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Essays on bankruptcy, credit risk and asset pricing

Min Jiang
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ESSAYS ON BANKRUPTCY, CREDIT RISK AND ASSET PRICING

by

Min Jiang

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Business Administration
in the Graduate College of
The University of Iowa

July 2012

Thesis Supervisors: Professor David Bates
Assistant Professor Redouane Elkamhi

ABSTRACT

In this dissertation, I consider a range of topics in bankruptcy, credit risk and asset pricing.

The first chapter proposes a structural-equilibrium model to examine some economic implications arising from voluntary filing of Chapter 11. The results suggest that conflict of interests (between debtors and creditors) arising from the voluntary filing option causes countercyclical losses in firm value. The base calibration shows that these losses amount to approximately 5% of the ex-ante firm value and are twice those produced by a model without incorporating the business cycles. Furthermore, besides countercyclical liquidation costs as in Chen (2010) and Bhamra, Kuehn and Strebulaev (2010), countercyclical pre-liquidation distress costs and the conflict of interests help to generate reasonable credit spreads, levered equity premium and leverage ratios. The framework nests several important models and prices the firm's contingent claims in closed-form.

The second chapter proposes a structural credit risk model with stochastic asset volatility for explaining the credit spread puzzle. The base calibration indicates that the model helps explain the credit spread puzzle with a reasonable volatility risk premium. The model fits well to the dynamics of CDS spreads and equity volatility in the data.

The third chapter develops a consumption-based learning model to study the interactions among aggregate liquidity, asset prices and macroeconomic variables in the economy. The model generates reasonable risk-free rates, equity premium, real yield curve, and asset prices in equity and bond markets. The base calibration implies a long-term yield spread of around 185 basis points and a liquidity premium of around 55 basis points for an average firm in the economy. The calibrated yield spread and liquidity premium are consistent with the empirical estimates.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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ABSTRACT

In this dissertation, I consider a range of topics in bankruptcy, credit risk and asset pricing.

The first chapter proposes a structural-equilibrium model to examine some economic implications arising from voluntary filing of Chapter 11. The results suggest that conflict of interests (between debtors and creditors) arising from the voluntary filing option causes countercyclical losses in firm value. The base calibration shows that these losses amount to approximately 5% of the ex-ante firm value and are twice those produced by a model without incorporating the business cycles. Furthermore, besides countercyclical liquidation costs as in Chen (2010) and Bhamra, Kuehn and Strebulaev (2010), countercyclical pre-liquidation distress costs and the conflict of interests help to generate reasonable credit spreads, levered equity premium and leverage ratios. The framework nests several important models and prices the firm's contingent claims in closed-form.

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CHAPTER 1
BUSINESS CYCLES AND THE BANKRUPTCY CODE: A STRUCTURAL APPROACH

1.1 Introduction

The U.S. Bankruptcy Code makes two types of relief available to bankrupt corporations: liquidation (Chapter 7) and reorganization (Chapter 11). Bankruptcy proceedings may be voluntary (instituted by the debtor) or involuntary (instituted by creditors). The mere filing of a voluntary petition for bankruptcy operates as a judicial order for relief, and allows the debtor immediate protection from creditors without the necessity of a hearing or other formal adjudication. In the U.S., the voluntary filing accounts for the overwhelming majority of bankruptcy cases even though it remains controversial among legislators, policymakers and economists. On one side, the debt relief helps the firm avoid or at least delay possible costly liquidation. On the other side, the debtors may time the voluntary filing of the Chapter 11 bankruptcy in bad faith, which creates conflict of interests between debtors and creditors.[1]

Is the voluntary filing option of Chapter 11 detrimental to the firm? And if yes, how much is the cost? The first aim of this study is to provide a quantitative framework and certainly a model-implied evidence to address these questions.[2]We begin by recognizing that since recessions are periods of systematic insolvencies and

[1]See Appendix 1 for a list of realistic cases, where the courts had to judge whether the debtors were filing for Chapter 11 bankruptcy in bad faith or not.

[2]Our focus is not on social impacts of the voluntary filing option of Chapter 11, which we believe is important but difficult-to-impossible to address quantitatively.

expensive state prices, accounting for changes in macroeconomic conditions is important in assessing the ex-ante implications of the voluntary filing option.[3] We develop a structural-equilibrium model with business cycle fluctuations to pin down the net effect of this option on the ex-ante firm value.[4] Second, we attempt to provide analytical expressions for the values of firms' securities (equity, asset and debt), and to examine the importance of integrating both business cycles and realistic features of the bankruptcy code in jointly explaining the credit spread, leverage and equity premium puzzles.

In sum, we find that the voluntary filing option allows equityholders to appropriate value ex-post at the expense of debtholders and causes countercyclical losses in the ex-ante firm value. Our base calibration suggests that the losses amount to 5% of the ex-ante value for a representative BAA-rated firm which is twice as large as those produced by a model that does not allow for business cycle fluctuations. These losses increase with leverage, business risk and long-run uncertainty in economic growth. The large magnitude and the strategic timing of the losses that this study illustrates provide new evidence against the voluntary filing option. Our model also implies that in addition to changing macroeconomic conditions and liquidation costs as in Chen (2010) and Bhamra, Kuehn and Strebulaev (2010 a&b) (henceforth BKS (2010)),

[3]See Appendix 1 for the graph illustrating how the Moody's global corporate default rates change with business cycles.

[4]Broadie, Chernov and Sundaresan (2007) introduce a risk-neutral model to assess Chapter 11 efficiencies. Their model is a special case of our framework. We describe in the literature review the differences between our model and theirs. We also show in the result section that business cycle fluctuation incorporated in our model is crucial to assess the ex-ante costs.

countercyclical distress costs prior to liquidation and the conflict of interests between debtors and creditors help significantly to jointly explain the credit spread, equity premium and leverage puzzles.

More precisely, in the model we assume that both consumption and earnings moments are stochastic and that the representative agent has Epstein-Zin-Weil preferences. We extend the static cases (without dynamic refinancing) of BKS (2010) and Chen (2010) to incorporate many features of the Chapter 11 process (e.g., exclusivity period, automatic stay, distress costs prior to liquidation, absolute priority in Chapter 7 and bargaining game in reorganization between debtors and creditors). We provide closed-form solutions for the firm's contingent claims. The model we propose embeds a list of important models in the literature. For example, the analytical expressions in Leland (1994), François and Morrelec (2004) or the static case of Chen (2010) and BKS (2010) can be obtained by adjusting some parameters in our framework. In particular, we obtain a closed-form solution for the model in Broadie, Chernov and Sundaresan (2007) (henceforth BCS (2007)), which corresponds to a risk-neutral version of our model with one economy and an exogenous strategic debt service. The analytical expressions of the firm's contingent claims in our setting constitute a platform to study other interesting issues related to Chapter 11 (e.g., the optimal grace period, the probability of emergence from Chapter 11, the expected time spent in Chapter 11, the term structure of reorganization probabilities versus liquidation probabilities, etc.) as well as to compare all nested models in terms of their asset pricing implications.

We apply our model to quantify the ex-ante implications arising from the voluntary filing option of Chapter 11. We find that management serving the best interest of the debtors has an incentive to file early to obtain debt relief. This early timing of default creates potential conflict of interests between debtors and creditors and leads to an ex-ante reduction in firm value. We carry on our analysis for a representative BAA-rated firm. For the base case, we calibrate liquidation costs to match the bond recovery rates available in Moody's report. The total expected default losses are matched to the median estimates provided by Andrade and Kaplan (1998). The model implies that the ex-ante reduction in firm value amounts to 2.9% in recession (2% in boom) relative to the value of the firm in the firm-value-maximization scenario (a scenario where management would be acting as firm value maximizers when making bankruptcy decisions rather than equity value maximizers).

To pin down what drives the ex-ante loss in firm value, we quantify the reduction in firm value with respect to other benchmark models. When we consider the Leland (1994) model as a benchmark, the ex-ante reduction in firm value is around 5% in normal economic conditions. This ex-ante loss is twice as large as the one that would be produced by a model that accounts for Chapter 11 but ignores changes in macroeconomic conditions. For example, BCS (2007) model generates a 2.5% reduction in firm value relative to the Leland (1994) model. Further, in order to disentangle the ex-ante costs that are uniquely due to the voluntary filing of Chapter 11 from the macroeconomic conditions effect, we use the BKS (2010) model as a benchmark. The ex-ante reduction in firm value in this case amounts to 2% in recession (or 1.5%

in boom).

One insight is common to all the benchmark cases considered above. The voluntary filing option of Chapter 11 engenders higher losses in firm value in recessions than in good economic times. The countercyclical nature of the loss is a drastic outcome considering that it is in recessions that the bankruptcy code should encourage more participation of debtors and creditors and reduce additional sources of conflicts of interest between them.

At this stage, some remarks are worth emphasizing. First, the percentage losses presented above are not incurred by firms in financial distress but rather by any BAA-representative firm in the economy and they are percentages of initial firm values. Second, these losses are higher for highly levered firms and for firms with higher business risks (cash flow volatility). Third, the magnitude of the ex-ante losses presented above are based on a conservative calibration. Our base case parameter values are calibrated to the empirical estimates of ex-post distress costs (around 10-23%) in Andrade and Kaplan (1998), whose analysis is based on a small sample of LBO firms. LBO firms may have lower distress costs than an average firm in the economy, which is probably why it is possible to lever them up so highly in the first place. Recently, using a large sample of firms, Korteweg (2010) shows that the ex-post distress costs are about 15-30% of firm value (at default). Using Korteweg's estimates would increase the costs borne by the voluntary filing option of Chapter 11 by a factor of 30%.

Another advantage of our model is the easiness to examine how the ex-ante losses co-vary with firm fundamentals and the parameters of the economy. Other things being equal, we find that a longer grace period increases the incentive of the debtors to file early because they reap more benefit from debt relief without the threat of liquidation. The liquidation cost influences the Chapter 11 filing time through two channels. First, a higher liquidation cost reduces the incentive of creditors to shut down the firm at the time when debtors propose a reorganization plan and consequently leads debtors to file early. Second, the liquidation cost increases the Chapter 11 boundary because creditors realize that the liquidation cost through a subsequent Chapter 7 is high and thus become optimally willing to accept a suspension of coupons. We also find that the losses decreases with the financial distress cost that the firm incurs in the reorganization process. This outcome is intuitive because these costs wash out part of the shareholders' benefit from debt relief. In addition, the model provides comparative statics with respect to the preference parameters and economic fundamentals. For example, we show that these ex-ante losses decrease with the EIS.[5]

The expected default loss generated by our model is around 3.4% of the initial firm value using the Andrade and Kaplan (1998) estimates (4.3% using the Kortweg (2010) estimates).[6] This loss is 20% higher than the one generated in a model that

[5]An increase in EIS leads to a lower risk-free rate and hence a higher discount factor and price-earnings ratio in the risk-neutral world. This translates to an increase in the endogenous asset-based default boundary. Ceteris paribus, since the shareholders suffer from distress costs that are proportional to the asset value at the Chapter 11 filing time, any increase in EIS brings up the loss to debtors and discourages an early filing for Chapter 11.

[6]Korteweg (2010) measures the net benefits to leverage by extending the constraints on

accounts for macroeconomic conditions but considers only liquidation costs (e.g. Chen (2010)). It is more than three times the one generated in a model that does not account for macroeconomic conditions (see Elkamhi, Ericsson and Parsons (2010)). We conclude that it is crucial to account for conflict of interests and countercyclical distress costs in order to generate reasonable expected default losses that help to counterbalance the expected tax benefit of debt.

This study also contributes to the literature of explaining the firm's capital structure choice. A common practice in the computation of expected default losses in the literature is to assume that all distress costs are incurred as a lump sum when the firm liquidates. Examples of reduced-form models that adopt this convention include Graham (2000), Molina (2005), and Almeida and Philippon (2007). This assumption is also common in the structural model literature (e.g., Leland (1994), Chen (2010), BKS (2010) and many others). In our model, rather than considering only default costs experienced at liquidation, we permit firms to experience debt-related value losses prior to liquidation. As soon as the firm enters the Chapter 11 process, the firm would suffer some sort of distress costs that are countercyclical. The financial distress prior to liquidation not only occurs with a higher probability, but also corresponds to a range of firm values larger than those for the liquidation only. Another novel and important effect in our framework is the conflict of interests introduced by the voluntary filing of Chapter 11. The voluntary filing option allows management to file

firm values and their betas implied by Modigliani and Miller. His estimates, as in the case of Binsbergen, Graham and Yang (2008), combine bankruptcy and distress losses and are on average very similar to ours (about 5% of firm value).

suboptimally or early for Chapter 11 in both states of the economy. This outcome leads to both an increase in expected ex-post distress costs and a reduction in the tax benefits of debt. Both effects cumulate to provide a significant improvement in understanding the observed capital structure even in a static framework.

Finally, and for completeness, we also attempt to provide some insights on a hypothetical alternative to the current voluntary filing option of Chapter 11. More specifically, we consider a setup in which debtors and creditors agree ex-ante on a debt-to-equity exchange.[7] We provide closed-form solutions for equity and debt values in this swap scenario and show that this option helps reduce the ex-ante losses in firm value. For low (high) shareholder bargaining power, this alternative produces a 2.2% (1.6%) increase in the ex-ante firm value in recession (boom) relative to the firm value produced under the best scenario of Chapter 11 (i.e., the timing of default is chosen in concordance with firm value maximization).

1.1.1 Literature Review

The structural approach to modeling the firm dates back to Merton (1974). Immediately after his seminal contribution, a series of simplifying assumptions are relaxed. Black and Cox (1976) and Brennan and Schwartz (1978) are examples of influential extensions. Two decades later, Leland (1994) extends this framework by

[7] To make it comparable to the voluntary filing option of Chapter 11, we assume that creditors agree that management can voluntarily initiate the exchange when it is optimal (operationalized by equity value maximization). This setup is a simple reduced-form adaptation of the excellent suggestion provided in the Squam Lake report - concerning financial firms.

incorporating both the tax advantage of debt and liquidation costs. His framework has been extended along many different dimensions.

BCS (2007) provide a substantial extension of the Leland (1994) model by introducing a characterization of Chapter 11 bankruptcy as well as the pure liquidation event (Chapter 7). The authors rely on a binomial lattice approach to solve for the firm's contingent claim values. They choose an *exogenous* strategic debt service - the firm in their model may emerge from the Chapter 11 process even if debtholders end up with less than the liquidation value. Thus, the implied ex-ante loss is inflated due to the extra compensation that the creditors would ex-ante require to offset such scenarios. In our model, we consider a Nash equilibrium and impose the equilibrium constraint that debtholders should get at least the liquidation value at the emergence time. Relaxing this constraint in our model augments the ex-ante reduction in total firm value by a factor of three for some exogenously chosen sharing rules. Our model extends the BCS (2007) model along other dimensions. We account for changing macroeconomic conditions and allow the long-run variation in growth rate to affect the timing of default and liquidation as well as optimal capital structure. We endogenously determine a pricing kernel that provides a unique mapping between the risk-neutral and objective probabilities. Finally, we derive closed-form solutions for equity, debt and firm values without relaxing any characteristic of the Chapter 11 process in their model.

Recently, special attention has been given to the importance of incorporating macroeconomic conditions in the analysis of firm financing and investment decisions.

For example, in a risk-neutral framework with a stochastic regime shift, Hackbarth, Miao, and Morellec (2006) generate notable implications of the economy on firm policy. Chen, Collin-Dufresne and Goldstein (2009) use a habit-formation consumption equilibrium framework and conclude that one of the necessary conditions to explain the credit spread puzzle is to allow for countercyclical default boundaries (exogenous in their model). Chen (2010) and BKS (2010) show that countercyclical liquidation boundaries and costs help explain the observed low leverage ratios as well as the credit spread puzzle. Our modeling of the business cycle builds on the work of the latter two papers. We extend the static versions of their models that consider only liquidation by introducing major features of the bankruptcy process and solving for the endogenous Chapter 11 and 7 boundaries.

The distinction of Chapter 11 and Chapter 7 boundaries is crucial to introduce conflict of interests concerning the expected timing of default. When only the liquidation boundary is modeled - as it is the case in the majority of structural models - the maximization of equity value while making default decision (smooth pasting condition) yields exactly the same default timing/boundary as the maximization of firm value does. Introducing Chapter 11 breaks this property and thus induces realistic conflict of interests. Doing so, first, allows us to analyze a completely different research question: the economic costs induced by the voluntary filing of Chapter 11 in the U.S. bankruptcy system for different states of the economy. Second, we demonstrate that countercyclical distress costs, conflict of interests and strategic debt service are crucial to simultaneously explain the optimal leverage ratios (static), credit spreads and equity premium.

This paper is organized as follows. Section 2 describes the economy and builds the model. Section 3 calibrates the parameters and analyzes the implications of the model. Finally, Section 4 concludes.

1.2 The Model

In this section, we develop a continuous-time consumption-based equilibrium model for the pricing of firms' contingent claims. We consider an economy in which agents are aware of economic cycles as well as the existence of major features of the US bankruptcy code. We solve for optimal filings of Chapters 11 and 7 in both states of the economy under different assumptions of the objective function (firm maximization and equity maximization). We cast the model in equilibrium because we want to analyze the effect of the aggregate investor's preferences and the economic uncertainty on the efficiency of the US bankruptcy code.

In what follows, we first start by describing the economy and the preferences. Then, we introduce a pricing model for the firm's equity, debt and asset values.

1.2.1 The Economy

The pricing kernel in the economy is determined by a representative agent with the standard continuous-time Epstein-Zin-Weil preference. The Epstein-Zin-Weil recursive preference with the Kreps-Porteus aggregator allows for a separation between the two different concepts of risk aversion (desire to stabilize consumption across states of nature) and elasticity of intertemporal substitution (willingness to smooth consumption over time). As a consequence, in addition to economic uncertainty within one economy, the risk of switching of the state of the economy or macroeconomic condition is priced.

We limit our study to two states of the economy (boom and recession). The aggregate consumption is continuous but its growth rate and volatility switch with the state of the economy. The real aggregate consumption is given by

$$\frac{dC_t}{C_t} = g_{s_t} dt + \sigma_{C,s_t} dW_{C,t} \tag{1.1}$$

where we use s_t to denote the state of the economy at time t. If the economy is in recession, then $s_t = 1$; if the economy is in boom, then $s_t = 2$. The expected consumption growth rate g_{s_t} and diffusion coefficient σ_{C,s_t} depend on the state of economy at time t. The state of economy switches according to a continuous-time Markov chain, which is defined by λ_{12} and λ_{21}, where λ_{ij} $(j \neq i)$ is the probability per unit time of switching from state i to state j. The specification in equation (1.1) has been used in several recent studies. Guo, Miao and Morellec (2005) study the impact of discrete changes in the growth rate and volatility of cash flows on firm's optimal investment policy. Hackbarth, Miao and Morellec (2006) introduce the regime shifts in macroeconomic shocks and analyze their impact on credit risk and dynamic capital structure choice. Similar to BKS (2010) and Chen (2010), we use this specification to model both real consumption process and firm's cash flows.

Given the continuous time Epstein-Zin-Weil preference and the real aggregate consumption process specified in equation (1.1), the stochastic pricing kernel of the representative agent in our economy is similar to those in BKS (2010) and Chen

(2010). We provide a concise and slightly different derivation of the pricing kernel in Supplementary Appendix A,

$$\frac{d\pi_t}{\pi_t} = -r(s_t)dt - \theta^B(s_t)dW_{C,t} + (\nu_{s_t,j} - 1)dN^P_{s_tj,t}, j \neq s_t \tag{1.2}$$

where $r(i)$ is the risk free rate when the state of economy is i.[8] Parameter $\theta^B(i)$ is the market price of risk for systematic Brownian shocks in economy i. Parameter $(\nu_{ij} - 1)$ is the market price of risk for large shocks or Poisson jumps of the economy due to the switching of economic states, and $N^P_{ij,t}$ is the compensated Poisson process. The expressions for $r(i)$, $\theta^B(i)$, $N^P_{ij,t}$ and ν_{ij} are given in Supplemental Appendix A. As shown in equation (1.2), the pricing kernel in our economy contains a jump component that results from considering the Epstein-Zin-Weil preference coupled with a hidden Markov chain process for the states of the economy.[9]

In what follows, we move to the modeling of the firm environment and the building blocks that we use to price all of the firm's contingent claims.

1.2.2 The Firm Process and the Valuation of its Contingent Claims

We consider a firm with one publicly traded consol bond, that continuously pays coupon cdt. We assume that managers' decisions are aligned with shareholders' best interests. Risk shifting or asset substitution problems are not modeled directly in this paper. These assumptions are made for convenience only, some of these frictions

[8]For clarity of exposition, we alternate sometimes between $r(i)$ and r_i. Similarly we do so for $\theta^B(i)$ used below.

[9]For the standard CRRA utility function, the parameter (ν_{ij}) influencing the market price of jump risk is equal to one. Consequently, the term containing the jump risk factor disappears from the pricing kernel.

would exacerbate the magnitude of economic inefficiency or economic losses in the ex-ante asset values. Keeping the model simple helps to isolate the ex-ante economic costs of the voluntary filing characteristics in both economic states. We choose our primitive modeling variable to be the operating cash flows or earnings before interest and taxes (EBIT). Firm i's operating cash flow is given by

$$\frac{dX_t^i}{X_t^i} = \theta_{s_t}^i dt + \sigma_X^{id,i} dW_{X,t}^{id} + \sigma_{X,s_t}^s dW_{C,t}, \tag{1.3}$$

where $\theta_{s_t}^i$ is the expected earnings growth rate of firm i in state s_t. Parameter $\sigma_X^{id,i}$ is the idiosyncratic volatility of firm i's earnings growth, which is constant over time. And, σ_{X,s_t}^s is the systematic volatility in state s_t. Given the stochastic discount factor in equation (1.2), we construct the unique risk-neutral measure or the pricing measure, which is denoted by Q.[10] Under the Q measure, firm i's cash flow process becomes

$$\frac{dX_t^i}{X_t^i} = \hat{\theta}_{s_t}^i dt + \sigma_X^{id,i} dW_{X,t}^{id} + \sigma_{X,s_t}^s dW_{C,t}, \tag{1.4}$$

where

$$\hat{\theta}_{s_t} = \theta_{s_t} - \gamma \sigma_{X,s_t}^s \sigma_{C,s_t}. \tag{1.5}$$

We model the firm earnings separately from the consumption process as suggested by Longstaff and Piazzesi (2004). They show that corporate cash flows have historically been far more volatile and sensitive to economic shocks than has aggregate consumption. Thus, we allow the representative firm in the economy to have

[10]We define the Radon-Nikodym derivative using a normalized version of our pricing. This construction defines a unique pricing / no-arbitrage measure that we denote by Q. For simplicity, we also denote the new Brownian motions under Q $W_{X,t}^{id}$ and $W_{C,t}$.

different total volatility than consumption through its idiosyncratic variation. However, consistent with empirical evidence, we allow the firm operating cash flow to be correlated with aggregate consumption. Given this setting, the unlevered value of the firm's asset in state i ($i\epsilon\{1,2\}$) is equal to the expected present value of future operating cash flows under the pricing measure defined by our pricing kernel. The unlevered firm's asset value is given by

$$V_{i,t} = E_t\left[\int_t^{\infty} \frac{\pi_s}{\pi_t}(1-\eta)X_s ds \,|s_t = i\right] = \frac{(1-\eta)X_t}{r_{V,i}}, \tag{1.6}$$

where η is the effective tax rate and

$$r_{V,i} = \bar{\mu}_i - \theta_i + \frac{(\bar{\mu}_j - \theta_j) - (\bar{\mu}_i - \theta_i)}{\tilde{\lambda}_{12} + \tilde{\lambda}_{21} + \bar{\mu}_j - \theta_j}\tilde{\lambda}_{ij}, \ j \neq i \tag{1.7}$$

with $\bar{\mu}_i = r_i + \gamma\sigma^s_{X,i}\sigma_{C,i}$. In equation (1.7), $\tilde{\lambda}_{ij}$ is the jump intensity of the Poisson process under the risk-neutral measure Q satisfying $\tilde{\lambda}_{ij} = \nu_{ij}\lambda_{ij}$, $\tilde{\lambda}_{ii} = -\tilde{\lambda}_{ij}(i \neq j)$. In equation (1.6), the formula for the unlevered asset value can be interpreted as a generalized Gordon growth model adjusted for changes in the states of economy. The formula suggests that if the expected growth rate of the cash flow is much lower in recession than in boom, the price earnings ratio will be highly procyclical. This is a desirable feature that mimics well-documented empirical facts.

In our model, when the firm operating cash flows is higher than the coupon amount ($X_t > c$), We say that the firm is in the liquid state. The amount $X_t - c$ is distributed to shareholders as dividends. When the total cash flow is less than the total amount due to creditors ($X_t < c$), we say that the firm is in the illiquid state. The shareholders may not necessarily file for Chapter 11 even if the firm is in the illiquid state. Instead, the firm can issue more equity to cover the coupon payment if it is optimal. The firm chooses to file for Chapter 11 at a stopping time τ_B when the unlevered asset value hits endogenously determined default boundary V_B[11]. The reorganization (Chapter 11) boundary is state dependent (recession or boom). Both these boundaries are endogenously determined in our model. Once in Chapter 11, the debtors stop servicing the debt during this period. The firm's cash flow is accumulated and reinvested in the firm's assets. However, the firm starts incurring some distress costs including legal fees, lost business and the loss of key employees. Under Chapter 11 provisions, all debt claims are frozen for at least 120 days during the exclusivity period. By law, the debtors remain in control of the firm for at least 180 days. During this period, the debtors are expected to formulate a reorganization plan, and no one else can propose a plan. The judge often extends this exclusive period multiple times. The 2005 reform of the US bankruptcy code limits these extensions to eighteen months. Only after that and only if acceptance has not been obtained can creditors propose a plan. We accommodate this practical time limit by imposing a threshold to the time spent in Chapter 11. More specifically, the Chapter 11 process in our model lasts until one of the following three events occur.

First, if the value of the firm's asset drops to an endogenously determined level V_L, then the firm is liquidated (under Chapter 7 of the U.S. bankruptcy code). In our framework, this liquidation boundary is endogenously determined and state

[11] We use the traditional notation V_B, as in Leland (1994) to denote the default asset level. However, it is important to make the distinction that in Leland's papers, it denotes the absorbing liquidation boundary, while in our study, it denotes the reorganization boundary.

dependent. It depends on both the liquidation and the distress costs conditional on the state of the economy at distress. At liquidation, the judge appoints a trustee who supervises the sale of the firm's asset so that debtors obtain nothing and creditors receive the value of the firm net of liquidation costs. The assumption that absolute priority rule applies at the time of liquidation is consistent with the recent empirical evidence in Bris, Welch and Zhu (2006).[12]

Second, we follow the recent practice of having a limit on the time the firm is allowed to spend in Chapter 11. We consider the common scenario that the firm is liquidated if it spends more time than the allowed grace period in Chapter 11. As described above, this grace period can be thought of as the maximum time spent in Chapter 11, which combines both the debtor-in-possession time limit as well as an eventual subsequent time spent by creditors to come up with a plan.

Third, if the firm proposes a plan and eventually emerges from Chapter 11. The plan is successful if management provides creditors with a value that is at least higher than the liquidation value. If not, the creditors may reject the plan and shut the firm down. We use a Nash equilibrium to solve the negotiation game between debtors and creditors.

1.2.3 The Valuation Of The Firm's Contingent Claims Before Chapter 11 Filing

The previous section details the economy and the firm's environment. This section deals with the pricing of the firm's levered assets, equity and debt. We only

[12] Allowing APR violation at liquidation would give managers more incentive to file early, resulting in an increase in the conflict of interests between debtors and creditors.

need to solve for the equity and the debt values since the levered asset value is the sum of these two values. Given the shareholders' payoff function, the value of equity $S_{i,t}$, conditional on current state being i at time t before default, is given by

$$S_{i,t} = (1-\eta)E_t\left[\int_t^{\tau_B} \frac{\pi_s}{\pi_t}\left(X_s - c\right)\, ds\,|s_t = i\right] + E_t\left[\frac{\pi_{\tau_B}}{\pi_t} S_{\tau_B}\,|s_t = i\right],\ i\epsilon\{1,2\} \quad (1.8)$$

where η is the effective tax rate.[13] In equation (1.8), the first term represents the present value of all the dividend payments before filing for Chapter 11 (or the after-tax value of the operating cash flows left after servicing the debt). The second term is the present value of the shareholders' claim upon default. Variable S_{τ_B} is the value of the shareholders' claim at the level of Chapter 11; it depends on the state of the economy at which the firm's unlevered asset value crosses the Chapter 11 boundary. Its expression is given in the next subsection.

The value of the debtholders' claim $D_{i,t}$, conditional on the state being i at time t before default, is given by

$$D_{i,t} = E_t\left[\int_t^{\tau_B} \frac{\pi_s}{\pi_t} c\, ds\,|s_t = i\right] + E_t\left[\frac{\pi_{\tau_B}}{\pi_t} D_{\tau_B}\,|s_t = i\right],\ i\epsilon\{1,2\}. \quad (1.9)$$

In equation (1.9), the first term is the present value of the coupon payments before filing for Chapter 11. The second term is the present value of the debtholders' claim upon default. Variable D_{τ_B} is the debt value at the endogenous Chapter 11 boundary. It also depends on the state of the economy at which the firm's unlevered asset value crosses the Chapter 11 boundary, and its expression is given in the next subsection.

[13]We investigate the assumption of state independence of effective tax rate in Supplementary Appendix M.

Given the equity and debt values at Chapter 11 boundary, we obtain the following result.

Proposition 1. *At any time t before entering or filing for the Chapter 11 process ($V_t > V_B$), with the state of economy being i, the firm's equity, debt and levered asset values are given, respectively by*

$$S_{i,t} = V_{i,t} - (1-\eta)\frac{c}{r_{C,i}} + \sum_{j=1}^{2} q_{ij}^{D}\left[\frac{(1-\eta)c}{r_{C,j}} - V_{B,j}\right] + \sum_{j=1}^{2}\left(q_{ij}^{D} S_{B,j}\right), \tag{1.10}$$

$$D_{i,t} = c\left(\frac{1}{r_{C,i}} - \sum_{j=1}^{2}\frac{q_{ij,t}^{D}}{r_{C,j}}\right) + \sum_{j=1}^{2}\left(q_{ij}^{D} D_{B,j}\right), \tag{1.11}$$

and

$$v_{i,t} = V_{i,t} + \frac{\eta c}{r_{C,i}} - \sum_{j=1}^{2}\frac{q_{ij,t}^{D}\eta c}{r_{C,j}} - \sum_{j=1}^{2} q_{ij}^{D}\left(V_{B,j} - v_{B,j}\right), \tag{1.12}$$

where $r_{C,i} = r_i + \frac{r_j - r_i}{\tilde{\lambda}_{12}+\tilde{\lambda}_{21}+r_j}\tilde{\lambda}_{ij}$, $j \neq i$. Variable q_{ij}^{D} is the time-t Arrow-Debreu price of a claim in state i that pays 1 unit of consumption conditional on the event that the firm files for Chapter 11 in state j. Variables $S_{B,j}$, $D_{B,j}$ and $v_{B,j}$ are the equity, debt and levered assets values at Chapter 11 boundary, conditional on the state at default being j.

For ease of comparison with the previous literature, we keep the same notation for the Arrow-Debreu price (q_{ij}^{D}) as in BKS (2010). However, there is a key difference that should be considered. Variable q_{ij}^{D} in BKS (2010) is the price of 1 unit consumption in liquidation since in their model there is no distinction between liquidation and reorganization. In our setting, q_{ij}^{D} is the price of 1 unit consumption at the reorganization boundary. It depends on the endogenous firm levels at liquidation and at default, both of which are conditional on the corresponding prevailing state of the economy. The solution for q_{ij}^D is shown in Supplementary Appendix B.

To complete the valuation of the firm's contingent claims, we need to solve for the values of the equity, debt and levered assets at Chapter 11 boundary (or $S_{B,j}$, $D_{B,j}$ and $v_{B,j}$), conditional on the state at default being j. This is what we address in the next subsection.

1.2.3.1 The Valuation of The Firm's Contingent Claims At Chapter 11 Filing

Our aim in this subsection is to price the firm's contingent claims at the Chapter 11 boundary, conditional on the state of the economy at the filing time.

The firm's value at τ_B (stopping time indicating the default time conditional on the state at default) with the state of economy being i is

$$v_{B,i} = E^Q\left[\int_0^\tau e^{-r_i t}\left(\delta V_t + \eta c 1_{V_t > V_{B,i}} - \omega_i V_t 1_{V_{L,i} < V_t < V_{B,i}}\right) dt\right] + \alpha_i E^Q\left[e^{-r_i \tau} V_{i,\tau}\right], \tag{1.13}$$

where V_t is the unlevered asset value. Variable δ is the payout ratio for the firm's asset. It accounts for both dividend and coupon payments if the firm value is above the endogenous Chapter 11 boundary. The stopping time τ is the liquidation time and it is conditional on the state of the economy at reorganization. As mentioned in section 2.2.1, it is defined as the minimum of two stopping times: the time of liquidation due to spending more than the limited grace period in default (denoted τ_2), and the time of liquidation due to the limited liability violation (denoted τ_4). We use the following mathematical notation for the liquidation stopping time: $\tau = \tau_2 \wedge \tau_4$. r_i is the risk-free rate for economy i. Parameter η is the tax rate. Parameter ω_i is the cost of financial distress that the firm incurs while staying under Chapter 11 protection in state i. It includes legal fees, lost business, and the loss of valuable employees. Parameter $V_{L,i}$ is the endogenous Chapter 7 boundary in state i. Parameter α_i is the recovery rate of the unlevered firm value at liquidation. Variable $V_{i,\tau}$ is the unlevered asset value at liquidation in state i. This is either equal to the value of the firm at the endogenous Chapter 7 boundary or the firm's value at the moment of exceeding the limited grace period. For parsimony, we compute the continuation value of the firm's contingent conditional on the state at the hitting time. The level of reorganization costs are conditional on the prevailing state of the economy. It is important also to note that the time counted is not the cumulated time (known as the occupation time). If the firm emerges from Chapter 11 and enters again, the clock is reset at 0 (known as excursion time). This is consistent with the regulation and the usual practice of the Chapter 11 process.

Equation (1.13) for the levered firm value at the Chapter 11 boundary contains four elements. The first term is the present value of all the expected cash flows generated by the firm until liquidation, regardless how liquidation is triggered. The second term is the present value of the expected tax benefits when the firm emerges from Chapter 11 (above the Chapter 11 boundary).[14] The third term accounts for the

[14]We assume in this term that the firm in our economy takes complete advantage of the tax benefit of the coupon payment until they file for Chapter 11. This is a simplification of our model, because the endogenous Chapter 11 (cash flow based) boundary in both states

present value of the cost of financial distress incurred whenever the firm is in Chapter 11. The last term represents the present value of the expected recovered firm value upon liquidation.

In solving equation (1.13), one need to solve for the distribution of the minimum of two stopping times with excursions. Another task is obtaining the distribution of unlevered asset value at liquidation. In a risk-neutral framework without regime switching for the economy, BCS (2007) rely on a sophisticated binomial lattice method to simulate the path of unlevered asset value and only obtain numerical solutions. In this study, we obtain the closed-form solution of our model in which the BCS (2007) model is a special case.

In summary, we address the first problem by relying on the notion of the excursion time of a semi-Markov process and applying it to the doubly perturbed Brownian motion. Doing so, we extend the results proposed in Dassios and Wu (2008) to a different stopping time exercise. To overcome the mathematical challenge of finding the distribution of unlevered asset value at liquidation, we use the concept of Brownian meander, which is first used in Chesney, Jeanblanc-Picque and Yor (1997). Solving all the terms in equation (1.13) yields the following result.

Proposition 2. *The firm's levered asset value at the hitting time of Chapter 11*

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of the economy is always lower than the coupon level. Under the US tax system, to take advantage of tax deductibility, the firm should have an operating income that exceeds the coupon amount. However, losses associated with EBIT being lower than the coupon amount can be carried forward. Thus, it is only the expected time value of loss carried forward that is included in extra in our derivation. Leland (1994) (in "section VI") considers a case in which the tax benefit can be lost even if the firm is solvent. He shows that this assumption yields minor differences.

boundary is given by

$$v_{B,i} = V_{B,i} + \frac{\eta c}{r_i} - P(\tau_2 < \tau_4)\left[V_{B,i}A(d) + \frac{\eta c}{r_i}B(d) + C(d)\right] - P(\tau_2 > \tau_4)\left[C(d) + D(d)\right], \tag{1.14}$$

where the expressions of $A(d)$, $B(d)$, $C(d)$, $D(d)$, $P(\tau_2 < \tau_4)$ and $P(\tau_2 > \tau_4)$ are given in Appendix 2.

The debt value at the Chapter 11 boundary is obtained by discounting all the expected cash flows at the state-dependent interest rates. Creditors are entitled to the coupon stream if the firm emerges from Chapter 11 at any moment above the corresponding reorganization boundary. In addition, creditors recoup a fraction of the cumulated and reinvested coupon stream that has not been distributed during the Chapter 11 process. This fraction is directly related to their bargaining power. In sum, given the shareholders' strategic debt service ϑ, we obtain the following result.

Proposition 3. *The firm's debt value $D_{B,i}$ at the hitting time of the Chapter 11 boundary in the state of the economy i is given by*

$$D_{B,i} = \vartheta c E(d) + (1 - \vartheta)cG(d) + \alpha_i V_{B,i} H(d), \tag{1.15}$$

where the expressions of $E(d)$, $G(d)$ and $H(d)$ are given in Appendix 2.

In equation (1.15), the solution for the strategic debt service ϑ is determined by the shareholders' bargaining power during the reorganization process. The procedure to solve for the equilibrium strategic debt service ϑ is shown in Supplementary Appendix E.

At this stage, given the solutions for the firm's value $v_{B,i}$ and the debt value $D_{B,i}$ at the Chapter 11 boundary, the equity value at the Chapter 11 boundary is simply the difference between them, or $S_{B,i} = v_{B,i} - D_{B,i}$. Plugging these expressions into the equations in Proposition 1 yields the complete expressions of equity, debt and levered asset values $E_{i,t}$, $D_{i,t}$, $v_{i,t}$ at any time t before filing for Chapter 11, conditional upon the economic state at t being i.

1.2.3.2 The Valuation of The Firm's Contingent Claims During Chapter 11 Filing

In this subsection, we study the valuation of the firm's securities during the Chapter 11 process. This is of particular interest for many practical applications. The equity value for the firm in the Chapter 11 process is given below.

Proposition 4. *When the firm's unlevered asset value is below the Chapter 11 boundary, the firm's equity value depends on the remaining time allowed still in Chapter 11 (d'), the accumulated cash flow position (M_0) as of the time of valuation, and the accumulated coupon (C_0) after filing for Chapter 11. The equity value is given by*

$$\begin{aligned} S(V, d', M_0, C_0) \quad = \quad & (M_0 - \vartheta C_0)A_1 + \tfrac{2(\delta-\omega)V}{\lambda^2-(\sigma+b)^2}\left[A_1 - e^{(\sigma+b)z'_B} f(\tfrac{\lambda^2}{2}, 0)\right] \\ & -\tfrac{2\vartheta c}{\lambda^2-b^2}\left[A_1 - e^{bz'_B} f(\tfrac{\lambda^2}{2}, 0)\right] + S_B f(r, b), \end{aligned} \tag{1.16}$$

where the expressions of A_1 and $f(x, y)$ are given in Appendix 2.

Similarly, we solve for the value of the distressed debt. To save space, we report the result and the derivation in Supplementary Appendix G. The levered asset value is then a simple sum of both the debt and equity values. Proposition 4 contains the expression for the survival probability (or the probability for the firm to emerge from the Chapter 11) given that the firm is in Chapter 11 already. In Appendix 2, we give the closed-form solution for the survival probability ($A1$ in Proposition 4), which we believe is a valuable metric to practitioners investing in distressed firms.

1.2.4 Special Cases

Our framework generalizes a myriad of well-known structural models in the literature. This property provides a platform that facilitates the understanding of the incremental benefits of each assumption in the previous models. For example, it is straightforward to compare the incremental contribution to the firm valuation by including macroeconomic conditions versus incorporating realistic reorganization process. Another useful exercise is to understand the influence of the negotiation in bankruptcy versus an exogenous sharing rule, and how these quantities get affected by changes in macroeconomic conditions. In addition to the easy implementation of analytical expressions, our model makes it straightforward to compute the sensitivities of the equity, debt and firm values to any risk parameters. Finally, the model can be used as a base to investigate other bankruptcy related questions (e.g., optimal grace period, expected excursion time conditional on hitting Chapter 11 boundaries, conditional likelihood of emergence from Chapter 11, etc.).[15]

We also point out that our methodology yields exactly the same close-form solution for the nested cases by using a different approach than the original one used

[15]Our analytical expressions also facilitates computation of many exiting and novel metrics. For example, the model provides Distance-to-Chapter11 instead of Distance-to-Liquidation. Also note that our model indicates a non-zero equity value at default, which is consistent with empirical facts.

in those studies. This property provides comfort that our derivation is sound. In what follows, we report some of the models in the literature that our model produces as special cases. We indicate which parameters need to be relaxed to attain these models.

Leland (1994)[16]: $V_L = 0$, $d = 0$ and $\lambda_{12} = 0$.

François and Morellec (2004): $V_L = 0$ and $\lambda_{12} = 0$.

Broadie, Chernov and Sundaresan (2007): $\lambda_{12} = 0$ and exogenous ϑ.

Static cases in Bhamra, Kuehn, and Strebulaev (2009) and Chen (2009): $d = 0$.

The variable d is the maximum time that the firm is allowed to stay in Chapter 11. Parameter λ_{ij} is the objective transition probability from state i to j, and V_L is the endogenous unlevered firm value at Chapter 7. In Supplementary Appendix H, we show detailed derivations of the analytical expressions for the special cases mentioned above.

1.3 Model Implications

In this section, we start first by discussing the state-dependent parameter choices for the economy, the firm and the legal environment in which it operates. Second, we assess the efficiency of the voluntary filing for Chapter 11 in the presence of business cycles. Third, we examine the implications of our model on the optimal capital structure, default probabilities and credit spreads for a BAA-representative

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[16] Without loss of generality, we assume that the state of economy is 1 for the cases with one state of economy.

firm in the economy. We present our findings for a cross section of firms that belong to different rating categories. Finally, we examine the ex-ante implication on firm value of debt-to-equity exchange, an alternative to Chapter 11. [17]

1.3.1 Parameter Calibration

The implementation of our model requires an identification of preference parameters , the economy variables and the firms' fundamentals. Because we have a business cycle setting, most of these parameter values have to be set in both states of the economy. We start with the parameters of the economy and preferences. Then, we proceed to cover the firm's specific variables.

1.3.1.1 The Economy

To estimate both the process of aggregate earnings and consumption, we rely on the maximum likelihood method in Hamilton (1989). We fit our parameters to the following aggregate US data from 1929-2007[18]: real non-durables plus service consumption expenditures from the Bureau of Economic Analysis and aggregate earnings data for all nonfinancial firms from the Compustat database. Through this, we estimate the state-dependent expected consumption growth rates, its volatilities, expected aggregate earning growth rates, its volatilities and the probabilities of switching between the states. Our results are reported in Panel A of Table 1. We find

[17]We also study the implications of our model on levered equity premia and risk-free rates. See Supplementary Appendix I for more details.

[18]We also estimate the model using real per-capita data from 1961 to 2007. The conclusion is qualitatively similar. Our estimates based on the shorter sample period are consistent with Le, Singleton and Dai (2010). See Supplementary Appendix M for details.

that the weighted averages of our state-dependent estimates are very similar to those reported Chen (2010).

We set time preference $\beta = 0.01$, and relative risk aversion $\gamma = 10$. For the risk premium analysis, we also use $\gamma = 7.5$. In the literature, researchers have different estimates for the EIS. Some studies (see Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002) and Guvenen (2006)) suggest that the EIS is higher than one, while others (see Hall (1988) and Campbell (1999)) find the opposite. We set the EIS $\psi = 1.5$ for the base case. However, we also provide our comparative statics for varying values of EIS.

1.3.1.2 The Firm

Panel B of Table 1 describes the firm's fundamental parameters as well as our parameter choices for the firm's environment. We do this for a representative firm in the economy (BAA-rated firm). [19] The parameters that we consider are: 1) expected growth rate of the firm's cash flow (g), 2) idiosyncratic volatility for the firm's cash flow ($\sigma_X^{id,i}$), 3) the effective tax rate (η), 4) the costs of financial distress (ω), 5) the liquidation costs (α), 6) the bargaining power (ς) and 7) the grace period (d) and 8) how to determine the optimal coupon (c^*). Our aim is to choose these parameter values as close as to those reported in the previous literature. Please refer to Appendix 2 for a detailed discussion of the rationale behind the parameter value choices.[20] We

[19] Later when we analyze cross-sectional model implications in this section, we also describe our methodology to obtain firm-specific variables for different credit rating categories.

[20] In Supplemental Appendix M, we provide the robustness of using a contant marginal tax rate and report the cost of financial distress and liquidation consistent with the estimates

also report in Appendix 2 an estimation methodology that allows separating distress from liquidation costs in both states of the economy. Our estimation methodology confirms that our parameter value choices for the unobserved variables are reasonable.

1.3.2 Quantitative Assessment of The Voluntary Filing of Chapter 11

In this subsection, we study the effect of Chapter 11 on the firm's asset, equity and debt values in different states of the economy. We examine two scenarios. In the first scenario (denoted Case 1 below), filing for Chapter 11 is endogenously determined to maximize the total firm value. In the second and more realistic scenario (denoted Case 2 below), Chapter 11 filing is endogenously determined in concordance with equity value maximization. In what follows, following BCS (2007), we refer to the first scenario as the first-best case and the second scenario as the second-best case. We begin by the first-best case because the security values for this case would serve as a benchmark for us to pin down the cost of conflict of interests arising from the voluntary filing of Chapter 11.

Figure 1 presents our results for the first-best case. It illustrates the impact of Chapter 11 on the optimal default boundary, equity value, firm value and bond yield in both states of economy for different grace periods and EIS. In order to only quantify the incremental benefit of Chapter 11 option in both states of economy we normalize all the values in the vertical axis by those from a model that considers macroeconomic conditions and liquidation costs but ignores Chapter 11 option. The

in Korteweg (2010) instead of those in Andrade and Kaplan (1998).

normalizing (or benchmark) model in this figure can be understood as the static case in BKS (2010).

Panel A of Figure 1 shows that the default boundaries (both in recession and in boom) are lower than the liquid-states of the firm (the level at which EBIT is equal to the required coupon payment at each state of the economy). We find that firms have incentives to issue equity in both states of the economy to service the debt and avoid costly distress and liquidation. The optimal default boundaries with the presence of Chapter 11 are higher than the optimal liquidation boundaries in a model without Chapter 11. This gap increases slightly with the grace period. The firm defaults earlier in our model than in the benchmark because the debt relief during the reorganization process helps the firm delay and even avoid possible liquidation.

Panel C of Figure 1 suggests that Chapter 11 under the first-best scenario leads to a higher firm value than the BKS-benchmark for both states of the economy. For a grace period limited to two years, the existence of Chapter 11 in the first-best case induces an ex-ante increase in the firm value by about 1% in recession and 0.7% in normal economic conditions. Intuitively, the reorganization process benefits the firm because the debt relief helps the firm avoid or at least delay possible costly liquidation. The benefit of this delay is higher in bad economic states simply because distress and liquidation costs are countercyclical. The firm benefits more from Chapter 11 with a longer grace period. This observation is consistent with the fact that the longer the grace period, the more likely the firm avoids liquidation. Panel D of Figure 1 shows that in this scenario the reorganization option enhances slightly the firm's equity value for both states of the economy and the increase in the equity value occurs more in recession than in boom. Panel B of Figure 1 illustrates that creditors get a large fraction from the increase in firm value in both states of the economy. The incremental debt value is higher when the firm is allowed to stay under Chapter 11 protection for a longer time.

The benefit of casting our model in equilibrium is that it permits to examine the impact of aggregate preferences on the firm's decisions. To save space, we focus only on how EIS affects the impact of Chapter 11 option on the firm's asset and equity values in Panels E and F. Other comparative statics concerning risk aversion, consumption and earning parameters can be similarly performed. Panel F shows that with a low EIS, the reorganization process almost does not enhance the equity value, indicating that most of the benefits from Chapter 11 goes to creditors. Furthermore, Panel E indicates that the higher the EIS, the lower the increase in firm value.[21]

Now we turn our attention to the second and more realistic scenario: the voluntary filing option of Chapter 11. In this case, bankruptcy decisions are assumed to be taken in concordance with equity value maximization. Our aim is to quantify the ex-ante implications that arise from the conflict of interests between debtors and creditors in the presence of regime-switching macroeconomic conditions. Figure 2 illustrates how the shareholders' voluntary filing for Chapter 11 influences the firm's

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[21] A higher EIS translates to a higher asset value-based default boundary because an increase in EIS largely enhances the price earnings ratio. With an earlier default (higher boundary), the firm suffers more from the proportional distress cost although the availability of Chapter 11 helps delay and possibly avoid the costly liquidation.

ex-ante asset, equity and debt values for different grace periods and EIS in both economic states. In order to isolate the ex-ante effects of the conflict of interests we normalize the values in the vertical axis in Figure 2 with the corresponding values from the first-best case (the ones determined to maximize the firm value).

Panel A documents that our model generates countercyclical default boundaries. The conflict of interests brings up the boundaries in both states of the economy compared to the first-best case. This increase is slightly higher in recession than in boom because debtors avoid higher distress costs in recession. Also, the state-dependent boundaries for both states of the economy are lower than the corresponding liquid-state levels, suggesting that management would be still willing to issue equity to avoid costly reorganization.

Panel C of Figure 2 illustrates the magnitude of the ex-ante reduction in firm value due to the conflict of interests between debtors and creditors. With a grace period of 2 years, the ex-ante reduction in firm value amounts to 2.9% in recession (2% in boom). The fact that the voluntary filing for Chapter 11 induces more losses in recession than in normal economic state is alarming. One reason for this is that it is in recession that defaults cluster and bankruptcy options should be designed to encourage more participation from economic agents. Later in this subsection we analyze what drives these ex-ante losses. In panel C, we also notice that these ex-ante losses increase with the grace period. This is intuitive, because when the grace period allotted to the firm under Chapter 11 protection is extended, the firm is more likely to emerge from bankruptcy, which indicates less liquidation threat due to early entry into the reorganization process. Thus, shareholders would file for Chapter 11 earlier to expropriate more rent from creditors. This translates to an increase in equity value at the expense of debt values (as shown in Panels D and B).

Panels E and F of Figure 2 focus on effects of the voluntary filing option on the firm's asset and equity values with respect to the EIS values. We find that for all the EIS values and both states of the economy, the conflict of interest induces significant ex-ante costs to the firm. These costs are more pronounced in recession than in boom. The higher the EIS, the smaller this induced reduction in firm value as well as the increase in equity value.The increase in EIS helps reduce the firm value losses because it leads to a lower cash-flow based default boundary (or delay of default). The EIS affects the timing of default through two channels. In the first channel, the aggregate investor with a higher EIS cares more about the intertemporal risk, and precautionary saving leads to a lower risk-free interest rate, which indicates a higher time discount factor in the risk-neutral framework. Hence, the present values of the tax benefit and the bankruptcy cost increase with EIS. In the absence of the distress cost, the net effect is that the firm lowers the reorganization boundary to reap more tax benefits. In the second channel, an increase in EIS decreases the average risk-neutral expected earnings growth rate since a higher EIS increases the risk-adjusted probability of being in the bad state of the economy in the risk-neutral world, which lifts the Chapter 11 boundary. We find that the net effect of the two channels is to lower the cash flow-based reorganization boundary.

Overall, Panels B-F of Figure 2 suggest that the conflict of interests due to the voluntary filing of Chapter 11 results in a significant reduction in the ex-ante firm value. These results are consistent with the fact that when shareholders rate debt relief more highly than the distress cost in the bankruptcy process, they tend to file for Chapter 11 earlier (suboptimally from the firm-value-maximization standpoint) to obtain partial debt relief. However, the earlier entry into Chapter 11 damages the firm value because the firm, as a whole, is more negatively affected by the distress cost and possible liquidation cost. Because of countercyclical distress and liquidation costs as well as countercyclical state prices, these ex-ante costs to the firm turn out to be more severe in recession than in boom.

To examine further what drives these ex-ante losses in firm value, we quantify the reduction in firm value due to the voluntary filing of Chapter 11 with respect to some other benchmark models in Figure 3. In our model, one source for the market price of risk is due to the change in the state of the economy. Hence, when shareholders choose the filing time for Chapter 11, their decision depends on the possible changes of the state of the economy in the future. To illustrate how business cycles influence the ex-ante loss in firm value, we compare the loss in firm value with business cycle risk (as in our model) with that for the case where we shut down the changes in the state of the economy. The latter case is similar to the BCS (2007) model. More specifically, Panel A of Figure 3 illustrates the ex-ante loss in firm value from our model relative to the firm value in the Leland (1994) framework. To save space, we only report the scenario with boom as the initial state. The ex-ante reduction in firm value is around 5% (for a grace period of 2 years) of the firm value from the Leland (1994) benchmark. Panel B of Figure 2 depicts the ex-ante loss in firm value generated by a model that captures the voluntary filing option of Chapter 11 but ignores changes in macroeconomic conditions (BCS (2007) framework). Comparing Panel B with Panel A yields the effect of changing macroeconomic conditions. The ex-ante loss in Panel A is more than twice that in Panel B for some choices of the grace periods. For example, for a grace period of 2 years, the BCS (2007) framework produces around 2.5% loss in firm value relative to the Leland (1994) benchmark rather than the 5% reduction as our model quantifies. Furthermore, in order to disentangle the ex-ante costs due to the voluntary filing of Chapter 11 from those due to the macroeconomic conditions effect, we use the static case in BKS (2010) as the benchmark in Panel D of Figure 3. The latter model allows for changes in macroeconomic conditions but only considers the liquidation option for the firm. With the BKS (2010) being the benchmark, the ex-ante reduction in firm value due to the voluntary filing of Chapter 11 amounts to 2% in recession (or 1.5% in boom) for a grace period of 2 years. These are slightly lower than the reduction reported in Panel C of Figure 3 (2.9% in recession and 2% in boom), where the benchmark is the first-best case.

Overall, Panel A to D gives a picture to the underpinnings and the corresponding magnitude of the ex-ante losses in firm values. It is also important to keep in mind that these ex-ante induced costs are not incurred by firms in financial distress but rather by a BAA-representative firm in the economy. The ex-ante loss in firm value is higher for highly levered firms.

Another important distinction between our setup and previous models of bankruptcy is that we impose the equilibrium constraint that creditors should get at least the liquidation value at the Chapter 11 emergence time. It is straightforward to relax this assumption in our model and account for exogenous sharing rules. Intuitively, relaxing this constraint would augment the ex-ante reduction in total firm value. In Panel A of Figure 4, we show the effect of this constraint on the ex-ante loss in firm value. To disentangle the effect of the constraint from the one due to the business cycle risk, we fix the state of the economy to be boom. The firm values in the vertical axis are normalized by the corresponding firm value under a model without Chapter 11. We can see that for all the EIS values, the ex-ante loss in firm value is multiple times higher for the case without constraint than that for the case with constraint. [22] The constraint on the debt value at the emergence time is consistent with the idea that the reorganization plan of the Chapter 11 process should be "fair and equitable". Mechanically, this constraint puts an upside limit on the amount of rent that shareholders can expropriate from creditors. Thus, shareholders choose not to file for Chapter 11 too early, which leads to less ex-ante loss in firm value.

We also analyze how distress and liquidation costs influence the gap in firm value between the first- and second- best cases. Panel B of Figure 4 shows that when the distress cost in recession rises, the difference in firm value between the two cases becomes smaller. In particular, when the distress cost is greater than 10%, the firm value for the second-best case is very close to that for the first-best case (less

[22]We find similar results when we perform this exercise for a constant EIS but different grace periods.

than 0.5% difference for both states of the economy). One reason for this is that when shareholders choose to file for Chapter 11, they weigh between the benefits of debt relief and distress costs. A very high distress cost discourages them from early filing for Chapter 11, which leads to a higher ex-ante firm value. Thus, a severe market punishment for firms' early entry into Chapter 11 would act as a disciplinary mechanism to prevent debtors to expropriate more rent from creditors.

In Panel C of Figure 4, we turn our attention to the impact of liquidation costs on the gap in the firm value between the first- and second- best scenarios of Chapter 11 filing. We find that the costs induced by conflict of interests are negatively related to the recovery at liquidation. This is not a surprising result because as previously discussed, early filing for Chapter 11 by shareholders damages the firm value through two channels. The first channel is the distress cost during the Chapter 11 process. The second channel is the possible liquidation cost if the firm stays below the Chapter 11 boundary longer than the grace period. Intuitively, the damage from the second channel diminishes if the firm can recover more at liquidation.

1.3.3 Further Analysis

The above reports and analyzes the effects of conflicts of interests between equityholders and creditors on the losses in ex-ante firm value. In this subsection, we conduct further analysis on the economic significance of these ex-ante losses. First, we discuss the sensitivity of our results on the firm's parameter value choices. Second, we extend our base model to analyze the agency costs due to asset substitution problems. Finally, we consider the difference in the firm's economic environment for the firm-value-maximization and equity-value-maximization cases and its effects on the ex-ante losses in firm value.

When we calibrate the models in the above, we set the firm's parameter values to match empirically observed moments. Although the parameter values we choose are consistent with the previous literature, we investigate whether our results are robust to different parameter value choices. This section discusses the sensitivity of our results on several important parameter values. We conduct sensitivity analysis of the ex-ante losses in firm value on six parameter values including liquidation cost and cost of financial distress in both states of the economy, strategic debt service, and effective tax rate.

First, we set the range for each parameter value to be from 20% below its value used in the base calibration to 20% above that number. Then we break the range for each parameter into five equal elements and choose one of the six possible values for each analysis. For example, since the base value for liquidation cost in recession is 30%, its range is from 24% to 36% and the value we choose is from {24%, 26.4%, 28.8%, 31.2%, 33.6%, 36%}. Since we have six parameter values to choose per round of computation, in total we obtain 46656 ($= 6^6$) results on the ex-ante firm value losses. Table 2 reports the mean and standard deviations of the 46656 numbers with respect to each benchmark model. Our sensitivity analysis shows that with respect to all the benchmark models, the average model implied ex-ante losses in firm value are comparable to the losses shown in Figures 2 and 3. The economic significance of our results is robust to different parameter value choices.

In the base model, we focus on modeling the negotiation cost in the bargaining game between equityholders and debtholders in the restructuring process. This subsection extends our base model to quantify the agency cost due to the asset substitution problem.

Jensen and Meckling (1976) introduce asset substitution and related agency cost: equityholders can potentially extract value from debtholders by increasing the investment risk. Leland (1998) investigates the effect of this agency cost on the firm's optimal capital structure and risk management. By extending our base model, we introduce the concept in Leland (1998) to study potential asset substitution problems in distressed firms. After Chapter 11 filing, the equityholders have an incentive to exploit the firm's option to avoid default. One way to achieve this is to increase the firm's operating risk and hence the chance to survive. Our purpose is to quantify the agency cost related to the firm's post-default risk shifting. We denote the firm's normal (before bankruptcy) operating risk level as σ_X^b and a higher level as σ_X^a (after bankruptcy). We conduct our analysis for two different levels of risk shifting which correspond to an increase of the firm's risk level by 20% ($\sigma_X^a = 1.2\sigma_X^b$) and 50% ($\sigma_X^a = 1.5\sigma_X^b$). We investigate the related losses in ex-ante firm value when we use the firm-value-maximization case as the benchmark.

Our results suggest the agency cost due to the asset substitution problem is economically significant. The higher the post-bankruptcy operating risk level, the higher the losses in the ex-ante firm value. Specifically, the losses increase by 12%

to 3.3% in recession (by 10% to 2.2% in boom) when the risk level increase by 20%. The losses increase by 39% to 4% in recession (by 34% to 2.7% in boom) when the risk level increases by 50%. The ex-post choice of higher risk level after bankruptcy gives equityholders an incentive to enter Chapter 11 even earlier (or more suboptimal for the firm) than the base case due to reduced chance of liquidation. In addition, the firm shifts down the liquidation boundary further than the base case given the higher chance of getting back to normal conditions. The combined effect is to induce more financial distress costs due to the expansion of the gap between the default and liquidation boundaries.

In the model calibration, to isolate out only the effect due to the conflict of interest due to voluntary filing, we use the same parameter values to calibration the firm-value-maximization and equity-value-maximization cases. We choose the parameter values (strategic debt service, liquidation and distress costs) to match empirically observed moments including bond recovery rate, magnitude of APR violation and default losses. However, these moments are likely to take different values for the two maximization cases. In the equity-value-maximization case, equityholders may use various ways like asset sales in the restructuring process to appropriate more rent from the creditors, which translates to a higher level of APR violation. Along the similar line of reasoning, it is reasonable to assume a higher bond recovery rate in the firm-value-maximization case.

Thus, we calibrate the parameter values to match different moment values for the firm-value-maximization case. Specifically, when we calibrate the firm-value-maximization case, we increase the bond recovery rates by 30% to 66% and decrease the magnitudes of APR violation in recession by 30% to 2.1%. Given the changes in the moment values, the model-implied liquidation costs are lower than those in the base case. In the new calibration, the voluntary filing for Chapter 11 generates 3.8% ex-ante reduction in firm value in recession and 2.6% in boom relative to the firm value in the firm-value-maximization case.

In all the above, we rely heavily upon our model to draw conclusions related to the voluntary filing of Chapter 11. Consequently, to feel comfortable concerning our results, one needs to show to what extent our model is capable of generating reasonable empirical moments of economic and firm's fundamentals at the first place. Thus, in what follows, we examine simultaneously our model implications on the optimal capital structure, default probabilities and credit spreads for a BAA-representative firm in the economy. We also present our findings for a cross-section of firms that belong to different rating categories. Finally, we study our model implications on levered equity premia and risk-free rates.

1.3.4 Capital Structure, Default Probabilities and Credit Spreads

In this subsection, we present the model implications on the optimal capital structure, default probabilities and credit spreads for a representative firm in the economy. Also, we discuss the model-implied default probabilities and credit spreads for different credit ratings by exogenously fitting our model to the empirically observed leverage ratios. We compare our results with the empirical data, which are shown in Table 3. We also present our results for both our model and a model without Chapter 11. The model without Chapter 11 and thus without the induced costs due to conflicts of interest can be viewed as the static BKS (2010) or two-states Chen (2010) models. Our aim is to present only the incremental contribution of accounting for countercyclical distress costs and the interaction between changes in macroeconomic conditions and the conflict of interests. One can refer to BKS (2010) and Chen (2010) to appreciate the role of incorporating macroeconomic conditions in the absence of the conflict of interests and pre-liquidation distress costs.

Table 4 shows the model-implied default probabilities, risk adjustments (which is defined in our setting, following the credit risk literature, as the ratio of risk neutral over the objective default probabilities), yield spread and leverage ratio for a representative firm in the economy (a BAA-rated firm) for the cases with and without Chapter 11.

In Panel A of Table 4, we fix the coupon level to solely study the effect of the Chapter 11 option on default probabilities and credit spreads for different initial states of the economy. We see that given the same initial state of the economy, the Chapter 11 option results in higher default probabilities and higher credit spreads. This is intuitive because shareholders always have the option to avoid costly liquidation by filing earlier for Chapter 11. In addition, the default probabilities and credit spreads are countercyclical for both the cases with and without Chapter 11. In bad economies, the higher conditional default losses and lower expected earnings growth rate imply lower continuation values for equityholders. Thus, the firm is more likely to default in recessions, and as a result, creditors charge a higher credit spread ex-ante.

Panel B of Table 4 illustrates the role of conflict of interests on the optimal capital structure, default probabilities and credit spreads at the optimal coupon level. We can see that the 10-year default probability (4.25) generated by the model with Chapter 11 is higher and closer to the historical default rate (4.39) than (3.86) generated by the model without Chapter 11. The Chapter 11 option generates a higher default boundary and hence, a higher default probability since management would choose to file for Chapter 11 earlier to expropriate rent from creditors by not fully servicing the debt. However, even for the model with Chapter 11, the 5-year default probability (0.36) is smaller than the historical average 5-year default rate (1.82). One reason for this underestimation is that our model solves the 5-year default probability for an *individual* BAA-rated firm at the optimal coupon level, while the historical default rate from the Moody's report is calculated as the *sample average* of firms that are not necessarily at their optimal. BKS (2010) provide an excellent discussion on this subject. Another reason for this underestimation can be related to the absence of a substantial jump in economic fundamentals.[23]

For the case with Chapter 11, the ratio of short term risk-neutral default probability over objective default probability (or risk adjustment) is equal to 3.45 for 5-year horizon and 3.03 for 10-year horizon. This is consistent with the empirical estimate of the risk adjustment in the literature. For example, using the instantaneous default intensities, Berndt et al. (2005) suggest that the average risk adjustment for

[23] See Du and Elkamhi (2011).

BAA-rated firms is equal to 2.76. Using the ratio of one year default probabilities, Elkamhi and Ericsson (2009) find that the risk adjustment was close to 3.5 in the third quarter of 2002. Also, as shown in Panel B of Table 4, with the Chapter 11 option, the firm's optimal leverage ratio (39.67%) is lower than that for the case (43.94%) without Chapter 11. Our model-implied optimal leverage ratio for the BAA rating is consistent with the empirical evidence (36.7%). When the voluntary filing of Chapter 11 is available, the early filing is suboptimal for firm value maximization since the firm experiences a higher distress cost. Ex ante, creditors charge a higher credit spread for a fixed level of coupon and the firm issues less debt to maximize firm value. In summary, the costs due to the conflict of interests help counterbalance the tax benefits of debt. Furthermore, the model-implied credit spreads are close to the empirical proxies. The difference can be due to the illiquidity component that our model does not explicitly consider.[24]

Panel C of Table 4 reports the results conditional on the initial state of the economy with the coupon level being optimally chosen. Similar to the results in Panel B of Table 4, for both states of the economy, the leverage ratio for the case with Chapter 11 is lower than that without Chapter 11. Furthermore, because of the countercyclical distress and liquidation costs, the optimal leverage ratio is procyclical

[24]We also observe that the yield spread for the case with Chapter 11 is lower than that for the case without Chapter 11. At first sight, this result can seem unreasonable since the early voluntary filing for Chapter 11 should engender a higher credit spread ex ante. However, this reasoning is based on the assumption that the coupon level is the same for both cases. The model-implied yield spread for the case without Chapter 11 is higher than that without Chapter 11 because the former case engenders a higher optimal leverage ratio than the latter case.

for both the cases with and without Chapter 11. When the initial state of economy is recession, the smaller expected cash flow growth rate and higher cash flow volatility engender a higher default probability than the case with the initial state of economy being boom. Conversely, the procyclical leverage ratio would induce a lower default probability in recession. Thus, it is not surprising that, for the case without Chapter 11, the default probability in recession is lower than that in boom due to the much smaller leverage in recession. The procyclical leverage choice also engenders a lower yield spread in recession for the case without Chapter 11. Thus, with a static capital structure, our model with Chapter 11 also generates more reasonable *conditional* default probabilities and yield spreads at the optimal capital structure.

Figure 5 illustrates how the presence of distress cost and the conflict of interests changes the firm's ex-ante capital structure choice. Panel A shows two optimal debt levels for the case without the conflict of interests. With liquidation cost only, the optimal debt level is at D^*_{old} when the firm's marginal tax benefit (slope of the dash-dotted curve) equals the marginal cost of debt (slope of the dotted curve). Essentially, D^*_{old} illustrates the leverage choice for the case in absence of pre-liquidation distress cost, a conventional scenario adopted in most capital structure studies and structural model literature.[25] Introducing distress cost enhances the cost of debt and shifts the optimal debt level leftwards from D^*_{old} to D^*_{new}. Panel B shows that the presence of the conflict of interests arising from the voluntary filing option would further shift

[25]See Graham(2000), Molina (2005), Almeida and Philippon (2007), Leland (1994), Chen (2010) , BKS (2010), and so on.

the optimal debt level leftwards to D^{**}. Intuitively, the voluntary filing option allows management to default at an early stage, which is suboptimal for the firm. The suboptimal filing causes the firm to incur a higher distress cost and to lose part of the tax benefits simultaneously. Both effects lead to a lower optimal leverage ratio for the firm. The countercyclical state prices endogenously generated in our model further amplifies these two effects.

To appreciate these ingredients and to permit comparability with the previous studies we also compute the present value of these expected losses (costs of debt) for the BAA-rated firm. We find that the losses amount to circa 3.4% of the initial firm value. This loss is higher than the one generated in a model that accounts for macroeconomic conditions but considers only liquidation costs (e.g. Chen (2010)). It is more than three times the one generated in a model that does not account for macroeconomic conditions (see Elkamhi et al (2009)). Thus, conflict of interests and countercyclical distress costs are crucial to generating reasonable expected default losses that can counterbalance the expected tax benefit of debt.

Table 5 reports the unconditional default probabilities and credit spreads for the six credit ratings (AAA, AA, A, BAA, BA and B). To obtain the expected cash flow growth rate and volatility for each credit rating other than BAA, we adjust the expected cash flow growth rate and volatility in Table 1 by the same proportion so that the model-implied leverage ratio matches the empirically observed leverage ratio (reported in Table 3) As well as three additional momements. Our calibration methodology is exactly similar to Huang and Huang (2003).[26] Panel A reports the results implied by our model when the Chapter 11 option is available, Panel B reports the results implied from the model without Chapter 11, and to ease comparison with observable proxies, Panel C lists the empirical data for the credit rating classes used in this exercise. As shown in Panel A of Table 5, after fitting the empirically observed leverage ratios, our model simultaneously generates reasonable default probabilities and credit spreads for investment-grade firms (AA, A and BAA). Furthermore, the risk adjustment is close to that in the empirical studies in the literature. For the BAA-rated firm, our model generates a risk adjustment of 3.6 for a 10-year horizon. More importantly, our model generates risk adjustments that are decreasing with credit quality. This is consistent with the empirical findings in Berndt et al (2009) and Elkamhi and Ericsson(2009). This is an intuitive result since highly rated firms are likely to default in bad state of the economy, which implies a higher risk adjustment for each unit of objective default probability.

We find that our model does not generate closer default probabilities for the short maturities and underestimates slightly the credit spreads for the junk firms (BA and B). One reason might be that we do not consider the liquidity component of the spread. However, this is beyond the scope of this study. It is worth noting that regardless of this weakness, when comparing Panel A and Panel B of Table 5, we see that the default probabilities generated by our model are higher and closer to

[26]We refer the readers to that paper for the details. To save space, we do not report it here.

the historical default rates than those generated by the model without Chapter 11.

1.3.5 Alternative to Chapter 11: Debt-equity Swap

As shown in Figures 2 and 3, voluntary filing for Chapter 11 leads to a reduction in firm value in both states of the economy, and the loss in firm value is more pronounced in recession than in boom. In this subsection, we study one hypothetical alternative to Chapter 11: debt-equity swap, which is a simple reduced-form representation of the excellent suggestion provided in the Squam Lake report. In the theoretical framework of the debt-equity swap, debtors and creditors negotiate how to split the firm. At an endogenously determined trigger point, the creditors are offered a proportion of the firm's equity to replace their original debt holdings. Following Fan and Sundaresan (2000), we model the distressed debt-equity swap with a simple Nash equilibrium under the assumption that the negotiation process is costless. At the debt-equity swap, the sharing rule of the firm value between the debtors and the creditors depends on exogenously specified bargaining power of the claimants, which is between 0 and 1. In Supplementary Appendix I, we provide closed-form solutions for the firm's security values in both states of the economy. Below, we first discuss the ex-ante economic implications of debt-equity swap in different macroeconomic conditions. Second, we present the effect of business cycle risk and shareholder bargaining power on the yield spreads in the debt-equity swap.

Panel A of Figure 6 shows how the firm value in debt-equity swaps changes with the shareholders' bargaining power. In the figure, the values in the vertical axis are relative to (or normalized by) the firm value for the first-best case with the Chapter 11 option (or the Chapter 11 filing time is chosen to maximize the ex-ante firm value). We observe that for any shareholder bargaining power, the firm value in debt-equity exchange is always higher than that for the first-best case with the Chapter 11 option. And, this increase in the firm value is more pronounced in recession than in boom and decreases with the shareholders' bargaining power in the negotiation process of the debt-equity swap. In the debt-equity swap, creditors prevent the liquidation of the firm by exchanging their debt for equity, and the firm does not suffer from the cost of the financial distress in the Chapter 11 process. Thus, the firm value in the debt-equity swap is higher than that for the case with Chapter 11. In fact, firm value increases more in recession since the possible liquidation and financial distress of the Chapter 11 option are more costly in contracted economic conditions. The more the shareholder bargaining power, the earlier shareholders propose the debt-equity swap since shareholder benefit is proportional to the firm value at the debt-equity swap time. However, the relatively early debt-equity swap is suboptimal for the firm since the firm as a whole loses the future tax benefit after the swap.

Different from that in Fan and Sundaresan (2000), the debt-equity-swap model in this paper allows us to study the effects of business cycles and investors' preferences. In Panels B and C of Figure 5, we illustrate the sensitivity of the yield spread to the business cycle risk and the aggregate investor's elasticity of intertemporal substitution. In Panel B, we fix the initial state of the economy to be boom, and normalize the yield spreads by those generated from the case in absence of variation in the state of the economy. For all the levels of shareholders' bargaining power, the yield spreads for the case with business cycle risk are higher than those for the case with fixed state of the economy. In the presence of business cycles, the pricing kernel incorporates the intertemporal risk due to the change in the economic states. The countercyclical marginal utilities and countercyclical liquidation costs make the present value of default losses with business cycles to be higher than for the case with the state of the economy being fixed as boom. Hence, business cycle risk induces creditors to charge a higher yield spread ex-ante. Panel B also shows that the yield spread increases with shareholders' bargaining power. Our results in Panel B suggest that given countercyclical shareholder bargaining, incorporating business cycles would further widen the yield spread. Panel C shows the effect of EIS on the yield spread for the case with business cycle risk. For each state of the economy, we normalize the yield spreads in the Y-axis by that for the case with a 1.5 EIS. We see that the yield spread increases with EIS in both states of the economy and is higher in recession than in boom. A higher EIS means a lower risk-free interest rate, which increases the present value of default losses more than the present value of coupon payments. Thus, creditors charge a higher yield spread for the case with a higher EIS. Furthermore, because of the countercyclical liquidation costs (hence, default losses), creditors require a higher yield spread in recession than in boom periods.

1.4 Conclusions

We propose a consumption-based structural equilibrium model to evaluate the firm's contingent claims in the presence of both business cycles and realistic features of the US bankruptcy code. Our model allows the long-run variations in growth rates to influence the timing of default and liquidation, strategic debt service, equity premium, credit spreads and the firm's capital structure decisions.

We find that when the timing of default is determined in concordance with total firm value maximization, Chapter 11 benefits the firm as a whole in both states of the economy and this benefit is higher in recessions than in good economic times. However, in the common case of voluntary filing for Chapter 11, shareholders file early to obtain debt relief. This creates potential conflict of interests between debtors and creditors and leads to a loss in the ex-ante firm value. Our model generates twice as much ex-ante reduction in firm value as does a model that ignores changes in economic conditions. We also find that the induced ex-ante firm value reduction in recessions is higher than in normal times. This is alarming since it is in recessionary periods that both economists and policy makers want the bankruptcy code to augment the participation incentives of debtors and creditors.

We provide closed-form solutions for the firm's contingent claim values before filing for Chapter 11 and during the reorganization period while considering a Nash equilibrium sharing rule in the reorganization plan confirmation. We also provide closed-form solutions for the firm's levered equity and debt in a setup in which debtors and creditors consider a simplistic debt-to-equity exchange as an alternative to common Chapter 11 features. The absence of the proportional distress and liquidation costs incurred during Chapters 11 & 7 renders the swap option a better mechanism to resolve distress. However, this setup fosters agency problems, in particular, information asymmetries. It would be worthwhile extending our model to account for these two frictions.

We propose a methodology to estimate the strategic debt service, the proportional cost of financial distress and the cost of liquidation from observed empirical target moments. Our estimation reveals that both distress and liquidation costs are, indeed, countercyclical and that distress costs are higher than those used in the literature. In addition, it is important to account for the reorganization option and countercyclical distress costs to explain simultaneously the equity premium, the credit spread and leverage puzzles. However, our model does not generate reasonable slope of the term structure of objective default probabilities and countercyclical leverage ratios for the aggregate firm. Our results are based on a static model and can be viewed as if our firms are always at the point of refinancing. Our model could be improved by extending our simplistic framework to accommodate a dynamic capital structure and multiple classes of debt. Further, it is interesting to examine the implication of DIP financing on the ex-ante firm value, the likelihood of emergence from Chapter 11 and dynamic capital structure. We leave these interesting questions to future work.

Table 1.1: Parameter Values of the Model.[1]

Panel A			
Parameter	Symbol	State 1	State 2
Expected consumption growth	g	0.0079	0.0236
Consumption growth volatility	σ_C	0.0331	0.0273
Expected earnings growth	θ	-0.0201	0.0389
Earnings growth volatility	σ_X^s	0.1901	0.1701
Probability of switching	λ	0.4892	0.2812
Panel B			
Parameter	Symbol	State 1	State 2
Tax rate	η	0.15	0.15
Recovery of firm value at liquidation	α	0.7	0.9
Grace period	d	2	2
Cost of reorganization	ω	0.03	0.01
Strategic debt service	ϑ	0.3	0.3

[1] This table reports the parameter values used in the empirical analysis. Panel A contains parameter values calibrated to quarterly real non-durable goods plus service consumption expenditure from the Bureau of Economic Analysis and quarterly nonfinancial firms' earnings data from the Compustat database. Panel B contains the remaining parameter values.

Table 1.2: Sensitivity Analysis of the Ex-ante Firm Value Losses on the Parameter Value Choices.[1]

benchmark	mean	std. dev
Leland (1994)	5.3%	1.9%
First-best: recession	3.1%	1.4%
First-best: boom	2.2%	0.9%
BKS (2010): recession	2.2%	0.8%
BKS (2010): boom	1.6%	0.6%

[1] This table reports the sensitivity of the ex-ante losses in firm value on six parameter values: liquidation cost and cost of financial distress in both states of the economy, strategic debt service, and effective tax rate. We set the range for each parameter value to be from 20% below its value used in the base calibration to 20% above that number. We break the range for each parameter into five equal elements and choose one of the six possible values for each analysis. In total, we obtain 46656 ($= 6^6$) results on the ex-ante firm value losses. We report the mean and standard deviations of the 46656 numbers with respect to each benchmark model.

Table 1.3: Empirical Data on Historical Default Rates, Leverage Ratios and Yield Spreads.[1]

	AAA	AA	A	BAA	BA	B
Default probability (%)						
5 years	0.107	0.234	0.612	1.824	9.639	24.175
10 years	0.508	0.551	1.752	4.397	18.276	41.088
Leverage ratio (%)	11.54	14.80	28.80	36.70	45.76	65.58
Long term yield spread (b.p.)	71	91	123	220	320	470

[1] This table reports the empirical evidence on default probabilities, leverage ratios and long-term yield spreads. We obtain the historical default rates for 1970-2008 from the Moody's report by Cantor, Emery, Matos, Ou and Tennant (2009), and the leverage ratios from Davydenko and Strebulaev (2007). We calculate the long-term yield spreads for both BAA and AAA ratings according to the Fed research data (30 years maturity). Since the Fed research data only cover BAA and AAA ratings, we obtain the yield spreads for other credit ratings from Huang and Huang (2003) (10 years maturity).

Table 1.4: Model-implied Default Probabilities, Yield Spreads and Optimal Capital Structure for BAA-rated firms.[1]

Panel A: Results with the Same Exogenous Coupon				
	with Chapter 11		without Chapter 11	
	Recession	Boom	Recession	Boom
Objective default probability (%)				
5-year	0.52	0.30	0.13	0.064
10-year	5.07	3.87	2.59	1.89
Risk adjustment				
5-year	3.38	3.43	4.23	4.38
10-year	2.94	3.05	3.67	3.79
Perpetuity yield spread (b.p.)	284.94	254.84	256.25	238.17

Panel B: Unconditional Results with the Optimal Coupon			
	with Chapter 11	without Chapter 11	Data
Objective default probability (%)			
5-year	0.36	0.27	1.82
10-year	4.25	3.86	4.39
Risk adjustment			
5-year	3.45	3.63	3
10-year	3.03	3.21	3
Perpetuity yield spread (b.p.)	243.79	267.03	220
Leverage ratio (%)	39.67	43.94	36.7

Panel C: Conditional Results with the Optimal Coupon				
	with Chapter 11		without Chapter 11	
	Recession	Boom	Recession	Boom
Objective default probability (%)				
5-year	0.48	0.30	0.23	0.29
10-years	4.94	3.87	3.5	4.06
Risk adjustment				
5-year	3.46	3.43	3.96	3.48
10-year	2.99	3.05	3.37	3.13
Perpetuity yield spread (b.p.)	247.34	241.51	260.57	270.59
Leverage ratio (%)	38.90	40.09	40.54	45.81

[1] This table reports the model-implied objective default probabilities, risk adjustments over 5- and 10-year horizons, yield spread and leverage ratio for a BAA-rated firm. The risk adjustment is calculated as the ratio of the risk-neutral default probability over the objective default probability. The yield spread is calculated as the difference between yield on BAA-rated perpetuity bond and yield on risk-free perpetuity bond. The leverage ratio is calculated as the ratio of the initial market value of debt over the initial market value of the firm. Panel A reports the results conditional on the initial state of the economy with the same coupon level. Panel B shows the unconditional results (or weighted average of the results for both states of the economy) for the cases with Chapter 11 (column 2) and without Chapter 11 (column 3) when the coupon level is endogenously chosen to maximize the firm value. The empirical data (column 4) are from Table 2. Panel C shows the results conditional on the initial state of the economy with optimal coupon level being chosen. All the parameter values are listed in Table 1.

Table 1.5: Default Probabilities and Yield Spreads for Different Credit Ratings.[1]

Credit rating	Objective default probability (%)		Risk-neutral default probability (%)		Yield spread (b.p.)
Panel A: Model-implied Results for the Case with Chapter 11					
	5-year	10-year	5-year	10-year	
AAA	0	0.04	0.007	0.38	82.53
AA	0.003	0.34	0.02	2	97.83
A	0.06	1.71	0.26	6.47	134.44
BAA	0.31	4.17	1.71	15.08	232.83
BA	2.9	13.12	7.79	28.08	292.13
B	11.77	26.1	21.69	44.55	421.33
Panel B: Model-implied Results for the Case without Chapter 11					
Credit rating	Objective default probability (%)		Risk-neutral default probability (%)		Yield spread (b.p.)
	5-year	10-year	5-year	10-year	
AAA	0	0.005	0	0.07	78.56
AA	0.0002	0.09	0.002	0.71	95.1
A	0.012	0.79	0.08	3.72	140.43
BAA	0.24	3.78	0.9	11.46	225.87
BA	1.2	8.89	3.12	22.35	272.2
B	2.9	13.22	7.15	29.36	360.5

Panel C: Empirical Data

Credit rating	Historical default rate (%)		Yield spread (b.p.)
	5-year	10-year	
AAA	0.107	0.508	71
AA	0.234	0.551	91
A	0.612	1.752	123
BAA	1.824	4.397	220
BA	9.639	18.276	320
B	24.175	41.088	470

[1] This table reports the objective and risk-neutral default probabilities over 5- and 10-year horizons and perpetuity yield spreads when we match the leverage ratios for six credit ratings (AAA, AA, A, BAA, BA and B) with the corresponding empirical leverage ratios in Table 2. The yield spread is calculated as the difference between yield on perpetuity bond for each credit rating and that for risk-free perpetuity bond. Panel A shows the results implied from our model when the Chapter 11 option is available. Panel B shows the results implied from our model when the Chapter 11 option is not available. All the parameter values are listed in Table 1.

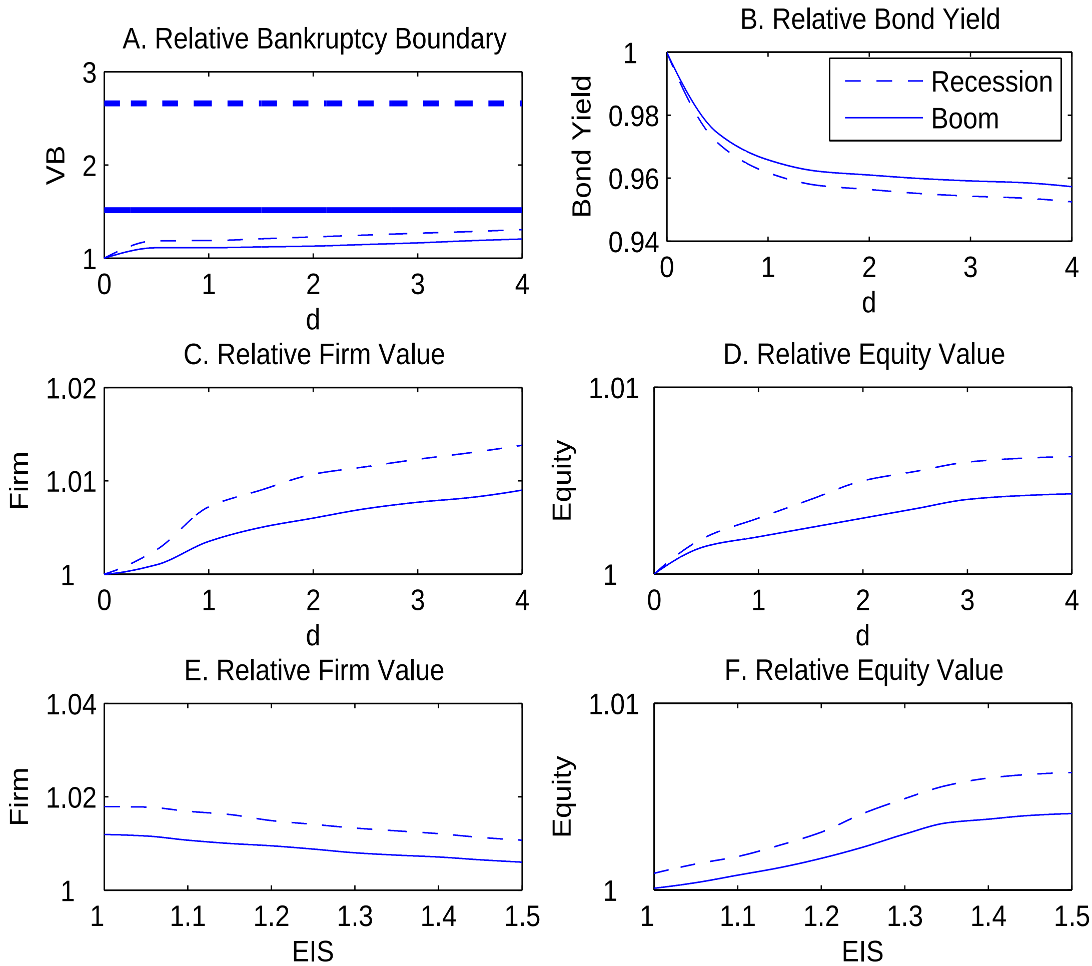

Figure 1.1: Firm Value Maximization with Different Grace Periods (d) and Elasticities of Intertemporal Substitution (EIS).

The six subplots correspond to the case in which Chapter 11 and Chapter 7 boundaries are endogenously determined to maximize the firm's value for the two states of the economy. The values for the Y-axis in each subplot are normalized by (or relative to) the corresponding values from the case without Chapter 11 (similar to BKS (2010) and Chen (2010)). Panel A plots the relative optimal default boundaries for different grace periods. Panel B plots the relative bond yields for different grace periods. Panel C plots the relative firm values for different grace periods. Panel D plots the relative equity values for different grace periods. Panel E plots the relative firm values for different EIS. Panel D plots the relative equity values for different EIS. The bold solid curve in Panel A corresponds to the position at which the firm's EBIT is equal to coupon payment ($X_t = c$) when the state of economy is boom. The bold dashed curve in Panel A corresponds to the position at which the firm's EBIT is equal to coupon payment ($X_t = c$) when the state of economy is recession. The other parameter values are provided in Table 1.

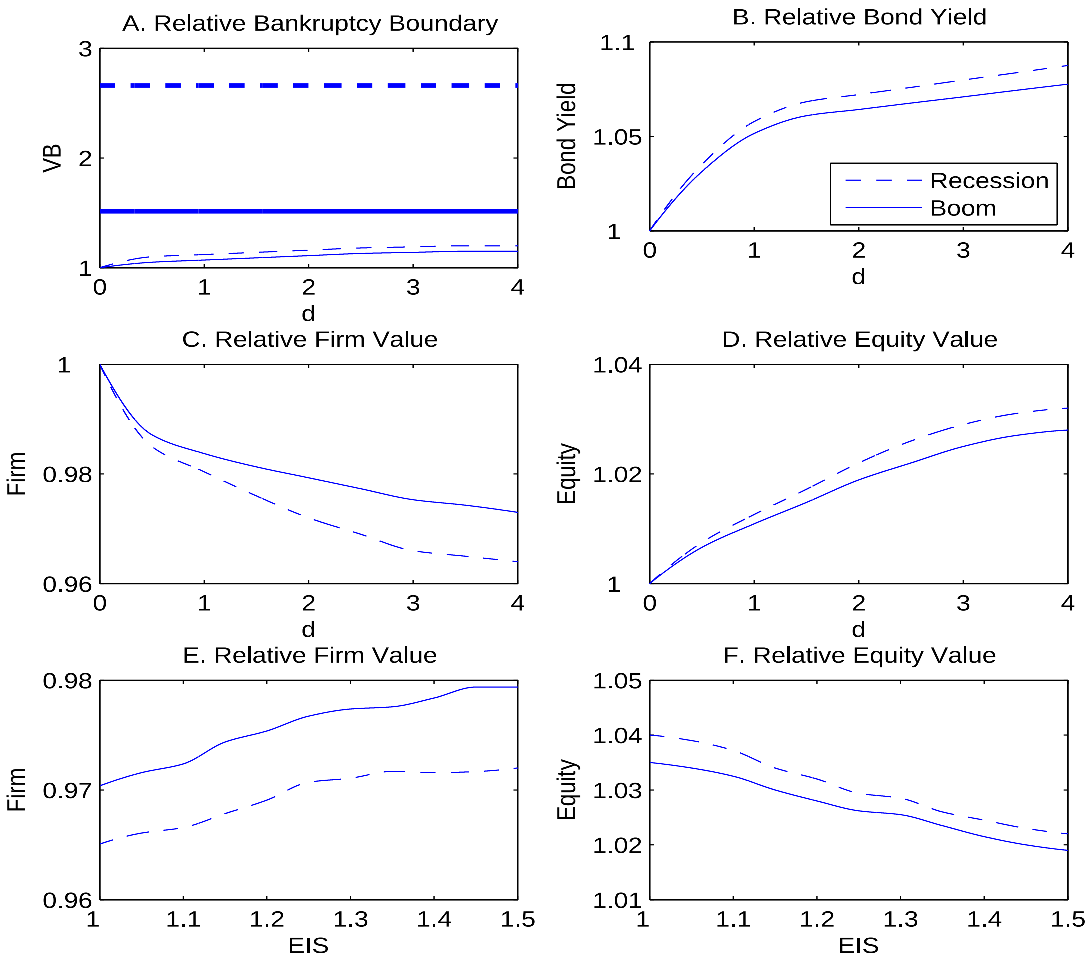

Figure 1.2: The Effect of Voluntary Filing for Chapter 11 for Different Grace Periods (d) and Elasticities of Intertemporal Substitution (EIS).
The six subplots correspond to the case in which Chapter 11 and Chapter 7 boundaries are endogenously determined to maximize the firm's equity value for the two states of the economy. The values for Y-axis in each subplot are normalized by (or relative to) the corresponding values from the firm-value-maximization case with Chapter 11. Panel A plots the relative optimal default boundaries for different grace periods. Panel B plots the relative bond yields for different grace periods. Panel C plots the relative firm values for different grace periods. Panel D plots the relative equity values for different grace periods. Panel E plots the relative firm values for different EIS. Panel D plots the relative equity values for different EIS. The bold solid curve in Panel A corresponds to the position at which the firm's EBIT is equal to coupon payment ($X_t = c$) when the state of economy is boom. The bold dashed curve in Panel A corresponds to the position at which the firm's EBIT is equal to coupon payment ($X_t = c$) when the state of economy is recession. The other parameter values are provided in Table 1.

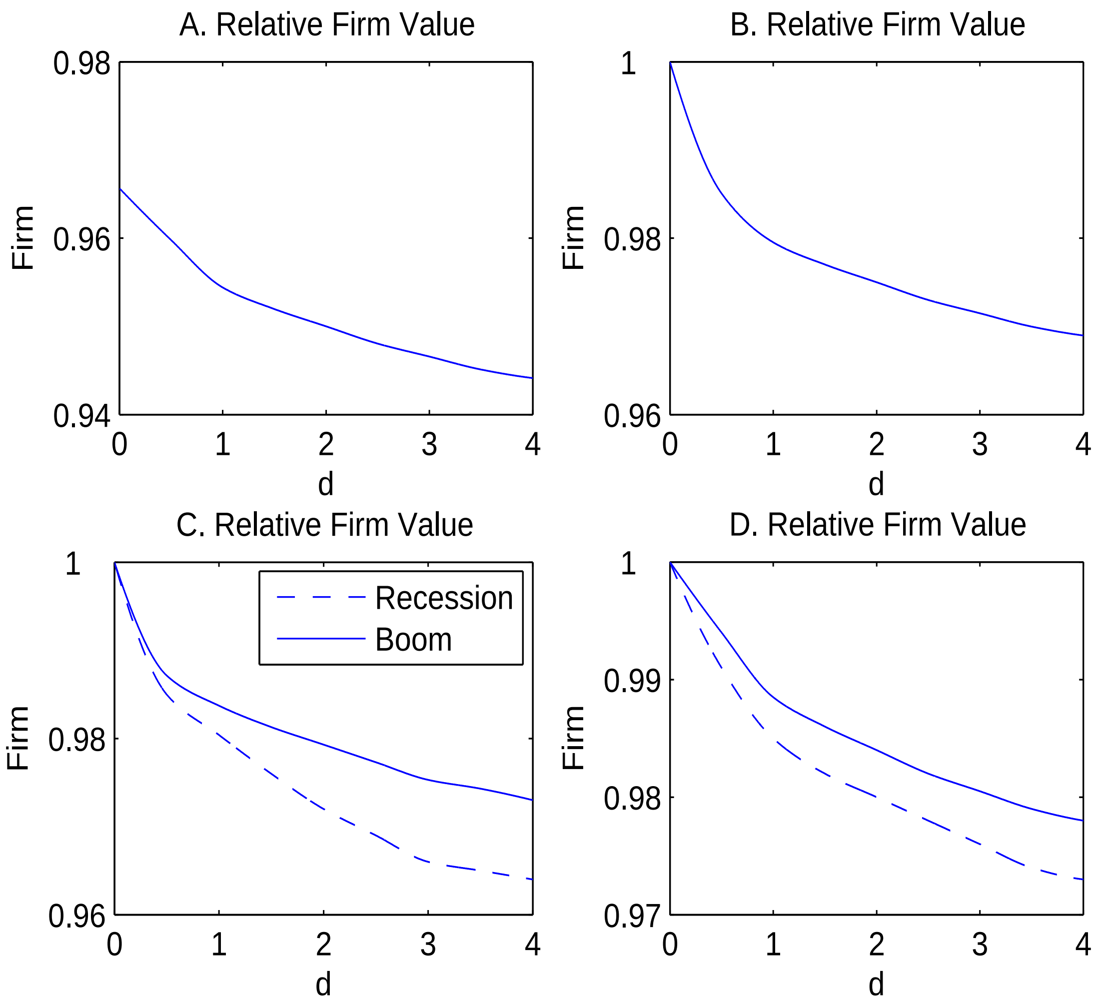

Figure 1.3: Macroeconomic Conditions and the Effect of Voluntary Filing for Chapter 11 on Firm Value for Different Grace Periods (d).

The four subplots correspond to the case in which Chapter 11 and Chapter 7 boundaries are endogenously determined to maximize the firm's equity value. The values for the Y-axis in each panel are normalized by the firm values from the benchmark case, which is different in each panel. Panel A plots the relative firm values from our model (voluntary filing for Chapter 11) relative to the Leland (1994) model as a benchmark. Panel B plots the firm values in a model that consider Chapter 11 voluntary filing without macroeconomic consideration (the BCS (2007) model) relative to the same benchmark as in Panel A. Panel C plots the relative firm values from our model (with voluntary filing for Chapter 11) when the benchmark is the firm-value-maximization case with Chapter 11 and two states of the economy. Panel D plots the relative firm values from our model (with voluntary filing for Chapter 11) when the benchmark is the case without Chapter 11 and with two states of the economy (similar to the static cases in BKS (2010) and Chen (2010)). The other parameter values are provided in Table 1.

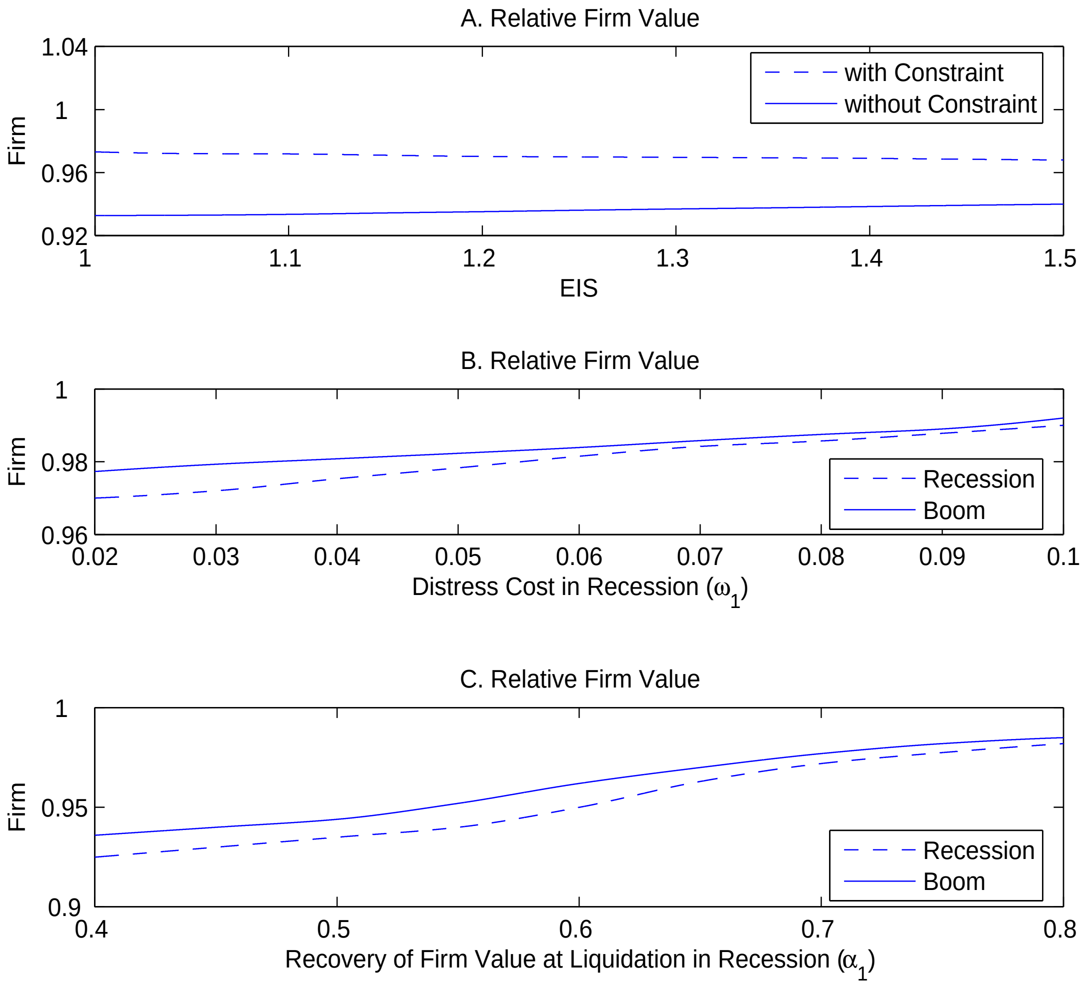

Figure 1.4: Constraint on Debt Value, Distress Costs in Recession (ω_1), Recovery at Liquidation in Recession (α_1) and Effect of Voluntary Filing for Chapter 11 on Firm Value.

The values for Y-axis in each subplot are normalized by (or relative to) the corresponding values from the firm-value-maximization case with Chapter 11. Panel A plots the effect of voluntary filing for Chapter 11 on the firm value with the state of the economy being fixed. The dashed curve illustrates the case when we constrain the debt value at emergence time to be at least equal to the firm's liquidation value. The solid curve illustrates the case when we do not constrain that. Panel B plots the effect of the distress cost in recession on the relative firm values for both states of the economy. Panel C plots the effect of the recovery at liquidation in recession on the relative firm values for both states of the economy. The other parameter values are provided in Table 1.

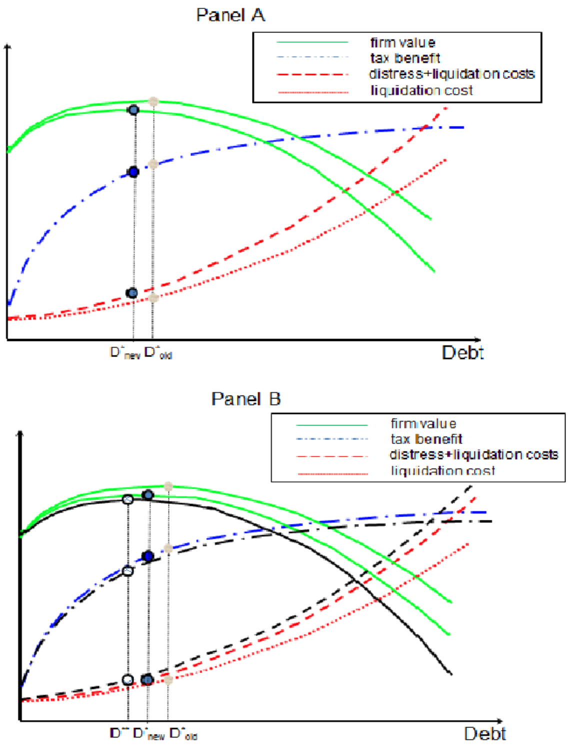

Figure 1.5: Optimal Capital Structure with and without the Conflict of Interests. Panel A illustrates the firm's optimal debt levels without the conflict of interests due to the voluntary filing of Chapter 11. D^*_{old} corresponds to the optimal debt level with liquidation cost only. D^*_{new} corresponds to the optimal debt level with distress and liquidation costs. Panel B illustrates the firm's optimal debt levels for three cases: the two cases without the conflict of interests as in Panel A and the case with the conflict of interests (D^{**}).

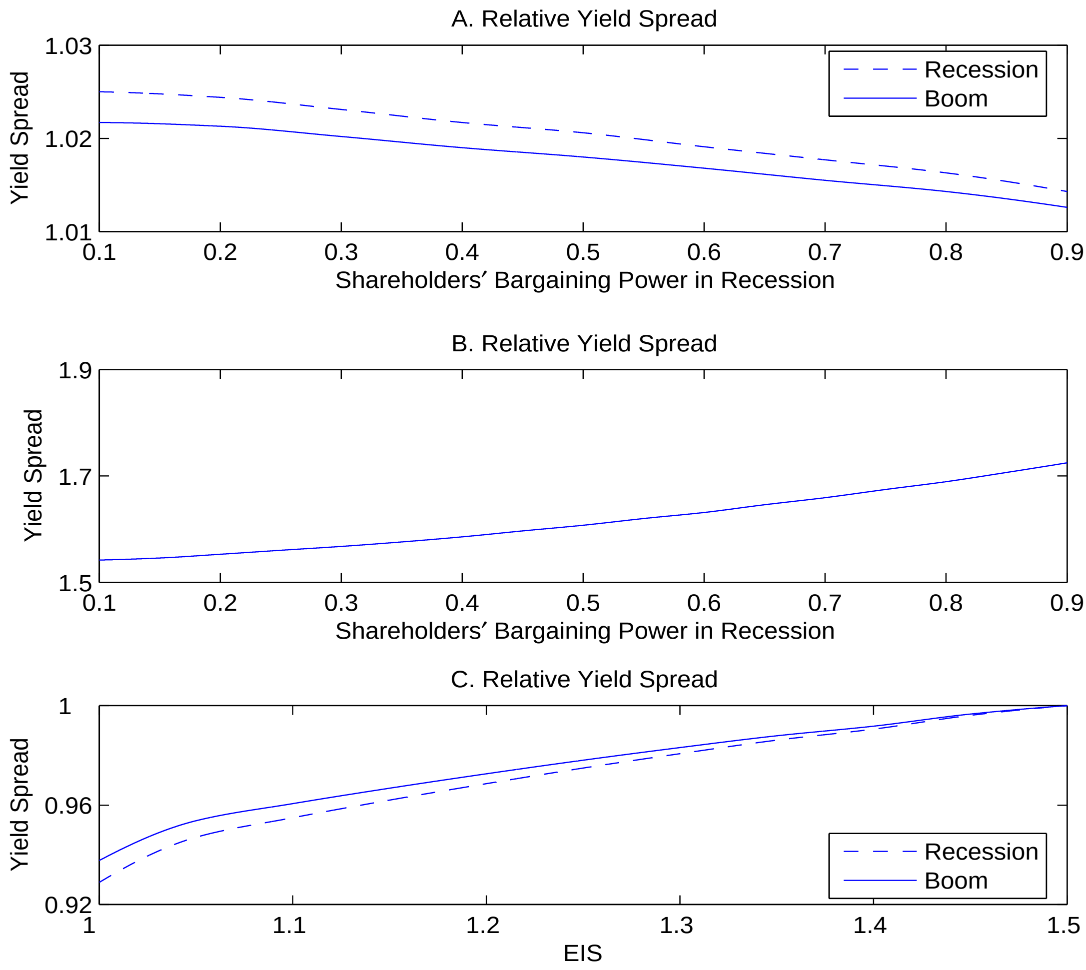

Figure 1.6: Illustration of the Debt-equity Swap Case.

Panel A shows the difference in firm value between the firm-value-maximization case with Chapter 11 and the debt-equity swap case for different shareholders' bargaining power (ζ). The Y-axis is the ratio of the firm value for the debt-equity swap case over the firm value for the firm-value-maximization case with Chapter 11. Panel B shows the effect of macroeconomic conditions and the shareholders' bargaining power on the yield spread for the debt-equity swap case. The Y-axis is the ratio of the yield spread for the case with business cycles over the yield spread for the case without business cycles (or when the state of the economy is fixed). Panel C shows the effect of EIS on the yield spreads for the debt-equity swap case with business cycles. The yield spreads in the Y-axis are normalized by (or relative to) the yield spread when EIS is equal to 1.5. The other parameter values are provided in Table 1.

CHAPTER 2
TIME-VARYING ASSET VOLATILITY AND THE CREDIT SPREAD PUZZLE

2.1 Introduction

Structural credit risk models have met with significant resistance in academic research. First, attempts to empirically implement models on individual corporate bond prices have not been successful.[1] Second, subsequent efforts to calibrate models to observable moments including historical default rates uncovered what has become known as the credit spread puzzle - the models are unable to match average credit spreads levels[2]. Finally, econometric specification tests further document the difficulties that existing models encounter in explaining the dynamics of credit spreads and equity volatilities[3]

In this paper, we develop a structural model with time-varying asset volatility in order to address both the levels and dynamics of credit spreads. Our first contribution is to show that the presence of a variance risk premium resolves the credit spread puzzle in terms of levels. Second, we show that the modelling of stochastic asset volatility allows the model to explain time series of equity volatilities while doing a better job at fitting time series of credit spreads at the individual firm level. Finally, we provide estimates of the size of variance risk premia required to explain

[1]See Jones Mason and Rosenfeld (1985) and Eom Helwege and Huang (2003).

[2]See Huang and Huang (2003)

[3]See Huang and Zhou (2008).

credit spread levels and benchmark these to existing empirical evidence.

The credit spread puzzle is defined as the inability of structural models, when calibrated to default probabilities, loss rates and Sharpe ratios, to predict spread levels across rating categories consistent with historical market spreads. Huang and Huang (2003), hereafter abbreviated HH, perform this calibration analysis for a broad and representative selection of models and find that, as an example, the latter never predict spreads in excess of a third of the observed levels for 4- and 10-year debt issued by A-rated firms. The performance is typically worse for more highly rated firms and somewhat better for low grade firms. [4]

Huang and Zhou (2008) test a broad set of structural models by designing a GMM-based specification test that confronts the models with panels of CDS term structures and equity volatilities. In addition to ranking the models by rejection frequency, their paper provides insights into the specific shortcomings of the models. One important weakness that emerges from their study is the models' inability to fit the dynamics of CDS prices *and* equity volatilities. In particular, the models find it difficult to generate time variation in the equity volatility of the same magnitude as is actually observed, suggesting that an extension to allow for stochastic asset volatility is desirable.

In addition to these two important findings, recent empirical work on default

[4]Chen Collin-Dufresne and Goldstein (2009) argue that if one accounts for time variation in Sharpe ratios over the business cycle, then spreads are more closely aligned with historical averages. Other papers have followed and reinforced the point that macroeconomic conditions can help explain spread levels.

swap spreads provides evidence suggestive of an important role for stochastic and priced asset volatility in credit risk modelling. Zhang Zhou and Zhu (2005) perform an empirical study of the influence of volatility and jumps on default swap prices. Although we abstract from jumps in this paper, their results point to the importance of modelling time-varying volatility.[5] Further evidence is provided in Wang Zhou and Zhou (2010) who show that in addition to volatility being important for the price of default protection, the variance risk premium is a key determinant of firm-level credit spreads. Both these studies provide evidence indicating that a structural credit model with time-varying and priced asset risk may be better poised to explain spreads than its constant volatility predecessors.

In addition to this recent work on credit markets, there is a significant body of literature documenting time variation in equity volatilities. Given this evidence, financial leverage would have to be the sole source of variation in stock return volatility in order for asset volatility to be constant, as it is assumed to be in the majority of structural credit risk models. In fact, recent empirical work by Choi and Richardsson (2009) clearly documents time variability in asset risk as well as a degree of asymmetry at the asset level, which complements the leverage effect generated even in a constant volatility model – equity volatility may increase when stock prices decline mechanically because leverage increases as asset values drop or because asset volatility

[5]The authors include both intra-day realized volatility and historical volatility as measures of short term and long-term volatility, consistent with the notion that equity volatility varies both because of changes in leverage and because of changes in asset volatility. Their results suggest that disentangling the two sources of variation is important for explaining default swap prices.

increases as asset values drop, or both.

Overall, stochastic asset volatility would appear to be a compelling extension to a class of models that has been around for more than thirty-five years. However, likely for technical reasons, it is one that has not garnered much attention.[6] We present, in closed form, solutions to debt and equity prices in stochastic asset volatility model framework where default is triggered by a default boundary, as in Black & Cox (1976), Longstaff & Schwartz (1995) and Collin-Dufresne Goldstein (2001). In doing so, we are, to the best of our knowledge, the first to solve the first passage time problem of stochastic volatility dynamics to a fixed boundary.

We first consider the comparative statics of our benchmark model, which can be thought of as an extension of the Black & Cox (1976) model. This permits us to study the channels through which stochastic asset volatility influences bond yields. The three important determinants of spreads are the volatility of volatility itself, the asymmetry of volatility, and the presence of a volatility risk premium. That the volatility itself is made an idiosyncratic risk source, uncorrelated with the level of a firm's asset value, does impact credit spreads in a model with intermediate default. While it does so in a modest way for longer-term credit spreads, it stands to make a significant impact on spreads for up to ten years to maturity. The same is true for the correlation between shocks to asset values and variances. A modest "leverage

[6] Huang (2005) describes the analytics of such a model, which in its simples form can be thought of as a Heston (1993) model augmented with jumps. In contrast to our model, this framework does not allow for a default threshold permitting default at any time; default occurs only at the maturity of the (zero-coupon) debt.

effect" at the unlevered firm value level leads to higher spreads with a pattern similar to that of the volatility of asset risk: a non-trivial increase in long-term spreads, but an important increase in relative spreads for maturities up to ten years. Finally, the dominant effect stems from the market price of asset volatility risk, which can increase spreads by an order of magnitude.

Of course, comparative statics are limited in that they do not reflect the constraints faced when taking a model to the data. To address this, we rely on the Huang and Huang (2003) calibration setting as a benchmark. This involves requiring the model to simultaneously fit four moment conditions: the historical probability of default, recovery rates, equity risk premia and leverage. We first confirm that, in the absence of stochastic asset risk, our model replicates the credit spread puzzle - that is, it is unable to generate credit spreads in line with historical averages. However, we find that for reasonable parameter values governing volatility dynamics and risk premia, our model resolves or significantly mitigates the underestimation that forms the basis for the credit spread puzzle. In other words, our framework is not subject to an inherent inability to explain historical credit spread levels while matched to the moments used in HH, where the puzzle was first documented.

One potential concern with this result is that several parameters remain free in our exercise.[7] To address this, we conduct an analysis to identify which of the three channels have the ability to significantly impact credit spreads. We find that the only means by which a stochastic asset risk model can influence spreads, given the four

[7]Note that this issue is present also in the Huang and Huang (2003) study.

chosen moments, is through the volatility risk premium. The other two channels - volatility of asset risk and an asset level "leverage effect" - are counteracted by the matching of empirical moments. Hence, it is the size of the volatility risk premium parameter that determines how reasonable our spread estimates are. Since we do not have rating-level data on the volatility risk parameters to use as inputs, we ask instead what level of volatility risk-adjustment is necessary, within each rating group, to match not only the previous four moments, but also the historical spread level. We express our risk adjustment as a ratio that can intuitively be thought of as analogous to the ratio between option-implied and historical equity volatilities. We find that risk-adjusted 3-month volatilities need to be between 20% and 60% higher than their physical measure counterparts to match credit spreads and the other four moments.

In addition, we find an interesting pattern across credit rating groups. Greater proportional risk premia are necessary for higher grade firms. This is consistent with recent findings by Coval Jurek and Stafford (2008) in structured credit markets. They find that although default risk is less important in an absolute sense for senior CDO tranches, systematic risk is extremely important as a proportion of total spreads for these tranches.

Like Huang and Huang (2003) , we find that the implied levels of asset volatilities are higher than historical estimates in the literature and are, in fact, more in line with levered equity volatility levels. To address this, we match our model to six moments: the four Huang and Huang moments, the spread level and the equity volatility. We find that our model with priced systematic volatility risk is able to match these moments quite easily. Although this model implies higher levels of lung run means for the risk-adjusted variances, the quantitative impact on risk-adjustment for the 3 month horizon is limited.

Since the premia may be biased by the presence of non-default components, we carry out one final calibration exercise. It has long been recognized that bond market illiquidity may be an important determinant of spreads. Given the magnitude of the credit spread puzzle documented in previous work, it is unlikely that illiquidity by itself would resolve the puzzle. However, it may well be the case that placing some of the burden of explaining total spread levels on liquidity will generate more accurate implied volatility risk premium levels. When using the level of a AAA short term spread index as a proxy for the level of illiquidity compensation, we find that required risk-adjustments as measured by the ratios of 3 month risk-adjusted to physical volatilities are reduced, in particular for higher grade firms, to a maximum of about 40%.

We then refocus our analysis on the ability of our model to fit the time-series of default swap spreads. We estimate our model firm by firm using GMM, relying on moment conditions matching default swap spreads across five maturities and realized equity volatility. For our sample of 49 firms, we document risk-adjusted mean reversion, a correlation between asset value and volatility shocks of -0.58, and asset risk volatility of 37% on average. The fit for CDS spreads is improved significantly as compared the constant volatility model. In addition, the average pricing errors are overall smaller than in four of the five models studied in Huang and Zhou (2008). While the constant volatility model generates a spread underestimation of 65%, stochastic asset risk reduces this to an underestimation of 10%. Absolute percentage pricing errors are smaller for 34 out of 35 rating / model combinations which they study and are approximately halved in comparison. The model with stochastic volatility could be rejected only for 3 firms out of 49 whereas for a constrained constant volatility version, 48 out of 49 firms lead to a rejection of the model.

This paper is organized as follows: Section 2 describes the model and explains how we derive closed-form solutions for a stochastic volatility credit risk model with fixed default boundary. Section 3 covers the comparative statics, while Section 4 discusses the various calibration exercises. Section 5 reports on our time-series specification tests, and finally Section 6 concludes.

2.2 The Model

We model the firm's unlevered asset value X as the primitive variable. Asset value dynamics can be described by the following two SDEs

$$\frac{dX_t}{X_t} = (\mu - \delta)dt + \sqrt{V_t}dW_1, \tag{2.1}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_2, \tag{2.2}$$

where δ is the firm's payout ratio and $E(dW_1 dW_2) = \rho dt$. Under the risk-neutral measure Q, X_t follows

$$\frac{dX_t}{X_t} = (r - \delta)dt + \sqrt{V_t}dW_1^Q, \tag{2.3}$$

$$dV_t = \kappa^*(\theta^* - V_t)dt + \sigma\sqrt{V_t}dW_2^Q, \tag{2.4}$$

with $\kappa^* = \kappa + \lambda_V$ and $\theta^* = \theta\kappa/\kappa^*$, where λ_V is the volatility risk premium.[8] As asset variance V_t follows Cox-Ingersoll-Ross (CIR) dynamics, the expected asset variance at t under the objective probability measure is, conditional on an initial variance V_0, given by

$$E(V_t) = V_0 e^{-\kappa t} + \theta \left(1 - e^{-\kappa t}\right). \tag{2.5}$$

Under the risk-neutral probability measure, it can be written

$$E^Q(V_t) = V_0 e^{-\kappa^* t} + \theta^* \left(1 - e^{-\kappa^* t}\right). \tag{2.6}$$

In what follows, we provide the solutions for the firm's equity value and equity volatility. To solve for the firm's equity value, we assume that the firm issues consol bonds.

Then the equity value can be written as the difference between the levered firm value (F) and the debt value (D), i.e., $E(X) = F(X) - D(X)$. The firm's levered asset value is given by

$$F(X) = X + \frac{\eta c}{r}(1 - p_D) - \alpha X_D p_D, \tag{2.7}$$

where X, η, c, α, X_D and p_D denote the initial unlevered asset value, the tax rate, the coupon rate, the liquidation cost, the default boundary and the present value of \$1 at default respectively. In equation (D.2.4), the first term is the unlevered asset value, the second term is the tax benefit and the third term is the bankruptcy cost. The

[8]Note we later assume that the market price of volatility risk, $\lambda_V V$, is proportional to the volatility of asset variance, $\lambda_V = k\sigma$, where k is a constant.

debt value is the present value of the coupon payments before default and recovered firm value at default, which is given by

$$D(X) = \frac{c}{r} + \left[(1-\alpha)X_D - \frac{c}{r}\right] p_D. \tag{2.8}$$

Thus, the equity value is given by

$$E(X) = X - \frac{(1-\eta)c}{r} + \left[(1-\eta)\frac{c}{r} - X_D\right] p_D. \tag{2.9}$$

Applying Itô's lemma, we obtain the stochastic process for the equity value as follows:

$$\frac{dE_t}{E_t} = \mu_{E,t} + \frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\sqrt{X_t}dW_{1t} + \frac{1}{E_t}\frac{\partial E_t}{V_t}\sigma\sqrt{V_t}dW_{2t}, \tag{2.10}$$

where $\mu_{E,t}$ is the instantaneous equity return. Given the specification in equation (2.10), we obtain the model-implied equity volatility as

$$\sigma_{E,t} = \sqrt{\left[\left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}\right] V_t}. \tag{2.11}$$

As is clear from equation (2.9), we can solve for the firm's equity value once p_D is found. Under the risk-neutral measure Q,

$$p_D = E^Q[e^{-r\tau}], \tag{2.12}$$

with $\tau = \inf\{s \geqslant 0, X_s \leqslant X_D\}$. To solve for p_D, we need to compute the probability density function of the stopping time τ under measure Q. We solve for the default probability by applying Fortet's lemma.

Recently, Longstaff and Schwartz (1995) introduce this approach into the finance literature to solve for the default probability in a stochastic interest rate setting.

Collin-Dufresne and Goldstein (2001) extend Fortet's equation to the case where the state variables (leverage ratio and interest rate) follow a general two-dimensional Gaussian Markov process. In both the Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001), the state variables are assumed not only to be Markov, but also Gaussian. However this is not the case given our volatility dynamics. In order to apply Fortet's equation to our framework, we first have to solve for the joint probability density of the asset value and asset variance. Next we briefly outline the steps involved.

First, define $z_t = \ln(X_t/X_D)$, a distance to default and $p(z_t, V_t, t|z_0, V_0, 0)$ the transition density function conditional on log asset value being z_0 and asset variance being V_0 at the outset. Further, denote $H(z_\tau, V_\tau, \tau|z_0, V_0, 0)$ the probability density that the first passage time of the log asset value to z_D is τ and the asset variance takes value V_τ at τ. Since z_t and V_t follow a two-dimensional Markov process in the stochastic volatility model, applying the Fortet's lemma, we obtain for $z_0 > z_D > z_t$,[9]

$$p(z_t, V_t, t|z_0, V_0, 0) = \int_0^t d\tau \int_0^\infty dV_\tau H(z_\tau = z_D, V_\tau, \tau|z_0, V_0, 0) p(z_t, V_t, t|z_\tau, V_\tau, \tau). \tag{2.13}$$

The probability density $H(z_\tau, V_\tau, \tau|z_0, V_0, 0)$ is implicit in equation (2.13), which we first discretize and then use a recursive algorithm to solve for numerically. We discretize time T into n_T equal subperiods and define $t_j = j\frac{T}{n_T} = j\triangle t$ with $j\epsilon\{1, 2, \cdots, n_T\}$. Let the maximum and minimum for the asset variance be $\bar{V}$ and $\underline{V}$, re-

[9]One main intuition behind the Fortets lemma is that given a continuous process, if it starts at z_0 which is higher than a fixed boundary (z_D), it has to cross the boundary to reach a point below the boundary (zt).

spectively. We discretize the variance V into n_V equal increments and denote $V_i = \underline{V} + i\triangle V$ with $i\epsilon\{1, 2, \cdots, n_V\}$ and $\triangle V = \frac{\bar{V}-\underline{V}}{n_V}$. Furthermore, we define $q(V_i, t_j) = \triangle t \cdot \triangle V \cdot H(z_{t_j=z_D}, V_{t_j} = V_i, t_j|z_0, V_0, 0)$. Note that $H(z_{t_j=z_D}, V_{t_j} = V_i, t_j|z_0, V_0, 0)$ is the probability density that the default time is t_j and asset variance is V_i at default. Then, the discretized version of equation (2.13) is

$$p(z_{t_j}, V_i, t_j|z_0, V_0, 0) = \sum_{m=1}^{j}\sum_{u=1}^{n_V} q(V_u, t_m)p(z_{t_j}, V_i, t_j|V_u, t_m), \forall i\epsilon\{1, 2, \cdots, n_V\}. \quad (2.14)$$

Given the joint transition density of z_t and V_t, we obtain $q(V_i, t_j)$ recursively as follows:

$$q(V_i, t_1) = \triangle V p(z_{t_1}, V_i, t_1|z_0, V_0, 0),$$

$$q(V_i, t_j) = \triangle V \left[p(z_{t_j}, V_i, t_j|z_0, V_0, 0) - \sum_{m=1}^{j-1}\sum_{u=1}^{n_V} q(V_u, t_m)p(z_{t_j}, V_i, t_j|V_u, t_m)\right], \forall j\epsilon\{2, 3, \cdots, n_T\}.$$

The probability that the default (first passage) time is less than T is given by

$$Q(z_0, V_0, T) = \sum_{j=1}^{n_T}\sum_{i=1}^{n_V} q(V_i, t_j). \quad (2.15)$$

Therefore, given the joint transition density function of z_t and V_t, we can apply Fortet's lemma to solve for the default probability. In the next subsection, we detail the procedure to solve for the joint transition density of z_t and V_t: first, by solving for the joint characteristic function and then using inverse Fourier to back out the transition density.

Define $\Psi(t)$ as the joint characteristic function of (z_T, V_T) conditional on (z_t, V_t) at $t < T$, i.e., $\Psi(t) \equiv E_t^Q\left[e^{i(\varphi_1 z_T + \varphi_2 V_T)}|z_t, V_t\right] = \Psi(\varphi_1, \varphi_2; z_t, V_t, h)$, where $h = T - t$. Zhylyevskyy (2010) shows that $\forall\sigma > 0$, the solution for $\Psi(t)$ is given by

$$\Psi(\varphi_1, \varphi_2; z_t, V_t, h) = e^{f_1(h;\varphi_1,\varphi_2)+f_2(h;\varphi_1,\varphi_2)V_t+i\varphi_1 z_t}, \quad (2.16)$$

where

$$f_1\left(h;\varphi_1,\varphi_2\right)=h\left(r-\delta-\frac{\kappa^*\theta^*\rho}{\sigma}\right)i\varphi_1+\frac{\kappa^*\theta^*}{\sigma^2}\left[h\kappa^*+h\sqrt{G}+2ln\frac{H+1}{He^{h\sqrt{G}}+1}\right], \tag{2.17a}$$

$$f_2\left(h;\varphi_1,\varphi_2\right)=\frac{1}{\sigma^2}\left[\kappa^*-i\rho\sigma\varphi_1-\sqrt{G}\frac{He^{h\sqrt{G}}-1}{He^{h\sqrt{G}}+1}\right], \tag{2.17b}$$

with

$$G(\varphi_1)=\sigma^2(1-\rho^2)\varphi_1^2+(\sigma^2-2\rho\sigma\kappa^*)i\varphi_1+{\kappa^*}^2,$$

$$H(\varphi_1,\varphi_2)=-\frac{i\rho\sigma\varphi_1-\kappa^*-\sqrt{G}+i\sigma^2\varphi_2}{i\rho\sigma\varphi_1-\kappa^*+\sqrt{G}+i\sigma^2\varphi_2}.$$

Let a_1, a_2, b_1 and b_2 be "large" in absolute value. Further, define $\triangle_1=\frac{b_1-a_1}{N_1}$, $\triangle_2=\frac{b_2-a_2}{N_2}$, $\varphi_{j_1}=a_1+j_1\triangle_1$ and $\varphi_{j_2}=a_2+j_2\triangle_2$, where $j_1=0,1,\cdots,N_1$ and $j_2=0,1,\cdots,N_2$. Zhylyevskyy (2010) applies a kernel-smoothed bivariate fast Fourier transformation and obtains the conditional joint density of (z_T,V_T) as follows:

$$f(z_T,V_T;z_t,V_t,h)\cong\frac{1}{4\pi^2}\triangle_1\triangle_2W(\triangle_1z_T,\triangle_2V_T)\sum_{j_2=0}^{N_2}\sum_{j_1=0}^{N_1}e^{-i(z_T\varphi_{j_1}+V_T\varphi_{j_2})}\Psi(\varphi_{j_1},\varphi_{j_2}), \tag{2.19}$$

where

$$W(\triangle_1z_T,\triangle_2V_T)=\int_{a_2}^{b_2}\int_{a_1}^{b_1}e^{-i(\triangle_1z_Tx+\triangle_2V_Ty)}K(x,y)dxdy,$$

and

$$K(x,y)=\begin{cases}\frac{(1-|x|)^2(1-|y|)^2}{x^2y^2+x^2(1-|y|)^2+(1-|x|)^2y^2+(1-|x|)^2(1-|y|)^2}, & \text{if } |x|\leq 1 \text{ and } |y|\leq 1,\\ 0 & \text{elsewhere.}\end{cases}$$

2.3 Comparative Statics

Introducing stochastic volatility into a credit risk model adds three new potential channels for asset risk to influence credit spreads. First, the very fact that volatility is random may impact spreads directly. As we shall see, this is particularly true for short-term credit spreads. Second, volatility may be correlated with shocks to asset value. For example, there may be a "leverage effect" or asymmetry at the unlevered volatility level - that is, the asset risk may increase as the value decreases. This would work over and above the traditional financial leverage effect that is already present in Merton (1974) and subsequent models. Third, volatility risk may be systematic and carry a risk premium that is eventually reflected in credit spreads. In what follows, we address each of these channels. First, we study them in a comparative statics setting. We acknowledge up front the limitations of such an exercise, which does not require the model to match empirical moments. However, it does help crystallize the economic intuition for the different effects. Later, we reconsider the impact of stochastic volatility in a calibration experiment akin to that designed by Huang and Huang (2003).

2.3.1 Stochastic Volatility and Term Structure of Credit Spreads

The most obvious potential channel through which our model can influence credit spreads relative to existing models is the randomness in the volatility itself. Uncertainty about the volatility level generates fatter tails in the asset value distribution, which, all else equal, increases the likelihood of distressed scenarios and thus spreads. Figure 1 demonstrates this effect by retracing the yield spread curves for different levels of the volatility of the asset variance, nesting the constant volatility case, corresponding to the original Black and Cox (1976) model. Panel A plots the yield spread curves in basis points, whereas Panel B plots the ratios of the spread curves relative to the constant volatility case. Note that the effect can be quite significant but is more so for maturities less than 10 years. This is even more noticeable in the lower panel of the figure that reproduces the same data in terms of ratios of spreads to the Black and Cox (1976) case. For maturities less than 7 years, it is quite straightforward for the model to more than double spreads. The relative effect dissipates further out on the term structure and seems to reach stable levels after the 10-year tenor - a spread increases in the range of 10% to 20% of the constant volatility spread.

2.3.2 Asymmetric Asset Volatility and Credit Spreads

Since Black (1976) and Christie (1982), the question remains whether the observed negative correlation between equity prices and equity volatilities is a purely financial effect. More recently, Choi and Richardsson (2009) study firm-level returns and document a degree of negative correlation between asset values and asset volatilities. We now consider the comparative statics of the parameter that governs this asymmetry in our setting, ρ the instantaneous correlation between shocks to asset value and volatility.

Figure 2 visualizes the relationship between asset volatility asymmetry and spreads. The second panel reports ratios of spreads for varying levels of ρ to the spread in the constant volatility level. For this experiment, we make conservative assumptions regarding the volatility of asset risk, setting $\sigma = 0.3$ and $\kappa = 4$. The case where $\rho = 0$ corresponds to a Black & Cox model extended to allow for asset-risk dynamics independent of asset value dynamics. The resulting spreads are barely higher than in the constant volatility case.

Setting the asymmetry parameter to $\rho = -0.3$, which implies positive shocks to asset risk on average when asset values suffer negative shocks, has a limited effect on spreads, in the range of 2-10 basis points. For short term spreads (less than five years), this amounts to a non-trivial relative increase - spreads are approximately doubled. The absolute size of the increase is relatively stable so that for longer tenors such as 15-20 years, the percentage stabilizes around 5%. Increasing the correlation between asset volatility and value shocks to $\rho = -0.6$ provides a more significant boost in spreads. With this level of correlation, 5-year spreads essentially triple as compared to the zero correlation case. At 15 years, spreads increase by about a fifth of the no-asymmetry spreads.

The pattern for the relative spread increases is quite similar to the one reported for the volatility of volatility parameter σ. It seems that the impact of asymmetry might be quantitatively slightly more important than volatility risk itself, although not dramatically. However, given that in a situation where a model is implemented empirically it is faced with matching several moments of the data, this result is limited to a ceteris paribus setting. We will return below to whether this holds in a calibration setting below. In addition, we will estimate the amount of asymmetry that best describes firm specific time series of equity volatilities and default swap spreads.

2.3.3 Asset Risk Premia

Finally, Figure 3 illustrates the impact of volatility risk premia on credit spreads without any asymmetry effect ($\rho = 0$). As can be seen, the effect of the risk premium parameter k is first-order. Spreads can be be increased dramatically by allowing for systematic asset volatility risk. For example, when $k = 3$, spreads more than triple for some maturity segments.

In a relative sense, it is clear from the lower panel of Figure 3 that, like in the case of volatility risk (σ) and asymmetry (ρ), the effect dominates the shorter part of the term structure, up to about 10 years. However, the sheer magnitude of the impact makes the effect significant for all maturities. While short-term spreads can be inflated tenfold, long-term spreads can easily double, if not triple

Obviously, the difficulty at this stage will be to determine reasonable values for the unlevered volatility risk premium. We address this below, and we will see that this third effect of stochastic volatility on credit spreads is in fact the dominant one, and will survive the calibrations to empirical moments.

In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\sigma = 0.3$, $\theta = 0.21^2$.

2.4 Stochastic Asset Volatility and the Credit Spread Puzzle

We have now documented that in a comparative static setting, three channels exist that may have an important effect on credit spreads: the volatility of asset risk itself, a "leverage" effect at the unlevered firm level and risk premia associated with shocks to asset risk. All three effects have the potential to help structural models achieve the levels of credit spreads necessary to address what has recently become known as the credit spreads puzzle. Huang and Huang (2003) calibrate a selection of different structural models to historical default rates, recovery rates, equity risk premia and leverage ratios. They find that all models are consistently incapable of simultaneously matching credit spreads while calibrated to these four moments. Their results are striking since they compare models with quite different features: stochastic interest rates, time-varying leverage, jump risk, counter-cyclical risk premia, endogenous default and strategic debt service. None of these extensions of the basic Merton (1974) framework is able to more than marginally bring market and model spreads closer to each other. This finding forms the basis for the credit spread puzzle.

We now ask whether the three channels through which stochastic asset volatility may influence spreads in our model, can help reconcile model with market spreads on average. In order to do so, we perform a calibration experiment closely following the methodology used in Huang and Huang (2003). In other words, we require our model to match, for different rating categories, the following observables

1. The historical default probability
2. The equity risk premium
3. The leverage ratio
4. The recovery rate

Table 1 reports on this exercise. We assume values for the additional parameters to be $\rho = -0.1, \sigma = 0.3, \kappa = 4$, and $k = 7$. The asymmetry is chosen to be modest as reported by Choi and Richardssson (2009). By means of comparison, in equity markets, Heston (1993) uses $\rho = -0.5$, while Broadie Chernov and Johannes (2009) use $\rho = -0.52$ and Eraker Johannes and Polson (2003) find values for ρ between -0.4 and -0.5. Pan (2002) uses a value for $\lambda_V = k\sigma$ equal to 7.6, while Bates (2006) uses a λ_V equal to 4.7. Our choice of k implies a volatility risk premium $\lambda_V = 2.1$. Bates (2006) documents estimates of κ in the range of 2.8 and 5.9 for a selection of models, whereas Pan (2002) estimates values ranging between 5.3 and 7.

For the Aaa category we are able to explain about 96% of average historical spread levels. For Aa to Baa we explain between 73% and 82% while for the two lowest we actually overestimate spreads by 13% and 24% respectively. This compares to 16% for Aaa, 29% for Baa and a maximum of 83% for B rated firms in Huang and Huang (2003). It is clear from this table that for this set of parameters, we can address the spread underestimation for high and low rating categories and reduce it significantly for the intermediate ones.[10] The exact numbers are sensitive to whether

[10]Note that there is always some value for the variance risk premium that will fit the

we consider 4 or 10 year spreads and we see that the hardest spreads to fit are high grade and short term. Nonetheless, the overall impression remains. Our framework, for reasonable and conservative inputs, does not suffer from the same systematic underestimation problem that all the models studied in Huang and Huang (2003) are subject to.

Unfortunately, we do not have rating-specific estimates of the new parameters related to our stochastic volatility model. We cannot claim that our model is able to match historical spreads given historical moment restrictions, only that it has little difficulty in reaching the required spread levels. To highlight the marginal importance of stochastic volatility in our model, the top panel in Table 2 repeats the calibration exercise in Table 1 with all parameters related to time-varying volatility set to zero ($\sigma = \rho = \kappa = k = 0$). The model thus recovered, corresponding to Black and Cox (1976), behaves very similarly to those studied in Huang and Huang (2003). For high grade bonds, the model cannot explain any significant part of the spread and reaches a maximum of 68% for the lowest grade bonds. This clearly highlights that various aspects of time-varying volatility are at the heart of the improved performance of our model in relation to historical spreads. This stands in sharp contrast to the previous literature.

However, the lack of granularity of our volatility parameter estimates remains. To address this, we carry out a "comparative statics" analysis of the calibration in Table 1. The intent is to understand the relative contribution of each of the three

spread exactly. We return to that exercise below.

parameters to the improved spread fitting ability of the model. As noted above, panel A in Table 2 defines the benchmark, by shutting down stochastic asset volatility altogether. This benchmark corresponds to the credit spread puzzle as presented in Huang and Huang (2003). Panel B opens up for stochastic volatility by setting $\sigma = 0.3$ but without any asymmetry (or "leverage" effect at the asset level) or risk premium for asset volatility risk. In this case, the predicted spreads remain very similar to the constant volatility case and the credit spread puzzle remains.

Thus uncorrelated and fully idiosyncratic asset risk is not the channel that allows our model to generate sufficient yield spreads. This may seem counterintuitive given that the comparative statics discussed above and depicted in Figure 2 appear to permit stochastic asset risk significant leeway in influencing credit spreads. The reason for this perhaps surprising result can be traced back to the design of the calibration experiment: the four moment conditions (default losses and rates, leverage and equity risk premia) work to cancel out the effect of more pronounced tails in the asset value distribution. For a given level of volatility risk, the increased spread that would result in a ceteris paribus exercise is mitigated by the requirement to fit the moments in the calibration. In particular, the model will tend to produce lower asset volatility levels for the high grade scenarios where the underestimation of spreads is the most severe.

Panel C of Figure 2, adds a modest amount of asymmetry to the scenario summarized in panel B. Again, the effect, which was significant in the comparative statics above (see Figure 1), is cancelled out by the requirements to fit the moments in the calibration. Thus a leverage effect at the asset value level with fully idiosyncratic asset risk does not help explain the puzzle documented in Huang and Huang (2003). Finally, Panel D of Figure 2 adds a risk premium to asset volatility. This effect is not constrained by the four moments used in the calibration. The spread explanation percentages increase significantly to between 75% and 128%, which, while not fitting spreads within each rating category accurately, does remove any systematic underestimation of spreads. In summary, it appears that the market price of asset volatility risk is one channel through which a structural credit risk model's ability to explain market spreads can be significantly improved.

So far, we have based our analysis very closely on the HH calibration as it forms the basis for the credit spread puzzle. However, although we have established that our framework is not subject to the limitation of generating insufficient spreads, we still face the problem of risk premium estimation. Although, at this stage, a full-fledged firm level estimation of risk premia is beyond our paper's scope, we will attempt to better understand the required volatility risk premia. In a first step, we simply ask what levels of asset volatility risk premia would be necessary to explain market spreads in the HH calibration.

Table 3 reports our results for an exercise where we augment the moment conditions used in HH by a requirement to also fit historical spreads (in addition to default losses, probabilities, equity risk premia and leverage ratios).[11] Rather than

[11]It is important to note that the two types of risk premia present in this calibration are not distinct. The level of the asset volatility premium impacts the asset return risk premium which is matched to historical equity risk premia. Thus, we are not merely adding a free parameter which trivially fits the new moment condition. All five moment conditions are satisfied simultaneously.

report the parameter k directly, which has no obvious intuitive empirical counterpart, we report λ_V as well as the square root of the ratio of the three month expected risk-adjusted and historical volatilities respectively. This ratio, $\frac{E^Q(V_t)}{E(V_t)}$, is intended to provide a quantity similar in spirit to observable ratios of option-implied and historical equity volatilities.

We find that 3 month risk adjusted volatilities need to be need to be between 22% and 53% higher that their historical counterparts, in order to fit historical credit spread data for ten year bonds across the rating categories. This wedge for 4 years bonds lies between 7% and 54%. The risk premium parameter λ_V ranges from -1.23 to -2.55 for the ten year spreads and between 0.45 and -2.55 for 4 year bonds. We are not aware of any empirical estimates of this quantity for individual firms. As mentionned above, for equity indices, λ_V has been found to be greater in absolute terms (-7.6 in Pan (2002) and -4.7 in Bates (2006)). Given that our estimates are for an unlevered volatility, risk premia should be lower in absolute terms. In addition, it has been shown that measures of implied volatilities tend to be lower relative to historical volatility for individual stocks (see e.g. Carr (2008)).

Another interesting finding that emerges from Table 3 is the pattern of the risk premia across ratings. The required risk premium is higher for the higher grade firms than for speculative grade firms. The ratio of risk-adjusted to historical volatilities is in fact monotonically increasing in credit quality. This implies that higher grade firms are relatively more sensitive to systematic shocks to volatility. A similar point has been made recently for the structured credit markets. Coval Jurek and Stafford

(2009) show that prices in long dated index options markets imply proportionally much higher risk premium components in senior than junior CDO tranches. For single-name securities grouped into credit rating categories, Huang and Huang (2003) document that it is harder to explain higher grade spreads with constant volatility structural models. Berndt et al (2008) and Elkamhi and Ericsson (2008) show that ratios of risk-adjusted to historical default probabilities are indeed increasing in credit quality. In sum, these are consistent with the average economic state in which a highly rated fixed income instrument defaults being worse than the average economic state in which a lower rated security defaults. Along the same line of reasoning, the higher the systematic risk of a firm, the greater the ratio of its risk-adjusted volatility over its historical volatility.

We note that the model fits spreads with limited impact on the most significant free parameter in the calibration we (like Huang and Huang (2003)) use - the asset volatility level. In panel A of Table 2, which corresponds to our version of the HH calibration with constant asset risk, implied asset volatilities range between 25% and 35%. This is comparable to the numbers reported in the base case by HH which range between 25% and 40%. It should be noted that these estimates are in fact quite high. Indeed, they are comparable to the estimates of *equity* volatilities across rating categories reported by Schaefer and Strebulaev (2008) which range from 25% for AAA firms to 42% (61%) for BB (B) firms respectively. In contrast, Schaefer and Strebulaev (2008) report asset volatilites averaging 22% for most rating categories and increasing to 28% for B firms. In other words, when confronted with the four moment conditions used so far, the model remains unable to produce reasonable asset and equity volatilities.

To address this problem, we modify the calibration by requiring that our model also fit historical equity volatilities, leaving the model with a total of six moments to match. Table 4 provides evidence from this modified calibration exercise. Our stochastic volatility model now simultaneously fits

1. default probabilities,
2. recovery rates,
3. leverage ratios,
4. equity risk premia,
5. equity volatilities,
6. and credit spreads.

The model is able to fit the four HH moments in addition to the historical equity volatilities and spreads, while retaining the values previously assumed for ρ and σ, while letting k and κ float. Again, instead of reporting the implied k values, we present the ratio of expected three month volatilities under the risk-adjusted and historical probability measures respectively.

The calibration produces quite reasonable implied asset volatility levels ranging from 21% for rating categories Aaa to Baa and 25% for Ba and 37% for B. These figures are much closer to those reported by Schaefer and Strebulaev although a little higher for B firms. Thus it appears that our model does not need to systematically suggest unrealistically high levels of asset volatility to fit the required moments, in contrast to the constant volatility models studied in HH. Furthermore, the link between ratings and risk-adjustment noted above survives. Ratios of expected volatilities and the risk premium parameter λ_V decrease as credit quality deteriorates. The levels remain similar.

We have thus shown that a stochastic volatility model with priced volatility risk is able to match all the moments in Huang and Huang (2003) in addition to credit spreads and historical equity volatilities for some level of $\frac{\theta^*}{\theta}$, the ratio of long term volatilities under the risk-adjusted and historical probability measures. However, we have so far placed the full burden of explaining spreads beyond a simple structural model on the presence of a variance risk premium. But corporate bond spreads may contain compensation for other risks not captured by our model. A strong candidate missing factor is the illiquidity of corporate bond markets. If illiquidity is an important determinant of bond spreads then our estimates of required volatility risk adjustments may be excessive. To understand how important such an effect might be, we follow Almeida and Phillippon (2007) in their liquidity correction of the Huang and Huang (2003) rating based scenario. We assume that the AAA rated one year yield spread contains negligible compensation for default risk and can be thought of mainly as compensating for illiquidity relative to a one-year government bond. We then substract this yield spread from our previous spreads to obtain rough estimates of default risk only spreads. Clearly this approach is simplistic as the liquidity spread may well depend on the credit rating (see. e.g. Ericsson and Renault (2006) and Xiong and He (2010)). However, the objective here is merely to gauge the quantitative impact on required variance risk premia of a reasonable level of liquidity adjustment, not a precise estimation of risk premia per rating category.

Table 5 reports our findings. There is a negligible impact on the implied asset volatilities. The significant, and expected, effect is to reduce the required variance risk premia. For 10 year bonds, the required ratio of expected variances now ranges from 1.22 to 1.39 (compared to the previous range of 1.23 to 1.56), while the range of lambda is -1.21 to -1.76 (compared to -1.26 to -2.34). The effect is strongest for higher rated bonds, which is intuitive since it is those spreads that are adjusted the most in a relative sense.

Table 6 summarizes the variance risk premium adjustments across ratings, calibration scenarios and risk adjustment metrics. The table reports on the ratio of long term means ($\frac{\theta^*}{\theta}$), ratios of risk-adjusted to historical expected variances with 1 and 3 month horizons, as well as the risk premium parameter λ_V directly. One clear pattern is that the risk-adjustment (as a ratio) depends critically on the horizon of the metric. The ratio $\frac{\theta^*}{\theta}$ has a perpetual horizon as it measures the wedge between the long run mean levels for the variance dynamics under the risk-adjusted and historical probability measures respectively. It tends to be higher than the ratio of expected variances at 3 months, which in turn is higher than that for a one month horizon.

Han and Zhou (2010) find ratios of risk-adjusted variances (measured as one month model-free option implied variances)in a large panel of individual firms to be on average 38% higher than their physical counterparts. This is higher than our typical risk adjustment which at the one month horizon lies between 10% and 20% for 10 year bonds (6% to 15% for 4 year bonds with the liquidity correction). On the other hand estimates of variance risk premia in Carr and Wu (2008), which are estimated using variance swap returns, come in lower at about 4% on average. Although the variance risk premia our model requires to explain the cross-section of historical credit spreads do not appear unreasonable, we leave further empirical work in this direction to future work.

All ratios decrease as credit quality deteriorates, as does λ_V, reflecting, as discussed above, that risk premia are proportionally more important to explain credit spreads for higher grade firms. Requiring the model to fit equity volatility has a positive effect on the required ratio $\frac{\theta^*}{\theta}$ for higher rated firms, while it has little effect on the other metrics.

2.5 Specification Tests - Constant vs. Stochastic Asset Volatility

2.5.1 Data

The CDS spread is the premium paid to insure the loss of value on the underlying realized at pre-defined credit events. This contrasts with the yield spread of a corporate bond, which reflects not only default risk but also the risk-free benchmark yield, the differential tax treatment and liquidity of corporate bonds vs. Treasury bonds. Further, while bonds age over time, CDS spreads are quoted daily for a fixed maturity. In addition, CDS contracts trade on standardized terms and while CDS and bond spreads are quite in line with each other in the long run, in the short run CDS spreads tend to respond more quickly to changes in credit conditions. For all these reasons, it is plausible that the CDS spread is a cleaner and more timely measure of the default risk of a firm than bond spreads. As a result, they may be better suited for specification tests of a structural credit risk model.

We collect single-name CDS spreads from a comprehensive database compiled by Markit. Daily CDS spreads reflect the average quotes contributed by major market participants. This database has already been cleaned to remove outliers and stale quotes. We require that two or more banks should have contributed spread quotes in order to include an observation (Cao, Yu, and Zhong, 2010). The data sample that is available to us include only the firms that constituted the CDX index from January 2002 to March 2008.

Our sample includes US dollar-denominated five-year CDS contracts written on senior unsecured debt of US firms. While CDS contracts range between six months and thirty years to maturity, we use the 1, 3, 5,7 and 10 years only because that are relatively more liquid than other maturities (6 months, 2 years, 20 years)

The range of restructurings that qualify as credit events vary across CDS contracts from no restructuring (XR) to unrestricted restructuring (CR). Modified restructuring (MR) contracts that limit the range of maturities of deliverable instruments in the case of a credit event are the most popular contracts in the United States. We therefore include only US dollar-denominated contracts on senior unsecured obligations with modified restructuring (MR), which also happen to be the most liquid CDS contracts in the US market (Duarte, Young, and Yu, 2007).

Together with the pricing information, the dataset also reports average recovery rates used by data contributors in pricing each CDS contract. In addition, an average rating of Moody's and S&P ratings as well as recovery rates are also included. Following Huang and Zhou (2008) we perform our test on monthly data. Their sample is restricted to 36 monthly intervals because their sample ends in 2004. Instead, we require that the CDS time series has at least 62 consecutive monthly observations to be included in the final sample. Another filter is that CDS data have to match equity price (CRSP), equity volatility computed from (TAQ) and accounting variables (COMPUSTAT). We also exclude financial and utility sectors, following previous empirical studies on structural models. After applying these filters, we are left with 49 entities in our study.

In testing structural models, the asset return volatility is unobserved and is usually backed out from the observed equity return volatility. Traditionally, researchers use a rolling window of daily returns volatility to proxy for equity volatility. In order to benchmark our results to the specification tests of alternative models covered in Huang and Zhou (2008) we use a more accurate measure of equity volatility from high-frequency data. Following Huang and Zhou(2008) we use bi-power variation to compute volatility. As shown by Barndorff-Nielsen and Shephard (2003), such an estimator of realized equity volatility is robust to the presence of rare and large jumps. The data on high frequency prices are provided by the NYSE TAQ (Trade and Quote) data base, which includes intra-day (tick-by-tick) transaction data for all securities listed on NYSE, AMEX, and NASDAQ. The monthly realized variance is the sum of daily realized variances, constructed from the squares of log intra-day 5-

minute returns. Then, monthly realized volatility is the square-root of the annualized monthly realized variance.

2.5.2 GMM Estimation of the Model

Let $cds(t, t+T)$ and $cds^{obs}(t, t+T)$ denote the model-implied and empirically observed CDS spreads of a CDS contract at time t for which the maturity date is $t+T$ respectively. Let $\sigma_{E,t}$ and $\sigma_{E,t}^{obs}$ denote the model-implied and empirically observed equity volatilities at time t respectively. Following the literature, the solution for the model-implied CDS spread is given by

$$cds(t, t+T) = \frac{(1-R)\sum_{i=1}^{4T} B(t, t+T_i)\left[Q(t, t+T_i) - Q(t, t+T_{i-1})\right]}{\sum_{i=1}^{4T} B(t, t+T_i)\left[1 - Q(t, t+T_i)\right]/4}, \tag{2.20}$$

where R is the recovery, $B(t, t+T_i)$ is the default-free discount function and $Q(t, t+T_i)$ is the risk-neutral default probability. As shown in the model section, the model-implied equity volatility is given by

$$\sigma_{E,t} = \sqrt{\left[\left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}\right] V_t}. \tag{2.21}$$

We define $f_t(\Theta)$ as the overidentifying restrictions, which is given by

$$f_t(\Theta) = \begin{bmatrix} cds(t, t+T_1) - cds^{obs}(t, t+T_1) \\ \cdots\cdots \\ cds(t, t+T_j) - cds^{obs}(t, t+T_j) \\ \sigma_{E,t} - \sigma_{E,t}^{obs} \end{bmatrix}, \tag{2.22}$$

where $\Theta = (\rho, X_D, \kappa, \theta, \sigma)$ is the parameter vector to be estimated. [12] The term structure of CDS spread includes five maturities:1, 2, 3, 5, 7 and 10 years. Thus

[12]There are two latent variables in the estimation: asset value and asset variance at each observation date. Following Huang and Zhou (2008), we back out the asset value from the observed leverage ratio, which is defined as the ratio of face value over the asset value. We estimate the initial asset variance in the two steps to be discussed later in this section.

we apply the seven moment conditions (from six CDS spreads and equity volatility) to estimate five parameter values in the GMM test. Given that the model is correctly specified, we obtain that $E\left[f_t(\Theta)\right] = 0$. We define the sample mean of the moment conditions as $g_{\bar{T}}(\Theta) = 1/\bar{T}\sum_{t=1}^{\bar{T}} f_t(\Theta)$, where $\bar{T}$ is the number of time series observations. Following Hansen (1982), the GMM estimator is given by

$$\hat{\Theta} = \arg\min g_{\bar{T}}(\Theta)' W(\bar{T}) g_{\bar{T}}(\Theta), \tag{2.23}$$

where $W(\bar{T})$ is the asymptotic covariance matrix of $g_{\bar{T}}(\Theta)$.[13] With some regularity conditions, the GMM estimator $\hat{\Theta}$ is $\sqrt{T}$ consistent and asymptotically normally distributed given that the model is correctly specified (null hypothesis). The J-statistics is given by

$$J = \bar{T} g_{\bar{T}}(\hat{\Theta})' W(\bar{T}) g_{\bar{T}}(\hat{\Theta}). \tag{2.24}$$

The J-statistics is asymptotically distributed as a Chi-square with the degree of freedom being equal to the difference between the number of moment conditions and the length of Θ, which is equal to one in our setup.

We estimate our model in two steps. First, we set the initial asset volatility ($\sqrt{V_t}$) at each observation date t to be $(1 - leverage_t)\sigma_{E,t}^{obs}$ and then obtain one GMM estimator ($\widehat{\Theta}$) for Θ. In the second step, we obtain the updated asset volatility ($\sqrt{V_t^{update}}$) at each observation date t such that the model implied equity volatility to be equal to the observed one, i.e., $\sigma_E^{model}(V_t^{update}, \hat{\Theta}) = \sigma_{E,t}^{obs}$. Then use V_t^{update} to obtain the updated GMM estimator ($\hat{\Theta}^{update}$) for Θ.

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[13]Following Newey and West (1987), we use a heteroskedasticity robust estimator for $W(\bar{T})$.

2.5.3 Results

Table 8 reports summary statistics for the 49 firms (4116 default swap quotes in total) in our sample which spans the period 2002-2008 and contains firms that are also part of the sample in the Huang and Zhou (2008) study we rely on as a benchmark. This choice of data is intentional to permit a better comparison - so that any differences in our results are more likely to indicate differences across models rather than data sample.

Rating-based averages for equity volatilities range from 22% to 41% and leverage ratios from 26% to 77%. Asset payout rates are also quite similar to those in Huang and Zhou (2008) typically just above 2%. CDS spreads are similar as well. Panel C in Table 8 reports on the standard deviations of CDS spreads.

Table 9 contains the parameter estimates resulting from the GMM implementation. We note first that the degree of volatility asymmetry - that is, the correlation between shocks to asset values and asset variances (ρ) is similar across firms and ratings and averages -0.58. This is similar to values reported in the literature on equity volatilities (see Eraker Johannes and Polson (2003) and the discussion in section 4 above). This is higher than the value we assumed in the comparative statics above (-0.1) and provides evidence of a “leverage” effect at the asset value level. In other words, the asymmetry observed in equity markets stems both from mechanical changes in financial leverage as stock prices fluctuate and the negative correlation between the levels of asset values and volatility. Table 12 converts the asset value asymmetry into an equity leverage effect (Appendix C provides the necessary derivations). We find that for most rating categories, the instantaneous correliton between asset value and variance shocks is lower than for equity and equity variance shocks. However, the magnitude of this difference is small. This suggests that financial leverage only plays a minor role in the asymmetry observed at the equity return level.

The estimated speed of mean reversion under the risk-adjusted probability measure (κ^*) ranges from 0.5 and 1.2 averaged within rating categories, lower than the levels we used in the comparative statics. Given that higher mean reversion speeds will tend to reduce variance volatility, our assumptions in the comparative statics, like those make for the correlation parameter, also appear conservative. The asset variance volatility parameter (σ), is estimated to values in the range of 24% to 45%, averaging 37%, somewhat higher than our choice of parameter value in the comparative statics (30%).

We find that the default boundary is estimated to between 62% and 75% of the book value of debt. This entails that a BBB firm, whose default boundary is 67% of debt, would default at an asset value level of about 32% of its current non-distressed value. This is broadly consistent with estimates in Davydenko (2007) and Warner (1977). Firms often operate at significantly negative net worth levels before defaulting, reflecting the valuable optionality of equity when faced with financial distress. Note that the ratio of the default point to liabilities is smaller (greater) for the lower (higher) grade firms - a B (AA) firm defaults at 75% (62%) of book debt where leverage is around 77% (26%). Thus they would default at an asset value level 42% (84%) lower than current value.

Long run risk-adjusted variance levels are estimated to lie between 3% and 9%, corresponding to volatility levels of 17% and 30% respectively.

The reported mean J-stats in Table 9 are well below the critical values at conventional significance levels. This stands in stark contrast with the findings in Huang and Zhou (2008) who find that almost all the models they study: the Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995) are consistently rejected whereas the Collin-Dufresne and Goldstein (2001) model is rejected in half of the cases. The last two columns of Table 9 report on the number of firms for which our model can be rejected. At the 1% (5%) level only 1 (3) firm out of 49 leads to a rejection of the model.

To provide a more specific benchmark by which to judge these results, Table 10 reports on the same exercise with the stochastic asset volatility channel turned off. These results are similar to the findings of Huang and Zhou (2008) for the same model. Here, the Black and Cox (1976) model can be rejected for 45 out of the 49 firms (compared to at best 87 out of 93, in HZ). Clearly, the addition of stochastic volatility renders the rejection of the model significantly harder. Note that the number of free parameters is greater when we introduce stochastic asset risk, and that this will make a rejection harder. A fairer comparison in this regard is the Collin-Dufresne Goldstein (2001) model (CDG) evaluated in Huang and Zhou (2008), which has the same number of additional parameters with respect to the Black and Cox (1976) model. In that case between and 67% and 75% of the firms lead to non rejections, compared to between 94% and 98% in Table 9. The CDG model yields somewhat lower pricing errors on the defaults swaps but, not surprisingly, faces more resistance in fitting the time series of equity volatilities.

The pricing errors are reported in Panels B and C of Table 9. The first finding is that spreads are underestimated by between 3 (A rated firms) and 61 (B rated firms) basis points with an average of 18. The direction of the bias is reminiscent of the findings across 29 of 35 rating model combinations in Huang and Zhou (2009). However, the level is significantly lower than for the Merton, Black and Cox, Longstaff and Schwartz models as reported by HZ. For the Collin-Dufresne Goldstein model, we have already noted a slightly better performance. For the BB (B) rating categories we underestimate spreads by 43 (171) basis points while Huang and Zhou find -143 (-518) basis points respectively. Moreover the dispersion of the errors is smaller. Huang and Zhou report absolute pricing errors in the range 13 to 1381 basis points across ratings (averaging 101 basis points) for the Black and Cox model. In contrast, we find a range between 7 and 96 basis points with average of 26 basis points.

A better understanding of the findings can be had by comparing to the Black & Cox (1976) model estimated on our sample (by shutting down stochastic asset risk). Our overall average underestimation with stochastic volatility is by 18 basis points (or 10 percent of the average total spread) as compared to 48 basis points (or 65% of the average total spread) with constant volatility. The average absolute pricing error is 26 basis points with stochastic volatility and 50 basis points without. Not surprisingly, the model with stochastic asset volatility does a much better job at fitting the time series of equity volatilities, generating average pricing errors (absolute pricing errors) of 15 (35) basis points as compared to -396 (1085) basis points.

Figure 4 summarizes average model implied and market spreads for the sample by rating groupings. In comparison to Huang and Zhou (2009), these figures provide a much more encouraging summary of the model's performance. Note also that one of the conclusions in Huang and Zhou is that their model finds it hard to fit both CDS *and* equity volatility time series. Our model, of course, matches equity volatilities by construction, but it appears that in doing so, it is also better able to fit the price of default insurance.

2.6 Concluding Remarks

We have developed and studied a first-passage time structural credit risk model with stochastic volatility as a means of addressing the credit spread puzzle documented in Huang and Huang (2003) and further studied in Collin Dufresne Goldstein (2009). We find that, in a comparative static setting, such a model has various ways of generating higher credit spreads than constant volatility models, but in a calibration setting, the key driver of spreads ends up being the volatility risk premium.

Having found that our model is able to generate sufficiently high credit spreads to not be subject to the credit spread puzzle, we consider the levels and patterns of volatility risk premia that are necessary to resolve the puzzle. The levels are quite plausible and the pattern is interesting. For high grade firms, the risk-adjustment needs to be proportionally higher than for lower grade firms. An Aaa firm will likely encounter financial difficulties only subsequent to a massive systematic shock to volatility echoing the findings of Coval Jurek and Stafford (2008). In the context of default swaps and corporate bonds respectively, Berndt et al (2008) and Elkamhi and Ericsson (2008) show that the risk adjustment ratios are indeed increasing in credit quality. Their results indicate that the average economic state in which a highly rated bond defaults is worse than the average economic state in which a lower rated security is likely to default. Similarly, the higher the systematic risk a firm has, the greater the ratio of its risk-adjusted volatility over its objective volatility. This translates to a downward sloping curve which links the risk adjustment ratio to the credit quality.

We extend the calibration method of Huang and Huang (2003) and find that our model, is able to fit their four moments as well as both spread levels *and* historical equity volatility levels quite easily, something earlier models have been incapable of.

Having thus evaluated the cross sectional properties of spreads implied by our model, we proceed to also study the ability of our model to explain jointly dynamics of credit spreads and equity volatilities, a task which has been shown to be out of the reach of constant volatility structural credit risk models. By construction, our model fits equity volatilities well while the fit for CDS prices is much improved relative to the findings for constant volatility models studied in Huang and Zhou (2009). In addition, this exercise provides interesting empirical evidence on the dynamics of firms' unlevered assets. We find evidence of a significant non-financial leverage effect - asset value and variances shocks are significantly negatively correlated.

The technical contribution of our paper, closed form analytics for a first passage time stochastic volatility model has many obvious applications in the credit risk literature. More generally, we believe there are numerous applications in the real options literature, where investment and volatility are closely related.

Table 2.1: Calibrated Yield Spreads for Different Credit Ratings and Maturities.[1]

Panel A: Maturity = 10 years							
	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.3	60.3	63	95.7
Aa	21.2	5.60	0.99	27.4	67.5	91	74.2
A	32.0	5.99	1.55	24.8	89.4	123	72.7
Baa	43.3	6.55	4.39	25.3	159.8	194	82.4
Ba	53.5	7.30	20.63	32.3	361.2	320	112.8
B	65.7	8.76	43.91	39.8	584.7	470	124.4

[1] This table reports the calibration results for the model in which the firm value follows the stochastic volatility process. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$.

Table 2.1 continued

Panel B: Maturity = 4 years

Credit Rating	Target: Leverage Ratio (%)	Target: Equity Premium (%)	Target: Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.04	35.9	20.9	55	38.0
Aa	21.2	5.60	0.23	33.4	53.5	65	82.3
A	32.0	5.99	0.35	28.9	70.3	96	73.2
Baa	43.3	6.55	1.24	28.5	150.1	158	95.0
Ba	53.5	7.30	8.51	34.7	434.3	320	135.7
B	65.7	8.76	23.32	40.5	796.3	470	169.4

Panel C: Maturity = 1 year

Credit Rating	Target: Leverage Ratio (%)	Target: Equity Premium (%)	Target: Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aa	21.2	5.60	0.03	53.5	12.4		
A	32.0	5.99	0.01	41.4	3.5		
Baa	43.3	6.55	0.12	40.7	57.8		
Ba	53.5	7.30	1.29	45.1	303.1		
B	65.7	8.76	6.47	50.1	944.8		

Table 2.2: Calibrated 10-year Yield Spreads for Different Credit Ratings and Parameter Values.[1]

Panel A: $\rho = 0$, $\sigma = 0$, $\kappa = 0$ and $k = 0$

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.9	0	63	0
Aa	21.2	5.60	0.99	28.1	0	91	0
A	32.0	5.99	1.55	25.0	5.9	123	4.8
Baa	43.3	6.55	4.39	24.7	32.9	194	16.9
Ba	53.5	7.30	20.63	30.1	148.3	320	46.3
B	65.7	8.76	43.91	35.2	320.8	470	68.3

[1] This table reports the calibration results for different parameter values of the stochastic volatility model. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean.

Table 2.2 continued

Panel B: $\rho = 0$, $\sigma = 0.3$, $\kappa = 4$ and $k = 0$

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.5	0	63	0
Aa	21.2	5.60	0.99	27.6	0	91	0
A	32.0	5.99	1.55	24.9	3.7	123	3.0
Baa	43.3	6.55	4.39	25.0	31.8	194	16.4
Ba	53.5	7.30	20.63	31.4	151.1	320	47.2
B	65.7	8.76	43.91	38.6	341.1	470	72.6

Panel C: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 0$

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.2	0	63	0
Aa	21.2	5.60	0.99	27.4	0	91	0
A	32.0	5.99	1.55	24.6	3.2	123	2.7
Baa	43.3	6.55	4.39	24.8	31.0	194	16.0
Ba	53.5	7.30	20.63	31.2	149.8	320	46.8
B	65.7	8.76	43.91	38.5	340.3	470	72.4

Panel D: $\rho = 0$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.6	61.7	63	97.9
Aa	21.2	5.60	0.99	27.7	69.1	91	75.9
A	32.0	5.99	1.55	25.1	91.8	123	74.6
Baa	43.3	6.55	4.39	25.3	157.1	194	81.0
Ba	53.5	7.30	20.63	32.8	371.3	320	116.0
B	65.7	8.76	43.91	40.6	599.3	470	127.5

Table 2.3: Calibrated Market Price of Volatility Risk for Different Credit Ratings and Maturities.[1]

Panel A: Maturity = 10 years							
	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Average Yld. Spread (bps)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	63	31.6	1.534	-2.55
Aa	21.2	5.60	0.99	91	27.4	1.503	-2.43
A	32.0	5.99	1.55	123	24.1	1.494	-2.40
Baa	43.3	6.55	4.39	194	25.3	1.486	-2.37
Ba	53.5	7.30	20.63	320	32.3	1.353	-1.83
B	65.7	8.76	43.91	470	39.8	1.222	-1.23

Panel B: Maturity = 4 years							
	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Average Yld. Spread (bps)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	55	35.6	1.535	-2.55
Aa	21.2	5.60	0.23	65	33.7	1.447	-2.22
A	32.0	5.99	0.35	96	28.6	1.436	-2.17
Baa	43.3	6.55	1.24	158	28.5	1.425	-2.12
Ba	53.5	7.30	8.51	320	34.6	1.285	-1.53
B	65.7	8.76	23.32	470	40.5	1.074	-0.45

[1] This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread(columns 2-4). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, and $\kappa = 4$.

Table 2.4: New Calibration Results for Different Credit Ratings and Maturities.[1]

Panel A: Maturity = 10 years

Credit Rating	Target: Lev. Ratio (%)	Target: Equity Prem. (%)	Target: Cumulative Def. Prob. (%)	Target: Average Yld. Sprd (bps)	Target: Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	63	24.5	21.6	1.560	-2.34
Aa	21.2	5.60	0.99	91	26.5	21.3	1.511	-2.14
A	32.0	5.99	1.55	123	28.5	21.3	1.508	-2.16
Baa	43.3	6.55	4.39	194	31.8	21.5	1.452	-1.92
Ba	53.5	7.30	20.63	320	42.7	25.8	1.387	-1.68
B	65.7	8.76	43.91	470	53.8	37.6	1.228	-1.26

Panel B: Maturity = 4 years

Credit Rating	Target: Lev. Ratio (%)	Target: Equity Prem. (%)	Target: Cumulative Def. Prob. (%)	Target: Average Yld. Sprd (bps)	Target: Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	55	24.5	21.8	1.481	-1.98
Aa	21.2	5.60	0.23	65	26.5	22.4	1.441	-1.88
A	32.0	5.99	0.35	96	28.5	22.1	1.424	-1.86
Baa	43.3	6.55	1.24	158	31.8	22.4	1.411	-1.80
Ba	53.5	7.30	8.51	320	42.7	25.4	1.234	-1.02
B	65.7	8.76	23.32	470	53.8	37.3	1.168	-0.84

[1] This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility (columns 2-5). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$ and $\sigma = 0.3$.

Table 2.5: New Calibration Results for Different Credit Ratings and Maturities with Liquidity Correction.[1]

Panel A: Maturity = 10 years

Credit Rating	Target Lev. Ratio (%)	Target Equity Prem. (%)	Target Cumulative Def. Prob. (%)	Target Average Credit Sprd (bps)	Target Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	12	24.5	21.6	1.391	-1.76
Aa	21.2	5.60	0.99	40	26.5	21.3	1.365	-1.64
A	32.0	5.99	1.55	72	28.5	21.3	1.358	-1.63
Baa	43.3	6.55	4.39	143	31.8	21.5	1.326	-1.47
Ba	53.5	7.30	20.63	269	42.7	25.8	1.309	-1.40
B	65.7	8.76	43.91	419	53.8	37.6	1.222	-1.21

Panel B: Maturity = 4 years

Credit Rating	Target Lev. Ratio (%)	Target Equity Prem. (%)	Target Cumulative Def. Prob. (%)	Target Average Credit Sprd (bps)	Target Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	4	24.5	21.8	1.447	-1.87
Aa	21.2	5.60	0.23	14	26.5	22.4	1.433	-1.86
A	32.0	5.99	0.35	45	28.5	22.1	1.420	-1.85
Baa	43.3	6.55	1.24	107	31.8	22.4	1.406	-1.78
Ba	53.5	7.30	8.51	269	42.7	25.4	1.278	-1.19
B	65.7	8.76	23.32	419	53.8	37.3	1.152	-0.77

[1] This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average credit spread and equity volatility (columns 2-5). The credit spread for each credit rating is calculated as the yield spread minus the 1-year AAA yield spread (51 bps) as in Almeida and Philippon (2007). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$ and $\sigma = 0.3$.

Table 2.6: Calibration Results for the Market Price of Volatility Risk for Different Credit Ratings and Maturities.[1]

Panel A: Maturity = 10 years								
	5-target Case				6-target Case			
Credit Rating	Ratio of LR Mean	Ratio of Exp. Asset Var. 3 Months	Ratio of Exp. Asset Var. 1 Month	λ_V	Ratio of LR Mean	Ratio of Exp. Asset Var. 3 Months	Ratio of Exp. Asset Var. 1 Month	λ_V
Aaa	2.756	1.534	1.200	-2.55	7.693	1.560	1.192	-2.34
Aa	2.548	1.503	1.190	-2.43	6.668	1.511	1.175	-2.14
A	2.496	1.494	1.187	-2.40	5.382	1.508	1.175	-2.16
Baa	2.454	1.486	1.184	-2.37	5.001	1.452	1.157	-1.92
Ba	1.843	1.353	1.139	-1.83	3.572	1.387	1.136	-1.68
B	1.444	1.222	1.092	-1.23	1.460	1.228	1.094	-1.26

[1] This table reports the calibrated results for the market price of volatility risk for the stochastic volatility model. In columns 2 and 6, we report the ratio of the risk-neutral long-run mean over the objective long-run mean for asset variance. In columns 3, 4, 7 and 8, we report ratios of the risk-neutral over the objective expected asset variance (of 3-month maturity in columns 3 and 7 and of 1-month maturity in columns 4 and 8). In the 5-target calibration, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread. In the 6-target calibration, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility. Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$ & $\kappa = 4$ for 5-target calibration, and $\rho = -0.1$ & $\sigma = 0.3$ for 6-target calibration.

Table 2.6 continued

Panel B: Maturity = 4 years

	5-target Case				6-target Case			
Credit	Ratio of	Ratio of Exp. Asset Var.		λ_V	Ratio of	Ratio of Exp. Asset Var.		λ_V
Rating	LR Mean	3 Months	1 Month		LR Mean	3 Months	1 Month	
Aaa	2.759	1.535	1.200	-2.55	9.819	1.481	1.163	-1.98
Aa	2.241	1.447	1.172	-2.21	4.665	1.441	1.153	-1.88
A	2.190	1.436	1.168	-2.40	3.475	1.424	1.150	-1.86
Baa	2.139	1.425	1.164	-2.13	3.472	1.411	1.145	-1.80
Ba	1.619	1.285	1.115	-1.53	2.480	1.234	1.083	-1.02
B	1.127	1.074	1.032	-0.45	1.455	1.168	1.065	-0.84

Panel C: Calibration results with liquidity correction

	Maturity = 10 years				Maturity = 4 years			
Credit	Ratio of	Ratio of Exp. Asset Var.		λ_V	Ratio of	Ratio of Exp. Asset Var.		λ_V
Rating	LR Mean	3 Months	1 Month		LR Mean	3 Months	1 Month	
Aaa	2.833	1.391	1.141	-1.76	6.194	1.447	1.154	-1.87
Aa	2.745	1.365	1.132	-1.64	4.207	1.433	1.151	-1.86
A	2.538	1.358	1.130	-1.63	3.372	1.420	1.149	-1.85
Baa	2.485	1.326	1.118	-1.47	3.372	1.406	1.144	-1.78
Ba	2.386	1.309	1.112	-1.40	3.204	1.278	1.097	-1.19
B	1.460	1.222	1.091	-1.21	1.393	1.152	1.059	-0.77

Table 2.7: Calibrated Yield Spreads for Different Credit Ratings and Maturities for the Stochastic Model with Merton Default.[1]

Panel A: Maturity = 10 years

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	33.6	54.9	63	87.1
Aa	21.2	5.60	0.99	30.1	63.3	91	69.6
A	32.0	5.99	1.55	27.1	77.1	123	62.7
Baa	43.3	6.55	4.39	28.9	144.6	194	74.5
Ba	53.5	7.30	20.63	39.8	355.1	320	111.0
B	65.7	8.76	43.91	57.6	665.2	470	141.5

Panel B: Maturity = 4 years

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.04	36.1	9.4	55	17.1
Aa	21.2	5.60	0.23	34.8	43.3	65	66.6
A	32.0	5.99	0.35	30.2	54.2	96	56.5
Baa	43.3	6.55	1.24	31.2	137.1	158	86.8
Ba	53.5	7.30	8.51	39.5	392.8	320	122.8
B	65.7	8.76	23.32	53.2	829.7	470	176.5

[1] This table reports the calibration results for the model in which the firm value follows the stochastic volatility process and the default only occurs at maturity. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$.

Table 2.8: Summary Statistics by Ratings.[1]

Panel A: Firm Characteristics				
Credit Rating	Firms	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)
AAA [2]	1	24.29	63.67	2.83
AA	1	21.99	25.81	1.36
A	16	27.19	37.95	2.33
BBB	22	27.72	48.51	2.12
BB	7	35.18	51.01	2.50
B	2	41.24	77.10	2.66

Panel B: CDS Spreads (%)						
Credit Rating	1-year	2-year	3-year	5-year	7-year	10-year
AAA	0.45	0.49	0.52	0.58	0.60	0.63
AA	0.12	0.14	0.17	0.22	0.25	0.30
A	0.28	0.33	0.37	0.45	0.51	0.57
BBB	0.40	0.47	0.54	0.67	0.75	0.85
BB	0.75	0.86	1.00	1.35	1.44	1.57
B	3.73	4.22	4.49	4.80	4.78	4.77

Panel C: CDS Spreads Std. Dev. (%)						
Credit Rating	1-year	2-year	3-year	5-year	7-year	10-year
AAA	1.16	1.12	1.08	0.98	0.90	0.85
AA	0.16	0.16	0.17	0.18	0.17	0.17
A	0.39	0.40	0.40	0.39	0.37	0.36
BBB	0.49	0.49	0.50	0.49	0.47	0.45
BB	0.86	0.89	0.88	0.88	0.83	0.85
B	7.78	7.02	6.40	5.70	5.17	4.70

[1] This table reports the summary statistics on the CDS spreads and the underlying firms from January 2002 to December 2008. Equity volatility is estimated using 5-minute intraday returns. Leverage ratio is calculated as the ratio of the total liabilities over the total asset, which is the sum of the total liability and equity market value. Asset payout ratio is the weighted average of dividend payout and interest expense over the total asset.

[2] Note that the only AAA-rated company in our sample is GE.

Table 2.9: Specification Test of the Stochastic Volatility Model.[1]

	Panel A: Parameter Estimates							
Credit Rating	ρ	κ^*	σ	$X_B/100$	$\theta^*(\%)$	J-stat	Proportion of Not Rejected	
							Sig. level=0.01	0.05
Overall	-0.58	0.98	0.37	0.67	5.34	1.637	48/49	46/49
	(0.007)	(0.008)	(0.007)	(0.013)	(0.011)			
AAA	-0.47	1.2	0.24	0.70	3.06	0.924	1/1	1/1
	(0.006)	(0.008)	(0.005)	(0.014)	(0.008)			
AA	-0.54	0.8	0.36	0.62	6.40	0.672	1/1	1/1
	(0.008)	(0.012)	(0.005)	(0.018)	(0.011)			
A	-0.56	1.2	0.42	0.68	4.58	1.008	17/17	16/17
	(0.006)	(0.009)	(0.007)	(0.014)	(0.007)			
BBB	-0.61	0.9	0.31	0.67	5.20	2.352	21/22	20/22
	(0.008)	(0.007)	(0.008)	(0.010)	(0.011)			
BB	-0.59	0.8	0.45	0.70	6.92	1.260	6/6	6/6
	(0.007)	(0.009)	(0.004)	(0.015)	(0.012)			
B	-0.65	0.5	0.32	0.75	9.24	1.092	2/2	2/2
	(0.012)	(0.009)	(0.007)	(0.018)	(0.017)			

[1] This table reports the GMM test results of the stochastic volatility model. Panel A reports the parameter estimates, p-values (in parentheses) and J-statistic. Panel B reports the average and absolute pricing errors of CDS spreads and equity volatility. Panel C reports the average and absolute percentage pricing errors of CDS spreads and equity volatility. The pricing errors are calculated as the time-series average, absolute, average percentage and absolute percentage differences between the model-implied and observed values.

Table 2.9 continued

Panel B: Average and Absolute Pricing Errors

Credit	Average Pricing Error (%)		Absolute Pricing Error (%)	
Rating	CDS Spreads	Equity Volatility	CDS Spreads	Equity Volatility
Overall	-0.18	0.15	0.26	0.35
AAA	-0.15	0.03	0.18	0.14
AA	-0.04	0.08	0.07	0.12
A	-0.03	0.12	0.17	0.06
BBB	-0.13	0.10	0.19	0.55
BB	-0.59	0.36	0.55	0.45
B	-0.61	0.23	0.96	0.34

Panel C: Average and Absolute Percentage Pricing Errors

Credit	Average Percentage Pricing Error (%)		Absolute Percentage Pricing Error (%)	
Rating	CDS spreads	Equity Volatility	CDS spreads	Equity Volatility
Overall	-10.16	0.40	29.61	0.96
AAA	-27.42	0.12	32.90	0.58
AA	-20.02	0.36	35.04	0.55
A	-7.16	0.41	36.65	0.20
BBB	-11.16	0.28	25.25	1.55
BB	-19.28	0.78	24.43	0.97
B	22.34	0.56	35.06	0.82

Table 2.10: Specification Test of the Black and Cox (1976) Model.[1]

Panel A: Parameter Estimates					
Credit Rating	Asset Vol(%)	$X_B/100$	J-stat	Proportion of Not Rejected	
				Sig. level=0.01	0.05
Overall	12.41	1.31	15.112	4/49	1/49
	(0.008)	(0.036)			
AAA	10.01	1.067	11.413	0/1	0/1
	(0.004)	(0.016)			
AA	11.75	1.795	8.487	1/1	0/1
	(0.010)	(0.051)			
A	10.15	1.586	16.396	0/16	0/16
	(0.008)	(0.042)			
BBB	12.45	1.201	15.263	2/22	0/22
	(0.008)	(0.034)			
BB	14.35	1.125	11.545	1/7	1/7
	(0.007)	(0.031)			
B	20.17	0.804	20.822	0/2	0/2
	(0.006)	(0.028)			

[1] This table reports the GMM test results of the Black and Cox (1976) model. Panel A reports the parameter estimates, p-values (in parentheses) and J-statistic. Panel B reports the average and absolute pricing errors of CDS spreads and equity volatility. Panel C reports the average and absolute percentage pricing errors of CDS spreads and equity volatility. The pricing errors are calculated as the time-series average, absolute, average percentage and absolute percentage differences between the model-implied and observed values.

Table 2.10 continued

Panel B: Average and Absolute Pricing Errors

Credit	Average Pricing Error (%)		Absolute Pricing Error (%)	
Rating	CDS Spreads	Equity Volatility	CDS Spreads	Equity Volatility
Overall	-0.48	-3.96	0.50	10.85
AAA	-0.27	3.04	0.29	10.35
AA	0.08	-1.56	0.27	8.41
A	-0.31	-5.01	0.31	11.22
BBB	-0.43	-1.81	0.43	8.14
BB	-0.74	-8.09	0.76	12.99
B	-1.77	-9.46	2.20	31.94

Panel C: Average and Absolute Percentage Pricing Errors

Credit	Average Percentage Pricing Error (%)		Absolute Percentage Pricing Error (%)	
Rating	CDS spreads	Equity Volatility	CDS spreads	Equity Volatility
Overall	-65.44	-6.56	67.31	29.51
AAA	-25.11	17.63	40.69	51.64
AA	-61.11	6.55	65.67	35.91
A	-73.02	-4.64	73.73	32.32
BBB	-68.09	-9.53	68.08	24.15
BB	-62.17	-3.74	64.16	32.61
B	-9.36	-17.69	32.64	41.08

Table 2.11: Summary Statistics of Individual Names.[1]

Company Name	Last Rating	5-year CDS(%)	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout(%)
Amgn Inc	A	0.392	26.94	15.99	0.42
Arrow Electronics Inc	BBB	1.357	35.48	55.89	1.76
Boeing Co	A	0.441	29.89	50.51	1.58
Baxter Intl Inc	A	0.343	26.66	26.42	1.59
Bristol Myers Squibb Co	A	0.297	27.28	25.09	3.95
Boston Scientific Corp	BB	0.816	35.41	24.53	0.75
Conagra Inc	BBB	0.440	21.10	41.21	3.56
Caterpillar Inc	A	0.359	26.43	54.47	2.45
Cigna Corp	BBB	0.680	30.36	77.62	0.27
Comcast Corp New	BBB	1.138	31.93	61.15	1.91
Campbell Soup Co	A	0.277	21.98	32.10	2.49
Computer Sciences Corp	BBB	0.613	28.78	46.44	1.17
Du Pont Co	A	0.275	24.43	36.84	2.81
Deere & Co	A	0.394	29.40	58.44	2.61
Disney Walt Co	A	0.492	27.27	34.68	1.49
Dow Chemical Co	A	0.658	27.37	45.31	3.21
Darden Restaurants Inc	BBB	0.689	32.41	31.42	1.69
Devon Energy Corp	BBB	0.553	33.31	44.82	1.87
Eastman Kodak Co	B	2.136	32.64	60.35	2.12
Eastman Chemical Co	BBB	0.658	29.04	53.05	3.13
Ford Motor Co	B	7.471	41.29	93.85	3.21
Fortune Brands Inc	BBB	0.570	21.48	36.96	2.31
General Electric Co	AAA	0.584	24.29	63.68	2.83
General Mills Inc	BBB	0.432	17.76	40.72	3.03
Goodrish Corp	BBB	0.780	28.43	52.39	2.55
Honeywell Intl Inc	A	0.359	28.39	40.17	2.10
Intl Paper Co	BBB	0.932	28.74	56.38	3.08
Ingersoll Rand Co	BBB	0.414	28.12	39.39	1.91
Kraft Foods Inc	BBB	0.464	21.12	52.20	2.76
Kroger Company	BBB	0.641	29.25	52.34	1.93
Lowes Corp.	A	0.641	31.32	79.34	0.60

[1] This table reports the ratings, 5-year CDS spread, equity volatility, leverage ratio and asset payout ratio for each of the 49 firms.

Table 2.11 continued

Company Name	Last Rating	5-year CDS(%)	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout(%)
Marriott Intl Inc New	BBB	0.894	27.06	32.10	1.29
Mcdonalds Corp	A	0.285	25.45	25.70	2.31
Mckesson Inc	BBB	0.564	27.67	53.14	0.76
Northrop Grumman Corp	BBB	0.459	20.99	46.26	2.08
Newell Rubbermaid Inc	BBB	0.572	26.50	41.99	3.03
Olin Corp	BB	1.247	37.83	48.51	3.11
Radioshack Corp	BB	1.136	37.90	29.09	1.64
Raytheon Co	BBB	0.599	28.09	43.06	2.33
Sherwin Williams Co	A	0.486	29.66	31.15	1.99
Supervalu Inc	BB	1.526	27.78	60.52	3.21
Safeway Inc	BBB	0.646	30.68	48.76	2.01
A T&T Corp	A	1.249	28.29	43.81	3.50
Target Corp	A	0.392	30.47	35.16	1.48
Temple Inland Inc	BB	1.411	31.01	81.40	3.75
Tyson Foods Inc	BB	1.254	36.37	62.01	3.11
Verizon Communications Inc	A	0.621	25.19	51.37	3.40
Whirlpool Corp	BBB	0.683	31.38	59.95	2.16
Wal Mart Stores Inc	AA	0.215	21.97	25.81	1.36

Table 2.12: Equity Leverage Effect (ρ_E).[1]

Credit Rating	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)	ρ	κ^*	σ	$X_B/100$	θ^* (%)	ρ_E
AAA	24.29	63.67	2.83	-0.47	1.2	0.24	0.70	3.06	-0.57
AA	21.99	25.81	1.36	-0.54	0.8	0.36	0.62	6.40	-0.53
A	27.19	37.95	2.33	-0.56	1.2	0.42	0.68	4.58	-0.69
BBB	27.72	48.51	2.12	-0.61	0.9	0.31	0.67	5.20	-0.68
BB	35.18	51.01	2.50	-0.59	0.8	0.45	0.70	6.92	-0.67
B	41.24	77.10	2.66	-0.65	0.5	0.32	0.75	9.24	-0.74

[1] This table reports the calculated leverage effect of equity (column 10) for each credit rating. We report the parameter values used to calculate the equity leverage effect in column 2-9.

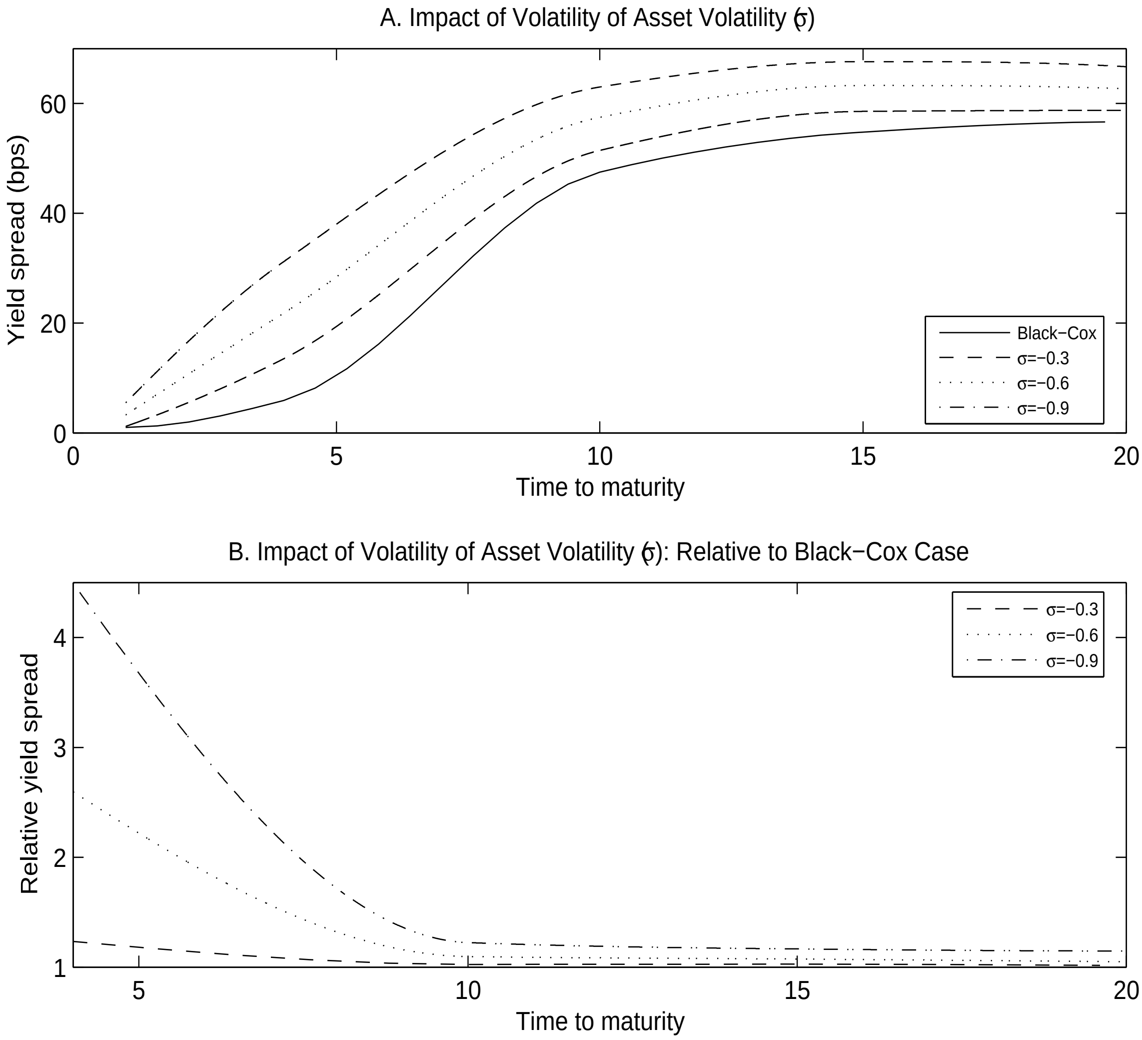

Figure 2.1: The impact of the volatility of volatility (σ) on the yield spread. In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\rho = -0.1$, $\theta = 0.21^2$.

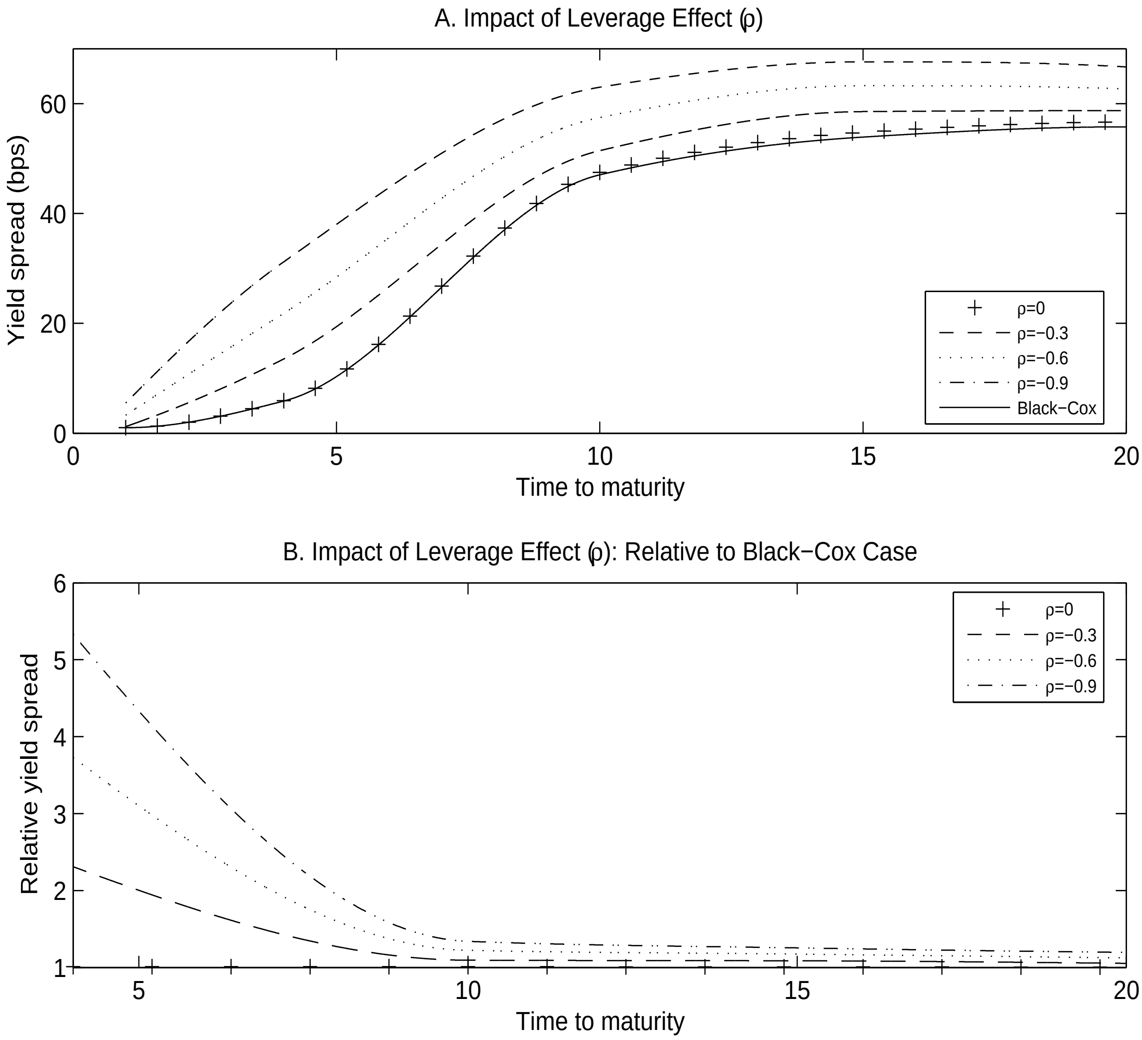

Figure 2.2: The impact of the leverage effect (ρ) on the yield spread.

This figure shows the impact of the leverage effect on the yield spread when the market price of volatility risk is zero. In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\sigma = 0.3$, $\theta = 0.21^2$.

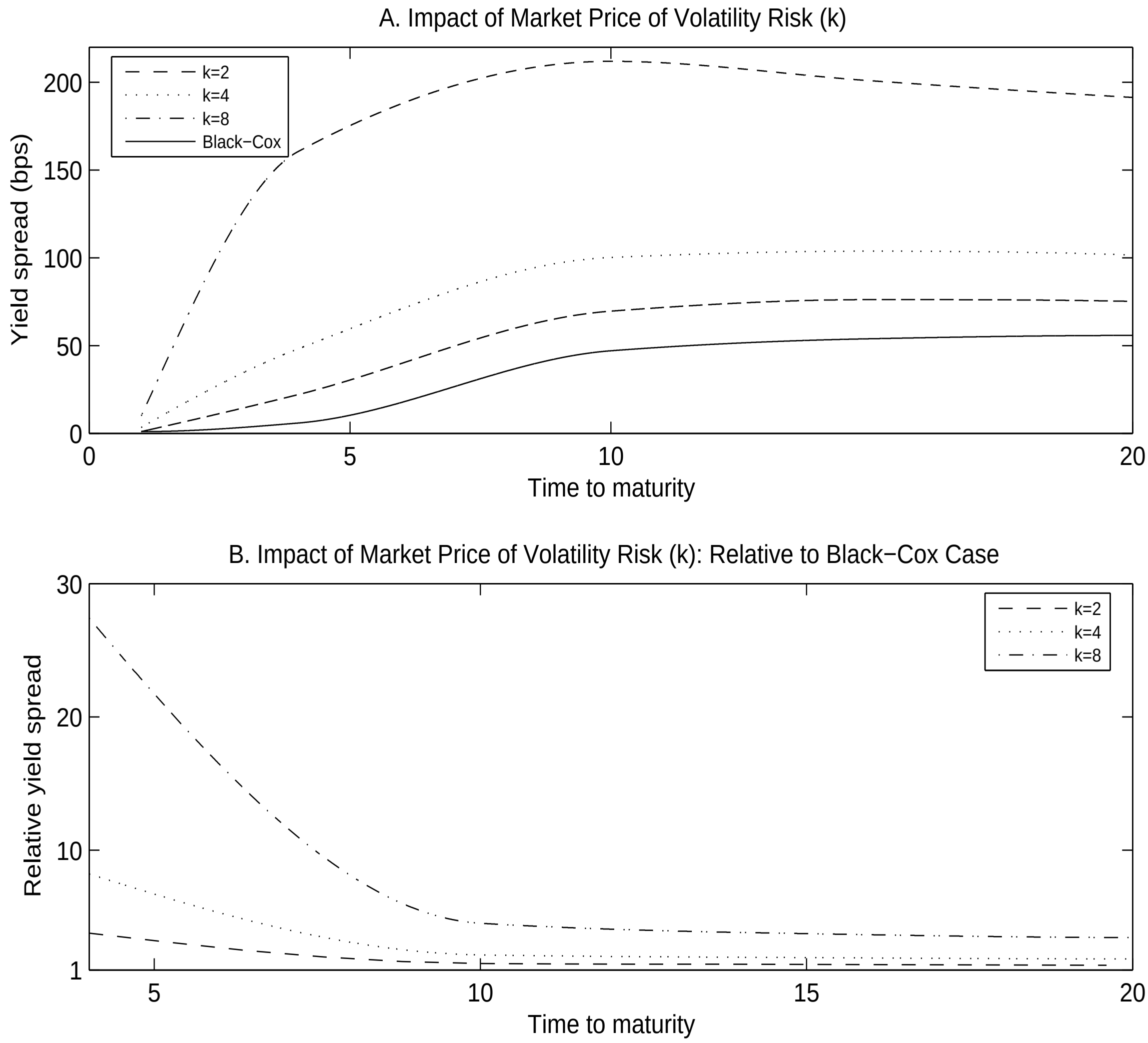

Figure 2.3: The impact of the market price of volatility risk (k) on the yield spread. This figure shows the impact of the market price of volatility risk on the yield spread in the absence of leverage effect ($\rho = 0$). In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\sigma = 0.3$, $\theta = 0.21^2$.

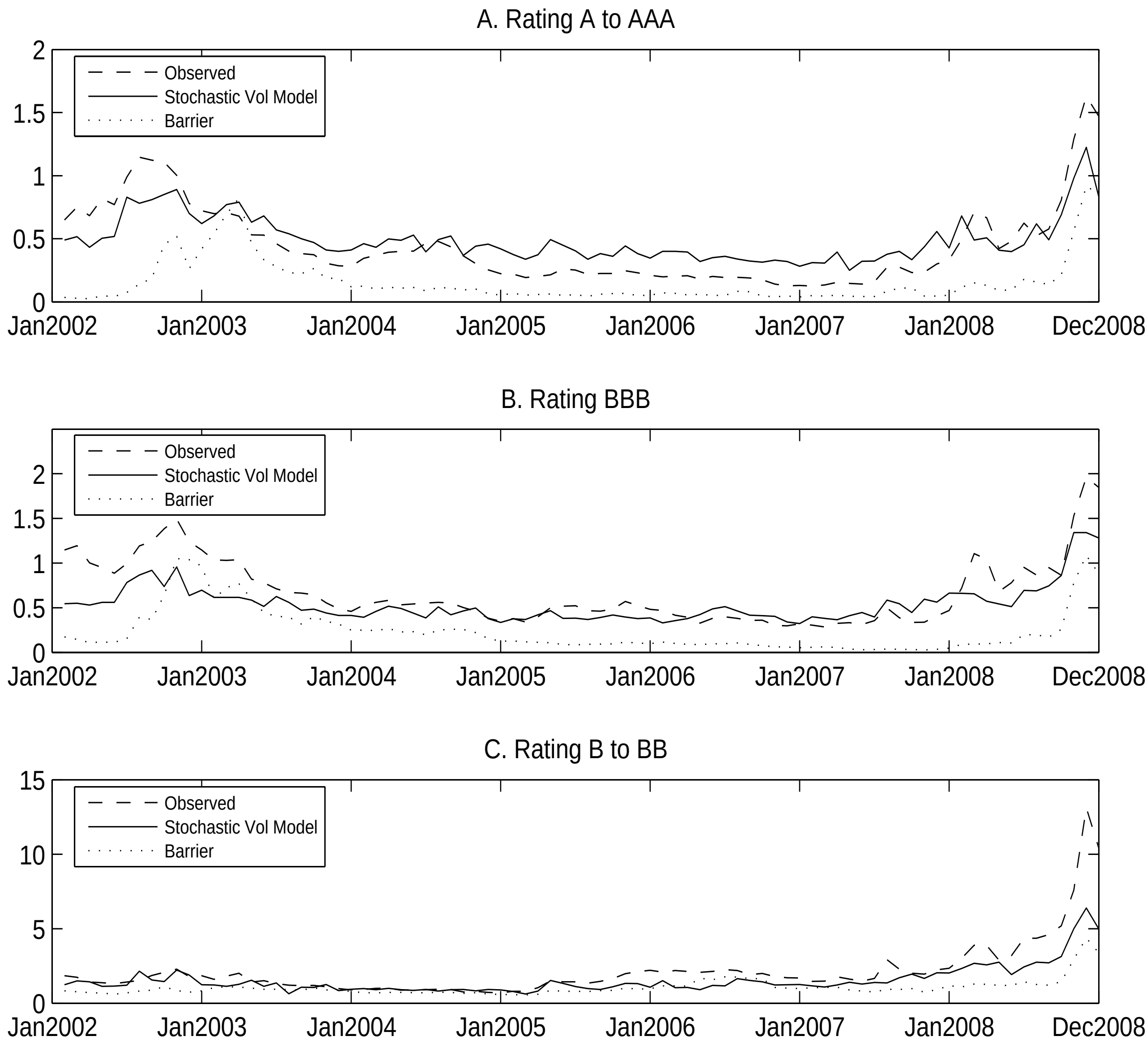

Figure 2.4: Observed and model-implied 5-year CDS spreads.

This figure shows the time series of observed 5-year CDS spreads and those estimated from the stochastic volatility model and the Black-Cox (1976) model. One unit in the Y axis corresponds to 100 basis points.

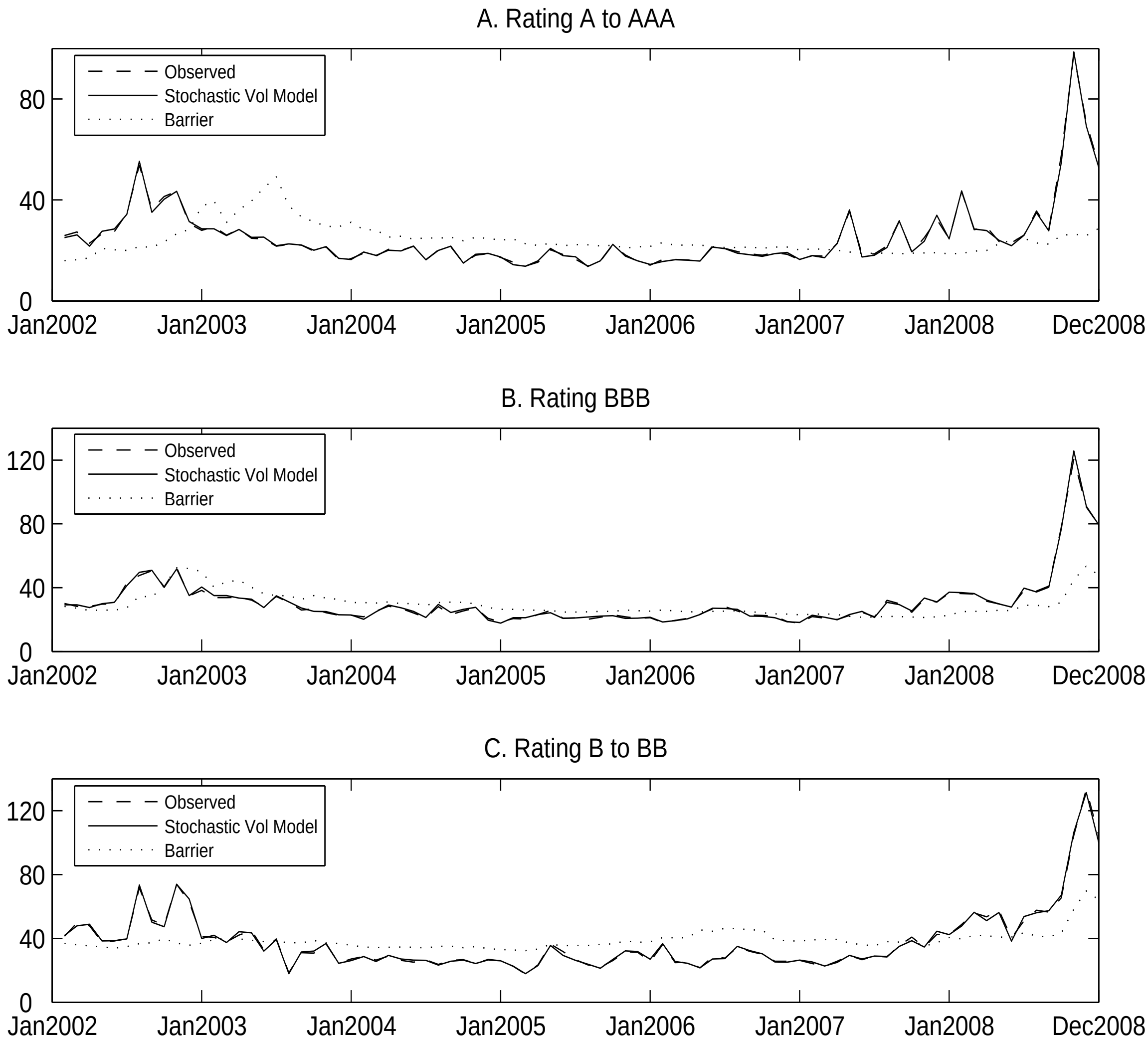

Figure 2.5: Observed and model-implied equity volatility.

This figure shows the time series of the realized volatility, which is estimated from 5-minute intraday stock returns, and the model-implied equity volatility from the stochastic volatility model and the Black-Cox (1976) model.

CHAPTER 3
LEARNING AND AGGREGATE LIQUIDITY

3.1 Introduction

Aggregate credit supply or liquidity is of interest to many economists, policymakers and practitioners because of its close relationship with monetary policy and asset returns in the financial market. In particular, after the recent financial crisis, many studies point to the importance in understanding the nature of liquidity. Some papers (Adrian and Shin (2008, 2009), Adrian, Moench and Shin (2010), etc.) find that the aggregate liquidity helps forecast real economic activity and inflation measured by the components of GDP such as durable consumption and housing investment. Furthermore, the aggregate liquidity contains strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios.[1] However, few papers in the literature provide a theoretical framework helping understand the interactions among aggregate liquidity, macroeconomic variables and asset returns.[2]

This paper aims to develop a quantitative framework to investigate those interactions. More specifically, this study focuses on quantifying the systematic liquidity risk premium and its connection with time-varying macroeconomic conditions and

[1]Longstaff and Wang (2008) also shows that aggregate credit supply helps forecast the equity premium.

[2]Acharya and Pedersen (2005) build a liquidity-adjusted capital asset pricing model to study the effects of liquidity risk on asset prices. But the macroeconomic dynamics is not present in their model. Longstaff and Wang (2008) use an equilibrium model with heterogeneous agents to study the role of credit market.

asset returns. Motivated by the information content (or high predictive power) of aggregate liquidity on the future economic growth (as shown in Adrian and Shin (2008, 2009), and Adrian, Moench and Shin (2010)), we develop a continuous-time consumption-based learning model, where aggregate liquidity works as an informational channel helping economic agents infer the unobserved economic growth rate.

In sum, the model implies a positive liquidity risk price, which is important to generate reasonable equity premium, risk-free rates and real yield curve. The paper provides a unified framework to explain many empirical facts in the literature, including procyclical risk-free rates and wealth consumption ratios as well as countercyclical equity premium and return volatility. Finally, we apply the model-implied pricing kernel to price the contingent claims of an average firm in the economy. The model generates reasonable levered equity premium and bond yield spread. More importantly, the model suggests that liquidity risk premium contributes significantly to the total yield spread of the corporate bonds. The magnitude and dynamics of the bond liquidity premium are consistent with the empirical evidence.

The model's working mechanism depends on two main ingredients. First, the expected consumption and liquidity growth rates follow a hidden Markov regime-switching model. The economic agents learn about the growth state from realized consumption and liquidity data. The time-varying uncertainty about the growth state depends on agents' posterior belief. Second, the economic agents in the exchange economy have recursive Epstein-Zin-Weil preferences and prefer to resolve uncertainty sooner.

With the recursive preferences, the agents are concerned about times of high uncertainty and demand an uncertainty premium for holding assets which pay off poorly in those times. In the learning model, the posterior expected consumption growth rate inherits the slow-moving and mean-reverting property in the prior. As explained by Bansal and Yaron (2004), with the recursive preferences, a small uncertainty about current economic growth translates to a large uncertainty about future consumption flows, hence amplifying the compensation for holding the assets with low payoff in times of high uncertainty. Also, the economic agents tend to save more in times of high uncertainty, leading to a lower risk-free rate in such times. A negative economic shock reduces the posterior belief of the high-growth state, hence brings up the uncertainty because the model-implied uncertainty is a hump-shaped function of the posterior state belief. This property results in the pattern of procyclical risk-free rates as well as countercyclical equity premium and return volatility.

As an informational channel, additional liquidity shocks make the posterior state belief be more volatile, hence bringing up the uncertainty about the growth state. This property helps generate a positive liquidity risk premium in the economy. Furthermore, the liquidity risk premium increases with the precision of the fluctuations of liquidity itself. In particular, at the long-run mean of the posterior belief, the model-implied equity premium increases by 7 times and the return volatility increases by 2 times when the liquidity growth volatility decreases 67%. With a high precision, a realized liquidity shock makes the economic agents adjust their belief of the growth state more strongly, hence increasing the volatility of the posterior and the uncertainty about the state of economic growth. This higher uncertainty eventually leads to larger return volatility and liquidity premium.

We find that the model-implied shape of real yield curve depends on magnitude of the posterior belief. When the investor puts a high belief on the high-growth/low-growth state, the model generates a downward/upward sloping real yield curve. With a high posterior belief, a negative consumption or liquidity shock raises the investor's uncertainty about the economic growth rate. Hence, the investor favors more the long-term real bonds to hedge the long run uncertainty in consumption. The relatively higher price in the long-term real bonds translates to a relatively lower yield in the long maturity.

Finally, the model implies a reasonable yield spread and bond liquidity risk premium for an average firm in the economy. At the long-run mean of the posterior belief, the model implies a total yield spread of around 185 basis points and a liquidity premium of around 55 basis points. These numbers are consistent with the empirical estimates for BAA-rated corporate bond. Furthermore, the model generates a countercyclical bond liquidity premium. This paper suggests that liquidity risk premium constitutes a significant proportion in the total yield spread.[3]

In the literature, learning mechanism has been extensively used to study vari-

[3] Many empirical papers in the literature (Longstaff, Mithal and Neis (2005), Bao, Pan and Wang (2011), Ericsson and Renault (2006), Friewald, Jankowitsch and Subrahmanyam (2011), Acharya, Amihud and Bharath (2010), etc.) study the liquidity premium in corporate bonds. In general, the calibrated liquidity premium in this paper is consistent with the related findings in the literature.

ous topics in financial market.[4] The basic setup of the learning mechanism is similar to Veronesi (2000). However, that paper suggests an opposite result (the higher the uncertainty, the lower the equity premium). The use of CRRA utility function is the main reason causing that seemingly surprising result. An increase in risk aversion raises the agents' hedging demand for the equity after bad news in dividends (or consumption), which counterbalances the negative pressure on prices from the negative dividend (or consumption) shock. However, the inverse relationship between risk aversion and elasticity of intertemporal substitution (EIS) in CRRA preferences leads to a lower EIS, strengthening the hedging demand. The dominance of the hedging demand results in a negative equity premium.

Recently, Ai (2010) study the effect of information quality on asset prices in a production economy. That paper suggests a positive relationship between uncertainty and equity premium. Weitzman (2007) and Lettau, Ludvigson and Wachter (2008) assume consumption volatility is unobservable and introduce learning to study equity premium from a different perspective. Bansal and Shaliastovich (2011) extends the learning model to study asset price jumps.

This paper is organized as follows. Section 2 describes the economy and the model. Section 3 calibrates the model and analyzes the implications of the model. Finally, Section 4 concludes.

[4] See Pastor and Veronesi (2009) for an excellent summary.

3.2 The Model

In this section, we develop a continuous-time equilibrium model with learning to price the aggregate consumption claim and firms' contingent claims. The model characterizes the aggregate liquidity as an information channel for the investors to learn about the true state. We first start by defining the economy and the preferences. Then, we describe the pricing model for the financial assets in detail.

3.2.1 The Economy

The representative agent has continuous time Epstein-Zin-Weil preference (Duffie and Epstein (1992a,b), Epstein and Zin (1989) and Weil (1990)). The utility index at time t for a consumption process c is

$$U_t = E_t\left[\int_t^{\infty} f(c_s, U_s)\, ds\right]. \tag{3.1}$$

The function $f(c, v)$ is the standard normalized Kreps-Porteus aggregator of consumption and continuation value in each period and takes the form

$$f(c,v) = \frac{\beta}{1-\frac{1}{\psi}} \cdot \frac{c^{1-\frac{1}{\psi}} - [(1-\gamma)v]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}{[(1-\gamma)v]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1}, \tag{3.2}$$

with β defined as time preference, γ defined as risk aversion and ψ defined as the elasticity of intertemporal substitution.

We assume that the expected consumption growth depends on state s_t, which follows a continuous-time Markov chain with 2 states ($s_t = 1$ or 2).[5] The generator

[5] Without loss of generality, we define $s_t = 1$ as the high-growth state.

matrix of the Markov chain (Λ) is defined by λ_{12} and λ_{21}, where λ_{ij} $(i \neq j)$ is the probability of switching from state i to state j. We assume that the investors do not observe the realizations of s_t but learn about the current state of s_t from observations of aggregate consumption and credit supply (or aggregate liquidity).

The real aggregate consumption is given by

$$\frac{dC_t}{C_t} = \theta_{C,s_t} dt + \sigma_C dz_{1,t}, \tag{3.3}$$

where $z_{1,t}$ is a standard Brownian motion. In equation (3.3), the expected consumption growth rate (θ_{C,s_t}) follows a regime-switching process and the consumption volatility (σ_C) is a constant.

The aggregate credit supply is given by

$$\frac{dL_t}{L_t} = \theta_{L,s_t} dt + \sigma_{L,1} dz_{1,t} + \sigma_{L,2} dz_{2,t}, \tag{3.4}$$

where $z_{2,t}$ is a standard Brownian motion uncorrelated with $z_{1,t}$. We assume that the expected growth rate for the credit supply (θ_{L,s_t}) depends on the state s_t and the diffusion coefficients ($\sigma_{L,1}$ and $\sigma_{L,2}$) are constants.

Let $I(t)$ denote the vector of the aggregate consumption and the credit supply, i.e., $I(t) = (C_t, L_t)'$. Then the dynamics of I_t is given by

$$\frac{dI(t)}{I_t} = \theta_{s_t} dt + \Sigma dz_t,$$

where $\theta_{s_t} = (\theta_{C,s_t}, \theta_{L,s_t})'$, $z_t = (z_{1,t}, z_{2,t})'$ and

$$\Sigma = \begin{pmatrix} \sigma_C & 0 \\ \sigma_{L,1} & \sigma_{L,2} \end{pmatrix}.$$

Note that in the model, the drifts of consumption growth and liquidity growth are not equal. More importantly, the shocks to the aggregate consumption growth and aggregate liquidity are correlated, which is consistent with the empirical evidence.[6] We define $\Im_t$ as the investors' information set at time t and π_t as the posterior belief that the state at t is the high-growth, i.e.,

$$\pi_t = prob\left(s_t = 1|\Im_t\right).$$

According to Lipster and Shiryayev (2001), the posterior probability π_t follows

$$d\pi = \left[\lambda_{11}\pi + \lambda_{21}(1-\pi)\right]dt + \pi(1-\pi)(\theta_1 - \theta_2)'(\Sigma')^{-1}d\widetilde{z}_t, \tag{3.5}$$

where

$$d\widetilde{z}_t = \Sigma^{-1}\cdot(\frac{dI_t}{I_t} - mdt) = \Sigma^{-1}(\theta_{s_t} - m)dt + dz_t, \tag{3.6}$$

with $m = \pi\theta_1 + (1-\pi)\theta_2$. In what follows, we denote μ_π as the drift term in equation (3.5), $\sigma_{\pi,1}$ as the diffusion coefficient for $z_{1,t}$, and $\sigma_{\pi,2}$ as the diffusion coefficient for $z_{2,t}$. Conditional on the investor's posterior belief, the expected consumption growth $\tilde{\theta}_C$ is equal to $\pi\theta_{C,1} + (1-\pi)\theta_{C,2}$ and the expected liquidity growth $\tilde{\theta}_L$ is equal to $\pi\theta_{L,1} + (1-\pi)\theta_{L,2}$. The covariance matrix (Σ_θ) of the vector θ conditional on the information set $\Im_t$ is given by

$$\Sigma_\theta = \pi(1-\pi)\begin{pmatrix} (\theta_{C,1} - \theta_{C,2})^2 & (\theta_{C,1} - \theta_{C,2})(\theta_{L,1} - \theta_{L,2}) \\ (\theta_{C,1} - \theta_{C,2})(\theta_{L,1} - \theta_{L,2}) & (\theta_{L,1} - \theta_{L,2})^2 \end{pmatrix}. \tag{3.7}$$

The investor's posterior uncertainty is proportional to $\pi(1-\pi)$. Equation (3.7) illustrates that the closer to the middle point of its range is the investor's posterior belief

[6] For simplicity, most models in the literature including Veronesi (2000) and Ai (2010) assume away the difference in the drift and correlation between the two shocks.

π, the higher the investor's uncertainty.

To solve for the pricing kernel, first we solve the consumption portfolio choice problem of the representative agent. Define ϕ as the fraction of the agent's wealth invested in the claim of the aggregate consumption. Then in the competitive equilibrium, the agent's objective is to maximize the utility function subject to the budget constraint. The value function J is a function of the aggregate wealth W and π. More specifically,

$$J(W, \pi) = \max_{C,\phi} E\left[\int_t^{\infty} f(C_s, J_s) ds | \Im_t\right]$$

subject to

$$dW_t = W_t \left[\phi(\mu_R - r) + r\right] dt + W_t \phi(\sigma_{R,1} d\tilde{z}_{1,t} + \sigma_{R,2} d\tilde{z}_{2,t}) - C_t dt, \tag{3.8}$$

where μ_R is the drift of the aggregate consumption claim. $\sigma_{R,1}$ and $\sigma_{R,2}$ are the diffusion coefficients of the aggregate consumption claim. Solving the optimal portfolio problem gives the following proposition.

Proposition 5. *(a) The real risk-free rate is given by*

$$\begin{aligned} r(\pi) = & \tilde{\theta}_C + \tfrac{H'}{H}\mu_\pi + \tfrac{1}{2}\tfrac{H''}{H}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \tfrac{H'}{H}\sigma_C\sigma_{\pi,1} + \tfrac{1}{H} \\ & - \gamma\left[\left(\sigma_C + \tfrac{H'}{H}\sigma_{\pi,1}\right)^2 + \left(\tfrac{H'}{H}\sigma_{\pi,2}\right)^2\right] + \tfrac{1-\gamma}{1-\frac{1}{\psi}}\tfrac{1}{\psi}\tfrac{H'}{H}\left[\left(\sigma_C + \tfrac{H'}{H}\sigma_{\pi,1}\right)\sigma_{\pi,1} + \tfrac{H'}{H}\sigma_{\pi,2}^2\right], \end{aligned} \tag{3.9}$$

where H is the equilibrium wealth-consumption ratio, which satisfies

$$\begin{aligned} 0 = & -\beta + \left(1 - \tfrac{1}{\psi}\right)\tilde{\theta}_C - \tfrac{1}{2}\gamma\left(1 - \tfrac{1}{\psi}\right)\sigma_C^2 + \tfrac{1}{H} + \left[(1-\gamma)\sigma_C\sigma_{\pi,1} + \mu_\pi\right]\tfrac{H'}{H} \\ & + \tfrac{1}{2}\left(\tfrac{1-\gamma}{1-\frac{1}{\psi}} - 1\right)(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2)\left(\tfrac{H'}{H}\right)^2 + \tfrac{1}{2}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2)\tfrac{H''}{H}. \end{aligned} \tag{3.10}$$

(b) The pricing kernel M satisfies

$$\frac{dM}{M} = -r(\pi)dt - \xi_1(\pi)d\tilde{z}_1 - \xi_2(\pi)d\tilde{z}_2. \tag{3.11}$$

The market prices for diffusion risks $\tilde{z}_{1,t}$ (consumption shock) and $\tilde{z}_{2,t}$ (liquidity shock) are respectively given by

$$\xi_1 = \gamma\sigma_C + \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,1}, \tag{3.12a}$$

$$\xi_2 = \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,2}. \tag{3.12b}$$

(c) The equity premium for the aggregate consumption claim is given by

$$\mu_W - r = \left(\sigma_C + \frac{H'}{H}\sigma_{\pi,1}\right)\left(\gamma\sigma_C + \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,1}\right) + \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}}\left(\frac{H'}{H}\sigma_{\pi,2}\right)^2. \tag{3.13}$$

3.2.2 Valuation of the Firm's Contingent Claims

To investigate the effect of aggregate liquidity on the yield spreads and levered equity premium at firm level, we develop a structural model to price the contingent claims for an average firm in the economy. More specifically, we consider a firm with one publicly traded consol bond, that continuously pays coupon cdt. We choose our primitive modeling variable to be the operating cash flows or earnings before interest and taxes (EBIT). The firm's operating cash flow is given by

$$\frac{dX_t}{X_t} = \tilde{\theta}_X dt + \sigma_{X,1}d\tilde{z}_{1,t} + \sigma_{X,2}d\tilde{z}_{2,t} + \sigma_{X,3}d\tilde{z}_{3,t}, \tag{3.14}$$

where $\tilde{z}_{1,t}$, $\tilde{z}_{2,t}$ and $\tilde{z}_{3,t}$ are mutually uncorrelated. $\tilde{\theta}_X$ is the firm's expected earnings growth rate under the investors' posterior belief. $\sigma_{X,1}$ and $\sigma_{X,2}$ are the systematic volatilities. $\sigma_{X,3}$ is the idiosyncratic volatility of the firm's earnings growth. The total volatility for the firm's earnings growth is given by

$$\sigma_X = \sqrt{\sigma_{X,1}^2 + \sigma_{X,2}^2 + \sigma_{X,3}^2}. \tag{3.15}$$

Given the pricing kernel from the economy, we apply the Girsanov theorem to define the risk-neutral or Q measure, under which

$$\frac{dX_t}{X_t} = \hat{\theta}_X dt + \sigma_{X,1} d\tilde{z}_{1,t}^Q + \sigma_{X,2} d\tilde{z}_{2,t}^Q + \sigma_{X,3} d\tilde{z}_{3,t}^Q, \tag{3.16}$$

where

$$\hat{\theta}_{X,s_t} = \tilde{\theta}_X - \sigma_{X,1}\xi_1 - \sigma_{X,2}\xi_2. \tag{3.17}$$

In our model, when the firm's operating cash flow is higher than the coupon amount $X_t > c$, the firm is in the liquid state. The amount $X_t - c$ is distributed to shareholders as dividends. When the total cash flow is less than the amount of required debt servicing ($X_t < c$), we say that the firm is in the illiquid state. The shareholders may not necessarily default even if the firm is illiquid as suggested by Leland (1994). Instead, the firm can issue more equity to cover the coupon payment. The firm defaults at the stopping time τ when the firm's operating cash flow hits an exogenously determined boundary X_B, i.e., $\tau = min\{s|X_s \leq X_B\}$. At default, the firm recovers α proportion of the unlevered asset value and the absolute priority rule applies.

Given this setting, the firm's unlevered asset value at time t (V_t) is equal to the expected present value of future operating cash flows under the pricing measure defined by our pricing kernel, i.e., $V_t = E\left(\int_t^\infty \frac{M_s}{M_t} X_s ds|\Im_t\right)$. The firm's equity value at time t is defined as $S_t = (1-\eta)E\left[\int_t^\tau \frac{M_s}{M_t}(X_s - c)ds|\Im_t\right]$, where η is the effective tax rate. The debt value at time t is $D_t = E\left[\int_t^{\tau} \frac{M_s}{M_t} cds | \Im_t\right] + E\left(\frac{M_\tau}{M_t} \alpha V_B | \Im_t\right)$, where α is the recovery rate of the firm value at default, and V_B is the unlevered asset value at default. The solutions for the firm's contingent claim prices are summarized in the following proposition.

Proposition 6. *(a) Under the investor's posterior belief, the unlevered asset value is given by*

$$V(X, \pi) = XG(\pi), \tag{3.18}$$

where $G(\pi)$ is the solution for the following ordinary differential equation,

$$\begin{aligned} &\tilde{\theta}_X + \tfrac{G'}{G}\mu_\pi + \tfrac{1}{2}\tfrac{G''}{G}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \tfrac{G'}{G}(\sigma_{X,1}\sigma_{\pi,1} + \sigma_{X,2}\sigma_{\pi,2}) - r(\pi) + \tfrac{1}{G} \\ &= \xi_1(\pi)\left(\sigma_{X,1} + \tfrac{G'}{G}\sigma_{\pi,1}\right) + \xi_2(\pi)\left(\sigma_{X,2} + \tfrac{G'}{G}\sigma_{\pi,2}\right). \end{aligned} \tag{3.19}$$

(b) Given the investor's posterior belief and the firm's cash flow level at time t, the firm's equity value is given by

$$S(X, \pi) = (1 - \eta)\left[V - qV_B - cF(1 - q)\right], \tag{3.20}$$

where V is the firm's unlevered asset value, q is the time-t Arrow-Debreu price of a contingent claim that pays one unit at default, and F is the present value at time t of a perpetuity with a constant payment being one. The solution for q is given in the appendix. F satisfies

$$\frac{F'}{F}\mu_\pi + \frac{1}{2}\frac{F''}{F}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \frac{1}{F} - r(\pi) = \xi_1(\pi)\frac{F'}{F}\sigma_{\pi,1} + \xi_2(\pi)\frac{F'}{F}\sigma_{\pi,2}. \tag{3.21}$$

(c) The firm's debt value is given by

$$D(X, \pi) = cF(1 - q) + \alpha q V_B. \tag{3.22}$$

3.3 Model Implications

In this section, we start first by describing the state-dependent parameter choices for the economy and the firm's cash flow processes. We then study properties of conditional real risk-free rates, wealth-consumption ratio, moments of returns for the aggregate consumption claim, and the real yield curve in equilibrium. Finally, we investigate the model-implied levered equity premium and yield spreads for an average firm in the economy.

3.3.1 Parameter Calibration

Our quantitative analysis requires an identification of preference parameters, the parameters defining the economy and the firm's fundamentals.

We proxy the average firm's expected earnings growth rates by the expected aggregate earning growth rates and proxy its volatilities by the aggregate earnings growth volatilities. To estimate both the process of aggregate earnings and consumption, we rely on the maximum likelihood method in Hamilton (1989). We fit our parameters to the following aggregate US data from 1947-2007: real non-durables plus service consumption expenditures from the Bureau of Economic Analysis and aggregate earnings data for all nonfinancial firms from the Compustat database. Through this, we estimate the expected consumption growth rates in both states, its volatilities, expected aggregate earning growth rates, its volatilities and the probabilities of switching between the states. We fit the idiosyncratic volatility of the firm's cash flow process to match the total volatility for an average firm in the economy (BAA-rated)

(25%). Hu, Pan and Wang (2011) suggests that the noise in the Treasury market measures well the liquidity in the financial market. We use the noise data in Hu, Pan and Wang (2011) to calibrate the aggregate liquidity process. Table 1 reports our parameter values used in the base model calibration. We find that the estimates are consistent with those reported in the related literature.

Insert Table 1 About Here

We set time preference $\beta = 0.02$, and relative risk aversion $\gamma = 10$. In the literature, researchers have different estimates for the EIS. Some studies (see Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002) and Guvenen (2006)) suggest that the EIS is higher than one, while others (see Hall (1988) and Campbell (1999)) find the opposite. We set the EIS $\psi = 2$ for the base case. However, we also provide our results for the case when EIS is smaller than one.

3.3.2 Equilibrium Asset Prices

This section discusses our model-implied equilibrium asset prices in the economy. We begin by analyzing the real risk-free rates. Then, we study the equity premium of the aggregate consumption claim. Finally, we investigate our model-implied term structure of government bond yields.

Real risk-free rates

Figure 1 plots the conditional equilibrium risk-free rate as a function of the posterior probability π of the high-growth state. To illustrate the effect of aggregate liquidity shocks on the risk-free rates, we draw the risk-free rate curves for two cases:

the case when both consumption and liquidity shocks are priced (solid curve) and the one when only consumption shock is priced (dashed curve).

For both cases, the risk-free rate is an inverse-hump-shaped function of the posterior probability (π). Note that the model implies that the investor's uncertainty is a hump-shaped function of the posterior (π).[7] As a result, the investor's uncertainty is higher as the posterior moves from either side of its range to the middle. With the recursive preference ($\gamma > 1/\psi$), the investor prefers to resolve the uncertainty sooner rather than later. This property makes the investor to save more when the uncertainty about the economic growth is high, leading to a lower risk-free rate. Comparing the two curves in figure 1 suggests that adding liquidity shocks further decreases the risk-free rate as π moves to the middle. The intuition is that adding liquidity shocks makes the posterior π be more volatile, hence bringing up the uncertainty about the growth state.

The inverse-hump-shaped curves in figure 1 seems to suggest that the correlation between the economic shocks (consumption and liquidity) and the change in risk-free rate may be positive or negative, depending on the position of π. However, the fact that the economy spends most of the time in the high-growth state translates to a highly skewed distribution of the posterior. Figure 1 shows that when the posterior is close to 1 (high-growth state), a negative consumption shock leads to a lower risk-free rate.[8] Thus, our model-implied risk-free rates are procyclical, which

[7]See equation (3.7).

[8]The dynamics of the posterior (π) in equation (3.5) suggests a positive correlation between economic shocks (consumption and liquidity) and the shock to the posterior. Hence,

is consistent with the data. At the long-run posterior probability ($\pi = 0.68$), the risk-free rate is equal to 0.009, which is very close to the data.[9]

Expected equity premium and return volatility (aggregate consumption claim)

This subsection reports the model-implied equity premium and return volatility for the aggregate consumption claim in the economy. To pin down the driving forces for the results, we investigate one special case of the model, where the economic agents have CRRA (constant relative risk aversion) preferences. Furthermore, we examine the effect of liquidity signal precision on the expected equity premium and return volatility.

Figure 2 presents the results for the base model. The plots in the figure show that both the conditional expected equity premium and the return volatility are hump-shaped function of the posterior probability π. Since the economy stays at the high-growth state in most of the time, those curves suggest that a negative economic shock brings up the equity premium and the return volatility. Thus, the model indicates countercylical equity premium and return volatility. Also calibrated to the aggregate dividend's data, the model generates reasonable equity premium and return volatility for the stock market, which are close to the data in Bansal and Yaron (2004) (See figure 9).

The posterior affects the equity premium through two channels. First, when

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a negative consumption shock reduces the posterior.

[9]Bansal and Yaron (2004) documents that the mean risk-free rate is equal to 0.0086 in their sample.

the posterior shifts from either side of its range to the middle, the uncertainty about the growth state rises. Given that the investor prefers an early resolution of uncertainty, the increase in uncertainty translates to a higher risk price. Second, the dynamics of the posterior (shown in equation (3.5)) shows that shifting the posterior to the middle brings up its variance. The higher volatility in the expected consumption growth indicates a more volatile return process for the aggregate consumption claim.[10] Panel B clearly illustrates this second effect. The combined effect of the two channels is to increase the equity premium with the posterior shifting to the middle of its range. The dashed curves in figure 2 give the proportions of the equity premium and return volatility solely due to the priced liquidity shock. Comparing the solid and dashed curves gives the fact that the liquidity shock contributes significantly to the total equity premium and return volatility. Furthermore, the effect of the liquidity strengthens with a higher uncertainty about the growth state.

In summary, the base model generates a positive risk premium for the investor's uncertainty. Liquidity shocks introduce an additional volatility to the posterior, raising the uncertainty about the growth state. This property helps generate a positive liquidity risk premium in the economy.

Contrary to other well known learning models in the literature like Veronesi (2000),[11] our base model generates a reasonable risk premium for the investor's un-

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[10]It is due to the fact that the payoff for aggregate consumption claim is the future consumption flow.

[11]In the Veronesi (2000) framework, the economic agents have CRRA utility function. That paper generates a seemingly surprising result: the higher the investor's uncertainty, the lower the risk premium.

certainty. In fact, when we set $\gamma = 1/\psi$ or $\gamma = 10, \psi = 0.1$, the general model degenerates to a framework similar to that in Veronesi (2000). Figure 3 plots the conditional equity premium and return volatility in that CRRA framework. Consistent with Veronesi (2000), a CRRA utility function leads to a negative equity premium and a negative correlation between the consumption shocks and the return for the aggregate consumption claim. More importantly, the CRRA framework does not price the liquidity shocks, as shown by the dashed curve in Panel A. The intuition is that in the Veronesi (2000) framework, a high risk aversion (γ) translates to a high agents' hedging demand for the equity after bad news in dividends (or consumption), which counterbalances the negative pressure on prices from the negative dividend (or consumption) shock. The dominance of the former effect results in a negative equity premium. Our model solution clearly illustrates this effect. With CRRA preference ($\gamma = 1/\psi$), equation (3.12a) suggests the risk price for consumption shock is equal to $\gamma\sigma_C$, which is a positive constant. Equation (3.12b) suggests that the liquidity shock is not priced. Thus, the equity premium is largely proportional to the return volatility (see equation (3.13)). The solutions for the return volatility are given by equations (D.1.11b) and (D.1.11c) in the appendix. The solutions suggest that when the wealth-consumption ratio is decreasing with the posterior π (or countercyclical), the return for the aggregate consumption claim co-varies negatively with the consumption and liquidity shocks. Furthermore, the increase in the variance of the posterior (or shifting of π to the middle point of its range) make the consumption flow (or cash flow of the claim) be more volatile, therefore raising the return variance. Thus, as shown in figure 3, when the posterior shifts from either side of its range to the middle, the return volatility and equity premium get more negative.

Figure 4 illustrates the difference in the wealth-consumption ratio generated by the base model (solid curve) and the CRRA framework (dashed curve). The base model implies a procyclical wealth-consumption ratio (or the wealth-consumption ratio is higher in the high-growth state), which is consistent with the empirical evidence. However,the CRRA framework implies a countercylical wealth-consumption ratio. Intuitively, the posterior (π) affects the wealth-consumption ratio through the interaction between the income effect and the substitution effect. First, the higher the posterior (π), the higher the expected growth rate. This property makes the economic agent be more wealthier and consume more, hence leading to a lower wealth-consumption ratio. This is the income effect. Second, the substitution effect makes the agent save today for consuming more tomorrow, indicating a higher wealth consumption ratio. When $\psi > 1$, the second effect dominates the first one, translating to an increasing wealth consumption ratio. When $\psi < 1$, we obtain the opposite result.

So far, we have shown that the base model generates a positive liquidity risk premium. The aggregate liquidity works as an informational channel which helps investors learn about the state of growth in the economy. The liquidity premium are positively related to the uncertainty about the growth rate. In the next, we quantitatively gauge the effect of the liquidity signal precision on the equity premium and return volatility. We calibrate the model with a high precision of the liquidity signal. In the new calibration, the volatility of the liquidity growth is one third of the one used in the base model calibration. Figure 5 plots the model-implied results with respect to different levels of the posterior.

Figure 5 shows that with a relatively high precision of the liquidity signal, the model generates a much higher equity premium and return volatility than those in figure 2. In particular, at the long-run posterior ($\pi = 0.68$), the equity premium and the return volatility are respectively around 7 and 2 times the correspondents generated in the base model calibration. The high precision helps amplify the positive effects of the investor's uncertainty on the equity premium and return volatility. Furthermore, comparing the solid and dashed curves in both panels shows that the liquidity shocks contribute to most of the total equity premium and the return volatility. Intuitively, with a higher precision, a realized liquidity shock makes the economic agents adjust their belief of the growth state more strongly, hence increasing the volatility of the posterior (see equation (3.5)). As discussed earlier, the higher volatility of the posterior translates to a higher uncertainty about the growth state, hence raising the return volatility and the equity premium.

Real Yield Curve

Figure 6 plots the model-implied real yield curve for the government bonds in the economy with respect to different levels of the posterior (π). The shape of the yield curve depends on the posterior. When the investor puts a high belief on the high-growth state ($\pi = 0.9$), the model generates a downward sloping real yield curve. When the investor believes more in the low-growth state ($\pi = 0.3$), the model generates an upward sloping yield curve.

Intuitively, when the posterior is high (or π is at the right side of its range), a negative consumption or liquidity shock raises the investor's uncertainty about the consumption growth rate. Hence, the investor favors more the long-term real bonds to hedge the long run uncertainty in consumption. The relatively higher price in the long-term real bonds translates to a relatively lower yield in the long maturity. When the posterior is low (or π is at the left side of its range), a negative consumption or liquidity shock brings down the investor's uncertainty about the consumption growth rate and the opposite reasoning follows.

3.3.3 Levered equity premium and yield spreads for an average firm

In this section, we study the model-implied levered equity premium and yield spreads for an average firm in the economy.

Figure 7 plots the conditional levered equity premium as a function of the posterior (π). The hump-shaped pattern is similar to that in figure 2. Along with similar line of reasoning in the section of aggregate consumption claim, the posterior affects the equity premium through two channels. First, shifting the posterior to the middle increases the uncertainty about the growth and hence the risk prices. Second, shifting the posterior to the middle brings up the return volatility for the firm's equity. The combined effect of the two channels is to increase the levered equity premium.

Figure 8 reports the model-implied yield spreads with respect to π for two cases: with (solid curve) and without (dashed curve) liquidity shocks. First, the plot suggests that in both cases, the yield spread is a hump-shaped function of the posterior

(π). The intuition is that the bond risk premium increases with the uncertainty about the state while the uncertainty rises with the posterior's (π) shift to the middle of its range. More importantly, the wide gap between the solid and dashed curve shows that the liquidity risk premium constitutes a significant part of the yield spread. At the long-run probability of high-growth ($\pi = 0.68$), the model indicates a total yield spread of around 185 basis points and a liquidity premium of around 55 basis points. These numbers are consistent with the empirical estimates for BAA-rated corporate bond.[12] Furthermore, the liquidity premium seems to decrease with the posterior (π). In the model, a low expected economic growth rate leads to a relatively high liquidity premium. The plot suggests a countercyclical bond liquidity premium. This feature is consistent with the empirical evidence in the literature.[13]

3.4 Conclusions

This study proposes a consumption-based learning model to study the interactions among aggregate liquidity, asset prices and macroeconomic variables in the economy. By incorporating aggregate liquidity as an informational channel for the economic agents to learn about hidden economic states, the model generates a positive market price for liquidity risk. This feature helps generate reasonable risk-free rates, equity premium, real yield curve, and asset prices in equity and bond markets.

We find that the model helps explain many empirical facts in the literature, including procyclical risk-free rates and wealth consumption ratios as well as counter-

[12]See Huang and Huang (2003) and Friewald, Jankowitsch and Subrahmanyam (2011).

[13]See Acharya, Amihud and Bharath (2010).

cyclical equity premium and return volatility. The liquidity risk premium increases with the precision of the fluctuations of liquidity itself. In particular, at the long-run mean of the posterior belief, the model-implied equity premium increases by 7 times and the return volatility increases by 2 times when the liquidity growth volatility decreases 67%.

We apply the model-implied pricing kernel to price the contingent claims of an average firm in the economy. The model generates reasonable levered equity premium and bond yield spread. More importantly, the model suggests that liquidity risk premium contributes significantly to the total yield spread of the corporate bonds. The magnitude and dynamics of the bond liquidity premium is consistent with the empirical evidence. At the long-run mean of the posterior belief, the model implies a total yield spread of around 185 basis points and a liquidity premium of around 55 basis points. The model also generates a countercyclical bond liquidity premium.

Table 3.1: Parameter Values.[1]

Parameter	Symbol	State 1	State 2
Expected consumption growth	θ_C	0.0386	0.0071
Consumption growth volatility	σ_C	0.0286	0.0286
Expected liquidity growth	θ_L	0.2046	-0.1683
Liquidity growth volatility 1	$\sigma_{L,1}$	0.0418	0.0418
Liquidity growth volatility 2	$\sigma_{L,2}$	0.1873	0.1873
Probability of switching	λ	0.0921	0.1843
Expected earnings growth	θ_X	0.0748	-0.0551
Earnings growth volatility 1	$\sigma_{X,1}$	0.0353	0.0353
Earnings growth volatility 2	$\sigma_{X,2}$	0.1513	0.1513
Earnings growth volatility 3	$\sigma_{X,3}$	0.1958	0.1958
Relative risk aversion	γ	10	10
Time preference	β	0.02	0.02
Elasticity of intertemporal substitution	ψ	2	2

[1] This table reports the parameter values used in the base model. We use quarterly real non-durable goods plus service consumption expenditure from the Bureau of Economic Analysis and the quarterly nonfinancial firms' earnings data from the Compustat database to calibrate $\theta_{C,1}$, $\theta_{C,2}$, σ_C, $\theta_{X,1}$, $\theta_{X,2}$, $\sigma_{X,1}$ and $\sigma_{X,2}$ and λ. We use the noise data in Hu, Pan and Wang (2011) to calibrate $\theta_{L,1}$, $\theta_{L,2}$, $\sigma_{L,1}$ and $\sigma_{L,2}$. The idiosyncratic volatility $\sigma_{X,3}$ is calibrated to fit the total volatility for an average firm in the economy (BAA-rated) (25%). State 1 denotes the state with high consumption growth.

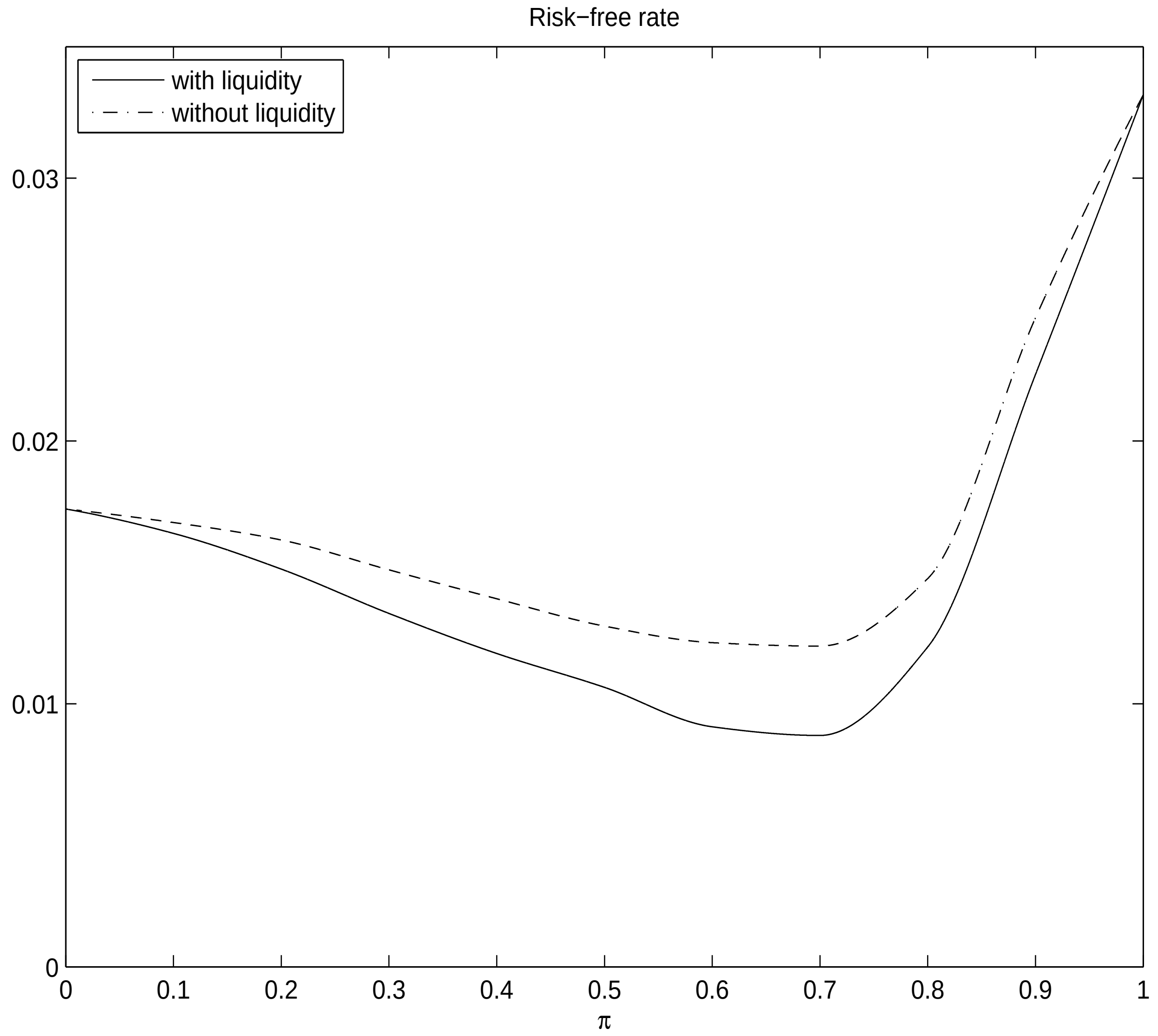

Figure 3.1: Conditional risk-free rate.
This figure plots the risk-free rates with respect to different posterior belief (π) that the state is good. The solid curve corresponds to the case with both consumption and liquidity shocks in the model and the dashed curve corresponds to the case with only consumption shocks. The other parameter values are provided in Table 1.

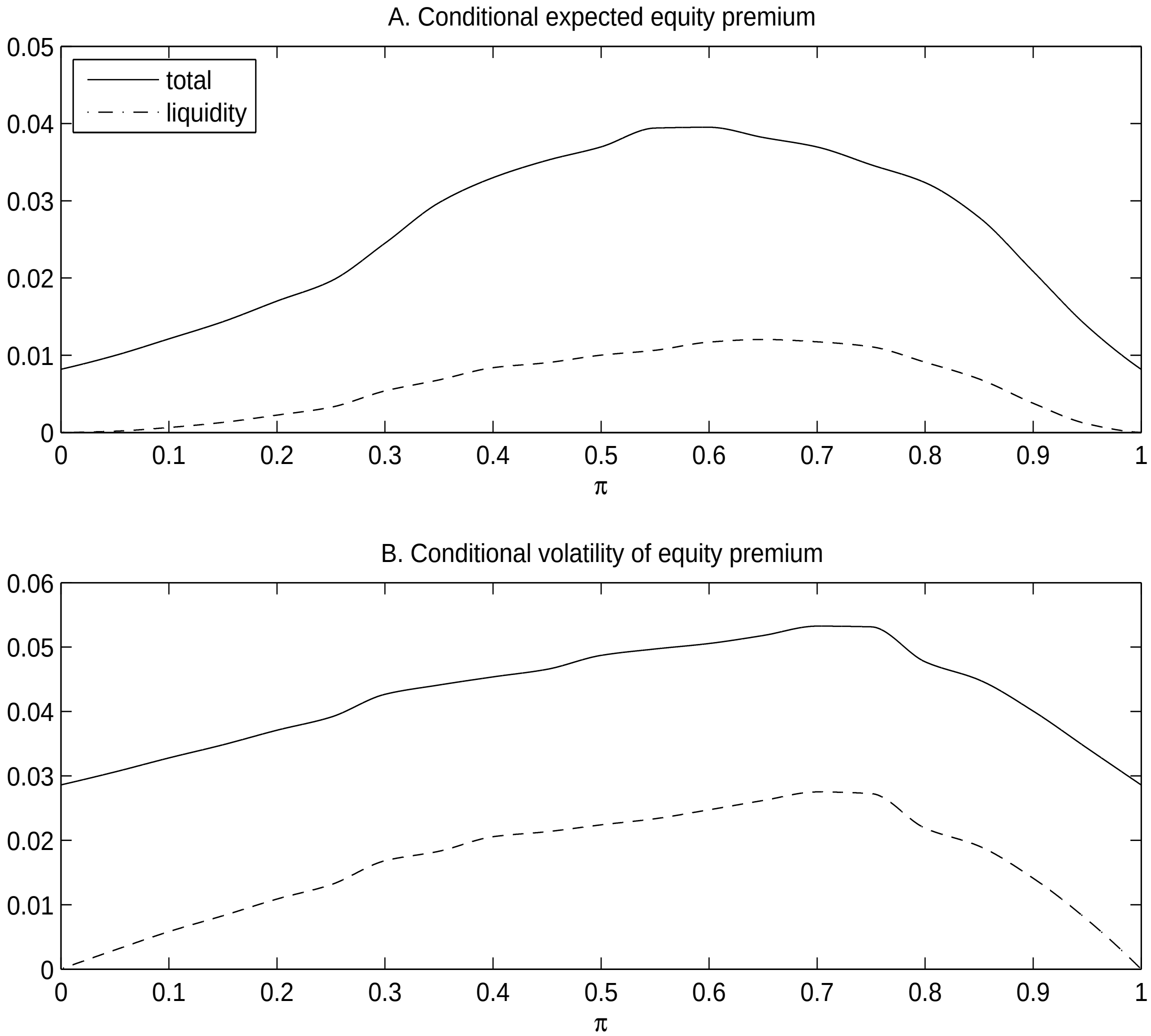

Figure 3.2: Conditional equity premium and conditional return volatility of the aggregate consumption claim in the base model.
This figure plots equity premium (Panel A) and volatility of aggregate consumption claim (Panel B) with respect to different posterior belief (π) that the state is good. In Panel A, the solid curve corresponds to the total equity premium with both consumption and liquidity shocks priced in the model and the dashed curve corresponds to the risk premium due to liquidity shocks. In Panel B, the solid curve corresponds to the total volatility of the return for the aggregate consumption claim with both consumption and liquidity shocks priced in the model and the dashed curve corresponds to the volatility due to liquidity shocks. The other parameter values are provided in Table 1.

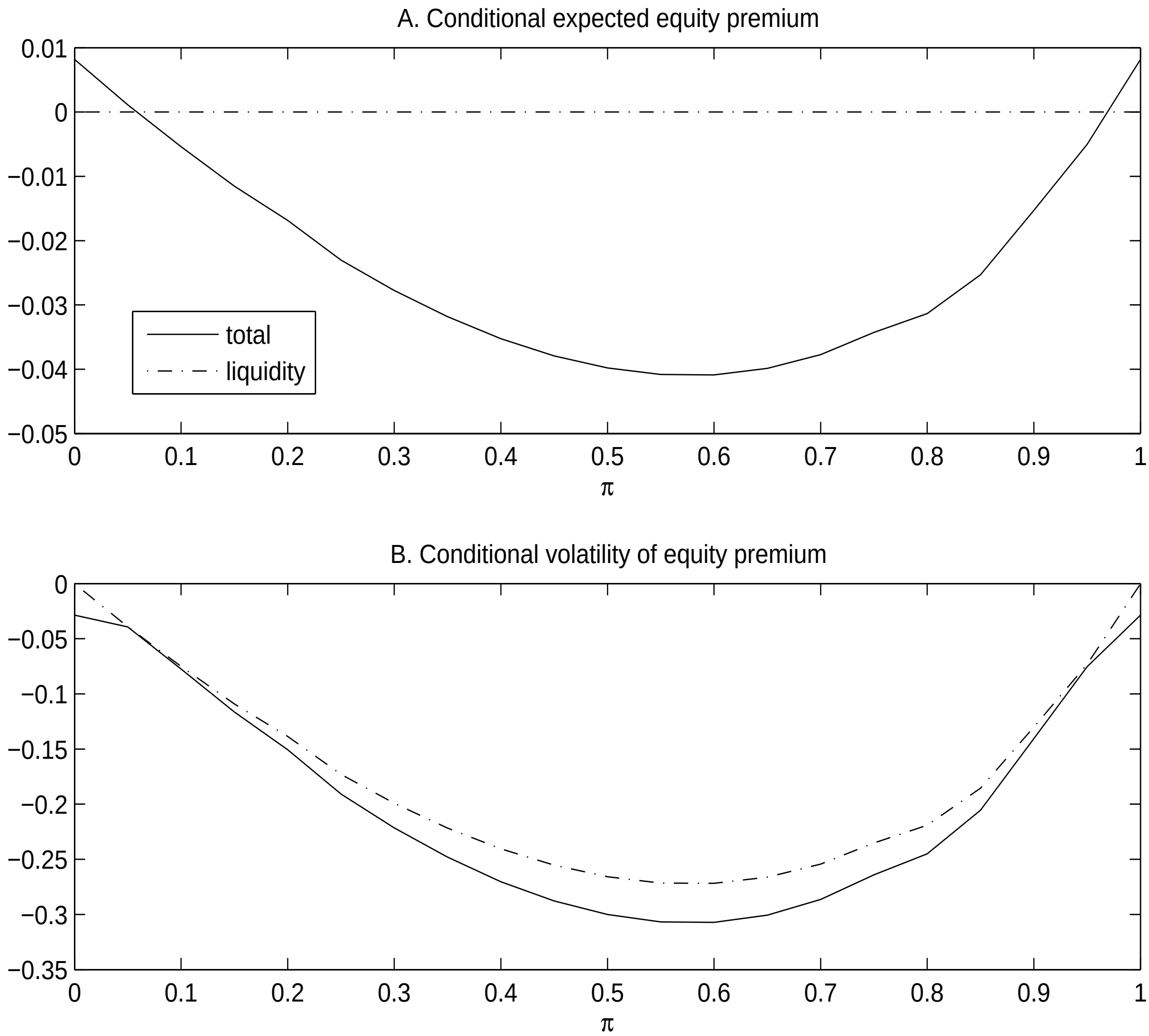

Figure 3.3: Conditional equity premium and conditional return volatility of the aggregate consumption claim in the Veronesi (2000) framework.
This figure plots equity premium (Panel A) and volatility of aggregate consumption claim (Panel B) with respect to different posterior belief (π) that the state is good. In Panel A, the solid curve corresponds to the total equity premium and the dashed curve corresponds to the risk premium due to liquidity shocks. In Panel B, the solid curve corresponds to the total volatility of the return for the aggregate consumption claim and the dashed curve corresponds to the volatility due to liquidity shocks. $\gamma = 10$ and $\psi = 0.1$. The other parameter values are provided in Table 1.

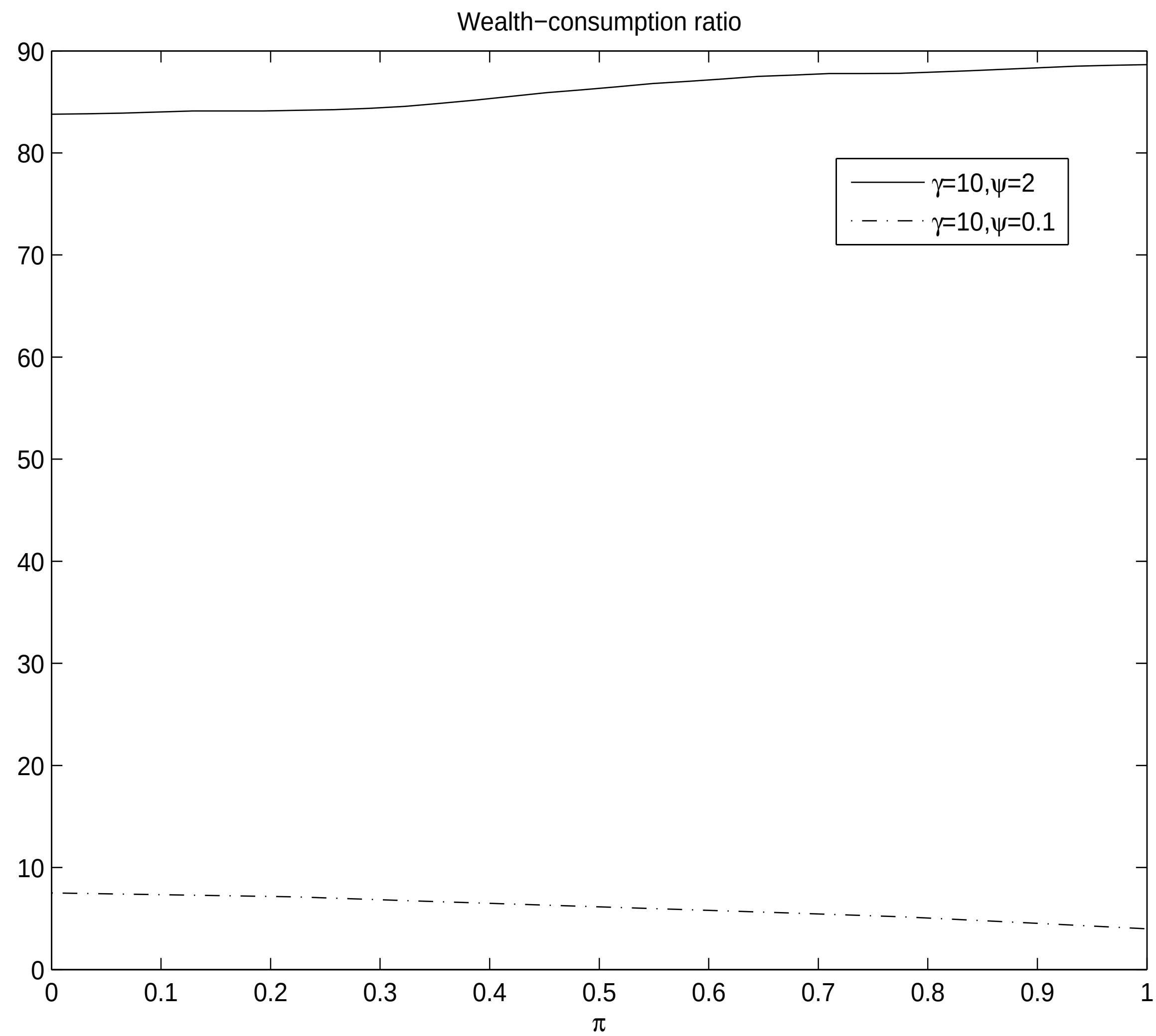

Figure 3.4: Conditional wealth-consumption ratio.
This figure plots the wealth-consumption ratios with respect to different posterior belief (π) that the state is good. The solid curve corresponds to the wealth-consumption ratios implied from the base model and the dashed curve corresponds to the Veronesi (2000) framework ($\gamma = 10$ and $\psi = 0.1$). The other parameter values are provided in Table 1.

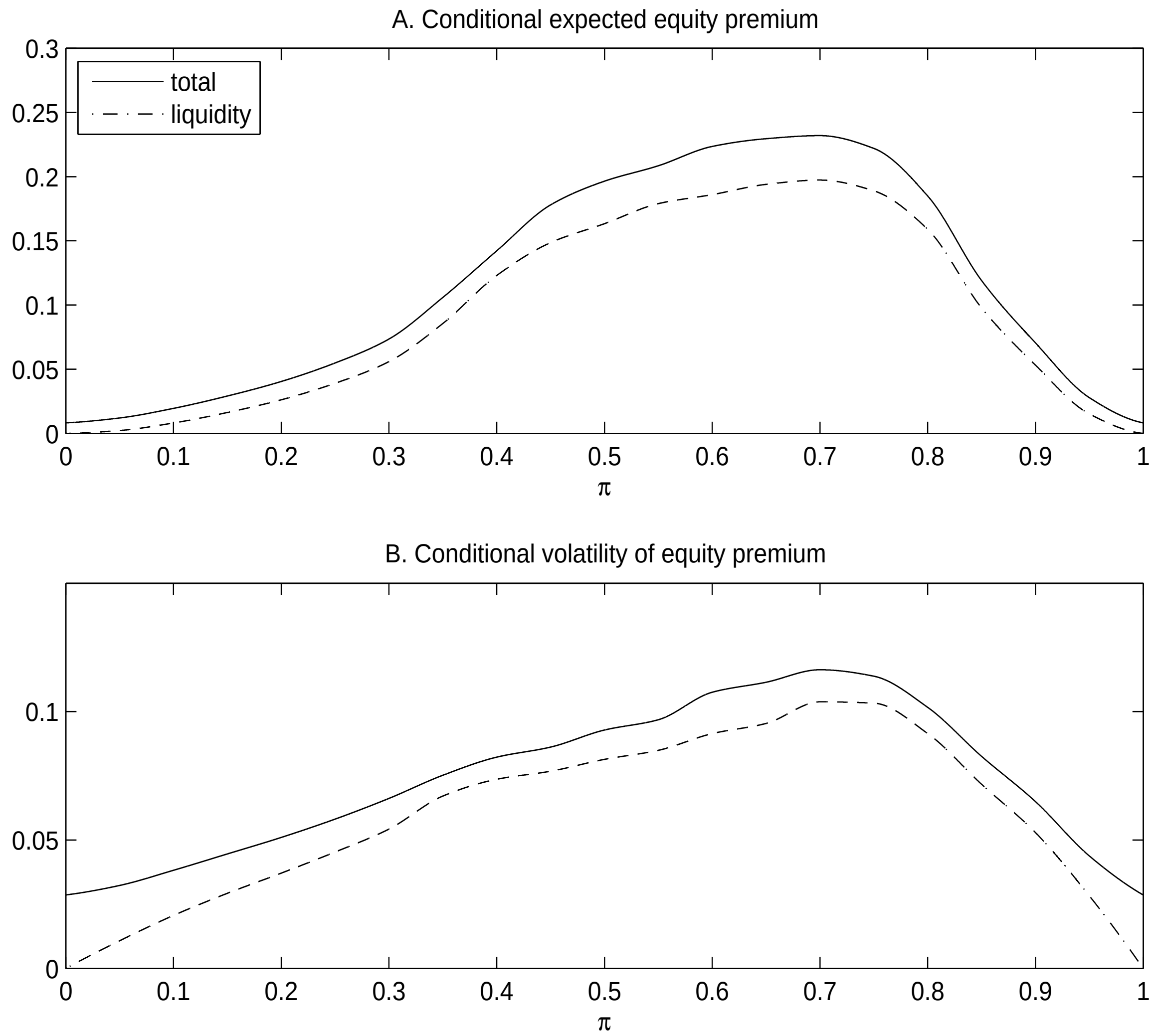

Figure 3.5: Conditional equity premium and conditional return volatility of the aggregate consumption claim with high precision of liquidity signals.
This figure plots equity premium (Panel A) and volatility of aggregate consumption claim (Panel B) with respect to different posterior belief (π) that the state is good. In Panel A, the solid curve corresponds to the total equity premium with both consumption and liquidity shocks priced in the model and the dashed curve corresponds to the risk premium due to liquidity shocks. In Panel B, the solid curve corresponds to the total volatility of the return for the aggregate consumption claim with both consumption and liquidity shocks priced in the model and the dashed curve corresponds to the volatility due to liquidity shocks. The liquidity growth volatilities are one third of those in Table 1 ($\sigma_{L,1} = 0.0125$ and $\sigma_{L,2} = 0.0562$). The other parameter values are provided in Table 1.

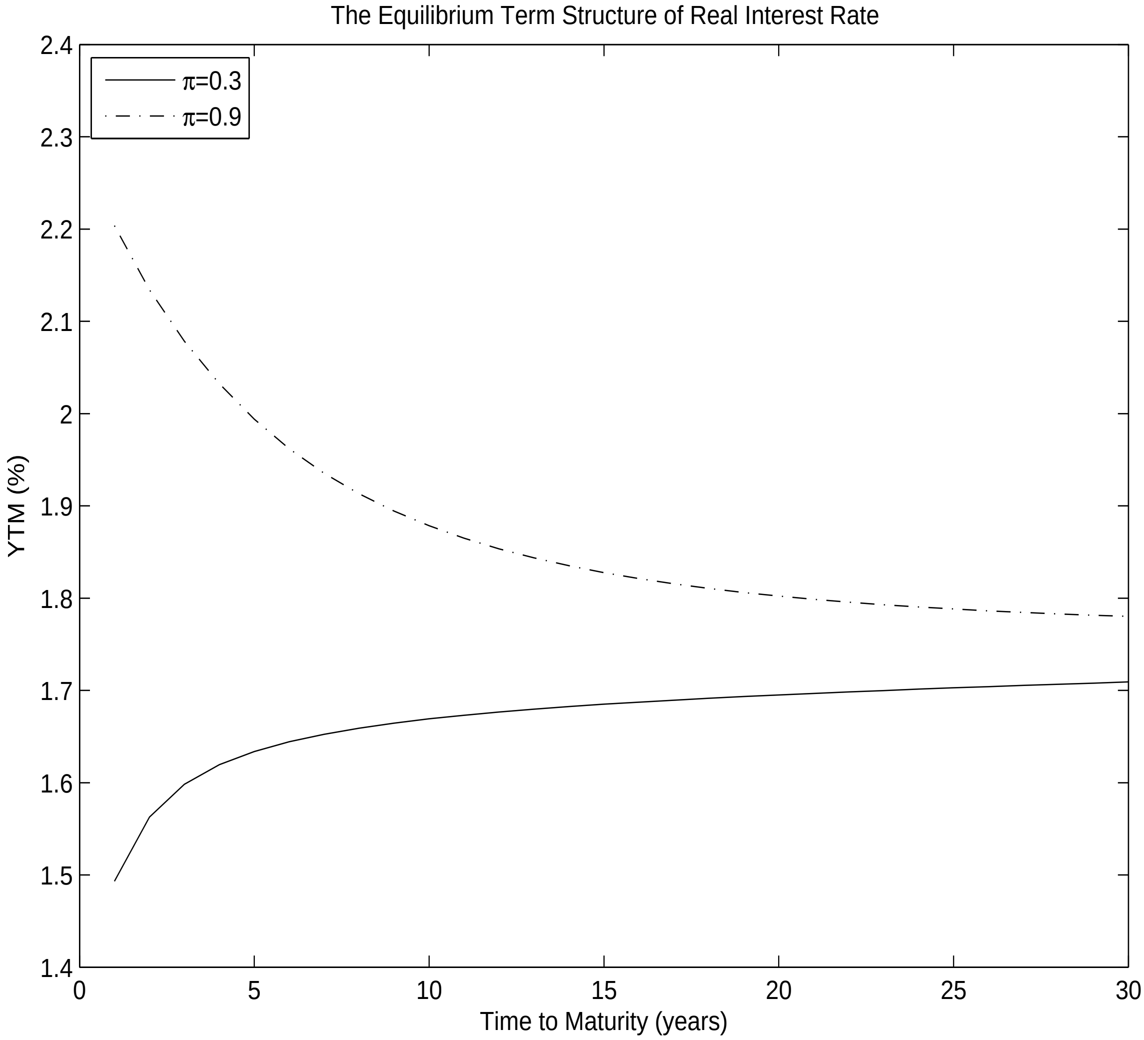

Figure 3.6: Equilibrium term structure of real interest rate.
This figure plots the real yield curve with respect to two different levels of posterior beliefs (π) that the state is good: $\pi = 0.3$ (solid) and $\pi = 0.9$ (dashed). The other parameter values are provided in Table 1.

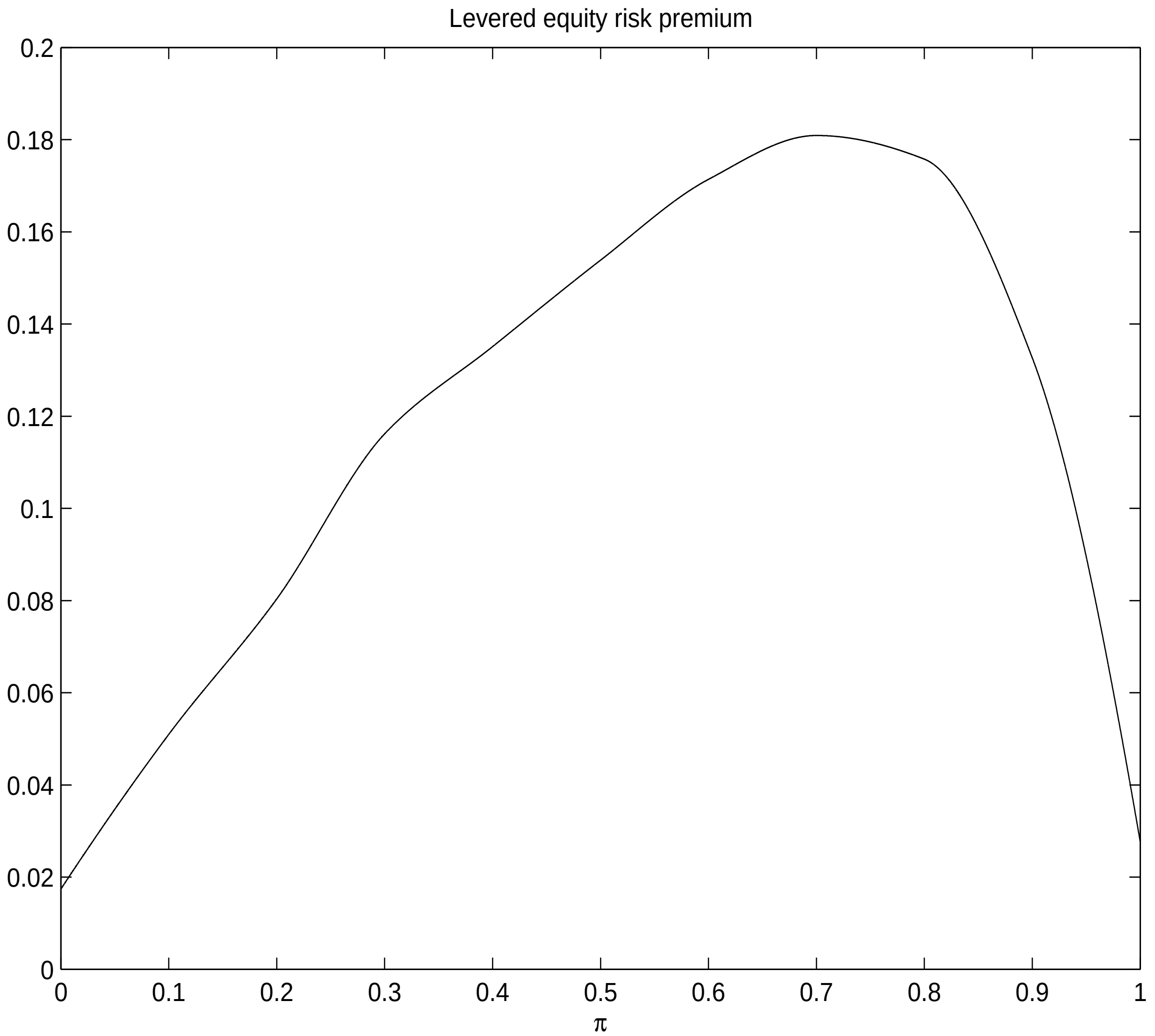

Figure 3.7: Levered equity premium for an average firm in the economy. This figure plots the levered equity premium of an average firm in the economy with respect to different posterior beliefs (π) that the state is good. The other parameter values are provided in Table 1.

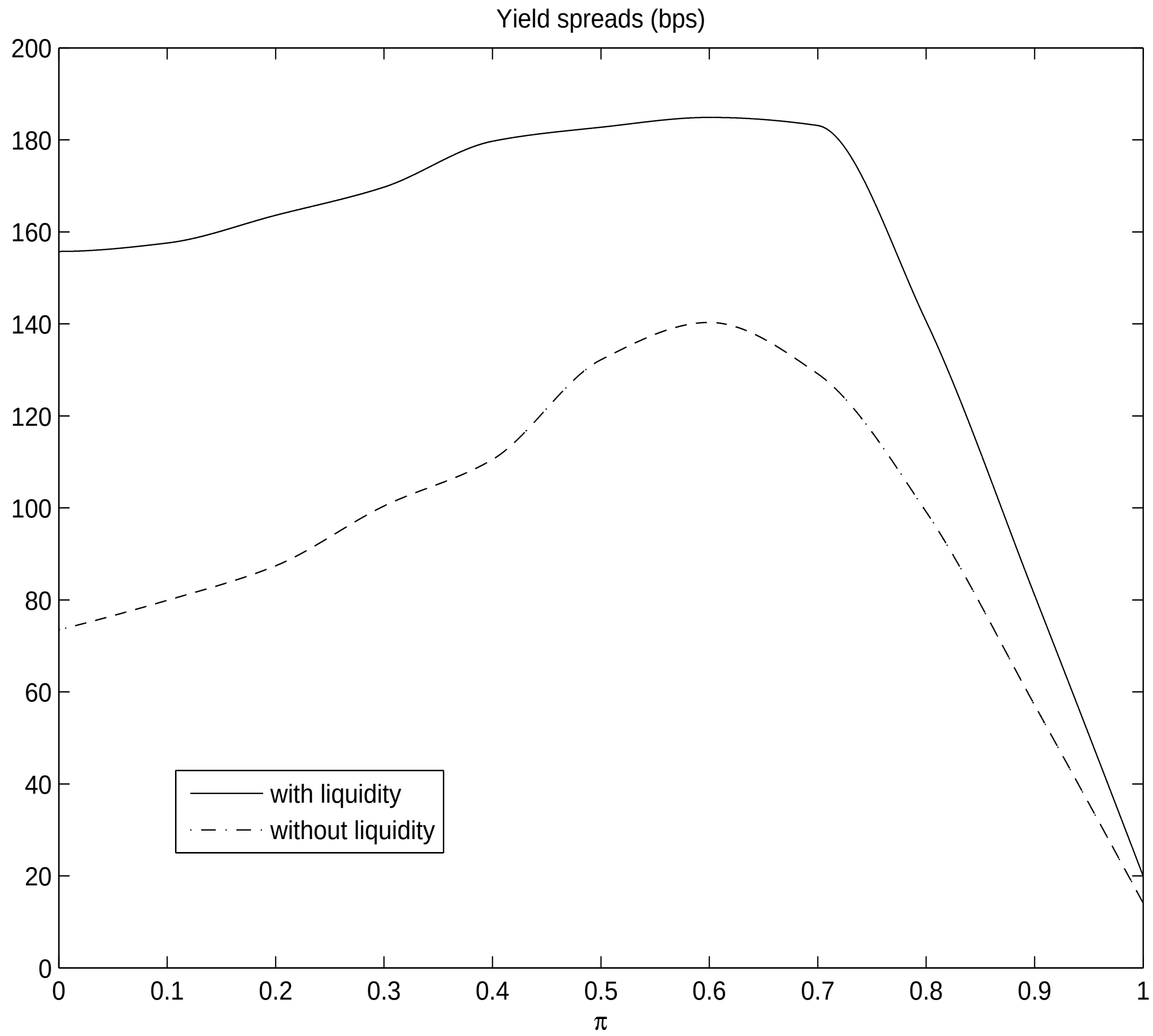

Figure 3.8: Yield spreads for an average firm in the economy.
This figure plots the yield spreads of an average firm in the economy with respect to different posterior beliefs (π) that the state is good. The solid curve corresponds to the case with both consumption and liquidity shocks in the model and the dashed curve corresponds to the case with only consumption shocks. The other parameter values are provided in Table 1.

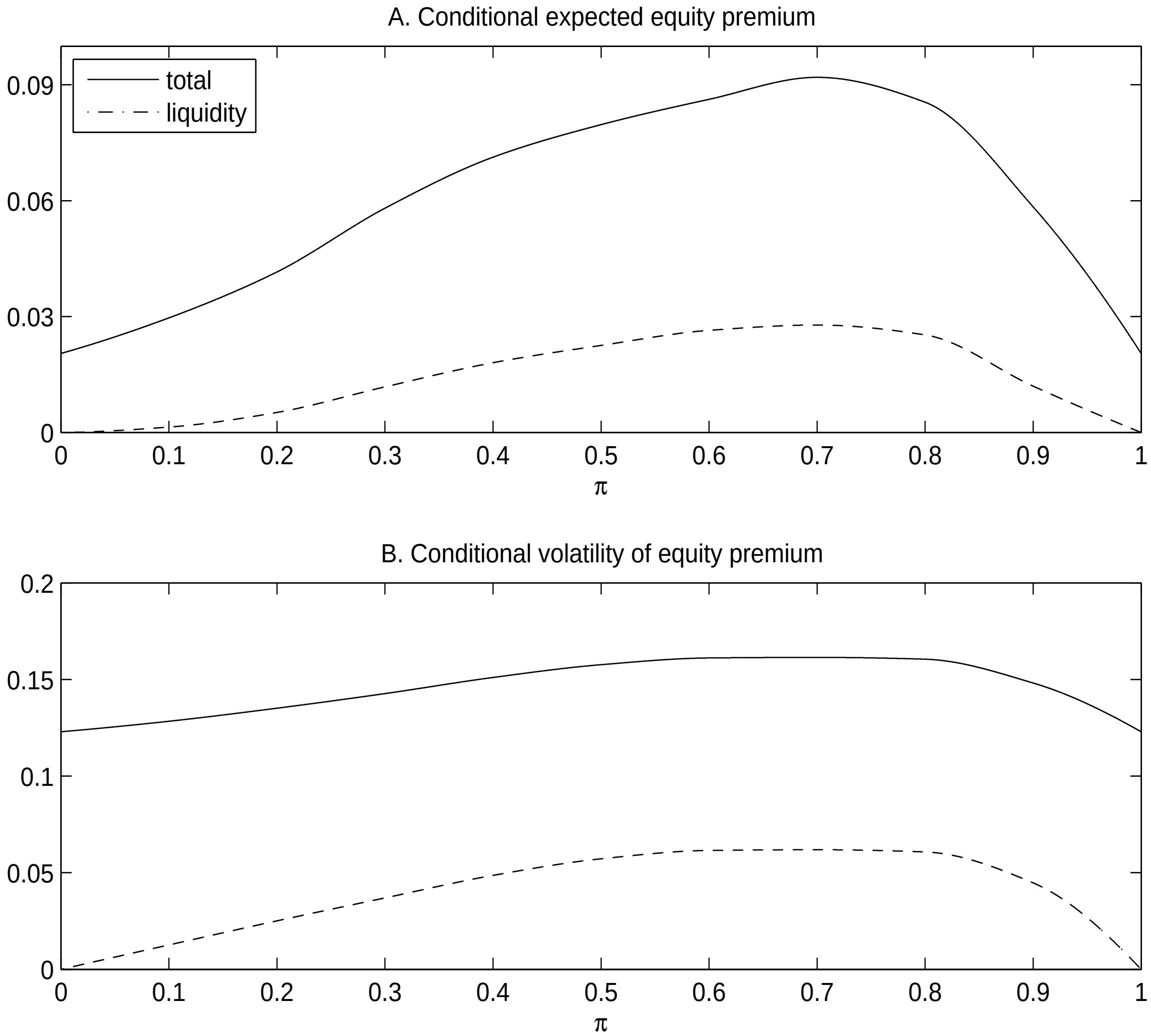

Figure 3.9: Equity premium of the stock market.

This figure plots the implied stock market equity premium and return volatility with respect to different posterior beliefs (π) that the state is good. The solid curve corresponds to the case with both consumption and liquidity shocks in the model and the dashed curve corresponds to the case with only consumption shocks. The other parameter values are provided in Table 1. Calibrated to the data, the dividend growth process Y_t follows $\frac{dY_t}{Y_t} = (\phi\theta_{C,s_t} - \omega)dt + \phi\sigma_C dz_{1,t} + \sigma_Y dz_{y,t}$ with $\phi = 2.5$, $\omega = 0.03$ and $\sigma_Y = 0.11$.

APPENDIX A
ADDITIONAL TABLE AND FIGURE FOR CHAPTER 1

This appendix provides a table with a list of six bankruptcy cases involving the judgement from the court on the filing intentions and a figure illustrating how the Moody's global corporate default rates change with business cycles.

Table A.1: Examples of Voluntary Filing of Chapter 11 Bankruptcy (in Bad/Good Faith).

Company Name	**Background**
Little Creek Development	Little Creek obtained a loan from Commonwealth for the purpose of developing town homes on two tracts of land. In March 1984, the parties signed promissory notes, deeds of trust, and other financing documents that would commit Commonwealth to fund up to $4.7 million for the project.In January 1985, Little Creek filed a petition for reorganization under Chapter 11 of the Bankruptcy Code. Commonwealth moved in the bankruptcy court for relief from the automatic stay, seeking to foreclose the property. The bankruptcy court lifted the automatic stay after concluding the bankruptcy petition was filed in order to escape the necessity of posting a substantial bond. The Fifth Circuit Court found that more evidence was necessary to support the bankruptcy court's conclusions and remanded the case.
Carolin Corporation	In the summer of 1986, Carolin defaulted on a purchase money promissory note. On December 3, 1986–fifty minutes before a scheduled foreclosure sale under the deed of trust–the company filed for Chapter 11 protection. The filing automatically stayed foreclosure. The following day, Carolin's secured creditor filed a motion in the bankruptcy court seeking relief from the automatic stay, adequate protection, conversion of the case to Chapter 7 or, in the alternative, dismissal of Carolin's Chapter 11 petition. On April 24, 1987, the bankruptcy court entered an order dismissing Carolin's Chapter 11 case for "lack of good faith in filing the petition."

Table A.1 continued

SGL Carbon	In 1997, the United States Department of Justice commenced an investigation of alleged price-fixing by graphite electrode manufacturers including SGL Carbon. Shortly thereafter, various steel producers filed class action antitrust lawsuits against SGL Carbon and other manufacturers. On December 16 of 1998, SGL Carbon filed a voluntary Chapter 11 bankruptcy petition in the U.S. District Court for Delaware. In January 1999, the Committee of Unsecured Creditors filed a motion to dismiss SGL Carbon's bankruptcy petition on the grounds that it was a "litigation tactic designed to frustrate the prosecution of the civil antitrust claims pending against SGL and preserve SGL's equity from these claims". The district court denied the motion to dismiss, but later on, the U.S. Third Circuit Court reversed the order because SGL Carbon's Chapter 11 petition lacked the requisite good faith.
Fraternal Composite Services, Inc.	On July 1, 1999, Karczewski filed a petition with the New York State Supreme Court, seeking the judicial dissolution of the corporation pursuant to New York Business Corporation Law. On August 3, 1999, the corporation elected to purchase Appellee's one-third interest rather than having the corporation dissolved. Before the court hearing on this issue, on April 29 of 2003, the corporation filed a voluntary petition pursuant to Chapter 11 of the Bankruptcy Code. On July 11 of 2003, Karczewski filed a motion to dismiss the Chapter 11 petition. On October 16 of 2003, the Bankruptcy court issued a Letter Decision and Order granting Karczewski's motion. The corporation's intent was to use the bankruptcy process solely as a means to delay, frustrate and relitigate the state-court issues.
Integrated Telecom, Inc.	Early in 2001, Integrated negotiated a lease of real property with NMSBPCSLDHB for a lease term of 10 years starting on Feb 23, 2001. Later on, the market for Integrated's products deteriorated and the firm faced a class action alleging claims in the amount of $93.24 million for various violations of the Securities Act of 1934 relating to the company's IPO. In Oct 2002, it filed a voluntary petition for relief under Chapter 11. Shortly thereafter, the landlord filed a motion to dismiss the Chapter 11 case alleging Integrated had not filed its voluntary petition in good faith and that the case was filed to interfere with the Landlord's claims and to increase any distributions to Integrated's shareholders. Both the bankruptcy court and the district court denied the landlord's motion to dismiss. The landlord then appealed the Third Circuit, who reversed the order of the district court and remanded the bankruptcy court to dismiss Integrated's bankruptcy petition.

Table A.1 continued

General Growth Properties	Since the fall of 2008, many of the GGP group properties faced covenant defaults. Some lenders began exercising cash control and other remedies over properties that generated sufficient cash flow to cover their own operating expenses. Certain other properties faced loan maturity or "hyperamortization" in time-frames ranging from the next few months to years. On April 16 of 2009, GGP filed voluntary petitions under Chapter 11 of the Bankruptcy code. Shortly thereafter, several secured creditors filed motions to dismiss the Chapter 11 cases. Six motions were filed, with one party subsequently withdrawing its motion. The primary ground on which dismissal is sought is that the cases were filed in bad faith in that there was no imminent threat to the financial viability of the debtors. On August 11 of 2009, the bankruptcy court at the south district of New York denied the motions to dismiss.

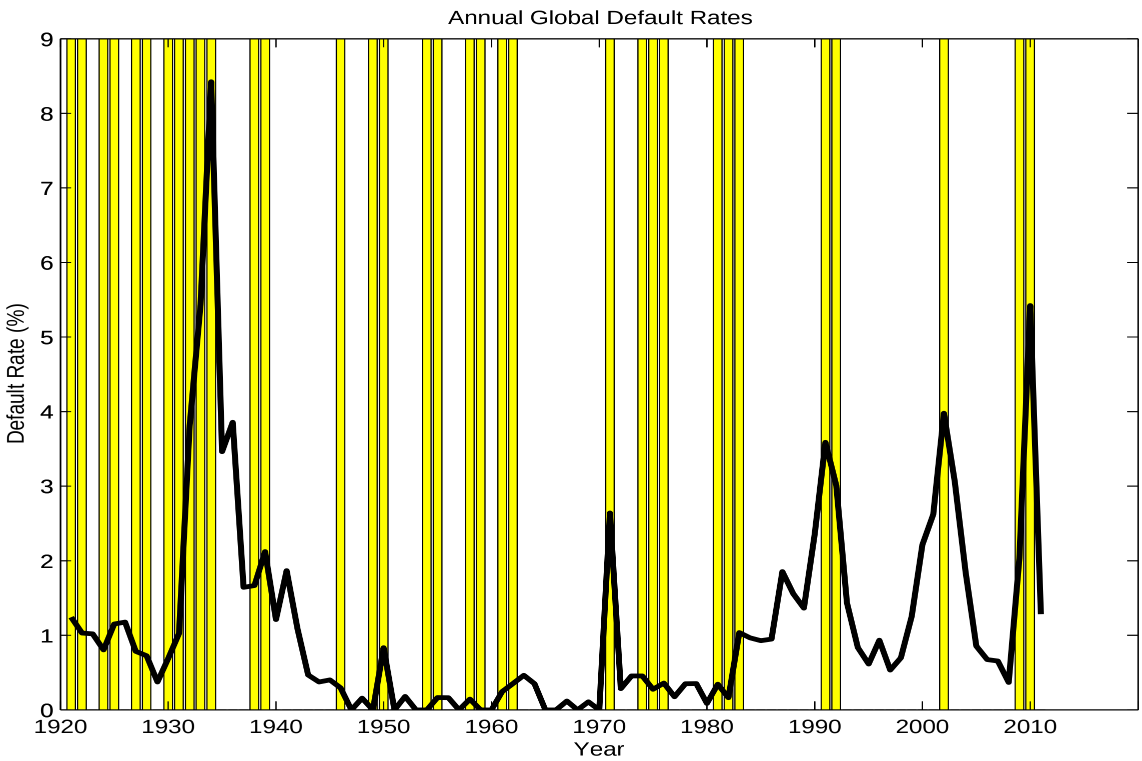

Figure A.1: Annual Moody's Global Corporate Default Rates.
This figure illustrates the time-series of the annual global corporate default rates. Shaded areas are NBER-defined recession periods. Data source: Moody's.

APPENDIX B
OVERVIEW OF THE PROOFS OF PROPOSITIONS AND CALIBRATION METHODOLOGY IN CHAPTER 1

In this appendix, we provide complete expressions of the terms that appear in the propositions. We only give the steps of the derivation in order to save space. We refer readers to Supplementary Appendix for details of the proofs, which is available upon request. In what follows, sections B.1-4 give an overview of the proofs for the propositions. Section B.5 provides the solution for the firm's security values for the case of debt-equity swap. Section B.6 gives the parameter choice for the representative firm and the bankruptcy environment and illustrates an estimation methodology of the strategic debt service, financial distress and liquidation costs.

B.1 Proof of Proposition 1

Given the shareholder's payoff function, the equity value $S_{i,t}$, conditional on current state being i at time t, is

$$S_{i,t} = (1-\eta)E_t\left[\int_t^{\tau_B} \frac{\pi_s}{\pi_t}\left(X_s - c\right)\,ds\,|s_t = i\right] + E_t\left[\frac{\pi_{\tau_B}}{\pi_t}S_{\tau_B}\,|s_t = i\right],\; i\epsilon\{1,2\} \quad \text{(B.1.1)}$$

where S_{τ_B} is the equity value at the Chapter 11 boundary. The right side of equation (B.1.1) is solved term by term. Specifically,

$$E_t\left[\int_t^{\tau_B} \frac{\pi_s}{\pi_t} X_s\,ds\,|s_t = i\right] = V_i(X_t) - \sum_{j=1}^{2} q_{ij}^D V_{B,j}, \quad \text{(B.1.2)}$$

$$E_t\left[\int_t^{\tau_B} \frac{\pi_s}{\pi_t} c\,ds\,|s_t = i\right] = c\left(\frac{1}{r_{C,i}} - \sum_{j=1}^{2}\frac{q_{ij,t}^D}{r_{C,j}}\right), \quad \text{(B.1.3)}$$

and

$$E_t\left[\frac{\pi_{\tau_B}}{\pi_t}S_{\tau_B}\,|s_t=i\right]=\sum_{j=1}^{2}\left(q_{ij,t}^{D}S_{B,j}\right), \tag{B.1.4}$$

where $q_{ij,t}^{D}$ denotes the time-t Arrow-Debreu price of a claim in state i that pays 1 unit of consumption conditional on the event that Chapter 11 will occur in state j. The solutions for $r_{C,i}$, $q_{ij,t}^{D}$ are given in Supplementary Appendix B. Plugging equations (B.1.2)-(B.1.4) into equation (B.1.1), yields

$$S_{i,t}=V_i(X_t)-\frac{(1-\eta)c}{r_{C,i}}+\sum_{j=1}^{2}q_{ij}^{D}\left[\frac{(1-\eta)c}{r_{C,j}}-V_{B,j}\right]+\sum_{j=1}^{2}\left(q_{ij,t}^{D}S_{B,j}\right) \tag{B.1.5}$$

where $S_{B,j}$ is the equity value at Chapter 11 boundary with the state of economy being j.

The debt value at time t, conditional on the state being i, is

$$D_{i,t}=E_t\left[\int_t^{\tau_B}\frac{\pi_s}{\pi_t}c\,ds\,|s_t=i\right]+E_t\left[\frac{\pi_{\tau_B}}{\pi_t}D_{\tau_B}\,|s_t=i\right],\ i\epsilon\{1,2\} \tag{B.1.6}$$

where D_{τ_B} is the debt value at the Chapter 11 boundary. The second term in equation (B.1.6) is

$$E_t\left[\frac{\pi_{\tau_B}}{\pi_t}D_{\tau_B}\,|s_t=i\right]=\sum_{j=1}^{2}\left(q_{ij,t}^{D}D_{B,j}\right). \tag{B.1.7}$$

Plugging equations (B.1.3) and (B.1.7) into equation (B.1.6) yields

$$D_{i,t}=c\left(\frac{1}{r_{C,i}}-\sum_{j=1}^{2}\frac{q_{ij,t}^{D}}{r_{C,j}}\right)+\sum_{j=1}^{2}\left(q_{ij,t}^{D}D_{B,j}\right), \tag{B.1.8}$$

where $D_{B,j}$ is the debt value at the Chapter 11 boundary with the state of economy being j. The levered asset value or firm value $v_{i,t}$ is equal to the sum of $S_{i,t}$ and $D_{i,t}$.

The details on the proof of equations (B.1.2)-(B.1.4) and equation (B.1.7) are given in Supplementary Appendix B.

B.2 Proof of Proposition 2

Given the Chapter 11 boundary $V_{B,i}$ and the Chapter 7 boundary $V_{L,i}$, with the state of economy being i at the Chapter 11 boundary, the firm value at default is

$$v_{B,i} = E^Q \left[\int_0^\tau e^{-r_i t} \left(\delta V_t + \eta c 1_{V_t > V_{B,i}} - \omega V_t 1_{V_{L,i} < V_t < V_{B,i}} \right) dt \right] + \alpha_i E^Q \left[e^{-r_i \tau} V_{i,\tau} \right], \tag{B.2.1}$$

where the liquidation time $\tau = \tau_2 \wedge \tau_4$ with τ_2 being the time of liquidation due to spending more than the grace period in default and τ_4 being the time of liquidation due to limited liability violation. We reorganize quation (B.2.1) as

$$\begin{aligned} v_{B,i} = & E^Q \left[\int_0^\tau e^{-r_i t} \left(\delta V_t 1_{V_t > V_{B,i}} + \eta c 1_{V_t > V_{B,i}} + (\delta - \omega) V_t 1_{V_{L,i} < V_t < V_{B,i}} \right) dt \right] \\ & + \alpha_i E^Q \left[e^{-r_i \tau} V_{i,\tau} \right]. \end{aligned} \tag{B.2.2}$$

We obtain the solution for each term in equation (B.2.2) as follows.

$$\begin{aligned} & E^Q \left(\int_0^\tau e^{-r_i t} V_t 1_{V_t > V_{B,i}} \, dt \right) \\ & = V_{B,i} \tfrac{1}{\lambda(\lambda - \sigma_{X,i} - b)} - V_{B,i} \left[\tfrac{1}{\lambda(\lambda - \sigma_{X,i} - b)} \cdot \tfrac{\Psi(-\lambda\sqrt{d}) - F(\lambda)}{\Psi(\lambda\sqrt{d})} P(\tau_2 < \tau_4) \right. \\ & \left. + \tfrac{e^{2\lambda z_L}}{\lambda(\lambda - \sigma_{X,i} - b)} P(\tau_2 > \tau_4) \right], \end{aligned} \tag{B.2.3}$$

where

$$\Psi(x) \equiv \int_0^\infty z e^{zx - \frac{z^2}{2}} dz = 1 + \sqrt{2\pi} x e^{\frac{x^2}{2}} \phi(x), \tag{B.2.4}$$

and

$$F(x) \equiv e^{\frac{dx^2}{2}} \left[e^{-\frac{(z_L - dx)^2}{2d}} - x\sqrt{2\pi d} \phi \left(\frac{z_L - dx}{\sqrt{d}} \right) \right], \tag{B.2.5}$$

with $\phi(\cdot)$ being the density function for standard normal distribution, $b = \left(\hat{\theta}_i - \frac{1}{2}\sigma_{X,i}^2 \right) / \sigma_{X,i}$, $\lambda = \sqrt{2r_i + b^2}$, $z_L = \frac{1}{\sigma_{X,i}} \ln \frac{V_{L,i}}{V_{B,i}}$ and $\sigma_{X,i} = \sqrt{\sigma_{X,i}^{s2} + \sigma_{X,i}^{id2}}$.

The solution for the second term in equation (B.2.2) is given by

$$E^Q\left[\int_0^\tau e^{-r_i t}\eta c 1_{X_t > X_{B,i}} dt\right] = \eta c\left[\frac{1}{\lambda(\lambda-b)} - \frac{1}{\lambda(\lambda-b)}\cdot\frac{\Psi(-\lambda\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})}P(\tau_2<\tau_4)\right.$$
$$\left.-\frac{e^{2\lambda z_L}}{\lambda(\lambda-b)}P(\tau_2>\tau_4)\right], \tag{B.2.6}$$

where $\Psi()$ and $F()$ are defined in equation (B.2.4) and equation (D.2.4), respectively.

The solution for the third term in equation (B.2.2) is given by

$$(\delta-\omega)E^Q\left(\int_0^\tau e^{-r_i t}V_t 1_{V_{L,i}<V_t<V_{B,i}}\right)$$
$$= V_{B,i}(\delta-\omega)\frac{1-e^{(\sigma_i+b+\lambda)z_L}}{\lambda(\lambda+\sigma_i+b)} - V_{B,i}(\delta-\omega)\left[\left(\frac{-2}{(\lambda+\sigma_i+b)(\sigma_i+b-\lambda)}\cdot\frac{\Psi\left(-(\sigma_i+b)\sqrt{d}\right)-F(\sigma_i+b)}{\Psi(\lambda\sqrt{d})}\right.\right.$$
$$\left.-\frac{e^{(\sigma_i+b+\lambda)z_L}}{\lambda(\lambda+\sigma_i+b)}\frac{\Psi(\lambda\sqrt{d})-F(-\lambda)}{\Psi(\lambda\sqrt{d})} + \frac{1}{\lambda(\sigma_i+b-\lambda)}\frac{\Psi(-\lambda\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})}\right)P(\tau_2<\tau_4)$$
$$\left.+\frac{e^{2\lambda z_L}-e^{(\sigma_i+b+\lambda)z_L}}{\lambda(\sigma_i+b-\lambda)}P(\tau_2>\tau_4)\right]. \tag{B.2.7}$$

The solution for the fourth term in equation (B.2.2) is given by

$$\alpha_i E^Q(e^{-r_i\tau}V_{i,\tau}) = \alpha_i V_{B,i}\left[\frac{\Psi\left(-(\sigma_i+b)\sqrt{d}\right)-F(\sigma_i+b)}{\Psi(\lambda\sqrt{d})}P(\tau_2<\tau_4)\right.$$
$$\left.+\, e^{(\sigma_i+b+\lambda)z_L}P(\tau_2>\tau_4)\right]. \tag{B.2.8}$$

Plugging equations (B.2.3), (B.2.6), (B.2.7) and (B.2.8) into equation (B.2.2) gives

$$v_{B,i} = V_{B,i}+\frac{\eta c}{r_i}-P(\tau_2<\tau_4)\left[V_{B,i}A(d)+\frac{\eta c}{r_i}B(d)+C(d)\right]-P(\tau_2>\tau_4)\left[C(d)+D(d)\right], \tag{B.2.9}$$

where

$$A(d) = \left[\frac{-2(\delta-\omega)}{(\lambda+\sigma_{X,i}+b)(\sigma_{X,i}+b-\lambda)}-\alpha_i\right]\frac{\Psi\left(-(\sigma_{X,i}+b)\sqrt{d}\right)-F(\sigma_{X,i}+b)}{\Psi(\lambda\sqrt{d})}$$
$$-(\delta-\omega)\frac{e^{(\sigma_{X,i}+b+\lambda)z_L}}{\lambda(\lambda+\sigma_{X,i}+b)}\frac{\Psi(\lambda\sqrt{d})-F(-\lambda)}{\Psi(\lambda\sqrt{d})}+\left[\frac{\delta-\omega}{\lambda(\sigma_{X,i}+b-\lambda)}+\frac{\delta}{\lambda(\lambda-\sigma_{X,i}-b)}\right]\frac{\Psi(-\lambda\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})},$$

$$B(d) = \frac{\lambda+b}{2\lambda}\cdot\frac{\Psi(-\lambda\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})},$$

$$C(d) = \frac{\eta c}{r_i}\cdot\frac{\lambda-b}{2\lambda}+\delta V_{B,i}\frac{e^{(\sigma_{X,i}+b+\lambda)z_L}}{\lambda(\lambda+\sigma_{X,i}+b)}+V_{B,i}\frac{\omega\left[1-e^{(\sigma_{X,i}+b+\lambda)z_L}\right]}{\lambda(\lambda+\sigma_{X,i}+b)},$$

$$\begin{aligned} D(d) &= V_{B,i}\tfrac{\delta e^{2\lambda z_L}}{\lambda(\lambda-\sigma_{X,i}-b)} + \tfrac{\eta c e^{2\lambda z_L}}{\lambda(\lambda-b)} + V_{B,i}(\delta-\omega)\tfrac{e^{2\lambda z_L}-e^{(\sigma_{X,i}+b+\lambda)z_L}}{\lambda(\sigma_{X,i}+b-\lambda)} \\ &\quad -\alpha_i V_{B,i} e^{(\sigma_{X,i}+b+\lambda)z_L}. \end{aligned}$$

Extending the methodology proposed by Dassios and Wu (2008) yields

$$P(\tau_2 > \tau_4) = \frac{\sum_{k=0}^{\infty}\phi'\left(\frac{(2k+1)(-z_L)}{\sqrt{d}}\right)}{\sum_{k=1}^{\infty}\phi'\left(\frac{2k(-z_L)}{\sqrt{d}}\right)+\frac{1}{2\sqrt{2\pi}}}. \tag{B.2.10}$$

A detailed proof of Proposition 2 is provided in Supplementary Appendix C.

B.3 Proof of Proposition 3

When the state of economy is i after filing for Chapter 11, given the shareholders' strategic debt service ϑ under Chapter 11, the firm's debt value at the Chapter 11 boundary is

$$\begin{aligned} D_{B,i} &= E^Q\left[\int_0^\tau e^{-r_i t}\left(c1_{V_t>V_{B,i}} + \vartheta c 1_{V_{L,i}<V_t<V_{B,i}}\right)dt\right] + \alpha_i E^Q\left[e^{-r_i\tau}V_{i,\tau}\right] \\ &= \vartheta c E^Q\left[\int_0^\tau e^{-r_i t}1_{V_t>V_{L,i}}dt\right] + (1-\vartheta)cE^Q\left[\int_0^\tau e^{-r_i t}1_{V_t>V_{B,i}}dt\right] + \alpha_i E^Q\left[e^{-r_i\tau}V_{i,\tau}\right]. \end{aligned} \tag{B.3.1}$$

The first term in equation (B.3.1) is given by

$$E^Q\left[\int_0^\tau e^{-r_i t}1_{V_t>V_{L,i}}dt\right] = E^Q\left[\int_0^\infty e^{-r_i t}1_{V_t>V_{L,i}}dt\right] - E^Q\left[\int_\tau^\infty e^{-r_i t}1_{V_t>V_{L,i}}dt\right], \tag{B.3.2}$$

where

$$E^Q\left[\int_0^\infty e^{-r_i t}1_{V_t>V_{L,i}}dt\right] = E^{Q'}\left[\int_0^\infty e^{bz_t}e^{-\frac{\lambda^2}{2}t}1_{z_t>z_{L,i}}dt\right] = \frac{1}{r_i} - \frac{e^{(b+\lambda)z_L}}{\lambda(b+\lambda)}$$

and

$$\begin{aligned} E^Q\left[\int_\tau^\infty e^{-r_i t}1_{V_t>V_{L,i}}dt\right] &= \left[\tfrac{1}{r_i}\cdot\tfrac{\Psi(-b\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})} - \tfrac{e^{(b+\lambda)z_L}}{\lambda(b+\lambda)}\cdot\tfrac{\Psi(\lambda\sqrt{d})-F(-\lambda)}{\Psi(\lambda\sqrt{d})}\right]P(\tau_2<\tau_4) \\ &\quad + \tfrac{1}{\lambda(\lambda-b)}e^{(b+\lambda)z_L}P(\tau_2>\tau_4). \end{aligned}$$

Plugging equations (B.3.2), (B.2.6) and (B.2.8) into equation (B.3.1) gives

$$D_{B,i} = \vartheta c E(d) + (1-\vartheta) c G(d) + \alpha_i V_{B,i} H(d), \tag{B.3.3}$$

where

$$\begin{aligned} E(d) \quad = \quad & \tfrac{1}{r_i} - \tfrac{e^{(b+\lambda)z_L}}{\lambda(b+\lambda)} - \left[\tfrac{1}{r_i} \cdot \tfrac{\Psi(-b\sqrt{d})-F(\lambda)}{\Psi(\lambda\sqrt{d})} - \tfrac{e^{(b+\lambda)z_L}}{\lambda(b+\lambda)} \cdot \tfrac{\Psi(\lambda\sqrt{d})-F(-\lambda)}{\Psi(\lambda\sqrt{d})}\right] P(\tau_2 < \tau_4) \\ & -\tfrac{1}{\lambda(\lambda-b)} e^{(b+\lambda)z_L} P(\tau_2 > \tau_4), \end{aligned}$$

$$G(d) = \frac{1}{\lambda(\lambda-b)}\left[1 - \frac{\Psi(-\lambda\sqrt{d}) - F(\lambda)}{\Psi(\lambda\sqrt{d})} P(\tau_2 < \tau_4) - e^{2\lambda z_L} P(\tau_2 > \tau_4)\right],$$

and

$$H(d) = \frac{\Psi(-(\sigma+b)\sqrt{d}) - F(\sigma+b)}{\Psi(\lambda\sqrt{d})} P(\tau_2 < \tau_4) + e^{(\sigma+b+\lambda)z_L} P(\tau_2 > \tau_4).$$

Supplementary Appendix D provides a detailed proof of Proposition 3. The solution for the strategic debt service ϑ is determined by the shareholders' bargaining power during the reorganization process. The procedure to solve for the equilibrium strategic debt service ϑ is shown in Supplementary Appendix E.

B.4 Proof of Proposition 4

When the firm's unlevered asset value is $V_L < V < V_B$ at time zero, we denote d' as the remaining time to the allowed grace period, τ'_B as the time for the firm's unlevered asset value to hit V_B, τ'_L as the time for the firm's unlevered asset value to drop to V_L, and M_0 and C_0 as the accumulated cash flow position and accumulated coupon after filing for Chapter 11. We define $\tau' \equiv d' \wedge \tau'_L$. Then, the firm's equity value at $V_L < V < V_B$ is

$$S(V,d',M_0,C_0) = (M_0 - \vartheta C_0)P(\tau'_B < \tau') + E^Q\left[\int_0^{\tau'_B} e^{-rt}\left((\delta-\omega)V_t - \vartheta c\right)dt 1_{\tau'_B<\tau'}\right]$$
$$+S_B E^Q\left(e^{-r\tau'_B}1_{\tau'_B<\tau'}\right). \tag{B.4.1}$$

We denote $A_1 = P(\tau'_B < \tau')$ and obtain that

$$\begin{aligned} A_1 &= P(\tau'_B < \tau'_L)P(\tau'_L < d') + P(\tau'_B < d')P(\tau'_L > d') \\ &= \tfrac{e^{bV'}-e^{b(V'_B-V'_L)}}{e^{bV'}-e^{-bV'}} \cdot \left[e^{-2bV'_L}N(b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}}) + N(-b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}})\right] + \left[N(b\sqrt{d'} - \tfrac{V'_B}{\sqrt{d'}})\right. \\ &\quad \left. + e^{2bV'_B}N(-b\sqrt{d'} - \tfrac{V'_B}{\sqrt{d'}})\right] \cdot \left[N(b\sqrt{d'} + \tfrac{V'_L}{\sqrt{d'}}) - e^{-2bV'_L}N(b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}})\right]. \end{aligned} \tag{B.4.2}$$

We also obtain that

$$E^Q\left[\int_0^{\tau'_B} e^{-rt}V_t dt 1_{\tau'_B<\tau'}\right] = \frac{2V}{\lambda^2 - (\sigma+b)^2}\left[A_1 - e^{(\sigma+b)z'_B}f(\frac{\lambda^2}{2},0)\right], \tag{B.4.3}$$

$$E^Q\left[\int_0^{\tau'_B} e^{-rt}dt 1_{\tau'_B<\tau'}\right] = \frac{2}{\lambda^2 - b^2}\left[A_1 - e^{bz'_B}f(\frac{\lambda^2}{2},0)\right], \tag{B.4.4}$$

and

$$E^Q\left(e^{-r\tau'_B}1_{\tau'_B<\tau'}\right) = f(r,b), \tag{B.4.5}$$

where the function f is defined as

$$\begin{aligned} f(x,y) \equiv\ & \left[e^{(y-\sqrt{2x+y^2})V'_B}N(\sqrt{(2x+y^2)d'} - \tfrac{V'_B}{\sqrt{d'}}) + e^{(y+\sqrt{2x+y^2})V'_B}\right. \\ & \left. N(-\sqrt{(2x+y^2)d'} - \tfrac{V'_B}{\sqrt{d'}})\right] \cdot \left[N(b\sqrt{d'} + \tfrac{V'_L}{\sqrt{d'}}) - e^{-2bV'_L}N(b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}})\right] \\ & + \frac{e^{V'_B y}\left[e^{\sqrt{2x+y^2}V'_L} - e^{-\sqrt{2x+y^2}V'_L}\right]}{e^{\sqrt{2x+y^2}V'} - e^{-\sqrt{2x+y^2}V'}} \cdot \left[e^{-2bV'_L}N(b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}}) + N(-b\sqrt{d'} - \tfrac{V'_L}{\sqrt{d'}})\right]. \end{aligned}$$

Plugging equations (B.4.2), (B.4.3)-(B.4.5) into equation (B.4.1) yields

$$\begin{aligned} S(V,d',M_0,C_0) &= (M_0 - \vartheta C_0)A_1 + \tfrac{2(\delta-\omega)V}{\lambda^2-(\sigma+b)^2}\left[A_1 - e^{(\sigma+b)z'_B}f(\tfrac{\lambda^2}{2},0)\right] \\ &\quad - \tfrac{2\vartheta c}{\lambda^2-b^2}\left[A_1 - e^{bz'_B}f(\tfrac{\lambda^2}{2},0)\right] + S_B f(r,b). \end{aligned}$$

Supplementary Appendix F provides a detailed proof of Proposition 4. Also we derive the solution for the debt value of a distressed firm in Supplementary Appendix G.

B.5 Debt-equity Swap

We use $V_{S,i}$ to denote the unlevered asset value at which the debt-equity swap occurs when the state of the economy is i. At the debt-equity swap boundary, the firm becomes an all-equity firm and the firm value is equal to the unlevered asset value since there are no tax benefits or bankruptcy costs afterwards. When the unlevered asset value hits V_S, the shareholders and the creditors share the firm and determine the sharing rule through a bargaining game. We denote the bargaining power of shareholders by ζ, the bargaining power of creditors by $1-\zeta$, and the proportion of firm value shared by the shareholders in state i by χ_i. Given that the state of the economy is i at the debt-equity swap, the incremental value for shareholders from the debt-equity swap is $\chi_i V_{S,i}$, and the incremental value for creditors is $(1-\chi_i)V_{S,i}-\alpha_i V_{S,i}$ or $(1-\alpha_i-\chi_i)V_{S,i}$. Following Fan and Sundaresan (2000), we use Nash equilibrium to solve the sharing rule χ_i^*, which is given by

$$\chi_i^* = argmax \left[(1-\alpha_i-\chi_i)V_{S,i}\right]^{1-\zeta}(\chi_i V_{S,i})^{\zeta}. \tag{B.5.1}$$

After taking the first order condition with respect to χ_i in equation (B.5.1), we obtain $\chi_i^* = (1-\alpha_i)\zeta$. Thus, at $V_{S,i}$, with the state of the economy being i, the equity and debt values are given, respectively, by

$$S_{S,i} = (1-\alpha_i)\zeta V_{S,i}, \tag{B.5.2a}$$

and

$$D_{S,i} = \left[1-(1-\alpha_i)\zeta\right]V_{S,i}. \tag{B.5.2b}$$

Similar to derivation of equation (B.1.5), the equity value at time t, given that the state of the economy at t is i, is

$$S_{i,t} = V_{i,t} - \frac{(1-\eta)c}{r_{C,i}} + \sum_{j=1}^{2} q_{ij}^{D} \left(\frac{(1-\eta)c}{r_{C,j}} - V_{S,j} \right) + \sum_{j=1}^{2} \left[q_{ij,t}^{D} S_{S,j} \right]. \tag{B.5.3}$$

Similar to derivation of equation (B.1.8), the debt value at time t, given the state of the economy at t is i, is

$$D_{i,t} = c \left(\frac{1}{r_{C,i}} - \sum_{j=1}^{2} \frac{q_{ij,t}^{D}}{r_{C,j}} \right) + \sum_{j=1}^{2} \left[q_{ij,t}^{D} D_{S,j} \right]. \tag{B.5.4}$$

In our model, the debt-equity swap boundary $V_{S,i}$ is endogenously determined by solving the following two standard smooth-pasting conditions:

$$\frac{\partial S_1(V, V_{S,1}, V_{S,2})}{\partial V} |_{V=V_{S,1}} = 0, \tag{B.5.5a}$$

and

$$\frac{\partial S_2(V, V_{S,1}, V_{S,2})}{\partial V} |_{V=V_{S,2}} = 0. \tag{B.5.5b}$$

B.6 Parameters Choice for the Representative Firm and the Bankruptcy Environment

B.6.1 Parameter Choice of the Firm

Our parameter choices concerning the firm's fundamental and its bankruptcy environment are reported in Panel B of Table 1 in the main content of this paper. In this section we provide a detailed discussion of our parameter choices. Our aim is to choose parameter values as close as to the previous findings in the literature and to make our results easily understood. In what follows, we describe all the eight parameters.

Expected growth rate of the firm's cash flow (θ): In the calibration for the representative firm in the economy (BAA-rated), we set the expected growth rate to be the same as that for the average nonfinancial firm in the economy. Our methodology for the latter proxy is described above in the economy section and their values are given in Panel A of Table 1.

Idiosyncratic earnings volatility for firm i ($\sigma_X^{id,i}$): Panel A of Table 1 provides our estimate for the systematic volatility in each state of the economy. We assume that the idiosyncratic component of earnings volatility is state independent. The reason for this is that there is contrasting evidence on the subject. On one hand, as documented in Engle and Ng (1993) $\sigma_X^{id,i}$ is high in recessionary times. On the other hand, a series of papers (among others, Campbell et al (2001)) document an increase in idiosyncratic volatility in the nineties, which was a period of unprecedented boom in the economy. Thus, in order to calibrate idiosyncratic volatility, we fit the total volatility to the average volatility for BAA-rated firm, 25%, as reported in Schaeffer and Strebulaev (2008). We find that ($\sigma_X^{id,i}$) is about 20% percent.

The tax rate (η): For tax rates, we use only one number for both economies. We have no reason to believe that effective tax rate is state dependent. We set the value as 15 percent, the number used in previous studies. We also checked this number with average forward-looking marginal effective tax rates from John Graham's website.

The liquidation cost ($1-\alpha$): There is a consensus that asset sales of distressed firms suffer from large discounts if the entire industry (economy by analogy) experiences liquidity constraints (see Shleifer and Vishny (1992), Pulvino (1998, 1999)and Maksimovic and Phillips (1998)). Thus, we choose to calibrate our model to a countercyclical liquidation costs. We use the numbers given in BKS (2010): 70% (90%) in bad (good) economic times. For an optimally levered firm, these numbers correspond to the recovery rates reported in Acharya et al (2007).

The cost of financial distress (ω): These costs include payments to lawyers, accountants, trustees, the loss of customers and strategic employees and important reduction in credit facilities. As in the case of liquidation costs, we choose countercyclical distress costs (our estimation in Supplementary Appendix J also confirms this assumption). One difficulty is that most previous studies do not report the total distress loss. Instead, they measure only the direct costs of bankruptcy. The other difficulty is that some researchers use market value as the denominator to compute the proportional rate, while others use book value. Some use the value at liquidation as the denominator, while others use the value at the onset (or one year prior) to default as the denominator. In this calibration, we choose to consider the measures as of the value at the onset of the distress and use the two numbers used in the BCS (2007) study ($\omega_1 = 0.03$, $\omega_2 = 0.01$).

The bargaining power (ζ): The bargaining power of the shareholders in the negotiation process is unobservable. Given the Nash equilibrium approach that we implement in this study, there is a one-to-one mapping between this parameter and the strategic debt service (see Supplementary Appendix E for detail). Instead of setting an ad hoc number for (ζ), we assume a state-independent exogenous $\vartheta = 0.3$. This translates to a countercyclical ζ in our model. This assumption sounds reasonable to us, because given a higher liquidation costs in recession, debtholders have much to lose if they insist on opposing the reorganization plan offered by equityholders. Holding everything else constant, this would implies a higher ζ in this state of the economy.

The grace period (d): Evidence suggests that Chapter 11 process may take only a few months in some cases, but in other situations, the process can be long and complex and may take several years. [1]. We choose the value of d as state independent and set its value to $d = 2$ because it is not clear whether the time spent in Chapter 11 should be longer or shorter in recessions than in booms. On one hand, during recessions, liquidity in the market is limited and firms may have hard time raising DIP financing that is known to be strongly correlated to a prompt survival. On the other hand, the cost of bankruptcy is much higher in recessions, which discourages managers to spend longer time in bankruptcy. In addition, liquidation costs are severe during economic downturns, which would induce a higher willingness of creditors to accept the plan.

The optimal coupon (c^*): This is not an exogenous parameter of the model. However, in multiple applications of our model for the BAA-rated firm as well as for the cross section analysis, we estimate the optimal coupon rate. The methodology

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[1] Historically, the average time from filing for the bankruptcy petition to resolution varies from 2.2 to 2.8 years (Franks and Torous (1994) and Weiss (1990)). Using a recent sample Bris, Welch, and Zhu (2006) report an average of 2.3 years. Bharath, Panchapegesan, and Werner (2009) show that the time to resolution in Chapter 11 has declined to 16 months on average in the period from 2000 to 2005.

to solve for the parameter c^* is as follows: given all the other parameters of the economy and the firm, we first initialize the amount of debt. Second, we compute debt and equity values for a fixed amount of debt outstanding and a fixed set of default boundaries. Third, we determine the optimal default and liquidation boundaries (4 boundaries: 2 for each states) for a fixed amount of debt outstanding. Finally, we determine the optimal amount of debt by maximizing the firm's value at time 0.

B.6.2 Estimation of the Strategic Debt Service and Separating Financial Distress from Liquidation Costs

For completeness we report in this subsection of the Supplementary Appendix an estimation methodology that permits a separability of distress costs (incurred in Chapter 11) from liquidation costs. One reason for doing this is to feel confident about our choices concerning the unobservable parameters. Our aim is to estimate the parameter values for the strategic debt service (ϑ), the cost of financial distress (ω_1, ω_2) and the firm's recovery rate at liquidation (α_1, α_2). To do so, we propose to fit the model to the following observed (or at least empirically quantified) data: 1) the average senior unsecured bond recovery rate, 2) the magnitude of APR violation, and 3) the total default losses in both economies. In what follows, we detail our motivation as well as the implied target equations.

(1) Bond recovery rate: The bond recovery rate is defined as the ratio of the debt value at default over the debt value at time 0. According to Huang and Huang (2003), the historical average recovery rate is 51.31%. We approximate the mean

bond recovery rate by

$$\frac{0.36\sum_{i=1}^{2}\left(q_{1i}^{D}D_{B,i}\right)}{D_1\sum_{i=1}^{2}q_{1i}^{D}}+\frac{0.64\sum_{i=1}^{2}\left(q_{2i}^{D}D_{B,i}\right)}{D_2\sum_{i=1}^{2}q_{2i}^{D}}=51.31\%. \tag{B.6.1}$$

(2) Magnitude of APR violation: We measure the magnitude of the absolute priority rule (APR) violation by the ratio of the equity value over the firm value at default. Empirical studies suggest that over the old sample period (before the 1990s), the magnitude of APR deviation is approximately 8 to 10% of the reorganized firm's value. Over the relatively new sample period (after the 1990s), the magnitude of the APR violation declines to less than 2% of the firm value. In Panels C and D of Table 2, the sensitivity analysis of the model shows that the APR violation is negatively related to the cost of financial distress and positively related to the liquidation cost. As discussed earlier, the higher the distress cost, the later the shareholders file for Chapter 11, which leads to a smaller amount of APR violation. We assume that the proportion of strategic debt service is the same in both states of the economy. We set a higher target for the magnitude of the APR violation in bad states of the economy than in good states. In particular,

$$\frac{v_{B,1}-D_{B,1}}{v_{B,1}}=3\%, \tag{B.6.2a}$$

$$\frac{v_{B,2}-D_{B,2}}{v_{B,2}}=0.05\%, \tag{B.6.2b}$$

where v_B is given in equation (B.2.9), and D_B is given in equation (B.3.1).

(3) Default losses: The default losses are equal to the sum of the reorganization and liquidation costs. Andrade and Kaplan (1998) suggest that the total net cost of

financial distress is around 10 to 23% of the firm value at the onset of distress. Since the credit facilities from suppliers to clients shrink more significantly in recession than in boom, the indirect cost of financial distress is higher in recession. Also, Shleifer and Vishny (1992) suggest that the liquidation cost is higher when the entire industry experiences liquidity constraints, which is what happens in recession. Thus, we set total default losses higher in recession than in boom. In particular,

$$\frac{E^Q\left[\int_0^\tau \omega_1 V_t 1_{V_{L,1}<V_t<V_{B,1}} dt\right] + (1-\alpha_1)V_{\tau,1}}{V_{B,1}} = 23\%, \tag{B.6.3a}$$

$$\frac{E^Q\left[\int_0^\tau \omega_2 V_t 1_{V_{L,2}<V_t<V_{B,2}} dt\right] + (1-\alpha_2)V_{\tau,2}}{V_{B,2}} = 10\%. \tag{B.6.3b}$$

Table B.1 provides our estimates for (ϑ, ω_1, ω_2, α_1, α_2, α). We find that both liquidation and distress costs are countercyclical. As discussed in Chen et al (2009), BKS (2010), and Chen (2010), the countercyclical liquidation boundary and costs are important to obtain reasonable optimal capital structure. We also show that distress costs are countercyclical, which implies a downward trend in optimal leverage. Chen (2010) reports estimates of liquidation costs for different states. Our estimates of liquidation costs are close to his results. His model only accounts for liquidation, and there is no room for any distress costs that are incurred prior to liquidation. Then he matches the liquidation level with empirical estimates that measure distress from a prior threshold. We argue that this setup biases his results slightly upward. More importantly, our estimates of distress costs and their relative dispersion between states are higher than those used in the study of BCS (2007), François and Morellec (2004) and others. Our results closely relate to the numbers reported in Altman

Table B.1: Estimating Absolute Priority Rule (APR) Violation and Default Losses.[1]

Panel A: Target Moments					
Mean bond recovery rate					51.31%
Magnitude of APR violation in recession					3%
Magnitude of APR violation in boom					0.05%
Default loss in recession					23%
Default loss in boom					10%
Panel B: Parameter Estimates					
ϑ	ω_1	ω_2	α_1		α_2
0.3	0.09	0.05	0.72		0.87
Panel C: APR Violation and Distress Cost					
ω_1	0.03	0.05	0.07	0.09	0.11
APR violation	6.12%	5.36%	4.45%	4.04%	2.98%
Panel D: APR Violation and Liquidation Cost					
$1-\alpha_1$	0.2	0.3	0.4		0.5
APR violation	2.24%	6.12%	12.5%		18.51%

[1] Panel A gives the five target moments used to estimate the following five parameters: the strategic debt service (ϑ), the distress costs (ω_1, ω_2) and the firm's recovery rates at liquidation (α_1, α_2). Panel B gives the estimates of the five parameters. Panel C gives the effect of distress cost on the magnitude of APR violation. Panel D gives the effect of liquidation cost on the magnitude of APR violation. All the other parameter values are listed in Table 1.

(1984) and when combined with the liquidation costs amounts to similar numbers as in Korteweg (2010).

In Panels C and D of Table B.1, we also implement a sensitivity analysis on these estimates. We document that the APR violation is negatively related to the cost of financial distress and positively related to the liquidation cost. As discussed earlier, the higher the distress cost, the later the shareholders file for Chapter 11, which leads to a smaller amount of APR violation.

APPENDIX C
CALIBRATION METHODOLOGY, ONE SPECIAL CASE AND DERIVATION OF VOLATILITY ASYMMETRY

C.1 Details on the calibration procedure

In chapter 2, the model is calibrated in three different ways. In the first one, we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability. In the second one, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread. In the third one, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility. For all the three calibration, we assume that the firm recovers 51.31% of the face value given default.

To calibrate our model, we need to specify the asset premium and the leverage ratio. The asset premium is given by

$$\pi_X = (1 - L)\pi_E + L\pi_D, \tag{C.1.1}$$

where π_E is the equity premium, π_D is the bond risk premium and L is the firm's leverage ratio. We use the yield spread of the corporate bond over a comparable default-free bond as a proxy for the bond risk premium. The leverage ratio is given by

$$L = P/X, \tag{C.1.2}$$

where P is the face value and X is the unlevered asset value.

Also to calibrate the model to the term structure of yield spreads, we need to price a corporate bond with finite maturity. For a corporate bond with maturity T and semi-annual coupon payment, the bond price is given by

$$D_{0,T} = \frac{c}{2}\sum_{i=1}^{2T-1}\left[1-\omega Q(0,T_i)\right]/(1+r)^{T_i} + \left(P+\frac{c}{2}\right)\left[1-\omega Q(0,T)\right]/(1+r)^{T}, \tag{C.1.3}$$

where P is the face value of the bond, c is the annual coupon payment, T_i is the ith coupon date, ω is loss rate given default, and $Q(0,T_i)$ is the risk-neutral default probability before time T_i. We assume that the corporate bond is priced at par. Thus we can back out the annual coupon payment from equation (C.1.3). Also the bond yield is the same as the coupon rate.

Finally, when we perform the second and third calibrations as mentioned earlier, we use equation (2.11) to calibrate the model to the historical average equity volatility for different credit ratings.

C.2 A solution for the default probability: one special case

We denote $S(z,V,h)$ as the risk-neutral probability that the log asset value z has never crossed the default boundary $z_D = 0$ before $T = t+h$, given that $z_t = z$ and $V_t = V$. Obviously, the default probability $Q(z,V,h) = 1 - S(z,V,h)$. Below we show the procedure to obtain the closed-form solution for S when S satisfies the smooth pasting condition at $z = 0$.

Define the default time as $\tau_t = inf\{s \geq t, z_s \leq z_D\}$. Then $P(\tau_t < T | z_t = z, V_t = V) = 1 - S(z, V, h)$. Since we can rewrite $S(z, V, h)$ as a conditional expectation, it follows a martingale and satisfies the following backward Kolmogorov equation.

$$S_h = \frac{1}{2} V S_{zz} + \rho \sigma V S_{zV} + \frac{1}{2} \sigma^2 V S_{VV} + (r - \frac{1}{2} V) S_z + \kappa^* (\theta^* - V) S_V, \qquad \text{(C.2.1)}$$

with the initial condition $S(z, V, 0) = 1$ and the boundary condition $S(0, V, h) = 0$. Define $\tilde{S}(\omega, V, h) = \int_0^\infty e^{-\omega z} S(z, V, h) dz$ with the real part of ω being positive. Given the solution for $\tilde{S}$, we obtain that $S(z, V, h) = \frac{1}{2\pi i} \int_{C - i\infty}^{C + i\infty} e^{z\omega} \tilde{S}(\omega, V, h) d\omega$ with C being a positive constant. To solve for $\tilde{S}(\omega, V, h)$, we use the following equations:

$$\tilde{S}(\omega, V, 0) = \int_0^\infty e^{-\omega z} dz = \frac{1}{\omega}, \qquad \text{(C.2.2a)}$$

$$\int_0^\infty e^{-\omega z} \frac{\partial S}{\partial z} dz = \omega \tilde{S}, \qquad \text{(C.2.2b)}$$

$$\int_0^\infty e^{-\omega z} \frac{\partial^2 S}{\partial z \partial V} dz = \omega \frac{\partial \tilde{S}}{\partial V}, \qquad \text{(C.2.2c)}$$

$$\int_0^\infty e^{-\omega z} \frac{\partial^2 S}{\partial z^2} dz = \omega^2 \tilde{S}. \qquad \text{(C.2.2d)}$$

When getting equation (C.2.2d), we assume that the "smooth pasting" (*we need to find the correct wording or the economic intuition here since it is the same as in Leland (1994)*) condition is satisfied by S at $z = 0$, i.e., $\frac{\partial S}{\partial z}|_{z=0} = 0$. Applying the transform $\int_0^\infty e^{-\omega z} \cdot dz$ on both sides of equation (C.2.1) and plugging equations (C.2.2a)-(C.2.2d), we obtain that

$$\frac{\partial \tilde{S}}{\partial h} = \frac{1}{2} \sigma^2 V \frac{\partial^2 \tilde{S}}{\partial V^2} + \left[\kappa^* \theta^* + (\rho \sigma \omega - \theta^*) V\right] \frac{\partial \tilde{S}}{\partial V} + \left[\omega r + \left(\frac{1}{2} \omega^2 - \frac{1}{2} \omega\right) V\right] \tilde{S}. \qquad \text{(C.2.3)}$$

Guessing that the solution for $\tilde{S}$ is $\tilde{S}(\omega, V, h) = \frac{1}{\omega} e^{-A(\omega, h) - B(\omega, h) V}$, we obtain

$$-A' - B' V = \frac{1}{2} \sigma^2 V B^2 - \left[\kappa^* \theta^* + (\rho \sigma \omega - \kappa^*) V\right] B + \omega r + \left(\frac{1}{2} \omega^2 - \frac{1}{2} \omega\right) V. \qquad \text{(C.2.4)}$$

Thus A and B satisfy

$$-A' = -\kappa^*\theta^* B + \omega r, \tag{C.2.5a}$$

$$-B' = \frac{1}{2}\sigma^2 B^2 - (\rho\sigma\omega - \kappa^*)B + \frac{1}{2}\omega^2 - \frac{1}{2}\omega, \tag{C.2.5b}$$

with $A(\omega, 0) = 0$ and $B(\omega, 0) = 0$. Note that we use $'$ to denote the first-order derivative. For example, $A' = \frac{dA}{dh}$ and $B' = \frac{dB}{dh}$. We first solve for B from equation (C.2.5b) and then solve for A from equation (C.2.5a). Essentially, equation (C.2.5b) is a Riccati equation. Let $B(h) = \frac{q'(h)}{q(h)} \cdot \frac{2}{\sigma^2}$ and plug it into equation (C.2.5b), we obtain

$$q'' - (\rho\sigma\omega - \kappa^*)q' - \frac{1}{2}\sigma^2 \left(-\frac{1}{2}\omega^2 + \frac{1}{2}\omega \right) q = 0.$$

Thus the general solution for q is $q(h) = C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h}$, where C_1 and C_2 are constants, and λ_1 and λ_2 solve

$$\lambda^2 - (\rho\sigma\omega - \kappa^*)\lambda - \frac{1}{4}\sigma^2(-\omega^2 + \omega) = 0.$$

Thus $\lambda_{1,2} = \frac{\rho\sigma\omega - \kappa^* \pm d}{2}$ with $d = \sqrt{(\rho\sigma\omega - \kappa^*)^2 + \sigma^2(-\omega^2 + \omega)}$. The solution for $B(\omega, h)$ is

$$B(\omega, h) = \frac{2}{\sigma^2} \cdot \frac{C_1\lambda_1 e^{\lambda_1 h} + C_2\lambda_2 e^{\lambda_2 h}}{C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h}}. \tag{C.2.6}$$

Since $B(\omega, 0) = 0$, we obtain that $C_2 = -C_1\lambda_1/\lambda_2$. Plugging equation the expression for C_2 into equation (C.2.6), we obtain

$$B(\omega, h) = \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot \frac{1 - e^{-dh}}{1 - ge^{-dh}}, \tag{C.2.7}$$

with $d = \sqrt{(\rho\sigma\omega - \kappa^*)^2 + \sigma^2(-\omega^2 + \omega)}$ and $g = \frac{\rho\sigma\omega - \kappa^* + d}{\rho\sigma\omega - \kappa^* - d}$. Now we solve for $A(\omega, h)$. First, plugging equation (C.2.7) into equation (C.2.5a) yields

$$A' = \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot \frac{1 - e^{-dt}}{1 - ge^{-dt}} - \omega r. \tag{C.2.8}$$

Let $u(h)=e^{-dh}$, then $u'=-du$. Plugging $A'=A'_u\cdot u'$ into equation (C.2.8) yields

$$A'_u\cdot u'=-A'_u du=\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2}\cdot\frac{1-e^{-dt}}{1-ge^{-dt}}-\omega r.$$

Thus

$$A'_u=\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2 d}\cdot\left(-\frac{1}{u}+\frac{1-g}{1-gu}\right)+\frac{\omega r}{du}. \tag{C.2.9}$$

The solution for equation (C.2.9) is

$$\begin{aligned} A &= \kappa^*\theta^*\cdot\tfrac{\rho\sigma\omega-\kappa^*+d}{\sigma^2 d}\cdot\left[-ln(u)+\tfrac{(1-g)ln(1-gu)}{-g}\right]+\tfrac{\omega r}{d}ln(u)+C_3\\ &=\kappa^*\theta^*\cdot\tfrac{\rho\sigma\omega-\kappa^*+d}{\sigma^2}\cdot h+\kappa^*\theta^*\cdot\tfrac{\rho\sigma\omega-\kappa^*+d}{\sigma^2 d}\cdot\tfrac{g-1}{g}ln\left(1-ge^{-dh}\right)-\omega rh+C_3, \end{aligned} \tag{C.2.10}$$

where C_3 is a constant. Since $A(0)=0$, we obtain $C_3=-\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2 d}\cdot\frac{g-1}{g}ln\left(1-g\right)$. Thus

$$A(\omega,h)=-\omega rh+\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2}\cdot h+\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2 d}\cdot\frac{g-1}{g}ln\left[\frac{1-ge^{-dh}}{1-g}\right]. \tag{C.2.11}$$

Since $(\rho\sigma\omega-\kappa^*+d)\frac{g-1}{g}=(\rho\sigma\omega-\kappa^*+d)\frac{2d}{\rho\sigma\omega-\kappa^*-d}\cdot\frac{\rho\sigma\omega-\kappa^*-d}{\rho\sigma\omega-\kappa^*+d}=2d$, we rewrite the solution for A as

$$A(\omega,h)=-\omega rh+\kappa^*\theta^*\cdot\frac{\rho\sigma\omega-\kappa^*+d}{\sigma^2}\cdot h+\frac{2\kappa^*\theta^*}{\sigma^2}ln\left[\frac{1-ge^{-dh}}{1-g}\right]. \tag{C.2.12}$$

Thus the solution for $S(z,V,h)$ is given by

$$S(z,V,h)=\frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty}e^{z\omega}\tilde{S}(\omega,V,h)d\omega, \tag{C.2.13}$$

with $\tilde{S}(\omega,V,h)=\frac{1}{\omega}e^{-A(\omega,h)-B(\omega,h)V}$, and the solutions for $A(\omega,h)$ and $B(\omega,h)$ are given by equations (C.2.12) and (C.2.7).

C.3 Volatility asymmetry at equity and asset value levels

In this appendix, we translate the correlation between asset value and variance shocks to a correlation between equity and equity variance shocks. The stochastic process that the unlevered asset value follows is given by

$$\frac{dX_t}{X_t} = (\mu - \delta)dt + \sqrt{V_t}dW_1, \tag{C.3.1}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_2, \tag{C.3.2}$$

where δ is the firm's payout ratio and $E(dW_1 dW_2) = \rho dt$. Applying Itô's lemma, we obtain that

$$\frac{dE_t}{E_t} = \mu_{E,t} + \frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\sqrt{X_t}dW_{1t} + \frac{1}{E_t}\frac{\partial E_t}{V_t}\sigma\sqrt{V_t}dW_{2t}, \tag{C.3.3}$$

where $\mu_{E,t}$ is the instantaneous equity return. The equity variance is given by

$$V_{E,t} = C \cdot V_t, \tag{C.3.4}$$

where $C = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}$. Applying Itô's lemma to $V_{E,t}$ given by equation (C.3.4), we obtain that

$$dV_{E,t} = \mu_{V_E,t}dt + \sigma V_{E,t}dW_{2t}. \tag{C.3.5}$$

Given the specification of the processes for equity and equity variance as in equations (C.3.3) and (C.3.5), we obtain that the correlation between equity and equity variance is

$$\rho_{E,t} = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\rho + \frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right)/C, \tag{C.3.6}$$

where $C = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}$.

APPENDIX D
OVERVIEW OF PROOFS OF PROPOSITIONS IN CHAPTER 3

This appendix provides a detailed proof of the propositions in chapter 3 and gives complete expressions of the terms that appear in the propositions. In what follows, section A describes the proof of the solutions for pricing kernel, risk-free rate, wealth-consumption ratio and the equity premium for the aggregate consumption claim. Section B derives the firm's contingent claim prices.

D.1 Proof of Proposition 1

In the competitive equilibrium, the agent's objective is to maximize the utility function subject to the budget constraint. The value function J is a function of the aggregate wealth W and π. More specifically,

$$J(W, \pi) = \max_{C,\phi} E\left[\int_t^\infty f(C_s, J_s) ds | \Im_t\right]$$

subject to

$$dW_t = W_t\left[\phi(\mu_R - r) + r\right]dt + W_t\phi(\sigma_{R,1}d\tilde{z}_{1,t} + \sigma_{R,2}d\tilde{z}_{2,t}) - C_t dt, \tag{D.1.1}$$

where μ_R is the drift of the aggregate consumption claim. $\sigma_{R,1}$ and $\sigma_{R,2}$ are the diffusion coefficients of the aggregate consumption claim. The Hamilton-Jacobi-Bellman (HJB) equation for the portfolio choice problem is given by

$$\begin{aligned} f(C,J) &+ J_W\left[rW + W\phi(\mu_R - r) - C\right] + \tfrac{1}{2}J_{WW}W^2\phi^2(\sigma_{R,1}^2 + \sigma_{R,2}^2) \\ &+ J_\pi\mu_\pi + \tfrac{1}{2}J_{\pi\pi}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + J_{W\pi}W(\sigma_{R,1}\sigma_{\pi,1} + \sigma_{R,2}\sigma_{\pi,2}) = 0, \end{aligned} \tag{D.1.2}$$

where we use μ_π, $\sigma_{\pi,1}$ and $\sigma_{\pi,2}$ to denote the drift and diffusion coefficients of the posterior probability π. We guess that the solution for J is given by

$$J(W,\pi) = \beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \cdot H(\pi)^{\frac{1-\gamma}{1-\frac{1}{\psi}}\cdot\frac{1}{\psi}} \cdot \frac{W^{1-\gamma}}{1-\gamma}. \tag{D.1.3}$$

Thus we obtain

$$J_W = (1-\gamma)\frac{J}{W}, \tag{D.1.4a}$$

$$J_{WW} = -\gamma(1-\gamma)\frac{J}{W^2}, \tag{D.1.4b}$$

$$J_{W\pi} = \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}(1-\gamma)\frac{J}{W}, \tag{D.1.4c}$$

$$J_\pi = \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}J, \tag{D.1.4d}$$

$$J_{\pi\pi} = \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\left[\frac{H''}{H} + \left(\frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi} - 1\right)\cdot\left(\frac{H'}{H}\right)^2\right]J. \tag{D.1.4e}$$

The first-order conditions of the HJB equation (D.1.2) are

$$f_C = J_W, \tag{D.1.5}$$

$$J_W(\mu_R - r) + J_{WW}W\phi^*(\sigma_{R,1}^2 + \sigma_{R,2}^2) + J_{W\pi}(\sigma_{R,1}\sigma_{\pi,1} + \sigma_{R,2}\sigma_{\pi,2}) = 0. \tag{D.1.6}$$

Plugging the first-order derivative of equation (3.2) and (D.1.4a) into equation (D.1.5) gives

$$\beta\frac{C^{-\frac{1}{\psi}}}{[(1-\gamma)J]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}} = (1-\gamma)\frac{J}{W}. \tag{D.1.7}$$

Plugging equation (D.1.3) into equation (D.1.7) gives

$$H(\pi) = \frac{W}{C}. \tag{D.1.8}$$

Thus

$$W = H(\pi)C. \tag{D.1.9}$$

Applying Ito's lemma gives

$$\begin{aligned}\frac{dW}{W} &= \frac{dC}{C} + \frac{H'}{H}d\pi + \frac{1}{2}\frac{H''}{H}(d\pi)^2 + \frac{H'}{H}\frac{dC}{C}d\pi \\ &= \tilde{\theta}_C dt + \sigma_C d\tilde{z}_{1,t} + \frac{H'}{H}(\mu_\pi dt + \sigma_{\pi,1}d\tilde{z}_1 + \sigma_{\pi,2}d\tilde{z}_2) + \frac{1}{2}\frac{H''}{H}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2)dt + \frac{H'}{H}\sigma_C\sigma_{\pi,1}dt \\ &= \left[\tilde{\theta}_C + \frac{H'}{H}\mu_\pi + \frac{1}{2}\frac{H''}{H}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \frac{H'}{H}\sigma_C\sigma_{\pi,1}\right]dt + \left(\sigma_C + \frac{H'}{H}\sigma_{\pi,1}\right)d\tilde{z}_1 + \frac{H'}{H}\sigma_{\pi,2}d\tilde{z}_2.\end{aligned} \tag{D.1.10}$$

Thus the expected return and diffusion coefficients for the aggregate wealth are given by

$$\mu_W = \tilde{\theta}_C + \frac{H'}{H}\mu_\pi + \frac{1}{2}\frac{H''}{H}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \frac{H'}{H}\sigma_C\sigma_{\pi,1} + \frac{1}{H}, \tag{D.1.11a}$$

$$\sigma_{W,1} = \sigma_C + \frac{H'}{H}\sigma_{\pi,1}, \tag{D.1.11b}$$

$$\sigma_{W,2} = \frac{H'}{H}\sigma_{\pi,2}. \tag{D.1.11c}$$

In equilibrium, the market clearing condition implies that $\phi^* = 1$, which indicates that the aggregate wealth is the same as the claim to the aggregate consumption. Thus, $\mu_R = \mu_W$, $\sigma_{R,1} = \sigma_{W,1}$, $\sigma_{R,2} = \sigma_{W,2}$ and equation (D.1.6) becomes

$$J_W(\mu_W - r) + J_{WW}W(\sigma_{W,1}^2 + \sigma_{W,2}^2) + J_{W\pi}(\sigma_{W,1}\sigma_{\pi,1} + \sigma_{W,2}\sigma_{\pi,2}) = 0.$$

Thus,

$$\mu_W - r = -\frac{J_{WW}W}{J_W}(\sigma_{W,1}^2 + \sigma_{W,2}^2) - \frac{J_{W\pi}}{J_W}(\sigma_{W,1}\sigma_{\pi,1} + \sigma_{W,2}\sigma_{\pi,2}). \tag{D.1.12}$$

Plugging equations (D.1.4a)-(D.1.4c) into equation (D.1.12) gives

$$\begin{aligned}\mu_W - r &= \gamma(\sigma_{W,1}^2 + \sigma_{W,2}^2) - \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}(\sigma_{W,1}\sigma_{\pi,1} + \sigma_{W,2}\sigma_{\pi,2}) \\ &= \sigma_{W,1}\left(\gamma\sigma_{W,1} - \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}\sigma_{\pi,1}\right) + \sigma_{W,2}\left(\gamma\sigma_{W,2} - \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}\sigma_{\pi,2}\right).\end{aligned} \tag{D.1.13}$$

We denote the diffusion coefficients for the pricing kernel M to be ξ_1 and ξ_2. More specifically, $\frac{dM}{M} = -r(\pi)dt - \xi_1(\pi)d\tilde{z}_1 - \xi_2(\pi)d\tilde{z}_2$. Then we back out the solutions for

ξ_1 and ξ_2 from the risk premium for the aggregate wealth, which is given by equation (D.1.13). In equilibrium,

$$\mu_W - r = -E\left(\frac{dM}{M}\frac{dW}{W}|\Im_t\right) = \sigma_{W,1}\xi_1 + \sigma_{W,2}\xi_2. \tag{D.1.14}$$

Equations (D.1.13) and (D.1.14) indicate that

$$\xi_1 = \gamma\sigma_{W,1} - \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}\sigma_{\pi,1}, \tag{D.1.15a}$$

$$\xi_2 = \gamma\sigma_{W,2} - \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}\sigma_{\pi,2}. \tag{D.1.15b}$$

Plugging equations (D.1.11b) and (D.1.11c) into equations (D.1.15a) and (D.1.15b) gives

$$\xi_1 = \gamma\sigma_C + \frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,1}, \tag{D.1.16a}$$

$$\xi_2 = \frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,2}. \tag{D.1.16b}$$

The equity premium is given by

$$\mu_W - r = \left(\sigma_C + \frac{H'}{H}\sigma_{\pi,1}\right)\left(\gamma\sigma_C + \frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\frac{H'}{H}\sigma_{\pi,1}\right) + \frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\left(\frac{H'}{H}\sigma_{\pi,2}\right)^2. \tag{D.1.17}$$

Now we solve for the risk-free rate. Plugging equation (D.1.11a) into equation (D.1.13) gives

$$r = \mu_W - \gamma(\sigma_{W,1}^2 + \sigma_{W,2}^2) + \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1}{\psi}\frac{H'}{H}(\sigma_{W,1}\sigma_{\pi,1} + \sigma_{W,2}\sigma_{\pi,2}). \tag{D.1.18}$$

Plugging equations (D.1.11a), (D.1.11b) and (D.1.11c) into equation (D.1.18) gives

$$\begin{aligned} r(\pi) = {} & \tilde{\theta}_C + \tfrac{H'}{H}\mu_\pi + \tfrac{1}{2}\tfrac{H''}{H}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \tfrac{H'}{H}\sigma_C\sigma_{\pi,1} + \tfrac{1}{H} \\ & - \gamma\left[\left(\sigma_C + \tfrac{H'}{H}\sigma_{\pi,1}\right)^2 + \left(\tfrac{H'}{H}\sigma_{\pi,2}\right)^2\right] + \tfrac{1-\gamma}{1-\frac{1}{\psi}}\tfrac{1}{\psi}\tfrac{H'}{H}\left[\left(\sigma_C + \tfrac{H'}{H}\sigma_{\pi,1}\right)\sigma_{\pi,1} + \tfrac{H'}{H}\sigma_{\pi,2}^2\right]. \end{aligned} \tag{D.1.19}$$

Note that now the only thing we need to solve for is the wealth-consumption ratio $H(\pi)$. First, plugging $\phi^* = 1$, equation (D.1.6), (D.1.4a)-(D.1.4e) into equation (D.1.2) gives

$$\begin{aligned} 0 = & \tfrac{\beta}{1-\frac{1}{\psi}}\left(\beta^{-1}H^{-1}-1\right)(1-\gamma)+(1-\gamma)(r-H^{-1})+\tfrac{1}{2}\gamma(1-\gamma)\left[\left(\sigma_C+\tfrac{H'}{H}\sigma_{\pi,1}\right)^2+\left(\tfrac{H'}{H}\sigma_{\pi,2}\right)^2\right] \\ & +\tfrac{1-\gamma}{1-\frac{1}{\psi}}\tfrac{1}{\psi}\tfrac{H'}{H}\mu_\pi+\tfrac{1}{2}\tfrac{1-\gamma}{1-\frac{1}{\psi}}\tfrac{1}{\psi}\left[\tfrac{H''}{H}+\left(\tfrac{1-\gamma}{1-\frac{1}{\psi}}\tfrac{1}{\psi}-1\right)\left(\tfrac{H'}{H}\right)^2\right](\sigma_{\pi,1}^2+\sigma_{\pi,2}^2). \end{aligned} \tag{D.1.20}$$

Plugging equation (D.1.19) into equation (D.1.20) and reorganizing the terms gives

$$\begin{aligned} 0 = & -\beta+\left(1-\tfrac{1}{\psi}\right)\tilde{\theta}_C-\tfrac{1}{2}\gamma\left(1-\tfrac{1}{\psi}\right)\sigma_C^2+\tfrac{1}{H}+\left[(1-\gamma)\sigma_C\sigma_{\pi,1}+\mu_\pi\right]\tfrac{H'}{H} \\ & +\tfrac{1}{2}\left(\tfrac{1-\gamma}{1-\frac{1}{\psi}}-1\right)(\sigma_{\pi,1}^2+\sigma_{\pi,2}^2)\left(\tfrac{H'}{H}\right)^2+\tfrac{1}{2}(\sigma_{\pi,1}^2+\sigma_{\pi,2}^2)\tfrac{H''}{H}. \end{aligned} \tag{D.1.21}$$

D.2 Proof of Proposition 2

We first solve for the firm's before-tax unlevered asset value. We guess that the solution for V is given by

$$V(X,\pi) = XG(\pi). \tag{D.2.1}$$

Applying Ito's lemma gives

$$\begin{aligned} \tfrac{dV}{V} & = \tfrac{dX}{X}+\tfrac{G'}{G}d\pi+\tfrac{1}{2}\tfrac{G''}{G}(d\pi)^2+\tfrac{G'}{G}\tfrac{dX}{X}d\pi \\ & = \tilde{\theta}_X dt+\sigma_{X,1}d\tilde{z}_1+\sigma_{X,2}d\tilde{z}_2+\sigma_{X,3}d\tilde{z}_3+\tfrac{G'}{G}(\mu_\pi dt+\sigma_{\pi,1}d\tilde{z}_1+\sigma_{\pi,2}d\tilde{z}_2)+\tfrac{1}{2}\tfrac{G''}{G}(\sigma_{\pi,1}^2 \\ & +\sigma_{\pi,2}^2)dt+\tfrac{G'}{G}(\tilde{\theta}_X dt+\sigma_{X,1}d\tilde{z}_1+\sigma_{X,2}d\tilde{z}_2+\sigma_{X,3}d\tilde{z}_3)(\mu_\pi dt+\sigma_{\pi,1}d\tilde{z}_1+\sigma_{\pi,2}d\tilde{z}_2) \\ & = \left[\tilde{\theta}_X+\tfrac{G'}{G}\mu_\pi+\tfrac{1}{2}\tfrac{G''}{G}(\sigma_{\pi,1}^2+\sigma_{\pi,2}^2)+\tfrac{G'}{G}(\sigma_{X,1}\sigma_{\pi,1}+\sigma_{X,2}\sigma_{\pi,2})\right]dt \\ & +\left(\sigma_{X,1}+\tfrac{G'}{G}\sigma_{\pi,1}\right)d\tilde{z}_1+\left(\sigma_{X,2}+\tfrac{G'}{G}\sigma_{\pi,2}\right)d\tilde{z}_2+\sigma_{X,3}d\tilde{z}_3. \end{aligned} \tag{D.2.2}$$

The standard pricing rule indicates that the asset risk premium is equal to $-E\left(\frac{dM}{M}\frac{dV}{V}\right)$. Thus we obtain that

$$\begin{aligned} & \tilde{\theta}_X+\tfrac{G'}{G}\mu_\pi+\tfrac{1}{2}\tfrac{G''}{G}(\sigma_{\pi,1}^2+\sigma_{\pi,2}^2)+\tfrac{G'}{G}(\sigma_{X,1}\sigma_{\pi,1}+\sigma_{X,2}\sigma_{\pi,2})-r(\pi)+\tfrac{1}{G} \\ = & \ \xi_1(\pi)\left(\sigma_{X,1}+\tfrac{G'}{G}\sigma_{\pi,1}\right)+\xi_2(\pi)\left(\sigma_{X,2}+\tfrac{G'}{G}\sigma_{\pi,2}\right). \end{aligned} \tag{D.2.3}$$

The above equation for $G(\pi)$ is a 2nd order ODE, which is solved using two boundary conditions at $\pi = 0$ and $\pi = 1$.

Next, we solve for the present value at time t of a perpetuity with a constant payment being one, which we define as $F(\pi) = E\left(\int_t^\infty \frac{M_s}{M_t} ds|\Im_t\right)$. Applying Ito's lemma gives

$$\begin{aligned}\frac{dF}{F} &= \frac{F'}{F}d\pi + \frac{1}{2}\frac{F''}{F}(d\pi)^2 \\ &= \left[\frac{F'}{F}\mu_\pi + \frac{1}{2}\frac{F''}{F}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2)\right]dt + \frac{F'}{F}(\sigma_{\pi,1}d\tilde{z}_1 + \sigma_{\pi,2}d\tilde{z}_2).\end{aligned}$$

The risk premium for the perpetuity asset is given by

$$\frac{F'}{F}\mu_\pi + \frac{1}{2}\frac{F''}{F}(\sigma_{\pi,1}^2 + \sigma_{\pi,2}^2) + \frac{1}{F} - r(\pi) = \xi_1(\pi)\frac{F'}{F}\sigma_{\pi,1} + \xi_2(\pi)\frac{F'}{F}\sigma_{\pi,2}. \tag{D.2.4}$$

We use the two boundary conditions at $\pi = 0$ and $\pi = 1$ to solve the above 2nd order ODE or equation (D.2.4). Next, we define $q(X_t, \pi_t)$ as the time-t Arrow-Debreu price of a contingent claim that pays one unit at default. No-arbitrage pricing rule implies

$$E^Q(dq|\Im_t) = r(\pi)qdt$$

Applying Ito's lemma gives

$$\frac{\partial q}{\partial X}\hat{\theta}_X X + \frac{1}{2}\frac{\partial^2 q}{\partial X^2}\sigma_X^2 X^2 + \frac{\partial q}{\partial \pi}\mu_\pi + \frac{1}{2}\frac{\partial^2 q}{\partial \pi^2}\sigma_\pi^2 + \frac{\partial^2 q}{\partial X \partial \pi}(\sigma_{\pi,1}\sigma_{X,1} + \sigma_{\pi,2}\sigma_{X,2})X = r(\pi)q. \tag{D.2.5}$$

In addition to the two boundary conditions at $\pi = 0$ and $\pi = 1$, the solution for q also satisfies $q(X_B, \pi) = 1$ and $\lim_{X\to\infty} q(X, \pi) = 0$.

Given the solutions for unlevered asset value $V(X, \pi)$, perpetuity asset value $F(\pi)$ and $q(X, \pi)$, the firm's equity value at time t (S_t) is given by

$$S_t = (1-\eta)E\left[\int_t^\tau \frac{M_s}{M_t}(X_s - c)ds|\Im_t\right]. \tag{D.2.6}$$

In equation (D.2.6),

$$\begin{aligned}
&(1-\eta)E\left[\int_t^{\tau}\tfrac{M_s}{M_t}X_s\,ds\,|\Im_t\right]\\
&=(1-\eta)E\left[\int_t^{\infty}\tfrac{M_s}{M_t}X_s\,ds\,|\Im_t\right]-(1-\eta)E\left[\int_{\tau}^{\infty}\tfrac{M_s}{M_t}X_s\,ds\,|\Im_t\right].\\
&=(1-\eta)V-(1-\eta)E\left[\int_{\tau}^{\infty}\tfrac{M_\tau}{M_t}\tfrac{M_s}{M_\tau}X_s\,ds\,|\Im_t\right]\\
&=(1-\eta)(V-qV_B)
\end{aligned} \tag{D.2.7}$$

We obtain the solution for the firm's equity value as

$$S_t=(1-\eta)\left[V-qV_B-cF(1-q)\right], \tag{D.2.8}$$

where η is the effective tax rate, c is the rate of continuous coupon payment and V_B is the unlevered asset value at default. The firm's debt value at time t (D_t) is given by

$$\begin{aligned}
D_t &= E\left[\int_t^{\tau}\tfrac{M_s}{M_t}cds|\Im_t\right]+E\left(\tfrac{M_\tau}{M_t}\alpha V_B|\Im_t\right)\\
&= cF(1-q)+\alpha qV_B.
\end{aligned} \tag{D.2.9}$$

where α is the recovery rate of the firm value at default.

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