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On Soft Function

in

Soft Topological Spaces

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By

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بسم الله الرحمن الرحيم

﴿رَبِّ أَوْزِعْنِي أَنْ أَشْكُرَ نِعْمَتَكَ الَّتِي أَنْعَمْتَ عَلَيَّ وَعَلَى
وَالِدَيَّ وَأَنْ أَعْمَلَ صَالِحًا تَرْضَاهُ وَأَصْلِحْ لِي فِي ذُرِّيَّتِي إِنِّي
تُبْتُ إِلَيْكَ وَإِنِّي مِنَ الْمُسْلِمِينَ﴾

صدق الله العظيم

سورة الأحقاف الآية ﴿ ١٥ ﴾

إِهداء

إلى شمس الحقيقة إلى بقيةِ الله التي لا تخلو من العترةِ الهادية ،

إلى المعد لقطع دابر الظلمة ، إلى المرتجى لإزالةِ الجور والعدوان ،

إلى باب الله الذي منه يُؤتى ، إلى وجه الله الذي إليه يتوجه الأولياء ،

إلى السبب المتصل بين الأرض والسماء ،إلى المضطر الذي يجاب إذا دعا

إلى صاحب يوم الفتح وناشر راية الهدى ،إلى مؤلف شمل الصلاح والرضا

إلى الطالب بذحول الانبياء وابناء الانبياء،إلى الطالب بدم المقتول بكربلاء

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Samer, 2016.

Supervisor Certification

I certify that the preparation of this thesis entitled " **On Soft Function in Soft Topological Spaces** " made under my supervision at the University of Al-Qadisiyah, College of Computer Science and Information Technology as a partial fulfillment of the requirements for the degree of Master of Science in Mathematics.

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List of Symbols

Symbols	Definitions
E	The set of parameters
P(X)	The power set of X
SS(X, E)	The collection of all soft set over **X**
(F,E)	The soft set
$\tilde{\subseteq}$	The soft subset
$\tilde{\cup}$	The soft union
$\tilde{\cap}$	The soft intersection
f_e	The e-parameter function of function $\boldsymbol{f}$ at $\boldsymbol{e}$
$x^{\boldsymbol{e}}$	The soft point **at** $\boldsymbol{e}$
$SP(X)$	The set of all soft points over **X**
Δ	Diagonal function
(F,E)×(G,E)	The product of two soft sets
(p_i, q_i)	The soft projection function on the factor $\boldsymbol{i}$
i	The inclusion function
$f\|_A$	The restriction function of $\boldsymbol{f}$ on $\boldsymbol{A}$
$(F,E)^c$	The soft complement of $(\boldsymbol{F},\boldsymbol{E})$
$(F,E)^\circ$	The soft interior of $(\boldsymbol{F},\boldsymbol{E})$
$\overline{(F,E)}$	The soft closure of $(\boldsymbol{F},\boldsymbol{E})$
$(F,E)^b$	The soft boundary of $(\boldsymbol{F},\boldsymbol{E})$
$(F,E)^`$	The soft derived of $(\boldsymbol{F},\boldsymbol{E})$

$sc-space$	Soft constant space
sts	Soft topological space
τ_e	The topology on X at $\boldsymbol{e}$
$\overset{s}{\cong}$	The soft homeomorphic
$\mathcal{N}_{x^e}$	The neighborhood system at $\boldsymbol{x^e}$
$\{\chi_d^{e_d}\}_{d\in D}$	A soft net
$\chi_d^{e_d} \longrightarrow x^e$	$\{\boldsymbol{\chi_d^{e_d}}\}_{\boldsymbol{d\in D}}$ converge to $\boldsymbol{x^e}$
τ_{sind}	The indiscrete soft topology
τ_{sd}	The discrete soft topology

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Introduction

One of the very important concepts in general topology is the concept of soft functions .The main aim of this work is to study certain types of soft functions in soft topological spaces, namely, soft continuous, soft compact ,soft coercive and soft proper functions .

In 1999,D. Molodtsov [12] introduced the soft set concept to solve complicated problems in economics ,engineering, environment, sociology, medical science , etc. He has shown several applications of these sets in solving many practical problems. Maji-Biswas-Roy[13],2003 defined and studied operations of soft sets.

The first practical application of soft sets given in[16]. In 2011, Shabir and Naz[21] defined soft topology by using soft sets and studied some basic notions of soft topological spaces such as soft open , soft closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms .

Aygunoglu and Aygun [1] ,2011 introduced soft product topology and generalized Alexander subbase theorem and Tychonoff theorem to the soft topological spaces .

Zorlutuna et al. [23] , 2012 studied some concepts in soft topological spaces such as interior point, interior, neighborhood, continuity, and compactness.

Banu and Halis in [4] ,2013 studied some properties of soft Hausdorff space.

Cigdem Gunduz Aras et al., [5] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters.

In 2015 Sabir Hussain and Bashir Ahmad [22] introduced the different concept of soft separation axioms in soft topological spaces.

Some other studies on soft topological spaces can be listed as [8,17,23] .

The present thesis consists of three chapters.

Chapter one is divided into two sections, in section one we recall some main soft concepts, and we review definitions and propositions, which we need in the next chapters. In section two, we recall the definition of soft topological space and its remarks, properties , theorem and propositions.

Chapter two consists of three sections. Section one, shows the definitions of soft (continuous, open, closed, homeomorphism) function . Moreover, we give examples, propositions, and remarks about these concepts Further mover, we study the product of this function and introduce some new concepts about soft interior . In Section two, we recall the definitions of convergence of nets . Also we introduce the definition of soft convergence of nets and give examples , remarks , and properties about this subject . Section three contain definition of soft separation axiom in soft topological spaces and we prove some results on this concept . Also, we study types of soft functions about a separation axioms.

Chapter three consists of three sections. In section one ,we review definition of soft compact space and give useful characterization on this concept .Some results about this subject are proved . Moreover, we introduce the definition of soft compact function and study some of its properties. In section two, we introduce a definition of soft coercive function and study some properties of this this concept. Moreover, we explain the relation between it and soft compact function.

In section three, we introduce a definition of soft proper function and study some of their remarks, proposition and properties.

Chapter one
Fundamental concepts

Introduction:

This chapter consists of two sections. In section one we recall some of the basic definitions which are needed in this thesis.

In section two we introduce the definition of soft constant space and review some basic definitions, propositions and theorems about soft topological spaces, which are defined over an initial universe set with a fixed set of parameters, they are needed throughout the work.

1.1 Soft Set and related Concepts:

This section will contain the definition of soft set, examples ,remarks ,propositions and some basic definitions that are needed throughout the work.

Also ,we introduce the concept of e-parameter function f_e of a function f at each parameter and we give some properties about it.

Definition (1.1.1):[12]

Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if only if F is a function from E into the set of all subsets of the set X ,i.e. F:E → P(X) ,where P(X) is the power set of X. The set of all soft set over X is denoted by SS(X, E).

Example(1.1.2):

Let X ={x_1, x_2, x_3, x_4} and E ={e, e`}.If F:E → P(X) is a function defined as ;
F(e) = {x_1, x_2, x_3} and F(e`) ={ x_2, x_4} , then (F,E) is a soft set written by
(F, E) = {(e, {x_1, x_2, x_3}),(e`, {x_2, x_4})}.

Definition (1.1.3):[12]

Let (F,E),(G,E)∈SS(X,E).We say that the pair (F,E) is a soft subset of (G,E) if F (e) ⊆ G(e), for every e ∈ E. Symbolically, we write (F,E) $\tilde{\subseteq}$ (G,E). Also, we say that the pairs (F,E) and (G,E) are soft equal if (F,E) $\tilde{\subseteq}$ (G,E) and (G,E) $\tilde{\subseteq}$ (F,E). Symbolically, we write (F,E)=(G,E).

Definition (1.1.4): [12,15]

A soft set (F, E) over X is said to be

i. null soft set denoted by $\tilde{\Phi}$ if ∀e ∈ E, F(e) = ∅.

ii. absolute soft set denoted by $\tilde{X}$, if ∀e ∈ E, F(e) = X .

Definition (1.1.5):[8]

Let A be a non-empty subset of X, then $\widetilde{A}$ denotes the soft set (F, E) over X for which F(e)=A ; for all $e \in$ E denoted by $\widetilde{A}$ or (A,E). In particular (X, E) will be denoted by $\widetilde{X}$.

Definition (1.1.6):[12]

Let I be an arbitrary index set and $\{(F_i, E) : i \in I\} \subseteq SS(X,E)$. The soft union of these soft sets is the soft set(F,E) ∈ SS(X,E), where the function F : E→ P(X) defined as follows: $F(e) = \cup\{ F_i(e) : i \in I\}$, for every $e \in E$. Symbolically, we write $(F,E) = \widetilde{\cup}\{(F_i, E) : i \in I\}$.

Definition (1.1.7): [12]

Let I be an arbitrary index set and $\{(F_i, E): i \in I\} \subseteq SS(X,E)$. The soft intersection of these soft sets is the soft set (F,E) ∈ SS(X,E), where the function F : E→ P(X) defined as follows: $F(e) = \cap\{ F_i(e) : i \in I\}$, for every $e \in E$. Symbolically, we write $(F,E) = \widetilde{\cap}\{(F_i, E): i \in I\}$.

Definition (1.1.8): [8]

The difference (H,E) of two soft sets (F, E) and (G, E) over X; denoted by (H,E) = (F, E)\(G, E); is defined by $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition (1.1.9):[24]

The complement of a soft set (F, E),denoted by $(F, E)^c$,is defined by $(F, E)^c = (F^c, E)$, $F^c : E \to P(X)$ is a function given by $F^c(e) = X - F(e)$, $\forall\, e \in E$. F^c is called the soft complement function of F. Clearly,$(F^c)^c$ is the same as F and $((F, E)^c)^c = (F, E)$. Clearly, $\widetilde{X}^c = \widetilde{\Phi}$ and $\widetilde{\Phi}^c = \widetilde{X}$.

Proposition(1.1.10):[13]

Let (F, E) be a soft set over X . Then

i. $(F, E)\ \tilde{\cup}\ (F, E) = (F, E)$, $(F, E)\ \tilde{\cap}\ (F, E) = (F, E)$

ii. $(F, E)\ \tilde{\cup}\ \tilde{\Phi} = (F, E)$, $(F, E)\ \tilde{\cap}\ \tilde{\Phi} = \tilde{\Phi}$

iii. $(F, E)\ \tilde{\cup}\ \tilde{X} = \tilde{X}$, $(F, E)\ \tilde{\cap}\ \tilde{X} = (F, E)$

iv. $(F, E)\ \tilde{\cup}\ (F, E)^c = \tilde{X}$, $(F, E)\ \tilde{\cap}\ (F, E)^c = \tilde{\Phi}$

Proposition(1.1.11) : [21]

Let (F, E), (G, E)and(H, E) are three soft sets over X. Then

i. $(F, E)\ \tilde{\cup}\ (G, E) = (G, E)\ \tilde{\cup}\ (F, E)$

ii . $(F, E)\ \tilde{\cap}\ (G, E) = (G, E)\ \tilde{\cap}\ (F, E)$

iii . $[(F, E)\tilde{\cup}\ (G, E)]^c = (F, E)^c\ \tilde{\cap}\ (G, E)^c$

iv . $[(F, E)\tilde{\cap}\ (G, E)]^c = (F, E)^c\ \tilde{\cup}\ (G, E)^c$

v . $(F, E)\ \tilde{\cup}\ [(G, E)\tilde{\cap}\ (H, E)] = [(F, E)\tilde{\cup}\ (G, E)]\ \tilde{\cap}\ [(F, E)\tilde{\cup}\ (H, E)]$

vi . $(F, E)\ \tilde{\cap}\ [(G, E)\tilde{\cup}\ (H, E)] = [(F, E)\tilde{\cap}\ (G, E)]\ \tilde{\cup}\ [(F, E)\ \tilde{\cap}\ (H, E)]$.

Definition (1.1.12): [16]

Let $\{(F_i, E): i \in I\}$be a nonempty family of soft sets over X. The intersection

$\tilde{\bigcap}_{i\in I}$ is defined by $\tilde{\bigcap}_{i\in I}(F_i , E) = (\tilde{\bigcap}_{i\in I} F_i , E)$, where $(\tilde{\bigcap}_{i\in I} F_i)(e) = \tilde{\bigcap}_{i\in I} (F_i\ (e))$

for all $e \in E$.

Definition (1.1.13): [16]

Let$\{(F_i , E); i \in I\}$be a nonempty family of soft sets over X. The union

$\tilde{\bigcup}_{i \in I}$ is defined by $\tilde{\bigcup}_{i \in I}(\mathrm{F}_i, \mathrm{E}) = (\tilde{\bigcup}_{i \in I} \mathrm{F}_i, \mathrm{E})$, where $(\tilde{\bigcup}_{i \in I} \mathrm{F}_i)(e) = \tilde{\bigcup}_{i \in I}(\mathrm{F}_i(e))$

for all $e \in$ E.

<u>Proposition(1.1.14) :</u> [10]

Let I be an arbitrary index set and $\{(\mathrm{F}_i, E)\}_{i \in \mathrm{I}}$ be a subfamily of SS(X, E) . Then:

i. $[\tilde{\bigcup}_{i \in I}(F_i, E)]^c = \tilde{\bigcap}_{i \in I}(F_i, E)^c$, and

ii. $[\tilde{\bigcap}_{i \in I}(F_i, E)]^c = \tilde{\bigcup}_{i \in I}(F_i, E)^c$.

<u>Definition (1.1.15):</u> [16]

Let X and Y be two nonempty sets and f: X → Y be a function, then the following are defined:

i. The image of a soft set (F,E) ∈ SS(X, E) under the function f is defined by f(F,E) = (f (F),E), where $[f(\mathrm{F})](e) = f[\mathrm{F}(e)]$,for all $e \in$ E.

ii. The inverse image of a soft set (G,E)∈ SS(Y ,E) under the function f is defined by f^{-1}(G,E)= (f^{-1}(G),E), where $[f^{-1}(\mathrm{G})](e) = f^{-1}[\mathrm{G}(e)]$,for all $e \in$E.

<u>Definition (1.1.16):</u> [16]

Let X and Y be two nonempty sets and f : X → Y be a function. If $(\mathrm{F}_1, \mathrm{E})$, $(\mathrm{F}_2, \mathrm{E})$ ∈ SS(X, E) ,and $(\mathrm{G}_1, \mathrm{E})$, $(\mathrm{G}_2, \mathrm{E})$, $(\mathrm{G}_3, \mathrm{E})$ ∈ SS(Y ,E) ,then

i. If $(\mathrm{F}_1, \mathrm{E}) \tilde{\subseteq} (\mathrm{F}_2, \mathrm{E})$ then $f[(\mathrm{F}_1, \mathrm{E})] \tilde{\subseteq} f[(\mathrm{F}_2, \mathrm{E})]$.

ii. If $(\mathrm{G}_1, \mathrm{E}) \tilde{\subseteq} (\mathrm{G}_2, \mathrm{E})$ then $f^{-1}[(\mathrm{G}_1, \mathrm{E})] \tilde{\subseteq} f^{-1}[(\mathrm{G}_2, \mathrm{E})]$.

iii. $(\mathrm{F}_2, \mathrm{E}) \tilde{\subseteq} f^{-1}[f(\mathrm{F}_2, \mathrm{E})]$,

$f^{-1}[f(\mathrm{G}_3, \mathrm{E})] = (\mathrm{G}_3, \mathrm{E})$, if f is injective.

iv. $f\,[f^{-1}(G_3, E)] \tilde{\subseteq} (G_3, E)$.

$f\,[f^{-1}(G_3, E)] = (G_3, E)$, if f is surjective.

Proposition(1.1.17):[16]

Let X and Y be two nonempty sets and $f : X \rightarrow Y$ be a surjective function. If $(F_1, E), (F_2, E) \in SS(X,E)$,and $(G_1, E), (G_2, E) \in SS(Y,E)$, then

i. $f\,[(F_1, E) \tilde{\cup} (F_2, E)] = f\,[(F_1, E)] \tilde{\cup} f\,[(F_2, E)]$.

ii. $f\,[(F_1, E) \tilde{\cap} (F_2, E)] \tilde{\subseteq} f\,[(F_1, E)] \tilde{\cap} f\,[(F_2, E)]$.

iii. $f\,[(F_1, E) \tilde{\cap} (F_2,E)] = f\,[(F_1,E)] \tilde{\cap} f\,[(F_2,E)]$, if f is injective.

iv. $f\,[(F_1, E)] \setminus f\,[(F_2, E)] \tilde{\subseteq} f\,[(F_1, E) \setminus (F_2, E)]$.

v. $f^{-1}[(G_1,E) \tilde{\cup} (G_2,E)] = f^{-1}[(G_1,E)] \tilde{\cup} f^{-1}[(G_2,E)]$.

vi. $f^{-1}[(G_1,E) \tilde{\cap} (G_2,E)] = f^{-1}[(G_1,E)] \tilde{\cap} f^{-1}[(G_2,E)]$.

Proposition(1.1.18):

Let $f : X \rightarrow Y$ be a function, if $(F,E) \in SS(X,E)$ and $(G,E) \in SS(Y,E)$, then

i. $f\,(F,E)^c = (f\,(F,E))^c$ if f bijective,

ii. $f^{-1}\,(G,E)^c = (f^{-1}\,(G,E))^c$

Proof: Clear.

Remark(1.1.19):

As a consequence of Definition(1.1.15) we can define a function f_e from a subset $F(e)$ of X into Y, i.e. f_e: $F(e) \rightarrow Y$, for all $e \in E$ as following:

i. $f_e\,[F(e)] = f[F(e)]$, for all $(F,E) \in SS(X, E)$,

ii. $f_e^{-1}[G(e)] = f^{-1}[G(e)]$,for all $(G,E) \in SS(Y, E)$, called it e-parameter function of function f at e.

Proposition (1.1.20):

Let $f: X \longrightarrow Y$, $g: Y \longrightarrow Z$ and $g \circ f: X \longrightarrow Z$ be functions, then for each $e \in E$,$(g \circ f)_e = g_e \circ f_e$.

Proof: Clear.

Proposition(1.1.21):

Let $f: X_1 \longrightarrow Y_1$, $g: X_2 \longrightarrow Y_2$ and $g \times f: X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ be functions , then for each $(e, e`) \in E \times E$, $(g_e \times f_{e`}) = (g \times f)_{(e,e`)}$

Proof: Clear.

Definition (1.1.22): [17,2]

Let (F, E) and (G, E) are two soft sets over X. Then

i. A soft set (F,E) over X is said to be a soft point if there exactly one $e \in E$ such that F(e) is a singleton, say $\{x\}$, and F $(e`) = \emptyset$, for all $e` \in E - \{e\}$. Such a soft point is denoted by x^e . Let $SP(X)$ be the set of all soft points over set X .

ii. The soft point x^e is said to be in the soft set (G, E) denoted by $x^e \tilde{\in}$(G,E), if $x \in G(e)$.

iii. Two soft points $x^e, y^{e`}$ are said to be equal if $e = e`$ and $x = y$.Thus, $x^e \neq y^{e`}$ $\Leftrightarrow x \neq y$ or $e \neq e`$.

Example(1.1.23):

Let X =$\{x_1, x_2, x_3\}$, E =$\{e, e`\}$ and(F, E) , (G, E) are two soft sets over X , where (F, E) = $\{(e, \{x_1\}), (e`, \emptyset)\}$ and (G,E)= $\{(e, \{x_1, x_2, x_3\}), (e`, \{x_2, x_4\})\}$. Then (F, E)=$x_1^e$ is a soft point and $x_1^e \tilde{\in}$(G,E).

Proposition(1.1.24) : [17]

$x^e \tilde{\in}$ (F, E) if and only if $x^e \tilde{\notin}$ (F, E)^c.

Proposition (1.1.25):[17]

For two soft sets (F, E) and (G, E),(F, E)$\tilde{\subseteq}$(G, E)if and only if if $x^e \tilde{\in}$

(F,E) then $x^e \tilde{\in}$ (G, E) and hence (F, E) = (G, E) if and only if $x^e \tilde{\in}$ (F, E) if and only if $x^e \tilde{\in}$ (G, E).

Definition (1.1.26): [21]

Let (F, E) be a soft set over X and A be a non-empty subset of X. Then the subsoft set of (F, E) over A denoted by (F_A ,E), is defined as follows $F_A(e) = A \cap F(e)$, for all $e \in E$. In other words $(F_A, E) = \tilde{A} \tilde{\cap} (F, E)$.

Remark (1.1.27) :

If $F(e) \subseteq A$, for all $e \in E$ then (F, E)= (F_A , E) .

Example(1.1. 28):

Let X={x ,y, z} ,E={ e, e`} and let A={ $x$ ,$y$} if (F,E)={($e$,{$x$}),($e$`,{ y})}

and $F_A(e) = A \cap F(e) = \{x\}$, $F_A(e`) = A \cap F(e`) = \{y\}$ then (F, E)= (F_A , E) .

Definition (1.1.29): [3]

Let (F,E_1) and(G,E_2) be two soft sets over X .Then the Cartesian product of (F, E_1) and (G, E_2) denoted by $(F, E_1) \times (G, E_2)$ is a soft set $(H, E_1 \times E_2)$ where $H: E_1 \times E_2 \rightarrow P(X \times X)$ and $H(e, e`) = F(e) \times G(e`)$ $\forall$ $(e, e`) \in E_1 \times E_2$.

Definition (1.1.30):[3]

Let (F_i, E_i) are soft sets over X_i,where $i \in I=\{1,2,\ldots,n\}$,and $p_i : \prod_{i=1}^{n} X_i \longrightarrow X_i$, $q_i : \prod_{i=1}^{n} E_i \longrightarrow E_i$ be projection functions in classical meaning. Then the soft functions (p_i, q_i), $i \in I$, is called soft projection function from $\prod_{i=1}^{n} X_i$to X_i and defined by:

$$(p_i, q_i)(\prod_{i=1}^{n}(F_i, E_i)) = (p_i, q_i)(\prod_{i=1}^{n} F_i, \prod_{i=1}^{n} E_i)$$
$$= (p_i(\prod_{i=1}^{n} F_i), q_i(\prod_{i=1}^{n} E_i)) = (F_i, E_i), \quad i \in I$$

In particular if i=1,2 then we have,

$$(p_i, q_i)((F_1,E_1)\times(F_2,E_2))$$
$$=(p_i, q_i)((F_1\times F_2),(E_1\times E_2))$$
$$=(p_i(F_1\times F_2))\times q_i(E_1\times E_2))$$
$$=(F_i, E_i)\quad, i=1,2.$$

Proposition(1.1.31) :

Let SS(X, E) and SS(Y,E) be families of soft sets. For two soft sets (F,E) and (G,E) over X and Y respectively . Then we have the following:

i. $\tilde{X}\times\tilde{Y}=\widetilde{X\times Y}$, where $\widetilde{X\times Y}$ denotes the absolute soft set X×Y with respect to parameter set E × E.

ii. $(F,E)\times\tilde{\Phi}=\tilde{\Phi}\times(G,E)=\tilde{\Phi}=\tilde{\Phi}\times\tilde{\Phi}$. [18]

iii. $[(F_1,E)\tilde{\cap}(F_2,E)]\times[(G_1,E)\tilde{\cap}(G_2,E)]=[(F_1,E)\times(G_1,E)]\tilde{\cap}[(F_2,E)\times(G_2,E)]$.[18]

Proof:

i. $\widetilde{X\times Y}=(X\times Y, E\times E)$, implies that by Definition(1.1.5)

$(X\times Y)(e,e`)=X\times Y=X(e)\times Y(e`)=\tilde{X}\times\tilde{Y}, \forall(e,e`)\in E\times E$.

On the other hand, $\tilde{X}\times\tilde{Y}=X(e)\times Y(e`)$,by Definition (1.1.5) we have $\forall e\in E$, $X(e)=X$ and $\forall\, e`\in E$, $Y(e`)=Y$ . So $\forall\, e\in E$ and $\forall e`\in E$, $X(e)\times Y(e`)=X\times Y$, which implies $\widetilde{X\times Y}=\tilde{X}\times\tilde{Y}$.

Remark(1.1.32):

By applying Definition(1.1.30)the inverse of soft projection function (p_i, q_i) can be defined as following : $(p_i, q_i)^{-1}(F_i, E_i)=(p_i^{-1}(F_i), q_i^{-1}(E_i))$. If $i=1,2$ we have:

$(p_1, q_1)^{-1}(F_1,E_1)=(p_1^{-1}(F_1), q_1^{-1}(E_1))$,where $\forall\, e\in E$

$p_1^{-1}[F_1(e)]=F_1(e)\times X_2=F_1\times X_2$ and $q_1^{-1}(E_1)=E_1\times E_2$,thus

$(p_1, q_1)^{-1}(F_1, E_1)=(p_1^{-1}(F_1), q_1^{-1}(E_1))$

$= (F_1 \times X_2), (E_1 \times E_2)$

$= (F_1 , E_1) \times (X_2 , E_2)$

$= (F_1 , E_1) \times \widetilde{X}_2$,

more ever $(F_1 ,E_1) \times \widetilde{X}_2$ is a soft set over $X_1 \times X_2$.

1.2 Soft topological spaces:

This section will contain the definition of soft topological space and its remarks, properties and propositions.

In addition, we introduce the definition of soft constant space which is needed in this work.

Definition(1.2.1):[21]

Let τ be the collection of soft sets over X with the fixed set of parameters E . Then τ is said to be a soft topology on X, if

i. $\widetilde{\Phi}$, $\widetilde{X}$ belong to τ,

ii. the intersection of any two soft sets in τ belongs to τ,

iii. the union of any number of soft sets in τ belongs to τ.

The triple (X , τ, E) is called a soft topological space, or *sts* for short over X .
The members of τ are called soft open sets. The complement of a soft open set is called the soft closed set.

Examples of soft topology (1.2.2) :[21]

i. The indiscrete soft topology on X is the family $\tau = \{\widetilde{\Phi}, \widetilde{X}\}$.

ii. The discrete soft topology on X is the family $\tau =$ SS(X,E).

Proposition (1.2.3):[21]

Let (X, τ_1, E) and (X, τ_2, E) be two $sts's$ over X, then (X, $\tau_1 \tilde{\cap} \tau_2$,E), where $\tau_1 \tilde{\cap} \tau_2 = \{$(F, E):(F, E) $\in \tau_1$ & (F, E) $\in \tau_2\}$ is a sts over X. But the union of two $sts's$ over X may not be a sts over X itself.

Definition (1.2.4):[16]

Let τ_1 and τ_2 be two $sts's$ over X. Then τ_2 is said to be soft finer than τ_1 if $\tau_1 \subseteq \tau_2$.

Proposition (1.2.5):[21]

Let (X, τ, E) be a sts over X. Then the collection $\tau_e = \{$F(e) : (F, E)$\in \tau\}$ defines a topology on X for each $e \in$ E .

Example(1.2.6):

Let X $= \{x_1, x_2, x_3\}$, E $= \{e, e`\}$ and let $\tau = \{\tilde{\Phi}$, $\tilde{X}$, ($F_1$,E),( $F_2$, E),( $F_3$, E) ,( $F_4$, E),( $F_5$, E)} be $sts$ over X . Here ($F_1$, E) ,( $F_2$, E),( $F_3$, E) ,( $F_4$, E), ( $F_5$, E) are soft sets over X, defined as follows: $F_1(e) = \{x_1\}$, $F_1(e`) = \{x_1, x_3\}$, $F_2(e) = \{x_2\}$, $F_2(e`) = \{x_1\}$, $F_3(e) = \{x_1, x_2\}$, $F_3(e`) = \{x_1, x_3\}$, $F_4(e) = \emptyset$, $F_4(e`) = \{x_1\}$, $F_5(e) = \{x_1, x_2\}$, $F_5(e`) =$ X .Then $\tau_e = \{\emptyset$, X,$\{x_1\}$,$\{x_2\}$,$\{x_1, x_2\}\}$ and $\tau_{e`} = \{\emptyset$, X,$\{x_1\}$,$\{x_1, x_3\}\}$ are topologies on X.

Remark(1.2.7):

The converse of Proposition(1.2.5) needs not true in general as the following example shows:

Let X $= \{x_1, x_2, x_3\}$, E $= \{e, e`\}$ and let $\tau = \{\tilde{\Phi}$, $\tilde{X}$, ($F_1$, E) ,($F_2$, E),($F_3$, E),($F_4$, E)} be a collection of soft sets , where ($F_1$, E) ,($F_2$, E),($F_3$, E)and($F_4$, E) are soft sets over X, defined as follows $F_1$(e)$= \{x_2\}$, $F_1$ (e`) $= \{x_1\}$, F_2(e) $= \{x_2, x_3\}$, F_2(e`) $= \{x_1, x_2\}$, $F_3$(e) $= \{x_1, x_2\}$, $F_3$(e`) $= \{x_1, x_2\}$, F_4(e) $= \{x_2\}$, F_4(e`) $= \{x_1, x_3\}$. Then $\tau_e = \{\emptyset$, X, $\{x_2\}$,$\{x_2, x_3\}$,$\{x_1, x_2\}\}$ and $\tau_{e`} = \{\emptyset$, X,$\{x_1\}$,$\{x_1, x_3\}$,$\{x_1,$

$x_2\}\}$ are topologies on X . But, τ is not a soft topology over X because (F_2, E) $\tilde{\cup}(F_3, E) = (F, E)$ where F(e)= X, and F(e\`) = $\{x_1, x_2\}$ and so (F, E)$\notin \tau$.

Remark(1.2.8):

i. The topology τ_e in Proposition(1.2.5) is called e-parameter topology and the pair (X, τ_e) is called e-parameter space.

ii. A soft set is called soft clopen if it is soft open and soft closed.

Definition (1.2.9) :

A soft constant space (briefly sc-space) over X is a soft topology (X, τ,E) ,whose members only $\tilde{A}$ for all A$\subseteq$X .

Examples(1.2.10):

i. Let X=$\{x , y\}$, E=$\{e,e\`\}$. Here $\emptyset$, X, $\{x\}$,$\{y\}$ all sub sets of X, if

$A_1=\emptyset \Longrightarrow \tilde{A}_1 = \tilde{\Phi}$, $A_2=X \Longrightarrow \tilde{A}_2 = \tilde{X}$

$A_3=\{x\} \Longrightarrow \tilde{A}_3 = \{(e, \{x\}), (e\`,\{x\})\}$,

$A_4=\{y\} \Longrightarrow \tilde{A}_4 = \{(e, \{y\}), (e\`,\{y\})\}$.

So the collection $\{(F,E)_{A_i}$, $A_i \subseteq X$, $i = 1,2,3,4\}$ is sc-space over X.

ii. The discrete soft topology is not sc-space over X .

Proposition (1.2.11):

Let X be a set, E be the set of parameters. If (X, τ, E) is a sc-space , then (X, τ_e) is a discrete space for all e∈E .

Proof: Clear.

Definition (1.2.12):[21]

Let (X, τ, E) be a sts over X and A be a non-empty subset of X . Then τ_A =$\{(F_A, A) | (F, E) \in \tau\}$ is said to be the soft relative topology on A and (A, τ_A, E) is called a soft subspace of (X, τ, E).

Theorem (1.2.13):[21]

Let(A, τ_A, E) be a soft subspace of *sts* (X, τ, E) and (F, E) be a soft set over X, then:

i. (F, E) is soft open over A if and only if $(F, E) = \widetilde{A} \widetilde{\sqcap} (G, E)$ for some $(G, E) \in \tau$

ii. (F, E) is soft closed over A if and only if $(F, E) = \widetilde{A} \widetilde{\sqcap} (G, E)$ for some soft closed set (G, E) over X.

Proposition (1.2.14):[21]

Let (A, τ_A, E) be a soft subspace of a *sts* (X, τ, E) and (F, E) be a soft open set over A . If $\widetilde{A} \in \tau$ then $(F, E) \in \tau$.

Definition (1.2.15):[8]

Let (X, τ, E) be a *sts* and let (G, E) be a soft set over X. Then the soft interior of (G,E) denoted by $(G, E)^\circ$ is the soft set defined as: $(G, E)^\circ = \widetilde{\cup} \{(F, E)$: (F, E) is soft open and $(F, E) \widetilde{\subseteq} (G, E)\}$. Thus, $(G, E)^\circ$ is the largest soft open set contained in (G, E).

Theorem (1.2.16):[8]

Let (X, τ, E) be a *sts* and let (F, E) and (G, E) be soft sets over X. Then :

i. $\widetilde{\Phi}^\circ = \widetilde{\Phi}$, $\widetilde{X}^\circ = \widetilde{X}$.

ii. $(F, E)^\circ \widetilde{\subset} (F,E)$.

iii. $((F, E)^\circ)^\circ = (F, E)^\circ$.

iv. (F,E) is a soft open set if and only if $(F, E)^\circ = (F,E)$.

v. If $(F,E) \widetilde{\subseteq} (G,E)$ implies $(F, E)^\circ \widetilde{\subseteq} (G, E)^\circ$.

vi. $(F, E)^\circ \widetilde{\sqcap} (G, E)^\circ = [(F,E) \widetilde{\sqcap} (G,E)]^\circ$.

vii. $(F, E)^\circ \widetilde{\cup} (G, E)^\circ \widetilde{\subseteq} [(F,E) \widetilde{\cup} (G,E)]^\circ$.

Proposition(1.2.17):

Let(X, τ,E)be a sts and (F,E)$\in$SS(X,E),then $x^e \tilde{\in}$ $(F, E)^\circ$ if and only if there exists a soft open set (H,E) over X, such that $x^e \tilde{\in}$ (H,E)$\tilde{\subseteq}$ (F,E), where $x^e \tilde{\in} \tilde{X}$.

Proof: Clear.

Definition (1.2.18):[8]

Let (X, τ, E) be a sts and let (G, E) be a soft set over X. Then the soft closure of (G, E) denoted $\overline{(G, E)}$ is the soft set ,defined as: $\overline{(G, E)} = \tilde{\bigcap}$ { (F, E) : (F, E) is soft closed and (G, E)$\tilde{\subseteq}$ (F, E)}. Note that $\overline{(G, E)}$ is the smallest soft closed set containing (G, E).

Theorem (1.2.19):[8]

Let (X, τ , E) be a s sts over X, (F, E) and (G, E) are soft sets over X. Then

i. $\overline{\tilde{\Phi}} = \tilde{\Phi}$ and $\overline{\tilde{X}} = \tilde{X}$.

ii. (F , E)$\tilde{\subset}$ $\overline{(F , E)}$.

iii. (F , E) is a soft closed set if and only if $\overline{(F , E)}$ = (F , E).

iv. $\overline{\overline{(F, E)}}$ = (F , E).

v. (F , E) $\tilde{\subset}$ (G, E) implies $\overline{(F , E)}$ $\tilde{\subset}$ $\overline{(G, E)}$.

vi. $\overline{(F , E) \tilde{\cup} (G, E)} = \overline{(F , E)} \tilde{\cup} \overline{(G, E)}$.

vii. $\overline{(F , E) \tilde{\cap} (G, E)} \tilde{\subset} \overline{(F , E)} \tilde{\cap} \overline{(G, E)}$.

Proposition(1.2.20):

Let (X, τ,E) be a sts and (F,E)$\in$SS(X,E) ,then $x^e \tilde{\in}$ $\overline{(F, E)}$ if and only if for each soft open set (H,E) over X, contain x^e then (H,E)$\tilde{\cap}$ (F,E) $\neq$ $\tilde{\Phi}$

Proof: clear.

Definition (1.2.21):[8]

Let (X , τ, E) be a sts over X. Then soft boundary of a soft set (F,E) is denoted by $(F,E)^b$ and is defined as: $(F,E)^b = \overline{(F,E)} \tilde{\cap} \overline{(F,E)^c}$.

Remark (1.2.22):

i. $(F,E)^b$ is soft closed set, because it's intersection of two soft closed set.

ii. $[(F,E)^c]^b = \overline{(F,E)^c} \tilde{\cap} \overline{[(F,E)^c]^c} = \overline{(F,E)^c} \tilde{\cap} \overline{(F,E)} = (F,E)^b$, this impels to $[(F,E)^c]^b = (F,E)^b$.

Definition(1.2.23):[2]

Let (X, τ, E) be a sts over X. A soft set (F, E) in (X, τ, E) is called a soft neighborhood of the soft point $x^e \tilde{\in}$ (F, E) if there exists a soft open set (G, E) such that $x^e \tilde{\in}$ (G, E) $\tilde{\subseteq}$ (F, E).

Definition (1.2.24):[17]

Let (X, τ, E) be a sts. A soft point x^e is said to be a limit soft point of a soft set (F, E) over X if every soft open set containing x^e contains at least one soft element of (F, E) other than x^e, i.e. if $\forall$(G, E) $\in \tau$ with $x^e \tilde{\in}$ (G, E), (F, E) $\tilde{\cap}$ $[(G, E) - x^e] \neq \tilde{\Phi}$. The union of all limiting soft points of (F, E) is called the derived soft set of (F, E) and is denoted by (F, E)` .

Remark(1.2.25):[17]

Let (F, E) be a soft set over X ,then $\overline{(F,E)} = (F,E) \tilde{\cup} (F,E)$`.

Definition (2.1.26) :[21]

Let (X, τ, E) be a sts over X . A subcollection $\mathfrak{B}$ of τ is called a soft basis for the soft topology τ if every member of τ can be expressed as a union of members of $\mathfrak{B}$.

Definition(1.2. 27) : [17]

A soft set (F, E) is said to be a soft neighborhood (briefly soft nbd) of the soft set (H, E) if there exists a soft set (G, E)$\in \tau$ such that (H, E) $\widetilde{\subseteq}$ (G, E) $\widetilde{\subseteq}$ (F, E) . If (H, E) $= x^e$, then (F, E) is said to be a soft nbd of the soft element x^e. The soft neighborhood system of a soft element x^e, denoted by $\mathcal{N}_{x^e}$,is the family of all its soft nbds.

Proposition(1.2.28):[17]

The neighborhood system $\mathcal{N}_{x^e}$ at x^e in a sts (X, τ, E) has the following properties:

i. $\mathcal{N}_{x^e} \neq \emptyset, \forall x^e \in SP$;

ii. $x^e \widetilde{\in}$ (F, E), $\forall$ (F, E) $\in \mathcal{N}_{x^e}$;

iii. (F, E) $\in \mathcal{N}_{x^e}$ and (F, E) $\widetilde{\subseteq}$ (G, E) then (G, E) $\in \mathcal{N}_{x^e}$;

iv. (F, E),(G, E) $\in \mathcal{N}_{x^e}$ then (F, E) $\widetilde{\cap}$ (G, E) $\in \mathcal{N}_{x^e}$;

v. (F, E) $\in \mathcal{N}_{x^e}$ then $\exists$(G, E) $\in \mathcal{N}_{x^e}$such that (G, E) $\widetilde{\subseteq}$ (F,E) and (G, E) $\in$ $\mathcal{N}_{x^{e`}}$, $\forall$ $\mathcal{N}_{x^{e`}}$ $\widetilde{\in}$ (G, E).

Proposition(1.2.29):[17]

A soft set (F, E) over X is soft open if and only if (F, E) is a nbd of each its soft points.

Proposition (1.2.30):[1]

Let (X, τ_1, E)and(Y, τ_2, E) be $sts's$. Let $\mathfrak{B}$ ={(F, E)×(G, E)|(F,E)$\in \tau_1$, (G, E) $\in\tau_2$} and τ be the collection of all arbitrary union of elements of $\mathfrak{B}$. Then τ is a soft topology over X × Y .

Definition (1.2.31):[1]

Let (X, τ_1,E)and(Y, τ_2,E) be $sts's$.Then the soft space (X ×Y, τ, E×E) as defined in proposition(1.2.30) is called product soft topological space over X × Y .Note that we use τ_{prod} to denote the product soft topology over X × Y .

Proposition (1.2.32):[1]

Let (F, E) and (G, E) be soft sets in SS(X,E) and SS(Y,E) , respectively. Then $[(F, E) \times (G, E)]^c = [(F, E)^c \times \widetilde{Y}] \widetilde{\cup} [\widetilde{X} \times (G, E)^c]$.

Definition(1.2.33):

Let(X , τ, E) and (Y , τ`,E) be $sts's$. We called the product soft space (X × Y , τ_{prod}, E×E) is S − product soft topological space, if for all soft closed set over X × Y can written as follow:

$$(H , E \times E) = \widetilde{\bigcup_{i \in I}} (F_i, E) \times (G_i, E) , I = \{1,2, \dots , n\}$$ where : (F_i , E) is soft closed set over X and (G_i, E) is soft closed set over Y, for all $i \in$ I .

Introduction:

In this chapter, we recall the definitions of soft continuous, soft open and soft closed and we give some properties for this concept . Also ,we introduce some concepts about soft interior and some new properties for product soft topology.

We introduce the definitions of soft net and soft cluster point, and we give remarks , theorems, examples, and propositions about these concepts.

Finally, we recall the definition of soft separation axioms and propositions, theorems, examples, remarks and corollaries related to it. Also, we study some types of soft functions about a separation axioms.

2.1 On Soft Functions:

In this section, we recall some basic definitions of soft continuous, soft open, soft closed, soft homeomorphism and we investigate the properties of the restriction , composition and product of these concepts .

In addition, we introduce some new concepts about soft interior which we need in our work .

Definition(2.1.1):[5]

Let (X, τ, E) and (Y, τ′, E) be sts's, f : X → Y be a function. The function f is soft continuous at x^e ∈̃ X̃ ,if for each soft neighbourhood (H,E) of f(x^e) there exists a soft neighborhood (F, E) of x^e such that f((F, E)) ⊂̃ (H,E) . If f is soft continuous function for all x^e, then f is called soft continuous function .

Theorem (2.1.2):[5]

Let (X, τ, E) and (Y, τ′, E) be sts's, f : X⟶ Y be a function. Then the following conditions are equivalent:

i. f : (X, τ, E) → (Y, τ′ , E) is a soft continuous function,

ii. For each soft open set (G, E) over Y , f −1 ((G, E)) is a soft open set over X,

iii. For each soft closed set (H, E) over Y,f −1((H, E)) is a soft closed set over X, iv. For each soft set (F, E) over X, f(((F,E))‾) ⊂̃ ((f(F,E)))‾,

v. For each soft set (G, E) over Y , (f -1((G,E)))‾⊂̃ f −1 (((G,E))‾),

vi. For each soft set (G, E) over Y , f −1 ((G, E)^°) ⊂̃ (f −1 (G, E))^° .

Definition(2.1.3):[5]

Let (X, τ, E) and (Y, τ′, E) be sts's, f : X → Y be a function .Then:

i. A function f is said to be a soft open ,if f(F, E) is soft open set over Y ,for each soft open set (F, E) over X ,.

ii. A function f is said to be a soft closed ,if f(F, E) is soft closed set over Y, for each soft closed set (F,E) over X.

Theorem (2.1.4):[5]

Let (X, τ, E) and (Y, τ′ , E) be sts's , f : X → Y be a function.

i. f is a soft open function if and only if for each soft set (F, E) over X, $f((F, E)^{\circ}) \tilde{\subset} (f(F, E))^{\circ}$ is satisfied.

ii. f is a soft closed function if and only if for each soft set (F, E) over X, $((f(F,E)))^{-} \tilde{\subset} f(((F,E))^{-})$ is satisfied.

Definition(2.1.5):[5]

Let (X, τ, E) and (Y, τ′, E) be sts's, f : X → Y be a function. If

i. f is bijective,

ii. f is soft continuous,

iii. f^{-1} is soft continuous .

Then f is said to be soft homeomorphism from X to Y. When a homeomorphism f exists between X and Y, we say that X is soft homeomorphic to Y, and we write $(X, \tau, E) \cong_s (Y, \tau', E)$.

Proposition(2.1.6):

Let (X, τ, E) and (Y, τ′, E) be sts's , f : X → Y be a bijective function. Then f is a soft open (soft closed) function if and only if f^{-1} is soft continuous function.

Proof: The proof is complete by using the fact $(f^{-1})^{-1}(G,E) = f(G,E)$.

Theorem (2.1.7) :[5]

Let (X, τ, E) and (Y, τ′ , E) be sts's, f : X → Y be a bijective function. Then the following conditions are equivalent:

i. f is a soft homeomorphism,

ii. f is a soft continuous and soft closed function,

iii. f is a soft continuous and soft open function.

Proposition(2.1.8):

The composition of soft continuous(soft open , soft closed) function is soft continuous (soft open ,soft closed) function.

Proof: Clear.

Proposition(2.1.9):

Let(X,τ,E),(Y,τ`,E) be sts's .If f:X→Y is a soft continuous (soft open) function and A be a subset of X,then the restriction function $f|_A$ is soft continuous (soft open) .

Proof: Clear.

Remark(2.1.10):

The restriction of soft closed function is not necessary being soft closed. As the following example shows:

Let $X = \{x_1, x_2, x_3\}$, $Y= \{y_1, y_2\}$, $E= \{ e, e`\}$ and $\tau =\{ \tilde{\Phi}, \tilde{X}, (F,E)\}$, $\tau'=\{\tilde{\Phi}, \tilde{Y}, (G,E)\}$ be two soft topologies defined over X and Y,(resp.). Here(F,E)and (G,E) are soft sets over X and Y(resp.) defined as follows: $F(e)=\{x_1\}$, $F(e`)=\{x_1,x_2\}$ and $G(e)=\emptyset$, $G(e`)=\{y_1\}$. Now we define the function $f: X \to Y$ as : $f(x_1)=f(x_2) = y_1$, $f(x_3)= y_2$. It is clear that f is soft closed. If $A= \{x1, x2\}$ then $\tau_A=\{\tilde{\Phi}, \tilde{A}, (FA, E)\}$ where $F_A(e)=\{x_1\}$ , $F_A(e`)= \{x_1,x_2\}$ then the restriction function $〖f|〗_A:A\longrightarrow Y$ is not soft closed function.

Now , we put a condition on set $\tilde{A}$ to satisfy the restriction function $〖f|〗_A$ is soft closed function .

Proposition(2.1.11):

Let (X,τ,E),$(Y,\tau`,E)$ be sts's and A ⊆X .If $f:X\to Y$ is soft closed function and $\tilde{A}$ is soft closed set over X ,then $〖f|〗_A:A\to Y$ is soft closed function.

Proof:

Let (F,E) be a soft closed set over A, then by Theorem (1.2.13,ii) we have $(F,E) = \tilde{A} \tilde{\cap} (G,E)$,for some soft closed (G,E) over X and $〖(f|〗_A)(F,E) =f(\tilde{A}) \tilde{\cap} f(G,E)$ since $f(\tilde{A})$, $f(G,E)$ are soft closed set over Y then $〖(f|〗_A)(F,E)$ is soft closed set over Y. Hence $〖f|〗_A$ is soft closed function.

Remark(2.1.12):

It is not necessary the constant function from a sts (X,τ,E) to a sts (Y,τ',E) be soft continuous. As the following example shows:

Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ and $E=\{e, e`\}$. Then $\tau = \{\tilde{\Phi}, \tilde{X}, (F,E)\}$ and $\tau' = \{\tilde{\Phi}, \tilde{Y}, (G, E)\}$ are sts^' s over X and Y(resp.). Here (F, E) and (G, E) are soft sets over X and Y (resp.) defined as follows:

$F(e) =\{x_1,x_2\}$,$F(e`) = \{x_3\}$ and $G1(e) = \{ y_1\}$, $G1(e`) = \{y2\}$. If we get the function $f:X \to Y$ defined as $f(x) = y_1, \forall x \in X$ then f is not a soft continuous function, since f^(- 1) (G,E) ⍰ τ, where f^(-1)[G (e)] = X, f^(-1)[G (e`)] = ∅.

Remark(2.1.13):

The identity function $f: (X, \tau,E) \longrightarrow (X, \tau`,E)$ is soft continuous when $\tau = \tau`$ and it is not necessary soft continuous when $\tau \neq \tau`$. As the following example shows:

Let $X = \{x_1, x_2, x_3\}$, $E = \{e, e`\}$ and $\tau = \{\tilde{\Phi}, \tilde{X}, (F1, E),(F2, E),(F3, E),(F4,E)\}$, $\tau' = \{\tilde{\Phi}, \tilde{X},(G1, E),(G2, E),(G3, E),(G4, E)\}$ be two soft topologies defined on X where (F1, E) ,(F2, E),(F3, E),(F4, E),(G1, E),(G2, E),(G3, E) and (G4, E) are soft sets over X, defined as follows:
$F1(e) = \{x_2\}$, $F1(e`) = \{x_1\}$, $F2(e) = \{x_2, x_3\}$, $F2(e`) = \{x_1, x_2\}$, $F3(e) = \{x_3\}$, $F3(e`) = \{x_1, x_2\}$, $F4(e) = \emptyset$, $F4(e`) = \{x_1\}$, $F5(e) = X$, $F5(e`) = \{x_1, x_2 \}$. And
$G1(e) = \{x_2\}$, $G1(e`) = \{x_1\}$, $G2(e) = \{x_2, x_3\}$, $G2(e`) = \{x_1, x_2\}$, $G3(e) = \{x_1, x_2\}$, $G3(e`) = X$, $G4(e) = \{x_2\}$, $G4(e`) = \{x_1, x_2\}$, Then (X, τ, E), (X, τ', E) are two sts's. Then the identity function $f: X \longrightarrow X$ is not soft continuous function.

Remark(2.1.14):

Let (X,τ,E) and $(Y,\tau`,E)$ be sc-spaces , then:

The constant function from X into Y is soft continuous.

The identity function is soft continuous,soft open,soft closed.

Proposition(2.1.15):

Let(X,τ,E) be a sts and A be a non-empty subset of X then the inclusion function $i:A\longrightarrow X$ is:

soft continuous;

soft closed if $\tilde{A}$ is soft closed;

soft open if $\tilde{A}$ is soft open.

Proof: Clear.

Proposition(2.1.16):

Let (X,τ,E) and $(Y,\tau`,E)$ be sts's and $f:X\longrightarrow Y$ be soft continuous function .If $\tilde{A}$ is any soft open(soft closed) set over Y, then $f_A:f^{-1}(A)\longrightarrow A$, which defined by $f_A(x)=f(x)$, for all $x\in f^{-1}(A)$ is soft continuous function.

Proof:

Let (F,E) be a soft open set over A . To prove $f_A^{-1}(F,E)$ is soft open set over $f^{-1}(A)$. Since $\tilde{A}$ is soft open set over Y, then (F,E) is soft open set over Y. Since f is soft continuous, then$f^{-1}(F,E)$, $f^{-1}(\tilde{A})$ are soft open sets over X, thus $f_A^{-1}(F,E)=f^{-1}(\tilde{A})\tilde{\cap} f^{-1}(F,E)$ is soft open set over $f^{-1}(A)$. Hence f_A is soft continuous function.

Proposition (2.1.17):

Let (X,τ,E) and $(Y,\tau`,E)$ be sts's, and $f:X\longrightarrow Y$ be surjective function . If f is soft open (soft closed),then $f_A:f^{-1}(A)\longrightarrow A$ is soft open (soft closed) where $A\subseteq Y$.

Proof: Clear.

Proposition (2.1.18):

Let (X, τ, E), $(Y, \tau`, E)$ and $(Z, \tau``, E)$ be sts's and $f:X\longrightarrow Y$, $g:Y\longrightarrow Z$ be two functions, then:

i. If gof is an soft closed and f is surjective soft continuous, then g is soft closed .

ii. If gof is soft closed and g is injective soft continuous, then f is soft closed .

Proof:

i. Let (G,E) be a soft closed set over Y, then $f^{-1}(G,E)$ is soft closed set over X .But $(g\circ f)(f^{-1}(G,E))= g(G,E)$ is soft closed set over Z. So g is soft closed function .

ii. Let (F,E) be a soft closed set over X, then $(g\circ f)(F,E)$, is soft closed set over Z. Since g is soft continuous then $g^{-1}((g\circ f)(F,E))$, is soft closed set over Y, but $g^{-1}((g\circ f)(F,E))= g^{-1}(g(f(F,E)) = f(F,E)$ .So f is soft closed function.

Proposition (2.1.19):

Let(X,τ,E),(Y,τ',E) be a sts' s and $f:X\longrightarrow Y$ be surjective function , and B be a base for τ', then f is soft continuous if and only if $f^{-1}(G,E)\in\tau, \forall (G,E)\in B$.

Proof: Clear.

Proposition(2.1.20):

Let(X,τ,E), (Y,τ',E) be a sts's and $f:X\longrightarrow Y$ be surjective function , and B be a base for τ, then f is soft open if and only if $f(F,E)\in\tau', \forall (F,E)\in B$.

Proof: Clear.

Now, we introduce the following definition about soft interior.

Definition(2.1.21):

Let(X, τ, E) be a sts and(G, E) be a soft set over X . Then we associate with(G,E) a soft set over X, denoted by (G°,E) and defined as $G^{\circ}(e)= (G(e))^{\circ}$ where $(G(e))^{\circ}$ is the interior of $G(e)$ in(X, τ_e) for each $e \in E$.

Theorem (2.1.22):

Let (X ,τ, E) be a sts and (G,E) be a soft set over X . Then $(G, E)^{\circ} \tilde{\subseteq} (G^{\circ},E)$.

Proof:

Let $x^e \tilde{\in} (G, E)^{\circ}$ then there exists a soft open set (H,E) such that $(H,E) \tilde{\subseteq} (G, E)$ and $x^e \tilde{\in} (H, E)$ thus $x \in H(e)$.Since $H(e) \in \tau_e$ and $H(e) \subseteq G(e)$then $x \in (G(e))^{\circ}=G^{\circ}(e)$ thus $x^e \tilde{\in} (G^{\circ}, E)$. Hence $(G, E)^{\circ} \tilde{\subseteq} (G^{\circ}, E)$.

The following example shows that $(G^{\circ},E) \neq (G, E)^{\circ}$.

Let $X = \{h_1, h_2, h_3\}$, $E = \{e, e`\}$.Then $\tau =\{\tilde{\Phi}, \tilde{X}, (F_1,E) ,( F_2, E) ,( F_3, E) ,( F_4, E), ( F_5, E)\}$ is a sts over $X$. Here $(F_1, E) ,( F_2, E),( F_3, E) ,( F_4, E),( F_5, E)$ are soft sets over $X$ defined as follows: $F_1(e) =\{h_1\}$, $F_1(e`)=\{h_1, h_3\}$, $F_2(e)= \{h_2\}$, $F_2(e`)=\{h_1\}$, $F_3(e)= \{h_1, h_2\}$, $F_3(e`)=\{h_1, h_3\}$, $F_4(e) = \emptyset$, $F_4(e`)=\{h_1\}$, $F_5(e)=\{h_1, h_2\}$, $F_5(e`)= X$. Here $\tau_e=\{\emptyset, X,\{h_1\},\{h_2\},\{h_1, h_2\}\}$, $\tau_{e`}=\{\emptyset, X,\{h_1\} ,\{h_1, h_3\} \}$ .Let $(G,E)$ be a soft set over $X$ ,where $G(e)= \{h_1, h_2\}$, $G(e`) = \{h_2, h_3\}$ then $G^{\circ}(e)= G(e)$ and $G^{\circ}(e`)=\emptyset$. So $(G^{\circ},E)=\{(e, \{h_1, h_2\}),(e`,\emptyset)\}$,on other hand $(G, E)^{\circ} = \tilde{\Phi}$.Thus $(G^{\circ},E) \neq (G, E)^{\circ}$.

Now, we give a condition to satisfy the equality.

Corollary(2.1.23):

Let (X, τ, E) be a sts over X and (G, E) be a soft set over X. Then $(G^{\circ}, E) = (G, E)^{\circ}$ if and only if $(G^{\circ}, E) \in \tau$.

Proof:

(⟹) Clear.

(⟸)To show that $(G^{\circ}, E) = (G, E)^{\circ}$.By Theorem (2.1.22) we have $(G, E)^{\circ}$ □̃ (G°,E). We will prove that (G°, E) □̃ $(G, E)^{\circ}$.Since $G^{\circ}(e)$⊆$G(e)$ for each e∈E then (G°,E) □̃ (G, E) and since $(G^{\circ}, E) \in \tau$ then (G°, E) □̃ $(G, E)^{\circ}$. Thus $(G^{\circ},E) = (G, E)^{\circ}$.

Theorem(2.1.24):[5]

Let $f : (X, \tau,E) \longrightarrow (Y, \tau',E)$ be a soft continuous function, then for each $e \in E$, $f_e:(X, \tau_e) \longrightarrow (Y, \tau'_e)$ is a continuous function.

Remark (2.1.25):

The converse of Theorem(2.1.24) does not hold. As the following example shows:

Let $X = \{h_1, h_2, h_3\}$,$Y = \{a, b, c\}$ and $E = \{e, e`\}$.Then $\tau =\{\tilde{\Phi}, \tilde{X}, (F_1,E) ,(F_2, E) ,(F_3, E),(F_4, E),(F_5, E)\}$ is a sts over X and $\tau' = \{\tilde{\Phi}, \tilde{Y} , (G_1, E),(G_2, E), (G_3, E)\}$ is a sts over Y . Here (F_1, E) ,(F_2, E),(F_3, E) ,(F_4, E),(F_5, E) are soft sets over X and (G_1, E),(G_2, E) ,(G_3, E) are soft sets over Y, defined as follows: $F_1(e) =\{h_1\}$, $F_1(e`)=\{h_1, h_3\}$, $F_2(e)= \{h_2\}$, $F_2(e`)=\{h_1\}$, $F_3(e)= \{h_1, h_2\}$, $F_3(e`) = \{h_1, h_3\}$, $F_4(e) = \emptyset$, $F_4(e`) = \{h_1\}$, $F_5(e)=\{h_1, h_2\}$, $F_5(e`)= X$ . And $G_1(e) = Y$, $G_1(e`) = \{b\}$, $G_2(e) = \{a\}$, $G_2(e`) = \{b\}$, $G_3(e) = \{a, b\}$, $G_3(e`)=\{b\}$. If we get the function $f : X \to Y$ defined as $f(h_1) = b$, $f(h_2) = a$, $f(h_3) = c$,then f is not a soft continuous function, since $f^{-1} (G_1,E) \notin \tau$, where $f^{-1} (G_1(e)) = X$, f^(-

$1)(G_1(e`)) = \{h_1\}$. Also, $f_e: (X, \tau_e) \to (Y, \tau_e^{'})$ and $f_{(e`)} : (X, \tau_{(e`)}) \to (Y, \tau_{(e`)}^{'})$ are continuous functions. Here $\tau_e = \{\emptyset, X, \{h_1\}, \{h_2\}, \{h_1, h_2\}\}$, $\tau_{(e`)} = \{\emptyset, X, \{h_1\}, \{h_1, h_3\}\}$ and $\tau_e^{'} = \{\emptyset, Y, \{a\}, \{a, b\}\}$, $\tau_{(e`)}^{'} = \{\emptyset, Y, \{b\}\}$.

Now, we give a condition to satisfy the converse of Theorem(2.1.24).

Theorem (2.1.26):

Let (G°,E) be a soft open set over Y, for each soft set (G, E) then $f :(X, \tau, E) \longrightarrow (Y, \tau', E)$ is a soft continuous function if and only if $f_e: (X, \tau_e) \longrightarrow (Y, \tau_e^{'})$ is continuous function, for each $e \in E$.

Proof:

($\Longrightarrow$)By Theorem(2.1.24) .

($\Longleftarrow$)Let $f_e : (X, \tau_e) \longrightarrow (Y, \tau_e^{'})$ be a continuous function, for each $e \in E$ and (G,E) be an arbitrary soft set over Y. Then $f_{(e)}^{-1} (G^{\circ}(e)) \subset (f_e^{-1} (G(e))^{\circ}$ is satisfied for each $e \in E$, thus $f^{-1} (G^{\circ}, E)$ □̃ $(f^{-1} (G, E))^{\circ}$. Since $(G^{\circ},E) \in \tau$ then by Corollary(2.1.23)$(G^{\circ},E)=(G,E)^{\circ}$ .This implies that by Theorem (2.1.2, vi)we have $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ is a soft continuous function.

Remark(2.1.27): Let (X, τ, E) be a sts , (X, τ_e) be a topological space and A□X .

i. If $(A, \tau A, E)$ is a soft subspace ,then the collection $(\tau A)_{(e)}=\{FA(e) :(FA, E) \in \tau A\}$ defines a topology on A for each $e \in E$.

ii. The subspace of (X, τ_e) will be denoted by (A, τ_{eA}),where $\tau_{eA} =\{A \cap F(e): F(e)$ □ $\tau_e\}$

Now, the following Proposition shows that $\tau_{eA}=(\tau A)_e$.

Theorem(2.1.28):

Let (X, τ, E) be a sts, and $A \subseteq X$ such that (A, τ_A, E) is a soft subspace, then

$\tau_{eA} = (\tau_A)_e$.

Proof:

$(\Longrightarrow)$ Let $U \in (\tau_A)_e$, then there exist soft open set (G_A, E) over A , such that $G_A(e)=U$, and since $(G_A, E)= A \tilde{\cap} (G, E)$ where (G, E) is soft open set over X, then $G_A(e) = A \cap G(e)$ for each $e \in E$, thus $U= A \cap G(e)$ and $G(e)$ is an open set in (X, τ_e). Therefore $U \in \tau_{eA}$.

$(\Longleftarrow)$ Let $W \in \tau_{eA}$, then $W=A \cap V$, where $V \in \tau_e$, then there exist soft open set (F,E) over X , such that $F(e)=V$, thus $W= A \cap F(e)$. Since $A \tilde{\cap} (F, E) \in \tau_A$ then $A \cap F(e) \in (\tau_A)_e$ which implies that $W \in (\tau_A)_e$. Hence $\tau_{eA}=(\tau_A)_e$.

Proposition(2.1.29): [5]

Let $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ be soft open (soft closed), then for each $e \in E$, $f_e : (X, \tau_e) \longrightarrow (Y, \tau'_e)$ is an open (closed) function.

Remark (2.1.30):

As a consequence of Proposition(2.1.8) and Proposition(2.1.29), if $f: (X, \tau, E) \longrightarrow (Y, \tau', E)$ and $g:(Y, \tau', E) \longrightarrow (Z, \tau'', E)$ are soft open (closed) functions ,then for each $e \in E$, $(g \circ f)_e = g_e \circ f_e$ is an open(closed)function.

Theorem (2.1.31)

Let $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ be a soft homeomorphism, then for each $e \in E$, $f_e : (X, \tau_e) \longrightarrow (Y, \tau'_e)$ is a homeomorphism .

Proof : By using Theorem (2.1.24) and proposition (2.1.29).

Theorem(2.1.32):

Let (X, τ, E) and (Y, τ', E) be two sts's, then $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ is a soft open function if and only if $f_e: (X, \tau_e) \longrightarrow (Y, \tau'_e)$ is open function, for each $e \in E$.

Proof:

($\Rightarrow$) By Proposition(2.1.29).

($\Leftarrow$) Let $f_e: (X, \tau_e) \longrightarrow (Y, \tau'_e)$ be a open function, for each $e \in E$, and (F, E) be an arbitrary soft set over X. Then $f_e (F(e))^{\circ} \subset (f)_e(F(e))^{\circ}$ is satisfied, for each $e \in E$. Thus $f(F^{\circ},E) \tilde{\subset} (f(F,E))^{\circ}$ by proposition(2.1.22) and Theorem (2.1.4) we have $f(F,E)^{\circ} \tilde{\subset} (f(F,E))^{\circ}$. This implies that $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is a soft open function .

Now, we study the concept of product soft functions

Theorem (2.1.33):

Let (X_1,τ_1, E) and (X_2,τ_2, E) be a sts's .Then soft projection functions (p_i, q_i): $(X_1,\tau_1, E) \times (X_2,\tau_2, E) \rightarrow (X_i,\tau_i, E)$ is soft continuous and soft open for each i=1,2.

Proof: Clear.

Theorem (2.1.34):

Let $(Y, \tau`, E)$, $\{(X_i,\tau_i, E)\}_{(i \in I)}$, I={1,2}, be a family of sts's and $(X_1 \times X_2, \tau_{prod}, E \times E)$ product sts.Then $f : (Y, \tau`, E) \rightarrow (X_1 \times X_2, \tau_{prod}, E \times E)$ is soft continuous if and only if $(p_i, q_i) \circ f$ is soft continuous for each $i \in I$.

Proof:

($\Rightarrow$) Clear .

(⟸) Let $(F,E)_1 \times (F,E)_2 \in B$ then

$f^{-1}[(F,E)_1 \times (F,E)_2] = f^{-1}[(F,E)_1 \times \tilde{X}_2) \tilde{\cap} (\tilde{X}_1 \times (F,E)_2]$

$= f^{-1}[(p_1, q_1)^{-1}(F,E)_1 \tilde{\cap} (p_2, q_2)^{-1}(F,E)_2]$

$= [((p_1, q_1) \circ f)^{-1}(F,E)_1 \tilde{\cap} ((p_2, q_2) \circ f)^{-1}(F,E)_2]$. Since $(p_i, q_i) \circ f$ is soft continuous then $((p_i, q_i) \circ f)^{-1}$ are soft open sets in τ'. Hence f is soft continuous.

Proposition (2.1.35):

Let (X,τ,E) and (Y,τ',E) be sts's. Then:

i. $(F,E)\times(G,E)$ is soft open set over $X\times Y$ if and only if (F,E) and (G,E) are soft open sets over X and Y (resp.).

ii. $(F,E)\times(G,E)$ is soft closed set over $X\times Y$ if and only if (F,E) and (G,E) are soft closed sets over X and Y (resp.).

Proof:

i. (⟹) Let $(F,E)\times(G,E)$ be a soft open set over $X\times Y$, then by Theorem (2.1.33), we have $(p_1, q_1)((F,E)\times(G,E)) = (F,E)$ and $(p_2, q_2)((F,E)\times(G,E)) = (G,E)$ are soft open sets over X, Y (resp.).

(⟸) clear.

ii. (⟹) Let $(F,E)\times(G,E)$ be a soft closed set over $X\times Y$, then $[(F,E)\times(G,E)]^c$ is soft open set over $X\times Y$. By Proposition (1.2.32) $[(F,E)\times(G,E)]^c = [(F,E)^c \times \tilde{Y}) \tilde{\cup} (\tilde{X} \times (G,E)^c]$ thus $(F,E)^c \times \tilde{Y}$ and $\tilde{X} \times (G,E)^c$ are soft open over $X\times Y$, by Proposition (1.2.30) we have $(F,E)^c$ and $(G,E)^c$ are soft open sets over X and Y (resp.). Hence (F,E) and (G,E) are soft closed set over X and Y (resp.)

(⟸) The proof in [16].

Proposition(2.1.36):

Let $f_i:(X_i ,\tau_i ,E)\longrightarrow(Y_i ,\tau_i^{'} ,E),i=1,2$ be a soft function then the product functions $f_1\times f_2: (X_1\times X_2 ,\tau_{prod} ,E\times E)\longrightarrow(Y1\times Y2, \tau'_{prod} ,E\times E)$ is soft continuous if and only if f_i is soft continuous functions , i=1,2.

Proof:

($\Longrightarrow$)To prove $f_1:(X_1 , \tau_1 ,E) \longrightarrow(Y_1 , \tau_1^{'} ,E)$ is soft continuous.Let (F, E) be an soft open set overY_1,then by Proposition(2.1.35,i) $(F, E)\times \tilde{Y}2$ is an soft open set over $Y1\times Y2$ since $f_1\times f_2$ is soft continuous, then $(f_1\times f_2)^{-1}((F, E)\times \tilde{Y}2)= f_1^{-1}(F, E) \times f_2^{-1} (\tilde{Y}2)$ is an soft open set, over $X_1\times X2$. Hence $f_1^{-1}(F, E)$ is an soft open set in (X_1 ,τ_1 ,E) , by Proposition(2.1.35 ,i).
In similar way $f_2:(X_2 , \tau2 ,E) \longrightarrow(Y2 , \tau_2^{'},E)$ is soft continuous function.

($\Longleftarrow$) Let $f_i:(X_i , \tau_i ,E) \longrightarrow(Y_i , \tau_i^{'} ,E),i=1,2$ be a soft continuous function. To prove $f_1\times f_2$ is soft continuous. Let $(F, E) \times(G, E)$ be a basic soft open set over $Y1\times Y2$,since$(f_1\times f_2)^{-1}[(F,E)\times(G,E)]= f_1{-1}(F,E)\times f_2{-1}(G,E)$ and f_1,f_2 are soft continuous functions ,then $f_1{-1}(F, E)$ and $f_2{-1}(G, E)$ are soft open sets over X1 and X2(resp.) ,by Proposition(2.1.35,i) we have $f_1{-1}(F, E) \times f_2{-1}(G, E)$ is soft open set over $X1\times Y1$ .Therefore $(f_1\times f_2)^{-1}((F, E) \times (G,E))$is soft open set over $X1\times Y1$.This implies that $f_1\times f_2:(X_1\times X_2, \tau_{prod} ,E\times E)\longrightarrow(Y1\times Y2, \tau`_{prod} , E)$ be soft continuous function .

Proposition(2.1.37):

Let $f_i:(X_i, \tau_i, E) \longrightarrow (Y_i, \tau_i^{'}, E)$, $i=1,2$ be a soft function, then the product functions $f_1\times f_2:(X_1\times X_2, \tau_{prod}, E\times E)\longrightarrow(Y_1\times Y_2, \tau'_{prod}, E\times E)$ is soft open if and only if f_i is soft open functions, $i=1,2$.

Proof: Clear.

Proposition (2.1.38):

Let $f_i:(X_i, \tau_i, E) \longrightarrow (Y_i, \tau_i^{'}, E)$, $i=1,2$ be a soft function such that $f_1\times f_2: (X_1\times X_2, \tau_{prod}, E\times E)\longrightarrow(Y_1\times Y_2, \tau`_{prod}, E\times E)$ be soft closed function then f_i is soft closed functions, $i=1,2$.

Proof:

To prove $f_1:(X_1, \tau_1, E) \longrightarrow (Y_1, \tau_1^{`}, E)$ is soft closed function, let (F, E) be a soft closed set over X_1 then $(F, E) \times \tilde{X}_2$ is a soft closed set over $X_1\times X_2$. So $(f_1\times f_2)((F,E) \times \tilde{X}_2)= f_1(F,E)\times f_2(\tilde{X}_2)$ is soft closed set over $Y_1\times Y_2$. Thus $f_1(F, E)$ is soft closed set over Y_1. By Proposition(2.1.35,ii).

In similar way $f_2:(X_2, \tau_2, E) \longrightarrow (Y_2, \tau_2^{'}, E)$ is soft closed function.

Remark (2.1.39):

The converse of proposition (2.1.38) is not true in general, and we put a condition on the product sts $(X_1\times X_2, \tau_{prod}, E\times E)$ to satisfy it.

Proposition(2.1.40):

Let $\{(X_i,\tau_i,E)\}$,$\{(Y_i, \tau`_i,E)\}$, $i=1,2$ be a families of sts's, where E is finite set and $X_1\times X_2$ is S-product soft topological space. If $f_i:X_i\longrightarrow Y_i$ is soft closed functions then $f_1\times f_2$ is soft closed.

Proof:

Let f_i , $i=1,2$ be a soft closed functions. To prove $f_1\times f_2$ is soft closed. Let $(D,E\times E)$ be a soft closed set over $X1\times X2$. Since $X_1\times X_2$ is S-product sts. Then

$(D,E\times E)=\tilde{\bigcup}_{i\in I} [(F_i,E)\times (G_i,E)]$, $I=\{1,2,\dots,n\}$ where (F_i,E) and (G_i,E)

are soft closed sets over X_1 and X_2 (resp.) for all $i\in I$. Since

$$(f_1\times f_2)[(D,E\times E)]=(f_1\times f_2)[\tilde{\bigcup}_{i\in I} [(F_i,E)\times (G_i,E)]]$$

$$= \tilde{\bigcup}_{i\in I} [(f_1\times f_2)[(F_i,E)\times (G_i,E)]]$$

$$= \tilde{\bigcup}_{i\in I} [[f_1(F_i,E)\times f_2(G_i,E)]]$$

Since $f_1(F_i,E)$, $f_2(G_i,E)$ are soft closed sets over Y1 and Y2 (resp.) then by Proposition(2.1.35,ii) ,we have $f_1(F_i,E)\times f_2(G_i,E)$ is soft closed set over $Y1\times Y2$. Therefore $f_1\times f_2$ is soft closed function .

Remark (2.1.41):

Let (X,τ,E) and (Y,τ',E) be sts's ,then:

i. If $(X\times Y, \tau_{prod}, E\times E)$ is the product soft topological space, then for all $(e,e`)\in E\times E$, we have $(X\times Y, (\tau_{prod})_{(e,e`)})$ is a product topological space.

ii. We will use $\tau_{(e,e`)-prod}$ to denote the product topology of $(X,\tau_e)$ and $(Y, \tau'_{e`})$ where $(e,e`)\in E\times E$.

Theorem (2.1.42)

Let (X,τ,E),(Y,τ',E) be sts's and $(X\times Y, \tau_{prod}, E\times E)$ is product soft topological space . Then $(\tau_{prod})_{(e,e`)}=\tau_{(e,e`)-prod}$, for each $(e,e`)\in E\times E$.

Proof:

($\Longrightarrow$) Let W ⟦(τ⟧ _prod)_((e,e`) ) then there exist a basic open set U × V such that W=U×V and there is a soft open set (F,E) ×(G,E) over X×Y, such that (F×G)(e,e`) = U×V .Since F(e)×G(e`)∈ τ_((e,e`)-prod), then W= U×V ∈ τ_((e,e`)-prod).

($\Longleftarrow$) Let W′ ∈ τ_((e,e`)-prod) then there exist a basic open set U′×V′ such that W′= U′×V′ and U′∈τ_e, V′∈ ⟦τ'⟧ _(e`) .So there is soft open sets (F,E)and(G,E) over X,Y (resp.), where F(e)=U′ ,G(e`)=V′ .By Proposition( 2.1.35,i) we have (F,E)×(G,E) is soft open set over X× Y. So (F,E)×(G,E)∈ τ_prod ⟹ F(e)×G(e`)∈ ⟦(τ⟧ _prod)_((e,e`) ) .Therefore U′×V′∈ ⟦(τ⟧ _prod )_((e,e`)), thus W′ ∈ ⟦(τ⟧ _prod)_((e,e`) ) . Hence ⟦ (τ⟧ _prod )_((e,e`))=τ_((e,e`)-prod).

Proposition (2.1.43):

Let (X_i , τ_i ,E) and(Y_i , τ_i^' ,E),i=1,2 be sts^' s. If ⟦ f⟧ _1×f_2:(X_1× X_2,τ_(prod), E×E)⟶(Y_1×Y_2, ⟦τ`⟧ _prod , E×E ) is soft continuous function,then for each (e, e`) ∈ E×E, (f_1×f_2)_((e,e`) ):(X_1×X_2, ⟦(τ⟧ _prod )_((e,e`)))⟶(Y_1×Y_2 , (⟦τ`⟧ _prod )_((e,e`))) is continuous function .

Proof:

By Theorem(2.1.24) f_1e ,f_(2e`) are continuous functions and by Proposition (1.1.21) ⟦(f⟧ _1e ×f_(2e`))=(f_1×f_2)_((e,e`) ) . Thus ( f_1×f_2 )_((e,e`)) is continuous function.

Proposition (2.1.44):

Let (X_i , τ_i ,E) and(Y_i, τ_i^' ,E),i=1,2 be sts's. If f_1×f_2:(X_1× X_2 ,τ_(prod),E×E) ⟶ (Y_1×Y_2 , 〖τ`〗_prod ,E×E) is soft open function then for each (e, e`)) 〖E×E_(f_1×f_2)_((e,e`) ) :(X_1× X_2 , 〖(τ〗_prod )_((e,e`))) ⟶ (Y_1×Y_2 , (〖τ`〗_prod )_((e,e`))) is open function.

Proof: clear.

Proposition (2.1.45):

Let (X_1,τ_1,E) and (X_2,τ_2,E) be s.c.spaces, then for each a∈X_1 the function f: X_2⟶X_1×X_2 , x⟼(a,x) is soft continuous.

Proof:

Since the constant function f_1: X_2⟶X_1, x⟼a is soft continuous and the identity function f_2: X_2⟶X_2 ,x⟼x is soft continuous. Therefor the function f is soft continuous.

Proposition(2.1.46):

Let (X,τ,E)be a sc-space.Then the diagonal function ∆:X⟶X×X,x⟼(x,x), is soft continuous.

Proof:

Since (p_i , q_i)∘∆=I_X for all i=1,2. And I_X is soft continuous,thus (p_i , q_i)∘∆ is soft continuous for all i=1,2. By Theorem(2.1.34) we have ∆ is soft continuous .

2.2 On Soft Convergence of Nets:

In this section , we recall the basic definitions , remarks and theorems about " convergence of net " . Also, we introduce the definition (to the best of our knowledge) ,of soft convergence of net and we give some results which are related with this subject .

Definition(2.2.1):[14]

A set D is called a directed set if there is relation $\geq$ on D satisfying :

i. $d \geq d$ for each $d \in D$.
ii. If $d_1 \geq d_2$ and $d_2 \geq d_3$ then $d_1 \geq d_3$.
iii. If $d_1 , d_2 \in D$ then there is some $d_3 \in D$, with $d_3 \geq d_1$ and $d_3 \geq d_2$.

Definition(2.2.2):[14]

A net in a set X is a function $\chi: D \to$ X , where D is directed set. The point $\chi(d)$is usually denoted by χ_d.

Definition (2.2.3):[20]

A subnet of a net $\chi: D \to$ X is the composition $\chi o \varphi$,where $\varphi: M \to D$ and M is directed set , such that :

i. $\varphi(m_1) \leq \varphi(m_2)$, where $m_1 \leq m_2$.
ii. For each $d \in D$there is some $m \in M$such that $d \leq \varphi(m)$. For $m \in M$ the point $\chi o \varphi(m)$ is often written χ_{dm} .

Definition(2.2.4) : [20]

Let $(\chi_d)_{d \in D}$be a net in a topological space X and $A \subseteq$ X , $x \in$ X then:

i. $(\chi_d)_{d \in D}$is called eventually in A if there is $d_0 \in D$ such that $\chi_d \in A$ for all $d \geq d_0$.
ii. $(\chi_d)_{d \in D}$is called frequently in A for each $d \in D$ there is $d_0 \in D$ with

$d_0 \geq d$ such that $\chi_{d_0} \in A$.

iii. $(\chi_d)_{d\in D}$ is said to be convergence to x if $(\chi_d)_{d\in D}$ eventually in each neighborhood of x (written$\chi_d \to x$) .The point x is called a limit point of $(\chi_d)_{d\in D}$.

iv. $(\chi_d)_{d\in D}$ is called have x as a cluster point if $(\chi_d)_{d\in D}$is frequently in each neighborhood of x(written $\chi_d \propto x$) .

Remark(2.2.5):[20]

Let f: X → Y be a function from a set X into a set Y,then:

i. If $(\chi_d)_{d\in D}$is a net in X , then $\{f(\chi_d)\}_{d\in D}$is a net in Y .

ii. If f is onto and $(y_d)_{d\in D}$be a net in Y, then there is a net $(\chi_d)_{d\in D}$in X such that $f(\chi_d) = y_d$ for each $d \in D$.

Theorem (2.2.6):[20]

Let X be a topological space and A ⊆ X , $x \in$ X then $x \in \overline{\mathrm{A}}$ if and only if there is a net $(\chi_d)_{d\in D}$in A which converges to x.

Theorem (2.2.7):[20]

Let X andY be topological spaces and f: X → Y be a function , $x \in$ X. Then f is continuous at x if and only if whenever a net $(\chi_d)_{d\in D}$ in X and$\chi_d \to x$, then $f(\chi_d) \to f(x)$ in Y .

Now, we introduce the following definition:

Definition(2.2.8):

Let D be a direct set, X be an ordinary set, and SP be the set of all the soft points in $\widetilde{X}$ .The function $\tilde{\chi}$: D ⟶ SP is called a soft net in $\widetilde{X}$. A soft net is often denoted by $\{\chi_d^{e_d}\}_{d\in D}$.

Example(2.2.9):

Let $(N, \geq)$ be a direct set, X be a set and E be a set of parameters. Then $\{\chi_n^{e_n}\}_{n\in N}$ is a soft net in $\widetilde{X}$, where $\tilde{\chi}: N \longrightarrow SP, \chi(n)=x^{e_n}$ for all $n \in N$, where x^{e_n} is soft point in $\widetilde{X}$.

Definition (2.2.10):

A soft net $\{y_h^{e_h}\}_{h\in H}$ in $\widetilde{X}$,is called a soft subnet of a soft net $\{\chi_d^{e_d}\}_{d\in D}$ in $\widetilde{X}$ if and only if there is a function $Z: H \longrightarrow D$ such that:

i. $\tilde{y} = \tilde{\chi} \circ Z$, that is, for each $i \in H, y_i = \chi_{Z(i)}$;

ii. For each $d \in D$,there exists some $h \in H$ such that, if $p \in H, p \leq h, Z(p) \leq d$.

Definition(2.2.11):

Let $\{\chi_d^{e_d}\}_{d\in D}$ be a soft net in a sts (X, τ, E) and (F, E) be a soft set over X ,then:

i. $\{\chi_d^{e_d}\}_{d\in D}$ is eventually in (F, E) if there is $d_\circ \in D$ such that $\chi_d^{e_d} \widetilde{\in} (F, E)$ for all $d \geq d_\circ$.

ii. $\{\chi_d^{e_d}\}_{d\in D}$ is frequently in (F, E) if for each $d \in D$ there is $d_\circ \in D$ with $d_\circ \geq d$ such that $\chi_{d_\circ}^{e_{d_\circ}} \widetilde{\in} (F, E)$.

Example(2.2.12):

Let $X=N$,$E=\{e_i, i \in N\}$ and let $\{\chi_n^{e_n}\}_{n\in N}$ be a soft net in a sts (X, τ, E) where $\tilde{\chi} : N \longrightarrow SP, \tilde{\chi}(n)= x_n^{e_n}$. $(F,E)=\{(e_i,\{x_i\}), i\in N\}$ is soft set over X and $\{\chi_n^{e_n}\}_{n\in N}$ is eventually in (F, E).

Remark(2.2.13):

As a consequence of definition (2.2.8) ,we have the following :

If $\tilde{\chi}: D \longrightarrow SP$ is a soft net in $\widetilde{X}$, then there is a net $\chi: D \longrightarrow X$, denoted by $\{\chi_d\}_{d\in D}$ and a net $\varrho: D \longrightarrow E$ denoted by $\{\varrho_d\}_{d\in D}$.

Example(2.2.14):

Let X=N ,E={$e_i, i \in$ N } and $\{\chi_n^{e_n}\}_{n\in N}$ be a soft net in $\widetilde{X}$ where $\tilde{\chi} : N \longrightarrow SP$, $\tilde{\chi}(n)$=($x_n^{e_n}$). Then $\chi: N \longrightarrow X, \chi(n)$=$x_n$ is a net in X and $\varrho: D \longrightarrow E$, $\varrho(n)$=e_n is a net in E .

Example(2.2.15):

Let X be a set ,E={$e_i, i \in$ N } and $\{\chi_n^{e_n}\}_{n\in N}$ be a soft net in $\widetilde{X}$ such that $\tilde{\chi} : N \longrightarrow SP$, $\tilde{\chi}(n)$=(x^{e_n}) $\forall\, n \in N, x \in X$. If (F,E)={($e_i$,{$x$}),$i$=1,...,n} be a soft set over X , then $\{\chi_n^{e_n}\}_{n\in N}$ is eventually in(F, E), and $\{\chi_n\}_{n\in \mathrm{N}}$ is eventually in F(e_i), i=1,...,n where $\chi: N \longrightarrow X, \chi(n)$=$x$.

The example in (2.2.12) shown that if $\{\chi_d^{e_d}\}_{d\in D}$is eventually in (F,E) need not be $\{\chi_d\}_{d\in \mathrm{D}}$ is eventually in F(e) for each $e\in$E .

Remark(2.2.16):

Every eventually soft net in a soft subset of a *sts* is frequently, but the converse is not true in general. As shows in following example:

Let X={x_1,x_2, x_3,x_4}and E={ e, e`}. Let H = {1,2,3,4}, then (H, ≥)be a direct set , if $\tilde{\chi}$: H $\longrightarrow SP$, $\chi(h)$= $x_h^e$ then $\{\chi_d^{e_d}\}_{d\in D}$is a soft net in *sts* (X, $\tau$ ,E). If (F,E)={( $e$,{$x_1$, $x_2$, $x_4$}),($e$`,{x_3})}be a soft set over X then $\{\chi_d^{e_d}\}_{d\in D}$ is frequently in (F, E), but not eventually in (F, E)

Definition(2.2.17):

Let $\{\chi_d^{e_d}\}_{d\in D}$be a soft net in a *sts* (X , τ ,E), then:

i. $\{\chi_d^{e_d}\}_{d\in D}$converge to a soft point x^e if $\{\chi_d^{e_d}\}_{d\in D}$is eventually in every soft neighborhood of x^e .written $\chi_d^{e_d} \longrightarrow x^e$, and x^e is called soft limit point .

ii. $\{\chi_d^{e_d}\}_{d\in D}$ is said to have x^e as a soft cluster point if $\{\chi_d^{e_d}\}_{d\in D}$ is frequently in every soft neighborhood of x^e (written $\chi_d^{e_d} \propto x^e$).

Proposition(2.2.18):

Let $\{\chi_d^{e_d}\}_{d\in D}$ be a soft net in sts (X, τ ,E) if $\{\chi_d^{e_d}\}_{d\in D}$ is a soft converge to x^e, then x^e is a soft cluster point of $\{\chi_d^{e_d}\}_{d\in D}$.

Proof:

Let (G,E) be a soft neighborhood of x^e,$d \in D$ .Since $\{\chi_d^{e_d}\}_{d\in D}$is converge to x^e then there exists $d_\circ \in$ D such that $\chi_{d_1}^{e_{d_1}} \tilde{\in}$ (G, E) for all $d_1 \geq d_\circ$.Since (D,≥) is a directed set ,then there is $d_2 \in$D such that $d_2 \geq d$, $d_2 \geq d_\circ$.
So $d_2 \in D$ with $d_2 \geq d$ such that $\chi_{d_2}^{e_{d_2}} \tilde{\in}$ (G, E) .Therefore x^e is a soft cluster point of $\{\chi_d^{e_d}\}_{d\in D}$.

Theorem(2.2.19):

Let(X, τ ,E) be a sts , (F,E) $\tilde{\subseteq}$ $\tilde{X}$ and $x^e \tilde{\in}$ $\tilde{X}$ .Then $x^e \tilde{\in} \overline{(F,E)}$ if and only if there is a soft net $\{\chi_d^{e_d}\}_{d\in D}$ in (F,E) such that $\chi_d^{e_d} \longrightarrow x^e$.

Proof:

(⟸) Suppose that there is a soft net $\{\chi_d^{e_d}\}_{d\in D}$in (F,E)such that $\chi_d^{e_d} \longrightarrow x^e$.
To prove $x^e \tilde{\in} \overline{(F,E)}$. Let(G, E)$\in \mathcal{N}_{x^e}$,since $\chi_d^{e_d} \longrightarrow x^e$,there is $d_\circ \in D$ such that $\chi_d^{e_d} \tilde{\in} (G,E)$ for all $d \geq d_\circ$. But $\chi_d^{e_d} \tilde{\in} (F,E)$ for all $d \in D$ .So (F,E)$\tilde{\cap}$(G,E) $\neq \tilde{\Phi}$ for all (G,E) $\in \mathcal{N}_{x^e}$, by Proposition(1.2.20) $x^e \tilde{\in} \overline{(F,E)}$.

(⟹) Let $x^e \tilde{\in} \overline{(F,E)}$,then by Proposition (1. 2.20) (F,E)$\tilde{\cap}$(G,E) $\neq \tilde{\Phi}$,for each (G,E) $\tilde{\in} \mathcal{N}_{x^e}$. Let D=$\mathcal{N}_{x^e}$ then($D, \tilde{\subset}$) is a directed set .Since for all (G,E)$\in D$,(F,E)$\tilde{\cap}$(G,E) $\neq \tilde{\Phi}$,there is $\tilde{\chi}_{(G,E)} \tilde{\in}$ (F,E)$\tilde{\cap}$(G,E). Define $\tilde{\chi}$: ($D, \tilde{\subset}$) ⟶ (F, E) by

$\tilde{\chi}$(G,E)= $\tilde{\chi}_{(G,E)}$ for all (G,E) $\tilde{\in}$ $\mathcal{N}_{x^e}$. Hence $\{\tilde{\chi}(G,E)\}_{(G,E)\in D}$ is a soft net in (F, E).To prove $\{\tilde{\chi}(G,E)\}_{(G,E)\in D}$ is a converges to x^e. Let (H,E) $\in \mathcal{N}_{x^e}$,then $\tilde{\chi}$(G, E) $\tilde{\in}$ (H,E) for all (G,E) $\tilde{\subseteq}$ (H,E).Hence $\{\tilde{\chi}(G,E)\}_{(G,E)\in D}$ converges to a soft point x^e .

Corollary(2.2.20):

Let $\{\chi_d^{e_d}\}_{d\in D}$ be a soft net in a sts (X, τ ,E) and x^e $\tilde{\in}$ $\tilde{X}$, then $\{\chi_d^{e_d}\}_{d\in D}$is said to have x^e as a soft cluster point if and only if there is a subnet of $\{\chi_d^{e_d}\}_{d\in D}$ converges to x^e.

Corollary(2.2.21):

Let(X, τ ,E) be a sts, and (F,E) $\tilde{\subseteq}$ $\tilde{X}$, x^e $\tilde{\in}$ $\tilde{X}$ .Then x^e $\tilde{\in}$ $\overline{(F,E)}$ if and only if there is a soft net $\{\chi_d^{e_d}\}_{d\in D}$ in (F,E) such that x^e is a soft cluster point of $\{\chi_d^{e_d}\}_{d\in D}$.

Remark(2.2.22):

Let f be a function from a set X into a set Y , then :

i. If $\{\chi_d^{e_d}\}_{d\in D}$ is a soft net in $\tilde{X}$, then $\{f(\chi_d^{e_d})\}_{d\in D}$ is a soft net in $\tilde{Y}$.

ii. If $\{Y_d^{e_d}\}_{d\in D}$ is a soft net in $\tilde{Y}$ then there is a soft net $\{\chi_d^{e_d}\}_{d\in D}$ in $\tilde{X}$ such that $f(\chi_d^{e_d})$ =$Y_d^{e_d}$ for each $d \in D$.

iii. If $\chi_d^{e_d} = x^e$ for all $d \in D$, then $\chi_d^{e_d} \longrightarrow x^e$.

Theorem (2.2.23):

Let (X, τ , E)and (Y, τ' , E) be two $sts's$ and f: X → Y be a function , x^e $\tilde{\in}$ $\tilde{X}$. Then f is soft continuous at x^e if and only if whenever a soft net $\{\chi_d^{e_d}\}_{d\in D}$ in $\tilde{X}$ and $\chi_d^{e_d} \to x^e$ then $f(\chi_d^{e_d}) \to f(x^e)$ in $\tilde{Y}$.

Proof:

($\Longrightarrow$) Let f be a soft continuous function ,and $\chi_d^{e_d} \longrightarrow x^e$.To prove $f(\chi_d^{e_d}) \longrightarrow f(x^e)$, let (F,E)$\in\mathcal{N}_{f(x^e)}$. Since f is soft continuous ,then f^{-1}(F,E)$\in\mathcal{N}_{x^e}$ for some $d_\circ \in$ D such that $\chi_d^{e_d} \,\widetilde{\in}\, f^{-1}(F,E)$ for all $d \geq d_\circ$. So for some $d_\circ \in D$ such that $f(\chi_d^{e_d}) \,\widetilde{\in}\, (F,E)$ for all $d \geq d_\circ$. Therefore $f(\chi_d^{e_d}) \longrightarrow f(x^e)$ in $\widetilde{\mathrm{Y}}$.

($\Longleftarrow$) Suppose that f is not soft continuous at x^e .Then there is (G,E)$\in \mathcal{N}_{f(x^e)}$ such that f(H,E)$\widetilde{\not\subset}$ (G,E) for any (H,E)$\in\mathcal{N}_{x^e}$ .Thus for all (H,E)$\in N_{x^e}$,we can defined $\tilde{\chi}: (\mathcal{N}_{x^e}, \widetilde{\sqsubseteq}) \longrightarrow (H,E)$, $\tilde{\chi}_{(\mathrm{H,E})} \in$ (H, E) such that $f(\tilde{\chi}_{(\mathrm{H,E})}) \notin$ (G,E). But $\{\chi_{(\mathrm{H,E})}\}_{(\mathrm{H,E})\in\mathcal{N}_{x^e}}$ is a soft net in $\widetilde{\mathrm{X}}$, with $\tilde{\chi}_{(\mathrm{H,E})} \longrightarrow x^e$, while $\{f(\chi_{(\mathrm{H,E})})\}_{(\mathrm{H,E})\in\mathcal{N}_{x^e}}$ is not converges to $f(x^e)$. This is a contradiction.

<u>Theorem (2.2.24)</u>:

Let (X, τ ,E) and (Y, τ`,E) be a $sts's$. A soft net $\{Z_d^{e_d}\}_{d\in D}$ in product sts (X×Y, τ_{prod}, E×E) is convergence to z^e in $\widetilde{\mathrm{X}\times\mathrm{Y}}$, if and only if $(p_i\, , q_i)(Z_d^{e_d}) \longrightarrow (p_i\, , q_i)(z^e)$ for all $i = 1,2$.

<u>Proof</u>:

($\Longrightarrow$) If $Z_d^{e_d} \longrightarrow z^e$ in $\widetilde{\mathrm{X}\times\mathrm{Y}}$, since $(p_i\, , q_i)$ are soft continuous , then by Theorem (2.2.23) we have $(p_i\, , q_i)(Z_d^{e_d}) \longrightarrow (p_i\, , q_i)(z^e)$.

($\Longleftarrow$) Suppose that $(p_i\, , q_i)(Z_d^{e_d}) \longrightarrow (p_i\, , q_i)(z^e)$ for all $i = 1,2$. Let $(\mathrm{F,E})_1 \times (\mathrm{F,E})_2$ be a soft basic open neighborhood of z^e in $\widetilde{\mathrm{X}\times\mathrm{Y}}$. Then there is $d_\circ \in D$ such that $(p_i\, , q_i)(Z_d^{e_d}) \,\widetilde{\in}\, (\mathrm{F,E})_i$ for all $d \geq d_\circ$ .It follows that for all $d \geq d_\circ$, $Z_d^{e_d} \,\widetilde{\in}\, (p_i\, , q_i)^{-1}(\mathrm{F,E})_i$. So we have $Z_d^{e_d} \longrightarrow z^e$.

<u>Corollary (2.2.25)</u>:

Let $(X \times Y, \tau_{prod}, E \times E)$ be product sts .A soft net $\{Z_d^{e_d}\}_{d \in D}$ in $\widetilde{X \times Y}$ having z^e as soft cluster point ,then for each $i \in I$, $((p_i, q_i)(Z_d^{e_d}))_{d \in D}$ has $(p_i, q_i)(z^e)$ soft cluster point .

2.3 On Soft Separation Axioms :

This section will contain the definition of soft separation axioms and propositions, theorems, examples, remarks and corollaries related to it.

In addition, we introduce some types of soft functions about a separation axioms.

Definition (2.3.1):[22]

A sts (X, τ ,E) is called soft $\tau_\circ$-space if for every $x^e, y^{e`} \tilde{\in} \tilde{X}$ such that $x^e \neq y^{e`}$ there exist soft open sets (F, E) and (G, E) such that either $x^e \tilde{\in}$ (F, E) and $y^{e`} \tilde{\notin}$ (F, E) or $y^{e`} \tilde{\in}$ (G, E) and $x^e \tilde{\notin}$ (G, E).

Examples(2.3.2) :

i. Let X ={x, y}, E ={ e, $e`$} and τ=τ_{sd}. Then (X, τ ,E) is soft $\tau_\circ$-space.
ii. Let X ={x, y}, E ={ e, $e`$} and τ=τ_{sind}. Then (X, τ ,E) is not soft $\tau_\circ$-space.

Proposition(2.3.3):[22]

Let (X, τ , E) be a sts and $x^e, y^{e`} \tilde{\in} \tilde{X}$ such that $x^e \neq y^{e`}$. If there exist soft open sets (F, E) and (G,E) such that either $x^e \tilde{\in}$ (F, E) and $y^{e`} \tilde{\in}$ $(F, E)^c$ or $y^{e`} \tilde{\in}$ (G, E) and $x^e \tilde{\in}$ $(G, E)^c$. Then (X, τ, E) is soft $\tau_\circ$-space.

Theorem(2.3.4): [22]

A soft subspace (A, τ_A , E) of a soft τ_0-space (X, τ , E) is soft τ_0-space.

Definition (2.3.5):[22]

A sts (X, τ ,E) is called soft τ_1-space if for every $x^e, y^{e`} \tilde{\in} \tilde{X}$ such that x^e

$\neq y^{e`}$ there exist soft open sets (F, E) and (G, E) such that $x^e \tilde{\in}$(F, E), $y^{e`} \tilde{\notin}$(F, E) and $y^{e`} \tilde{\in}$ (G, E) , $x^e \tilde{\notin}$ (G, E).

Examples(2.3.6):

Let X ={a, b}, E ={e, e`} and τ =τ_{sd}.Then (X, τ ,E) is soft τ_1-space.

Proposition(2.3.7):[22]

Let (X, τ , E) be a sts and x^e , $y^{e`} \tilde{\in}$ $\tilde{X}$ such that $x^e \neq y^{e`}$. If there exist soft open sets (F, E) and (G, E) such that $x^e \tilde{\in}$ (F, E) , $y^{e`} \tilde{\in}$ (F, E)$^c$ and $y^{e`} \in$ (G, E) , $x^e \in$ (G, E)^c . Then (X, τ , E) is soft τ_1-space.

Theorem (2.3.8) : [22]

A soft subspace (A, τ_A, E) of a soft τ_1-space (X, τ , E) is soft τ_1-space .

Theorem (2.3.9):[22]

Let (X, τ , E)be a sts then (X, τ , E) is soft τ_1-space if and only if every soft point over X , is soft closed .

Definition (2.3.10):[22]

A sts (X, τ,E) is called a soft Hausdorff space or soft τ_2-space if for every x^e, $y^{e`} \tilde{\in}$ $\tilde{X}$ such that $x^e \neq y^{e`}$ there exist soft open sets (F, E) and (G, E) such that $x^e \tilde{\in}$ (F, E), $y^{e`} \tilde{\in}$ (G, E) and(F, E) $\tilde{\cap}$ (G, E) = $\tilde{\Phi}$.

Theorem (2.3.11) :[22]

For a sts,(X, τ , E) we have: soft τ_2-space ⇒ soft τ_1-space ⇒ soft $\tau_\circ$-space.

Remark (2.3.12):

The converse of Theorem(2.3.11) is not true in general, as shown in the following examples.

Examples(2.3.13) :

i. Let X = { x, y }, E = {e, e`} and τ = {$\tilde{X}$, $\tilde{\Phi}$, (F$_1$, E),(F$_2$, E),(F$_3$, E)} where(F$_1$,E)

,(F_2, E),(F_3, E) are soft sets over X defined as follows: $F_1(e) = X$, $F_1(e`) = \{x\}$, $F_2(e) = \{y\}$, $F_2(e`) = X$, $F_3(e) = \{y\}$, $F_3(e`) = \{x\}$. Then $\tau$ defines a soft topology over X. Also (X, $\tau$, E) is soft $\tau_1$-space , but it is not a soft $\tau_2$-space, for $x^e, y^{e`} \widetilde{\in} \widetilde{X}$ and $x^e \neq y^{e`}$, but there is no soft open sets ($F_1$, E) and($F_2$, E) such that $x^e \widetilde{\in}$ ($F_1$,E) , $y^{e`} \widetilde{\in}$ (F_2, E) and (F_1, E) $\widetilde{\cap}$ (F_2, E) = $\widetilde{\Phi}$.

ii. Let X = { x, y }, E = { $e, e`$} and $\tau$= { $\widetilde{\Phi}$, $\widetilde{X}$, ($F_1$, E)} where($F_1$,E) is soft set over X, defined as follows: $F_1(e) = X$, $F_1(e`) = \emptyset$. Then τ defines a soft topology over X. Also (X, τ , E) is soft $\tau_\circ$-space , but it is not a soft τ_1-space, since $x^e, y^{e`} \widetilde{\in} \widetilde{X}$ and $x^e \neq y^{e`}$, such that $x^e \widetilde{\in}$ (F_1, E),$y^{e`} \widetilde{\notin}$ ($F_1$, E) and $x^e \widetilde{\in} \widetilde{X}$ , $y^{e`} \widetilde{\in} \widetilde{X}$.

Theorem (2.3.14) : [22]

A soft subspace (A, τ_A , E) of a soft τ_2-space (X, τ, E) is soft τ_2-space.

Theorem(2.3.15):

Let (X, τ ,E) be a *sts*, then (X, τ,E) is soft τ_2-space if and only if every soft net has unique soft limit point .

Proof:

($\Longrightarrow$) Let (X, τ ,E) be soft τ_2-space and $\{\chi_d^{e_d}\}_{d\in D}$ be a soft net in $\widetilde{X}$ such that $\chi_d^{e_d} \longrightarrow x^e$, $\chi_d^{e_d} \longrightarrow y^{e`}$ and $x^e \neq y^{e`}$.Since $\chi_d^{e_d} \longrightarrow x^e$ then for all (F,E)$\in \mathcal{N}_{x^e}$ there is $d_\circ \in D$ such that $\chi_d^{e_d} \widetilde{\in} (F, E)$ for all $d \geq d_\circ$. And since $\chi_d^{e_d} \longrightarrow y^{e`}$ then for all (G,E) $\in \mathcal{N}_{y^{e`}}$ there is $d_1 \in D$ such that $\chi_d^{e_d} \widetilde{\in} (G, E)$ for all $d \geq d_1$. But (F,E) $\widetilde{\cap}$ (G,E)$\neq \widetilde{\Phi}$ for all (F,E)$\in \mathcal{N}_{x^e}$,(G,E)$\in \mathcal{N}_{y^{e`}}$ which is a contradiction since (X, $\tau$ ,E) is soft $\tau_2$-space .Hence $x^e = y^{e`}$.

($\Longleftarrow$) Suppose (X, τ ,E) is not soft τ_2-space , then there is x^e , $y^{e`} \widetilde{\in} \widetilde{X}$ such that $x^e \neq y^{e`}$ and (F,E)$\widetilde{\cap}$(G,E)$\neq \widetilde{\Phi}$ for all (F,E) $\in \mathcal{N}_{x^e}$, (G,E) $\in \mathcal{N}_{y^{e`}}$.

Let $\mathcal{N}_{x^e}^{y^{e`}}$ ={ (F,E)$\widetilde{\cap}$(G,E):(F,E)$\in \mathcal{N}_{x^e}$ , (G,E)$\in \mathcal{N}_{y^{e`}}$ } suppose D= $\mathcal{N}_{x^e}^{y^{e`}}$,then D is directed set by inclusion .For all (H,E)$\in$ D there is $\widetilde{\chi}_{(H,E)} \in$ (H,E) . Hence

$\{\tilde{\chi}_{(H,E)}\}_{(\text{H,E}) \in D}$ is a soft net in $\widetilde{\text{X}}$. To prove $\tilde{\chi}_{(H,E)} \longrightarrow x^e$, let (I,E) $\in \mathcal{N}_{x^e}$ then (I,E)$\in$ D, so $\tilde{\chi}_{(H,E)} \in$(I,E) for all (H,E) $\widetilde{\subseteq}$ (I,E) ,thus $\tilde{\chi}(H,E) \in$(H,E) $\widetilde{\subseteq}$(I,E) , which implies that $\tilde{\chi}_{(H,E)} \longrightarrow x^e$. By the same way we can prove $\tilde{\chi}_{(H,E)} \longrightarrow y^{e`}$, which is a contradiction .

<u>Definition (2.3.16):</u> [22]

Let (X, τ , E) be *sts*, (G, E) be a soft closed set over X, and $x^e \widetilde{\in} \widetilde{\text{X}}$ such that $x^e \widetilde{\notin}$ (G, E). If there exist soft open sets (F_1, E) and (F_2, E) such that $x^e \widetilde{\in}$ (F_1, E), (G,E) $\widetilde{\subseteq}$ (F_2, E) and (F_1, E)$\widetilde{\cap}$(F_2, E) $= \widetilde{\Phi}$, then (X, τ , E) is called a soft regular space. A soft regular τ_1-space is called a soft τ_3-space.

<u>Proposition(2.3.17) :</u>

Let (X, τ , E) be *sts*, (G, E) be a soft closed set over X and $x^e \widetilde{\in} \widetilde{\text{X}}$ such that $x^e \widetilde{\notin}$ (G, E). If (X, τ , E) is soft regular space, then there exists a soft open set (F, E) such that $x^e \widetilde{\in}$ (F, E) and (F, E)$\widetilde{\cap}$(G, E) $= \widetilde{\Phi}$.

<u>Proof:</u> It is clear from definition (2.3.16).

<u>Theorem (2.3.18) :</u>[22]

A soft subspace (A, τ_A , E) of a soft τ_3-space (X, τ, E) is soft τ_3-space.

<u>Definition (2.3.19):</u>[22]

Let (X, τ , E) be a *sts*,and (F_1, E),(F_2, E) be soft closed sets over X such that (F_1, E) $\widetilde{\cap}$(F_2, E) $= \widetilde{\Phi}$. If there exist soft open sets (G_1, E) and (G_2, E) such that (F_1, E) $\widetilde{\subseteq}$ (G_1, E), (F_2, E) $\widetilde{\subseteq}$ (G_2, E) and (G_1, E) $\widetilde{\cap}$ (G_2, E) $= \widetilde{\Phi}$, then (X, τ , E) is called a soft normal space. A soft normal τ_1-space is called a soft τ_4-space.

<u>Remark (2.3.20):</u>

It is not necessary that the soft subspace of a soft τ_4-space is soft τ_4-space. As the following example shows:

Example(2.3.21):

Let X = {x, y, z}, E = {$e, e`$} and $\tau$ = { $\widetilde{\Phi}$, $\widetilde{X}$, ($F_1$, E),($F_2$, E),($F_3$, E),($F_4$, E) ,($F_5$, E)} where $F_1(e)$ = {$x, y$}, $F_1(e`)$ = {x, z}, $F_2(e)$ = {y, z}, $F_2(e`)$= {$y, x$}, $F_3(e)$ = {$y$}, $F_3(e`)$ = {x} $F_4(e)$ = {z}, $F_4(e`)$ = {$y$} ,$F_5(e)$ = {$x$}, $F_5(e`)$ = {z} .Therefore, (X, τ , E) is a soft τ_4-space. If A={x, z}and τ_A={ $\widetilde{\Phi}$, $\widetilde{A}$, (F_{1A}, E),(F_{2A}, E),(F_{3A}, E), (F_{4A}, E) ,(F_{5A}, E)} where $F_{1A}(e)$ = {x }, $F_{1A}(e`)$ = {$x, z$}, $F_{2A}(e)$ = { $z$}, $F_{2A}(e`)$= { x}, $F_{3A}(e) = \emptyset$, $F_{3A}(e`)$ = {$x$}, $F_{4A}(e)$ = {$z$}, $F_{4A}(e`) = \emptyset$,$F_{5A}(e)$ = {x}, $F_{5A}(e`)$ = {z}. But the soft subspace (A, τ_A , E) is not normal space, since (F_{1A}, E)^c ,(F_{4A}, E)^c are soft closed sets and (F_{1A}, E)$^c \widetilde{\sqcap}$(F_{4A}, E)^c =$\widetilde{\Phi}$, there exist soft open sets $\widetilde{A}$ and (F_{2A}, E) such that (F_{1A}, E)^c $\widetilde{\sqsubseteq}$ (F_{2A}, E), (F_{4A}, E)^c $\widetilde{\sqsubseteq}$ $\widetilde{A}$ and $\widetilde{A}$ $\widetilde{\sqcap}$ (F_{2A}, E) $\neq \widetilde{\Phi}$.

Proposition(2.3.22) :

A soft subspace (A, τ_A , E) of a soft normal space (X, τ, E) is soft normal if $\widetilde{A}$ is soft closed set.

Proof:

Let (G_1, E),(G_2, E) be soft closed sets over $\widetilde{A}$ such that (G_1, E) $\widetilde{\sqcap}$ (G_2, E) = $\widetilde{\Phi}$. Then (G_1, E) = $\widetilde{A}$ $\widetilde{\sqcap}$ (F_1, E) and (G_2, E) = $\widetilde{A}$ $\widetilde{\sqcap}$ (F_1, E) for some soft closed sets (F_1, E),(F_2, E) over X from Theorem(1.2.13,ii). Since $\widetilde{A}$ is soft closed subset of $\widetilde{X}$. Then (G_1, E),(G_2, E) are soft closed sets over X ,such that (G_1,E) $\widetilde{\sqcap}$ (G_2, E) = $\widetilde{\Phi}$. Hence by soft normality there exist soft open sets (H_1, E) and (H_2, E) such that (G_1, E)$\widetilde{\sqsubseteq}$ (H_1, E), (G_2, E) $\widetilde{\sqsubseteq}$ (H_2, E) and (H_1, E) $\widetilde{\sqcap}$ (H_2, E)= $\widetilde{\Phi}$. Since (G_1, E),(G_2, E) $\widetilde{\sqsubseteq}$ $\widetilde{A}$, then (G_1, E)$\widetilde{\sqsubseteq}$ $\widetilde{A}$ $\widetilde{\sqcap}$ (H_1, E), (G_2, E) $\widetilde{\sqsubseteq}$ $\widetilde{A}$ $\widetilde{\sqcap}$ (H_2, E) and [$\widetilde{A}$ $\widetilde{\sqcap}$(H_1, E)] $\widetilde{\sqcap}$[$\widetilde{A}$ $\widetilde{\sqcap}$(H_2, E)] = $\widetilde{\Phi}$, where $\widetilde{A}$ $\widetilde{\sqcap}$(H_1,E) and $\widetilde{A}$ $\widetilde{\sqcap}$ (H_2, E) are soft open sets over A . Therefore, (A, τ_A , E) is soft normal space.

Remark (2.3.23):

A soft τ_4- space need not be a soft τ_3-space as shown in example(2.3.21).

Now we introduce some types of soft functions about a separation axioms

Proposition(2.3.24) :

Let (X, τ, E) and (Y, τ', E) be $sts's$ and f : X → Y be a function which is bijective and soft open. If (X, τ , E) is soft $\tau_\circ$-space, then (Y, τ', E) is soft $\tau_\circ$-space.

Proof:

Let $y_1^e, y_2^{e`} \widetilde{\in} \widetilde{Y}$ such that $y_1^e \neq y_2^{e`}$. Since f is surjective, then ∃ $x_1^e, x_2^{e`} \widetilde{\in} \widetilde{X}$ such that $f\ (x_1^e) = y_1^e$ , $f\ (x_2^{e`})$ =$y_2^{e`}$ and $x_1^e \neq x_2^{e`}$. Since (X, τ , E) is soft $\tau_\circ$-space , there exist soft open sets (F, E) and (G, E) over X such that either $x_1^e \widetilde{\in}$ (F, E) and $x_2^{e`} \widetilde{\notin}$ (F, E) or $x_2^{e`} \widetilde{\in}$ (G, E) and $x_1^e \widetilde{\notin}$ (G, E). So, either y_1^e=$f(x_1^e)$ $\widetilde{\in} f$ (F, E) and $y_2^{e`}$ =$f\ (x_2^{e`}) \widetilde{\notin}\ f$(F, E) or $y_2^{e`}$=$f\ (x_2^{e`}) \widetilde{\in} f$(G, E) and y_1^e=$f\ (x_1^e)$ $\widetilde{\notin} f$(G, E). Hence either $y_1^e\ \widetilde{\in} f$ (F, E) and $y_2^{e`} \widetilde{\notin}\ f$(F, E) or $y_2^{e`} \widetilde{\in} f$(G, E) and $y_1^e \widetilde{\notin} f$(G, E). Since f is soft open function, then f(F, E), f(G, E) are soft open sets over Y . Hence (Y, τ′, E) is also a soft $\tau_\circ$-space.

Proposition(2.3.25) :

Let (X, τ, E) and (Y, τ', E) be $sts's$ and f:X → Y be a function which is bijective and soft open. If (X, τ , E) is soft τ_1-space, then (Y, τ', E) is also a soft τ_1-space

Proof: It is similar to the proof of proposition(2.3.24).

Proposition(2.3.26) :

Let (X, τ, E) and (Y, τ', E) be $sts's$ and f: X → Y be soft function which is bijective and soft open. If (X, τ, E) is soft τ_2-space, then (Y, τ',E)is soft τ_2-space.

Proof:

Let $y_1^e, y_2^{e`} \tilde{\in} \tilde{Y}$ such that $y_1^e \neq y_2^{e`}$. Since f is surjective, then $\exists\ x_1^e, x_2^{e`} \tilde{\in} \tilde{X}$ such that $f(x_1^e) = y_1^e$ , $f(x_2^{e`}) = y_2^{e`}$ and $x_1^e \neq x_2^{e`}$ .Since (X, τ, E) is soft τ_2-space , there exist soft open sets (F, E) and (G, E) over X such that either $x_1^e \tilde{\in}$ (F, E) and $x_2^{e`} \tilde{\in}$ (G, E) and (F, E) $\tilde{\cap}$ (G, E) $= \tilde{\Phi}$. So, $y_1^e = f(x_1^e) \tilde{\in} f$ (F, E) and $y_2^{e`} = f(x_2^{e`}) \tilde{\in} f$(G, E). Hence $y_1^e \tilde{\in} f$ (F, E), $y_2^{e`} \tilde{\in} f$(G, E) and f(F, A)$\tilde{\cap} f$(G, A) $=$ f[(F, A) $\tilde{\cap}$ (G, A)] $= f[\tilde{\Phi}] = \tilde{\Phi}$ from proposition(1.1.17,iii). Since f is soft open function, then f(F, A), f(G, A) are soft open sets over Y . Thus (Y, τ', E) is also a soft τ_2-space.

Proposition(2.3.27) :

Let (X, τ , E) and (Y, τ', E) be $sts's$ and f: X → Y be soft function which is bijective, soft continuous and soft open. If (X, τ , E) is soft regular space, then (Y, τ', E) is a soft regular space.

Proof:

Let (G, E) be a soft closed set over Y and $y^e \tilde{\in} \tilde{Y}$ such that $y^e \tilde{\notin}$ (G, E). Since f is surjective and soft continuous , then $\exists\ x^e \tilde{\in} \tilde{X}$ such that $f(x^e) = y^e$ and f^{-1} (G, E) is soft closed set over X such that $x^e \tilde{\notin} f^{-1}$(G, E). Since (X, τ, E) is soft regular space, there exist soft open sets (F, E) and (H, E) over X such that $x^e \tilde{\in}$ (F, E), f^{-1}(G, E) $\tilde{\subseteq}$ (H, E) and (F, E) $\tilde{\cap}$ (H, E) $= \tilde{\Phi}$. It follows that $y^e =$ $f(x^e) \tilde{\in} f$ (F, E) and (G, E) $= f$ [f^{-1}(G, E)]$\tilde{\subseteq} f$(H, E) from Proposition (1.1.16, iv). Hence $y^e \tilde{\in} f$ (F, E) and (G, E) $\tilde{\subseteq} f$(H, E) and f(F, E) $\tilde{\cap} f$(H, E) $= f$[(F, E) $\tilde{\cap}$ (H, E)] $= f[\tilde{\Phi}] = \tilde{\Phi}$. Since$f$ is soft open function. Then f (F, E), f(H, E) are soft open sets over Y . Thus, (Y, τ', E) is also a soft regular space.

Proposition(2.3.28) :

Let (X, τ, E) and (Y, τ', E) be $sts's$ and f: X → Y be soft function which is bijective, soft continuous and soft open. If (X, τ, E) is soft τ_3-space, then (Y, τ', E) is also a soft τ_3- space.

Proof:

Since (X, τ, E) is soft τ_3-space, then (X, τ, E) is soft regular τ_1-space. It follows that (Y, τ', E) is also a soft τ_1-space from proposition(2.3.25) and soft regular space from proposition(2.3.27). Hence (Y, τ', E) is also a soft τ_3-space.

Proposition(2.3.29) :

Let (X, τ , E) and (Y, τ', E) be $sts's$ and f: X → Y be soft function which is bijective, soft continuous and soft open. If (X, τ , E) is soft normal space, then (Y, τ', E) is a soft normal space.

Proof:

Let (F, E),(G, E) be soft closed sets over Y such that (F, E) $\tilde{\sqcap}$ (G, E) = $\tilde{\Phi}$. Since f is soft continuous, then f^{-1}(F, E) and f^{-1}(G, E) are soft closed sets over X such that f^{-1}(F, E)$\tilde{\sqcap}$ f^{-1}(G, E) = f^{-1}[(F, E) $\tilde{\sqcap}$ (G, E)] = f^{-1} [$\tilde{\Phi}$] = $\tilde{\Phi}$ from proposition (1.1.18) (vi). Since (X, τ , E) is normal space, then there exist soft open sets (K, E) and (H, E) over X such that f^{-1}(F, E) $\tilde{\sqsubseteq}$ (K, E), f^{-1}(G,E) $\tilde{\sqsubseteq}$ (H, E) and (K, E) $\tilde{\sqcap}$ (H, E) = $\tilde{\Phi}$. It follows that (F, E) = f [f^{-1}(F, E)]$\tilde{\sqsubseteq}$ f(K, E) ,(G, E) = f [f^{-1} (G, E)]$\tilde{\sqsubseteq}$ f (H, E) from proposition (1.1.16 ,iv) and f (K, E) $\tilde{\sqcap}$ f (H, E) = f[(K,E)$\tilde{\sqcap}$(H, E)] = f [$\tilde{\Phi}$] = $\tilde{\Phi}$. Since f is soft open function. Then f(K, E), f(H, E) are soft open sets over Y . Thus, (Y, τ', E) is also a soft normal space.

Corollary (2.3.30):

Let (X, τ, E) and (Y, τ', E) be $sts's$ and f: X → Y be soft function which is bijective, soft continuous and soft open. If (X, τ,E) is soft τ_4-space, then(Y, τ',E) is also a soft τ_4- space.

Proof: It is clear from proposition(2.3.25) and proposition(2.3.29).

Introduction:

Chapter three is divided into three sections .In section one, we recall definition of soft compact space and give the definition of soft compact function on this concept and we prove some results on this concept.

In section two we introduce definitions of soft coercive function and we prove some results on this concept ,and explain the relation between it and soft compact function.

After that , we construct the definition of soft proper function and study some of their properties .

3.1 On Soft Compact Function:

In this section, we recall the definition of soft compact space and prove some of their properties . After that , we introduce definition of soft compact function by using the concept of soft compact set.

Definition(3.1.1) : [24]

A family Ψ of soft sets is a cover of a soft set (F, E) if (F, E) $\tilde{\subseteq}$ $\tilde{\cup}$\{ (F_i,E) : (F_i,E) ∈ Ψ, i ∈ I\}. It is a soft open cover if each member of Ψ is a soft open set. A sub cover of Ψ is a subfamily of Ψ which is also a cover.

Definition(3.1.2):[24]

A sts (X, τ , E) is soft compact if each soft open cover of $\tilde{X}$, has a finite subcover.

Theorem (3.1.3) :[23]

Let (A, τ_A ,E) be a soft subspace of a soft space (X, τ , E). Then (A, τ_A ,E) is soft compact if and only if every cover of $\widetilde{A}$ by soft open sets in X contains a finite sub cover.

Proposition(3.1.4): [19]

Let (X, τ_2, E) be a soft compact space and $\tau_1 \subseteq \tau_2$. Then (X, τ_1, E) is soft compact.

Theorem (3.1.5): [19]

i. Every soft compact subspace of a soft τ_2- space is soft closed.

ii. Every soft closed subset of a soft compact space is soft compact.

Theorem (3.1.6):

Let f be surjective ,soft continuous function from the soft compact space (X, τ ,E) into the soft space(Y, τ',E).Then(Y, τ', E)is soft compact.

Proof: Clear.

Proposition(3.1.7):

Let (X, τ ,E)be a sts, A⊆X where $\widetilde{A}$ is soft open set over X and (F,E) $\widetilde{\subseteq}$ $\widetilde{A}$. Then (F ,E) is soft compact set over X if and only if (F ,E)is soft compact set over A .

Proof: Clear.

Proposition(3.1. 8):

Let (X, τ ,E) be a sts ,where X ,E are finite sets , then (X, τ ,E) is soft compact space.

Proof: Clear.

Proposition(3.1.9):

The intersection of soft compact subset with soft closed set is soft compact.

Proof:

Let (G,E) be a soft compact and (F,E) is soft closed set and Ψ ={(H_i,E): $i\in$ I} be a soft open cover of (F,E)$\tilde{\cap}$(G,E) such that (F,E) $\tilde{\cap}$(G,E)$\tilde{\subseteq}$ $\tilde{\cup}${(H_i, E): $i\in$I} $\Longrightarrow$ (G,E) $\tilde{\subseteq}$ $\tilde{\cup}${(H_i, E): $i\in$ I }$\tilde{\cup}$(F,E)^c since (F,E) is soft closed then (F,E)^c is soft open set so {(H_i, E): $i\in$ I }$\tilde{\cup}$ (F,E)^c is soft open cover of (G,E) and since (G,E) is soft compact then there is finite sub cover {(H_{ij}, E): $i\in$ I }$\tilde{\cup}$ (F,E)^c ; j=1,...,n of (G,E) such that (G,E) $\tilde{\subseteq}$ {(H_i, E): $i\in$ I}$\tilde{\cup}$ (F,E)^c so(F,E)$\tilde{\cap}$(G,E) $\tilde{\subseteq}$ $\tilde{\cup}${(H_{ij}, E): $i\in$ I} . Hence (F,E) $\tilde{\cap}$ (G,E) is soft compact .

Theorem (3.1.10):[24]

A *sts* is soft compact if and only if each family of soft closed sets with the finite intersection property has a non-null intersection.

Recall that if X is a topological space , then X is compact if and only if every net in X has a cluster point in X . **[25]**

Simple verification shows that this result remain valid when X is a soft topological space as following:

Theorem(3.1. 11)

Let (X, τ , E) be a *sts*, then (X, τ, E)is soft compact if and only if every soft net in $\tilde{X}$ has a soft cluster point in $\tilde{X}$.

Now, we introduce the definition of soft compact function.

Definition(3.1.12):

Let (X , τ,E), (Y , τ',E) be $sts's$ and f :X⟶ Y be a function .Then f is called soft compact if f^{-1}(G,E) is a soft compact set over X for all soft compact set (G,E) over Y.

Examples (3.1.13):

i. Let $X = \{h_1, h_2, h_3\}$, $Y = \{y_1, y_2\}$, $E = \{e, e`\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}$, ($F_1$, E),($F_2$, E) , ($F_3$, E)}, $\tau' = \{\widetilde{\Phi}, \widetilde{Y}$, (G_1, E),(G_2, E)} be two soft topologies defined on X and Y , (resp.). Here (F_1, E),(F_2, E),(F_3, E),(G_1, E),(G_2, E) are soft sets over X and Y , (resp.). The soft sets are defined as follows: $F_1(e) = \{h_1, h_3\}$, $F_1(e`) = \{h_2\}$, $F_2(e) = X$, $F_2(e`) = \{h_2, h_3\}$, $F_3(e) = \{h_3\}$, $F_3(e`) = \{h_2\}$ and $G_1(e) = \emptyset$, $G_1(e`) = \{ y_1\}$, $G_2(e) = \{ y_1\}$, $G_2(e`) = Y$. Now we define the function f: X → Y as $f(h_1) = f(h_2) = y_1$, $f(h_3) = y_2$. It is clear that f^{-1}(G, E) is a soft compact over X, for all soft compact set(G,E) over Y. Thus f is soft compact function.

ii. Every surjective function from a sts (X, τ ,E) where X,E are finite sets into any space is soft compact function.

Proposition (3.1.14):

Let (X, τ, E) be a sts and A be a non-empty subset of X .If $\widetilde{A}$ is soft closed then the inclusion function i:A⟶ X is soft compact .

Proof:

Let (G,E) be a soft compact set over X .Since i^{-1}(G, E)=(G,E) $\widetilde{\cap}$ $\widetilde{A}$ then by Proposition(3.1.9) (G,E) $\widetilde{\cap}$ $\widetilde{A}$ is soft compact over A .Hence i is soft compact function.

Proposition(3.1.15):

Let (X , τ, E), (Y , τ',E) and (Z, τ'',E) be $sts's$. If f: X ⟶ Y and g: Y ⟶ Z are soft functions, then

i. If f and g are soft compact functions then $g \circ f: \mathrm{X} \longrightarrow \mathrm{Z}$ is a soft compact function.

ii. If $g \circ f$ is soft compact function and g is soft continuous , bijective function then f is soft compact.

Proof:

i. clear.

ii. Let (H, E) be a soft compact over Y then by Theorem (3.1.6) we have $g(\mathrm{H}, \mathrm{E})$ is soft compact set over Z thus $(g \circ f)^{-1}(g(\mathrm{H}, \mathrm{E}))$ is a soft compact set over X .Since g is injective then $(g \circ f)^{-1}\big(g(\mathrm{H}, \mathrm{E})\big) = f^{-1}(\mathrm{H}, \mathrm{E})$ thus $f^{-1}(\mathrm{H}, \mathrm{E})$ is a soft compact set over X . Hence f is soft compact function.

Remark(3.1.16):

The restriction function of soft compact function is not necessary being soft compact and the following proposition give a condition to satisfy it .

Proposition (3.1.17):

Let $(\mathrm{X}, \tau, \mathrm{E})$ and$(\mathrm{Y}, \tau', \mathrm{E})$ be $sts's$. If $f: \mathrm{X} \longrightarrow \mathrm{Y}$ is a soft compact function and $\widetilde{\mathrm{A}}$ is a soft closed set over X. Then the restriction function $f_{|\mathrm{A}}: \mathrm{A} \longrightarrow \mathrm{Y}$ is soft compact.

Proof:

Let $g = f|_{\mathrm{A}}$, since $\widetilde{\mathrm{A}}$ is soft closed subset of X, then by Proposition (3.1.14)the inclusion function $i: \mathrm{A} \longrightarrow \mathrm{X}$ is a soft compact but $g = f \circ i$ then by Proposition (3.1.15 ,i) we have g is soft compact function .

Now, we discuss the properties of product of soft compact functions.

Lemma (3.1.18):[4]

Let (X, τ ,E) and (Y, τ',E) be two $sts's$.Then, X and Y are homeomorphic to a subspace of X × Y.

Lemma(3.1.19):

Let (X, τ, E)be a sts . If (F,E× $\{e\}$)is a soft compact set over X × $\{a\}$ then there exists an soft compact set (G,E) over X, such that (F, E× $\{e\}$) = (G,E)× $\{a^e\}$, where a^e is any soft point which does not belong to $\widetilde{X}$.

Proof:

Let (F,E× $\{e\}$)be a soft compact set over X × $\{a\}$ then there exists an soft compact set (G,E) over X, such that (F, E× $\{e\}$)=(G,E)× $\{a^e\}$ .To prove (G,E) is an soft compact set over X. Let {(H_i,E):i∈I}be an soft open cover of (G,E), then (G,E) $\widetilde{\subseteq}$ $\widetilde{\cup}$(H_i, E); i∈I thus(G,E)×{ a^e}$\widetilde{\subseteq}$ $\widetilde{\cup}$ (H_i, E) × $\{a^e\}$ so(F, E × $\{e\}$) $\widetilde{\subseteq}$ $\widetilde{\cup}$ (H_i,E) × $\{a^e\}$ .Since (F, E× $\{e\}$) is soft compact then there is ,j=1,…,n such that (F, E× $\{e\}$)$\widetilde{\subseteq}$ $\widetilde{\cup}$(H_{ij}, E) × $\{a^e\}$ thus (G, E) ×$\{a^e\}$ $\widetilde{\subseteq}$ $\widetilde{\cup}$(H_{ij}, E) × $\{a^e\}$ then (G,E) $\widetilde{\subseteq}$ $\widetilde{\cup}$(H_{ij}, E) .Hence (G,E) is an soft compact set over X. Therefore there exists an soft compact set (G,E) over X, such that (F, E× $\{e\}$)= (G,E) × $\{a^e\}$.

Lemma (3.1.20):

Let (X×Y, τ_{prod} , E × E)be a soft compact space ,then (X, τ,E) and (Y, τ',E) are soft compact spaces.

Proof: By using Theorem (2.1.33) and Theorem(3.1.6) .

Corollary(3.1.21):

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be soft compact functions, then:

i. $f_1^{-1}\{y_1^e\}$ is a soft compact set over $X_1, \forall\ y_1^e \tilde{\in} \tilde{Y}_1$.

ii. $f_2^{-1}\{y_2^e\}$ is a soft compact set over $X_2, \forall\ y_2^e \tilde{\in} \tilde{Y}_2$.

Theorem (3.1.22):

Let f_i: $(X_i ,\tau_i, E) \rightarrow (Y_i ,\tau'_i, E), i=1,2$ be a soft functions .If $f_1 \times f_2$:($X_1 \times X_2$, τ_{prod}, E×E) ⟶ ($Y_1 \times Y_2$,τ'_{prod} , E×E) is soft compact function then f_i is soft compact function ,for all i=1,2 .

Proof:

To prove f_1 is soft compact function. Let (G,E) be a soft compact set over Y_1 and $y^e \tilde{\in} \tilde{Y}_2$ then $\{y^e\}$ is soft compact over Y_2 .Thus by Lemma(3.1.19) (G,E)× $\{y^e\}$ is soft compact set over $Y_1 \times Y_2$ .Since $f_1 \times f_2$is soft compact function , then $(f_1 \times f_2)^{-1}$ ((G,E)× $\{y^e\}$) = f_1^{-1} (G,E)× $f_2^{-1}\{y^e\}$ is soft compact over $X_1 \times X_2$,thus by lemma(3.1.20) f_1^{-1}(G,E) is soft compact over X_1 . Hence f_1 is soft compact function.
In the same way , we can prove f_2 is soft compact function.

Remark(3.1. 23):

If (X, τ,E) is soft compact space ,then it's not necessary (X,τ_e) is compact space, $e \in$E . As the following example shows:

Let X=Z , E = $\{e, e`\}$ and $\tau$ =$\{\tilde{\Phi}$ ,$\tilde{X}$ $\}\tilde{\cup}\{(e$,A),( $e`$,$\emptyset$):$\emptyset \neq$ A$\subseteq$ X} .Then (X, τ,E) is soft compact but (X,τ_e) is not compact space.

Also, by Remark (1.2.7) the converse of Remark(3.1.23) is not true .
Now ,when E = $\{e\}$ it's easy to prove the following Proposition.

Proposition (3.1. 24) :

Let (X, τ, E) be a sts, where $E = \{e\}$. Then (X, τ, E) is soft compact if and only if the topological space (X, τ_e) is compact .

Proof: clear.

Proposition (3.1.25):

Let (X, τ, E) be a sts and $E=\{e\}$ .Then (G,E) is soft compact set , if and only if , $G(e)$ is compact set .

Proof: Clear.

Theorem(3.1.26):

Let $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft compact function, where $E = \{e\}$, such that every compact set in (Y, τ_e') is an open set. Then $f_e : (X, \tau_e) \rightarrow (Y, \tau_e')$ is a compact function .

Proof:

Let U be a compact set in Y, then there exists soft open set (G,E) over Y such that $U = G(e)$ .By Proposition(3.1.25) we have (G, E)is a soft compact set. Since f:$(X, \tau, E) \rightarrow (Y, \tau', E)$ is a soft compact function, then $f^{-1}(G, E)$ is a soft compact set over X and $f^{-1}(G, E)(e) = f^{-1}(G(e)) = {f_e}^{-1}(U)$ is an compact set. This implies that f_e is a compact function.

3.2 On Soft Coercive Function:

In this section , we introduce a definition of soft coercive function and we give some result ,which are related with this subject .

Definition(3.2.1):[9]

Let X and Y be spaces .A function $f: \mathrm{X} \to \mathrm{Y}$ is called an coercive if for every compact set $\mathrm{J} \subseteq \mathrm{Y}$ there exists compact set $\mathrm{K} \subseteq \mathrm{X}$ such that $f(\mathrm{X} \setminus \mathrm{K}) \subseteq \mathrm{Y} \setminus \mathrm{J}$.

Definition(3.2.2):

Let $(\mathrm{X}, \tau, \mathrm{E})$ and $(\mathrm{Y}, \tau', \mathrm{E})$ be $sts's$. A function $f: \mathrm{X} \to \mathrm{Y}$ is called a soft coercive if for every soft compact set (G, E) over Y , there exists soft compact set (F,E) over X such that $f\left(\widetilde{\mathrm{X}} \setminus (\mathrm{F}, \mathrm{E})\right) \widetilde{\subseteq} \widetilde{\mathrm{Y}} \setminus (\mathrm{G}, \mathrm{E})$.

Example(3.2.3):

If $(\mathrm{X}, \tau, \mathrm{E})$ is an soft compact space then the function $f: \mathrm{X} \to \mathrm{Y}$ is an soft coercive. Let (G, E) be an soft compact set over Y since $(\mathrm{X}, \tau, \mathrm{E})$is soft compact space and $f(\widetilde{\mathrm{X}} \setminus \widetilde{\mathrm{X}}) = f(\widetilde{\Phi}) \widetilde{\subseteq} \widetilde{\mathrm{Y}} \setminus (\mathrm{G}, \mathrm{E})$ then f is soft coercive function.

Proposition(3.2.4):

Every soft compact function is a soft coercive .

Proof:

Let $f: \mathrm{X} \to \mathrm{Y}$ be a soft compact function .To prove f is a soft coercive. Let (G,E) be a soft compact set over Y. Since f is soft compact function ,then f^{-1}(G,E) is soft compact set over X . Thus $f\left(\widetilde{\mathrm{X}} \setminus f^{-1}(\mathrm{G}, \mathrm{E})\right) \widetilde{\subseteq} \widetilde{\mathrm{Y}} \setminus (\mathrm{G}, \mathrm{E})$. Hence $f: \mathrm{X} \to \mathrm{Y}$ is a soft coercive function .

Note that, the convers of Proposition(3.2.4) is not true in general as the following example shows:

Let X=[0,1],Y=$\{y_1, y_2\}$and $\mathrm{E} = \{e\}$.If τ=$\{(\mathrm{F},\mathrm{E}):\mathrm{F}(e)=(a, b);a, b \in \mathrm{X}\} \widetilde{\cup} \{\widetilde{\Phi}, \widetilde{\mathrm{X}}\}$ and $\tau' = \tau_{sd}$. Then $(\mathrm{X}, \tau, \mathrm{E})$ and$(\mathrm{Y}, \tau', \mathrm{E})$ are $sts's$ over X and Y(resp.). Now we define the function $f : \mathrm{X} \to \mathrm{Y}$ as :

$f(x)=\begin{cases} y_1 & \forall \quad x \in (o,1) \\ y_2 & \forall \quad x \in \{o,1\} \end{cases}$. Then f is a soft coercive function, but it's not soft compact function.

Proposition(3.2.5):

Let (X , τ ,E) and(Y , τ' ,E) be $sts's$, such that (Y , τ',E) is a soft τ_2-space and f: X $\longrightarrow$ Y is soft continuous function .Then f is a soft coercive if and only if f soft compact.

Proof:

($\Longrightarrow$) Let (G, E) be an soft compact set over Y. To prove f^{-1}(G, E) is soft compact set over X. Since (Y , τ',E) is a soft τ_2-space and f is soft continuous function, so f^{-1}(G, E)is soft closed set over X. Since f is soft coercive function , then there exists soft compact set (K, E) over X such that $f(\widetilde{X} \setminus (K, E))$ $\widetilde{\subseteq} \widetilde{Y} \setminus (G, E)$ .Then $f(K, E)^c \widetilde{\subseteq} (G, E)^c$,therefore $f^{-1}(G, E) \widetilde{\subseteq} (K, E)$. So by proposition(3.1.5,ii), we have f^{-1}(G, E) is soft compact set over X. Hence f is a soft compact function.

($\Longleftarrow$) By Proposition(3.2.4) .

Proposition (3.2. 6):

Let (X, τ, E) be a sts and A be a non-empty subset of X.If $\widetilde{A}$ is soft closed then the inclusion function i:A$\longrightarrow$ X is soft coercive .

Proof:

Let (G,E) be a soft compact set over X. Since $\widetilde{A}$ is soft closed and (G,E) is soft compact then i^{-1}(G, E)=(G,E)$\widetilde{\cap}\, \widetilde{A}$ is soft compact over A. But $i(\widetilde{A} \setminus i^{-1}(G,E)) = i(\widetilde{A} \setminus (G, E) \widetilde{\cap}\, \widetilde{A}) = i(\widetilde{A} \setminus (G,E)) \widetilde{\subseteq} \widetilde{X} \setminus (G, E)$. Thus i is soft coercive function.

Proposition(3.2.7):

Let (X, τ, E), (Y, τ', E) and (Z, τ'', E) be $sts's$. If $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ are soft coercive functions, then $g \circ f: X \longrightarrow Z$ is a soft coercive function.

Proof:

Let (G, E) be an soft compact set over Z then there exists soft compact set (K, E) over Y such that $g(\widetilde{Y} \setminus (K, E)) \widetilde{\subseteq} \tilde{Z} \setminus (G, E)$.Since f is an soft coercive function then there exists soft compact set (F, E) over X such that $f(\widetilde{X} \setminus (F, E)) \widetilde{\subseteq} \widetilde{Y} \setminus (K, E)$ then $g(f\left(\widetilde{X} \setminus (F, E)\right)) \widetilde{\subseteq} g(\widetilde{Y} \setminus (K, E))) \widetilde{\subseteq} \tilde{Z} \setminus (G, E)$. Then $(g \circ f)(\widetilde{X} \setminus (F, E)) \widetilde{\subseteq} \tilde{Z} \setminus (G, E)$. Hence $g \circ f: X \to Z$ is an soft coercive function.

Proposition (3.2.8):

Let (X, τ, E) ,(Y, τ', E) and (Z, τ'', E) be $sts's$, such that $f: X \to Y$ is bijective, soft compact function and $g: Y \to Z$ is an soft coercive function. Then $g \circ f: X \to Z$ is an soft coercive function.

Proof:

Let (H, E)be an soft compact set over Z. Since $g: Y \to Z$ is an soft coercive then there exists soft compact(G, E)over Y,such that $g\left(\widetilde{Y} \setminus (G, E)\right) \widetilde{\subseteq} \tilde{Z} \setminus (H, E)$ .Let $(F, E) = f^{-1}(G, E)$, since $f: X \to Y$ is an soft compact function then (F, E) is an soft compact set on X. Thus

$$
\begin{aligned}
(g \circ f)\left(\widetilde{X} \setminus (F, E)\right) &= g\left(f\left(\widetilde{X} \setminus (F, E)\right)\right) \\
&= g\left(f\left(\widetilde{X} \widetilde{\cap} (F, E)^c\right)\right) \\
&= g(f\left(\widetilde{X}\right) \widetilde{\cap} f(\, (F, E)^c) \\
&= g\left(\widetilde{Y} \widetilde{\cap} (f(f^{-1}((G, E)))^c\right), \text{ since } f \text{ bijective,} \\
&= g\left(\widetilde{Y} \widetilde{\cap} (G, E)^c\right) = g(\widetilde{Y} \setminus (G, E)) \widetilde{\subseteq} \tilde{Z} \setminus (H, E).
\end{aligned}
$$

Thus $(g \circ f)\left(\widetilde{X} \setminus (F,E)\right) \widetilde{\subseteq} \tilde{Z} \setminus (H,E)$. Hence $g \circ f: X \to Z$ is an soft coercive function.

Proposition (3.2.9):

Let (X , τ , E) and(Y , τ', E) be $sts's$. If f: X $\longrightarrow$ Y is soft coercive function, and $\widetilde{A}$ is a soft closed set over X. Then the restriction function $f_{|_A}$: A $\longrightarrow$ Y is soft coercive function.

Proof: Clear.

Proposition(3.2.10):

Let (X, τ, E) , (Y, τ', E) be $sts's$ and f: X $\longrightarrow$ Y be soft coercive , soft continuous function. If $\widetilde{A}$ is any soft clopen over Y, then $f_A: f^{-1}(A) \longrightarrow A$, is soft coercive function.

Proof :

Let (F, E) be an soft compact set over A , since $\widetilde{A}$ is soft open over Y ,then by proposition(3.1.7)we have (F,E)is soft compact set over Y. Since f is soft coercive , then there exists soft compact set (K ,E) over X such that

$$f\left(\widetilde{X} \setminus (K,E)\right) \widetilde{\subseteq} \widetilde{Y} \setminus (F,E).$$

Since $\widetilde{A}$ is soft closed set over Y and f is soft continuous, then $f^{-1}(\widetilde{A})$ soft closed set over X , by proposition(3.1.9) , $f^{-1}(\widetilde{A}) \widetilde{\cap} (K,E)$ is soft compact set over X . Since $f^{-1}(\widetilde{A})$ is soft open set over X , then by proposition (3.1.7) , $f^{-1}(\widetilde{A}) \widetilde{\cap}(K,E)$is soft compact set over $f^{-1}(\widetilde{A})$. Since$f^{-1}(\widetilde{A})\setminus(K,E) \widetilde{\subseteq} \widetilde{X} \setminus(K ,E)$ then:

f_A $(f^{-1}(\widetilde{A}) \setminus (K,E)) \widetilde{\subseteq} f_A(\widetilde{X} \setminus (K,E))$

$= \widetilde{A} \widetilde{\cap} f(\widetilde{X} \setminus (K,E))$

$\widetilde{\subseteq} \widetilde{A} \widetilde{\cap} (\widetilde{Y} \setminus (F,E)) = \widetilde{A} \setminus (F,E)$.

Thus f_A $(f^{-1}(\widetilde{A}) \setminus f^{-1}(\widetilde{A}) \widetilde{\cap} (K,E)) \widetilde{\subseteq} \widetilde{A} \setminus (F,E)$. Therefore f_A: $f^{-1}(A) \longrightarrow$ A soft coercive function .

Theorem(3.2.11):

Let f_i: $(X_i, \tau_i, E) \rightarrow (Y_i, \tau'_i, E)$, i=1,2 be injective , soft functions. If $f_1 \times f_2$ is soft coercive function ,then f_i is soft coercive functions ,for all i=1,2 .

Proof:

($\Longrightarrow$) To prove f_i is soft coercive functions ,for all i=1,2. Let (G_1,E) be a soft compact set over Y_1 and $y^e \widetilde{\in} \widetilde{Y}_2$ .Thus by Lemma(3.1.19) , $(G_1,E) \times \{y^e\}$ is soft compact set over $Y_1 \times Y_2$ .Since $f_1 \times f_2$ is soft coercive function then there exists soft compact set $(F_1, E) \times (F_2, E)$ over $X_1 \times X_2$,such that:

$(f_1 \times f_2)[(\widetilde{X}_1 \times \widetilde{X}_2) \setminus ((F_1, E) \times (F_2, E))] \widetilde{\subseteq} \widetilde{Y}_1 \times \widetilde{Y}_2 \setminus (G_1, E) \times \{y^e\}$,so
$f_1[\widetilde{X}_1 \setminus f_1(F_1, E)] \times f_2[\widetilde{X}_2 \setminus f_2(F_2, E)] \widetilde{\subseteq} [\widetilde{Y}_1 \setminus (G_1, E)] \times [\widetilde{Y}_2 \setminus \{y^e\}]$,thus
$f_1[\widetilde{X}_1 \setminus (F_1, E)] \widetilde{\subseteq} [\widetilde{Y}_1 \setminus (G_1, E)]$. Hence f_1 is soft coercive function .

In the same way , we can prove f_2 is soft coercive function.

Proposition(3.2.12) :

Let f : $(X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft coercive function where E = {e} ,such that every compact set in (Y, τ'_e) is an open set .Then for each $e \in E$, f_e : $(X, \tau_e) \rightarrow (Y, \tau'_e)$ is a coercive function .

Proof:

Let f be a soft coercive .To prove f_e is a coercive function. Let U be a compact set in (Y, τ'_e) ,then by Proposition(3.1.25) there exists a soft compact set (G, E) over Y such that U = G(e). Since f is a soft coercive function, then there exists soft compact set (F, E) over X such that $f(\widetilde{X} \setminus (F, E)) \tilde{\subseteq} \widetilde{Y} \setminus (G, E)$. This implies that $f(X \setminus F(e)) \subseteq Y \setminus G(e)$, where $F(e)$ is a compact set on X. Thus f_e is a coercive function.

<u>3.3 On Soft Proper Function</u>:

In this section, we introduce the definition of soft proper function .Also, we give some propositions, examples and theorems about this subject .

<u>Definition (3.3.1)</u>: [7]

Let X and Y be topological spaces and $f: X \longrightarrow Y$ be a function, then f is called a proper function if :

i. f is continuous function ;
ii. f is closed function ;
iii. $f^{-1}\{y\}$ is compact set in X , for every $y \in Y$.

Now, we introduce the following definition .

<u>Definition (3.3.2)</u> :

Let(X, τ,E) and (Y, τ',E) be a $sts's$,and $f: X \longrightarrow Y$ be a function , then f is called soft proper function if:

i. f is soft continuous function ;

ii. f is soft closed function ;

iii. $f^{-1}\{y^e\}$ is soft compact, for all $y^e \tilde{\in} \widetilde{Y}$.

<u>Example(3.3.3)</u>:

Let $X = \{x\}$ and $E = \{e, e`\}$, such that $\tau = \tau_{sind}$. Let $f: X \longrightarrow X$ be the identity function . Then f is soft proper function.

The following example shows not every function is soft proper function.

Example(3.3.4):

Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ and $E=\{e, e`\}$. Then $\tau = \{\widetilde{\Phi}, \widetilde{X}, (F,E)\}$ and $\tau' = \{\widetilde{\Phi}, \widetilde{Y}, (G,E)\}$ are $sts's$ over X and Y(resp.). Here (F, E) a soft sets over X and (G, E) a soft sets over Y defined as follows:

$F(e) = \{x_1, x_2\}$, $F(e`) = \{x_3\}$, and $G(e) = \{y_1\}$, $G(e`) = \{y_2\}$. If we get the function $f: X \rightarrow Y$ defined as $f(x) = y_1, \forall x \in X$ then f is not soft proper.

Remark(3.3.5).

The restriction of soft proper function is not necessary being a soft proper function.

Proposition(3.3.6):

Let $f: (X, \tau, E) \rightarrow (Y, \tau', E)$be a soft proper function and $\widetilde{A}$ is a soft closed set over X.Then the restriction function $f_{|A}: (A, \tau_A, E) \longrightarrow (Y, \tau', E)$is soft proper function.

Proof:

By Proposition(2.1. 9) and by proposition (2.1.11)then $f|_A$ is soft continuous and soft closed function.

Now, to prove $(f|_A)^{-1}\{y^e\}$ is a soft compact set over A, for all $y^e \widetilde{\in} \widetilde{Y}$. Since f is soft proper function , then $f^{-1}\{y^e\}$ is a soft compact set over X . Thus by proposition (3.1.9) , $f^{-1}\{y^e\} \widetilde{\cap} \widetilde{A} = (f|_A)^{-1}\{y^e\}$ is a soft compact set over A , for every $y^e \widetilde{\in} \widetilde{Y}$. Hence $f|_A$ is a soft proper.

Proposition(3.3.7):

Let (X, τ, E) , (Y, τ', E) be $sts's$ and $f: X \longrightarrow Y$ be soft proper function. If $\widetilde{A}$ is any soft closed set over Y, then $f_A: f^{-1}(A) \longrightarrow A$ is soft proper function.

Proof:

i. Since f is a soft continuous function , then by proposition (2.1.16) f_A is a soft continuous.

ii. Since f is soft closed function , then by proposition (2.1. 17), f_A is soft closed function .

iii. Now, to prove $f_A^{-1}\{y^e\}$ is soft compact set for every $y^e \widetilde{\in} \widetilde{A}$. Since f is a soft proper function ,then $f^{-1}\{y^e\}$ is a soft compact set over X . Since $f^{-1}(\widetilde{A})$ is soft closed set then by proposition(3.1.9), $f^{-1}(\widetilde{A}) \widetilde{\cap} f^{-1}\{y^e\} = f_A^{-1}\{y^e\}$ is soft compact set over $f^{-1}(A)$. Hence f_A is soft proper.

Proposition(3.3.8):

Let (X , τ, E), (Y , τ', E) and (Z, τ'', E) be $sts's$. If $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ are soft proper functions, then $g \circ f: X \longrightarrow Z$ is soft proper function.

Proof:

i. Since f and g are soft continuous functions , then $g \circ f$ *is* soft continuous. By Proposition(2.1. 8) .

ii. Since$f$ and g are soft closed then $g \circ f$ is soft closed by proposition(2.1.8).

iii. Let $z^e \widetilde{\in} \tilde{Z}$, since g is soft proper, then $g^{-1}\{z^e\}$ is soft compact over Y, for every $z^e \widetilde{\in} \tilde{Z}$, and since f is soft proper function ,then $f^{-1}(g^{-1}\{z^e\})$ is soft compact over X . But $f^{-1}(g^{-1}\{z^e\})=(g \circ f)^{-1}\{z^e\}$, thus $(gof)^{-1}\{z^e\}$ is soft compact set over X . Therefor by (i), (ii) and (iii) we have $g \circ f$ is soft proper function.

Proposition(3.3.9):

Let (X , τ,E), (Y , τ',E) and (Z, τ'',E) be $sts's$. If f: X ⟶ Y and g: Y ⟶ Z be soft continuous functions, such that $g \circ f$: X ⟶ Z is soft proper function. If f is surjection, then g is soft proper function.

Proof:

i. g is soft continuous, by hypothesis .

ii. By proposition (2.1. 18, i) we have and g: Y→ Z is a soft closed .

iii. Let $z^e \tilde{\in} \tilde{Z}$ .To prove $g^{-1}\{z^e\}$ soft compact set over Y for $z^e \tilde{\in} \tilde{Z}$. Since $g \circ f$ is soft proper function, then $(g \circ f)^{-1}\{z^e\} = (f^{-1}(g^{-1}\{z^e\})$ is soft compact over X . Now, since f is soft continuous ,by Theorem(3.1.6), we have $f(f^{-1}(g^{-1}\{z^e\})$ is soft compact over Y. But $f(f^{-1}(g^{-1}\{z^e\}) = g^{-1}\{z^e\}$. Hence $g^{-1}\{z\}$ is soft compact. By (i),(ii) and (iii) , the function g: Y→ Z is soft proper .

Proposition (3.3.10):

Let (X , τ,E), (Y , τ',E) and (Z, τ'',E) be $sts's$. If f: X ⟶ Y and g: Y ⟶ Z are soft continuous functions, such that $g \circ f$: X ⟶ Z is soft proper function. If g is injective , then f is soft proper function.

Proof:

i. f is soft continuous. "by hypothesis "

ii. By proposition (2.1.18 ,ii) we have f: X→ Y is a soft closed function .

iii. Let $y^e \tilde{\in} \tilde{Y}$, then $g(y^e) \tilde{\in} \tilde{Z}$,let $z^e = g(y^e)$. To prove $f^{-1}\{y^e\}$ is a soft compact over X. Since gof: X→Z is soft proper and g is injective function . Then the set$(gof)^{-1}\{z^e\}) = (f^{-1}(g^{-1}\{z^e\}) = f^{-1}\{y^e\}$ is soft compact over X for every $y^e \tilde{\in} \tilde{Y}$. Hence by (i),(ii) and (iii) , the function f: X→ Y is soft proper .

Remark(3.3.11):

It's clear that $f(\widetilde{X}) = \widetilde{f(X)}$,where $f(\widetilde{X})= f(X,E) = (f(X),E)$ and $\widetilde{f(X)}= (f(X),E)$.

Proposition(3.3.12):

Let (X,τ,E) and (Y,τ',E) be $sts's$, and $f\colon X \longrightarrow Y$ be an bijective soft continuous function, (X,τ,E) is soft compact and (Y,τ',E) is soft τ_1-space . Then the following statement are equivalent :

- **i.** f is soft proper function ,
- **ii.** f is soft homeomorphism of (X,τ,E) into soft closed subset of Y .

Proof:

(i→ ii) Since $\widetilde{X}$ is a soft closed set and f is soft proper, then $f(\widetilde{X})$ is a soft closed over Y, and by remark (3.3.11) we have $f(\widetilde{X}) = \widetilde{f(X)}$. Since f is an injective soft continuous and soft closed onto $f(X)$. Then $f : X \to f(X)$ is a soft homeomorphism .

(ii → i) To prove that f is a soft proper function .We will prove that $f^{-1}\{y^e\}$ is soft compact set over X , for every $y^e \widetilde{\in} \widetilde{Y}$. Now ,by Theorem(2.3.9)for each $y^e \widetilde{\in} \widetilde{Y}$, $\{y^e\}$ is soft closed set over Y , since f be a soft continuous . Then $f^{-1}\{y^e\}$ is soft closed set over X . But X is a soft compact space , by Theorem (3.1.5 ,ii) , $f^{-1}\{y^e\}$ is a soft compact set over X . Thus f is soft proper .

Corollary(3.3.13):

Every soft homeomorphism from soft compact, onto soft τ_2-space is proper function.

Proof: Clear.

Theorem (3.3.14):[7]

Let $f\colon X \longrightarrow Y$ be a continuous function . Then the following statements are equivalent:

i. f is proper function .

ii. If $(\chi)_{d\in D}$ is a net in X and $y \in Y$ is a cluster point of the net $f(\chi_d)$ then there is a cluster point $x \in X$ of $(\chi)_{d\in D}$ such that $f(x) = y$.

Simple verification shows that this result remain valid when X and Y are soft spaces as following:

Let (X, τ, E) and (Y, τ', E) be a soft topological spaces, and $f\colon X \longrightarrow Y$ be a soft continuous function .Then the following statements are equivalent:

i. f is soft proper function .

ii. If $\{\chi_d^{e_d}\}_{d\in D}$ is a soft net in $\widetilde{X}$ and $y^e \widetilde{\in} \widetilde{Y}$ is a soft cluster point of the soft net $f(\chi_d^{e_d})$then there is a soft cluster point $x^e \widetilde{\in} \widetilde{X}$ of $\{\chi_d^{e_d}\}_{d\in D}$such that $f(x^e)=y^e$.

Proposition(3.3.15):

Let (X, τ, E) be an soft τ_2-space and $f\colon X \longrightarrow \{a\}$ be a function. Then f is an soft proper if and only if (X, τ, E) is a soft compact space , where a^e is any soft point which does not belong to $\widetilde{X}$.

Proof:

(⟹) Let $\{\chi_d^{e_d}\}_{d\in D}$ be a soft net in $\widetilde{X}$, then $f(\chi_d^{e_d}) = a^e$ for all $d \in D$. Then $f(\chi_d^{e_d}) \propto a^e$. Since f is a soft proper , then there is $x^e \widetilde{\in} \widetilde{X}$ such that $\chi_d^{e_d} \propto x^e$, and $f(x^e) = a^e$.Thus by Theorem(3.1.11), (X, τ, E) is a soft compact space.

(⟸) Since (X, τ, E) is a soft τ_2-space and $f^{-1}\{a^e\} = \widetilde{X}$ is a soft compact ,then by Theorem(3.1.5,i) is an soft closed . Hence f is a soft continuous, soft closed and $f^{-1}\{a^e\}$ soft compact. Thus f is a soft proper function.

Corollary(3.3.16):

Let (X, τ, E) and (Y, τ', E) be a $sts's$, and $f: X \longrightarrow Y$ be a soft proper function where (X, τ, E) is soft compact, soft τ_2-space then $f_{\{y\}}: f^{-1}\{y\} \longrightarrow \{y\}$ is a soft proper function.

Proposition(3.3.17):

Let (X, τ, E), (Y, τ', E)be $sts's$ and f: X $\longrightarrow$ Y be surjective ,soft continuous function such that (X, τ, E) is soft compact and (Y, τ', E) is soft τ_2-space then f is soft closed.

Proof: Clear.

Proposition(3.3.18):

Let (X, τ, E),(Y, τ', E)be $sts's$ and f: X $\longrightarrow$ Y be surjective ,soft continuous function such that (X, τ, E) is soft compact and(Y, τ', E) is a soft τ_2 -space then f is soft proper.

Proof:

By using proposition (3.3.17), f is a soft closed .To prove that $f^{-1}\{y^e\}$ is soft compact set overX ,for every $y^e \widetilde{\in} \widetilde{Y}$. Since (X, τ, E) is soft compact space . then by Theorem (3.1.5,ii), $f^{-1}\{y^e\}$ is soft compact . Thus f is soft proper.

Proposition(3.3.19):

Let (X, τ, E) be a soft compact space , soft τ_1-space and $\widetilde{A}$ be a soft closed set over X. Then the inclusion function i: A $\rightarrow$ X is soft proper.

Proof:

By Theorem (2. 1.15) , i: A $\rightarrow$ X is soft continuous and soft closed function .

To prove $i^{-1}\{x^e\}$ is soft compact set over A for every $x^e \widetilde{\in} \widetilde{X}$. Since$\{x^e\}$ is a soft closed set over X and i is a soft continuous , then $i^{-1}\{x^e\}$ is soft closed set over

A . Since (X, τ, E) is soft compact space, by Theorem (3.1.5,ii), (A, τ_A, E) is soft compact . Hence $i^{-1}\{x^e\}$ is soft compact set over A for every $x^e \widetilde{\in} \widetilde{X}$.
Thus $i: A \to X$ is soft proper.

Proposition(3.3.20):

Let $(X, \tau, E)$and (Y, τ', E) be $sts's$ and $f: X \to Y$ be a soft homeomorphism from soft compact ,soft τ_2-space(X, τ, E) into soft space (Y, τ', E), then $f^{-1}: Y \to X$ is soft proper function .

Proof:

It's obvious that f^{-1} is soft continuous and soft closed .To prove $(f^{-1})^{-1}\{x^e\}$ is soft compact set over Y for every $x^e \widetilde{\in} \widetilde{X}$ .Since$\{x^e\}$ is soft compact set over X , then by Theorem (3.1.6),$f(\{x^e\})$ is soft compact set over Y ,but $f(\{x^e\}) = (f^{-1})^{-1}\{x^e\}$. Thus $f^{-1}$is soft proper function.

Proposition(3.3.21):

Every injective soft proper function is soft compact function.

Proof:

Let $f: X \to Y$ be soft proper function from a soft space (X, τ, E) into a soft space (Y, τ', E) and (F,E) be a soft compact subset over Y. To prove that $f^{-1}(F, E)$ is soft compact set over X.

Let $\{(G_i, E): i \in I\}$ be a soft open cover of $f^{-1}\widetilde{(F, E)}$,then by definition of soft proper $f^{-1}(y^e)$ is soft compact for all $y^e \widetilde{\in}$(F,E). But $f^{-1}(y^e) \widetilde{\subseteq} f^{-1}(F, E) \widetilde{\subseteq} \widetilde{\cup}_{i\in I}(G_i, E)$,thus there exists finite subset I` of I such that $f^{-1}(y^e) \widetilde{\subseteq} \widetilde{\cup}_{i\in I`}(G_i, E)$.

Put $(G_i, E)_{y^e} = \widetilde{\cup}_{i\in I`}(G_i, E)$for all $y^e \widetilde{\in}$ (F,E) . So $f^{-1}(y^e) \widetilde{\subseteq} (G_i, E)_{y^e}$ then $y^e \widetilde{\in} f(G_i, E)_{y^e}$ and then $y^e \widetilde{\notin} f(\widetilde{X} - (G_i, E)_{y^e})$ thus $y^e \widetilde{\in} \widetilde{Y} - f(\widetilde{X} - (G_i, E)_{y^e})$. Therefore $y^e \widetilde{\subseteq} \widetilde{\cup} [\widetilde{Y} - f(\widetilde{X} - (G_i, E)_{y^e}]$ for all $y^e \widetilde{\in}$ (F,E) ,since $(G_i, E)_{y^e}$ is soft open set over X, then $\widetilde{X} - (G_i, E)_{y^e}$ is soft closed set thus $f(\widetilde{X} - (G_i, E)_{y^e})$ is soft closed set over Y. Therefore $\widetilde{Y} - f(\widetilde{X} - (G_i, E)_{y^e})$ is an soft open cover of (F,E) then (F,E) $\widetilde{\subseteq} \widetilde{\cup} [\widetilde{Y} - f(\widetilde{X} - (G_i, E)_{y^e})]$,for all $y^e \widetilde{\in}$ (F,E) .

Thus f^{-1}(F,E) $\tilde{\subseteq}$ $\tilde{\bigcup}$ $(G_i, E)_{y^e}$, $y^e \tilde{\in}$ (F,E) .Therefore f^{-1}(F, E) is soft compact set over X . Hence f is soft compact function.

Now, we introduce the product of soft proper function.

<u>Theorem(3.3.22)</u>:

Let f_i: (X_i ,τ_i, E)→ (Y_i , τ'_i,E) , i=1,2 be soft functions, if $f_1 \times f_2$ is soft proper function and(Y_i ,τ'_i, E),i=1,2 are soft compact spaces, then f_i is soft proper function ,for all i=1,2 .

<u>Proof</u>:

To prove f_1 is soft proper function, by Proposition (2.1.36) and Proposition(2.1.38), f_1 is soft continuous and soft closed .

Now, we prove that ${f_1}^{-1}\{y_1^e\}$ is soft compact set over X_1 for every $y_1^e \tilde{\in} \tilde{Y}_1$. Let $y_1^e \tilde{\in} \tilde{Y}_1$, then $\{y_1^e\} \times \tilde{Y}_2$ is soft compact set over $Y_1 \times Y_2$.Since $f_1 \times f_2$ is soft proper. Then $(f_1 \times f_2)^{-1}(\{y_1^e\} \times \tilde{Y}_2) = f_1^{-1}\{y_1^e\} \times f_2^{-1}(\tilde{Y}_2)$ is soft compact set over $X_1 \times X_2$. By Lemma(3.1.20) $f_1^{-1}\{y_1^e\}$ is soft compact set over X_1 for every $y_1^e \tilde{\in} \tilde{Y}_1$. Hence f_1 is soft proper function.

In a similar way we can prove f_2 is soft proper.

<u>Proposition(3.3.23)</u>:

Let f : (X, τ, E) → (Y, τ', E) be a soft proper function, then for each $e \in$ E, f_e : (Y, τ_e) → (Y, τ'_e) is a proper function.

<u>Proof</u>: Clear .

<u>Remark (3.3.24)</u>:

As a consequence of proposition (3.3.23) and proposition(3.3.8), if f: (X , τ, E) ⟶ (Y , τ', E) , g: (Y , τ',E) ⟶ (Z, τ'',E) are soft proper functions ,then for each $e \in$ E, $(g \circ f)_e = g_e \circ f_e$ is an proper function.

<u>Proposition</u>(3.3.25):

Let(X_i, τ_i, E) and (Y_i, τ'_i, E), i=1,2 be $sts's$. If $f_1 \times f_2$: $(X_1 \times X_2, \tau_{prod}, E \times E) \rightarrow (Y_1 \times Y_2, \tau'_{prod}, E \times E)$ is a soft proper function then for each $(e, e`) \in E \times E$, $(f_1 \times f_2)_{(e,e`)}$: $(X_1 \times X_2, (\tau_{prod})_{(e,e`)}) \rightarrow (Y_1 \times Y_2, (\tau'_{prod})_{(e,e`)})$ is proper function.

<u>Proof</u>: Clear.

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رقم الايداع في دار الكتب و الوثائق ببغداد ٣٤٨١.............لسنة 2016

الملخص

الهدف الأساسي من هذه الرسالة هو دراسة انواع جديدة من الدوال الواهنة في الفضاءات التبولوجية الواهنة ودراسة خواص هذه الدوال والعلاقة فيما بينها .

ظهرت خلال الرسالة بعض المفاهيم الجديدة منها التي تم شرحها تتضمن(التقارب الواهن ،الفضاء الثابت الواهن، الدالة المرصوصة الواهنة، الدالة الاضطرارية الواهنة، الدالة السديدة الواهنة) ووضحنا خواص هذهِ المفاهيم.

لقد قدمنا تعريف دالة المستوى f_e للدالة f عند كل متغير ودرسنا العلاقة بينها وبين الدوال(المفتوحة الواهنة ، المستمرة الواهنة، الاضطرارية الواهنة ، المرصوصة الواهنة، السديدة الواهنة).

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كلية علوم الحاسوب والرياضيات

قســـــــم الرياضيات

حول الدالة الواهنة

في

الفضاءات التبولوجية الواهنة

رسالة مقدمة

إلى

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كجزء من متطلبات نيل درجة ماجستير في علوم الرياضيات

من قبل

سامر عدنان جبير

بإشراف

أ. م .د. ستار حميد حمزة

١٤٣٨هـ **٢٠١٦ م**