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Teaching algebra-based concepts to students with learning disabilities: the effects of preteaching using a gradual instructional sequence

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TEACHING ALGEBRA-BASED CONCEPTS TO STUDENTS WITH LEARNING DISABILITIES: THE EFFECTS OF PRETEACHING USING A GRADUAL INSTRUCTIONAL SEQUENCE

by

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<u>An Abstract</u>

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for the Doctor of Philosophy degree in Teaching and Learning
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Thesis Supervisor: Associate Professor William J. Therrien

ABSTRACT

Research to identify validated instructional approaches to teach math to students with LD and those at risk for failure in both core and supplemental instructional settings is necessary to assist teachers in closing the achievement gaps that exist across the country. The concrete-to-representational-to-abstract instructional sequence (CRA) has been identified through the literature as a promising approach to teaching students with and without math difficulties (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, & Jackson, 2003; Flores, 2010). The CRA sequence transitions students from the use of concrete manipulatives to abstract symbols through the use of explicit instruction to increase computational and conceptual understanding.

The main purpose of this study was to assess the effects of preteaching essential pre-algebra skills on the overall algebra achievement scores for students with disabilities and those at risk for failure in math. Specifically the study examined the following research questions: (1) What are the effects of preteaching math units using the CRA instructional sequence on the algebra achievement of students with LD and those at risk for math failure? (2) What are the effects of preteaching math units using the CRA instructional sequence on the transfer of algebra-based skills of students with LD and those at risk for math failure to the general education setting? (3) What are the effects of preteaching math units using the CRA instructional sequence on the maintenance of algebra-based skills for students with LD and those at risk for math failure?

Summary of Study Design and Findings

Thirty-two students enrolled in one of four 6th grade classrooms across two elementary schools participated in this study. Sixth grade students who currently receive tier 2 or tier 3 supplemental and intensive instruction in math; and those identified as having a math learning disability will be participants. A treatment and control, pre/post experimental design was used to examine the effect of the intervention on students' math achievement. The intervention was replicated across two math units related to teaching algebra-based concepts: Solving Equations and Fractions. The treatment condition consisted of a combination of preteaching and the use of the CRA instructional sequence. Prior to each unit, Solving Equations and Fractions, researchers pretaught students 3 essential prerequisite skills necessary for success in the upcoming unit, at the concrete, representational, and abstract levels of learning. Each preteaching session lasted for ten days, 30 minutes each day. Immediate, delayed, and follow-up measures were used to support the examination of the research questions and hypotheses.

Overall findings indicate that the combination of preteaching using the CRA gradual sequence is effective at improving the overall algebra performance for students with disabilities.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph. D. thesis of

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To Corey, Emily, Julie, and Megan

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CHAPTER ONE

INTRODUCTION

Math Statistics

Over the last decade a renewed emphasis on math education for all students was heightened by the No Child Left Behind Act (NCLB, 2001), the reform efforts of the National Council of Teachers of Mathematics (NCTM, 2000), the National Math Advisory Panel (NMAP, 2008), the creation and adoption of the Common Core State Standards (CCSS, 2010), and the Individuals with Disabilities Education Act (IDEA, 2004). Aligning with new standards and levels of accountability, the country has seen an increased trend in average math performance (see the National Center for Educational Statistics Web site, http://nces.ed.gov/pubsearch/search.asp). In 2011, the National Assessment of Educational Progress (NAEP) scores for 4th and 8th graders were the highest that they had ever been with the most notable improvements in NAEP scores taking place between the years 2001 to 2003.

Despite improvements in overall scores, there continue to be large achievement gaps for subgroup populations. Large percentages of low-income students, African American students, Hispanic students, and students with disabilities score below basic performance on the NAEP (U.S. Department of Education, 2011). In 2009 and 2011, fourth grade students with disabilities scored significantly lower than their grade level peers on the national assessment in mathematics (National Center for Education Statistics, 2011); more than 50% of these students failing to reach proficiency.

Special Education

In 2006, the U.S. Department of Education reported that students with LD on average spend at least 80% of their day in general education; and that most of these students are receiving their core math instruction in an inclusive environment. The curriculum often used in general education settings is closely aligned with the NCTM standards; shifting focus away from skill development to conceptual meanings and application of big ideas in mathematics (Sayeski & Paulsen, 2010). The overall goal of standards-based curricula is to increase higher-level thinking skills through real-life problem solving and collaborative learning opportunities. The use of explicit explanations, adequate feedback, and sufficient amount of guided and independent practice to master content skills are not used frequently within standards-based curriculum (Sood & Jitendra, 2007). These areas are components of strong instruction for students with LD and those at risk for failure in math (Baker, Gersten, & Lee, 2002). Without an emphasis on requisite skill mastery, the gaps in learning between students with and without math disabilities will increase (Sayeski & Paulsen, 2010), particularly when learning algebraic and other abstract math concepts (Maccini, McNaughton, & Ruhl, 1999; Witzel, 2005).

NCLB (2001) has identified math as an area that all students need to reach proficiency, including students with LD. In addition, the National Council of Teachers of Mathematics Principles and Standards (NCTM, 2000) emphasized the need to teach all students geometry and algebra skills. The NCTM considers these to be crucial building blocks that can alter success in mathematics. These renewed standards have led to increasing changes in graduation requirements. The completion of Algebra 1 is mandatory in most school districts for students to receive a diploma (Witzel, 2005). As a result, sixth and seventh grade students are now receiving pre-algebra and many eighth graders are enrolled in Algebra I. There is not an exception for students with LD. To be successful in algebra, research has shown students need to have a strong foundation of knowledge of basic math computations, but should also engage in activities that promote more conceptual understandings of math concepts and learning with an emphasis on problem-solving (Thornton, Langrall, & Jones, 1997). Bottge (1999) indicated that teachers do not feel students with LD have a consistent knowledge of these basic facts or a meaningful understanding of how to apply them to algebra-based problems. An emphasis on creating an understanding of mathematical concepts, combined with a strong computational foundation, is necessary for students with LD to have success in more advanced math classes (Bottge, 1999) such as algebra.

The completion of algebra is critical for many reasons, one of which is that it serves as a gate-keeper to postsecondary education (Maccini et al., 1999). More jobs, including vocations that do not require two and four year degrees, are hiring applicants with a strong background in mathematics (U.S. Department of Labor, 2007). It is, therefore, necessary to provide evidenced-based instructional approaches that support students with LD in math, increasing their postsecondary options.

Research to identify validated instructional approaches to teach students with LD and those at risk for failure in both core and supplemental instructional settings is necessary to assist teachers in closing the achievement gaps that exist across the country. The concrete-to-representational-to-abstract instructional sequence has been identified through the literature as a promising approach to teaching students with and without math difficulties (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, &

Jackson, 2003; Flores, 2010). The CRA sequence transitions students from the use of concrete manipulatives to abstract symbols through the use of explicit instruction to teach computational and conceptual understandings.

Purpose of and Instructional Approach used in the Study

The main purpose of this study was to assess the effects of preteaching essential pre-algebra skills on the overall algebra math achievement scores for students with disabilities and those at risk for failure in math. The students in the study were enrolled in a general education math class that had adopted and used a standards-based curriculum (Everyday Mathematics; Bell et al., 2004) as the foundation of mathematical planning and implementation. The intervention (preteaching) used the concrete-to-representational-to-abstract gradual teaching sequence. Research examining the effects of supplemental interventions, such as the use of preteaching and CRA, to teach students with math difficulties algebra-based concepts is necessary to increase the potential for success within settings using standards-based curricula. Utilizing effective supplemental instruction with students who struggle in math will increase performance, reducing achievement gaps.

CHAPTER TWO

LITERATURE REVIEW

Chapter Overview

Higher standards and expectations for all students in mathematics, and continually large achievement gaps for students with learning disabilities (LD) and other subgroup populations (NAEP; U.S. Department of Education, 2011) has led to an increased emphasis on math research. Particular emphasis has been placed on the skills, content, and instructional practices that create strong math education for struggling learners. This literature review will examine the characteristics of students with LD, essential components of math instruction, effective interventions for teaching algebra-based concepts, the use of preteaching, and discussion of the impacts of the instructional setting.

Characteristics of Students with Mathematics Disabilities

Between 5-8% of the school-age population has been identified with math-related learning disabilities (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974). There are two different subgroups of students with LD, those with only difficulties in math and those who also struggle with reading and/or attention related disabilities (i.e., attention deficit hyperactivity disorder; Geary, 2003). Regardless of whether or not students struggle in other academic areas, the computational and problem solving strengths and weaknesses are consistent among students with LD who struggle in math.

Students with LD experience difficulty with both the procedural and conceptual aspects of mathematics (Barron, Bransford, Kulewixz, & Hasslebring, 1989; Mastropieri, Bakken, & Scruggs, 1991). Procedures, or algorithms, in mathematics are the set of rules or steps used to find solutions to problems. Procedural errors often are the result of a weak understanding of the underlying concepts of mathematics (Geary, 2003), but can also be a result of difficulties with memory, attention, and organization. As problems become more difficult and involve more operations (e.g., fractions and algebra), students with LD begin making more procedural errors, and often fail to detect errors once they have been made. Areas such as algebra and fractions that involve multiple computations and procedures are among the most difficult for students with math disabilities (Jordan, Miller, & Mercer, 1999). The abstract nature of both skill areas contributes to the difficulty.

The conceptual learning of mathematics refers to the understanding of the underlying ideas, or concepts, that make up algorithms. As students enter higher level math courses, such as algebra, these conceptual understandings become more abstract (Witzel, Mercer, & Miller, 2003). Abstract thinking requires a person to think beyond what they can see or touch (Hawker & Cowley, 1997). This is particularly difficult for students with LD. Emphasis on teaching the precursor skills to algebra using concrete manipulatives can help to support this abstract understanding (Witzel, 2005). When students develop strong conceptual ideas, the essence of mathematical learning, they are more likely to become accurate in their procedures used to solve problems (Geary, 2003).

As students begin to learn the rules and operations for various problem types they must also be able to generalize the solutions to other similar and more complex problem types. This too is a difficult task for both students with and without LD (Fuchs & Fuchs, 2003). Research indicates that students with LD have narrow schemas, or conceptual frameworks, in which to connect or relate novel problems compared to their peers

(Cooper & Sweller, 1987; Fuchs, L.S., Fuchs, Finelli, Courey, & Hamlett, 2004). In order for students with LD to develop stronger conceptual frameworks to increase transfer of skills; and to advance the procedural understandings of mathematics, teachers must increase the use of effective, research-based, instruction in math.

Essential Components of Effective Math Instruction

The National Mathematics Advisory Panel (2008) was commissioned to make instructional recommendations informed by high-quality math education research. In preparation of the Panel's recommendations and final report, the Center for Instruction created a document synthesizing the existing quantitative intervention research for students with LD (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2008). The technical report found five essential components for effective math curriculum and instruction: (1) explicit instruction, (2) the use of heuristics, (3) student verbalizations of their mathematical reasoning, (4) the use of visual representations to solve problems, and (5) sequencing or providing a range of examples (i.e.,, easy to hard, concrete to abstract). In addition to these five components the use of formative assessment, adequate feedback, and peer learning were found to be highly effective practices within math education.

Explicit Instruction

Explicit instruction is a systematic approach to teaching that incorporates the use of clear modeling, teacher think alouds, guided practice, corrective feedback, and frequent practice and review (Gersten et al., 2009; NMAP, 2008). The research on explicit instruction incorporates the use of step-by-step modeling to teach operations and procedures, gradually moving from introductory concepts to more complex problem types. In addition to this gradual sequence of instruction, interventionists model each step through the use of think alouds, allowing students to think about the reasons behind the procedures (Fuchs et al, 2004; Tournaki, 2003). The research also encourages students to use this type of thinking during guided practice allowing for immediate feedback (Shunk & Cox, 1986). The recommended use of explicit instruction for teaching math to students with LD is supported by a strong research base (Darch, Carnine, & Gersten, 1984; Fuchs et al., 2004; Shunk & Cox, 1986; Wilson & Sindelar, 1991).

Use of Heuristics

A heuristic is “a method or strategy that exemplifies a generic approach for solving a problem,” (Gersten et al., 2008). A heuristic, or strategy, is not problem specific but can be used to guide students through multiple problem types within a content or skill area. For example, a highly researched heuristic (STAR) for problem solving includes such steps as “**S**earch the word problem, **T**ranslate the words into an equation in picture form, **A**nswer the problem, **R**eview the solution,” (Maccini & Hughes, 2000). A moderate research base supports the use of multiple heuristics to assist students with LD in acquisition and retention of math skills (Woodward, Monroe, & Baxter, 2001; Van Luit & Naglieri, 1999).

Verbalization of Mathematical Reasoning

The modeling and encouragement of student’s thinking aloud about their approach to problem solving is a critical component of effective math instruction (Gersten et al., 2008). The process of discussing mathematical reasoning with peers and /or teachers assists students, particularly those with learning disabilities, to evaluate their solutions or procedures. The amount of prompts and specificity of verbalizations varies within the research. For example, Hutchinson (1993) provided students with specific questions to answer in order to help them verbalize the problem steps and solutions. Other researchers offer a broader approach to the verbalization component by simply instructing students to talk with peers about the problem solutions (Schunk & Cox, 1986).

Visual Representations

Researchers show that the use of visual representations to support the conceptual and procedural understandings of mathematics may lead to significant achievement gains for students with disabilities (Gersten et al., 2009). Visual representations are concrete manipulatives or pictorial representations that help students understand the abstract concepts in mathematics. The use of number lines, base ten blocks, arrays, and strip diagrams are among a few commonly used visual representations in mathematics. A moderate research base supports the use of these objects to teach foundational skills and assist in overall understandings of math procedures (Fuchs et al., 2004; Fuchs et al., 2008; Fuchs, Fuchs, Craddock, Hollenbeck, Hamlett, & Schatchneider, 2008; Witzel, 2005; Witzel et al., 2003). The research on the use of visual representations to teach students with LD emphasizes the importance of systematically fading from the concrete to the abstract level (Witzel, 2005; Witzel et al., 2003). The literature emphasizes the importance of using concrete materials as a way to understand the underlying concepts of mathematics, but stresses not to create student reliance on the materials (Fuchs et al., 2005; Fuchs et al., 2008; Witzel et al., 2003).

Sequencing Examples

Research on effective math instruction emphasizes the importance of using well-sequenced examples that provide a range of experiences from easy to difficult, or from concrete to abstract examples (Witzel, 2005; Witzel et al. 2003). A gradual movement from simple to more complex learning should be used each time a new problem type (e.g., reducing fractions or adding and subtracting fractions) is introduced. This type of approach strengthens both the procedural and conceptual knowledge of each problem type versus relying on students to generalize one skill to a variety of problems. The strong research base for the use of instructional sequences (e.g., Maccini & Hughes, 2000; Scheurermann, Deshler, & Schumaker, 2009; Witzel, 2005) suggests that teaching one skill or concept (e.g., one digit multiplication) within the range of examples significantly improves student understanding and skill retention compared to the introduction of multiple skills (e.g., one digit multiplication and two digit multiplication).

Principal Findings of Effective Instructional Components

The results found in the literature on effective instructional interventions cannot be attributed to just one instructional component, but rather a combination of them (e.g., explicit instruction + visual representations). For example, the use of teacher modeling, well-sequenced examples, guided practice, adequate feedback, and verbalization from students are components consistently observed in combination with explicit instruction (e.g., Owen & Fuchs, 2002; Ross & Braden, 1991; Tournaki, 2003; Xin, Jitendra, & Deatline-Buchman, 2005). Furthermore, many of the studies on explicit instruction include the use of heuristics and other forms of cognitive strategy instruction that support the acquisition and retention of skills (Van Luit & Naglieri, 1991; Woodward et al., 2001). Overall these findings suggest the combined use of all components when planning math instruction for students with LD will enhance acquisition, retention, and long term math success.

In addition to the findings by Gersten and colleagues (2008), and the research supporting the NMAP (2008) recommendations, two meta-analyses have addressed effective math interventions for both students with and at risk for LD (Baker, Gersten, & Lee, 2002; Kroesbergen & Van Luitt, 2003). The authors of both reviews also emphasize the importance of systematic and explicit instruction, visual representations, and the opportunity for students to verbalize mathematical thinking.

Many interventions containing one or more of these instructional components have been well documented as potential ways to support the learning, transfer, and maintenance of skills for students with and at risk for LD. The concrete-representational-abstract (CRA) gradual teaching sequence (e.g., Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass et al., 2003; Flores, 2010) has an extensive research base that supports the improvement of overall math achievement across many different domains. As recommended by the NMAP (2008) and through the continued research in the field of special education, the CRA sequence encompasses several essential instructional components; visual representations, explicit and systematic instruction, multiple opportunities for student verbalization and teacher feedback, ample opportunities for guided and independent practice, and sequenced examples (Baker et al., 2002; Gersten et al., 2008; Kroesbergen & Van Luitt, 2003).

Concrete-Representational-Abstract Instructional Sequence

Purpose and Rationale for the Use of CRA

The CRA sequence has been shown to be an effective instructional model that provides students with a systematic instructional path. It begins with students practicing mathematical skills using concrete objects or manipulatives. The second phase of instruction replaces the concrete objects with pictorial or visual representations of the concept. Often this representational phase is used in conjunction with a mnemonic device or other learning heuristic. The purpose of this additional scaffold is to provide support for remembering procedural skills that will transition the student from the representational to abstract phase with less difficulty (Flores, 2010; Hutchinson, 1993). The final phase, the abstract component of the sequence, replaces drawings and other visual representations with numbers and/or symbols. At each phase of instruction the teacher's role is to provide explicit instruction and guided practice; as well as appropriate amounts of feedback and formative assessment of skill mastery (Riccomini, Witzel, & Robbins, 2008).

As the focus of mathematical standards shifts from rule-driven computations and memorization of algorithms to higher-level conceptual understandings (NCTM 2000), it is increasingly important that students with disabilities are taught using interventions that support this type of learning. Historically, students with LD in math fail to develop a strong conceptual understanding of the underlying theories and operations necessary for success (Butler et al., 2003). Research has consistently documented teaching using concrete objects and developing representations of important concepts as a way to enhance the acquisition of these abstractions among students with math difficulties (Allsopp, 1999; Butler et al., 2003; Miller & Mercer, 1993; Witzel, 2005).

Effectiveness of the CRA Sequence

The CRA sequence is a well researched intervention and it is documented as an effective practice for a wide range of students. Studies including low-achieving students (Flores, 2010; Mercer & Miller, 1992), students with LD (Cass et al., 2003; Maccini &

Ruhl, 2000), students with emotional and behavioral disorders (EBD; Mercer & Miller, 1992; Riccommini et al., 2008) and students with intellectual disabilities (ID; Morin & Miller, 1998) all showed significant gains for students who received the CRA intervention. The gradual sequence has been documented as effective from grades as early as second (Harris, Miller, & Mercer, 1995), to late elementary (Riccomini et al., 2008; Ketterlin-Geller, Chard, & Fien, 2008), to junior high level students (Butler et al., 2003; Cass et al., 2003; Witzel, 2005), and throughout high school math classes (Maccini & Ruhl, 2000).

Among these various settings and populations of students, the CRA sequence has been used effectively for teaching a variety of math concepts. Peterson, Mercer, & O'Shea (1988) examined the effects of the CRA sequence on the understanding of place value for students with LD at the elementary and middle school levels. Students receiving the CRA instruction outperformed their grade level peers and also maintained their performance over time.

Researchers have also explored the use of the CRA sequence on the acquisition and retention of multiplication facts (Harris et al., 1995; Morin & Miller, 1998). Harris and colleagues (1995) studied the effects of teaching second graders with learning and behavioral disabilities basic multiplication facts using the gradual instructional sequence. The concrete phase consisted of counting groups of discs on plates, reinforcing that multiplication involves adding groups of objects. At the representational level, students were replacing the concrete materials with pictures of larger circles to symbolize groups and dots to replace the discs. Students participating in the CRA instruction all met the performance criterion of 80% accuracy or higher and performed similarly to their grade level peers following intervention. Morin and Miller (1998) also explored the effects of using the CRA sequence to teach multiplication. Using similar manipulatives to Harris and colleagues (1995), Morin and Miller (1998) investigated the use of the gradual sequence on three seventh grade students with ID. Intervention data on the three subjects indicated excellent progress and consistent performance of 80% accuracy or higher during intervention lessons (Morin & Miller, 1998).

Flores (2010) continued the research on using the CRA sequence to teach computations. Her research focused on the subtraction performance of six, third-grade students. Combined with the use of base ten blocks and pictorial representations of the concrete manipulatives, Flores used the DRAW (**D**iscover the sign, **R**ead the problem, **A**nswer or draw and check, **W**rite the answer) cognitive strategy along with a gradual teaching sequence. Results indicated that all six students met performance criteria of 80% accuracy or higher, and four of the six students maintained criterion level after six weeks of no instruction. Mancl, Miller, and Kennedy (2012) continued with similar research exploring the effects of teaching subtraction with regrouping using the CRA sequence to five students with disabilities in an elementary school. Similar to Flores (2010), they also combined the CRA sequence with the use of cognitive strategies (i.e.,, BBB; **B**igger number on **B**ottom means **B**reak down and trade & RENAME; **R**ead the problem, **E**xamine the ones column, **N**otes ones in the ones column, **A**ddress the tens column, **M**ark tens in the tens column & **E**xamine and note hundreds). Results reported by the authors indicated that all five students met performance criteria of 80% or higher and maintained skills at the criterion level one week later.

Miller and Mercer (1993) extended the research on CRA suggesting that the combination of instruction on word problems with computational concepts within the sequence would enhance the ability for students with LD to improve on word problem applications. At each phase of instruction the amount of information provided in the word problem gradually increased, including the use of extraneous information at the abstract level. After conducting two validation experiments, Miller and Mercer (1993) observed significant improvements in accuracy of word problem solutions among students with LD. In particular they noted that following treatment students with LD were able to consistently eliminate extraneous information from word problems.

In addition to using the CRA sequence to teach computations and problem solving skills, more recent research has examined the use of the CRA sequence to teach algebra (Maccini & Ruhl, 2000; Witzel, 2005) and algebra-based concepts (fractions; Butler et al., 2003; geometry, Cass et al., 2003) to students with LD.

Effectiveness of CRA for Teaching Algebra-Based Concepts

A total of 7 studies examined using CRA to teach algebra-based concepts to students with disabilities. See Table 1 for an overview of these studies.

Table 1

Summary of Studies using CRA to Teach Algebra

Article	Sample	Grade	Study Design	Math Skill Area	Results
Cass, Cates, Smith, & Jackson, 2003	3 students with LD	6-8	Multiple baseline across subjects and behaviors	Problem solving using formulas for area and perimeter	All students met performance criteria of 80% accuracy or better within an average of six days for instruction on perimeter. All students met performance criteria of 80% accuracy or better within an average of five days for instruction on area.
Jordan, Miller, & Mercer, 1999	125 students 5 with LD, 1 with EBD, 5 with VI or S/HI*	4	Repeated measures analysis of variance	Fraction concepts: Computation, equivalence, & comparison	The results favored CRA group compared to those in control. $p < 0.0001$
Maccini & Hughes, 2000	6 students with LD	9-12	Multiple probe across subjects	Addition, subtraction, multiplication, & division of integers	All students increased mean percent accuracy on problem representations ($M = 60.62$-86.96) and mean percent accuracy on problem solutions ($M = 38.87$ – 57.89)

Table 1---continued.

Maccini & Ruhl, 2000	3 students with LD	8	Multiple probe across subjects	Subtraction of integers	All students increased mean percent accuracy on problem representation (M = 57.33) and generalization measures of problem representation (M = 54.33). All students increased mean percent accuracy on problem solutions (M = 67) and generalization measures of problem solutions (M = 28.7).
Scheurermann, Deshler, & Schumaker, 2009	14 students with LD	6-8	Multiple baseline across subjects	Understanding and manipulating one-variable equations in word problems	All students exhibited medium to large growth in their computations of one-variable equations on posttest (ES = 2.32) and generalization measures (ES = .67; ES =.54).
Witzel, 2005	358 students 49 students with LD	6-7	Pre-post-follow-up design	Solving multiple-step linear functions (variable on both sides of equal sign)	The results favored CRA group compared to those in control instruction. $p < 0.01$

Table 1---continued.

Witzel, Mercer, & Miller, 2003	68 students with LD	6-7	Pre-post-follow-up design	Solving for a single variable in multiple-variable equations	The results favored CRA group compared to those in control who learned through traditional abstract instruction. $p < 0.01$

*VI=visually impairment; S/HI = speech/hearing impairment

A literature review by Maccini, McNaughton, and Ruhl (1999) examined effective strategies for teaching introductory level algebra skills to students with LD. This research review identified six studies that addressed this issue. Among these six studies only one study, a published dissertation, (Huntington, 1994) examined the effectiveness of the CRA sequence to teach pre-algebra skills to students with LD. The results of Huntington’s study showed significant gains in overall relational word problem solving and strong results on generalization measures. In the review by Maccini and colleagues the authors calculated high effects for the use of CRA when teaching algebra, but findings were somewhat limited to generalization due to the fact that only one study using this CRA was included in the review.

Initially following the review by Maccini and colleagues (1999) a study was published by Maccini and Hughes (2000) examining the effects of incorporating the STAR learning strategy, (Maccini, 1998), within the CRA sequence to teach students with LD to solve word problems involving integers. A multiple-probe design across six secondary students was used to investigate individual performance over time. During each instructional phase of the CRA sequence the research would (a) model 2-3 problems while thinking aloud, (b) provide up to five problems with guided practice, and (c) present five problems for independent practice. All participants improved on mean percentage accuracy on immediate measures from baseline (subtraction, M=38.87; addition, M=57.89; multiplication, M=41.48; division, M=42.98) to the abstract instructional phase of CRA (range across integer problem types, M=90.1-98.9). In addition, all students showed increased ranges across integer problem types on near- and far-transfer generalization measures (Maccini & Hughes, 2000). The implications from this study indicate that students need more time to learn addition and subtraction of integers opposed to multiplication and division.

Maccini and Ruhl (2000) replicated a similar study design involving the use of the STAR strategy and the CRA sequence to teach word problems involving integers. Participants were three secondary students with LD. Essential components of the instructional procedures included (a) the use of teacher modeling and think alouds, (b) inclusion of students in the think-aloud process, (c) guided and independent practice, (d) corrective feedback, and (e) the use of an advanced organizer. Findings from the study yielded similar results as that of Maccini and Hughes (2000) with all three students increasing their mean percent accuracy on problem solutions by a mean of 50.3 percentage points from baseline to the abstract phase of instruction. Results indicate that students were able to master the mathematical objectives that were taught in a relatively short amount of time with the ability to generalize what they learned to more complex math problems.

Cass and colleagues (2003) continued to explore the use of the CRA sequence to teach algebra-based concepts to students with LD. Their study examined the use of the gradual instructional sequence to teach area and perimeter word problems to three junior high students. A multiple-baseline across participants and two behaviors (area and perimeter) design was utilized to determine individual growth and to compare the effectiveness of the sequence across two types of word problems. The following instructional techniques were combined with the use of the CRA sequence to teach both area and perimeter: (a) teacher modeling, (b) verbal prompting, and (c) guided and independent practice. The authors reported that all three students following the abstract phase of treatment solved both types of word problems with 100% accuracy. All participants remained at a high level of accuracy on six maintenance measures administered three weeks following the intervention. The findings from this study add to the growing body of literature on the use of the CRA to teach algebra-based concepts to students with LD.

As the research base for the CRA approach began to grow, Butler and colleagues (2003) compared the effectiveness of using the approach with and without the concrete phase of instruction. Through random assignment, 50 middle school students were placed in a CRA or RA (representational-abstract) instructional group. A series of ten scripted lessons were completed focusing on solving word problems with fractions. Each lesson incorporated the use of (a) teacher modeling, (b) guided and independent practice, (c) routine feedback, (d) problem-solving practice, and (e) visual prompts and cues (i.e., note cards and advance organizers). Students in both treatment groups scored higher than a comparison group of general education eighth graders. However, students in the CRA group outperformed the RA group in the area of quantity fractions. Implications from their findings stress the importance of using concrete manipulatives when teaching initial fraction concept development.

As more students with LD begin to receive core mathematics in the general education setting, researchers are investigating practices that address the needs of students with or at risk for math disabilities in the general education setting. Witzel, Mercer, and Miller (2003) investigated the effectiveness of the CRA instructional sequence on thirty-four matched pairs of sixth and seventh grade students with LD and those at risk for failure in algebra. Unique to this study was that the intervention was conducted in an inclusive setting where the interventionist was the general education math teacher. A pre-post-follow-up design was used to determine the effectiveness of using the CRA sequence to teach more advanced algebra concepts including inverse operations and transformations. Instruction in both control and treatment groups included the use of teacher modeling and guided and independent practice. While the comparison group was taught the same content, instruction was limited to repeated abstract lessons as opposed to the use of pictorial representations and concrete objects. Both groups showed significant growth from pretest to posttest, but those students in the CRA group outperformed the comparison group on follow-up tests. An error pattern analysis also indicated that students in the CRA group made fewer computational errors. The implications reported in the study suggest that the use of concrete objects and pictorial representations can be effective at the secondary level, in addition to benefiting elementary-age students. Furthermore, the researchers concluded that the CRA approach can easily be implemented within general education settings and that it may be appropriate for students with and without disabilities in mathematics.

Witzel (2005) continued to examine the use of this strategy in general education settings. His study involved six general education math teachers and 231 middle school students, both general and special education. Forty-nine of the students were identified as having a learning disability. Each teacher taught one of the inclusion math classes using the CRA method and the other class using traditional abstract instruction. The algebra unit they taught consisted of 19 lessons ranging from solving inverse operations to solving linear functions with unknowns on one or both sides of the equal sign. To be consistent with research on effective practices both control and treatment group lessons consisted of (a) teacher modeling, (b) guided and independent practice, and (c) the use of visual cues (i.e.,, advanced organizer). While both groups made adequate growth from pretest to posttest, students in the CRA instructional group scored significantly higher on both immediate and follow-up tests. Similar to the findings by Witzel and colleagues (2003), the authors of this study concluded that the CRA model shows promise for inclusive settings and for the instruction of students with and without disabilities.

Scheuermann and colleagues (2009) explored the effects of the CRA sequence on middle school students with LD. The use of an explicit inquiry routine (inquiry dialogue & the CRA model) was used to develop the understanding of solving one-variable equations imbedded in word problems. The most important instructional variable examined was the verbalization of mathematical understandings taking place among students and teachers during the instructional phase. The study was designed to examine a student's ability to (a) abstractly solve problems over time, (b) concretely solve problems over time, (c) transfer skills to more complex equations, and (d) transfer skills learned in word problems to those found in a student's algebra textbook. The researchers indicated that students were able to accurately solve problems over time and the more familiar students became with the instructional format and dialogue the easier it was for them to generalize skills to more complex equations. The students maintained increased levels of performance for eleven weeks following the instructional intervention.

Theoretical Rationale

The CRA instructional sequence appears to be an effective intervention for teaching students with disabilities a range of math skills and concepts. According to Bruner's theory of learning (1960) the sequence contains several components that contribute to the success of the intervention. First, the gradual instructional sequence incorporates the use of explicit and systematic instruction imbedded with strong modeling and guided practice opportunities. Bruner's theory states that children develop strong conceptual understandings through this type of instruction and are able to then make more connections with future learning (Kuchey, 2010). Thus, students build confidence and accuracy in their ability to solve more complex math problems. Second, the CRA sequence uses appropriate visual representations that support, not only the conceptual, but also procedural understanding of math concepts. Bruner identifies these visual representations as "thinking through action." Actions then lead to "thinking through images and through language (abstract thought)" which creates strong conceptual knowledge. This supports student retention and transfer of knowledge. Finally CRA uses purposeful sequenced examples that allow students to move from easy to difficult concepts. Following this model, students receive adequate feedback and verbalization through the transition from concrete to abstract instruction, allowing students the opportunity to evaluate the process and solutions for each problem type. The evaluation process promotes success in higher level and more complex skill sets. Bruner's work supports the notion that mathematical learning should engage the mind in sequenced action and thought in order to develop and apply meaning (Kuchey, 2010).

Preteaching

Purposes and Rationale for Preteaching

Preteaching is instruction that takes place prior to the initial exposure of academic material. Students with disabilities often receive small group interventions in the form of preteaching or reteaching. Preteaching has been used for two main purposes. The first purpose is to give students, typically those who are struggling in the content area, the opportunity to master necessary requisite skills for the upcoming academic unit or lesson (Carnine, 1980; Kameenui & Carnine, 1986) and build a background knowledge to connect to the topic (Munk, Gibb, & Caldarella, 2010). Students with LD and at risk students struggle to organize and retain information which ultimately slows down their ability to apply prior knowledge to new, more complex problems and information (Miller & Mercer, 1997). A second purpose, or benefit, of preteaching is the improvement of self-concept and motivation among students (Lalley & Miller, 2006). Those who often struggle in the classroom, due to a lack of understanding and background knowledge, typically become passive learners and develop poor self-concepts which lead to further academic failure (Miller & Mercer, 1997).

Effectiveness of Preteaching

Preteaching has been documented as an effective intervention for a diverse population of students. English language learners (Chung, 2002; Fitzgerald & Graves, 2005), students with LD (Burns, Dean, & Foley, 2004; Hawkins, Hale, Sheely, & Ling, 2011; Munk et al., 2010), students with EBD (Anderson-Inman, 1981; Beck, Burns, & Lau, 2009), students with ID (Rose, 1984), and at risk students (Carnine, 1980; Lalley & Miller, 2006) have all benefited from preteaching. Studies examining the use of preteaching range in grade level from first grade (Carnine, 1980) to students in eleventh grade (Hawkins et al., 2011). The research base on preteaching extends across multiple content areas including science vocabulary instruction (Koury, 1996; Munk et al., 2010), reading comprehension (Burns et al., 2004; Graves & Cook, 1983), reading fluency (Hawkins et al., 2010; Rose, 1984), and math computations (Carnine, 1980; Lalley & Miller, 2006).

The majority of research on strategic preteaching activities has been examined in the area of reading comprehension and fluency. A meta-analysis on sight word research by Browder and Xin (1998) suggests preteaching has a significant impact on the acquisition of sight words for students with disabilities. In addition to these findings Burns, Dean, & Foley (2004) investigated the effects of preteaching unknown keywords on the reading comprehension and fluency of students with disabilities. More recent research (Munk et al., 2010) has examined the effects of preteaching science vocabulary on overall science performance among students with LD. The authors concluded that students receiving preteaching intervention were observed participating more frequently during the unit instruction and made significant improvements in their overall science performance.

The Effect of Preteaching on Math Performance

A total of 3 studies examined preteaching for Math. See Table 2 for an overview of these studies.

Table 2
Summary of Studies on Preteaching for Math

Article	Sample	Grade	Study Design	Skill Area	Results
Carnine, 1980	15 low achieving students	1	Pretest-posttest experimental design	Multiplication Facts	Students in the preteaching group met performance criteria in significantly less time than those in control group. No differences in accuracy of problem solutions.
Kameenui & Carnine, 1986	20 low achieving students	2	Repeated measures experimental design	Subtraction with regrouping	Students in the preteaching group outperformed those in control during instruction ($p < .05$), but there were no differences on posttest or follow up measures.
Lalley & Miller, 2006	24 low achieving students	3	Pretest-posttest experimental design	Third grade math concepts (i.e., number sense, multiplication, problem solving)	Students in both the preteaching and reteaching group significantly outperformed students in the control group ($p < .001$).

While a particularly large body of research supports the effectiveness of preteaching with the purpose of increasing academic performance, very few studies exist reporting the effects preteaching has specifically on math performance (Carnine, 1980; Kameenui & Carnine, 1986; Lalley & Miller, 2006). The first study to examine the effects of preteaching on math performance was used to teach basic multiplication computations prior to a lesson involving a more complex and conceptual understanding of the skill area. Carnine (1980) found that preteaching basic computation skills was time efficient and increased math performance for at risk first graders significantly more than similar peers' performance who received concurrent small group tutoring. Results indicated that students in the preteaching group reached class criterion for math performance at a faster rate than those in the concurrent group. The authors also reported that students receiving preteaching were better able to transfer skills from simple to more complex problems than peers in the concurrent teaching group.

A similar study by Kameenui & Carnine (1986) evaluated the effectiveness of preteaching one critical component skill for solving multi-step subtraction problems compared to teaching all component steps at the same time to at risk 2nd graders. The findings from the authors suggest that students who received the preteaching intervention were able to acquire the multi-step subtraction procedures with mastery at a faster rate than those who learned all components at the same time.

Lalley and Miller (2006) incorporated the preteaching intervention into the math curriculum in a slightly different manner. Their study analyzed the effects of preteaching on academic performance and increased self-concept among twenty-four at risk 3rd graders. The researchers compared the effects of preteaching to supplemental support that took place following initial instruction (i.e., reteaching). Lalley and Miller suggest that both interventions equally support increased academic performance. However, results indicate that students who received the preteaching intervention reported higher self-concept and increased motivation during academic instruction compared to peers in the re-teaching group.

Among the few studies evaluating the effects of preteaching on increased math performance, no studies report the effects for students with LD. However, the criteria for inclusion within these studies indicates that participants fell below the average range on standardized math assessments and displayed continuous poor performance on classroom math tasks (Carnine, 1980; Kameenui & Carnine, 1986; Lalley and Miller, 2006).

Essential Components of Preteaching

The essential components of preteaching include planning, instruction, and evaluation (Munk et al., 2010). Identifying students who will benefit from this preparatory instruction is critical. Effective preteaching models incorporate the use of formative assessments and continual student evaluation (Gersten et al., 2002; National Math Panel, 2008). Criterion-referenced screening tools may help determine who will benefit most from the supplemental instruction, but they could also potentially misidentify students (Munk et al., 2010). Using formative assessment tools in addition to other screeners to continually evaluate students' performance and progress is necessary when determining which students should participate in preteaching sessions (Munk et al., 2010). Formative assessment can also be beneficial when incorporated throughout instruction. The use of student and teacher verbalizations within guided practice and modeling opportunities are among many ways to collect these continuous data. Increasing the use of these effective instructional components will strengthen students' ability to evaluate their own performance and teachers' ability to adapt and/or modify the teaching and learning continuum.

Gersten and colleagues (2008) and Kroesbergen and Van Luitt (2003) suggest that combining effective instructional elements within classroom instruction can increase the performance of students with disabilities. These elements of instruction should align not only with classroom instructional practices but also with supplemental teaching, including the use of preteaching. Burns, Hodgson, Parker, and Fremont (2011) discuss the importance of effective intervention packages (i.e., preteaching combined with effective instructional strategies) to create opportunity for greater growth among students with or at risk for disabilities. Their research suggests that combining preteaching with effective instructional components (Gersten et al., 2008; Kroesbergen & Van Luitt, 2003) will increase the overall performance of students receiving the intervention.

Theoretical Rationale

A moderate sized research base suggests that preteaching skills to students with disabilities can support long term student success in content area learning. Preteaching has been identified as a strategy to build background knowledge and to support the retention of information (Munk et al., 2010). Bruner's theory of learning supports the use and growth of schemas in order for students to acquire and transfer new knowledge (Kuchey, 2010). While Bruner often supported student centered, activities-based learning; he found that students also needed guided practice and modeling found in explicit teaching. Research continues to support this theory, that the use of explicit instruction combined with small group instruction (i.e., preteaching) benefits students, particularly those who are struggling (e.g. Bottge, Rueda, Serlin, Hung, & Kwon, 2007; NMAP, 2008; Sayeski & Paulsen, 2010).

Providing students with LD concrete and tangible experiences assists them in making connections between skills learned in preteaching with other learning settings and contexts (Burns et al., 2004; Fuchs et al., 2004). Students with LD often lack the prior knowledge necessary to make these connections (Miller & Mercer, 1992). Preteaching can support students in building necessary conceptual frameworks that in turn enhance overall learning of new concepts. Students should be introduced to these visual representations during the initial preteaching sessions, however gradually moving from the use of concrete materials to abstract learning that is more aligned to other instructional contexts or settings (i.e., general education classroom).

Preteaching using the CRA Instructional Sequence

The strong research base supporting the use of the CRA instructional sequence to teach mathematical concepts to students with disabilities has shown that it is effective in many instructional settings. The research on preteaching interventions also lends itself as an effective intervention for students with LD in many content areas. While a small amount of literature examines the use of preteaching in mathematics, the findings suggest its promise. Imbedding a highly effective intervention, such as CRA, within a preteaching model may result in evidence supporting the use of preteaching for the purpose of math education and extend the research on using CRA within various content and instructional settings.

Research has shown the type of instruction that takes place within preteaching models is important (Burns et al., 2011). After examining the CRA sequence it is apparent that the use of many effective instructional components is imbedded within this well-sequenced model of instruction. No research to date has examined the combination of preteaching using the CRA sequence to teach mathematics. In addition, no research has examined the effects of the generalization of skills from preteaching instruction to general education classrooms. Furthermore, there has been limited research on the use of the CRA sequence to enhance core instruction for students with disabilities included in general education math settings (Witzel, 2005; Witzel et al., 2003).

Theoretical Rationale

Grounded in Bruner's theories on learning and cognitive development, research on the CRA instructional sequence and preteaching suggest the combination of the two will be effective for students with disabilities in math. The use of both strategies has been effective for students at risk for or with disabilities across a variety of settings and grade levels. In addition, the combination of preteaching and the CRA sequence contain many elements of the instructional components (i.e., explicit instruction, modeling, guided practice, sequenced examples, repetition, verbalization, and visual representations) advocated for by the National Math Panel (2008) and found to be effective in quality, research studies in the field of special education (e.g., Fuchs et al., 2008; Gersten et al., 2002; Gersten et al, 2008; Kroesbergen & Van Luit, 2003).

Instructional Context

It is necessary that the combination of preteaching and the CRA sequence be provided within the context of current instructional practices. This section will discuss the implications that tiered prevention models and curriculum may have on the implementation of effective math instruction.

Response to Intervention

Researchers advocate for the use of mathematics interventions that are delivered within a tiered prevention model in order to mitigate increasingly large numbers of students identified with math disabilities (NMAP, 2008; Gersten et al., 2009). The Response to Intervention (RTI) framework has been documented as an effective prevention model that ensures students receive high quality instruction and interventions that meet their unique learning needs (Fuchs, Fuchs, & Vaughn, 2008). The RTI model is a three tier model. Tier 1 is core instruction that all students receive in the general education setting. Tier 2 interventions are typically 20-40 minutes in length and provide additional instruction to students struggling with the core content. Tier 3 is a more comprehensive intervention that is provided to students not benefiting from Tier 2 support. Tier 2 and 3 support can take place as pull-out instruction or in a small group setting within the classroom. The use of preteaching and CRA could potentially be effective as Tier 2 or Tier 3 interventions.

The original premise behind the implementation of RTI models in schools was to provide early intervention for struggling students. In addition, according to the Center on Instruction Report in 2009, a major goal of the RTI process is to increase the collaboration between general and special education (Newman-Gonchar, Clarke, & Gersten, 2009). This coordination of instruction provides students with and at risk for disabilities support in content-area learning which helps to close the achievement gap. RTI varies in how it is implemented from district to district, and often from school to school. However, the components at its core are the same; (a) the use of evidence-practices, (b) regular screening, (c) preventative methods (i.e., small group interventions), (d) progress monitoring, and (e) the use of valid diagnostic tests to guide instructional planning. The first, but often neglected, step to implementing RTI is to evaluate the current instructional practices being used at the tier 1 level (i.e., general education classroom instruction).

Standards-Based Curriculum

With increasing numbers of students identified with or at risk for math disabilities, evaluating general education math instruction is important. Textbooks largely influence the content, skills, and manner in which students receive daily math instruction (Sayeski & Paulsen, 2010). However, research has shown that some of the most widely used textbooks do not align with practices that are effective for students with or at risk for learning disabilities (Doabler, Fien, Nelson-Walker, & Baker, 2012). The NMAP (2008) and the Common Core State Standards' (CCSS) Initiative (2010) recommend a stronger emphasis on conceptual understandings between the concrete and abstract relationships in math. In addition, the NMAP and CCSS advocate the use of collaborative problem solving and real life experiences to teach fundamental content. While many researchers (Fuchs et al., 2008; Gersten, Jordan, & Flojo, 2005; NMAP, 2008) strongly recommend a balance between conceptual and procedural knowledge instruction, many publishing companies now market *standards-based* or *reform* programs, that shift the focus away from basic skill development to a focus on application of the big ideas in mathematics (Parmer & Cawley, 1997). The CCSS continue to support the use of big ideas, but also encourage the depth of instruction compared to fast-paced curricula that cover many big ideas in short periods of time. The majority of reform programs are designed as a spiral curriculum, short units on topics that are continuously revisited throughout the school year (Sayeski & Paulsen, 2010). Created based on the NCTM (2000) standards, these curricula spend little time on fluency and computation; and the practice of teaching "depth" is not incorporated within the intended flow of the program. The overall goal of reform curricula, teaching students to approach math with greater understanding, is beneficial to increasing conceptual math skills. However, without the mastery of the basic skills students with learning disabilities and those at risk often end up with larger gaps in their overall learning of math concepts.

Students with disabilities have difficulty applying procedures correctly without frequent repetition and practice (Miller & Mercer, 1992). As problems become more difficult, the lack of procedural fluency and conceptual understanding leads to more frequent errors for students with disabilities (Geary, 2003). Research on students with math disabilities also indicates the need for developing strong schematic frameworks which can be done by connecting easy problems with those that are more difficult (Fuchs et al., 2004). Developing these connections can be difficult for students with disabilities within a spiral curriculum design.

With a heavy reliance on district curriculum for educational planning and instruction it is important to understand the instructional elements of the reform curricula (Sayeski & Paulsen, 2010). Sood and Jitendra (2007) completed a comparative analysis of the instruction in a reform-based textbook (EM; Bell et al., 2004) compared to that of a traditional textbook. While findings indicated that the reform curriculum provided many of the hands-on and real-world problem solving approaches advocated by the NCTM (2000) and NMAP (2008) it lacked in areas such as providing adequate feedback to students, explicit explanations, and sufficient amount of guided and independent practice (Sood & Jitendra, 2007), all of which are essential components of strong instruction for students with LD and those at risk for failure in math (Baker et al., 2002).

Overview and Hypothesis

Overall the results of studies involving the use of the CRA to teach mathematics indicate increased math performance for students with LD or those at risk for math failure. However, while CRA consistently increased overall math performance on immediate and maintenance measures, no studies included the use of measures to indicate students' ability to transfer skills to different settings. Baker and colleagues (2002) refer to the lack of transfer and generalization measures used in study designs as "the most troublesome areas of special education research and practice." Furthermore, few studies examined the efficacy of the CRA approach on students with disabilities within inclusive settings. Last, no studies incorporated the use of CRA instruction as Tier 2 or 3 interventions in the form of preteaching.

The overall aim of this study is to examine these components. First, the study examines the effectiveness of preteaching algebra-based concepts using the CRA approach on the acquisition and maintenance of algebra computations for students with LD and those at risk for failure in math. Second, the study examines the effectiveness of students receiving the treatment on the overall math performance in core instruction that utilizes a standards-based curriculum. Students receiving the intervention should increase both fluency and maintenance of algebra computations as they develop a stronger conceptual understanding of the specific skills. Once these skills are mastered at the abstract phase students will then be able to transfer these skills to more contextualized instruction used in the EM reform curricula, emphasized by the revised NCTM framework (2000).

Specifically, this study will investigate three null hypotheses:

Students with LD and those at risk for failure in math who receive CRA instruction in combination with preteaching as a supplemental intervention in math will:

(a) Not differ significantly in acquisition of algebra-based concepts following treatment compared to the control group.

(b) Not differ significantly in the maintenance of skills over a two week period similarly to their peers in the control group.

(c) Not differ significantly on the generalization of skills in the core instruction compared to the control group.

CHAPTER THREE

METHODS

Chapter Overview

The purpose of this study was to examine the effects of preteaching using the concrete-representational-abstract instructional sequence (CRA) has on the fluency, maintenance, and generalizability of algebra-based math skills for students with learning disabilities (LD) and students at risk for failure in math. Students were randomly assigned to one of two preteaching conditions (CRA or no treatment). A retired teacher and a doctoral student with previous teaching experience conducted the intervention. Students' math achievement scores, including fluency, maintenance, and follow-up measures; and treatment integrity data were collected throughout the study.

Participants and Setting

Students enrolled in one of four 6th grade classrooms across two elementary schools participated in the study. The state in which this research was conducted uses the Instructional Decision Making (IDM) model for the identification of students with disabilities. Similar to other tiered prevention models, the IDM model ensures students are receiving viable curriculum and that continuous assessment data are gathered and informed instructional decisions are made on a regular basis. Decisions based on diagnostic and formative assessments determine if students benefit from the tiered prevention model or if further support through special education services are required. This model does not identify students with specific disabilities but rather provides students with services that best address their individual needs. The learning needs of the students participating in this study are similar to those of students diagnosed with learning disabilities. Sixth grade students who were receiving tier 2 or tier 3 supplemental and intensive instruction in math; and those identified as having a math disability (based upon the State of Iowa guidelines) were participants.

A total of 40 students who were receiving tier 2 or tier 3 supplemental support for mathematics, or who were identified as having an IEP were invited to be participants in this study. Consent was given for 35 of the students to participate. Scores from three of the 35 participants were not included in final analysis due to high absenteeism. All three of the excluded students missed three or more of the lessons during their participation in the ten-lesson treatment phase. Therefore a total of 32 student scores were analyzed for the purpose of this study. Table 3 includes participant demographic information.

Table 3
Student Demographics

Descriptors	**Percentage* of Students (#)**	**Percentage of Students (#) at School One**	**Percentage of Students (#) at School Two**
Social Economic Status:			
Student receives free/reduced lunch	56 (18)	12 (4)	44 (14)
Race:			
African American	34 (11)	22 (7)	12 (4)
White	44 (14)	22 (7)	22 (7)
Hispanic	19 (6)	12 (4)	7 (2)
Middle-Eastern	3 (1)	0 (0)	3 (1)

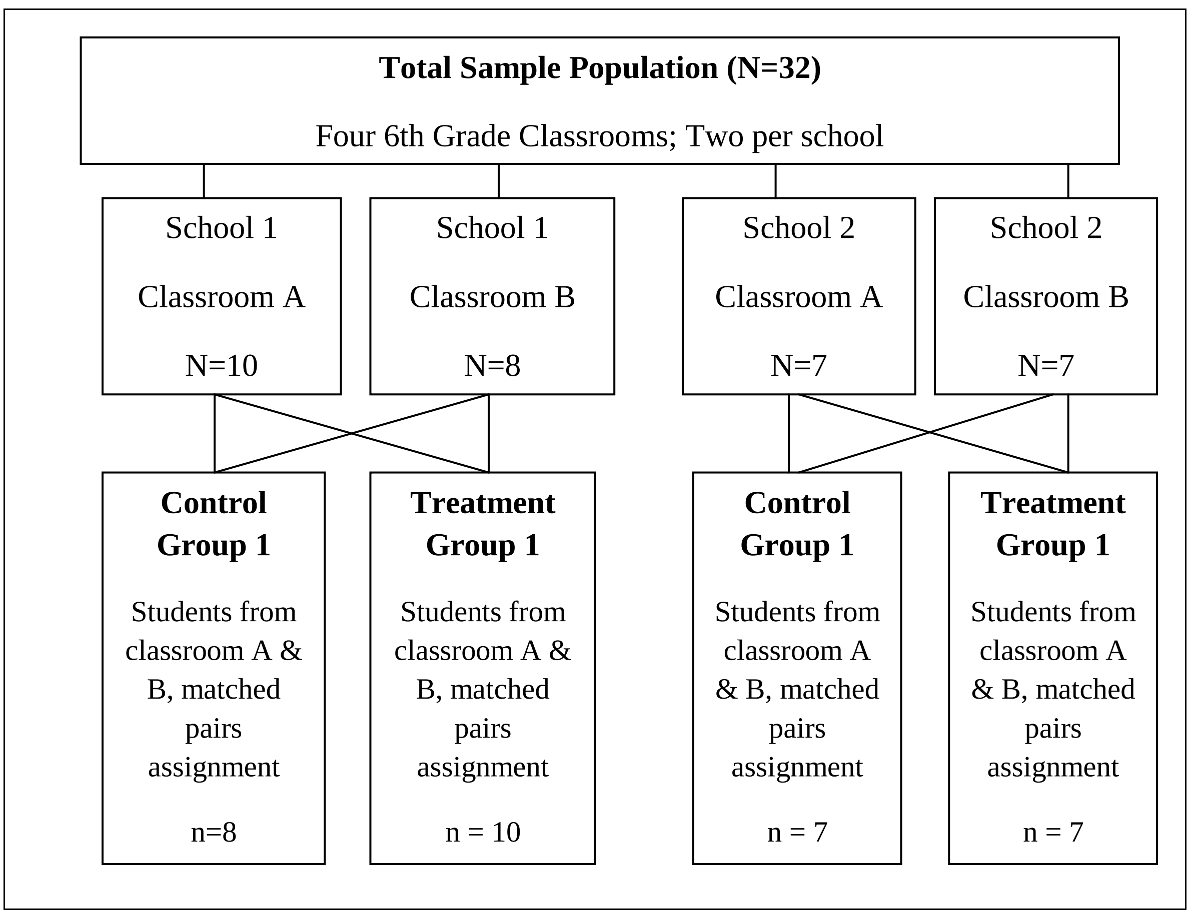
Table 3---continued.

Gender:			
Female	40 (13)	22 (7)	18 (6)
Male	60 (19)	34 (11)	26 (8)
Level of Instruction Support:			
Tier 2 Supplemental	59 (19)	32 (10)	27 (9)
Tier 3 Comprehensive	26 (8)	22 (7)	3 (1)
Special Education (IEP)	15 (5)	3 (1)	12 (4)

Note: *Percentages rounded to the nearest whole number.

Figure 1 shows how students were placed within study design.

Figure 1
Student selection within study design

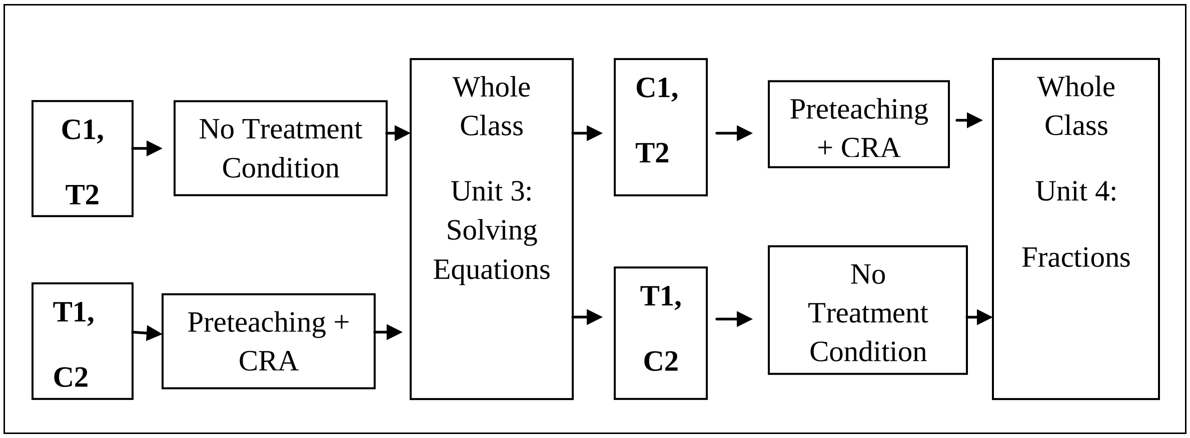


Study Design

A pre-post-follow-up experimental design was used to examine the effect of the intervention on students' math achievement. Students were randomly assigned to treatment or control. A block on classrooms was made to ensure an equal number of students from each classroom were represented in the control and treatment groups. This assisted in controlling for confounding effects due to variations in routines and instruction. Next students were stratified based on students' pretest scores and intervention level criteria (i.e., supplemental, intensive, or students with individual education plans (IEPs)), to ensure the two groups were comparable in ability levels and that student populations were equally represented in the control and treatment groups. This assisted in controlling for confounding effects due to variations in student ability and previous levels of math support.

The intervention was replicated across two math units related to teaching algebra-based concepts: Solving Equations and Fractions. The same students selected for the first math unit remained as participants for the replication during the second unit. However, students selected for the treatment condition during the Solving Equations unit were reassigned to the control condition for the second unit, Fractions. Likewise, those assigned to control for the first unit of instruction were reassigned to the treatment condition for the replication. This allowed for potential replication of the effects across two math content areas. Figure 2 provides an overview of the treatment intervention and assigned conditions.

Figure 2
Treatment Intervention and Conditions



Note: C=control group, T = treatment group

Intervention

The treatment condition was a combination of preteaching and the use of the CRA instructional sequence. Prior to each unit, Solving Equations and Fractions, researchers pretaught students these essential prerequisite skills necessary for success in the upcoming unit at the concrete, representational, and abstract levels of learning. Instruction was provided following the guidelines in the unit intervention sections below.

Solving Equations Unit intervention. Two weeks prior to the beginning of the Everyday Math (EM) algebra unit (Solving Equations), students in the treatment group received 10 sessions (5 per week for 30 minutes) that addressed three prerequisite skills. These three skills were: (1) adding, subtracting, and multiplying integers; (2) simplifying equations; and (3) solving equations with one unknown variable. During the first session, students were instructed on how to use the manipulatives and were taught new vocabulary used throughout the CRA condition (i.e., variables, divisor line, and coefficients). The next day of instruction students learned the first skill through modeling and practice using concrete manipulatives. The third day of instruction taught the same skill area through modeling and practice using pictorial representations of the previously used concrete materials. The fourth day of instruction again taught this same skill using abstract guided and independent practice worksheets. These steps were repeated for the remaining two skill areas. This took place days 5-10 of the treatment condition.

During intervention implementation, students in the control group attended study skills but did not work on activities related to mathematics. Students identified as having Individual Education Plans (IEPs) did not receive supplemental instruction in the area of algebra-based concepts throughout the duration of the study but still received support from the special education teacher in other areas of mathematics. See Table 4 for Solving Equations preteaching schedule.

Table 4
Solving Equations Preteaching Schedule

Intervention Day	**Stage of CRA Sequence**	**Skill**
1	Training using manipulatives.	Understanding key vocabulary (i.e., variable, coefficient, and divisor line)
2	Concrete (C)	Adding, subtracting, and multiplying integers.
3	Representational (R)	Adding, subtracting, and multiplying integers.
4	Abstract (A)	Adding, subtracting, and multiplying integers.
5	C	Simplifying equations.
6	R	Simplifying equations.
7	A	Simplifying equations.
8	C	Solving equations with one unknown variable.
9	R	Solving equations with one unknown variable.
10	A	Solving equations with one unknown variable.

Fraction Unit intervention. Similar to the equations unit, two weeks prior to the beginning of the EM fraction unit, students in the treatment group received 10 sessions (5 per week for 30 minutes) that addressed three prerequisite skills. These three skills were: (1) comparing two fractions with the symbols >, =, or <; (2) reducing fractions to simplest form; and (3) adding and subtracting fractions with like and unlike denominators. During the first day of instruction, students were trained to use the manipulatives and were taught new vocabulary used throughout the CRA condition (i.e., equivalent, numerator, and denominator). The next day of instruction students learned the first skill through modeling and practice using concrete manipulatives. The third day of instruction taught the same skill area through modeling and practice using pictorial representations of the previously used concrete materials. The fourth day of instruction again taught this same skill using abstract guided and independent practice worksheets. These steps were repeated for the remaining two skill areas. This took place days 5-10 of the treatment condition.

During intervention implementation, students in the control group attended study skills but did not work on activities related to fractions. See Table 5 for Fractions preteaching schedule.

Table 5
Fractions Preteaching Schedule

Intervention Day	**Stage of CRA Sequence**	**Skill**
1	Training using manipulatives.	Understanding key vocabulary (i.e., variable, coefficient, and divisor line)
2	Concrete (C)	Comparing two fractions with the symbols >, =, or <

Table 5---continued.

3	Representational (R)	Comparing two fractions with the symbols >, =, or <
4	Abstract (A)	Comparing two fractions with the symbols >, =, or <
5	C	Reducing fractions to simplest form
6	R	Reducing fractions to simplest form
7	A	Reducing fractions to simplest form
8	C	Adding and subtracting fractions with like and unlike denominators
9	R	Adding and subtracting fractions with like and unlike denominators
10	A	Adding and subtracting fractions with like and unlike denominators

Material

Three instructional workbooks were used for the preteaching intervention phase; *Solving Equations* (Witzel & Riccomini, 2011), *Computation of Fractions* (Witzel & Riccomini, 2008), *and Computation of Integers* (Riccomini & Witzel, 2009). These workbooks contain step-by-step, scripted lessons for teaching algebra-based concepts using the CRA sequence. Descriptions of manipulatives and explanations for their use are included in the workbooks. For the purpose of this study, materials from each book that address the target skills taught in preteaching groups were used. The recommended manipulatives (i.e., Popsicle sticks, string, tongue depressors, and cups) were provided in individual student kits for use in the intervention groups.

Treatment integrity checklists were created that contained the essential components for each instructional phase of the gradual teaching sequence (see Appendix

A). All lessons were videotaped to collect these data. These data were collected during the preteaching condition only.

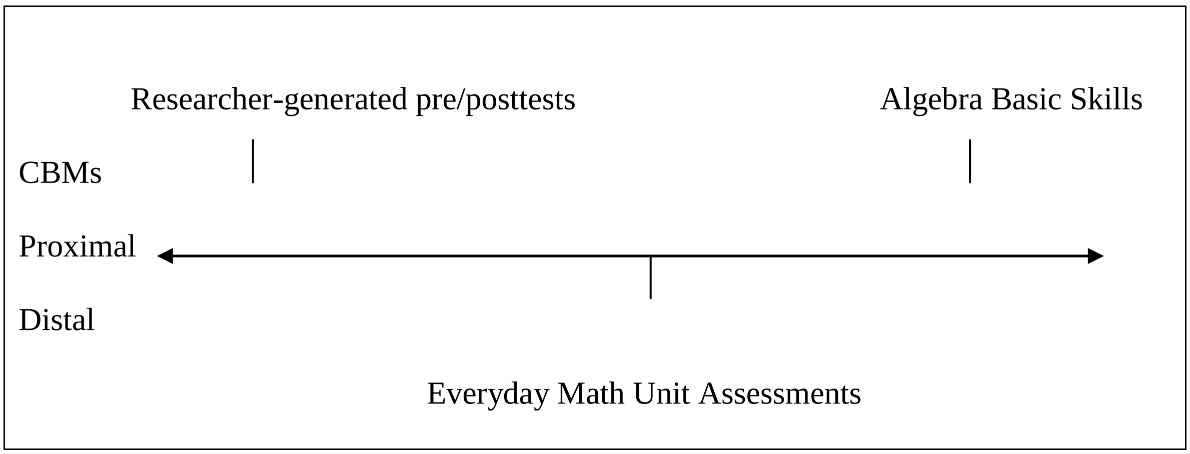
Teacher Training

There were two interventionists for all preteaching groups, one at each school. One of the interventionists was the primary investigator and female graduate student with previous teaching experience. The second interventionist was a retired, female teacher with previous teaching experience in special and general education. Both teachers were trained to implement the strategy through webinars on CRA instruction and by reading related instructional materials (Witzel & Riccomini, 2008; WWC Intervention Report, 2010). Following initial training, teachers were observed teaching one 30-minute lesson. Specifically, these teachers were observed for the following criteria: (1) Using appropriate explanations consistent with the purpose of the instructional stage, (2) providing appropriate student feedback, (3) monitoring student performance during guided and independent practice, and (4) following sequential order of the lessons consistent with instructional manual. Both teachers were able to implement the intervention consistently and fluently after two 2-hour sessions.

Dependent Variables

The Basic Skills Algebra curriculum-based measure (CBM) (Foegen & Lind, 2009) was used for the pretest, posttest, and follow-up measures. The recommended administration time of 3 minutes, compared to other CBMs typically administered within a 1 minute time frame, was used due to the complex nature of the problem types (i.e.,, evaluating equations and simplifying fractions). The Basic Skills Algebra CBM has an alternate form reliability of .83 and a test-retest reliability of .86. The measure also has a strong correlation (.73) with the Iowa Test of Basic Skills. In addition, the use of a researcher generated pre- and post-test consisting of 18 problems, 6 related to each of the 3 identified skill areas was used. This measure is more closely aligned to the EM instructional units. See Figure 3 for proximal-distal continuum.

Figure 3
Proximal-Distal Continuum



Following whole class unit instruction, students in both treatment and control groups completed the EM unit assessment. This measure was used to evaluate the transfer of skills from the preteaching setting into whole class unit instruction. This test contained 25 items, 15 of which required use of the requisite skills taught during preteaching treatment to solve. While requiring students to use the pretaught content to solve these 15 problems, the selected problems were much more complex in nature. For example, a word problem requiring students to simplify and solve for unknown variables may have also contained exponents or other difficult content not addressed during preteaching. The other ten items incorporated the use of the skill areas taught during this phase of the experiment, but did not require knowledge of the pretaught skills to solve. Therefore only the 15 selected problems were included in the analysis. A follow-up measure similar to the researcher generated posttest (18 items; six related to each of the 3 skill areas taught during preteaching) was administered at the same time as the unit test. Table 6 provides a schedule of assessment administration.

Table 6
Schedule of Assessment Administration

	Pretest	Posttest	Unit Assessment	Follow-up Assessment
Assessment	Algebra Basic Skills CBM & RG pretest	Algebra Basic Skills CBM & RG posttest v.1	Everyday Math Unit Assessment	RG posttest v. 2
Timeline	Prior to treatment beginning.	Immediately following treatment.	Two weeks following the end of treatment.	Two weeks following the end of treatment (same as unit assessment).

Note: RG = researcher generated

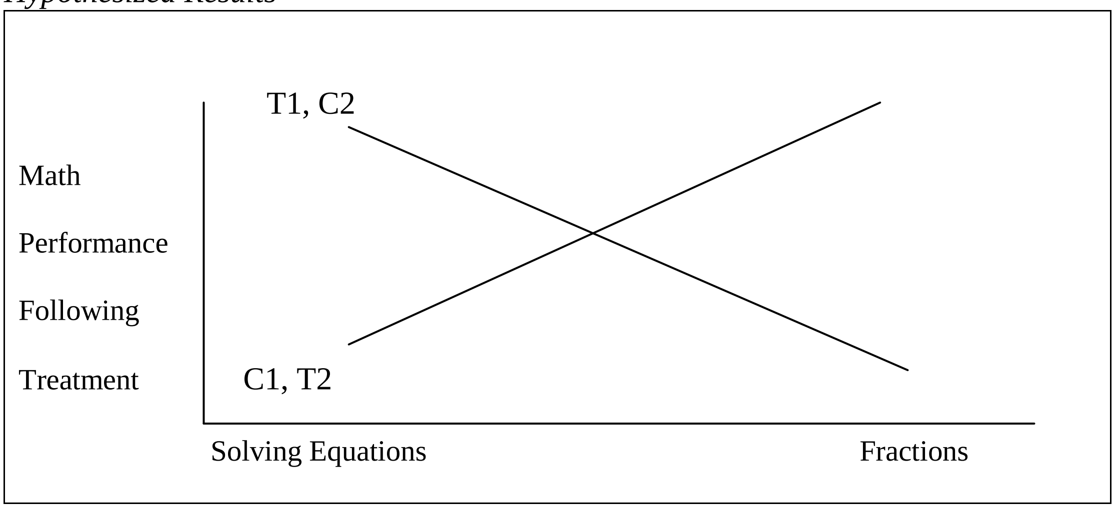
Interrater Reliability

All student worksheets and assessments were scored concurrently by the researcher, second interventionist (retired teacher), and one of the four classroom teachers. The three scorers agreed 100% on the outcome results.

Analysis of Results

This study's aim was to test the null hypothesis that no significant difference was present in math performance scores between control and treatment groups. Based on the theories of learning and reported literature described in Chapter 2, it was hypothesized that students with LD and those at risk for failure in math would both show increased performance in fluency, maintenance, and generalization of algebra-based computations following intervention compared to their peers in the control group. Figure 4 displays these hypothesized results for treatment and control groups for both math units, Solving Equations and Fractions.

Figure 4
Hypothesized Results



Note: T= treatment group, C= control group

Procedures

The study was conducted in 10 steps. (1) Researchers were trained following the criteria under the teacher training section. (2) Students were selected for participation in the study from the four classrooms based on eligibility for supplemental, intensive, or special education support in the area of math. (3) Once students were selected for participation and parent consent and student assent were collected, students took a pretest assessment. (4) Students were randomly assigned to a preteaching condition (CRA or no treatment). (5) Two weeks prior to the beginning of the EM unit, Solving Equations, students worked with trained interventionists to master these identified requisite skill areas. Students were instructed following the guidelines in the intervention section. (6) Following treatment students were administered posttests. (7) All students in both conditions participated in Unit 3, Solving Equations, in the general education classroom. (8) Upon completion of the unit, all students took the EM unit assessment. (9) Four weeks after the completion of the unit students in both conditions were administered follow-up assessments. (10) Steps 5-9 were replicated for unit instruction on Fractions. During this replication control 1 and treatment 1 students changed condition.

CHAPTER FOUR

RESULTS

Chapter Overview

The purpose of this study was to examine the effects of preteaching algebra-based concepts on overall algebra acquisition, maintenance, and generalization of students with or at risk for learning disabilities. Students in this study were randomly assigned to the treatment or control group. The treatment group received small group instruction in the form of preteaching using the concrete-representational-abstract (CRA) instructional approach. The intervention took place prior to whole-class (i.e., general education) unit instruction of solving equations and fractions. Researchers used a pre-post-follow-up experimental design to draw inferences about research questions and hypotheses related to treatment.

This chapter contains the Preliminary Testing, Hypothesis Testing, and Summary of Results sections. The Preliminary Testing section includes the types of analyses used in the study, the results of the tests necessary to determine that the underlying assumptions of t-tests and ANCOVA tests were met, and the treatment integrity data. The Hypothesis Testing section includes solving equations and fraction student performance data and the results of statistical comparisons of interest. The Summary of Results section contains a brief, descriptive summary of the statistical analyses related to the initial research questions and hypotheses.

Preliminary Testing

Group Comparisons

Prior to the beginning of the intervention an ANOVA between treatment and subgroups (i.e., groups, schools) was performed. This test was run to ensure that significant differences in performance between subgroups did not exist. The stratified sampling procedures discussed in chapter 3 assisted in creating equivalent groupings. The effect of school however could not be controlled for through the assignment. The results of the ANOVA indicate that significant differences did not occur between schools or groups. See Appendix B-C for summary of results.

Types of Analyses

An independent samples t-test was used to analyze the results of the preteaching + CRA treatment using the scores from the researcher generated (RG) posttest and Algebra Basic Skills CBM (Foegen & Lind, 2009). An analysis of covariance (ANCOVA) using pretest scores as the covariate was used to make comparisons between groups using the Everyday Math (EM) Unit Assessment and RG follow-up measure.

Assumptions of T-tests and ANCOVAs

There are general assumptions that apply when running t-tests and an analysis of covariance (ANCOVA), both parametric techniques. The first assumption is that the dependent variable is at the interval or ratio level, meaning it uses a continuous versus discrete scale. The second assumption is that participants were randomly assigned to groups from the population. The third assumption is that the scores that make up the collection of data are independent of one another, and not influenced by other observations. None of these assumptions were violated within this study design.

The fourth assumption when using parametric techniques is that the population from which the sample is drawn is normally distributed. While this assumption is often violated within educational research, with sample sizes of thirty or more students, this violation does not typically cause problems with the use of statistical techniques such as the t-test or ANCOVA (Glass, Peckham, & Sanders, 1972). The sample size for this particular study included 32 students, 15 in one cell and 17 in the other, therefore eliminating issues related to potential violations of normality. The last assumption when using parametric techniques is that samples are obtained from populations of equal variance. To determine if this assumption was violated, a Levene's test of equality of variance was performed for each of the analyses included in this chapter. All tests were found insignificant thereby the assumption of homogeneity of variances was not violated. Results are summarized in Appendix D-E.

Similar to the assumption of normal distribution, if the assumption of homogeneity of variance is violated, ANCOVA techniques are robust provided that the sample size of each group is similar (Feldt, 1993; Stevens, 1996) making this violation inconsequential. When using t-test analyses, if the assumption of homogeneity is violated, the t-value will be corrected to reduce potential Type 2 error.

In addition to the assumptions mentioned above, ANCOVA tests specifically require that the assumption of linear homogeneity be met (Feldt, 1993). A scatter plot was created of the distribution of scores for each group (see Appendix F-I). After inspecting each scatter plot it was clear that each relationship was linear, so the assumption of a linear relationship was not violated.

Treatment Integrity Data

Treatment integrity data were collected during all instructional sessions. Following both interventions the interventionists (researcher and teacher) completed the treatment fidelity checklists while watching the recorded sessions. The total number of agreements at each stage (i.e., concrete, representational, and abstract) were divided by the total possible points on the checklist and multiplied by 100. Agreement between the researchers was at a 94.5% criterion. The average percentage of treatment fidelity for the Solving Equations Unit between schools and researchers was 94.3% over the nine instructional settings. The average percentage of treatment fidelity for the Fractions Unit between schools and researchers was 97.9% over the nine instructional settings. Table 7 contains the summary of treatment fidelity data for both units over each instructional phase.

Table 7

Treatment Integrity Data

Unit of Instruction	Teacher	Concrete Phase	Representational Phase	Abstract Phase	Total
Solving Equations	1	93(3)*	100(3)	93(3)	95.3(9)
Solving Equations	2	100 (3)	93(3)	87(10)	93.3(9)
Fractions	1	100(3)	100(3)	93(3)	97.7(9)
Fractions	2	100(3)	100(3)	93(3)	97.7(9)

Note: Percentage (Number of sessions data in which data was collected).

Hypothesis Testing

This study tests three null hypotheses across two instructional contexts (i.e., solving equations and fractions). The first comparison made within each context examined the immediate mean differences on the initial preteaching condition between control and treatment groups. Two measures, a proximal, researcher generated (RG) test and a distal measure (Algebra Basic Skills CBM) were administered for the purpose of this comparison. The second comparison made within each instructional context analyzed the difference in mean performance on the generalizability of skills to general education (Everyday Math Unit Assessment) taking into account initial pretest scores. The third comparison made across both contexts analyzed the mean performance difference on a two-week follow-up measure.

Solving Equations Unit Comparisons

Immediate comparison of treatment. On the pretest, students in the treatment group performed on average slightly higher than students in the control group. Preliminary testing indicated these differences were not significant. Students in the control group did not make any growth from pretest to posttest. However, following the intervention students in the treatment group scored on average higher than the control group. Inspection of the descriptive data in Table 8 revealed a mean difference of 4.29 in treatment student performance on the posttest measure.

Table 8
Descriptive Statistics of Posttest Math Performance for Solving Equations

Group	Pretest Mean (SD)	Posttest Mean (SD)	Mean Difference
Control	1.47(1.77)	1.47(1.77)	0.00
Treatment	2.06(1.86)	6.35(3.62)	4.29

An independent samples t-test was used to make three comparisons and to determine if observed differences on RG posttest measure could be inferred to the population. This analysis revealed a significant difference between control and treatment groups on the RG posttest measure (t (30) = 4.96; p < .001) and a large effect (Effect Size

(ES) = 1.71). The null hypothesis that there would be no difference between control and treatment groups on posttest measures was rejected.

A similar analysis was run to determine if the observed differences in CBM post treatment performance could be inferred to the population. The results of the independent samples t-test found no significant difference between groups this measure (*t* (30) = 1.35, *p* = .19) and small effects (ES = .34). See Table 9 for descriptive statistics of CBMs following Solving Equations instruction.

Table 9
Descriptive Statistics of Algebra Basic Skills CBM

Group	Pretest Mean (SD)	Posttest Mean (SD)	Mean Difference
Control	4.27(2.43)	5.13(2.33)	.86
Treatment	4.35(2.23)	6.12(3.46)	1.77

The third comparison made immediately following treatment, was the comparison between students receiving special education support or in the process of being referred for services, compared to those students receiving tier 2 supplemental supports. An independent samples t-test analysis was used to compare performance between these groups on RG posttest for treatment group students only. The results indicated no difference between intervention levels on the RG posttest measure (*t* (15) = .88; *p* = .95) and medium effect (ES = .56). See Table 10 for descriptive statistics of posttest math performance of treatment grouped by intervention level.

Table 10
Descriptive Statistics of Treatment Posttest Performance by Intervention Level

Group	Mean	Standard Deviation
IEP/Intensive	5.70	1.09
Supplemental	7.29	3.90

Generalization measure of treatment comparison. The EM Unit assessment served as a generalized measure of treatment across settings (i.e., small group to general education) as well as from simple to more complex content instruction. Inspection of the mean scores in Table 11 revealed that students in the treatment group performed lower than those in the control group.

Table 11
Descriptive Statistics of EM Unit Test Performance for Solving Equations

Group	EM Mean Score	EM Standard Deviation
Control	11.00	2.42
Treatment	10.41	2.67

To determine, if this difference was significant an ANCOVA test was run using the pretest as the covariate. The results found in Table 12 indicate that there was no significant difference between group performance on the EM Unit measure (F (1, 29) = .53; p =.47) and small effect (ES = -.23) in favor of the control group. The null hypothesis that groups would not differ on the generalization of math performance from the preteaching setting to general education instruction was not rejected.

Table 12

ANCOVA Results for EM Unit between Groups Comparisons

Source	df	Sum of Squares	Mean Square	F	P
Pre-test	1	2.16	2.16	.32	.57
Group	1	3.54	3.54	.53	.47
Error	29	193.961	6.69		

Follow-up measure to treatment comparison. A comparison between groups on the follow-up measure, administered two weeks following treatment, was made to determine if the mean differences observed could be inferred to the population. Upon inspection of the mean scores in Table 13, it was noted that the treatment group scored an average of 1.79 points higher than students in the control group.

Table 13

Descriptive Statistics of Follow-Up Performance for Solving Equations

Group	Follow-up Mean Score	Follow-up Standard Deviation
Control	4.33	2.25
Treatment	6.12	2.59

An ANCOVA test was run to determine the significance of this mean difference and if it could be inferred to the population. The results indicate a significant difference between groups on the follow-up measure (F (1, 29) = 4.31; $p < .05$) and a large effect (ES = .74). Table 14 summarizes these results. Thus, the null hypothesis that groups would not differ in performance on follow-up measures was rejected.

Table 14
ANCOVA Results for Follow-Up between Groups Comparisons

Source	df	Sum of Squares	Mean Square	F	P
Pre-test	1	1.11	1.11	.18	.67
Group	1	26.437	26.437	4.31	.05
Error	29	177.99	6.14		

Fraction Unit Comparisons

Immediate comparison of treatment. Students in the control group performed slightly higher on the pretest; however preliminary testing indicated these differences were not significant. As indicated in Table 15, students in the control group performed lower on the posttest than the pretest, with a mean difference of -.04. Following treatment, students in the experimental group performed an average of 4.05 points higher than their pretest score.

Table 15
Descriptive Statistics of Posttest Math Performance for Fractions

Group	Pretest Mean (SD)	Posttest Mean (SD)	Mean Difference
Control	8.80(4.52)	8.76(4.68)	-.04
Treatment	9.35(4.37)	13.40(3.94)	4.05

An independent samples t-test was used to determine if observed differences on RG posttest measure could be inferred to the population. This analysis revealed a significant difference between control and treatment groups on the RG posttest measure (t (30) = 3.69; $p < .001$) and a large effect (ES = .67). The null hypothesis that there would be no difference between control and treatment groups on posttest measures was rejected.

A similar analysis was run to determine if the observed differences in CBM post treatment performance could be inferred to the population. The results of the independent samples t-test found no significant difference between groups on this measure (t (30) = .25, $p = .80$) and a very small effect (ES = .09) in favor of students in the treatment group. Table 16 provides the descriptive statistics for CBMs following Fraction preteaching.

Table 16
Descriptive Statistics of Algebra Basic Skills CBM

Group	Pretest Mean (SD)	Posttest Mean (SD)	Mean Difference
Control	4.35(2.23)	6.53(3.54)	2.18
Treatment	4.27(2.43)	6.80(2.46)	2.53

The third comparison made immediately following treatment, was the comparison between students receiving special education support or in the process of being referred for services compared to those students receiving tier 2 supplemental supports. An independent samples t-test analysis was used to compare performance between these groups on RG posttest for treatment group students only. The results indicate no difference between intervention levels on the RG posttest measure (*t* (15) = .67; *p* = .51) on performance and a medium effect (ES = .45) in favor of students receiving supplemental support. See Table 17 for descriptive statistics of posttest math performance of treatment grouped by intervention level.

Table 17
Descriptive Statistics of Treatment Posttest Performance by Intervention Level

Group	Mean	Standard Deviation
IEP/Intensive	12.00	3.61
Supplemental	13.75	4.10

Generalization measure of treatment comparison. Students in both control and treatment groups performed similarly on the EM Unit assessment. However, inspection of the mean scores in Table 18 reveals that students in the treatment group performed slightly higher on average than those in the control group.

Table 18
Descriptive Statistics of EM Unit Test Performance for Fractions

Group	EM Unit Mean Score	EM Unit Standard Deviation
Control	8.29	2.82
Treatment	9.73	2.81

To determine, if this difference was significant an ANCOVA test was run using the pretest as the covariate. The results found in Table 19, indicate that there was no significant difference between group performance on the EM Unit measure (F (1, 29) = 2.24, p =.15) and a medium effect (ES = .51) in favor of treatment group students. The null hypothesis that groups would not differ on the generalization of math performance from the preteaching setting to general education instruction was not rejected.

Table 19
ANCOVA Results for EM Unit between Groups Comparisons

Source	df	Sum of Squares	Mean Square	F	P
Pre-test	1	28.38	28.38	3.29	.08
Group	1	19.32	19.32	2.24	.145
Error	29	250.081	8.623		

Follow-up measure to treatment comparison. A follow-up measure of the fraction skills taught during preteaching was administered two weeks following completion of intervention. Upon inspection of the descriptive statistics summarized in

Table 20, it was noted that the treatment group scored on average lower than the control group.

Table 20
Descriptive Statistics of Follow-Up Performance for Fraction Unit

Group	Follow-up Mean Score	Follow-up Standard Deviation
Control	5.41	3.08
Treatment	4.86	3.76

An ANCOVA test was run to determine if these results could be inferred to the population. The results indicate no significant difference between groups on the follow-up measure (F (1, 29) = .18; p =.67) and a small effect (ES = .16) in favor of treatment group students. Table 21 summarizes these results. Thus, the null hypothesis that groups would not differ in performance on follow-up measures for fractions was not rejected.

Table 21
ANCOVA Results for Follow-Up between Groups Comparisons

Source	df	Sum of Squares	Mean Square	F	P
Pre-test	1	.56	.56	.05	.83
Group	1	2.21	2.21	.18	.67
Error	29	349.294	12.05		

Summary of Results

Two sets of null hypotheses were investigated in this study. The first set examined the effects preteaching algebra using CRA instruction had on the immediate algebra performance; and generalization and maintenance of these skills for students with or at risk for math disabilities. Findings indicate that the mean growth for students in the treatment was statistically significant compared to those who did not receive the intervention (t (30) = 4.96; $p < .001$). The EM Unit assessment was the generalization measure for this study. The mean scores in performance on this measure were not significantly different between groups (F (1, 29) = .53; p =.47). The results of the analyses performed on the follow-up measure (i.e., measure of skill maintenance) were found to be statistically significant (F (1, 29) = 4.31; $p < .05$). See Table 22 for a summary of the first set of null hypotheses.

Table 22
Summary of Null Hypotheses for Solving Equations

Null Hypotheses	Retain or Reject
Groups will not differ significantly on the acquisition of algebra skills immediately following treatment.	Reject
Groups will not differ significantly on the generalization of algebra skills to core instruction as measured by the EM Unit Assessment.	Fail to reject
Groups will not differ significantly on the maintenance of algebra skills two weeks following treatment.	Reject

The second set of null hypotheses examined the effects preteaching fractions using CRA instruction had on the immediate fraction performance; and generalization and maintenance of these skills for students with or at risk for math disabilities. Findings indicate that the mean growth for students in the treatment was statistically significant compared to those who did not receive the intervention (t (30) = 3.69; p < .001). The mean scores in performance on the EM Unit measure were not significantly different between groups (F (1, 29) = 2.24, p = .15). The results of the analyses performed on the follow-up measure were also not statistically significant (F (1, 29) = .18; p = .67). See Table 23 for a summary of the first set of null hypotheses.

Table 23
Summary of Null Hypotheses for Fractions

Null Hypotheses	Retain or Reject
Groups will not differ significantly on the acquisition of fraction skills immediately following treatment.	Reject
Groups will not differ significantly on the generalization of fraction skills to core instruction as measured by the EM Unit Assessment.	Fail to reject
Groups will not differ significantly on the maintenance of fraction skills two weeks following treatment.	Fail to reject

CHAPTER FIVE

DISCUSSION

Chapter Overview

Results from the analyses in CHAPTER FOUR indicate the concrete-representational-abstract sequence (CRA) is effective for improving math performance for students with and without learning disabilities (LD). The findings are consistent with previous literature on the use of the CRA instructional sequence (e.g. Cass et al., 2003; Scheurermann et al., 2009; &Witzel et al., 2003). The results indicate that sixth graders with and without LD can successfully learn to represent and solve algebra-based problems. The following chapter will include a Summary of Findings by Unit of Instruction, Summary of Findings for Preteaching + CRA, Implications, Limitations, and Future Research sections.

Summary of Findings by Unit of Instruction

Overall findings from both the Solving Equations and Fractions instruction indicate that the combination of preteaching using CRA instruction was beneficial for students with and without LD at increasing overall math performance in the identified skill areas. Students in both instructional contexts increased their ability to concretely represent complex, higher level math concepts and move to mastery of the skills at the abstract level of learning.

Solving Equations

Posttest. The largest effects were seen on the immediate posttest among the Solving Equations treatment group. At pretest the majority of students did not attempt to answer any problems involving simplifying or solving one variable equations; and few attempted to add or subtract with integers. Students in the control group did not attempt any answers on the posttest. Students in the treatment group however, significantly improved on this assessment ($t(30) = 4.96$; $p < .001$). A large effect size (1.71) was calculated in favor of students in the treatment group. Students made the most improvement in their ability to simplify equations by combining like variables. In addition, students in the treatment group showed increased growth in their ability to solve one-variable equations. Students in the treatment group continued to have difficulty with adding and subtracting integers. Most students were able to accurately represent the equation using tally marks but made procedural errors in solving the problems correctly. These findings align with previous studies where authors indicated students with LD struggled with aspects of math as a result of weak procedural knowledge (Jordan et al., 1999; Mastropieri et al., 1991).

Following preteaching, students were also administered the Basic Skills Algebra CBM. Despite the nonsignificant difference between groups, students in the treatment group attempted more problems than students in the control group. This may suggest an increased confidence in attempting higher level math problems. While the treatment group scored on average slightly higher on this measure (ES = .56), the growth rate should be interpreted with caution given such a small sample.

Generalization. Results of the generalization measure for the Solving Equations Unit are similar to those reported in the review of literature by Maccini and colleagues (1999) that indicated students with LD typically score low on generalization measures because the targeted-problems are often too dissimilar from those explicitly taught during the intervention phase of the design. Consistent with their findings, students in the treatment group did not differ in performance from students in the control group on the Everyday Math Unit Assessment. The spiraling design of reform-based curriculum is evident in the format of the textbook assessments. In addition to the inclusion of skills from the current unit, these assessments also contain problems from previous "spirals," or units. For the purpose of this study only the 15 problems related to both preteaching instruction and the Solving Equations Unit were included in the final analyses. The nonsignificant difference between groups and small effect (ES = .23) in favor of control students may be attributed to the limited number of problem types and opportunities to generalize the skills.

The prerequisite skills determined for use in the current study aligned with state standards and the Common Core; and in addition were outlined as essential prerequisites in the textbook administrator's manual. While practice problems relating to the skill areas were included in over 80% of the lessons taught during the classroom unit, they were not equally represented on the unit assessment. Of the 15 related problems on the unit assessment, only one related to adding and subtracting integers and two related to evaluating expressions. The remaining items related specifically to simplifying equations, the skill area that students in the treatment group mastered quickly (see Table 24 for the number of items per skill area). The lack of significance on the generalization measure and the high mean performance of both groups (M = 11.00 and M = 10.41) could be a result of the lack of complexity of the problems represented compared to the average lesson difficulty within the unit.

Table 24
Number of Aligned Problem Types on Solving Equations EM Unit Assessment

Solving Equations Preteaching Skills	Adding, Subtracting, & Multiplying Integers	Simplifying Equations	Solving Equations with One Unknown Variable
# of aligned items on EM Assessment	1	12	2

Follow-up. Two weeks following treatment students in the Solving Equations treatment group were able to maintain skills significantly greater than their peers. The follow-up measure was administered at the end of whole class unit instruction. During the whole class instruction time the identified skill areas were taught in greater depth through problem solving activities and peer learning games that reinforced representational and abstract learning. Given that all students had exposure to the content, it would be expected that growth in performance for both groups would be observed on the follow-up measure. Following the additional instruction, students in the treatment group still differed significantly in their overall performance in the identified skill areas. A large effect (ES=.74) was calculated in favor of students in the treatment group. These findings suggest that students in the treatment group entered the class with an increased knowledge of the predetermined skill areas compared to those in the control, and maintained this difference throughout the two week instructional period.

Fractions

Posttest. The Fractions treatment group also made significant gains on the posttest measure; reaffirming the effectiveness of using the CRA sequence to teach higher level math skills. At pretest it was observed that students in both treatment and control groups had a basic knowledge of fractions and were able to accurately add and subtract fractions with like denominators. However, most students did not attempt to answer problem types that involved adding and subtracting fractions with unlike denominators. Most students attempted to compare fractions but with low accuracy. Students in the control group performed slightly lower on the posttest, while students in the treatment group scored significantly higher (ES = .67). Students in the treatment group were able to add and subtract fractions with like denominators and compare fractions 90% of the time, but students continued to struggle when fraction computations had unlike denominators. Students were however, based on observed tally marks, able to accurately represent problems but made computational errors when solving these problem types. The findings from the Fraction Unit align with results from the Solving Equations Unit assessment, that students struggled most with procedural understandings on the experimenter-generated measure.

Similar to the Solving Equations Unit, groups did not differ significantly on the Basic Skills Algebra CBM, but mean scores were slightly higher among treatment group students (ES = .09) and treatment students also attempted more problems.

Generalization. Similar to the Solving Equations Unit Assessment, results of the generalization measure for the Fractions Unit were also nonsignificant. However, students in the treatment did score slightly higher on average (ES = .51). In addition, students who received preteaching instruction using the CRA sequence attempted more problems versus students in the control group who left more blanks. This may suggest students in the treatment felt more confident with their conceptual understanding of the skills but had difficulty with computations. Unlike the Solving Equations Unit Assessment, the 15 problems, selected for analysis, that required students to have knowledge of pretaught material to solve were equally distributed throughout the measure (see Table 25).

Table 25
Number of Aligned Problem Types on Fractions EM Unit Assessment

Fractions Preteaching Skills	Comparing Fractions with the symbols >, <, or =.	Reducing fractions to simplest form.	Adding and Subtracting Fractions with like and unlike denominators.
# of aligned items on EM Assessment	5	5	5

Follow-up. While the follow-up measure administered at the end of whole class unit instruction indicated no significant difference between control and treatment groups, students in the treatment scored on average slightly higher (ES = .16). Classroom instruction during this unit focused on complex fraction computations including adding and subtracting mixed numbers. The abstract nature of the classroom instruction during the fraction unit and no reinforcement of representational models to support this advanced content may have attributed to the nonsignificant results. The findings may suggest that students in the treatment group entered the class with an increased knowledge of the predetermined skill areas compared to those in the control, but without adequate practice and reinforcement of the representational models were not able to adequately maintain the pretaught skills.

Summary of Findings for Preteaching + CRA

The present study adds to the literature in three ways:

(1) Teaching algebra-based skills to students with disabilities

(2) Providing explicit instruction within a reform-based mathematics approach

(3) Strengthening deficit areas for learners with and at risk for disabilities within a tiered model of intervention.

First, the current study adds to the sparse literature on teaching students with disabilities higher level math skills (i.e., algebra) using the CRA instructional sequence. Despite the presence of the CRA sequence among current special education literature, only seven previous studies examined the use of the CRA sequence to teach algebra to students with LD (Cass et al., 2003; Jordan et al., 1999; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheurermann et al., 2009; Witzel, 2005; and Witzel et al., 2003). The use of this sequence is particularly important because it improves students' conceptual as well as procedural knowledge; two tasks often taught in isolation within general education settings (Scheuermann et al., 2009). The increased use of representations observed in student work and on assessments; in addition to the increased accuracy on the posttest, is an indicator that both elements were successfully taught during the preteaching sessions.

Second, this study adds to the literature by suggesting a practical and feasible intervention that merges explicit instruction within a reform-based mathematics classroom. For more than a decade, researchers in math education have supported the reform-based mathematics approach and the field of special education has continued to support more explicit instructional approaches (Hudson, Miller, & Butler, 2006). As more students with disabilities receive core math in general education settings it has become increasingly important to find ways to support a combination of these practices. Strong posttest scores from both units of instruction suggest the use of the CRA explicit approach in addition to the use of reform-based activities may support students with and without disabilities in mastering the requisite skills essential for higher level learning.

Last, this study effectively incorporated the CRA instruction within a tiered intervention model. Often supplemental or intensive interventions occur concurrently with core instruction, but occasionally preteaching is used to provide tiered instruction. Previous research on preteaching indicates that students prefer this to concurrent supplemental instruction because they enter the general education class with increased confidence and knowledge to succeed (Munk et al., 2010). The significant differences on posttest measures between students that received preteaching compared to those in the control group indicates that students in the treatment group entered core instruction with an increased knowledge of the foundational content necessary for success.

Most notable about the dramatic increase in treatment scores is the relatively short time in which student improvement was seen. Each preteaching lesson was only 20-30 minutes for ten sessions. Given that three skills were taught over the ten sessions, the average amount of time needed to make significant gains in each skill area was approximately 90 minutes. The short instructional time needed to see benefits increases the feasibility for implementing the CRA instructional sequence as a supplement to core instruction.

The results indicated that the intervention was equally effective for students requiring tier 2 and 3 support as well as for students receiving special education support in math. Therefore, the study corroborates previous research that the CRA sequence, regardless of instructional tier, is a feasible and effective intervention to facilitate understanding of difficult math content for struggling students (Witzel, Mink, & Riccomini, 2011). A report on highly effective intervention frameworks for struggling math learners (Newman-Gonchar et al., 2009) emphasizes the importance of aligning instruction at various tiers to maximize student performance and growth. Research supporting interventions that are effective for students requiring differing levels of support to maximize learning are critical.

In addition, in only two short training sessions, the researcher and teacher were able to master the instructional sequence, and had come to common understandings and agreements about how to successfully implement and represent various problem types. A high treatment integrity average of 94.3% for Solving Equations instruction and 97.9% for Fraction instruction was obtained. The report by Newman-Gonchar and colleagues (2009) also emphasizes the importance of high quality instruction with strong fidelity of implementation to support student growth in math.

Limitations

Three main limitations were identified within this study. First, the sample size of only 32 students was a limitation to the results of this study. The small sample limited the power of the study increasing the likelihood of a Type 2 error occurring. This may have attributed to nonsignificant findings on generalization and follow-up measures. In addition, the small sample makes it difficult to generalize findings to a variety of settings, grade levels, and student populations. With the exception of the Solving Equations Unit assessment where mean scores were extremely close (ES= -.23), the ESs for all measures included in the current study were in favor of treatment group students and the calculated ESs in the medium to large range.

Second, the curriculum unit assessment did not provide a large number of items that directly related to the requisite skills. This limited number of opportunities for students to demonstrate their knowledge of the skills may have resulted in nonsignificant findings on generalization measures. In addition the lack of alignment between the unit test and pretest make it difficult to accurately assess student growth.

Third, the use of the researcher as one of the instructors brings into question the feasibility of the intervention. It is important for the overall maintenance and sustainability of an effective practice that it can be implemented by a classroom teacher. It is apparent through the high level of treatment fidelity data interventionists were highly effective given a short training period. This suggests classroom teachers could feasibly be trained to effectively implement the intervention. However, given limited time and resources found among most schools, it may not be possible to sustain preteaching over multiple instructional units.

Implications

It is becoming increasingly difficult for teachers to effectively implement math programs as students' diverse learning needs increase in general education classrooms. This is particularly true in schools where the philosophy and methodology of special education and math education collide with the adoption of reform-based curriculum. The National Math Advisory Panel (2008) acknowledges the presence of these opposing pedagogical beliefs and advocates for the integration of practices from both. This study presents a feasible intervention that provides students with or at risk for disabilities the opportunity to learn through explicit demonstration with the support of adequate guided and independent practice opportunities, while also participating in reform-based activities.

The results of this study indicated large effects (ES = 1.71 and ES = 0.67) on immediate measures suggesting that the use of the explicit instructional sequence was beneficial for students. However, students did not perform significantly different on classroom measures. Previous research on the CRA sequence suggest continued and methodical use of the sequence over longer periods of time will result in greater accuracy and reasoning (Witzel et al., 2011). Continued use of pretest data or other progress monitoring data to determine student needs and, in turn, place them in preteaching groups may result in students with disabilities receiving the intervention multiple times throughout the year. There are many skills sets that are essential to success in math, particularly algebraic reasoning, and growth in these areas requires continued use of evidenced-based practices (i.e.,, preteaching + CRA). Long term exposure and use of the sequence to learn requisite grade level skills will most likely result in significant increases on classroom measures and increase in maintenance of skills.

In addition to increased time and exposure to the explicit teaching of essential skills, research also suggests that students benefit from preteaching groups or other supplemental groups that align with validated classroom instructional practices (Fuchs, et al., 2008). As the diversity of general education classrooms increase, it is important for teachers to incorporate a balance of explicit and reform-based activities to meet the needs of all students. Previous findings suggest that the CRA sequence used within whole class instruction will benefit normal and high achieving students as well as those who struggle in mathematics (Witzel, 2005; Witzel et al., 2011). The collaboration of the special and general education teachers to align tiered interventions with core instruction through the use of common language, similar representations, and visual models will ensure students are successful in core learning.

Future Research

In summary the findings of this study add to the growing body of literature on effective math methods and applications within tiered intervention models, such as RTI. A strong need for research that addresses areas of strength and weakness within RTI approaches to math are beneficial as the field moves towards embracing RTI whole-heartedly (Vaughn, 2011). It is particularly important that the use of such approaches is feasible for teachers to implement and that they provide the essential elements of learning critical for students with disabilities.

Further research studies that examine the feasibility of teacher implementation are necessary for the approach to be sustainable. Research that aligns the preteaching intervention with validated classroom instruction and curriculum should also be explored to determine the effects on generalization and maintenance. The inclusion of dependent measures that align with the three stages of the CRA sequence should be used within future study designs. In addition, given the large amount of procedural errors on posttests, research examining the use of the CRA sequence compared to instruction focusing on procedural knowledge is warranted. Last, the replication of this study with a larger more diverse sample population will allow results to be more generalizable.

APPENDIX A

TREATMENT FIDELITY CHECKLIST

Concrete Phase:	☐ Researcher modeling of use of manipulatives. ☐ Researcher modeling the steps of the algorithm. ☐ Guided practice for each student is provided. ☐ Researcher provides immediate and positive feedback when appropriate. ☐ Students successfully complete at least 5 problems independently at this phase before moving to representational phase.
Representational Phase:	☐ Researcher modeling of use of pictorial representations. ☐ Researcher modeling the steps of the algorithm. ☐ Guided practice for each student is provided. ☐ Researcher provides immediate and positive feedback when appropriate. ☐ Students successfully complete at least 5 problems independently at this phase before moving to abstract phase.
Abstract Phase:	☐ Researcher modeling the steps of the algorithm. ☐ Guided practice for each student is provided. ☐ Researcher provides immediate and positive feedback when appropriate. ☐ Researcher encourages students to try solving problem without the use of pictorial representations when appropriate. ☐ Students successfully complete 6 of the 8 abstract problems independently at this phase.

APPENDIX B

ANOVA RESULTS FOR PRETEST COMPARISONS BETWEEN GROUPS

Source	df	Sum of Squares	Mean Square	F	P
Algebra pretest Between Groups	1	2.79	2.79	.85	.36
Algebra Pretest Within Groups	30	98.68	3.29		
Total	31	101.47			
Fraction Pretest Between Groups	1	2.44	2.44	.12	.72
Fraction Pretest Within Groups	30	590.28	19.67		
Total	31	592.72			

APPENDIX C

ANOVA RESULTS FOR PRETEST COMPARISONS BETWEEN SCHOOLS

Source	df	Sum of Squares	Mean Square	F	P
Algebra pretest Between Groups	1	6.11	6.11	1.92	.18
Algebra Pretest Within Groups	30	95.36	3.18		
Total	31	101.47			
Fraction Pretest Between Groups	1	69.01	69.01	3.95	.06
Fraction Pretest Within Groups	30	523.71	17.46		
Total	31	592.72			

APPENDIX D

LEVENE'S TESTS FOR COMPARISONS BETWEEN SOLVING EQUATIONS DEPENDENT MEASURES

Data Set for Solving Equations Unit	Levene's Test	P-value
CBM posttest between groups comparison	1.30	.26
Researcher generated posttest between groups comparison	.86	.36
Everyday Math Unit Test between groups comparison	.37	.70
Researcher generated follow-up test between groups comparison	2.16	.13
Researcher generated posttest comparison within treatment group between intervention levels	.004	.948

Note: Ho = Variance for the data pair is equal

APPENDIX E

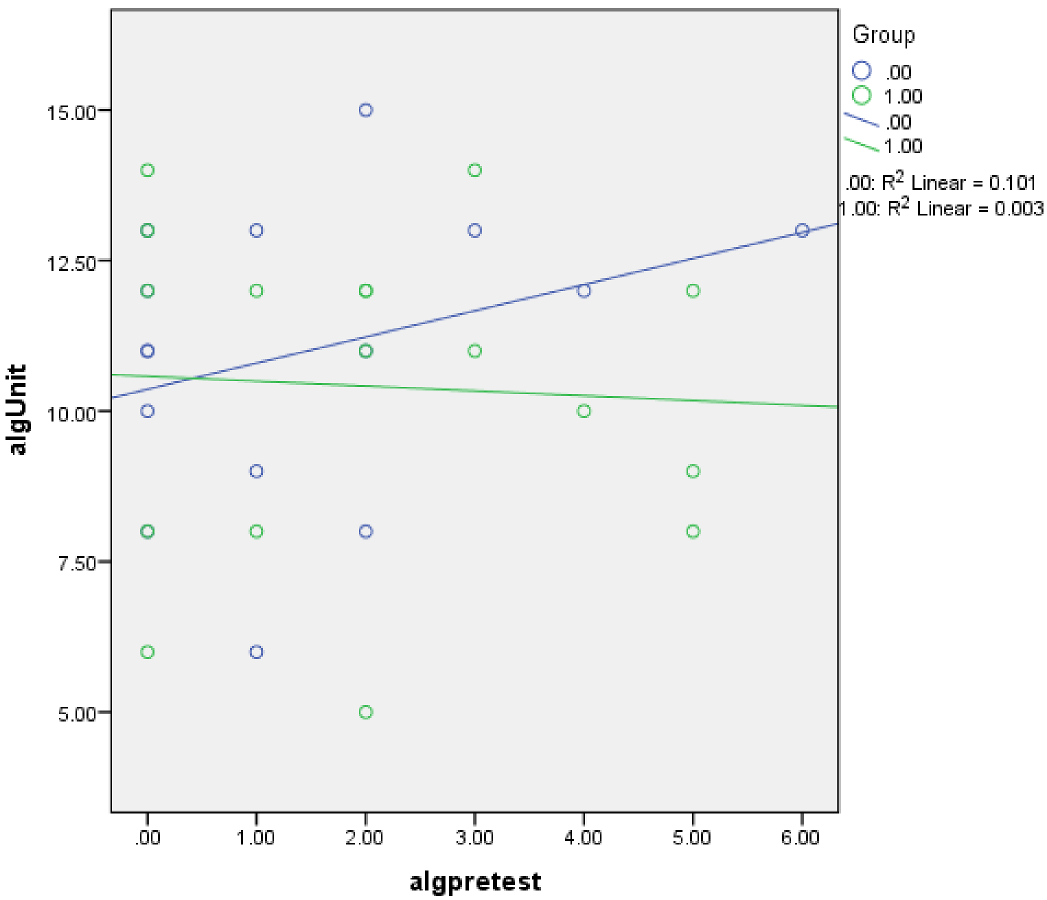
LEVENE'S TESTS FOR COMPARISONS BETWEEN FRACTION DEPENDENT MEASURES

Data Set for Fractions Unit	**Levene's Test**	**P-value**
CBM posttest between groups comparison	2.25	.14
Researcher generated posttest between groups comparison	1.60	.22
Everyday Math Unit Test between groups comparison	2.60	.10
Researcher generated follow-up test between groups comparison	.12	.89
Researcher generated posttest comparison within treatment group between intervention levels	.21	.66

Note: Ho = Variance for the data pair is equal

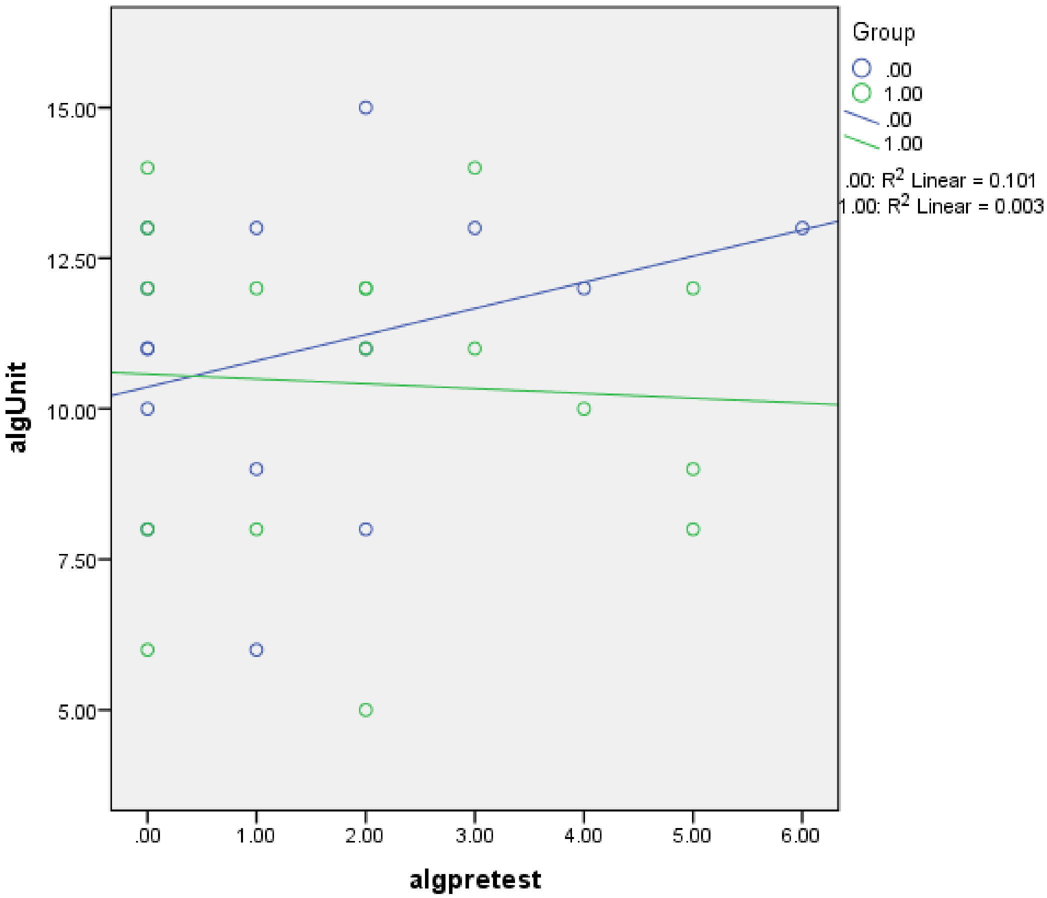
APPENDIX F

SCATTERPLOT OF ALGEBRA UNIT ASSESSMENT AND PRETEST



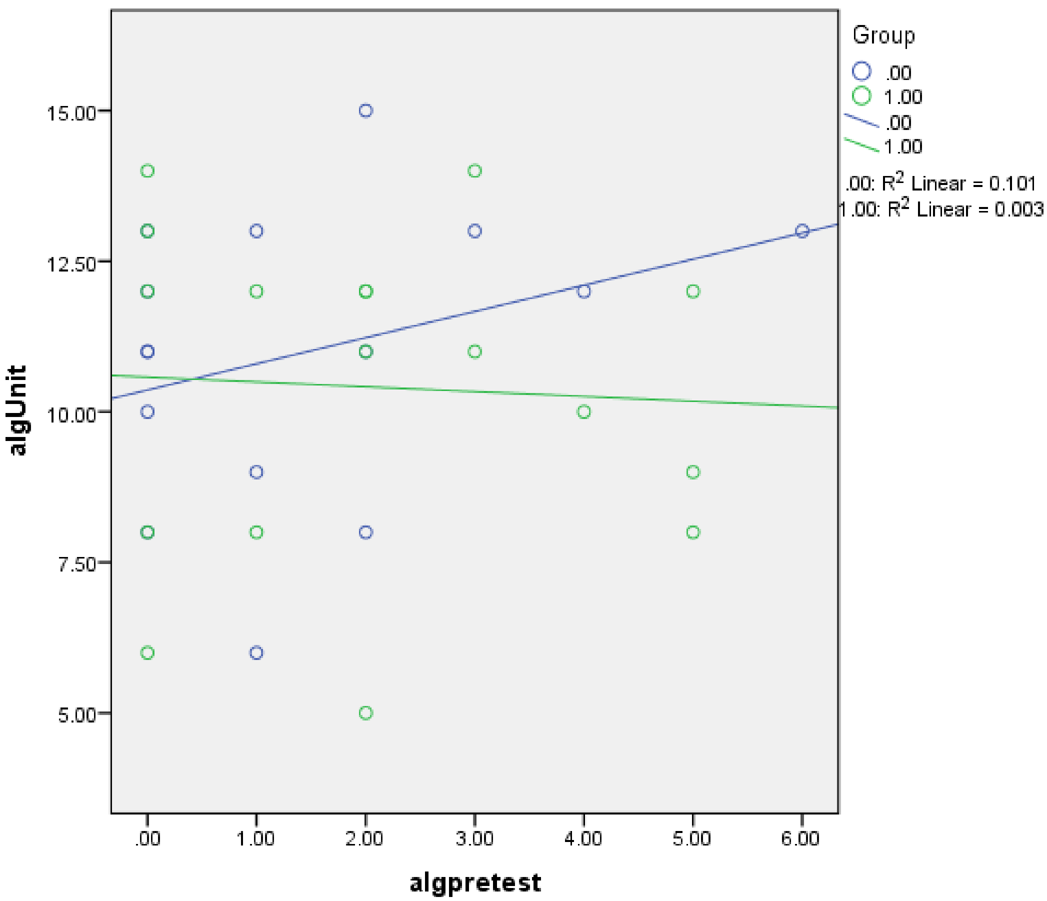
APPENDIX G

SCATTERPLOT OF ALGEBRA FOLLOW-UP MEASURE AND PRETEST



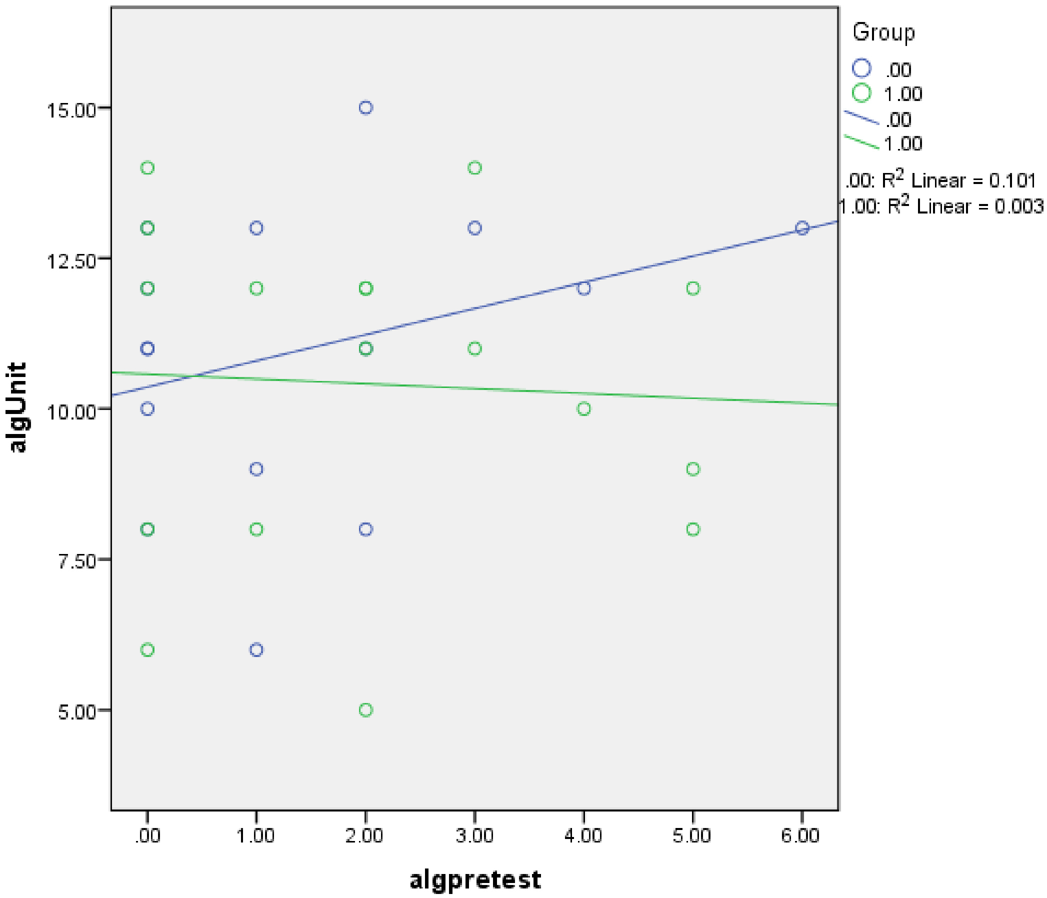
APPENDIX H

SCATTERPLOT FOR FRACTION UNIT ASSESSMENT AND PRETEST



APPENDIX I

SCATTERPLOT FOR FRACTION FOLLOW-UP AND PRETEST



REFERENCES

Allsopp, D. H. (1999). Using modeling, manipulatives, and mnemonics with eighth-grade math students. *Teaching Exceptional Children*, *32*(2), 74-81.

Anderson-Inman, L. (1981). Transenvironmental programming: Promoting success in the regular class by maximizing the effect of resource room instruction. *Journal of Special Education Technology*, *4*(4), 3-12.

Badian, N. A. (1983). Dyscalculia and nonverbal disorders of learning. *Progress in learning disabilities*, *5*, 235-264.

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *Elementary School Journal*, *103*, 51-73. doi: 10.1086/499715

Barron, B., Bransford, J., Kulewicz, S., & Hasselbring, T. (1989, March). Uses of macrocontexts to facilitate mathematical thinking. In *annual meeting of the American Educational Research Association, San Francisco, CA*.

Beck, M., Burns, M. K., & Lau, M. (2009). Preteaching unknown items as a behavioral intervention for children with behavioral disorders. *Behavior Disorders*, *34*, 91-99.

Bell, M., Bell, J., Bretzlauf, J., Dellard, A., Hartfield, R., Isaacs, A., et al. (2004). *Everyday Mathematics.* Chicago: SRA/McGraw-Hill.

Berends, M., Lucas, S.R., Sullivan, T., & Briggs, R.J., (2005). *Examining gaps in math achievement among racial-ethnic groups 1972-1992.* Santa Monica, CA: RAND.

Bost, L., & Riccomini, P. J. (2006). Effective Instruction: An Inconspicuous Strategy for Dropout Prevention. *Remedial and Special Education*, *27*(5), 301-311. doi: 10.1177/07419325060270050501

Bottge, B. A. (1999). Effects of contextualized math instruction on problem solving of average and below-average achieving students. *Journal of Special Education*, *33*, 81-92. doi: 10.1177/002246699903300202

Bottge, B. A., Rueda, E., Serlin, R. C., Hung, Y. H., & Kwon, J. M. (2007). Shrinking Achievement Differences with Anchored Math Problems Challenges and Possibilities. *The Journal of Special Education, 41,* 31-49. doi: 10.1177/00224669070410010301

Browder, D. M., & Xin, Y. P. (1998). A meta-analysis and review of sight word research and its implications for teaching functional reading to individuals with moderate and severe disabilities. *The Journal of Special Education*, *32*, 130-153. doi: 10.1177/002246699803200301

Bryant, D., Bryant, B. R., & Hammill, D. D. (2000). Characteristic behaviors of students with LD who have teacher-identified math weaknesses. *Journal of Learning Disabilities, 33*, 168-77, 99. doi: 10.1177/002221940003300205

Burns, M. K., Dean, V. J., & Foley, S. (2004). Preteaching unknown key words with incremental rehearsal to improve reading fluency and comprehension with children identified as reading disabled. *Journal of school psychology*, *42*, 303-314. doi: 10.1016/j.jsp.2004.04.003

Burns, M. K., Hodgson, J., Parker, D. C., & Fremont, K. (2011). Comparison of the effectiveness and efficiency of text previewing and preteaching keywords as small-group reading comprehension strategies with middle-school students. *Literacy Research and Instruction*, *50*(3), 241-252.

Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice, 18,* 99-111. doi: 10.1111/1540-5826.00066

Carnine, D. (1980). Preteaching versus concurrent teaching of the component skills of a multiplication algorithm. *Journal for research in Mathematics Education*, *11*, 375-379. doi: 10.2307/748628

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice*, *18*, 112-120. doi: 10.1111/1540-5826.00067

Chung, J. M. (2009). The effects of using two advance organizers with video texts for the teaching of listening in English. *Foreign Language Annals*, *35*, 231-241. doi: 10.1111/j.1944-9720.2002.tb03157.x

Common Core State Standards Initiative. (2010). Common core state standards for mathematics. *Retrieved September*, *15*, 2010.

Cooper, G., & Sweller, J. (1987). Effects of schema acquisition and rule automation on mathematical problem-solving transfer. *Journal of educational psychology*, *79*(4), 347. doi: 10.1037//0022-0663.79.4.347

Darch, C., Carnine, D., & Gersten, R. (1984). Explicit instruction in mathematics problem solving. *The Journal of Educational Research*, 351-359.

Doabler, C. T., Fien, H., Nelson-Walker, N. J., & Baker, S. K. (2012). Evaluating three elementary mathematics programs for presence of eight research-based instructional design principles. *Learning Disability Quarterly, 35*, 200-211. doi: 10.1177/0731948712438557

Fitzgerald, J., & Graves, M. (2005). Reading supports for all. *Educational Leadership, 62*(4), 68-71.

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education*, *31*(3), 195-207. doi: 10.1177/0741932508327467

Fuchs, L.S. & Fuchs, D. (2003). Enhancing the mathematical problem solving of students with mathematics disabilities. In Swanson, H., Harris, K., & Graham, S. *Handbook of learning disabilities* (pp. 306-322).

Fuchs, L. S., Fuchs, D., Craddock, C., Hollenbeck, K. N., Hamlett, C. L., & Schatschneider, C. (2008). Effects of small-group tutoring with and without validated classroom instruction on at risk students' math problem solving: Are two tiers of prevention better than one? *Journal of Educational Psychology, 100*, 491-509. doi:10.1037/0022-0663.100.3.491

Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., & Hamlett, C. L. (2004). Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. *American Educational Research Journal*, *41*, 419-445. doi: 10.3102/00028312041002419

Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R., & Schroeter, K. (2003). Enhancing third-grade student's mathematical problem solving with self-regulated learning strategies. *Journal of educational psychology, 95*(2), 306.

Fuchs, L. S., Seethaler, P. M., Powell, S. R., Fuchs, D., Hamlett, C. L., & Fletcher, J. M. (2008). Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. *Exceptional Children, 74*(2), 155-173.

Fuchs, D., Fuchs, L., & Vaughn, S. (Eds.). (2008). *Response to intervention: A framework for reading educators*. International Reading Assn.

Geary, D.C. (2003). Learning disabilities in arithmetic: Problem-solving differences and cognitive deficits. In Swanson, H., Harris, K., & Graham, S. *Handbook of Learning Disabilities* (pp. 199-212).

Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development*, *2*(3), 249-269. doi: 10.1016/S0885-2014(87)90075-X

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to intervention (RtI) for elementary and middle schools. *Assisting students struggling with mathematics: Response to intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Services, Institute of Education Sciences, US Department of Education*.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., Flojo, J., & Center on, I. (2008). Mathematics instruction for students with learning disabilities or difficulty learning mathematics: A synthesis of the intervention research. *Center on Instruction*.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*(3), 1202-1242. doi: 10.3102/0034654309334431

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of learning disabilities*, *38*(4), 293-304. doi: 10.1177/00222194050380040301

Glass, G. V., Peckham, P. D., & Sanders, J. R. (1972). Consequences of failure to meet assumptions underlying the fixed effects analyses of variance and covariance. *Review of educational research*, *42*(3), 237-288. doi: 10.2307/1169991

Graves, M. F., Cooke, C. L., & Laberge, M. J. (1983). Effects of previewing difficult short stories on low ability junior high school students' comprehension, recall, and attitudes. *Reading Research Quarterly*, 262-276. doi: 10.2307/747388

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine & Child Neurology*, *38*(1), 25-33. doi: 10.1111/j.1469-8749.1996.tb15029.x

Harris, C. A., Miller, S.P., & Mercer, C.D. (1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research and Practice, 10*(3), 180-95.

Hawker, B. & Cowley, C. (Ed.). (1997). *The Oxford Minireference Dictionary and Thesaurus*. Oxford University Press, USA.

Hawkins, R. O., Hale, A., Sheeley, W., & Ling, S. (2011). Repeated reading and vocabulary-previewing interventions to improve fluency and comprehension for struggling high-school readers. *Psychology in the Schools*, *48*(1), 59-77. doi: 10.1002/pits.20545

Huntington, D. J. (1994). *Instruction in concrete, semi-concrete, and abstract representation as an aid to the solution of relational problems by adolescents with learning disabilities* (Doctoral dissertation, University of Georgia).

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly*, 34-63.

Jordan, L., Miller, M. D., & Mercer, C. D. (1999). The effects of concrete to semiconcrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal*, *9*(3), 115-22.

Kameenui, E. J., & Carnine, D. W. (1986). Preteaching versus concurrent teaching of component skills of a subtraction algorithm to skill-deficient second graders: A components analysis of direct instruction. *The Exceptional Child*, *33*(2), 103-115. doi: 10.1080/0156655860330203

Keselman, H. J., Huberty, C. J., Lix, L. M., Olejnik, S., Cribbie, R. A., Donahue, B. & Levin, J. R. (1998). Statistical practices of educational researchers: An analysis of their ANOVA, MANOVA, and ANCOVA analyses. *Review of Educational Research*, *68*(3), 350-386. doi: 10.2307/1170601

Ketterlin-Geller, L., Chard, D. J., & Fien, H. (2008). Making connections in mathematics: Conceptual mathematics intervention for low-performing students. *Remedial and Special Education*, *29*(1), 33-45. doi:10.1177/0741932507309711

Kortering, L. J., & Braziel, P. M. (1999). School dropout from the perspective of former students: Implications for secondary special education programs. *Remedial and Special Education*, *20*(2), 78-83. doi: 10.1177/0741932507309711

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities*, *7*(3), 164-177. doi: 10.1177/002221947400700309

Koury, K. A. (1996). The impact of preteaching science content vocabulary using integrated media for knowledge acquisition in a collaborative classroom. *Journal of Computing in Childhood Education*, *7*(3-4), 179-197.

Kroesbergen, E. H., & Van Luit, J. E. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education*, *24*(2), 97-114. doi: 10.1177/07419325030240020501

Kuchey, D. (2010). Bruner, Jerome S. (1915–). In T. Hunt, J. Carper, T. Lasley, & C. Raisch (Eds.), *Encyclopedia of educational reform and dissent.* (pp. 118-119). Thousand Oaks, CA: SAGE Publications, Inc. doi: 10.4135/9781412957403.n57

Lalley, J. P., & Miller, R. H. (2006). Effects of pre-teaching and re-teaching on math achievement and academic self-concept of students with low achievement in math. *Education*, *126*(4), 747-755.

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *15*(1), 10-21. doi: 10.1207/SLDRP1501_2

Maccini, P., McNaughton, D., & Ruhl, K. L. (1999). Algebra instruction for students with learning disabilities: Implications from a research review. *Learning Disability Quarterly*, *22*(2), 113-26. doi: 10.2307/1511270

Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23*(4), 465-489.

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice*, *27*(4), 152-166. doi: 10.1111/j.1540-5826.2012.00363.x

Mastropieri, M. A., Bakken, J. P., & Scruggs, T. E. (1991). Mathematics instruction for individuals with mental retardation: A perspective and research synthesis. *Education & Training in Mental Retardation*, 26, 115.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education*, *13*(3), 19-35. doi: 10.1177/074193259201300303

Miller, S. P., Butler, F. M., & Lee, K. H. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of literature. *Focus on Exceptional Children*, *31*(1), 1-24.

Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice*, *22*(1), 47-57.

Miller, S. P., & Mercer, C. D. (1997). Educational aspects of mathematics disabilities. *Journal of learning disabilities*, *30*(1), 47-56. doi: 10.1177/002221949703000104

Montague, M. (1993). Mathematical problem-solving characteristics of middle school students with learning disabilities. *Journal of Special Education*, *27*(2), 175. doi: 10.1177/002246699302700203

Morin, V. A., & Miller, S. P. (1998). Teaching multiplication to middle school students with mental retardation. *Education and Treatment of Children*, *21*(1), 22-36.

Munk, J. H., Gibb, G. S., & Caldarella, P. (2010). Collaborative preteaching of students at risk for academic failure. *Intervention in School and Clinic*, *45*(3), 177-185. doi: 10.1177/1053451209349534

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Boston. VA: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. US Department of Education.

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. *Center on Instruction*.

Owen, R. L., & Fuchs, L. S. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education*, *23*(5), 268-278. doi: 10.1177/07419325020230050201

Parmer, R. S., Cawley, J. F., & Miller, J. H. (1994). Difference in mathematics performance between students with learning disabilities and students with mental retardation. *Exceptional Children, 60,* 549-563.

Peterson, S. K., & Mercer, C. D. „& O'Shea, L.(1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research*, *4*, 52-56.

Riccomini, P. J., Witzel, B., & Robbins, K. (2008). Improving the mathematics instruction for students with emotional and behavioral disorders: Two evidenced-based instructional approaches. *Beyond Behavior*, *17*(2), 24-30.

Rose, T. L. (1984). The effects of two prepractice procedures on oral reading. *Journal of Learning Disabilities*, *17*(9), 544-548. doi: 10.1177/002221948401700908

Ross, P. A., & Braden, J. P. (1991). The effects of token reinforcement versus cognitive behavior modification on learning-disabled students' math skills. *Psychology in the Schools*, *28*(3), 247-256. doi: 10.1002/1520-6807(199107)28:3<247::AID-PITS2310280310>3.0.CO;2-E

Sayeski, K. L., & Paulsen, K. J. (2010). Mathematics reform curricula and special education: identifying intersections and implications for practice. *Intervention in School and Clinic*, *46*(1), 13-21. doi: 10.1177/1053451210369515

Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, 103-120.

Schunk, D. H., & Cox, P. D. (1986). Strategy training and attributional feedback with learning disabled students. *Journal of Educational Psychology*, *78*(3), 201. doi 10.1037//0022-0663.78.3.201

Sood, S., & Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *Journal of Special Education*, *41*(3), 145-157. doi: 10.1177/00224669070410030101

Swanson, H. L., Harris, K. R., & Graham, S. (Eds.). (2005). *Handbook of learning disabilities*. The Guilford Press.

Thornton, C.A., Langrall, C.W., & Jones, G.A. (1997). Mathematics instruction for elementary students with learning disabilities. *Journal of Learning Disabilities, 30(*2), 142. doi: 10.1177/002221949703000202

Tournaki, N. (2003). The differential effects of teaching addition through strategy instruction versus drill and practice to students with and without learning disabilities. *Journal of Learning Disabilities*, *36*(5), 449-458. doi: 10.1177/00222194030360050601

U.S. Department of Labor. (2007) *Includes information on specific jobs, working conditions, training and education requirements, earnings projections and job prospects*. Retrieved from http://www.dol.gov.

Vaughn, S. (2011). Foreward. In Gersten, R. & Newman-Gonchar, R. *Understanding RTI in mathematics.* (pp.xv-xvi).

Van Luit, J. E., & Naglieri, J. A. (1999). Effectiveness of the MASTER program for teaching special children multiplication and division. *Journal of Learning Disabilities*, *32*(2), 98-107. doi: 10.1177/002221949903200201

Wilson, C. L., & Sindelar, P. T. (1991). Direct instruction in math word problems: Students with learning disabilities. *Exceptional Children*, *57*(6), 512-519.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal*, *3*(2), 49-60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, *18*(2), 121-131. doi: 10.1111/1540-5826.00068

Woodward, J., Monroe, K., & Baxter, J. (2001). Enhancing student achievement on performance assessments in mathematics. *Learning Disability Quarterly*, 33-46. doi: 10.2307/1511294

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of Mathematical Word Problem—Solving Instruction on Middle School Students with Learning Problems. *The Journal of Special Education*, *39*(3), 181-192.