagner, David Pimm , p. 17-16.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.

Hiebert, J., Morris, A.K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58,* 47-61.

Hill, H. C., Rowan, B., Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal,* 42(2), 371-406.

Kalchman, M. & Koedinger, K. R. (2005) Teaching and learning functions. In *How students learn: Mathematics in the classroom.* Washington, DC: National Academies Press.

Kennedy, M. M. (1999). Approximations to indicators of complex student learning. *Educational Evaluation and Policy Analysis*, 21(4) 345-363.

Kieran, C. (2007) Learning and teaching algebra at the middle school through college levels. In F. K. Lester (Ed.) *Second Handbook of Research on Mathematics Teaching and Learning* (405-458) Charlotte, NC: Information Age Publishing.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press

Koedinger, K R.& Nathan, M.J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences, 13 (2),* 129-164.

Loveless, T. (2008) *The misplaced math student: Lost in 8th-grade algebra*. Washington DC: Brown Center On Education Policy at Brookings.

Matsumura, L. C., Slater, S. C., Junker, B., Peterson, M., Boston, M., Steele, M., & Resnick, L. (2006). *Measuring reading comprehension and mathematics instruction in urban middle* *schools: A pilot study of the Instructional Quality Assessment* (CSE Technical Report 681). Los Angeles: Center for the Study of Evaluation, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review,* 13(2), 125-145.

Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Organizing math literacy in America 's schools.* Boston: Beacon Press.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Oakes, J. (1990). *Multiplying inequalities: The effects of race, social class and tracking on opportunities to learn mathematics and science.* Santa Monica, CA: Rand.

Pianta, R., & Hamre, B. (2009). Conceptualization, measurement, and improvement of classroom processes: Standardized observation can leverage capacity. *Educational Researcher*, 38(2), 109-119.

Rampey, B.D., Dion, G.S., & Donahue, P.L. (2009). *NAEP 2008 trends in academic progress* (NCES Report No. 2009-479). Washington, DC: National Center for Education Statistics, U.S. Department of Education.

Rogers, J., Oakes, J., Fanelli, S., Medina, D., Valladares, S. & Terriquez, V. (2007). *California Educational Opportunity Report 2007*. Los Angeles, CA: Institute for Democracy, Education, and Access & University of California All Campus Consortium on Research for Diversity.

Rowan, B., Camburn, E., & Correnti, R. (2004). Using teacher logs to measure the enacted curriculum: A study of literacy teaching in third-grade classrooms. The Elementary School Journal, 105(1), 75-101.

Rowan, B. & Correnti, R. (2009). Studying reading instruction with teacher logs: Lessons from the study of instructional improvement." *Educational Researcher, 38*(2), 120-131.

Rowan, B., Harrison, D. M., & Hayes, A. (2004). Using instructional logs to study mathematics curriculum and teaching in the early grades. *The Elementary School Journal,* 105(1), 103-127.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15 (2),* 4-14.

Spielhagen, F. (2006). Closing the Achievement Gap in Math: 8th grade Algebra for all students. *American Secondary Education*, *34*(3), 29.

Steele, D. F. (2001). The interfacing of pre-service and inservice experiences of reform based teaching: A longitudinal study. *Journal of Mathematics Teacher Education*, 4, 139-172.

Steele, D. F., & Widman, T. F. (1997). Practitioner's Research: A study in Changing Preservice Teachers' Conceptions about Mathematics. *School Science and Mathematics*, *97*(4), 184-192.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College, Columbia University.

Stump, S.L. (2001). Developing preservice teachers' knowledge of slope. *Journal of Mathematical Behavior, 20*, 207-227.

Wilson, S.M., Floden, R.E. & Ferrini-Mundy, J. (2001). *Teacher Preparation Research: Current knowledge, gaps and recommendations*. Seattle, WA: Center for the Study of Teaching and Policy.

Windschitl, M., Thompson, J., Braaten, M., Stroupe, D., Chew, C., & Wright, B. (2010, April). *The beginner's repertoire: A core set of instructional practices for teacher preparation.* Paper presented at the annual meeting of the American Educational Research Association, Denver, CO.