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Essays on supply contracts and dynamic pricing

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ESSAYS ON SUPPLY CONTRACTS AND DYNAMIC PRICING

by

Dengfeng Zhang

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Business Administration
in the Graduate College of
The University of Iowa

December 2008

Thesis Supervisors: Professor Timothy J. Lowe
Associate Professor Renato de Matta

ABSTRACT

Successful retailers like eBay and Amazon.com utilize consignment and/or wholesale contractual arrangements with their suppliers (also referred to as sellers) to manage the flow of goods, information and funds in their supply chains. Furthermore, today's pricing environment demands better, faster, and more frequent pricing decisions than ever before, and sellers' management of their inventory through dynamic pricing has contributed significantly to their financial success. The purpose of this study is two-fold. First, to investigate the decisions and channel performance under a consignment contractual arrangement and how they differ from a wholesale contractual arrangement. Second, to study a revenue management problem involving a seller of perishable products in an e-commerce setting where buyers are sensitive to both price and seller reputation.

Research issues addressed by this study fall under four themes: evaluation of contract preferences of retailer and supplier; coordination of retailer and supplier decisions (i.e. channel coordination); investigation of the effects of product competition on channel decisions and profits; and finally, development of a dynamic pricing strategy.

Our contributions to the existing body of literature are as follows: develop a better understanding of the factors contributing to the growing success of virtual retailers as a result of the increasing number of products that are being sold on consignment contractual arrangements; provide some insights on contractual preferences of managers to learn why they prefer consignment over wholesale contractual arrangement with certain sellers; develop incentives to promote better coordination between the supplier and retailer in a consignment contractual arrangement, thus creating a win-win situation for both players; develop a better understanding of the formation of reservation price, i.e. the amount a consumer is willing to pay for a product/service, from the perspective of consumer behavior. This work will unveil the underlying dynamics and relationships between reservation price and customer feedback rating of seller product/service quality which sellers can use to set product prices and maximize revenues.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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To my family

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CHAPTER 1
THE RETAILER-SUPPLIER PREFERENCE TO SELL ON CONSIGNMENT OR WHOLESALE

1.1 Introduction

Within a wide category of products with uncertain customer demand, the contracts utilized in the market place can vary significantly. For example, books (e.g. school textbooks and novels), canned goods (Campbell soup), beverages (beer) are frequently sold on consignment. However, contractual arrangements involving fashion items (sunglasses and clothing), produce, flowers are generally sold on wholesale contracts. Both consignment and wholesale contracts are employed in auto parts (Auto-Zone and NAPA), office equipment (OfficeMax), large department stores merchandise purchasing (Walmart and Target), and movie distribution and electronic marketplaces (eBay and Amazon.com). Contract choices may change over time as in the case of K-mart urging toy manufacturers to sell on consignment (Wall Street Journal, 1993).

Consider two independently managed parties, a retailer and a supplier, that have before them consignment and wholesale contracts. Both parties have interests in reaching an agreement, but their choices of contract type may not be entirely identical. For example, the retailer may prefer consignment while the supplier may prefer wholesale. Assuming both parties behave rationally, i.e., they seek to maximize their own profits, Rubinstein (1982) poses the question “what will be the agreed contract?” The outcome is largely dependent on how the negotiation process is to be conducted. However, as Cachon (2004) states, “it is valuable to make some prediction of the contract that the retailer and the supplier are likely to choose and that is independent of the details of the negotiation process."

We define the retailer and supplier contract preference before negotiation as their "original contract choice," and the agreed contract after negotiation as their "final contract choice." The final contract choice is the ultimate output of the negotiation process, which stipulates the profit and risk shares of the retailer and supplier. In this study, our primary concern is the supplier's and retailer's original contract choice. We assume that profitability is a major factor affecting the retailer and supplier original contract choices. Specifically, we measure the strength of retailer and supplier preference for a particular contract based on the contract's profit making potential relative to that of another contract.

Businessweek (2007) reports that parties enter negotiations lacking knowledge about the other party's contract preference. Highlighted in that report was two university professors' negotiation experiment involving 266 MBA students. The students were divided between parts suppliers and parts buyers. Results of this experiment show that each side underestimated how much the other was willing to bend, with the result that each party reckoned it got the better end of the negotiation. The buyers, for instance, thought they had hit the sellers' bottom figure, when in fact they overpaid by a wide margin. So knowing the supplier and retailer original contract preferences and how much profits they are likely to make from different contracts can help not only in developing final contract terms that are attractive and acceptable, but also it can help minimize offering "over sweetened" contract terms which can potentially create dissatisfaction between trading parties.

In this chapter, we try to identify the market factors that strongly influence a consignment and wholesale contract's potential to yield higher profits for the retailer and supplier. Assuming each individual party makes optimal decisions (e.g. pricing and stocking decisions), we also try to measure the sensitivity of retailer and supplier profits to changes in those market factors that we have identified.

Formally, we assume in this study two general contract types: consignment and wholesale contracts. Consignment, as described in Hacket (1993), is a unique contract where the retailer, over a given period, takes possession of the goods owned by the seller (supplier), promotes the sale of these goods to buyers, and receives a share of the sales revenue (sales commission). The supplier owns those goods until they are sold. With wholesale, an upstream entity (supplier) sells a product to a downstream party (retailer) who in turn serves market demand (Tsay, 1999).

We assume a single retailer whom we designate as the Stackelberg leader and one supplier as the follower. Consider the sequence of events for the Consignment Contractual Arrangement (CCA), and Wholesale Contractual Arrangement (WCA). In a CCA, the retailer offers a take-it-or-leave-it consignment contract stipulating the revenue and cost shares. In response to the retailer, the supplier makes her price and stocking decisions upon accepting the offer. However, in a WCA, the retailer sets his margin which the supplier utilizes to decide the wholesale price to charge the retailer. The retailer determines his order quantity and retail price based on the supplier wholesale price.

Ignoring profits that the retailer and supplier can extract from CCA and WCA, it is reasonable to find the retailer eliciting a strong preference for CCA over WCA because with CCA he does not face any inventory risk. Moreover, CCA frees the retailer from administrative work such as setting product prices and managing product inventories. The supplier showing strong preference for WCA may not be surprising for the same reasons the retailer would choose CCA. However, as will be demonstrated in this study, with profit considerations, the relative strength of the retailer and supplier preference levels for CCA and WCA (which we define shortly) can vary widely with different market settings. In this study, the market settings we investigate have different customer demand functions and levels of demand variability and price elasticity.

We assume the customer demand function to be either a Multiplicative Demand (MD), i.e. $\widetilde{D}(p) = d(p)\varepsilon$, where $d(p)$ is an iso-price elastic demand function and ε is a random term, or an Additive Demand (AD), i.e. $\widetilde{D}(p) = d(p) + \varepsilon$. The application of MD and AD is abundant in the literature. Papers by Whitin (1955), Mills (1959), and Thowsen (1975) employ AD in their study for price decision while other researchers investigate pricing behavior under MD (see Karlin and Carr (1962), Zabel (1970, 1972)). Petruzzi and Dada (1999) examine the price relationship between the two models and explain the difference in pricing behavior as a result of how price decisions are incorporated into demand uncertainty. As suggested by Granot and Yin (2005), AD would be an appropriate model wherein the variance of demand is unaffected by the expected demand level. Conversely, MD would be suitable for the situation where the variance of demand increases with expected demand in a manner which leaves the coefficient of variation unaffected. In terms of demand functional forms, $d(p)$ can be linear or nonlinear. The choice of linear and nonlinear affects the vertical strategic interaction and thus pricing behavior as shown in Lee and Staelin (1997). We have seen both functional forms in the literature. However, the nonlinear form is generally favored due to its nonnegativity. By contract parameter values, we are referring to the channel cost structure and price markups. Specifically, we assume zero fixed costs and linear variable costs. For WCA, retail price markup is either an Additive Price Markup (APM) or a Multiplicative Price Markup (MPM).

Assuming the retailer and supplier make optimal pricing and stocking level decisions according to the sequence of events we briefly described earlier for CCA and WCA, we determine the ratio of retailer (supplier) expected profit for CCA (WCA) to retailer (supplier) expected profit for WCA (CCA), i.e. ρ_R (ρ_S) for every market setting. We use this ratio as a measure of retailer (supplier) contract preference level. We interpret the ratio as follows. When ρ_R $(\rho_S) > 1$, it means the retailer (supplier) has a stronger preference for CCA (WCA). $\rho_R = 1$ $(\rho_S = 1)$ means that the retailer (supplier) is indifferent between WCA and CCA. We note that since we are working with expected profits, our metric for preference level implicitly takes into consideration the retailer and supplier inventory risks. Intuitively, when demand uncertainty goes up, the retailer and supplier inventory risks increase and channel profits decline.

Our study shows the following results: When customer demand function is MD (AD), the retailer and supplier preference levels are independent of (dependent on) demand uncertainty. Also for MD, but not for AD, we obtain lower and/or upper bounds of the retailer's and supplier's preference levels. These bounds can be used to guide parties in developing cost effective and cost efficient incentives. For both MD and AD, a decrease in the retailer cost share results in a decrease in the retailer (supplier) preference for CCA (WCA). In particular, with MD and MPM, when the retailer cost share approaches zero, the retailer and supplier optimal profits under CCA and WCA converge, and both retailer and supplier become indifferent of the contract type. Finally, the impact of price elasticity depends on market price markup. Under MPM, an increase in price elasticity raises the retailer's CCA and supplier's WCA preference levels. However, under APM, their preference levels diminish when price elasticity increases.

The remainder of this chapter is organized as follows. In Section 1.2, we review the relevant literature and highlight our contributions. Section 1.3 presents our model formulations with MD setting. Also, this section studies the pricing and stocking level decisions under CCA and WCA, and the differences in the retailer, supplier and channel profits between CCA and WCA. For WCA, we perform a comparative analysis of MPM and APM. We repeat the study, but this time assuming an AD market setting in Section 1.4. Finally, we conclude this chapter and discuss some future research directions in Section 1.5.

1.2 Relevant Literature

Contract selection between WCA and CCA has been a frequent topic of discussion in numerous trade and practitioner journal articles, giving reasons why retailers and suppliers prefer one contract over the other contract (for example see Pellegrini (1986), Marvel (1993), Butz (1997), Wallin et al. (2006), Rungtusanatham and Rabinovich (2007)). They give anecdotes from individuals and firms citing what they perceive as advantages and disadvantages of choosing a particular contract. They highlight the easy and difficult aspects of executing a contract and the downside risks trading parties face. These studies generally do not give any indication how strongly trading parties adhere to their original choices under different market conditions and contract terms.

In contrast, the body of academic literature on supply chain contracts provides sufficient evidence about the strong influence of external market factors, channel cost structure and price markup policies on profit allocations between the supplier and retailer, and overall channel profits for wholesale and consignment contracts. First, it has been widely accepted that the chosen customer demand model affects the profit allocation between the retailer and supplier. The two demand models we use in our study are multiplicative and additive demand models. Many studies, such as Petruzzi and Dada (1999), Liu et al. (2007), and Wang et al. (2004), show that the application of these two models often lead to different pricing and stocking behaviors and thus profit allocation. Petruzzi and Dada interpret this as a result of how price decisions affect demand uncertainty.

Second, in addition to the choice of demand models, we have witnessed the co-existence of two different retail price markup policies (e.g. Levy et al. (2004)). Choi (1991) uses an additive retail price markup policy, and Liu et al. (2007) employs a multiplicative retail price markup policy. It has been shown in the accounting literature that multiplicative and additive markup policies have different impacts on supplier profit when the supplier sets the wholesale price (Sahay, 2003).

Third, other market parameters, such as demand variability, product price elasticity, channel cost structure, have a far-reaching impact on the profit allocation between the retailer and supplier as shown in numerous studies. For example, Lariviere and Porteus (2001) examine a price-only WCA between a manufacturer and a retailer. They investigate the impact of market size, variability and the retailer's price sensitivity on the manufacturer's wholesale price decision and channel performance. Wang et al. (2004) study a single-product CCA with revenue sharing between a supplier and a retailer. Assuming uncertain and price-sensitive demand, they assess the impact of demand-price elasticity and retailer cost share on channel profits.

Finally, a body of literature has been devoted to study the implications of who makes the first move to offer a contract. Choi (1991) studies a WCA of two manufacturers and a common retailer under various game settings: Retailer Stackelberg, Manufacturer Stackelberg, and Vertical Nash. Liu et al. (2007) study the multiplicative markup policy in WCA where either the retailer or the supplier is the leader setting the markup. These studies show that the final allocation of channel profits depends on the contract offer and action of the first mover who is assumed to have the negotiating power between trading parties.

We note that, individually, the aforementioned academic papers narrowly focus on a single contractual arrangement, either wholesale or consignment, which is assumed to be the absolute contract choice of the supplier and retailer. When the contract reveals some channel inefficiencies, they propose incentives to coordinate channel decisions and eliminate those inefficiencies. (We will discuss this issue later in this section). A potential weakness of these studies is they ignore the fact that the supplier and retailer original contract preferences may be non identical, and it is only through bargaining that trading parties reach an agreement.

Studies have shown that the relative advantage of one contract over the other can be narrowed down by offering incentives. A stream of literature on WCA addresses the "double marginalization" issue in a decentralized channel where they develop different methods to coordinate channel decisions and eliminate inefficiencies. Some of these methods include incentives such as quantity flexibility (Tsay, 1999), options (Barnes-Schuster et al., 2002), backup (Eppen and Iyer, 1997), buyback (Pasternack, 1985), quick response (Iyer and Bergen, 1997), revenue sharing (Cachon and Lariviere, 2005), channel rebates (Taylor, 2002), bonus (Burer et al., 2008), lump-sum side payment (Zhao and Wang, 2002) or some combination. For a comprehensive review of these contracts, readers are referred to Cachon and Lariviere (2005).

Kandel (1996) distinguishes between CCA and WCA according to the allocation of responsibility for unsold inventory. With CCA, the supplier takes full responsibility to dispose the unsold inventory, while with WCA, the retailer purchases the products outright and assumes responsibility for unsold inventory. Interestingly, this distinction between WCA and CCA wanes when WCA has a buyback provision which gives the retailer the option to return any unsold units at a buyback price. Of course for the retailer to return unsold items, the buyback price should be strictly larger than the retailer scrap price. We note that marketing promotional efforts can also change the landscape of risk distribution between the retailer and supplier, and their decision making and profit allocation accordingly (Hacket (1993); Krishnan et al. (2004)).

The aforementioned studies show that one can optimize the profit and risk allocations between trading parties by offering incentives. This is true not only for WCA, but also for CCA (see Wang et al. (2004)). In practice, however, trading parties may have different interests and as will be investigated by this study, market factors and contract terms can have a profound influence on their original choice of contract. Since our interest lies in the original contract choices of trading parties, we consider supply chain scenarios where no coordination mechanisms are introduced for either CCA or WCA. As mentioned earlier, our knowledge of the retailer and supplier preferences is valuable in developing cost efficient and cost effective incentives. For certain market settings, we state in this chapter whether generous or modest incentives may be necessary to reach an agreement. However, we defer for future work how to formulate and develop incentives that are implementable. We shall conduct our study in a market setting where the retailer is the Stackelberg leader, i.e. it is the retailer that offers a take-it-or leave it contract to the supplier, not only for CCA, but also for WCA. We reserve for future research the case where the supplier is the Stackelberg leader.

In this study, we provide an analytical framework to obtain the profit maximizing pricing and stocking level decisions of the retailer and supplier. We determine the retailer and supplier preference levels for their original contract choices. Specifically, for the retailer (supplier), we utilize the ratio of the total optimal profit with CCA (WCA) to that of WCA (CCA) as our measure of retailer (supplier) preference level for CCA (WCA). Finally, we investigate how external market factors, channel cost structure and price markup policies influence those levels.

1.3 Model Formulations with Multiplicative Demand Model

Consider a supplier S and a retailer R. The supplier has unlimited production capacity to manufacture quantity q of a product. The manufacturing cost of the product is c_S per unit, while the retailer incurs a selling cost c_R per unit. Let the total unit cost be

$c = c_R + c_S$ and let α be the retailer's share of unit cost so that $\alpha = \frac{c_R}{c}$. Note that the cost information is known to the supplier and the retailer beforehand. Therefore, α is given rather than being a decision variable. Suppose the demand for the product is $\widetilde{D}(p) = d(p)\varepsilon$ where $d(p)$ is an iso-price-elastic demand function given by $d(p) = ap^{-\beta}$ with $a > 0$ and $\beta > 1$. β represents the self price elasticity of demand while ε is a random term independent of the price p with cumulative density function $F(\cdot)$ and probability density function $f(\cdot)$, known to both the retailer and the supplier. We assume demand is always nonnegative and thus ε has support $[A, B]$ with $B > A \geq 0$. Later on we find that it is difficult to derive exact analytical results describing the optimal pricing and stocking level decisions for WCA without assuming a specific distribution for ε. We then assume the demand distribution follows a uniform distribution [1].

We assume no salvage revenues and no product shortage costs. In this study, the superscript cs stands for CCA, ws for WCA and cc for the centralized channel. The proof of every major proposition in this chapter can be found in the Appendix. We now present the retailer and supplier decisions under CCA.

1.3.1 Consignment Contractual Agreement (CCA)

In CCA, we assume the retailer is the leader who offers the supplier a take-it-or-leave-it consignment contract that stipulates his revenue share r. The supplier is the fol-

[1]Perakis and Roels (2008) and Roels and Perakis (2006) show that if the demand distribution has a known finite support or is known to be symmetric, then the robust minmax regret order quantity is the same as the optimal order quantity for the traditional newsvendor profit maximization problem with a uniform distribution. For our purpose, we note that the demand variability usually can be inferred from historical sales.

lower that makes the price and stocking decisions (p, q). The sequence of events is: (1) the supplier conveys her intent to sell the product through the retailer; (2) the retailer offers a take-it-or-leave-it consignment contract to the supplier that stipulates the revenue share he will capture for each unit sold; (3) the supplier reviews the contract, accepts it as long as it yields a positive expected profit, and makes the production quantity and price decisions; (4) the product is delivered immediately at the beginning of a single selling season, and the retailer offers it to customers at the supplier's price; (5) demand is realized and at the end of the selling season, the retailer remits the supplier's share of the revenues and sends every unsold unit back to the supplier.

Given the retailer's revenue share r, the supplier's expected profit is:

$$\begin{aligned} \pi_S^{cs} &= (1-r)pE\{min[q, \widetilde{D}(p)]\} - (1-\alpha)cq \\ &= (1-r)pE\{q - [q - d(p)\varepsilon]^+\} - (1-\alpha)cq. \end{aligned} \tag{1.1}$$

We express the supplier's stocking decision as $z \equiv q/d(p)$ (Petruzzi and Dada (1999)) where z is the "stocking factor". Letting $\Lambda(z)$ be the expected overage factor given by $\Lambda(z) = \int_A^z (z-x)f(x)dx$, equation (1.1) becomes

$$\pi_S^{cs} = d(p)\{(1-r)p[z - \Lambda(z)] - (1-\alpha)cz\}. \tag{1.2}$$

Let $h(x) = f(x)/[1-F(x)]$ represent the failure rate of the random term of demand. We assume $\partial h(x)/\partial x > 0$. Note that the condition $\partial h(x)/\partial x > 0$ is known as an increasing failure rate (IFR, Lariviere and Porteus (2001)), a property satisfied by a broad family of distributions such as normal, uniform, gamma and Weibull with mild parameter restrictions. Proposition 1.3.1 provides the supplier's optimal decisions (Wang et al., 2004).

Proposition 1.3.1 *For any given r, and $z \in [A, B]$, the unique optimal price is determined by*

$$p^{cs*}(z) = \left(\frac{\beta c}{\beta - 1}\right)\left(\frac{z}{z - \Lambda(z)}\right)\left(\frac{1-\alpha}{1-r}\right). \tag{1.3}$$

Furthermore, the optimal z^{cs*} that maximizes $\pi_S^{cs}\left[p^{cs*}(z), z\right]$ is uniquely determined by

$$F(z) = \frac{z + (\beta - 1)\Lambda(z)}{\beta z}\,. \tag{1.4}$$

For any given r, the retailer observes the supplier's optimal price and stocking responses (p^{cs*}, z^{cs*}). Then, the retailer optimizes expected profit which is given by

$$\pi_R^{cs} = d(p^{cs*})\left\{rp^{cs*}\left[z^{cs*} - \Lambda(z^{cs*})\right] - \alpha c z^{cs*}\right\}. \tag{1.5}$$

Consider the optimal revenue share for the retailer in the following proposition (Wang et al., 2004).

Proposition 1.3.2 *The retailer's optimal revenue share is uniquely determined by:*

$$r^* = \frac{\alpha(\beta - 2) + 1}{(\beta - \alpha)}. \tag{1.6}$$

Substituting r^* in (1.3) yields $p^{cs*} = \frac{\beta c}{\beta-1}\frac{z^{cs*}}{z^{cs*}-\Lambda(z^{cs*})}\frac{\beta-\alpha}{\beta-1}$. Under the centralized channel, a central planner makes both pricing and stocking decisions. Hence $r = 0$ and $\alpha = 0$ in (1.1). Based on Proposition 1.3.1, it is quite straightforward to obtain the optimal solution (p^{cc*}, z^{cc*}) under the centralized channel. That is,

$$p^{cc*} = \frac{\beta c}{\beta - 1}\frac{z^{cc*}}{z^{cc*} - \Lambda(z^{cc*})}. \tag{1.7}$$

By (1.4), we obtain z^{cc*}. Comparing p^{cs*} and p^{cc*}, we find the consignment channel is not coordinated unless the retailer incurs all the cost and extracts all the channel profit, i.e.

$\alpha = 1$. Obviously, this is not the case in the real world. It is possible to develop incentive schemes that will divide the cost and channel profit between the supplier and retailer and improve channel coordination. However, we leave this feature for future research.

As mentioned earlier, it is difficult to derive exact analytical results for p^{cs*} and z^{cs*} without assuming a specific distribution for ε. So consider $\varepsilon \sim U[0, M]$. It satisfies the IFR condition. The supplier's optimal price and stocking decisions as well as the retailer's revenue share decisions are: $p^{cs*} = \frac{(\beta-\alpha)(\beta+1)c}{(\beta-1)^2}$, $z^{cs*} = \frac{2M}{\beta+1}$ and $r^* = \frac{\alpha(\beta-2)+1}{(\beta-\alpha)}$. Substituting p^{cs*}, z^{cs*} and r^* in the supplier's and retailer's profit functions, the optimal profits are $\pi_S^{cs*} = (1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta} \pi^{cc*}$ and $\pi_R^{cs*} = \left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)} \pi^{cc*}$ where

$$\pi^{cc*} = \frac{2Mac}{(\beta-1)(\beta+1)} \left[\frac{(\beta+1)c}{(\beta-1)}\right]^{-\beta} \tag{1.8}$$

is the centralized channel profit. By (1.7) and (1.4), the price and stocking decisions of the centralized channel are:

$$p^{cc*} = \frac{(\beta+1)c}{(\beta-1)}, \quad z^{cc*} = \frac{2M}{(\beta+1)}. \tag{1.9}$$

1.3.2 Wholesale Contractual Agreement (WCA)

In a WCA, the retailer buys certain amount of products from the supplier at a wholesale price w, and sells them to customers at a retail price p after applying a markup to his total cost. This retail pricing method is easy to implement. It is widely used by retailers to price products (Monroe, 1990). In this study, we use two retail price markup policies: the Additive Retail Price Markup (APM) and the Multiplicative Retail Price Markup (MPM). More details are provided in the following sections regarding these two policies. We continue employing the same Stackelberg game setting where the retailer is the leader. The decision-making process consists of three stages: (1) Retailer announces his margin (APM or MPM), i.e., his response function; (2) Supplier decides the wholesale price based on the retailer's margin (or response function); (3) Retailer makes the stocking and pricing decisions based on the wholesale price and offers the product to customers. We consider WCA with APM in the next section.

1.3.2.1 Additive Retail Price Markup (APM)

For any given retail margin m, the supplier sets the wholesale price w^* that maximizes her profit function

$$\pi_S^{ws} = d(p)z\left\{w - (1-\alpha)c\right\}. \tag{1.10}$$

The retailer observes the wholesale price w^* and sets the stocking factor z such that he maximizes his profit function given by

$$\pi_R^{ws} = d(p)\left\{p\left[z - \Lambda(z)\right] - (w + \alpha c)z\right\}. \tag{1.11}$$

Finally, the retailer sets the retail price to $p = w^* + \alpha c + m$. Note that with APM, also known as "cost plus", the margin is a fixed amount that is added to the wholesale price and retailer cost share.

Consider the retailer decisions. Given the retailer margin m and the supplier wholesale price w, the expected demand is $d(p) = ap^{-\beta} = a(w+\alpha c+m)^{-\beta}$. We find the retailer stocking level as follows:

$$\frac{\partial \pi_R^{ws}}{\partial z} = d(p)\left\{p\left[1 - F(z)\right] - (w + \alpha c)\right\}. \tag{1.12}$$

Because π_R^{ws} is a concave function of z, setting $\frac{\partial \pi_R^{ws}}{\partial z} = 0$ yields z^*. Specifically, we obtain

$$F(z^*) \;=\; 1 - \frac{w + \alpha c}{p} = \frac{m}{w + \alpha c + m}. \tag{1.13}$$

Assuming ε is uniformly distributed, i.e. $\varepsilon \sim U[0, M]$, the stocking factor is

$$z^* = \frac{Mm}{w + \alpha c + m} \tag{1.14}$$

The supplier profit is:

$$\begin{aligned} \pi_S^{ws} &= ap^{-\beta} z^* \left\{ w - (1-\alpha)c \right\} \\ &= Ma(w + \alpha c + m)^{-(\beta+1)} m \left\{ w - (1-\alpha)c \right\}. \end{aligned} \tag{1.15}$$

The optimal wholesale price is given next.

Proposition 1.3.3 *For any given m and assuming $\varepsilon \sim U[0, M]$, the supplier's optimal wholesale price w^* is uniquely determined by*

$$w^* = \frac{m + c}{\beta} + (1 - \alpha)c. \tag{1.16}$$

Expression (1.16) implies that the supplier raises her wholesale price when the retailer sets a high margin and/or when his cost share $(1 - \alpha)c$ increases.

The retailer sets the retail price to

$$\begin{aligned} p &= w^* + \alpha c + m \\ &= \frac{\beta + 1}{\beta} (m + c). \end{aligned} \tag{1.17}$$

Interestingly, not only can the retailer raise the retail price when m and/or c increase, but he can also lower it when customers are more price sensitive. Consider now how we obtain

m. The retailer sets m such that he maximizes his profit. Substituting (1.17) and (1.14) into the retailer's profit function (1.11), the retailer profit is

$$\begin{aligned}\pi_R^{ws} &= d(p)\left\{p\left[z^* - \Lambda(z^*)\right] - (w^* + \alpha c)z^*\right\} \\ &= K_2\,(m+c)^{-(\beta+1)}\,m^2, \qquad (1.18)\end{aligned}$$

where $K_2 = \frac{Ma}{2}(\frac{\beta}{\beta+1})^{\beta+1}$.

Optimizing π_R^{ws} with respect to m yields the next proposition.

Proposition 1.3.4 *The retail optimal margin m is uniquely determined by*

$$m^* = \frac{2c}{\beta - 1}. \qquad (1.19)$$

Intuitively, the retailer increases m when the unit cost c increases, and decreases m when consumers are more price sensitive. We now outline our final results as follows: Substituting m^* in (1.17), the optimal retail price can be rewritten as $p^{ws*} = \frac{(\beta+1)^2 c}{\beta(\beta-1)}$. Likewise, substituting m^* in (1.16) yields the optimal wholesale price $w^* = \frac{(\beta+1)c}{(\beta-1)\beta} + (1-\alpha)c$. By (1.14), $z^{ws*} = \frac{2M\beta}{(\beta+1)^2}$. Substituting p^{ws*}, z^{ws*} and w^* in the supplier's profit function (1.10) and the retailer's profit function (1.11), the profits are $\pi_S^{ws*} = (\frac{\beta+1}{\beta})^{-\beta}\pi^{cc*}$ and $\pi_R^{ws*} = (\frac{\beta+1}{\beta})^{-(\beta+1)}\pi^{cc*}$ where $\pi^{cc*} = \frac{2Mac}{(\beta-1)(\beta+1)}\left[\frac{(\beta+1)c}{(\beta-1)}\right]^{-\beta}$.

1.3.2.2 Comparison Between CCA and APM

Let π^{cc*} be the centralized channel profit given in (1.8). Table 1.1 summarizes the supplier, retailer, and total channel profits under CCA and APM which are closely related to price elasticity β, unit cost c, and M. Further, for CCA, the retailer's cost share α also affects their profits. The next proposition compares CCA and APM profits.

Table 1.1: Retailer, supplier and channel profits under CCA and APM

	CCA	APM
π_S^*	$(1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\pi^{cc*}$	$\left(\frac{\beta+1}{\beta}\right)^{-\beta}\pi^{cc*}$
π_R^*	$\left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)}\pi^{cc*}$	$\left(\frac{\beta+1}{\beta}\right)^{-(\beta+1)}\pi^{cc*}$
π^*	$\left[\frac{\beta-\alpha}{\beta-1}+(1-\alpha)\right]\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\pi^{cc*}$	$\frac{2\beta+1}{\beta+1}\left(\frac{\beta+1}{\beta}\right)^{-\beta}\pi^{cc*}$

Note: α - Retailer cost share, β - Price elasticity

Proposition 1.3.5 *The supplier and the retailer profits satisfy* $\pi_S^{cs*} < \pi_S^{ws*}$ *and* $\pi_R^{cs*} > \pi_R^{ws*}$. *The channel profit satisfies* $\pi^{cs*} > \pi^{ws*}$.

By Proposition 1.3.5, the retailer (supplier) is better off with CCA (WCA). As mentioned earlier, we measure the retailer (supplier) contract preference level by taking the ratio of retailer (supplier) profit for CCA (WCA) to retailer (supplier) profit for WCA (CCA). Formally, we define $\rho_S = \frac{\pi_S^{ws*}}{\pi_S^{cs*}}$ and $\rho_R = \frac{\pi_R^{cs*}}{\pi_R^{ws*}}$. A profit ratio ρ_R (ρ_S) larger than 1.0 means the retailer (supplier) has a strong preference for CCA (WCA). A profit ratio of 1.0 means indifference between WCA and CCA. Next we present the effects of demand uncertainty, retailer cost share and price elasticity on preference levels in the following propositions.

Proposition 1.3.6 *Given an MD market setting and* $\varepsilon \sim U[0, M]$, ρ_S *and* ρ_R *do not change with demand variability.*

The proof of Proposition 1.3.6 is straightforward. Demand variability only affects π^{cc*} (as shown in Table 1.1) which is a common term in the retailer and supplier optimal profits.

Proposition 1.3.7 *ρ_S and ρ_R increase with the retailer cost share α.*

The proof of Proposition 1.3.7 is part of that for Proposition 1.3.5.

Proposition 1.3.8 *ρ_S and ρ_R decrease with the product price elasticity β.*

Proposition 1.3.9 *For any given retailer cost share α, ρ_S is no less than $\frac{e^{-\alpha}}{1-\alpha}$, and ρ_R is within* $[e^{\alpha}, 4]$.

Proposition 1.3.9 defines the boundaries of the supplier and retailer preference levels. A high price elasticity gives values of ρ_S and ρ_R that are close to their lower boundaries. Further, it is easy to show that $\rho_R > \rho_S$ when $\alpha < 0.8$. In this range, reaching a CCA is likely when the retailer offers the supplier a modest incentive. If $\alpha \geq 0.8$, then $\rho_S > \rho_R$ and ρ_S increases faster than ρ_R which would suggest that the retailer must offer a more generous incentive to reach a CCA. We provide additional insights for the preceding propositions in the concluding section of this chapter.

1.3.2.3 Multiplicative Retail Price Markup (MPM)

In this section, we present WCA with MPM. We assume ε follows a distribution with IFR. The retailer sets the retail price to $p = (w + \alpha c)(1 + u)$ where u is the percentage markup. The supplier and retailer profit functions are given by (1.10) and (1.11). For any given u, the supplier determines her wholesale price w^*. After observing w^*, the retailer sets the retail price to $p = (w^* + \alpha c)(1 + u)$. The retailer optimizes u based on his profit function (1.11). Following the same method in Liu et al. (2007), we obtain the price and stocking decisions as: $p^{ws*} = \frac{\beta}{\beta-1} p^{cc*}$ and $z^{ws*} = z^{cc*}$. The optimal wholesale price is $w^* = \frac{\beta c}{\beta-1} - \alpha c$. The retailer, supplier and channel profits under CCA and MPM are summarized in Table 1.2, where $\pi^{cc*} = \frac{acz^{cc*}}{\beta-1}\left[\left(\frac{\beta c}{\beta-1}\right)\left(\frac{z^{cc*}}{z^{cc*}-\Lambda(z^{cc*})}\right)\right]^{-\beta}$. The profits under

Table 1.2: Retailer, supplier and channel profits under CCA and MPM

	CCA	MPM
π_S^*	$(1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\pi^{cc*}$	$\left(\frac{\beta}{\beta-1}\right)^{-\beta}\pi^{cc*}$
π_R^*	$\left(\frac{\beta-\alpha}{\beta-1}\right)^{(1-\beta)}\pi^{cc*}$	$\left(\frac{\beta}{\beta-1}\right)^{(1-\beta)}\pi^{cc*}$
π^*	$\left[\frac{\beta-\alpha}{\beta-1}+(1-\alpha)\right]\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\pi^{cc*}$	$\frac{2\beta-1}{\beta-1}\left(\frac{\beta}{\beta-1}\right)^{-\beta}\pi^{cc*}$

Note: α - Retailer cost share, β - Price elasticity

MPM are a function of price elasticity, unit cost, and demand distribution. Interestingly, Proposition 1.3.5 also applies to MPM. Finally, we now formally present four results as propositions which describe the effects of demand uncertainty, cost share, price elasticity and provide bounds on ρ_S and ρ_R.

Proposition 1.3.10 *ρ_S and ρ_R are independent of the probability distribution of ε.*

The proof of Proposition 1.3.10 is omitted as it is straightforward to show that ρ_S and ρ_R only depend on the cost share and price elasticity. Proposition 1.3.10 implies that demand variability does not affect the retailer and supplier contract choices. The reason is under MD, the demand coefficient of variation is constant for all price policies. We show later that this is not true under the Additive Demand (AD) setting.

Proposition 1.3.11 *ρ_S and ρ_R increase with the retailer cost share α. $\rho_S = 1$ and $\rho_R = 1$ when $\alpha = 0$.*

The proof of Proposition 1.3.11 is similar to that of Proposition 1.3.7. The relationship between the preference level and the cost share under MPM is the same as in APM. However, an interesting finding is when the retailer cost share $\alpha = 0$, the retailer and supplier under MPM obtain the same profits as in CCA , and thus they are indifferent between CCA and MPM.

Proposition 1.3.12 *ρ_S and ρ_R increase with product price elasticity β.*

The impact of price elasticity on retailer and supplier preference levels under MPM is opposite that of APM's. Under APM, both the retailer and supplier preference levels diminish as price elasticity increases.

Proposition 1.3.13 *For a given α, ρ_S is no more than $\frac{e^{-\alpha}}{1-\alpha}$, and ρ_R is less than e^{α}.*

The proof of Proposition 1.3.13 is similar to that of Proposition 1.3.9. Surprisingly, however, we find that the lower bounds on preference levels in APM to be the upper bounds in MPM. This means that for any given value of α, the preference levels are significantly weaker in MPM compared to those in APM. Note however that in both MPM and APM, if the retailer incurs all the channel cost (i.e. $\alpha = 1$) and extracts all channel profits under CCA, then the supplier preference level for MPM goes to infinity. This suggests that the retailer would need to share a significant portion of his profits with the supplier as incentive to reach a CCA.

1.4 Model Formulations with Additive Demand (AD) Model

In the previous sections, we consider the retailer and the supplier price and stocking decisions with multiplicative customer demand (MD). In this section, we investigate the retailer and supplier preference levels when customer demand is linear and additive. In particular, we define the customer demand as $\tilde{D}(p) = d(p) + \varepsilon$ where $d(p) = a - \beta p$ and ε is a random term with positive support on $[A, B]$ with $B > A \geq 0$. Again let β denote the price elasticity. In order to guarantee a positive demand and avoid trivial results, we assume $a - \beta c + A > 0$. Unlike MD, finding analytical solutions to problems with AD is often formidable (Granot and Yin, 2005). We resort to a numerical study to obtain some meaningful results and compare them with MD's. Our analysis starts with CCA and then we proceed to WCA with APM and MPM.

1.4.1 Consignment Contractual Arrangement (CCA)

Under CCA, for any given revenue share r, the supplier profit function is

$$\begin{aligned} \pi_S^{cs} &= (1-r)p \min\{q, \widetilde{D}(p)\} - (1-\alpha)cq \\ &= (1-r)p\{q - [q - d(p) - \varepsilon]^+\} - (1-\alpha)cq. \end{aligned} \tag{1.20}$$

Utilizing the stocking factor $z \equiv q - (a - \beta p)$, we can rewrite (1.20) as

$$\pi_S^{cs} = (1-r)p[a - \beta p + z - \Lambda(z)] - (1-\alpha)c(a - \beta p + z). \tag{1.21}$$

For any given retailer revenue share r, the supplier sets her price and stocking factor that maximizes her profit. For any fixed z, π_S^{cs} is concave in p. The optimal price is

$$p^{cs*}(z) = \frac{(1-\alpha)c}{2(1-r)} + \frac{a + z - \Lambda(z)}{2\beta}. \tag{1.22}$$

Given $p^{cs*}(z)$, we find the optimal z via (1.21). As shown in Wang et al. (2004), it is extremely difficult to derive the closed-form solution for z even with mild restrictions on the revenue share r. However, they are able to establish the following result.

Proposition 1.4.1 *When $h(z)^2 + h'(z) > 0$, for any $0 < r < 1 - (1-\alpha)\beta c/(a + A)$, p^{cs*} and z^{cs*} exist and are unique.*

The restriction on revenue share r in Proposition 1.4.1 guarantees the supplier a positive profit. The retailer optimizes his revenue share by maximizing his total profit based on the supplier response function. The retailer's profit function is

$$\pi_R^{cs} = rp^{cs*}[a - \beta p^{cs*} + z^{cs*} - \Lambda(z^{cs*})] - \alpha c(a - \beta p^{cs*} + z^{cs*}) \tag{1.23}$$

where (p^{cs*}, z^{cs*}) are the optimal price and stocking decisions for any given r. Because (1.23) is a complex function, we are unable to find a closed-form expression for the revenue share r^* that maximizes (1.23). However, it is possible to show that a unique r^* exists within a certain range. Consider the next proposition (see Wang et al. (2004) for details).

Proposition 1.4.2 *When $h'(z) > 0$ i.e.,$F(\cdot)$ has an increasing failure rate, there exists an r^* that maximizes the retailer's profit. Furthermore, $\alpha < r^* < 1 - (1-\alpha)\beta c/(a + A)$.*

The retailer has to satisfy the condition $r^* < 1 - (1-\alpha)\beta c/(a + A)$ in order to induce the supplier to produce a positive quantity (Wang et al., 2004). If $\varepsilon \sim U[0, M]$, then $h(z) = \frac{1}{M-z}$ and $h(z)$ satisfies the conditions in Propositions 1.4.1 and 1.4.2. Hence, the supplier and retailer optimal decisions exist.

1.4.2 WCA with Additive Retail Price Markup (APM)

Under WCA, the supplier's profit function is

$$\pi_S^{ws} = q[w - (1-\alpha)c]. \tag{1.24}$$

The stocking factor is defined by $z \equiv q - (a - \beta p)$. We can rewrite (1.24) as

$$\pi_S^{ws} = (a - \beta p + z)[w - (1-\alpha)c]. \tag{1.25}$$

The supplier maximizes (1.25) given the retailer gross margin m. Meanwhile, the supplier knows the retailer stocking factor z for any given wholesale price w. We can determine the relationship between w and z by optimizing the retailer profit function which is given by

$$\begin{aligned} \pi_R^{ws} &= pE\min\{q, \widetilde{D}(p)\} - (w + \alpha c)q \\ &= p[a - \beta p + z - \Lambda(z)] - (w + \alpha c)(a - \beta p + z). \end{aligned} \tag{1.26}$$

If the retailer announces m and the supplier decides to charge w for each unit, then the retailer offers the product to consumers at a price $p = w + \alpha c + m$.

$$\frac{\partial \pi_R^{ws}}{\partial z} = p\,[1 - F(z)] - (w + \alpha c). \tag{1.27}$$

π_R^{ws} is concave in z. We obtain the retailer optimal response as follows: Set $\frac{\partial \pi_R^{ws}}{\partial z} = 0$. The stocking factor z^* satisfies

$$F(z^*) \;=\; 1 - \frac{w + \alpha c}{p} = \frac{m}{w + \alpha c + m}. \tag{1.28}$$

As before, we assume the random term in demand function follows a uniform distribution $\varepsilon \sim U[0, M]$. With this assumption we have

$$z^* = \frac{Mm}{w + \alpha c + m}. \tag{1.29}$$

Substituting (1.29) into the supplier's profit function leads to

$$\begin{aligned}\pi_S^{ws} &= (a - \beta p + z^*)[w - (1-\alpha)c] \\ &= \left[a - \beta(w + \alpha c + m) + \frac{Mm}{(w + \alpha c + m)}\right][w - (1-\alpha)c]. \qquad (1.30)\end{aligned}$$

Now consider the next proposition.

Proposition 1.4.3 *For a given retail margin m, there exists a unique wholesale price w^* that maximizes the supplier's profit. Further, $(1-\alpha)c < w^* < \frac{a+\sqrt{a^2+4M\beta m}}{2\beta} - m - \alpha c$.*

The range for w^* given in Proposition 1.4.3 guarantees a positive order quantity from the retailer (see proof in Appendix for details). Next, we determine the retailer margin m given the supplier optimal response function $w^*(m)$. Substituting (1.29) into the retailer's profit function (1.26) gives

$$\begin{aligned}\pi_R^{ws} &= p[a - \beta p + z^* - \Lambda(z^*)] - (w^* + \alpha c)(a - \beta p + z^*) \\ &= am - \beta(w^* + \alpha c + m)m + \frac{Mm^2}{2(w^* + \alpha c + m)}. \qquad (1.31)\end{aligned}$$

The next proposition states our results.

Proposition 1.4.4 *There exists at least one m^* that maximizes the retailer's profit. Moreover, $0 < m^* < \frac{4a+3M}{6\beta}$.*

Propositions 1.4.3 and 1.4.4 show that optimal decisions exist for both retailer and supplier. They provide lower and upper bounds on the optimal wholesale price and margin. The lower bounds on wholesale price and retail margin are obvious since the retailer and the supplier must recover their costs. The upper bounds are more complicated and depend on a set of parameters. In the following section, we carry out a numerical study to compare the retailer, supplier and channel profits under CCA and APM.

1.4.3 Comparison Between CCA and WCA with APM

AD has fewer tractable properties than MD (see Granot and Yin (2005)). We are unable to derive closed-form solutions for CCA and APM, so we employ a numerical study to compare channel profits under CCA and APM. The details of this numerical study are as follows: As before, we assume the random term to be uniformly distributed, i.e. $\varepsilon \sim U[0, M]$. Also, we let $a = 50$ and the unit cost $c = 10$. First, we study the impact of demand uncertainty. We varied M within $[2, 30]$ to simulate different levels of demand uncertainty. We set α to 0.2 and β to 3. Figure 1.1 presents our results. With low demand

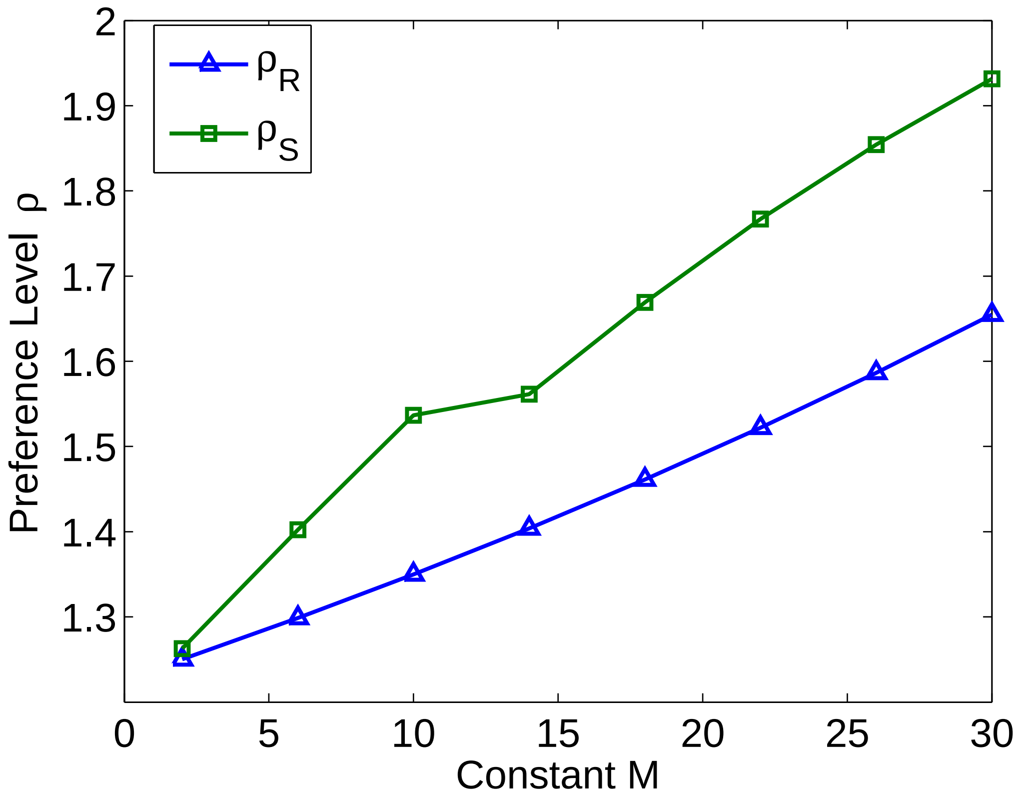


Figure 1.1: The impact of demand uncertainty on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus APM, where $\varepsilon \sim U[0, M]$.

uncertainty (i.e. small M), ρ_S and ρ_R were small and roughly equal to each other. However, ρ_S and ρ_R increased as demand became more uncertain. In other words, the supplier and retailer's preferences with their original contract choices, i.e., WCA and CCA respectively, turned stronger. Moreover, the difference (gap) between ρ_S and ρ_R widened.

Next we examine the impact of price elasticity. We varied β within $[0.1, 4.1]$ and fixed M to 2. Figure 1.2 presents the impact of price elasticity on the retailer's and supplier's preference levels under CCA and APM. With β fixed, $\rho_S > \rho_R$. However, the gap between ρ_S and ρ_R diminished as β increased. Finally, we investigated the impact of

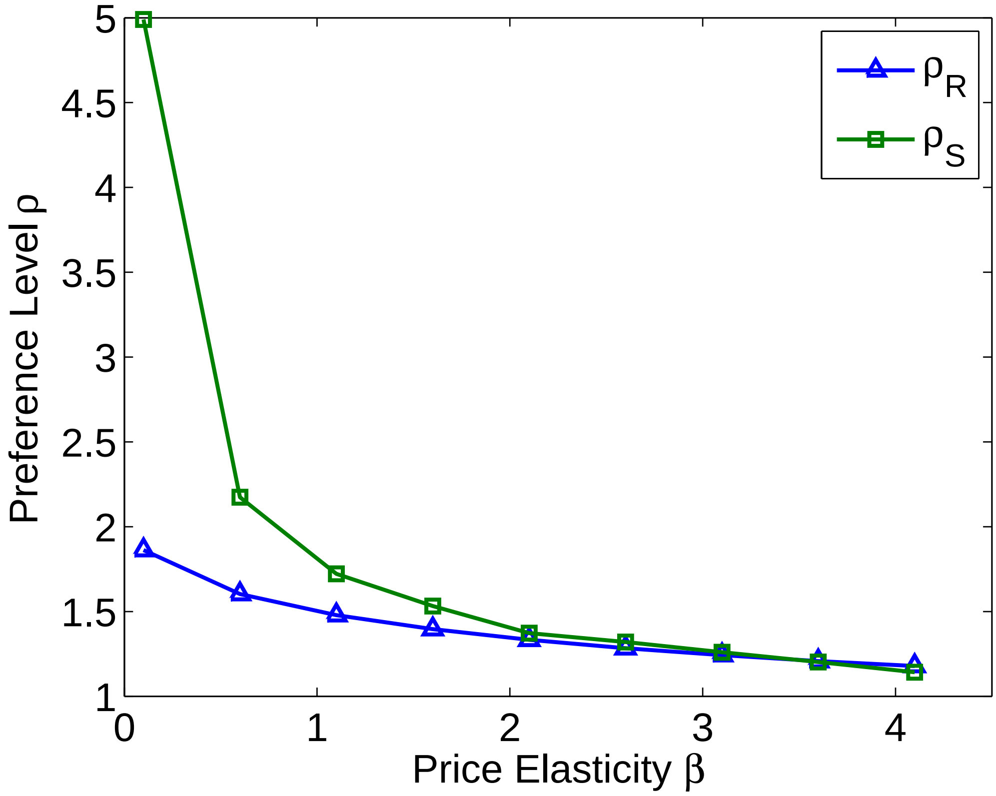


Figure 1.2: The impact of price elasticity β on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus APM, where $\varepsilon \sim U[0, M]$.

the retailer's cost share on the retailer and supplier preference levels for CCA and APM, respectively. We fixed β to 3 and varied α between $[0.1, 0.9]$. Figure 1.3 shows the gap $\rho_S - \rho_R$ increasing as α increases. These results are consistent with that of MD. In terms of total supply chain profit, CCA outperformed APM.

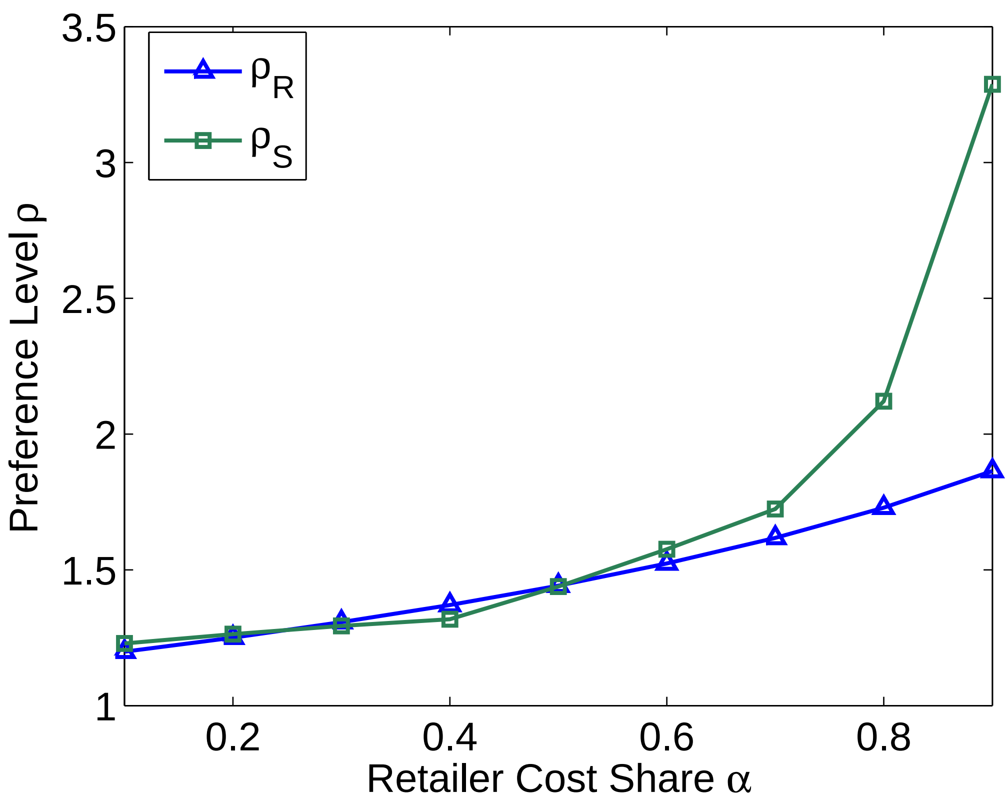


Figure 1.3: The impact of retailer cost share α on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus APM, where $\varepsilon \sim U[0, M]$.

1.4.4 WCA with MPM and how it compares with CCA

With MPM, the supplier and retailer profit functions are given by (1.25) and (1.26). Liu et al. (2007) show that the optimal price and stocking decisions exist, but it is impossible to obtain closed-form solutions. Hence, we study the impact of demand uncertainty, price elasticity and cost share on the retailer and supplier preference levels numerically. We assumed ε follows a gamma distribution with shape parameter $k = 2$, which guarantees the IFR property. To study the effect of demand uncertainty on retailer and supplier preferences, we varied the scale parameter θ within $[0.5, 3]$. To study the impact of price elasticity, we fixed k to 2 and θ to 2 and varied β within $[0.1, 4.1]$. Finally, we examined the impact of cost share by also fixing k to 2 and θ to 2, and varying α within $[0.1, 0.9]$. The results are shown in Figures 1.4, 1.5 and 1.6. Our results show that the retailer and supplier contract preference levels and the gap between them increased with demand variability. Compared with APM, the behavior of preference levels with MPM was totally different. The gap between preference levels was large for the entire range of β with both preference levels increasing with β. Although the preference levels show similar upward trend as in APM where we varied the retailer cost share, the gap between them was larger for MPM.

1.5 Conclusions

In this chapter, we define the relative strength of retailer and supplier preferences between consignment and wholesale contracts as the ratio of their optimal expected profits for their first choice of contract over that for the competing alternative contract. We determine the optimal expected profits for the case where the retailer offers the supplier a

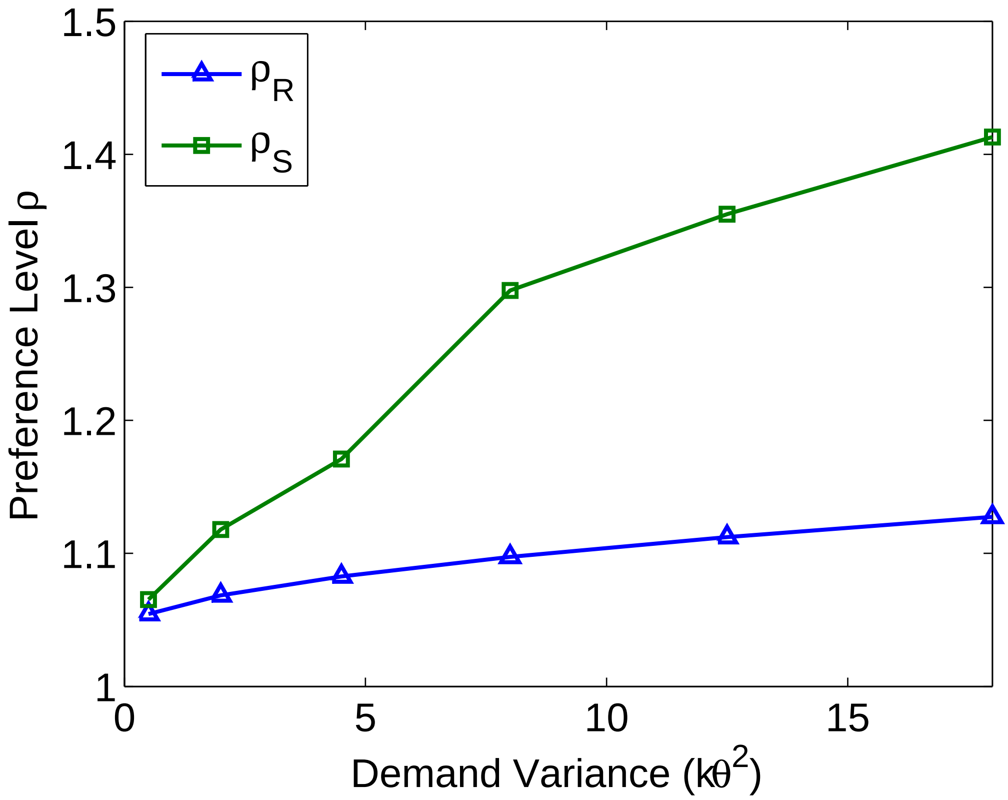


Figure 1.4: The impact of demand uncertainty on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus MPM, where demand follows a gamma distribution with shape parameter $k = 2$ and scale parameter $\theta \in [0.5, 3]$.

take-it-or-leave-it consignment or wholesale contract. Our contribution is that we study the retailer and supplier's original contract choices as well as their alternative contract choices. We examine their preference levels between these contract choices and how they are significantly affected by a number of salient factors, including consumer demand (additive demand (AD) or multiplicative demand (MD)), price markup (additive price markup (APM) or multiplicative price markup (MPM)), market price elasticity, demand uncertainty and cost share (see Table 1.3). Consider our results when consumer demand is Multiplicative

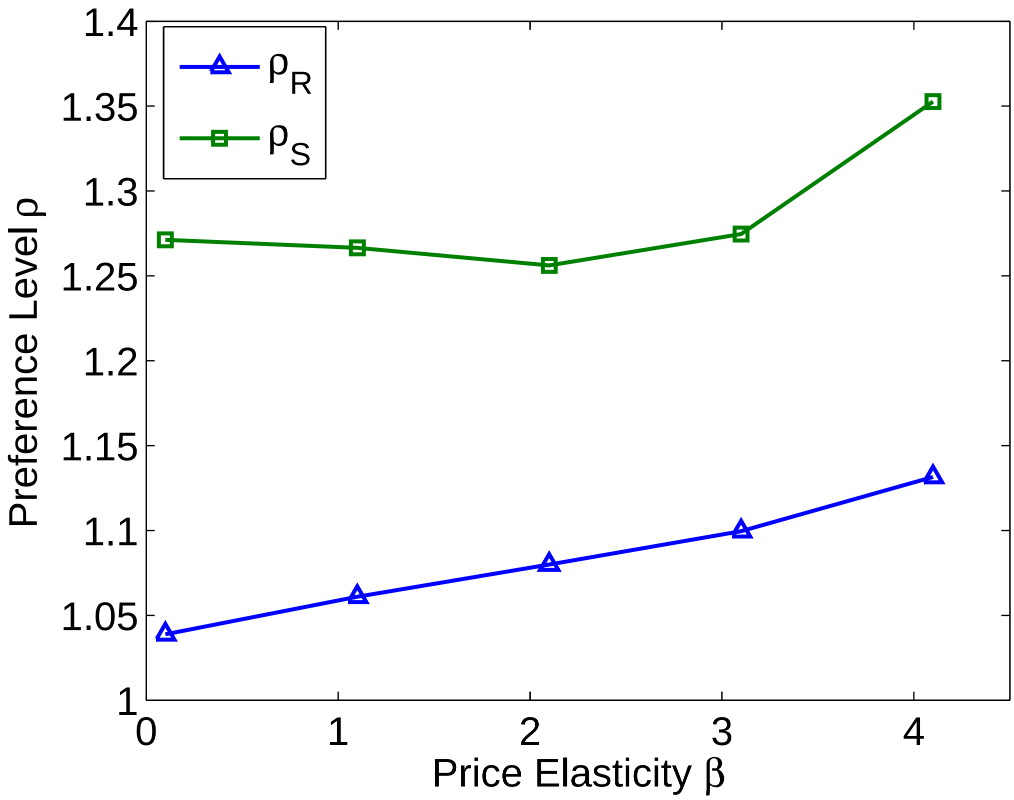


Figure 1.5: The impact of price elasticity β on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus MPM, where demand follows a gamma distribution with shape parameter $k = 2$ and scale parameter $\theta = 2$.

Demand. We obtain lower and upper bounds on the supplier and retailer preference levels which are functions of the retailer cost share. Using those bounds, we find that when the retailer cost share approaches zero, the retailer and supplier optimal expected profits under consignment and wholesale with Multiplicative and Additive Price Markups converge, with the preference levels for Multiplicative Price Markup noticeably weaker than that for Additive Price Markup's. That is, the retailer and supplier become increasingly indifferent between consignment and wholesale contractual arrangements. However, if the retailer in-

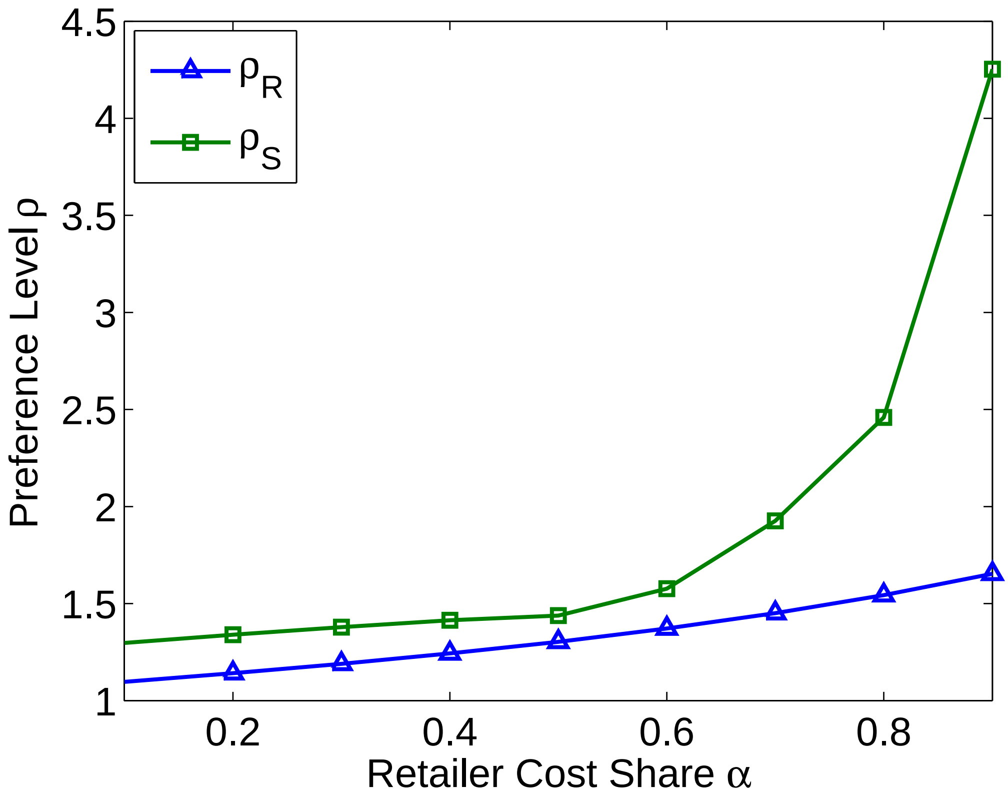


Figure 1.6: The impact of retailer cost share α on retailer and supplier preference levels (ρ_R and ρ_S) for CCA versus MPM, where demand follows a gamma distribution with shape parameter $k = 2$ and scale parameter $\theta = 2$.

curred all the channel costs and extracted all channel profits under consignment, then the supplier preference level for wholesale with Multiplicative Price Markup increases without bound. This suggests that, to reach agreement on a consignment, the retailer will have to share a significant portion of his profits with the supplier. Finally with Multiplicative Demand, demand variability only affected the channel's total expected profit, but not the retailer and supplier preference levels for consignment and wholesale with either Additive Price Markup or Multiplicative Price Markup.

With Additive Demand, we find that an increasing demand uncertainty can significantly raise the retailer and supplier preferences for their first choice of contract type. Not only is the supplier preference level for wholesale larger than that of the retailer's for consignment, the gap between them increases rapidly as demand variability increases. Intuitively, as demand becomes uncertain, neither retailer nor supplier will take more inventory risks which explains why the retailer and supplier preferences for their choices of contract becomes stronger. Numerically, we also show that the gap also increases with the retailer cost share. Finally, the retailer preference level increases with price elasticity, but unlike in Multiplicative Demand, it is non-monotonic, i.e., it decreases and then increases as price elasticity increases.

Our research can be extended in several ways. In a centralized channel, a central planner makes the profit maximizing decisions for both the retailer and the supplier. In a decentralized channel, "double marginalization" often exists which reduces channel profits unless certain forms of incentive are offered to induce coordination in decision-making (see Tirole (1988)). A potential topic for future research is to develop incentives to promote better coordination between supplier and retailer decentralized decisions in a consignment contractual arrangement. One can also extend our work to a single retailer and two suppliers of substitutable products to study the effects of product competition on contract preferences.

Finally, one might want to find other robust preference measures which can be employed along with the profit ratios utilized in this chapter, for example, a preference level that is rooted on inventory risk assessments. Other than the market factors and contract parameter terms presented in this chapter, other factors that can affect inventory risks can be investigated, such as the rapid development of new technology (e.g. obsolescence risk in electronic products) and changes in supply chain and logistics practices, standards, laws and taxation.

Table 1.3: The behavior of retailer and supplier preference levels and preference level bounds for multiplicative and additive demands with increasing price elasticity, retailer cost share and demand uncertainty.

	Price Elasticity $\beta \uparrow$		Cost Share $\alpha \uparrow$		Demand Uncertainty $\uparrow$		Preference Level Bounds	
	APM	MPM	APM	MPM	APM[a]	MPM	APM	MPM
Multiplicative Demand (MD)	$\rho_R \downarrow \rho_S \downarrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \rightarrow \rho_S \rightarrow$	$\rho_R \rightarrow \rho_S \rightarrow$	$e^{\alpha} \leq \rho_R \leq 4$ $\rho_S \geq \frac{e^{-\alpha}}{1-\alpha}$	$\rho_R < e^{\alpha}$ $\rho_S < \frac{e^{-\alpha}}{1-\alpha}$
Additive Demand (AD)	$\rho_R \downarrow \rho_S \downarrow$	$\rho_R \uparrow \rho_S \downarrow\uparrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \uparrow \rho_S \uparrow$	$\rho_R \uparrow \rho_S \uparrow$	–	–

$\rho_R = \frac{\pi_R^{*cs}}{\pi_R^{*ws(APM\ or\ MPM)}}$ and $\rho_S = \frac{\pi_S^{*ws(APM\ or\ MPM)}}{\pi_S^{*cs}}$

$\uparrow$ increase, $\downarrow$ decrease, $\rightarrow$ no effect, $\uparrow\downarrow$ mixed effects

[a] Results are derived where $\varepsilon \sim U[0, M]$

CHAPTER 2
CHANNEL COORDINATION IN A CONSIGNMENT CONTRACT

2.1 Introduction

The online marketplace is thriving at an unprecedented pace. According to its annual report, eBay's 2007 total gross sales volume was \$7.67 billion, a record high and a 28% increase over 2006. Amazon.com, experiencing the same unprecedented growth, reported \$14.84 billion net sales in 2007, a 39% hike over 2006. Of the business models being used by virtual retailers, consignment selling has become a very successful and significant part of their business. At Amazon.com, consignment contributes 28% of total units sold in 2007, an unprecedented growth from 6% in 2000. A typical consignment, as described in Hacket (1993), is a unique contract where the retailer, over a given period, takes possession of the goods owned by the supplier, promotes the sale of these goods to buyers, and receives a share of the sales revenue (sales commission). The supplier owns those goods until they are sold. For instance, consigning an art work to an art gallery stipulates in the contract that the artist entrusts her work to the art gallery over a certain period with the promise to share the proceeds upon sale (Crawford and Mellon, 1981). Other forms of consignment agreements do not require the retailer to take physical possession of goods, but instead the retailer has the exclusive right to sell the goods such as in real estate sales and virtual markets. The sales commission also varies. Amazon.com, for example, stipulates a 60-day free listing of an item. At sale, Amazon collects \$0.99 closing fee plus a variable commission depending on the item's category, e.g., computers (6%), camera and photo and electronics items (8%), items in the everything else store (15%). Sales commission on real estate sales ranges from 3% to 7% of sales price.

When operational and marketing decisions in a consignment channel are made separately, an independent assessment of customer demand by the supplier may be insufficient to make profit maximizing pricing and stocking decisions, and coordination with the retailer in making those decisions is imperative. Consequently, poor coordination can lead to low stocking levels, high prices, and low channel profits. It has been shown that poor coordination in a consignment channel can reduce channel profits by as much as 26.4 % (see Wang et al. 2004). In this chapter, we investigate, in a consignment contract, how effective bonus and Revenue Sharing with Side Payment (RSSP), when utilized individually as an incentive, can induce the supplier to increase her stocking level and lower her price such that channel profits are maximized.

A bonus is an amount granted and paid *ex gratia* to the supplier based on supplier sales to end customers. For example, Cafepress.com, an online marketplace selling customized products such as t-shirts, posters and mugs, has a sales volume bonus program, where if a supplier's monthly sales exceed a certain threshold, then the supplier receives a bonus. Consider Cafepress's incremental bonus policy: "As your total base price sales increase, you earn bonuses based on the amount you sell over certain thresholds. The first threshold is at \$250. Once you sell over \$250 in base price sales, you will receive a bonus of 7% for each dollar of base price sales earned over \$250 but less than \$1,000. Each dollar of base prices sales over \$1,000 but less than \$5,000 earns 15% bonus..." (refer to *www.cafepress.com* for complete details). Notice that the supplier earns a bonus when his sales surpasses a threshold T. In this study, we first investigate a two-tier bonus (r, r_0, T) where the retailer reduces his revenue share from r to r_0 for sales made above T and present the insights behind our results. Using the same analytic modeling approach that we employ for the two-tier bonus, we can optimize a bonus with multiple tiers. As will be shown later, however, the multi-tier bonus gives an insignificant improvement in channel profit over the two-tier bonus.

With RSSP, the supplier pays a fee (side payment) to join a program which entitles her not only to sell her products through the retailer, but also a discount of the retailer's original commission. For example, Amazon.com's UK pro-merchant program utilizes a RSSP in a consignment contract. The program charges a $\pounds 28.75$ monthly fee and the supplier receives a 30% discount on retailer commission on sales of electronics and photo items (see *www.amazon.co.uk* for details). The contract, in general, can be described using two parameters (r, Φ) where r is the retailer revenue share per unit sold and Φ is the side payment. In this study, we show why it is optimal for the retailer to charge a fee. We also show how to find the optimal RSSP parameters.

In studying the role of bonus and RSSP in coordinating the consignment distribution channel, we assume a single period. This is a reasonable assumption since the products that we consider typically have short life cycles (e.g. fashion items and electronics). Moreover, this is consistent with the supply chain literature on contracts (see Lariviere 1999). We also assume a stochastic demand. We show in this study that the two-tier bonus cannot fully coordinate the consignment channel. However, we are able to demonstrate that fine tuning the bonus parameters (r, r_0, T) can significantly increase channel profits. In contrast, we show that RSSP in a consignment contract can fully coordinate the consignment channel only if the retailer's revenue share is equal to his cost share. Moreover, the RSSP contract is pareto improving for a particular range of side payment values. We study the effects of changes in price elasticity and cost share on the optimal contract parameters for bonus and RSSP.

Finally, we present a variation of RSSP called Cost Sharing with Side Payment (CSSP) which is exemplified by Amazon.com's Advantage program for book, music and movie suppliers (see *advantage.amazon.com* for details). The CSSP parameters consist of the retailer commission r, the retailer additional cost share δ and the side payment Φ. With CSSP, in addition to r, Amazon.com charges a payment for providing additional services (e.g. storage, shipping and handling, payment processing). Similar to RSSP, when the retailer's revenue share r is set equal to his total cost share (including extra cost share δ), the channel is coordinated. As demonstrated by Cafepress's supplier incentive program and Amazon.com's pro-merchant and advantage programs, the bonus, RSSP and CSSP presented in this chapter are implementable contracts. We reserve for future research to study the benefits of having a combination of these incentives in a consignment contract.

The remainder of this chapter is organized as follows. Section 2.2 reviews the literature. Section 2.3 presents our model formulation and related assumptions. Also, we study the centralized and decentralized consignment channels of a single supplier and a single retailer. Coordination mechanisms are investigated in Section 2.4. Finally, we conclude this chapter and discuss some future research directions in Section 2.5.

2.2 Literature Review

In a decentralized market channel where the supplier and retailer make independent decisions, practitioners and researchers have proposed various ways to coordinate channel decisions and eliminate inefficiencies. Some of the proposed mechanisms include quantity flexiblility contracts (Tsay, 1999), options (Barnes-Schuster et al., 2002), backup (Eppen and Iyer, 1997), buyback (Pasternack, 1985), quick response (Iyer and Bergen, 1997), revenue sharing (Cachon and Lariviere, 2005), channel rebates (Taylor, 2002), bonus (Burer et al., 2008), lump-sum side payment (Zhao and Wang, 2002) or a combination of them. The terms of contracts in these studies are unique to a wholesale distribution channel where the supplier offers her products at a take-it-or-leave wholesale price to the retailer who, in response to the offer, determines his order quantity and retail price.

In the course of our work, we investigated the potential of every aforementioned incentive to coordinate a decentralized consignment channel. The bonus and revenue sharing proved promising in significantly reducing channel inefficiencies. Burer et al. (2008) study a wholesale contract for the corn seed industry employing mixed incentives that include a bonus and a penalty to coordinate the channel. Taylor (2002) investigates wholesale channel rebates in the auto industry where the manufacturer, through the channel rebates, rewards the retailer for exceeding the manufacturer's sales goal. His work shows that between linear and target rebates, only target rebates can coordinate the channel. Zhao and Wang (2002) explore a multi-period problem where an upstream manufacturer outsources both distribution and retailing to a retailer. The manufacturer adds a side payment to the wholesale price to coordinate the channel. Our work differs because we consider, in a consignment channel, the retailer making reward payments to the supplier.

Under a revenue sharing agreement, the retailer pays the supplier a portion of the retail price for each unit sold to the consumer to induce the supplier to produce more. Studies exploring the revenue sharing capability to coordinate a wholesale channel in a newsvendor setting include Cachon and Lariviere (2005), and Donahue (2000) and Taylor (2001) where there is a second buying opportunity. Dana and Spier (2001) consider competing newsvendors under perfect competition while Gerchak and Wang (2004) consider the problem in assembly systems. In every cited revenue sharing study, the retailer makes the stocking level and retail pricing decisions. However, in our work, it is the supplier that makes these decisions, and revenue sharing is an inducement to the supplier not only to produce more, but also to lower his price. Also, unlike the previous studies which show that revenue sharing on its own can coordinate a wholesale channel, it is not true for a consignment channel. To achieve full coordination, we show the need to complement it with a side payment which explains why charging a membership fee in Amazon.com's pro-merchant program is optimal.

The paper by Wang et al. (2004) is closely related to our work in that they study a single-product consignment contract with revenue sharing between a supplier and a retailer. Assuming uncertain and price-sensitive demand, they assess the impact of demand-price elasticity and the retailer's cost share on channel profits. They conclude that the profit loss in a decentralized supply chain increases with the demand-price elasticity, and decreases with the retailer's cost share. However, they do not develop any coordination mechanisms to reduce the channel loss. A follow-up work by Li and Hua (2008) employs a cooperative game approach to coordinate the decentralized consignment channel. However, their approach has some limitations when applied in the real world, such as the lack of efficiency in the bargaining process (Cachon, 2004). With RSSP and CSSP, we show that it is possible to eliminate all the channel losses. Further, players do not need to bargain and thus it is more efficient and easier to implement. Wang (2006) extends the consignment contract to a supply chain with multiple sources of complementary parts and a single manufacturer. The suppliers can decide the price and production decisions simultaneously or sequentially. Wang derives closed-form results for both situations, but offers no mechanisms to coordinate the consignment channel. Our work focuses on a single product consignment case. We reserve for future work to extend our results to coordinate the retailer decisions with that of multiple suppliers of competing products.

2.3 Modeling Centralized and Decentralized Channels

Consider a supplier S and a retailer R. The supplier has unlimited production capacity to manufacture quantity q of a product where the per unit manufacturing cost is c_S. The retailer incurs a per unit selling cost c_R. Let the total unit cost be $c = c_R + c_S$, and let α be the retailer's share of unit cost so that $\alpha = \frac{c_R}{c}$. Set by supplier S, the product price is p. The retailer offers the supplier a take-it-or-leave-it consignment contract that stipulates his revenue share r, $0 < r < 1$. Suppose the random demand for the product is $\widetilde{D}(p)$ which has cumulative density function $\Psi(\cdot)$ and probability density function $\psi(\cdot)$. We restrict ourselves to the case where demand uncertainty takes a multiplicative form. More specifically, $\widetilde{D}(p) = d(p)\varepsilon$, where $d(p)$ is an iso-price-elastic demand function given by $d(p) = ap^{-\beta}$ with $a > 0$ and $\beta > 1$. β represents the self price elasticity of demand. ε is a random term independent of the price p with cumulative density function $F(\cdot)$ and probability density function $f(\cdot)$, known to both the retailer and the supplier. We have:

$$\Psi(x) = F\left[\frac{x}{d(p)}\right]. \tag{2.1}$$

We assume demand is always nonnegative and thus ε has support $[A, B]$ with $B > A \geq 0$. We assume no salvage revenues and no product shortage costs. In this study, the superscript cc represents a centralized channel, cs stands for a simple consignment contract, bs for a consignment contract with bonus and sp for a consignment contract with side payment.

In this section, we investigate consignment in a decentralized channel where the retailer offers the supplier a take-it-or-leave-it revenue share contract. The supplier in response makes the price and stocking level decisions. We also investigate a centralized channel where a central planner makes the decision for both the retailer and the supplier. Finally, we compare the performance of the decentralized channel with that of the centralized channel.

2.3.1 Decentralized Channel

Define $[x]^+ = max\{x, 0\}$. Given the retailer's revenue share r, the supplier's expected profit is:

$$\begin{aligned}
\pi_S^{cs} &= (1-r)pE\{min[q, \widetilde{D}(p)]\} - (1-\alpha)cq \\
&= (1-r)pE\{q - [q - \widetilde{D}(p)]^+\} - (1-\alpha)cq \\
&= (1-r)pE\{q - [q - d(p)\varepsilon]^+\} - (1-\alpha)cq. \qquad (2.2)
\end{aligned}$$

We express the supplier's stocking decision as $z \equiv q/d(p)$ (Petruzzi and Dada, 1999) where z is the "stocking factor". Letting $\Lambda(z)$ be the expected overage factor where $\Lambda(z) = \int_A^z (z-x)f(x)dx$, (2.2) becomes:

$$\pi_S^{cs} = d(p)\{(1-r)p[z-\Lambda(z)] - (1-\alpha)cz\}. \tag{2.3}$$

Let $h(x) = f(x)/[1-F(x)]$ represent the hazard rate function of the demand distribution, and $xh(x)$ as the general failure rate (GFR). Proposition 2.3.1 provides the supplier's optimal decisions (Wang et al., 2004).

Proposition 2.3.1 *For any given r and $z \in [A, B]$, the unique optimal price is determined by*

$$p^{cs*}(z) = \left(\frac{\beta c}{\beta - 1}\right)\left(\frac{z}{z-\Lambda(z)}\right)\left(\frac{1-\alpha}{1-r}\right). \tag{2.4}$$

Furthermore, if $\partial[xh(x)]/\partial x = h(x) + x\partial[h(x)]/\partial x > 0$, the optimal z^{cs*} that maximizes $\pi_S^{cs}\left[p^{cs*}(z), z\right]$ is uniquely determined by

$$F(z) = \frac{z + (\beta - 1)\Lambda(z)}{\beta z}. \tag{2.5}$$

We note that our demand distribution has an Increasing General Failure Rate (IGFR), a property satisfied by a broad family of distributions such as normal, uniform, gamma and Weibull with mild parameter restrictions (see Lariviere and Porteus (2001) for more details).

For any given r, the retailer observes the supplier's optimal price p^{cs*} and stocking factor z^{cs*}. Then, the retailer optimizes his profit based on p^{cs*} and z^{cs*}. The retailer's expected profit is given by

$$\pi_R^{cs} = d(p^{cs*})\left\{rp^{cs*}\left[z^{cs*} - \Lambda(z^{cs*})\right] - \alpha c z^{cs*}\right\}. \tag{2.6}$$

The optimal revenue share for the retailer is given by the following proposition (see Wang et al. 2004 for details).

Proposition 2.3.2 *The retailer's optimal revenue share is uniquely determined by:*

$$r^{cs*} = \frac{\alpha(\beta-2)+1}{(\beta-\alpha)}. \tag{2.7}$$

Substituting r^{cs*} in (2.4) yields $p^{cs*} = \frac{\beta c}{\beta-1}\frac{z^{cs*}}{z^{cs*}-\Lambda(z^{cs*})}\frac{\beta-\alpha}{\beta-1}$. When $\varepsilon \sim U[0,B]$, we have $\Lambda(z) = \int_0^z (z-x)f(x)dx = \int_0^z (z-x)\frac{1}{B}dx = \frac{z^2}{2B}$. Solving (2.5) gives $z^{cs*} = \frac{2B}{(\beta+1)}$. Thus, $p^{cs*} = \frac{(\beta-\alpha)(\beta+1)c}{(\beta-1)^2}$.

2.3.2 Centralized Channel

Under the centralized channel, a central planner makes price and stocking decisions. Hence $r = 0$ and $\alpha = 0$ in (2.2). Based on Proposition 2.3.1, it is straightforward to obtain the optimal solution (p^{cc*}, z^{cc*}) under the centralized channel. That is,

$$p^{cc*} = \left(\frac{\beta c}{\beta-1}\right)\left(\frac{z^{cc*}}{z^{cc*}-\Lambda(z^{cc*})}\right), \tag{2.8}$$

and z^{cc*} can be obtained by (2.5). When $\varepsilon \sim U[0,B]$, $z^{cc*} = \frac{2B}{(\beta+1)}$. By (2.8), we derive $p^{cc*} = \frac{(\beta+1)c}{(\beta-1)}$. Comparing the pricing and stocking decisions of the supplier in the centralized and decentralized channels, we conclude that unless the retailer absorbs all of the costs in the decentralized channel, the decentralized channel remains uncoordinated since $p^{cs*} = \frac{\beta-\alpha}{\beta-1}p^{cc*} > p^{cc*}$, while for the production quantity, $q^{cs*} = a\left(p^{cs*}\right)^{-\beta} z^{cc*} < q^{cc*} = a\left(p^{cc*}\right)^{-\beta} z^{cc*}$. Thus, under the decentralized channel, supplier overpricing and understocking of the product leads to channel inefficiency. Next, we investigate the gap between the decentralized and centralized channel profits.

2.3.3 Channel Performance of the Decentralized Channel

The supplier's profit under a decentralized channel is:

$$\begin{aligned}
\pi_S^{cs*} &= d(p^{cs*})\{(1-r^{cs*})p^{cs*}[z^{cs*}-\Lambda(z^{cs*})]-(1-\alpha)cz^{cs*}\} \\
&= ac\frac{1-\alpha}{\beta-1}(p^{cs*})^{-\beta}z^{cs*} \\
&= (1-\alpha)\left(\frac{\beta-1}{\beta-\alpha}\right)^{\beta}\pi^{cc*}, \qquad (2.9)
\end{aligned}$$

where π^{cc*} is the centralized channel profit given by:

$$\pi^{cc*} = \frac{ac}{\beta-1}(p^{cc*})^{-\beta}z^{cc*}. \qquad (2.10)$$

Similarly, the retailer's profit under a decentralized channel is:

$$\begin{aligned}
\pi_R^{cs*} &= d(p^{cs*})\left\{r^{cs*}p^{cs*}\left[z^{cs*}-\Lambda(z^{cs*})\right]-\alpha cz^{cs*}\right\} \\
&= ac\frac{\beta-\alpha}{(\beta-1)^2}(p^{cs*})^{-\beta}z^{cs*} \\
&= \left(\frac{\beta-1}{\beta-\alpha}\right)^{\beta-1}\pi^{cc*}. \qquad (2.11)
\end{aligned}$$

Note that when $\alpha=1$ in (2.9) and (2.11), the supplier, as expected, obtains zero profit while the retailer captures all the channel profits. Although the channel profits are maximized under this situation, the supplier has no incentive to accept the contract. The total channel profit loss ($\Delta\pi$) is the gap between centralized channel profit and decentralized channel profit, which can be expressed as:

$$\begin{aligned}
\Delta\pi &= \pi^{cc*}-(\pi_S^{cs*}+\pi_R^{cs*}) \\
&= \left[1-\left(\frac{\beta-1}{\beta-\alpha}\right)^{\beta}\frac{\beta(2-\alpha)-1}{\beta-1}\right]\pi^{cc*}. \qquad (2.12)
\end{aligned}$$

As shown in Wang et al. (2004), $\Delta\pi$ can be as much as 26.4% of the centralized channel profit. This large bound on channel profit loss motivates us to find coordination mechanisms. We introduce them in the next section.

2.4 Coordination Mechanisms

In order to induce the supplier to reduce the product price and increase the stocking level, the retailer must offer incentives. In this section, we propose two mechanisms: a bonus mechanism with multi-tier revenue share structure and a side payment scheme. The advantage of the bonus mechanism lies in its simplicity since it does not require any upfront fee. Its downside is that 100% channel efficiency is unattainable. A side payment can fully coordinate the channel, but it requires the supplier to pay a fee upfront. Detailed proofs of our main results are in the Appendix.

2.4.1 Bonus Mechanism

Cafepress's multi-tier bonus structure motivates the supplier to sell more in each period. The bonus is equivalent to the retailer reducing his commission when the supplier's sales exceeds a threshold. In this study, we use a two-tier bonus mechanism (r, r_0, T) where the retailer reduces his revenue share from r to r_0 for every dollar made beyond the threshold sales T. The bonus scheme gives the supplier the incentive to reduce her price and capture more sales.

It can be shown easily that this two-tier bonus mechanism shares some qualities and features of target rebate system (Taylor, 2002) and a bonus system (Burer et al., 2008) that have proven, in certain instances, to be effective in coordinating a decentralized channel in a wholesale contract setting. Later, we show that the bonus mechanism benefits the whole channel as well as the supplier and the retailer if the right contractual arrangement is in place.

We first investigate how the supplier makes her decision given the retailer's contract (r, r_0, T). The supplier's expected profit function is

$$\begin{aligned}\pi_S^{bs} &= [(1-r)p-(1-\alpha)c]q-(1-r)pE[q-\tilde{D}(p)]^+ \\ &\quad +(r-r_0)E\left[p\min(q,\tilde{D})-T\right]^+. \end{aligned} \tag{2.13}$$

π_S^{bs} agrees with π_S^{cs} when $pq \leq T$. In what follows, we denote a maximizer of (2.13) as (p^{bs*}, q^{bs*}). Also, let z^{bs*} denote an optimal stocking factor, defined in the usual way. The difference between equations (2.2) and (2.13) is the bonus term $(r-r_0)E\left[p\min(q,\tilde{D})-T\right]^+$ which represents the additional revenue that the supplier can acquire if her sales surpassed the threshold set by the retailer.

The following lemma is straightforward.

Lemma 2.4.1 *With r fixed, for any consignment contract $\{(r, r_0, T) : r > r_0\}$, the supplier's profit is decreasing in both r_0 and T.*

Next, we develop the necessary condition for an optimal solution to (2.13). Note that when $pq \geq T$, the supplier's expected profit with the bonus system is:

$$\begin{aligned}&q\left[(1-r)p-(1-\alpha)c\right]-(1-r)pE[q-\tilde{D}(p)]^+ \\ &+(r-r_0)\left\{(pq-T)[1-\Psi(q)]+\int_{\frac{T}{p}}^{q}(p\xi-T)\psi(\xi)d\xi\right\}.\end{aligned} \tag{2.14}$$

Thus (2.14) is a relaxation of (2.13) that agrees with (2.13) when $pq \geq T$. Let $z_T = \frac{T}{d(p)p}$ denote the threshold level, i.e., the ratio between the threshold sales T and the expected sales $d(p)p$. The higher the value of z_T is, the more difficult for the supplier to earn a bonus. Also, to avoid some trivial cases, we restrict T such that $\frac{T}{ac^{-(\beta-1)}} > A$ so that the sales threshold T is always above the minimum sales $ap^{-(\beta-1)}A$ for any $p > c$, i.e., $z_T > A$. Using stocking factor z and threshold level z_T, we rewrite $pq \geq T$ as $z \geq z_T$. Along with (2.1), we can transform (2.14) to:

$$\begin{aligned} \pi_S^{br} &= d(p)\left\{(1-r)p\left[z-\Lambda(z)\right]-(1-\alpha)cz\right\} \\ &\quad +d(p)\left\{(r-r_0)p\left[z-\Lambda(z)-(z_T-\Lambda(z_T))\right]\right\}. \end{aligned} \tag{2.15}$$

In (2.15) note that the first term, $d(p)\left\{(1-r)p\left[z-\Lambda(z)\right]-(1-\alpha)cz\right\}$, is π_S^{cs}, the supplier's expected profit without the bonus, and the second term, $d(p)(r-r_0)p[z-\Lambda(z)-(z_T-\Lambda(z_T))]$, is the expected bonus.

Letting $\Delta = d(p)(r-r_0)p\left[z-\Lambda(z)-(z_T-\Lambda(z_T))\right]$, we have $\pi_S^{br} = \pi_S^{cs} + \Delta$.

Lemma 2.4.2 *With r fixed, for any given (p, z), we have:*

(a) $\Delta \leq 0$, *and* $\pi_S^{br} \leq \pi_S^{bs} = \pi_S^{cs}$, *when* $z \leq z_T$*;*

(b) $\Delta \geq 0$, *and* $\pi_S^{br} = \pi_S^{bs} \geq \pi_S^{cs}$, *when* $z > z_T$.

By Lemma 2.4.2, $\pi_S^{bs} \geq \pi_S^{br}$ and $\pi_S^{bs} \geq \pi_S^{cs}$ always hold. Thus, π_S^{br} and π_S^{cs} are the lower bounds of π_S^{bs}. To maximize her expected profit (2.13), the supplier first maximizes the unconstrained problem (2.15). Let Ω denote the set of decisions that maximize (2.15).

Proposition 2.4.1 *Any $(\hat{p}, \hat{z}) \in \Omega$ satisfies the following:*

$$\begin{aligned} &F(\hat{z}) = \tfrac{(1-r_0)\hat{p}-(1-\alpha)c}{(1-r_0)\hat{p}}, \quad and \\ &(1-r_0)G(\hat{z}) + (r-r_0)H(\hat{z_T}) = 0, \end{aligned} \tag{2.16}$$

where $\hat{z_T} = \frac{T}{d(\hat{p})\hat{p}}$, $G(x) = (1-\beta)[x-\Lambda(x)] + \beta x[1-F(x)]$, *and* $H(x) = (\beta - 1)\int_A^x \xi f(\xi)d\xi$.

Since $z_T > A$, we have $H(z_T) = (\beta - 1)\int_A^{z_T} \xi f(\xi)d\xi > 0$. Recall that (p^{cs*}, z^{cs*}) uniquely maximizes π_S^{cs}.

Lemma 2.4.3 *For any* $(\hat{p}, \hat{z}) \in \Omega$, $(\hat{p}, \hat{z}) \neq (p^{cs*}, z^{cs*})$.

We summarize in Proposition 2.4.2 the supplier's decisions that maximize (2.13). Note that $z_T^{cs*} = \frac{T}{d(p^{cs*})p^{cs*}}$.

Proposition 2.4.2 $(p^{bs*}, z^{bs*}) \in \{(p^{cs*}, z^{cs*})\} \cup \Omega$. *Furthermore, if there exists* $(\hat{p}, \hat{z}) \in \Omega$ *where* $\hat{z} \leq \hat{z_T}$, *then* $(p^{bs*}, z^{bs*}) = (p^{cs*}, z^{cs*})$ *is the unique maximizer of* π_S^{bs}. *Finally, if* $z^{cs*} \geq z_T^{cs*}$, *then* (p^{cs*}, z^{cs*}) *is not optimal to* π_S^{bs}.

Also, we have:

Proposition 2.4.3 *For any* $(\hat{p}, \hat{z}) \in \Omega$, $\frac{(1-\alpha)}{(1-r_0)}p^{cc*} \leq \hat{p}$, *and* $\hat{z} > z^{cc*}$. *Furthermore, if* $(\hat{p}, \hat{z})$ *maximizes* π_S^{bs}, *then* $\hat{p} \leq \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$.

Denote $\Omega' = \left\{(p', z') : \frac{(1-\alpha)}{(1-r_0)}p^{cc*} \leq p' \leq \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}, z' > z^{cc*}, (p', z') \in \Omega\right\}$. Ω' is a subset of Ω. We know from Proposition 2.4.3 that the solutions in $\Omega \setminus \Omega'$ will never be optimal to π_S^{bs}. Hence, in Proposition 2.4.2, we can substitute Ω' for Ω. Since p^{cc*} and z^{cc*} are known for a given demand distribution via (2.8) and (2.5), Proposition 2.4.3 can be used to establish additional bounds for $(\hat{p}, \hat{z}) \in \Omega$. These bounds, in combination with properties established in Proposition 2.4.1, can be utilized to expedite the search for the supplier's optimal price and stocking decisions. Moreover, note that the upper bound of $\left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$ on the pricing decision decreases with sales threshold T and increases with the upper support of the distribution of ε, while the lower bound $\frac{(1-\alpha)}{(1-r_0)}p^{cc*}$ increases with the bonus revenue share r_0 and decreases with the retailer cost share α. As price elasticity β goes up, both the lower bound and upper bound become tighter.

We now address the issue of channel coordination. To reach a full channel coordination, we must have $p^{bs*} = p^{cc*}$ and $z^{bs*} = z^{cc*}$. We have:

Proposition 2.4.4 *No bonus mechanism (r, r_0, T) coordinates the consignment channel, if demand is positive and $r \neq \alpha$.*

When $r = \alpha$, we have $(p^{cs*}, z^{cs*}) = (p^{cc*}, z^{cc*})$ according to Proposition 2.3.1. The bonus mechanism (α, r_0, T), where $r_0 < \alpha$, may coordinate the channel, depending on whether the supplier makes a decision by (p^{cs*}, z^{cs*}) or by $(\hat{p}, \hat{z}) \in \Omega$. However, the retailer will not find (α, r_0, T) attractive and thus will not propose it to the supplier, since the retailer himself, is better off to offer the supplier the revenue sharing contract without bonus, i.e., r^{cs*}, where $r^{cs*} > \alpha$, than the bonus mechanism (α, r_0, T).

Although Propositions 2.4.1, 2.4.3 and 2.4.2 characterize the supplier's optimal choices in the more general case, additional properties are available when the random term ε is uniformly distributed. Proposition 2.4.5 characterizes the solution that maximizes π_S^{bs} under this assumption.

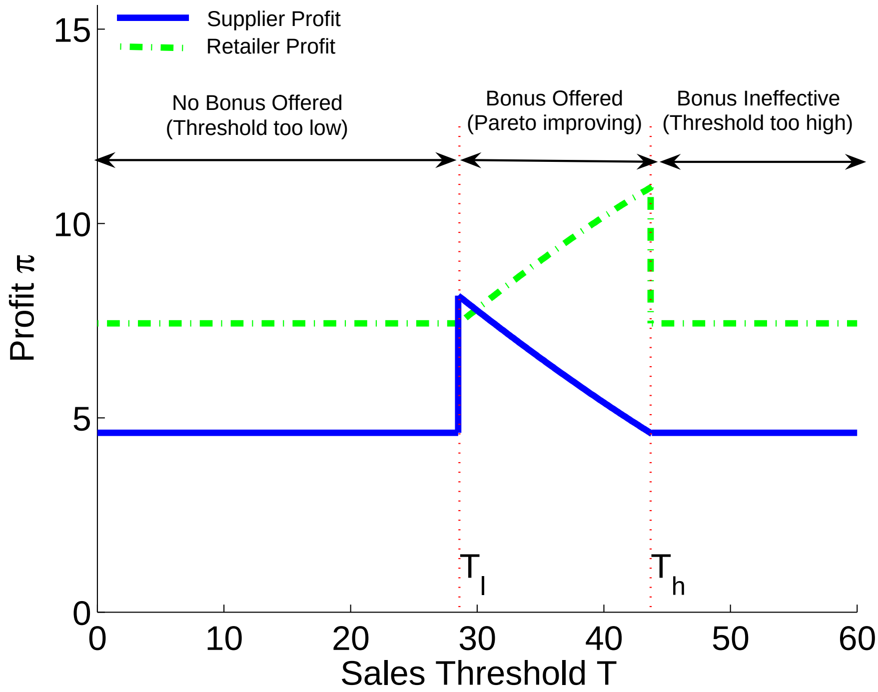
Let $M = \frac{2(1-\alpha)c}{(1-r_0)}$. Denote $N_1(p) = \frac{(1-r_0)B}{2}\left[\frac{M^2(\beta+1)}{2p^3} - \frac{M\beta}{p^2}\right]$, $N_2(p) = (r - r_0)(\beta-1)^2\frac{T^2}{Ba^2}p^{2\beta-3}$, and $\Theta(p) = (1-r_0)G(z^*)+(r-r_0)H(z_T)$ where $z^* = F^{-1}\left[1 - \frac{(1-\alpha)c}{(1-r_0)p}\right]$. Let $p_l = \frac{M(\beta+1)}{2(\beta-1)}$, $p_u = min\left\{\left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}, p_0\right\}$ where p_0 is the largest p value such that $N_1(p) + N_2(p) = 0$, and $p_m = \frac{3M(\beta+1)}{4\beta}$.

Proposition 2.4.5 *Assuming* $\varepsilon \sim U[0, B]$*, if* $\Theta(p_u) < 0$ *and* $p_u > p_l$*, then there exists a unique* $p' \in (p_l, p_u)$ *such that* $\Theta(p') = 0$ *when (a)* $1.5 \leq \beta \leq 3$ *or (b)* $\beta > 3$ *and* $N_1(p_l) + N_2(min\{p_m, p_u\}) < 0$*. Moreover,* (p', z') *is the only element in* Ω'*.*

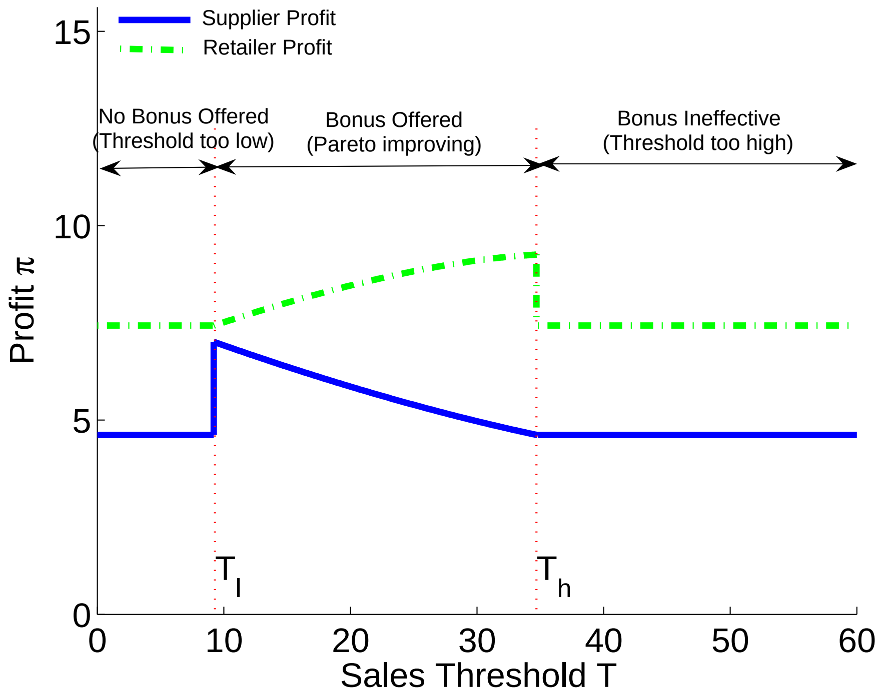
Proposition 2.4.5 can be further utilized to accelerate the search process of the supplier's optimal price and stocking decision since the supplier's profit function between (p_l, p_u) is quasi-concave as shown in the proof. Once (p', z') is found, the supplier selects the price and stocking factor between (p', z') and (p^{cs*}, z^{cs*}) according to Proposition 2.4.2. Next we provide a numerical study to present that a pareto-improving bonus mechanism exists in certain parameter settings where at least one party is better off while nobody is worse off. Also, we show that even though full coordination is impossible, a bonus can bring the channel significantly close to full coordination.

In our numerical study, the random term in the demand function $\varepsilon \sim U[0, 2]$; demand function constant $a = 1000$; product price elasticity $\beta = 3$; total unit cost $c = 2$; and the retailer's cost share $\alpha = 0.1$.

Under the no-bonus scenario, the retailer, based on (2.7), sets his revenue share r to 0.38, and according to Proposition 2.3.1, the supplier's optimal decisions are $p^{cs*} = \frac{(\beta-\alpha)(\beta+1)c}{(\beta-1)^2}$ and $z^{cs*} = \frac{4}{(\beta+1)}$, i.e., $(p^{cs*}, z^{cs*}) = (5.8, 1)$. The profits for the retailer and the supplier are $\pi_R^{cs*} = 7.43$ and $\pi_S^{cs*} = 4.61$ respectively. With the bonus mechanism (r, r_0, T), we assume the retailer still sets r according to (2.7), i.e. $r = 0.38$. We set the bonus revenue share r_0 to either 0.05 or 0.25, and then vary the sales threshold T between 0 and 60. The results of our numerical study reveal that a pareto-improving bonus mechanism exists. Consider $r = 0.38$. We found the threshold ranges $[28, 43]$ where $r_0 = 0.05$ (see



(a) $r = 0.38$ and $r_0 = 0.05$



(b) $r = 0.38$ and $r_0 = 0.25$

Figure 2.1: The retailer and supplier expected profit as a function of sales threshold T for a two-tier bonus.

Figure 2.1(a)) and $[9, 34]$ where $r_0 = 0.25$ (see Figure 2.1(b)) are pareto-improving regions. Because the retailer is the leader, he can actually design the contract (r, r_0, T) in such a way that T is slightly less than T_h, e.g., $T_h = 42$ for $r_0 = 0.05$ and $T_h = 33$ for $r_0 = 0.25$. At this value of T, the supplier has the least incentive to go for the bonus, and the retailer gets most of the profits brought by channel improvement. However, in order to extract the maximum profit, the retailer has to keep r_0 at a reasonable level. Later we will show how the retailer should choose r_0 and T as price elasticity β and retailer cost share α change.

Another observation is that with r fixed, r_0 and T move in opposite directions. That is, when r_0 increases, the pareto-improving region $[T_l, T_h]$ shifts to the left (see Figure 2.1(b)). In other words, if the retailer sets a higher sales threshold, then the retailer should reward the supplier with higher revenue share for crossing the threshold. In terms of overall channel performance, when $T \in [T_l, T_h]$, the channel yields higher profits because both the retailer and the supplier are better off. In particular, the centralized price and stocking level decisions are $p^{cc*} = \frac{(\beta+1)c}{(\beta-1)}$ and $z^{cc*} = \frac{4}{(\beta+1)}$. Thus, $(p^{cc*}, z^{cc*}) = (4, 1)$. By (2.10), the centralized channel profit is $\pi^{cc*} = 15.63$. With $r = 0.38$, the decentralized channel profit without the bonus is $\pi^{cs*} = 12.04$, i.e., 77.08% of the centralized channel profit. Table 2.1 presents the total channel profit and efficiency under a decentralized channel with bonus. The results are based on the same parameters in the previous numerical study. We chose $r = 0.38$ via (2.7), set $r_0 = 0.2$, and varied T between 16 and 28 (note the other combinations of r_0 and T also yield significant channel profit improvement as shown in Figures 2.1(a) and 2.1(b)). Table 2.1 demonstrates that even though full coordination is impossible, our numerical results reveal that a bonus package can bring the channel sig-

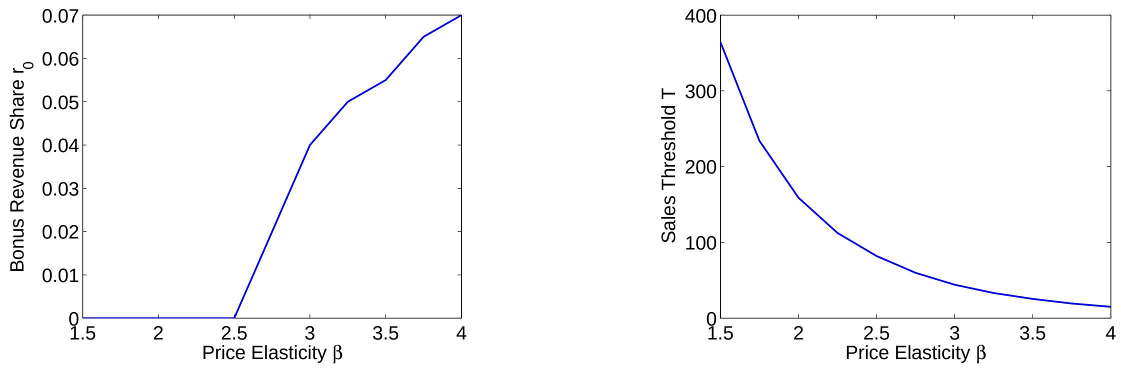
Table 2.1: Channel performance with bonus

(r, r_0, T)	π^{bs*}	π^{bs*}/π^{cc*}
$(0.38, 0.2, 16)$	15.02	96.12%
$(0.38, 0.2, 20)$	14.97	95.83%
$(0.38, 0.2, 24)$	14.91	95.42%
$(0.38, 0.2, 28)$	14.82	94.87%

nificantly close to full coordination (see the column labeled π^{bs*}/π^{cc*}). The channel loss is less than 6% which is significantly lower than the more than 22% loss without bonus. Finally, as the retailer increases the sales threshold T, the channel efficiency deteriorates slightly. Since the retailer is the leader, he can increase T to a level where the supplier profit is barely more than that when bonus is not offered without significantly degrading the overall channel performance.

Another interesting scenario is when the price elasticity and retailer cost share change. Assume the retailer is greedy and always sets r_0 and T such that he maximizes his profits and the supplier gets the least benefit from the bonus mechanism. We also assume $r = r^{cs*}$. Using the same parameter setting, we first examine the impact of β on retailer's decisions.

Figure 2.2 shows that as price elasticity β increases, the retailer should lower the sales threshold T while gradually increasing the bonus revenue share r_0. This result is straightforward since the expected sales diminishes as β increases. To encourage the sup-



(a) The impact of β on r_0

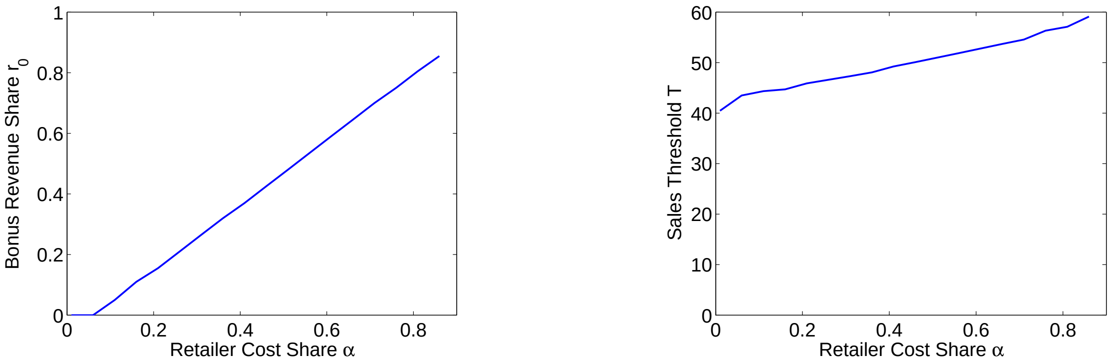
(b) The impact of β on T

Figure 2.2: The impact of β on r_0 and T.

plier to get the bonus, the retailer has to reduce the sales threshold T. Meanwhile, as sales threshold decreases, the retailer should reduce the supplier's marginal benefit for sales over T, i.e., increase r_0. An interesting observation from Figure 2.2(a) is when price elasticity $\beta \in [1.5, 2.5]$, the retailer is better off forgoing his commission for sales exceeding the threshold T, i.e., set $r_0 = 0$.

We now explore the impact of retailer cost share α on r_0 and T. Figure 2.3 reveals that the retailer should increase his bonus revenue share r_0 and his sales threshold T as his cost share α goes up. The impact of the cost share α on r_0 is straightforward. The retailer needs to match up his revenue share with his cost share. For T, as the retailer cost share α increases, the channel is closer to the centralized channel and more sales are generated. Therefore, the retailer raises the sales threshold T when α increases.

We know the two-tier bonus (r, r_0, T) can significantly improve the decentralized channel performance. However, it cannot fully coordinate the decentralized channel. Next,



(a) The impact of α on r_0 (b) The impact of α on T

Figure 2.3: The impact of α on r_0 and T.

we investigate a more general case where an n-tier bonus is introduced ($n > 2$). We represent an n-tier bonus as $(r, r_0, T_0, ..., r_{n-1}, T_{n-1})$ with $r > r_0 > ...r_{n-1}$ and $T_0 < T_1 < ... < T_{n-1}$. T_i is the sales threshold for bonus tier i, and r_i is the corresponding bonus revenue share, where $i = 0, ..., n-1$. Given this bonus scheme offered by the retailer, the supplier's expected profit function is:

$$\begin{aligned}
\pi_S^{bs} &= [(1-r)p - (1-\alpha)c]q - (1-r)pE[q - \tilde{D}(p)]^+ \\
&\quad +(r - r_0)E\left[p\min(q, \tilde{D}) - T_0\right]^+ + ... \\
&\quad +(r_{n-2} - r_{n-1})E\left[p\min(q, \tilde{D}) - T_{n-1}\right]^+. \qquad (2.17)
\end{aligned}$$

Using stocking factor z and sales threshold level $z_{T_i} = \frac{T_i}{d(p)p}(i = 0, ..., n-1)$, we transform (2.17) to:

$$\begin{aligned}
\pi_S^{bs} &= d(p)\left\{(1-r)p\left[z - \Lambda(z)\right] - (1-\alpha)cz\right\} \\
&\quad +d(p)(r - r_0)p\left[z - \Lambda(z) - (z_{T_0} - \Lambda(z_{T_0}))\right]^+ + ... \\
&\quad +d(p)(r_{n-2} - r_{n-1})p\left[z - \Lambda(z) - \left(z_{T_{n-1}} - \Lambda(z_{T_{n-1}})\right)\right]^+. \qquad (2.18)
\end{aligned}$$

The supplier's decision process under the n-tier bonus is similar to that of the two-tier bonus. First, the supplier makes decisions as if no bonus is present. Second, the supplier determines price and stocking factor by targeting bonus tier i. Finally, the supplier chooses the price and stocking factor in the previous two stages that yields the highest expected profit. However, the n-tier bonus is also unable to coordinate the decentralized channel.

Proposition 2.4.6 *The n-tier bonus* $(r, r_0, T_0, ..., r_{n-1}, T_{n-1})$ *cannot coordinate the consignment channel, when demand is positive and* $r \neq \alpha$.

It remains to be shown whether the n-tier bonus increases channel profits more than the two-tier bonus, and whether channel profit difference between them is significant. Our numerical study shows a profit improvement for both the retailer and supplier with the n-tier bonus. However, the profit improvement over the two-tier bonus is insignificant. Table 2.2 presents the total channel profit and efficiency under a decentralized channel with three-tier bonus. Compared with the results of two-tier bonus in Table 2.1, the three-tier bonus showed about 2% channel improvement over the two-tier bonus.

2.4.2 Revenue Sharing with Side Payment (RSSP)

We saw in the previous section that a bonus mechanism cannot fully coordinate a consignment channel. In this section, we present a revenue sharing with side payment contract. We show that this contract setting can fully coordinate the channel and maximize channel profit. Moreover, the profit can be arbitrarily split between the retailer and the supplier. A side payment in a consignment contract context is analogous to a membership fee. For example, Amazon.com offers a Pro-merchant program where enrolled suppliers

Table 2.2: Channel performance with three-tier bonus

(r, r_0, T_0, r_1, T_1)	π^{bs*}	π^{bs*}/π^{cc*}
$(0.38, 0.2, 16, 0.15, 40)$	15.39	98.48%
$(0.38, 0.2, 20, 0.15, 43)$	15.35	98.21%
$(0.38, 0.2, 24, 0.15, 46)$	15.29	97.84%
$(0.38, 0.2, 28, 0.15, 49)$	15.20	97.30%

pay a monthly membership fee plus commissions on transactions. Amazom in Britain (UK) even offers discounted commissions to pro-merchants. Enrolled suppliers only pay 8.05% commission on electronics and photo items after paying a $\pounds 28.75$ monthly membership fee while the regular commission is 11.5%. The contract in general can be described using two parameters (r, Φ) where r is the retailer revenue share for each unit of product sold and Φ is the supplier side payment to the retailer. Next, we develop a family of revenue sharing with side payment schemes.

For the supplier, her profit under side payment is given as

$$\pi_S^{sp} = d(p)\{(1-r)p[z - \Lambda(z)] - (1-\alpha)cz\} - \Phi. \tag{2.19}$$

Since Φ is a side payment independent of price and stocking factor, the sole option to coordinate the channel is to set $r = \alpha$ in (2.19), yielding a supplier price and stock level equal to that in the centralized channel. Hence, the supplier profit is $\pi_S^{sp*} = (1-\alpha)\pi^{cc*} - \Phi$. If we let $\Phi = \phi\pi^{cc*}$ where ϕ is a proportion of centralized channel profit, then $\pi_S^{sp*} =$

$(1-\alpha-\phi)\pi^{cc*}$ and $\pi_R^{sp*} = (\alpha+\phi)\pi^{cc*}$. We now show that the supplier and retailer are better off with a side payment.

A consignment contract with side payment will not work unless it benefits both parties. In other words, $\pi_S^{sp*} > \pi_S^{cs*}$ and $\pi_R^{sp*} > \pi_R^{cs*}$. We generalize our findings in the following proposition.

Proposition 2.4.7 *A consignment contract* (r, Φ) *can coordinate the channel and arbitrarily divide the channel profit between the supplier and the retailer when* $r = \alpha$ *and* $\Phi = \phi\pi^{cc*}$*. Furthermore, the contract is pareto improving when* $(\frac{\beta-1}{\beta-\alpha})^{\beta-1} - \alpha < \phi < (1-\alpha)\left[1-(\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$.

Since the retailer is the leader, he can increase the side payment ϕ to $\phi_H = (1-\alpha)\left[1-(\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$ without discouraging supplier's participation. Assume the retailer is greedy and always charges ϕ_H. This yields the following result:

Proposition 2.4.8 ϕ_H *decreases with the price elasticity* β *and the retailer's cost share* α*.*

Proposition 2.4.8 implies that when product is more elastic, the retailer should be less greedy on the side payment so that the supplier will not be deterred from participating. The notion that side payment decreases with the retailer's cost share may seem counterintuitive. However, since, by (2.7), the retailer's revenue share r increases with α, one can expect a net increase in the retailer's profit.

In addition to the Pro-merchant program, Amazon (UK) is also running another program named "Advantage", which is currently for books, CDs, DVDs and VHSs. The difference between "Advantage" and regular consignment selling is that under "Advantage"

Amazon takes more responsibility in inventory handling, customer service and payment processing. For example, Amazon warehouses supplier's items in their own distribution center and delivers them to customers. In other words, Amazon takes a larger share of the cost in selling the items. However, Amazon charges a larger commission. For example, Amazon takes away 60% of sales proceeds instead of the regular 17.25% on book items. The program also charges an annual fee of $\pounds 23.50$, which is similar to the Pro-merchant program monthly membership fee. We can represent the "Advantage" program as (r, δ, Φ) where r is the revenue share, δ is the extra cost share the retailer bears in addition to the regular cost share α and Φ is the side payment.

Following the same logic as in the basic consignment contract with side payment (r, Φ), we now show our findings for the Advantage program:

Proposition 2.4.9 *A consignment contract (r, δ, Φ) can coordinate the channel and arbitrarily allocate the channel profit between the supplier and the retailer when $r = \alpha + \delta$ and $\Phi = \phi\pi^{cc*}$. Moreover, the contract is pareto improving when $(\frac{\beta-1}{\beta-\alpha})^{\beta-1} - \alpha < \phi + \delta < (1-\alpha)\left[1 - (\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$.*

The proof for Proposition 2.4.9 is similar to that of Proposition 2.4.7. The advantage program offers more flexibility to the retailer because of the additional cost δ and side payment Φ. That is, when $\delta = 0$, the contract reverts back to the Pro-merchant program. On the other hand, the retailer can take more cost $\delta > 0$ and set $r > \alpha$ while lowering Φ. Also, this can be beneficial to a small or medium sized supplier lacking the economy of scale that large suppliers have to handle inventory, payment processing and customer inquiries. Finally, the pro-merchant program benefits experienced volume suppliers more than small or medium suppliers.

2.5 Conclusions

Virtual retailers are offering a variety of programs to online suppliers to generate more sales. In this chapter, we study coordination in a consignment channel with multi-tier bonus structure and Revenue Sharing with Side Payment (RSSP). We show with a numerical example that although the bonus cannot fully coordinate the channel, fine tuning its parameters can significantly improve channel performance. In comparison, we find RSSP can fully coordinate the consignment channel. Moreover, it is easy to customize RSSP to target different types of suppliers because with side payment, channel profits can be arbitrarily divided between the supplier and the retailer. However, RSSP's downside is the upfront fee which can discourage participation from low volume suppliers. We present a variation of RSSP called Cost Sharing with Side Payment (CSSP) in which, in addition to a side payment, the retailer is able to charge additional points on top of his regular commission to cover a portion of the expense for extra services such as product storage, shipping and handling which otherwise would have been incurred by the supplier. CSSP can be attractive to small scale suppliers while RSSP can benefit medium and large scale suppliers.

Our study can be extended to a consignment channel with two suppliers of competing products (duopoly). Interesting questions include: does competition affect the channel performance as well as channel members' welfare? Are bonus and revenue sharing incentives as effective as in the single retailer and single supplier setting? What if demand uncertainty is additive? We reserve these topics for future study.

CHAPTER 3
PRICE COMPETITION IN A CONSIGNMENT CONTRACT BETWEEN A SINGLE RETAILER AND TWO SUPPLIERS

3.1 Introduction

In previous chapters, we have considered contractual arrangements between a single retailer and a single supplier. In this chapter, we extend our work to CCA with single retailer and two suppliers of substitutable products (also referred to as the common retailer CCA case). Our goal is to study the effects of product competition. The suppliers in this study compete on product price. Hence, product demand is not only dependent on the price of the product (i.e. self price elasticity), but also on the price of the competing product (i.e. cross price elasticity). Bernstein and Federgrunen (2005) study a WCA setting with single supplier and competing retailers to find the channel equilibrium price and stocking decisions. They employ a price-discount sharing scheme to coordinate a decentralized channel with a single supplier and multiple retailers. Choi (1991) investigates the impact of competition in a single retailer, two suppliers WCA setting. In a CCA setting with one retailer and multiple suppliers, Wang (2006) investigates the price and stock level decisions of suppliers of complementary products and the revenue share decision of the retailer.

Our work will seek the suppliers' optimal prices as well as the retailer's optimal revenue share in a CCA setting. We study the effects of product competition on channel members' decisions and profits. In addition, we will investigate the impact of adding another retailer to the market channel such that a supplier can have an exclusive retailer for her product (exclusive retailer CCA case). More specifically, we aim to answer the following questions:

(a) How should suppliers make their price decisions in a competing environment? How much should the retailer charge for each unit sold? What are the decision outcomes for different customer demand functions?

(b) When competition intensifies or diminishes, how should the suppliers and the retailer respond to those changes?

(c) Does competition improve total channel profits or does it benefit only one party?

(d) If another retailer enters the market channel so that each supplier has an exclusive retailer, how does this affect the suppliers' price decisions and profit, and the retailer revenue share?

3.2 Preliminaries

Consider two independent suppliers S_i $(i = 1, 2)$ and a retailer R. Each supplier S_i has unlimited production capacity to manufacture product i where this product can be a substitute for the competing product. The manufacturing cost of product i is c_{S_i}. The retailer cost is c_{R_i}. Let the total unit cost be $c_i = c_{R_i} + c_{S_i}$. Supplier S_i's product price is p_i. The retailer offers the suppliers a single take-it-or-leave-it consignment contract that stipulates his revenue share r. Note that we assume symmetric revenue shares which is quite common in consignment markets for the products in the same category. For instance, Amazon.com charges 8% commission for any brand of digital cameras. It may not be the best strategy, but potentially cost effective for Amazon to manage considering Amazon's 97,500 active seller accounts (Vogels, 2005).

We study two demand models. The first is a nonlinear demand model. Letting $\boldsymbol{p} = (p_1, p_2)$, we define the demand function as $d_i(\boldsymbol{p}) = ap_i^{-\beta}p_j^{\delta}$ with $a > 0, \beta > 2,\ 1 \leq \delta < (\beta - 1),\ i, j = 1, 2,\ i \neq j$. β represents the self price elasticity of demand while δ is the cross price elasticity of demand. We assume symmetry on price elasticity β and cross price elasticity δ. It is necessary to assume a range for β and δ in order to guarantee a Nash equilibrium solution that we will derive later. $d_i(\boldsymbol{p})$ is continuous and twice differentiable in the specified region defining the demand parameters. We also consider a linear demand model $d_i(\boldsymbol{p}) = a - \beta p_i + \delta p_j$ where $a > 0$ and $\beta > \delta > 0$ for $i, j = 1, 2$ and $i \neq j$.

For each model, the sequence of events is as follows:

(a) The suppliers convey their intent to sell products through the retailer.

(b) The retailer, who is the Stakelberg leader, offers each supplier a take-it-or-leave-it consignment contract.

(c) The suppliers review the contract, accept it as long as it yields a positive profit, and make independent production quantity and price decisions.

(d) The products are available immediately at the beginning of a single selling season, and the retailer offers them to customers at the suppliers' prices.

(e) Demands are realized. At the end of the selling season, the retailer remits the suppliers' share of the revenues to the suppliers.

In the following, we first study the retailer and the suppliers' optimal decisions under a nonlinear demand model.

3.3 Nonlinear Demand Model

There is competition between suppliers when the product demand of supplier S_1 depends not only on p_1, but also on the other supplier's price p_2. The cross price elasticity δ is a measure of product substitutability (Moorthy, 1988). A large δ implies high product substitutability which causes strong competition between suppliers. Next, we investigate the decision outcomes for the decentralized channel, which is followed by exclusive retailing in a consignment channel.

3.3.1 Decentralized Channel

We evaluate S_i's price decisions in a decentralized consignment channel as follows: Given the revenue share $(1-r)$, knowing S_2's selling price p_2, S_1 chooses a price p_1 to maximize her profit. In response, S_2 may recalibrate her price to maximize her profit and so on until prices reach equilibrium. For S_i, the expected profit is as follows:

$$\begin{aligned} \pi_{S_i}^{cs} &= d_i(\boldsymbol{p})\left[(1-r)p_i - c_{S_i}\right] \\ &= a p_i^{-\beta} p_j^{\delta}\left[(1-r)p_i - c_{S_i}\right] \end{aligned} \tag{3.1}$$

We present our results in the following proposition.

Proposition 3.3.1 *For given r, S_i's optimal decision to (3.1) is given by:*

$$p_i^{cs*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r)}. \tag{3.2}$$

Proposition 3.3.1 implies that the suppliers make independent price decisions which is one of the properties of a nonlinear demand function. Moorthy (1988) and Choi (1991) make a similar observation as this. The optimal price depends upon the self price elasticity β, the retailer's revenue share r and the supplier's cost c_{S_i}. Next, we show how to find the optimal revenue share. Consider the retailer's problem shown below.

Knowing the suppliers' pricing decisions (3.2) for any given revenue share, the retailer finds the optimal revenue share r. The retailer's profit function is given by:

$$\pi_R^{cs} = \sum_{i=1}^{2} d_i(\boldsymbol{p}^{cs*})\,(rp_i^{cs*} - c_{R_i}). \tag{3.3}$$

Substituting p_i^{cs*} into (3.3), we obtain

$$\begin{aligned}
\pi_R^{cs} &= a(p_1^{cs*})^{-\beta}(p_2^{cs*})^{\delta}\,(rp_1^{cs*} - c_{R_1}) + a(p_2^{cs*})^{-\beta}(p_1^{cs*})^{\delta}\,(rp_2^{cs*} - c_{R_2}) \\
&= (1-r)^{(\beta-\delta)}\left[M_3\left(\frac{r}{1-r}M_1 - c_{R_1}\right) + M_4\left(\frac{r}{1-r}M_2 - c_{R_2}\right)\right] \\
&= (1-r)^{(\beta-\delta)}\left[\frac{r}{1-r}(M_1M_3 + M_2M_4) - (M_3c_{R_1} + M_4c_{R_2})\right] \\
&= (1-r)^{(\beta-\delta)}\left(\frac{r}{1-r}K_1 - K_2\right),
\end{aligned} \tag{3.4}$$

where $M_1 = \frac{\beta c_{S_1}}{(\beta-1)}$, $M_2 = \frac{\beta c_{S_2}}{(\beta-1)}$, $M_3 = aM_1{}^{-\beta}M_2{}^{\delta}$, $M_4 = aM_2{}^{-\beta}M_1{}^{\delta}$, $K_1 = (M_1M_3 + M_2M_4)$ and $K_2 = (M_3c_{R_1} + M_4c_{R_2})$. Note that K_1 and K_2 are constants independent of r. Consider the next proposition.

Proposition 3.3.2 *The retailer's optimal revenue share r^{cs*} is uniquely determined by*

$$r^{cs*} = 1 - \frac{(\beta-\delta-1)}{(\beta-\delta)}\frac{1}{1+\frac{K_2}{K_1}}. \tag{3.5}$$

From (3.5), we can observe that r^{cs*} depends on the following parameters: the retailer's cost on each product, the suppliers' cost, self price elasticity and cross price elasticity. Their impact on r^{cs*} is:

Proposition 3.3.3 *Assuming $\beta > 2,\ 1 \leqslant \delta < (\beta-1)$, r^{cs*} has the following properties:*

(a) r^{cs*} *increases in* c_{R_i}*;*

(b) when $c_{R_1} = c_{R_2}$*,* r^{cs*} *increases in* δ *for any given* α *and* β*.*

Property (a) is rather intuitive. The retailer's revenue share increases with his cost share in either product. Property (b) shows that the retailer can exploit the competition between suppliers by increasing the revenue share. This holds when the retailer shares the same cost on both products. It is a reasonable assumption, considering virtual retailers such as Amazon.com may not physically carry products, but merely provide facilitating services. The impact of self price elasticity β on the revenue share is not monotonic. The relationship is more complex, depending on self price elasticity, cross price elasticity, and cost structure.

Substituting r^{cs*} in (3.2) yields the next proposition.

Proposition 3.3.4 *The suppliers' equilibrium prices are uniquely determined by:*

$$p_i^{cs*} = \frac{\beta(\beta - \delta)(1 + \frac{K_2}{K_1})c_{S_i}}{(\beta - 1)(\beta - \delta - 1)}, i = 1, 2. \tag{3.6}$$

The suppliers' price decisions are also a function of the retailer's cost, the suppliers' cost, price elasticity and cross price elasticity. We now investigate their impacts on the suppliers' price decisions.

Proposition 3.3.5 *When* $\beta > 2,\ 1 \leqslant \delta < (\beta - 1)$*, the suppliers' equilibrium price* p_i^{cs*} *has the following properties:*

(a) p_i^{cs*} *increases in the retailer's cost* c_{R_i}*;*

(b) when $c_{R_1} = c_{R_2}$*,* p_i^{cs*} *increases in the cross price elasticity* δ *for any given* α *and* β*.*

3.3.2 Exclusive Retailing

We are also interested in investigating the scenario where each supplier consigns her product to an exclusive retailer (i.e. exclusive retailer CCA case). For example, consider a supplier of an Olympus digital camera on eBay.com, and another supplier of an identical camera on Amazon.com. It is not unusual to find that interactions exist between channels. For example, if eBay charges a higher sales commission, the eBay supplier likely will increase her retail price which may make the camera being sold on Amazon relatively more attractive. In our analysis, we shall use the same market setting where the retailers are the Stackelberg leaders, while the suppliers are the followers. Denote $\pi_{S_i}^{es}$ as supplier S_i's profit, $\pi_{R_i}^{es}$ as retailer R_i's profit and r_i as retailer R_i's revenue share.

Supplier S_i's profit function is the same as in the common retailer case (see (3.1)), and therefore p_i^{es*} can be determined by (3.2), i.e., $p_i^{es*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r_i)}$. For retailer R_i, knowing the suppliers' price decisions, his profit function is:

$$\begin{aligned}
\pi_{R_i}^{es} &= d_i(\boldsymbol{p}^{es*})\left(r_i p_i^{es*} - c_{R_i}\right) \\
&= a(p_i^{es*})^{-\beta}(p_j^{es*})^{\delta}\left(r_i p_i^{es*} - c_{R_i}\right) \\
&= M \cdot g(r_i), \qquad (3.7)
\end{aligned}$$

where $g(r_i) = (1-r_i)^{(\beta-1)}\left[\frac{\beta c_{S_i}}{(\beta-1)} - \left(c_{R_i} + \frac{\beta c_{S_i}}{\beta-1}\right)(1-r_i)\right]$ and $M = a\left(\frac{\beta}{\beta-1}\right)^{-(\beta-\delta)} {c_{S_i}}^{-\beta}$ ${c_{S_j}}^{\delta}(1-r_j)^{-\delta}$ is a constant independent of R_i's revenue share decision r_i. The retailer maximizes $g(r_i)$.

Proposition 3.3.6 *Retailer R_i's optimal revenue share under the exclusive retailing is de-*

termined by:

$$r_i^{es*} = 1 - \frac{(\beta-1)}{\beta}\frac{1}{1+\frac{\beta-1}{\beta}\frac{c_{R_i}}{c_{S_i}}}, i = 1, 2. \tag{3.8}$$

Substituting r_i^{es*} into p_i^{es*}, we derive supplier S_i's optimal price decision.

$$p_i^{es*} = \frac{\beta\left[\beta(c_{S_i} + c_{R_i}) - c_{R_i}\right]}{(\beta-1)^2}, i = 1, 2. \tag{3.9}$$

Proposition 3.3.7 *$\forall i, j = 1, 2, i \neq j$, when $\frac{c_{R_i}}{c_{S_i}} \leq \frac{c_{R_j}}{c_{S_j}}$, $r_i^{es*} \leq r_j^{es*}$, $r_i^{es*} < r^{cs*}$, and $p_i^{es*} < p_i^{cs*}$.*

Proposition 3.3.7 demonstrates that when one retailer incurs a smaller portion of the unit cost than the other retailer, the retailer with lower cost share has to charge a lower commission. Also, his commission needs to be set below the revenue share under the common retailer setting. For price decision, the supplier who subscribes to the retailer with lower cost share tends to price lower than in a common retailer setting.

3.3.3 A Special Case: $c_{S_1} = c_{S_2}$ and $c_{R_1} = c_{R_2}$

In this section, we consider the case where the products are identical and are merely distinguished by packaging or brand name label (e.g. digital cameras and DVD players). As the actual production of these products could be outsourced to a third party, it is not unusual for the suppliers to have the same manufacturing costs $c_{S_1} = c_{S_2} = c_S$. Furthermore, an electronic market place for these products, such as Amazon.com, will incur the same product handling and storage costs since Amazon.com may not even carry the physical products, but merely provide the virtual marketplace services, thus incurring only a single cost $c_R = c_{R_1} = c_{R_2}$ to retail the products of either supplier.

Under symmetric cost structure, $M_1 = M_2 = \frac{\beta c_S}{(\beta-1)}$ and $M_3 = M_4 = 1$. Therefore, we have $K_1 = \frac{2\beta c_S}{(\beta-1)}$ and $K_2 = 2c_R$. Substituting them into the optimal revenue share decision (3.5), we obtain

$$r^{cs*} = 1 - \frac{(\beta-\delta-1)}{(\beta-\delta)} \frac{1}{1+\frac{\beta-1}{\beta}\frac{c_R}{c_S}}. \tag{3.10}$$

Also, supplier S_i's price decision is:

$$p_i^{cs*} = \frac{(\beta-\delta)\left[\beta(c_S+c_R)-c_R\right]}{(\beta-1)(\beta-\delta-1)}, i = 1,2. \tag{3.11}$$

Because the cost structure is symmetric, the condition $\frac{c_{R_i}}{c_{S_i}} \le \frac{c_{R_j}}{c_{S_j}}$ in Proposition 3.3.7 is satisfied. Hence we have $r_i^{es*} < r^{cs*}, \forall i = 1,2$. In other words, both retailers have to reduce their revenue shares. This is intuitive, since channel competition becomes intense compared with the common retailer scenario. Reduction of revenue shares enables suppliers to reduce price and retain customers.

We now investigate supplier and retailer welfare under the common retailer and exclusive retailing settings. Supplier S_i's profit under the common retailer setting is

$$\begin{aligned} \pi_{S_i}^{cs*} &= a(p_i^{cs*})^{-\beta}(p_j^{cs*})^{\delta}\left[(1-r^{cs*})p_i^{cs*}-c_{S_i}\right] \\ &= \frac{ac_S}{\beta-1}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta)}\left[\frac{(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta)}. \end{aligned} \tag{3.12}$$

Also, Retailer R's profit under the common retailer scenario is:

$$\begin{aligned} \pi_R^{cs*} &= \sum_{i=1}^{2} d_i(\boldsymbol{p}^{cs*})\left(r^{cs*}p_i^{cs*}-c_{R_i}\right) \\ &= \frac{2a}{(\beta-\delta)}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta-1)}\left[\frac{(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta-1)}. \end{aligned} \tag{3.13}$$

Proposition 3.3.8 *π_R^{cs*} and $\pi_{S_i}^{cs*}$ have the following properties:*

(a) Let δ_0^R *be such that* $\frac{[\beta(c_S+c_R)-c_R](\beta-\delta)}{(\beta-1)(\beta-\delta-1)}|_{\delta=\delta_0^R}=1$*. When* $\delta>\delta_0^R$*,* π_R^{cs*} *increases with* δ *while* π_R^{cs*} *decreases with* δ *when* $\delta<\delta_0^R$*;*

(b) For given c_R*,* c_S*, and* β*, if there exists* δ_0^S *such that* $\{ln\frac{[\beta(c_S+c_R)-c_R](\beta-\delta)}{(\beta-1)(\beta-\delta-1)}-\frac{1}{\beta-\delta-1}\}|_{\delta=\delta_0^S}=0$*, then* $\pi_{S_i}^{cs*}$ *increases with* δ *when* $\delta<\delta_0^S$*, and decreases with* δ *when* $\delta>\delta_0^S$*.*

Proposition 3.3.8 states that when product substitutability is strong (exceeds δ_0^R), the retailer benefits from the increasing substitutability. The suppliers are in opposite positions. The suppliers only benefit from the increasing substitutability when product substitutability is not too strong (below δ_0^S).

Next, we obtain retailer R_i and supplier S_i's profit under exclusive retailing:

$$\begin{aligned}\pi_{S_i}^{es*} &= a(p_i^{es*})^{-\beta}(p_j^{es*})^{\delta}\left[(1-r_i^{es*})p_i^{es*}-c_{S_i}\right] \\ &= \frac{ac_S}{\beta-1}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta)}\left[\frac{\beta}{(\beta-1)^2}\right]^{-(\beta-\delta)}. \end{aligned} \tag{3.14}$$

and,

$$\begin{aligned}\pi_{R_i}^{es*} &= a(p_i^{es*})^{-\beta}(p_j^{es*})^{\delta}\left[r_i^{es*}p_i^{es*}-c_{R_i}\right] \\ &= \frac{a}{\beta}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta-1)}\left[\frac{\beta}{(\beta-1)^2}\right]^{-(\beta-\delta-1)}. \end{aligned} \tag{3.15}$$

We have the following proposition:

Proposition 3.3.9 *The supplier's profit under exclusive retailing is strictly larger than that in the common retailer setting, i.e.* $\pi_{S_i}^{es*}>\pi_{S_i}^{cs*}$*. However, the retailer level profit is less in an exclusive retailing setting, i.e.* $\pi_{R_1}^{es*}+\pi_{R_2}^{es*}<\pi_R^{cs*}$*.*

Proposition 3.3.9 shows that suppliers make more money under exclusive retailing because of the introduction of retailer level competition, which reduces the retailer's ability to exploit suppliers. Meanwhile, due to the competition, the retailers have to reduce their revenue shares and thus adversely affect their profits. Next, we investigate the channel profits under these two settings. Let $\rho_C \equiv \frac{\pi_{S_i}^{es*}+2\pi_{R_i}^{es*}}{\pi_{S_i}^{cs*}+\pi_R^{cs*}}$ represent the total channel profit ratio between the exclusive retailing and common retailer settings. Substituting the retailers and suppliers' profits into ρ_C, we obtain

$$\rho_C = \frac{1+\frac{K}{\beta-1}}{1+\frac{K}{\beta-\delta-1}} \cdot \left[\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]^{-(\beta-\delta)}, \tag{3.16}$$

where $K = \frac{2[\beta(c_S+c_R)-c_R]}{c_S}$.

We found that for given self price elasticity β, the value of ρ_C can be smaller, larger, or equal to one, depending on the value of the cross price elasticity δ. In other words, there is no absolute dominance between the exclusive retailing and common retailer settings in terms of total channel efficiency. Our numerical study shows that when the product competition is weak, i.e., δ is small, exclusive retailing has an advantage over the common retailer setting ($\rho_C > 1$). As competition becomes more fierce, i.e., δ is larger, this advantage diminishes and the common retailer setting starts to dominate ($\rho_C < 1$).

3.4 Future Studies on Other Demand Models

As shown in previous studies, the application of different demand models often leads to different pricing and stocking behaviors as well as profit allocation. In addition to the nonlinear demand model, the linear demand function $d_i(\boldsymbol{p}) = a - \beta p_i + \delta p_j$ where $a > 0$ and $\beta > \delta > 0$ for $i, j = 1, 2$ and $i \neq j$ has also been used in the literature to study the retailer and the suppliers' decision-making processes. We will continue to use the game setting as that in the nonlinear demand model, i.e., the retailer is the leader where the suppliers are followers, and we will follow the same methodology as in the nonlinear demand model and will address the issues outlined in the preceding sections in our future study.

CHAPTER 4
DYNAMIC PRICING IN REPUTATION-BASED E-COMMERCE RETAILING

4.1 Introduction

In today's competitive retailing business, sellers must regularly review prices they charge customers to ensure those prices match the customers' willingness to pay. Consider for example O'Neill Inc (seller). O'Neill designs and produces apparel, wetsuits, and accessories for water sports such as surfing, diving, waterskiing, and wind surfing. There are two seasons for these products, spring season which spans from the beginning of February to the end of July, and fall season starting August to the end of November (Cachon and Terwiesch, 2003). While certain products stay popular among customers in both seasons and can be carried and sold for many years, other products sell only in one season. For example, O'Neill switches color patterns of surf suits between seasons to attract more customers. The value of seasonal products drops sharply at the end of the season, and clearance sales of unsold products at basement price (below costs) are often typical. Because these products have long order lead times, O'Neill must make inventory decisions months before the season starts. Further, during the season, price adjustment may be necessary as product demand changes to maximize expected revenues during the season.

The problem confronting O'Neill in pricing seasonal products is a Revenue Management (RM) decision. The objective of RM is to maximize the seller's revenue by dynamically changing the product price as the season evolves. RM has started gaining recognition after the deregulation of the U.S. airline industry in the 1970s. Pioneering work had been done by Rothstein (1971) and Littlewood (1972) which dealt with the airline overbooking problem. The remarkable success of RM at American Airlines – a 2% to 8% improvement in revenues (Smith et al., 1992), enticed researchers to fully exploit the possibility of introducing RM to other industries such as hotel, retailing, car rental and cruise lines.

Traditional RM literature assumes that customer arrivals follow a stochastic process, e.g. Poisson process. A customer purchases a product when her reservation price (the maximum price she is willing to pay) is greater than the price set by the seller. We employ here the same concept of reservation price; however, our study goes deeper into the process of how a customer forms her reservation price. Recent studies show three major determinants of customer satisfaction: product quality, service quality and price (Gustafsson et al., 2005, and references therein). Product quality is closely related to physical attributes of a product. Service quality consists of intangible dimensions. Empirical study reveals that a seller's "responsiveness", "attentiveness", "ease of use" and "access" are most important dimensions in online retailing service (Jun et al., 2004). In this study, we assume customer reservation price depends on product quality and service quality.

In the past, customer perceptions of product and service quality are solicited through market surveys which are expensive and time consuming. Today, the Internet brings a platform for people to express their shopping experience electronically and their impression of a seller's product and service quality through well established reputation systems. According to Dellarocas (2003), reputation systems are mechanisms "using the Internet's bidirectional communication capabilities to artificially engineer large-scale, word-of-mouth networks in which individuals share opinions and experiences on a wide range of topics including companies, products, services, and even world events". Visiting shopping websites such as Yahoo.com and Bizrate.com and typing in a product keyword yield the ratings of product quality and service quality for sellers. For example, on Yahoo's marketplace for sandals, stars next to a product's listing represent the ratings for product quality and seller service quality, e.g., 5 stars for best product and service performance, and 1 star for worst. The rating scores are average evaluations from customers that bought the product. During the selling season, we assume product quality is fixed which is reasonable since the stocking decision is made before the season starts and no reordering is allowed during the season. However, service quality can change during the selling season.

Undoubtedly, a seller's service rating has been an important indicator of customer confidence on seller service performance. A high rating creates the impression that the seller is doing a good job in delivering what he promises. Tsay and Agrawal (2000) argue that customer purchase decisions depend not only on selling price, but also on service level. A customer may refuse to buy a product from a seller with poor service rating even though the price of the product is low. Conversely, the customer may be willing to pay a premium to a seller who has a good service track record. The study by Ba and Pavlou (2002) shows that a high rating will induce a high level of trust and confidence which in turn increases the customer's willingness to pay the price premium. Hence, it is important for the seller to change the price of his products according to his service ratings.

The major contributions of our study are: First, we offer a model-based pricing policy in a RM framework. Our model is more general than the classic RM model which does not consider reputation in the seller's pricing decision. Second, we investigate the revenue-making potentials of various pricing policies that a seller may adopt in the real world and compare them to that of a model-based pricing policy. We show the dominance of model-based policy over other policies in a variety of market scenarios. Also, we demonstrate how our model can be employed as a decision-making tool for reputation investment.

The remainder of this chapter is organized as follows. In Section 4.2, we review the revenue management, dynamic pricing and performance rating literature. Section 4.3 presents our assumptions. In Section 4.4, we study a periodic version of Reputation-based Revenue Management (RBRM) model. We simulate a variety of market scenarios and investigate the relative performance of different pricing policies in Section 4.5. Finally, we conclude this chapter with future research directions in Section 4.6.

4.2 Literature Review

Over the last decade, RM has been introduced in different industries such as airline, hotel, retail, car rental, cruise line and internet services. Although their operating environments differ, these industries share a basic RM framework with the following features: (1) perishable products; (2) finite selling season; and, (3) stochastic demand with price-sensitive customers. We refer readers to Bitran and Caldentey (2003) for a comprehensive overview of the RM literature. What differentiates our study from existing literature is the reservation price. We define reservation price as the maximum price a customer is willing to pay for one unit of a product. In the literature, it has been generally assumed that the reservation price follows a distribution which could be time-variant or time-invariant.

However, the literature does not explicitly model the underlying drivers of the reservation price. Our work studies the formation of reservation price from the perspective of consumer behavior, and unveils the underlying dynamics between the reservation price and product and service quality.

Gallego and van Ryzin (1994) formulate a continuous-time revenue maximization problem with homogeneous demand intensity and derive the structural properties of the price and expected revenue for the problem, which are: (1) expected revenue strictly increases in the number of items on inventory (stock level) and remaining selling time; (2) price decreases in the stock level and increases in the remaining selling time. They obtain a closed-form pricing policy for an exponential family of demand functions, and develop heuristic-based pricing policies for other demand functions. The authors extend the basic problem to other scenarios such as compound Poisson process, discrete pricing policies and time-varying demand intensity.

Gallego and van Ryzin (1997) extend the single product revenue maximization problem to a multi-product case. They formulate the multi-product deterministic problem in a similar fashion as they did in their previous paper (Gallego and van Ryzin, 1994), and identify the relationship between stochastic and deterministic problems. As analytic solutions to the multi-product RM problem are extremely difficult to obtain, the authors develop two heuristics which are asymptotically optimal to the deterministic case. Maglaras and Meissner (2006) convert the problem of choosing an optimal pricing policy to that of choosing an aggregate consumption rate. Given the rate, the product prices are determined. Talluri and van Ryzin (2004) use a general customer choice model to solve the RM problem in a multi-product environment where product prices are known. They develop the optimal policy characterizing a set of products to offer.

Zhao and Zheng (2000) exploit the RM problem assuming demand follows a more general nonhomogeneous Poisson process, i.e., both customer arrival rate and the reservation price distribution change over time. Their work is an extension to Gallego and van Ryzin (1994), where only homogeneous demand and a special type of nonhomogeneous demand are studied. They develop two structural properties, inventory-monotonicity and time-monotonicity, which characterize the optimal solution to their problem. The time-monotonicity property only holds under a sufficient condition, i.e., a customer's willingness to pay a premium decreases as time elapses. Their model setting is closest to ours because we also assume a time-variant reservation price distribution. However, a major distinction in our work is that we model the factors that drive the changes of reservation price explicitly.

In addition to developing the optimal pricing condition for a continuous-time model, Bitran and Mondschein (1997) investigate the optimal price and expected profits under a more realistic situation where prices change periodically. They perform a comparative study of the pricing policies and expected profits of different scenarios, i.e., variable or fixed discount rates on product price and a reservation price distribution with either large or small standard deviations. An extension of the work to a retail chain with multiple geographically dispersed stores can be found in Bitran et al. (1998). They investigate two scenarios which either allow or do not allow inventory transfers between stores. They develop a heuristic procedure for each scenario and compare the results with a real-world case.

Elmaghraby and Keskinocak (2003) discuss the possibilities of dynamic pricing in today's business environment. Also they review the dynamic pricing strategies studied in the literature and being adopted in practice. They research two types of problems in great detail. They are the dynamic pricing with and without inventory replenishment. To some degree, the latter problem follows the general framework of a RM problem.

We observe that the aforementioned body of RM literature captures the impact of inventory availability on product price by introducing the reservation price. The distribution of reservation price is assumed to be either time-invariant or time-variant. Absent in these studies are other important factors such as service quality and product quality which also contribute to the formation of reservation price. Intuitively, when customer ratings of seller product and service quality change, prices should respond to those changes. A body of marketing and economics literature has studied the relationship between rating, quality and price. However, they do not consider the perishability and availability of inventory in the models (Shapiro 1982; Ba and Pavlou 2002; Zacharia et al. 2001). To the best of our knowledge, nothing has been done to combine seller rating and the availability of product inventory as determinants of product prices in a RM framework. Our study unifies these two bodies of literature by simultaneously taking the seller's rating and inventory effects into consideration. We formulate our RBRM model in the following section.

4.3 Preliminaries

Consider a seller of a perishable product in an online marketplace that faces a short selling season of length L. At the beginning of the season, the seller has I units on hand. The unsold units have zero salvage value at the end of the season. We assume product quality is fixed as the product does not deteriorate and no new product deliveries are allowed during the season. We assume an initial seller service rating, which can improve or deteriorate as the season moves forward. Define $\Omega = \{1, 2, ...h\}$ as an ordinal set of seller performance ratings with increasing order (e.g. Likert scale). Let $r_t \in \Omega$ denote the seller rating at time t. Suppose the seller reviews his price periodically. We let T represent the total number of review periods. T is backward indexed, i.e., the first period is T and the last period is 1. We assume negligible administrative cost to change the product price, e.g., the seller can post the change on the store Webpage at no cost. Customer arrivals for each period follow a time-homogeneous Poisson process J with time-homogeneous arrival rate a. Each customer purchases at most one unit. Let ϕ denote the customer unit valuation of seller rating, and ϕ does not vary with time. Also, we denote υ as the intrinsic value of the product where υ is a function of product quality. υ is fixed throughout the season as product quality does not change. We use an additive model to define a customer's reservation price as

$$u = \phi r + \upsilon + \epsilon, \tag{4.1}$$

where ϵ is a random term that represents customer idiosyncrasy in forming her reservation price. The CDF and pdf of ϵ are $F_\epsilon(\cdot)$ and $f_\epsilon(\cdot)$ respectively. We note that other reservation price functions exist that also warrant investigation. We reserve this for future research.

At time t, when a customer arrives and observes product quality and the seller's service rating, she forms a reservation price u_t based on (4.1) and compares it with the seller retail price p_t. The consumer surplus is $u_t - p_t$. If the consumer surplus is greater than zero, i.e., $u_t > p_t$, the customer purchases the product; otherwise, she leaves the store without buying the product. Let $P(X)$ be the probability of event X. The demand intensity is therefore:

$$\lambda(p_t, r_t) = aP[u_t > p_t] = a[1 - F_\epsilon(p_t - \phi r_t - \upsilon)] \tag{4.2}$$

Based on the number of units on hand c_t and his rating r_t, the seller's decision at the beginning of each period $t \in \{1, 2, ..., T\}$ is to determine the price p_t which maximizes his total expected revenue from period t onwards. Figure 4.1 illustrates the seller's decision path. For convenience, we omit the time index "t" in variables and parameters for the remainder of this chapter.

Figure 4.1: The seller's decision path in our RBRM model.

4.3.1 Customer Rating of Seller Service Quality

Generally, every customer buying a product online has the opportunity to evaluate the sellers' service quality. For instance, on Amazon.com, customers rate a seller on a 5-point scale with 1 representing the worst service. In comparison, Priceline uses a 10-point scale, while eBay uses categorical ratings: positive, neutral and negative. In making a purchase decision, we assume that every individual customer only pays attention to the K most recent seller service ratings. Also, we assume that, before the new selling season starts, the seller has a selling history which contains at least K customer service ratings. Let $w \in \Omega$ be the customer's rating of the seller given after the customer makes a purchase. Denote Ξ as the K ordered set of most recent seller ratings, i.e., $\Xi = \{w_K, ..., w_1\}$ with w_K being the oldest rating. Note that every new customer rating replaces the oldest rating in Ξ. We define the seller's current service rating r to be a function of the arithmetic average of the customer ratings in Ξ, i.e., $r = g\left(\frac{w_1+...+w_K}{K}\right)$ where $g(\cdot)$ is a rounding function.

We assume customers give their ratings via a Markov chain with stationary transition probability matrix $Z_{h\times h}$:

$$P\{w = j|r = i\} = z_{ij},\ i, j \in \Omega,\ z_{ij} \in [0, 1]\,. \tag{4.3}$$

Given the seller's current rating is i, z_{ij} in (4.3) is the probability that a customer, after buying a product, gives the seller a rating of j. Obviously, the seller does not receive a new rating when no purchases occur in a given period. Consider our first assumption.

Assumption 4.3.1 *For all $k \in \Omega$, $\sum\limits_{j=k}^{h} z_{ij}$ is nondecreasing in $i \in \Omega$, and $\sum\limits_{j\in\Omega} z_{ij} = 1, \forall i \in \Omega$.*

Assumption 4.3.1 states that the rating mechanism is history dependent, and the chance of evolving to a higher rating increases with the seller's current rating i. Mathematically, z_{ij} is stochastically increasing with the current rating i.

4.3.2 Seller Rating Transition Probabilities

We now determine the seller's rating transition probability matrix $M_{h \times h}$ given the customer rating transition probability matrix $Z_{h \times h}$ and K most recent customer ratings. For simplicity, consider hereafter a two-tier rating scheme, i.e., $\Omega = \{1, 2\}$. That is, the seller performance is low (represented by state 1) or high (represented by state 2). Later we also refer to the 2 states simply as $r = 1$ and $r = 2$. Also, define the rounding function $g(x)$ as $g(x) = 1, \forall x < 1.5$, and $g(x) = 2$ otherwise. When $\Omega = \{1, 2\}$, the customer's rating transition matrix is expressed as:

$$Z_{2\times 2} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}.$$

Assumption 4.3.1 is equivalent to $z_{22} \geq z_{12}$ and $z_{11} \geq z_{21}$ under the two-tier rating scheme.

We first investigate a case where $K = 3$. The combinations that yield a low seller rating are $(w_3, w_2, w_1) = \{(1, 1, 1), (2, 1, 1), (1, 2, 1), (1, 1, 2)\}$. Denote w_0 as the new customer rating. When w_0 enters Ξ, w_3 leaves. For $(1, 1, 1)$ and $(2, 1, 1)$, regardless of $w_0 = 1$ or 2, the seller rating stays in state 1. While for $(1, 2, 1)$ and $(1, 1, 2)$, the seller rating goes to state 2 when $w_0 = 2$. Note that the following conditions are sufficient for the seller rating to change from states 1 to 2, i.e., (1) $w_3 = 1$; (2) the sum of ratings in Ξ is equal to 4; (3) the new rating is $w_0 = 2$. The stationary transition probability of seller rating going from state 1 to state 2 is $m_{12} = \frac{2}{4} * z_{12}$. Thus, $m_{11} = 1 - m_{12}$. Similarly, we can derive m_{21} and m_{22}.

We now derive M for any K. Let b be the number of 2's in Ξ. Denote C_K^b as the number of possible combinations of b 2's in Ξ. If $r = 1$, then the total number of possible combinations of K customer ratings in Ξ is $C_K^0 + ... + C_K^{n_m}$, where n_m is the largest b. We have $n_m = \lceil 1.5K \rceil - 1 - K$ where $\lceil 1.5K \rceil$ is the smallest integer such that $\lceil 1.5K \rceil \geq 1.5K$. The conditions which result in $r = 2$ moving to $r = 1$ are: (1) $w_K = 1$; (2) the sum of the ratings in Ξ is $\lceil 1.5K \rceil - 1$; (3) $w_0 = 2$. $C_{K-1}^{n_m}$ is the number of combinations that satisfy (1) and (2), and z_{12} is the probability that $w_0 = 2$ given $r = 1$. Hence, m_{12} is given by $m_{12} = \frac{C_{K-1}^{n_m}}{C_K^0 + ... + C_K^{n_m}} z_{12}$ and $m_{11} = 1 - m_{12}$.

Similarly, when $r = 2$, the number of 2s in Ξ is at least $n_m + 1$. Now, the total number of combinations in Ξ that yield $r = 2$ is $C_K^{n_m+1} + ... + C_K^K$. The conditions that will move the seller's rating to $r = 1$ from $r = 2$ are: (1) $w_K = 2$; (2) the sum of the ratings in Ξ is $\lceil 1.5K \rceil$; (3) $w_0 = 1$. $C_{K-1}^{n_m}$ is the number of combinations that satisfy the first two conditions, and z_{21} is the probability that $w_0 = 1$ given $r = 2$. Thus $m_{21} = \frac{C_{K-1}^{n_m}}{C_K^{n_m+1} + ... + C_K^K} z_{21}$ and $m_{22} = 1 - m_{21}$.

Hence, the stationary transition probability matrix for seller's rating is

$$M_{2\times 2} = \begin{pmatrix} 1 - \frac{C_{K-1}^{n_m}}{C_K^0 + ... + C_K^{n_m}} z_{12} & \frac{C_{K-1}^{n_m}}{C_K^0 + ... + C_K^{n_m}} z_{12} \\ \frac{C_{K-1}^{n_m}}{C_K^{n_m+1} + ... + C_K^K} z_{21} & 1 - \frac{C_{K-1}^{n_m}}{C_K^{n_m+1} + ... + C_K^K} z_{21} \end{pmatrix}. \tag{4.4}$$

It is reasonable to assume that $M_{2\times 2}$ is stationary since seller rating is always computed from K most recent customer ratings. Although in what follows we restrict our attention to a two-tiered rating scheme, the above procedures can be used to derive the seller's rating transition matrix $M_{h\times h}$ with $h > 2$. Under the two-tier scheme, given Assumption 4.3.1, we have:

Proposition 4.3.1 *For all $k \in \Omega$, $\sum_{j=k}^{h} m_{ij}$ is nondecreasing in $i \in \Omega$.*

Proposition 4.3.1 states that when the customer's rating transition probability z_{ij} is stochastically increasing with the seller's current rating i, the seller's rating transition probability m_{ij} has the same property. Next, we formulate the periodic review model.

4.4 Periodic Pricing Review Policies

In this section, we assume the seller updates his product price periodically such as once a day, week or month, and his overall rating is updated and released to customers at the end of each review period. In the periodic model, we can have multiple arrivals in a given period. Each customer purchases a product only when $u > p$. We first present our mathematical model under the periodic review policy. Later, we utilize the model to examine how the seller price and revenue change under various parameter settings.

4.4.1 Mathematical Formulation

As defined earlier, T is the total number of reviews undertaken during the entire selling season L. We let ΔT_t denote the length of the tth review period. Hence, $\sum_{t=1}^{T} \Delta T_t = L$. Let l_t represent the starting time of period t. Therefore,

$$l_t = \sum_{i=t+1}^{T} \Delta T_i \text{ and } l_T = 0. \tag{4.5}$$

As the seller's rating r is updated and is made available to customers at the end of each review period, the demand intensity remains unchanged for that period. The average number of customer arrivals during period t is

$$k_t(p, r) = \lambda(p, r)\Delta T_t, \tag{4.6}$$

where $\lambda(p, r)$ is the demand intensity defined in (4.2). Let $J_t(p, r)$ be the product demand in period t given the price p and seller rating r. Assuming Poisson arrivals, the probability that product demand is equal to x in period t is

$$P[J_t(p, r) = x] = \frac{\exp^{-k_t(p,r)}[k_t(p, r)]^x}{x!}. \tag{4.7}$$

At the end of period t, a new seller rating r is computed by averaging the K most recent customer ratings. Define $m(x)$ as

$$m(x) = \begin{cases} m_{ij}, x > 0 \\ 1, \quad x = 0, \end{cases}$$

where m_{ij} is the stationary transition probability of seller service rating given by (4.4). We can formulate the dynamic programming equation for the periodic review policy as follows:

$$V_t(c, i) = \max_{p \geq 0} \left\{ \sum_{x=0}^{\infty} \left[\min(c, x)p + \sum_{j \in \Omega} V_{t-1}(c - \min(c, x), j)m(x) \right] P[J_t(p, i) = x] \right\}, \tag{4.8}$$

with boundary conditions:

$$V_0(c, i) = 0; \quad \forall c, i \in \Omega$$

$$V_t(0, i) = 0; \quad \forall t, i \in \Omega.$$

In the above formulation, $V_t(c, i)$ is the expected revenue-to-go function starting from period t, given c units on hand and the current seller service rating i. For example, $V_T(\cdot)$ is the total expected revenue throughout the selling season L. The first term $\min(c, x)p$ is the revenue realized in period t, given that x customers want to buy the product. The second term $\sum_{j \in \Omega} V_{t-1}(c - \min(c, x), j)m(x)$ is the expected revenue from period $t-1$ onwards, given $\{c - \min(c, x)\}$ units ending inventory in period t. The objective is to determine the price at the beginning of each period so as to maximize the total expected revenue.

In (4.8), we can transform $\sum_{x=0}^{\infty} \sum_{j \in \Omega} [V_{t-1}(c - \min(c, x), j)m(x)] \, P[J_t(p, i) = x]$ to $\sum_{x=0}^{c} \sum_{j \in \Omega} [V_{t-1}(c - x, j)m(x)] \, P[J_t(p, i) = x]$, since when $x > c$, $V_{t-1}(c - \min(c, x), j) = 0$. Also, it is easy to show that

$$\sum_{x=0}^{\infty} [\min(c, x)p] \, P[J_t(p, i) = x] \quad = \quad \sum_{x=0}^{c} [(x - c)p] \, P[J_t(p, i) = x] + pc.$$

Therefore, we can recast (4.8) as

$$V_t(c, i) = \max_{p \geq 0} \left\{ \exp^{-k_t(p,i)} \sum_{x=0}^{c} \frac{[k_t(p, i)]^x}{x!} \left[p(x - c) + \sum_{j \in \Omega} V_{t-1}(c - x, j)m(x) \right] + pc \right\}. \tag{4.9}$$

4.4.2 An Illustrative Example

We illustrate the application of (4.9) where $\Omega = \{1, 2\}$ and $I = 2$. The dynamic programming problem has six possible states: $(c, r) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$. We assume the initial state of the seller is $(c, r) = (2, 1)$. Assume $K = 2$. The total number of review periods is $T = 3$, and the average arrival rate is $a = 5$. Also, we assume that the customer rating transition probability matrix $Z_{2 \times 2} = \{z_{ij}\}$ is:

$$Z = \begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{pmatrix}. \tag{4.10}$$

We can derive the seller rating transition matrix M according to (4.4) which yields

$$M = \begin{pmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \end{pmatrix}. \tag{4.11}$$

In addition, we assume consumer valuation on service quality $\phi = 10$, and the intrinsic value of the product $\upsilon = 20$. The random term of reservation price follows a normal distribution with zero mean and a variance of 4, i.e., $\epsilon \sim N(0, 4)$. The seller reviews and fine tunes his price every period to maximize his total expected profit over the entire selling season. Table 4.1 presents the solution.

Table 4.1: The optimal price policy

(c,r)	Period 1	Period 2	Period 3
(0,1)	-	-	-
(0,2)	-	-	-
(1,1)	-	31.31	29.16
(1,2)	-	41.23	38.91
(2,1)	31.87	30.98	28.36

Entering the selling season, the seller prices the product at $\$31.87$. If one unit is sold in the 1st period and seller rating improves (i.e. r goes up to 2), the seller should increase the product price to $\$41.23$ in the 2nd period. But if seller rating stays at 1, then the seller should lower the price to $\$31.31$ in the 2nd period. (Note if the last unit is sold in the 2nd period, then selling ceases.) If no units are sold in the 2nd period, then in the 3rd period, the seller should lower the price further to $\$38.91$ if the current seller rating is 2, and lower it to $\$29.16$ if the current seller rating is 1. If no units are sold in the 1st period, then the seller should lower the price to $\$30.98$ in the 2nd period. If nothing was sold in the 2nd period, the seller should lower the price again to $\$28.36$ in the 3rd period. The total expected profit of this solution is $\$64.97$ which is also the optimal profit.

4.5 Pricing Policies

In this section, we study different seller pricing policies and compare their revenues. RBRM is the optimal policy where the seller dynamically updates his price by taking the on-hand inventory and possible rating transitions into consideration, i.e., the seller maximizes his revenue by (4.8). We call it as the "optimal pricing policy" or π^o. Another pricing policy which has been used in practice is the "classic pricing policy" or π^c (see Bitran and Mondschein (1997) and references therein). In π^c, the seller sets his price based on (4.8), but with the seller rating fixed at its initial value. The third policy is the seller sets a fixed price for the entire selling season (see Gallego and van Ryzin (1994)). We denote it as π^d-"deterministic pricing policy". Finally, we consider a myopic policy where the seller examines his rating at the beginning of each review period and presumes the rating will stay the same for the rest of the selling season. We denote this as π^m-"myopic pricing policy". Note that π^m differs from π^c because π^c ignores service rating changes.

We study the revenue generating potentials of these policies across 5 dimensions, i.e., (1) consumer heterogeneity in valuation; (2) consumer valuation level; (3) initial on-hand inventory level; (4) the K value; and (5) the product intrinsic value. We examine 5 different scenarios under each dimension and fix the selling period to 12 days. We assume the seller reviews his price every three days. Therefore, the total number of review periods is 4. The average arrival rate in each period is $a = 50$. We assume a two-tier rating scheme $\Omega = \{1, 2\}$. The customer rating transition matrix is either

$$Z_{(1)} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}, Z_{(2)} = \begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{pmatrix}, or\ Z_{(3)} = \begin{pmatrix} 0.3 & 0.7 \\ 0.2 & 0.8 \end{pmatrix}.$$

We set the probabilities in $Z_{(1)}$ such that customers randomly give ratings to the seller regardless of the seller's current rating. In $Z_{(2)}$ and $Z_{(3)}$, however, we set the probabilities such that the seller's current rating is positively related to the next customer's rating. We make this correlation stronger in $Z_{(3)}$. Note that by Proposition 4.3.1, every Z and corresponding M are stochastically increasing with seller's current rating.

Overall, we have 15 cases per dimension (i.e., 3 transition matrices times 5 scenarios) and 25 replications per case for a total of 375 runs. For every case, we use Algorithm 4.1 to generate random customer arrivals and their corresponding valuations. Algorithm 4.2 simulates the customer purchase decisions and compute the average revenue. The results are presented in the following sections.

4.5.1 Consumer Heterogeneity

In this section, we study the effect of consumer heterogeneity on the expected revenues of different pricing policies. In this experiment, we fix the initial inventory I to 60 and the initial rating to $r = 1$. In addition, the product intrinsic value is $v = 20$, and consumer valuation on rating $\phi = 10$. We assume the random term ϵ is normally dis-

Algorithm 4.1 Generate random customer arrivals and their valuations

```
for replication =1 To 25 do
  for each review period do
    Generate Poisson arrivals
    for each arrival do
      Generate ϵ value for ϵ distributions
      Generate a value τ from U[0,1]
    end for
  end for
end for
```

Algorithm 4.2 Simulate customer purchase decision

```
Generate pricing policies π^i where i = {o, c, d, m}
Set V^i = 0
for replication =1 To 25 do
  for each review period do
    for each arrival do
      Compute reservation price u
      for each pricing policy do
        Retrieve price p
        Compute consumer surplus (CS)=u − p
        if CS > 0 then
          Customer purchases a product
          Customer gives a rating by τ, and update Ξ
          V^i = V^i + p
        else
          Customer refuses to purchase
        end if
      end for
    end for
    Update the seller rating score r
  end for
end for
Average V^i
```

tributed with zero mean, but with different variances. In particular, we study five scenarios: $N(0,1)$,$N(0,4)$,$N(0,9)$,$N(0,16)$ and $N(0,25)$. Table 4.2 reports the results for the 5 scenarios.

In Table 4.2, V^{π} denotes the expected revenues under pricing policy π where $\pi = \{\pi^o, \pi^c, \pi^d, \pi^m\}$. The results demonstrate the advantage of employing π^o. In particular, when consumer valuation has small variance, π^o's dominance over other policies is more profound. For example, when $\sigma^2 = 1$ under $Z_{(2)}$, π^c only achieves 79.3% of the revenues realized under π^o. This is not surprising because with wide dispersion in consumer valuation, product price matters less. Bitran and Mondschein (1997) made a similar observation. Among the policies, π^m is closest to π^o on average. In other words, when one cannot forecast the seller's rating transition matrix, we can adopt π^m to achieve the best possible results. π^c's low performance reveals the downside of ignoring rating changes. Moreover, note that under $Z_{(3)}$, the seller achieves higher expected revenues than those of $Z_{(1)}$ and $Z_{(2)}$. However, achieving this may require more seller investment to improve his service quality. Our model can be utilized as a decision-making tool for reputation investment. For example, in Table 4.2, when $\sigma^2 = 9$, the difference of expected revenues under $Z_{(2)}$ and $Z_{(3)}$ is $\$2340.49 - \$2275.35 = \$65.14$. If improving service quality can change the customer's rating transition to $Z_{(3)}$ from $Z_{(2)}$ for less than $\$65.14$, then it is worth investing on improving seller service quality. Otherwise, the seller should not pursue the program.

4.5.2 Consumer Valuation Level

In the real world, some customers are sensitive to seller service rating, and their purchase decision is likely to be less sensitive to price increases. In contrast, some customers are not concerned about the seller's service rating and their purchase decision is likely to be more sensitive to price increases. In this section, we segment customers into groups with each group showing a different valuation ϕ of seller's service rating. Specifically, we let $\phi = \{1, 5, 10, 15, 20\}$ with 1 representing least valuation on the worthiness of rating and 20 highest valuation. The initial inventory, initial rating, and the product intrinsic value are the same as those in the previous section, i.e., $I = 60$, $r = 1$ and $\upsilon = 20$. The random term ϵ is $N(0, 4)$. Table 4.3 presents the simulation results.

Again π^o shows dominance over all other pricing policies. The dominance is more profound as customers become more sensitive to seller service rating. For instance, when $\phi = 1$ and customer rating transition matrix is $Z_{(2)}$, π^c is able to attain 98.7% of π^o's revenues and 73.6% when $\phi = 20$. This follows since as seller service rating becomes more important in customer purchasing decision, a good pricing policy is one that adjusts the price according to the rating transition. Generally, when consumer valuation on rating increases, the seller can increase sales because he can charge a higher price. π^m can be a good policy when the transition probabilities are unknown.

4.5.3 Initial On-hand Inventory

We now study the impact of initial inventory on the expected revenues of different pricing policies. We set the initial inventories to either 40%, 80%, 120%, 160% or 200%

of the arrival rate, i.e., $I = \{20, 40, 60, 80, 100\}$. The initial rating is $r = 1$. For consumer valuation on rating, we fix it at $\phi = 10$ while $\upsilon = 20$ and $\epsilon \sim N(0, 4)$. The relative performance of pricing policies is shown in Table 4.4. An interesting phenomenon is the consistent relative performance of pricing policies across different levels of initial inventory, e.g., under transition matrix $Z_{(2)}$, π^c and π^d were approximately 82% of π^o, and π^m was 98% of π^o. This is different from our previous results where the revenue ratios show larger variations across different consumer valuations and heterogeneity levels.

4.5.4 The K Value

We also investigate the impact of K on the expected revenues of various pricing policies. Our results in Table 4.5 show that K can also affect the relative performance of different pricing policies. But under $Z_{(3)}$, the results for $K = 5$ through $K = 20$ show little change in ratios across pricing policies. Note that under $Z_{(3)}$, the seller in our experiment is likely to have a high seller service rating. When seller service rating is consistently high, then increasing K will not affect their performance significantly.

4.5.5 Product Intrinsic Value Level

In this section, we examine the impact of product intrinsic value on the revenues of different pricing policies. We set the product intrinsic value level to either $1, 5, 10, 15$ or 20. The other parameter settings are $I = 60, r = 1, \phi = 10$, and $\epsilon \sim N(0, 4)$. We summarize the results in Table 4.6. Notice that higher intrinsic values yield higher revenues. Also, Table 4.6 shows that π^o's performance is more profound when υ is low. This is reasonable since at low υ's, the seller's service rating carries more weight in the formation of consumer reservation price. However, we acknowledge the fact that there may exist a positive relationship between the intrinsic value and the consumer valuation of seller rating, i.e., customers might put a low valuation on the worthiness of the seller's rating when v is low. We reserve the exploration of this issue for future study.

4.6 Conclusions

Dynamic pricing has abundant applications in the operations, economics and finance fields. However, it has never been more important and necessary than now, an era in which internet and e-commerce have been experiencing a rapid growth, for two reasons. First, in the electronic world, it is much easier to access demand information, consumer preferences, and inventory levels, which offers sellers the flexibility to change their pricing policies dynamically corresponding to real-time information. Furthermore, the cost associated with modifying price is negligible on the Internet or corporate webpage compared with that of traditional brick and mortar retailing businesses. Second, from the customer standpoint, they have better knowledge of the price offered by different sellers and their service ratings through online search portals or market places such as Bizrate.com and Amazon.com. The search cost today is small. Therefore, customers are more knowledgeable than before which gives them the ability to optimize their purchase decisions. As a result, new potential applications for RM are emerging (see Bitran and Caldentey (2003) for more details).

Our study extends the traditional RM by not only taking the seller's inventory on-hand into price decisions, but also the seller's service rating. We formulated the seller's pricing problem in a RM framework. Also, we examined other pricing policies that are utilized in the real world, such as “classic pricing policy”, “deterministic pricing policy” and “myopic pricing policy”. In particular, we investigated the relative performance of those pricing policies by simulating their applications in a variety of marketing scenarios. Our study demonstrated the dominance of the optimal pricing policy over all other pricing policies. In particular, we found the downside of “classic pricing policy” which did not incorporate rating changes and “deterministic pricing policy” which fixed one price throughout the selling season. The close performance of “myopic pricing policy” (more than 90% of the expected revenues of “optimal pricing policy”) seems to imply that we can employ it in lieu of “optimal pricing policy” when rating transition information is unavailable.

Our research can be extended in a few avenues. First, it would be interesting to estimate customer valuation on service and product quality based on historical data available, and to set retail price dynamically based on the updated information. Second, in this study we presented a two-tier rating scheme. Further research is possible to capture a multi-tier rating scheme and explore the corresponding structures and properties of the problem to better aid the seller’s price decisions. Finally, we assumed the rating transition matrix is static. Future study can be done by relaxing this assumption and allowing the rating transition matrix to change over time.

Table 4.2: Consumer heterogeneity's impact on revenues

Z	σ^2	V^{π^o}	V^{π^c}/V^{π^o}	V^{π^d}/V^{π^o}	V^{π^m}/V^{π^o}
(1)	1	2156.31	84.9%	84.4%	95.6%
	4	2193.09	84.5%	84.2%	95.5%
	9	2202.55	85.1%	84.4%	96.0%
	16	2095.02	91.0%	90.9%	101.5%
	25	2199.91	89.4%	87.9%	100.6%
(2)	1	2297.14	79.7%	79.3%	94.4%
	4	2281.43	81.2%	81.0%	95.5%
	9	2275.35	82.5%	82.1%	98.2%
	16	2228.80	85.6%	85.5%	100.4%
	25	2290.97	85.6%	84.6%	100.9%
(3)	1	2332.60	78.4%	82.6%	94.5%
	4	2327.84	79.6%	81.7%	97.2%
	9	2340.49	80.2%	81.8%	98.1%
	16	2240.37	85.2%	83.8%	101.8%
	25	2434.88	80.5%	82.9%	96.3%

Table 4.3: Consumer valuation level's impact on revenues

Z	ϕ	V^{π^o}	V^{π^c}/V^{π^o}	V^{π^d}/V^{π^o}	V^{π^m}/V^{π^o}
(1)	1	1333.43	98.9%	98.1%	100.2%
	5	1669.56	93.1%	92.7%	100.2%
	10	2193.09	84.5%	84.2%	95.5%
	15	2678.99	80.3%	80.1%	93.9%
	20	3199.58	76.6%	76.4%	91.8%
(2)	1	1339.55	98.7%	97.8%	100.4%
	5	1733.22	89.6%	89.3%	99.0%
	10	2281.43	81.2%	81.0%	95.5%
	15	2808.49	76.6%	76.4%	94.1%
	20	3333.29	73.6%	73.3%	93.2%
(3)	1	1347.53	98.3%	96.9%	100.4%
	5	1773.88	87.6%	88.2%	99.0%
	10	2327.84	79.6%	81.7%	97.2%
	15	2883.46	74.7%	77.6%	96.0%
	20	3435.97	71.4%	74.7%	95.3%

Table 4.4: Intial on-hand inventory's impact on revenues

Z	I	V^{π^o}	V^{π^c}/V^{π^o}	V^{π^d}/V^{π^o}	V^{π^m}/V^{π^o}
(1)	20	739.71	87.7%	87.0%	96.6%
	40	1472.12	85.3%	84.3%	95.8%
	60	2193.09	84.5%	84.2%	95.5%
	80	2819.87	86.1%	85.7%	96.8%
	100	3416.50	87.4%	87.1%	98.8%
(2)	20	765.59	84.7%	84.1%	97.9%
	40	1527.52	82.3%	81.6%	97.3%
	60	2281.43	81.2%	81.0%	95.5%
	80	2970.85	81.9%	81.4%	96.7%
	100	3593.81	83.4%	82.8%	99.5%
(3)	20	795.73	81.5%	82.3%	98.4%
	40	1576.84	79.8%	82.0%	97.1%
	60	2327.84	79.6%	81.7%	97.2%
	80	3058.05	79.7%	82.4%	96.2%
	100	3749.80	80.1%	81.3%	97.8%

Table 4.5: The K value

Z	K	V^{π^o}	V^{π^c}/V^{π^o}	V^{π^d}/V^{π^o}	V^{π^m}/V^{π^o}
(1)	1	2194.21	84.4%	84.4%	94.6%
	5	2193.09	84.5%	84.2%	95.5%
	10	2125.12	87.2%	86.8%	98.7%
	15	2071.31	89.4%	88.9%	99.1%
	20	2020.50	91.7%	91.2%	102.8%
(2)	1	2223.75	83.3%	80.3%	96.3%
	5	2281.43	81.2%	81.0%	95.5%
	10	2278.29	81.4%	81.0%	99.2%
	15	2264.77	81.8%	81.4%	97.7%
	20	2271.78	81.6%	81.2%	98.0%
(3)	1	2230.50	83.1%	85.3%	98.3%
	5	2327.84	79.6%	81.7%	97.2%
	10	2349.13	78.9%	80.8%	97.3%
	15	2343.58	79.1%	80.7%	97.6%
	20	2345.23	79.1%	80.9%	97.2%

Table 4.6: The product intrinsic value's impact on revenues

Z	υ	V^{π^o}	V^{π^c}/V^{π^o}	V^{π^d}/V^{π^o}	V^{π^m}/V^{π^o}
(1)	1	1052.21	68.0%	67.9%	91.3%
	5	1290.66	74.0%	73.7%	92.9%
	10	1528.13	82.1%	81.7%	98.0%
	15	1886.96	82.3%	82.0%	95.2%
	20	2193.09	84.5%	84.2%	95.5%
(2)	1	1128.04	63.5%	63.5%	92.6%
	5	1376.26	69.4%	69.3%	93.2%
	10	1668.75	75.2%	74.9%	94.8%
	15	1968.44	78.9%	78.7%	95.5%
	20	2281.43	81.2%	81.0%	95.5%
(3)	1	1164.34	61.6%	63.8%	96.7%
	5	1431.08	66.8%	70.0%	95.4%
	10	1701.94	73.8%	75.3%	97.8%
	15	2003.82	77.6%	79.0%	98.0%
	20	2327.84	79.6%	81.7%	97.2%

APPENDIX A
PROOFS OF CHAPTER ONE

A.1 Proof of Proposition 1.3.3

Proof: Taking the derivative of π_S^{ws} with respect to w yields

$$\begin{aligned}\frac{\partial \pi_S^{ws}}{\partial w} &= -K(\beta+1)(w+\alpha c+m)^{-(\beta+2)}\{w-(1-\alpha)c\}+K(w+\alpha c+m)^{-(\beta+1)} \\ &= K(w+\alpha c+m)^{-(\beta+2)}\{-\beta w+\alpha c+m+(\beta+1)(1-\alpha)c\},\end{aligned}$$

where $K = Mam$. Setting $\frac{\partial \pi_S^{ws}}{\partial w} = 0$, we obtain $w^* = (1-\alpha)c + \frac{m+c}{\beta}$. It is easy to see that when $w < w^*$, $\frac{\partial \pi_S^{ws}}{\partial w} > 0$, while $w > w^*$ implies $\frac{\partial \pi_S^{ws}}{\partial w} < 0$. Thus π_S^{ws} is maximized at w^*.

A.2 Proof of Proposition 1.3.4

Proof: The derivative of π_R^{ws} with respect to m is:

$$\begin{aligned}\frac{\partial \pi_R^{ws}}{\partial m} &= -K_2(\beta+1)\left(m+c\right)^{-(\beta+2)} m^2 + 2K_2\left(m+c\right)^{-(\beta+1)} m \\ &= K_2\left(m+c\right)^{-(\beta+2)} m\left[-(\beta-1)m+2c\right].\end{aligned}$$

Setting $\frac{\partial \pi_R^{ws}}{\partial m} = 0$, we obtain $m^* = \frac{2c}{\beta-1}$. Again, for $m < m^*$, $\frac{\partial \pi_R^{ws}}{\partial m} > 0$ while $m > m^*$ implies $\frac{\partial \pi_R^{ws}}{\partial m} < 0$. Thus m^* is the unique maximizer of the retailer's profit function.

A.3 Proof of Proposition 1.3.5

Proof: For retailer profit, $\pi_R^{cs*} = \left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)} \pi^{cc*}$ is increasing in α while π_R^{ws*} is constant with respect to α. Therefore, to prove the ratio $\rho_R(\alpha) \equiv \frac{\pi_R^{cs*}}{\pi_R^{ws*}} > 1$ for any $0 < \alpha < 1$, we only need to prove that $\rho_R(\alpha = 0) > 1$. Note that

$$\rho_R(\alpha = 0) = \left(\frac{\beta}{\beta-1}\right)^{-(\beta-1)}\left(\frac{\beta+1}{\beta}\right)^{(\beta+1)}$$

$$= e^{ln\left[\left(\frac{\beta}{\beta-1}\right)^{-(\beta-1)}\left(\frac{\beta+1}{\beta}\right)^{(\beta+1)}\right]}$$
$$= e^{-(\beta-1)ln\frac{\beta}{\beta-1}+(\beta+1)ln\frac{\beta+1}{\beta}}.$$

The limit of $\rho_R(\alpha = 0)$ as $\beta \to \infty$ is:

$$\begin{aligned}\lim_{\beta\to\infty} \rho_R(\alpha = 0) &= \lim_{\beta\to\infty} \left[\left(\frac{\beta}{\beta-1}\right)^{-(\beta-1)} \left(\frac{\beta+1}{\beta}\right)^{(\beta+1)}\right] \\ &= \lim_{\beta\to\infty} \left[\left(\frac{\beta}{\beta-1}\right)^{-(\beta-1)} \left(\frac{\beta+1}{\beta}\right)^{\beta} \frac{\beta+1}{\beta}\right] \\ &= e^{-1} \cdot e^1 \cdot 1 = 1.\end{aligned}$$

If we can prove $\frac{\partial\rho_R(\alpha=0)}{\partial\beta} < 0$, then we can show $\rho_R(\alpha = 0) > 1$ for any $\beta < \infty$. The following is the proof of $\frac{\partial\rho_R(\alpha=0)}{\partial\beta} < 0$. Let $y(\beta) = -(\beta-1)ln\frac{\beta}{\beta-1} + (\beta+1)ln\frac{\beta+1}{\beta}$. Hence,

$$\begin{aligned}\frac{\partial\rho_R(\alpha = 0)}{\partial\beta} &= e^{y(\beta)} \left[-ln\frac{\beta}{\beta-1} - (\beta-1)\left(\frac{1}{\beta} - \frac{1}{\beta-1}\right)\right] + \\ &\quad e^{y(\beta)} \left[ln\frac{\beta+1}{\beta} + (\beta+1)\left(\frac{1}{\beta+1} - \frac{1}{\beta}\right)\right] \\ &= e^{y(\beta)} \left[-ln\frac{\beta}{\beta-1} + \frac{1}{\beta} + ln\frac{\beta+1}{\beta} - \frac{1}{\beta}\right] = e^{y(\beta)} ln\frac{\beta^2-1}{\beta^2} < 0.\end{aligned}$$

This concludes our proof $\rho_R(\alpha) \equiv \frac{\pi_R^{cs*}}{\pi_R^{ws*}} > 1$ for any $0 < \alpha < 1$.

The supplier profit ratio is $\rho_S(\alpha) \equiv \frac{\pi_S^{ws*}}{\pi_S^{cs*}}$, which can be expressed as:

$$\begin{aligned}\rho_S(\alpha) &= \frac{1}{(1-\alpha)} \left[\frac{(\beta-\alpha)\beta}{(\beta-1)(\beta+1)}\right]^{\beta} \\ &= \frac{(\beta-\alpha)^{\beta}}{(1-\alpha)} \left[\frac{\beta}{(\beta-1)(\beta+1)}\right]^{\beta}.\end{aligned}$$

In what follows, we prove $\rho_S(\alpha = 0) > 1$ and $\frac{\partial\rho_S(\alpha)}{\partial\alpha} > 0$ to conclude that $\rho_S(\alpha) > 1$ for any $0 < \alpha < 1$. First, $\rho_S(\alpha = 0) = \left[\frac{\beta^2}{(\beta+1)(\beta-1)}\right]^{\beta} > 1$. Then, we have:

$$\begin{aligned}\frac{\partial\rho_S(\alpha)}{\partial\alpha} &= \left[\frac{(\beta-\alpha)^{-\beta}}{(1-\alpha)^2} - \frac{\beta(\beta-\alpha)^{\beta-1}}{(1-\alpha)}\right] \left[\frac{\beta}{(\beta-1)(\beta+1)}\right]^{\beta} \\ &= \frac{\alpha(\beta-1)}{(1-\alpha)^2}(\beta-\alpha)^{\beta-1} \left[\frac{\beta}{(\beta-1)(\beta+1)}\right]^{\beta} > 0.\end{aligned}$$

Therefore, $\rho_S(\alpha) > 1$ for any $0 < \alpha < 1$. Hence we have the following relationship between the supplier and the retailer profits under CCA and WCA. $\pi_R^{cs*} > \pi_R^{ws*}$ and $\pi_S^{cs*} < \pi_S^{ws*}$.

For total channel profit, the ratio $\rho_C(\alpha) \equiv \frac{\pi^{cs*}}{\pi^{ws*}}$, which can be written as:

$$\begin{aligned}\rho_C(\alpha) &= \left[\frac{\beta-\alpha}{\beta-1}+(1-\alpha)\right]\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\left(\frac{\beta+1}{2\beta+1}\right)\left(\frac{\beta}{\beta+1}\right)^{-\beta}\\ &= \left[\frac{\beta-\alpha}{\beta-1}+(1-\alpha)\right](\beta-\alpha)^{-\beta}\left(\frac{\beta+1}{2\beta+1}\right)\left[\frac{\beta}{(\beta+1)(\beta-1)}\right]^{-\beta}.\end{aligned}$$

Now we prove $\rho_C(\alpha=0) > 1$ and $\frac{\partial \rho_C(\alpha)}{\partial \alpha} > 0$. First, we prove $\rho_C(\alpha=0) > 1$.

$$\rho_C(\alpha=0) = \frac{(\beta+1)(2\beta-1)}{(\beta-1)(2\beta+1)}\left(\frac{\beta}{\beta+1}\right)^{-\beta}\left(\frac{\beta}{\beta-1}\right)^{-\beta}.$$

The limit of $\rho_C(\alpha=0)$ as $\beta \to \infty$ is:

$$\begin{aligned}\lim_{\beta\to\infty}\rho_C(\alpha=0) &= \lim_{\beta\to\infty}\left[\left(\frac{\beta}{\beta-1}\right)^{-(\beta-1)}\left(\frac{\beta+1}{\beta}\right)^{\beta}\left(\frac{\beta+1}{\beta}\right)\left(\frac{2\beta-1}{2\beta+1}\right)\right]\\ &= e^{-1}\cdot e^{1}\cdot 1\cdot 1 = 1.\end{aligned}$$

We need to show $\frac{\partial \rho_C(\alpha)}{\partial \beta} < 0$ to prove $\rho_C(\alpha=0) > 1$. Let $x(\beta) = ln\left[\rho_C(\alpha=0)\right]$. Hence, $\rho_C(\alpha=0) = e^{x(\beta)}$, and $\frac{\partial \rho_C(\alpha)}{\partial \beta}$ is given by:

$$\frac{\partial \rho_C(\alpha=0)}{\partial \beta} = e^{x(\beta)}\frac{\partial x(\beta)}{\partial \beta} = e^{x(\beta)}\left[ln\frac{\beta^2-1}{\beta^2}+\frac{4}{4\beta^2-1}\right].$$

Let $z(\beta) = \left[ln\frac{\beta^2-1}{\beta^2}+\frac{4}{4\beta^2-1}\right]$. To show $\frac{\partial \rho_C(\alpha=0)}{\partial \beta} < 0$, we need to prove $z(\beta) < 0$. $\lim_{\beta\to\infty} z(\beta) = \lim_{\beta\to\infty}\left[ln\frac{\beta^2-1}{\beta^2}+\frac{4}{4\beta^2-1}\right] = 0$. Also,

$$\begin{aligned}\frac{\partial z(\beta)}{\partial \beta} &= \frac{2}{\beta(\beta^2-1)}-\frac{32\beta}{(4\beta^2-1)^2}\\ &= \frac{16\beta^2+1}{\beta(\beta^2-1)(4\beta^2-1)^2} > 0.\end{aligned}$$

Hence, we have $z(\beta) < 0$, which leads to $\frac{\partial \rho_C(\alpha=0)}{\partial \beta} < 0$. This concludes the proof of $\rho_C(\alpha = 0) > 1$. Next we prove $\frac{\partial \rho_C(\alpha)}{\partial \alpha} > 0$.

$$\begin{aligned}\frac{\partial \rho_C(\alpha)}{\partial \alpha} &= \left[1 + \frac{\alpha(1-\beta)}{\beta-\alpha}\right](\beta-\alpha)^{-\beta}\left(\frac{\beta+1}{2\beta+1}\right)\left[\frac{\beta}{(\beta+1)(\beta-1)}\right]^{-\beta} \\ &= \left[\frac{\beta(1-\alpha)}{\beta-\alpha}\right](\beta-\alpha)^{-\beta}\left(\frac{\beta+1}{2\beta+1}\right)\left[\frac{\beta}{(\beta+1)(\beta-1)}\right]^{-\beta} > 0.\end{aligned}$$

since $\frac{\partial \rho_C(\alpha)}{\partial \alpha} > 0$ and $\rho_C(\alpha = 0) > 1$. It follows that $\rho_C(\alpha) > 1$ for any $0 < \alpha < 1$. Therefore, $\pi^{cs*} > \pi^{ws*}$.

A.4 Proof of Proposition 1.3.8

Proof: $\rho_R = \frac{\pi_R^{cs*}}{\pi_R^{ws*}} = \frac{y_1(\beta)}{y_2(\beta)}$, where $y_1(\beta) = \left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)}$ and $y_2(\beta) = \left(\frac{\beta+1}{\beta}\right)^{-(\beta+1)}$. In order to prove $\frac{\partial \rho_R}{\partial \beta} < 0$, we only need to prove that $\frac{\partial y_1(\beta)}{\partial \beta} < 0$ and $\frac{\partial y_2(\beta)}{\partial \beta} > 0$. We first prove $\frac{\partial y_1(\beta)}{\partial \beta} < 0$.

$$y_1(\beta) = \left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)} = e^{-(\beta-1)ln\frac{\beta-\alpha}{\beta-1}}.$$

The derivative of $y_1(\beta)$ with respect to β is:

$$\frac{\partial y_1(\beta)}{\beta} = e^{-(\beta-1)ln\frac{\beta-\alpha}{\beta-1}}\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right].$$

If we can prove that $\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right] < 0$, then we have $\frac{\partial y_1(\beta)}{\beta} < 0$. We now prove $\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right] < 0$. $\lim_{\alpha\to 1}\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right] = 0$, and the derivative of $\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right]$ with respect to α is:

$$\frac{\partial\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right]}{\partial \alpha} = \frac{1}{\beta-\alpha} - \frac{\beta-1}{(\beta-\alpha)^2} = \frac{1-\alpha}{(\beta-\alpha)^2} > 0.$$

Hence, $\left[-ln\frac{\beta-\alpha}{\beta-1} + \frac{1-\alpha}{\beta-\alpha}\right] < 0$ for $0 < \alpha < 1$.

Second, we prove $\frac{\partial y_2(\beta)}{\partial \beta} > 0$.

$$y_2(\beta) = \left(\frac{\beta+1}{\beta}\right)^{-(\beta+1)} = e^{-(\beta+1)ln\frac{\beta+1}{\beta}}.$$

The derivative of $y_2(\beta)$ with respect to β is:

$$\frac{\partial y_2(\beta)}{\beta} = e^{-(\beta+1)ln\frac{\beta+1}{\beta}}\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right].$$

If we can prove that $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right] > 0$, then we have $\frac{\partial y_2(\beta)}{\beta} > 0$. We now prove $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right] > 0$. $\lim_{\beta\to\infty}\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right] = 0$, and the derivative of $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right]$ with respect to β is:

$$\frac{\partial\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right]}{\partial\beta} = -\frac{1}{\beta+1}+\frac{1}{\beta}-\frac{1}{\beta^2}=\frac{1}{\beta(\beta+1)}-\frac{1}{\beta^2}<0.$$

Therefore, $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta}\right] > 0$ for $\beta < \infty$.

This concludes the proof of $\frac{\partial\rho_R}{\partial\beta} < 0$, because $\frac{\partial y_1(\beta)}{\beta} < 0$ and $\frac{\partial y_2(\beta)}{\beta} > 0$.

We now prove $\frac{\partial\rho_S}{\partial\beta} < 0$. $\rho_S = \frac{\pi_S^{ws*}}{\pi_S^{cs*}} = \frac{x_1(\beta)}{x_2(\beta)}$, where $x_1(\beta) = \left(\frac{\beta+1}{\beta}\right)^{-\beta}$ and $x_2(\beta) = (1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}$. In order to prove $\frac{\partial\rho_S}{\partial\beta} < 0$, we only need to prove that $\frac{\partial x_1(\beta)}{\partial\beta} < 0$ and $\frac{\partial x_2(\beta)}{\partial\beta} > 0$. We first prove $\frac{\partial x_1(\beta)}{\partial\beta} < 0$.

$$x_1(\beta) = \left(\frac{\beta+1}{\beta}\right)^{-\beta} = e^{-\beta ln\left(\frac{\beta+1}{\beta}\right)}.$$

The derivative of $x_1(\beta)$ with respect to β is:

$$\frac{\partial x_1(\beta)}{\beta} = e^{-\beta ln\left(\frac{\beta+1}{\beta}\right)}\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right].$$

If we can prove that $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right] < 0$, then we have $\frac{\partial x_1(\beta)}{\beta} < 0$. We now prove $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right] < 0$. $\lim_{\beta\to\infty}\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right] = 0$, and the derivative of $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right]$ with respect to β is:

$$\frac{\partial\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right]}{\partial\beta} = \frac{\beta}{\beta+1}\frac{1}{\beta^2}-\frac{1}{(\beta+1)^2}=\frac{1}{\beta(\beta+1)}-\frac{1}{(\beta+1)^2}>0.$$

Therefore, $\left[-ln\frac{\beta+1}{\beta}+\frac{1}{\beta+1}\right]<0$ for $\beta<\infty$.

Second, we prove $\frac{\partial x_2(\beta)}{\partial\beta}<0$.

$$x_2(\beta) = (1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta} = e^{\left[ln(1-\alpha)-\beta ln\left(\frac{\beta-\alpha}{\beta-1}\right)\right]}.$$

The derivative of $x_2(\beta)$ with respect to β is:

$$\frac{\partial x_2(\beta)}{\beta} = e^{\left[ln(1-\alpha)-\beta ln\left(\frac{\beta-\alpha}{\beta-1}\right)\right]}\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right].$$

If we prove $\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]>0$, then we know $\frac{\partial x_2(\beta)}{\beta}>0$.

We now prove $\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]>0$. $\lim_{\alpha\to 1}\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]$ $=0$, and the derivative of $\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]$ with respect to α is:

$$\frac{\partial\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]}{\partial\alpha} = \frac{1}{\beta-\alpha}-\frac{\beta}{(\beta-\alpha)^2}=\frac{-\alpha}{(\beta-\alpha)^2}<0.$$

Hence, $\left[-ln\left(\frac{\beta-\alpha}{\beta-1}\right)+\frac{\beta(1-\alpha)}{(\beta-\alpha)(\beta-1)}\right]>0$ for $0<\alpha<1$.

We now conclude the proof of $\frac{\partial\rho_S}{\partial\beta}<0$, because $\frac{\partial x_1(\beta)}{\beta}<0$ and $\frac{\partial x_2(\beta)}{\beta}>0$.

A.5 Proof of Proposition 1.3.9

Proof: $\rho_S=\frac{\pi_S^{ws*}}{\pi_S^{cs*}}=\frac{x_1(\beta)}{x_2(\beta)}$, where $x_1(\beta)=\left(\frac{\beta+1}{\beta}\right)^{-\beta}$ and $x_2(\beta)=(1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}$.

The limit of $x_1(\beta)$ as $\beta\to\infty$ is:

$$\lim_{\beta\to\infty}x_1(\beta) = \lim_{\beta\to\infty}\left(\frac{\beta+1}{\beta}\right)^{-\beta}=\lim_{\beta\to\infty}\left(1+\frac{1}{\beta}\right)^{-\beta}=e^{-1}.$$

The limit of $x_2(\beta)$ as $\beta\to\infty$ is:

$$\begin{aligned}\lim_{\beta\to\infty}x_2(\beta) &= \lim_{\beta\to\infty}(1-\alpha)\left(\frac{\beta-\alpha}{\beta-1}\right)^{-\beta}\\ &= \lim_{\beta\to\infty}(1-\alpha)\left[\left(1+\frac{1-\alpha}{\beta-1}\right)^{\frac{\beta-1}{1-\alpha}}\right]^{\frac{-\beta(1-\alpha)}{\beta-1}}=(1-\alpha)e^{-(1-\alpha)}.\end{aligned}$$

Hence, The limit of ρ_S as $\beta \to \infty$ is:

$$\lim_{\beta\to\infty} \rho_S = \frac{\lim_{\beta\to\infty} x_1(\beta)}{\lim_{\beta\to\infty} x_2(\beta)} = \frac{e^{-1}}{(1-\alpha)e^{-(1-\alpha)}} = \frac{e^{-\alpha}}{(1-\alpha)}.$$

Because $\frac{\partial \rho_S}{\partial \beta} < 0$, ρ_S is thus no less than $\frac{e^{-\alpha}}{(1-\alpha)}$.

$\rho_R = \frac{\pi_R^{cs*}}{\pi_R^{ws*}} = \frac{y_1(\beta)}{y_2(\beta)}$, where $y_1(\beta) = \left(\frac{\beta-\alpha}{\beta-1}\right)^{-(\beta-1)}$ and $y_2(\beta) = \left(\frac{\beta+1}{\beta}\right)^{-(\beta+1)}$. Similarly, we derive $\lim_{\beta\to\infty} y_1(\beta) = e^{-(1-\alpha)}$ and $\lim_{\beta\to\infty} y_2(\beta) = e^{-1}$. Hence, $\lim_{\beta\to\infty} \rho_R = \frac{\lim_{\beta\to\infty} y_1(\beta)}{\lim_{\beta\to\infty} y_2(\beta)} = e^{\alpha}$. When $\beta \to 1^+$, $\lim_{\beta\to 1^+} y_1(\beta) = 1$ and $\lim_{\beta\to 1^+} y_2(\beta) = \frac{1}{4}$. Hence, $\lim_{\beta\to 1^+} \rho_R = \frac{\lim_{\beta\to\infty} y_1(\beta)}{\lim_{\beta\to\infty} y_2(\beta)} = 4$. Because $\frac{\partial \rho_R}{\partial \beta} < 0$, this completes the proof that ρ_R is within $[e^{\alpha}, 4]$.

A.6 Proof of Proposition 1.3.12

Proof:

$$\rho_R = \left(\frac{\beta-\alpha}{\beta}\right)^{1-\beta} = e^{(1-\beta)ln\frac{\beta-\alpha}{\beta}}.$$

The derivative of ρ_R with respect to β is:

$$\frac{\partial \rho_R}{\beta} = e^{(1-\beta)ln\frac{\beta-\alpha}{\beta}} \left[-ln\frac{\beta-\alpha}{\beta} + (1-\beta)\left(\frac{1}{\beta-\alpha} - \frac{1}{\beta}\right)\right].$$

$\frac{\partial \rho_R}{\beta}(\alpha = 0) = 0$. If we can prove that $\frac{\partial\left[-ln\frac{\beta-\alpha}{\beta} + (1-\beta)\left(\frac{1}{\beta-\alpha} - \frac{1}{\beta}\right)\right]}{\partial \alpha} > 0$ for any $0 < \alpha < 1$, then we can conclude $\frac{\partial \rho_R}{\beta} > 0$.

$$\frac{\partial\left[-ln\frac{\beta-\alpha}{\beta} + (1-\beta)\left(\frac{1}{\beta-\alpha} - \frac{1}{\beta}\right)\right]}{\partial \alpha} = \frac{1}{\beta-\alpha} + \frac{1-\beta}{(\beta-\alpha)^2} = \frac{1-\alpha}{(\beta-\alpha)^2} > 0.$$

$$\rho_S = \frac{1}{1-\alpha}\left(\frac{\beta}{\beta-\alpha}\right)^{-\beta} = e^{-ln(1-\alpha)-\beta ln\frac{\beta}{\beta-\alpha}}.$$

The derivative of ρ_S with respect to β is:

$$\frac{\partial \rho_S}{\beta} = ee^{-ln(1-\alpha)-\beta ln\frac{\beta}{\beta-\alpha}} \left[-ln\frac{\beta}{\beta-\alpha} - \beta\left(\frac{1}{\beta} - \frac{1}{\beta-\alpha}\right)\right].$$

$\frac{\partial \rho_S}{\beta}(\alpha = 0) = 0$. If we can prove that $\frac{\partial\left[-ln\frac{\beta}{\beta-\alpha}-\beta\left(\frac{1}{\beta}-\frac{1}{\beta-\alpha}\right)\right]}{\partial\alpha} > 0$ for any $0 < \alpha < 1$, then we can conclude $\frac{\partial \rho_S}{\beta} > 0$.

$$\frac{\partial\left[-ln\frac{\beta}{\beta-\alpha}-\beta\left(\frac{1}{\beta}-\frac{1}{\beta-\alpha}\right)\right]}{\partial\alpha} = -\frac{1}{\beta-\alpha}+\frac{\beta}{(\beta-\alpha)^2} = \frac{\alpha}{(\beta-\alpha)^2} > 0.$$

This completes the proof.

A.7 Proof of Proposition 1.4.3

Proof: The derivative of π_S^{ws} with respect to w is:

$$\begin{aligned}
\frac{\partial \pi_S^{ws}}{\partial w} &= \left[-\beta - \frac{Mm}{(w+\alpha c+m)^2}\right][w-(1-\alpha)c] + \\
&\quad \left[a - \beta(w+\alpha c+m) + \frac{Mm}{(w+\alpha c+m)}\right] \\
&= -2\beta(w+\alpha c+m) + K_3(w+\alpha c+m)^{-2} + K_4,
\end{aligned}$$

where $K_3 = Mm\,(m+c)$ and $K_4 = a + \beta(m+c)$. The second derivative of π_S^{ws} with respect to w is:

$$\frac{\partial^2 \pi_S^{ws}}{\partial w^2} = -2\beta - 2K_3(w+\alpha c+m)^{-3}.$$

Since K_3 is a constant greater than zero, $\frac{\partial^2 \pi_S^{ws}}{\partial w^2} < 0$. Therefore, the supplier's profit function is a concave function of w. Furthermore,

$$\begin{aligned}
\frac{\partial \pi_S^{ws}}{\partial w} &= -2\beta(w+\alpha c+m) + K_3(w+\alpha c+m)^{-2} + K_4 \\
&= a - \beta(w+\alpha c+m) + \frac{Mm}{(w+\alpha c+m)}\frac{(m+c)}{(w+\alpha c+m)} - \beta[w-(1-\alpha)c] \\
&< a - \beta(w+\alpha c+m) + \frac{Mm}{(w+\alpha c+m)}
\end{aligned}$$

when $w = \frac{a+\sqrt{a^2+4M\beta m}}{2\beta} - m - \alpha c$, the retailer's order quantity $q = a - \beta(w + \alpha c + m) + \frac{Mm}{(w+\alpha c+m)} = 0$. Since q is a decreasing function of wholesale price w, $q =$

$a - \beta(m+c) + \frac{Mm}{(m+c)} > 0$ at $w = (1-\alpha)c$. Therefore, we have $\frac{\partial \pi_S^{ws}}{\partial w} < 0$ at $w = \frac{a+\sqrt{a^2+4M\beta m}}{2\beta} - m - \alpha c$. When $w = (1-\alpha)c$, $\frac{\partial \pi_S^{ws}}{\partial w} = a - \beta(m+c) + \frac{Mm}{(m+c)} > 0$. Hence, we know there exists $(1-\alpha)c < w^* < \frac{a+\sqrt{a^2+4M\beta m}}{2\beta} - m - \alpha c$ such that $\frac{\partial \pi_S^{ws}}{\partial w} = 0$. Due to the concavity of π_S^{ws}, w^* uniquely maximizes the supplier's profit.

A.8 Proof of Proposition 1.4.4

Proof: The derivative of π_R^{ws} with respect to m is:

$$\begin{aligned}
\frac{\partial \pi_R^{ws}}{\partial m} &= a - \beta\left(\frac{\partial w^*}{\partial m} + 1\right)m - \beta(w^* + \alpha c + m) \\
&\quad - \frac{Mm^2}{2(w^* + \alpha c + m)^2}\left(\frac{\partial w^*}{\partial m} + 1\right) + \frac{Mm}{(w^* + \alpha c + m)} \\
&= a - \left(\frac{\partial w^*}{\partial m} + 1\right)\left[\beta m + \frac{Mm^2}{2(w^* + \alpha c + m)^2}\right] + \frac{Mm}{(w^* + \alpha c + m)} \\
&\quad - \beta(w^* + \alpha c + m).
\end{aligned}$$

By implicit differentiation, $\frac{\partial w^*}{\partial m}$ is given by

$$\frac{\partial w^*}{\partial m} = \frac{-\partial^2 \pi_S^{ws}(w^*, m)}{\partial w \partial m} \Big/ \frac{\partial^2 \pi_S^{ws}(w^*, m)}{\partial w^2},$$

where $\frac{\partial^2 \pi_S^{ws}(w^*, m)}{\partial w^2}$ is given in the proof of Proposition 1.4.3. $\frac{\partial^2 \pi_S^{ws}(w^*, m)}{\partial w \partial m}$ can be derived as follows.

$$\frac{\partial^2 \pi_S^{ws}(w^*, m)}{\partial w \partial m} = -2\beta - 2K_3(w^* + \alpha c + m)^{-3} + (w^* + \alpha c + m)^{-2} M(2m + c) + \beta.$$

Therefore, we can derive $\frac{\partial w^*}{\partial m}$ as follows.

$$\begin{aligned}
\frac{\partial w^*}{\partial m} &= \frac{-\partial^2 \pi_S^{ws}(w^*, m)}{\partial w \partial m} \Big/ \frac{\partial^2 \pi_S^{ws}(w^*, m)}{\partial w^2} \\
&= \frac{2\beta + 2K_3(w^* + \alpha c + m)^{-3} - (w^* + \alpha c + m)^{-2} M(2m + c) - \beta}{-2\beta - 2K_3(w^* + \alpha c + m)^{-3}} \\
&= -1 + \frac{\beta + M(2m + c)(w^* + \alpha c + m)^{-2}}{2\beta + Mm(m + c)(w^* + \alpha c + m)^{-3}}.
\end{aligned}$$

Substituting $\frac{\partial w^*}{\partial m}$ into $\frac{\partial \pi_R^{ws}}{\partial m}$, we obtain

$$\frac{\partial \pi_R^{ws}}{\partial m} = a - \frac{(\beta p^2 + 2Myp + Mc)(\beta p + \frac{My}{2})y}{2\beta p^2 + My^2 p + Myc} + My - \beta p,$$

where $y = \frac{m}{p}$ and $p = w^* + \alpha c + m$. When $m = 0$, i.e. $y = 0$, $\frac{\partial \pi_R^{ws}}{\partial m} = a - \beta(w^* + \alpha c) > 0$. When $m = \frac{4a+3M}{6\beta}$, $\frac{\partial \pi_R^{ws}}{\partial m} < 0$, because $y < 1$ and $\frac{(\beta p^2+2Myp+Mc)}{2\beta p^2+My^2p+Myc} > \frac{(\beta p^2+2My^2p+Myc)}{2\beta p^2+My^2p+Myc} > \frac{1}{2}$. Hence, we have

$$\begin{aligned}
\frac{\partial \pi_R^{ws}}{\partial m} &< a - \frac{(\beta p + \frac{My}{2})y}{2} + My - \beta p \\
&= a - \frac{\beta py}{2} - \beta p - \frac{My^2}{4} + My \\
&< a - \frac{\beta py}{2} - \beta p + \frac{3M}{4} \\
&< a - \frac{3\beta py}{2} + \frac{3M}{4}.
\end{aligned}$$

$-\frac{My^2}{4} + My$ is a concave function of y and reaches maximum at $y^* = 2$. Because $y < 1$, the maximum value that $-\frac{My^2}{4} + My$ can reach is $\frac{3M}{4}$. $py = m$. Set $a - \frac{3\beta py}{2} + \frac{3M}{4} = a - \frac{3\beta m}{2} + \frac{3M}{4} = 0$ leads to $m = \frac{4a+3M}{6\beta}$. Hence, we know there exists at least one $0 < m^* < \frac{4a+3M}{6\beta}$ that maximizes the retailer's profit.

APPENDIX B
PROOFS OF CHAPTER TWO

B.1 Lemma 2.4.2

Proof of Lemma 2.4.2:

Since $\frac{d[x-\Lambda(x)]}{dx} = 1 - F(x) \geq 0$, we have $\Delta \leq 0$ when $z \leq z_T$, and $\Delta \geq 0$ when $z > z_T$. When $z \leq z_T$, $\pi_S^{bs} = \pi_S^{cs}$, and $\pi_S^{br} = \pi_S^{cs} + \Delta \leq \pi_S^{cs}$. Therefore, $\pi_S^{br} \leq \pi_S^{bs} = \pi_S^{cs}$. When $z > z_T$, $\pi_S^{br} = \pi_S^{bs}$, and $\pi_S^{br} = \pi_S^{cs} + \Delta \geq \pi_S^{cs}$. Therefore, $\pi_S^{br} = \pi_S^{bs} \geq \pi_S^{cs}$.

B.2 Proposition 2.4.1

Proof of Proposition 2.4.1:

First, for given p, we show below that (2.15) is strictly concave in z.

$$\begin{aligned}\frac{\partial \pi_S^{br}}{\partial z} &= d(p)\left\{(1-r)p\left[1-F(z)\right] - (1-\alpha)c + (r-r_0)p\left[1-F(z)\right]\right\} \\ &= d(p)\left\{(1-r_0)p\left[1-F(z)\right] - (1-\alpha)c\right\}.\end{aligned}$$

Setting $\frac{\partial \pi_S^{br}}{\partial z} = 0$ leads to:

$$F(\hat{z}) = \frac{(1-r_0)p - (1-\alpha)c}{(1-r_0)p}.$$

Furthermore,

$$\frac{\partial^2 \pi_S^{br}}{\partial z^2} = -d(p)(1-r_0)pf(z) < 0.$$

Thus given p,

$$\hat{z} = F^{-1}\left[\frac{(1-r_0)p - (1-\alpha)c}{(1-r_0)p}\right], \tag{B.1}$$

is the unique maximizer to (2.15). Substituting $\hat{z}$ into (2.15), we have:

$$\begin{aligned}
\pi_S^{br}(\hat{z}) &= d(p)\left\{(1-r)p\left[\hat{z}-\Lambda(\hat{z})\right]-(1-\alpha)c\hat{z}+(r-r_0)p\left[\hat{z}-\Lambda(\hat{z})-(z_T-\Lambda(z_T))\right]\right\} \\
&= d(p)\left\{(1-r_0)p\left[\hat{z}-\Lambda(\hat{z})\right]-(1-\alpha)c\hat{z}\right\}-d(p)\left\{(r-r_0)p\left[z_T-\Lambda(z_T)\right]\right\} \\
&= h(p)-h_T(p),
\end{aligned}$$

where $h(p)=d(p)\left\{(1-r_0)p\left[\hat{z}-\Lambda(\hat{z})\right]-(1-\alpha)c\hat{z}\right\}$ and $h_T(p)=d(p)(r-r_0)p\left[z_T-\Lambda(z_T)\right]$.

Taking the derivative of $h(p)$ w.r.t. p, we have:

$$\begin{aligned}
\frac{dh(p)}{dp} &= \frac{\partial h(p)}{\partial p}+\frac{\partial h(p)}{\partial z}|_{z=\hat{z}}\frac{\partial \hat{z}}{\partial p} \\
&= \frac{\partial h(p)}{\partial p}+0\cdot\frac{\partial \hat{z}}{\partial p} \\
&= a(-\beta)p^{-\beta-1}\left\{(1-r_0)p\left[\hat{z}-\Lambda(\hat{z})\right]-(1-\alpha)c\hat{z}\right\}+ap^{-\beta}\left\{(1-r_0)\left[\hat{z}-\Lambda(\hat{z})\right]\right\} \\
&= (1-r_0)ap^{-\beta}\left\{(1-\beta)\left[\hat{z}-\Lambda(\hat{z})\right]+\frac{\beta(1-\alpha)c}{(1-r_0)p}\hat{z}\right\} \\
&= (1-r_0)d(p)\left\{(1-\beta)\left[\hat{z}-\Lambda(\hat{z})\right]+\beta\hat{z}\left[1-F(\hat{z})\right]\right\}.
\end{aligned}$$

Similarly, we have:

$$\begin{aligned}
\frac{dh_T(p)}{dp} &= \frac{\partial h_T(p)}{\partial p}+\frac{\partial h_T(p)}{\partial z_T}\frac{\partial z_T(p)}{\partial p} \\
&= (r-r_0)a(1-\beta)p^{-\beta}\left[z_T-\Lambda(z_T)\right]+(r-r_0)ap^{1-\beta}\left[1-F(z_T)\right]\frac{T}{a}(\beta-1)p^{\beta-2} \\
&= (r-r_0)ap^{-\beta}(\beta-1)\left\{-\left[z_T-\Lambda(z_T)\right]+z_T\left[1-F(z_T)\right]\right\} \\
&= -(r-r_0)d(p)\left[(\beta-1)\int_A^{z_T}\xi f(\xi)d\xi\right].
\end{aligned}$$

Let $G(x)=(1-\beta)[x-\Lambda(x)]+\beta x[1-F(x)]$ and $H(x)=(\beta-1)\int_A^x\xi f(\xi)d\xi$. Setting $\frac{\partial \pi_S^{br}(\hat{z})}{\partial p}|_{p=\hat{p}}=0$, we have:

$$\begin{aligned}
\frac{\partial \pi_S^{br}(\hat{z})}{\partial p}|_{p=\hat{p}} &= \frac{\partial h(\hat{p})}{\partial p}|_{p=\hat{p}}-\frac{\partial h_T(p)}{\partial p}|_{p=\hat{p}} \\
&= d(\hat{p})[(1-r_0)G(\hat{z})+(r-r_0)H(\hat{z_T})]=0. \qquad \text{(B.2)}
\end{aligned}$$

Thus if $(\hat{p}, \hat{z})$ is an optimal solution, it must satisfy (B.1) and (B.2). This concludes the proof.

B.3 Lemma 2.4.3

Proof of Lemma 2.4.3:

Any $(\hat{p}, \hat{z}) \in \Omega$ must satisfy the necessary condition in Proposition 2.4.1, i.e., $(1-r_0)G(\hat{z})+(r-r_0)H(\hat{z_T}) = 0$. Since $H(\hat{z_T}) = (\beta-1)\int_A^{\hat{z_T}} \xi f(\xi)d\xi > 0$, it follows that $G(\hat{z}) < 0$. As shown in Wang (2006), when $z = z^{cc*}$, $G(z) = 0$. Also, $G(z) > 0$ when $z < z^{cc*}$ and $G(z) < 0$ when $z > z^{cc*}$. Thus, $\hat{z} > z^{cc*}$. Both z^{cc*} and z^{cs*} are uniquely determined by (2.5) giving $z^{cc*} = z^{cs*}$, and so $\hat{z} > z^{cs*}$. This completes the proof of $(\hat{p}, \hat{z}) \neq (p^{cs*}, z^{cs*})$.

B.4 Proposition 2.4.2

Proof of Proposition 2.4.2:

We first prove $(p^{bs*}, z^{bs*}) \in (p^{cs*}, z^{cs*}) \cup \Omega$. Assume $(p^{bs*}, z^{bs*}) \neq (p^{cs*}, z^{cs*})$ and $(p^{bs*}, z^{bs*}) \notin \Omega$. Denote $z_T^{bs*} = \frac{T}{d(p^{bs*})p^{bs*}}$. We now have two cases:

(1)$z^{bs*} \leq z_T^{bs*}$

By Lemma 2.4.2, $\pi_S^{bs}(p^{cs*}, z^{cs*}) \geq \pi_S^{cs}(p^{cs*}, z^{cs*})$ and $\pi_S^{cs}(p^{bs*}, z^{bs*}) = \pi_S^{bs}(p^{bs*}, z^{bs*})$. Since (p^{cs*}, z^{cs*}) is the unique maximizer of π_S^{cs} and $(p^{bs*}, z^{bs*}) \neq (p^{cs*}, z^{cs*})$, we have $\pi_S^{cs}(p^{cs*}, z^{cs*}) > \pi_S^{cs}(p^{bs*}, z^{bs*})$. It now follows that $\pi_S^{bs}(p^{cs*}, z^{cs*}) \geq \pi_S^{cs}(p^{cs*}, z^{cs*}) > \pi_S^{cs}(p^{bs*}, z^{bs*}) = \pi_S^{bs}(p^{bs*}, z^{bs*})$, which contradicts that (p^{bs*}, z^{bs*}) is optimal to π_S^{bs}.

(2) $z^{bs*} > z_T^{bs*}$

For any $(\hat{p}, \hat{z}) \in \Omega$, by Lemma 2.4.2, we have $\pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{br}(\hat{p}, \hat{z})$ and $\pi_S^{br}(p^{bs*}, z^{bs*}) =$

$\pi_S^{bs}(p^{bs*}, z^{bs*})$. Since $(p^{bs*}, z^{bs*}) \notin \Omega$, we have $\pi_S^{br}(\hat{p}, \hat{z}) > \pi_S^{br}(p^{bs*}, z^{bs*})$. Thus, $\pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{br}(\hat{p}, \hat{z}) > \pi_S^{br}(p^{bs*}, z^{bs*}) = \pi_S^{bs}(p^{bs*}, z^{bs*})$, which again contradicts that (p^{bs*}, z^{bs*}) is optimal to π_S^{bs}.

We now prove, if there exists $(\hat{p}, \hat{z}) \in \Omega$ where $\hat{z} \leq \hat{z_T}$, then (p^{cs*}, z^{cs*}) is the unique maximizer of π_S^{bs}. By Lemma 2.4.2, $\pi_S^{bs}(p^{cs*}, z^{cs*}) \geq \pi_S^{cs}(p^{cs*}, z^{cs*})$ always holds, and $\pi_S^{cs}(\hat{p}, \hat{z}) = \pi_S^{bs}(\hat{p}, \hat{z})$ when $\hat{z} \leq \hat{z_T}$. Since (p^{cs*}, z^{cs*}) is the unique maximizer of π_S^{cs}, and from Lemma 2.4.3 $(p^{cs*}, z^{cs*}) \neq (\hat{p}, \hat{z})$, we have $\pi_S^{cs}(p^{cs*}, z^{cs*}) > \pi_S^{cs}(\hat{p}, \hat{z})$. Thus,

$$\pi_S^{bs}(p^{cs*}, z^{cs*}) \geq \pi_S^{cs}(p^{cs*}, z^{cs*}) > \pi_S^{cs}(\hat{p}, \hat{z}) = \pi_S^{bs}(\hat{p}, \hat{z}). \tag{B.3}$$

Denote $\bar{z_T} = \frac{T}{d(\bar{p})\bar{p}}$. If there also exists $(\bar{p}, \bar{z}) \in \Omega$ where $\bar{z} > \bar{z_T}$, we will show $\pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{bs}(\bar{p}, \bar{z})$. Since $(\hat{p}, \hat{z}), (\bar{p}, \bar{z}) \in \Omega$, we have:

$$\pi_S^{br}(\hat{p}, \hat{z}) = \pi_S^{br}(\bar{p}, \bar{z}). \tag{B.4}$$

By Lemma 2.4.2, when $\bar{z} > \bar{z_T}$, we have:

$$\pi_S^{br}(\bar{p}, \bar{z}) = \pi_S^{bs}(\bar{p}, \bar{z}), \tag{B.5}$$

and with $\hat{z} \leq \hat{z_T}$, we have:

$$\pi_S^{br}(\hat{p}, \hat{z}) \leq \pi_S^{bs}(\hat{p}, \hat{z}). \tag{B.6}$$

(B.3) through (B.6) give

$$\pi_S^{bs}(p^{cs*}, z^{cs*}) > \pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{bs}(\bar{p}, \bar{z}).$$

Thus (p^{cs*}, z^{cs*}) is the unique maximzer of π_S^{bs}.

We now prove, when $z^{cs*} \geq z_T^{cs*}$, (p^{cs*}, z^{cs*}) is not optimal to π_S^{bs}. By Lemma 2.4.2, for any $(\hat{p}, \hat{z}) \in \Omega$, $\pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{br}(\hat{p}, \hat{z})$ always holds, and $\pi_S^{br}(p^{cs*}, z^{cs*}) = \pi_S^{bs}(p^{cs*}, z^{cs*})$

when $z^{cs*} \geq z_T^{cs*}$. By Lemma 2.4.3, $(p^{cs*}, z^{cs*}) \neq (\hat{p}, \hat{z})$. Thus, $\pi_S^{br}(\hat{p}, \hat{z}) > \pi_S^{br}(p^{cs*}, z^{cs*})$. It now follows that $\pi_S^{bs}(\hat{p}, \hat{z}) \geq \pi_S^{br}(\hat{p}, \hat{z}) > \pi_S^{br}(p^{cs*}, z^{cs*}) = \pi_S^{bs}(p^{cs*}, z^{cs*})$.

B.5 Proposition 2.4.3

Proof of Proposition 2.4.3:

From the proof of Lemma 2.4.3, we have $\hat{z} > z^{cc*}$. Thus, $F(\hat{z}) \geq F(z^{cc*})$. By Proposition 2.4.1, $F(\hat{z}) = \frac{(1-r_0)\hat{p}-(1-\alpha)c}{(1-r_0)\hat{p}}$. Similarly, under the decentralized channel where $r_0 = \alpha$, the supplier makes price and stocking factor decisions as in the centralized channel. We now have $F(z^{cc*}) = \frac{p^{cc*}-c}{p^{cc*}}$. Thus $\frac{(1-r_0)\hat{p}-(1-\alpha)c}{(1-r_0)\hat{p}} \geq \frac{p^{cc*}-c}{p^{cc*}}$ giving $\hat{p} \geq \frac{(1-\alpha)}{(1-r_0)}p^{cc*}$. Also, according to Proposition 2.4.2, when maximizing π_S^{bs}, we consider the solutions in Ω only if $\hat{z} > \hat{z_T}, \forall(\hat{p}, \hat{z}) \in \Omega$. Thus any $(\hat{p}, \hat{z}) \in \Omega$ must satisfy $a\hat{p}^{-\beta}B\hat{p} \geq T$ where $a\hat{p}^{-\beta}B\hat{p}$ is the maximum sales that the supplier can obtain for any $(\hat{p}, \hat{z}) \in \Omega$. This gives $\hat{p} \leq \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$.

B.6 Proposition 2.4.4

Proof of Proposition 2.4.4:

When demand is zero, both centralized channel profit and decentralized channel profit are zero. Thus channel is coordinated. When demand is positive, based on Proposition 2.4.2, $(p^{bs*}, z^{bs*}) \in (p^{cs*}, z^{cs*}) \cup \Omega$. We have two cases to consider.

(1) $(p^{bs*}, z^{bs*}) = (p^{cs*}, z^{cs*})$. By Proposition 2.3.1, we know that the channel is not coordinated unless the retailer sets $r = \alpha$.

(2) $(p^{bs*}, z^{bs*}) \in \Omega$. Assume that a contract (r, r_0, T) coordinates the channel, i.e., $p^{bs*} = p^{cc*}$ and $z^{bs*} = z^{cc*}$. By (2.5), $G(z^{bs*}) = G(z^{cc*}) = 0$. Furthermore, $H(z_T^{bs*}) = (\beta - 1)\int_A^{z_T^{bs*}} \xi f(\xi) d\xi > 0$. It now follows that $d(p^{bs*})[(1-r_0)G(z^{bs*}) + (r - r_0)H(z_T^{bs*})] > 0$,

which violates the first necessary condition in Proposition 2.4.1.

B.7 Proposition 2.4.5

Proof of Proposition 2.4.5:

When $\varepsilon \sim U[0,B]$, $p^{cc*} = \frac{(\beta+1)c}{(\beta-1)}$ and $z^{cc*} = \frac{2B}{(\beta+1)}$. For given p, $z_T = \frac{T}{d(p)p} = \frac{T}{a}p^{\beta-1}$. When $p \le \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$, we have $z_T \le \frac{T}{a}\cdot\frac{aB}{T} = B$, and

$$H(z_T) \;=\; (\beta-1)\int_A^{z_T}\xi f(\xi)d\xi = (\beta-1)\int_0^{z_T}\xi\frac{1}{B}d\xi = \frac{(\beta-1)z_T^2}{2B}.$$

Also, for given p, $F(z^*) = \frac{(1-r_0)p-(1-\alpha)c}{(1-r_0)p}$. When $0 \le z^* \le B$, $F(z^*) = \frac{z^*}{B}$. Thus, $z^* = B\left[1-\frac{(1-\alpha)c}{(1-r_0)p}\right]$. $0 \le z^* \le B$ now requires $p \ge \frac{(1-\alpha)c}{(1-r_0)}$. When $p \ge \frac{(1-\alpha)c}{(1-r_0)}$, we have

$$\begin{aligned} G(z^*) &= (1-\beta)[z^*-\Lambda(z^*)]+\beta z^*[1-F(z^*)] \\ &= (1-\beta)\left[z^*-\frac{(z^*)^2}{2B}\right]+\beta z^*\left(1-\frac{z^*}{B}\right) = z^*-\frac{(z^*)^2(\beta+1)}{2B}. \end{aligned}$$

Let $M = \frac{2(1-\alpha)c}{(1-r_0)}$. $p_l = \frac{M(\beta+1)}{2(\beta-1)} = \frac{(1-\alpha)c(\beta+1)}{(1-r_0)(\beta-1)} > \frac{(1-\alpha)c}{(1-r_0)}$ and $p_u = min\left\{\left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}, p_0\right\} \le \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$. By assumption, $p_u > p_l$. Thus, when $p \in [p_l, p_u]$, we can rewrite $\Theta(p)$ as

$$\begin{aligned} \Theta(p) &= (1-r_0)G(z^*)+(r-r_0)H(z_T) \\ &= (1-r_0)\left[z^*-\frac{(z^*)^2(\beta+1)}{2B}\right]+(r-r_0)(\beta-1)\frac{{z_T}^2}{2B} \\ &= \frac{(1-r_0)B}{2}\left[-\frac{M^2(1+\beta)}{4p^2}+\frac{M\beta}{p}+(1-\beta)\right]+(r-r_0)(\beta-1)\frac{T^2}{2Ba^2}p^{2(\beta-1)}. \end{aligned}$$

Taking the derivative of π_S^{br} w.r.t. p, we have:

$$\frac{\partial\pi_S^{br}}{\partial p} \;=\; d(p)[(1-r_0)G(z^*)+(r-r_0)H(z_T)] = d(p)\Theta(p).$$

If we have $\Theta(p_l) > 0$, $\Theta(p_u) < 0$, and $\frac{\partial\Theta(p)}{\partial p} < 0$, $\forall p \in (p_l, p_u)$, then there exists a unique $p' \in (p_l, p_u)$ such that $\Theta(p') = 0$. Furthermore, when $p_l < p < p'$, $\frac{\partial\pi_S^{br}}{\partial p} = d(p)\Theta(p) > 0$, and when $p' < p < p_u$, $\frac{\partial \pi_S^{br}}{\partial p} = d(p)\Theta(p) < 0$. Thus, π_S^{br} is quasi-concave in $p \in (p_l, p_u)$. p' such that $\Theta(p') = 0$ is the unique maximizer of π_S^{br} in (p_l, p_u).

$\Theta(p_u) < 0$ is one of the assumptions. We now show $\Theta(p_l) > 0$. When $p = p_l$, $z^*(p_l) = B\left[1 - \frac{(1-\alpha)c}{(1-r_0)p_l}\right] = \frac{2B}{(\beta+1)} = z^{cc*}$. $G(z^*(p_l)) = G(z^{cc*}) = 0$, and $H(z_T(p_l)) = (\beta-1)\int_A^{z_T(p_l)} \xi f(\xi) d\xi > 0$. Thus, $\Theta(p_l) = (1-r_0)G(z^*(p_l)) + (r-r_0)H(z_T(p_l)) > 0$.

We next prove $\frac{\partial \Theta(p)}{\partial p} < 0$, $\forall p \in (p_l, p_u)$, when (a) $1.5 \le \beta \le 3$, or (b) $\beta > 3$ and $N_1(p_l) + N_2(min\{p_m, p_u\}) < 0$.

$$\begin{aligned}\frac{\partial \Theta(p)}{\partial p} &= \frac{(1-r_0)B}{2}\left[\frac{M^2(\beta+1)}{2p^3} - \frac{M\beta}{p^2}\right] + (r-r_0)(\beta-1)^2\frac{T^2}{Ba^2}p^{2\beta-3} \\ &= N_1(p) + N_2(p) = N(p).\end{aligned}$$

(a) $1.5 \le \beta \le 3$.

$\frac{\partial N_1(p)}{\partial p} = \frac{(1-r_0)B}{2}\left[\frac{-3M^2(\beta+1)}{2p^4} + \frac{2M\beta}{p^3}\right]$. Setting $\frac{\partial N_1(p)}{\partial p} = 0$, we have $p = p_m = \frac{3M(\beta+1)}{4\beta}$. When $p > p_m$, $\frac{\partial N_1(p)}{\partial p} > 0$. Also, when $\beta \le 3$, we have $p_l = \frac{M(\beta+1)}{2(\beta-1)} \ge \frac{3M(\beta+1)}{4\beta} = p_m$. Thus, when $\beta \le 3$, for any $p > p_l$, $\frac{\partial N_1(p)}{\partial p} > 0$ (see Figure B.1(a)). $\frac{\partial N_2(p)}{\partial p} = (r-r_0)(\beta-1)^2(2\beta-3)\frac{T^2}{Ba^2}p^{2\beta-4} \ge 0$ when $\beta \ge 1.5$. Overall, when $1.5 \le \beta \le 3$, $\frac{\partial N(p)}{\partial p} = \frac{\partial N_1(p)}{\partial p} + \frac{\partial N_2(p)}{\partial p} > 0$, for any $p > p_l$. If we can prove $N(p_u) \le 0$, then we conclude $\frac{\partial \Theta(p)}{\partial p} = N(p) < 0$, $\forall p \in (p_l, p_u)$ (see Figure B.1(b)).

We now prove $N(p_u) \le 0$. $p_u = min\left\{\left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}, p_0\right\}$. If $p_0 > \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$, then $p_u = \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$. Since $N(p_0) = 0$ and $\frac{\partial N(p)}{\partial p} > 0$, for any $p > p_l$, we have $N(p_u) < 0$. If $p_0 \le \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$, $p_u = p_0$. $N(p_u) = N(p_0) = 0$.

(b)$\beta > 3$ and $N_1(p_l) + N_2(min\{p_m, p_u\}) < 0$.

When $\beta > 3$, $p_m = \frac{3M(\beta+1)}{4\beta} > \frac{M(\beta+1)}{2(\beta-1)} = p_l$. We have two cases to consider.

(1) $p_u \le p_m$ (see Figure B.2(a)). $\forall p \in (p_l, p_u)$, $\frac{\partial N_1(p)}{\partial p} < 0$ and $\frac{\partial N_2(p)}{\partial p} > 0$. Thus,

$N_1(p_l) + N_2(p_u)$ is the upper bound of $N(p)$. If $N_1(p_l) + N_2(p_u) < 0$, then $\forall p \in (p_l, p_u)$, $N(p) = N_1(p) + N_2(p) < 0$.

(2) $p_u > p_m$ (see Figure B.2(b)). $\forall p \in (p_l, p_m)$, similar to the argument in (1), $N(p) = N_1(p) + N_2(p) < 0$ if $N_1(p_l) + N_2(p_m) < 0$. For $p \in [p_m, p_u)$, as proven in (a), since $\frac{\partial N(p)}{\partial p} = \frac{\partial N_1(p)}{\partial p} + \frac{\partial N_2(p)}{\partial p} > 0$ and $N(p_u) \leq 0$, we have $N(p) < 0$, for any $p \in [p_m, p_u)$. Thus, $\forall p \in (p_l, p_u)$, $N(p) = N_1(p) + N_2(p) < 0$.

We now prove (p', z') is the only element in Ω'. $p_u = min\left\{\left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}, p_0\right\}$. When $p_0 \geq \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$, $p_u = \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$. p_u and p_l agree with the price bounds in Proposition 2.4.3. Hence, (p', z') is the only element in Ω'. When $p_0 < \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}$, then $p_u = p_0$. We now consider whether there exists $p \in [p_0, \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}]$ that is potentially optimal to π_S^{br}. Since p_0 is the largest p value such that $N(p) = 0$, $N(p)$ increases with p after certain point (e.g. $p > p_m$) and $\lim_{p\to\infty} N(p) = \infty$, it follows that $\frac{\partial \Theta(p)}{\partial p} = N(p) \geq 0$, $\forall p \geq p_0$. Otherwise, if there exists $N(p) < 0$ where $p > p_0$. Since $N(p)$ eventually increases with p and $\lim_{p\to\infty} N(p) = \infty$, there must at least exist a $p > p_0$ where $N(p) = 0$, which contradicts that p_0 is the largest p value such that $N(p) = 0$. Since for any $p'' \in [p_0, \left(\frac{aB}{T}\right)^{\frac{1}{\beta-1}}]$ where $\Theta(p'') = 0$, the second necessary condition

$$\frac{\partial^2 \pi_S^{br}}{\partial p^2} |_{p=p''} = \left[\frac{\partial d(p)}{\partial p}\Theta(p)\right] |_{p=p''} + \left[d(p)\frac{\partial \Theta(p)}{\partial p}\right] |_{p=p''} = 0 + \left[d(p)\frac{\partial \Theta(p)}{\partial p}\right] |_{p=p''} \geq 0.$$

This concludes that p'' is not optimal to π_S^{br}.

B.8 Proposition 2.4.6

Proof of Proposition 2.4.6:

Denote (p^{bs*}, z^{bs*}) as the supplier's optimal price and stocking factor decisions to (2.18).

When demand is zero, the channel is coordinated, because both centralized channel and decentralized channel have zero profit. When demand is positive, similar to the two-tier bonus, $(p^{bs*}, z^{bs*}) \in (p^{cs*}, z^{cs*}) \cup \Omega_0 \cup ... \cup \Omega_{n-1}$, where $\Omega_i (i = 0, ..., n-1)$ is the supplier's set of optimal decisions by targeting bonus tier i. When the supplier targets bonus tier $i = 0$, the supplier's problem becomes the two-tier bonus system. We proved in Proposition 2.4.4 that the channel cannot be coordinated under this bonus system. Any $(\hat{p_i}, \hat{z_i}) \in \Omega_i (i \geq 1)$ must satisfy the following equations:

$$F(\hat{z_i}) = \tfrac{(1-r_i)\hat{p_i} - (1-\alpha)c}{(1-r_i)\hat{p_i}}, \quad and$$
$$(1-r_i)G(\hat{z_i}) + (r - r_0)H(\hat{z_{T_0}}) + \textstyle\sum_{j=1}^{i}(r_{j-1} - r_j)H(\hat{z_{T_j}}) = 0,$$

where $\hat{z_{T_k}} = \frac{T_k}{d(\hat{p_i})\hat{p_i}}, \forall k \leq i$, $G(x) = (1-\beta)[x - \Lambda(x)] + \beta x[1 - F(x)]$ and $H(x) = (\beta - 1)\int_A^x \xi f(\xi) d\xi$.

We have two cases to consider.

(1) (p^{bs*}, z^{bs*})=(p^{cs*}, z^{cs*}). The channel is not coordinated unless the retailer sets $r = \alpha$ as shown in Proposition 2.4.4.

(2) $(p^{bs*}, z^{bs*}) \in \Omega_i (i \geq 1)$. Assume that the channel is coordinated, we have $p^{bs*} = p^{cc*}$ and $z^{bs*} = z^{cc*}$. Since $G(z^{bs*}) = G(z^{cc*}) = 0$ and $H(z^{bs*}_{T_j}) = (\beta - 1)\int_A^{z^{bs*}_{T_j}} \xi f(\xi) d\xi > 0$, $\forall j \leq i$, we have

$$d(p^{bs*})\left[(1-r_i)G(z^{bs*}) + (r - r_0)H(z^{bs*}_{T_0}) + \sum_{j=1}^{i}(r_{j-1} - r_j)H(z^{bs*}_{T_j})\right] > 0,$$

which violates the first necessary condition.

B.9 Proposition 2.4.7

Proof of Proposition 2.4.7:

When $r = \alpha$, $\pi_S^{sp} = (1-\alpha)\pi^{cc} - \Phi$. Since Φ is a constant independent of price and stocking factor, the optimal solution to π_S^{sp} is $p^{sp*} = p^{cc*}$ and $z^{sp*} = z^{cc*}$. Hence, the channel is coordinated. When $\Phi = \phi\pi^{cc*}$, $\pi_S^{sp*} = (1-\alpha-\phi)\pi^{cc*}$ and $\pi_R^{sp*} = (\alpha+\phi)\pi^{cc*}$. In order to make (r, Φ) acceptable for both the retailer and the supplier, we need to have $\pi_S^{sp*} > \pi_S^{cs*}$ and $\pi_R^{sp*} > \pi_R^{cs*}$. According to (2.9) and (2.11), we can derive the range of ϕ that meets the above requirement. $\pi_S^{sp*} > \pi_S^{cs*}$, i.e., $(1-\alpha-\phi)\pi^{cc*} > (1-\alpha)(\frac{\beta-1}{\beta-\alpha})^{\beta}\pi^{cc*}$, gives $\phi < (1-\alpha)\left[1-(\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$. Also, $\pi_R^{sp*} > \pi_R^{cs*}$, i.e., $(\alpha+\phi)\pi^{cc*} > (\frac{\beta-1}{\beta-\alpha})^{\beta-1}\pi^{cc*}$, gives $\phi > (\frac{\beta-1}{\beta-\alpha})^{\beta-1} - \alpha$.

B.10 Proposition 2.4.8

Proof of Proposition 2.4.8:

$\phi_H = (1-\alpha)\left[1-(\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$. Let $y = (\frac{\beta-1}{\beta-\alpha})^{\beta}$. Proving ϕ_H decreases with β is equivalent to proving y increases with β, i.e., $\frac{\partial y(\beta)}{\partial \beta} > 0$. We have $y = e^{lny} = e^{\beta[ln(\beta-1)-ln(\beta-\alpha)]}$.

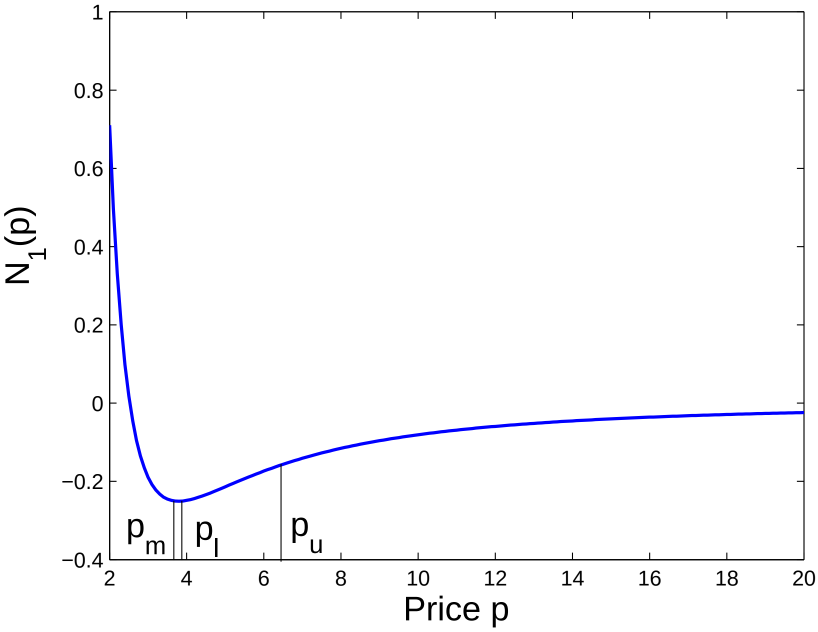
$$\frac{\partial y(\beta)}{\partial \beta} = e^{lny}\left[ln\frac{\beta-1}{\beta-\alpha} + \beta(\frac{1}{\beta-1} - \frac{1}{\beta-\alpha})\right] = e^{lny}x,$$

where $x = ln\frac{\beta-1}{\beta-\alpha} + \beta(\frac{1}{\beta-1} - \frac{1}{\beta-\alpha})$. Now, to prove $\frac{\partial y(\beta)}{\partial \beta} > 0$, we only need to prove $x > 0$.

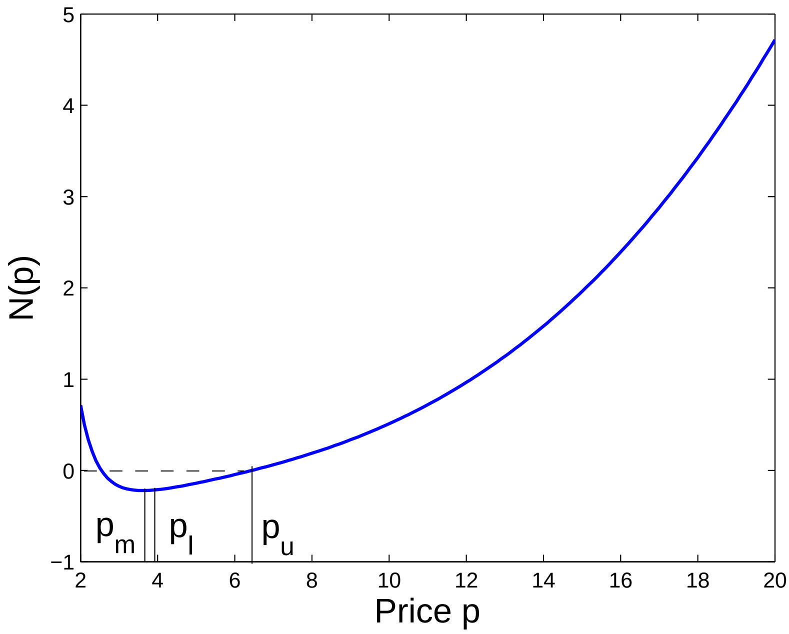
We know $x(\alpha = 1) = \left[ln\frac{\beta-1}{\beta-\alpha} + \beta(\frac{1}{\beta-1} - \frac{1}{\beta-\alpha})\right] = 0$. Furthermore,

$$\frac{\partial x(\alpha)}{\partial \alpha} = \frac{1}{\beta-\alpha} - \frac{\beta}{(\beta-\alpha)^2} = \frac{-\alpha}{(\beta-\alpha)^2} < 0.$$

Hence, for $0 < \alpha < 1$, $x > 0$. We now have $\frac{\partial y(\beta)}{\partial \beta} > 0$. $\phi_H = (1-\alpha)\left[1-(\frac{\beta-1}{\beta-\alpha})^{\beta}\right]$ decreases with β for given α. For given β, when α increases, it is obvious that $y = (\frac{\beta-1}{\beta-\alpha})^{\beta}$ increases. Thus, $\phi_H = (1-\alpha)(1-y)$ decreases with α.

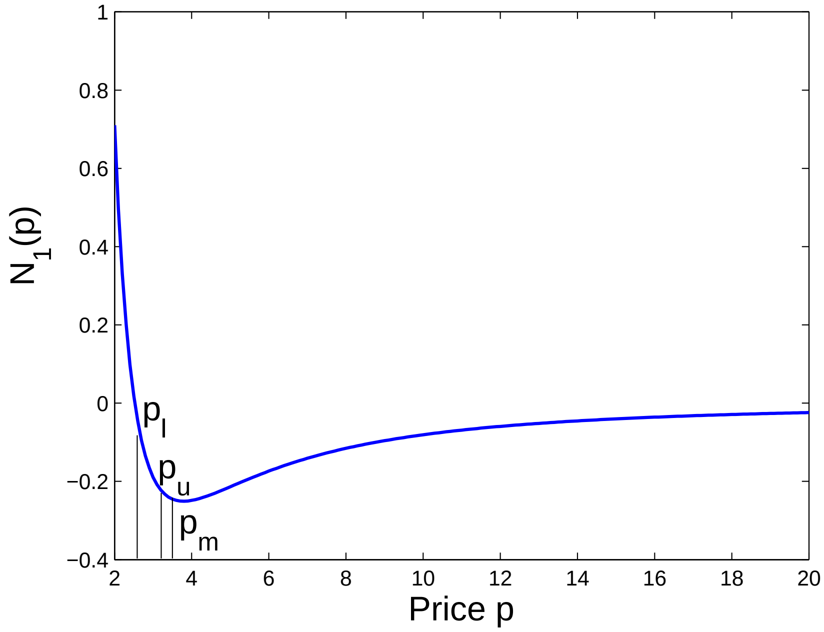


(a) $N_1(p)$

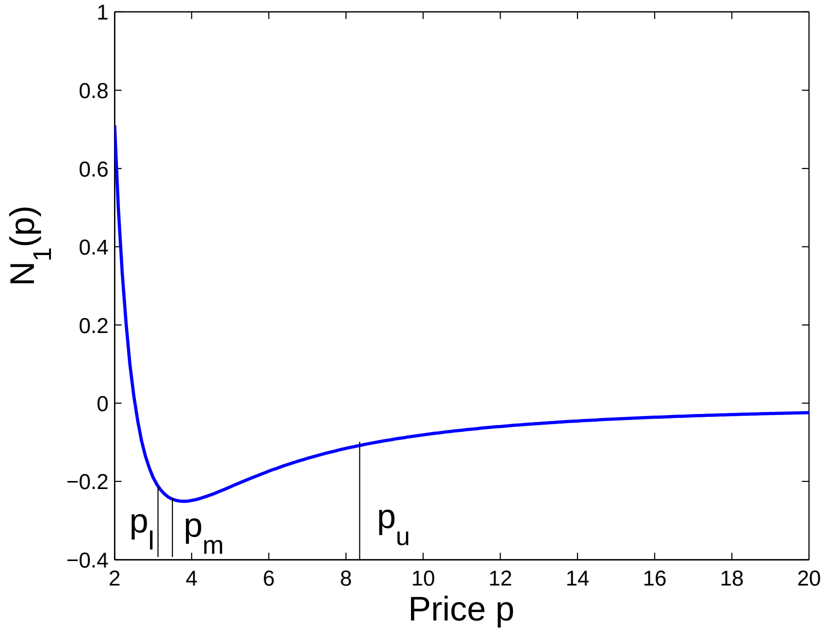


(b) $N(p)$

Figure B.1: $N_1(p)$ and $N(p)$, $1.5 \leq \beta \leq 3$



(a) $N_1(p)$



(b) $N_1(p)$

Figure B.2: The two cases for $N_1(p)$, $\beta \geq 3$.

APPENDIX C
PROOFS OF CHAPTER THREE

C.1 Proof of Proposition 3.3.1

Proof: The derivative of S_i's profit function with respect to p_i is

$$\begin{aligned}\frac{\partial \pi_{S_i}^{cs}}{\partial p_i} &= -\beta a p_i^{-\beta-1} p_j^{\delta} \left[(1-r)p_i - c_{S_i}\right] + a p_i^{-\beta} p_j^{\delta}(1-r) \\ &= a p_i^{-\beta-1} p_j^{\delta} \left[-(\beta-1)(1-r)p_i + \beta c_{S_i}\right].\end{aligned}$$

Setting $\frac{\partial \pi_{S_i}^{cs}}{\partial p_i} = 0$ gives $[-(\beta-1)(1-r)p_i + \beta c_{S_i}] = 0$. Note that $p_i^{cs*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r)}$. When $p_i > p_i^{cs*}$, $\frac{\partial \pi_{S_i}^{cs}}{\partial p_i} < 0$. We have $\frac{\partial \pi_{S_i}^{cs}}{\partial p_i} > 0$ when $p_i < p_i^{cs*}$. So, $\pi_{S_i}^{cs}$ is a pseudo-concave function of p_i, and p_i^{cs*} is the unique maximizer.

C.2 Proof of Proposition 3.3.2

Proof: The derivative of R's profit function (3.4) with respect to the revenue share r is

$$\begin{aligned}\frac{\partial \pi_R^{cs}}{\partial r} &= (\beta-\delta)(1-r)^{(\beta-\delta-1)}(-1)\left(\frac{r}{1-r}K_1 - K_2\right) + (1-r)^{(\beta-\delta)}\left[\frac{r}{(1-r)^2}K_1 + \frac{K_1}{(1-r)}\right] \\ &= (1-r)^{(\beta-\delta-1)}\left[-(\beta-\delta)\frac{r}{1-r}K_1 + (\beta-\delta)K_2 + K_1 + \frac{r}{1-r}K_1\right] \\ &= (1-r)^{(\beta-\delta-2)}\left[-(\beta-\delta)(K_1+K_2)r + K_1 + (\beta-\delta)K_2\right].\end{aligned}$$

Setting $\frac{\partial \pi_R^{cs}}{\partial r} = 0$, we derive r^{cs*}. Since π_R^{cs} is a pseudo-concave function of r, r^{cs*} uniquely maximizes the retailer's profit.

C.3 Proof of Proposition 3.3.3

Proof: By (3.5), we know r^{cs*} increases with K_2/K_1. Therefore, to obtain the relationship between r^{cs*} and c_{R_i}, we only need to know the relationship between K_2/K_1 and c_{R_i}.

K_2/K_1 is:

$$\begin{aligned}\frac{K_2}{K_1} &= \frac{M_3 c_{R_1} + M_4 c_{R_2}}{M_1 M_3 + M_2 M_4} \\ &= \frac{\frac{M_3}{M_4} c_{R_1} + c_{R_2}}{\frac{M_3}{M_4} M_1 + M_2} \\ &= \frac{\left(\frac{c_{S_1}}{c_{S_2}}\right)^{-(\beta+\delta)} c_{R_1} + c_{R_2}}{\left(\frac{c_{S_1}}{c_{S_2}}\right)^{-(\beta+\delta)} \frac{\beta c_{S_1}}{\beta-1} + \frac{\beta c_{S_2}}{\beta-1}} \\ &= \frac{\beta-1}{\beta}\left[\frac{c_{R_1}\left(\frac{c_{S_1}}{c_{S_2}}\right)^{-(\beta+\delta)} + c_{R_2}}{c_{S_1}\left(\frac{c_{S_1}}{c_{S_2}}\right)^{-(\beta+\delta)} + c_{S_2}}\right].\end{aligned}$$

Let $x = \left(\frac{c_{S_1}}{c_{S_2}}\right)$. The derivative of $\frac{K_2}{K_1}$ with respect to c_{R_1} is:

$$\frac{\partial\left(K_2/K_1\right)}{\partial c_{R_1}} = \frac{\beta-1}{\beta}\frac{x^{-(\beta+\delta)}}{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right]}.$$

The derivative of $\frac{K_2}{K_1}$ with respect to c_{R_2} is:

$$\frac{\partial\left(K_2/K_1\right)}{\partial c_{R_2}} = \frac{\beta-1}{\beta}\frac{1}{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right]}.$$

$\frac{\partial(K_2/K_1)}{\partial c_{R_1}} > 0$ and $\frac{\partial(K_2/K_1)}{\partial c_{R_2}} > 0$. Since r^{cs*} is positively related to K_2/K_1, we conclude that r^{cs*} increases in c_{R_i}.

We now prove part (b).

$$\frac{\partial r^{cs*}}{\partial \delta} = \frac{1}{1+\frac{K_2}{K_1}}\frac{1}{(\beta-\delta)^2} + \frac{(\beta-\delta-1)}{(\beta-\delta)}\frac{1}{\left(1+\frac{K_2}{K_1}\right)^2}\frac{\partial\left(K_2/K_1\right)}{\partial \delta}.$$

The derivative of $\frac{K_2}{K_1}$ with respect to δ is:

$$\begin{aligned}\frac{\partial\left(K_2/K_1\right)}{\partial \delta} &= \frac{\beta-1}{\beta}\frac{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right] c_{R_1} x^{-(\beta+\delta)}(-1) ln x}{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right]^2} \\ &\quad - \frac{\beta-1}{\beta}\frac{\left[c_{R_1} x^{-(\beta+\delta)} + c_{R_2}\right] c_{S_1} x^{-(\beta+\delta)}(-1) ln x}{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right]^2} \\ &= \frac{\beta-1}{\beta}\frac{x^{-(\beta+\delta)} ln x\left(c_{S_1} c_{R_2} - c_{S_2} c_{R_1}\right)}{\left[c_{S_1} x^{-(\beta+\delta)} + c_{S_2}\right]^2}.\end{aligned}$$

Since we assume the retailer shares the same cost on each product, we have $c_{R_1} = c_{R_2} = c_R$. When $c_{S_1} > c_{S_2}$, $(c_{S_1}c_{R_2} - c_{S_1}c_{R_2}) > 0$. Also, $x > 1$. Hence, $lnx > 0$. Therefore, $\frac{\partial(K_2/K_1)}{\partial\delta} > 0$. We have $(c_{S_1}c_{R_2} - c_{S_2}c_{R_1}) < 0$ and $lnx < 0$ when $c_{S_1} < c_{S_2}$. So, $\frac{\partial(K_2/K_1)}{\partial\delta} > 0$ holds as well. This concludes that $\frac{\partial r^{cs*}}{\partial\delta} > 0$ when $c_{R_1} = c_{R_2} = c_R$.

C.4 Proof of Proposition 3.3.5

Proof: By Proposition 3.3.1, we know:

$$p_i^{cs*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r)}.$$

Therefore, p_i^{cs*} is positively related to r. For part (a), when c_{R_i} increases, r also goes up according to Proposition 3.3.3. So, p_i^{cs*} increases in c_{R_i}. The proof of part (b) is the same and is thus omitted here.

C.5 Proof of Proposition 3.3.6

Proof: Let $g(r_i) = (1-r_i)^{\beta-1}\left[K_3 - K_4(1-r_i)\right]$, where $K_3 = \frac{\beta c_{S_i}}{(\beta-1)}$ and $K_4 = \frac{\beta c_{S_i}}{\beta-1}(1-r_i)$.

$$\begin{aligned}
\frac{\partial g(r_i)}{\partial r_i} &= -(\beta-1)(1-r_i)^{\beta-2}\left[K_3 - K_4(1-r_i)\right] + (1-r_i)^{\beta-1}K_4 \\
&= -(\beta-1)K_3(1-r_i)^{\beta-2} + K_4\beta(1-r_i)^{\beta-1} \\
&= (1-r_i)^{\beta-2}\left[-(\beta-1)K_3 + K_4\beta(1-r_i)\right].
\end{aligned}$$

Setting $\frac{\partial g(r_i)}{\partial r_i} = 0$, we derive $r_i^{es*} = 1 - \frac{\beta-1}{\beta}\frac{K_3}{K_4}$. Since when $r > r_i^{es*}$, $\frac{\partial g(r_i)}{\partial r_i} < 0$ and when $r < r_i^{es*}$, $\frac{\partial g(r_i)}{\partial r_i} > 0$, $g(r_i)$ is pseudo-concave in r_i. r_i^{es*} is the unique maximizer. Substituting K_3 and K_4 into $r_i^{es*} = 1 - \frac{\beta-1}{\beta}\frac{K_3}{K_4}$ leads to (3.8). This completes the proof.

C.6 Proof of Proposition 3.3.7

Proof: When $\frac{c_{R_i}}{c_{S_i}} \leq \frac{c_{R_j}}{c_{S_j}}$, $r_i^{es*} \leq r_j^{es*}$ is rather obvious. Since $1 \leq \delta \leq (\beta-1)$, we have $\frac{\beta}{\beta-1} > \frac{\beta-\delta}{\beta-\delta-1} > 0$. Also, when $\frac{c_{R_i}}{c_{S_i}} \leq \frac{c_{R_j}}{c_{S_j}}$, we have $\frac{\beta-1}{\beta}\frac{c_{R_i}}{c_{S_i}} \leq \frac{K_2}{K_1} = \frac{\beta-1}{\beta}\left[\frac{c_{R_i}\left(\frac{c_{S_i}}{c_{S_j}}\right)^{-(\beta+\delta)}+c_{R_j}}{c_{S_i}\left(\frac{c_{S_i}}{c_{S_j}}\right)^{-(\beta+\delta)}+c_{S_j}}\right]$. So, $r_i^{es*} < r^{cs*}$ and $p_i^{es*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r_i^{es*})} < p_i^{cs*} = \frac{\beta c_{S_i}}{(\beta-1)(1-r^{cs*})}$. This completes the proof.

C.7 Proof of Proposition 3.3.8

We first prove the property of the retailer's profit with regard to δ.

$$\begin{aligned}
\pi_R^{cs*} &= \frac{2a}{(\beta-\delta)}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta-1)}\left[\frac{(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta-1)} \\
&= 2ay(\delta),
\end{aligned}$$

where $y(\delta) = \frac{1}{\beta-\delta}\left[\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta-1)}$ and $K_5 = [\beta(c_S+c_R)-c_R]$.

Taking the derivative of $y(\delta)$ with respect to δ leads to

$$\begin{aligned}
\frac{\partial y(\delta)}{\partial\delta} &= \frac{\partial e^{ln(y(\delta))}}{\partial\delta} = \frac{\partial e^{\left[-ln(\beta-\delta)-(\beta-\delta-1)ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]}}{\partial\delta} \\
&= e^{ln(y(\delta))}\left[\frac{1}{\beta-\delta}+ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}-\frac{1}{\beta-\delta}\right] \\
&= e^{ln(y(\delta))}ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}.
\end{aligned}$$

Let $h_R(\delta) = \frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}$. It is obvious that $h_R(\delta)$ increases with δ. Since $h_R(\delta_0^R) = 1$, when $\delta > \delta_0^R$, we have $h_R(\delta) > 1$ and $\frac{\partial y(\delta)}{\partial\delta} > 0$. When $\delta < \delta_0^R$, we have $h_R(\delta) < 1$ and $\frac{\partial y(\delta)}{\partial\delta} < 0$. This completes the proof.

Next we prove the property of the supplier's profit with regard to δ.

$$\begin{aligned}
\pi_{S_i}^{cs*} &= \frac{ac_S}{(\beta-1)}\left[\beta(c_S+c_R)-c_R\right]^{-(\beta-\delta)}\left[\frac{(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta)} \\
&= \frac{ac_S}{(\beta-1)}x(\delta),
\end{aligned}$$

where $x(\delta) = \left[\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}\right]^{-(\beta-\delta)}$ and $K_5 = [\beta(c_S + c_R) - c_R]$.

Taking the derivative of $x(\delta)$ with respect to δ leads to

$$\begin{aligned}\frac{\partial x(\delta)}{\partial \delta} &= \frac{\partial e^{ln(x(\delta))}}{\partial \delta} = \frac{\partial e^{-(\beta-\delta)ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)}}}{\partial \delta} \\ &= e^{ln(x(\delta))}\left[ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)} - \frac{1}{\beta-\delta-1}\right].\end{aligned}$$

Let $h_S(\delta) = ln\frac{K_5(\beta-\delta)}{(\beta-1)(\beta-\delta-1)} - \frac{1}{\beta-\delta-1}$. We can prove $h_S(\delta)$ decreases with δ as follows.

$$\begin{aligned}\frac{\partial h_S(\delta)}{\partial \delta} &= -\frac{1}{\beta-\delta} + \frac{1}{\beta-\delta-1} - \frac{1}{(\beta-\delta-1)^2} \\ &= \frac{1}{(\beta-\delta)(\beta-\delta-1)} - \frac{1}{(\beta-\delta-1)^2} < 0.\end{aligned}$$

Since $h_S(\delta_0^S) = 0$, when $\delta > \delta_0^S$, we have $h_S(\delta) < 0$ and $\frac{\partial x(\delta)}{\partial \delta} < 0$. When $\delta < \delta_0^S$, we have $h_S(\delta) > 0$ and $\frac{\partial x(\delta)}{\partial \delta} > 0$. This completes the proof.

C.8 Proof of Proposition 3.3.9

Proof: Since $\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)} = \frac{\beta^2-\beta\delta-\beta+\delta}{\beta^2-\beta\delta-\beta} > 1$ and $\delta < (\beta-1)$, we have $\frac{\pi_{S_i}^{es*}}{\pi_{S_i}^{cs*}} = \left[\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]^{\beta-\delta} > 1$. Also, $\rho_R(\delta) = \frac{\pi_{R_1}^{es*}+\pi_{R_2}^{es*}}{\pi_R^{cs*}} = \frac{\beta-\delta}{\beta}\left[\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]^{\beta-\delta-1} < 1$, since we can show the ratio decreases with δ for given β. When $\delta = 0$, $\rho_R(\delta)$ reaches maximum which is 1. We now prove $\rho_R(\delta)$ decreases with δ. Let $y_1(\delta) = \left[\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]^{\beta-\delta-1}$.

$$\begin{aligned}\frac{\partial \rho_R(\delta)}{\partial \delta} &= \frac{\partial e^{ln(y_1(\delta))}}{\partial \delta}\frac{\beta-\delta}{\beta} - e^{ln(y_1(\delta))}\frac{1}{\beta} \\ &= e^{ln(y_1(\delta))}\left\{-\left[ln\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]\left(\frac{\beta-\delta}{\beta}\right) + \frac{1}{\beta}\right\} - e^{ln(y_1(\delta))}\frac{1}{\beta} \\ &= e^{ln(y_1(\delta))}\left\{-\left[ln\frac{(\beta-1)(\beta-\delta)}{\beta(\beta-\delta-1)}\right]\left(\frac{\beta-\delta}{\beta}\right)\right\} < 0.\end{aligned}$$

This completes the proof.

APPENDIX D
PROOFS OF CHAPTER FOUR

D.1 Proof of Proposition 4.3.1

Proof: To prove the proposition, we only need to show $m_{11} \geq m_{21}$ and $m_{12} \leq m_{22}$.

(1) When K is an odd number, $C_K^{n_m+1} + ... + C_K^K = C_K^0 + ... + C_K^{n_m}$. Let $\delta = \frac{C_{K-1}^{nm}}{C_K^0+...+C_K^{nm}} = \frac{C_{K-1}^{nm}}{C_K^{nm+1}+...+C_K^K}$. We have $m_{11} = 1 - \delta z_{12}$ and $m_{21} = \delta z_{21}$. By Assumption 4.3.1, $z_{11} \geq z_{21}$. Therefore, $m_{11} = 1 - \delta + \delta(1 - z_{12}) = 1 - \delta + \delta z_{11} \geq 1 - \delta + \delta z_{21} = 1 - \delta + m_{21}$. Since $\delta \leq 1$, $m_{11} \geq m_{21}$. Thus, $m_{12} = 1 - m_{11} \leq 1 - m_{21} = m_{22}$.

(2) When K is an even number, $C_K^{n_m+1} + ... + C_K^K > C_K^0 + ... + C_K^{n_m}$. Let $\delta_1 = \frac{C_{K-1}^{nm}}{C_K^0+...+C_K^{nm}}$ and $\delta_2 = \frac{C_{K-1}^{nm}}{C_K^{nm+1}+...+C_K^K}$. Hence, $\delta_1 > \delta_2$. We have $m_{11} = 1 - \delta_1 z_{12}$ and $m_{21} = \delta_2 z_{21}$. $m_{11} = 1 - \delta_1 z_{12} = 1 - \delta_1 + \delta_1(1 - z_{12}) \geq 1 - \delta_1 + \delta_2(1 - z_{12}) = 1 - \delta_1 + \delta_2 z_{11} \geq 1 - \delta_1 + \delta_2 z_{21} = 1 - \delta_1 + m_{21}$. Since $\delta_1 \leq 1$, we have $m_{11} \geq m_{21}$, and $m_{12} = 1 - m_{11} \leq 1 - m_{21} = m_{22}$.

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